Multiquark systems in nuclear processes

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Fiz. Elem. Chastits At. Yadra **15**, 1249–1295 (November–December 1984)

Experimental indications of the existence of multiquark systems in nuclei are discussed. A theory of the coupling of the hadron and quark channels in nuclei is formulated and used to calculate the amplitudes of six-quark states in the deuteron and their widths. The influence of multiquark states on the deuteron and ^{3}He form factors and on deuteron polarization is analyzed. A study is made of the part played by multiquark systems in deep inelastic lepton scattering, in particular, the A dependence of the structure function and its behavior at large values of the scale variable x. Comparisons with the appropriate experimental data are made.

INTRODUCTION

The atomic nucleus is a system of interacting nucleons; nucleons are colorless three-quark clusters. Thus, in nuclei and nuclear processes there may arise nontrivial multiquark systems, in the first place $6q$, $9q$, and $12q$ systems. It is natural to expect that in the ground and weakly excited states of nuclei ($E\approx M_A \cong AM$, where E is the energy of the system and M is the nucleon mass) they are small admixtures to the main nucleon channel in the total wave function of the nucleus:

$$\Psi = \Psi\,(AN) + C\Psi\,(3Aq), \qquad (1)$$

i.e., $|C(E\approx AM)|^2 \ll 1$. The probability of such admixtures can be estimated as the probability that k nucleons (a flucton) of a "nucleon gas" of particles are in a small volume V_ξ (Ref. 1):

$$\beta_k^A = \binom{A}{k}(V_\xi/AV_0)^{k-1} = \binom{A}{k}(r_\xi/r_0)^{3(k-1)}/A^{k-1}. \qquad (2)$$

Here, $r_0 = 1.2$ F is the mean range of the nucleon–nucleon interaction in the nucleus, and the parameter $r_\xi = 0.75$ F is of the order of the core radius on the NN forces. It is natural to relate the latter to the confinement radius. The flucton idea proved to be helpful in the description of nuclear reactions at high energies. For example, it made it possible to interpret for the first time deuteron knockout from nuclei at large momentum transfers.[2] Later, the idea of cumulative reactions, which was put forward in Ref. 3, was realized using flucton ideas (see, for example, Refs. 4 and 5); on the same basis, using quark counting rules,[6] nuclear form factors at large momentum transfers and deep inelastic scattering by nuclei were calculated (for example, Ref. 7).

The nature of fluctons can be understood if one introduces the concept of multiquark systems. However, the first calculations of multiquark bags[8,9] showed that their masses greatly exceed the masses of the corresponding nuclei, i.e., $E_\lambda = M_A + \Delta_\lambda$, where $\Delta_\lambda \geqslant 0.2$ GeV. Thus, these states of multiquark systems are specific and must be manifested as resonances in the corresponding scattering amplitudes. In this region of energies, the total nuclear wave function must have the form

$$\Psi \approx C\Psi\,(3Aq), \qquad (3)$$

where $|C(E\approx E_\lambda)|^2 \approx 1$. At the present time there are indications of a resonance behavior of the $^1D_2$ and $^3F_3$ nucleon–nucleon phase shifts in the region of total energy of the system around $E_\lambda \approx 2.2$ GeV.[10]

Thus, the ground states of nuclei may have small admixtures of multiquark systems, and pure multiquark systems may be manifested with appreciable probability at energies 0.2 GeV and more above the masses of the corresponding nuclei in the form of dibaryons, tribaryons, etc.

1. EXPERIMENTAL INDICATIONS OF THE EXISTENCE OF MULTIQUARK SYSTEMS

Cumulative processes

Cumulative nuclear reactions have by now been well studied (see, for example, Refs. 11 and 12). Recently,[13] a large body of experimental material was used to make an overall analysis of the cross sections for cumulative meson production in the reactions $p(8.9\ \text{GeV}/c) + A \to c + X$ ($c = \pi^\pm, K^\pm$). It was found, first, that in such processes limiting fragmentation of the nuclei begins at $E \gtrsim 4$ GeV, and the corresponding cross section can be parametrized in the factorized form

$$E_c\frac{d\sigma}{d\mathbf{p}_c} = f\,(A, p_{\perp c}^2, X) = A^n\,\Phi\,(p_{\perp c}^2)\,G\,(x), \qquad (4)$$

where A is the atomic number of the target nucleus, and

$$\Phi\,(p_\perp^2) = 0.9\exp\,(-2.7p_\perp^2) + 0.1; \qquad (5)$$

$$G\,(x) = G_0\exp\,(-x/\langle x\rangle);\ \langle x\rangle = 0.14. \qquad (6)$$

At high energies, the scale variable is $x = (p_\parallel^c + E_\parallel^c)/(p+E)$.

Assuming that the momentum P is distributed in a multiquark system consisting of k nucleons of the nucleus, i.e., $P = kP_0(E\approx kE_0)$, where P_0 is the momentum per nucleon of the multiquark system, we obtain the usual longitudinal scale variable for one target nucleon, $x = kX$. From this it can be seen, second, that the cumulative region is the region

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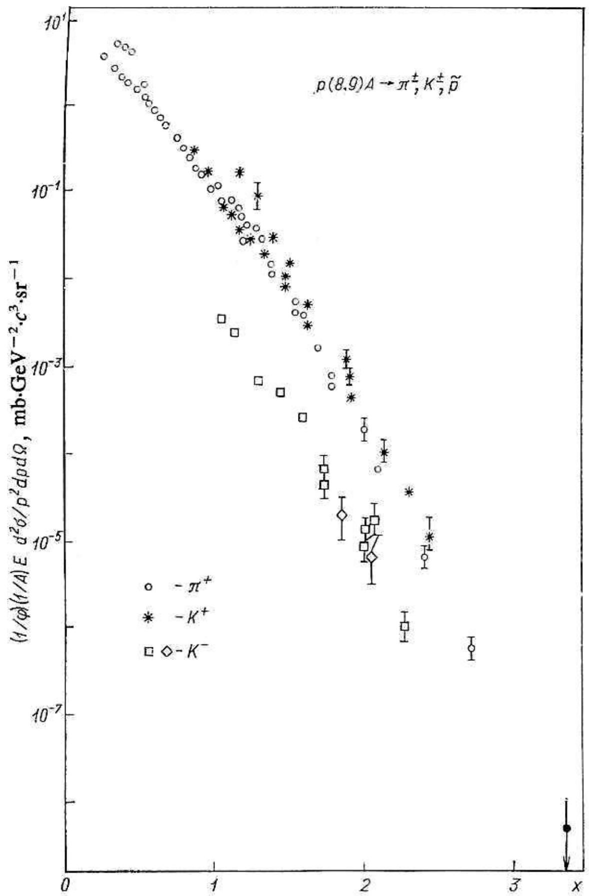

FIG. 1. Invariant cross sections for the production of cumulative mesons in pA collisions at $E_0 = 9.54$ GeV.

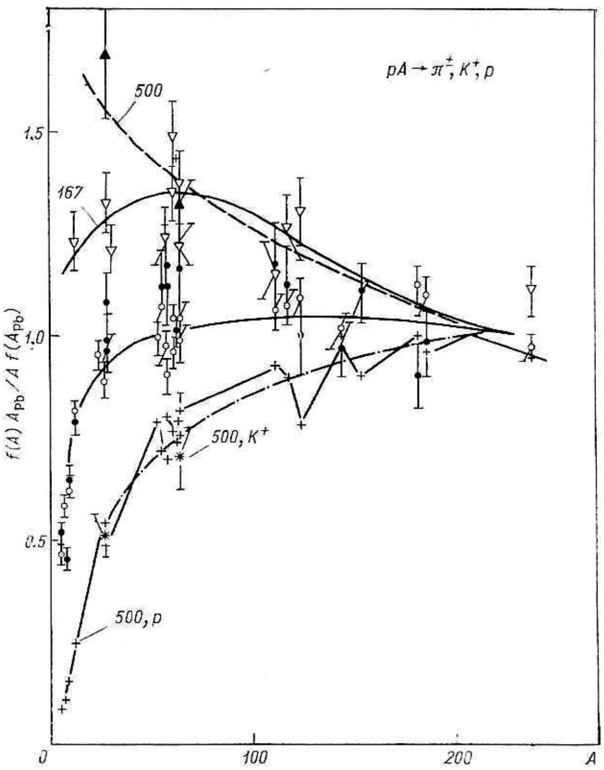

FIG. 2. The ratios $f(A)A_{\rm Pb}/Af(A_{\rm Pb})$ as functions of the atomic number A for different momenta and emission angles of particles produced in pA interactions. The open circles and the black circles are the data for π^- (168°) and π^+ (180°) with momentum 500 MeV/c; the black triangles are for pions with momentum 500 MeV/c and emission angle 90°; the open inverted triangles are for pions with momentum 167 MeV/c and angle 168°; the pluses are for protons with momentum 500 MeV/c, 180°; and the stars are for pions with $p = 500$ MeV/c, 168°.

kinematically forbidden for interaction with one nucleon of the nucleus, i.e., $x > 1$ (in principle $k > 1$ now and in the limit $k = A$ we have $x \leqslant A$). We can now say that x has the meaning of the part of the mass of the object that corresponds to the nucleon mass. From Fig. 1 it can be seen[13] that the process is observed at x values ranging up to 3 and above. This means that the incident proton interacts with a multiquark system of mass $M_k = kM\,(k \gtrsim 3)$. At the same time, the large momentum transfer in the reaction ($\sim 2\ ({\rm GeV}/c)^2$) means that the multiquark system is distributed in a small volume of radius $r_\xi \lesssim 1$ F. Third, it is found that the nuclear structure function $G(x)$ is universal and that its slope is determined by the constant $\langle x \rangle = 0.14$. Fourth, there is a "volume dependence" of the cross section: $Ed\sigma/d\mathbf{p} \approx A^1$, i.e., $n = 1$ in (4) (Fig. 2, Ref. 13). This indicates that a multiquark system is not a "tube" in the nuclear matter (in this case there would be an $A^{2/3}$ dependence) but a three-dimensional formation of strongly compressed nucleons of a cluster type that exists in the nucleus independently of the reaction in which it participates. Finally, the individual properties of the nucleus are manifested in small deviations from the trivial A^1 dependence. This can be seen in Fig. 3 (Ref. 14), in which we have plotted the ratios of the cross sections for different nuclei, divided by the corresponding mass number. Since in a deuteron there cannot be more than six quarks, and in other nuclei there can also be multiquark systems of 9–12 quarks, the deuteron cross section in the kinematic region $x \gtrsim 2$ vanishes, while the others will be nonzero. This gives a sharp rise in the ratios of these last cross sections relative to the deuteron cross section in the region $x > 1$ (Fig. 3).

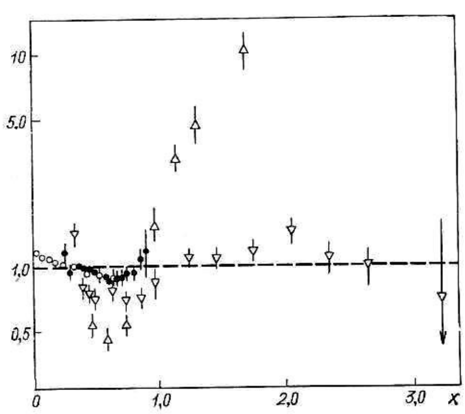

FIG. 3. Ratios of invariant cross sections: open circles: $\sigma^{\rm Fe}/\sigma^{\rm D}(\mu A \to \mu' + \ldots)$; black circles: $\sigma^{\rm Fe}/\sigma^{\rm D}(eA \to e' + \ldots)$; open triangles: $\sigma^{\rm Pb}/\sigma^{\rm AL}(pA \to \pi + \ldots)$; open inverted triangles: $\sigma^{\rm Pb}/\sigma^{\rm D}(pA \to \pi + \ldots)$.

Elastic and deep inelastic lepton scattering

The mechanism of electromagnetic scattering of leptons is well known, and therefore investigation of processes with $x > 1$ can give very important information about multiquark systems. There are currently available SLAC data on deep inelastic electron scattering on ^2D (Ref. 15) and $^{3,4}He$ (Ref. 16) and preliminary data on $(\mu\mu')$ on the nucleus ^{12}C up to $x \lesssim 1.4$.[17] We may mention that predictions of the behavior of the ^{12}C structure function in the region $x \approx 1$ were made in Ref. 18 on the basis of the dependence (6) obtained from hadronic reactions. For muon deep inelastic scattering there was also found to be a small deviation from the A^1 dependence of the type shown in Fig. 3 in the ratio of the structure functions of the nucleons of iron nuclei and the deuteron (EMC effect[19]).

With regard to the nuclear form factors, the idea of, for example, $6q$ admixtures in the deuteron is rather attractive in attempts to explain the behavior of the deuteron form factor at large $q^2 \gtrsim 2$ (GeV/c)^2.[20] It appears that $9q$ admixtures are also needed to describe the $^{3,4}He$ form factors.[21] Six-quark components in the deuteron wave function were also needed in the description of the momentum distribution of the nucleons obtained in deuteron–nucleus collisions with protons emitted at small angles.[22]

In conclusion, it can be said that in nuclei one expects the existence of multiquark systems irrespective of the reaction in which the nucleus itself participates. Their mass is $M_k = kM$ and they have a linear dimension of the order of the core radius of the NN forces, so that these systems are best studied in processes with large momentum transfer. It is of interest to establish the specific features of the distributions of the quark–parton functions of the multiquark systems, their q^2 dependence, the internal structure, and the part played by such systems in the understanding of nuclear forces (QCD for multiquark systems at short and long distances). It is possible that analysis of the details in the behavior of these systems will also be helpful in studying the fundamental properties of elementary particles.

2. HADRON AND QUARK CHANNELS IN NUCLEI

The mutual influence of the nucleon and multiquark degrees of freedom in nuclei can be investigated theoretically in different ways. One way is the phenomenological, in which it is assumed that in the wave function (1) of the nucleus the first nucleon nuclear part of the function is known, while the second—the quark part— is specified on the basis of some model with weight $|C|^2$, which is a parameter of the problem. In principle, the probability $|C|^2$ can be estimated using the fluctuation formula (2), i.e., one can assume $|C_k|^2 = \beta_k^A$ (see, for example, Ref. 4).For the problems in which it is necessary to know the function (1) only in the nucleon channel (for example, to study the NN phase shifts at energies below 1 GeV), one can attempt to introduce information about the "internal," multiquark region into the logarithmic derivative of the function of the nucleon channel. (This direction was investigated, for example, in Ref. 23.) Below, we shall obtain a system of equations that couple these two channels. Their solution makes it possible to calculate the probability amplitudes C_k of the admixtures of the multiquark systems in nuclei and the widths for the decay of these systems to the hadron channel.

Formulation of the system of equations

The problem is therefore as follows: to add to the known nuclear function with frozen three-quark clusters—nucleons—a multiquark function corresponding to the distribution of quarks within a common volume (multiquark bag) in which there are no separated nucleon clusters.

To find the nuclear and quark parts of the function, it is necessary to know the interactions within the nucleon and quark phases of nuclear matter, and also between them. We shall assume that the interaction potentials V_{NN} and V_{qq} are known from nuclear-physics and elementary-particle models, respectively, while the interaction V_{qN}—between the nucleons and the quarks—is the subject of investigation.

If we initially ignore V_{qN}, then the two phases—the nuclear and the quark phases—exist independently. The nucleon interaction V_{NN} determines the usual problem of nuclear physics, i.e., the behavior of a system of A nucleons, while the quark interaction V_{qq} gives states of bag type for the multiquark systems—a new problem for nuclear physics. If in some manner we introduce an interaction between the quark and nucleon channels, we can obtain an estimate of the admixtures of the multiquark states in nuclei and the widths of decay of the multiquark states to the nucleon and meson channels.

As a concrete problem, we consider the $6q$ system for the example of two interacting nucleons.[24–26] In principle, such a system may also be formed within a nucleus. The generalization to the case of $9q$ and $12q$ systems does not present fundamental difficulties.

We shall develop the approach in the spirit of the so-called interpolation method of nuclear physics.[27] For this, we construct from the original $3n-3$ relative coordinates of the n quarks a "global coordinate" ρ, which is widely used in the method of hyperspherical functions[28]:

$$\boldsymbol{\rho} = (\rho,\ \Omega_{3n-4}), \tag{7}$$

$$\rho^2 = \frac{1}{n}\sum_{i>j}^{n}(\mathbf{r}_i-\mathbf{r}_j)^2 = \sum_{i=1}^{n}(\mathbf{r}_i-\mathbf{R})^2 = n\mathbf{r}^2;\ \mathbf{R} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{r}_i, \tag{8}$$

where $\mathbf{r}$ is the mean distance between the quarks in the system. If the sytem is divided into two clusters, $n_1 + n_2 = n$, with corresponding global coordinates ρ_1 and ρ_2, then

$$\rho^2 = \rho_1^2 + \rho_2^2 + z_{12}^2, \tag{9}$$

$$\mathbf{z}_{12} = \sqrt{n_1 n_2/n}\,\mathbf{R}_{12}, \tag{10}$$

and at relatively large distances R_{12} between the centers of mass of the clusters the quantity ρ can be associated with this distance.

Thus, we represent the wave function of the multiquark system in the form

$$\Psi = \Psi_{\rm ext} + \Psi_{\rm in}. \tag{11}$$

Here, Ψ_{in} is a function of compound-system type that determines the multiquark system in the interior region (bag):

$$\Psi_{\text{in}} = \sum_\lambda C_\lambda \Psi_\lambda, \tag{12}$$

where Ψ_λ are the eigenfuctions of the part H_0 of the total Hamiltonian H that acts in the interior region and contains the potential $V_0 = \Sigma V_{ij}$ of the interaction of quarks i and j, i.e.,

$$H_0 \Psi_\lambda = E_\lambda \Psi_\lambda. \tag{13}$$

We shall define the interior region (nominally $\rho < \rho_0$) as the region of localization of all the quarks of the system; the function Ψ_{ext} describes well the multiquark system in the "exterior" region, where, for example, the $6q$ system is separated into nucleon clusters consisting of colorless quark triplets:

$$\Psi_{\text{ext}} = \omega(\boldsymbol{\rho})\, \Phi(\rho), \tag{14}$$

$$\omega(\boldsymbol{\rho}) = \hat{A}_{12}\{Z^L Y_{LM}(\hat{z})\, \varphi_1(\boldsymbol{\rho}_1)\, \varphi_2(\boldsymbol{\rho}_2)\}. \tag{15}$$

Here, $\hat{A}_{12}$ is the operator of permutation of quarks belonging to the different cluster functions φ_1 and φ_2. These last functions are solutions of an equation of the type (3) for the three-quark system, i.e., the nucleon.

