Microscopic description of pre-equilibrium processes on the basis of the unified theory of nuclear reactions

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Microscopic (stationary) approaches to the description of pre-equilibrium emission of particles from excited states of nuclei are discussed in the framework of the unified theory of nuclear reactions. Two types of statistical multistage emission processes are considered: processes through a compound nucleus (SMCP) and direct processes (SMDP). Examples of the description of reactions in the scheme SMDP + SMCP are given. Analysis of experimental data justifies a combined nuclear-reaction mechanism in accordance with the scheme SMDP + SMCP. A generalized model of the decay of giant multipole resonances is discussed on the basis of the quasiparticle–phonon model of the nucleus and the microscopic theory of pre-equilibrium reactions for the description of experiments with energy resolution $\Delta E < W_{\text{opt}}$.

INTRODUCTION

For the description of experimental data on pre-equilibrium nuclear reactions with light ions ($a \leqslant 4$) phenomenological models are used with greater or lesser success.[1,2] The main shortcoming of the majority of such models is that they are not derived from the many-particle Schrödinger equation in the scattering problem but rather are based merely on physically intuitive postulates. The diverse derivations of the models means that it is not possible to establish connections between them or restrictions on their regions of applicability. In addition, in each model there are corresponding adjustable parameters, which are chosen in order to obtain the best agreement with experiment. However, despite the fact that experiment is the criterion for correctness of a theory, the successes of phenomenological models in describing experimental data cannot in themselves serve as a justification, since there is always the danger of satisfactory description of experiment by an incorrect theory. It is of fundamental importance to establish connections between the phenomenological models and fundamental microscopic theory,[3] since this will permit a deeper understanding of the physical meaning of the phenomenological models and determine their regions of applicability. The most consistent description of pre-equilibrium processes can be obtained in the framework of the microscopic theory of nuclear reactions, which has the same concepts as the shell model, namely, a self-consistent potential and a residual interaction between the particles. An advantage of such an approach is the possibility of making calculations of the nuclear-reaction mechanisms with the same parameters as are used to describe nuclear structure.

The phenomenological models (all possible modifications of the hybrid model and exciton model) based on the use of a phenomenological kinetic equation[1,2] and the microscopic models based on a quantum kinetic equation for open finite systems[3] are suitable only for an averaged description of experiments with a poor energy resolution $\Delta E \gg W_{\text{opt}}$ (W_{opt} is the imaginary part of the optical potential). This situation is a consequence of the transition from the complicated many-particle Schrödinger equation to the simpler kinetic equations.[3,4] However, quantitative description of experiments with $\Delta E < W_{\text{opt}}$ in the framework of the hybrid and exciton phenomenological models is possible only if an unrealistic nucleon mean free path in the nucleus is used.[5] This internal inconsistency deprives them of physical significance in the description of experiments with $\Delta E < W_{\text{opt}}$. In addition, all modifications of the exciton (hybrid) model suffer from the following basic shortcomings:

1) the use of classical mechanics, which is valid for large masses of the particles and sufficiently high energies;

2) the assumption of equal probability of all configurations for a given type of intermediate state, an assumption that is not satisfied when allowance is made for the contributions from the collective modes (phonons);

3) uncertainty in the value of the mean-square transition matrix element, this having the consequence that it is not possible to obtain the absolute value of the cross section, and also the unphysical nature of this parameter in a simultaneous treatment of multistage direct processes and processes through a compound nucleus;

4) incorrect allowance for the continuum, in particular, the use of the cross section of the inverse reaction on the excited nucleus (σ_{inv}), which cannot be rigorously estimated or measured;

5) the impossibility of systematic allowance for a single-stage direct reaction with composite particles and, therefore, the uncertainty in the initial number of excitons, which is fitted experimentally.

At the present time, phenomenological models are used basically to parametrize experimental data. Completeness of the description of all possible nuclear-process mechanisms and relative simplicity of the calculations are the main requirements imposed in the construction of a unified theory of pre-equilibrium nuclear reactions with light ions.

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1. STATISTICAL DESCRIPTION OF MULTISTAGE PROCESSES IN FESHBACH'S UNIFIED THEORY OF NUCLEAR REACTIONS

Multistage processes and general methods of the unified theory of nuclear reactions

The question of the connection between pre-equilibrium models and the unified theory of nuclear reactions was posed by Feshbach.[6] In the unified theory of nuclear reactions[7] (in the stationary approach) the problem of the interaction of an incident particle with a nucleus described by the total Hamiltonian of the system

$$H\psi = E\psi, \quad H = H_0 + V,$$

where V is the potential-energy operator, is solved by separating from the total wave function ψ by means of the projection operators $\hat{P}$ and $\hat{Q}$ the wave function $\hat{P}\psi$ of the open channels (continuum) and the wave function $\hat{Q}\psi$ of the closed channels (discrete spectrum):

$$(E - H_{PP} - H_{PQ}(E - H_{QQ})^{-1}H_{QP})\,\hat{P}\psi = 0,$$
$$(E - H_{QQ} - H_{QP}(E^{+} - H_{PP})^{-1}H_{PQ})\,\hat{Q}\psi = H_{QP}\chi_i^{(+)},$$

where we have used the notation $H_{PP} = \hat{P}H\hat{P}$, $H_{QQ} = \hat{Q}H\hat{Q}$, $H_{PQ} = \hat{P}H\hat{Q}$, etc.; $\chi_i^{(+)}$ is an eigenfunction of the effective Hamiltonian H_{PP} corresponding to the entrance channel i; E^{+} means $E + i\eta$ with $\eta \to +0$. Using the well-known technique, we can write the T matrix of the reaction in the form[7]

$$T_{fi} = T_{fi}^{\mathrm{MDP}} + T_{fi}^{\mathrm{MCP}}. \tag{1}$$

Here, the matrix T_{fi}^{MDP} describes the multistage direct processes, since in the framework of such an approach the direct processes are defined as transitions due to the coupling of different open configurations (channels), whereas the matrix T_{fi}^{MDP} describes multistage processes with the formation of a compound nucleus, to which there correspond transitions from open to closed configurations and vice versa:

$$T_{fi}^{\mathrm{MCP}} = \langle\psi_f^{(-)}|V_{PQ}(E - h_{QQ})^{-1}V_{QP}|\psi_i^{(+)}\rangle, \tag{2}$$

where

$$h_{QQ} = H_{QQ} + W_{QQ} = H_{QQ} + V_{QP}(E - H_{\mathrm{opt}})^{-1}V_{PQ}, \tag{3}$$

$\psi_f^{(-)}$, $\psi_i^{(+)}$ are the wave functions of the exit and entrance channels, respectively, these being eigenfunctions of the effective Hamiltonian

$$H_{\mathrm{opt}} = H_{PP} + H_{PQ}(E - H_{QQ} + i\,\Delta E/2)^{-1}H_{QP}, \tag{4}$$

and ΔE is the averaging interval. The operators V_{PQ} and V_{QP} are determined by the relations

$$V_{QP} = \left[\frac{i\,\Delta E/2}{E - H_{QQ} + i\,\Delta E/2}\right]^{1/2} H_{QP},$$
$$V_{PQ} = H_{PQ}\left[\frac{i\,\Delta E/2}{E - H_{QQ} + i\,\Delta E/2}\right]^{1/2}.$$

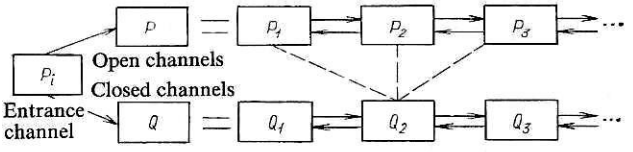
Allowance for intermediate states of any order (states of increasing complexity) in the framework of the unified theory is made by representing the projection operators $\hat{Q}$ and $\hat{P}$ in the form

$$\left.\begin{aligned}\hat{Q} &= \sum_n \hat{Q}_n, \quad \hat{Q}_n\hat{Q}_m = \hat{Q}_n\delta_{n,m},\\ \hat{P} &= \sum_n \hat{P}_n, \quad \hat{P}_n\hat{P}_m = \hat{P}_n\delta_{n,m}.\end{aligned}\right\} \tag{5}$$

At the same time, a subsidiary condition is imposed:

$$\hat{Q}_n h_{QQ}\hat{Q}_m = 0, \quad \hat{P}_n H_{\mathrm{opt}}\hat{P}_m = 0, \quad \text{if} \quad |n - m| \geqslant 2. \tag{6}$$

The condition (6) is a fundamental assumption, the basis of the analysis. Each intermediate state (eigenfunction of the effective Hamiltonian H_{QQ} or H_{opt}) is determined by a configuration of a given complexity, it being assumed that an intermediate state of order n is more complicated than one of order $n - 1$ and less complicated than one of order $n + 1$. Of course, the complexity of an intermediate state is determined in accordance with the chosen representation. Thus, in the exciton model of pre-equilibrium decay increasing complexity of an intermediate state corresponds to an increasing number of excitons. However, other representations are possible, these being determined by the physics of the particular reactions. The condition (6) presupposes that the residual (effective) interaction causes only transitions from an intermediate state of order n (n-th stage) to one of order $n \pm 1$ [$(n \pm 1)$-th stage]. The successive intermediate states can be regarded as the corresponding stages in the development of the intermediate system. The set of all continuum intermediate states of order n (configurations of the system with one or several particles in the continuum) forms the corresponding space P_n of open channels. The set of intermediate states of order n of the discrete (quasidiscrete) spectrum—bound (quasibound) configurations of the system—form the corresponding space Q_n of closed channels. In accordance with the above, a multistage nuclear process can be represented graphically in the form of the following scheme, in which the arrows indicate the direction of the transitions that are taken into account:



From the assumption that the residual interaction V is determined solely by two-particle forces it follows automatically that the condition (6) holds for H_{QQ}. However, for h_{QQ} determined by the expression (3) the condition (6) is not strictly satisfied, since for the operator W_{QQ}

$$\hat{Q}_n W_{QQ}\hat{Q}_m \neq 0 \quad \text{for} \quad m > n + 1.$$

A similar violation of the condition (6) occurs for the operator H_{opt} [see (4)]. In the following exposition, we ignore these violations, assuming that they are small by virtue of the statistical hypothesis of a random distribution of the signs of the matrix elements. In what follows, we shall assume that the conditions (6) hold for the operators H_{opt} and h_{QQ}.

