Solution of problems in the microscopic theory of the nucleus using the technique of generalized coherent states

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A general approach to the solution of problems in the microscopic theory of the nucleus using the technique of generalized coherent states (GCSs) is discussed. The GCSs constructed explicitly for the lightest p-shell nuclei generate a complete basis of many-particle functions convenient for investigating the dynamics of collective and cluster degrees of freedom. These GCSs have been used to calculate the spectra of the collective excitations of the nuclei ^{5}He, ^{5}Li, ^{6}He, ^{6}Li, ^{7}Li, ^{7}Be, and ^{8}Be, and also the phase shifts and parameters of the centrifugal resonances in the αt system. In addition, these nuclei are shown to have very narrow resonances with a well-defined collective nature and due to the weak coupling of the cluster and collective modes, the collective motion having a large amplitude. The wave functions obtained for the ground state and the continuum states have made possible to study photodisintegration of the nuclei ^{6}He, ^{6}Li, ^{7}Li, and ^{7}Be and radiative capture of deuterons, tritons, and ^{3}He nuclei by α particles.

INTRODUCTION

The technique of generalized coherent states[1] opens up great possibilities for developing the microscopic theory of nuclear systems on the basis of systematic use of a many-particle oscillator basis. The advantages of a microscopic theory based on generalized coherent states are not solely in the significant simplification of the computational algorithms used in the traditional approaches, but also in the fact that the oscillator representation generated by coherent states is convenient for the construction of explicit expressions for the wave functions of nuclear systems and an analysis of the properties of compact nucleon configurations. At the same time, the main thrust of the calculations is shifted to the determination of generating matrix elements of various operators and then matrix elements between basis functions of the oscillator representation. Ultimately, it is possible to find a way to solve problems that previously were outside the scope of the microscopic theory or were amenable to investigation only with great effort.

In the review of Ref. 1, the main attention was concentrated on the problems that arise in the construction of generalized coherent states and matrix elements of various operators between generalized coherent states. In this review, we present the main results of studies in which the technique of generalized coherent states has been used to solve a number of topical problems in microscopic nuclear theory.

In Sec. 1, we calculate generating matrix elements of a many-particle Hamiltonian that are capable of generating the matrix elements of the Hamiltonian between basis functions of the microscopic collective $Sp(2, R)$ nuclear model, and also between the basis functions of the algebraic version of the resonating-group method. In the same section, we construct the matrix elements of operators of various electric multipole transitions, these being needed to calculate the cross sections of photodisintegration of nuclei and radiative capture.

In Sec. 2, we study the wave equation of the microscopic model $Sp(2, R)$. It has become clear in recent years that the $Sp(2, R)$ model reproduces the dynamics of the axial quadrupole nonsphericity of nuclei and the rotational mode strongly coupled to it. The authors of Ref. 2, and even earlier those of Ref. 3, used the Hill–Wheeler integral equation as the dynamical equation of the $Sp(2, R)$ model. However, it was shown in Ref. 4 that it is convenient to use a system of linear algebraic equations for the wave-function expansion amplitudes instead of the Hill–Wheeler equation. The GCS technique greatly simplifes the problem of constructing the system of linear algebraic equations.[5]

The algebraic version of the resonating-group method can compete successfully with the traditional variants of this method. It is shown in Sec. 3 of this review that the algebraic version can be implemented by the reduction to diagonal form of the matrix of the many-particle Hamiltonian in the oscillator representation.

The first theoretical calculations of monopole and quadrupole nuclear excitations already showed[2,3,6] that these excitations are in the continuum and are therefore resonance states. However, the systematic calculation of the widths of the resonance states in the framework of the known microscopic models of collective excitations proved to be unrealistic, since these models do not take into account open channels of nuclear breakup except, perhaps, those that correspond to complete disintegration of the nucleus into independently and freely moving nucleons.

At the same time, interest in collective resonances arises naturally from the point of view of the resonating-group method when it is taken as the framework to pose the problem of taking into account monopole (volume) and quadrupole polarization of interacting clusters. In attempting to understand the part played by polarization effects in collisions of light nuclei, we immediately arrive at the problem of collective (polarization) resonances of the compound nucleus formed by the coalescing clusters.

The topicality of the problem of unifying the microscopic theory of collective excitations with the resonating-group method was recognized by those who originated the K-harmonic method, which is intended in the first place for the study of volume collective vibrations of nuclei, as well as by those who created the resonating-group method in its

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modern form. The upshot was the proposal of two different variants for solving this problem. Baz' and Zhukov, working on the basis of the well-known papers of Ref. 7, constructed the interpolation model,[8] and Wildermuth and Tang[9] explained, how, in principle, polarization effects can be taken into account on the background of the effects of the open cluster channels. Regrettably, neither generalization was carried through to constructive completion, and therefore they did not answer problems relating to collective resonances. In Sec. 4 we show how to solve the problem of collective resonances using the technique of generating invariants by systematic inclusion in the complete basis of the cluster mode of basis states of the quadrupole collective mode.

Photo- and electrodisintegration of light nuclei, radiative capture, and also γ bremmstrahlung by nuclei are phenomena whose nature is predetermined by the behavior of the wave function of the nuclear system in the intrinsic region at short distances between the clusters that occur in a nucleus. These phenomena are amenable to systematic microscopic investigation in the framework of the algebraic version of the resonating-group method that uses the technique of generalized coherent states. The microscopic theory of photonuclear reactions is discussed in Sec. 5.

1. GENERALIZED COHERENT STATES OF COLLECTIVE AND CLUSTER EXCITATIONS OF THE LIGHTEST p-SHELL NUCLEI

Before we give the results of calculations of the matrix elements of various operators between generalized coherent states (GCSs), we explain the basic ideas of the algorithm for solving problems of nuclear physics by means of GCSs, referring the reader to Refs. 1, 4, 10, and 11 for more details about the subject.

Recently, nuclear systems have very often been investigated by means of bases of many-particle oscillator functions $\{|n\rangle\}$ which reproduce the many-particle correlations important for the description of the given process. In this approach, the wave function of the system is represented in the form of a series with respect to the basis functions $|n\rangle$:

$$\Psi = \sum_n C_n \mid n\rangle, \tag{1}$$

and the coefficients C_n, i.e., the generalized Fourier components of the wave function Ψ in the oscillator representation, are found by solving a system of linear algebraic equations:

$$\sum_{n'} (\langle n \mid \hat{H} \mid n'\rangle - E\delta_{nn'})\, C_{n'} = 0. \tag{2}$$

It is simpler to find a numerical solution of this system than a numerical solution of the multidimensional partial differential equations (or systems of equations) and integro-differential and integral equations that arise in various microscopic methods such as the resonating-group method, the generator coordinate method, etc.

The generating-function method, or the GCS method, was proposed in order to overcome the difficulties associated with the construction of the matrix elements $\langle n|\hat{H}|n'\rangle$ of the Hamiltonian $\hat{H}$ and the matrix elements of other operators between basis oscillator functions. The generalized coherent states $\Phi(\beta)$ used in this method depend on the coordinates (spatial, spin, and isospin) of all the nucleons in the system, and on a certain number of parameters β which are called generator parameters. Expansion of the GCSs in powers of these parameters generates the functions $|n\rangle$ in which we are interested:

$$\Phi(\beta) = \sum_n a_n \beta^n \mid n\rangle, \tag{3}$$

where a_n are certain structure constants. In order to separate from this expansion the function $|n\rangle$, it is necessary to calculate the nth derivative of the GCS at the point $\beta = 0$:

$$\frac{\partial^n}{\partial \beta^n} \Phi(\beta)\Big|_{\beta=0} = a_n n! \mid n\rangle.$$

If we have calculated the matrix element of some operator $\hat{F}$ between the GCSs $\Phi(\beta)$ and $\Phi(\tilde{\beta})$,

$$\langle \Phi(\beta) \mid \hat{F} \mid \Phi(\tilde{\beta})\rangle = \sum_{n,\,n'} a_n a_{n'} \beta^n \tilde{\beta}^{n'} \langle n \mid \hat{F} \mid n'\rangle,$$

then, differentiating this matrix element (which we shall call the generating matrix element and abbreviate to GME) with respect to the parameters β and $\tilde{\beta}$, we obtain the matrix element $\hat{F}$ between the oscillator functions $|n\rangle$ and $|n'\rangle$:

$$\langle n \mid \hat{F} \mid n'\rangle = (a_n a_{n'} n! n'!)^{-1} \frac{\partial^n}{\partial \beta^n} \frac{\partial^{n'}}{\partial \tilde{\beta}^{n'}} \langle \Phi(\beta) \mid \hat{F} \mid \Phi(\tilde{\beta})\rangle \Big|_{\beta=\tilde{\beta}=0}. \tag{4}$$

In the algorithm proposed in Refs. 4, 10, and 11, the GCSs are constructed in the form of Slater determinants from single-particle orbitals. The actual form of these orbitals and the physical meaning of the generator parameters that occur in them are determined by the many-particle correlations which reproduce the generated oscillator basis. To describe the collective excitations of nuclei, the GCSs are constructed from the orbitals of an anisotropic oscillator, and the combinations of parameters that determine the deformation of the oscillator field serve as the generator parameters. The GCSs constructed from the orbitals of Brink's model[12] (cluster orbitals) are suitable for investigating reactions induced by nuclear collisions. The generator parameters of these GCSs are identical to the parameters that specify the position of the clusters in space.

Besides the orbitals just mentioned, there was proposed in Refs. 10, 13, and 14 a new form of orbital containing not only deformation parameters but also cluster parameters. The GCSs constructed from such combined orbitals make it possible to generate oscillator basis functions which reproduce excitations and both collective and cluster modes of motion. As was noted in Refs. 1 and 13, the simultaneous allowance for these modes is above all necessary for the correct description of resonance states. In addition, the inclusion of the cluster mode with the collective mode makes it possible to take into account more accurately the interaction of the collective and intrinsic excitations in calculations of the spectra of collective states and, in the description of nuclear reactions, the dynamic polarization of interacting clusters.

To construct GCSs in what follows, we shall use in fact such combined orbitals,[1,10,14] or rather, a certain simplified form of them. The simplification arises from the group of problems considered below and listed in the Introduction,

and also from the choice of the subject of our investigations. The orbitals which we use have the form

$$\varphi(\mathbf{r}) = \exp\left\{-\frac{1}{2}r^2 - \frac{\beta}{1-\beta}(\mathbf{pr})^2 + 2\frac{(\mathbf{R}_i\mathbf{r})^2}{1-\beta} - \frac{R_i^2}{1-\beta}\right\}, \quad (5)$$

where $\mathbf{R}_i = R_i\mathbf{p}$ is the cluster parameter that determines the position of the ith cluster center, β is the deformation parameter, and $\mathbf{p}$ is the unit vector which specifies the deformation axis and simultaneously the direction of the vector $\mathbf{R}_i$. This orbital makes it possible to generate from the complete manifold of collective modes just the one mode of quadrupole excitations, and in the nucleus that is considered, to take into account one cluster branch, describing decay of the nucleus into two s-shell clusters.

We give below the results of calculations of various matrix elements between GCSs for p-shell nuclei with the number A of nucleons in the interval $5 \leqslant A \leqslant 8$. These nuclei are regarded as systems composed of two interacting clusters, one of which is an α particle ($A_1 = 4$), while the second is formed by A_2 nucleons ($A_2 \leqslant 4$, $A = 4 + A_2$). It was shown in Ref. 15 that the GCSs constructed from the orbitals (5) generate two different bases. The first is formed by the functions of a definite irreducible representation $\sigma = 1/2f_1 + (A-5)/4$ of the group $Sp(2, R)$, while the second is formed by functions with the different $O(A-1)$ symmetry $[f] = [f_1 f_2 f_3]$ (for p-shell nuclei, $[f] = [f00]$, $f = f_0, f_0+2, f_0+4, \ldots$), which reproduce the relative motion of the clusters. We recall that the basis functions of the irreducible representation σ of the group $Sp(2, R)$ belong to the part of the complete basis of the minimal approximation of the method of generalized hyperspherical functions (MA MGHF) that reproduces the longitudinal vibrations of the ellipsoid of inertia of the nucleus and is dominant[16–18] in the ground-state wave function and lowest collective excitations of p-shell nuclei. The parameter β of the orbital (5) is a generator coordinate of this basis. In turn, a basis of functions with a different $O(A-1)$ symmetry can be generated by means of the generator parameter R.

Calculation of the properties of the nucleus in the $Sp(2, R) \otimes O(A-1)$ basis, which takes into account both these modes, makes it possible, by lifting the restriction on the choice of the intrinsic functions inherent in the MA MGHF, to consider collective excitations of the SU(3) symmetry (λ 0) simultaneously with different intrinsic excitations [f00].

Omitting the details of the calculations, we give at once the explicit form of the overlap integrals of the GCSs $\Phi(R, \beta)$ and $\Phi(\widetilde{R}, \widetilde{\beta})$ and the generating matrix elements (GMEs) of the Hamiltonian of the nucleus. For the chosen clustering ($A = 4 + A_2$) the overlap integral has the form[1)]

$$F \equiv \langle \Phi(R, \beta) \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle = \Delta^{-\frac{A-1}{2}} \operatorname{sh}^{A_2}\xi \times \exp\left\{-\frac{A_2}{A}[4\mathcal{A}_{R\widetilde{R}} - (4-A_2)\xi]\right\}, \quad (6)$$

$$\Delta = 1 - \beta\widetilde{\beta}t^2; \quad t = (\mathbf{pq}); \quad \xi = \frac{1}{\Delta}R\widetilde{R}(\mathbf{pq});$$

$$\mathcal{A}_{R\widetilde{R}} = \Delta^{-1}t^2[\beta\widetilde{R}^2 + \widetilde{\beta}R^2].$$

The generating matrix element of the kinetic-energy operator is

$$\langle \Phi(R, \beta) \mid \hat{T} \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle = \frac{\hbar^2}{2mr_0^2}\left\{\frac{A-1}{2}P_0 - \frac{4A_2}{A}(P_1R^2 + P_2\widetilde{R}^2) + P_3\xi\frac{\partial}{\partial\xi}\right\}\langle \Phi(R, \beta) \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle;$$

$$\pi = \Delta^{-1}(2 + \beta + \widetilde{\beta}); \quad P_0 = 1 + \pi; \quad P_1 = \Delta^{-1}(1 + \pi\widetilde{\beta}t^2);$$

$$P_2 = \Delta^{-1}(1 + \pi\beta t^2); \quad P_3 = -1 + \pi.$$

The GME of the operator of the central exchange NN interaction with a Gaussian radial dependence has the form

$$\langle \Phi(R, \beta) \mid \hat{V} \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle = V_{ss}f_{ss} + V_{sp}^{(+)}f_{sp}^{(+)} - V_{sp}^{(-)}f_{sp}^{(-)} + V_{pp}f_{pp}, \quad (7)$$

where

$$f_{ss} = z^{3/2}(\Delta/\Delta_\gamma)^{1/2}\langle \Phi(R, \beta) \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle;$$

$$f_{sp}^{(\pm)} = \frac{f_{ss}}{2\operatorname{sh}\xi}\{\Omega_{R\widetilde{R}}e^{\pm\eta} - (\Omega_{R0} + \Omega_{0\widetilde{R}} - 1)e^{-\xi}\};$$

$$f_{pp} = \frac{f_{ss}}{4\operatorname{sh}^2\xi}\{4\Omega_{R\widetilde{R}}e^{-\xi} - 2(\Omega_{R0} + \Omega_{0\widetilde{R}})(1 + e^{-2\xi}) + e^{2\xi} + 3e^{-2\xi}\};$$

$$\Omega_{R\widetilde{R}} = \exp\left\{-\frac{R^2}{2}\left[\frac{a + b\widetilde{\beta}}{\Delta_\gamma} - \frac{\widetilde{\beta}t^2}{\Delta}\right] - \frac{\widetilde{R}^2}{2}\left[\frac{a + b\beta}{\Delta_\gamma} - \frac{\beta t^2}{\Delta}\right]\right\};$$

$$\Delta_\gamma = 1 - a(\beta + \widetilde{\beta}) - b\beta\widetilde{\beta}; \quad a = 1 - z; \quad b = t^2z^2 - a^2;$$

$$\eta = \frac{zR\widetilde{R}(\mathbf{pq})}{\Delta_\gamma}; \quad z = \left(\frac{2r_0^2}{s_0^2} + 1\right)^{-1},$$

and the constants V_{ss}, $V_{sp}^{(\pm)}$, and V_{pp} are formed from the intensities $V_{2S+1,2T+1}$ of the Gaussian potential and are equal to $V_{ss} = 3(V_{13} + V_{31})$, $V_{sp}^{\pm} = A_2(9V_{33} \pm 3V_{31} \pm 3V_{13} + V_{11})/4$ for $A_2 = 1, 2, 3, 4$, and V_{pp} is equal to 0 for $A_2 = 1$, V_{13} (for ^{6}He) or V_{31} (for ^{6}Li) for $A_2 = 2$, $V_{ss}/2$ for $A_2 = 3$, and V_{ss} for $A_2 = 4$.

The GME of the operator of the spin-orbit coupling of the nucleons is

$$\langle \Phi(R, \beta) \mid \hat{V}_{ls} \mid \Phi(\widetilde{R}, \widetilde{\beta}) \rangle = \overline{V}_{sp}^{(+)}\mathcal{F}_{sp}^{(+)} - \overline{V}_{sp}^{(-)}\mathcal{F}_{sp}^{(-)} + \overline{V}_{pp}\mathcal{F}_{pp}, \quad (8)$$

where the constants $\overline{V}_{sp}^{(\pm)}$ and $\overline{V}_{pp}$ are formed from the corresponding components of the spin-orbit part of the NN potential: $\overline{V}_{sp}^{(\pm)} = -(3V_3^{1s} \pm V_1^{1s})$ for $A_2 = 1, 2, 3, 4$; $\overline{V}_{pp} = (1 - A_2)V_1^{1s}$ for $A_2 \geqslant 2$, and the functions $\mathcal{F}_{sp}^{(\pm)}$, $\mathcal{F}_{pp}$ have the form

$$\mathcal{F}_{sp}^{(\pm)} = \alpha f_{sp}^{(\pm)} + \frac{f_{ss}}{2\operatorname{sh}\xi}\{\Omega_{R\widetilde{R}}(\mathcal{K}_{R\widetilde{R}} \mp PR\widetilde{R})e^{\pm\eta} - (\mathcal{K}_{R0}\Omega_{R0} + \mathcal{K}_{0\widetilde{R}}\Omega_{0\widetilde{R}})e^{-\xi}\};$$

$$\mathcal{F}_{pp} = \alpha f_{pp} + \frac{f_{ss}}{2\operatorname{sh}^2\xi}\{2\Omega_{R\widetilde{R}}(\mathcal{K}_{R\widetilde{R}}\operatorname{ch}\eta - PR\widetilde{R}\operatorname{sh}\eta)e^{-\xi} - (\mathcal{K}_{R0}\Omega_{R0} + \mathcal{K}_{0\widetilde{R}}\Omega_{0\widetilde{R}})(1 + e^{-2\xi})\},$$

where

$$\alpha = -\beta\widetilde{\beta}\Delta_\gamma^{-1}z^2t\sqrt{1-t^2}; \quad P = \Delta_\gamma^{-2}z\sqrt{1-t^2}(\Delta_\gamma + 2\beta\widetilde{\beta}z^2t^2);$$

$$\mathcal{K}_{R\widetilde{R}} = \Delta_\gamma^{-2}z^2t\sqrt{1-t^2}\{\widetilde{\beta}R^2(1 - \widetilde{\beta}a) + \beta\widetilde{R}^2(1 - \beta a)\}.$$

[1)] *Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the articles in the displayed equations.

