Quasiparticle–phonon model of the nucleus. V. Odd spherical nuclei

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The formalism of the quasiparticle–phonon model of the nucleus for odd spherical nuclei is presented. The exact commutation relations of the quasiparticle and phonon operators together with the anharmonic corrections for the phonon excitations are taken into account in the derivation of equations for the energies and structure coefficients of the wave functions of excited states, which include quasiparticle–phonon and quasiparticle–two-phonon components. The influence of various physical effects and of the dimension of the phonon basis on the fragmentation of the single-quasiparticle and quasiparticle-phonon states is investigated.

INTRODUCTION

The behavior of the single-particle levels of the average field is revealed in the low-lying states of odd nuclei. As the excitation energy increases, the structure of the states becomes more complicated. The strength of the single-particle states is fragmented (distributed) over many nuclear levels, and many-quasiparticle components begin to become predominant in the wave functions. In recent years, great progress has been made in the study of deep hole and highly excited particle states in odd spherical nuclei. As a result, rich experimental information has been obtained on the fragmentation of single-quasiparticle states.

Study of the radiative strength functions is of great interest. Much experimental material is available on partial $E1$ and $M1$ transitions from neutron resonances to the ground and low-lying states of odd spherical nuclei. The time has come to study γ decays of deep hole and highly excited particle states. The widths of the partial radiative transitions in odd nuclei are determined by the fragmentation of the single-quasiparticle and quasiparticle-phonon states.

The fragmentation of these states can be successfully described in the framework of the quasiparticle-phonon model of the nucleus. This was formulated on the basis of a development and generalization of the superfluid nuclear model.[1-3] The basic ideas were developed in 1971–1974 in Refs. 4–7. The fundamentals of the model and some of the mathematical formalism have been presented in reviews,[8-11] conference proceedings, and lectures.[12-16] The present paper continues the series of reviews of Refs. 8–11. It is devoted to an exposition of the mathematical formalism of the model for odd spherical nuclei and an investigation of the influence of the various components of the wave functions of highly excited states on the spectroscopic characteristics and radiative strength functions.

1. HAMILTONIAN OF THE MODEL

A general characterization of the Hamiltonian of the quasiparticle-phonon model is given in Refs. 8–10. Here, we shall only briefly consider this question in order to make the arguments in the main text of the paper more clear. In addition, in this section we shall introduce the main notation.

The Hamiltonian of the model includes phenomenological average fields for the protons and neutrons, these taking the form of the Woods-Saxon potential, monopole pairing proton-proton and neutron-neutron interactions, and separable multipole and spin-multipole forces, which have isoscalar and isovector components, in the particle-hole channel. Effects associated with multipole pairing[17] will not be considered in this paper, and the corresponding terms are not included in the Hamiltonian of the model (see Ref. 10).

Thus, in the notation of Refs. 10 and 11, the Hamiltonian has the form

$$H = H_{\rm av} + H_{\rm PAIR} + H_M^{ph} + H_{SM}^{ph}. \tag{1}$$

After a Bogolyubov canonical transformation from the nucleon operators a_{jm}^+, a_{jm} to the quasiparticle operators α_{jm}^+, α_{jm},

$$a_{jm} = u_j\alpha_{jm} + (-)^{j-m}\, v_j\alpha_{j-m}^+,$$

in which the coefficients u_j, v_j are chosen to minimize the expectation value of $H_{\rm av} + H_{\rm PAIR}$ with respect to the ground (quasiparticleless) state of the even-even nucleus, this part of the Hamiltonian is given by the following expression (Refs. 1–3 and 18)[1)]:

$$H_{\rm av} + H_{\rm PAIR} \to \sum_{jm} \varepsilon_j \alpha_{jm}^+\alpha_{jm}. \tag{2}$$

In (2), ε_j is the energy of the quasiparticle in the level of the average field with quantum numbers nlj (for brevity, denoted by the single letter j). The summation in (2) is over all levels of both the proton (p) and neutron (n) systems, the isotopic index $\tau = \{n, p\}$ being understood in this case. In what follows, the substitution $\tau \to -\tau$ will signify the substitution $n \leftrightarrow p$.

The separable multipole and spin-multipole interactions have the form[11]

$$\left.\begin{aligned} H_M^{ph} &= \sum_\lambda H_M^{ph}(\lambda) = -\frac{1}{2}\sum_{\lambda\mu}\sum_{\substack{\tau\\ \rho=\pm1}} (\varkappa_0^{(\lambda)} + \rho\varkappa_1^{(\lambda)}) M_{\lambda\mu}^+(\tau)\, M_{\lambda\mu}(\rho\tau),\\ H_{SM}^{ph} &= \sum_L \sum_{\substack{\lambda=L,\\ L\pm1}} H_{SM}^{ph}(\lambda, L) = \sum_{L,\,M}\sum_{\substack{\lambda=L,\\ L\pm1}}\sum_{\substack{\tau,\\ \rho=\pm1}} (\varkappa_0^{(\lambda L)}\\ &+ \rho\varkappa_1^{(\lambda L)})\,[S_{LM}^\lambda(\tau)]^+ S_{LM}^\lambda(\rho\tau). \end{aligned}\right\} \tag{3}$$

The constants of the isoscalar and isovector components of the interactions are denoted by $\varkappa_0^{(\lambda)}$, $\varkappa_0^{(\lambda L)}$ and $\varkappa_1^{(\lambda)}$, $\varkappa_1^{(\lambda L)}$, respectively.

Although, strictly speaking, the structure of states of electric type λ^π is formed under the influence of the $H_M^{\rm ph}(\lambda)$

[1)] Here and in what follows, monopole excitations are not considered.

$+H^{\rm ph}_{SM}(\lambda,\lambda)$ forces and that of states of magnetic type under the influence of the $H^{\rm ph}_{SM}(\lambda-1,\lambda)+H^{\rm ph}_{SM}(\lambda+1,\lambda)$ forces, and in certain cases the contribution of *both* components is important for the adequate description of the structure and properties of nuclear excitations,[18,19] we shall restrict ourselves in the present paper to the variant in which the Hamiltonian (1) contains only the terms $H^{\rm ph}_M$ and $\Sigma_L H^{\rm ph}_{SM}(L-1,L)$.

After the Bogolyubov transformation, the operator $M^+_{\lambda\mu}(\tau)$ takes the form

$$M^+_{\lambda\mu}(\tau)=\frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda+1}}\sum_{jj'}^{\tau} f^{\lambda}_{j'j}\left\{\frac{1}{2}u^{(+)}_{j'j}[A^+(j'j,\lambda\mu)+(-)^{\lambda-\mu}A(j'j,\lambda-\mu)]+v^{(-)}_{j'j}B(j'j\lambda-\mu)\right\},\qquad(4)$$

where we have introduced the notation

$$A^+(j'j\lambda\mu)=\sum_{mm'}\langle j'm'jm|\lambda\mu\rangle\alpha^+_{j'm'}\alpha^+_{jm};$$

$$B(j'j\lambda\mu)=\sum_{mm'}(-)^{j+m}\langle j'm'jm|\lambda\mu\rangle\alpha^+_{j'm'}\alpha_{j-m};$$

$$f^{(\lambda)}_{j'j}=\langle j'\,||\,R_\lambda(r)\,i^\lambda Y_{\lambda\mu}\,||\,j\rangle;\quad u^{(+)}_{j'j}=u_{j'}v_j+u_jv_{j'};$$

$$v^{(-)}_{j'j}=u_ju_{j'}-v_jv_{j'}.$$

The form of the operator $S^{L-1}_{LM}(\tau)$ can be found in Refs. 10 and 11.

We now go over from the creation and annihilation operators of fermion pairs with angular momentum λ, i.e., $A^+(j'j\lambda\mu)$, $A(j'j\lambda\mu)$, to creation and annihilation operators for phonons with angular momentum λ, i.e., $Q^+_{\lambda\mu i}$, $Q_{\lambda\mu i}$:

$$Q^+_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\psi^{\lambda i}_{jj'}A^+(jj'\lambda\mu)-(-)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}A(jj'\lambda-\mu).\qquad(5)$$

The summation in (5) is over the proton and neutron single-quasiparticle states. The operators $Q_{\lambda ui}$ and $Q^+_{\lambda' u' i'}$ satisfy the commutation relations

$$[Q_{\lambda\mu i},Q^+_{\lambda'\mu' i'}]=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\frac{1}{2}\sum_{jj'}(\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'}-\varphi^{\lambda i}_{jj'}\varphi^{\lambda i'}_{jj'})-\sum_{jj'j_2}\sum_{mm'm_2}\{\psi^{\lambda i}_{j'j_2}\psi^{\lambda' i'}_{jj_2}\times\langle j'm'j_2m_2|\lambda\mu\rangle\langle jmj_2m_2|\lambda'\mu'\rangle-(-)^{\lambda+\lambda'-\mu-\mu'}\varphi^{\lambda i}_{jj_2}\varphi^{\lambda' i'}_{j'j_2}\langle jmj_2m_2|\lambda-\mu\rangle\times\langle j'm'j_2m_2|\lambda'-\mu'\rangle\}\alpha^+_{jm}\alpha_{j'm'}.\qquad(6)$$

Assuming that the ground-state wave function Ψ_0 of an even-even nucleus is a phonon vacuum (i.e., $Q_{\lambda\mu i}\Psi_0=0$), and that the simplest excited state is a single-phonon state $Q^+_{\lambda\mu i}\Psi_0$, we can find the energies of the single-phonon states by means of a variational principle. In the nuclei in which the number of quasiparticles in the ground state is small, i.e., $\langle\Psi_0|\alpha^+_{jm}\alpha_{jm}|\Psi_0\rangle\simeq 0$, the expectation value with respect to Ψ_0 of the commutator (6) takes the form

$$\langle\Psi_0|[Q_{\lambda\mu i},Q^+_{\lambda'\mu' i'}]|\Psi_0\rangle=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\frac{1}{2}\sum_{jj'}(\psi^{\lambda i}_{jj'}\psi^{\lambda' i'}_{jj'}-\varphi^{\lambda i}_{jj'}\varphi^{\lambda' i'}_{jj'}).$$

Then, requiring orthonormality of the single-phonon wave functions, we obtain conditions on $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$:

$$\frac{1}{2}\sum_{jj'}\psi^{\lambda i}_{jj'}\psi^{\lambda' i'}_{jj'}-\varphi^{\lambda i}_{jj'}\varphi^{\lambda' i'}_{jj'}=\delta_{\lambda\lambda'}\delta_{ii'}.\qquad(7)$$

In this approximation (i.e., in the random-phase approximation), the equation for the energies $\omega_{\lambda i}$ of the single-phonon states (of electric type or multipole states) is obtained in the form

$$(\varkappa^{(\lambda)}_0+\varkappa^{(\lambda)}_1)(X^{\lambda i}_M(n)+X^{\lambda i}_M(p))-4\varkappa^{(\lambda)}_0\varkappa^{(\lambda)}_1X^{\lambda i}_M(n)X^{\lambda i}_M(p)=1;\qquad(8)$$

$$X^{\lambda i}_M(\tau)=\frac{1}{2\lambda+1}\sum_{jj'}^{\tau}\frac{(f^{(\lambda)}_{jj'}u^{(+)}_{jj'})^2(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2_{\lambda i}}.\qquad(9)$$

We also write down the expressions for the amplitudes $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$:

$$\psi^{\lambda i}_{jj'}(\tau)=\frac{1}{\sqrt{2\mathcal{Y}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{jj'}u^{(+)}_{jj'}}{\varepsilon_{jj'}-\omega_{\lambda i}};\quad \varphi^{\lambda i}_{jj'}(\tau)=\frac{1}{\sqrt{2\mathcal{Y}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{jj'}u^{(+)}_{jj'}}{\varepsilon_{jj'}+\omega_{\lambda i}};\qquad(10)$$

$$\mathcal{Y}^{\lambda i}_\tau=Y^{\lambda i}_\tau+Y^{\lambda i}_{-\tau}\left\{\frac{1-(\varkappa^{(\lambda)}_0+\varkappa^{(\lambda)}_1)X^{\lambda i}_M(\tau)}{(\varkappa^{(\lambda)}_0-\varkappa^{(\lambda)}_1)X^{\lambda i}_M(-\tau)}\right\}^2;\qquad(11)$$

$$Y^{\lambda i}_\tau=\frac{1}{2}\frac{\partial}{\partial\omega}X^{\lambda i}_M(\tau)\Big|_{\omega=\omega_{\lambda i}}=+\frac{1}{2\lambda+1}\sum_{jj'}^{\tau}\frac{(f^{(\lambda)}_{jj'}u^{(+)}_{jj'})^2\varepsilon_{jj'}\omega_{\lambda i}}{(\varepsilon^2_{jj'}-\omega^2_{\lambda i})^2}.$$

Here $\varepsilon_{jj'}=\varepsilon_j+\varepsilon_{j'}$.

Using the expressions (8)–(10) and ignoring the terms $\sim B^+(jj'\lambda-\mu)B(jj'\lambda\mu)$, we can rewrite the multipole part of the interaction of the nucleons in the form

$$H^{ph}_M=H^{ph}_{Mv}+H^{ph}_{Mvq}=-\frac{1}{4}\sum_{\substack{\lambda\mu ii'\\ \tau}}\frac{X^{\lambda i}_M(\tau)+X^{\lambda i'}_M(\tau)}{\sqrt{\mathcal{Y}^{\lambda i}_\tau\mathcal{Y}^{\lambda i'}_\tau}}\times(Q^+_{\lambda\mu i}(-)^{\lambda-\mu}+Q_{\lambda-\mu i})(Q^+_{\lambda-\mu i'}(-)^{\lambda-\mu}+Q_{\lambda\mu i'})-\frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\Big\{(Q^+_{\lambda\mu i}(-)^{\lambda-\mu}+Q_{\lambda-\mu i})\times\sum_{jj'\tau}\frac{f^{(\lambda)}_{jj'}v^{(-)}_{jj'}}{\sqrt{\mathcal{Y}^{\lambda i}_\tau}}B(jj'\lambda-\mu)+\text{h.c.}\Big\}.\qquad(12)$$

A similar expression is obtained for the spin-multipole interaction $H^{\rm ph}_{SM}$.[11] It can be constructed from the expressions (8)–(12) by means of the substitutions

$$u^{(+)}_{jj'}\to u^{(-)}_{jj'}=u_jv_{j'}-u_{j'}v_j;\quad v^{(-)}_{jj'}\to v^{(+)}_{jj'}=u_{j'}u_j+v_{j'}v_j;$$

$$f^{(\lambda)}_{jj'}\to f^{(\lambda L)}_{jj'}=\langle j\,||\,R_\lambda(r)\,i^\lambda[\sigma Y_{\lambda\mu}]_{LM}\,||\,j'\rangle.$$

In addition, in the term $H^{\rm ph}_{Mvq}$ it is necessary to take the Hermitian conjugate of the phonon factor and to reverse in it the signs of the projections. The sign of the amplitude $\varphi^{Li}_{jj'}$ of a spin-multipole phonon is also reversed compared with (10).

The isovector part of the interactions $H^{\rm ph}_M$ and $H^{\rm ph}_{SM}$ acts not only in the neutral but also in the charge-exchange channel. For the charge-exchange terms of the Hamiltonian (1), one can make similar transformations and obtain not only secular equations for the energies of the charge-exchange phonons but also the form of the interaction of these phonons with the quasiparticles. This was done in Refs. 11 and 20, to which we refer the reader for the necessary information.

Thus, in what follows we shall work with the Hamiltonian

$$H' = \sum_{jm} \varepsilon_j \alpha^+_{jm}\alpha_{jm} + H^{ph}_{Mv} + H^{ph}_{SMv} + H^{ph}_{Mvq} + H^{ph}_{SMvq}. \quad (13)$$

The terms $H^{\rm ph}_{Mvq} + H^{\rm ph}_{SMvq}$ are responsible for the interaction of the quasiparticles with the phonons. In even-even nuclei, this interaction leads to fragmentation of the single-phonon states, the giant resonances in medium and heavy nuclei acquiring widths.[11] In the following sections, we shall consider the part that this interaction plays in odd nuclei.