Statistical theory of multistage processes through a compound nucleus

A statistical description of multistage processes in the framework of the unified theory of nuclear reactions was

given in Ref. 8. We consider now multistage processes through a compound nucleus. In accordance with the scheme of a multistage process given above, the T_{fi}^{MCP} matrix of the reaction [see (2)] can be represented as a sum of the partial contributions from the individual stages:

$$T_{fi}^{\mathrm{MCP}} = \sum_{n=1}^{\bar{n}} T_{fi\,(n)}^{\mathrm{MCP}}, \tag{7}$$

where

$$T_{fi\,(n)}^{\mathrm{MCP}} = \langle \psi_f^{(-)} | V_{P,\,n} G_n^{(b)} V_{n,\,n-1} G_{n-1}^{(b)} V_{n-1,\,n-2} \ldots V_{2,1} G_1^{(b)} V_{1,\,P} | \psi_i^{(+)} \rangle,$$

$V_{n,\,n-1} \equiv \hat{Q}_n V \hat{Q}_{n-1}$, $G_n^{(b)}$ is the Green's operator of the corresponding intermediate state of order n (in the Q space):

$$G_n^{(b)} = \hat{Q}_n (E - h_{n,\,n} - V_{n,\,n+1} G_{n+1}^{(b)} V_{n+1,\,n})^{-1} \hat{Q}_n. \tag{8}$$

We define the effective potential $\omega_{n,\,n}$ for each space Q_n by

$$\begin{aligned} \omega_{n,\,n} &= V_{n,\,n+1}\, G_{n+1}^{(b)} V_{n+1,\,n} \\ &= V_{n,\,n+1} (E - h_{n+1,\,n+1} - \omega_{n+1,\,n+1})^{-1} V_{n+1,\,n}. \end{aligned} \tag{9}$$

This potential $\omega_{n,\,n}$ determines the transitions from Q_n to the more complicated space Q_{n+1}, and the effect of the continuum is manifested through the operators $W_{n,\,n} = h_{n,\,n} - H_{n,\,n}$ [see (3)] by virtue of their coupling to the open channels. The end of the chain of intermediate states is determined by the boundary conditions

$$G_{\bar{n}+1}^{(b)} \equiv 0 \quad \text{or} \quad \omega_{\bar{n},\,\bar{n}} = 0.$$

If the conditions $\Gamma_n \gg D_{n-1} > D_n$ hold for $n \leqslant \bar{n}$, where $\bar{n}$ is the evaporation stage, $\Gamma_n = \langle \Gamma_{n,\beta} \rangle$ is the mean width, and D_n is the mean distance between the levels in Q_n, then the effects of transition to more complicated configurations in each Green's operator $G_n^{(b)}$ give rise to a weak energy dependence of the operators $\omega_{n,\,n}$. Then for $\omega_{n,\,n}$ and $G_n^{(b)}$ we have

$$\omega_{n,\,n} = \sum_{\alpha} V_{n,\,n+1} | \varphi_{n+1,\,\alpha} \rangle (E - e_{n+1,\,\alpha})^{-1} \langle \bar{\varphi}_{n+1,\,\alpha} | V_{n+1,\,n}; \tag{10}$$

$$G_n^{(b)} = \sum_{\alpha} \varphi_{n,\,\alpha} (E - e_{n,\,\alpha})^{-1} \bar{\varphi}_{n,\,\alpha}^{*}, \tag{11}$$

where $\varphi_{n,\,\alpha}$ are the eigenfunctions of the operator $(h_{n,\,n} + \omega_{n,\,n})$,

$$\left.\begin{aligned} (e_{n,\,\alpha} - h_{n,\,n} - \omega_{n,\,n})\, \varphi_{n,\,\alpha} = 0, \\ (e_{n,\,\alpha}^{*} - h_{n,\,n}^{+} - \omega_{n,\,n}^{+})\, \bar{\varphi}_{n,\,\alpha} = 0. \end{aligned}\right\} \tag{12}$$

The imaginary parts of the energy eigenvalues $e_{n,\,\alpha}$ are determined by

$$2\ \mathrm{Im}\ e_{n,\,\alpha} = \Gamma_{n,\,\alpha} = \Gamma_{n,\,\alpha}^{(1)} + \Gamma_{n,\,\alpha}^{(2)}, \tag{13}$$

where

$$\Gamma_{n,\,\alpha}^{(1)} = -2\ \mathrm{Im}\ \langle \bar{\varphi}_{n,\,\alpha} | h_{n,\,n} | \varphi_{n,\,\alpha} \rangle, \tag{14}$$

$$\Gamma_{n,\,\alpha}^{(2)} = -2\ \mathrm{Im}\ \langle \bar{\varphi}_{n,\,\alpha} | \omega_{n,\,n} | \varphi_{n,\,\alpha} \rangle. \tag{15}$$

We assume that the extent to which h_{QQ} is complex as a result of the coupling to the open channels P is weak. In other words, we assume that the transition probability for $n < \bar{n}$ from Q_n to Q_{n+1} is much greater than the probability of decay of Q_n to open channels. Under these restrictions, one can make in the transition matrix element ($\omega_{n,\,n}$ is assumed to be almost independent of the energy) the substitution $\bar{\varphi}_{n,\,\alpha} = \varphi_{n,\,\alpha}$. Then, using (14) and (15) and averaging over the quantum numbers of the states (n, α), we obtain

$$\Gamma_n^{(1)} = \Gamma_n^{\uparrow} = \sum_{c} \Gamma_n^{(c)}, \tag{16}$$

$$\Gamma_n^{(2)} = \Gamma_n^{\downarrow}, \tag{17}$$

where

$$\Gamma_n^{(c)} = 2\pi \langle | \langle \varphi_{n,\,\alpha} | V_{n,\,P} | \psi_c^{(+)} \rangle |^2 \rangle,$$

$\{\psi_c^{(+)}\}$ is the complete set of admissible open channels $\{c\}$, and

$$\Gamma_n^{\downarrow} = \frac{2\pi}{D_{n+1}} \langle | \langle \varphi_{n+1,\,\beta} | V_{n+1,\,n} | \varphi_{n,\,\alpha} \rangle |^2 \rangle.$$

It follows from (13) that the total width of an intermediate state of order n is

$$\Gamma_n = \Gamma_n^{\downarrow} + \Gamma_n^{\uparrow}. \tag{18}$$

Using the fundamental statistical assumption[4] that the phases of the matrix elements are random under the condition that the number of states in the averaging interval is large, we obtain for $|T_{fi}^{\mathrm{MCP}}|^2$ after averaging over the quantum numbers of the intermediate states

$$\langle | T_{fi}^{\mathrm{MCP}} |^2 \rangle = \langle \sum_{n,\,n'}^{\bar{n}} T_{fi\,(n')}^{*\,\mathrm{MCP}} T_{fi\,(n)}^{\mathrm{MCP}} \rangle \simeq \sum_{n=1}^{\bar{n}} | T_{fi\,(n)}^{\mathrm{SMCP}} |^2,$$

where $|T_{fi\,(n)}^{\mathrm{SMCP}}|^2$ is determined by

$$|T_{fi\,(n)}^{\mathrm{SMCP}}|^2 = \frac{\Gamma_n^{(f)}}{\Gamma_n} \Big[\prod_{k=1}^{n-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k} \Big] \frac{1}{2\pi} \frac{\Gamma_1^{(i)}}{D_1}$$

$$\Gamma_n^{(f)} = 2\pi \langle | \langle \varphi_f^{(-)} | V_{P,\,n} | \varphi_{n,\,\alpha} \rangle |^2 \rangle,$$

$$\Gamma_1^{(i)} = 2\pi \langle | \langle \varphi_{1,\,\alpha} | V_{1,\,P} | \psi_i^{(+)} \rangle |^2 \rangle.$$

Accordingly, for the averaged reaction cross section $\sigma_{fi}^{\mathrm{SMCP}}$ we have

$$\sigma_{fi}^{\mathrm{SMCP}} = \frac{\pi}{k_i^2} \sum_{n=1}^{\bar{n}} \frac{\Gamma_n^{(f)}}{\Gamma_n} \Big[\prod_{k=1}^{n-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k} \Big] \frac{2\pi \Gamma_1^{(i)}}{D_1} \tag{19}$$

It should be noted that the expression (19) for $\sigma_{fi}^{\mathrm{SMCP}}$ is obtained only under the condition $\Gamma_n \gg D_{n-1} > D_n$, which determines the dynamics of the process, and after averaging over the quantum numbers of the intermediate states, except for the spin and parity.

The partial contribution $\sigma_{fi}^{\mathrm{SMCP}}$ [see (19)] from the final evaporation stage $\bar{n}$ of the multistage process is determined by the expression

$$\sigma_{fi(\bar{n})}^{\mathrm{SMCP}} = \frac{\pi}{k_i^2} \frac{T_f T_i}{\sum_{c} T_c}, \tag{20}$$

where

$$T_c = 2\pi \Gamma_{\bar{n}}^{(c)} / D_{\bar{n}}, \quad T_f = 2\pi \Gamma_{\bar{n}}^{(f)} / D_{\bar{n}},$$

$$T_i = 2\pi \frac{\Gamma_1^{(i)}}{D_1} \Big[\prod_{k=1}^{\bar{n}-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k} \Big].$$

The expression (20) for the final stage $\bar{n}$ (evaporation stage) is very close to the Hauser–Feshbach expression, which determines the contribution of the equilibrium processes, i.e., the

emission from the equilibrium state of the nucleus.