In conclusion, we give the generating matrix elements of the operator

$$\hat{F}=\frac{1}{2}\sum_{i=1}^{A}(1+\hat{\tau}_{iz})\exp\{i\mathbf{k}(\mathbf{r}_i-\mathbf{R}_A)\}, \tag{9}$$

($\mathbf{R}_A=(1/A)\Sigma_{i=1}^{A}\mathbf{r}_i$ is the center-of-mass coordinate), knowledge of which is needed to describe photo- and electroexcitation of nuclei. Depending on the process investigated, the wave number $\mathbf{k}$ will determine either the momentum of the absorbed or emitted photon or the momentum transferred by the electron to the nucleus. In addition, using the standard device, one can obtain from the operator $\hat{F}$ the operators of the electric multipole $E\lambda$ transitions:

$$\tilde{Q}_{\lambda\mu}\equiv\frac{1}{2}\sum_{i=1}^{A}(1+\hat{\tau}_{iz})\tilde{r}_i^{\lambda}Y_{\lambda\mu}(\tilde{\Theta}_i)$$

$$=\frac{(-i)^{\lambda}}{\lambda!A_{\lambda}^{\lambda}}\frac{d^{\lambda}}{dk^{\lambda}}\int d\Theta_{\mathbf{k}}Y_{\lambda\mu}^{*}(\Theta_{\mathbf{k}})\hat{F}\,|_{|\mathbf{k}|=0}. \tag{10}$$

Here, $\{\tilde{r}_i,\tilde{\theta}_i\}$ are the spherical components of the radius vector of the ith particle in the center-of-mass system, $\theta_{\mathbf{k}}$ are the angles that specify the direction of the vector $\mathbf{k}$ in space, and

$$A_l^{\lambda}=4\pi\frac{\lambda!}{(\lambda-l)!!(\lambda+l+1)!!}. \tag{11}$$

Moreover, it is convenient to make this transition from $\hat{F}$ to the operators $\hat{Q}_{\lambda\mu}$ in the generating matrix elements, i.e., to use the relation

$$\langle\Phi\,|\,\hat{Q}_{\lambda\mu}\,|\,\tilde{\Phi}\rangle=\frac{(-i)^{\lambda}}{\lambda!A_{\lambda}^{\lambda}}\frac{d^{\lambda}}{dk^{\lambda}}\int d\Theta_{\mathbf{k}}Y_{\lambda\mu}^{*}(\Theta_{\mathbf{k}})\langle\Phi\,|\,\hat{F}\,|\,\tilde{\Phi}\rangle\,|_{k=0}.$$

The order in which the operations—differentiation with respect to the modulus of the vector $\mathbf{k}$ and integration over the angles $\Theta_{\mathbf{k}}$—is performed is arbitrary.

The generating matrix element of the operator $\hat{F}$ can be represented in the form

$$\langle\Phi\,|\,\hat{F}\,|\,\tilde{\Phi}\rangle=\{2f_0+(Z-2)f_1\}\langle\Phi\,|\,\tilde{\Phi}\rangle, \tag{12}$$

where

$$f_0=\exp\Big\{-\frac{A-1}{4A}k^2+\frac{A-1}{4A\Delta}[\beta(\mathbf{kp})^2$$

$$+\tilde{\beta}(\mathbf{kq})^2-2\beta\tilde{\beta}(\mathbf{pq})(\mathbf{pk})\mathbf{qk})]$$

$$+\frac{iA_2}{A}R[(\mathbf{pk})-\beta(\mathbf{pq})(\mathbf{kq})]+\frac{iA_2}{A}\tilde{R}[(\mathbf{kq})-\tilde{\beta}(\mathbf{pq})(\mathbf{kq})]\Big\};$$

$$f_1=\frac{f_0}{2\operatorname{sh}\xi}\{e^{\xi}\Omega(R,\tilde{R})-e^{-\xi}[\Omega(R,0)+\Omega(0,\tilde{R})-1]\};$$

$$\Omega(R,\tilde{R})=\exp\Big\{-\frac{iR}{\Delta}[(\mathbf{pk})-\beta(\mathbf{pq})\mathbf{kq})]$$

$$-\frac{i\tilde{R}}{\Delta}[(\mathbf{kq})-\tilde{\beta}(\mathbf{pq})(\mathbf{pk})]\Big\}.$$

The first of the two terms in Eq. (12), f_0, determines the contribution to the energy of the interaction of the electromagnetic field with the nucleus of the protons that form the α particle, while the second term, f_1, corresponds to the contribution of the protons of the second cluster.

The GMEs (6)–(8) generate the matrix elements of the many-particle Hamiltonian between functions of the $Sp(2R)\otimes O(A-1)$ irreducible representation, which take into account simultaneously two modes of the nucleon motion: the mode of the longitudinal vibrations of the ellipsoid of inertia of the nucleus and the "cluster" mode, the relative motion of the clusters. At the same time, in a number of problems in microscopic theory each of these branches is of independent interest. In this connection, it is necessary to construct GMEs that make it possible to investigate excitations of the cluster as well as—independently—the collective degrees of freedom. Such GMEs can be readily separated from (6)–(8).

Sp(2,R) basis. As we have already noted, the $Sp(2,R)$ basis separates in the complete space of the collective functions of the MA MGHF a certain part of the collective functions with fixed $O(A-1)$ symmetry. Therefore, to go over to the $Sp(2,R)$ basis in the function $\Phi(R,\beta)$, it is necessary to separate states $\Phi(\beta)$ with given $O(A-1)$ symmetry. If $[f]=[f00]$, then for arbitrary values of f such a problem can be solved by means of the projection operator $\hat{P}_{[f]}$ introduced in Ref. 10. However, it is required to include only the simplest intrinsic functions, i.e., the intrinsic functions of the MA MGHF, the required GMEs can be obtained without preliminary projection of the functions $\Phi(R,\beta)$. In this case, it is necessary to use the results of Ref. 15, which gives the connection between the quantum numbers of the basis functions of the irreducible representation σ of the group $Sp(2,R)$ and the powers of the parameter R in the expansion of $\Phi(R,\beta)$ in a series in powers of R. It can be shown that to solve the problem it is sufficient to expand the GMEs (6)–(8) in series in powers of R and $\tilde{R}$ and to retain in this expansion first the terms of order $(R\tilde{R})^{\bar{f}}$ and then, independently, the terms of order $(R\tilde{R})^{\bar{\bar{f}}}$, where $\bar{f}=A-4$, $\bar{\bar{f}}=A-3$, $\bar{\bar{\bar{f}}}=\bar{f}+1$. Such a procedure makes it possible to separate from (6)–(8) the GMEs of the Hamiltonian of the nucleus between functions with fixed $O(A-1)$ symmetry ($[f]=[\bar{f}00]$ or $[\bar{\bar{f}}00]$) and SU(3) symmetry $(\lambda\mu)=(f+2n,0)$, $n=0,1,2,\ldots$. These functions occur in two nonintersecting irreducible representations

$$\bar{\sigma}=\frac{1}{2}\bar{f}+\frac{A-5}{4}\quad\text{and}\quad\bar{\bar{\sigma}}=\frac{1}{2}\bar{\bar{f}}+\frac{A-5}{4} \tag{13}$$

of the group $Sp(2,R)$. The functions of the $\bar{\sigma}$ basis correspond to spatial parity $\pi_1=(-1)^{\bar{f}}$ (referred to below as normal parity), and the functions of the $\bar{\bar{\sigma}}$ basis to spatial parity $\pi_2=(-1)^{\bar{\bar{f}}}$ (anomalous parity).

The GMEs constructed in this manner depend on the generator parameters β and $\tilde{\beta}$ that generate the $Sp(2,R)$ basis, on the parameter t, and on the parameters of the NN interaction: $V_{2S+1,2T+1}$ and z. We give the most important GMEs.

Generating matrix elements of a central NN interaction:

$$\langle\Phi^{[f]}(\beta)\,|\,\hat{V}_c\,|\,\Phi^{[f]}(\tilde{\beta})\rangle=V_{ss}f_{ss}^{[f]}+V_{sp}^{(+)}f_{sp}^{[f](+)}$$

$$-V_{sp}^{(-)}f_{sp}^{[f](-)}+V_{pp}f_{pp}^{[f]}, \tag{14}$$

where for normal parity

$$f_{ss}^{[\bar{f}]}=z^{3/2}(\Delta/\Delta_\gamma)^{1/2}\langle\Phi^{[\bar{f}]}(\beta)\,|\,\Phi^{[\bar{f}]}(\tilde{\beta})\rangle;$$

$$f_{sp}^{[\bar{f}](\pm)}=1/2f_{ss}^{[\bar{f}]}(1\pm z\Delta/\Delta_\gamma);$$

$$f_{pp}^{[\bar{f}]} = 1/4 f_{ss}^{[\bar{f}]} \left\{ 3 - \frac{\Delta}{\Delta_\gamma} \left(2z + a^2 \frac{t^2-1}{t^2} \right) + 3z^2\Delta^2/\Delta_\gamma^2 \right\},$$

and for anomalous parity

$$f_{ss}^{[\bar{\bar{f}}]} = z^{3/2} (\Delta/\Delta_\gamma)^{1/2} \langle \Phi^{[\bar{f}]}(\beta) \mid \Phi^{[\bar{\bar{f}}]}(\tilde{\beta}) \rangle;$$

$$f_{sp}^{[\bar{\bar{f}}](\pm)} = \frac{t}{\Delta} \left\{ f_{sp}^{[\bar{f}](\pm)} + \frac{A}{2A_2(4-A_2)} (f_{pp}^{[\bar{f}]} - f_{ss}^{[\bar{f}]}) \right\};$$

$$f_{pp}^{[\bar{\bar{f}}]} = \frac{t}{\Delta} \left\{ f_{pp}^{[\bar{f}]} - \frac{A}{A_2(4-A_2)} (f_{pp}^{[\bar{f}]} - f_{ss}^{[\bar{f}]}) \right\}.$$

We also give the integral of the overlapping of $\Phi^{[f]}$ and $\tilde{\Phi}^{[f]}$ with the identity operator. For the $O(A-1)$ symmetry $[f] = [f00]$ it has the form

$$\langle \Phi^{[f]}(\beta) \mid \Phi^{[f]}(\tilde{\beta}) \rangle = t^f \Delta^{-J}, \quad J = f + (A-1)/2. \tag{15}$$

Generating matrix elements of the operator of the spin-orbit interaction.

$$\langle \Phi^{[f]}(\beta) \mid \hat{V}_{ls} \mid \Phi^{[f]}(\tilde{\beta}) \rangle = \bar{V}_{sp}^{(+)} \mathcal{F}_{sp}^{[f](+)} - \bar{V}_{sp}^{(-)} \mathcal{F}_{sp}^{[f](-)} + \bar{V}_{pp} \mathcal{F}_{pp}^{[f]}, \tag{16}$$

where for normal parity

$$\mathcal{F}_{sp}^{[\bar{f}](\pm)} = \alpha_1 f_{sp}^{[\bar{f}](\pm)} \pm \alpha_2 f_{ss}^{[\bar{f}]};$$

$$\mathcal{F}_{pp}^{[\bar{f}]} = 3\alpha_1 f_{pp}^{[\bar{f}]} + \frac{1}{2} z^2 \frac{\Delta}{\Delta_\gamma} f_{ss}^{[\bar{\bar{f}}]}$$

$$\times \left[\frac{3\beta\tilde{\beta}t^2}{\Delta} \left(1 - z^2 \frac{\Delta^2}{\Delta_\gamma^2} \right) + \frac{3\Delta}{\Delta_\gamma} - 1 \right] \frac{\sqrt{1-t^2}}{t};$$

$$\alpha_1 = -\frac{z^2\beta\tilde{\beta}t}{\Delta_\gamma} \frac{\sqrt{1-t^2}}{t};$$

$$\alpha_2 = -z \frac{\Delta}{\Delta_\gamma} \left(\frac{z^2\beta\tilde{\beta}t^2}{\Delta_\gamma} + 1/2 \right) \frac{\sqrt{1-t^2}}{t},$$

and for anomalous parity

$$\mathcal{F}_{sp}^{[\bar{\bar{f}}](\pm)} = \frac{t}{\Delta} \left\{ \mathcal{F}_{sp}^{[\bar{f}](\pm)} + \frac{A}{2A_2(4-A_2)} (\mathcal{F}_{pp}^{[\bar{f}]} - \alpha_1 f_{ss}^{[\bar{f}]}) \right\};$$

$$\mathcal{F}_{pp}^{[\bar{\bar{f}}]} = \frac{t}{\Delta} \left\{ \mathcal{F}_{pp}^{[\bar{f}]} - \frac{A}{A_2(4-A_2)} (\mathcal{F}_{pp}^{[\bar{f}]} - \alpha_1 f_{ss}^{[\bar{f}]}) \right\}.$$

The next step is the transition from the GMEs (14)–(16) to the matrix elements of the corresponding operators between the oscillator functions $|nLM\rangle$, where $n = 0, 1, 2, \ldots$ determines the number of the oscillator function, and $2n$ gives the number of excitation quanta (collective or cluster) above the ground-state configuration $|OLM\rangle$; L and M are the orbital angular momentum and its projection onto one of the axes of the laboratory coordinate system. We shall make this transition in two stages. In the first, using the differential relation (4), we calculate the matrix elements $\langle n|\hat{F}|n'\rangle$ of the operator $\hat{F}$ between the functions $|n\rangle$ and $|n'\rangle$, which are superpositions of the states $|nLM\rangle$ and $|n'LM\rangle$ with fixed values of L and its projection M. The projection onto states with given L and M of the resulting elements $\langle n|F|n'\rangle$ is done in the second stage. Before we give the relations necessary for this, we recall that each of the functions and the collective and cluster bases have a definite SU(3) symmetry $(\lambda\mu)$, where for a given group of nuclei and chosen clustering $\lambda = 2n + f$, $\mu = 0$. Besides these quantum numbers and L and M, the functions of the collective basis are also characterized by the indices $[f00]$ of the irreducible representation of the group $O(A-1)$, which determine the intrinsic motion of the system. In the cluster basis, the clustering which we have taken into account serves as this additional label. At the same time, the function of the cluster basis can, as is shown in Refs. 10 and 15, be represented in the form of a series of collective functions with the same SU(3) symmetry as for the cluster function but with different irreducible-representation indices $[f00]$ of the group $O(A-1)$.

The projection onto states with given L and M can be done conveniently by means of the relation

$$\langle nLM \mid \hat{F} \mid n'LM \rangle = [A_L^{2n+f} A_L^{2n'+f}]^{-1/2} \int_{-1}^{1} \langle n \mid \hat{F} \mid n' \rangle P_L(t)\, dt, \tag{17}$$

where $P_L(t)$ is a Legendre polynomial. The expression in front of the integral in (17) is a normalizing factor. It appears because the state $|nLM\rangle$ occurs in the function $|n\rangle$ with the weight $\sqrt{A_L^{2n+f}}$.

On the other hand, the projection can be done in a different form. We recall that the matrix element $\langle n|\hat{F}|n'\rangle$ is a function of the unit vectors $\mathbf{p}$ and $\mathbf{q}$, i.e.,

$$\langle n \mid \hat{F} \mid n' \rangle = \langle n\mathbf{p} \mid \hat{F} \mid n'\mathbf{q} \rangle. \tag{18}$$

Therefore, the projection can be done by integrating the matrix element (18) over the angular variables of the vectors $\mathbf{p}$ and $\mathbf{q}$ with weight $Y_{L'M'}^*(\Omega_\mathbf{q}) Y_{LM}(\Omega_\mathbf{p})$:

$$\langle nLM \mid \hat{F} \mid n'L'M' \rangle = [A_L^{2n+f} A_{L'}^{2n'+f}]^{-1/2}$$
$$\times \int d\Omega_\mathbf{q}\, d\Omega_\mathbf{p} Y_{LM}(\Omega_\mathbf{p}) Y_{L'M'}^*(\Omega_\mathbf{q}) \langle n\mathbf{p} \mid \hat{F} \mid n'\mathbf{q} \rangle. \tag{19}$$

In such a form, the projection is convenient for tensor operators.

With regard to the structure constants a_n (3), for their calculation it is necessary to expand the overlap integral in powers of the generator parameters. Forthe basis of the collective functions, the coefficients a_n are, as follows from (15), equal to

$$a_n = \sqrt{\frac{\Gamma(J+n)}{n!\,\Gamma(J)}}, \quad J = f + \frac{1}{2}(A-1), \tag{20}$$

while for the cluster basis they will be determined below.

Thus, if we have as a preliminary found all the necessary GMEs, then the relations (4), (17), and (19) give a general scheme for constructing the matrix elements of the various operators between the oscillator functions.