We emphasize that in considering odd nuclei we assume that the structure, energies, and other properties of the phonons of the even-even nuclei are already known. They are calculated by means of the equations and relations (8)–(11) and the analogous expressions for the spin-multipole phonons, i.e., in the random-phase approximation. In this sense, all the parameters of the Hamiltonian (1) have already been fixed so as to describe the properties of the excitations of the even-even nuclei with the possible accuracy. On the transition to the odd nuclei, no new parameters appear.

2. SYSTEM OF BASIC EQUATIONS OF THE MODEL IN ODD SPHERICAL NUCLEI

It can be seen from the structure of the Hamiltonian (13) that it includes terms whose eigenstates are single-quasiparticle states and states of the quasiparticle–N-phonon type ($N=1, 2,\ldots$). The interaction $H^{\rm ph}_{Mvq} + H^{\rm ph}_{SMvq}$ of the quasiparticles and the phonons will mix these states, and in the lowest order the matrix elements of the interaction between states differing by one phonon will be nonzero. Therefore, in the quasiparticle-phonon model the wave functions of the odd nuclei are expressed in the form of expansions in the basis formed by the above states. We are interested in the distribution over the nuclear spectrum of the few-quasiparticle components of the wave function, so that components not more complicated than quasiparticle–two-phonon components will be included in the model wave function.

Thus, the model wave function of an odd spherical nucleus has the form

$$\Psi_\nu(JM) = C_{J\nu}\{\alpha^+_{JM} + \sum_{\lambda i j} D^{\lambda i}_j(J\nu)\,[\alpha^+_{jm}Q^+_{\lambda\mu i}]_{JM}$$
$$+ \sum_{\substack{\lambda_1 i_1\lambda_2 i_2\\ j\lambda}} F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)\,[\alpha^+_{jm}[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{\lambda\mu}]_{JM}\}\,\Psi_0. \quad (14)$$

Strictly speaking, the phonon operators are not ideal boson operators, and it is necessary to take into account their fermion structure in calculating the various matrix elements. It is also necessary to take into account the nonvanishing commutators of the phonon and quasiparticle operators. Besides the commutation relations (6), in what follows we shall also need

$$[\alpha^+_{jm},\ Q_{\lambda\mu i}] = [Q^+_{\lambda\mu i},\ \alpha_{jm}]^+ = \sum_{j_1 m_1}\langle j_1 m_1 jm|\lambda\mu\rangle\,\psi^{\lambda i}_{j_1 j}\alpha_{j_1 m_1}. \quad (15)$$

Using (6) and (15), we obtain the normalization condition for the wave function (14) in the form[21]

$$\langle\Psi^*_\nu(JM)\,\Psi_\nu(JM)\rangle = 1 = C^2_{J\nu}\{1 + \sum_{\lambda i j}[D^{\lambda i}_j(J\nu)]^2$$
$$+ \sum_{j_1 j_2\lambda_1 i_1\lambda_2 i_2}\mathscr{L}_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2)\,D^{\lambda_1 i_1}_{j_1}(J\nu)\,D^{\lambda_2 i_2}_{j_2}(J\nu)$$
$$+ 2\sum_{\lambda_1 i_1\lambda_2 i_2 j\lambda}[F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)]^2 + \sum_{\substack{\lambda_1 i_1\lambda_2 i_2 j\lambda\lambda'\\ \lambda'_1 i'_1\lambda'_2 i'_2}}$$
$$F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)\,F^{\lambda'_1 i'_1\lambda'_2 i'_2}_{j\lambda'}(J\nu)$$
$$\times\mathscr{K}^J_j(\lambda';\ \lambda'_2 i'_2\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2;\ \lambda)$$
$$+ \sum_{\substack{\lambda_1 i_1\lambda_2 i_2 jj'\\ \lambda'_1 i'_1\lambda\lambda'}} F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)\,F^{\lambda'_1 i'_1\lambda_2 i_2}_{j'\lambda'}(J\nu)$$
$$\times\mathscr{M}^J_{\lambda_2}(j'\lambda'\lambda'_1 i'_1|\lambda_1 i_1 j\lambda) + \sum_{\substack{\lambda_1 i_1\lambda_2 i_2 jj'\\ \lambda'_2 i'_2\lambda\lambda'}} F^{\lambda_2 i_2\lambda_1 i_1}_{j\lambda}(J\nu)\,F^{\lambda'_2 i'_2\lambda_1 i_1}_{j'\lambda'}(J\nu)$$
$$\times\mathscr{M}^J_{\lambda_1}(j'\lambda'\lambda'_2 i'_2|\lambda_2 i_2 j\lambda) + \sum_{\substack{\lambda_1 i_1\lambda_2 i_2\\ \lambda'_1 i'_1\lambda'_2 i'_2}}\sum_{\substack{jj'\lambda\lambda'\\ \lambda_3 i_3\lambda''}} F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)\,F^{\lambda'_1 i'_1\lambda'_2 i'_2}_{j'\lambda'}(J\nu)$$
$$\times[\mathscr{K}^J_j(\lambda''\lambda'_2 i'_2\lambda_3 i_3|\lambda_1 i_1\lambda_2 i_2\lambda)\,\mathscr{M}^J_{\lambda'_2}(j'\lambda'\lambda'_1 i'_1|\lambda_3 i_3 j\lambda'')$$
$$+ \mathscr{K}^J_{j'}(\lambda''\lambda_3 i_3\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2\lambda)\,\mathscr{M}^J_{\lambda'_1}(j'\lambda'\lambda'_2 i'_2|\lambda_3 i_3 j\lambda'')]\}. \quad (16)$$

The functions $\mathscr{L}_J$, $\mathscr{M}^J_\lambda$, and $\mathscr{K}^J_j$ are fairly complicated combinations of the phonon amplitudes $\psi^{\lambda i}_{jj'}$, $\varphi^{\lambda i}_{jj'}$ with geometrical factors. The complete expressions for them are given in Refs. 21–24. If the Pauli principle is not taken into account, these functions vanish. As is shown in Refs. 21–24, they are of variable sign, and the diagonal functions $\mathscr{L}_J$, $\mathscr{M}^J_\lambda$ and $\mathscr{K}^J_j$ are maximal. As an example, Table I gives the values of the function $\mathscr{L}_J$ for the states $J^\pi = 1/2^-$ and $3/2^-$ in ^{59}Ni.[24] From the physical point of view, the maximality of the diagonal functions $\mathscr{L}_J$, $\mathscr{M}^J_\lambda$, and $\mathscr{K}^J_j$ is natural, since the Pauli principle will be violated with the greatest probability in the configurations constructed from identical quasiparticles. Using this circumstance, we restrict ourselves in the following calculations to the diagonal approximation. In this variant, (16) takes the form

$$\langle\Psi^*_\nu(JM)\,\Psi_\nu(JM)\rangle = 1 = C^2_{J\nu}\Big\{1 + \sum_{\lambda i j}[D^{\lambda i}_j(J\nu)]^2$$
$$\times(1 + \mathscr{L}(Jj\lambda i)) + 2\sum[F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)]^2\Big[1$$
$$+ \frac{1}{2}\mathscr{K}^\lambda(\lambda_2 i_2\lambda_1 i_1|\lambda_1 i_1\lambda_2 i_2)\Big][1 + \mathscr{M}(Jj\lambda i|\lambda_1 i_1\lambda_2 i_2)]\Big\}. \quad (17)$$

The diagonal functions $\mathscr{L}_J$ $(Jj\lambda i)$, $\mathscr{K}^J$ $(\lambda_2 i_2\lambda_1 i_1|\lambda_1 i_1\lambda_2 i_2)$ and $\mathscr{M}$ $(Jj\lambda|\lambda_1 i_1\lambda_2 i_2)$ have the form

$$\mathscr{L}(Jj\lambda i) = \sum_{j_1}(2\lambda+1)\begin{Bmatrix} j & j_1 & \lambda\\ j & J & \lambda\end{Bmatrix}(\psi^{\lambda i}_{j_1 j})^2; \quad (18)$$

$$\mathscr{K}^J(\lambda_2 i_2\lambda_1 i_1|\lambda_1 i_1\lambda_2 i_2) = \sum_{j_1 j_2 j_3 j_4}(-)^{j_2+j_4-J}$$
$$\times(2\lambda_1+1)(2\lambda_2+1)\begin{Bmatrix} j_1 & j_2 & \lambda_2\\ j_4 & j_3 & \lambda_1\\ \lambda_1 & \lambda_2 & J\end{Bmatrix}[\psi^{\lambda_1 i_1}_{j_3 j_4}\psi^{\lambda_1 i_1}_{j_1 j_4}$$
$$\times\psi^{\lambda_2 i_2}_{j_3 j_2}\psi^{\lambda_2 i_2}_{j_1 j_2} - \varphi^{\lambda_1 i_1}_{j_3 j_4}\varphi^{\lambda_1 i_1}_{j_1 j_4}\varphi^{\lambda_2 i_2}_{j_3 j_2}\varphi^{\lambda_2 i_2}_{j_1 j_2}]; \quad (19)$$

$$\mathscr{M}(Jj\lambda|\lambda_1 i_1\lambda_2 i_2) = -(2\lambda+1)\sum_{j_3}\Bigg[\begin{Bmatrix} J & \lambda & j\\ \lambda & \lambda_2 & \lambda_1\\ j & \lambda_1 & j_3\end{Bmatrix}$$
$$\times(2\lambda_1+1)(\psi^{i_1\lambda_1}_{jj_3})^2 + \begin{Bmatrix} J & \lambda & j\\ \lambda & \lambda_1 & \lambda_2\\ j & \lambda_2 & j_3\end{Bmatrix}(2\lambda_2+1)(\psi^{\lambda_2 i_2}_{jj_3})^2\Bigg]. \quad (20)$$

For configurations with a quasiparticle and a phonon, which are strictly forbidden by the Pauli principle,

TABLE I. Values of the coefficients $\mathscr{L}_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2)$ for states with $J^\pi = 1/2^-$ and $3/2^-$ of the nucleus ^{59}Ni.

J^π	$n_1l_1j_1$	$\lambda^\pi_{1i_1}$	$n_2l_2j_2$	$\lambda^\pi_{2i_2}$	$\mathscr{L}_J(j_1\lambda_1 i_1 \| j_2\lambda_2 i_2)$
1/2⁻	$1f_{5/2}$	2^+_5	$1f_{5/2}$	2^+_5	−0.95
	$2p_{3/2}$	2^+_1	$2p_{3/2}$	2^+_1	−0.67
	$2p_{3/2}$	2^+_1	$1p_{3/2}$	2^+_1	−0.004
	$2p_{3/2}$	2^+_1	$2p_{3/2}$	2^+_3	−0.04
	$2p_{3/2}$	2^+_1	$1f_{5/2}$	2^+_1	0.17
	$2p_{3/2}$	2^+_{11}	$1d_{5/2}$	3^-_1	−0.05
3/2⁻	$1f_{7/2}$	2^+_{10}	$1f_{7/2}$	2^+_{10}	−0.36
	$1f_{7/2}$	2^+_1	$2p_{3/2}$	2^+_2	−0.13
	$1f_{7/2}$	2^+_2	$2p_{1/2}$	2^+_2	−0.1
	$1f_{7/2}$	2^+_8	$1d_{5/2}$	3^-_4	0.01

$\mathscr{L}(Jj\lambda i) = -1$, and they are automatically eliminated from the normalization and the subsequent expressions.[24] If the Pauli principle is violated in the two-phonon components, then $\mathscr{K}^\lambda = -2$, and such components are also eliminated.[22,23] Finally, if the Pauli principle is violated in a quasiparticle–two-phonon component between the quasiparticle and one of the phonons, then $\mathscr{M} = -1$, and such a component vanishes from the wave function (14). Thus, allowance for the rigorous commutation relations between the phonon and quasiparticle operators guarantees fulfillment of the Pauli principle.

We calculate the expectation value of H' (13) with respect to the wave functions (14), obtaining

$$\begin{aligned}
\langle\Psi^*_\nu(JM)\,H'\Psi_\nu(JM)\rangle &= C^2_{J\nu}\Big\{\varepsilon_J + \sum_{\lambda ij}[D^{\lambda i}_j(J\nu)]^2(\varepsilon_j+\omega_{\lambda i})\\
&+\frac{1}{2}\sum_{\substack{\lambda_1 i_1 j_1\\ \lambda_2 i_2 j_2}} D^{\lambda_1 i_1}_{j_1}(J\nu)\,D^{\lambda_2 i_2}_{j_2}(J\nu)\,[\mathscr{L}_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2)\\
&\times(\varepsilon_{j_1 j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}) - R_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2)]\\
&-\sqrt{2}\sum_{\lambda ij} D^{\lambda i}_j(J\nu)\Big[\Gamma(Jj\lambda i)+\sum_{\lambda' j' i'}\Gamma(Jj'\lambda' i')\\
&\times\mathscr{L}_J(j\lambda i|j'\lambda' i')\Big] + 2\sum_{\lambda_1 i_1\lambda_2 i_2 jJ_1}\Big\{\Big[F^{\lambda_1 i_1\lambda_2 i_2}_{jJ_1}(J\nu)\Big]^2\\
&\times(\varepsilon_j+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}) + \sum_{\lambda'_1 i'_1\lambda'_2 i'_2 j' J'_1} F^{\lambda_1 i_1\lambda_2 i_2}_{jJ_1}(J\nu)\\
&\times F^{\lambda'_1 i'_1\lambda'_2 i'_2}_{j'J'_1}(J\nu)\Big[\frac{1}{2}\mathscr{K}^J_j(J'_1\lambda'_2 i'_2\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2 J_1)\,\delta_{jj'}\\
&+\mathscr{M}^J_{\lambda_2}(j'J'_1\lambda'_1 i'_1|\lambda_1 i_1 jJ_1)\,\delta_{\lambda_2\lambda'_2}\delta_{i_2 i'_2}\\
&+\frac{1}{2}\sum_{J'\lambda_3 i_3}\mathscr{K}^J_j(J'\lambda'_2 i'_2\lambda_3 i_3|\lambda_1 i_1\lambda_2 i_2 J_1)\\
&\times\mathscr{M}^J_{\lambda'_2}(j'J'_1\lambda'_1 i'_1|\lambda_3 i_3 jJ')\Big](\varepsilon_j+\omega_{\lambda_1 i_1}+\omega_{\lambda_3 i_2})\\
&+S_J(j'J'_1\lambda'_2 i'_2\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2 jJ_1)\Big)\Big\} + 2\sum_{\substack{jj'\lambda i\lambda' i'\\ \lambda_1 i_1\lambda_2 i_2}} D^{\lambda i}_j(J\nu)F^{\lambda_1 i_1\lambda_2 i_2}_{j'\lambda'}(J\nu)\\
&\times[\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}+\mathscr{L}_J(j'\lambda' i'|j\lambda i)]\\
&\times\Big[U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda' i') + \frac{1}{2}U^{\lambda'_2 i'_2}_{\lambda'_1 i'_1}(\lambda' i')\,\mathscr{K}^J_{j'}(\lambda'\lambda'_2 i'_2\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2\lambda)\Big]\\
&+2\sqrt{2}\sum_{\substack{\lambda ij\lambda_1 i_1\lambda_2 i_2,\\ \lambda' ij'\lambda'_1 i'_1\lambda'_2 i'_2}}\ \sum_{\lambda_3 j_3} D^{\lambda i}_j(J\nu)\,F^{\lambda_1 i_1\lambda_2 i_2}_{j'\lambda'}(J\nu)\\
&\times[(2j_3+1)(2\lambda_3+1)]^{1/2}(-)^{j'+\lambda'_1+\lambda'_2-J}\begin{Bmatrix}\lambda'_1 & \lambda'_2 & \lambda_3\\ J & j' & j_3\end{Bmatrix}\\
&\times\Big[\delta_{i'_1 i_1}\delta_{j_3 j}\delta_{\lambda_2\lambda}\delta_{i_2 i}\delta_{\lambda'_1\lambda_1}\delta_{\lambda'_2\lambda_2}\delta_{i'_2 i_2}\delta_{\lambda_3\lambda'}\Big(\Gamma(j_3 j'\lambda'_1 i'_1)\\
&+\sum_{j'_3\lambda'_3 i'_3}\Gamma(jj'_3\lambda'_3 i'_3)\,\mathscr{L}_J(j'_3\lambda'_3 i'_3|j'\lambda_1 i_1)\Big)\\
&+\Gamma(j_3 j'\lambda'_1 i'_1)\,(\delta_{\lambda'_1\lambda_1}\delta_{i'_1 i_1}\delta_{\lambda'_2\lambda_2}\delta_{i'_2 i_2}\delta_{\lambda_3\lambda'}\\
&\times\mathscr{L}_J(j_3\lambda_2 i_2|j\lambda i) + \frac{1}{2}\mathscr{K}^J_{j'}(\lambda_3\lambda i\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2\lambda')\\
&\times(\delta_{jj_3}\delta_{\lambda'_2\lambda}\delta_{i'_2 i}+\mathscr{L}_J(j_3\lambda'_2 i'_2|j\lambda i))\Big)\\
&+\frac{1}{2}\sum_{j'_3\lambda'_3 i'_3}\Gamma(jj'_3\lambda'_3 i'_3)\,\mathscr{K}^J_{j'}(\lambda_3\lambda i\lambda'_3 i'_3|\lambda_1 i_1\lambda_2 i_2)\\
&\times\mathscr{L}_J(j'_3\lambda'_1 i'_1|j'\lambda_1 i_1)\,\delta_{\lambda'_2\lambda_2}\delta_{i'_2 i_2}\delta_{j_3 j}\Big]\Big\}. \qquad (21)
\end{aligned}$$

