Asymptotic multiplicities

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The phenomena most characteristic of processes in which an asymptotically large number n of particles are produced (when the hadronic matter is strongly excited and the collision of elementary particles can be expected to be central and nonperipheral) are discussed. The expansion with respect to correlation functions made for this purpose is convenient for formulating the phenomenology of the production of a very large number of particles. The expansion is used to separate the contributions from "hard" and "soft" processes, this being necessary for the description of hadron production cross sections in QCD. It is shown that if the contributions from the soft processes to the topological cross sections σ_n^{ab} decrease sufficiently rapidly, $\sigma_n^{ab} < O(e^{-n})$, then at high energies hard processes will be dominant in the asymptotic behavior with respect to n, irrespective of the species of the colliding particles a and b. This makes it possible to obtain experimentally verifiable QCD predictions using perturbation theory.

INTRODUCTION

In this paper, we consider the description of multiparticle productions of hadrons when their number n is very large:

$$n \gg \bar{n}(s). \qquad (1)$$

Here, $\bar{n}(s)$ is the mean multiplicity, a natural scale of the n values. It may depend on the species of the colliding particles, the type of process, etc. Below, we shall say more precisely how the condition (1) arises; for the moment, it is to be understood as the definition of the region of asymptotic values of n in which we are interested. Such processes have been studied neither theoretically nor experimentally, and therefore, in this review paper, in which we shall not go into too many details, we shall discuss as fully as possible only the most characteristic features of these processes.

Interest in processes with $n \gg \bar{n}(s)$ is stimulated in the first place by the fact that at a very high multiplicity many degrees of freedom of the colliding particles must be excited. If this is the case, one must in the first place consider which excitations are to be considered at given multiplicity n and given energy $\sqrt{s}$. We wish to show why central (nonperipheral) interactions begin to be predominant with increasing n. In the final section, we give predictions that can be tested experimentally.

When $n \gg \bar{n}$, the most characteristic features are the following: a) failure of KNO scaling (because the nature of the interaction changes); b) a growth in the mean transverse momentum of the produced particles must be observed (since central interactions are dominant).

From the beginning, it must be clear that in selecting in experiments processes with high multiplicity of the produced baryons we do not necessarily investigate many-parton excitations of the colliding particles—since the ground state in Yang–Mills field theory is degenerate,[1] the hadron wave function cannot, strictly speaking, be expanded solely with respect to parton [quark (q) and gluon (g)] states. In other words, particle collisions may result in the excitation of degrees of freedom which cannot be classified in terms of particles (partons) but which influence the dynamics of the produced hadrons.

The extent to which such contributions, which cannot be taken into account in canonical perturbation theory, are important can, for example, be estimated by noting that the mean number of produced hadrons, $\bar{n}(s) = O(\ln s)$ (Fig. 1), is appreciably less than the largest possible number of hadrons (actually pions),

$$\bar{n}(s) \ll n_{\max} \equiv \sqrt{s}/m_\pi, \qquad (2)$$

that could be produced at high energies $\sqrt{s} \to \infty$. At the same time, it should be noted that the mean multiplicity $\bar{n}_j$ of the partons (q and g) calculated in the framework of QCD perturbation theory is fairly large[6]:

$$\bar{n}_j(s) = O(e^{\sqrt{\ln s}}) \gg \bar{n}(s) \qquad (3)$$

(Fig. 2).

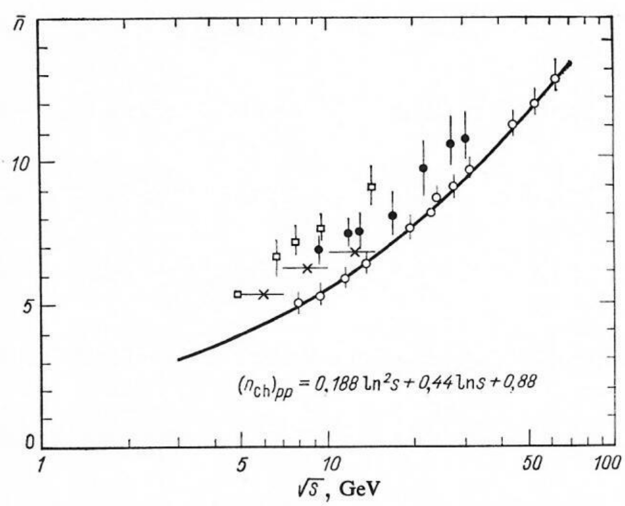

FIG. 1. Dependence of the mean multiplicity of charged particles on the energy $\sqrt{s}$ in various processes. The black circles are for e^+e^- annihilation (PLUTO), with $K_s^0 \to \pi^+\pi^-$ eliminated,[2] the open circles are for the pp interaction,[3] the open squares are for $p\bar{p}$ annihilation,[4] and the crosses are for the $p\nu$ interaction.[5]

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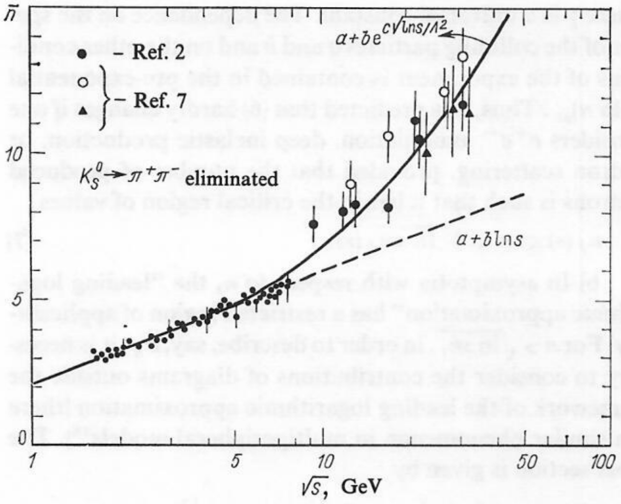

FIG. 2. Mean multiplicity of charged particles in e^+e^- annihilation[2]: $a = 2.38 \pm 0.09$, $b = 0.04 \pm 0.01$, $c = 1.92 \pm 0.07$, $\Lambda = 0.5$ GeV.

It can be seen from this that the canonical form of perturbation theory, which is designed precisely to describe fluctuations that do not conserve the particle number, is not valid in the case of field theories of Yang–Mills type.[8,9] It is the allowance for these important—as follows from Ref. 3—nonperturbative effects that is the main difficulty of the modern theory.

Behind this somewhat formal-appearing complication there is a huge number of studies, attempts to calculate in one way or another observable quantities on the basis of the canonically poorly defined[8,9] Yang–Mills field theory. In the light of what has been said above, our aim in the section that follows is to show how, without going into the details of the hadron production dynamics, it is possible to describe the cross sections of the processes [both the topological, $\sigma_n(s)$, and the differential, $d\sigma_n/dp$...].

In Sec. 2 we show how to separate the phenomena associated with the hadron production process from the background of the preceding parton production process. We assume that these two processes can be separated, since they correspond to different spatial scales.

At least in the early stage of investigations in the framework of QCD, there is a natural desire to avoid formal difficulties, i.e., to consider processes in which the details of the dynamics of the production of the hadrons (or, rather, their formation) can be ignored. One usually regards as of this kind processes with the participation of hard photons or processes involving the production of hadrons with large transverse momenta, i.e., processes in which the interaction at short distances is guaranteed by the actual condition of the experiment (in this connection, see the reviews of Refs. 10 and 11). This makes it possible to use the asymptotic freedom in QCD and thereby simplify the calculations.

To the "hard" processes listed above, one could add processes involving the production of a very large number of particles. Indeed, at the first glance the dominance of hard processes in the asymptotic behavior with respect to n is guaranteed by the inequality (3). In other words, if we assume that the number of partons increases in proportion to the number of hadrons, then at high multiplicities processes involving the decay of partons with large virtuality must be predominant,[12] and then the particle production process must of necessity be hard.

However, it should be noted that the number n of produced hadrons does not correspond, by virtue of the nonperturbative effects, to the number ν of produced hadrons; it is obvious, at least in the framework of the "leading logarithmic approximation," that $n \gtrsim \nu$ even if a process is initially hard (an example of how hadrons can be produced "nonperturbatively" is given in Ref. 13). Thus, choosing $n \geq \bar{n}_j(s) \gg \bar{n}(s)$, we do not yet guarantee that ν as well is asymptotically large. The aim of Sec. 2 is to discuss theoretically the conditions under which a process in asymptotia with respect to n can be regarded as hard.

The result is more readily formulated if one introduces the mean number $\bar{\nu}(n,s)$ of partons produced for given n and s. Then it can be asserted[12] that if the cross section for hadron production in the soft process, $(\sigma_n)^{\text{soft}}$, satisfies the condition

$$(\sigma_n)^{\text{soft}} < O\,(e^{-n}) \tag{4}$$

then irrespective of the experimental conditions and the species of the colliding particles,

$$\frac{\bar{\nu}\,(n,\,s)}{n} \xrightarrow[n\to\infty]{} O\,(1). \tag{5}$$

Here, $n \to \infty$ is to be understood literally (see the following section).

To prove (5), we use the decomposition into "far" and "near" correlations in the momentum space explained in Sec. 1, and this makes it possible to formulate quantitatively the problem of calculating cross sections with the participation of hadrons. At the same time, however, the possibility of separating the soft processes is included in the condition of the assertion formulated above and is introduced into the scheme from without.

The model considerations advanced in Sec. 3 show that the condition (4) is apparently satisfied.

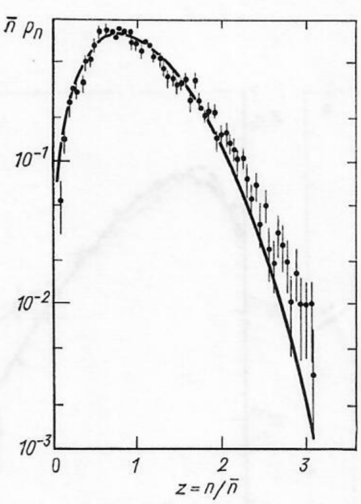

FIG. 3. Multiplicity distributions $\bar{n}P_n = \sigma_n/\sigma_{\text{tot}}$.[14]

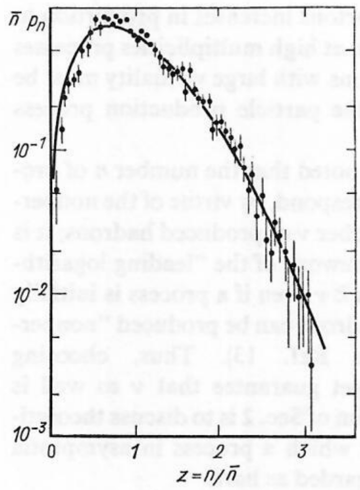

FIG. 4. The same as in Fig. 3. Data taken from Ref. 15.

Essentially, we consider models based on the following assumptions: a) the produced particles have, on the average, bounded transverse momenta; b) in a first approximation, the correlations between the produced particles can be ignored.