It should be noted that even for nuclei with a comparatively small number of nucleons ($A = 5$–8) the matrix elements of the operators of the central and spin–orbit interactions are cumbersome. However, as can be seen from (14)–(16), the GMEs of both operators are superpositions of fairly simple blocks. For example, the GMEs (14) of the operator $\hat{V}_c$ contain only the standard constructions

$$\mathcal{W}_\lambda^{(r,\,s)} = t^\lambda \Delta^{-r} \Delta_\gamma^{-s}, \tag{21}$$

which depend on $\beta, \tilde{\beta}$, and t, and the GMEs (16) of the operator $\hat{V}_{ls}$ contain only the blocks

$$\mathcal{W}_\lambda^{(r,\,s)}(n_1, n_2) = \frac{\beta^{n_1}\tilde{\beta}^{n_2}}{\Delta^r \Delta_\gamma^s} t^\lambda \sqrt{1-t^2}; \quad n_1, n_2 = 0, 1, 2 \ldots \tag{22}$$

Therefore, the matrix elements of the operators $\hat{V}_c$ and $\hat{V}_{ls}$ are superpositions of the matrix elements

$$\langle m_1 L \mid \mathscr{V}_\lambda^{(r,\,s)} \mid m_2 L\rangle, \tag{23}$$

$$\langle m_1 L \mid \mathscr{W}_\lambda^{(r,\,s)}(n_1,\,n_2) \mid m_2 L\rangle, \tag{24}$$

calculated for definite values of the numbers r, s, and λ. We obtain the explicit form of (23) by using (17):

$$\langle m_1 L \mid \mathscr{V}_\lambda^{(r,\,s)} \mid m_2 L\rangle = \sum_{n=0}^{\overline{m}} G_{m_1 m_2}^{\lambda f L}(n) \sum_{m=0}^{n} B_{m_1 m_2}^{(r,\,s)}(n,\,m),$$

$$\overline{m} = \min\{m_1,\,m_2\}, \tag{25}$$

where

$$G_{m_1 m_2}^{\lambda f L}(n) = A_L^{\lambda+2n}/\sqrt{A_L^{2m_1+f} A_L^{2m_2+f}};$$

$$B_{m_1 m_2}^{(r,\,s)}(n,\,m) = \frac{z^{2n} a^{m_1+m_2-2n}\,\Gamma(r+n-m)\,\Gamma(s+m+m_1-n)\,\Gamma(s+m+m_2-n)}{\sqrt{a_{m_1}^{[f]} a_{m_2}^{[f]}}\, m!\,(n-m)!\,(m_1-n)!\,(m_2-n)!\,\Gamma(r)\,\Gamma(s)\,\Gamma(s+m)}$$

the matrix elements (24) can be calculated similarly:

$$\langle m_1 L \mid \mathscr{W}_\lambda^{(r,\,s)}(n_1,\,n_2) \mid m_2 L\rangle = \sum_{n=0}^{\overline{m}} G_{m_1 m_2}^{1,\,\lambda f L}(n) \sum_{m=0}^{n} B_{i_1,\,i_2}^{(r,\,s)}(n,\,m), \tag{26}$$

$$i_1 = m_1 - n_1;\ i_2 = m_2 - n_2;\ \overline{m} = \min\{i_1, i_2\},$$

where

$$G_{m_1 m_2}^{1,\,\lambda f L}(n) = A_L^{1,\,\lambda+2n}/\sqrt{A_L^{2m_1+f} A_L^{2m_2+f}};$$

$$A_L^{1,\,\lambda} = \int_{-1}^{1} t^\lambda \sqrt{1-t^2}\, P_L^1(t)\, dt = -\frac{L(L+1)}{\lambda} A_L^{\lambda+1},$$

and P_L^1 is an associated Legendre function.

The matrix elements (25) and (26) solve the problem of constructing the matrix elements of the operators of the central and *ls* interactions in nuclei in the first half of the *p* shell ($A = 5, 6, 7, 8$) between orthogonal functions of the $Sp(2, R)$ basis that correspond to an arbitrary number of excitation quanta.

Cluster basis. The GMEs for this basis can be obtained with comparative ease, namely, in the GMEs (6)–(8) of the $Sp(2, R) \otimes O(A-1)$ basis it is sufficient to set the parameters β and $\tilde{\beta}$ equal to zero. Therefore, we give the explicit form of these GMEs directly. (Clustering $A = 4 + A_2$, $A_2 \leqslant 4$.) The overlap integral is

$$\langle \Phi(\mathbf{R}) \mid \Phi(\tilde{\mathbf{R}})\rangle = \mathrm{sh}^{A_2}\xi \exp\left\{\frac{A_2(4-A_2)}{A}\xi\right\};\quad \xi = (\mathbf{R}\tilde{\mathbf{R}}). \tag{27}$$

The GME of the operator of the central *NN* interaction is

$$\langle \Phi(\mathbf{R}) \mid \tilde{V}_C \mid \Phi(\tilde{\mathbf{R}})\rangle = V_{ss} f_{ss}^R + V_{sp}^{(+)} f_{sp}^{R(+)} - V_{sp}^{(-)} f_{sp}^{R(-)} + V_{pp} f_{pp}^R, \tag{28}$$

where

$$f_{ss}^R = z^{3/2}\langle \Phi(\mathbf{R}) \mid \Phi(\tilde{\mathbf{R}})\rangle;$$

$$f_{sp}^{R(\pm)} = z^{3/2} \sum_{r=0}^{A_2-1} D_r^{A_2-1} \{\Omega_{R\tilde{R}} M[\xi(\mathscr{E}_r - 1 \pm z)] - (\Omega_{R0} + \Omega_{0\tilde{R}} - 1) M[\xi(\mathscr{E}_r - 2)]\};$$

$$f_{pp}^R = f_{ss}^R + 2z^{3/2} \sum_{r=0}^{A_2-2} D_r^{A_2-2} \{\Omega_{R\tilde{R}} M[\xi(\mathscr{E}_r - z - 3)] + \Omega_{R\tilde{R}} M[\xi(\mathscr{E}_r + z - 3)] - (\Omega_{R0} + \Omega_{0\tilde{R}} - 1) M[\xi(\mathscr{E}_r - 2)] - (\Omega_{R0} + \Omega_{0\tilde{R}} - 1) M[\xi(\mathscr{E}_r - 4)]\};$$

$$D_r^l = \frac{(-1)^r\, l!}{r!\,(l-r)!};\quad \mathscr{E}_r = \frac{8A_2}{A} - 2r,$$

and

$$M(x) = \begin{cases} \mathrm{sh}\, x, & \pi = -1; \\ \mathrm{ch}\, x, & \pi = +1. \end{cases}$$

To calculate the normalizing factors in the cluster basis, we expand the overlap integral in powers of R and $\tilde{R}$:

$$\langle \Phi(\mathbf{R}) \mid \Phi(\tilde{\mathbf{R}})\rangle = \sum_{n=0}^{\infty} \frac{B_N}{N!} (\mathbf{R}\tilde{\mathbf{R}})^N$$

$$= \sum_{n=0}^{\infty} \frac{B_N}{N!} (R\tilde{R})^N \sum_{LM} A_L^N Y_{LM}(\Omega_{\mathbf{R}})\, Y_{LM}^*(\Omega_{\tilde{\mathbf{R}}}). \tag{29}$$

Here

$$B_N = \sum_r D_r^{A_2} \mathscr{E}_r^N;\quad N = 2n + f,\quad A_2 = A - 4, \tag{30}$$

f has the previous meaning, and the functions A_L^N are determined by (11). The coefficients of $(RR)^N$ in (29) then determine the normalizing factors:

$$a_n = \sqrt{B_N/N!}. \tag{31}$$

As can be seen from (28), the GMEs of the potential-energy operator consist of constructions of the form

$$\mathscr{V}(C_1,\,C_2,\,C_3) = \exp\{-C_1 R^2 - C_2 \tilde{R}^2 + C_3(\mathbf{R}\tilde{\mathbf{R}})\},$$

where C_1, C_2, C_3 are certain constants containing the parameters of the *NN* interaction; therefore, the matrix elements of this operator between the basis functions of the cluster model have the form of a linear superposition of the terms

$$\langle n_1 L \mid \mathscr{V}(C_1,\,C_2,\,C_3) \mid n_2 L\rangle = \sqrt{\frac{N_1!}{B_{N_1}}\frac{N_2!}{B_{N_2}}} \sum_{k=0}^{\min(n_1,\,n_2)} \frac{A_L^{2k+f}}{\sqrt{A_L^{N_1} A_L^{N_2}}} \times \frac{C_1^{n_1-k} C_2^{n_2-k} C_3^{2k+f}}{(n_1-k)!\,(n_2-k)!\,(2k+f)!};\quad N_i = 2n_i + f. \tag{32}$$

The coefficients of this superposition are combinations of the quantities $V_{2S+1,2T+1}$, which determine the intensity of the interaction of two nucleons in different spin (S) and isospin (T) states.

By means of the limiting process (10) we can readily obtain from the GMEs of the operator $\hat{F}$ (12), having first set $\beta = \tilde{\beta} = 0$, the cluster GMEs of the dipole-transition operator.

$$\langle \Phi(\mathbf{R}) \mid \hat{Q}_{1\mu} \mid \Phi(\tilde{\mathbf{R}})\rangle = \frac{-\mathrm{i}}{A_1^1} \int d\Omega_{\mathbf{k}} Y_{1\mu}^*(\Omega_{\mathbf{k}}) \frac{2(2Z-A)}{A} \times [(\mathbf{Rk}) + (\mathbf{k}\tilde{\mathbf{R}})]\langle \Phi(\mathbf{R}) \mid \Phi(\tilde{\mathbf{R}})\rangle \tag{33a}$$

and the GMEs

$$\langle \Phi(\mathbf{R}) \mid \hat{Q}_{\lambda\mu} \mid \Phi(\tilde{\mathbf{R}})\rangle = -\frac{1}{A_\lambda^\lambda} \int d\Omega_{\mathbf{k}} Y_{\lambda\mu}^*(\Omega_{\mathbf{k}}) \times \left\{\frac{Z}{2}\frac{A-1}{A} k^2 + 2\frac{A_2^2 + 8(Z-2)}{A^2}[(\mathbf{Rk})^2 + (\mathbf{k}\tilde{\mathbf{R}})^2] + (\mathbf{Rk})(\mathbf{k}\tilde{\mathbf{R}})\left[\frac{4(A-2Z)A_2}{A^2} + \frac{(Z-2)}{A_2}\frac{\partial}{\partial\xi}\right]\right\}$$

$$\times \langle \Phi(\mathbf{R}) \mid \Phi(\tilde{\mathbf{R}}) \rangle; \quad \xi = (\mathbf{R}\tilde{\mathbf{R}}), \tag{33b}$$

from which it is possible to separate the monopole ($\lambda = 0$) and quadrupole ($\lambda = 2$) transitions (Z is the number of protons). From the resulting GMEs (33), we calculate the matrix elements of the operator of the monopole transition and the reduced matrix elements of the dipole and quadrupole transitions between the oscillator functions of the cluster model. We give their explicit form separately for the isoscalar and isovector components, which we shall distinguish by means of the indices (0) and (1), respectively.

For the isoscalar monopole transition we have

$$\left.\begin{aligned} &\langle nL^{\pi} \mid Q_0^{(0)} \mid nL^{\pi} \rangle = \frac{1}{2}\left\{N + \frac{3}{2}(A-1)\right\}; \\ &\langle n+1L^{\pi} \mid Q_0^{(0)} \mid nL^{\pi} \rangle = \langle nL^{\pi} \mid Q_0^{(0)} \mid n+1L^{\pi} \rangle \\ &= \frac{2A_2}{A}\sqrt{\frac{B_N}{B_{N+2}}(N-L+2)(N+L+3)}. \end{aligned}\right\} \tag{34a}$$

For the isovector monopole transition

$$\left.\begin{aligned} &\langle nL^{\pi} \mid Q_0^{(1)} \mid nL^{\pi} \rangle = \frac{(2Z-A)}{A} \\ &\times \left\{N\left(\frac{A}{2A_2} - \frac{4A_2}{A}\frac{B_{N-1}}{B_N}\right) + \frac{3}{4}(A-1)\right\}; \\ &\langle n+1L^{\pi} \mid Q_0^{(1)} \mid nL^{\pi} \rangle \\ &= \frac{8(2Z-A)}{A^2}\sqrt{\frac{B_N}{B_{N+2}}(N-L+2)(N+L+3)}. \end{aligned}\right\} \tag{34b}$$

In the long-wave approximation, the dipole transition has only an isovector component and is determined by four reduced matrix elements:

$$\left.\begin{aligned} &\langle n(L+1)^{\pi_2} \| Q_1^{(1)} \| nL^{\pi_1} \rangle \\ &= \frac{2(2Z-A)}{A}\sqrt{\frac{3}{4\pi}(L+1)\frac{B_N}{B_{N+1}}(N+L+3)}; \\ &\langle (n-1)(L+1)^{\pi_2} \| Q_1^{(1)} \| nL^{\pi_1} \rangle \\ &= \frac{2(2Z-A)}{A}\sqrt{\frac{3}{4\pi}(L+1)\frac{B_{N-1}}{B_N}(N-L)}; \\ &\langle n(L-1)^{\pi_2} \| Q_1^{(1)} \| nL^{\pi_1} \rangle \\ &= -\frac{2(2Z-A)}{A}\sqrt{\frac{3}{4\pi}L\frac{B_N}{B_{N+1}}(N-L+2)}; \\ &\langle (n-1)(L-1)^{\pi_2} \| Q_1^{(1)} \| nL^{\pi_1} \rangle \\ &= -\frac{2(2Z-A)}{A}\sqrt{\frac{3}{4\pi}L\frac{B_{N-1}}{B_N}(N+L+1)}. \end{aligned}\right\} \tag{35}$$

Here, $\pi_2 = -\pi_1$. Finally, the quadrupole isoscalar transition is represented by the following matrix elements:

a) $L_2 = L_1 + 2$:

$$\left.\begin{aligned} &\langle n+1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{A} C_{L_1}^{(1)}\sqrt{\frac{B_N}{B_{N+2}}(N+L_1+5)(N+L_1+3)}; \\ &\langle nL_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle = \frac{1}{2} C_{L_1}^{(1)}\sqrt{(N+L_1+3)(N-L_1)}; \\ &\langle n-1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{A} C_{L_1}^{(1)}\sqrt{\frac{B_{N-2}}{B_N}(N-L_1)(N-L_1-2)}; \end{aligned}\right\} \tag{36a}$$

b) $L_2 = L_1$:

$$\left.\begin{aligned} &\langle n+1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{A} C_{L_1}^{(2)}\sqrt{\frac{B_N}{B_{N+2}}(N-L_1+2)(N+L_1+3)}; \\ &\langle nL_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle = \frac{1}{4} C_{L_1}^{(2)}(2N+3); \\ &\langle n-1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{4} C_{L_1}^{(2)}\sqrt{\frac{B_{N-2}}{B_N}(N-L_1)(N+L_1+1)}; \end{aligned}\right\} \tag{36b}$$

c) $L_2 = L_1 - 2$

$$\left.\begin{aligned} &\langle n+1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{A} C_{L_1}^{(3)}\sqrt{\frac{B_N}{B_{N+2}}(N-L_1+4)(N-L_1+2)}; \\ &\langle nL_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle = \frac{1}{2} C_{L_1}^{(3)}\sqrt{(N+L_1+1)(N-L_1+2)}; \\ &\langle n-1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle \\ &= \frac{2A_2}{A} C_{L_1}^{(3)}\sqrt{\frac{B_{N-2}}{B_N}(N+L_1+1)(N+L_1-1)}, \end{aligned}\right\} \tag{36c}$$

where

$$C_L^{(1)} = \sqrt{\frac{15}{8\pi}\frac{(L+1)(L+2)}{(2L+3)}};$$

$$C_L^{(2)} = -\sqrt{\frac{5}{4\pi}\frac{L(L+1)(2L+1)}{(2L+3)(2L-1)}};$$

$$C_L^{(3)} = \sqrt{\frac{15}{8\pi}\frac{L(L-1)}{(2L-1)}}.$$

the isovector quadrupole transitions can be readily obtained from the corresponding matrix elements of the isoscalar transition by means of the relations

$$\langle n \pm 1L_2^{\pi} \| Q_2^{(1)} \| nL_1^{\pi} \rangle = \frac{4(2Z-A)}{A_2 A}\langle n \pm 1L_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle;$$

$$\langle nL_2^{\pi} \| Q_2^{(1)} \| nL_1^{\pi} \rangle = \frac{2(2Z-A)}{A^2}\left[\frac{A^2}{2A_2} - 4A_2\frac{B_{N-1}}{B_N}\right]\langle nL_2^{\pi} \| Q_2^{(0)} \| nL_1^{\pi} \rangle. \tag{37}$$

The matrix elements given in this section make it possible to calculate the spectra of collective excitations and to describe scattering and reactions in nuclear systems, and also processes due to the interaction of photons and electrons with nuclei. The following sections of the present paper are devoted to the solution of these problems.

2. SPECTRA OF COLLECTIVE EXCITATIONS OF THE LIGHTEST *p*-SHELL NUCLEI. THE *Sp(2, R)* BASIS

In the theoretical investigation of the collective excitations of odd nuclei in the framework of microscopic theory it is impossible not to take into account the interaction of the spin and collective degrees of freedom. This greatly complicates calculations of the spectra of the excited states of odd and particularly doubly odd nuclei as well as the theoretical interpretation of the nature of the various excitations.

These difficulties can be eliminated if the theory is constructed on the basis of the method of generalized hyperspherical functions (MGHF) and the concept of central and spin-orbit nucleon-nucleon interactions. In the framework of the minimal approximation of the MGHF one can take

into account successfully the influence of the spin degrees of freedom on quadrupole excitations of collective type.

The present section is devoted to the solution of this problem; in it, we study the interaction of the spin and collective degrees of freedom in the lightest odd nuclei of the p shell, and also the effect of this interaction on the spectra of the investigated nuclei.

If the wave function projected onto the state with given total angular momentum I $\Psi_{\alpha I}^{[f]}$, is expanded in the series

$$\Psi_{\alpha I}^{[f]} = \sum_{n=0}^{\infty} C_n^{I\alpha} |n,\ I\rangle \tag{38}$$

with respect to the functions

$$|n,\ I\rangle \equiv |(\lambda\mu)\ IM\ [f]\rangle,$$
$$[f] = [f_1 f_2 f_3],\ (\lambda\mu) = (f_1 - f_2 + 2n,\ f_2 - f_3), \tag{39}$$

of the irreducible representation $1/2 f_1 + (A-5)/4$ of the group $Sp(2, R)$, then we obtain the homogeneous system of linear algebraic equations (2) for the expansion coefficients $C_n^{I\alpha}$ (the index $\alpha = 1, 2, 3, \ldots$ lists in order of increasing energy all states with total angular momentum I of the nucleus and spatial parity π).

The matrix elements of the Hamiltonian $\hat{H}$ needed for the following calculations are given in Sec. 1.

The NN interaction has been represented by the Volkov[20] potential (V) and the Brink-Boeker[21] potential (BB), augmented by the spin-orbit interaction of Gogny, Pires, and de Tourreil (GPT).[22] Allowance for the coupling between the spin and collective degrees of freedom made it possible to calculate the spin-orbit splitting of states with definite total orbital angular momentum.

For each nucleus at the start of the p shell ($A = 5, 6, 7, 8$), apart from ^{8}Be, two branches of collective excitations of different spatial parity ($\pi = \pm 1$) were calculated. The excitation branch is completely determined by the $O(A-1)$ symmetry of the intrinsic function $\{\chi_\nu^{[f]}\}$ on which it is constructed.[1] In Table I, we give the indices of the $O(A-1)$ symmetry of the intrinsic functions used in the calculation, and also the orbital angular momenta and parities of the states of the different branches.

For the example of the nucleus ^{7}Li, Fig. 1 illustrates the convergence to the limit of the approximate energies of the collective excitations of negative and positive parity calculated with n basis functions without allowance for the spin-orbit interaction; the limiting value of the energy corresponds to an infinitely large number of basis functions. For the ground state, the limit is reached more rapidly than for the excited states—the higher the excitation energy, the more basis functions are needed to approximate with a given accuracy the limiting energy of the excitation and its wave function [or, which is the same thing, the amplitudes $C_n^{I\alpha}$ of the expansion of the wave function of the collective excitation with respect to the basis of the $Sp(2, R)$ irreducible representation].