The main differences are associated with: 1) the difference between T_c and T_i due to the presence of the depletion factor

$$D(\bar{n}) = [\prod_{k=1}^{\bar{n}-1} \Gamma_k^{\downarrow}/\Gamma_k] < 1$$

in (20); 2) the dependence of the expression (20) on the nature of the entrance channel P_i. The contribution $\sigma_{fi(\bar{n})}^{\text{SMCP}}$ up to the final evaporation stage $\bar{n}$ is almost independent of the particular value of $\bar{n}$ under the assumption that the $P_{\bar{n}}$ configurations are sufficiently complicated. Therefore, $\bar{n}$ can be chosen on the basis of the condition $\rho_{\bar{n}-1} \ll C\rho_{\bar{n}}$, where $C \sim 10$, and ρ_n is the density of states in the space Q_n. This then determines a finite limit of the chain of intermediate states. Summing $\sigma_{fi(\bar{n})}^{\text{SMCP}}$ determined by (20) over all the final states f, we obtain the total cross section for emission from the evaporation stage:

$$\sigma_{i(\bar{n})}^{\text{SMCP}} = \sum_f \sigma_{fi(\bar{n})}^{\text{SMCP}} = \pi T_i/k_i^2.$$

Accordingly, for the total reaction cross section σ_i^{SMCP} we obtain

$$\sigma_i^{\text{SMCP}} = \frac{\pi}{k_i^2} \frac{2\pi\Gamma_1^{(i)}}{D_1}, \tag{21}$$

where

$$\sigma_{i(\bar{n})}^{\text{SMCP}}/\sigma_i^{\text{SMCP}} = \Big[\prod_{k=1}^{\bar{n}-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k}\Big].$$

From (21) there follows the important fact that the total reaction cross section σ_i^{SMCP} is determined by the strength function $2\pi\Gamma_1^{(1)}/D_1$ for the formation of the doorway state.

The expression (19) determines the reaction cross section $\sigma_{fi}^{\text{SMCP}}$, it being assumed that the exit channel f is described by the wave function $\varphi_f^{(-)}$ (isolated state). However, in experiments it is not in general possible to observe isolated final states because of the averaging over an energy interval due to the experimental resolution. The total wave function of the exit channel f satisfies the equation

$$(E^- - H_{\text{opt}})\,\psi_f^{(-)} = 0.$$

We introduce eigenvectors of the state in each space P_ν. It is assumed that the final nucleus has excitation energy U, with

$$[E^- - (H_{\text{opt}})_{\nu\nu}]\,\varphi_{\nu,\gamma,\mu}^{(-)}(U) = 0,$$

where H_{opt} is determined by the expression (4) and $(H_{\text{opt}})_{\nu\nu} = \hat{P}_\nu H_{\text{opt}} \hat{P}_\nu$, γ is the set of quantum numbers that describe the exit channel, and μ are the remaining quantum numbers that describe the state in P_ν. The corresponding width of decay of the state (n, α) to the final state $\varphi_{\nu,\gamma,\mu}^{(-)}(U)$ is

$$\Gamma_n^{(\nu,\gamma,\mu)}(U) = 2\pi\,\langle|\langle\varphi_{\nu,\gamma,\mu}^{(-)} \mid V_{P,n} \mid \varphi_{n,\alpha}\rangle|^2\rangle,$$

where we understand averaging over the index α, and $\nu = n$, $n \pm 1$. The total wave function $\psi_f^{(-)}$ is a superposition of isolated states $\varphi_{\nu,\gamma,\mu}^{(-)}$:

$$\psi_f^{(-)} = \sum_{\nu,\mu} C_{\nu,\gamma,\mu}^{(f)} \varphi_{\nu,\gamma,\mu}^{(-)}(U).$$

In the given case, we obtain for $\Gamma_n^{(f)}$ (using the assumption that the phases of the matrix elements are random)

$$\Gamma_n^{(f)} = 2\pi \sum_{\nu,\nu',\mu,\mu'} \langle C_{\nu,\gamma,\mu}^{(f)} C_{\nu',\gamma,\mu'}^{(f)} \langle\varphi_{\nu',\gamma,\mu'}^{(-)}(U) \mid V_{P,n}\mid \varphi_{n,\alpha}\rangle \times \langle\varphi_{n,\alpha} \mid V_{n,P}\mid \varphi_{\nu,\gamma,\mu}^{(-)}(U)\rangle\rangle = \sum_{\nu} |C_{\nu,\gamma}^{(f)}|^2\, \Gamma_n^{(\nu,\gamma)}(U) \equiv \Gamma_n^{(\gamma)}(U).$$

If we assume strong coupling between the different states P_ν, then

$$|C_{\nu,\gamma}^{(f)}|^2 \simeq \rho^{(\nu,\gamma)}(U)/\rho^{(\gamma)}(U),$$

where $\rho^{(\nu,\gamma)}(U)$ is the density of states of type γ in the space P_ν, and $\rho^{(\gamma)}(U)$ is the total density of states of type γ with excitation energy U of the final nucleus. In this case, $\Gamma_n^{(\gamma)}(U)$ is determined by the expression

$$\Gamma_n^{(\gamma)}(U) = \sum_{\nu=n-1}^{n+1} \frac{\rho^{(\nu,\gamma)}(U)}{\rho^{(\gamma)}(U)} \Gamma_n^{(\nu,\gamma)}(U). \tag{22}$$

Substituting (22) in (19), we finally obtain for the cross section averaged over the energy

$$\frac{d\sigma_{fi}^{\text{SMCP}}}{d\varepsilon_f} = \frac{\pi}{k_i^2} \sum_{n=1}^{\bar{n}} \sum_{\nu=n-1}^{n+1} \frac{2\pi\Gamma_1^{(i)}}{D_1} \Big[\prod_{k=1}^{n-1} \frac{\Gamma_k^{\downarrow}}{\Gamma_k}\Big] \rho_f^{(\nu,\gamma)}(U)\, \Gamma_n^{(\nu,\gamma)}(U)/\Gamma_n. \tag{23}$$

where $\rho_f^{(\nu,\gamma)}(U)$ are the densities of the allowed states of the final nucleus.

We classify the entrance and exit channels by the quantum numbers (l, s) and $\gamma \equiv (J\pi, l', s')$, respectively, where s and s' are the total spins of the channel, the vector sums of the spins of the colliding and outgoing fragments are $\mathbf{s} = \mathbf{j} + \mathbf{I}$ and $\mathbf{s}' = \mathbf{j}' + \mathbf{I}'$ (j and j' are the spins of the incident and emitted particle, and I and I' are the spins of the target nucleus and the final nucleus), l and l' are the orbital angular momenta of the colliding and outgoing fragments, and J is the total spin of the system, $\mathbf{J} = \mathbf{s} + \mathbf{l}$. Then the angular distribution of the emitted particles is determined by the expression

$$\frac{d^2\sigma_{fi}^{\text{SMCP}}}{d\varepsilon_f\, d\Omega_f} = \sum_{L,J,l,l',s,s'} \frac{(-1)^{s'-s}}{(2I+1)(2j+1)} \times Z(lJlJ;\, sL)\, Z(l'Jl'J;\, s'L)\, P_L(\cos\vartheta_f)\, d\sigma_{l's'J\pi,ls}^{\text{SMCP}}/d\varepsilon_f, \tag{24}$$

where $Z(lJlJ;\, sL)$ are the Blatt–Biedenharn coefficients.

It can be seen from the structure of the expression (24) that the angular distributions of the emitted particles are symmetric with respect to the angle $\vartheta_{\text{cms}} = 90°$. In Eq. (24), the cross section $d\sigma_{l's'J\pi,ls}^{\text{SMCP}}/d\varepsilon_f$ is determined by the expression (23), and the excitation energy of the compound nucleus is $E = \varepsilon_i + B_i$ and the excitation energy of the final nucleus is $U = E - B_f - \varepsilon_f$, where ε_i and ε_f are the kinetic energies of the particles in the entrance and exit channels, respectively, and B_i and B_f are their binding energies in the compound nucleus.

Statistical theory of multistage direct processes

We now consider the statistical theory of multistage direct processes developed in Ref. 8. The T matrix due to the

contributions from the multistage direct processes is given by the equation

$$T^{\mathrm{MDP}} = v + v\,(E^{+} - H_{\mathrm{opt}})^{-1} v, \tag{25}$$

where the interaction operator v is determined by

$$v = H_{\mathrm{opt}} - \sum_{\nu} (H_{\mathrm{opt}})_{\nu\nu}.$$

It follows from (25) that

$$T_{fi}^{\mathrm{MDP}} = \langle \varphi_f^{(-)} | \, v \, | \varphi_i^{(+)} \rangle + \langle \varphi_f^{(-)} | \, v\,(E^{+} - H_{\mathrm{opt}})^{-1} v \, | \varphi_i^{(+)} \rangle,$$

where the first term determines the contribution of the single-stage direct process in the distorted-wave Born approximation (DWBA), T_{fi}^{1DP}, whereas the second term represents the contributions of the multistage direct processes (two-stage, T_{fi}^{2DP}, three-stage, T_{fi}^{3DP}, etc.).

In accordance with the considered scheme of the multistage process, T_{fi}^{MDP} can be represented in the form

$$T_{fi}^{\mathrm{MDP}} = \langle \varphi_f^{(-)} | \, v_{f,\,i} \, | \varphi_i^{(+)} \rangle + \sum_{\nu} T_{fi\,(\nu)}^{\mathrm{MDP}}, \tag{26}$$

where

$$T_{fi\,(\nu)}^{\mathrm{MDP}} = \langle \varphi_f^{(-)} | \, v_{f,\,\nu} G_{\nu}^{(u)} v_{\nu,\,\nu-1} G_{\nu-1}^{(u)} \ldots v_{2,\,1} G_1^{(u)} v_{1,\,i} \, | \varphi_i^{(+)} \rangle;$$

$$v_{f,\,\nu} = \hat{P}_f v \hat{P}_{\nu};\ v_{1,\,i} = \hat{P}_1 v \hat{P}_i;\ v_{\nu,\,\nu-1} = \hat{P}_{\nu} v \hat{P}_{\nu-1};$$

$$G_n^{(u)} = \hat{P}_n\,(E^{+} - \hat{P}_n H_{\mathrm{opt}} \hat{P}_n - v_{n,\,n+1} G_{n+1}^{(u)} v_{n+1,\,n})^{-1} \hat{P}_n$$

is the Green's operator corresponding to the intermediate state of order n (in P space).