Thus, the region of asymptotic values of the multiplicity can apparently be described in the framework of the parton concepts of QCD. However, it must be emphasized that the model considerations do not make it possible to predict the region of n values in which the nature of the interactions changes (the process becomes hard). Moreover, the experimental data in the region $n \gg \bar{n}$ are so sparse that they do not permit determination of the transitional region of values of n (see Figs. 3–5, in which different models are fitted).

Having foundations no more serious than the model considerations, only the qualitative conclusions of the calculations can be stressed (see Sec. 4). Here we list the main ones.

a) At high energies, there is a bounded region of "critical" values of n in which the cross sections are universal:

$$\ln \frac{\sigma_n^{ab}(s)}{\sigma_{\text{tot}}^{ab}(s)} = -\gamma \frac{n}{\bar{n}_j(s)} + O(\ln n)_{ab}, \tag{6}$$

where γ is a universal constant. The dependence on the species of the colliding particles a and b and on the other conditions of the experiment is contained in the pre-exponential $O(\ln n)_{ab}$. Thus, it is predicted that (6) hardly changes if one considers e^+e^- annihilation, deep inelastic production, or hadron scattering, provided that the number of produced hadrons is such that it lies in the critical region of values

$$\bar{n}_j(s) \leqslant n \leqslant \sqrt{\ln s}\, \bar{n}_j(s). \tag{7}$$

b) In asymptotia with respect to n, the "leading logarithmic approximation" has a restricted region of applicability: For $n > \sqrt{\ln s}\bar{n}_j$, in order to describe, say, σ_n, it is necessary to consider the contributions of diagrams outside the framework of the leading logarithmic approximation (there is a similar phenomenon in multiperipheral models[18]). The cross section is given by

$$\sigma_n^{ab}(s) = O\left\{\exp\left[-\frac{n}{\bar{n}_j(s)} C_{ab}(n, s)\right]\right\}, \tag{8}$$

where $C_{ab}(n,s)$ is an increasing function of n. This indicates a unique possibility of investigating the contributions of diagrams more complicated than those considered in the leading logarithmic approximation without going beyond the logic of this approximation.[19] In this we see a significant difference between processes involving the production of a very large number of particles and other hard processes.

In Sec. 5 we make predictions that can be verified experimentally for both the integrated and the differential cross sections.

The ideas used in the paper are based on the proposition that $\sigma_n/\sigma_{\text{tot}}$ is the partition function of the gas of (produced) particles. However, if the multiplicity n is very large, it may be more convenient to use the different picture of the particle production process as a stationary nonequilibrium process (when the interference between the different contributions to $\sigma_n/\sigma_{\text{tot}}$ is negligible[20]).[21]

1. EXPANSION WITH RESPECT TO CORRELATION FUNCTIONS

The quantitative side of the problem of describing hadron production is as follows. We represent the interactions of elementary particles in terms of color fields $Q_\alpha(x)$, i.e.,

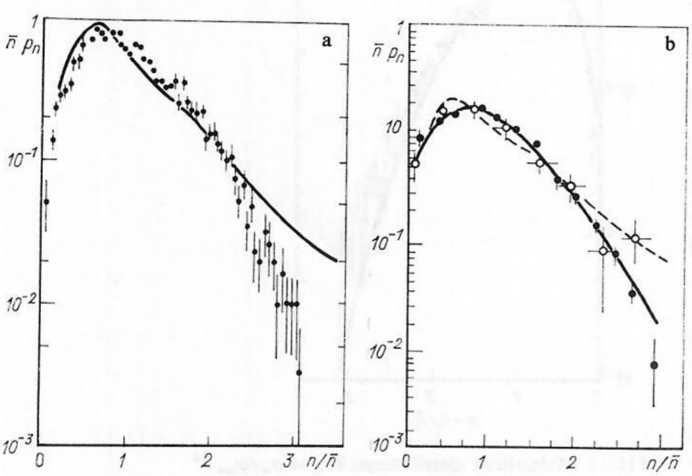

FIG. 5. The same as in Fig. 3. Data taken from Ref. 16 (a) and Ref. 17 (b).

vector fields $B^a_\mu(x)$ and quark fields $q_i(x)$. Their coupling to the asymptotic states (say, pions) can be determined as follows (by analogy with Ref. 22). From $Q_\alpha(x)$, we form a colorless gauge functional $O(x|Q)$. Then the expectation value with respect to the fields $Q_\alpha(x)$ has the form

$$\langle O(x|Q)\,O(y|Q)^*\rangle_Q = \int d^4p\, e^{ip(x-y)}\left(\frac{C_0^\pi(p)^2}{p^2-m_\pi^2}+\dots\right)+\dots, \tag{9}$$

where the pion production "vertex" $C_0^\pi(p)$ depends functionally on the choice of $O(x|Q)$ and determines the given "content" of the fields Q_α in the pion. In (9), we have not written out the contributions of the remaining (including many-particle) hadron states.

In (9), we must, first of all, define the operation of averaging $\langle\rangle_Q$ over all configurations of the fields $Q_\alpha(x) = (B^a_\mu(x), q_i(x))$. A problem here is that, because of the conformal invariance of non-Abelian gauge theories, two points of space separated by a spacelike interval are mutually dependent.[23,24] Allowance for these constraints in the canonical formalism presents very great difficulty,[25] which we attempt to avoid in the first stage in the description of the hadron production processes. (A further discussion of the field theory of multiple production is given in Sec. 2.)

To avoid these difficulties of the microscopic theory, we express the required exclusive cross sections in terms of the correlation functions of the produced particles, i.e., we wish to use the connection between the inclusive and exclusive quantities and thus extend the theory in such a way as to make maximal use of the empirically accumulated material. A very simple form of such a construction of the theory was given in Ref. 26. At the same time, as we shall see below, this approach also has purely theoretical advantages, making it possible to discuss collective phenomena in the gas of the produced particles, something which is important in the case of production of a very large number of hadrons.

We choose the following method for constructing the phenomenology of multiparticle production. We introduce a "trial" assumption (which is justified in what follows) that the topological cross sections of hadron production satisfy

$$\sigma_n < O(e^{-n}) \tag{10}$$

as $n \to \infty$. We are mainly interested in the production of pions, and, to eliminate the restrictions associated with the finiteness of the phase space, we consider the theoretical limit $m_\pi = 0$ [then in (2) $n_{\max} = \infty$]. All the expressions that follow can be written down with allowance for the conservation laws, but, having taken $m_\pi = 0$, upper bounds on the cross sections in asymptotia with respect to n can have only a dynamical origin. At the same time, it should be recalled that the σ_n which we calculate satisfy $\sigma_n \leq (\sigma_n)^{\exp}$, where $(\sigma_n)^{\exp}$ is the experimental value of the cross section.

In what follows, a somewhat different expression of (10) will be more convenient. We consider the generating functions

$$T_{ab}(z, s) = \sum_{n=0}^{\infty} z^n \sigma_n^{ab}(s), \tag{11}$$

where z is an arbitrary number. We shall be interested in only real positive z. (If T_{ab} is regarded as the grand partition function, then the physical meaning of the activity z is discussed in detail in Ref. 27.) From the positivity of σ_n we can deduce the inequalities

$$T_{ab}(z, s) < \sigma_{\text{tot}}^{ab}(s) \quad \text{for} \quad z < 1; \tag{12a}$$

$$\frac{\partial^k}{\partial z^k} T_{ab}(z, s) > 0, \quad k = 0, 1, 2, \dots \tag{12b}$$

These inequalities mean that in (11) the series must converge for all $z \leq 1$. However, augmenting (12) by the inequality (10), we find that the series converges for $|z| < \infty$.

For $m_\pi \neq 0$, the sum in (11) terminates at $n = n_{\max} < \infty$ and, thus, the $m_\pi = 0$ limit in a certain sense corresponds to transition to the "thermodynamic limit," in which the number of particles and the volume in which the system is placed are unbounded. Then if the number of particles is sufficiently large, as in the considered case, the energy $\sqrt{s}$ determines the temperature of the gas of the produced particles. For this, it is necessary to introduce an equivalent of the density matrix $\rho_{ab}(\beta, z)$ (see, for example, Ref. 28) such that

$$T_{ab}(z, s) = \frac{1}{2\pi i\sqrt{s}} \int \frac{d\beta}{2\pi} \beta^2 I_1(\beta\sqrt{s})\, \rho_{ab}(\beta, z). \tag{13}$$

Here, β is inversely proportional to the temperature T of the gas of the produced particles, and the contour of integration is taken parallel to the imaginary axis (I_1 is a Bessel function of an imaginary argument). Finally, using (11), we obtain

$$\sigma_n(s) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} T(z, s). \tag{14}$$

The representation (13) is not entirely convenient, being Lorentz noncovariant. It can be extended if $\rho(\beta, z)$ is replaced by $\rho(\alpha, z)$, where α is a 4-vector. Then instead of (13) we have

$$T(z, s) = \int \frac{d^4\alpha}{(2\pi)^4} e^{i\alpha P_{ab}} \rho(\alpha, z), \tag{15}$$

where P_{ab} is the total 4-momentum of the colliding particles a and b, and $P_{ab} \equiv p_a + p_b$. The representation (13) can be obtained by choosing a coordinate system in which $\alpha = (\beta, 0)$ (at the same time, however, the total energy $\sqrt{s}$ must be fairly large[29]).

Further, if we wish to calculate the exclusive spectra of the produced particles, z in $\rho(\alpha, z)$ must be assumed to depend on the momentum of the produced particle: $z = z(k)$. It follows from what was said above that

$$\rho(\alpha, z) = \sum_n \int \prod_{j=1}^{n} \left\{ \frac{d^3k_j e^{i\alpha k_j}}{(2\pi)^3\, 2\sqrt{k_j^2+m_\pi^2}} z(k_j) \right\} |A_n|^2, \tag{16}$$

where A_n are the multiparticle production amplitudes. The differential cross sections of the particles are calculated in accordance with the standard expressions.[30]

The density matrix $\rho(\alpha, z)$ was discussed in such detail in order to emphasize its universal significance for describing the experimental data. In terms of statistical physics, $\rho(\rho, z)$ is the grand partition function in this approach. Therefore, singularities of $T(z,s)$ indicate the presence of a phase transition in the system of produced particles.[31] However, as is readily seen [see (13) and (14)], the asymptotic behavior with respect to σ_n is controlled by the positions of the singularities of $T(z,s)$ with respect to z.[32] Thus, to elucidate the asymptotic behavior of σ_n with respect to n it is desirable to use the experience accumulated in the theory of condensed media (see, for example, Ref. 33). The restriction (10) presupposes that $T(z,s)$ is a regular function for all $|z| < \infty$. The absence of a phase transition in the gas of the produced particles appears natural and is confirmed by the models considered in Sec. 3.