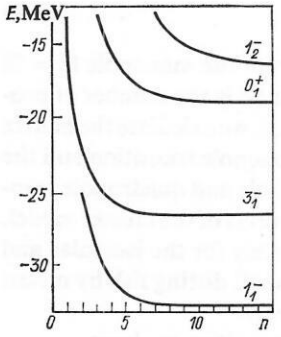

FIG. 1. Dependence of the energies E_α of the lowest collective excitations ($I_\alpha^\pi = 1_1^-, 3_1^-, 0_1^+, 1_2^-, 2_1^+$) of the ^{7}Li nucleus on the number n of basis functions in the case of the BB potential. The calculated energy eigenvalues are determined at the points of the curves that correspond to integral values of n.

Fixing the orbital angular momentum L of the state and the parity, π, and also the number of basis functions n, and diagonalizing the Hamiltonian, we sought then the value of the oscillator radius r_0 at which there is a minimum of the energy of the lowest state of the investigated nucleus with the quantum numbers I^π. It was found that r_0 depends on I^π and in all cases decreases monotonically with increasing n. To understand this dependence, it is convenient to consider the properties of the $Sp(2, R)$ basis.

The basis of the irreducible representation $1/2 f_1 + (A-5)/4$ of the group $Sp(2, R)$ consists of oscillator functions whose excitation quanta are constructed along the direction distinguished in three-dimensional space by the tensor $(f_1 - f_2, f_2 - f_3)$, i.e., by all the oscillator functions that have the minimal (allowed by the Pauli principle) number of quanta and belong to the irreducible representation $(f_1 - f_2, f_2 - f_3)$ of the group SU(3). Therefore, the basis of the irreducible representation of the group $Sp(2, R)$ is anisotropic, and its anisotropy axis after projection of the basis

TABLE I. Indices of $O(A-1)$ symmetry of the intrinsic functions $\chi_\nu^{[f]}$ included in the calculation.

Nucleus	[f00]	I^π	[f00]	I^π
^{5}He, ^{5}Li	[400]	$1^-, 3^- \ldots$	[200]	$0^+, 2^+ \ldots$
^{6}He, ^{6}Li	[200]	$0^+, 2^+ \ldots$	[300]	$1^-, 3^- \ldots$
^{7}Li, ^{7}Be	[300]	$1^-, 3^- \ldots$	[400]	$0^+, 2^+ \ldots$
^{8}Be	[400]	$0^+, 2^+ \ldots$	—	—

functions onto a state with definite angular momentum becomes one of the characteristic axes of the system of nucleons.

Solving the system of equations for the coefficients $C_n^{I\alpha}$ of the expansion of the wave function of the nucleus in a series with respect to the $Sp(2, R)$ basis, we obtain the possibility of studying collective vibrations of the nucleons along the anisotropy axis of the basis. With regard to the transverse collective vibrations (vibrations in the direction perpendicular to the distinguished axis), they can be described within the $Sp(2, R)$ basis only on the average, this being done by the choice of the optimal value of the oscillator radius r_0.

The results of the calculation of the coefficients of the expansion of the wave function with respect to the $Sp(2, R)$ basis for the nuclei in the first half of the p shell actually show that the mean slope of the surface of the effective potential energy in the space of the collective variables in the direction distinguished by the $Sp(2, R)$ basis is less than in the transverse direction,[2)] and therefore the ratio of the amplitude of the longitudinal collective vibrations to the amplitude of the transverse vibrations exceeds unity and, moreover, increases with increasing energy. The large value of this ratio indicates a relative softness of the longitudinal vibrations and is a qualitative justification for the choice of the $Sp(2, R)$ basis from the complete space of collective basis functions of the irreducible representation of the group $Sp(6, R)$. It should be noted that in reality the longitudinal and transverse vibrations are coupled. Quantitative estimates of the effects of taking into account this coupling and, in particular, its influence on the energy of the longitudinal collective excitations are intended in later calculations on the basis of an extension of the $Sp(2, R)$ basis to the $Sp(6, R)$ basis. However, we can already estimate how this extension will influence the energy of the first collective excitation I_2^π, which has the same parity and total angular momentum I as the ground state I_1^π. To estimate the change in the energy of the level I_2^π on the transition from $Sp(2, R)$ to $Sp(6, R)$, we can use the difference

$$\Delta E\,(I_2^\pi) = E_{I_2^\pi}(r_2) - E_{I_1^\pi}(r_1),$$

where r_1 and r_2 are the points of the minima of the energies of the levels I_1^π and I_2^π, respectively (Fig. 2). This difference reaches 1.3 MeV for $A = 5$, 0.9 MeV for $A = 6$, 0.5 MeV for $A = 7$, and 0.3 MeV for $A = 8$ and does not exceed 2–7% of the energy of the I_2^π level.

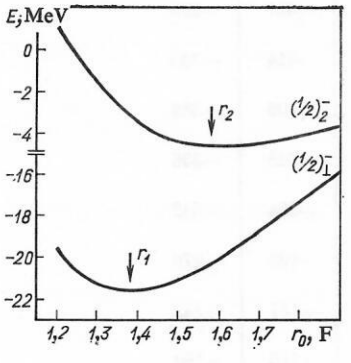

FIG. 2. Dependence of the energies of the levels $I_\alpha^\pi = (1/2)_1^-$ and $I_\alpha^\pi = (1/2)_2^-$ in the ^{7}Li nucleus on the oscillator radius r_0. The BB + GPT potential.

Earlier investigations of the spectra of the collective excitations of the light magic nuclei ^{4}He, ^{16}O, and ^{40}Ca (see Refs. 17 and 23) found a small dispersion of the amplitudes $C_n^{I\alpha}$ of the eigenvectors of the ground state ($I^\pi = 0^+, \alpha = 1$): The amplitude C_0^{01} was found to be near unity and the weight corresponding to all the remaining amplitudes C_n^{01}, $n = 1, 2, \ldots$, does not exceed a few percent, provided the optimal radius r_0 is found in each case. Thus, at least for NN potentials of Brink-Boeker or Volkov type, the basis function $|0, 0\rangle$ is a good approximation for the ground-state wave function of a magic nucleus under the condition that the solution of the Schrödinger equation is constructed in the form of the superposition (38) of the basis functions $|n, 0\rangle$.

But in the excited states of the same nuclei a large dispersion of the amplitudes $C_n^{I\alpha}$ is observed—in order of magnitude, the amplitudes $C_n^{I\alpha}$ are the same for the first values of n.

The dispersion of the amplitudes $C_n^{I\alpha}$ is greater, the larger the amplitude of the vibrations of the collective variable that characterizes the elongation of the ellipsoid of inertia of the nucleus along one of its three principal axes. Therefore, the appreciable increase in the dispersion of the amplitudes $C_n^{I\alpha}$ of the eigenvectors of the collective excitations of magic nuclei compared with the dispersion of the amplitudes of the ground-state eigenvector means that the effective potential of the nucleus in the space of the collective variables becomes shallower with increasing distance from the center of the nucleus along one of the three principal axes of its ellipsoid of inertia, and the amplitude of the collective vibrations in the excited states increases together with the excitation energy more rapidly than in the case of the oscillator effective potential.

For excited states, such a situation persists on the transition to near-magic[6] and nonmagic nuclei. The dependence of the behavior of the amplitudes $C_n^{I\alpha}$ of the three lowest collective excitations ($\alpha = 1, 2, 3$) constructed over the vibrational state $I^\pi = 1_1^-$ on the number n of basis functions is shown in Fig. 3. Each point of these curves that corresponds to an integer value of n determines the weight of the nth basis function in the expansion (38).

It is important to note that for nonmagic nuclei the dis-

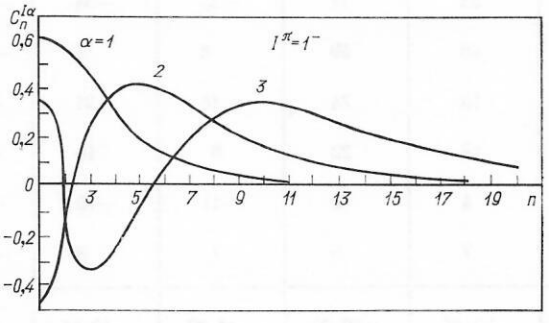

FIG. 3. Dependence of the amplitudes $C_n^{I\alpha}$ of the three lowest collective excitations ($\alpha = 1, 2, 3$, $I^\pi = 1^-$) of the ^{7}Li nucleus on the number n of basis functions in the case of the BB potential.

[2)] Essentially, the slope in the transverse direction is characterized by the optimal radius r_0, and the slope is greater, the smaller is $r_0 = r_0(I^\pi)$.

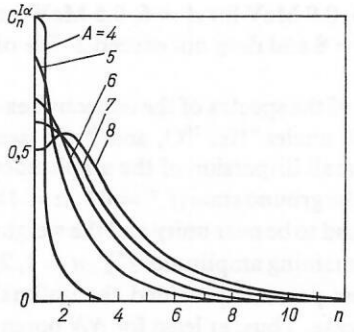

FIG. 4. Comparison of the amplitudes $C_n^{I\alpha}(\alpha = 1)$ of the ground states of nuclei with the number of nucleons A equal to 4, 5, 6, 7, 8. The values of $C_n^{I\alpha}$ are determined by the given curves at integral values of n. The BB potential.

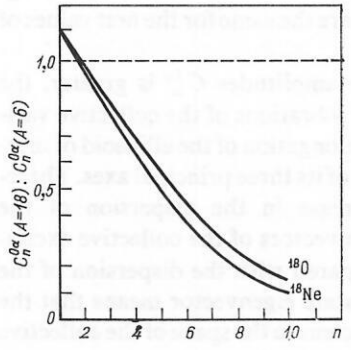

FIG. 5. Dependence of the ratio $C_n^{I\alpha}(A = 18) : C_n^{I\alpha}(A = 6)$ $(\alpha = 1, I^\pi = 0^+)$ on the number n of basis functions.

persion of the amplitudes $C_n^{I\alpha}$ becomes appreciable not only for excited states but also for the ground state and increases with increasing number A of nucleons. This can be seen from Fig. 4, which shows not only the amplitude C_n^{I1} of the ground state $I^\pi = 0^+$ of the nucleus ^{4}He (see Ref. 17) but also the ground-state amplitudes of the nuclei with the number of nucleons A equal to 5, 6, 7, 8. It can be seen that on the transition from the magic nucleus ^{4}He to the heavier nonmagic nuclei a growth in the dispersion of the ground-state amplitudes is observed. For this reason, the longitudinal vibrations already have a large amplitude in the ground state of nonmagic nuclei, and the amplitude increases with increasing number of nucleons.

In this connection, it is interesting to compare the amplitudes C_n^{01} $(A = 18)$ of the near-magic ^{18}O and ^{18}Ne nuclei[6] with the analogous amplitudes C_n^{01} $(A = 6)$ of the ^{6}He nucleus. All these nuclei each have two nucleons above a magic core (^{16}O for $A = 18$ and ^{4}He for $A = 6$) and differ in the mass of the core. Figure 5 shows the ratio of C_n^{01} $(A = 18)$ to C_n^{01} $(a = 6)$ as a function of the number n of basis functions. It can be seen that the values of this ratio are always, except in the case $n = 0$, less than unity and decrease with increasing n. This indicates a decrease in the amplitude of the collective vibrations in the ground state of the nuclei with the heavier core; this can evidently be attributed to the greater inertia of the core, which for $A = 6$ is more strongly polarized than for $A = 18$.

The amplitudes in Tables II–VI and in Figs. 3–4 completely reproduce the behavior of the nuclear wave function when allowance is made for a comparatively small number of basis functions. At the same time, information is needed on the asymptotic behavior of the nuclear wave function (or,

TABLE II. Amplitudes $C_n^{I\alpha}$ ($\times 10^3$), energies E_α, and rms radii R of I_α^π states of the ^{5}Li and ^{5}He nuclei. The BB + GPT potential.

n	$(3/2)_1^-$	$(1/2)_1^-$	$(7/2)_1^-$	$(5/2)_1^-$	$(1/2)_1^-$	$(5/2)_1^+$	$(3/2)_1^+$	$(3/2)_2^-$	$(1/2)_2^-$	$(1/2)_2^+$
1	871	812	901	908	—692	695	626	374	403	447
2	421	500	—430	—413	—528	511	540	—309	—185	37
3	197	229	—17	—45	—355	363	390	—479	—451	—235
4	441	172	—43	—37	—260	253	286	—420	—414	—330
5	52	71	—23	—34	—159	176	202	—388	—406	—384
6	48	59	3	6	—118	121	143	—286	—313	—356
7	15	24	—16	—21	—70	85	101	—237	—264	—332
8	18	22	8	10	—54	58	71	—169	—196	—279
9	4	9	—11	—12	—31	41	51	—135	—157	—242
10	7	8	7	8	—25	28	36	—96	—116	—196
E, MeV	—22,36	—20,20	—14,67	—12,55	—11,31	—7,03	—6,12	—3,69	—2,45	0,41
R, F	1,96	2,04	2,05	2,06	2,57	2,64	2,74	2,83	2,92	3,61

TABLE III. The same as in Table II but for the ^{6}He nucleus.

n	0_1^+	2_1^+	4_1^+	1_1^-	0_2^+	2_2^+	3_1^-	1_2^-	3_2^-
1	701	767	906	568	459	458	555	457	488
2	561	519	—418	544	—2	—126	530	—181	185
3	336	292	—51	420	—339	—400	422	—102	—81
4	230	193	—37	320	—401	—419	325	—256	—236
5	131	109	—30	226	—414	—400	244	—341	—314
6	87	71	0	163	— 353	—328	179	—356	—340
7	50	41	—15	113	—296	—267	131	—343	—331
8	33	27	3	80	—232	—206	95	—308	—305
9	19	16	—7	55	—182	—158	69	—268	—270
10	13	10	3	39	—138	—119	50	—226	—233
E, MeV	—25,27	—21,01	—13,43	—11,32	—8,24	—6,93	—5,98	—0,92	1,52
R, F	2,29	2,30	2,31	—2,83	2,97	3,04	3,00	3,68	3,89

which is the same thing, the asymptotic behavior of the amplitudes $C_n^{I\alpha}$). Knowledge of the asymptotic behavior of $C_n^{I\alpha}$ at large values of n is helpful not only as a check on the convergence of the results when the basis is extended but also for simplifying the technique of calculating the eigenvectors of the Hamiltonian (see Ref. 24), this simplification being achieved by replacing the exact values of the coefficients $C_n^{I\alpha}$ at large values of n by their asymptotic behavior. To consider the asymptotic behavior of the coefficients $C_n^{I\alpha}$, we consider the matrix elements of the kinetic-energy operator.

TABLE IV. The same as in Table II but for the ^{6}Li nucleus.

n	1_1^+ ($I=0$)	3_1^+ ($I=2$)	2_1^+ ($I=2$)	1_1^+ ($I=2$)	5_1^+ ($I=4$)	4_1^+ ($I=4$)	2_1^- ($I=1$)	1_1^- ($I=1$)	0_1^- ($I=1$)	3_1^+ ($I=4$)	1_2^+ ($I=0$)
1	701	793	743	666	—905	—908	577	559	534	—912	—459
2	561	469	536	579	420	411	543	545	546	392	2
3	336	276	307	353	45	63	417	423	432	98	339
4	230	179	205	243	38	37	316	324	334	38	400
5	131	99	119	149	27	34	222	230	240	47	414
6	87	65	77	97	0	0	160	166	174	3	353
7	50	37	46	61	14	16	110	115	122	21	296
8	33	24	30	39	—3	—3	78	82	87	—1	232
9	19	14	18	25	1	8	53	56	61	9	182
10	13	9	12	16	—3	—3	38	40	43	—2	138
E, MeV	—25,27	—21,97	—20,58	—18,76	—14,63	—13,10	—11,46	—11,18	—10,78	—10,47	—8,25
R, F	2,29	2,25	2,31	2,40	2,27	2,28	2,82	2,84	2,88	2,29	2,97

TABLE V. The same as in Table II but for the ^{7}Li and ^{7}Be nuclei.

n	$(3/2)_1^-$	$(1/2)_1^-$	$(7/2)_1^-$	$(5/2)_1^-$	$(1/2)_1^-$	$(11/2)_1^-$	$(3/2)_1^-$	$(5/2)_1^+$	$(9/2)_1^-$	$(1/2)_2^-$	$(3/2)_1^+$
1	618	599	695	636	−482	906	478	492	908	474	−467
2	578	581	547	568	−535	−414	110	532	−405	135	−528
3	396	406	355	388	−453	−70	−247	449	−91	−226	−455
4	275	284	235	264	−360	−37	−373	355	−39	−363	−365
5	175	183	149	174	−267	−30	−407	266	−37	−405	−277
6	114	120	95	113	−195	−5	−371	195	−7	−374	−214
7	72	76	61	74	−139	−12	−315	140	−14	−321	−148
8	46	49	38	47	−98	0	−254	100	−1	−260	−106
9	29	31	25	31	−69	−5	−199	70	−6	−205	−76
10	18	20	16	20	−48	0	−152	50	0	−158	−54
E, MeV	−32,7	−32,19	−27,14	−25,79	−19,11	−18,41	−16,63	−16,60	−16,52	−16,24	−16,17
R, F	2,42	2,44	2,42	2,48	2,87	2,37	2,99	2,90	2,38	3,01	2,93

In the general case, when $f_1 \neq f_2 \neq f_3$, the matrix element of the kinetic-energy operator between the GCSs of the irreducible representation $1/2\,f_1 + (A-5)/4$ of the group $Sp(2, R)$ has the form

$$\langle \Phi^{[f]} \mid \hat{T} \mid \tilde{\Phi}^{[f]} \rangle = \frac{1}{2}\left\{ -f_1 + f_2 + f_3 + \frac{A-1}{2} + \left(f_1 + \frac{A-1}{2}\right)\frac{2+\beta+\tilde{\beta}}{\Delta}\right\} \langle \Phi^{[f]} \mid \tilde{\Phi}^{[f]} \rangle, \tag{40}$$

where

TABLE VI. The same as in Table II for the ^{8}Be nucleus (calculation with the potential V; see Ref. 20).