Thus, the unknown parts of the total wave function Ψ are the coefficients C_λ, which determine the admixture of the truly $6q$ states of the system, and $\Phi(\rho)$, which play the part of the wave functions of the relative motion of the nucleons. These quantities are determined as the solutions of the corresponding system of coupled equations, which can be obtained by using the equation for the total function

$$(H - E)\Psi = [H_0 + (V - V_0) - E]\Psi = 0. \tag{16}$$

Here, V is the exact potential, which acts in the complete region of the space $\boldsymbol{\rho}$; in the interior region, we assume that it is equal to V_0. Substituting in (16) the expressions (11) and (12), multiplying it from the left by Ψ_λ^* and integrating over $d\boldsymbol{\rho} = \boldsymbol{\rho}^\Lambda d\Omega d\rho$, where $\Lambda = 3n - 4$, and using (13), we obtain for the admixture coefficient

$$C_\lambda = C_\lambda^0 - \gamma_\lambda = -\frac{\alpha_\lambda}{E_\lambda - E} - \gamma_\lambda, \tag{17}$$

where

$$\gamma_\lambda = \langle \Psi_\lambda | \Psi_{\text{ext}} \rangle, \tag{18}$$

$$\alpha_\lambda = \langle \Psi_\lambda | V - V_0 | \Psi_{\text{ext}} \rangle. \tag{19}$$

It can be seen that C_λ are determined by the overlap of the functions of the interior and exterior motions in the system, γ_λ, and by the matrix element α_λ of the transition between the nucleon and quark phases of the nuclear matter. Moreover, both matrix elements form the region of space near the surface $\rho \approx \rho_0$, where the functions C_λ *and* Ψ_{ext} overlap maximally.

The expression (17) is the first of the system of equations that determine the mutual dependence of C_λ and Ψ_{ext}. For what follows, it is convenient to put it into a different form. To this end, in the function $\Psi_{\text{ext}} = \omega(\boldsymbol{\rho})\Phi(\rho)$ we make the change of variable

$$\Phi(\rho) = \varphi(\rho)/\sqrt{\mu}, \tag{20}$$

where

$$\mu = \rho^\Lambda \int d\Omega\, \omega^+(\boldsymbol{\rho})\, \omega(\boldsymbol{\rho}). \tag{21}$$

Then Eq. (17) becomes

$$-(E_\lambda - E)\, C_\lambda = \int d\rho\, \varphi(\rho)\, \tilde{D}_\lambda^*(\rho), \tag{22}$$

where

$$\tilde{D}_\lambda = (E_\lambda - E)\, B_\lambda + D_\lambda. \tag{23}$$

$$B_\lambda = \int d\Omega \rho^\Lambda \omega^+ \Psi_\lambda / \sqrt{\mu}, \tag{24}$$

$$D_\lambda = \int d\Omega \rho^\Lambda \omega^+ (V - V_0)\, \Psi_\lambda / \sqrt{\mu}. \tag{25}$$

Thus, the first of the coupled equations (22) determines the coefficient of admixture of the $6q$ component in the total wave function of the system.

The second equation, which determines the wave function $\Phi(\rho)$ of the relative motion of the clusters in the hadron channel, is obtained by multiplying (16) by ω^+ and integrating over the hyperangles $d\,\boldsymbol{\Omega}$. It can be shown that it takes the form[26]

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + U_{\text{eff}}(\rho) - \varepsilon\right)\varphi(\rho) = -\sum_\lambda C_\lambda \tilde{D}_\lambda, \tag{26}$$

where the effective potential is

$$U_{\text{eff}} = \bar{U} - \frac{\hbar^2}{2m} X = \bar{U} + \frac{\hbar^2}{2m\sqrt{\mu}}\left(\frac{\mu'}{\sqrt{\mu}}\right)', \tag{27}$$

and the potential

$$\bar{U} = \frac{\rho^\Lambda}{\mu}\int d\Omega\omega^+ (T + V - 2M)\, \omega = \frac{\int d\Omega\omega^+ (H - 2M)\, \omega}{\int d\Omega \omega^+ \omega} \tag{28}$$

depends on ρ and for clusters separated in space is the potential of their relative motion. The energy of the relative motion of the clusters is

$$\varepsilon = E - 2M, \tag{29}$$

where M is the nucleon mass. The second term in the potential U_{eff} of Eq. (26) gives rise to a soft repulsion of the nucleons at short distances of the form $\mathscr{L}(\mathscr{L} + 1)/\rho^2$, where $\mathscr{L}$ is a large number ($\mathscr{L} \geqslant 3n/2 - 1$). This is analogous to the repulsion of nuclei shifted to short distances that arises on account of the Pauli principle for the nucleons as Fermi particles.[27] The quark approaches now make it possible to calculate as well the mean potential $\bar{U}$ of the NN interaction, which is usually also repulsive at short distances.[9,29,30]

Discussing the system (22), (26) overall, we note that because the functions of the multiquark system and the hadron channel are nonorthogonal the right-hand sides of these equations contain a term proportional to $(E_\lambda - E)B_\lambda$. It plays an important part in estimating the "admixture" C_λ. In particular, if the nonorthogonality is large, then it alone determines, for example, the admixture of the $6q$ system in the deuteron. Then

$$|C_\lambda^D|^2 = \left|\int \Psi_\lambda^* \Psi^D d\tau\right|^2. \tag{30}$$

Since the quark function in the case of the s state does not change sign in the interior region and decreases rapidly at its boundary r_0, we can write approximately

$$|C_0^D|^2 \simeq \left(\frac{4}{3}\pi r_0^3\right)^{-1} \left|\int_0^{r_0} \Psi^D 4\pi r^2 dr\right|^2 \simeq 4\pi \int_0^{r_0} |\Psi^D|^2 r^2 dr. \quad (31)$$

Such an expression was proposed for the first time by Blokhintsev[1] in 1957 to calculate the deuteron fluctuations in the region of the core of the NN forces. It has now become clear that the "Blokhintsev nuclear flucton" is essentially a multiquark system. Later, $|C^D|^2$ was calculated in accordance with Eq. (30) but with allowance for quark antisymmetrizations.[31,32] The admixtures of multiquark systems with $n \leqslant 12$ quarks in the lightest nuclei were calculated in Refs. 4 and 29 in accordance with an expression analogous to (30).

The position of quark–nuclear physics. The choice of the interactions

We shall analyze the equations that couple the nucleon and multiquark channels in nuclei:

$$-(E_\lambda - E)\, C_\lambda = \int d\rho\varphi(\rho)\, \tilde{D}_\lambda^*(\rho), \quad (32)$$

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + U_{\text{eff}} - \varepsilon\right)\varphi(\rho) = -\sum_\lambda C_\lambda \tilde{D}_\lambda. \quad (33)$$

In principle, if the quark–quark interaction in a multiquark system is assumed known (as yet, it has been studied only for $q\bar{q}$ and $3q$ systems), one can find the functions Ψ_λ of the bag states and, hence, the effective interaction potential U_{eff} in the nucleon channel and the interaction of the channels $\tilde{D}_\lambda$. This completely determines the system (32)–(33), making it possible to find the wave function $\varphi(\rho)$ of the nucleons of the nucleus and the admixture $C_\lambda \Psi_\lambda$ of the multiquark state. We have already noted that in the ground and weakly excited states of the nucleus $|C_\lambda|^2 \ll 1$. This means that for such energies the right-hand side in Eq. (33) can be ignored, and the equation itself can, after introduction of the effective nucleon mass in the nucleus, $m_{\text{eff}} = 3m(A-1)/A$, and transition to "nuclear distances" between the nucleons, $r \approx \rho\sqrt{A/3(A-1)}$, be put in the form typical of nuclear physics:

$$\left(-\frac{\hbar^2}{2m_{\text{eff}}}\frac{d^2}{dr^2} + U(r) - \varepsilon\right)\varphi(r) = 0. \quad (34)$$

Here, U is the nuclear potential, which is usually specified phenomenologically.

The inclusion of the quark–nuclear interaction has qualitative consequences. Thus, if there exists a spectrum of multiquark systems—dibaryon, tribaryon, etc.—then in the hadron channels at these energies resonances or other features must be observed in the cross sections. Further, if it is assumed that multiquark systems exist in a region bounded by the bag radius r_0, then they will be manifested at large momentum transfers $q > q_0 \sim \hbar/r_0$. Finally, the bag functions Ψ_λ will, in principle, determine the quark–parton distributions in the multiquark systems, i.e., the structure functions of the nuclei will, in particular, depend on them.

A few words about the choice of r_0. It may be that the choice of a large bag radius of the order of the nucleon radius (or the mean internucleon distances in a nucleus) nevertheless makes it possible to imitate the periphery of the NN forces, whose behavior there is apparently determined by meson exchanges. But if these exchanges are taken into account explicitly, by specifying, for example, phenomenological realistic forces at "large distances," then it is natural to identify r_0 with the radius of the quark core of the nucleon. This concept corresponds to the ideas of the so-called little bags surrounded by a cloud of pions.[33,34] Below, for estimates and calculations we shall take in the nucleon–nucleon channel a core radius $r_0 = 0.8$ F, which corresponds to a quark core $r_0^N = 0.4$ F of the nucleon.

Concretely, we choose the interaction potential in the two-nucleon system on the basis of the following considerations. We assume that the potential V_0, which determines the interior $6q$ state in the nucleon–nucleon system, is a sum of two-body qq potentials[35]:

$$V_0 = \sum_{i>j=1}^{6} V_{ij};\; V_{ij} = V_{ij}^c + V_{ij}^r; \quad (35)$$

$$V_{ij}^c = -\frac{\varkappa_c}{2}\lambda_i^c\lambda_j^c(\mathbf{r}_i - \mathbf{r}_j)^2;\; V_{ij}^r = -\mu(\lambda^a\sigma)_i(\lambda^a\sigma)_j. \quad (36)$$

Here, λ^a are the Gell-Mann matrices of the $SU(3)$ color group. The parameter $\varkappa_c$ is related to Ω by $\varkappa_c = \Omega^2/(288 m_q)$ ($\hbar = c = 1$) and for $m_q = 0.3$ GeV and $\Omega = 1$ (GeV/c)^2 has the value 1.16×10^{-2} GeV^3. The parameter $\mu = 75/4$ MeV is chosen to reproduce the color magnetic splitting of the N–Δ masses.

The true potential V that acts in the complete region of space must be equal to the quark potential V_0 for $r < r_0$ and in the exterior region $r > r_0$ must determine the motion of the separated nucleons. Thus,

$$V = V_0\theta(r_0 - r) + \left[V^M(r) + \sum_{i>j=1}^{3} V_{ij} + \sum_{i>j=4}^{6} V_{ij}\right]\theta(r - r_0), \quad (37)$$

where V^M is the "tail" of the meson-exchange NN potential. We shall take the radius of the $6q$ bag equal to twice the radius of the quark core of the nucleon: $r_0 \approx 2r_0^N = 0.8$ F.

Thus, all the interactions are determined, and we can make estimates of the admixtures of the $6q$ system in the deuteron and the widths of dibaryon decay to the nucleon–nucleon channel.

3. SIX-QUARK ADMIXTURES IN TWO-NUCLEON SYSTEMS

Amplitude of six-quark states in the deuteron

To estimate the $6q$ admixtures in the deuteron, we use the experimental fact that these admixtures are small, i.e., $|C_\lambda|^2 \ll 1$. Then the right-hand side of Eq. (33) is negligibly small, and the system (32)–(33) decouples. In this case, Eq. (33) describes the ordinary weakly bound state of the deuteron, and Eq. (32) or (17) is used to calculate the admixture C_λ on the basis of the deuteron function obtained from (33). For practical calculations, it is convenient to specify the function of the NN system not in its dependence on the global coordinate and the hyperangles but in the usual form

$$\Psi_{\text{ext}}^M = \hat{A}\{\varphi_n\varphi_p\Psi_{np}^M(\mathbf{r})\}, \quad (38)$$

where the deuteron function in the field of the phenomenological potential Ψ^M_{np} depends on the distance **r** between the centers of mass of the nucleons, and the antisymmetrization operator is

$$\hat{A}=\frac{1}{\sqrt{10}}\left(1-\sum_{\substack{i=1,2,3\\ j=4,5,6}}\hat{P}_{ij}\right). \quad (39)$$

Here, $\hat{P}_{ij}$ exchanges quarks belonging to different nucleons in the color, spin–isospin, and coordinate spaces, so that

$$\hat{P}_{ij}=\hat{P}^c_{ij}\hat{P}^{SI}_{ij}\hat{P}^X_{ij}. \quad (40)$$

The antisymmetrization of Ψ^M_{ext} with respect to permutation of the nucleons themselves is taken into account in the function of the relative motion,

$$\Psi^M_{np}(\mathbf{r})=\frac{u(r)}{r}\frac{1}{\sqrt{4\pi}}\chi_M+\frac{w(r)}{r}\sum_{m,\mu}(2m1\mu|1M)Y_{2m}\chi_\mu, \quad (41)$$

where

$$\chi_\mu=\sum(1/2\,\sigma_1\,1/2\,\sigma_2|J_\mu)(1/2\tau_1\,1/2\,\tau_2|I\tau)\Phi_{\sigma_1\tau_1}\Phi_{\sigma_2\tau_2}, \quad (42)$$

and $\Phi_{\sigma\tau}$ are the spin–isospin functions of the nucleons with spin and isospin projections σ and τ, respectively. The nucleon functions $\varphi_{n,p}$ include a symmetric spatial part and an antisymmetric color part:

$$\varphi_n=\Phi_n(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\Phi^c_n;\ \varphi_p=\Phi_p(\mathbf{r}_4,\mathbf{r}_5,\mathbf{r}_6)\Phi^c_p \quad (43)$$

where $\mathbf{r}_k$ are coordinates of the constituent quarks. We choose the wave functions of these quarks in the form of harmonic-oscillator functions,

$$\Phi_n=\left(\frac{\Omega}{\pi}\right)^{3/2}\exp\left(-\frac{\Omega}{2}(\xi_1^2+\xi_2^2)\right);$$
$$\Phi_p=\left(\frac{\Omega}{\pi}\right)^{3/2}\exp\left(-\frac{\Omega}{2}(\eta_1^2+\eta_2^2)\right), \quad (44)$$

where the Jacobi coordinates are defined as

$$\left.\begin{array}{l}\xi_1=(\mathbf{r}_2+\mathbf{r}_3-2\mathbf{r}_1)/6;\eta_1=(\mathbf{r}_5+\mathbf{r}_6-2\mathbf{r}_4)/6;\\ \xi_2=(\mathbf{r}_3-\mathbf{r}_2)/2\sqrt{3};\ \eta_2=(\mathbf{r}_6-\mathbf{r}_5)/2\sqrt{3}.\end{array}\right\} \quad (45)$$

Below, we choose the parameter $\Omega=1$ (GeV/c)^2, this giving a "quark radius" of the nucleon $r_0^N\simeq 2/\sqrt{\Omega}\simeq 0.4$ F and permitting description of the data on elastic pp (Ref. 36) and ep (Ref. 37) scattering at large momentum transfers [$q^2>1$ (GeV/c)^2] for a wave function relativized in a definite manner. For this value of Ω, the rms charge radius of the nucleon is less than the experimental value. For correct calculation of it, it is necessary to take into account the contribution of the meson fields, which must be dominant at comparatively large distances.[33,34] In our case, in the solution of the nucleon–nucleon interaction problem, the meson fields are taken into account by specifying the realistic nucleon–nucleon potential in the exterior region, so that in the interior region only the quark–quark interaction remains. To calculate the admixture C_λ, we use Eq. (17), where in the total deuteron energy $E=2M+\varepsilon$ we ignore the small deuteron binding energy $\varepsilon=-2.23$ MeV.

We consider the case when all the quarks belong to the s^6 configuration. Then the function of the interior motion of the quarks can be written as

$$\Psi_{\lambda_1}=\Phi_n\Phi_p\Phi_1\,|\,cSI\rangle_{\lambda_1}, \quad (46)$$

where $\Phi_{n,p}$ are given by the relations (44), and

$$\Phi_1=\left(\frac{\Omega}{4\pi}\right)^{3/4}\exp(-\Omega r^2/8). \quad (47)$$

As hitherto, we shall assume that the antisymmetrization with respect to the quarks within the nucleons is taken into account, and therefore we write out explicitly only the part of the operator of antisymmetrization with respect to quarks belonging to different $3q$ clusters; then

$$|cSI\rangle_{\lambda_1}=\frac{3}{\sqrt{10}}\frac{1}{\sqrt{10}}\left(1-\sum_{\substack{i=1,2,3\\ j=4,5,6}}\hat{P}^c_{ij}\hat{P}^{SI}_{ij}\right)|cSI\rangle_{np}. \quad (48)$$

Note that for the $6q$ system the parameter $\Omega=\Omega_6$ differs somewhat from the corresponding parameter $\Omega=\Omega_3$ for the $3q$ system because of the presence of the factor $\lambda_i\lambda_j$ in the potential (36), namely, $\Omega_6=\sqrt{4/5}\Omega_3\simeq 0.9\Omega_3$. However, we shall ignore this difference, assuming $\Omega_6=\Omega_3=\Omega$. We specify similarly the function Ψ_{λ_2} for the $6q$ configuration [s^4p^2], where Φ_2^2 contains polynomial corrections.[26] Then for the first term of the $6q$ amplitude (22) we obtain explicit but cumbersome expressions, which we shall not give here (they are given in Ref. 26). The expressions for the nonorthogonality factor γ_λ in (17) are

$$\gamma_{\lambda_1}=\gamma(s^6)=\sqrt{\frac{10}{9}}\sqrt{4\pi}\int_0^\infty\Phi_1 ur\,dr;\ \gamma_{\lambda_2}$$
$$=\gamma(s^4p^2)=\sqrt{\frac{3}{2}}\sqrt{4\pi}\int_0^\infty(1-\Omega r^2/6)\Phi_1 ur\,dr. \quad (49)$$

The function $u(r)$ has been calculated in the field of phenomenological NN potentials: the Hulthén potential, the Reid potential with a hard core,[38] and the Feshbach–Lomon potential.[39] The nonrelativistic oscillator model with a large parameter Ω (small bag radius) does not pretend to a correct description of the masses M and E_λ. Therefore, the gap separating the hadron and quark phases of the system was chosen in accordance with the calculations of realistic models, namely, $\Delta\simeq\Delta_1\simeq\Delta_2=300$ MeV. The results of the calculations are given in Table I.