We introduce eigenstates $\tilde{\varphi}_{n,\alpha}^{(-)}$ of the effective Hamiltonian $[(H_{\mathrm{opt}})_{nn} + v_{n,n+1} G_{n+1}^{(u)} v_{n+1,n}]$ associated with the Green's operator $G_n^{(u)}$,

$$[(H_{\mathrm{opt}})_{nn} + v_{n,\,n+1} G_{n+1}^{(u)} v_{n+1,\,n}]\,\tilde{\varphi}_{n,\,\alpha}^{(-)} = [(\hbar^2 k_n^2/2m) + e_{n,\,\alpha} - \mathrm{i}\eta]\,\tilde{\varphi}_{n,\,\alpha}^{(-)}, \tag{27}$$

where $\tilde{\varphi}_{n,\alpha}^{(-)}$ in the plane-wave representation ($\mathbf{k}$ representation) is asymptotically the product of the wave function of the final nucleus in state α with energy $e_{n,\alpha}$ and excited-state configuration corresponding to the space P_n and the distorted-wave function describing a particle in the continuum with wave number $\mathbf{k}_n$. In accordance with (27), for $G_n^{(u)}$ we have

$$G_n^{(u)} = \sum_{\alpha} \int \frac{d\mathbf{k}_n}{(2\pi)^3}\, \tilde{\varphi}_{n,\,\alpha}^{(-)\,*}\,[E - (\hbar^2 k_n^2/2m) - e_{n,\,\alpha} - \mathrm{i}\eta]^{-1}\,\tilde{\varphi}_{n,\,\alpha}^{(-)}. \tag{28}$$

For simplicity we consider in (28) only open configurations with one particle in the continuum. In deriving the required expression for $|T_{fi\,(\nu)}^{\mathrm{MDP}}|$ from (26) and (28) we make the following approximations: 1) the hypothesis of random phases of the matrix elements in the summation and averaging over the quantum numbers $\{\alpha\}$ of the intermediate states and the states of the final nucleus; 2) under the assumption of a weak energy dependence of the matrix elements near the singularities of the Green's operators in the case $e_{n,\alpha}^{*} = e_{n,\alpha}^{*}$ integrals of the type

$$\int de_{n,\,\alpha}\,[E - (\hbar^2 k_n^2/2m) - e_{n,\,\alpha} - \mathrm{i}\eta]^{-1}\,[E - (\hbar^2 k_n'^2/2m) - e_{n,\,\alpha} + \mathrm{i}\eta]^{-1}$$

are approximated by the expressions

$$2\pi^2 \delta\,[(\hbar^2 k_n^2/2m) - (\hbar^2 k_n'^2/2m)];$$

3) the integrals over the angles are calculated under the condition

$$\int d\Omega_n d\Omega_n' \to \int d\Omega_n d\Omega_n' \delta\,(\Omega_n - \Omega_n').$$

In the framework of these approximations, the averaged cross section determined by the contributions of the multistage direct processes is described by the expression

$$\frac{d^2\sigma_{fi}^{\mathrm{SMDP}}(\mathbf{k}_f,\,\mathbf{k}_i)}{d\varepsilon_f d\Omega_f} \sum_{\nu}^{\nu+1} \sum_{m=\nu-1} \int \frac{d\mathbf{k}_1}{(2\pi)^3} \cdots \int \frac{d\mathbf{k}_{\nu}}{(2\pi)^3} \left[\frac{d^2 W_{m,\,\nu}(\mathbf{k}_f,\,\mathbf{k}_{\nu})}{d\varepsilon_f d\Omega_f}\right] \frac{d^2 W_{\nu,\,\nu-1}(\ _{\nu},\,\mathbf{k}_{\nu-1})}{d\varepsilon_{\nu} d\Omega_{\nu}} \cdots \left[\frac{d^2 W_{2,1}(\mathbf{k}_2,\,\mathbf{k}_1)}{d\varepsilon_2 d\Omega_2}\right] \frac{d^2\sigma_{1i}^{1\,\mathrm{SDP}}(\mathbf{k}_1,\,\mathbf{k}_i)}{d\varepsilon_1 d\Omega_1}, \tag{29}$$

where

$$\frac{d^2 W_{n,\,n-1}(\mathbf{k}_n,\,\mathbf{k}_{n-1})}{d\varepsilon_n d\Omega_n} = 2\pi^2 \rho(\mathbf{k}_n)\,\rho_n^{(b)}(U_n)\,\langle |\, v_{n,\,n-1}(\mathbf{k}_n,\,\mathbf{k}_{n-1})\,|^2 \rangle;$$

$$\frac{d^2\sigma_{1i}^{1\,\mathrm{SDP}}(\mathbf{k}_1,\,\mathbf{k}_i)}{d\varepsilon_1 d\Omega_1} = \frac{2\pi m}{\hbar^2 k_i} \rho(\mathbf{k}_1)\,\rho_1^{(b)}(U_1)\,\langle |\, v_{1,\,i}(\mathbf{k}_1,\,\mathbf{k}_i)\,|^2 \rangle$$

is the cross section of the first collision process, in which the incident particle with wave number $\mathbf{k}_i$ goes over to the state with $\mathbf{k}_1$ and U_1 ($U_1 = E - \varepsilon_1$, where ε_1 is the kinetic energy of the particle in the continuum), $\rho(\mathbf{k}_n)$ is the density of states of a particle in the continuum with wave number $\mathbf{k}_n$, $\rho(\mathbf{k}_n) = mk_n/(2\pi)^3\hbar^2$, and $\rho_n^{(b)}(U_n)$ is the density of bound states of the residual nucleus in the n-th stage with energy $U_n = E - \varepsilon_n$, where ε_n is the kinetic energy of the particle in the continuum for the intermediate stage of n-th order. The single-stage direct reaction determined by the matrix T_{fi}^{1DP} can be described by the well-known expression in the DWBA:

$$\frac{d^2\sigma_{fi}^{1\,\mathrm{SDP}}(\mathbf{k}_f,\,\mathbf{k}_i)}{d\varepsilon_f d\Omega_f} = \frac{2\pi m}{\hbar^2 k_i} \rho(\mathbf{k}_f)\,\rho_1^{(b)}(U)\,\langle |\, v_{f,\,i}(\mathbf{k}_f,\,\mathbf{k}_i)\,|^2 \rangle.$$

It is readily seen from the structure of the expression (29) that the angular distributions of the emitted particles have an asymmetry, the degree of which increases with increasing energy of the emitted particles. It should be noted that the statistical theory of multistage compound and direct processes proposed in Ref. 8 is restricted to single-nucleon reactions. The practical realization of such a formalism in the description of reactions with composite particles comes up against serious mathematical difficulties in the expansion of one basis set with respect to another when it is necessary to taken into account in each of them the continuum states.

Application of the statistical theory of multistage processes to the description of experiments

Example 1. We demonstrate the calculations in accordance with the statistical theory of multistage processes in the description of the nuclear reaction ^{181}Ta(p,n) with energy $\varepsilon_p = 18$ MeV of the incident protons.[8] Since the experimental angular distributions are symmetric with respect to

the angle $\vartheta_f = 90°$, the experiment is described in the framework of the statistical theory of compound processes under the assumption that the direct contributions are small. To simplify the calculations, the following scheme for describing the reaction is proposed:

target nucleus [spin 0, mass number $A-1$])	+	incident nucleon [total angular momentum J, energy ϵ_i]
↓		
final nucleus [spin s, excitation energy U]	+	emitted nucleon [orbital angular momentum l, energy ϵ_f]

In this description of the reaction, the nucleons are assumed to be identical spinless fermions and the target nucleus is assumed to have spin 0.

In such a scheme, the expression for the SMCP cross section (24) takes the form

$$\frac{d^2\sigma^{\text{SMCP}}}{d\varepsilon_f d\Omega_f} = \pi\lambda_i^2 \sum_J (2J+1)\Big[\sum_{n=1}^{\bar{n}}\Big\{\sum_{\nu=n-1}^{n+1}\sum_{l,s,\lambda} C_{lsJ}^{\lambda} \times P_\lambda(\cos\vartheta_f)\,\Gamma_{n,J}^{(\nu,l,s)}(U)\,\rho_f^{(\nu,s)}(U)\Big\} \times \prod_{m=1}^{n-1}(\Gamma_{m,J}^{\downarrow}/\Gamma_{m,J})\,2\pi\Gamma_{1,J}^{(i)}/D_{1,J}\Big],$$

where

$$C_{lsJ}^{\lambda} = (-1)^s\left(\frac{2\lambda+1}{4\pi}\right)^{1/2}\begin{pmatrix} J & J & \lambda \\ 0 & 0 & 0\end{pmatrix} Z(lJlJ;\ s\lambda),$$

$\lambda_i = \hbar/(2m\varepsilon_i)^{1/2}$, and m is the nucleon mass. In the proposed scheme for describing the compound process the intermediate states of order n (bound states of the space Q_n) are classified by the number of particles, $p_n = n+1$ and the number of holes, $h_n = n$ (total number of excitons $N = p_n + h_n = 2n+1$); accordingly, the scattering states of the space P_n are classified by an n-particle–n-hole bound configuration of the final nucleus and a nucleon in the continuum.