Here we have introduced the notation

$$\Gamma(Jj\lambda i) = \sqrt{\frac{2\lambda+1}{2J+1}}\,\frac{f^{(\lambda)}_{Jj}v^{(-)}_{Jj}}{\sqrt{\mathscr{Y}^{\lambda i}_\tau}}; \qquad (22)$$

$$\begin{aligned}
R_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2) &= \sum_{i'}{}^{\tau}\{X^\tau(\lambda_2 i_2\lambda_2 i')\,\mathscr{L}_J(j_1\lambda_1 i_1|j_2\lambda_2 i')\\
&+X^\tau(\lambda_1 i_1\lambda_1 i')\mathscr{L}_J(j_2\lambda_2 i_2|j_1\lambda_1 i') + \sum_{j_3\lambda i} X^\tau(\lambda i\lambda i')\,[\mathscr{L}_J(j_1\lambda_1 i_1|j_3\lambda i)\\
&\times\mathscr{L}_J(j_2\lambda_2 i_2|j_3\lambda i') + \mathscr{L}_J(j_2\lambda_2 i_2|j_3\lambda i)\,\mathscr{L}_J(j_1\lambda_1 i_1|j_3\lambda i')]\}; \qquad (23)
\end{aligned}$$

$$X^\tau(\lambda_1 i_1\lambda_2 i_2) = \frac{1}{4}\,\frac{X^{\lambda_1 i_1}(\tau)+X^{\lambda_2 i_2}(\tau)}{\sqrt{\mathscr{Y}^{\lambda_1 i_1}_\tau\mathscr{Y}^{\lambda_2 i_2}_\tau}};$$

$$\begin{aligned}
S_J(j'J'_1\lambda'_2 i'_2\lambda'_1 i'_1|\lambda_1 i_1\lambda_2 i_2 jJ_1) &= \frac{1}{2}\sum_{J'\lambda ii'}{}^{\tau}\Big\{[\mathscr{M}^J_{\lambda_2}(j'J'_1\lambda'_1 i'_1|\lambda ijJ')\\
&+\delta_{jj'}\delta_{\lambda\lambda'_1}\delta_{ii'_1}\delta_{J'J'_1}]\Big[X^\tau(\lambda i\lambda i')\,\mathscr{K}^J_j(J'\lambda'_2 i'_2\lambda i'|\lambda_1 i_1\lambda_2 i_2 J_1)\\
&+X^\tau(\lambda'_2 i'_2\lambda_2 i_2)\,\mathscr{K}^J_j(J'\lambda'_2 i'\lambda i|\lambda_1 i_1\lambda_2 i_2 J_1)\\
&+\sum_{J_2\lambda_3 i_3\lambda_4 i_4}{}^{\tau} X^\tau(\lambda_3 i_3\lambda_3 i')\,\mathscr{K}^J_j(J_2\lambda_4 i_4\lambda_3 i'|\lambda_1 i_1\lambda_2 i_2 J_1)\\
&\times\mathscr{K}^J_j(J'\lambda'_2 i'_2\lambda i|\lambda_3 i_3\lambda_4 i_4 J_2)]\\
&-\sum_{J_2 j_3\lambda_3 i_3}{}^{\tau} X^\tau(\lambda i\lambda i')\,\mathscr{M}^J_{\lambda_2}(jJ_1\lambda_1 i_1|\lambda i' j_3 J')
\end{aligned}$$

$$\times \Big[\mathcal{M}^J_{\lambda_2}(j_3J'\lambda i|\lambda_3 i_3 j'J_2)(\delta_{\lambda_3\lambda_1'}\delta_{i_3 i_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'}\delta_{J_1'J_2} + \frac{1}{2}\mathcal{K}^J_{j'}(J_1'\lambda_2' i_2'\lambda_1' i_1'|\lambda_3 i_3\lambda_2 i_2 J_2)) + (-)^{J'+J_2+\lambda_2+\lambda_3} \times \mathcal{M}^J_{\lambda}(j_3J'\lambda_2 i_2|\lambda_3 i_3 j'J_2)\Big(\delta_{\lambda\lambda_1'}\delta_{ii_1'}\delta_{\lambda_3\lambda_2'}\delta_{i_3 i_2'}\delta_{J_1'J_2} + \frac{1}{2}\mathcal{K}^J_{j'}(J_1'\lambda_2' i_2'\lambda_1' i_1'|\lambda i\lambda_3 i_3 J_2)\Big)\Big] - 2\Big(\delta_{\lambda\lambda_1'}\delta_{ii_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'}\delta_{J_1'J} + \frac{1}{2}\mathcal{K}^J_{j'}(J_1'\lambda_2' i_2'\lambda_1' i_1'|\lambda i\lambda_2 i_2 J')\Big)X^{\tau}(\lambda i\lambda i')\,\mathcal{M}^J_{\lambda_2}(jJ_1\lambda_1 i_1|\lambda i' j'J')\Big\};\tag{24}$$

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i) = \langle|Q_{\lambda\mu i}(H^{ph}_{Mvq} + H^{ph}_{SMvq})[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{\lambda\mu}|\rangle$$

$$= (-)^{\lambda_1+\lambda_2-\lambda}\frac{1}{\sqrt{2}}[(2\lambda_1+1)(2\lambda_2+1)]^{1/2}\sum_{j_1j_2j_3}{}^{\tau}\Big[\frac{f^{\lambda}_{j_1j_2}v^{(\mp)}_{j_1j_2}}{\sqrt{\mathcal{Y}^{\lambda i}_{\tau}}}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_1 & j_2 & j_3\end{Bmatrix} \times(\psi^{\lambda_1 i_1}_{j_3 j_1}\varphi^{\lambda_2 i_2}_{j_2 j_3} + (-)^{b_\lambda}\psi^{\lambda_2 i_2}_{j_2 j_3}\varphi^{\lambda_1 i_1}_{j_3 j_1}) + \frac{f^{\lambda_1}_{j_1j_2}v^{(\mp)}_{j_1j_2}}{\sqrt{\mathcal{Y}^{\lambda_1 i_1}_{\tau}}}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_3 & j_2 & j_1\end{Bmatrix}(\varphi^{\lambda i}_{j_2 j_3}\varphi^{\lambda_2 i_2}_{j_3 j_1} + (-)^{b_1}\psi^{\lambda i}_{j_2 j_3}\psi^{\lambda_2 i_2}_{j_3 j_1}) + \frac{f^{\lambda_2}_{j_1j_2}v^{(\mp)}_{j_1j_2}}{\sqrt{\mathcal{Y}^{\lambda_2 i_2}_{\tau}}} \times \begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_1 & j_3 & j_2\end{Bmatrix}(\psi^{\lambda i}_{j_3 j_1}\psi^{\lambda_1 i_1}_{j_2 j_3} + (-)^{b_2}\varphi^{\lambda i}_{j_3 j_1}\varphi^{\lambda_1 i_1}_{j_2 j_3})\Big],\tag{25}$$

where $v^{(-)}_{j_1j_2}$ appears in combination with the multipole matrix elements $f^{\lambda_i}_{j_1j_2}$, and $v^{(+)}_{j_1j_2}$ with the spin-multipole elements; the numbers b_λ and b_k take the value 0 in the case of the multipole matrix elements $f^{\lambda_i}_{j_1j_2}$ and 1 in the case of the spin-multipole elements. In the diagonal approximation for the functions $\mathcal{L}_J$, $\mathcal{K}^J_j$, and $\mathcal{M}^J_\lambda$ [see the expressions (18)–(20)], the mean value of the Hamiltonian has the simpler form

$$\langle\Psi^*_\nu(JM)H'\Psi_\nu(JM)\rangle = C^2_{J\nu}\Big\{\varepsilon_J + \sum_{\lambda ij}[(D^{\lambda i}_j(J\nu))^2(\varepsilon_j + \omega_{\lambda i} - R(Jj\lambda i)) - \sqrt{2}D^{\lambda i}_j(J\nu)\Gamma(Jj\lambda i)] \times[1+\mathcal{L}(Jj\lambda i)] + 2\sum_{\lambda_1 i_1\lambda_2 i_2 j\lambda}[F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)]^2 \times(\varepsilon_j + \omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + S(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2)) \times\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{M}(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2)] + 2\sum_{\substack{\lambda_1 i_1\lambda_2 i_2\\ \lambda ij}}D^{\lambda i}_j(J\nu)F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) \times\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{L}(Jj\lambda i)] - 2\sqrt{2}\sum_{\substack{\lambda i\lambda_1 i_1\\ jj'\lambda'}}D^{\lambda i}_j(J\nu)F^{\lambda_1 i_1\lambda i}_{j'\lambda'}(J\nu)[(2j+1) \times(2\lambda'+1)]^{1/2}(-)^{j'+\lambda_1+\lambda+J} \times\begin{Bmatrix}\lambda_1 & \lambda & \lambda'\\ J & j' & j\end{Bmatrix}\Gamma(jj'\lambda_1 i_1)\Big[1+\frac{1}{2}\mathcal{K}^{\lambda'}(\lambda i\lambda_1 i_1\mid\lambda_1 i_1\lambda i)\Big] \times[1+\mathcal{L}(Jj'\lambda_1 i_1)+\mathcal{L}(Jj\lambda i)]\Big\};\tag{26}$$

$$\left.\begin{aligned} R(Jj_1\lambda_1 i_1) &= \frac{R_J(j_1\lambda_1 i_1\mid j_1\lambda_1 i_1)}{1+\mathcal{L}(Jj_1\lambda_1 i_1)};\\ S(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2) &= \frac{S(j\lambda\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2 j\lambda)}{\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{M}(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2)]}.\end{aligned}\right\}\tag{27}$$

As can be seen from (26), the states forbidden by the Pauli principle do not contribute to the expectation value of the Hamiltonian.

We use the variational principle

$$\delta\{\langle\Psi^*_\nu(JM)H'\Psi_\nu(JM)\rangle - \eta_{J\nu}[\langle\Psi^*_\nu(JM)\Psi_\nu(JM)\rangle - 1]\} = 0\tag{28}$$

and obtain the following system of three equations:

$$\Big\{\varepsilon_J - \eta_{J\nu} + \sum_{\lambda ij}[(D^{\lambda i}_j(J\nu))^2(\varepsilon_j + \omega_{\lambda i} - R(Jj\lambda i) - \eta_{J\nu}) - \sqrt{2}D^{\lambda i}_j(J\nu)\Gamma(Jj\lambda i)][1+\mathcal{L}(Jj\lambda i)] + 2\sum_{\lambda_1 i_1\lambda_2 i_2}[F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)]^2 \times[\varepsilon_J + \omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + S(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2) - \eta_{J\nu}] \times\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{M}(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2)] + 2\sum_{\lambda ij\lambda_1 i_1\lambda_2 i_2}D^{\lambda i}_j(J\nu)F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu) \times U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{L}(Jj\lambda i)] - 2\sqrt{2}\sum_{\lambda ijj'\lambda'\lambda_1 i_1}D^{\lambda i}_j(J\nu)F^{\lambda_1 i_1\lambda_2 i_2}_{j'\lambda'}(J\nu)(-)^{j'+\lambda_1+\lambda+J} \times[(2j+1)(2\lambda'+1)]^{1/2} \times\begin{Bmatrix}\lambda_1 & \lambda & \lambda'\\ J & j' & j\end{Bmatrix}\Gamma(jj'\lambda_1 i_1)\Big[1+\frac{1}{2}\mathcal{K}^{\lambda'}(\lambda i\lambda_1 i_1\mid\lambda_1 i_1\lambda i)\Big] \times[1+\mathcal{L}(Jj'\lambda_1 i_1)+\mathcal{L}(Jj\lambda i)]\Big\} = 0;\tag{29}$$

$$D^{\lambda i}_j(J\nu)\Big[\varepsilon_j + \omega_{\lambda i} - R(Jj\lambda i) - \eta_{J\nu} - \frac{1}{\sqrt{2}}\Gamma(Jj\lambda i)\Big] \times[1+\mathcal{L}(Jj\lambda i)] + \sum_{\lambda_1 i_1\lambda_2 i_2}F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big] \times[1+\mathcal{L}(Jj\lambda i)] - \sqrt{2}\sum_{\lambda' j'\lambda_1 i_1}F^{\lambda_1 i_1\lambda i}_{j'\lambda'}(J\nu)[(2j+1)(2\lambda'+1)]^{1/2} \times(-)^{j'+\lambda_1+\lambda+J}\Gamma(jj'\lambda_1 i_1)\begin{Bmatrix}\lambda_1 & \lambda & \lambda'\\ J & j' & j\end{Bmatrix} \Big[1+\frac{1}{2}\mathcal{K}^{\lambda'}(\lambda i\lambda_1 i_1\mid\lambda_1 i_1\lambda i)\Big] \times[1+\mathcal{L}(Jj'\lambda_1 i_1)+\mathcal{L}(Jj\lambda i)] = 0;\tag{30}$$

$$F^{\lambda_1 i_1\lambda_2 i_2}_{j\lambda}(J\nu)[\varepsilon_j + \omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + S(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2) - \eta_{J\nu}] \times\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{M}(Jj\lambda\mid\lambda_1 i_1\lambda_2 i_2)] + \frac{1}{2}\sum_i D^{\lambda i}_j(J\nu)U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big] \times[1+\mathcal{L}(Jj\lambda i)] - \frac{1}{2\sqrt{2}}\sum_{j_2}\Big[D^{\lambda_2 i_2}_{j_2}(J\nu)\Gamma(j_2 j\lambda_1 i_1) \times[(2j_2+1)(2\lambda+1)]^{1/2}(-)^{j+\lambda_1+\lambda_2+J}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ J & j & j_2\end{Bmatrix} \times\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big][1+\mathcal{L}(Jj\lambda_1 i_1)+\mathcal{L}(Jj_2\lambda_2 i_2)] + D^{\lambda_1 i_1}_{j_2}(J\nu)\Gamma(j_2 j\lambda_2 i_2)[(2j_2+1)(2\lambda+1)]^{1/2} \times(-)^{j+J-\lambda}\begin{Bmatrix}\lambda_2 & \lambda_1 & \lambda\\ J & j & j_2\end{Bmatrix}\Big[1+\frac{1}{2}\mathcal{K}^{\lambda}(\lambda_2 i_2\lambda_1 i_1\mid\lambda_1 i_1\lambda_2 i_2)\Big]$$

$$\times [1+\mathcal{L}(Jj\lambda_2 i_2)+\mathcal{L}(Jj_2\lambda_1 i_1)]\Big]=0. \qquad (31)$$