The connection between the asymptotic behavior with respect to n and the structure of the singularities with respect to z follows directly from (14). Thus, at large n the integral in (14) can be conveniently calculated by the method of steepest descent. The equation for the saddle-point values of z is

$$z \frac{d}{dz} \ln T(z, s) = n. \tag{17}$$

By virtue of (12), this equation always has a solution $z_0 = z_0(n,s)$ that increases with increasing n. If z_0 is known, one can show that in the limit $n \to \infty$

$$\ln \frac{\sigma_n^{ab}(s)}{\sigma_{\text{tot}}^{ab}(s)} = -n \ln z_0^{ab}(n, s) + O(\ln n)_{ab}. \tag{18}$$

We note that if T_{ab} is singular at $z = z_c \geq 1$, then the solution of (17) has the form

$$z_0^{ab}(n, s) = z_c(s) + O(1/n)_{ab}, \tag{19}$$

where the n-dependent correction is determined by the nature of the singularity. Substituting (19) in (18), we find that

$$\sigma_n \geqslant O(e^{-Cn}), \quad C \equiv \ln z_c \geqslant 0. \tag{20}$$

Otherwise, if the singularity is at $z = \infty$,

$$\sigma_n < O(e^{-n}), \tag{21}$$

which is what we wanted to prove.

We can now discuss in more detail the limit $m_\pi \to 0$. As we pointed out above, the limit $m_\pi = 0$ is chosen in order to eliminate a significant influence of the boundaries of the phase space. Instead of this, we could consider $n \ll n_{\max}$, which is obviously the same thing. However, it may then happen that at a given energy $\sqrt{s}$ the difference $|z_0 - z_c|$ is so large that the singularity of $T(z,s)$ does not affect the asymptotic behavior of σ_n. Therefore, in what follows we shall assume that the energy is sufficiently high and $n \ll n_{\max}$. It is then obvious that we do not introduce a large error by taking $m_\pi = 0$.

We are interested in the limit of very large multiplicities, and the temperature of the pion gas must be low, $\beta \to \infty$, and the density high, $z > 1$. Under these conditions, the calculation of $\rho(\beta, z)$ can be reduced to the taking into account of the correlations between the produced particles, i.e., it is necessary to construct a phenomenology of the collective phenomena in the gas of the produced particles.

If between the produced particles there are correlations, the phase space cannot be uniformly populated. Let the probability for the formation of an inhomogeneity of $n_i \geq 1$ particles with total 4-momentum p_i be $\omega_{ni}(p_i)$. It is clear that $\Sigma n_i = n$ and $\Sigma p_i = P_{ab}$. Introducing the density matrix $\rho(\beta, z)$, one can avoid complications with these conservation laws. Then instead of ω_n it is necessary to consider

$$t(z, p) = \sum_{n=1}^{\infty} z^n \omega_n(p) \tag{22}$$

and with each such group of particles one must associate a Boltzmann factor $\exp(-\beta p_{i0})$, where p_{i0} is the total energy of the group of particles, and $p_i = (p_{i0}, \mathbf{p}_i)$. The next step is to take into account the correlations between the groups of particles distinguished in this manner. For this, we introduce the probability $\widetilde{N}_l(p_1, ..., p_l)$ that l groups of particles are produced with correlation. At the same time, $\Sigma_{i=1}^{l} p_i = P_{ab}$.

As a result, using the standard methods of cluster expansion from statistical mechanics, we obtain

$$\rho(\beta, z) = \rho_1(\beta) \exp\left\{\sum_{l=1}^{\infty} \frac{1}{l!} \int \prod_{i=1}^{l} \left[\frac{d^4 p_i}{(2\pi)^4} e^{-\beta p_{i0}} \theta(p_{i0}) \times (t(p_i, z) - 1)\right] \widetilde{N}_l(p_1, \ldots, p_l)\right\}, \tag{23}$$

where $t(p, z)$ is normalized by

$$t(p, z = 1) = 1 \tag{24}$$

and, accordingly, $\rho_1(\beta) = \rho(\beta, 1)$. In (23), it is more convenient to go over to an integration over the momenta $\mathbf{p}_i$ and the masses $m_i > 0$ of the groups of particles. Since in what follows we shall be interested in only large multiplicities, we restrict ourselves to the approximation in which $|\mathbf{p}_i| \ll m_i$. Then the representation (23) can be rewritten in the form

$$\rho(\beta, z) = \rho_1(\beta) \exp\left\{\sum_{l=1}^{\infty} \frac{1}{l!} \int \prod_{i=1}^{l} \left[\frac{dm_i}{m_i} e^{-\beta m_i} (t(m_i, z) - 1)\right] \times N_l(m_1, \ldots, m_l; \beta)\right\}. \tag{25}$$

The convenience of the representation (23) is to a large degree by how "well" the groups of correlated particles have been separated. For example, groups of particles may arise through the decay of resonances or in hard processes—through the decay of high-virtuality partons. In any case, as follows from (23), $\rho(\beta, z)$ is actually a functional of $t(p, z)$, and in this sense (23) must be regarded as a definition,

$$\widetilde{N}_l(p_1, \ldots, p_l) = e^{\beta (P_{ab})_0} \prod_{i=1}^{l} \frac{\delta}{\delta t(p_i, z)} \ln \rho(\beta, z) \Big|_{z=1}, \tag{26}$$

and then, using the definition (16), we can find $\widetilde{N}_l$.

The representation (23) reflects the assumption that it is possible to distinguish the correlations between the produced particles on the basis of their distances in the phase space. We shall assume nominally that the contributions to $t(p, z)$ and N_l correspond to "near" and "far" correlations, respectively. In other words, we assume the existence of a certain dimensional parameter with respect to which the near and far correlations can be distinguished. There may be several such parameters, and it will then be necessary to discuss the hierarchy of correlation lengths. Here, we consider the simplest case when there is just one such parameter.

It can be concluded from (18) that the asymptotic behavior with respect to n depends to a large degree on $z_0(n,s)$. Moreover, in what follows, we shall be interested in only the z_0 that are nearest to unity, and then in the first approximation $t(m, z)$ can be represented in the form

$$\ln t(mz) = \bar{n}_r(m)(z-1) + O((z-1)^2), \tag{27}$$

where $\bar{n}_r(m)$ is the mean multiplicity in a group of closely correlated particles.

It follows from (27) that $t(m, z)$ is an increasing function of m, at least in the region $z>1$ when the estimate (27) is valid. This means that with increasing n the important region of values in the intervals (25) correspond to $m \simeq \bar{m}(z_0)$ and increase with increasing z, or, which is the same thing, with increasing n [if, of course, $N_l(m_1, ..., m_l; \beta)$ do not decrease so rapidly that near correlations are almost absent and $\bar{m} \simeq m_\pi$]. Note also that by virtue of the conservation laws the values of m must decrease in (25) with increasing l, and it follows from this that for $0 < z_0 - 1 \ll 1$ the higher correlation functions N_l, $l>1$, can be ignored.

When $z_0 - 1 \ll 1$, we retain in (25) only the first term of the expansion with $l=1$. Then from (17) we find that

$$z_0(n, s) = 1 + \frac{1}{\bar{n}_r(s)} O(\ln n) \tag{28}$$

and as a result we obtain from (18)

$$\ln \frac{\sigma_n}{\sigma_{\text{tot}}} = -\frac{n}{\bar{n}_r(s)} O(\ln n). \tag{29}$$

This estimate (which in what follows will be confirmed by the model considerations) makes it possible to determine the region of asymptotic values of n in which we are interested. Since with increasing n it is necessary to take into accoutn the growth of $z_0(n,s)$ and, accordingly, $t(z_0,s)$, it is readily seen from (25) that for $n > \bar{n}_r$ it is also necessary to take into account the correlations between the separated groups of particles, i.e., it follows from this that the parameter which determines the characteristic scale of multiplicity values is the mean multiplicity in a group of closely correlated particles (at the same time, it can be expected that the l th term in the expansion of the argument of the exponential in (25) will be important at $n \sim l\bar{n}_r$; see Ref. 18).

Thus, we have determined the asymptotic behavior with respect to n using the condition, justified from the point of view of the physics of multiparticle production, that in the given region (1) of values of n all possible correlations between the groups of particles must be important [for example, if $\bar{n}_r(m)$ is the mean multiplicity into which a resonance of mass m decays, then for $n \gg \bar{n}_r$ we must take into account correlations between the resonances when calculating the cross sections].

It follows from (18) that σ_n is the larger, the weaker the dependence of $z_0(n,s)$ on n; i.e., σ_n is the larger in asymptotia with respect to n, the more rapid the increase in $T(z,s)$ with increasing z. On the basis of this, we assume that the main contribution to $T(z,s)$ is made by the correlations for which the corresponding correlation functions, say $\widetilde{N}_l(p_1, ..., p_l)$, are positive definite. In other words, this means dominance of the "attraction" between the produced particles in asymptotia with respect to n and explains why we construct the phenomenology by separating first groups of closely correlated particles.

It is intuitively clear that the correlation functions $\widetilde{N}_l$ will be positive definite in the complete phase space, provided interference between the different contributions to the amplitudes A_n of multiparticle production can be ignored. This question is discussed below from the point of view of perturbation theory (see Secs. 3 and 4).

To conclude this section, we discuss the following question: Can the fluctuations corresponding to the near correlations dominate the fluctuations corresponding to far correlations to such an extent that the latter cannot be observed experimentally? This question requires further discussion, since the fluctuations of each type are determined by corresponding mean multiplicities and, as a consequence of this, determine contributions to different regions of n values. However, even if one cannot expect interference between these fluctuations, suppression is still possible.

The suppression is due to the need to take into account the conservation laws, something that is important if the multiplicity of the produced particles is very high. At the same time, the multiplicity distribution to which allowance for only the near correlations leads must be maximally broad. It is only in this case that the measure that determines the probability of occurrence of another type of fluctuation (which, we emphasize, is not dynamically forbidden) will be small. We note that this phenomenon can be detected only in a calculation of the density matrix $\rho(\beta, z)$ and is not noted if the many-particle amplitudes are considered.

Bearing in mind that we assumed regularity of $T(z,s)$ on the circle $|z| = 1$, we shall have a maximally broad distribution only if $t(z,\bar{s})$ is singular at $z = z_c > 1$. For example, suppose $t(z,s) = (z_c(s) - z)^{-\varkappa}$, $\varkappa > 0$. If

$$z_c(s) = 1 + \frac{\text{const}}{\bar{n}(s)} > 1, \tag{30}$$

then from (25), taking into account the conservation laws, i.e., the fact that $\beta \to \infty$ with increasing n, only the first term of the expansion with $l = 1$ will be important, since $z_c(s)$ in (30) is a decreasing function of s. As a result, we find that $T(z,s) \sim t(z,s)$. In Secs. 3 and 4 we shall give examples that illustrate the occurrence of such singularities.