It can be seen that the values of C_λ for the Reid and Feshbach–Lomon potentials are close to each other and differ fairly strongly from the result that the Hulthén potential gives, especially for [s^6] states. Further, the contribution γ_λ to the amplitude C_λ is large and may exceed by several times the contribution C^0_λ, particularly for the more complicated configurations. The total probability $C^2=C_1^2+C_2^2$ of the two configurations is 6.6×10^{-2}, this agreeing in general with what is obtained from the analysis of data on elastic eD scattering[40] and on cumulative processes with large momentum transfer,[4] and also with theoretical estimates obtained in other approaches.[9,31,32] Note that if no allowance is made for the nonorthogonality of the quark and nucleon components of the total deuteron function, the $6q$ admixture is much less: $C^2\approx 0.5\times 10^{-2}$.

To conclude this subsection, we obtain an estimate of η, which characterizes the strength of the transition between the hadron and quark phases in the NN system. For this, using (17) and (22), we obtain

TABLE I. Amplitudes of 6q states.

Configuration	Amplitude of 6q admixtures	Hulthén	Reid	Feshbach–Lomon
$[s^6]$	C_1^0 $-\gamma_1$ C_1	−0.082 −0.29 −0.37	−0,058 −0,088 −0.147	−0,070 −0,062 −0.132
$[s^4p^2]$	C_1^0 $-\gamma_2$ C_2	+0,01 +0.25 0.26	−0,033 +0.24 0,21	−0.036 +0,22 0,18

$$C_\lambda^0 = -\frac{1}{\Delta_\lambda}\int d\rho D_\lambda^* \varphi. \tag{50}$$

We have already noted that such a phase transition takes place predominantly in the region of the surface $\rho \approx \rho_0$ or $r \approx r_0$, i.e., the matrix element D_λ can be represented as

$$D_\lambda = \eta_\lambda \delta(r - r_0). \tag{51}$$

Then, substituting (51) in (50) and making the substitutions $\rho \to r$ and $\varphi \to u(r)$, we obtain from (41) an estimate of the parameter η_λ, which determines the transition strength:

$$\eta_\lambda = -C_\lambda^0 \Delta_\lambda / u(r_0).$$

Choosing $r_0 \approx 0.8$ F and substituting here $\Delta_\lambda = 300$ MeV and C_λ^0 from Table I for the Reid potential with a hard core, and also the numerical value of the deuteron function $u(r_0)$ at the point r_0 for this potential, we obtain

$$\eta_{\lambda_1} = 12.8 \cdot 10^{-2} (\text{GeV})^{1/2}; \quad \eta_{\lambda_2} = 7.26 \cdot 10^{-2} (\text{GeV})^{1/2}. \tag{52}$$

This estimate will be needed in what follows in calculating the widths of decay of the 6q system to the NN channel.

Widths of six-quark states in the nucleon–nucleon channel

The system of equations that couple the nucleon and quark channels make it possible to estimate the widths of decay of the 6q states to the NN channel. This is very important for studying the possible existence of a (quasi)stable dibaryon as a 6q system. We recall that ordianry quark spectroscopy gives zero widths of the 6q states, whereas analysis of the experiments leads to widths of the order of several tens of mega-electron-volts.

The solution of the system (22), (26) in the continuum $\varepsilon > 0$ is such that near the resonance $E \approx E_\lambda$ the smallness of the quantities $(E - E_\lambda)$ on the left-hand side of Eq. (22) must be compensated by a large (in general, complex) C_λ. Thus, from the very beginning it is impossible to decouple Eqs. (22) and (26) as in the case of the bound state of the NN system. To solve the system, we again proceed in the spirit of the interpolation method.[27,28] In Refs. 25 and 26, examples were given of such a solution in a more general case. Here, we shall consider the special case of a δ interaction between the quark and nucleon channels [see (51)] under the assumption that one 6q state λ exists. Then we write the general solution of Eq. (26) in the form ($\hbar = 1$)

$$\varphi_l = \chi_l(r) + 2mC_\lambda \int dr' G_l^{(+)}(r, r') \tilde{D}_\lambda(r')$$
$$= \chi_l(r) + 2mC_\lambda \eta_\lambda G_l^{(+)}(r, r_0), \tag{53}$$

where $G_l^{(+)}$ is the Green's function of Eq. (26). Here, instead of ρ we use the variable r of the relative distance between the nucleons, and m is the reduced mass of the colliding nucleons. In the expression (22) for $\tilde{D}_\lambda$, we have ignored terms of the type $\gamma_\lambda(E - E_\lambda)$, which are small in the resonance region, so that $\tilde{D}_\lambda \approx D_\lambda = \eta_\lambda \delta(r - r_0)$. Substituting (53) in Eq. (22), we obtain in the framework of the original assumptions the following expression for the amplitude:

$$C_\lambda = -\frac{\eta_\lambda^* \bar{\chi}_l}{(E_\lambda - E) + 2m|\eta_\lambda|^2 \bar{G}_l^{(+)}}, \tag{54}$$

where

$$\bar{\chi}_l = \chi_l(r_0); \quad \bar{G}_l^{(+)} = G_l^{(+)}(r_0, r_0). \tag{55}$$

The Green's function has the form

$$G_l^{(+)}(r, r') = -\sqrt{\frac{\pi}{2}} \frac{1}{k} \chi_l^{(+)}(r_>) \chi_l(r_<), \tag{56}$$

where $\chi_l^{(+)} = \sqrt{(\pi/2)}\,(y_l + i\chi_l)$, and $\chi_l(r)$ and $y_l(r)$ are, respectively, the solutions of the homogeneous equation (26) regular and irregular at the origin and with asymptotic behavior as $r \to \infty$

$$\left.\begin{aligned} \chi_l &\approx \sqrt{\frac{2}{\pi}} \sin(kr - \pi l/2 + \delta_l); \\ y_l &\approx \sqrt{\frac{2}{\pi}} \cos(kr - \pi l/2 + \delta_l); \\ \chi_l^{(+)} &\approx \exp[i(kr - \pi l/2 + \delta_l)]. \end{aligned}\right\} \tag{57}$$

Here, δ_l is the phase shift in the potential U_{eff}. Using (56) and (57), we write the general solution of (53) in the case of s-wave scattering ($l = 0$) for the region asymptotic with respect to r in the form

$$\varphi_0 \underset{r\to\infty}{\simeq} i/\sqrt{2\pi}\, e^{-i\delta_0} [e^{-ikr} - S_R e^{2i\delta_0} e^{ikr}], \tag{58}$$

where the part S_R of the scattering matrix has the form

$$S_R = 1 - 2i \frac{m\pi}{k} C_\lambda \eta_\lambda \bar{\chi}_0. \tag{59}$$

Substituting here (54), we find that S_R at $E \approx E_\lambda$ has the resonance form

$$S_R = [(E_\lambda - E) - \delta E + i\Gamma/2]/[(E_\lambda - E) - \delta E - i\Gamma/2], \tag{60}$$

with resonance width

$$\Gamma = 2\frac{m\pi}{k} \mid \eta_\lambda \mid^2 \bar{\chi}_0^2, \tag{61}$$

and level shift

$$\delta E = \frac{m\pi}{k} \mid \eta_\lambda \mid^2 \bar{\chi}_0 \bar{y}_0. \tag{62}$$

To estimate the width Γ of the decay of the six-quark level λ to the nucleon–nucleon channel, we substitute in (61) the numerical values of η_λ obtained at the end of the previous section, and for the value of the function $\bar{\chi}_0^2$ at the point r_0 we take the estimate $\langle\chi^2\rangle \simeq 1/\pi$ of its mean value. Then we obtain

$$\Gamma\ (s^6) = 29\ \text{MeV};\ \Gamma\ (s^4p^2) = 9.3\ \text{MeV}.$$

These values are less than the experimental estimates, which, depending on the method used to analyze the data, are in the range $\Gamma_{\text{exp}} \sim 50$–150 MeV. However, as will be seen in the following section, the calculations give a mean distance between the 6q levels of the same order of magnitude as the widths Γ themselves. It is then entirely possible that the "observed" Γ_{exp} is in a certain sense a quantity that is averaged by the experiment itself or its analysis.

Spectrum of states of the 6q system. Dibaryons

It is natural to pose the problem of finding the spectrum of 6q states on the same basis as in elementary-particle physics. There, to obtain the total energy (mass) of an elementary particle, models of interacting quarks are used. For this, for example, one specifies the V_{qq} forces on the basis of the confinement principle, ensuring that free quarks do not exist at distances greater than the hadron diameter. Then, naturally, in our problem too we must take the V_{qq} forces that have been verified in elementary-particle physics and calculate the 6q system by means of them. For the solution of this problem it is important that the quarks have an additional quantum number—color—besides spin 1/2. Then, having constructed an antisymmetric color part of the function, we can put all six quarks in the s^6 shell, which is no longer forbidden by the Pauli principle. For nucleons consisting of three quarks this state is the lowest. One might think that for six quarks this state is also the lowest. As the basis for such a calculation, one generally uses the model of the MIT quark bag. Such a calculation was made in Ref. 8, and it was found that for the 6q system with the quantum numbers of the deuteron the total energy $E_{6q}(s^6)$ exceeds the deuteron mass by about 270 MeV. It was shown later[41] that even if one pair of the six quarks is transferred to the p shell, the strong residual qq interaction leads to an energy $E_{6q}(s^4p^2) = 2.39$ GeV, which is close to $E_{6q}(s^6)$. A similar result is given by calculations at the 6q system under the assumption that it forms clusters of colored groups of quarks.[9] We give below the results of a calculation of the 6q system in the MIT bag with canonized qq forces for configurations of the type s^5p^1, $s_1^{n_1}s_2^{n_2}$ (Fig. 4).[42] Such states are called dibaryons. A number of conclusions can be drawn with regard to their properties:

1. The dibaryon spectrum lies very high in the energy E, in the region 200–300 MeV and greater above the mass of a pair of nucleons. Thus, the deuteron is not a 6q system but predominantly a $3q + 3q$ system. The difference $E_\lambda - 2M = U_c \gtrsim 200$–300 MeV thus determines the order of magni-

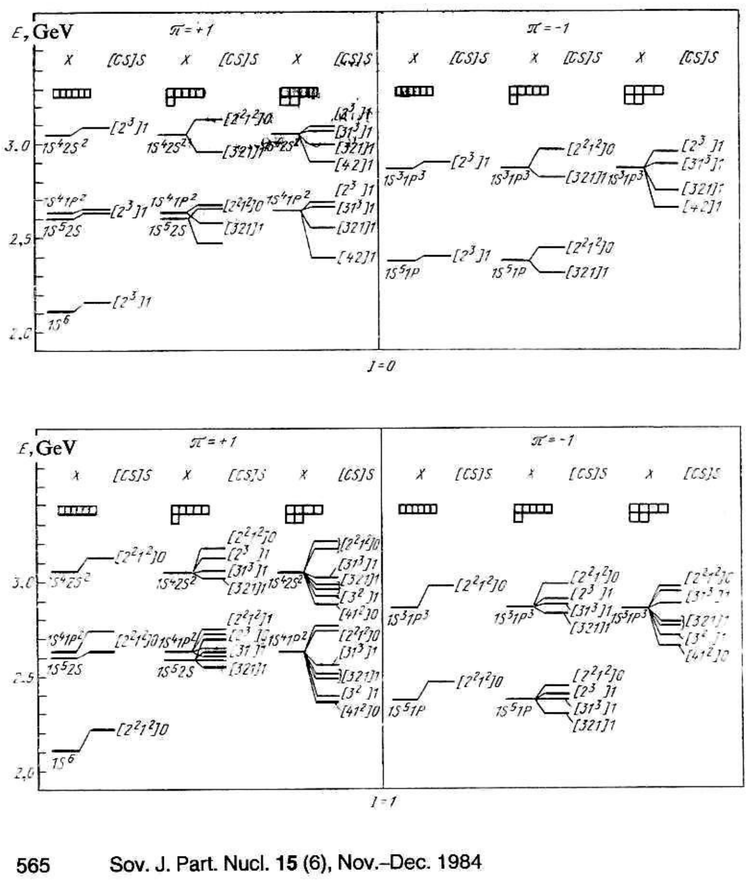

FIG. 4. Spectrum of dibaryon states for two values of the isotopic spin I of the system. The spin is S; $[CS]$ is the symmetry (Young diagram) of the color–spin part of the wave function; X is the spatial symmetry of the wave function; π is the parity of the state.

tude of the core, or barrier, that separates the two phases in the nucleus—the nucleon and the quark phases.

2. The s^6 spatial configuration of quarks does not have serious advantages over the other $s^{n_1}p^{n_2}$ and $s_1^{n_1}s_2^{n_2}$ configurations. This gives grounds for speaking of nodes in the *NN* scattering wave function at short distances.[43,44]

3. The dibaryon density of states is comparatively high, and it is therefore very probable that the resonances in the channels will overlap. In this connection, it should be noted that the experiments do not as yet indicate clear peaks in the corresponding cross sections of the *NN* channels, which could be interpreted as resonances. However, there have already been tens of experiments that "see" a so-called structure in the cross sections, polarization characteristics, etc., in the indicated region of energies. It is assumed[10] that the $^1D_2$ and $^3F_3$ phase shifts exhibit behavior of resonance type.

We note that the energy and radius of the 6*q* system can be related to the parameters of the logarithmic derivative of the wave function of the nucleon–nucleon channel on the bag surface, while the logarithmic derivative in its turn determines the behavior of the *NN* function in the complete region outside the bag. This provides the basis for constructing the so-called *P* scattering matrix,[23,45,46] which explains the dynamics of nucleon scattering without the introduction of a core at short distances.

4. INFLUENCE OF SIX-QUARK ADMIXTURES ON THE DEUTERON AND ³He FORM FACTORS

Measurements of the form factors of the lightest nuclei, ^{2}D, ^{3,4}He, have been made at large momentum transfers $q^2 > 1$ (GeV/c)^2.[20,21] This means that the electron penetrates into a very small region of space measuring

$$r \sim q^{-1} \sim 0.2 \ \text{F},$$

where according to modern ideas the quark structure of nuclear matter must be manifested. According to these ideas, the form factors at asymptotically large momentum transfers have a power-law behavior (quark counting rules)[6]:

$$(q^2)^{N-1} F_N(q^2) = C^2_{Nq}, \tag{63}$$

where N_q is the number of quarks that constitute the system, and the constant $C^2_{N_q}$ characterizes the weight of the *N*-quark configuration in the total wave function of the system.[7]

Figure 5 shows how the quantity (63) reaches a plateau at $q^2 > 1$ (GeV/c)^2 for hadrons and the lightest nuclei. It can be seen that for deuterons we only approach the asymptotic region, while for the ^{3}He and ^{4}He nuclei this region is not yet reached. Moreover, the contribution of the *N*-quark admixture $C^2_{N_q}$ depends on the approximation of the form factors in the pre-asymptotic region of momentum transfers and is several percent.[7] This means that the nucleon channel must be taken into account as the main part of the nuclear wave function.

We shall consider below how the six-quark admixtures in the wave function of a nucleus influence the behavior of its form factor. Models in which not only the nucleon part of the wave function but also multiquark admixtures occur

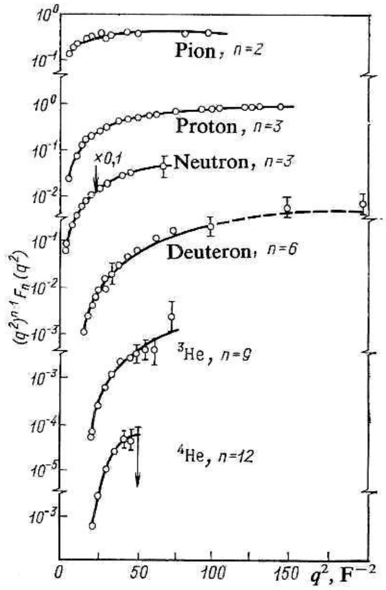

FIG. 5. Dependence of the experimental elastic form factors of the pion, proton, neutron, deuteron, and ^{3}He and ^{4}He nuclei,[20,21] multiplied by $(q^2)^{n-1}$, on q^2. The curves are drawn through the points.

have been used to study the deuteron and ^{3,4}He form factors on various occasions (Refs. 20, 40, and 47–52). We shall present below the results of the theoretical investigations of the deuteron and ^{3}He form factors in accordance with Refs. 40, 51, and 52, in which particular attention is devoted to the effects of antisymmetrization with respect to the quarks and the interference of the nucleon and six-quark parts of the wave function—questions that are very important but little studied in other investigations. We shall also consider the question of deuteron polarization in elastic *e*D scattering and the influence of an admixture of 6*q* components on it.[51]

Deuteron form factor

As before, we write the deuteron wave function in the form

$$\Psi^M = C_1\hat{A}\,(\varphi_n\varphi_p\Psi^{M}_{np}(\mathbf{r})) + C_{6q}\Psi_{6q}. \tag{64}$$

In what follows, we shall consider the simplest s^6 configuration of the six-quark system, and give only estimates for the contribution of the others.