Substituting the expression for $F_{j\lambda}^{\lambda_1 i_1 i_2}(J\nu)$ from Eq. (31) in Eqs. (29) and (30), we obtain the fundamental system of equations of the quasiparticle-phonon model of the nucleus for odd spherical nuclei:

$$\mathcal{F}(\eta_{J\nu}) \equiv \varepsilon_J - \eta_{J\nu} - \frac{1}{\sqrt{2}} \sum_{\lambda i j} D_j^{\lambda i}(J\nu)\,\Gamma(Jj\lambda i) \times [1+\mathcal{L}(Jj\lambda i)] = 0; \qquad (32)$$

$$D_{j_1}^{\lambda_1 i_1}(J\nu)\,[\varepsilon_{j_1}+\omega_{\lambda_1 i_1}-R(Jj_1\lambda_1 i_1)-\eta_{J\nu}]\,[1+\mathcal{L}(Jj_1\lambda_1 i_1)]$$
$$-\frac{1}{2}\sum_{\lambda_2 i_2 j_2 \lambda_3 j_3} \frac{(2\lambda_3+1)\,[(2j_1+1)(2j_3+1)]^{1/2} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ j_2 & J & j_1 \end{Bmatrix} \Gamma(j_1 j_2 \lambda_2 i_2) \times \left[1+\frac{1}{2}\mathcal{K}^{\lambda_3}(\lambda_1 i_1 \lambda_2 i_2 \mid \lambda_2 i_2 \lambda_1 i_1)\right]}{[\varepsilon_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+S(Jj_2\lambda_3 \mid \lambda_2 i_2 \lambda_1 i_1)-\eta_{J\nu}] \times [1+\mathcal{M}(Jj_2\lambda_3 \mid \lambda_3 i_3 \lambda_1 i_1)]}$$
$$\times [1+\mathcal{L}(Jj_2\lambda_2 i_2)+\mathcal{L}(Jj_1\lambda_1 i_1)]$$
$$\times \Big\{ D_{j_3}^{\lambda_1 i_1}(J\nu)\,\Gamma(j_3 j_2 \lambda_2 i_2) \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ j_1 & J & j_3 \end{Bmatrix}$$
$$\times [1+\mathcal{L}(Jj_2\lambda_2 i_2)+\mathcal{L}(Jj_3\lambda_1 j_1)]$$
$$+ D_{j_3}^{\lambda_2 i_2}(J\nu)\,\Gamma(j_3 j_2 \lambda_1 i_1)\,(-)^{\lambda_1+\lambda_2-\lambda_3}$$
$$\times \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ J & j_2 & j_3 \end{Bmatrix} [1+\mathcal{L}(Jj_2\lambda_1 i_1)+\mathcal{L}(Jj_3\lambda_2 i_2)]\Big\}$$
$$-\frac{1}{2}\sum_{\lambda_2 i_2 \lambda_3 i_3 i_1'} \frac{D_{j_1}^{\lambda_1 i_1'}(J\nu)\,U_{\lambda_2 i_2}^{\lambda_3 i_3}(\lambda_1 i_1)\,U_{\lambda_2 i_2}^{\lambda_3 i_3}(\lambda_1 i_1') \times \left[1+\frac{1}{2}\mathcal{K}^{\lambda_1}(\lambda_3 i_3 \lambda_2 i_2 \mid \lambda_2 i_2 \lambda_3 i_3)\right]}{[\varepsilon_{j_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}+S(Jj_1\lambda_1 \mid \lambda_2 i_2 \lambda_3 i_3)-\eta_{J\nu}] \times [1+\mathcal{M}(Jj_1\lambda_1 \mid \lambda_2 i_2 \lambda_3 i_3)]}$$
$$\times [1+\mathcal{L}(Jj_1\lambda_1 i_1')]\,[1+\mathcal{L}(Jj_1\lambda_1 i_1)]$$
$$+\frac{1}{\sqrt{2}}\sum_{\lambda_2 i_2 \lambda_3 i_3 j_3} D_{j_3}^{\lambda_3 i_3}(J\nu)\,\Gamma(j_3 j_1 \lambda_2 i_2)$$
$$\times \sqrt{(2j_3+1)}\,(-)^{j_1+\lambda_1+J} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ j_3 & J & j_1 \end{Bmatrix}$$
$$\times \Big\{ \frac{\sqrt{(2\lambda_1+1)}\,U_{\lambda_3 i_3}^{\lambda_2 i_2}(\lambda_1 i_1)\left[1+\frac{1}{2}\mathcal{K}^{\lambda_1}(\lambda_3 i_3 \lambda_2 i_2 \mid \lambda_2 i_2 \lambda_3 i_3)\right]}{[\varepsilon_{j_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}+S(Jj_1\lambda_1 \mid \lambda_2 i_2 \lambda_3 i_3)-\eta_{J\nu}]}$$
$$\times [1+\mathcal{M}(Jj_1\lambda_1 | \lambda_2 i_2 \lambda_3 i_3)]^{-1}\,[1+\mathcal{L}(Jj_1\lambda_1 i_1)]$$
$$\times [1+\mathcal{L}(Jj_1\lambda_2 i_2)+\mathcal{L}(Jj_3\lambda_3 i_3)]$$
$$+\frac{\sqrt{(2\lambda_3+1)}\,U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda_3 i_3)\left[1+\frac{1}{2}\mathcal{K}^{\lambda_3}(\lambda_1 i_1 \lambda_2 i_2 \mid \lambda_2 i_2 \lambda_1 i_1)\right] \times [1+\mathcal{L}(Jj_3\lambda_3 i_3)]}{[\varepsilon_{j_3}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+S(Jj_3\lambda_3 \mid \lambda_2 i_2 \lambda_1 i_1)-\eta_{J\nu}] \times [1+\mathcal{M}(Jj_3\lambda_3 \mid \lambda_2 i_2 \lambda_1 i_1)]}$$
$$\times [1+\mathcal{L}(Jj_3\lambda_2 i_2)+\mathcal{L}(Jj_1\lambda_1 i_1)]\Big\}$$
$$=\frac{1}{\sqrt{2}}\Gamma(Jj_1\lambda_1 i_1)\,[1+\mathcal{L}(Jj_1\lambda_1 i_1)]. \qquad (33)$$

Finding the coefficients from the system of equations (33) and substituting them in (32), we can determine the energies $\eta_{J\nu}$ of the states $\Psi_\nu(JM)$. Note that Eq. (32) still has the same form when more complicated configurations are taken into account in the wave function (14). It is in fact a matrix form of the Dyson equation for the quasiparticle–phonon model. Using (21), we can derive an analogous system of equations in the nondiagonal approximation too. If we assume that the odd quasiparticle has a weak effect on the structure of the phonon excitations of the even-even core and ignore the corrections due to the Pauli principle, then the functions $\mathcal{L}(Jj\lambda i)$, $\mathcal{K}^{\lambda}(\lambda_1 i_1 \lambda_2 i_2 | \lambda_2 i_2 \lambda_1 i_1)$, $\mathcal{M}(Jj_1\lambda_1 | \lambda_2 i_2 \lambda_3 i_3)$ vanish. We then obtain the system of equations

$$\mathcal{F}(\eta_{J\nu}) \equiv \varepsilon_J - \eta_{J\nu} - \frac{1}{\sqrt{2}}\sum_{\lambda i j}\Gamma(Jj\lambda i)\,D_j^{\lambda i}(J\nu) = 0; \qquad (34)$$

$$\sum_{\lambda i j} D_j^{\lambda i}(J\nu)\Big[\Big(\varepsilon_j+\omega_{\lambda i}-\eta_{J\nu}-\frac{1}{2}\sum_{\lambda_2 i_2 j_2}\frac{\Gamma^2(jj_2\lambda_2 i_2)}{\varepsilon_{j_2}+\omega_{\lambda i}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}\Big)$$
$$\times \delta_{\lambda\lambda_1}\delta_{ii_1}\delta_{jj_1}$$
$$-\sum_{j_3}\frac{\Gamma(j_3 j_1 \lambda i)\,\Gamma(j_3 j \lambda_1 i_1)}{\varepsilon_{j_3}+\omega_{\lambda i}+\omega_{\lambda_1 i_1}-\eta_{J\nu}}[(2j+1)(2j_1+1)]^{1/2}\begin{Bmatrix} \lambda_2 & j_3 & j_1 \\ \lambda_1 & J & j \end{Bmatrix}$$
$$-\frac{1}{2}\sum_{\lambda_2 i_2 \lambda_3 i_3}\frac{U_{\lambda_3 i_3}^{\lambda_2 i_2}(\lambda_1 i_1)\,U_{\lambda_3 i_3}^{\lambda_2 i_2}(\lambda_1 i)}{\varepsilon_{j_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}-\eta_{J\nu}}\delta_{\lambda\lambda_1}\delta_{jj_1}$$
$$+\frac{1}{\sqrt{2}}\sum_{\lambda_2 i_2}(-)^{j+\lambda_1+J}\sqrt{2j+1}\begin{Bmatrix} \lambda_2 & \lambda & \lambda_1 \\ J & j_1 & j \end{Bmatrix}\Gamma(jj_1\lambda_2 i_2)$$
$$\times\Big\{\frac{\sqrt{2\lambda_1+1}\,U_{\lambda i}^{\lambda_2 i_2}(\lambda_1 i_1)}{\varepsilon_{j_1}+\omega_{\lambda i}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}+\frac{\sqrt{2\lambda+1}\,U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i)}{\varepsilon_j+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}\Big\}\Big]$$
$$=\frac{1}{\sqrt{2}}\Gamma(Jj_1\lambda_1 i_1). \qquad (35)$$

Comparing Eqs. (33) and (35), we see that allowance for the effects due to the Pauli principle leads to a renormalization of the matrix elements and a shift of the energy poles. In Sec. 4, taking the example of a simpler model wave function in which the quasiparticle–two-phonon components are not taken into account, we demonstrate the part played by the Pauli-principle corrections.

If in Eq. (35) we ignore the terms containing $U_{\lambda_3 i_3}^{\lambda_2 i_2}(\lambda i)$, which are the most important for describing the fragmentation of quasiparticle–one-phonon states,[25] we can readily obtain the equation derived for the first time in Ref. 7:

$$\sum_{\lambda i j} D_j^{\lambda i}(J\nu)\Big[\Big(\varepsilon_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}-\frac{1}{2}\sum\frac{\Gamma^2(jj_2\lambda_2 i_2)}{\varepsilon_{j_2}+\omega_{\lambda i}+\omega_{\lambda_2 i_1}-\eta_{J\nu}}\Big)$$
$$\times \delta_{\lambda\lambda_1}\delta_{ii_1}\delta_{jj_1}$$
$$+\frac{1}{2}\sum_{j_3}\frac{\Gamma(j_3 j_1 \lambda i)\,\Gamma(j_3 j \lambda_1 i_1)}{\varepsilon_{j_3}+\omega_{\lambda_1 i_1}+\omega_{\lambda i}-\eta_{J\nu}}[(2j_1+1)(2j+1)]^{1/2}\begin{Bmatrix} \lambda & j_3 & j_1 \\ \lambda_1 & J & j \end{Bmatrix}\Big]$$
$$=\frac{\Gamma(Jj_1\lambda_1 i_1)}{\sqrt{2}}. \qquad (36)$$

Equation (36) has been used to calculate the distribution of the strength of deep hole and highly excited particle states[15,26-28] and neutron strength functions.[27,29,30] Neglect of the terms with U corresponds to the usual assumptions of the phenomenological models.[31]

If in the excited-state wave function of the odd nucleus we do not take into account the quasiparticle–two-phonon components, i.e., we restrict ourselves to a wave function of the form

$$\Psi_\nu(JM) = C_{J\nu}\{\alpha_{JM}^{+} + \sum_{\lambda i j} D_j^{\lambda i}(J\nu)\,[\alpha_{jm}^{+} Q_{\lambda\mu i}^{+}]_{JM}\}\,\Psi_0, \qquad (37)$$

then in the diagonal approximation for the functions $\mathscr{L}_J(j_1\lambda_1 i_1|j_2\lambda_2 i_2)$ we obtain for $D_j^{\lambda i}(J\nu)$ the simple expression[24]

$$D_j^{\lambda i}(J\nu)=\frac{1}{\sqrt{2}}\frac{\Gamma(Jj\lambda i)}{\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta_{J\nu}}. \quad (38)$$

Substituting (38) in (32), we arrive at a secular equation for determining the energy eigenvalues:

$$\mathscr{F}(\eta_{J\nu})=\varepsilon_J-\eta_{J\nu}-\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]}{\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta_{J\nu}}=0. \quad (39)$$

If we do not take into account the corrections for the Pauli principle, the factors $\mathscr{L}_J$ and R vanish. Then (39) goes over into the well-known secular equation for odd nuclei of the superfluid nuclear model.[3] The more general equations (38) and (39) were obtained for the first time in Ref. 24.

The system of equations (32) and (33) has not yet been solved. Only some special cases of it have been studied in detail. The results of these investigations are given in the following sections.

3. STRENGTH FUNCTIONS. SOLUTION OF THE EQUATIONS OF THE MODEL

In typical cases, the rank of the system of equations (33) is 100–200, and in the region of nuclei with $A\sim 210$ and/or for subshells with excitation energy $E_x \gtrsim 10$ MeV it may reach 1000. When allowance is made for the nonlinear dependence of the coefficients in (33) on the state energy η, it is very difficult to solve such a system.

The method of strength functions greatly simplifies the problem.[32,33] Using it, one can directly, without solving the system of equations (32) and (33), calculate the values, averaged over a given energy interval Δ, of the coefficients of the wave function (14) or physical quantities that depend on them, such as spectroscopic factors or excitation probabilities. The strength-function method is presented in detail in Refs. 9, 11, and 34. Here, we shall only use the results of these studies.

The general form of the solution of the system of linear equations (33) can be written down by using Cramer's rules[26]:

$$D_j^{\lambda i}(J\nu)=\frac{1}{\sqrt{2}}\frac{\Gamma(Jj\lambda i)}{\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta_{J\nu}}K_j^{\lambda i}(\eta_{J\nu}). \quad (40)$$

The function $K_j^{\lambda i}(\eta_{J\nu})$ is the ratio of two determinants and includes all poles of the type $\varepsilon_j+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+S(Jj\lambda i|\lambda_1 i_1\lambda_2 i_2)-\eta_{J\nu}$. If (40) is taken into account, Eq. (32) takes the form

$$\mathscr{F}(\eta_{J\nu})\equiv\varepsilon_J-\eta_{J\nu}-\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}_J(Jj\lambda i)]}{\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta_{J\nu}}K_j^{\lambda i}(\eta_{J\nu})=0. \quad (41)$$

At the points of the solution of the system of equations (32) and (33), the condition

$$\left.\frac{d\mathscr{F}(\eta)}{d\eta}\right|_{\eta=\eta_{J\nu}}=-\frac{1}{C_{J\nu}^2} \quad (42)$$

is satisfied.