To understand the extent to which this suppression mechanism corresponds to reality, we must make use of the experience in the description of phase transitions. If we give up the assumption that the system of produced particles can be realized from event to event at a given energy in different phase, i.e., if the correlation radius, which is $\sim \langle |\ln p_i^2 / p_j^2| \rangle$, where p_i and p_j are the momenta of the particles, is for given $n \gg \bar{n}(s)$ as well-defined quantity (this presupposes that the fluctuations in the neighborhood of $\langle |\ln p_i^2 / p_j^2| \rangle$ are Gaussian), then we should discuss only a "phase transition of zeroth order." In such a phase transition, the latent energy of the transition is infinite, and therefore there exists only one phase, the same for all n. Then, as can be seen from (20), since the nature of the singularity does not affect the asymptotic behavior of σ_n (the nature of the singularity determines only the pre-exponential factor), it can, for example, be assumed that

$$t(z, s) = \frac{f(z, s)}{(z_c(s) - z)^{\varkappa}}, \quad \varkappa > 0, \tag{31}$$

where $f(z,s)$ is an arbitrary smooth function of z.

In (31), it must be assumed that $z_c > 1$, since otherwise, for $z_c = 1$, $\sigma_{\text{tot}} = T(z,s)|_{z=1}$ is not defined in the limit $m_\pi = 0$, and this contradicts the ideas of current algebra.

In (30) we implicitly use the fact that it is only in theories in which

$$z_c(s) \to 1 \quad \text{as} \quad s \to \infty \tag{32}$$

that the singularity affects the asymptotic behavior with respect to n. Then, if we take (32) in conjuction with (12a) and use the fact that

$$\bar{n}(s) = \frac{d}{dz} \ln T(z, s)|_{z=1}, \tag{33}$$

we find (30) from (31).

This phenomenon is the well-known dominance of "critical fluctuations"[33] in the neighborhood of the phase-transition point $z_c(s)$. We shall call the region of n values in which the singularity with respect to z affects the multiplicity distribution the region of "critical values of n." The assumption (32) means that the critical region of values of n becomes larger with increasing s.

2. ROLE OF HARD PROCESSES IN MULTIPARTICLE PRODUCTION

The successes in the description of deep inelastic processes[34,35] indicates that at short distances a theory can be constructed on the basis of parton ideas.[10,11] At the same time, we can evidently omit the correlations associated with the confinement forces.

The observed hadrons are the products of interactions at large distances, of the order of some scale parameter μ. The numerical value of this parameter, which is important for the description of experiment, can be found only if there is a quantitative theory of confinement. Here, we shall assume that such a parameter exists and that its value is such that at short distances $\langle 1/\mu$ parton ideas can be used.

Since color fields are not observed in the free state but are "hidden" in colorless hadrons, the question of the terms in which the theory of multiparticle hadron production is to be formulated is acute. In other words, in describing the production of hadrons as the result of the interaction of the color fields $Q_\alpha(x) = (B^a_\mu(x), q_i(x))$, we must determine the functional dependence of the vertices $C_0(p)$ on $O(x|Q)$ (see Ref. 9), but this is actually one of the aspects of the confinement problem.

We assume that there exists a functional $O^\pi(x|Q)$ such that (9) can be written in the form

$$O^\pi(x|Q)\, O^\pi(y|Q)^*\rangle_Q = \int d^4p e^{ip(x-y)} \frac{C_\pi^2(p)}{p^2 - m_\pi^2}. \tag{34}$$

Having determined in this manner the "observed" functional O^π, we can express the amplitudes A_n for multiple pion production in the form

$$A_n(x_1, \ldots, x_{n+2}) = \left\langle \prod_{i=1}^{n+2} O^\pi(x_i|Q) \right\rangle_Q + \ldots$$

$$\equiv \int DQ e^{iS(Q)} \prod_{i=1}^{n+2} Q^\pi(x_i|Q) + \ldots \tag{35}$$

Here, we have not written out the contributions corresponding to the possibility of pion production in pairs through the observable $O^{2\pi}(x_1, x_2|Q)$, etc. Following (35), we represent the total amplitude of multiple pion production in all possible channels in the form

$$A_n(x_1, \ldots, x_{n+2}) = \int DQ e^{iS(Q)} F_n(x_1, \ldots, x_{n+2} \mid Q), \tag{36}$$

where

$$F_n(x_1, \ldots, x_{n+2}|Q) = \prod^{n+2} O^\pi + \Sigma O^{2\pi} \prod^{n} O^\pi + \ldots \tag{37}$$

and the first term in this sum gives the contribution written down in (34).

(One may note a certain analogy between this problem and the problem of describing helium-II,[36] in which the superfluid component is not directly observable. The expression "observable variables" is taken from Ref. 37.)

Thus, we assume that at distances $> 1/\mu$ it is necessary to take into account in the integral (36) not only the contributions calculated perturbatively but also the nonperturbative contributions.[38,39] Simultaneously, we assume that at short distances $< 1/\mu$ it is sufficient to take into account only the interactions of the partons, and here perturbation theory can be used. We note that at the present level of knowledge this scheme of calculations is effectively uncontrollable (see Ref. 20) and is based, as was said above, on the experience of describing hard processes.

The existence of the scale-fixing parameter μ having been assumed, the further calculations are fairly simple—it is necessary to take into account the different nature of the interactions at large and small distances. For this, it is simplest to divide the fields $Q_\alpha(x)$ into two parts:

$$Q_\alpha(x) = \eta_\alpha(x) + \xi_\alpha(x), \tag{38}$$

in such a way that, say, the fields $\eta_\alpha(x)$ correspond to interactions at only small distances,

$$\left.\begin{aligned} \eta_\alpha(x) &= \int dy K_+(x-y)\, Q_\alpha(y), \\ K_+(x-y) &= \int \frac{d^4q}{(2\pi)^4} e^{iq(x-y)}\, \theta(|q| - \mu), \end{aligned}\right\} \tag{39}$$

and the fields ξ_α to those at large distances,

$$\xi_\alpha(x) = \int dy K_-(x-y)\, Q_\alpha(y), \tag{40}$$

where

$$K_+(x) + K_-(x) = 1. \tag{41}$$

This ensures the following gauge-invariant decomposition of the element DQ:

$$DQ = \prod_{\alpha, x} dQ_\alpha(x) = \prod_{\alpha, x} d\eta_\alpha(x)\, d\xi_\alpha(x) = D\eta\, D\xi. \tag{42}$$

Here, we have used the fact that the parton virtuality is $|q| \simeq |q_\perp|$,[19] and therefore the Jacobian of the transformation in (42) is $O(1)$. Such divisions are frequently used to describe systems with many degrees of freedom in order to separate the necessary types of interactions.[40]

We note also that Eqs. (38) and (41) guarantee that the decomposition does not violate the gauge symmetry of the theory. At the same time, since $K_+(x)$ decreases rapidly as $|x| \to \infty$, and $K_-(x)$ as $|x| \to 0$, there is a certain arbitrariness in the choice of the gauge fields η_α and ξ_α.

Substituting (38) in the Langrangian, we rewrite the action $S(Q)$ in the form of the sum

$$S(\eta+\xi)=S(\eta)+\overline{S}(\eta,\xi). \quad (43)$$

This equation is the definition of the action $\overline{S}(\eta,\xi)$, in which we take into account not only the self-interaction of the fields ξ_α but also their interaction with the parton fields η_α. The explicit form of $\overline{S}$ is for us not important here.

The proposed separation of the interactions at short distances includes implicitly the assumption that the fields $\eta_\alpha(x)$ can be represented in the form of plane-wave expansions, and it thus rules out participation of the fields $\eta_\alpha(x)$ in the confinement of the color fields in hadrons.

Since, by definition, the functional $F_n(|Q)$ describes pion production, and since the field η_α does not participate in hadron production, we introduce the basic assumption that

$$F_n(|\eta+\xi)\simeq F_n(|\xi). \quad (44)$$

We emphasize once more that the only justification of this hypothesis is agreement with experiment. A similar idea for describing production of hadrons in a hard jet-production process, in which a "preconfinement" stage was separated naturally, was discussed in Ref. 41.

Substituting (42)–(44) in (36), we find that the pion production amplitude can be written in the form

$$A_n=\int D\eta e^{iS(\eta)}\int D\xi e^{i\overline{S}(\eta,\xi)}F_n(|\xi). \quad (45)$$

The insertion corresponding to gauge fixing is assumed to be included in the measure of integration.

Using (45), we can calculate the density matrix $\rho(\alpha,z)$ in accordance with (16):

$$\rho(\alpha,z)=\int D\eta^+D\eta^- e^{iS(\eta^+)-iS(\eta^-)}e^{\chi(\eta^+|\alpha,z)}, \quad (46)$$

where

$$\chi(\eta^\pm|\alpha,z)=\ln\int D\xi^+D\xi^- e^{i\overline{S}(\eta^+,\xi^+)-i\overline{S}(\eta^-,\xi^-)}$$
$$\times\sum_n\int\prod_{j=1}^{n}\frac{d^3k_j\, e^{i\alpha k_{jz}(k_j)}}{(2\pi)^3 2\sqrt{k_j^2+m_\pi^2}}\widetilde{F}_n(k_1,\ldots,k_n|\xi^+)$$
$$\widetilde{F}_n^*(k_1,\ldots,k_n|\xi^-). \quad (47)$$

In (47),

$$\delta\left(P_{ab}-\sum_1^n k_j\right)\widetilde{F}_n(k_1,\ldots,k|)$$
$$=\int\prod_{j=1}^{n}e^{ix_jk_j}dx_j\, e^{ix_{n+1}k_{n+1}}dx_{n+1}\, e^{ix_{n+2}k_{n-2}}dx_{n+2}$$
$$F_n(x_1,\ldots,x_{n+2}|).$$

In the derivation of (46) it was assumed that the order of the integrations over the fields η_α and over the phase space can be interchanged and that any order can be taken. (This is always possible by virtue of (12a) for $z\leq 1$. The region $z>1$ must be determined as a result of analytic continuation from the circle $|z|=1$; see also Ref. 26.)

Thus, by the division of the fields (38) we have "hidden" in $\chi(\eta^\pm|\alpha,z)$ all the problems of the dynamics of hadron production. At the same time, since the integration over the fields η_α is to be done in the framework of standard perturbation theory, making an expansion in the neighborhood of $\eta_\alpha^\pm=0$, it is convenient to represent $\chi(\eta^\pm|)$ as a sum in powers of $\eta_\alpha^\pm$

$$\chi(\eta^\pm|\alpha,z)=\sum_{k_1,k_2}\int\prod_j^{k_1}\eta_{\alpha_j}^+(x_j)\,dx_j\prod_j^{k_2}\eta_{\alpha_j}^-(x_j')\,dx_j'$$
$$\times\gamma_{k_1k_2}(x_1,\ldots,x_{k_1};x_1',\ldots,x_{k_2}'|\alpha,z), \quad (48)$$

where, by definition, $\gamma_{k_1k_2}$ are connected vertex functions that cannot be expressed in factorized form. Further, the expansion (48) contains only colorless combinations of the fields η_α^+ and η_α^-. In fact, the vertices $\gamma_{k_1k_2}$ describe the correlations between the partons due to the confinement forces.