The total width for configurations in Q_n with angular momentum J is, in accordance with (16)–(18),

$$\Gamma_{n,J} = \Gamma_{n,J}^{\downarrow} + \sum_{\nu=n-1}^{n+1}\sum_{l,s}\int_0^{\varepsilon_p}\Gamma_{n,J}^{(\nu,l,s)}(U)\,\rho_f^{(\nu,s)}(U)\,dU,$$

where the width $\Gamma_{n,J}^{\downarrow}$ for disintegration into more complicated configurations (relaxation width) in the approximation $h_{n,n+1} \simeq H_{n,n+1}$ is

$$\Gamma_{n,J}^{\downarrow} = 2\pi\,\langle|H_{n,n+1}^{J}|^2\rangle/D_{n+1,J}.$$

In the microscopic approach, all the required quantities $\{\Gamma_{n,J}\}$ are calculated using only the potential of the residual two-body interaction v. Figure 1 shows the results of calculating $d^2\sigma_{fi}^{\text{SMCP}}/d\varepsilon_f d\Omega_f$ for the reaction ^{181}Ta(p, n) at the energy $\varepsilon_p = 18$ MeV and $\vartheta_f = 120°$ and the comparison with experiment. The figure shows the total contribution of the compound process (including all stages $n < \bar{n}$) and the contribution from the evaporation stage $\bar{n}$. In the soft part of the spectrum, allowance has been made for multiple emission of neutrons in the evaporation stage ($2n$ and $3n$).

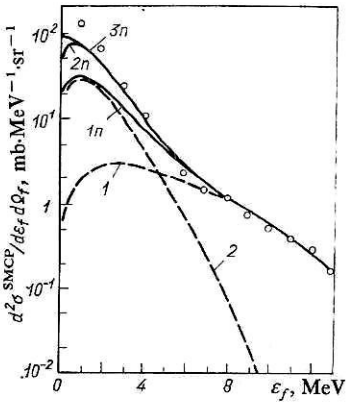


FIG. 1. Calculated differential cross section of the reaction ^{181}Ta(p, n) at $\varepsilon_p = 18$ MeV at angle $\vartheta_f = 120°$ (the open circles represent the experimental points). The curves show the contribution of multiple neutron emission: 1) the SMCP contribution; 2) the evaporation stage (stage $\bar{n}$) contribution.

Example 2. We demonstrate the scheme of SMDP calculation with strong asymmetry of the angular distributions for the example of the reaction ^{120}Sn(p, n) at $\varepsilon_p = 45$ MeV.[9] In accordance with the statistical theory of multistage direct processes, the cross section of such a reaction is determined by the expression (29). It can be seen from the structure of this expression that the first step in the SMDP investigation is the calculation of the single-stage interaction. Under the assumptions of the model formulated in Example 1, $(d\sigma_{fi}/d\Omega_f)_L^{DWBA}$ was calculated by the DWUCK program. The calculation showed that the reaction cross section depends very weakly on the microscopic details of the shell-model description and is basically a function of the spin and excitation energy of the final nucleus. Then the differential cross section for emission of a neutron in the first SMDP stage can be obtained from the relation

$$\frac{d^2\sigma_{fi}^{1\,\text{SDP}}}{d\varepsilon_f d\Omega_f} = \sum_L \left\langle\left(\frac{d\sigma_{fi}}{d\Omega_f}\right)_L^{\text{DWBA}}\right\rangle \rho_1^{(b)}(U)\,R_1(L)\,(2L+1),$$

where $R_1(L)$ is the spin distribution function for the final nucleus.

The relation obtained in this manner for the single-stage process was used to calculate the SMDP contributions. Figure 2 shows the calculated cross sections $d^2\sigma_{fi}^{\text{SMDP}}/dUd\Omega_f$ with allowance for the DP and SMDP contributions for different excitation energies U of the final nucleus (continuous curves). The figure gives the calculations of the cross section for different emission angles, the partial contributions from the individual stages ($n = 1$–6) being shown. It can be seen from the figure that at different angles the main contribution to the differential cross section is made by different stages of the process in different regions of the excitation energy U of the final nucleus. With increasing angle ϑ_f the contribution of the first stage decreases and the contribution to the cross section from the following stages increases accordingly.

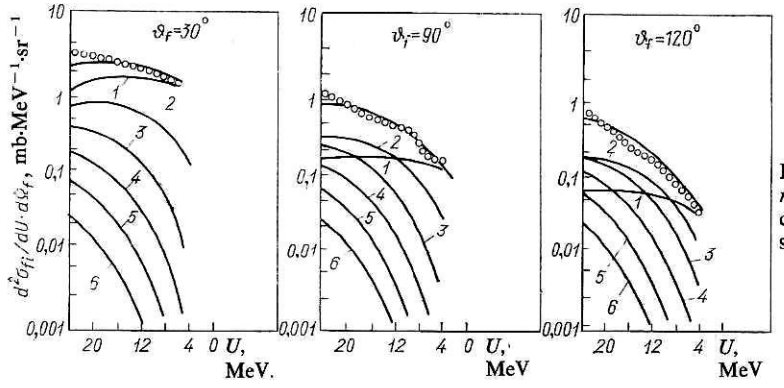


FIG. 2. Experimental neutron spectra from the reaction ^{120}Sn(p, n) at $\varepsilon_p = 45$ MeV (open circles) at different emission angles, compared with the calculated SMDP contributions of different stages (the figures give the number of the stage).

2. QUANTUM-FIELD FORMALISM OF THE UNIFIED THEORY OF PRE-EQUILIBRIUM NUCLEAR REACTIONS WITH COMPOSITE PARTICLES

Use of the method of quantum Green's functions to describe nuclear reactions with composite particles

The success achieved in recent years in a number of problems of nuclear theory is associated primarily with the application to the description of a nonrelativistic system of many interacting particles of methods taken from quantum field theory. Most important has been the use of causal Green's functions. Using the method of quantum Green's functions, one can obtain not only varied static information about nuclear structure (excitation spectrum, expectation values of the operators of the physical quantities, etc.) but also varied dynamical information about the mechanisms of the nuclear processes. We shall consider the quantum-field formalism of the unified theory of nuclear reactions with composite particles at energies up to 100 MeV and higher using the method of quantum Green's functions and the ideas of the theory of Fermi liquids[10] for a nuclear reaction of the form $A + a \to B + b$, where A is the target nucleus, a is the composite particle in the initial state i, B is the residual nucleus, and b is the composite particle in the final state f. The system of colliding fragments is described by a Hamiltonian with two-particle forces. To simplify the description, we shall assume that A is an even–even nucleus, and we shall ignore the emission of several particles in the final state. In the second-quantization representation, the creation operator for a nuclear fragment of type c (c also denotes the number of nucleons in the nuclear fragment) is determined as follows[11]:

$$a_c^+(t) = \mathrm{S}d^3x_{[c]} \times \exp(-iE_c t)\,\varphi_c(x_{[c]})\,\tilde{\psi}^+(x_c, t)\ldots\tilde{\psi}^+(x_1, t), \tag{30}$$

where $\tilde{\psi}^+(x,t)$ is the nucleon creation operator in the Heisenberg representation, $\varphi_c(x_{[c]})$ are the wave functions of the c-particle Hamiltonian, $x_{[c]} = x_1, \ldots, x_c$, E_c is the total energy of the nuclear fragment, $x = (\mathbf{r}, s, \tau)$, s is the spin variable, τ is the isotopic variable, and $\mathrm{S}d^3x = \Sigma_{s,\tau}\int d\mathbf{r}$.

Accordingly, the scattered states in the system of the two interacting nuclear fragments a and A (as $t \to \pm\infty$) and the S_{fi} matrix for the reaction $A + a \to B + b$ are determined by

$$\left.\begin{aligned} |a,\ A\rangle^{\mathrm{in(out)}} &= \lim_{t\to\pm\infty} a_a^+(t)\,a_A^+(t)\,|0\rangle; \\ S_{fi} &= \lim_{\substack{t\to\infty \\ t'\to-\infty}} \langle 0|\,a_B(t)\,a_b(t)\,a_a^+(t')a_A^+(t')\,|0\rangle. \end{aligned}\right\} \tag{31}$$

After simple manipulations, ignoring the recoil of the final nucleus B, we readily find from (31) that the S_{fi} reaction matrix can be expressed in terms of the generalized Green's function

$$G_{ba}(x_{[b]}, t; x'_{[a]}, t') = \langle B|\,T\{\tilde{\psi}(x_b, t)\ldots \tilde{\psi}(x_1, t)\,\tilde{\psi}^+(x'_1, t')\ldots\tilde{\psi}^+(x'_a, t')\,|A\rangle$$

by a relation of the form[12]

$$\left.\begin{aligned} S_{fi}(E_a) = \lim_{\varepsilon,\,\eta\to0} \varepsilon\eta\,\mathrm{S}\,d^3x_{[b]}d^3x_{[a]} \int_0^\infty dt \int_{-\infty}^0 dt' \exp(iE_b t - \eta t) \\ \times \exp(-iE_a t' + \varepsilon t')\,\varphi_b^*(x_{[b]})\,G_{ba}(x_{[b]}, t; x_{[a]}, t')\varphi_a(x_{[a]}), \end{aligned}\right\} \tag{32}$$

where $E_a = \varepsilon_a + B^0_a$, $E_b = \varepsilon_b + B^0_b$, ε_a and ε_b are the kinetic energies, B^0_a and B^0_b are the binding energies, and φ_a and φ_b are the wave functions of the free composite particles a and b, respectively.