We now define the strength function that describes the distribution of the single-quasiparticle component of the wave function (14) over the spectrum of the nucleus by means of the relation

$$C_J^2(\eta)=\frac{\Delta}{2\pi}\sum_\nu\frac{C_{J\nu}^2}{(\eta-\eta_{J\nu})^2+\Delta^2/4}. \quad (43)$$

Using the relation (42) and residue theory, we can show[32,33] that

$$C_J^2(\eta)=\frac{1}{\pi}\operatorname{Im}\frac{1}{\mathscr{F}(\bar\eta)}, \quad (44)$$

where $\bar\eta=\eta+i\Delta/2$ is a complex variable. The actual form of $C_J^2(\eta)$ can be readily obtained[26] from (44) and (41):

$$C_J^2(\eta)=\frac{1}{\pi}\frac{\frac{\Delta}{2}[1+\Gamma_2(\eta)]}{[\varepsilon_J-\eta-\gamma_2(\eta)]^2+\frac{\Delta^2}{4}[1+\Gamma_2(\eta)]^2}; \quad (45)$$

$$\gamma_2(\eta)=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]\left\{L_j^{\lambda i}(\eta)(\varepsilon_j+\omega_{\lambda i}-\eta-R(Jj\lambda i))-\frac{\Delta^2}{4}M_j^{\lambda i}(\eta)\right\}}{(\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta)^2+\Delta^2/4};$$

$$\Gamma_2(\eta)=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]\{M_j^{\lambda i}(\eta)(\varepsilon_j+\omega_{\lambda i}-\eta-R(Jj\lambda i))+L_j^{\lambda i}(\eta)\}}{(\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta)^2+\Delta^2/4}.$$

We have introduced the new notation

$$K_j^{\lambda i}(\bar\eta)=L_j^{\lambda i}(\eta)+i\frac{\Delta}{2}M_j^{\lambda i}(\eta).$$

If in the wave function we ignore the quasiparticle–two-phonon components, then $K_j^{\lambda i}(\bar\eta)=L_j^{\lambda i}(\eta)+i\Delta/2M_j^{\lambda i}(\eta)=0$, i.e., $L_j^{\lambda i}(\eta)=1$, and $M_j^{\lambda i}(\eta)=0$. In this case, we obtain for the function $C_J^2(\eta)$ the expression[24]

$$C_J^2(\eta)=\frac{1}{\pi}\frac{\frac{\Delta}{2}[1+\Gamma_1(\eta)]}{[\varepsilon_J-\eta-\gamma_1(\eta)]^2+\Delta^2/4[1+\Gamma_1(\eta)]^2}; \quad (46)$$

$$\gamma_1(\eta)=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)](\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta)}{(\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta)^2+\Delta^2/4};$$

$$\Gamma_1(\eta)=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]}{(\varepsilon_j+\omega_{\lambda i}-R(Jj\lambda i)-\eta)^2+\Delta^2/4}.$$

If, further, we set $[\alpha_{jm}^+, Q_{\lambda\mu i}]=[\alpha_{jm}, Q_{\lambda\mu i}^+]=0$, the functions $\gamma_1(\eta)$ and $\Gamma_1(\eta)$ take the form given for the first time in Ref. 35 (see also Refs. 30, 32, and 33):

$$\left.\begin{aligned}\gamma_1(\eta)&=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)(\varepsilon_j+\omega_{\lambda i}-\eta)}{(\varepsilon_j+\omega_{\lambda i}-\eta)^2+\Delta^2/4};\\ \Gamma_1(\eta)&=\frac{1}{2}\sum_{\lambda ij}\frac{\Gamma^2(Jj\lambda i)}{(\varepsilon_j+\omega_{\lambda i}-\eta)^2+\Delta^2/4}.\end{aligned}\right\} \quad (47)$$

The expression for the strength function $C_J^2(\eta)$ in which the effect of the Pauli principle between the odd quasiparticle and the phonon is not taken into account is identical to (46).

We can similarly obtain an expression for the strength function of the quasiparticle–one-phonon component, which we denote by $d^2(\eta)$.[15,36] The strength function $d^2(\eta)$ is defined as follows:

$$d^2(\eta) = \frac{\Delta}{2\pi} \sum_{\nu} [C_{J\nu} D_j^{\lambda i}(J\nu)]^2 \frac{1}{(\eta-\eta_{J\nu})^2+\Delta^2/4}. \tag{48}$$

Using the arguments given in Ref. 34, one can show that the term $\sim [D_j^{\lambda i}(J\nu)]^2$ causes the function $d^2(\eta)$ to acquire additional poles, and this makes it difficult to use the strength-function method. But the product $D_j^{\lambda i}(J\nu) \times D_{j'}^{\lambda' i'}(J\nu)$, where $\lambda ij \neq \lambda' i' j'$ does not contain additional poles. Therefore, we transform (48) as follows. We express $D_j^{\lambda i}(J\nu)$ in terms of other $D_{j'}^{\lambda' i'}(J\nu)$, using (41):

$$\left.\begin{aligned} D_j^{\lambda i}(J\nu) &= \Big\{\varepsilon_J - \eta_{J\nu} - \frac{1}{\sqrt{2}} \sum_{\lambda' i' j' \neq \lambda ij} \Gamma(Jj'\lambda' i') \\ &\times [1+\mathscr{L}(Jj'\lambda' i')] D_{j'}^{\lambda' i'}(J\nu)\Big\} \{\Gamma(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]\}^{-1}; \\ [D_j^{\lambda j}(J\nu)]^2 &= \frac{D_j^{\lambda i}(J\nu)}{\Gamma(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]} \\ &\times \Big\{\varepsilon_J - \eta_{J\nu} - \frac{1}{\sqrt{2}} \sum_{\lambda' i' j' \neq \lambda ij} \Gamma(Jj'\lambda' i')[1+\mathscr{L}(Jj'\lambda' i')] D_{j'}^{\lambda' i'}(J\nu)\Big\}. \end{aligned}\right\} \tag{49}$$

Substituting (49) in (48) and applying the procedure of Refs. 32 and 33, we obtain

$$d^2(\eta) = \frac{1}{\pi}\,\mathrm{Im}$$

$$\times \frac{D_j^{\lambda i}(\bar\eta)\Big\{\varepsilon_J - \eta_{J\nu} - \frac{1}{\sqrt{2}} \sum{}' \Gamma(Jj'\lambda' i')[1+\mathscr{L}(Jj'\lambda' i')] D_{j'}^{\lambda' i'}(\bar\eta)\Big\}}{\Gamma(Jj\lambda i)[1+\mathscr{L}(Jj\lambda i)]\mathscr{F}(\bar\eta)}. \tag{50}$$

However, to calculate $C_J^2(\eta)$ or $d^2(\eta)$, it is necessary to know how to calculate the coefficients $D_j^{\lambda i}(\bar\eta)$ for different values of the complex variable $\bar\eta$. A method of solving the system for complex $\bar\eta$ was proposed and tested in Ref. 37 for the example of a special case of the system (33), namely, the system (36). However, it can also be applied to the general system (33).

The iterative method proposed in Ref. 37 starts from the empirical fact that the diagonal elements of the coefficient matrix of the unknowns $D_j^{\lambda i}(J\nu)$ (we denote it by $\mathfrak{J}$, and its matrix elements by g_{ik}) are, as a rule, greater than the nondiagonal elements. This is due, in particular, to the fact that the nondiagonal g_{ik} are formed by incoherent sums, and the diagonal g_{ii} by coherent sums. The nondiagonal elements become large only in the cases when the energy η is near a pole. The part they play consists mainly of eliminating the redundant poles of the system, and the roots $\eta_{J\nu}$ and the fragmentation are basically determined by the diagonal elements g_{ii}.

This property of the system (36) was noted long ago.[38] In Refs. 7 and 38, an attempt was made to solve the system (36) in the coherent approximation, i.e., by retaining only diagonal terms. Unfortunately, spurious, unphysical solutions appeared. To eliminate them, an attempt was made to take into account the nondiagonal terms nearest the root being sought.[39,40] However, because of the fluctuations of $\Gamma(Jj\lambda i)$ it is difficult to formulate a general criterion for choosing such nondiagonal terms. The requirement that the poles of these nondiagonal terms be near the root being sought was found to be insufficient. Allowance for one or two nondiagonal poles is sufficient for calculating the wave functions of only those solutions that have a simple structure. But when many components made comparable contributions to the wave function of the solution, the artificial limitation in the number of nondiagonal poles taken into account gave rise to serious distortions in the structure of the solutions. Increasing the number of nondiagonal poles taken into account leads to an excessively rapid complication of the expressions, rendering them of little use in practice.

We write the system (36) in the matrix form

$$\mathfrak{J}\mathbf{D} = \mathbf{\Gamma}. \tag{51}$$

We have already defined the matrix $\mathfrak{J}$; $\mathbf{D}$ is the vector of the unknown coefficients $D_j^{\lambda i}(J\nu)$, and $\mathbf{\Gamma}$ is the vector formed by the right-hand sides of (36). The fact that in the matrix $\mathfrak{J}$ the elements $|g_{ii}|$ are, as a rule, greater than $|g_{ik}|$ $(i \neq k)$ makes it possible to use Jacobi's iterative method to solve Eq. (51). Then the nth approximate solution of the system is found by means of the relation

$$\mathbf{D}^{(n)} = \mathbf{D}^{(n-1)} + \mathscr{H}(\mathbf{\Gamma} - \mathfrak{J}\mathbf{D}^{(n-1)}).$$

The matrix $\mathscr{H}$ is diagonal and has the elements

$$h_{ii} = g_{ii}^{-1}.$$

The initial approximation $\mathbf{D}^{(0)}$ is taken to be the vector $\mathbf{\Gamma}$. The iterative process converges if for all rows of the matrix $\mathfrak{J}$

$$k_i = |g_{ii}|^{-1} \sum_{i \neq i'} |g_{i'i}| < 1. \tag{52}$$

If the condition (52) is not satisfied, the matrix $\mathscr{H}$ must be taken in a different form. Suppose

$$k_i > 1, \quad i = 1, 2, \ldots, m;$$
$$k_i < 1, \quad i = m+1, m+2, \ldots, n$$

(n is the rank of the matrix $\mathfrak{J}$). Then we define $\mathscr{H}$ as follows:

$$\mathscr{H} = \begin{cases} \mathfrak{J}'^{-1} & i = 1, 2, \ldots, m; \\ g_{ii}^{-1} & i = m+1, m+2, \ldots, n. \end{cases} \tag{53}$$

The matrix $\mathfrak{J}'$ is formed by the first m rows and columns of $\mathfrak{J}$. To each value of $\bar\eta$ there will correspond a matrix $\mathfrak{J}'$, and its dimension will vary from point to point.

It is here helpful to make an analogy with the method of the many-pole approximation described above. The choice of the matrix $\mathscr{H}$ in the form (53) means that in the first approximation we separate from the complete matrix $\mathfrak{F}$ a submatrix corresponding to the poles most strongly coupled to the state whose fragmentation we calculate. From the remaining elements g_{ik} we take into account only the diagonal ones. This in fact corresponds to the many-pole approximation.[39,40] But if the condition (52) is satisfied for all rows of the matrix $\mathscr{H}$, then the first approximation to the exact solution will be the result obtained by keeping only the coherent terms.[7,38] However, if we use the iterative method, we have a numerical criterion for choosing the nondiagonal elements whose contribution is most important at the given point, and the number of elements taken into account can change from point to point. In addition, the following iterations also take into account the contribution of the nondiagonal elements which introduce small corrections and eliminate the redundant solutions at a large distance.

Compared with direct inversion of the matrix $\mathfrak{F}$, the method works less effectively the lower the dimension of the matrix $\mathfrak{F}'$, i.e., the closer to unity is $(n-m)/n$. A certain part in raising the efficiency of the iterative method is played by the calculation of, not the exact values of $C^2_{J\nu}$ or $[D^{\lambda i}_j(J\nu)]^2$, but their values averaged by means of a weight function, since an increase in the parameter Δ decreases the value of the nondiagonal matrix elements and increases the rate of convergence of the iterative process.

The fragmentation of the hole neutron state $1g_{9/2}$ of the nucleus ^{119}Sn was considered in Ref. 37 as an example. All the quasiparticle-phonon and quasiparticle–two-phonon states in the interval $0 \leqslant E_x \leqslant 10$ MeV were taken into account. The rank of the matrix $\mathfrak{F}$ was $n = 276$. At all points $\bar{\eta}$, the rank of the matrix $\mathfrak{F}'$ did not exceed 20 ($m \leqslant 20$), i.e., it was in order of magnitude less than n. The influence of the value of Δ on the rate of convergence of the iterations can be seen from the following example. The degree of proximity of the kth approximation $\mathbf{D}^{(k)}$ to the exact solution is characterized by the absolute magnitude of the so-called discrepancy vector $\mathbf{r}^{(k)}$. A value $\mathbf{r}^{(k)} \leqslant 10^{-4}$ was obtained after seven or eight iterations for $\Delta = 0.1$ MeV but after two or three iterations for $\Delta = 0.5$ MeV.

4. INFLUENCE OF THE PAULI PRINCIPLE ON THE QUASIPARTICLE-PHONON INTERACTION

In expressing the wave function of an odd nucleus in the language of quasiparticle and phonon operators and forgetting the fermion structure of the phonons, we violate the Pauli principle. The consequences of this are twofold. First, we obtain a large number of spurious states that would not be there if we made a correct antisymmetrization of the many-quasiparticle components having the same complexity as the corresponding quasiparticle–n-phonon components. Second, depending on the structure of the phonons and the quantum numbers of the quasiparticles, the interaction between them will be renormalized.

In the first studies[30,41] in which the quasiparticle-phonon model was used to investigate the fragmentation of single-quasiparticle states of spherical nuclei at intermediate and high excitation energies, an empirical procedure for selecting correctly antisymmetrized components of the wave functions (37) was used in order to take into account at least crudely the Pauli principle. A criterion (to some degree arbitrary) of phonon "collectivity" was introduced; in accordance with this, all phonons were divided into two classes—"collective" and "noncollective." Phonons for which the contribution of the maximal two-quasiparticle component to the normalization did not exceed some given upper limit (in practice 40–50%) were assumed to be collective. All the remaining phonons were assumed to be noncollective. The components of the wave functions (14) or (37) which contained only collective phonons were retained in the wave function without any further analysis. The components with noncollective phonons were analyzed from the point of view of their structure, and some of them, those not satisfying trivial symmetry requirements, were rejected. For example, this was done for the states in which the odd quasiparticle and the quasiparticles from the principal component of the phonon formed the combination

$$\alpha^+_{j_1}\psi^{\lambda i}_{j_1 j_2}[\alpha^+_{j_1}\alpha^+_{j_2}]_\lambda,$$

whereas combinations of the type $\alpha^+_{j_2}\psi^{\lambda' i}_{j_1 j_2}[\alpha^+_{j_1}\alpha^+_{j_1}]_{\lambda'}$ were retained. All components with noncollective phonons with three or more quasiparticles in one level, and so forth, were rejected "in any case."

This empirical procedure made it possible to eliminate the majority of the spurious states, i.e., to take into account in some manner the "kinematic" effects, if one may put it that way. However, the procedure made no allowance at all for the "dynamical" effects, i.e., the renormalization of the quasiparticle-phonon interaction.

Then, in Ref. 24, a systematic procedure was proposed for taking into account the Pauli principle. It was investigated in most detail for the wave function (37), which contains components not more complicated than those of the quasiparticle–one-phonon type.[2)] The corresponding equations are given in Sec. 2, and we here consider only some numerical examples.