If the expansion (48) is augmented by the assumption that the integrals over the fields ξ_α can be calculated in the neighborhood of gauge-invariant expectation values, say, $\langle G_{\mu\nu}^2\rangle$, $\langle\bar{q}q\rangle$, etc., then (46)–(48) can become the point of departure for the operator expansion used in Ref. 39. In our case, in which we consider multiparticle production, it is also necessary to know the local values of the expectation values, their dependence on the coordinates.

Instead of (46), it is more convenient to consider here a representation in which the conservation laws are taken into account explicitly. Expanding (46) in powers of χ and substituting this expansion in (15), we find that the generating function T has the form

$$T(z,s)=\sum_{k=1}^{\infty}T_k(z,s)=\sum_{k=1}^{\infty}\frac{1}{k!}\int\sum_{i=1}^{k}\frac{d^4p_i}{(2\pi)^4}\delta\left(P_{ab}-\sum_{i=1}^{k}p_i\right)$$
$$\times\int D\eta^+D\eta^- e^{iS(\eta^+)-iS(\eta^-)}\prod_{i=1}^{k}\widetilde{\chi}(\eta^\pm|p_i,z), \quad (49)$$

where

$$\widetilde{\chi}(\eta^\pm|p,z)=\int\frac{d^4\alpha}{(2\pi)^4}e^{i\alpha p}\chi(\eta^\pm|\alpha,z). \quad (50)$$

By definition [see (47)], $\widetilde{\chi}(\eta^\pm|p,z)$ is a complex functional of the fields $\eta_\alpha^\pm$. At the same time, by virtue of (12b), $\widetilde{\chi}(\eta^\pm|p,z)$ must be in magnitude increasing functions of z. Then the mean number

$$\bar{k}(z,s)=\frac{1}{T(z,s)}\sum_{k=1}^{\infty}kT_k(z,s) \quad (51)$$

must also be an increasing function of z, i.e., of n if we take into account the equation of state (17).

For this reason, it follows by virtue of the law of conservation of the 4-momentum P_{ab} that there exists in (49) z values such that $p_i^2\simeq m_\pi^2$ if $\chi(|\alpha,z)$ is a regular function of z. In this case, the hadron production process is actually described by the interactions of the fields $\eta_\alpha(x)$, i.e., it is basically a hard process. Thus, if $\chi(\eta^\pm|\alpha,z)$ are regular functions of z, then, expanding $\chi(\eta^\pm|)$ in powers of the fields $\eta_\alpha^\pm$ [see (48)] and noting that the effective parton coupling constant satisfies $\alpha_s(\mu^2)\ll 1$, we can assume that the mean number of partons is $\bar{\nu}\sim\bar{k}$. Then

$$\frac{\bar{\nu}(n,s)}{n}\to O(1) \quad (52)$$

as $n\to\infty$.

Thus, if $z_0(n,s)$ increases unboundedly with increasing

n, (52) must necessarily be satisfied. We can now use Sec. 2 to formulate the assertion made in the Introduction.

3. MODEL CONSIDERATIONS

The expansion (48) makes it possible to separate the correlations between the partons due to the confinement forces. At the same time, the scale parameter μ is assumed to be such that the effective parton coupling constant satisfies $\alpha_s(\mu^2) \ll 1$, and therefore in (48) it is sufficient to consider only the first term in the expansion $\sim \gamma_{22}$, which determines the generating function

$$\widetilde{\gamma}(k_1, k_2; k_1', k_2' \mid \beta, z)$$
$$= \int \prod_1^2 dx_j dx_j' e^{ik_j x_j - ik_j' x_j} \gamma_{22} \mid x_1, x_2; x_1', x_2' \mid \beta, z),$$

this describing hadron production as a result of parton interactions. We here use model considerations to elucidate the problem of the singularities of $\widetilde{\gamma}$ with respect to z.

We expand the density matrix $\widetilde{\gamma}$ defined in this manner with respect to the correlation functions [see (25)]:

$$\widetilde{\gamma}(\mid \beta, z) = \widetilde{\gamma}_1(\mid \beta) \exp\left\{\sum_{l=1}^{\infty} \frac{1}{l!} \int \prod_{i=1}^{l} \left[\frac{dm_i}{m_i} e^{-\beta m_i} (t(z, m_i) - 1)\right] \times n_l(m_1, \ldots, m_l; \beta)\right\}. \quad (53)$$

In what follows, we shall not be interested in the dependence of the momenta k_i and k_i'; it is sufficient to know that $\beta \to \infty$ as $n \to \infty$.

We shall construct models for calculating $t(z, m)$ and $n_l(m_1, \ldots, m_l; \beta)$ on the basis of the following assumptions:

A) the produced hadrons have, on the average, bounded transverse momenta;

B) in a first approximation, the correlations between the produced hadrons can be ignored.

These assumptions are the basis of every description of hadron interactions (see also Ref. 42) and, evidently, every consistent theory of strong interactions must agree with these ideas.

The simplest form of theory corresponding to conditions A and B is the statistical model in which resonances of arbitrary mass are produced but the (background) correlations between the resonances are ignored, i.e., the model in which only the first term of the expansion is retained in the exponential in (53),[32]

$$\widetilde{\gamma}(\mid \beta, z) = \widetilde{\gamma}_1(\mid \beta) \exp\left\{\int \frac{dm}{m} (t(2, m) - 1) e^{-\beta m} n_1(m, \beta)\right\}. \quad (54)$$

Here, n_1 is the mass spectrum of the resonances.

On the basis of condition A, we must assume the multiperipheral mechanism of resonance decay. Therefore, in (27) we must set $n_r \sim \ln m$, so that

$$\ln t(z, m) = r_0'(z-1) \ln \frac{m}{m_0} + O((z-1)^2). \quad (55)$$

For the mass spectrum of the resonances, we take in accordance with the chosen normalization[43]

$$n_1(m; \beta) = n_1(m_0) e^{\beta^* m}, \quad \beta^* = \text{const} > 0. \quad (56)$$

Note that a different choice of n_1 leads to a more rapid decrease in the particle production cross sections.

If we substitute (55) and (56) in (54), the equation of state (17) takes the form

$$\frac{n}{r_0' z} = n_1 \frac{\partial}{\partial r} \frac{\Gamma(\Delta, r)}{\Delta r}, \quad (57)$$

where $\Delta \equiv m_0(\beta - \beta^*)$ and $r = r_0'(z-1)$; $\Gamma(\Delta, r)$ is the incomplete Γ function.

We are interested in low temperatures, when $\Delta \to 0$ and the particle density is high, $r > 0$. Then Eq. (17) for $\ln 1/\Delta \ll n \ll \ln^2 1/\Delta$ has the solution

$$z_0 = 1 + 1/r_0' \ln 1/\Delta. \quad (58)$$

As a result,

$$\ln \frac{\widetilde{\gamma}(\mid \beta, n)}{\widetilde{\gamma}_1(\mid \beta)} \simeq -\frac{n}{r_0' \ln 1/\Delta}. \quad (59)$$

The temperature of the gas of the produced particles can be found from an equation equivalent to (33):

$$\bar{n}_r(s) = \frac{d}{dz} \ln \widetilde{\gamma}(\mid \beta, z) \mid_1. \quad (60)$$

From this we find that

$$\bar{n}_r(s) \sim \ln 1/\Delta. \quad (61)$$

Substituting (61) in (59), we obtain

$$\ln \frac{\gamma(\mid s, n)}{\widetilde{\gamma}_1(\mid s)} = -\gamma \frac{n}{\bar{n}_r(s)} + O(\ln n) \quad (62)$$

for $\bar{n}_r \ll n \ll \bar{n}_r^2$.

If $n \gg \bar{n}_r^2$, Eq. (57) leads to

$$\ln \frac{\widetilde{\gamma}(\mid s, n)}{\widetilde{\gamma}_1(\mid s)} \simeq -\frac{n}{\bar{n}_r(s)} \ln \frac{n}{\bar{n}_r^2(s)}. \quad (63)$$

Thus, in the region, $\bar{n}_r(s) \ll n \ll \bar{n}_r^2(s)$ the multiplicity distribution in the model can imitate a singularity with respect to z, so that in this region of multiplicity values, fluctuations of purely resonance type must suppress all the remainder (both those of background type corresponding to Pomeron exchanges and those corresponding to the parton degrees of freedom).

We mention here the part played by condition A. If we give up this condition and assume instead of (55) that

$$\ln t(z, m) = (z-1) r_0 m, \quad (64)$$

then

$$\widetilde{\gamma}(\mid \beta, z) \sim (\Delta + r_0(1-z))^{-n_1(m_0)}, \quad (65)$$

i.e., in this case the suppresion by the resonance fluctuations of all the remainder must be observed in the complete region of n values (however, it is at the same time predicted that $\bar{n} \sim \sqrt{s}$).

It can be seen from the example considered above that the cutoff condition with respect to the transverse momentum plays the part of the suppression mechanism of the resonance fluctuations. However, this does not yet prove that in the theory constructed with allowance for cutoff with respect to the transverse momentum it is impossible to obtain cross sections satisfying $\sigma_n \geq O(e^{-n})$ in asymptotia with respect to n.

To eliminate fluctuations associated with the production of heavy resonances, which can, as was shown above, suppress all the remaining fluctuations, we assume that $n_1(m;\beta)$ decrease sufficiently rapidly with increasing m. Then, expanding $t(z,m)$ in powers of $(z-1)$ and ignoring all correlations, we find that $\tilde{\gamma}(|\beta,z)\sim\exp[\bar{n}(\beta)(z-1)]$, where $\bar{n}(\beta)$ is the mean number of particles with given temperature $\sim 1/\beta$, i.e., going over to the energy representation,

$$\gamma(z,\ s)=\gamma_1(s)\,e^{\bar{n}(s)(z-1)}, \tag{66}$$

and this leads to a Poisson distribution with respect to the multiplicity.

We choose a model in which there is no dependence of γ_1 on s and assume in accordance with condition A that

$$\bar{n}(s)=\frac{1}{2}r\ln s/s_0, \tag{67}$$

this corresponding to the contribution to $\gamma(z,s)$ of one Pomeron exchange.

On the basis of the Born contribution (66), one can develop a diagrammatic technique in which $\exp[\bar{n}(s)(z-1)]$ plays the part of a propagator of a ("cut") Pomeron[17] in order to include fluctuations in the density of the produced particles. Such a model of "strong coupling" of Pomerons[44] was investigated in Ref. 45.

This model can be interpreted as describing the motion of a Brownian particle in the plane of the impact parameter **b** with z determined by the density of (infinitely heavy) impurities. Allowance for the action of this particle on itself is equivalent to taking into account the fluctuations in the density of the produced particles and leads to the following instability.

If the mobility $D(z,s)$ is calculated, it is found that at high energies D is singular with respect to z at $z=1$ (Fig. 6). This instability has the consequence that $T(z,s)$ is singular at $z=1$, and therefore the corresponding multiparticle production cross section will decrease as a power $(\ln\sigma_n\sim-\ln n)$.