n	0_1^+	2_1^+	4_1^+	0_2^+	2_2^+	6_1^+	4_1^+	0_3^+	2_3^+	4_3^+	6_2^+
1	527	539	579	−469	−485	−904	541	378	387	−4 3	228
2	577	575	563	−221	−202	412	128	−359	−364	−161	301
3	447	442	427	137	155	98	−215	−307	−321	350	434
4	324	318	300	323	333	41	−360	−290	−282	232	440
5	220	216	206	394	394	34	−389	−149	−128	47	399
6	146	143	137	384	380	10	−359	26	46	−120	342
7	95	94	91	338	331	11	−304	168	183	−235	279
8	61	60	59	278	272	3	−245	262	271	−297	223
9	39	39	39	220	214	4	−191	309	312	−317	174
10	25	25	25	169	165	1	−145	320	318	−309	134
E, MeV	−48,39	−45,28	−37,76	−31,25	−29,46	−26,43	−25,43	−17,67	−16,68	−14,69	−12,22
R, F	2,48	2,49	2,52	2,94	2,95	2,40	2,96	3,43	3,45	3,48	3,58

$$\langle \Phi^{[f]} \mid \tilde{\Phi}^{[f]} \rangle = d_{11}^{\lambda_0} d_{33}^{\mu_0} \Delta^{-f_1 - \frac{A-1}{2}}; \quad \Delta = 1 - \beta\tilde{\beta} d_{11}^2;$$
$$\lambda_0 = f_1 - f_2; \quad \mu_0 = f_2 - f_3,$$

and d_{ij} are the matrix elements of the rotation matrix in three-dimensional space (they depend on three Eulerian angles). Here, as before, the oscillator radius r_0 is taken as the unit of length, the nucleon mass m as the unit of mass, and Planck's constant $\hbar$ has the unit of angular momentum. If $f_2 = f_3 = f$, then from (40), after projection onto a state with a definite total orbital angular momentum I of the system, it follows that the matrix elements of the kinetic-energy operator between the basis functions of the irreducible representation $(\lambda_0 + 2n, 0)$ of the group SU(3) are nonzero only when the quantum number n of the basis functions $|n, I\rangle$ is the same or differs by unity:

$$\langle n, I \mid \hat{T} \mid n, I \rangle = \frac{1}{2}\left[f_1 + 2f + 2n + \frac{3}{2}(A-1) \right]; \quad (41)$$

$$\langle n+1, I \mid \hat{T} \mid n, I \rangle = \langle n, I \mid \hat{T} \mid n+1, I \rangle = \frac{1}{2}$$
$$\sqrt{\frac{(n+1)\left(f_1 + n + \frac{A-1}{2}\right)[(\lambda_0 + 2n + 2)(\lambda_0 + 2n + 3) - I(I+1)]}{(\lambda_0 + 2n + 1)(\lambda_0 + 2n + 2)}}. \quad (42)$$

Thus, the matrix of the kinetic-energy operator is tridiagonal, and the elements of this matrix at large values of the quantum number n are proportional to n. Since the matrix elements of the potential-energy operator of the short-range NN forces decrease with increasing quantum numbers n_1 and n_2 of the basis functions $|n_1, I\rangle$ and $|n_2, I\rangle$ between which they are constructed, at large values of n_1 and n_2 in the matrix of the collective Hamiltonian it is necessary to retain only the matrix elements of the kinetic-energy operator and the matrix elements of the potential-energy oprator diagonal with respect to n_1 and n_2. It is these that determine the asymptotic (for $n \gg 1$) behavior of the coefficients $C_n^{I\alpha}$ of the expansion of the wave function of the system with respect to the basis functions $|n, I\rangle$.

We note that there is a possibility of inverting the matrix

$$\| \langle n_1, I \mid \hat{T} \mid n_2, I \rangle - E\delta_{n_1 n_2} \| \quad (43)$$

and obtaining its inverse $\mathcal{G}_{n_1, n_2}(E)$, the matrix of the Green's function, so that instead of the usual equation

$$\sum_{n_2=0}^{\infty} [\langle n_1, I \mid \hat{T} \mid n_2, I \rangle + \langle n_1, I \mid \hat{V} \mid n_2, I \rangle - E\delta_{n_1 n_2}] C_{n_2}^{I\alpha} = 0,$$
$$n_1 = 0, 1, 2 \ldots, \quad (44)$$

for the eigenvalues and eigenvectors of the collective Hamiltonian in the representation of the basis functions $|n, I\rangle$ one obtains in the same representation a discrete analog of the homogeneous Lippmann-Schwinger equations,

$$C_n^{I\alpha} + \sum_{n_1 n_2} \mathcal{G}_{n n_1}(E) \langle n_1, I \mid \hat{V} \mid n_2, I \rangle C_{n_2}^{I\alpha} = 0, \quad (45)$$

and this may be helpful not only for numerical realization of the solution of the Sturm-Liouville problem but also for studying the asymptotic behavior of the coefficients $C_n^{I\alpha}$.

To find the eigenvalues and eigenvectors of the infinite system of linear algebraic equations (44), it is necessary to retain in this system only the first $N+1$ equations, setting $n_1 < N$, and simultaneously truncating the summation over n_2 at the $(N+1)$-term. The operation of truncating the system of equations is well founded, and the process converges for the states for which the amplitudes $C_n^{I\alpha}$ decrease sufficiently rapidly with increasing n. The law of this decrease can be established *a priori* by using semiclassical estimates for $C_n^{I\alpha}$ (see Ref. 24), these being based essentially on the limiting (at large n_1 and n_2) expressions for the matrix elements of the Hamiltonian H.

An asymptotic expression can be obtained for the coefficients $C_n^{I\alpha}$ by going to the limit of large n in the expressions for the matrix elements of the Hamiltonian $\langle n_1, I | H | n_2, I \rangle$.

In the construction of the limiting expressions for the matrix elements of the tridiagonal matrix of the kinetic-energy operator problems do not arise. With regard to the matrix elements of the potential-energy operator, with increasing n_1 and n_2 they all, except, perhaps, the diagonal ones, tend to zero. We denote the limiting values of the diagonal matrix elements of the potential-energy operator by $V_0 = V_0(I^\pi)$.

In the limit of large n, the equations that relate the expansion coefficients $C_n^{I\alpha}$ of the state I_α^π take the simple form

$$-\frac{1}{2}\left(n + \frac{1}{2} f_1 + \frac{A+3}{4} - \frac{\nu^2}{8n}\right) C_{n+1}^{I\alpha}$$
$$-\left[n + \frac{1}{2} f_1 + f + \frac{3}{4}(A-1) + V_0 - E\right] C_n^{I\alpha}$$
$$-\frac{1}{2}\left(n + \frac{1}{2} f_1 + \frac{A-1}{4} - \frac{\nu^2}{8n}\right) C_{n-1}^{I\alpha} = 0, \quad n \gg 1.$$

These equations are equivalent to an ordinary differential equation for the function $C(x)$:

$$x \frac{d^2}{dx^2} C + \frac{d}{dx} C + 2\left(E - f - \frac{A-2}{2} - \frac{\nu^2}{8x} - V_0\right) C = 0, \quad (46)$$

where

$$x = n + \frac{1}{2} f_1 + \frac{A+1}{4}.$$

A solution of the differential equation (46) that decreases with increasing x is the Macdonald function $K_\nu(\sqrt{2x\mathscr{E}})$, where

$$\mathscr{E} = -E + f + V_0 + \frac{A-2}{2}. \quad (47)$$

Therefore, at large values of n

$$\left.\begin{aligned} C_n^{I\alpha} &= \text{const} \cdot K_\nu(\sqrt{2x\mathscr{E}}), \\ \nu^2 &= \left(f_1 + \frac{A-5}{2}\right)^2 + (I + 1/2)^2 + 2(f_1 - f) - \frac{9}{4}. \end{aligned}\right\} \quad (48)$$

To find the coefficient of $\mathscr{E}$ in (48), it is necessary to substitute in (47) $E = E(I_\alpha^\pi)$, the energy of the αth state with total angular momentum I and parity π, and choose the constant factor in front of the Macdonald function in accordance with the normalization condition.

Finally, it follows from (47) and (48) that the threshold enegy E_0 of the system of nucleons separating the states of the discrete and continuous spectra of collective excitations is determined by the energy of the zero-point collective vi-

brations in the direction perpendicular to the one in which the excitation quanta of the oscillator functions of the $Sp(2, R)$ basis are aligned and by the limiting value of the diagonal matrix element of the potential-energy operator:

$$E_0(I^\pi) = f + \frac{A-2}{2} + V_0. \quad (49)$$

Note also that since h^2/mr_0^2 is the unit of measurement, the threshold energy in the approximation of the $Sp(2, R)$ basis depends on the oscillator radius r_0, which occurs both in the common factor and in the function $V_0(I^\pi)$.

It can be seen directly from (48) that the choice of N must depend on $\mathscr{E}$: The smaller $\mathscr{E}$, the more equations we must retain in the system (44) in order to guarantee a given accuracy for the eigenvalue.

The results of calculation of the energy E and the rms radius R of the ground and lowest collective excitations of the nuclei ^{5}He, ^{5}Li, ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, and ^{8}Be are given in Tables II–VI. The calculation was made with the Brink-Boeker potential (BB) augmented by the spin-orbit part of the Gogny, Pires, and de Tourreil (GPT) potential.

The spectra of the lowest collective excitations of nuclei with the numbers of nucleons A equal to 5, 6, 7 are given in Figs. 6–8 [a) calculation with BB central forces; b) calculation with a BB potential augmented by the spin-orbit part of the GPT potential (BB + GPT). In the experimental spectrum (c), which is taken from Ref. 25, we have retained only

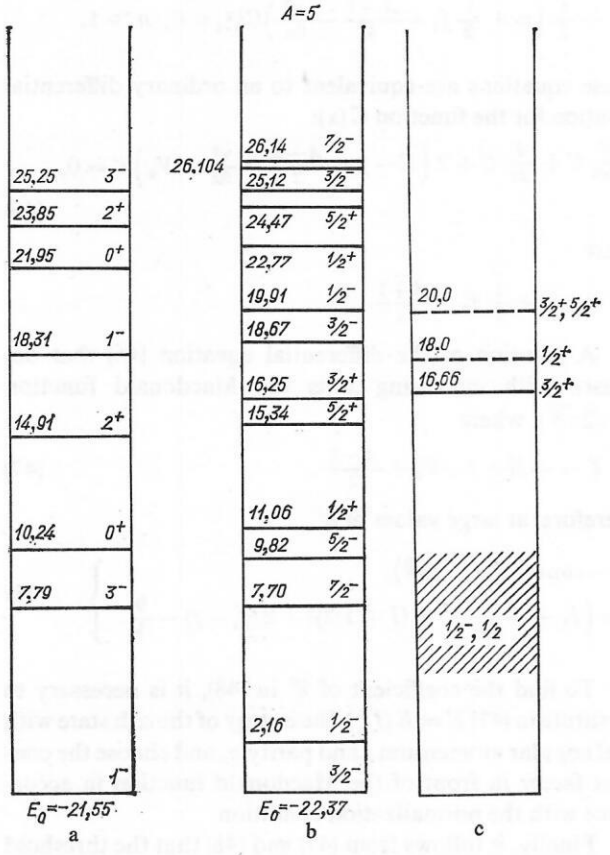

FIG. 6. Spectra of the lowest collective excitations of the ^{5}He and ^{5}Li nuclei: a) calculation with centrifugal BB forces; b) calculation with the BB + GPT potential; c) experiment.[25]

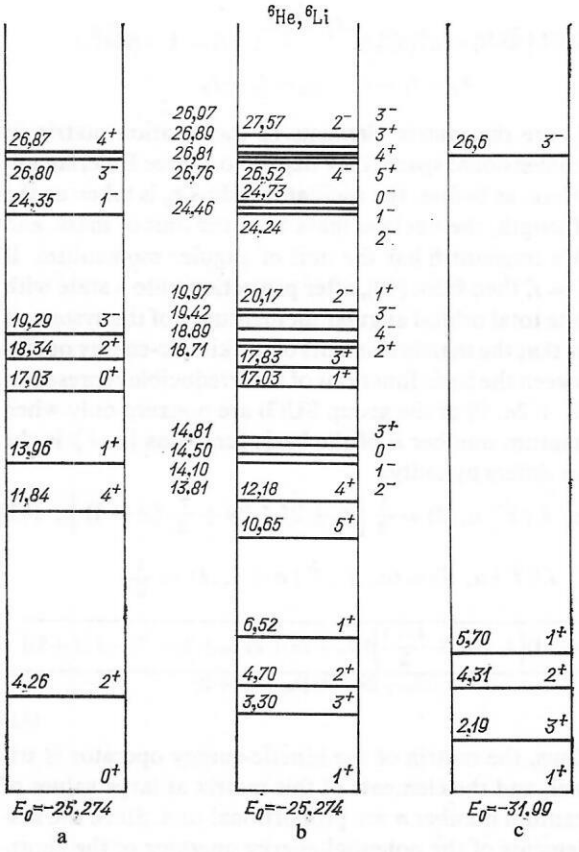

FIG. 7. Spectra of the lowest collective excitations of the nuclei ^{6}He (a) and ^{6}Li (b). In the experimental spectrum (c), only the levels with $T = 0$ are given.

the states that correspond to isospin $T = 1/2$ for nuclei with A equal to 5, 7 and $T = 0$ for the nucleus ^{6}Li]. Since the Coulomb interaction of the protons was not taken into account, and the even components of the BB potential are the same, spectrum (c) in Fig. 6 refers to ^{5}He and ^{5}Li, and in Fig. 8 to ^{7}Li and ^{8}Be. For $A = 6$, spectrum (a) of Fig. 7 reproduces the ^{6}He states with isospin $T = 1$, and spectrum (b) gives the ^{6}Li states with $T = 0$.

It can be seen from Figs. 6–8 that in all cases the ground state is the lowest state of the collective branch that corresponds to normal parity, and the states of anomalous parity lie 10–15 MeV above the ground state.

The energy of the collective excitations was measured from the binding energy, which for the BB + GPT potential is -22.37 MeV for $A = 5$; -25.27 MeV for $A = 6$; -35.70 MeV for $A = 7$; and -48.39 MeV for $A = 8$. The levels of the lowest rotational band, 1^+, 3^+, 2^+, 1^+ in ^{6}Li and $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-$ in ^{7}Li, have the correct sequence, and the energies of these levels agree satisfactorily with experiment. The same applies to the lowest $3/2^-$ and $1/2^-$ levels (2.16 MeV) in ^{5}Li, but as yet there is no reliable experimental information about the position of the $1/2^-$ level.

The levels $5/2^-$ (7.47 MeV) and $7/2^-$ (9.61 MeV) in ^{7}Li, which are absent in the theoretical spectrum, have a pronounced cluster structure, allowance for which in our calculation requires the use of a more complicated intrinsic func-

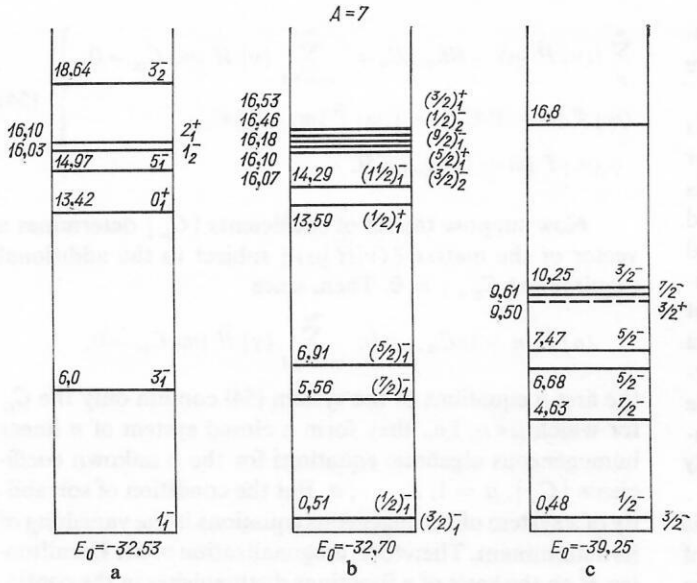

FIG. 8. Spectra of the lowest collective excitations of the ^{7}Li and ^{7}Be nuclei: a) calculation with the BB potential; b) with the BB + GPT potential; c) experiment.[25]

tion $\chi_\nu^{[f]}$. Such allowance is very topical and makes it necessary to go beyond the framework of the MA MGHF.

The spin-orbit splittings of the $3/2^-$ and $1/2^-$ levels, and also of the $7/2^-$ (5.56 MeV) and $5/2^-$ (6.91 MeV) levels in ^{7}Li and the 3^+ (3.30 MeV) and 1^+ (6.52 MeV) levels in ^{6}Li, are close to the experimental values—the calculated values for the BB + GPT potential are, respectively, 0.51, 1.35, 3.22 MeV, and the experimental values are 0.48, 2.0, 3.51 MeV.

Attention should also be drawn to the positions of the single-quantum collective excitations. All these states, $3/2^-$ (18.67 MeV) and $1/2^-$ (19.91 MeV) in ^{5}He, ^{5}Li, 1^+ (17.03 MeV) in ^{6}Li, and $3/2^-$ (16.07 MeV) and $1/2^-$ (16.48 MeV) in ^{7}Li, lie somewhat above the lowest states of anomalous parity but, in contrast to the latter, exhibit a tendency to sink with increasing number of nucleons.

The rms radius R of the ground state (see Tables II–VI) of the nucleus ^{7}Li ($R = 2.42$ F) is equal to the experimental value $R_{\text{exp}} = 2.405 \pm 0.020$ F,[26] while for the nucleus ^{6}Li ($R = 2.29$ F for BB + GPT and $R = 2.19$ F for the potential V) it is somewhat too low: $R_{\text{exp}} = 2.505 \pm 0.020$ F.[26] With regard to the nuclei ^{5}He, ^{5}Li, and ^{8}Be,the theoretical estimates (respectively, $R = 1.96$ F and $R = 2.48$ F; BB + GPT potential) are somewhat below the values obtained in the calculations of Ref. 27: $R = 2.31$ F and $R = 2.71$ F.

3. CONTINUUM STATES AND PHASE SHIFTS OF ELASTIC SCATTERING OF LIGHT CLUSTERS

As was shown by calculations made using the resonating-group method,[9,28,29] the nuclei ^{5}He, ^{6}He, ^{6}Li, ^{7}Li, and ^{8}Be have a clearly expressed cluster structure. The clustering of these nuclei is a direct consequence of the nature of the forces that act between the nucleons. The nucleon–nucleon interaction couples nucleons within each of the clusters more strongly than nucleons from different clusters. In turn, the clustering greatly influences the formation of the structure of the ground-state wave functions of light nuclei, the position of the lowest disintegration thresholds, and the nature of the resonance states in the above-barrier region. Therefore, many properties of light nuclei can be explained by assuming that the wave function Ψ of the states of their discrete and continuous spectra can be represented in the form

$$\Psi = \hat{A}\,[\varphi\,(1)\,\varphi\,(2)\,f], \tag{50}$$

where $\hat{A}$ is the antisymmetrization operator, $\varphi\,(1)$ is the wave function of an alpha cluster, $\varphi\,(2)$ is the wave function of the second fragment, whose mass number is $A_2 = A - 4$, and f is the wave function of the relative motion of the two fragments. As a rule, $\varphi\,(1)$ and $\varphi\,(2)$ are taken to be the well-known translationally invariant wave functions of the oscillator shell model for the corresponding fragments. With regard to the function f, it is found by solving the integro-differential equation of the resonating-group method or by means of some equivalent procedure.