The exact commutation relations between the operators α^+_{jm} and $Q_{\lambda\mu i}$ lead to the appearance in Eqs. (38) and (39) of two new factors $\mathscr{L}(Jj\lambda i)$ and $R(Jj\lambda i)$, which are absent in the simpler and long known equations of the superfluid nuclear model.[3] The factor $\mathscr{L}(Jj\lambda i)$ regulates the contribution of the given quasiparticle-phonon component to the wave-function normalization, this depending on the "degree of violation" of the Pauli principle for this component. The same factor determines the renormalization of the interaction of the quasiparticle α^+_{JM} with the component $[\alpha^+_{jm}Q^+_{\lambda\mu i}]_{JM}$, this depending on the internal structure of the phonon $Q^+_{\lambda\mu i}$ and on the quantum numbers of the quasiparticle α^+_{jm}. In the case of "maximal" violation of the Pauli principle $\mathscr{L}(Jj\lambda i) = -1$

[2)] For the numerical solution of Eqs. (38) and (39) and the calculation on their basis of the structure of the excited states of odd spherical nuclei and some of their electromagnetic characteristics, the program PHOQUS was created.[42]

TABLE II. Values of the factor $\mathscr{L}(1/2, j\lambda i)$ for $1/2^-$ states of the nucleus ^{59}Ni in the case of strong violation of the Pauli principle. The structure of the single-phonon wave functions is also given.

$nlj \otimes \lambda_i^\pi$	Structure of phonon λ_i^π		$\mathscr{L}$ (1/2, $j\lambda i$)
	Two-quasiparticle components $[n_1l_1j_1, n_2l_2j_2]$	Contribution to the normalization, %	
$1f_{5/2} \otimes 2_5^+$	$[1f_{5/2}, 1f_{5/2}]_{2^+}$	94,9	−0,95
$2p_{3/2} \otimes 2_1^+$	$[2p_{3/2}, 2p_{3/2}]_{2^+}$	65,4	−0,67
	$[2p_{3/2}, 2p_{1/2}]_{2^+}$	4,3	—
	$[1f_{7/2}, 2p_{3/2}]_{2^+}$	2,8	—
	$[2p_{3/2}, 1f_{5/2}]_{2^+}$	1,4	—
$1g_{9/2} \otimes 5_1^-$	$[1f_{7/2}, 1g_{9/2}]_{5^-}$	41,2	−0,16

in the component $[\alpha_{jm}^+ Q_{\lambda\mu i}^+]_{JM}$, and this component is removed from the wave function (37) automatically, so to speak. It is readily seen that the empirical procedure described above carried out this function of the factor $\mathscr{L}(Jj\lambda i)$. Besides the "renormalization" of the interaction, there is also a shift of the pole $\varepsilon_j + \omega_{\lambda i}$, the magnitude of which is determined by the factor $R\,(Jj\lambda i)$.

Tables II and III give the values of $\mathscr{L}(Jj\lambda i)$ and $R\,(Jj\lambda i)$ for some states of the nucleus ^{59}Ni. States for which the corrections for the Pauli principle are large were chosen. Let us consider, for example, the state whose structure is given in the first row of Table II. For it, the factor $\mathscr{L}(1/2^-, 5/2^-, 2_5^+)$ is near -1 because three quasiparticles in the shell $1f_{5/2}$ cannot have total angular momentum 1/2. The deviation, albeit weak, of the factor $\mathscr{L}$ from -1 is explained by the fact that the contribution of the configuration $[1f_{5/2}, 1f_{5/2}]_{2^+}$ to the structure of the 2_5^+ phonon is less than 100%. It can be seen from Table II that there is a considerable weakening of the interaction of the quasiparticle in the level $2p_{3/2}$ with the lowest quadrupole phonon, since a large contribution to its structure is made by the configuration $[2p_{3/2}, 2p_{3/2}]_{2^+}$, and for three quasiparticles in the level $2p_{3/2}$ the total angular momentum 1/2 is also forbidden. We note that both these states would be rejected if the wave function were analyzed by means of the empirical procedure. When the Pauli principle is taken into account consistently, the second state will remain.

The shift of the pole is greatest $[R\,(1/2, 3/2, 2_1^+) \sim 0.8$ MeV] for the component $[2p_{3/2} \otimes 2_1^+]_{1/2}$. Its example demonstrates a general feature—strong shifts of the poles occur, as a rule, in the components containing collective phonons. From Table III it can be seen that the factor $R\,(Jj\lambda i)$ can take both positive and negative values, i.e., the poles may be shifted either upward or downward relative to their unperturbed position. At the same time, when the same procedure for taking into account the Pauli principle is used in odd deformed nuclei, the poles are always shifted upward in the energy scale.[43]

We now consider how the structure of the most excited states is changed by the exact allowance for the Pauli principle. As an example, we have chosen the states $1/2^-$ of the nucleus ^{59}Ni and $5/2^+$ of ^{119}Sn. Table IV compares the results of calculations of the structure of these states with consistent allowance for the Pauli principle and using the empirical procedure for selecting the correctly symmetrized quasiparticle-phonon components. The individual components of the wave functions are changed appreciably, although abrupt changes in the structure of the states do not occur. An example of a "dramatic" rearrangement of the structure of a state due to the effect of the Pauli principle is given in Ref. 44. It was found that allowance for the Pauli principle leads to the appearance in ^{131}Ba of a low-lying state with $J^\pi = 9/2^-$, $E_x \simeq 150$ keV and structure $[1h_{11/2} \otimes 2_1^+]_{9/2^-}$, in complete agreement with the experimental data. Without allowance for the Pauli principle, the appearance of this state cannot be explained. It is interesting that in the neighboring isotope ^{133}Ba such a state does not occur (again in agreement with the experimental data).

There is no doubt that the Pauli principle must be taken into account in the analysis of the low-lying part of the spec-

TABLE III. Values of the factors $R\,(Jj\lambda i)$ of some poles $j \otimes \lambda_i^\pi$ of the $J^\pi = 1/2^-$, $3/2^-$ states of the nucleus ^{59}Ni.

J^π	Structure of the pole $j \otimes \lambda_i^\pi$	$\varepsilon_j + \omega$, MeV	$\varepsilon_j + \omega - R\,(Jj\lambda i)$, MeV	$-R\,(Jj\lambda i)$, MeV	J^π	Structure of the pole $j \otimes \lambda_i^\pi$	$\varepsilon_j + \omega$, MeV	$\varepsilon_j + \omega - R\,(Jj\lambda i)$, MeV	$-R\,(Jj\lambda i)$, MeV
$1/2^-$	$2p_{3/2} \otimes 2_1^+$	4,076	4,857	0,781	$3/2^-$	$2p_{3/2} \otimes 2_1^+$	4,076	3,311	−0,765
	$2p_{3/2} \otimes 2_2^+$	5,116	4,829	−0,287		$2p_{3/2} \otimes 2_2^+$	5,116	5,926	0,810
	$1f_{5/2} \otimes 2_1^+$	5,120	5,131	0,011		$1f_{5/2} \otimes 2_1^+$	5,120	5,096	−0,024
	$1f_{5/2} \otimes 2_2^+$	6,160	6,163	0,003		$1p_{1/2} \otimes 2_1^+$	5,456	5,446	−0,010
						$1f_{5/2} \otimes 2_2^+$	6,160	6,154	−0,006

TABLE IV. Energies and structures of the states with $J^\pi = 1/2^-$ of the nucleus ^{59}Ni and $J^\pi = 5/2^+$ of the nucleus ^{119}Sn.

Nucleus	Empirical allowance for the Pauli principle				Consistent allowance for the Pauli principle				
	$\eta_{J\nu}$, MeV	$C^2_{J\nu}$, %	$nlj\otimes\lambda^\pi_i$	$(D^{\lambda i}_j)^2$, %	$\eta_{J\nu}$, MeV	$C^2_{J\nu}$, %	$nlj\otimes\lambda^\pi_i$	$\mathscr{L}(Jj\lambda i)$	$(1+\mathscr{L})[D^{\lambda i}_j]^2$, %
^{59}Ni $1/2^-$	1,810	85,16	$1f_{5/2}\otimes 2^+_1$	3,50	1,868	86,91	$1f_{5/2}\otimes 2^+_1$	–0,001	3,67
			$2p_{3/2}\otimes 2^+_1$	2,54			$2p_{3/2}\otimes 1^+_3$	–0,08	2,24
			$2p_{3/2}\otimes 1^+_1$	2,17			$1f_{5/2}\otimes 2^+_2$	0,014	1,66
			$1f_{5/2}\otimes 2^+_2$	1,57			$1g_{9/2}\otimes 4^-_6$	0,054	1,23
			$1g_{9/2}\otimes 4^-_6$	1,44					
	5,117	0,01	$2p_{3/2}\otimes 2^+_2$	80,02	4,840	0,11	$2p_{3/2}\otimes 2^+_2$	–0,291	80,11
			$1f_{5/2}\otimes 2^+_1$	19,98			$2p_{3/2}\otimes 2^+_1$	–0,668	19,16
	4,128	1,69	$2p_{3/2}\otimes 2^+_{11}$	96,98	4,873	0,40	$2p_{3/2}\otimes 2^+_1$	–0,668	79,25
							$2p_{3/2}\otimes 2^+_2$	–0,291	17,47
							$1f_{5/2}\otimes 2^+_1$	–0,001	2,73
^{119}Sn $5/2^+$	2,130	87,09	$3s_{1/2}\otimes 2^+_1$	5,96	2,139	90,91	$2d_{5/2}\otimes 2^+_1$	–0,022	2,70
			$2d_{5/2}\otimes 2^+_1$	2,79			$3s_{1/2}\otimes 2^+_1$	–0,287	1,70
	3,120	2,41	$2d_{3/2}\otimes 2^+_1$	81,86	3,016	0,66	$2d_{3/2}\otimes 2^+_1$	0,087	99,17
			$3s_{1/2}\otimes 2^+_1$	15,29					
	3,073	3,22	$3s_{1/2}\otimes 2^+_1$	78,63	3,650	1,32	$3s_{1/2}\otimes 2^+_1$	–0,287	97,51
			$2d_{3/2}\otimes 2^+_1$	17,60					

trum of odd nuclei. In a number of cases, the corrections obtained will be small, but it is not always possible to predict this *a priori*. It is different when the averaged characteristics of the nuclear spectra are described, especially at intermediate and high excitation energies. In these cases, it appears to us that the empirical procedure for taking into account the Pauli principle works entirely satisfactorily. Such a conclusion is supported by the following facts. It can be seen from Table IV that the *total values of the principal components* of the wave functions of the excited states hardly change over the interval 1 MeV after exact allowance has been made for the Pauli principle. Further, for the states described by the wave functions (14) or (37) the importance of the Pauli principle decreases with increasing excitation energy, since *the same* number of quasiparticles will be distributed over an *ever increasing* number of single-particle states. These considerations are confirmed by the calculations made in Refs. 45 and 46. As an example, we show in Fig. 1 the strength function of the $2p_{1/2}$ states of the nucleus ^{59}Ni calculated with the consistent allowance for the Pauli principle and using the empirical procedure.[3)]

The fact that the empirical procedure takes into account the Pauli principle quite well at intermediate and high excitation energies indicates that the occurrence of spurious states due to its violation is relatively more important than the renormalization of the quasiparticle-phonon interaction and the shifts of the poles. This is well demonstrated by the calculation of the strength function of the $1h_{11/2}$ state in the nucleus ^{205}Pb made in Ref. 46 (Fig. 2). In the variant in which the Pauli principle is ignored altogether the strongest fragmentation of the $1h_{11/2}$ subshell is obtained, since the appearance of a large number of spurious states effectively enhances the interaction with the complicated configurations. But the strength functions calculated using the exact and the empirical procedures for taking into account the Pauli principle are very similar.

[3)] It is here worth mentioning that calculations with consistent allowance for the Pauli principle require an order of magnitude more computing time.

5. INFLUENCE OF THE QUASIPARTICLE–TWO-PHONON COMPONENTS AND THE DIMENSION OF THE PHONON SPACE ON THE DISTRIBUTION OF THE STRENGTH OF THE SINGLE-QUASIPARTICLE STATES AT INTERMEDIATE AND HIGH EXCITATION ENERGIES

The dimension of the system of equations (36) is determined by the dimensions of the single-particle basis and the

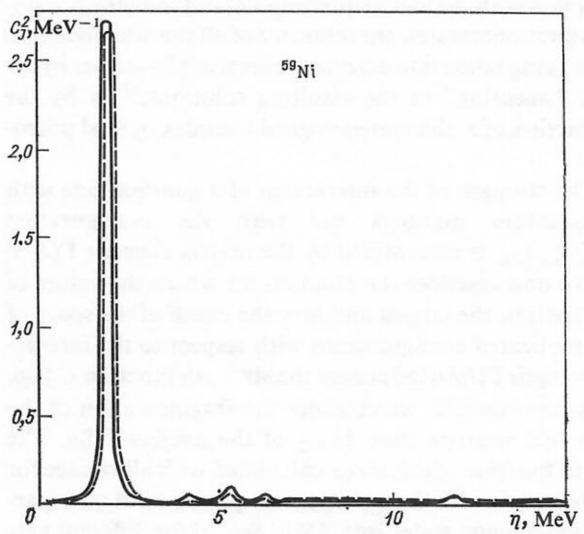

FIG. 1. Strength function $C^2_j(\eta)$ of the $2p_{1/2}$ state of the nucleus ^{59}Ni. The continuous curve is the calculation with consistent allowance for the Pauli principle; the broken curve is the calculation using the empirical procedure.

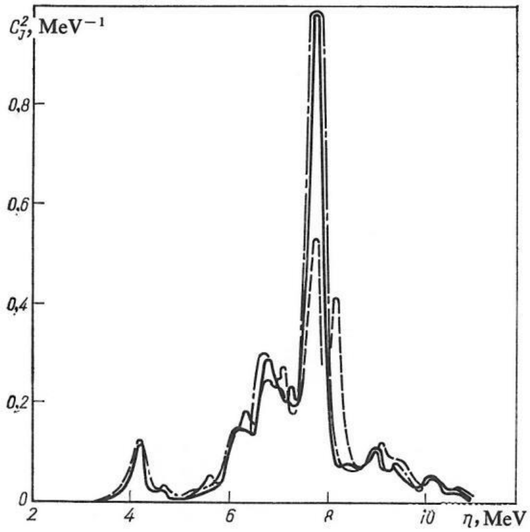

FIG. 2. Strength function of the state $1h_{11/2}$ of the nucleus ^{205}Pb. The continuous curve represents the calculation with the empirical procedure for taking into account the Pauli principle; the chain curve, the calculation with consistent allowance for the Pauli principle; and the broken curve, the calculation without allowance for the Pauli principle.

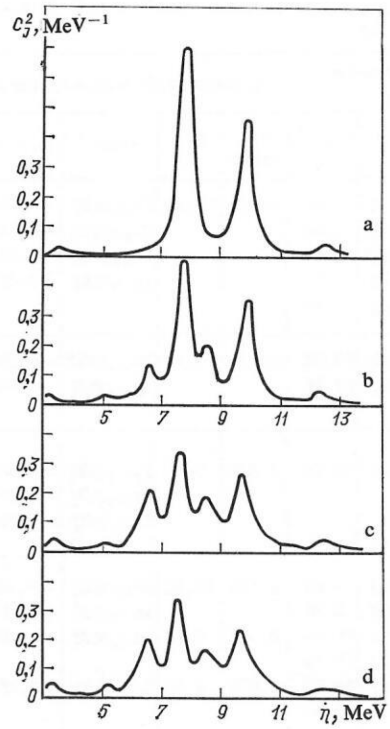

FIG. 3. Strength function of the neutron quasibound state $1h_{9/2}$ of the nucleus ^{117}Sn for different truncations of the space of quasiparticle-phonon and quasiparticle–two-phonon states: a) cutoff at the level $\Gamma_{min} \geqslant 0.5\Gamma_{max}$; b) cutoff at the level $\Gamma_{min} \geqslant 0.25\Gamma_{max}$; c) cutoff at the level $\Gamma_{min} \geqslant 0.1\Gamma_{max}$; d) cutoff at the level $\Gamma_{min} \geqslant 0.05\Gamma_{max}$.

phonon space used in the calculations. For a state with excitation energy $E_x \sim 10$–15 MeV it becomes catastrophically large, and, even using the strength-function method and effective numerical methods of calculation, the losses of computer time are very appreciable. Therefore, in calculations of the characteristics of nuclear spectra the problem naturally arises of selecting the most important components of the quasiparticle-phonon basis.