In the calculation of this result, the so-called ε expansion was used. Formally, the resulting distribution was calculated in a space of dimension $d=4$, in which ultraviolet divergences are important and, by virtue of this, there is actually no cutoff with respect to the transverse momentum. At the same time, if the theoretical expressions depend weakly on ε, we can, taking $d=4-\varepsilon$, choose $\varepsilon=2$ in order to go over to the description of real processes. However, it can be seen that for $z\neq 0$ this theory is singular at $\varepsilon=0$, and this indicates inadmissibility of the ε expansion.

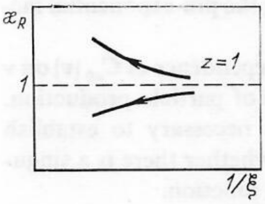

FIG. 6. Dependence of the normalized diffusion coefficient $\varkappa_R=D(z,s)/D(1,s)$ on the energy; $\xi=\ln s$. The upper and lower curves correspond to $z>1$ and $z<1$, respectively.

This model has been given here in order to demonstrate once more the part played by the cutoff with respect to the transverse momentum.

Much more popular is the model in which

$$\gamma_1(s)\sim s^{\delta}, \tag{68}$$

where $\delta\cong 0.070\pm 0.005$.[46] In this model of strong interactions, the total cross section increases in proportion to the square of the logarithm of the energy, $\sigma_{tot}\sim\ln^2 s$. Such behaviro of σ_{tot} corresponds to the picture of an interacting hadron as a disk whose profile function $\Phi_0(\mathbf{b},s)$ is expressed in the form

$$\Phi_0(\mathbf{b},\ s)=i\theta\,(a_0^2\xi^2-\mathbf{b}^2), \tag{69}$$

where $\xi=\ln s/s_0$ and **b** is the impact parameter.

One can regard Φ_0 as the propagator determined in the Born approximation and attempt to establish the extent to which the solution (69) is stable with respect to quantum corrections. This question is discussed in Ref. 47. The problem here is in the need to take into account the diffuseness of the edge of the disk. As was shown in Ref. 48, if the disk does not have a diffuse edge and the profile function has the form (69), it is impossible to describe consistently particle production in such a model. In its turn, the diffuseness of the edge of the disk may lead to complications with s-channel unitarity. In Ref. 48, it was shown, however, that if the Pomeron interaction vertices are sufficiently small, the model can be reconciled with the s-channel unitarity condition.

Having obtained in this manner a noncontradictory model of multiparticle production, we can use it to describe $\gamma(s,n)$. Since the Pomeron interaction vertex is small, in the first approximation we can use the eikonal approximation, which corresponds to an expansion in reciprocal powers of the hadron interaction range. The effective expansion parameter is

$$X=\frac{2\lambda\alpha}{R^2+\alpha_{\mathrm{P}}'\xi}\,e^{\delta\xi}, \tag{70}$$

where $\alpha=3.64+0.05\,(\mathrm{GeV}/c)^{-2}$, $R^2=3.56\pm 0.06\,(\mathrm{GeV}/c)^{-2}$, $\lambda=1.5$, and $\alpha_{\mathrm{P}}'=0.25\pm 0.05\,(\mathrm{GeV}/c)^{-2}$. Using these parameters, we can achieve good agreement with experiment[17] (see Fig. 5b). In this approximation, ignoring the possible oscillations,

$$\tilde{\gamma}(|\ s,\ n)\sim\exp\Big(-\frac{n}{\bar{n}(s)}\ln\frac{n}{\bar{n}(s)}\Big), \tag{71}$$

if $n\gg\bar{n}(s)$ (which covers the complete range of values of n in which we are interested).

However, the eikonal approximation has a restricted region of applicability. Despite the smallness of the Pomeron coupling constant r_{P}, for

$$n\gg\bar{n}(s)\ln\frac{8}{r_{\mathrm{P}}R^2} \tag{72}$$

it is necessary to take into account interactions of Pomerons (numerically, $r_{\mathrm{P}}R^2\simeq 10^{-4}$). In Ref. 48, the following equation was obtained for $T(\mathbf{b}\xi,z)$:

$$T(\xi,\ z)=\int d^2\mathbf{b}\,T(\mathbf{b};\ \xi,\ z). \tag{73}$$

In general form, the equation can be expressed as

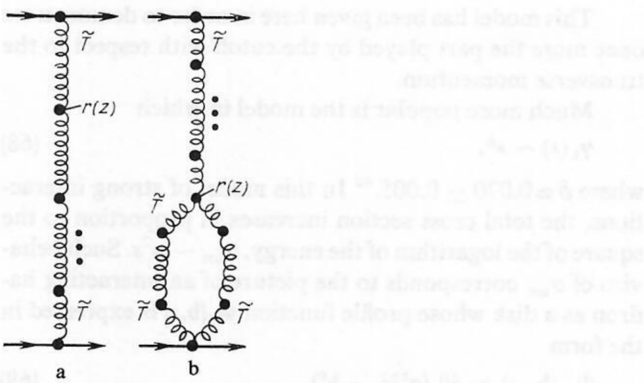

FIG. 7. Typical diagrams determining the contributions to $\hat{\chi}$ (b,ξ, z); $r(z)$ is the Froissaron interaction vertex.

$$T(\mathbf{b};\ \xi,\ z) = \frac{1}{2}\,\hat{\chi}\,(\mathbf{b}\xi,\ z \mid T), \tag{74}$$

where $\hat{\chi}$ is determined by the diagrams of Fig. 7. For example, the contribution of the diagram in Fig. 7a to $\hat{\chi}$ can be expressed in the form

$$\hat{\chi} = \sum_{k=2}^{\infty} (r_{\mathbb{P}})^{k-1} \int \prod_{i=1}^{k} \{d^2\mathbf{b}_i\, d\xi_i\, T\,(\mathbf{b}_i;\ \xi_i,\ z)\}\ \delta\left(\sum_{1}^{k} \xi_i - \xi\right) \delta^{(2)}\left(\sum_{i}^{k} \mathbf{b}_i - \mathbf{b}\right). \tag{75}$$

We are interested in singular solutions of Eq. (74). At the same time, in asymptotia with respect to n, we shall, for $z > 1$, be interested in only positive contributions to $\hat{\chi}$ (**b**; $\xi, z|T$) (which corresponds to positive definite correlation functions $\widetilde{N}_l$).

A standing, not dependent on **b** and ξ, singularity is to be rejected immediately, since in this case there will be singularities of different orders on the left- and right-hand sides of (74).

The singularity cannot depend on the impact parameter, since then Eq. (73), which is a consequence of assumption A, is not defined in this case.

Thus, if there is a singularity, then $z_c = z_c(\xi)$, and by virtue of (32) we shall be interested only in $z_c \to 1$ as $\xi \to \infty$. It can be shown that in this case allowance for the quantum corrections (say, (75)] shifts the singularity to the point $z = 1$, but this, as was said above, contradicts the original equation (74).

4. PRODUCTION OF PARTICLES IN HARD PROCESSES

It can be seen from the foregoing model analysis that

$$\ln \frac{\tilde{\gamma}\,(\mid s,\, n)}{\tilde{\gamma}_1\,(\mid s)} < -an,\quad a > 0,\quad n \to \infty.$$

This inequality is evidently a consequence of the boundedness of the transverse momenta of the produced particles, i.e., a consequence of the quasi-one-dimensionality of the phase space of the system.

Thus, since (4) is satisfied, the asymptotic behavior with respect to n can be described in the framework of QCD partons. We shall calculate the distributions with respect to the multiplicity in the following sequence: First we consider particle production as a result of the decay of a parton with large virtuality; then, using this result, we consider the multiplicity distribution in deep inelastic processes. Finally, we shall consider below hadron–hadron interactions.

In order not to introduce additional uncertainties associated with the model considerations regarding $\gamma_{k_1k_2}(|\alpha, z)$ into the predictions of the parton model, we simply use the fact that as $n \to \infty$ the number n of produced hadrons must be proportional to the number ν of produced partons.

Before we turn to the actual calculations, we should, however, note that in rejecting effects associated with hadron production we introduce a certain error at finite n. It is therefore sensible to consider first the degree of accuracy and the detail of the calculations that must be maintained in what follows (bearing in mind also the complete absence of experimental data in the range of values of n of interest to us).

Below, we shall limit ourselves to the leading logarithmic approximation, and we therefore use the convenient expansion with respect to correlation functions, distinguishing the correlations between partons in a jet produced by the decay of a heavy parton from correlations between jets. This is convenient in the first place because in the leading logarithmic approximation Markov processes play an important part[19,49] and the correlations between the jets must be small.

Bearing in mind the restriction (10), it is to be expected that the corresponding distribution will have the form

$$\sigma_n^{ab} = \varphi_n^{ab} \exp\left(-\frac{n}{\bar{n}}\, C_{ab}^h\,(n)\right). \tag{76}$$

Here, C_{ab}^h is a nondecreasing function of n, and $\bar{n}$ is the mean multiplicity of the hadrons produced in the given process. The pre-exponential factor φ_n^{ab} must, following (12b), be an increasing function of n. It follows from (76), in accordance with Sec. 1, that the important values of n are $n \sim \bar{n}$, i.e., the scale of values of n is determined by the mean multiplicity.

A similar representation can be obtained by considering the production of ν partons (we shall mainly be interested in the production of a large number of gluons):

$$\sigma_\nu^{ab} = \bar{\varphi}_\nu^{ab} \exp\left(-\frac{\nu}{\bar{\nu}}\, C_{ab}\,(\nu)\right), \tag{77}$$

where $\bar{\nu}$ is the mean number of partons produced in the given process, and $C_{ab}(\nu)$ is again a nondecreasing function of ν, which can also depend on the nature of the process.

Comparing these two expressions, we can conclude that the subsequent calculations are to be made up to the accuracy of the pre-exponential $\bar{\varphi}_\nu^{ab}$, for in ignoring the effects associated with confinement, we introduce an error so large that there can be no point in considering the pre-exponential factor.

Further, as we have seen, the dependence of $C_{ab}(\nu)$ on ν significantly influences the picture of particle production. Therefore, in the first place it is necessary to establish whether $C_{ab}(\nu)$ depends on ν, i.e., whether there is a singularity at $z = z_c < \infty$ in the partition function.

In accordance with (39), the parton virtualities are large, $|q|^2 \gtrsim \mu^2$, so that the corresponding effective coupling constant satisfies $\alpha_s(\mu^2) \ll 1$. Then simple kinematic considerations lead to a natural infrared cutoff in QCD.[50] This circumstance can significantly influence the dynamics, provided the infrared divergences are important in the theory (otherwise, these corrections associated with the infrared cutoff are unobservable, as in the $(\lambda\varphi^3)_6$ theory, since they go beyond the framework of the leading logarithmic approximation). In QCD, it is to be expected that the condition $|q^2| \gtrsim \mu^2$ will lead to a significant reduction of the phase space, and this must affect the asymptotic behavior with respect to ν.