To study exchange effects, we retain in this function for the time being only the first term. Then, using the definition of the form factor (Q_j is the charge of the *j*-th quark),

$$F^{MM'}(q^2) = \langle \Psi^M_{\text{ext}} \,|\, \sum_j Q_j \exp(i\mathbf{q}\mathbf{r}_j) \,|\, \Psi^{M'}_{\text{ext}} \rangle, \tag{65}$$

where Ψ_{ext} is the wave function (38), and the properties of the permutation operator,[31,53] we obtain[40]

$$F^{MM'}(q^2) = (F_C + F^e_C)\,\delta_{MM'}$$
$$- (F_Q + F^e_Q)\,(2m_l 1M \mid 1M')\,Y_{2m_l}(\hat{q})\sqrt{4\pi}, \tag{66}$$

where $F_{C,Q}$ are the ordinary "direct" charge and quadrupole form factors, and $F^e_{C,Q}$ are their exchange parts:

$$F_C(q^2) = F_p(q^2) \int dr\,(u^2 + w^2)\, j_0(qr/2); \tag{67}$$

$$F_Q(q^2) = F_p(q^2)\int dr\,(2uw - w^2/\sqrt{2})\, j_2(qr/2); \tag{68}$$

$$F_C^e(q^2) = B\left\{\frac{1}{3}F_p(q^2/2)\,[I_{00}^{0(+)}(q) + I_{22}^{0(+)}(q)] + \frac{2}{3}F_p(3q^2/4)\,[I_{00}^{0(-)}(q) + I_{22}^{0(-)}(q)]\right\}; \tag{69}$$

$$F_Q^e(q^2) = B\left\{\frac{1}{3}F_p(q^2/2)\,[2I_{20}^{2(+)}(q) - I_{22}^{2(+)}(q)/\sqrt{2}] + \frac{2}{3}F_p(3q^2/4)\,[2I_{20}^{2(-)}(q) - I_{22}^{2(-)}(q)/\sqrt{2}]\right\}. \tag{70}$$

Here, $F_p(q^2)$ is the charge form factor of the proton, j_0 and j_2 are spherical Bessel functions, and $B = (3/64)(\Omega/2\pi)^{3/2}$. The expressions for the exchange integrals are

$$I_{LL'}^{l(\pm)} = 4\pi \sum_{l_1 l_2 \lambda} (2\lambda+1)\sqrt{(2l_1+1)(2l_2+1)(2l+1)/(2L'+1)} \times (\pm i)^{l_1-l_2}\begin{Bmatrix} l_2\lambda L \\ L' l l_1 \end{Bmatrix}(l_1 0\lambda 0\mid L'0)(l_2 0\lambda 0\mid L0) \times (l0l_20\mid l_10)\int dr\,dr' rr'\Phi_L(r)\Phi_{L'}(r') \times \exp(-5\Omega(r^2+r'^2)/32)\, j_{l_1}\left(\frac{3/4}{3/8}qr\right) j_{l_2}\left(\frac{3/4}{3/8}qr'\right) i_\lambda\left(\frac{3\Omega}{16}rr'\right), \tag{71}$$

where i_l are spherical Bessel functions of an imaginary argument, and $\Phi_0 = u$, $\Phi_2 = w$.

A number of simplifications can be made in the calculation of the exchange form factors because the radial cluster fuctions have Gaussian form. However, in the investigation of the region of large momentum transfers the corresponding Gaussian expression for the proton form factor must be replaced by a realistic expression obtained by analyzing experiments (dipole formula):

$$F_p(q^2) = \langle \varphi_p \mid \sum_j Q_j \exp(i\mathbf{q}\mathbf{r}_j) \mid \varphi_p\rangle = \exp(-q^2/\Omega) \Rightarrow (1 + q^2/0.71\,(\mathrm{GeV}/c)^2)^{-2}. \tag{72}$$

The exchange terms are calculated for three realistic potentials: the Reid potential with a soft core, RSC,[38] and with a hard core, RHC,[38] and the Paris potential P.[54] The ratio

$$R = \frac{(F_C + F_C^e)^2 + (F_Q + F_Q^e)^2}{F_C^2 + F_Q^2},$$

which is shown in Fig. 6, reveals that the contribution of the exchange terms to the cross section is only a few percent at all q^2. For the RSC and P potentials it does not exceed 5%. Evidently the realistic potentials are such that the presence of the core in them imitates the contribution of the exchange terms. Thus, explicit allowance for them does not introduce significant changes and does not make it possible to improve the agreement between theory and experiment significantly.

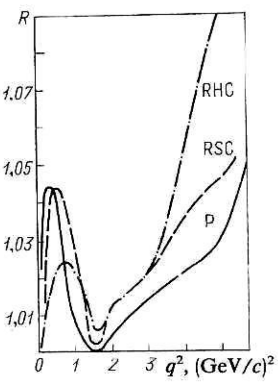

FIG. 6. Contribution of exchange form factors to the cross section of eD scattering without allowance for $6q$ configurations.

We now investigate the part played by the $6q$ admixtures. Since our aim is to study the region of large momentum transfers, it is desirable whenever possible to take into account relativistic effects. In particular, we take Ψ_{6q}^x in the form given by the model of a relativistic harmonic oscillator for the quarks[40,47,51]:

$$\Psi_{Nq}^X(\xi_1, \ldots, \xi_{N-1}, P) = \left(\frac{\alpha_N}{\pi N}\right)^{N-1}\exp\left\{\frac{\alpha_N}{2N}\sum_{i=1}^{N-1}[\xi_i^2 - 2(P\xi_i)^2/M_{Nq}^2]\right\}. \tag{73}$$

Here, $\xi_1, \ldots, \xi_{\nu-1}$ are the Jacobi 4-coordinates of the N-quark system, and P is the total 4-momentum.

Then, using the two-channel wave function and ignoring the contribution of the small exchange terms, we obtain the following expression for the form factor[51]:

$$F_{MM'} = (C_1^2 F_C + 2C_1C_{6q}F_{\mathrm{int}}^{l=0} + C_{6q}^2 F_{6q})\,\delta_{MM'} + (C_1^2 F_Q + 2C_1C_{6q}F_{\mathrm{int}}^{l=2})\,S_{MM'} \equiv G_C\delta_{MM'} + G_Q S_{MM'}, \tag{74}$$

where

$$S_{MM'}(\hat{q}) = \sqrt{12\pi/5}\,(-1)^M (1-M1M' \mid 2M'')\, Y_{2M''}^*(\hat{q}). \tag{75}$$

The form factor of an N-quark system in the relativistic harmonic-oscillator model was obtained in Ref. 48:

$$F_{Nq}(q^2) = (1 + q^2/2M_{Nq}^2)^{-N+1} \times \exp\{-(N-1)\,q^2/[4\alpha_N(1 + q^2/2M_{Nq}^2)]\}. \tag{76}$$

Here, the parameters of the problem are $\alpha_N = N^{3/2}\mathcal{K}$ ($\mathcal{K}$ is the oscillator mass) and the mass M_{Nq}^2. For the proton form factor F_{3q} ($N=3$) the parameter values obtained in Ref. 48 were $\alpha_3 \approx 0.5\,(\mathrm{GeV}/c)^2$ and $M_{3q} \approx 1$ GeV. Then for the form factor F_{6q} ($N=6$) of the $6q$ system $\alpha_6 = 2\sqrt{2}\alpha_3 \approx 1.4\,(\mathrm{GeV}/c)^2$. The value $M_{6q} \approx 1.2$ GeV was obtained in Ref. 48 by comparing F_{6q} with experimental data at large q^2. Note that we have set the electric form factor of the neutron equal to zero. Finally, the interference form factor

$$F_{\mathrm{int}}^{l=0,\,2} = \langle \varphi_n\varphi_p\Psi_{np}^M \mid \sum_j Q_j \exp(i\mathbf{q}\mathbf{r}_j) \mid \Psi_{6q}\rangle \tag{77}$$

arises because of the overlap of the wave functions of the NN and $6q$ channels.[40]

Ignoring the square of the magnetic form factor,[48,55] which at small q^2 is appreciably less than the sum $F_C^2 + F_Q^2$, and at large q^2 is less than F_{6q}^2, we write the structure function in the form

$$A\,(q^2) = G_C^2\,(q^2) + G_Q^2\,(q^2). \tag{78}$$

We determine the weight of the s^6 configuration C_{6q}^2 by requiring that $A(q^2)$ agree with the experimental data at large q^2, and we take C_1 from the normalization condition $A(q^2=0) = 1$.

The cross section of elastic eD scattering, or the struc-

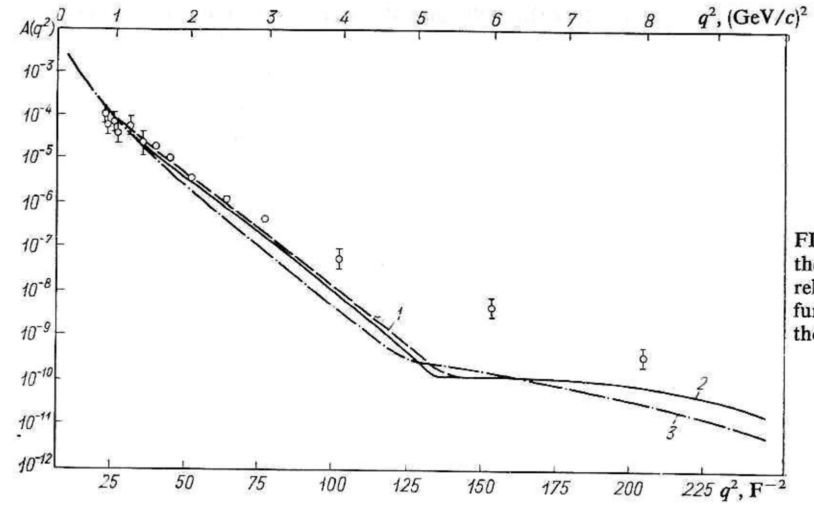

FIG. 7. The structure function $A(q^2)$. Curve 1 is calculated with the Reid soft-core (RSC) wave function[38] without allowance for relativistic effects; curve 2 is calculated with the Paris wave function[54] without allowance for relativistic effects; curve 3 is the same as curve 1 but with allowance for relativistic effects.

ture function $A(q^2)$, have been measured in a wide range of momentum transfers up to $q^2 \approx 200$ F^{-2}.[20] Nonrelativistic calculations with the ordinary deuteron wave functions[38,54] for $q^2 > 100$ F^{-2} lead to a discrepancy with the experimental data (Fig. 7, curves 1 and 2). Allowance for relativistic effects[56] (curve 3) leads to an increase in the discrepancy between theory and experiment. The reason for this is evidently that the realistic *NN* potentials are themselves usually chosen for calculation with a nonrelativistic equation, and such potentials effectively take into account the possible relativistic corrections to the various observable quantities. As can be seen from Fig. 8 (curves 1, 2, 3), allowance for the quark structure makes it possible to describe the cross section of elastic *e*D scattering. Here, to demonstrate the contribution of the interference of the 6*q* and *NN* channels, we give the results of calculations of $A(q^2)$ with both a positive (curve 1) and a negative (curve 2) interference form factor. It can be seen that allowance for the interference changes $A(q^2)$ by a few tens of percent, and the agreement with the experimental data in the region $q^2 \sim 25$ F^{-2} is improved if the contribution of the interference to $A(q^2)$ is negative, i.e., $C_{6q} = -\sqrt{0.07}$. At the same time, the behavior of $A(q^2)$ at large momentum transfers $q^2 > 25$ F^{-2} hardly depends on the choice of the *NN* forces. To show the nature of the relativistic effects, we give in Fig. 8 (curve 3) a calculation of $A(q^2)$ taken from Ref. 40. The *NN* functions were relativized in accordance with the prescription of Ref. 56, and the 6*q* admixture was taken to be $C^2_{6q} \approx 7\%$. Comparison of this calculation with the analogous nonrelativistic one shows that with simultaneous allowance for the 6*q* contribution the relativization of the *NN* function improves the agreement with the experimental data. One can show that in the case of the choice $C^2_{6q} = 7$–8.5% good agreement with experiment is maintained.

To summarize, it can be said that the probability for the existence of the six-quark system is of the order $C^2_{6q} \sim 7$–8.5%, and its rms radius $r_{6q} = 0.77$ F, determined from the condition $F_{6q}(q^2) = 1 - (1/6)r^2_{6q}\, q^2$, is close to the value obtained from the data on cumulative reactions[57] and deep inelastic *eA* scattering.[7,58] It is important that the interference of the 6*q* and *NN* channels and allowance for the relativistic effects, which change $A(q^2)$ within a few tens of percent, improve the agreement with the experimental data in the region $q^2 \approx 25$ F^{-2}.

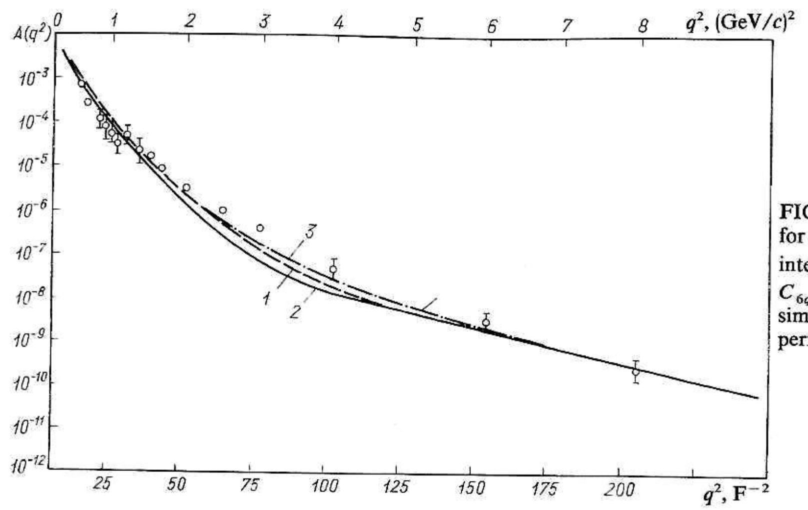

FIG. 8. The structure function $A(q^2)$ calculated with allowance for the quark structure of the deuteron. Curve 1 is with positive interference $C_{6q} = \sqrt{0.07}$(P, RSC), curve 2 with negative $C_{6q} = -\sqrt{0.07}$(P, RSC), and curve 3 with $C_{6q} = -\sqrt{0.07}$ and simultaneous allowance for relativistic effects (RSC). The experimental data are taken from Ref. 20.

Polarization in elastic eD scattering

In the investigation of the polarization of the recoil deuterons it is planned to measure the tensor polarization component $T_{20} = p_{zz}/\sqrt{2}$ and the ratio R of the vector, P_x, and tensor, P_{xz}, polarizations.[55] It is a feature of these characteristics that they do not include a dependence on the form factors of the nucleons that form the deuteron, and in the framework of the usual ideas about the deuteron their behavior is determined solely by the wave function of the relative motion of the nucleons. But if a $6q$ admixture in the deuteron is assumed, its effect on T_{20} and R is nontrivial and requires special analysis. Expressions relating the vector and tensor polarizations of the deuteron to its charge quadrupole form factor and magnetic form factor are given in Refs. 51, 55, 56, and 59–65. Proceeding analogously, we can obtain the corresponding expressions with allowance for the contribution of the $6q$ admixtures[51]:

$$T_{20} = \frac{1}{\sqrt{2}} \frac{1+X}{1+X^2/8}, \tag{79}$$

$$R = \frac{P_x}{P_{xz}} = \frac{2\sqrt{1+\eta}\sin\theta/2}{9\sqrt{1+\eta\sin^2\theta/2}}(1+X). \tag{80}$$

Here

$$X = \frac{2\sqrt{2}G_C}{G_Q} = \frac{(C_1^2F_C + C_{6q}^2F_{6q} + 2C_1C_{6q}F_{\rm int}^{l=0})}{(C_1^2F_Q + 2C_1C_{6q}F_{\rm int}^{l=2})}, \tag{81}$$

$\eta = q^2/4M_{\rm D}^2$, and θ is the electron scattering angle.

We have already noted that T_{20} and R do not include a dependence on the nucleon form factor. This means that in approaches (for example, Refs. 56 and 66) in which the behavior of the deuteron form factor at large q^2 is explained by introducing the neutron form factor it will always be found that T_{20} and R depend only on the functions Ψ_{np} of the relative motion of the nucleons. In this approach, the different methods of taking into account the relativization of the function of the relative motion will be manifested differently. Thus, Fig. 9 gives the results of nonrelativistic calculations of T_{20} (curves 1 and 2) and calculations of T_{20} in the framework of the relativistic approaches of Ref. 56 (curve 3) and Refs. 65 and 67 (curve 4). Although these models of Refs. 56 and 65–67 describe the structure function $A(q^2)$ equally well in the complete region of momentum transfers, it is found that in the calculation of the behavior of T_{20} and R they lead to different results. Thus, for variation of the momentum transfer q_3 in the interval from 1 to 18 F^{-1} the T_{20} calculated in Ref. 56 vanish three times (curve 3). The first vanishing is due to the change in the sign of the charge form factor of the deuteron at $q^2 \sim 25\ \mathrm{F}^{-2}$ associated with the existence of the repulsive NN core; the second is due to the zero of the quadrupole form factor at $q^2 \sim 120\ \mathrm{F}^{-2}$.

The polarization calculated in Refs. 65 and 67 has just one zero, at $q^2 \sim 25\ \mathrm{F}^{-2}$ (curve 4). The absence of the second zero at $q^2 \sim 120\ \mathrm{F}^{-2}$ is explained by the fact that in this approach the allowance for the relativistic effects strongly displaces the zero of the quadrupole form factor to larger q^2. For comparison, the same figure (curves 5 and 6) gives the calculation of T_{20} with allowance for the $6q$ admixtures. All the parameters are obtained from the analysis of $A(q^2)$. It can be seen that allowance for the $6q$ structure of the deuteron gives a behavior of T_{20} in the region of momentum transfers $25 < q^2 < 120\ \mathrm{F}^{-2}$ very different from what is given by the models considered above. We recall that in the interpretation of the deuteron form factor $G_{\rm C}(q^2)$ itself the contribution of the $6q$ admixtures is decisive in the q region that begins at $q^2 > 25\ \mathrm{F}^{-2}$, and $F_{6q}(q^2)$ decreases with increasing q much more slowly than $F_{\rm C}(q^2)$.[40,51] This has the consequence that the minimum in the charge form factor of the deuteron is filled by virtue of the contribution of the $6q$ system, while the actual position of the minimum is determined by the size of the core of the NN forces. The same effect is manifested in the behavior of T_{20}, for which it is found that the "zero" at $q^2 \sim 25\ \mathrm{F}^{-2}$ associated with the structure of $F_{\rm C}(q^2)$ is filled by virtue of the contribution of the $6q$ admixture, the upshot of which is the observation of a "plateau" in the region of momentum transfers $25 < q^2 < 120\ \mathrm{F}^{-2}$, this extending to the zero of the quadrupole form factor $G_{\rm Q}(q^2)$.

To exhibit the dependence of T_{20} on C_{6q}^2, Fig. 10 (curves 1, 2, 3) shows calculations with different values of C_{6q}^2. It can be seen that when C_{6q}^2 is decreased a "dip" appears in T_{20} in the interval $25 < q^2 < 120\ \mathrm{F}^{-2}$. At the same time, T_{20} is very sensitive to a change in C_{6q}^2. In the same figure we show calculations of T_{20} with allowance for the interference form factors $F_{\rm int}^{l=0.2}$ and with allowance for the contribution of the relativistic effects.[51] It can be seen that these effects change the behavior of T_{20} in a 20% range.