In the framework of the shell model, the excited states of the final nucleus $\{\tilde{\varphi}_{B^*}(x_{[B^*]})\}$ are classified by a given number of excitons $B^* = p_B + h_B$ (p_B is the number of quasiparticles, h_B the number of quasiholes) and by the excitation energy $U_B = E_B - E^0_A - E^0_B$, where E^0_A and E^0_B are the ground-state energies of the nuclei A and B, respectively. The quasiparticles and quasiholes are defined relative to the Fermi energy of the target nucleus A. In such a description, we obtain for the S_{fi} matrix from (32)

$$\begin{aligned} S_{fi}(E_a) = \lim_{\varepsilon,\,\eta\to0} \varepsilon\eta \int_0^\infty dt \exp(iE_b t - \eta t) \\ \times \int_{-\infty}^0 dt' \exp(iE_a t' + \varepsilon t') \\ \times \lim_{\gamma\to0} \gamma \int_{t_1}^\infty dt_2 \exp\{i(E_B - E^0_A)\,t_2 - \gamma t_2\}\,\mathrm{S}\,d^3x_{[a+b+B^*]} \\ \times \varphi_b^*(x_{[b]})\,\tilde{\varphi}^*_{B^*}(x_{[B^*]})\,G[b + B^*, a]\,\varphi_a(x_{[a]})\,i^F, \end{aligned} \tag{33}$$

where $G[b + B^*, a]$ is the causal many-particle Green's

function, determined in the general case by the expression

$$G[b+B^*,\ a] = -\mathbf{i}^F \langle A | T\{\tilde{\psi}(x_1, t_1)\ldots\tilde{\psi}(x_F,\ t_F)\tilde{\psi}^+(x_1', t_1')\ldots \ldots\tilde{\psi}^+(x_F',\ t_F')\} | A\rangle,$$

where a and b are the numbers of nucleons in the initial and final particles a and b, respectively, the averaging is over the ground state of the target nucleus A, and $F=(a+b+B^*)/2$.

We introduce the total (reducible) interaction amplitudes $\tilde{\Gamma}_{N,N'}$, which are determined in the quantum-field formulation of the many-body problem as the symmetrized sums of all Feynman graphs that begin and end with an interaction event by means of the relations ($N \neq N'$)

$$G[N,\ N'] = \tilde{G}[N,\ N]\,\tilde{\Gamma}_{N,N'}\tilde{G}[N',\ N'], \tag{34}$$

where $\tilde{G}[N,N]$, the Green's function of the system of N interacting quasiparticles and quasiholes, satisfies the equation

$$\tilde{G}[N,\ N] = G^{0s}[N,\ N] + G^0[N,\ N]\,\tilde{I}_{N,N}\tilde{G}[N,\ N]. \tag{35}$$

The blocks $\tilde{I}_{N,N}$ are determined by the relations

$$\tilde{I}_{N,N} = I_{N,N} + I_{N,\ N+2}\tilde{G}[N+2,\ N+2]\,I_{N+2,N}, \tag{36}$$

in which $\tilde{I}_{N,N}$ are the irreducible symmetrized interaction amplitudes (irreducible vertex parts) corresponding to summation of Feynman graphs not containing blocks connected by ap, $(a+1)p1h$, $(a+2)p2h$, ..., $(b+p_B)ph_Bh$ lines, $G^0[N,N]=G(x_1, t_1; x_1', t_1')\ldots G(x_N, t_N; x_N', t_N')$ is the product of N single-particle Green's functions,

$$G^{0s}[N,\ N] = \{G(x_1,\ t_1;\ x_1',\ t_1')\ \ldots\ G(x_N,\ t_N;\ x_N',\ t_N')\}_{\text{symm}}$$

is the symmetrized product of single-particle Green's functions, and $\{G[1,1]\equiv G(x,t;x',t')\}$. Substituting (33) in (34) and ignoring the effects of the bound states of the intermediate system in the entrance and exit channels, making some simple transformations, and evaluating $\{\lim\}$, we obtain for the T_{fi} matrix of the reaction $A+a\to B+b$ the expression

$$T_{fi}(E_a) = \mathrm{S}d^3x_{[B^*+b+a]}\tilde{\varphi}^{(-)*}_{b,B}(x_{[b+B^*]}) \times\tilde{\Gamma}_{B^*+b,\,a}(x_{[b]};\ x_{[B^*]};\ x_{[a]};\ E_a)\,\tilde{\varphi}^{(+)}_a(x_{[a]}), \tag{37}$$

where the wave functions $\tilde{\varphi}^{(+)}_a(x_{[a]})$ and $\tilde{\varphi}^{(-)}_{b,B^*}(x_{[b+B^*]})$ for the composite particles a and b in the entrance and exit channels satisfy the corresponding equations

$$\tilde{\varphi}^{(*)}_a(x_{[a]}) = \varphi_a(x_{[a]}) + \mathrm{S}\,d^3x'_{[a]}\tilde{G}^0[a,a]V^{(a)}(x'_{[a]})\tilde{\varphi}^{(+)}_a(x'_{[a]}); \tag{38}$$

$$\tilde{\varphi}^{(-)}_{b,B^*}(x_{[b+B^*]}) = \hat{A}\{\varphi_b(x_{[b]})\,\tilde{\varphi}_{B^*}(x_{[B^*]})\} + \mathrm{S}d^3x'_{[b+B^*]}\tilde{G}^0[b,b]\tilde{G}[B^*,B^*] \times V^{(b)}(x'_{[b+B^*]})\,\tilde{\varphi}^{(-)}_{b,B^*}(x'_{[b+B^*]}); \tag{39}$$

$$V^{(a)} = \sum_{i=1}^{a} M_i + \tilde{I}_{a,\,a} - V_{a,\,a};$$

$$V^{(b)} = \sum_{i=1}^{b} M_i\tilde{G}^{-1}[B^*,B^*] + \tilde{I}_{b+B^*,\,b+B^*} - V_{b,\,b}\tilde{G}^{-1}[B^*,\ B^*] - \tilde{I}_{B^*,\,B^*}\tilde{G}^{0-1}[b,\ b].$$

Here, M_i is the nucleon mass operator, $V_{N,N}$ is the interaction operator of N nucleons in vacuum, $\tilde{G}^0[N,N]$ is the Green's function of a free nuclear fragment of N nucleons, and $\tilde{\varphi}_{B^*}$ is an eigenfunction of the effective Hamiltonian $H_0+\tilde{I}_{B^*,B^*}$, a weak dependence of $\tilde{I}_{B^*,B^*}$ on the energy being assumed. It follows from the system of equations (34)–(36) that $\tilde{\Gamma}_{b+B^*,a}$ is determined by the expression (transitions with $\Delta N=+2$)

$$\tilde{\Gamma}_{b+B^*,a} = I_{b+B^*,a}\delta_{b+B^*,a+2} + I_{b+B^*,N}\delta_{N+2,b+B^*}\tilde{G}[N,\ N] \times I_{N,\,N-2}\tilde{G}[N-2,\ N-2]\ \ldots\ I_{a+4,\,a+2} \times\tilde{G}[a+2,\ a+2]\,I_{a+2,a}. \tag{40}$$

We now renormalize Eq. (40). As a result of the renormalization,[10] the regular parts of the Green's functions $\{\tilde{G}[N,N]\}$ occur in equations that determine the effective amplitudes $\{I'_{N,N'}\}$ with $\Delta N\geqslant+2$ (instead of the "bare" amplitudes $\{I_{N,N'}\}$ with $\Delta N=+2$). Therefore, in (40) one can use for the Green's functions $\tilde{G}[N,N]$ their pole parts, for which we can write down an expansion (spectral decomposition) with respect to the eigenfunctions of a system of N excitons:

$$\tilde{G}[N,N] = \sum_{\beta}\frac{\tilde{\varphi}_{N,\beta}(x_{[N]})\,\tilde{\varphi}^*_{N,\beta}(x'_{[N]})}{E-e_{N,\beta}} + \sum_{c,\alpha}\mathrm{S}\,d\varepsilon_{N,c,\alpha}\,d\alpha\rho_N(\varepsilon_{N,c,\alpha};\alpha) \times\frac{\tilde{\varphi}^{(+)}_{N,c,\alpha}(x_{[N]})\,\tilde{\varphi}^{(+)*}_{N,c,\alpha}(x'_{[N]})}{E-\varepsilon_{N,c,\alpha}+\mathrm{i}\gamma_{N,c,\alpha}/2} = \tilde{G}^{(b)}[N,\ N]+\tilde{G}^{(u)}[N,N], \tag{41}$$

where $e_{N,\beta}=E_{N,\beta}-\mathrm{i}\Gamma_{N,\beta}/2$, $\Gamma_{N,\beta}=\Gamma^{\downarrow}_{N,\beta}+\Gamma^{\uparrow}_{N,\beta}$, $\{\tilde{\varphi}_{N,\beta}(x_{[N]})\}$ describe the quasibound states of the interacting excitons belonging to the quasidiscrete spectrum of the Hamiltonian $H_0+\tilde{I}_{N,N}$; $\{\tilde{\varphi}_{N,c,\alpha}(x_{[N]})\}$ describe the unbound states of the N interacting excitons (scattering states in channel c) belonging to the continuum. Here, α denotes the set of physical quantities that have definite values simultaneously with the energy $\varepsilon_{N,c,\alpha}$; $\rho_N(\varepsilon_{N,c,\alpha};\alpha)$ is the density of the states $\tilde{\varphi}^{(+)}_{N,c,\alpha}$ per unit interval of the energy and unit interval of α; $\gamma_{N,c,\alpha}$ determines the damping of the spreading wave packet of exact scattering states in channel c. In the general case, the energy and damping of the intermediate states of order N are determined by the real and imaginary parts of the simple poles of the Green's function $\tilde{G}[N,N]$ in

the lower half-plane of the complex variable. To simplify the treatment, we shall in what follows restrict ourselves to a basis of states $\tilde{\varphi}^{(+)}_{N,c,\alpha}$ that asymptotically ($r \to \infty$) describe one quasiparticle of type c in the continuum with wave number k_c and energy $\hbar^2 k_c^2/2m_c$ and also a residual nucleus C with energy $e_{N,C,\alpha} = U_{C,\alpha} - i\Gamma_{N,c,\alpha}/2$ and quantum numbers $\alpha(\varepsilon_{N,c,\alpha} = e_{N,c,\alpha} + \hbar^2 k_c^2/2m_c$. The function $\tilde{\varphi}^{(+)}_{N,c,\alpha}$ satisfies an equation of the type (39).