It is, however, convenient to give up the attempt to find an explicit expression for f, especially since it occurs, in all calculations made with it, in complicated constructions to which the antisymmetrization operator is applied. Instead of this, writing for f an expansion with respect to the basis of oscillator functions f_ν of the relative motion of the clusters,

$$f = \sum_\nu C_\nu f_\nu, \tag{51}$$

where ν is the number of oscillator excitation quanta, it is expedient to construct the wave function in the form of the generalized Fourier series

$$\Psi = \sum_\nu C_\nu\,|\nu\rangle;\quad |\nu\rangle = \hat{A}\,[\varphi\,(1)\,\varphi\,(2)\,f_\nu]. \tag{52}$$

The Fourier coefficients C_ν must satisfy the system of linear differential equations

$$\sum_{\nu'} \langle\nu\,|\,\hat{H}\,|\,\nu'\rangle\,C_{\nu'} - EC_\nu = 0, \tag{53}$$

where $\hat{H}$ is the many-particle Hamiltonian of the nucleus. The representation of the wave function of the nucleon system in the form of the series (52) with respect to the many-

particle oscillator basis functions $|\nu\rangle$ generated by the cluster GCSs is the principal point of the algebraic version of the resonating-group method.[24,30,31]

The basis $|\nu\rangle$ of many-particle oscillator functions is adapted to the description of the dynamics of the cluster mode of light nuclei. At the same time, the states in this basis with the minimal number ν of oscillator quanta correspond to the shell-model configurations that are usually associated with the ground states of light nuclei. Therefore, the expansion (52) actually takes into account not only the cluster but also the shell configurations and, therefore, opens a way to a generalization of the concepts of the translationally invariant shell model. The aim of this generalization is to include in the treatment the cluster modes of nucleon systems without going beyond the means adopted in the translationally invariant shell model.

It is well known that the resonating-group method is used not only to study the continuum states of systems of interacting nucleon clusters but also states of the discrete spectra of the same systems (see Refs. 9, 28, 29, and 32). In the latter case, the algebraic version of the resonating-group method reduces to diagonalization of the many-particle microscopic Hamiltonian of the system of clusters between the functions $|\nu\rangle$ of the oscillator basis generated by the generator function of the resonating-group method (see Sec. 1). Restricting the matrix of the Hamiltonian to the order ν_0, where ν_0 is the number of basis functions employed, and solving the eigenvalue problem for this matrix, we obtain ν_0 different eigenvalues and as many corresponding eigenvectors. Among the eigenvalues, only the first few (k, $k<\nu_0$) will, in general, be negative. The remaining eigenvalues are positive, and, as a rule, they are usually not considered (see, however, Refs. 33 and 34), since, at the first glance, it is hard to believe that the eigenvectors corresponding to them, found in the representation of the restricted oscillator basis, can have any relation to the continuum wave functions. Nevertheless, the first impression of the eigenvectors with positive eigenvalues, which suggested that there was no point in investigating them, was wrong. Indeed, as will be shown later, the eigenvectors with positive energy, no less than those with negative energy, tend to an exact solution as the oscillator basis is extended, and even for not particularly large ν_0 give information about the S matrix. It should be noted that the possibility of extracting data about the S matrix (reaction cross sections, energies of resonances, widths of resonances) using the positive eigenvalues of the matrix of the Hamiltonian in the truncated oscillator basis was demonstrated in Ref. 35. However, these studies considered only the simplest model systems, in which the interaction potential has a barrier and core at short distances. We shall consider the case of a real cluster model of two interacting subsystems, for which it is possible to find the phase shifts and the eigenvectors corresponding to the continuum states.

To understand the significance of the approximate solutions obtained by diagonalizing the microscopic Hamiltonian on the truncated oscillator basis, we transform the original system of linear algebraic equations (53) for the coefficients $\{C_\mu\}$ to the form (for more details, see Ref. 31)

$$\left.\begin{aligned}
&\sum_{\mu}^{n}(\langle\nu|\hat{H}|\mu\rangle - E\delta_{\nu\mu})C_\mu + \sum_{m=n+1}^{\infty}\langle\nu|\hat{H}|m\rangle C_m = 0,\\
&\langle m|\hat{T}|m-1\rangle C_{m-1} + (\langle m|\hat{T}|m\rangle - E)C_m\\
&+\langle m|\hat{T}|m+1\rangle C_{m+1} = 0.
\end{aligned}\right\} \quad (54)$$

Now suppose the set of coefficients $\{C_\mu\}$ determines a vector of the matrix $\|\langle\nu|\hat{H}|\mu\rangle\|$ subject to the additional requirement $C_{n+1}=0$. Then, since

$$\langle n|\hat{T}|n+1\rangle C_{n+1} = 0, \quad \sum_{m=n+1}^{\infty}\langle\nu|\hat{H}|m\rangle C_m = 0,$$

the first n equations of the system (54) contain only the C_μ for which $\mu\leqslant n$, i.e., they form a closed system of n linear homogeneous algebraic equations for the n unkown coefficients $\{C_\mu\}$, $\mu = 1, 2, \ldots, n$. But the condition of solvability of a system of homogeneous equations is the vanishing of its determinant. Therefore, diagonalization of the Hamiltonian $\hat{H}$ on the basis of n functions distinguishes in the continuum of the Hamiltonian the eigenvectors $\{C_\mu^\alpha\}$ with vanishing $(n+1)$th coefficient and positive energy E_α, measured from the threshold of breakup of the nucleus into two clusters.

From the condition $C_{n+1}^\alpha(E_\alpha, I) = 0$, where I is the angular momentum of the state with energy E_α, and the asymptotic expression C_{n+1}^α (see Refs. 1 and 24) it follows that

$$\left.\begin{aligned}
&j_I(k_\alpha x_{n+1}) - \operatorname{tg}\delta_I(E_\alpha)\, n_I(k_\alpha x_{n+1}) = 0,\\
&x_n = \sqrt{4(n+n_0)+2I+3},\ k_\alpha = \sqrt{2E_\alpha},
\end{aligned}\right\} \quad (55)$$

where j_I and n_I are spherical Bessel and Neumann functions, respectively, n_0 is the minimal admissible number of oscillator excitation quanta of the relative-motion wave function of the two clusters, and $\delta_I(E_\alpha)$ is the phase shift in the state with angular momentum I and energy E_α. The units of measurement in (55) are chosen to make $\hbar = m = r_0 = 1$, where $\hbar$ is Planck's constant, m is the nucleon mass, and r_0 is the oscillator radius. If the energy E_α of the state with angular momentum I is known, then (55) makes it possible to obtain the phase shift $\delta_I(E_\alpha)$.

For the complete description of the eigenvector $\{C_\nu(E_\alpha, I)\}$ of the Hamiltonian of the resonating-group method, it is necessary to give the values of the coefficients C_ν with numbers $\nu > n+1$. This is easy to do, since for C_ν when $\nu > n+1 \gg 1$ the following asymptotic expression holds

$$C_\nu(E_\alpha, I) = B\sqrt{x_\nu}\,[j_I(k_\alpha x_\nu) - \operatorname{tg}\delta_I(E_\alpha)\, n_I(k_\alpha x_\nu)], \quad (56)$$

where the factor B is determined from the condition that the coefficient $C_n(E_\alpha, I)$, calculated in accordance with (56), is equal to the coefficient $C_n(E_\alpha, I)$ of the eigenvector $\{C_\nu(E_\alpha)\}$ obtained by diagonalizing the Hamiltonian between n basis functions.

To illustrate the general propositions formulated above, we use the example of a system of two interacting nuclei: ^{4}He and ^{3}H. These nuclei are capable of forming a bound system, ^{7}Li, in states $J^\pi = 3/2^-$ and $J^\pi = 1/2^-$. We

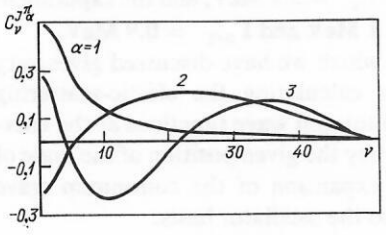

FIG. 9. Dependence of the coefficients $C_\nu^{J^\pi\alpha}$ of the expansion (51) on the number ν of excitation quanta of the function of the relative motion of the clusters.

write the bound-state wave functions of ^{7}Li, and also the states J^π belonging to the continuum, in the form

$$\Psi_{J^\pi}^{\alpha} = \sum_{\nu=1}^{\infty} C_\nu^{J^\pi\alpha} |\nu,\ J^\pi\rangle, \tag{57}$$

where $\{|\nu, J^\pi\rangle\}$ is the set of orthonormalized configurations of an isotropic harmonic oscillator generated by the wave function of the resonating-group method for the ^{4}He ^{3}H channel. The index α labels in the order of increasing energy the states with quantum numbers J^π. The coefficients $C_\nu^{J^\pi\alpha}$, which belong to different eigenvectors ($\alpha = 1, 2, \ldots$), are calculated by diagonalizing the Hamiltonian of the system of seven nucleons between the n basis functions $|\nu, J^\pi\rangle$. They exhibit the very simple dependence on ν shown in Fig. 9. The eigenvectors shown in the figure were obtained in a calculation with 50 basis functions. The nucleon-nucleon interaction was simulated by the Brink-Boeker potential[21] and the spin-orbit GPT interaction.[22]

The coefficients $C_\nu^{3/2^-,1}$ of the amplitude of the ground state of the system, decreasing monotonically with increasing ν, tend exponentially to zero. As already noted, the coefficients $C_\nu^{3/2^-,\alpha}$ of the continuum states vanish for $\nu = n+1$ and, in addition, change sign $\alpha - 1$ times as ν varies from 1 to $n+1$. With increasing n, the energies of all the continuum eigenvectors decrease, approaching zero, and the nodes of the coefficients $C_\nu^{3/2^-,\alpha}$ move to the right along the ν axis. However, the first node of the coefficients $C_\nu^{3/2^-,\alpha}$ of the continuum state with minimal energy in the limit when $E \to 0$ ($n \to \infty$) tends to its limiting value $\nu_0 \sim 8$. Since the energy of this eigenvector is near zero ($E_2^{3/2^-} = 7.05 \times 10^{-2}$ if $n = 50$), the position of the first node of its coefficients can be related to the coefficient a in the asymptotic expression for the amplitudes of the state with zero energy,

$$\lim_{n\to\infty} C_\nu^{3/2^-,\,2} = \text{const}\,[(4\nu+9)^{3/2} - 3a^3]. \tag{58}$$

This relationship is determined by

$$a = \sqrt{4\nu_0+9}/\sqrt[3]{3}, \tag{59}$$

where ν_0 is the limiting position of the first node as $n \to \infty$. Figure 10 shows the dependence of ν_0 on the energy $E = E_\alpha^{J^\pi} - E_{\text{th}}$ (E_{th} is the threshold energy). Also shown there is the behavior of the function (59), whose limiting value as $\nu \to \nu_0$ gives the $t\alpha$ elastic scattering length. Setting $\nu_0 = 8$, we find that $a = 4.43$ (in units of r_0).

Note that

$$a^3 = \lim_{k\to 0} \frac{\delta_{3/2^-}(k)}{k^3}. \tag{60}$$

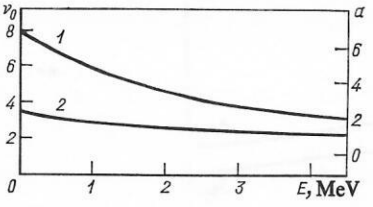

FIG. 10. Dependence of the position of the first node ν_0 of the function $C_\nu^{J^\pi\alpha}$ (curve 1) and of the function a (curve 2) on the energy.

Figure 11 shows $\delta_{3/2^-}$ and $\delta_{1/2^-}$, the phase shifts of ^{3}H elastic scattering on ^{4}He in the states $3/2^-$ and $1/2^-$, respectively, as functions of the energy. To calculate the phase shifts, we used the relation (55) and the energies $E_\alpha^{J^\pi}$ of the eigenstates obtained by diagonalizing the Hamiltonian with different numbers n of basis functions.

We can now again estimate the value of the coefficient a if we calculate it in accordance with Eq. (60), using the phase shift $\delta_{3/2^-}$—at the point with minimal energy—which we now know:

$$E_2^{3/2^-} = 4.948 \cdot 10^{-2},\ \delta_{3/2^-} = 2.78 \cdot 10^{-2}.$$

The formula leads to the estimate $a = 4.28$, in good agreement with the value obtained above.

The qualitative behavior of the phase shifts obtained by the theoretical calculation and of the magnitude of the splitting of the phase shifts agrees well with the phase-shift analysis of the experiments of Refs. 36–38.

Figure 12 shows the dependence of the elastic-scattering phase shifts in the states $J^\pi = 7/2^-$ and $J^\pi = 5/2^-$. The experimental values of the phase shifts at energy (in the center-of-mass system of the nuclei participating in the reaction) less than 3 MeV are taken from Ref. 37, and at energies above 3 MeV from Ref. 39. The theoretical energy dependence of the phase shift was determined by means of the relation (55). The energy is measured in mega-electron-volts, and the phase shift in degrees.

Both the theoretical and the experimental phase shifts increase sharply at energies 2–5 MeV, passing through the point $\delta = \pi/2$. At the same time, the calculated phase shift increases somewhat more slowly than the experimental one. We note, however, that in the region of the resonance the phase shift of the $J^\pi = 7/2^-$ state passes through the point $\delta = \pi/2$ with a larger derivative than the phase shift of the $J^\pi = 5/2^-$ state, in good agreement with the experimental dependence of the phase shift.

From the calculated phase shifts, we recovered the par-

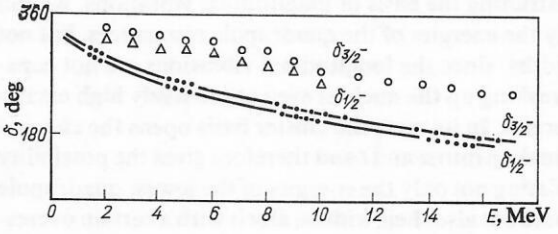

FIG. 11. Behavior of the phase shifts $\delta_{3/2^-}$ and $\delta_{1/2^-}$ ($I^\pi = 1^-$). The open circles and open triangles are, respectively, the experimental phase shifts $\delta_{3/2^-}$ and $\delta_{1/2^-}$. The points joined by the smooth curves reproduce the calculated dependence of the same phase shifts.

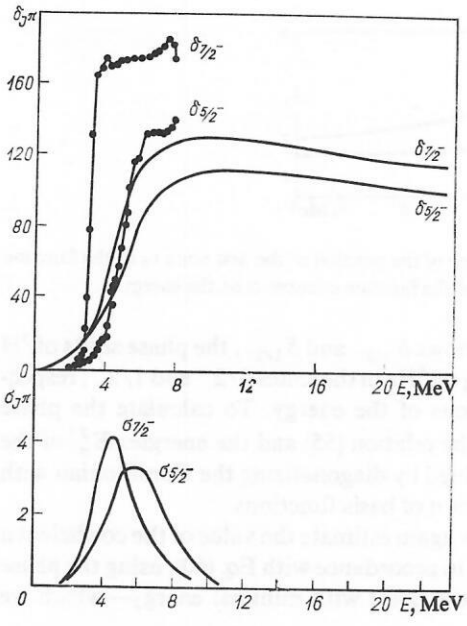

FIG. 12. Energy dependence of the phase shifts $\delta_{7/2^-}$ and $\delta_{5/2^-}$ ($I^\pi = 3^-$). The individual points reproduce the experimental values of the phase shifts, and the smooth curves give the calculated values. The lower part of the figure shows the cross sections of αt elastic scattering.

tial-wave cross sections $\sigma_{J^\pi}(E)$ of elastic $t\alpha$ scattering, which are also shown in Fig. 12. The maximum of the cross section $\sigma_{7/2^-}(E)$ is at $E_{\text{res}} \cong 4.6$ MeV (the experimental resonance energy is $E_{\text{res}} = 4.63$ MeV), while in the case of the $J^\pi = 5/2^-$ resonance the maximum of the cross section is at $E_{\text{res}} \cong 6.3$ MeV (experimentally, $E_{\text{res}} = 6.68$ MeV).

The positions of the $7/2^-$ and $5/2^-$ resonances of ^{7}Li agree very satisfactorily with the positions of the $7/2^-$ and $5/2^-$ collective quadrupole excitations that we found earlier[5] (see Sec. 2) by diagonalizing the Hamiltonian on the basis of the oscillator functions of the longitudinal vibrations of the ellipsoid of inertia [$Sp(2, R)$ basis]. This agreement is not fortuitous but is predetermined, first, by the fact that there is a value near unity of the overlap integral of the oscillator functions of the "cluster" basis and the oscillator functions of the basis of the longitudinal vibrations of the ellipsoid of inertia as long as there is a small number ν of their oscillator excitation quanta and, second, by the fact that the lowest collective excitations are formed primarily on basis functions of the longitudinal vibrations with small ν, i.e., on functions that to a certain degree are also represented in the cluster basis.

Restricting the basis of longitudinal vibrations, we can find only the energies of the quadrupole resonances, but not their widths, since the longitudinal vibrations are not capable of breaking up the nucleus even at relatively high excitation energies. In its turn, the cluster basis opens the channel of ^{7}Li breakup into α and t and therefore gives the possibility of calculating not only the energies of the lowest quadrupole resonances but also their widths, albeit with a certain overestimation due to the inadequate reproduction by the cluster basis of the collective quadrupole mode, which stabilizes the resonance. The theoretical estimates are, respectively, $\Gamma_{7/2^-} = 2.6$ MeV and $\Gamma_{5/2^-} = 3.1$ MeV, and the experimental values are $\Gamma_{7/2^-} \simeq 0.1$ MeV and $\Gamma_{5/2^-} = 0.9$ MeV.

Thus, the method which we have discussed gives very simple possibilities for calculating the elastic-scattering phase shifts and the continuum wave functions at the energies that are determined by the given position of the node of the coefficients of the expansion of the continuum wave functions with respect to the oscillator basis.

4. INTERACTION OF COLLECTIVE AND QUASIMOLECULAR DEGREES OF FREEDOM

In light and medium nuclei, not a few resonances have been found whose energies are tens of mega-electron-volts above the ground state while the widths do not exceed a few hundred kilo-electron-volts.[40] Resonances that lie below the Coulomb barrier are usually associated with quasimolecular (cluster) structures,[41,42] while resonances above the Coulomb barrier are frequently called shape resonances,[43] their interpretation being excitations of collective modes of the nucleus. However, the currently existing assumptions about the nature of these resonances have not as yet received sufficiently convincing confirmation. At the same time, at least for the nuclei of the first half of the p shell a microscopic approach taking into account simultaneously the degrees of freedom of both the collective modes and the modes of the open nuclear-breakup channels makes it possible to investigate the situation and to obtain quantitative answers to questions about the nature of the high-lying narrow resonance states.