We here investigate this question for the problem of calculating the strength functions of single-quasiparticle states with excitation energies 5–10 MeV. The additional interest in this problem is due to the fact that attempts can be found in the literature to describe the distribution of the strength of highly excited single-particle modes in the spectra of odd nuclei by taking into account "exactly" only the interaction with the lowest quadrupole (and sometimes octupole) vibrational states, the influence of all the other configurations being taken into account "effectively"—either by artificial "smearing" of the resulting solutions,[47] or by the introduction of a phenomenological complex optical potential.[48]

The strength of the interaction of a quasiparticle with the quantum numbers nlJ with the configuration $[\alpha_{jm}^+ Q_{\lambda\mu i}^+]_{JM}$ is determined by the matrix element $\Gamma(Jj\lambda i)$ (22). We now consider the phonons for which the values of $\Gamma(Jj\lambda i)$ will be the largest and how the cutoff of the space of the complicated configurations with respect to the interaction strength $\Gamma(Jj\lambda i)$ influences the strength function $C_J^2(\eta)$.

As an example, we consider the fragmentation of the quasibound neutron state $1h_{9/2}$ of the nucleus ^{117}Sn. The strength function of this state, calculated with allowance for the interaction with the quasiparticle-phonon and quasiparticle–two-phonon states [see (45) in Sec. 3] for different variants of the cutoff with respect to the matrix element of the interaction $\Gamma(Jj\lambda i)$, is shown in Fig. 3. The maximal matrix element Γ_{max} is the matrix element $\Gamma(1h_{9/2}, 1h_{9/2}2_1^+)$. We note that it is precisely such matrix elements [i.e., $\Gamma(J, J2_1^+)$] that are maximal for the majority of states and nuclei, though there are exceptions. For example, in the nuclei neighboring on ^{208}Pb the largest matrix elements are associated with a 3_1^- phonon. In Table V we have given the number of quasiparticle-phonon and quasiparticle–two-phonon components that occur in the wave function (14) for the given cutoff with respect to $\Gamma(Jj\lambda i)$ and also some characteristics of the distributions of the single-particle strength that are obtained.

It can be seen from Table V that only six $\Gamma(1h_{9/2}, j\lambda i)$ matrix elements are less than Γ_{max} by not more than a factor 2. Of them, two are associated with a 2_1^+ phonon, two more with a 3_1^- phonon, and one each with 3_{33}^+ ($\omega = 17.2$ MeV) and 2_{25}^- ($\omega = 11.4$ MeV) phonons. Thus, although the main role in the formation of the gross structure of the strength function is played by the lowest quadrupole and the lowest octupole phonon, the influence of the collective states of intermediate energies also cannot be ignored. Moreover, in estimating their possible importance one cannot be guided by characteristics such as the probability of excitation from the ground state. For example, the 2_{25}^- phonon mentioned above belongs to the $M2$ resonance,[49] but does not have the maximal value of $B(M2, 0_{g.s}^+ \to 2^-)$.

It was shown in Ref. 26 that another intermediate-energy collective state—a low-energy octupole resonance[50]—influences the shape of the strength function of the neutron hole state $1g_{9/2}$ of the ^{119}Sn nucleus. Figure 4 shows the strength function $C_{1g_{9/2}}^2(\eta)$ of this state calculated with

TABLE V. Energy centroid $\overline{E}_x$, second moment σ, and exhaustion of the total strength S of the state over an interval 3–14 MeV, and also the number of different components in the wave function (14) for the state $1h_{9/2}$ of the nucleus ^{117}Sn for different truncations of the space of complicated configurations

Cutoff with respect to $\Gamma(Jj\lambda i)$ of Γ_{max}, %	Number of quasi-particle-photon configurations	Number of quasi-particle–two-photon configurations	$\overline{E}_x$, MeV	σ, MeV	S, %
50	7	34	8,69	1,66	95
25	25	289	8,55	1,82	93
10	108	4552	8,37	1,98	91
5	238	18 858	8,31	2,06	90

allowance for the quasiparticle-phonon components and with different phonon bases. In the simplest case, when the interaction with only the 2_1^+ and 3_1^- phonons is taken into account, the overwhelming part of the $1g_{9/2}$ strength is divided between three states (in Fig. 4a, these correspond to the three peaks of the strength function). After the addition of the interaction with the 2^+ and 3^- phonons of higher energies (Fig. 4b), the principal peak of the strength function is split. The immediate cause of this is the low-energy octupole resonance, which is coupled to the $1g_{9/2}$ state by a fairly large matrix element $\Gamma(1g_{9/2},\ 1h_{11/2}2_3^{3-})$. In these calculations, the octupole resonance was itself obtained as a single-phonon 3^- state with number $i=3$ (energy $\omega\simeq 5$ MeV) and probability of excitation from the ground state $B(E3)\simeq 0.25B(E3, 0_{g.s}^+\to 3_1^-)$. The splitting of the principal peak produced by the interaction with the resonance increases with further extension of the phonon basis (Fig. 4c), and the principal peak itself becomes broader.

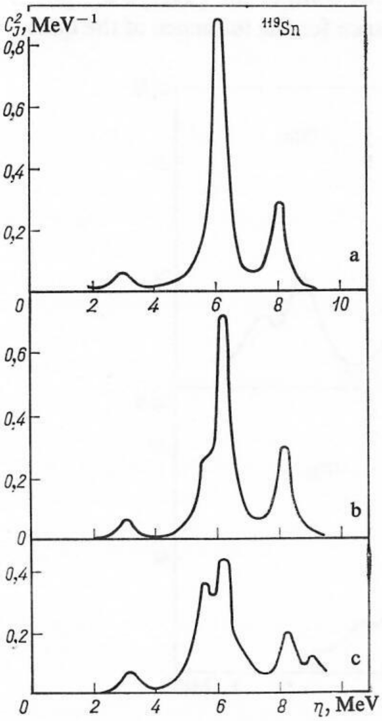

FIG. 4. Strength function of the neutron hole state $1g_{9/2}$ of the nucleus ^{119}Sn: a) calculation with only the lowest 2^+ and 3^- phonons; b) calculation with the 2^+ and 3^- phonons having energy $\omega\leqslant 11$ MeV; c) calculation with a large phonon basis.

We now return to Fig. 3. With a decreasing lower bound for $\Gamma(Jj\lambda i)$, the complexity of the variational wave function (14) increases catastrophically. The cutoff with respect to Γ at the level $5\times10^{-4}\Gamma_{max}$ corresponds to a wave function having 919 quasiparticle-phonon components and 38 218 quasiparticle–two-phonon components. Initially, the shape of the strength function changes perceptibly, but already on the transition from a cutoff at the level $0.1\Gamma_{max}$ to a cutoff at the level $0.05\Gamma_{max}$ the changes in the strength function become slight, although the number of components taken into account increases very strongly. The gross structure of the strength function is formed when allowance is made for all the matrix elements at the level $0.25\Gamma_{max}$. For this cutoff, the main contribution to the structure of the $1h_{9/2}$ state is still made by the quadrupole and octupole phonons. Twelve of the 25 quasiparticle–phonon components are formed with their participation. In the total wave function (14), their relative contribution is less. Here, the number of components with the participation of $3^-, 4^+, 5^-, 6^+$ phonons is greatest. For states with other angular momenta J the multipolarity of the phonons most frequently encountered in the wave function (14) changes. For example for the state $13/2^+$ we have the phonons with $\lambda^\pi=4^+, 6^+$; for the state $3/2^-$, the phonons with $\lambda^\pi=3^-, 5^-$. However, in all cases the contribution of the spin-multipole phonons is appreciably less than that of the multipole phonons.

We now consider how the quasiparticle–two-phonon components influence the fragmentation of the single-quasiparticle states. Although Eqs. (32) and (33) indicate that the matrix element of the interaction operator $H_{Mvq}^{ph}+H_{SMvq}^{ph}$ between the single-quasiparticle component and the quasiparticle–two-phonon components is zero, the high density of the quasiparticle-two-phonon states at energies $E_x>10$ MeV may indirectly lead to appreciable changes in the strength function. Histograms of the number of quasiparticle-phonon and quasiparticle–two-phonon states with $J^\pi=9/2^+$ over an interval $\Delta E_x=1$ MeV in ^{123}Te are given in Fig. 5. At excitation energies $E_X\simeq 6$–8 MeV, the density of these states with two phonons exceeds the density of those with one phonon by 3–5 times, and with increasing E_x this predominance is enhanced. Accordingly, at energies $E_x>7$ MeV there begin to be appreciable changes in the strength function of the neutron hole state $1g_{9/2}$ of the nucleus ^{123}Te due to the inclusion in the wave function of quasiparticle–two-phonon components (Fig. 6). The $1g_{9/2}$ strength is shifted to higher excitation energies, as a result of which the second peak of the strength function $C_{1g_{9/2}}^2(\eta)$ increases. The

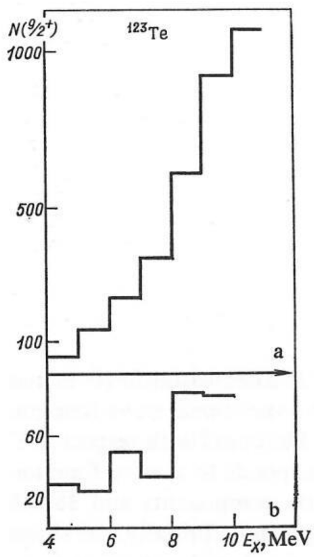

FIG. 5. Histogram of the number of states of different structure with $J^\pi = 9/2^-$ on an interval $\Delta E_x = 1$ MeV as a function of the excitation energy E_x in ^{123}Te: a) quasiparticle–two-phonon states; b) quasiparticle-phonon states.

majority of the deep hole and highly excited particle shells hitherto studied have excitation energy $E_x \leqslant 10$ MeV. Allowance for the quasiparticle–two-phonon components will have a strong effect on the results of calculations of the high-energy part of their strength functions.

As numerous calculations in the framework of the quasiparticle–phonon model show, and as can be seen from the results given in the present section, the strength functions of the single-quasiparticle states have a fairly complicated shape. A significant part is played in the formation of this "fine" structure of the strength functions by the collective phonons of intermediate energies and the quasiparticle–two-phonon components. When the contribution of the large number of weakly interacting configurations is taken into account effectively in the spirit of Refs. 47 and 48, one can hardly hope for a description of the fine structure. But the characteristic scale of the inhomogeneities in the strength functions of the single-quasiparticle states is ~ 1 MeV, and they can be discovered experimentally very readily. In a number of cases, such structure has already been observed[51] and reproduced quite well in the framework of the quasiparticle-phonon model.[36,52]

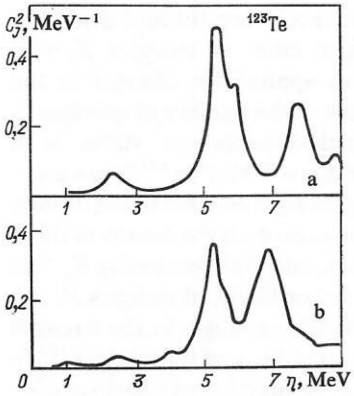

FIG. 6. Strength function of the neutron hole state $1g_{9/2}$ of the nucleus ^{123}Te: a) calculation with the wave function (37); b) calculation with the wave function (14).

6. FRAGMENTATION OF STATES WITH A QUASIPARTICLE AND A PHONON

Hitherto, the fragmentation of states with a quasiparticle and a phonon has been investigated in much less detail than the fragmentation of single-quasiparticle states. However, the importance of studying this problem is in no doubt. If one does not know how to calculate correctly these components of the wave functions of the excited states, one cannot hope for an adequate description of the probabilities of γ transitions from highly excited states to low-lying states or their electric and magnetic moments. The key to the description of the radiative strength functions in odd nuclei lies in the correct calculation of these components.[25,53]

Correct calculation of the quasiparticle–one-phonon components is possible only with the variational wave function (14), since they are coupled by the matrix elements of the interaction operator $H^{\text{ph}}_{Mvq} + H^{\text{ph}}_{SMvq}$ to both the single-particle and the quasiparticle–two-phonon components, the matrix elements for the two cases being of the same order of magnitude. This last circumstance can be most clearly seen from (36), where we have ignored the corrections due to the Pauli principle and the anharmonicity of the phonons.

We show in Fig. 7 the strength function $d^2(\eta)$ of the state $[1g_{9/2} \otimes 2_1^+]_{9/2+}$ of the nucleus ^{119}Sn.[15,36] The coefficients $D_j^{\lambda i}(\eta)$ were found from (36). In the same figure, we show the distribution of the component $[1g_{9/2} \otimes 2_1^+]_{9/2+}$, calculated without allowance for the influence of the quasi-

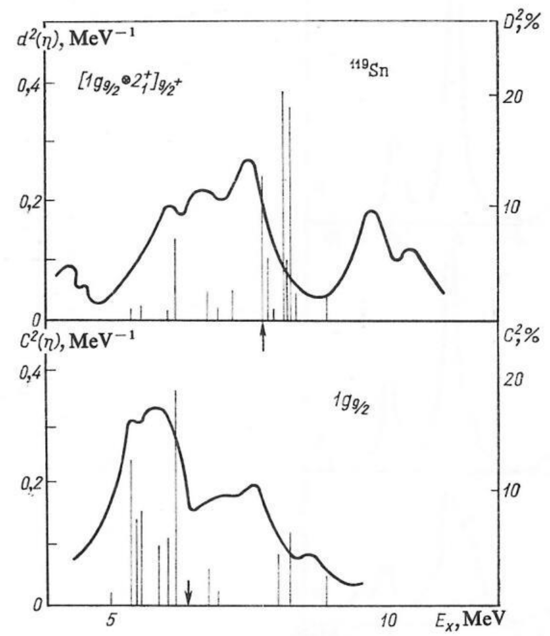

FIG. 7. Strength functions $d^2(\eta)$ and $C_j^2(\eta)$ of the states $[1g_{9/2} \otimes 2_1^+]_{9/2+}$ and $1g_{9/2}$ of the nucleus ^{119}Sn (left-hand scale). The vertical lines show the distribution of the strength of the components $[1g_{9/2} \otimes 2_1^+]_{9/2+}$ and $1g_{9/2}$ in the spectrum of ^{119}Sn when the interaction with the quasiparticle–two-phonon states is not taken into account (right-hand scale).

particle–two-phonon components. The two distributions differ radically. For comparison, we show in the lower part of the figure the strength function calculated in the same approximations for the hole state $1g_{9/2}$ of the nucleus ^{119}Sn and the distribution of $C^2_{1g_{9/2},\nu}$ over the states of the nucleus without allowance for the influence of the quasiparticle–two-phonon components. These components do not change the qualitative picture of the distribution of the $1g_{9/2}$ strength.

In a certain sense, the quasiparticle–two-phonon components have an even greater influence on the fragmentation of the quasiparticle–one-phonon components in which the phonon is noncollective. Because the phonon is noncollective, this component is coupled to the single-quasiparticle component by a small interaction matrix element and in the absence of the quasiparticle–two-phonon components is hardly fragmented. Figure 8, in which we show the strength function $d^2(\eta)$ of the state $[2d_{5/2}\otimes 2_7^+]_{9/2+}$ of the nucleus ^{119}Sn, demontrates well the changes that occur in such a component on the addition of an interaction with the quasiparticle–two-phonon components.