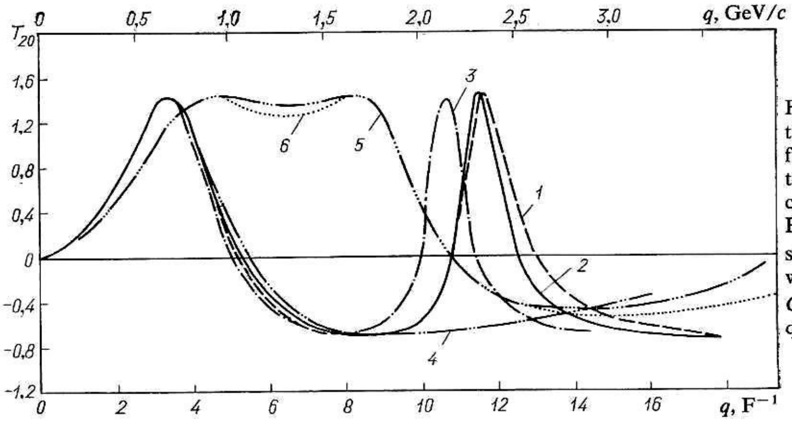

FIG. 9. The tensor polarization T_{20}. Curve 1 is calculated with the soft-core Reid wave function,[38] curve 2 with the Paris wave function of the deuteron,[54] curve 3 with allowance for relativistic effects in the approach of Ref. 56 (RSC), curve 4 is the same as curve 3 but in the approach of Refs. 65 and 67, curve 5 is with the Paris deuteron wave function[54] with allowance for the quark structure $C_{6q} = -\sqrt{0.07}$, and curve 6 is with the soft-core Reid wave function[38] with allowance for the quark structure $C_{6q} = -\sqrt{0.07}$ (RSC); curves 1–4 are without allowance for the quark structure.

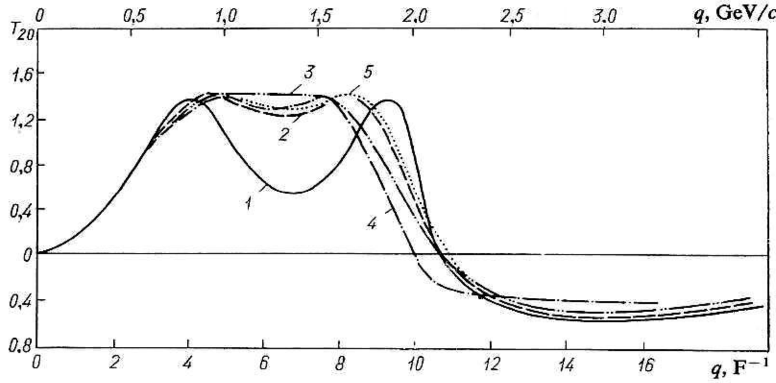

FIG. 10. The tensor polarization T_{20}. Curve 1 is calculated with the soft-core Reid wave function[38] with allowance for the deuteron quark structure $C_{6q} = -\sqrt{0.05}$ with interference, curve 2 is the same as curve 1 but with $C_{6q} = -\sqrt{0.07}$, curve 3 is also the same as curve 1 for $C_{6q} = -\sqrt{0.085}$, curve 4 is with allowance for relativistic effects[56] and $C_{6q} = -\sqrt{0.07}$, and curve 5 is without interference and with $C_{6q} = -\sqrt{0.07}$.

Hitherto, we restricted treatment to the s^6 configuration when taking into account the $6q$ admixture. In Ref. 50, the s^4p^2 configuration was considered and it was shown that the form factor $F_{6q}^{s^4p^2}(q^2)$ behaves in the region $q^2 < 200\ \mathrm{F}^{-2}$ in approximately the same way as $F_{6q}^{s^6}(q^2)$ and has a minimum at $q^2 \sim 200\ \mathrm{F}^{-2}$. Assuming that the s^4p^2 configuration makes the main contribution to the channel with $l = 0$, one can expect a strong influence of this configuration on T_{20} only in the region $q^2 \sim 200\ \mathrm{F}^{-2}$, where $F_{6q}^{s^4p^2}(q^2)$ changes sign. If it is assumed that $F_{6q}^{s^4p^2}(q^2)$ contributes to both the $l = 0$ and the $l = 2$ channel, then changes must be observed in the behavior of T_{20} at smaller q^2 in the region of the minimum of the quadrupole NN deuteron form factor $F_Q(q^2)$, i.e., at $q^2 \sim 120\ \mathrm{F}^{-2}$. It is therefore to be expected that inclusion of the s^4p^2 configuration will not greatly change the behavior of T_{20}, at least when $q^2 \lesssim 120\ \mathrm{F}^{-2}$.

Thus, we arrive at the following conclusions:

1. The assumption of the existence in the deuteron with probability $C_{6q}^2 \approx 7$–8.5% of the six-quark s^6 configuration with radius $r_{6q} = 0.77$ F makes it possible to describe the cross section of elastic eD scattering at large momentum transfers q^2. At the same time, the behavior of the polarization tensor T_{20} in the region of momentum transfers $25 < q^2 < 120\ \mathrm{F}^{-2}$ is changed qualitatively.

2. The inclusion of relativistic effects in the framework of Gross's model[56,59] with simultaneous allowance for $6q$ admixtures, and also the interference of the $6q$ and NN channels change $A(q^2)$ and $T_{20}(q^2)$ by 10–20%, the case when the amplitude of the admixture has negative sign being the preferred one.

3. The antisymmetrization of the deuteron wave function with respect to the quarks that constitute the nucleons does not play an important part if realistic NN potentials are used.

Form factor of ³He

In principle, to study this question it is necessary to take into account not only the $6q$ but also the $9q$ admixtures in the wave function. In general, for a nucleus of A nucleons one can write

$$\Psi_A = C_1\Psi_1 + C_2\Psi_2 + \ldots = \sum_{k=1}^{A} C_k\Psi_k. \qquad (82)$$

Here, Ψ_1 is the nuclear wave function of the model of independent particles, Ψ_2 is a nuclear function with a distinguished two-nucleon cluster that with probability C_2^2 has an admixture of a $6q$ component, Ψ_3 is the function of a nucleus with a distinguished three-nucleon cluster that with probability C_3^2 is present in the form of a $9q$ configuration, etc. The representation of these functions in the form of products of single-particle and cluster functions without allowance for exchange terms ultimately makes it possible to express the form factor in terms of the experimentally known distribution of the charge density of the nucleus and the form factors of simpler systems. In this connection, we recall that the contribution of the exchange terms to the deuteron was indeed found to be negligibly small.[40] For Ψ_k we shall choose Gaussian functions in the model of independent particles, as well as more realistic nuclear wave functions.

We first consider the simpler case of Gaussian wave functions, when we can write

$$\Psi_1 = \overline{\Psi}_1(\mathbf{r}_1^{(1)}, \mathbf{r}_2^{(1)}, \mathbf{r}_3^{(1)}) \times \overline{\Psi}_2(\mathbf{r}_1^{(2)}, \mathbf{r}_2^{(2)}, \mathbf{r}_3^{(2)}) \ldots \overline{\Psi}_A(\mathbf{r}_1^{(A)}, \mathbf{r}_2^{(A)}, \mathbf{r}_3^{(A)}), \qquad (83)$$

where $r_j^{(i)}$ are the coordinates of quarks $j = 1, 2, 3$ that constitute nucleon i. In the functions

$$\Psi_2 = \overline{\Psi}_{12}(\mathbf{r}_1^{(1)}, \ldots \mathbf{r}_3^{(2)}) \times \overline{\Psi}_3(\mathbf{r}_1^{(3)}, \mathbf{r}_2^{(3)}, \mathbf{r}_3^{(3)}) \ldots \overline{\Psi}_A(\mathbf{r}_1^{(A)}, \mathbf{r}_2^{(A)}, \mathbf{r}_3^{(A)}), \qquad (84)$$

$$\Psi_3 = \overline{\Psi}_{123}(\mathbf{r}_1^{(1)}, \ldots \mathbf{r}_3^{(3)}) \times \overline{\Psi}_4(\mathbf{r}_1^{(4)}, \mathbf{r}_2^{(4)}, \mathbf{r}_3^{(4)}) \ldots \overline{\Psi}_A(\mathbf{r}_1^{(A)}, \mathbf{r}_2^{(A)}, \mathbf{r}_3^{(A)}) \text{, etc.,} \qquad (85)$$

allowance is made for the possibility of quark clustering, for example, the possibility that two nucleons are in the $6q$ state (84), three nucleons are in the $9q$ state (85), etc. Introducing Jacobi coordinates, we reduce in the oscillator basis the wave functions of the "multiquark clusters" (fluctons) of the $3q$ states (nucleons) $\overline{\Psi}_1$, $6q$ states $\overline{\Psi}_{12}$, $9q$ states, $\overline{\Psi}_{123}$, etc., to the form

$$\overline{\Psi}_1(\mathbf{r}_1^{(1)}, \mathbf{r}_2^{(1)}, \mathbf{r}_3^{(1)}) = \varphi_1(\mathbf{R}^{(1)})\,\Phi_{3q}(\xi_1, \eta_1), \qquad (86)$$

$$\overline{\Psi}_{1\ldots k}(\mathbf{r}_1^{(1)}, \ldots \mathbf{r}_3^{(k)}) = \varphi_{1\ldots k}(\mathbf{R}^{(1\ldots k)}) \times \Phi_{3kq}(\xi_1, \eta_1, \xi_2, \eta_2 \ldots). \qquad (87)$$

where $\mathbf{R}^{(1\ldots k)}$ are the coordinates of the center of mass of the k-th flucton, which contains $3k$ quarks, and ξ, η are the relative coordinates of the quarks within it. The functions $\varphi_{1\ldots k}$ of the flucton center-of-mass motion can now be represented in the form of products of single-particle functions of the nucleons:

$$\varphi_{1\ \ldots\ k} = (\varphi_1)^k. \qquad (88)$$

We generalize this relation, introducing the phenomenological charge-distribution density of point nucleons of the nucleus taken from analysis of the form factors of electron–nucleus scattering at relatively small momenta—up to the first minimum of the form factor. Such a density is nearly Gaussian, so that

$$|\ \varphi_{1\ \ldots\ k}\ |^2 = (\rho\ (R))^k. \qquad (89)$$

With regard to the determination of the coefficients of admixture C_k^2 of the multiquark states in the nuclear function, this has so far been achieved only for the deuteron, and for heavier nuclei only qualitative estimates have been obtained.[24] It is, however, possible to parametrize C_k^2 by means of the expression for the probability of finding k nucleons of the nucleus in the correlation volume $V_\xi = \frac{4}{3}\pi r_\xi^3$ (in the "flucton" volume):

$$C_k^2 = \binom{A}{k} (V_\xi / A V_0)^{k-1} B_k;$$
$$B_k = \binom{N}{k_n}\binom{Z}{k_z}\binom{A}{k}^{-1} \Big/ Z, \qquad (90)$$

where B_k takes into account the isotopic composition of the flucton with k_n neutrons and k_p protons. Here, $V_0 = (4/3)\pi r_0^3$ and $r_0^3 = 1.2$ F. From analysis of cumulative processes the value $r_\xi = 0.75$ F was obtained earlier.[4,57,68]

Using now the definition of the charge form factor of the nucleus,

$$F_A = \sum_{ij} Q_j^{(i)} \int |\ \Psi_A\ |^2 \exp(i\mathbf{qr})\ \delta(\mathbf{r} - \mathbf{r}_j^{(i)})\ dr \prod_{ij} d\mathbf{r}_j^{(i)}, \qquad (91)$$

where $Q_j^{(i)}$ are the charges of the quarks, we obtain the expression[52]

$$F_A = \sum_{k=1}^{A} C_k^2 F_k + 2 \sum_{i \neq k} C_i C_k F_{ik}. \qquad (92)$$

Here, F_k are the form factors of the fluctons, and F_{ik} are the corresponding interference parts of the form factor. It is easy to find that

$$F_k = F_k^{\text{cm}}\ \overline{F}_k F_k^q = \widetilde{F}_k F_k^q. \qquad (93)$$

$$\overline{F}_k = \int \rho^k (R) \exp(i\mathbf{qR})\ d\mathbf{R}, \qquad (94)$$

$$F_k^q = \int |\ \Phi_{3kq}(\xi_1, \eta_1 \ldots)\ |^2 \exp(iq\{\xi, \eta, \ldots\})\ d\xi_1\, d\eta_1 \ldots \qquad (95)$$

Here, F_k is the form factor of the center-of-mass motion of the k-th flucton, and F_k^q is the form factor of the multiquark system itself. The presence of the redundant variables in the nuclear function (83) made it necessary to introduce a correction for the center-of-mass motion: $F_k^{\text{cms}} = \exp(q^2/4\alpha_A A)$. The parameter α_A is determined by comparing (93) with the experimentally known form factor of the ^{3}He nucleus for $k = 1$ (the nucleon part of the form factor without the multiquark admixtures) in the region of small q^2, where one can use the Gaussian density $\rho = (\pi/\alpha_A)^{3/2} e^{\alpha_A r^2}$. This gives

$$\widetilde{F}_k = \exp[-q^2 (1 - k/A)/4\alpha_A k];$$
$$\alpha_A^{-1} = A\ (b_A^2 - a_p^2)/(A - 1), \qquad (96)$$

where, for example, for the ^{3}He nucleus

$$b_{^3\text{He}}^2 = 1.823\ \ \text{F}^2;\quad a_p^2 = 0.36\ \ \text{F}^2.$$

We again take the form factor of the multiquark system in the form (76), which is obtained in the relativistic harmonic-oscillator model. In the calculations of the form factor of the ^{3}He nucleus below, the contribution of the $6q$ and $9q$ impurities was taken into account. We have already said that $M_{6q} = 1.2$ GeV (Ref. 48) and $\mathscr{K} = 0.096$ GeV^2. With regard to the parameter M_{9q}, it was found from comparison with the expression for F_{9q} obtained in the model of independent quarks,[69] which gives $M_{9q} \simeq 1.4$ GeV.

We first consider only the influence of the $6q$ configuration on $F_{^3He}$. In this case,

$$F_{^3\text{He}} = C_1^2 F_1 + C_2^2 F_2 + 2C_1 C_2 F_{12}. \qquad (97)$$

We take the value $C_2^2 = 0.085$ from Refs. 40 and 51.

We now obtain an expression for the interference term:

$$F_{12} = \sum_{ij} Q_j^{(i)} \int \Psi_1^* \Psi_2 \exp(i\mathbf{qr})\ \delta(\mathbf{r} - \mathbf{r}_j^{(i)})\ dr \prod_{ij} d\mathbf{r}_j^{(i)}. \qquad (98)$$

Substituting here (83) and (84), we find

$$F_{12} = F_{12}^q \overline{F}_{12} F^{\text{cm}}, \qquad (99)$$

where the form factor F_{12} appears because of the overlap of the wave functions of the $3q$ and $6q$ clusters[70]:

$$F_{12}^q = \left[\left(\frac{2\alpha_3\alpha_6}{\alpha_3 + \alpha_6}\right)^2 \Big/ g\right]^2 \exp(-f/2g). \qquad (100)$$

Here,

$$f = (\alpha_3 M_{6q}^2 + \alpha_6 M_{3q}^2)(\gamma^2 - 1)$$
$$- M_{6q} M_{3q} (\alpha_3 + \alpha_6)\gamma + (M_{3q}^2 + M_{6q}^2)(\alpha_3 + \alpha_6)/2; \qquad (101)$$

$$g = \alpha_3\alpha_6\gamma^2 + (\alpha_6 - \alpha_3)^2/4, \qquad (102)$$

$$\gamma = (q^2 + M_{3q}^2 + M_{6q}^2)/2M_{3q}M_{6q}, \qquad (103)$$

$$\alpha_6 = \sqrt{2}\,\alpha_3 = \sqrt{2}\cdot 3^{3/2}\mathscr{K}. \qquad (104)$$

The form factor $\overline{F}_{12}$ arises because of the overlap of the wave functions of the center-of-mass motion of the $3q$ and $6q$ clusters and has the form

$$\overline{F}_{12} = \left[\frac{8\alpha_A\alpha_6}{(\alpha_A + \alpha_6)^2}\right]^{3/4} \exp(-q^2 d^2/8\alpha_A), \qquad (105)$$

where

$$d^2 = (3\alpha_A + \alpha_6)/(\alpha_A + \alpha_6). \qquad (106)$$

Before we turn to the calculations, we examine the behavior and order of magnitude of the interference form factor F_{12}. We note that the parameter α_A, which determines the center-of-mass motion of the $3q$ and $6q$ clusters in the nucleus, is appreciably less than α_6, the parameter that characterizes the motion of the quarks in the strongly bound $6q$

system. Then $d^2 \approx 1$, and the argument of the exponential in (105) agrees with the one obtained in calculating F_2 of the form factor (94) with Gaussian nuclear functions. Then

$$\bar{F}_{12}(q^2) \approx \eta \bar{F}_2(q^2), \tag{107}$$

where $\eta \simeq (8\alpha_A/\alpha_6)^{3/4}$. Substituting in the expression for η the values $\alpha_6 = 18$ F^{-2} and $\alpha_A = 0.45$ F^{-2}, we obtain $\eta = 0.089$, i.e., numerically η is practically equal to the modulus of the $6q$ admixture, $|C_2| = \sqrt{0.085}$, this value corresponding to what is given by the analysis of the deuteron form factor.[40,51] Moreover, if it is borne in mind that in our case $M_{3q} \simeq M_{6q}$ and $\alpha_3 \sim \alpha_6$, then it can be shown that F_2^q and F_{12}^q are actually identical. Thus, we finally have

$$F_{12}(q) \approx \eta F_2^q(q)\, \tilde{F}_2(q) = \eta F_2(q). \tag{108}$$

This approximate equality was tested by numerical calculations using the expressions for $C_2^2 F_2$ and $2C_1C_2F_{12}$. The results of the calculations are given in Fig. 11 (curves 1 and 2). It can be seen that Eq. (108) does indeed hold; in particular, the slopes of the interference and $6q$ form factors are very nearly equal. Then the form factor $F_{^3\mathrm{He}}$ can be represented in the form

$$F_{^3\mathrm{He}} = C_1^2 F_1 + C_2^2 F_2 + 2C_1C_2 \mid C_2 \mid F_2. \tag{109}$$

It is immediately clear from this that the interference of the NN and $6q$ channels here plays a very important part. Thus, since $C_1 \approx 1$, we have for $C_2 > 0$

$$F_{^3\mathrm{He}} = C_1^2 F_1 + 3C_2^2 F_2, \tag{110}$$

and for $C_2 < 0$

$$F_{^3\mathrm{He}} = C_1^2 F_1 - C_2^2 F_2. \tag{111}$$

Calculations of $|F_{^3\mathrm{He}}^{(C_2>0)}|$ and $|F_{^3\mathrm{He}}^{(C_2>0)}|$ in accordance with the exact expressions (98)–(106) are given in Fig. 12. It can be seen that the variant with $C_2 > 0$ must be immediately rejected (chain curve), since in this case the first minimum of the form factor, observed experimentally at $q \simeq 3.2$–3.6 F^{-1}, is

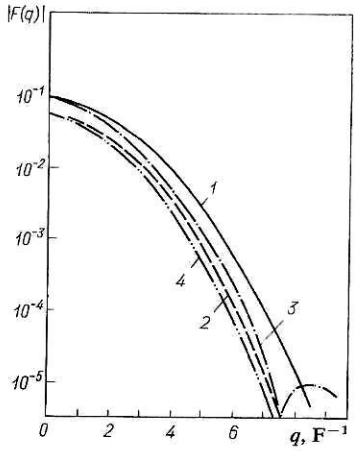

FIG. 11. Contribution to the form factor of the ^{3}He nucleus of the six-quark, $|C_2^2F_2|$ (curves 2 and 4), and interference, $|2C_1C_2F_{12}|$ (curves 1 and 3), form factors calculated with Gaussian (1, 2) and realistic (curves 3 and 4) wave functions of the nucleons of the ^{3}He nucleus.