Resonances of cluster degrees of freedom are clearly manifested in the cases when after transfer by the excited nucleus of all the energy it has received to one of the clusters this cluster cannot immediately leave the nucleus. It is prevented from doing this by the barriers produced either by the Coulomb interaction or by centrifugal effects.

Another limiting situation obtains if the excitation energy of the nucleus is transferred to collective degrees of freedom. Since all the nucleons are then involved in the collective synchronous motion, and only a small fraction of the total excitation energy is appportioned to each nucleon of the nucleus, the nucleus will remain in such states until the excitation energy passes from the collective to the cluster modes of the open nuclear-breakup channels. The time of this transition must be regulated by the strength of the coupling between the collective and the cluster channels. In the case of weak coupling, the collective excitations will be observed as narrow resonances in the cluster-interaction channel. In the case of strong coupling, the resonances of the collective mode can become too broad and can be effectively unobservable in nuclear reactions (but not in photonuclear reactions); at the same time, however, the widths of the resonances due to barrier effects in the cluster channels can be greatly reduced.

To confirm all these qualitative ideas, we must include in a quantitative computational scheme the cluster and collective modes simultaneously. We now consider how to solve these problems on the basis of a generalization of the ideas of the algebraic version of the resonating-group method.

To study the part played by the interaction of the collective and cluster degrees of freedom of the composite nucleon system in a collision of two light nuclei (clusters), we seek the wave function of the nucleon system in the form of a series

$$\Psi = \sum_{n,\nu} C_{n,\nu} |n,\ \nu\rangle, \tag{61}$$

where $C_{n,\nu}$ are expansion coefficients to be determined, and $|n,\nu\rangle$ are many-particle oscillator translationally invariant functions of the basis states with a definite number ν of oscillator vibrational quanta of the two clusters relative to one another and a definite number n of oscillator quanta of the longitudinal vibrations of the mass quadrupole of the nucleon system formed from the two clusters.

The representation of the basis functions $|n,\nu\rangle$ makes it possible to follow both the relative motion of the two clusters of the compound nucleus and the longitudinal vibrations of its mass quadrupole. The basis $\{|n,\nu\rangle\}$ contains the basis of the collective $Sp(2, R)$ model and the basis of the standard variant of the resonating-group method. Thus, we obtain the basis $\{|n\rangle\}$ of the collective $Sp(2, R)$ model if we set $\nu = 0$:

$$|n\rangle \equiv |n,\ 0\rangle.$$

In turn, the basis $\{|\nu\rangle\}$ of the standard variant of the resonating-group method corresponds to a zero value of n:

$$|\nu\rangle \equiv |0,\ \nu\rangle.$$

It is well known that the functions $|0, \nu\rangle$ abd $|n, 0\rangle$ are not orthogonal if $n = \nu$ (orthogonality of these functions for $n \neq \nu$ is guaranteed by the fact that they belong to states with different total numbers of oscillator quanta). The degree of nonorthogonality of the functions $|0, \nu\rangle$ and $|n, 0\rangle$ for $n = \nu$ is determined by the magnitude of their overlap integral $\langle 0, \nu|n, 0\rangle|_{\nu=n}$.

The overlap integrals of the basis functions of the alpha-cluster and collective channels were analyzed in Refs. 44 and 45 with a view to explaining the large probability for the decay of collective excitations through the alpha-cluster channel. However, these studies considered only the qualitative aspect of the problem, since the wave functions of the excited states of the nucleus were chosen in a highly schematic form.

The values of the overlap integrals of the collective and cluster basis functions can be readily calculated by means of Eqs. (6), (21), and (31) given in Sec. 1. For this, it is necessary to calculate the fth derivative with respect to R, the $(2n+f)$th derivative with respect to $\widetilde{R}$, and the nth derivative with respect to β of the overlap integral (6) at the point $R = \widetilde{R} = \beta = 0$, having first set $\widetilde{\beta} = 0$ in (6). [We recall that $f = A - 4$ for states of normal parity $\pi = (-1)^A$, while $f = A - 3$ for states of anomalous parity $\pi = (-1)^{A+1}$.] The result must be projected onto the state with the given orbital angular momentum and divided by the normalization factors (21) and (31). The upshot is that we arrive at the following expression for the overlap integrals:

$$\langle 0,\ \nu \mid n,\ 0\rangle = \delta_{n\nu} \left(\frac{A_1 A_2}{A}\right)^{\nu} \sqrt{\frac{B_f}{B_N} \frac{(2n+f)!}{f!} \frac{\Gamma(J)}{\Gamma(J+\nu)\,\nu!}}, \quad N = 2n + f. \tag{62}$$

As before, it is assumed that a nucleus of the p shell consists of an alpha cluster ($A_1 = 4$) and a cluster in which the number of nucleons is $A_2 = A - 4$. The values of the overlap integrals (62) for the states of normal parity of the nuclei ^{5}He, ^{6}He, ^{7}Li, and ^{8}Be are given in Table VII.

It follows from Table VII directly that the states of the collective basis with small n hardly differ from the states of the cluster basis with small ν. Therefore, in the case of the excitation of quadrupole vibrations of small amplitude, when basis states with small n are dominant, vibrations of the clusters relative to one another are also excited. But apart from the Coulomb and centrifugal barriers, the cluster channel does not have any other reasons preventing the system from breaking up into two clusters, provided the energy of the system exceeds the threshold. As a result, the collective excitations of small amplitude decay into two clusters without hindrance, and it is difficult to believe that they will be manifested openly in the scattering of one cluster by another.

At the same time, in the excitation of collective vibrations of large amplitude basis states with large n which do not overlap with the basis states of the cluster channels begin to be dominant. Therefore, the collective quadrupole excita-

TABLE VII. Overlap integrals of the functions of the collective and cluster bases.

n	$A=5$	$A=6$	$A=7$	$A=8$
0	1	1	1	1
1	0,840	0,843	0,910	0,894
2	0,726	0,710	0,812	0,792
3	0,640	0,605	0,721	0,670
4	0,574	0,524	0,649	0,619
5	0,521	0,462	0,570	0,551
6	0,479	0,413	0,511	0,492
7	0,443	0,373	0,461	0,443
8	0,413	0,340	0,419	0,401
9	0,385	0,313	0,383	0,365
10	0,365	0,289	0,352	0,334
11	0,346	0,269	0,325	0,307
12	0,329	0,252	0,301	0,283
13	0,313	0,236	0,281	0,262
14	0,299	0,222	0,262	0,244
15	0,287	0,210	0,246	0,227

TABLE VIII. Oscillator radii and threshold energies.

A	^{5}He ($\alpha+n$) ^{5}Li ($\alpha+p$)	^{6}Li ($\alpha+d$) ^{6}He ($\alpha+2n$)	^{7}Li ($\alpha+t$) ^{7}Be ($\alpha+^3$He)	^{8}Be ($\alpha+\alpha$)
r_0, F	1,41	1,47	1,45	1,41
E_{th}, MeV	−28,19	−25,36	−34,68	−56,38

tions are isolated from the nuclear breakup channels and can be manifested in nuclear reactions in the form of narrow resonances, as is confirmed by theoretical calculations.

To investigate resonance states in nuclei of the first half of the p shell, we used 25 functions $|n, 0\rangle \equiv |n, \text{col}\rangle$, $0 \leqslant n \leqslant 25$, of the basis of collective excitations (the calculations of Refs. 4, 5, and 17 showed that 25 functions are quite sufficient for a correct description of the four or five lowest collective excitations with definite orbital angular momentum) and 100 functions $|0, \nu\rangle \equiv |\nu, \text{cl}\rangle$, $1 \leqslant \nu \leqslant 100$, of the basis of cluster states. The nucleon–nucleon interaction was simulated by the first variant of the Brink-Boeker potential.[21] Noncentral forces and the Coulomb interaction between the protons were not taken into account. The oscillator radius was chosen to minimize the energy of the breakup threshold. The actual values of r_0 for each nucleus are given in Table VIII, which also contains the threshold energy.

Among the different methods for orthogonalizing the basis $\{|n, \text{col}\rangle, |\nu, \text{cl}\rangle\}$ used in the calculations, we chose a variant of Schmidt orthogonalization in which the functions $|\nu, \text{cl}\rangle$ of the cluster basis are not changed, while the functions $|n, \text{col}\rangle$ of the collective basis are rearranged to make them orthogonal to the functions $|\nu, \text{cl}\rangle$. The new rearranged functions $\overline{|\nu, \text{col}\rangle}$ of the collective basis are related to the original functions $|\nu, \text{col}\rangle$ of the same basis and the functions $|\nu, \text{cl}\rangle$ of the cluster basis by the simple relation

$$\overline{|\nu, \text{col}\rangle} = (|\nu, \text{col}\rangle - a_\nu |\nu, \text{cl}\rangle)/\sqrt{1-a_\nu^2},$$

where $a_\nu \equiv \langle \nu, \text{col}|\nu, \text{cl}\rangle$. The system of basis functions $\{\overline{|\nu, \text{col}\rangle}, |\nu, \text{cl}\rangle\}$ is very convenient for describing the continuum states. The generalized Fourier series of the wave function Ψ of the nucleus has in the rearranged basis the form

$$\Psi = \sum_{\nu, \alpha} C_{\alpha, \nu} |\nu, \alpha\rangle,$$

where the index α can be equal to either 1 or 2. If $\alpha = 1$, then

$$|\nu, 1\rangle \equiv \overline{|\nu, \text{col}\rangle},$$

but if $\alpha = 2$, then

$$|\nu, 2\rangle = |\nu, \text{cl}\rangle.$$

The case of minimal ν is an exception, since then the functions of the cluster and collective bases are identical. The coefficients $C_{\alpha,\nu}$ satisfy the wave equation of the investigated model, written down in the representation of the basis $\{|\nu, \alpha\rangle\}$.

We consider first the continuum states of the nucleon system that can be obtained by diagonalizing its Hamiltonian on the complete basis $\{|\nu, \alpha\rangle\}$ or on part of this basis. A particularly interesting question is the evolution of the spectrum of collective excitations when the number of basis functions of the cluster channel taken into account is varied (Fig. 13). The left-hand side of Fig. 13 shows the spectrum of collective 0^+ excitations (i.e., excitations with zero angular momentum and positive parity) of ^{8}Be obtained in a calculation with 25 basis functions $|\nu, 0\rangle \equiv |\nu, \text{col}\rangle$. To this basis there were then added 1, 2, 3, etc., cluster functions $|0, \nu\rangle \equiv |\nu, \text{cl}\rangle \equiv |\nu, 2\rangle$. The curves in Fig. 13 show how the eigenvalues of the many-particle Hamiltonian vary as the cluster basis is enlarged. Of course, physical meaning attaches to only those points of the curves that correspond to integer values of μ, where μ is the number of basis functions $|\nu, 2\rangle$ employed. With increasing μ, the energies of the ground state and of the first excited 0^+ state decrease monotonically, the latter approaching the threshold slowly. Note that for $\mu \geqslant 50$ the ground state sinks below the threshold. (We recall that in the calculation the Coulomb interaction was not taken into account, and therefore the ^{8}Be nucleus is bound in the ground state.) The second, third, and other excited states reach a plateau at certain values of μ. The energies corresponding to this plateau are the energies of the resonance states formed by the interaction of the quadrupole and cluster modes. On the basis of an analysis of the data shown in Fig. 13, it can be concluded that when the cluster mode is included the first excited state of the quadrupole degrees of freedom "dissolves" in the continuum, while the second, third, and remaining collective excitations lower their energy and become resonances. The lowering of the energies of the collective resonances occurs simultaneously with the decrease in the energy of the ground state, and therefore, the ground state and resonance states are situated at a few megaelectron-volts ($\sim$8 MeV for ^{8}Be) below the corresponding states obtained in the calculations on the basis of the collective functions. (Note that the energy is measured from the

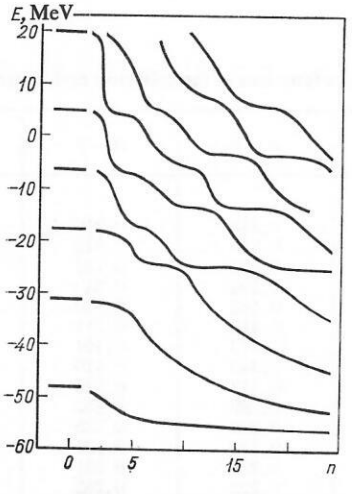

FIG. 13. Dependence of the energy of the collective 0^+ excitations of the nucleus ^{8}Be on the number of cluster functions included.

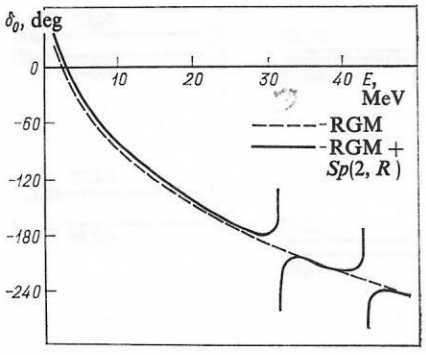

FIG. 14. The S-wave phase shift of $\alpha\alpha$ scattering calculated by the resonating-group method (broken curve) and by the same method with allowance for the collective degrees of freedom (continuous curve).

threshold of complete disintegration of the nucleus into free nucleons.) This conclusion is also confirmed by the results of calculation of the phase shifts of elastic $\alpha\alpha$ scattering in the channel with zero angular momentum (Fig. 14). When the eigenvalues reach the plateau, the elastic-scattering phase shift exhibits resonance behavior. At the same time, in the neighborhood of the first collective excited 0^+ state [its energy relative to the threshold of $\alpha\alpha$ decay is 15.03 MeV] the phase shift behaves monotonically, illustrating the conclusion that this state is "dissolved" in the continuum.[3)]

To elucidate the causes of this result, we consider the wave functions of the collective model. As already noted above, the overlap integrals of the basis functions of the collective and cluster modes are nonzero, so that in any function of a collective excitation $\{C_\nu^{\mathrm{col},\chi}\}$ (here, χ is the set of quantum numbers characterizing the collective state, and $C_\nu^{\mathrm{col},\chi}$ are generalized Fourier coefficients) functions of the cluster basis are present with a certain weight. This weight in the given state χ can be calculated in accordance with the formula

$$W^\chi = \sum_\nu [\langle \nu,\ \mathrm{cl}\ |\ \nu,\ \mathrm{col}\rangle C_\nu^{\mathrm{col},\chi}]^2.$$

Comparing the weights W^χ for different states χ, one can estimate the degree of coupling of the collective and cluster modes in these states.

Table IX gives the values of W^χ for the five lowest collective excitations of a number of the lightest p-shell nuclei. The weight W^0 of the cluster basis functions in the ground state of these nuclei is 75–90%, while in the first excited state it is between 40 and 54%. The remaining states, which become resonances after inclusion of the cluster mode, are represented by cluster functions of not more than 35%. Thus, the larger the amplitude of the collective vibrations, the weaker their coupling to the cluster mode. And this, in its turn, leads to the formation of narrow resonances with high excitation energy ($E > 20$ MeV). By means of Table IX it is possible to establish the maximal weight of the cluster functions in a collective state for which the collective state does not yet "dissolve" in the continuum. This weight is about 36% (the cluster weight of the second excited 3^- state in the ^{7}Li nucleus). It also follows from Table IX that a collective state dissolves in the continuum if its cluster weight exceeds 40%.

Figures 15–17 show the spectra of the collective excitations of the lightest p-shell nuclei obtained in calculations with a basis of longitudinal, quadrupole vibrations and the spectra of the resonances formed by the interaction of the collective and cluster degrees of freedom. The same figures show the resonances obtained in calculations with the cluster basis. The data given in the figures make it possible to answer the question concerning the effect of the collective mode on the parameters of the resonances generated by the centrifugal barriers. The coupling of the cluster mode to the collective mode appreciably reduces (by 1.5–2 times) the widths of these resonances and, to a lesser extent, their energies.

The energies of the collective resonances exceed 25 MeV, but their widths are not more than 1 MeV. At the same time, the widths of the resonances due to the centrifugal barriers are almost an order of magnitude greater than the mean width of the collective resonances and are 0.9–3.2 MeV. With increasing energy and orbital angular momentum, the widths of the collective resonances undergo small changes, this being explained by the weak coupling of the collective and cluster modes at the energies corresponding to these resonances.

TABLE IX. Weights of cluster states, in %, in the wave functions of the collective excitations of p-shell nuclei.

A	I^π	n				
		0	1	2	3	4
5	1^-	89,04	39,69	21,24	17,62	18,72
6	0^+	83,62	45,38	26,92	21,38	22,49
	2^+	84,13	45,66	27,29	22,06	23,14
7	1^-	83,59	53,11	34,61	27,54	28,53
	3^-	82,61	54,08	35,83	28,92	29,68
8	0^+	76,21	49,37	33,66	26,69	26,97
	2^+	75,46	49,55	34,11	27,33	28,73

[3)]However, a detailed investigation of the situation shows that in the region of the continuum energies where the collective states "vanished" the theory predicts well-defined broad maxima in the cross sections of photodisintegration and radiative capture, so that in fact the first collective excitations are reborn in giant quadrupole resonances.

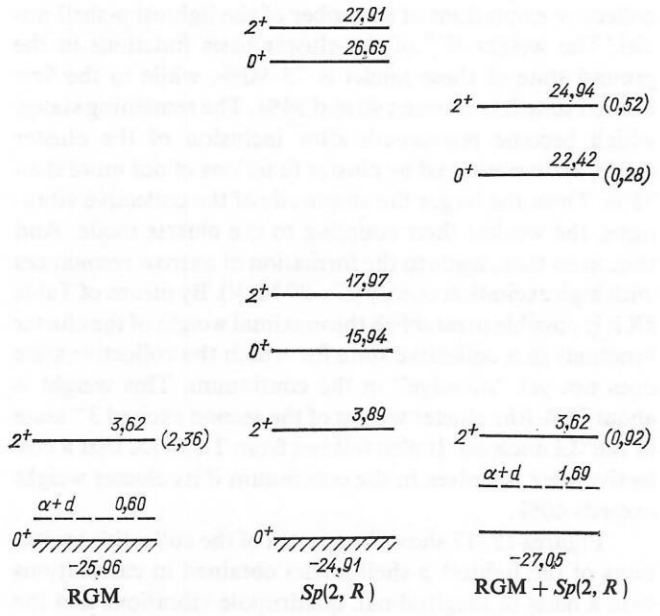

FIG. 15. Spectrum of collective excitations and resonance states of the nucleus ^{6}Li (^{6}He).