Among the excited states of the system of N interacting excitons there are states completely determined by specifying the set of quantum numbers $\beta = \{\lambda\}$, where λ characterizes a single-particle state. We shall say that such configurations are exciton states and classify them by the number p_N of quasiparticles and the number h_N of quasiholes. However, more complicated states are also possible, these being classified by not only the number of quasiparticles and quasiholes but also the number N_{ph} of collective phonon excitations. In the general case, the spectral decomposition also contains terms describing all possible cluster excitations of the nucleus. In the traditional exciton model, only the quasiparticle–quasihole configurations $\{|p_N p h_N h\rangle\}$, $N = p_N + h_N$, are taken into account. In a consistent model of pre-equilibrium processes it is necessary to take into account the configurations $\{|\text{phonons}\rangle; |ph + \text{phonons}\rangle\}$ and transitions with $\Delta N = \pm 2$, $\Delta N_{\text{ph}} = \pm 1$, this corresponding to the quasiparticle–phonon model of pre-equilibrium decay. Such an extension of the basis of intermediate states makes the description more correct and capable of reproducing more accurately the experimental data (when realistic residual nuclear forces are used).

The representation of the Green's functions by means of Feynman graphs makes it possible to sum the graphs that are reducible with respect to the internal lines containing at least one continuum quasiparticle and irreducible with respect to the lines of the bound quasiparticles; as a result, we obtain for $N > N'$ for the total amplitudes $\tilde{\Gamma}_{N,N'}$ (when only transitions with $\Delta N = +2$ are taken into account)

$$\left.\begin{aligned}
&\tilde{\Gamma}^{(u)}_{N,N'} = I'_{N,N'}\delta_{N,N'+2} + I'_{N,N-2} \times \\
&\times \tilde{G}^{(u)}[N-2, N-2]\, I'_{N-2,N-4}\tilde{G}^{(u)}[N-4, N-4]\ldots \\
&\ldots I'_{N'+4,N'+2}\tilde{G}^{(u)}[N'+2, N'+2]\, I'_{N+2,N'}; \\
&\tilde{\Gamma}_{N,N'} = \tilde{\Gamma}^{(u)}_{N,N'} + \sum_{\{N_i\}} \tilde{\Gamma}^{(u)}_{N,N_1} G^{(b)}[N_1, N_1]\, \tilde{\Gamma}^{(u)}_{N_1,N_2}\ldots \\
&\ldots \tilde{G}^{(b)}[N_i, N_i]\, \tilde{\Gamma}^{(u)}_{N_i,N'},
\end{aligned}\right\} \tag{42}$$

where the sum over $\{N_i\}$ is taken subject to $N_1 > N_2, \ldots, N_{i-1} > N_i > N'$. It follows from (37) and (42) that

$$T_{fi}(E_a) = T^{\text{MDP}}_{fi}(E_a) + T^{\text{MCP}}_{fi}(E_a),$$

where

$$\begin{aligned}
T^{\text{MDP}}_{fi}(E_a) = &\text{S}\, d^3x_{[b+B^*+a]}\, \tilde{\varphi}^{(-)*}_{b,B^*}(x_{[b+B^*]}) \\
&\times \tilde{\Gamma}^{(u)}_{b+B^*,a}(x_{[b]}; x_{[B^*]}; x_{[a]}; E_a)\, \tilde{\varphi}^{(+)}_a(x_{[a]})
\end{aligned} \tag{43}$$

describes the multistage direct processes, and

$$\begin{aligned}
T^{\text{MCP}}_{fi}(E_a) = &\text{S}\, d^3x_{[b+B^*+a]}\, \tilde{\varphi}^{(-)*}_{b,B^*}(x_{[b+B^*]}) \times \\
&\times \Big\{ \sum_{\{N_i\}} \tilde{\Gamma}^{(u)}_{b+B^*,N_1} \tilde{G}^{(b)}[N_1, N_1]\, \tilde{\Gamma}^{(u)}_{N_1,N_2}\ldots \\
&\ldots \tilde{G}^{(b)}[N_i, N_i]\, \tilde{\Gamma}^{(u)}_{N_i,a} \Big\} \tilde{\varphi}^{(+)}_a(x_{[a]})
\end{aligned} \tag{44}$$

describes the combined multistage processes in which the system of continuum intermediate states ($\tilde{G}^{(u)}$) goes over at some stage into the quasibound intermediate states ($\tilde{G}^{(b)}$), etc. The specific features of the reactions with composite particles are contained in the functions $\tilde{\varphi}^{(-)}_{b,B^*}$ and $\tilde{\varphi}^{(+)}_a$. Introducing wave functions $\chi^{(+)}_a$ and $\chi^{(-)}_b$ (distorted waves) satisfying the equations of the optical model with $V^{(a)}_{\text{opt}}(\varepsilon_a)$ and $V^{(b)}_{\text{opt}}(\varepsilon_b)$ ($V^{(a)}_{\text{opt}}$ and $V^{(b)}_{\text{opt}}$ are the phenomenological optical potentials for particles a and b, respectively), we finally obtain for T^{MDP}_{fi} (there is a similar expression for T^{MCP}_{fi})

$$\begin{aligned}
T^{\text{MDP}}_{fi}(E_a) = &\text{S}\, d^3x_{[b+B^*+a]}\, \chi^{(-)}_b(x_{[b]}) \\
&\times \tilde{\varphi}_{B^*}(x_{[B^*]}) \\
&\times \{1 + [V^{(b)} - V^{(b)}_{\text{opt}}(\varepsilon_b)]\, \tilde{G}^{(u)}[b+B^*, b+B^*]\}\, \tilde{\Gamma}^{(u)}_{b+B^*,a} \\
&\times \{1 + \tilde{G}^{(u)}[a, a]\, [V^{(a)} - V^{(a)}_{\text{opt}}(\varepsilon_a)]\}\, \chi^{(+)}_a(x_{[a]}).
\end{aligned} \tag{45}$$

To derive formulas for the averaged cross sections of the reaction $A + a \to B + b$ on the basis of the expressions (43)–(45) we make approximations similar to those in the procedure of the statistical calculations described in Sec. 1:

1) we take into account only the pole parts of the Green's functions $\tilde{G}[N, N]$ in the intermediate states;

2) we assume that the phases of the matrix elements are random in performing the statistical averaging for the multistage compound and direct processes (in the plane-wave approximation) by analogy with the averaging procedure described in Sec. 1;

3) we ignore rearrangement processes in the entrance and exit channels:

$$[V^{(a)} - V^{(a)}_{\text{opt}}] \simeq 0;\ [V^{(b)} - V^{(b)}_{\text{opt}}] \simeq 0.$$

In the framework of these approximations, we obtain the required expressions for the statistical description of multistage nuclear reactions with composite particles[13,14]:

1. Statistical multistage processes through quasibound intermediate states of the compound nucleus (SMCP). The SMCP cross section is determined by an expression of the type (24) with extension of the description to reactions with composite particles.

2. Statistical multistage direct processes (SMDP).

First Case (the widths $\Gamma_{N,C} \to 0$ in the intermediate states of the residual nucleus C). If it is assumed that the energy $e_{N,\alpha}$ of the residual nucleus C in the intermediate state described by the function $\varphi^{(+)}_{N,C,\alpha}$ is real, then for this case the procedure of the calculations is analogous to the scheme described in Sec. 1. Then for the SMDP cross section we obtain an expression that is the same as (29) with extension to the reaction ($A + a \to B + b$) and additional allowance for all possible rearrangement processes in the intermediate states, i.e., $c + C \to c' C'$.

Second Case (widths $\Gamma_{N,C} \neq 0$ in the intermediate states

of the residual nucleus C). In this case, after statistical averaging, we obtain for $d^2\sigma^{\mathrm{SMDP}}/d\varepsilon_b\,d\Omega_b$ an expression[14] that has a structure close to the expressions of the hybrid model with allowance for the direction of the particles.[15]

3. Combined statistical multistage processes (SMDP + SMCP). It follows from (44) that the cross section of the combined (SMDP + SMCP) process is determined by the expression[14]

$$\frac{d^2\sigma^{\mathrm{SMDP+SMCP}}(\mathbf{k}_b,\ \mathbf{k}_a)}{d\varepsilon_b\,d\Omega_b} = \sum_c \int \frac{d\mathbf{k}_c}{(2\pi)^3} \times \frac{d^2W^{\mathrm{SMCP}}(\mathbf{k}_b,\ \mathbf{k}_c)}{d\varepsilon_b\,d\Omega_b}\,\frac{d^2\sigma^{\mathrm{SMDP}}(\mathbf{k}_c,\ \mathbf{k}_a)}{d\varepsilon_c\,d\Omega_c}, \tag{46}$$

where $d^2\sigma^{\mathrm{SMDP}}(\mathbf{k}_c\ \mathbf{k}_a)/d\varepsilon_c\,d\Omega_c$ is determined by an expression of the type (29) that describes the first stage (SMDP) in the nuclear process, which ends in the stage N_{ub}, when the intermediate system of continuum states goes over to quasibound states of the compound nucleus; $d^2W^{\mathrm{SMCP}}(\mathbf{k}_b, \mathbf{k}_c)/d\varepsilon_b\,d\Omega_b$ is determined by an expression of the type (24), which describes the second (SMCP) stage.

It should be noted that in Ref. 8 only two types of independent mechanisms of the nuclear process (SMCP and SMDP) were determined. In contrast to the cascade–exciton model,[16] which combines the mechanisms of different models (semiclassical cascade and the traditional exciton model), the expression (46) for the statistical multistage direct and compound processes is obtained in the framework of the quantum-field approach in the complete region of energies on the basis of unified closed physical methods of the microscopic description. Thus, our systematic microscopic analysis of the multistage processes makes it possible to formulate a combined model of pre-equilibrium nuclear reactions that take place in accordance with the SMDP + SMCP scheme. Analysis of experiments[13,17] with a view to determining the contributions of the symmetric (SMCP) and asymmetric (SMDP) components in their dependence on the development of the hierarchy of the process confirms the validity of the scheme of the combined nuclear-reaction mechanism.