To demonstrate this, we use the equation obtained in Ref. 50 to determine the probability σ_ν^a of decay of parton a into ν partons. Since we are interested in only the asymptotic behavior with respect to the multiplicity, for which the extreme left-hand singularities with respect to z are important, since the position of the singularity is determined by the mean multiplicity of the partons, and since $\bar{\nu}_g > \bar{\nu}_q$, i.e., the mean gluon multiplicity exceeds that of the quarks,[51] we can ignore quark jets in the first approximation. We then arrive at the equation ($\tau \sim \ln q^2/\Lambda^2$):

$$\tau \frac{d}{d\tau} \sigma_\nu^g = (1-\tau+\tau_0)\,\sigma_\nu^g + (1+\bar{\delta}) \sum_{\nu_1=0}^{\nu} \sigma_\nu^g \sigma_{\nu-\nu_1}^g + \sum_{\nu_1=0}^{\nu} \int_{\tau_0}^{\tau} d\tau_1\, \sigma_{\nu_1}^g \sigma_{\nu-\nu_1}^g. \tag{78}$$

We shall be interested in the following solution of this equation:

$$\sigma_\nu^g = O(1)\, \nu^{\varkappa}\, e^{-\nu \ln z_c}, \quad \varkappa = \text{const}. \tag{79}$$

Bearing this in mind, we introduce

$$\alpha_k(\tau) = \frac{1}{k!} \sum_{\nu=1}^{\infty} \nu^k \sigma_\nu^g(\tau). \tag{80}$$

The corresponding partition function is

$$t_g(\tau, z) = \sum_{k=0}^{\infty} (\ln z)^k \alpha_k(\tau), \quad z > 1. \tag{81}$$

Substituting (79) in (80), we find that $\sigma_\nu^g(\tau)$ has the asymptotic behavior (79), provided

$$\alpha_k(\tau) = \beta_k (\bar{\nu}_j(\tau))^k, \quad \bar{\nu}_j(\tau) \equiv (\ln z_c(\tau))^{-1}, \tag{82}$$

where the positive coefficients β_k can depend weakly on τ [we have decided not to consider quark jets, and therefore $\alpha_0 = O(1)$].

Using (78), we find that

$$\bar{\nu}_j(\tau) = O(e^{\sqrt{\tau}}) \tag{83}$$

[see (3)]. At the same time, the coefficient β_k satisfies the recursion relation

$$\frac{k^2-1}{k} \beta_k = \sum_{m=1}^{k-1} \left(\frac{1}{m} - \frac{4(1+\bar{\delta})}{\sqrt{\tau}} \right) \beta_m \beta_{k-m}. \tag{84}$$

Since they β_k are positive, we are forced to assume that for arbitrary but bounded $k \gg 1$ and $\tau \to \infty$ the negative term on the right-hand side of (84) can be ignored. Then it is easy to find that

$$\beta_k = 2k\,(1 + O(1/k)). \tag{85}$$

Substituting this solution in (84), we find that (85) is justified, provided

$$k \ll \sqrt{\tau}. \tag{86}$$

This condition restricts the region of applicability of (85). In terms of the multiplicity ν, the restriction (86) means that the "critical region" of ν values is determined by the inequalities

$$\bar{\nu}_j(\tau) \ll \nu \ll \sqrt{\tau}\, \bar{\nu}_j(\tau). \tag{87}$$

Thus, $t_g(\tau, z)$ cannot be singular at $z = z_c < \infty$, although in the restricted region (87) of values of ν the multiplicity distribution σ_ν^g may imitate such a singularity.

A singularity of the partition function may be a consequence of only the unbounded growth of the fluctuations as $z \to z_c$. In the example considered above, the effective reduction of the phase space (due to the finiteness of μ) prevents this phenomenon. Mathematically, this is reflected in the fact that the right-hand side of (84) acquires a negative term $\sim 1/\sqrt{\tau}$, and this plays the part of a cutoff factor (for comparison, see the calculations in the $(\lambda\varphi^3)_6$ theory,[52] in which this factor is absent).

To calculate the multiplicity distribution outside the critical region of ν values, in which the reduction of the phase space is appreciable and therefore the fluctuations are suppressed, we must, as follows from (84), take into account in σ_ν^g the corrections $\sim 1/\tau$. But then Eq. (78), obtained in the leading logarithmic approximation, is incorrect—it is necessary to take into account the following corrections, i.e., to go beyond the framework of the leading logarithmic approximation. It should be noted that in the framework of the logic of this approximation allowance for the contributions of more complicated diagrams does not lead to a singular partition function (this would mean divergence of the perturbation-theory series). From this it may be concluded that outside the region (87)

$$\sigma_\nu^g(\tau) = O\left(e^{-\frac{\nu}{\bar{\nu}_j(\tau)} C_g(\tau, \nu)}\right). \tag{88}$$

Here, $C_g(\tau,\nu)$ is an increasing function of ν.

Let the structure function $D_{ab}(x,Q^2,\nu)$ describe the production of ν partons,

$$\sum_\nu D_{ab}(x, Q^2; \nu) = D_{ab}(x, Q^2), \tag{89}$$

the quantity on the right being the structure function that determines the probability of finding parton b with virtuality $Q^2 < 0$ in parton a. For $-Q^2 \gg \mu^2$, to determine $D_{ab}(x,Q^2)$ it is sufficient to use the leading logarithmic approximation, this corresponding to the contributions of the simple ladder diagrams.[10,11] In other words, in the deep inelastic region $D_{ab}(x,Q^2)$ corresponds to a simple Markov process-"Brownian motion with respect to the coordinate $\sim \ln(1/x)$, the role of the time being played by $\ln\ln|Q^2|$." It is clear that such a process takes its simplest form if the mobility, which is of order $\ln(1/x)/\ln\ln|Q^2|$, is large. We shall assume that

$$\ln 1/x \gg \ln\ln|Q^2|, \tag{90}$$

but, simultaneously,

$$\ln 1/x \ll \ln Q^2| \tag{91}$$

(this last restriction is a specific one for the considered approximation[49]).

In the leading logarithmic approximation, the main contributions to $D_{ab}(x,Q^2)$ that compensate the smallness of $\alpha_s(\mu^2)$ arise from a wide region of integration with respect to k_i^2, the (positive) virtuality of the produced partons, the important values of this variable satisfying $\mu^2 \ll k_i^2 \ll -Q^2$. If it is remembered that the time needed for capture of a parton in a hadron is $\gtrsim 1/\mu$ (in the self-frame), then a parton with virtuality $k_i^2 \gg \mu^2$ must have time to decay before it is captured in a hadron. This leads to dominance of the jet production of partons in the hard process.

Now suppose ν is large. Then, in the first place, many-partons jets must be produced, i.e., $\langle k_i^2 \rangle$ must increase with increasing ν. But this amounts to an effective decrease in the region of integration over k_i and, ultimately, to a decrease in the fraction of the contributions calculated in the leading logarithmic approximation due to the decrease in the mobility (i.e., the decrease in the phase space in which the jets are produced). As a result, we find that it is necessary to take into account many-ladder diagrams (similar to those considered in Ref. 53). This solution, the alternative to the process of production in asymptotia with respect to ν of one jet, is a consequence of the boundedness of the region (87) of critical values of ν.

Thus, we have shown earlier in this section that in a certain bounded region (87) of values of ν it can be assumed that the fluctuations in the parton density are so large in a jet that they lead to a corresponding singularity in the partition function. Then, using the results of Sec. 1, we can assume that

$$D_{ab}(x, Q^2, \nu) = O(e^{-\nu/\bar{\nu}_j}), \tag{92}$$

if ν belongs to (87). For values of ν outside this interval, Eq. (88) holds; then, in accordance with Sec. 1, many-jet events must be important. However, in deep inelastic processes it is necessary to take into account in asymptotia with respect to ν the decrease in the mobility, and this makes it necessary to calculate the contributions of many-ladder diagrams.

To show this quantitatively, we can consider the Bethe-Salpeter equation for the generating function $T_{ab}(x,Q^2;z)$ corresponding to $D_{ab}(x,Q^2;\nu)$ $(\tau \sim \ln Q^2/\Lambda^2)$:

$$\tau \frac{d}{d\tau} T_{ab}(x, \tau; z) = \sum_c \int_x^1 dy\, P_{ac}(y)\, \hat{t}_{ac}(\tau, y; z)\, T_{cb}(y, \tau; z). \tag{93}$$

This equation is normalized in such a way that at $z = 1$ Eq. (93) becomes the equation for the structure functions $D_{ab}(x,\tau)$,[49,54] i.e.,

$$\hat{t}_{ab}(\tau, y; z)\,|_{z=1} = 1. \tag{94}$$

The kernels P_{ab} were found in Refs. 49 and 54.

The functions $\hat{t}_{ab}$ describe gluon production when parton a is scattered by parton b and are actually identical to the partition function for one jet found above.

Equation (93) can be simplified somewhat by using (91), namely, in the first approximation the dependence of $\hat{t}_{ab}$ on y can be ignored. Then, since a Markov process is considered, ignoring the correlations between the jets, we can find that

$$\ln T_{ab} \sim \sqrt{L(\tau, z) \ln 1/x}, \tag{95}$$

where

$$L(\tau, z) \equiv \int_{\tau_0}^{\tau} \frac{d\tau_1}{\tau_1} \hat{t}(\tau_1, x=1; z). \tag{96}$$

The estimate (95) was found when the effective mobility (which here depends on z) is fairly large:

$$\ln 1/x \gg L(\tau, z). \tag{97}$$

However, as can be seen from (96), with increasing ν, i.e., z_0, the mobility decreases, since $\hat{t}_{ab}$ is, by definition, an increasing function of z.

Thus, following Sec. 1, outside the critical region (87) of values of ν it is necessary to take into account the correlations between the jets, and in this case

$$T_{ab}(x, \tau; z) = \int \frac{dj}{2\pi} \left(\frac{1}{x}\right)^j d_{ab}(j, \tau) \times \exp\Big\{\sum_{l=1}^{\infty} \int \prod_{i=1}^{l} \Big[\frac{d\tau_i}{\tau_i} (\hat{t}(\tau, x=1; z) - 1)\Big] \times N_{ab}^l(\tau_1, \ldots, \tau_l; \tau \mid j)\Big\}, \tag{98}$$

where N_{ab}^l describe the correlations between the jets. If we follow the logic of the leading logarithmic approximation, it follows that, since the perturbation series in QCD must converge, outside the critical region

$$D_{ab}(x, \tau; \nu) = O\left(\exp\left(-\frac{\nu}{\bar{\nu}_j} C_{ab}(x, \tau; \nu)\right)\right), \tag{99}$$

where $C_{ab}(x,\tau;\nu)$ are increasing functions of ν whose explicit form is determined by $N_{ab}^l(\tau_1, ..., \tau_l; \tau \mid j)$.

Using the preceding results, we can now consider the production of an asymptotically large number of particles in hadron–hadron interactions. More precisely, we shall consider the scattering of two fast partons a and b of small virtuality. This is sufficient for our purposes, since we are not interested in the pre-exponential factor in (77).