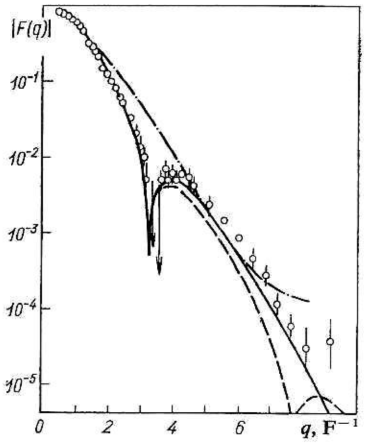

FIG. 12. Form factor of the ^{3}He nucleus, calculated in a model using Gaussian wave functions of the nucleons. The continuous curve is with a negative amplitude of the six-quark admixture, $C_2 = -\sqrt{0.085}$, without allowance for C_3^2: $C_3^2 = 0$; the broken curve is with the allowance, $C_3^2 = 0.003$, for the nine-quark admixture; and the chain curve is with a positive amplitude $C_2 = \sqrt{0.085}$ of the $6q$ admixture and $C_3^2 = 0$. The experimental data are taken from Ref. 21.

not even reproduced. We recall that the same sign of the $6q$ amplitude, $C_2 < 0$, is obtained from the analysis of elastic eD scattering. Considering the overall results of the calculation of the form factor of the ^{3}He nucleus using Gaussian nuclear NN and $6q$ functions, we conclude that agreement with the experimental data can here be achieved. However, the agreement is immediately lost if we add to the total function a $9q$ admixture as well (broken curve) (without allowance for the interference form factors F_{13} and F_{23}). In such a case, we obtain a second minimum in the region of momentum transfers $q \sim 7$–8 F^{-1}, which, however, is not as yet clearly found in the available experimental data.

To discover the reasons for the appearance of the second minimum, we make the model more accurate. It must be noted here that the first term in the form factor (97) is a purely nuclear form factor, for which the nucleons are not transformed into multiquark configurations at short distances. It is therefore natural to take it in the form obtained in exact calculations using realistic models for the nucleon–nucleon forces. For the ^{3}He nucleus, we have taken as such the calculations from Ref. 71, in which the three-body problem was solved exactly in the framework of the Faddeev equations using realistic nucleon–nucleon potentials and with allowance for the contribution of the meson exchange currents and relativistic effects. Figure 13 shows the results of calculations of $|F_{^3\mathrm{He}}^{(C_2<0)}|$ in which F_1 was taken from Ref. 71 and the contribution of the $6q$ components F_2 and F_{12} was calculated on the basis of Gaussian nucleon functions. It can be seen that in such a calculation the second minimum remains even when the $6q$ configurations are taken into account. Addition of the $9q$ configuration changes the qualitative picture little, leading merely to a shift of the second minimum toward smaller q. Thus, the second minimum, which appears in calculations of the form factor of the ^{3}He nucleus based on the solution of the problem of three interacting nucleons, still remains when allowance is made for the

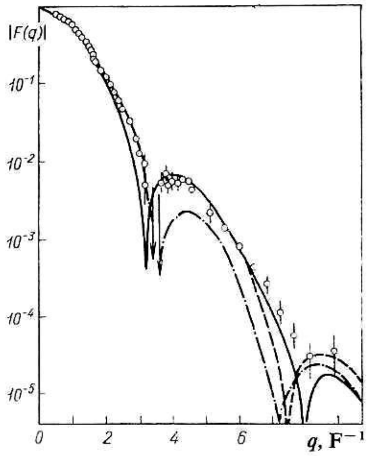

FIG. 13. Form factor of the ^{3}He nucleus, calculated with Gaussian wave functions of the nucleons in F_2 and F_{12} and with a realistic three-body nuclear form factor F_1.[71] The continuous curve is a calculation with $C_2 = -\sqrt{0.085}$ and $C_3^2 = 0$; the broken curve is with $C_2 = -\sqrt{0.085}$ and $C_3^2 = 0.003$; and the chain curve is for a realistic three-body form factor F_1. The experimental data are taken from Ref. 21.

$6q$ and $9q$ admixtures, although its position and shape may change. It is here evidently necessary to have a more detailed measurement of the experimental ^{3}He form factor in order to establish definitively whether this second minimum does exist in the region $q \sim 7$–$8\ \mathrm{F}^{-1}$. The existing data do not as yet provide an unambiguous answer to this question. At the same time, it is also interesting to analyze whether three-body theory itself is capable of taking into account correctly nuclear effects in this same region of q. As regards making the model more accurate, it is here possible to take one further step forward by using more realistic functions instead of the simple Gaussian wave functions of the ^{3}He nucleus. For this, we write the wave function of the ^{3}He nucleus in Jacobi coordinates:

$$\Psi_1 = \Psi_{3q}(\xi_1\eta_1)\,\Psi_{3q}(\xi_2\eta_2)\,\Psi_{3q}(\xi_3\eta_3)\,\Psi(\rho, r), \qquad (112)$$

$$\Psi_2 = \Psi_{6q}(\xi_1, \eta_1, \xi_2, \eta_2, r/\sqrt{2})\,\Psi_{3q}(\xi_3\eta_3)\,\varphi(\rho), \qquad (113)$$

$$\Psi_3 = \Psi_{9q}(\xi_1, \eta_1, \xi_2, \eta_2, \xi_3, \eta_3, r/\sqrt{2}, \sqrt{2/3}\rho). \qquad (114)$$

We choose the wave function $\Psi(\rho, r)$ of the relative motion of the nucleons in the factorized form

$$\Psi(\rho, r) = u(r)\,\Phi(\rho), \qquad (115)$$

so that

$$u(r) = \int \Psi(\rho, r)\,\Phi(\rho)\,d\boldsymbol{\rho} \qquad (116)$$

is determined as the overlap integral of the three-particle function $\Psi(\rho, r)$ and the function $\Phi(\rho)$ of the motion of the correlated pair of nucleons with respect to the third nucleon in the ^{3}He nucleus. At the end, we again obtain Eq. (97) (the case $C_3 = 0$), in which the individual terms have the form[52]

$$F_1(q^2) = F_{3q}(q^2) \int |\Phi(\rho)|^2 \exp(i2\mathbf{q}\boldsymbol{\rho}/3)\,d\boldsymbol{\rho} \equiv F_{3q}(q^2)\,\tilde{\tilde{F}}_1(q^2), \qquad (117)$$

$$F_2(q^2) = \frac{2}{3} F_{6q}(q^2)\,\tilde{\tilde{F}}_2(q^2/4) + \frac{1}{3} F_{3q}(q^2)\,\tilde{\tilde{F}}_2(q^2), \qquad (118)$$

$$F_{12}(q^2) = \frac{2}{3} F_{12}^q(q^2)\,\tilde{\tilde{F}}_{12}(q^2/4)\,\bar{\bar{F}}_{12}(q^2) + \frac{1}{3} F_{3q}(q^2)\,\tilde{\tilde{F}}_{12}(q^2). \qquad (119)$$

Here

$$\tilde{\tilde{F}}_2(q^2) = \int |\varphi(\rho)|^2 \exp(i2\mathbf{q}\boldsymbol{\rho}/3)\,d\boldsymbol{\rho}, \qquad (120)$$

$$\tilde{\tilde{F}}_{12}(q^2) = \int \Phi^*(\rho)\,\varphi(\rho) \exp(i2\mathbf{q}\boldsymbol{\rho}/3)\,d\boldsymbol{\rho}, \qquad (121)$$

$$\bar{\bar{F}}_{12}(q^2) = \int u(r)\,\tilde{\Psi}_{6q}(r\sqrt{2}) \exp(i\mathbf{q}\mathbf{r})\,d\mathbf{r}. \qquad (122)$$

It is natural to assume that the wave function of the motion of the $6q$ cluster with respect to the $3q$ cluster in ^{3}He is the same as the wave function of the motion of the correlated pair of nucleons with respect to the third nucleon. This means that in Eqs. (120) and (121) we must set $\Phi(\rho) = \varphi(\rho)$. We then obtain

$$\tilde{\tilde{F}}_2(q^2) = \tilde{\tilde{F}}_{12}(q^2) = \tilde{\tilde{F}}_1(q^2). \qquad (123)$$

These relations make it possible to simplify the expressions (118) and (119) by combining in them the second terms with the form factor $F_1(q^2)$:

$$F_2(q^2) = \frac{2}{3} F_{6q}(q^2)\,\tilde{\tilde{F}}_1(q^2/4), \qquad (124)$$

$$F_{12}(q^2) = \frac{2}{3} F_{12}^q(q^2)\,\tilde{\tilde{F}}_1(q^2/4)\,\bar{\bar{F}}_{12}(q^2). \qquad (125)$$

To calculate $\bar{F}_{12}(q^2)$, it is necessary to know $u(r)$ at small r, since $\tilde{\Psi}_{6q}(r/\sqrt{2})$, the part of the wave function of the $6q$ system that depends only on r, is determined within the range of the core of the NN forces. One expects that with a Gaussian form of the wave functions one can satisfactorily describe the structure of the ^{3}He nucleus at intermediate and large distances between the nucleons but not well at short distances, since they do not take into account the Jastrow type of damping as $r \to 0$. Therefore, it will be helpful to find other functions that for otherwise identical conditions work well at short distances. We shall choose $u(r)$ in the form

$$u(r) = N' \exp(-\tilde{\gamma}^2 r^2)\,(1 - \exp(-\beta^2 r^2)), \qquad (126)$$

where the value of β is kept the same as it is given in Ref. 72, i.e., $\beta = 1.9\ \mathrm{F}^{-1}$, and the parameter $\tilde{\gamma}$ is chosen to make the form factor F_{12} at small q^2 agree with what is given by (105) (this leads to the value $\tilde{\gamma} = 0.71\ \mathrm{F}^{-1}$; see Fig. 11, curves 3 and 4). Finally, the coefficient N' is chosen on the basis of the normalization condition of the function (126). Figure 14 shows calculations of the form factor of the ^{3}He nucleus, made in the framework of this model. The results are very close to those obtained using the Gaussian form of the nuclear wave functions (see Fig. 13).

We consider briefly the results of analysis of the ^{3}He form factor in the framework of other approaches[73,74] that take into account the quark structure of the nucleus. In Ref. 73, it was assumed that $F_{^3\mathrm{He}} = F_1 + C_{9q}F_{9q}$, where F_1 is the ordinary three-body form factor,[71] and F_{9q} is the form factor of a nine-quark system. In Ref. 74, $C_{6q}F_{6q}$ was added to it in addition. In both approaches, the interference between the

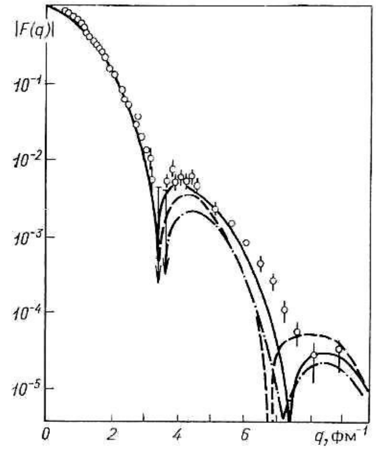

FIG. 14. The same as in Fig. 13 but for realistic wave functions of the nucleons in F_2 and F_{12}.

nucleon and quark channels was ignored. It was then found that agreement with the experimental data can be obtained if one assumes a negative sign of C_{6q} and C_{9q}; moreover, in $|F_{^3\mathrm{He}}|$ the second minimum is absent. In the language of wave functions, this means that the weight of the 6q and 9q admixtures must be negative, i.e., C_2^2, $C_3^2 < 0$. Comparing these approaches with the one considered here, we can readily see that a negative sign of C_{6q} simply imitates the contribution of the interference of the nucleon and 6q channels. With regard to the negative sign of C_{9q}, our preliminary estimates indicate that the contribution of $F_{13} + F_{23}$ to $F_{^3\mathrm{He}}$ can be ignored, but then the form factor of the 9q system must definitely occur with positive weight C_3^2. The final answer to this question can be obtained only after numerical calculations of F_{13} and F_{23}, and these are difficult to make because of the large model uncertainty of the problem.

Thus, analysis of elastic e^3He scattering shows that:

1. The interference of the nucleon and 6q channels is very important.

2. Allowance for the 6q admixtures in the ^{3}He wave function leads overall to better agreement with the experimental data in the complete range of momentum transfers to $q \lesssim 6\ \mathrm{F}^{-1}$ as compared with purely nuclear models.

3. In the calculated $|F_{^3\mathrm{He}}|$ in the region $q \approx 7$–$8\ \mathrm{F}^{-1}$ there is a second minimum, which is evidently due to the presence of such a minimum in the form factor $|F_1|$ calculated in the framework of the three-nucleon problem.

4. For a more definite conclusion about the nature of the multiquark admixtures in ^{3}He we require: a) a more detailed experimental investigation of $F_{^3\mathrm{He}}$ at $q \approx 7$–$8\ \mathrm{F}^{-1}$; b) more accurate calculations of F_1 in the framework of the three-body problem. The contribution of the 9q admixtures to the ^{3}He form factor must be taken into account with inclusion of the interference of the nucleon and the 9q channels, and also of the 6q and 9q channels. It may be concluded that for the nuclei ^{2}D and ^{3}He the measurements of the form factors have not yet reached the asymptotic region, so that the quark counting rules cannot be used directly. Further, in the preasymptotic region the calculations of the form factors depend very critically on the choice of the parameters of the models of the multiquark systems, the relativization procedure, and so forth. In this connection, it is of great interest to consider other processes with allowance for the quark structure of the nuclei (see, for example, Refs. 75–79). We shall here consider deep inelastic scattering.

5. SIX-QUARK ADMIXTURES IN DEEP INELASTIC SCATTERING

Deep inelastic scattering of leptons by nuclei with large momentum transfer makes it possible to determine the momentum distribution of the quarks in the nucleus. Thus, the cross section of the reaction $\mu A \to \mu' X$ in the case of muon scattering at small angles is determined by the so-called structure function F_2^A and has the form

$$\frac{1}{A}\frac{d^2\sigma^{\mu A}}{dq^2\,dx} = \frac{4\pi\alpha^2}{q^4}\frac{F_2^A(x,\,q^2)}{x}, \tag{127}$$

where $x = q^2/2M\nu$ is the scale variable ($t = -q^2$ is the square of the 4-momentum transfer, $\nu = E_\mu - E'_\mu$ is the energy transfer, and M is the nucleon mass). The $x > 1$ region[17] is kinematically forbidden for scattering by individual nucleons of the nucleus. Therefore, events with $x \geqslant 1$ can be attributed to multiquark objects in the nucleus.[80,81] Data have recently been published on measurement of the ratio $\tilde{R}$ of the structure functions of iron and deuterium[19,82]:

$$\tilde{R}(x) = \frac{2\sigma^A}{A\sigma^D} = \frac{F_2^A(x)}{F_2^D(x)};\quad A = 56. \tag{128}$$

This ratio is also very sensitive to assumptions about the distribution of the quarks in the nuclei. Thus, if it is assumed that a nucleus consists of only A nucleons, the $\tilde{R}$ must be equal to unity. However, the experiments showed that $\tilde{R}$ has a rise in the region of small x, a minimum at $x \simeq 0.65$, and a sharp rise at $x \simeq 1$, i.e., additional non-nucleon components are manifested in the nuclear structure functions. For the moment, we will refer to them as "exotic." In principle, the neutron structure function, which is determined as the difference between the deuteron and hydrogen data, already takes into account phenomenologically part of the "exotic nature" of the deuteron. Therefore, the result of the experimental data under discussion can be reformulated as follows: A heavy nucleus is exotic in a different way from the deuteron. Indeed, because of the difference, extracted from the deuteron data,[83] between the proton and neutron structure functions, $F_2^p(x)$ and $F_2^n(x)$, we obtain an effect $\tilde{R} \simeq 1.3 \times 10^{-2}$, which is an order of magnitude less than the experimental value ($\sim 1.7 \times 10^{-1}$) and can be ignored. In this section, we discuss the reasons that lead to the observed behavior of $\tilde{R}$; we investigate the influence of the 6q admixtures and the part played by the pion degrees of freedom. Calculations have been made for a fixed value of q^2 ($q^2 = 30\ \mathrm{GeV}^2/c^2$).[84] The effect of the q^2 dependence of the structure function in the cumulative region is discussed separately.