5. MICROSCOPIC THEORY OF PHOTONUCLEAR REACTIONS

One of the most important characteristics of excited nuclear states in the continuum is the probability of electromagnetic transitions to these states from the ground state of the nucleus. It is well known that the reduced matrix elements of the operators of the electric dipole and quadrupole transitions are related in a simple manner to the effective

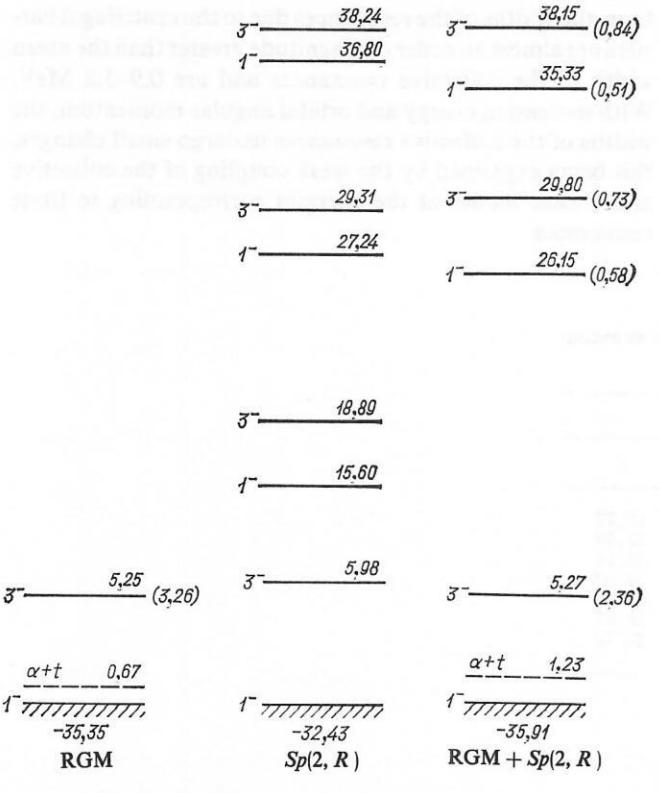

FIG. 16. The same as in Fig. 15 but for the nucleus ^{7}Li (^{7}Be).

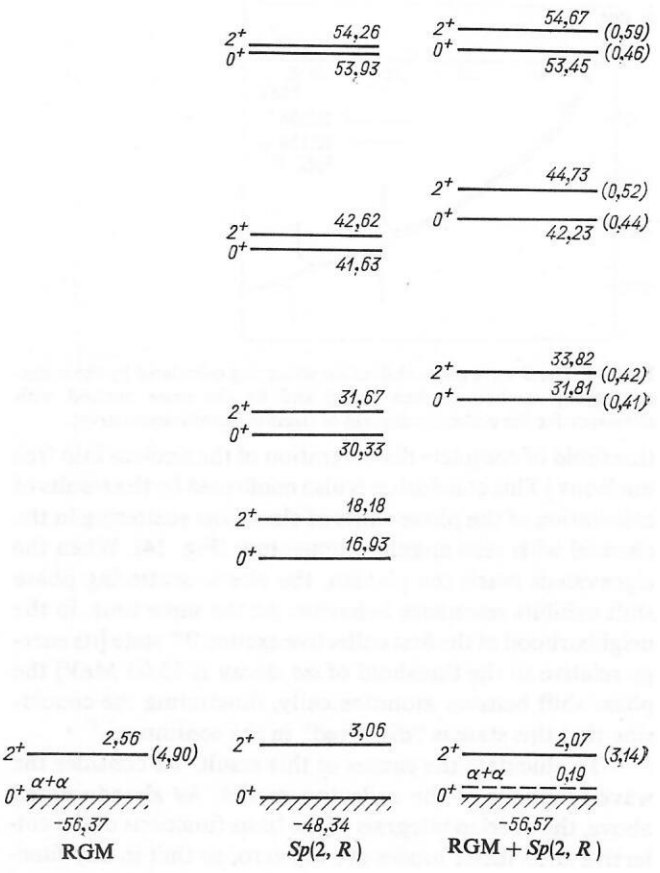

FIG. 17. The same as in Fig. 15 but for the nucleus ^{8}Be.

cross sections of photo- and electrodisintegration, and also to the radiative-capture cross sections. Therefore, knowing the wave functions of the ground state and the continuum states and, in addition, calculating the reduced matrix elements of $E1$ and $E2$ transitions, we can obtain by means of theory information directly from experimental data on photonuclear reactions at energies above the threshold.

Antisymmetrized wave functions of states of the discrete and continuous spectra were calculated for a system of two interacting clusters in Sec. 3, and all the reduced matrix elements needed to calculate the transition probabilities were calculated in Sec. 1. In this section, using the well-known relation

$$B(E\lambda;\ L_1^{\pi_1}\varepsilon \to L_2^{\pi_2}E) = \frac{1}{2L_1+1}\Big[\sum_{n,\,n'} C_{n'}^{L_2^{\pi_2}E}\langle n' L_2^{\pi_2} \| \hat{Q}_\lambda \| n L_1^{\pi_1}\rangle\, C_n^{L_1^{\pi_1}\varepsilon}\Big]^2 \qquad (63)$$

we calculate the probabilities of $E0$, $E1$, and $E2$ transitions in the nuclei ^{6}Li, ^{6}He, and ^{7}Li. Here, ε and E are, respectively, the energies, measured from the breakup threshold, of the ground state and the continuum state, and the functions of these states $C_{n'}^{L_2^{\pi_2}E}$ and $C_n^{L_1^{\pi_1}\varepsilon}$ have the usual normalization:

$$\sum_n C_n^{L^\pi\varepsilon} C_n^{L^\pi\varepsilon} = 1;\quad \sum_n C_n^{L^\pi E} C_n^{L^\pi E'} = \delta(k-k'); \qquad (64)$$

$$k = \sqrt{2mE/\hbar^2}.$$

We note that the algebraic version of the resonating-group method makes it possible, by virtue of the simplicity of

the computational algorithm, to investigate not only the probabilities of electromagnetic transitions to the continuum but also the dependence of these probabilities on the energy of the electromagnetic radiation and the distribution of the energy-weighted sum rules over the continuum states with different energies E.

An investigation of the monopole isoscalar transition from the ground state ($E_{g.s}$) to the state with energy E was made in Ref. 46, in which the matrix elements of this transition were calculated for two nuclei: ^{7}Li and ^{8}Be. It was found that the probabilities $B(E0, E_{g.s} \to E)$ have a clear maximum about 2–3 MeV above the threshold of the breakup of these nuclei into $\alpha + t$ and $\alpha + \alpha$, respectively, and the contribution to the energy-weighted sum rule from transitions to this region reaches about 50–70%. At the same time, as we have already noted above, in a study of photodisintegration and radiative-capture reactions it is not only the probabilities of the $E1$ and $E2$ transitions but also their dependence on E that is of interest.

To construct this dependence, we need the wave functions $\{C_n^{L^\pi E}\}$ of states with arbitrary energy E exceeding the energy threshold. Such functions were found by solving the system (53), the Brink-Boeker potential containing only central components being chosen as the NN-interaction potential. This last circumstance makes it somewhat harder to compare directly the properties of the nuclear states characterized by the quantum numbers L^π with the corresponding experimental data; however, even in this simple approximation one can establish important properties of the wave functions $\{C_n^{L^\pi E}\}$ and, in particular, the ones that determine the most characteristic features in the behavior of $B(E\lambda)$ with increasing energy E.

The results of calculations of $B(E\lambda)$ as functions of E are shown in Figs. 18–21. For all multipolarities that we considered, the probabilities of transitions from the ground state to continuum states with energy E exhibit a strong dependence on E. Increasing strongly from zero at the point $E = 0$, they reach their maximal value at energies not exceeding 2 MeV

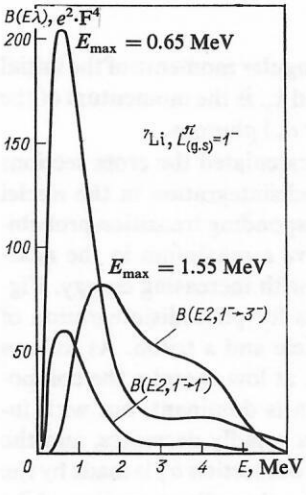

FIG. 18. Isoscalar monopole $B(E0)$ and quadrupole $B(E2)$ transitions in the nucleus ^{7}Li coupling the 1^- ground state and the continuum states, as functions of the energy of the relative motion of the fragments α and t.

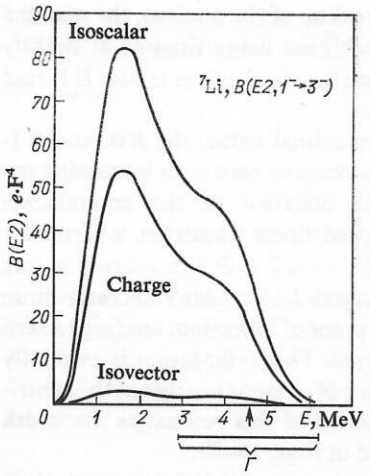

FIG. 19. Isoscalar, isovector, and charge quadrupole $E2$ transitions $B(E2, 1^- \to 3^-)$ in the ^{7}Li nucleus. The arrow at the bottom indicates the energy ($E = 4.58$ MeV) and the width ($\Gamma = 3.26$ MeV) of the 3^- centrifugal resonance.

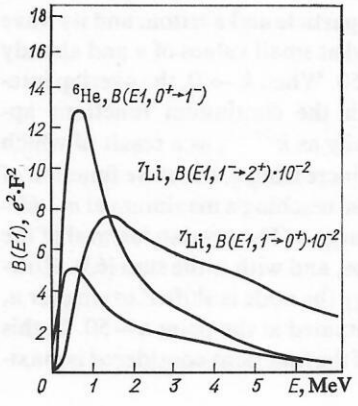

FIG. 20. Dipole transition probability $B(E1)$ in the ^{6}He and ^{7}Li nuclei as a function of the energy of the relative motion of the fragments $\alpha + 2n$ and $\alpha + t$, respectively.

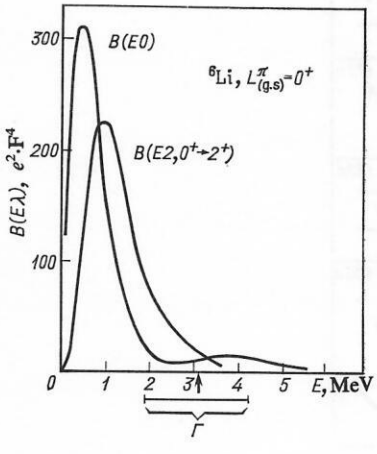

FIG. 21. Probabilities of monopole and quadrupole charge transitions in the ^{6}Li nucleus. The arrow at the bottom indicates the energy ($E = 3.04$ MeV) and width ($\Gamma = 2.36$ MeV) of the 2^+ centrifugal resonance.

above the threshold for breakup of the nucleus, the maxima in the $E0$-transition probabilities being situated at slightly lower energies than the analogous maxima of the $E1$- and $E2$-transition probabilities.

Having reached the maximal value, the $E0$- and $E1$-transition probabilities decrease to zero with increasing energy. There is a similar behavior of the components $B(E2, L_1^{\pi} \to L_1^{\pi})$ of the quadrupole transition, whereas for the components $B(E2, L_1^{\pi} \to L_1^{\pi}+2)$ the corresponding curves in the region of energies $3 \leqslant E \leqslant 4$ MeV decrease more slowly, passing through a point of inflection, tending to zero only after this has happened. This retardation is evidently associated with the presence of a resonance due to the centrifugal barrier. The parameters of this resonance (its width and position) are indicated in Figs. 18–21.

The causes of the appearance of the resonance peak in the dependences of $B(E\lambda)$ on E can be clarified by analyzing the wave functions $\{C_n^{L^{\pi}\varepsilon}\}$ and $\{C_n^{L^{\pi}E}\}$ for different values of the energy, and also their overlap integral with the reduced matrix elements of the operator $\hat{Q}_{\lambda\mu}$. Figure 22 shows the functions of the ground state ($L^{\pi}=1^-$) and four states of ^{7}Li ($L^{\pi}=2^+$) lying in the continuum with different energies. The ^{7}Li ground state lies 0.67 MeV below the threshold of disintegration into an α particle and a triton, and its wave function is mainly localized at small values of n and already almost vanishes at $n \sim 40$–50. When $k \to 0$, the overlap integral of this function with the continuum functions approaches zero asymptotically as k^{2l+2}, as a result of which $B(E\lambda)=0$ at $k=0$. With increasing n, the wave function of the E_1 state slowly increases, reaching a maximum at $n \sim 60$–70, its node lying at fairly large n. The overlap integral of the two functions also increases, and with it the sum (63). However, with increasing energy the node is shifted to smaller n, and at $E=1.5$ MeV it is situated at the point $n \sim 50$. In this case, the overlap integral of the functions considered is maximal, since in the region in which the function $\{C_n^{2+E}\}$ is negative ($n>50$) the ground-state function is practically equal to zero. Therefore, by virtue of the fact that the reduced matrix elments have a fixed sign, the value of $B(E1, 1^- \to 2^+)$ is also maximal at $E=1.5$ MeV (see Fig. 20). With further increase in the energy, the node of the continuum-state function is shifted to the region in which the function $\{C_n^{1-\varepsilon}\}$ is nonzero, and negative terms begin to make an appreciable contribution to the sum (63), this ultimately leading to a decrease in this sum. It is obvious that in the limit of very large E, when the continuum function begins to oscillate rapidly, the sum (63) must tend to zero.

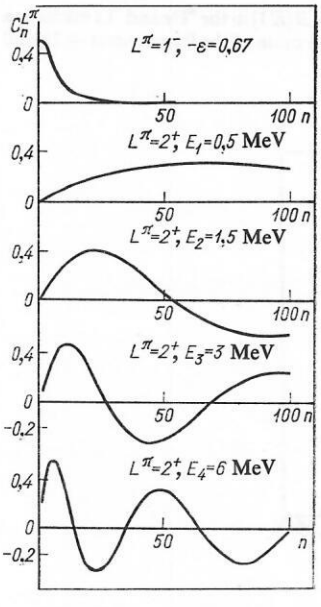

FIG. 22. Wave functions $\{C_n^{I^{\pi}E}\}$ of the 1^- ground state of the ^{7}Li nucleus and 2^+ states in the $\alpha+t$ channel.

Thus, the maximum in the dependences of $B(E\lambda)$ on E is due to the general properties of the functions $\{C_n^{L^{\pi}\varepsilon}\}$ and $\{\tilde{C}_n^{L^{\pi}E}\}$, and therefore it must be observed in all the $E\lambda$ transitions which we consider. Besides $B(E\lambda)$, we calculated the isoscalar monopole and quadrupole sum rules exhausted by the continuum states. We found that the near-threshold region exhausts an appreciable fraction of not only the monopole sum rule, as was shown for the first time in Ref. 46, but also the quadrupole sum rule. For example, in the ^{6}He(^{6}Li) nucleus the energy interval from 0 to 4 MeV above the $\alpha+d$ threshold corresponds to about 75% of the quadrupole sum rule, of which only 15% is exhausted by the region of the centrifugal resonance discussed above. The near-threshold region ($0 \leqslant E \leqslant 7$ MeV) in ^{7}Li exhausts about 60% of the quadrupole sum rule, of which about 20% corresponds to the centrifugal 3^- resonance. In the same region of energies, the $E2$ transition $1^- \to 1^-$ exhausts about 30% of the sum rule.

The cross sections of photodisintegration, σ_γ, and radiative capture, σ_c, are connected by simple relations to the $E\lambda$-transition probabilities:

$$\sigma_c = \frac{16\pi^3 e^2}{\hbar v} \frac{(\lambda+1)\, k_\gamma^{2\lambda+1}}{\lambda\,[(2\lambda+1)!!]^2} (2I_i+1) \frac{1}{k^2} B(E\lambda;\ I_i E \to I_f \varepsilon);$$

$$\sigma_\gamma = \frac{8\pi^3 e^2}{\hbar v} \frac{(\lambda+1)\, k_\gamma^{2\lambda-1}}{\lambda\,[(2\lambda+1)!!]^2} B(E\lambda;\ I_i \varepsilon \to I_f E),$$

where

$$v = \frac{\hbar \bar{k}}{\mu} = \frac{\hbar k}{m} \sqrt{\frac{A_1+A_2}{A_1 A_2}},$$

where I_i and I_f are the orbital angular momenta of the initial and final states, respectively, and k_γ is the momentum of the incident (for σ_γ) or emitted (for σ_c) photons.

Using these relations, we calculated the cross sections of radiative capture and photodisintegration in the nuclei ^{6}He, ^{6}Li, and ^{7}Li. Like the corresponding transition probabilities, all the cross sections have a maximum in the near-threshold region and decrease with increasing energy. Figure 23 shows the cross sections for photodisintegration of the ^{7}Li nucleus into an α particle and a triton. As follows from the reduced dependences, at low energies the component of the $1^+ \to 0^+$ transition is dominant, but with increasing energy this component rapidly decreases, and the main contribution to the total cross section σ_γ^{Σ} is made by the $1^- \to 2^+$ transition. Thus, in the photodisintegration of ^{7}Li the yield of fragments with low energy of the relative motion will be maximal. The same can be said of the process of radia-

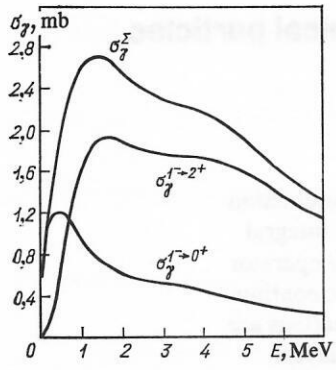

FIG. 23. Cross section for photodisintegration ^{7}Li(γ, α)t. The indices $L^\pi = 0^+$ and $L^\pi = 2^+$ of the cross section $\sigma_\gamma^{1^-\to L^\pi}$ determine the angular momentum and parity of the exit $(\alpha + t)$ channel, and $\sigma_\gamma^\Sigma = \sigma_\gamma^{1^-\to 0^+} + \sigma_\gamma^{1^-\to 2^+}$.

tive capture in this nucleus—the formation of a compound ^{7}Li nucleus is most probable at low energies of the relative motion of the colliding particles.

CONCLUSIONS

Thus, the technique of generalized coherent states is effective in the solution of many problems of microscopic nuclear theory. The examples given above only partly demonstrate the possibilities of this technique. The next step in the development of the approach discussed in the review may be associated with the investigation of other problems of microscopic nuclear theory. Among the problems whose solution has become realistic on the basis of the technique of generalized coherent states we can mention many-channel reactions with the participation of several nuclei in the exit channels, then nuclear and photonuclear reactions accompanied by the excitation of various collective modes, and, finally, the problem of the complete solution of the wave function of the $Sp(6, R)$ symplectic model. The importance of these problems for the microscopic theory of the nucleus is at the present time in no doubt.

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