The corrections for the Pauli principle and the anharmonicity of the phonon excitations can have a strong effect on the fragmentation of the states with a quasiparticle and a phonon. The part played by the first of these effects is demonstrated to a certain degree by the calculations discussed in Sec. 4 (see, for example, Table IV). We now turn to a discussion of the influence of the anharmonic effects.

Their importance was revealed in calculations of the radiative strength functions in the odd isotopes of Fe and Ni[25]. We shall not give here the expressions for the strength functions $b(E1,\eta)$ and $b(M1,\eta)$, since they are cumbersome, and all that is important here is the qualitative result, and comparison with experiment does not form part of our task. Suffice it to say that $E1$ and $M1$ transitions from states in the neighborhood of the neutron binding energy B_n to low-lying single-quasiparticle states were calculated. The calculation was made with the wave function (14), and allowance was made for the contribution to the transition from the single-quasiparticle and quasiparticle-phonon components.

Calculation of the strength function $b(E1,\eta)$ of the tran-

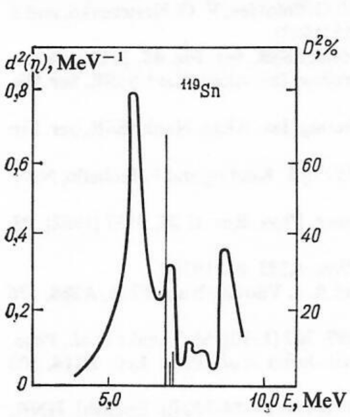

FIG. 8. Strength function $d^2(\eta)$ of the state $[2d_{5/2}\otimes 2^+]_{9/2+}$ of the nucleus ^{119}Sn (left-hand scale). The vertical lines show the distribution of the strength of this state in the spectrum of ^{119}Sn when the interaction with the quasiparticle-phonon states is not taken into account (right-hand scale).

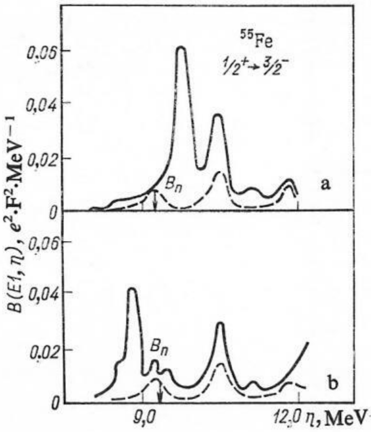

FIG. 9. Radiative strength function $b(E1,\eta)$ of the transitions $1/2^+\to 3/2^-_{g.s}$ in the nucleus ^{55}Fe: a) calculation without allowance for anharmonic corrections $[U^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)=0]$; b) calculation with anharmonic corrections. The broken curves show the single-quasiparticle component of the strength function. The arrows indicate the position of the neutron binding energy B_n.

sitions in ^{55}Fe from highly excited $1/2^+$ states to the $3/2^-$ ground state showed that $b(E1,\eta)$ near $B_n=9.29$ MeV strongly influences the fragmentation of the component $[2p_{3/2}\otimes 1_5^-]_{1/2+}$, which has energy $\varepsilon(2p_{3/2})+\omega(1_5^-)-\eta_{g.s}=11.0$ MeV. The distinguished role of this component is due to the relatively large value of $B(E1, 0^+_{g.s}\to 1_5^-)$, the excitation probabilities of the neighboring single-phonon 1^- states being much less. If the fragmentation of the component $D^{1_5^-}_{2p_{3/2}}(1/2^+)$ is calculated by means of (35) without allowance for the terms $\sim\Gamma U$ and UU, then the value of $b(E1,\eta)$ at $\eta\simeq B_n$ is small (Fig. 9a). Inclusion in the calculation of the terms $\sim UU$,[4)] which are responsible for fragmentation of the 1_5^- phonon, strongly changes $b(E1,\eta)$, increasing its value at $\eta\simeq B_n$. The inclusion of anharmonic corrections hardly affected the behavior of the single-quasiparticle component of the strength function $b(E1,\eta)$ (shown in Fig. 9 by the broken curve).

Similar results were obtained for the strength function of the $M1$ transitions $1/2^-\to 3/2^-_{g.s}$ in ^{61}Ni (Fig. 10). In this nucleus, the behavior of $b(M1,\eta)$ near B_n is significantly influenced by allowance for the anharmonic effects in the distribution of the strength of the state $[2p_{3/2}\otimes 1_3^+]_{1/2-}$, which is about 3 MeV above B_n. As for $E1$ transitions in ^{55}Fe, the distinguished part played by the component $[2p_{3/2}\otimes 1_3^+]_{1/2-}$ in the description of the $M1$ transitions in ^{61}Ni is explained by the high probability of the $M1$ transition $0^+_{g.s}\to 1_3^+$ in ^{60}Ni.

As can be seen from the results of the present section, for the correct description of the fragmentation of the quasiparticle-phonon states it is necessary to take into account effects that are not important in calculations of the strength functions of the single-quasiparticle states. This greatly complicates the problem. But its solution opens up great possibilities for systematic microscopic analysis of a large body of experimental data on the properties of nuclei at intermediate and high excitation energies. Soon, the data on the radia-

4) Terms of order ΓU were not taken into account in the calculation.[25]

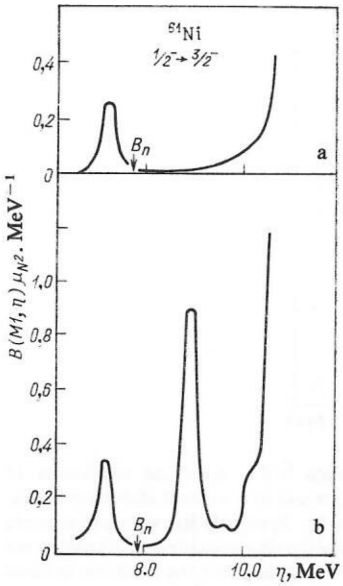

FIG. 10. Radiative strength function $b(M1, \eta)$ of the transitions $1/2^- \to 3/2^-_{g.s}$ in the nucleus ^{61}Ni: a) calculation without allowance for anharmonic corrections $[U^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0]$; b) calculation with anharmonic corrections. The arrows indicate the position of the neutron binding energy B_n.

tive strength functions of nuclei at excitation energies $\sim B_n$ will be augmented by the results of experiments being prepared in a number of laboratories in the world on the γ decay of deep hole states. These facts make the study of the fragmentation of the quasiparticle-phonon states highly topical.

We should like to thank Dao Tien Khoa for collaboration and assistance.

[1] V. G. Solov'ev, Vliyanie parnykh korrelyatsiĭ sverkhprovodyashchego tipa na svoĭstva atomnykh yader (Influence of Pairing Correlations of Superconducting Type on the Properties of Nuclei), Gosatomizdat, Moscow (1963); V. G. Soloviev, Selected Topics in Nuclear Theory, IAEA, Vienna (1963), p. 233.

[2] V. G. Soloviev, At. Energy Rev. **3**, 117 (1965); V. G. Soloviev, in: Nuclear Structure, Dubna Symposium 1968, IAEA, Vienna (1968), p. 101.

[3] V. G. Solov'ev, Teoriya slozhnykh yader, Nauka, Moscow (1971); English translation: Theory of Complex Nuclei, Pergamon Press, Oxford (1976).

[4] V. G. Solov'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. **35**, 666 (1971); **38**, 1580 (1974).

[5] V. G. Solov'ev, Teor. Mat. Fiz. **17**, 90 (1973).

[6] A. I. Vdovin, G. Kyrchev, and Ch. Stoyanov, Teor. Mat. Fiz. **21**, 137 (1974); G. Kyrchev and V. G. Solov'ev, Teor. Mat. Fiz. **22**, 244 (1975).

[7] A. I. Vdovin and V. G. Solov'ev, Teor. Mat. Fiz. **19**, 275 (1974).

[8] V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **9**, 860 (1978) [Sov. J. Part. Nucl. **9**, 343 (1978)].

[9] L. A. Malov and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **11**, 301 (1980) [Sov. J. Part. Nucl. **11**, 111 (1980)].

[10] A. I. Vdovin and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **14**, 237 (1983) [Sov. J. Part. Nucl. **14**, 99 (1983)].

[11] V. V. Voronov and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **14**, 1380 (1983) [Sov. J. Part. Nucl. **14**, 583 (1983)].

[12] V. G. Soloviev, Nukleonika **23**, 1149 (1978).

[13] V. G. Solov'ev, in: Izbrannye voprosy struktury yadra (Selected Problems of Nuclear Structure), Vol. 2, D-9920, JINR, Dubna (1976), p. 146; Struktura yadra (Nuclear Structure), D4-80-385, JINR, Dubna (1980), p. 57; Mezhdunarodnaya shkola-seminar po fizike tyazhelykh ionov (International Seminar School on Heavy-Ion Physics), D7-83-644, JINR, Dubna (1983), p. 182.

[14] V. V. Voronov, in: Neutron Induced Reactions (eds. I. Ribansky. and E. Betak), Bratislava (1980), p. 291; A. I. Vdovin, Izv. Akad. Nauk SSSR, Ser. Fiz. **43**, 2018 (1979).

[15] Ch. Stoyanov, Izv. Akad. Nauk SSSR, Ser. Fiz. **45**, 1820 (1981).

[16] Ch. Stoyanov, in: IV Shkola po neĭtronnoĭ fizike (Fourth School on Neutron Physics), D3,4-82-704, JINR, Dubna (1982), p. 87; V. V. Voronov, *ibid.*, p. 105.

[17] S. T. Belyaev, Yad. Fiz. **4**, 936 (1966) [Sov. J. Nucl. Phys. **4**, 671 (1967)].

[18] V. G. Solov'ev, Teoriya atomnogo yadra. Yadernye modeli (Theory of the Atomic Nucleus. Nuclear Models), Énergoizdat, Moscow (1981).

[19] A. I. Vdovin and V. Yu. Ponomarev, Preprint R4-80-392 [in Russian], JINR, Dubna (1980); Nguen Dinh Dang and V. Yu. Ponomarev, Izv. Akad. Nauk SSSR, Ser. Fiz. **47**, 2147 (1983); V. Ju. Ponomarev, Ch. Stoyanov, and A. I. Vdovin, Z. Phys. A **308**, 157 (1982); A. I. Vdovin *et al.*, Preprint R4-84-32 [in Russian], JINR, Dubna (1984).

[20] V. G. Solov'ev, Teor. Mat. Fiz. **57**, 438 (1983); V. A. Kuzmin and V. G. Soloviev, Preprint E4-83-786 [in English], JINR, Dubna (1983).

[21] Dao Tien Khoa *et al.*, Preprint E4-84-501 [in English], JINR, Dubna (1984).

[22] V. V. Voronov and V. G. Solov'ev, Teor. Mat. Fiz. **57**, 75 (1983).

[23] V. G. Solov'ev, Ch. Stoyanov, and R. Nikolaeva, Izv. Akad. Nauk SSSR, Ser. Fiz. **47**, 2082 (1983).

[24] Chan Zuy Khuong, V. G. Soloviev, and V. V. Voronov, J. Phys. G **7**, 151 (1981).

[25] V. G. Soloviev and Ch. Stoyanov, Nucl. Phys. **A382**, 206 (1982).

[26] V. G. Soloviev, Ch. Stoyanov, and A. I. Vdovin, Nucl. Phys. **A342**, 261 (1980).

[27] V. G. Soloviev, Ch. Stoyanov, and V. V. Voronov, Nucl. Phys. **A399**, 141 (1983).

[28] Ch. Stoyanov and A. I. Vdovin, Phys. Lett. **B130**, 134 (1983); Nguyen Dinh Thao *et al.*, Yad. Fiz. **37**, 43 (1983) [Sov. J. Nucl. Phys. **37**, 23 (1983)]; Nguyen Dinh Thao *et al.*, J. Phys. G **10**, 517 (1984).

[29] V. V. Voronov and Chan Zuy Khuong, Preprint E-4-13005 [in English], JINR, Dubna (1980).

[30] D. Dambasuren *et al.*, J. Phys. G **2**, 25 (1976).

[31] G. Alaga and G. Ialongo, Nucl. Phys. **A97**, 600 (1967); K. Heyde and P. J. Brussard, Nucl. Phys. **A104**, 81 (1967); Z. Phys. **259**, 15 (1973).

[32] L. A. Malov and V. G. Soloviev, Nucl. Phys. **A270**, 87 (1976); L. A. Malov and V. G. Solov'ev, Yad. Fiz. **23**, 53 (1976); **26**, 729 (1977) [Sov. J. Nucl. Phys. **23**, 27 (1976); **26**, 384 (1977)].

[33] G. Ochirbat, Preprint R4-8774 [in Russian], JINR, Dubna (1975).

[34] L. A. Malov, Preprint R4-81-228 [in Russian], JINR, Dubna (1981).

[35] A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. 1, Benjamin, New York (1969) [Russian translation published by Mir, Moscow (1971)].

[36] Ch. Stoyanov, Avtor. dis. na soisk. uchen. stepeni d-ra fiz.-mat. nauk (Author's Abstract of Doctoral Dissertation), 4-82-885, JINR, Dubna (1982).

[37] Ch. Stoyanov, Teor. Mat. Fiz. **40**, 422 (1979).

[38] V. G. Soloviev and L. A. Malov, Nucl. Phys. **A196**, 433 (1972).

[39] L. A. Malov and V. G. Solov'ev, Yad. Fiz. **21**, 502 (1975) [Sov. J. Nucl. Phys. **21**, 263 (1975)]; Teor. Mat. Fiz. **25**, 265 (1975).

[40] L. A. Malov and G. Ochirbat, Preprints R4-8447, R4-8492 [in Russian], JINR, Dubna (1974).

[41] A. I. Vdovin, Ch. Stoyanov, and Chan Zuy Khuong, Izv. Akad. Nauk SSSR, Ser. Fiz. **43**, 999 (1979).

[42] Ch. Stoyanov and Chan Zuy Khuong, Preprint R4-81-234 [in Russian], JINR, Dubna (1981).

[43] S. I. Bastrukov, V. O. Nesterenko, and V. G. Solov'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. **46**, 2144 (1982); V. G. Soloviev, V. O. Nesterenko, and S. I. Bastrukov, Z. Phys. A **309**, 353 (1983).

[44] B. A. Alikov *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **45**, 2111 (1981).

[45] Chan Zuy Khuong and V. V. Voronov, Izv. Akad. Nauk SSSR, Ser. Fiz. **45**, 837 (1981).

[46] V. V. Voronov and Chan Zuy Khuong, Izv. Akad. Nauk SSSR, Ser. Fiz. **45**, 1910 (1981).

[47] P. Doll, Nucl. Phys. **A292**, 165 (1977); T. Koeling and F. Iachello, Nucl. Phys. **A295**, 45 (1978).

[48] S. P. Klevansky and R. H. Lemmer, Phys. Rev. C **25**, 3137 (1982); **28**, 1763 (1983).

[49] V. Ju. Ponomarev *et al.*, Nucl. Phys. **A323**, 61 (1979).

[50] V. G. Soloviev, Ch. Stoyanov, and A. I. Vdovin, Nucl. Phys. **A288**, 376 (1977).

[51] A. Stuirbrink *et al.*, Z. Phys. A **297**, 307 (1980); M. Tanaka *et al.*, Phys. Lett. **B78**, 221 (1978); H. Langevin-Joliot *et al.*, Phys. Lett. **B114**, 103 (1982).

[52] Ch. Stoyanov and A. I. Vdovin, Preprint E4-84-330 [in English], JINR, Dubna (1984).

[53] V. V. Voronov and V. G. Solov'ev, Yad. Fiz. **23**, 942 (1976) [Sov. J. Nucl. Phys. **23**, 496 (1976)].

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