Six-quark admixtures and the structure function of a nucleus

The structure function of a 6q system of mass $M_f \simeq 2M$ depends on its scale variable $x' = q^2/2M_f\nu \simeq x/2$, where x is the nucleon scale variable, and it contributes to the structure function of μA scattering:

$$F_2^A(x) = (1 - P^A)\, F_2^N(x) + P^A F_2^{6q}(x), \tag{129}$$

where P^A is the probability that a $6q$ system is present in the nucleus (the deuteron), and F_2^N is the nucleon structure function:

$$F_2^N(x) \equiv \frac{1}{2}(F_2^p(x) + F_2^n(x)) = \frac{5}{18}(xu_v(x) + xd_v(x)) + \frac{12}{9} xu_S(x). \tag{130}$$

We write the structure function of the $6q$ system in the form

$$F_2^{6q}(x) = \frac{5}{18}\left[\frac{x}{2} u_v^f\left(\frac{x}{2}\right)\right] + \frac{6}{9}\left[\frac{x}{2} u_S^f\left(\frac{x}{2}\right)\right], \tag{131}$$

where $u_S = \bar{u}_S = d_S = \bar{d}_S = S_S = \bar{S}_S$, $u_S^f = \bar{u}_S^f = d_S^f = \bar{d}_S^f = S_S^f = \bar{S}_S^f$, and $u_v^f = d_v^f$. Analysis of cumulative reactions (Refs. 4, 57, 80, and 81) and theoretical calculations[85] show that the probability of finding the $6q$ system in a heavy nucleus is approximately twice what it is in the deuteron: $P^A \approx 2P^{\rm D}$. This can be understood qualitatively by estimating the ratio $P^A/P^{\rm D}$ by means of Eq. (2). Indeed, the probability divided by the atomic weight of the nucleus is $P^A = \frac{1}{2}\beta_6^A$, and, hence, $P^A/P^{\rm D} = (r_0^{\rm D}/r_0^A)^3$, where $r_0^{\rm D}$ and r_0^A are the effective nucleon radius in the deuteron and nucleus, respectively. Since the deuteron is not such a tightly bound system, $r_0^{\rm D} > r_0^A$ and $P^A > P^{\rm D}$. In what follows, we take $P^{\rm D} = 7\times 10^{-2}$. The parameters that determine $F_2^N(x)$ are chosen to reproduce the deuteron structure function. The parameters in $F_2^{6q}(x)$ are determined from the normalization condition $\int u_v^f(x)dx = 3$ and the assumption that the mean quark momenta in the nucleon and in the $6q$ system are the same: $\int xu_S^f(x)dx = \int xu_S(x)dx$. At the same time, we assume that $u_v^f(x)$ decreases as $x \to 1$ as $(1-x)^7$, i.e., slower than is required by the quark counting rules (preasymptotic region). This last assumption means that there is incomplete "unfreezing" of the color degrees of freedom in the $6q$ system.[4,80,81] Thus, if $P^A \neq P^{\rm D}$, the ratio $\tilde{R} = F_2^A/F_2^{\rm D}$ will differ from unity:

$$\tilde{R} = 1 + (P^A - P^{\rm D})\frac{F_2^{6q}(x) - F_2^N(x)}{F_2^{\rm D}(x)}. \tag{132}$$

At $x \sim 1$, $F_2^{\rm D} \simeq 0$, $F_2^{\rm D} \approx P^{\rm D} F_2^{6q}$, and $\tilde{R}(x \simeq 1) - 1 \simeq P^A/P^{\rm D} - 1 \simeq 1$, in qualitative agreement with the experiments. Overall, however, the expression (132) cannot explain the experiments in the complete region of x [the calculation in accordance with (132) is shown by curve 1 in Fig. 15]. For example, at $x \simeq 0$ we have $\tilde{R}_{\rm th} - 1 = -7\times 10^{-2} \times 0.27 = -1.9\times 10^{-2}$, whereas experiment gives $\tilde{R}_{\rm exp} - 1 \simeq 1.8 \times 10^{-1}$. Thus, it is clear that the rise in $\tilde{R}$ at small x is due to other causes. A qualitative growth of $\tilde{R}$ as $x \to 1$ is also predicted by other approaches which take into account the dynamics of the nucleon–nucleon interaction at short distances,[19,82] for example, the "nucleon-correlation" model.[86] However, in addition to the difficulties in describing the rise of $\tilde{R}$ in the region of small x in these models, they all predict too rapid growth of $\tilde{R}$ as $x \to 1$; for example, the calculation of Ref. 86 gives $\tilde{R}_{\rm th}(0.8) - 1 \simeq 1[\tilde{R}_{\rm exp}(0.8) - 1 \simeq -3 \times 10^{-2}]$. And a decrease in $\tilde{R}$ at $x \sim 1$ achieved by reducing the contribution of the high-momentum components leads these models to difficulties in the interpretation of $F_2^A(x)$ in the interval $0 \leqslant x \leqslant 1.4$.[17]

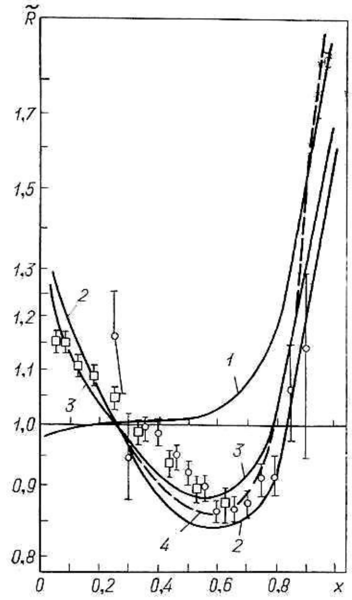

FIG. 15. Calculation and comparison with experiment of the ratio $\tilde{R} = (2/A)\sigma^{\rm Fe}/\sigma^{\rm D}$. 1) Allowance for only $6q$ states; 2) and 3) allowance for $6q$ states and pion fields: 2) $\rho_c/\rho_0 = 3.9$; 3) $\rho_c/\rho_0 = 4.2$, $P^A = 2P^{\rm D}$; 4) $\rho_c/\rho_0 = 3.9$, $p^A = 2.3P^{\rm D}$. The open squares are the data of Ref. 18, and the open circles are the data of Ref. 82.

Structure function and meson fields in nuclei

Allowance for pion degrees of freedom—"pion exchange currents"—leads to a rise in the ratio $\tilde{R}$ at small x. Qualitatively, this can be understood as follows. Suppose that in a nucleus there are n pions. Then, ignoring their internal motion in the nucleus, we expect the corresponding contribution to the structure function to be proportional to $nF^\pi(x_\pi)$, where $F^\pi(x_\pi)$ is the pion structure function and x_π is the pion scale variable: $x_\pi = q^2/2m_\pi\nu = Mx/m_\pi$. It follows from the condition $0 \leqslant x_\pi \leqslant 1$ that the region of the pion contribution to the structure function is restricted to small x: $0 \leqslant x < m_\pi/M \simeq 0.15$. Allowance for the internal motion of the pions in the nucleus somewhat extends this region.

We emphasize that we are here speaking of soft pions, which must be distinguished not only from hard pions, which are produced on the fragmentation of a fast quark (Fig. 16a), but also from "hard exchange pions," described by the exchange diagrams in Figs. 16b and 16c. The former make a contribution to the structure function that decreases in the limit $x \to 1$ as $(1-x)^{3-4}$ and are taken into account in the nucleon structure function. The latter are taken into account in $F_2^{6q}(x)$, since such diagrams contribute only at short nucleon–nucleon distances and imitate the short-range part of the NN forces.[26] In contrast, the soft pions are responsible for the long-range part of the NN forces, and they cannot be decribed by quark exchange diagrams of the type shown in Figs. 16b and 16c.

To estimate the "effective number" of pions in the nuclear medium, the process whose diagram is shown in Fig. 17 was considered in Ref. 84. In this case, the contribution of the pions to the μA scattering cross section has the form

$$d\sigma_\pi^{\mu A}(x) = \int d\sigma^{\mu\pi}(x(k))\, n_A^\pi(\mathbf{k}, \rho_c)\, d\mathbf{k}. \tag{133}$$

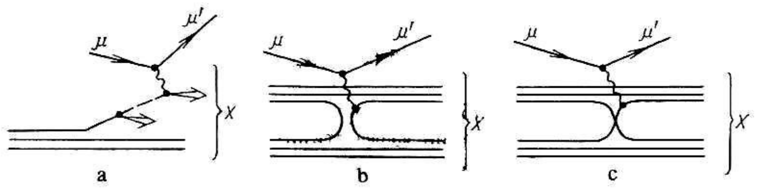

FIG. 16. Contribution of hard pions.

Here, $d\sigma^{\mu\pi}$ is the cross section of deep inelastic $\mu\pi$ scattering, and the function $n_A^\pi(\mathbf{k}, \rho_c)$ is the distribution of the "effective pions" in the nucleus; it depends on the parameter ρ_c, which includes all the original information about the nuclear medium and the interactions of its constituents[87–89] (pions, nucleons, Δ isobars, etc.). The effective number of pions in the medium is determined by the integral

$$n_A^\pi = \int n_A^\pi(\mathbf{k}, \rho_c)\, d\mathbf{k}. \tag{134}$$

Note that an additional contribution to the structure function $F_2^A(x)$ will be determined by the difference $\delta n_A^\pi = n_A^\pi - n_0^\pi$, where n_0^π corresponds to the contribution from the diagrams in Fig. 17 in the "vacuum," i.e., in the deuteron, this contribution being taken into account already in the nucleon structure function. As an illustration, Fig. 18 shows the dependence of δn_A^π on the ratio ρ_c/ρ_0, where ρ_0 is the normal nuclear density. The theoretical estimates predict $\rho_c \approx 3.2\rho_0$.[88] Allowance for the pion fields leads to a renormalization of the momentum distributions of the quarks in the nucleons and the $6q$ systems of the nucleus. To find the new distributions, it is necessary to take into account:

a) the conservation of the momentum carried by the quarks in the nucleus:

$$\langle xq^{\mathrm{D}}(x)\rangle = \langle xq^A(x)\rangle = \langle x\tilde{q}^A(x)\rangle + \langle xq^\pi(x)\rangle; \tag{135}$$

b) the conservation of the baryon charge:

$$\int q_v^{\mathrm{D}}(x)\, dx = \int \tilde{q}_v^A(x)\, dx, \tag{136}$$

where $\tilde{q}^A(x)$ is the distribution of the nucleon quarks and $6q$ quarks, $q_A^\pi(x)$ is the distribution of the pion quarks,

$$q_A^\pi(x) = \int q^\pi(x/y)\, \delta n_A^\pi(y)\, dy, \tag{137}$$

q^π is the distribution of the quarks in the pion, and q_v is the distribution of the valence quarks. The conditions (135) and (136) lead to a certain deformation of the quark distributions in a heavy nucleus, namely, the probability distributions of the valence u quarks are somewhat increased at small $x(x \lesssim 0.05)$ and decreased for $x \gtrsim 0.05$; the distributions of the sea quarks are reduced in the complete region of x.

Thus, the final expression for F_2^A has the form

$$F_2^A(x) = (1-P^A)\left[\frac{5}{18}(xu_v^A(x) + xd_v^A(x)) + \frac{12}{9}xu_S^A(x)\right] + P^A\left[\frac{5}{18}\left(\frac{x}{2}u_v^{A,f}\left(\frac{x}{2}\right)\right) + \frac{6}{9}\left(\frac{x}{2}u_S^{A,f}\left(\frac{x}{2}\right)\right)\right] + \delta F_2^{A(\pi)}(x), \tag{138}$$

The calculation indicates that the pion contribution, $\delta F_2^{A(\pi)}(x)$, has the behavior $(1-x)^{6-7}$ and imitates an enhancement of the sea quarks in the nucleus. The contribution $\delta F_2^{A(\pi)}$ is proportional to δn_A^π and depends on the parameter ρ_c. Figure 15 gives the result of calculation of $\tilde{R}$ for two values of ρ_c. Curve 2 is calculated for $\rho_c = 3.9\rho_0$ ($\delta n_A^\pi \simeq 0.83$), and curve 3 for $\rho_c = 4.2\rho_0$ ($\delta n_A^\pi \simeq 0.63$); $P^A = 2P^{\mathrm{D}}$. The renormalization of the quark distributions leads to a small decrease in $\tilde{R}$ in the cumulative region. If this ratio is to remain equal to 2 at $x \sim 1$,[80,81] it is necessary to increase somewhat P^A. The corresponding calculation is shown in Fig. 15 by the broken curve: $P^A \simeq 2.3P^{\mathrm{D}}$, $\rho_c = 3.9\rho_0$. It can be seen that the theoretical curves agree with experiment. In Fig. 19, we compare theoretical calculations of the structure functions $F_2^A(x)$ and $F_2^{\mathrm{D}}(x)$ themselves with the experiments[17,19,83] for $0 \leqslant x \leqslant 1.4$. It can be seen that the calculated curves basically reproduce the experiments. Further, the calculation of F_2^{D} for $x \geqslant 1$ is a prediction. Some of the change in the behavior of the theoretical curves in the region $x = 0.8$–1.0 is due to our complete neglect of the Fermi motion of the nucleons in the nucleus (deuteron), which makes a contribution for $0.9 \leqslant x \leqslant 1.1$.

As an illustration, Fig. 20 gives a test calculation of the deuteron structure function with allowance for relativistic Fermi motion in the "pair-correlation" model of Ref. 86. It can be seen from the figure that allowance for the Fermi motion leads only to a smoothing of the curves in the region

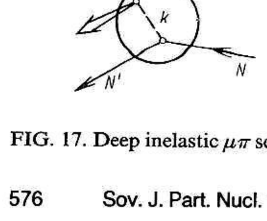

FIG. 17. Deep inelastic $\mu\pi$ scattering in a nucleus.

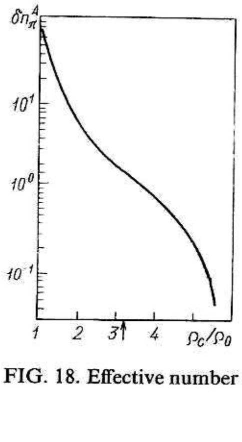

FIG. 18. Effective number of pions in a nucleus.

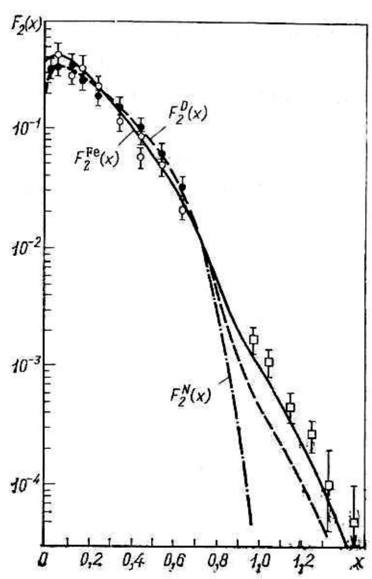

FIG. 19. Calculation and comparison with experiment of structure functions. The continuous curve is for a heavy nucleus, the broken curve for the deuteron, and the chain curve for the nucleon. The experimental data are as follows: black circles from Ref. 83, open squares from Ref. 17, and open circles from Ref. 91.

$0.9 \leqslant x \leqslant 1.1$ and to a small renormalization of the probability P^D, all the main results remaining unchanged.

Thus, the rise in $\tilde{R}$ in the cumulative region $x \approx 1$ is due to the dynamics of the NN interaction at short distances, namely, the admixture of $6q$ states in the nucleus. The rise for $0 \leqslant x \leqslant 0.2$ is due to the admixture of "long-range" pion fields, which are enhanced in heavy nuclei; this is a purely nuclear effect. A rearrangement of the pion field occurs in sufficiently heavy nuclei, and in this sense nuclei with $A = 9$–16 are "transitional."

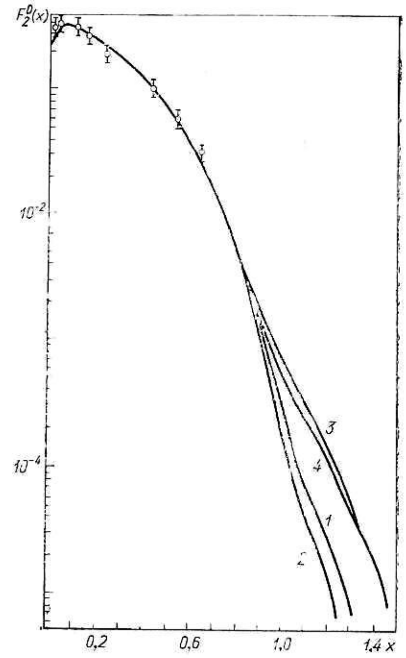

FIG. 20. Fermi motion in the deuteron: 1) calculation with the Hamada–Johnston potential; 2) with the Paris potential; 3) calculation with the Paris potential plus 7% $6q$ component; 4) with allowance for 7% $6q$ component without Fermi motion; the points are the data of Ref. 82.

The distribution of the pion quarks in the nucleus recalls the distribution of the sea quarks in the nucleon and $6q$ systems. Therefore, at small x the effect could be treated phenomenologically by enhancing significantly the contribution of the corresponding sea quarks in the hadrons of the nucleus. In this sense, our approach particularizes such a phenomenology, giving it physical content. In our view, the true enhancement of the "sea" in the $6q$ systems will, if it arises, be only a correction to the main effect.

We note finally that a growth in $\tilde{R}$ at small x due to enhancement of the pion field in heavy nuclei was also discussed in Refs. 90 and 91.

The q^2 dependence of the nuclear structure function

Effects of scaling violation are manifested in the q^2 dependence of the structure functions. It is interesting to consider to what extent these effects are strengthened or weakened in multiquark systems; the conclusion can, in its turn, give new information about the chromodynamic constant Λ. Naturally, for these purposes it is necessary to investigate the region of large q^2.

The functional dependence of the structure function on q^2 is found by solving the evolutionary chromodynamic equations of Lipatov, Altarelli, and Parisi.[92] To solve these equations, it is necessary to specify the initial condition, namely, the form of the structure function for fixed $q^2 = q_0^2$. For sufficiently large $x (x > 0.3)$, $F_2^N(x, q_0^2)$ in (130) and $F_2^{6q}(x)$ in (131) are determined by the distributions of the valence and u and d quarks:

$$q_i\,(x,\,q^2) = C_i x^{-\alpha_i}\,(1-x)^{\gamma_i}\,. \tag{139}$$

The parameters C_i, α_i, γ_i in (139) are found by comparison with experiment at the point $q^2 = q_0^2$, in our case $q_0^2 = 30\ (\mathrm{GeV}/c)^2$. To establish the main qualitative effects, we shall use an approximate analytic solution of the evolution equations that is exact in the limit $x \to 1$ (Ref. 93):

$$q_i\,(x,\,q^2) = q_i\,(x,\,q_0^2)\varphi_i\,(x,\,q^2), \tag{140}$$

$$\varphi_i\,(x, q^2) = (\tau/\tau_0)^{0,11}\,[(1-x)/(1+\gamma_i)]^{\tilde{P}}, \tag{141}$$

and for other x in the region $x \geqslant 0.3$ agrees with a numerical

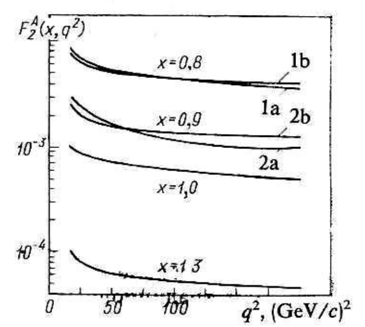

FIG. 21. The q^2 dependence of the structure function of the carbon nucleus.

solution with error not exceeding 10%.[94] Here,

$$\tau_0 = \ln (q_0^2/\Lambda^2); \quad \tau = \ln (q^2/\Lambda^2), \tag{142}$$

where Λ is the scale parameter of quantum chromodynamics.