In the leading logarithmic approximation, the main contribution is made by processes in which the virtualities k_i^2 and the longitudinal momenta of the partons, which are $\sim x_i$, are "aligned" as follows. If, say, $k_i^2 \ll k_{i+1}^2$, then accordingly $x_i \gtrsim x_{i+1}$. Using the convenient terminology of Ref. 5, one can then regard parton $i + 1$ as a "partonometer" of parton i. Then, considering the scattering of two fast partons of small virtuality $|\sim\mu^2|$, we must assume in the leading logarithmic approximation that the interaction occurs as a result of the collision of partonometers whose virtualities are appreciably greater than the virtualities of the colliding partons a and b, since it is only in this case that the smallness of α_s can be compensated by a large value of $\ln s$. This kinematics is similar to the kinematics of "Drell–Yan processes."[55] In our case, the heavy parton r that initiates the jet plays the part of the photon with large (positive) virtuality.

The corresponding partition function $H_{ab}(z,s)$ has the form

$$H_{ab}(z, s) = \sum_{c,d} \int_0^1 dx_1\, dx_2 \int_{\mu^2}^{s} \frac{dk^2}{k^2} \alpha_s(k^2)\, P_{cd}\left(\frac{x_1}{x_2}\right) \times \theta(x_2 - x_1)\, \delta(x_1 x_2 s - k^2)\, \hat{t}_r\left(k^2, \frac{x_1}{x_2}; z\right) \times T_{ac}(x_1, k^2; z)\, T_{db}(x_2, k^2; z) + (x_2 \leftrightarrow x_1). \tag{100}$$

Here, all quantities have been determined eariler. At the same time, partons c and d are regarded as partonometers of partons a and b, respectively.

Analysis of (100) leads to the conclusion that this expression, obtained in the leading logarithmic approximation, does not have a region of applicability; for in the critical region (87) of ν values "single-jet" events are important, and therefore

$$\ln \frac{\sigma_{\nu}^{ab}(s)}{\sigma_{\text{tot}}^{ab}(s)} \sim -\frac{\nu}{\bar{\nu}_j(s)}. \qquad (101)$$

Outside the critical region, it is necessary to take into account the correlations between the jets, and this makes it necessary to take into account many-ladder diagrams.

As follows from what was said above, at high energies and for a multiplicity outside the critical region of ν values the production of slow (infrared) gluons plays an important part in QCD. Therefore, to calculate the contributions outside the critical region (87), we must: a) take into account the contributions of the many-ladder diagrams (retaining, however, the logic of the leading logarithmic approximation); b) take into account more accurately the kinematic restrictions in the infrared region. These two conditions complicate the expressions to such an extent that analytic methods of calculations are ineffective.

There is, however, a circumstance that somewhat simplifies the calculations. As we said above, we are interested in the case $n \to \infty$, since at such values of n it can be expected that the effects associated with hadron production are unimportant. Now the variable z plays the part of the activity, i.e., to each parton production vertex we ascribe $\sqrt{z} > 1$. This means that we thereby artificially increase the contribution of the diagrams that describe the production of "real" partons. But then, at least for $\alpha_s \ll 1$, it is necessary to consider the contributions of only those diagrams that lead to parton production. This means that all the contributions to the amplitudes for the production of a very large number of particles do not interfere with each other, i.e., all contributions to the corresponding partition functions (or density matrices) must be positive. This somewhat restricts the class of diagrams that must be taken into account outside the critical region of values $\nu \sim n$.

CONCLUSIONS

We give some consequences of the picture described above, for the production of a very large number of hadrons, that appear interesting from the point of view of experimental investigation.[56] As we have seen, the present state of the theory is such that detailed quantitative predictions on the basis of a unified Lagrangian field theory cannot yet be obtained. Therefore, it is not possible to ascertain the value of the parameter μ or, which is the same thing, to determine the region of values of the energy $\sqrt{s}$ and the multiplicity n in which hard processes play the main role. Because of this, we make only qualitative predictions.

a) *Failure of* KNO *scaling*. In asymptotia with respect to n a new, "hard" particle production mechanism comes into play. Whereas for $n \sim \bar{n}$ KNO scaling means that $\bar{n}(\sigma_n/\sigma_{\text{tot}}) \sim \exp(-\gamma n/\bar{n})$ [we have used here the fit (62)]. for $n \gg \bar{n}$ we must expect [see (79)]

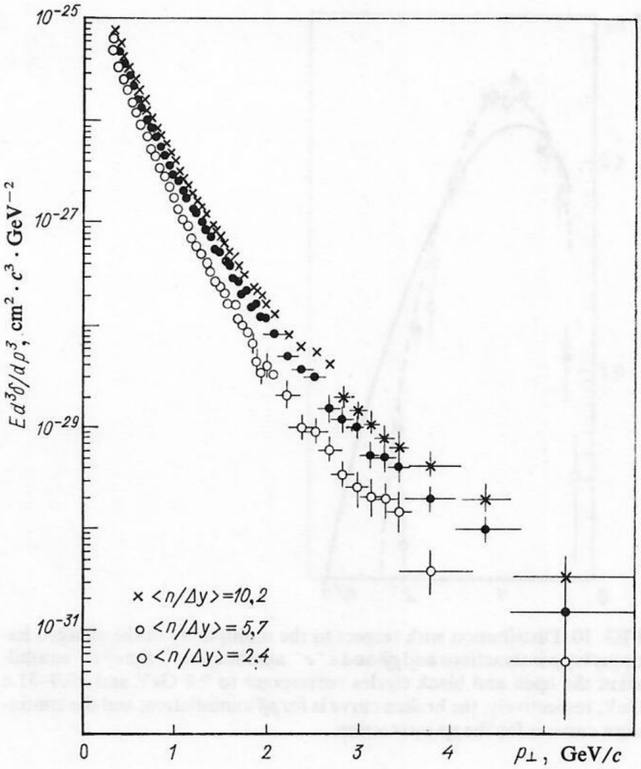

FIG. 8. Distribution with respect to the transverse momenta for different values of the multiplicity over the rapidity interval Δy.[58]

$$\bar{n}\frac{\sigma_n}{\sigma_{\text{tot}}} \sim \exp\left(-\gamma'\left(\frac{\bar{n}}{\bar{n}_j}\right)\frac{n}{\bar{n}}\right), \qquad (102)$$

where $\bar{n}/\bar{n}_j$ is the quantity that determines the deviation from KNO scaling in the exponential in (102).

b) *Growth of the mean transverse momentum*. In soft processes, the mean transverse momentum of the produced hadrons is almost independent of the multiplicity (if one ignores the decrease in the phase space with increasing n) and $\langle p_\perp \rangle = 0.3$–$0.4$ GeV/c. At asymptotic multiplicities, theoretically at superhigh energies, production of a small number of jets is dominant in the critical region $n \sim \bar{n}_j$, and therefore particles are produced by the decay of partons with virtuality $k^2 \sim s$. Since the parton virtualities are $k^2 \sim k_\perp^2$,[9] we find that a growth in n must lead to an appreciable growth in the transverse momenta of the particles. The experimental data given in Fig. 8 indicate a certain growth of the mean transverse momentum with increasing multiplicity.

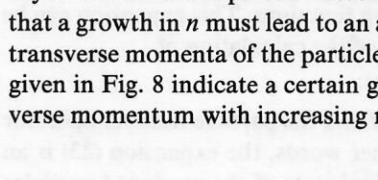

FIG. 9. Expected multiplicity distribution at asymptotic values of the energy.

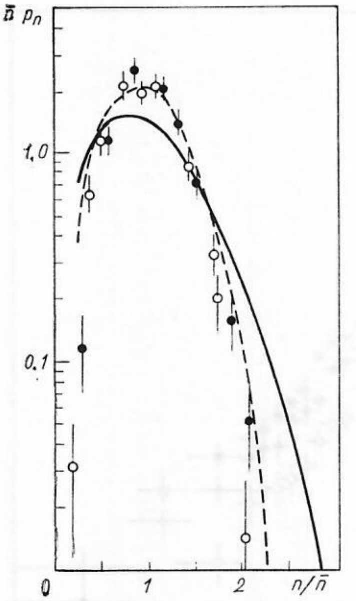

FIG. 10. Distribution with respect to the multiplicity of the charged hadrons in pp interactions and $p\bar{p}$ and e^+e^- annihilation[59]: for e^+e^- annihilation the open and black circles correspond to 9.4 GeV and 29.9–31.6 GeV, respectively; the broken curve is for $p\bar{p}$ annihilation, and the continuous curve is for the pp interaction.

c) *"Flattening" of the tail of the multiplicity distribution.* At high energies, when

$$\bar{n}_j(s) \gg \bar{n}(s), \tag{103}$$

one must see a change in the dependence of σ_n on n [see (102)]. If we use the fit made in Ref. 7, the contributions of the hard processes will be dominant at $\sqrt{s} > 10^2$ GeV. A qualitative picture is shown in Fig. 9.

d) *Universality of the distributions.* Because collective phenomena are important in the production of a very large number of particles, the cross sections are universal in the critical region of multiplicity values. Thus, for $\bar{n}(s) \ll n \ll \bar{n}^2(s)$ the cross section σ_n does not depend on the type of process if the spectrum of resonances is determined by (56). For example, the multiplicity distributions in hadron–hadron interactions and in e^+e^- annihilation must be the same (Fig. 10). A similar picture must be observed in the region (87).

The expression (23), which we have used widely, is an expansion of the density matrix $\rho(\beta, z)$ introduced in (16) with respect to correlation functions. This expansion can be represented as the result of the calculation of

$$\mathrm{Tr}\,(e^{-\beta H_{Z^N}}) = \rho\,(\beta,\ z),$$

where the Hamiltonian H and the particle-number operator N must commute. In other words, the expansion (23) is an attempt to describe the final state of the produced particles in terms of equilibrium statistical mechanics. As we have seen, this representation is convenient, since it enables us to follow the most characteristic collective phenomena of the process of formation of the final state.

As was shown above, in the limit $n \to \infty$, when $z > 1$, if $\tilde{N}_l(p_1, ..., p_l) > 0$ for all p_i, then σ_n will be large. In Sec. 1 it was noted that this is possible, provided the interference between the different contributions to the amplitudes A_n is ignored in the calculation of the cross sections. In terms of perturbation theory, this presupposes dominance of the contributions of the diagrams corresponding to particle production.

Thus, if we restrict ourselves to fluctuations of the fields that lead to particle production, all contributions to the cross section will be positive. In this approximation, the process of particle production has features of a stationary nonequilibrium process, which must be described by the analog of a kinetic equation.

It then turns out that the terms used to describe the degrees of freedom of the interacting hadrons are not important. Thus, it is possible to obtain a self-consistent quantization of Yang–Mills fields that is similar to the expansion in the neighborhood of the trajectory corresponding to the stationary phase of the integrand in (47) with respect to the fields ξ_α. A preliminary exposition of this question is given in Refs. 20 and 21.

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