Figure 21 shows the q^2 dependence of the nuclear structure function for different fixed x.[81] All curves except 1*a* and 2*a* are calculated for $P_2 = 0.14$. Curves 1*a* and 2*a* are calculated for $P_2 = 0$, i.e., without allowance for the 6*q* admixtures, and, for convenience, curves 1*a*, 1*b* and 2*a*, 2*b* are normalized at the point $q^2 = q_0^2$. It can be seen that the violation of scaling is stronger when the 6*q* admixtures are not taken into account. This means that if the scale parameter is extracted from data on the nuclear structure functions in the region $x \simeq 1$, then neglect of the 6*q* states will give an underestimate of Λ.

CONCLUSIONS

The question of the existence of a multiquark phase in nuclear matter is one of the most topical questions in modern nuclear and elementary-particle physics. We have considered it in the aspect of the manifestation of multiquark admixtures in the ground states of nuclei and the possible existence of "pure" (resonance-type) multiquark states, for example, dibaryons, which estimates suggest also lie at comparatively low energies. (In principle, there could also be a situation in which all nuclear matter becomes a quark–gluon plasma. For this, of course, special conditions are necessary—strong compression of nuclei in nucleus–nucleus collisions and, thus, the concentration of a high energy in the nuclear volume.) The basis of the theoretical treatment of the group of problems which we have outlined must evidently be provided by a relativistic approach that takes into account the coupling of the hadron and multiquark channels and the dynamical existence and interaction of two (or more) phases.

In the review, we have formulated a nonrelativistic approach that takes into account the coupling of two channels—a multiquark and a nucleon–nucleon channel. The interaction in the latter is given by realistic nucleon–nucleon forces with the region of the core eliminated; in it, the picture of the two-body *NN* interaction is replaced by a representation of a multiquark interaction. In the framework of this approach, it is possible to calculate both the probability of the multiquark admixture in the nuclei and the widths of the multiquark resonances in *NN* scattering. Its advantage is that it does not require a revision of the basic propositions of traditional nuclear physics. The point is that the multiquark components are concentrated in small spatial regions, and at low energies their effect can be imitated in the nucleon channel by introducing the core of the *NN* forces. Therefore, the conclusions of the traditional nuclear models, which presuppose the presence of only nucleon channels in the investigation of the ground and excited states of nuclei and nuclear reactions with small [$q^2 \ll 1$ (GeV/c)^2] momentum transfer, remain unchanged. At large momentum transfers [$q^2 > 1$ GeV/c)^2] the main contribution is made by the region of short distances, i.e., the region of the multiquark states, and the manifestation of their structural details may be very important. In particular, the momentum distributions of the quarks in such systems are very specific and depend on the number of quarks and on the momentum transfer to the system. The assumption of the presence of multiquark systems in nuclei is now necessary for the understanding of lepton–nucleus processes at large momentum transfers in the region forbidden kinematically for the corresponding processes involving free nucleons. In a number of cases, this contribution is decisive, as has been demonstrated by specific examples of allowance for multiquark systems in calculations of the electromagnetic form factors, the polarization characteristics, and the structure functions of nuclei.

An interesting problem of the further theoretical investigation is the construction of a relativistic theory of multiquark systems and the calculation on its basis of definite nuclear processes.

An important experimental task at the present time is the measurement of the deuteron structure function in the region $1 < x < 2$ in a large interval of q^2. In addition, measurement of the polarization characteristics of the deuteron, for example, the tensor polarization, at large q^2 will permit significant advance in our understanding of the physics of multiquark systems.

We thank S. M. Dorkin, V. N. Dostovalov, L. P. Kaptar', and B. L. Reznik for a major contribution to the papers that were used in the preparation of the review.

[1] D. I. Blokhintsev, Zh. Eksp. Teor. Fiz. **33**, 1295 (1957) [Sov. Phys. JETP **6**, 995 (1958)].
[2] L. S. Azhgireĭ *et al.*, Zh. Eksp. Teor. Fiz. **33**, 1185 (1957) [Sov. Phys. JETP **6**, 911 (1958)].
[3] A. M. Baldin, Kratk. Soobshch. Fiz. **1**, 35 (1971); Fiz. Elem. Chastits At. Yadra **8**, 429 (1977) [Sov. J. Part. Nucl. **8**, 175 (1977)].
[4] V. V. Burov, V. K. Luk'yanov, and A. I. Titov, in: Trudy mezhdunarodnoĭ konferentsii po izbrannym voprosam struktury yadra (Proc. of the Intern. Conf. on Selected Questions of Nuclear Structure), D-9920, Vol. 2, JINR, Dubna (1976), p. 432. V. K. Luk'yanov and A. I. Titov, Fiz. Elem. Chastits At. Yadra **10**, 815 (1979) [Sov. J. Part. Nucl. **10**, 321 (1979)].
[5] A. V. Efremov, Yad. Fiz. **24**, 1208 (1976) [Sov. J. Nucl. Phys. **24**, 633 (1976)]; Fiz. Elem. Chastits At. Yadra **13**, 613 (1982) [Sov. J. Part. Nucl. **13**, 254 (1982)].
[6] V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973); S. Brodsky and G. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
[7] V. V. Burov *et al.*, Yad. Fiz. **28**, 321 (1978) [Sov. J. Nucl. Phys. **28**, 162 (1978)].
[8] V. A. Matveev and P. Sorba, Lett. Nuovo Cimento **20**, 145 (1977); V. A. Matveev, in: Proc. of the CERN–JINR School of Physics, Hondo (Finland), 1981, Geneva (1982), p. 306.
[9] A. T. Aerts, R. J. Mulders, and J. J. de Swart, Phys. Rev. D **17**, 260 (1978).
[10] M. M. Makarov, Usp. Fiz. Nauk **136**, 185 (1982) [Sov. Phys. Usp. **25**, 366 (1982)]; T. Kamae, in: Proc. of the Ninth Intern. Conf. on High Energy Physics and Nuclear Structure, Versailles, France (1981), p. 25.
[11] G. A. Leksin, in: Trudy XVIII Mezhdunarodnoĭ konferentsii po fizike vysokikh énergiĭ. Tbilisi, 1976 (Proc. of the 18th Intern. Conf. on High Energy Physics, Tbilisi, 1976), D1,2-10400, Vol. 1, JINR, Dubna (1977), p. A6-3.
[12] V. S. Stavinskiĭ, Fiz. Elem. Chastits At. Yadra **10**, 949 (1979) [Sov. J. Part. Nucl. **10**, 373 (1979)].
[13] A. M. Baldin *et al.*, Preprint E1-82-472 [in English], JINR, Dubna (1982).
[14] A. M. Baldin, Preprint E2-83-415 [in English], JINR, Dubna (1983).
[15] W. P. Schütz *et al.*, Phys. Rev. Lett. **38**, 259 (1977).
[16] D. Day *et al.*, Phys. Rev. Lett. **43**, 1143 (1979); S. Rock *et al.*, Phys. Rev. C **26**, 1592 (1982).

[17] I. A. Savin, in: Trudy Mezhd. seminara po problemam fiziki vysokikh énergiĭ (Proc. of the Intern. Symposium on Problems of High Energy Physics), D1,2-81-728, Dubna (1981), p. 223.
[18] A. M. Baldin, Preprint E1-80-545 [in English], JINR, Dubna (1981).
[19] J. J. Aubert, Phys. Lett. **123B**, 275 (1983).
[20] R. G. Arnold *et al.*, Phys. Rev. Lett. **35**, 776 (1975) and references therein; R. G. Arnold *et al.*, Contributed Paper at the Ninth Intern. Conf. on High Energy Physics and Nuclear Structure, Versailles, France (1981), p. 94.
[21] J. S. McCarthy, I. Sick, and R. R. Whitney, Phys. Rev. C **15**, 1396 (1977); M. Bernheim *et al.*, Lett. Nuovo Cimento **5**, 431 (1972); R. G. Arnold *et al.*, Phys. Rev. Lett. **40**, 1429 (1978).
[22] V. G. Ableev *et al.*, Nucl. Phys. **A393**, 491 (1983).
[23] Yu. A. Simonov, Phys. Lett. **107B**, 1 (1981).
[24] V. K. Luk'yanov, B. L. Reznik, and A. I. Titov, Preprint R2-12754 [in Russian], JINR, Dubna (1979).
[25] S. M. Dorkin, V. K. Lukyanov, and A. I. Titov, Preprint E2-80-43 [in English], JINR, Dubna (1980).
[26] S. M. Dorkin, V. K. Luk'yanov, and A. I. Titov, Preprint R2-82-913 [in Russian], JINR, Dubna (1982).
[27] A. I. Baz' and M. V. Zhukov, Yad. Fiz. **16**, 958 (1972) [Sov. J. Nucl. Phys. **16**, 529 (1973)].
[28] A.I. Baz', in: Trudy Mezhd. Shkoly po strukture yadra (Proc. of the Intern. School on Nuclear Structure), D2-6465, JINR, Dubna (1972), p. 7.
[29] V. K. Lukyanov and A. I. Titov, in: Few Body Systems and Nuclear Forces (eds. H. Zingl *et al.*), Springer-Verlag, Berlin (1978), p. 397.
[30] T. D. Babutsidze, T. I. Kopaleĭshvili, I. Z. Mochabeli, and M. Sh. Chachkhunashvili, Yad. Fiz. **33**, 1406 (1981) [Sov. J. Nucl. Phys. **33**, 754 (1981)].
[31] Yu. F. Smirnov and Yu. M. Tchuvilsky, J. Phys. G **4**, L1 (1978).
[32] V. M. Dubovik and I. T. Obukhovsky, Z. Phys. A **299**, 341 (1981).
[33] V. Vento, M. Rho, E. M. Muman *et al.*, Nucl. Phys. **A345**, 413 (1980).
[34] M. M. Musakhanov, Yad. Fiz. **33**, 810 (1981) [Sov. J. Nucl. Phys. **33**, 422 (1981)].
[35] D. A. Liberman, Phys. Rev. D **16**, 1542 (1977).
[36] J. Orear, Phys. Rev. D **18**, 1484 (1978).
[37] A. L. Licht and A. Pagnamenta, Phys. Rev. D **2**, 1150 (1970).
[38] R. V. Reid, Ann. Phys. (N. Y.) **50**, 411 (1968).
[39] E. Lomon and H. Feshbach, Ann. Phys. (N. Y.) **48**, 94 (1968).
[40] V. V. Burov, S. M. Dorkin, V. K. Luk'yanov, and A. I. Titov, Z. Phys. A **306**, 1491 (1982).
[41] I. T. Obukhovskiĭ *et al.*, Yad. Fiz. **31**, 516 (1980) [Sov. J. Nucl. Phys. **31**, 269 (1980)].
[42] S. M. Dorkin, B. L. Reznik, and A. I. Titov, Yad. Fiz. **36**, 1244 (1982) [Sov. J. Nucl. Phys. **36**, 725 (1982)]; Preprint R4-81-791 [in Russian], JINR, Dubna (1981).
[43] V. G. Neudatchin, Yu. F. Smirnov, and R. Tamagaki, Prog. Theor. Phys. **58**, 1 (1977).
[44] Yu. F. Smirnov, I. T. Obukhovskiĭ, and V. G. Neudachin, Yad. Fiz. **27**, 860 (1978) [Sov. J. Nucl. Phys. **27**, 456 (1978)].
[45] R. L. Jaffe and M. P. Shultz, Preprint CALT-68-775 (1980).
[46] V. N. Efimov, Preprint R4–82–202 [in Russian], JINR, Dubna (1972).
[47] M. A. Markov, J. Phys. **3**, 452 (1940); H. Yukawa, Phys. Rev. **77**, 219 (1950); R.P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).
[48] Y. Kizukuri *et al.*, Prog. Theor. Phys. **64**, 1478 (1980); Y. Kizukuri, M. Namuki, and K. Okano, Prog. Theor. Phys. **61**, 559 (1979).
[49] A. P. Kobushkin, Yad. Fiz. **28**, 495 (1978) [Sov. J. Nucl. Phys. **28**, 252 (1978)]; A. P. Kobushkin and V. P. Shelest, Fiz. Elem. Chastits At. Yadra **14**, 1146 (1983) [Sov. J. Part. Nucl. **14**, 483 (1983)].
[50] I. T. Obukhovskiĭ and E. V. Tkalya, Yad. Fiz. **35**, 288 (1982) [Sov. J. Nucl. Phys. **35**, 164 (1982)].
[51] V. V. Burov *et al.*, Preprint R2-83-439 [in Russian], JINR, Dubna (1983); V.V. Burov *et al.*, Z. Phys. A **315**, 205 (1984).
[52] V. V. Burov, V. K. Luk'yanov, and A. I. Titov, Preprint R2-83-749 [in Russian], JINR, Dubna (1983).
[53] V. G. Neudachin and Yu. F. Smirnov, Nuklonnye assotsiatsii v legkikh yadrakh (Nucleon Associations in Light Nuclei), Nauka, Moscow (1969).
[54] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
[55] R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. C **23**, 363 (1981); M. I. Haftel, L. Mathelitsch, and H. F. K. Zingl, Phys. Rev. C **22**, 1285 (1980).
[56] R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. C **21**, 1426 (1980).
[57] V. V. Burov, V. K. Luk'yanov, and A. I. Titov, Izv. Akad. Nauk SSSR, Ser. Fiz. **42**, 38 (1978).
[58] H. J. Pirner and J. P. Vary, Phys. Rev. Lett. **46**, 1376 (1981).
[59] F. Gross, Nucl. Phys. **A358**, 215 (1981).
[60] H. S. Song, F. L. Ridener, Jr., and R. H. Good, Jr., Phys. Rev. D. **25**, 61 (1982).
[61] M. Gourdin and C. A. Piketty, Nuovo Cimento **32**, 1137 (1964).
[62] J. S. Levinger, Acta Phys. Acad. Sci. Hung. **33**, 135 (1973).
[63] T. Brady, E. Tomusiak, and J. S. Levinger, Can. J. Phys. **52**, 1322 (1974).
[64] M. J. Moravcsik and J. Ghosh, Phys. Rev. Lett. **32**, 321 (1974).
[65] R. S. Bhalerao, Nucl. Phys. **A389**, 557 (1982).
[66] V. K. Mitrjushkin, P. K. Rashidov, and S. V. Trubnikov, Preprint E2–82–156 [in English], JINR, Dubna (1982).
[67] R. S. Bhalerao and S. A. Gurvitz, Phys. Rev. Lett. **47**, 1815 (1981).
[68] V. V. Burov, V. K. Luk'yanov, and A. I. Titov, Phys. Lett. **67B**, 46 (1977).
[69] S. Brodsky and B. Chertok, Phys. Rev. D **14**, 3003 (1976).
[70] K. Fujimura, T. Kobayashi, and M. Namiki, Prog. Theor. Phys. **44**, 193 (1970).
[71] R. A. Brandenburg, Y. E. Kim, and A. Tubis, Phys. Rev. C **12**, 1368 (1975); A. E. L. Dieperink *et al.*, Phys. Lett. **63B**, 261 (1976); J. A. Tjon, Phys. Rev. Lett. **40**, 1239 (1978).
[72] F. C. Khanna, Nucl. Phys. **A165**, 475 (1971).
[73] B. Chertok, Phys. Rev. Lett. **41**, 1155 (1978).
[74] M. Namiki, K. Okano, and N. Oshimo, Phys. Rev. C **25**, 2157 (1982).
[75] S. J. Brodsky and B. T. Chertok, Phys. Rev. Lett. **37**, 269 (1976).
[76] R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. Lett. **38**, 1516 (1977).
[77] S. V. Goloskokov *et al.*, Preprint R2-10142 [in Russian], JINR, Dubna (1976).
[78] B. Povh, Nucl. Phys. **A358**, 111 (1981).
[79] I. L. Grach and L. A. Kondratyuk, Preprint No. 59, Institute of Theoretical Physics, Moscow (1983).
[80] E. A. Boundarchenko and A. V. Efremov, Preprint E2-82-927 [in English], JINR, Dubna (1982).
[81] A. I. Titov, Preprint E2-83-72 [in English], JINR, Dubna (1983).
[82] A. Bodek *et al.*, SLAC-PUB-3041 (1983).
[83] J. J. Aubert *et al.*, Phys. Lett. **123B**, 123 (1983).
[84] A. I. Titov, Preprint E2-83-460 [in English], JINR, Dubna (1983).
[85] V. K. Lukyanov and A. I. Titov, in: Proc. of the Intern. Conf. on Extreme States in Nuclear Systems, Vol. 2, Dresden (1980), p. 60; K. A. Gridnev *et al.*, Izv. Akad. Nauk Kaz. SSR, Ser. Fiz. Mat. **2**, 74 (1979).
[86] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 215 (1981).
[87] A. B. Migdal, Fermiony i bozony v sil'nykh polyakh (Fermions and Bosons in Strong Fields), Nauka, Moscow (1978).
[88] G. G. Bunatyan, Yad. Fiz. **31**, 1186 (1980) [Sov. J. Nucl. Phys. **31**, 613 (1980)].
[89] G. G. Bunatyan and N. N. Mishustin, Yad. Fiz. **36**, 1121 (1982) [Sov. J. Nucl. Phys. **36**, 656 (1982)].
[90] C. Llewellyn-Smith, Phys. Lett. **128B**, 107 (1983).
[91] M. Ericson and A. N. Thomas, CERN/TH-3553 (1983).
[92] L. N. Lipatov, Yad. Fiz. **20**, 181 (1984) [Sov. J. Nucl. Phys. **20**, 94 (1975)].
[93] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[94] G. Altarelli, Phys. Rep. **81**, 1 (1982).
[95] D. J. Gross, Phys. Rev. Lett. **32**, 1071 (1974).

Translated by Julian B. Barbour