The expressions obtained above for the three (SMCP, SMDP, and SMDP + SMCP) cases with composite particles taking into account only transitions with $\Delta N = +2$ were obtained with neglect of the rearrangement processes in the entrance and exit channels.[14] If this approximation is not made, it is necessary to split the interaction blocks $[V^{(a)} - V^{(a)}_{\mathrm{opt}}]$ and $[V^{(b)}_{\mathrm{opt}} - V^{(b)}_{\mathrm{opt}}]$ into two terms[43]:

$$[V^{(a)} - V^{(a)}_{\mathrm{opt}}] = \Delta V^{\mathrm{reg}}_{a,\,a} + \Delta V^{\mathrm{fl}}_{a,\,a},$$
$$[V^{(b)} - V^{(b)}_{\mathrm{opt}}] = \Delta V^{\mathrm{reg}}_{b+B^*,\,b+B^*} + \Delta V^{\mathrm{fl}}_{b+B^*,\,b+B^*},$$

where ΔV^{reg} is the regular part of the interaction block $[V - V_{\mathrm{opt}}]$ and depends weakly on the energy, and ΔV^{fl} is the fluctuation part of the interaction, this vanishing after statistical averaging: $\langle \Delta V^{\mathrm{fl}} \rangle = 0$. It is here assumed that in ΔV^{fl} allowance is made only for the contributions due to the effect of polarization of the nuclear medium, i.e., the difference between the interaction between the quasiparticles (in the nucleus) and the interaction of free nucleons. Allowance for the block ΔV^{reg} corresponds to the consideration of all possible processes (stripping, disintegration, pickup, direct knockout, coalescence) with $\Delta N = 0$ in the framework of the theory of direct interactions using effective interaction potentials.

Thus, for the systematic description of nuclear reactions with composite particles when $V^{(a)} \neq V^{(a)}_{\mathrm{opt}}$, $V^{(b)} \neq V^{(b)}_{\mathrm{opt}}$, it is necessary to take into account the specific nature of the process of formation of the free composite particle in the exit channel and the disintegration of the incident free composite particle in the entrance channel, i.e., transitions of the type (quasiparticle $\rightleftarrows$ free composite particle). If the nuclear polarization effects are ignored, we arrive at a description of the multistage nuclear processes in the framework of a Born series in which the free composite particles occur in the intermediate states.[18] It must be borne in mind that the theory of multistage direct reactions (TMDR) developed in Ref. 18 describes only the multistage direct processes in the approximation of random phases of the matrix elements in the l representation for the distorted waves; this gives rise to appreciable difficulties in the calculations of the contributions of the third, fourth, etc., stages of the multistage direct processes. In the SMDP + SMCP formalism, the statistical treatment is done in the k representation, and in the integration over the energy the factors $(E - \varepsilon_{k_c} + i\eta)^{-1}$ are replaced by $i\pi\delta(E - \varepsilon_{k,c})$, and in the integral over $d\Omega'_c d\Omega''_c$ the factor $\delta(\Omega'_c - \Omega''_c)$ is introduced. These approximations in the SMDP + SMCP formalism greatly simplify the calculations of the partial contributions of the individual SMDP stages ($n = 1, 2, 3$, etc.). Despite the differences mentioned above, the TMDR and the SMDP + SMCP formalism give results that are in practice similar when the first and second SMDP stages are described.[9,18]

As in the TMDR approach of Ref. 18, Ignatyuk[19] used in the DWBA approximation microscopic wave functions with allowance for one- and two-phonon excitations to describe the high-energy component of the cross section due to one- and two-stage direct processes. He calculated the angular and energy distributions of the neutrons in (p, n) reactions and obtained good agreement with the experiments. It should be noted that the SMDP + SMCP formalism (see Refs. 8, 13, 14, 18, and 19) is essentially a generalization of traditional coupled-channel calculations in the framework of the shell model in the continuum[20] to the case when there is a large number of open channels and intermediate states excited in the nuclear reactions (statistical multistage processes).

Investigation of nuclear reactions with composite particles in the microscopic model of pre-equilibrium decay

As we have already mentioned, in pre-equilibrium nuclear reactions with nucleons and composite particles the following types of nuclear-reaction mechanisms are possible in the framework of the microscopic approach: direct single-stage processes (DP) and statistical multistage direct processes (SMDP), which determine the asymmetry in the angular distributions of the reaction products, and statistical multistage processes through a compound system (SMCP), including the equilibrium component (EC), which determine

the symmetric part of the angular distributions. For the interpretation of the pre-equilibrium nuclear reactions it is necessary to determine the densities of states that occur in their description. These are densities of states determined for a given number of excitons $N = p_N + h_N$ at excitation energy E, $\rho_N(E) \equiv \rho(p_N, h_N, E)$, $E = E_a$ or U_B; the densities of the admissible states in the intermediate system: $\rho_+(p_N, h_N, E_a)$; and the densities of the admissible states of the final nucleus: $\rho_f(p_N, h_N, U_B)$. It is obvious that for the description of statistical multistage compound processes we need the densities $\rho^{(b)}(p_N, h_N, E)$ constructed on the basis of the quasibound configurations of the compound system, and the single-particle states must correspond to the discrete (quasidiscrete) spectrum of the shell model. To calculate such densities, the equidistant approximation of Strutinsky and Ericson is generally used. Thus, the densities of the quasibound excited exciton states for the multistage compound processes will be determined by the single-particle quasiparticle states in the potential well of the nucleus above the Fermi energy ε_F up to an energy S equal to the sum of the binding energy β, the energy of the centrifugal barrier U_{cf}, and the energy of the Coulomb barrier U_C for the charged particles ($S = B + U_{cf} + U_C$), and by the single-particle quasihole states from the bottom of the well up to ε_F. To describe the statistical multistage direct processes we need the densities $\rho^u(m, p_N - m, h_N, E)$ of the unbound states constructed on the basis of the configurations of the intermediate system in which m particles are in a continuum state:

$$\rho^{(u)}(m,\ p_N - m,\ h_N,\ E) = \int_0^{E-S} dE_m \rho^{(b)}(p_N - m,\ h_N,\ E - E_m) \times \rho(m,\ E_m),$$

where $\rho(m, E_m)$ is the density of states of m particles with total kinetic energy E_m in the continuum,

$$\rho(m,\ E_m) = \int_0^{E_m} \dots \int_0^{E_m} d\varepsilon_1 \dots d\varepsilon_m \rho(\varepsilon_1) \dots \rho(\varepsilon_m) \times \delta\left(E - \sum_{i=1}^{m} \varepsilon_i\right),$$

and $\rho(\varepsilon_i)$ is the density of states of particle i with kinetic energy ε_i in the continuum measured from the zero of the potential energy.

In the framework of the SMDP + SMCP formalism, the matrix elements of the intranuclear transitions are determined by the properties of the wave functions within a finite region of radius R (the distance at which the nuclear forces can be taken to be zero). Using the method of factorizing the single-particle functions in the potential, it is possible to show that the radial part of the wave function for a continuum particle in the region $r \lesssim R$ can be represented in the form[21]

$$\left\{\frac{1}{\pi}\left[R + \frac{d\delta_{lj}}{dk} + \frac{1}{2k}\sin(2kR + \delta_{lj})\right]\right\}^{1/2} u_{lj}(r), \qquad (47)$$

where δ_{ij} are the scattering phase shifts, and $u_{lj}(r)$ is a function that depends weakly on the energy and is normalized in accordance with $\int_0^R u_{lj}^2(r)dr = 1$. Thus, the density of continuum single-particle states can be expressed for $kR \gg 1$ in the form[22]

$$\rho(\varepsilon) = \sum_{lj} \frac{(2j+1)}{\pi}\left[R\frac{dk}{d\varepsilon} + \frac{d\delta_{lj}}{d\varepsilon}\right]. \qquad (48)$$

In the region outside the potential resonances, the term $d\delta_{lj}/d\varepsilon$ can be ignored, and for $\rho(\varepsilon)$ we obtain[3]

$$\rho(\varepsilon) = \left(\frac{4\pi}{3}R^3\right)4\pi(2m)^{3/2}\varepsilon^{1/2}/(2\pi\hbar)^3. \qquad (49)$$

The introduction of the two types of densities $\rho^{(u)}$ and $\rho^{(b)}$ corresponds to the two types of mechanism of the pre-equilibrium reactions: SMDP and SMCP. The total density of the states with the given number of excitons is determined by

$$\rho(p_N,\ h_N,\ E) = \rho^{(b)}(p_N,\ h_N,\ E) + \sum_m \rho^{(u)}(m,\ p_N - m,\ h_N,\ E).$$

The microscopic approach to the calculation of the densities[23] is associated with the use of realistic single-particle state schemes in a definite potential, for example, Seeger–Howard or Woods–Saxon. The shell structure of the distribution of the single-particle states in the nuclei leads to fluctuations in the densities of the excited states,[23] and this will be manifested in the spectra of the secondary particles emitted in the reaction. Such fluctuations in the realistic (microscopic) densities are particularly pronounced for low excitation energies and a small number of excitons. At a higher excitation energy, the fluctuations are damped. These microfluctuations are manifested over an energy interval of order 1 MeV. The calculations show that with increasing mass number A the fluctuations are damped. In principle, it is preferable to use realistic densities in the calculations in place of the equidistant densities. As an illustration, Fig. 3 shows realistic (microscopic) densities $\rho^{(b)}$ of the quasibound states calculated for the ^{60}Ni nucleus on the basis of a realistic level scheme in the Woods–Saxon potential, and also the densities of the quasibound states in the equidistant approximation as functions of the excitation energy E of the compound system for $S = 8$ MeV and $\varepsilon_F = 40$ MeV. In the cal-

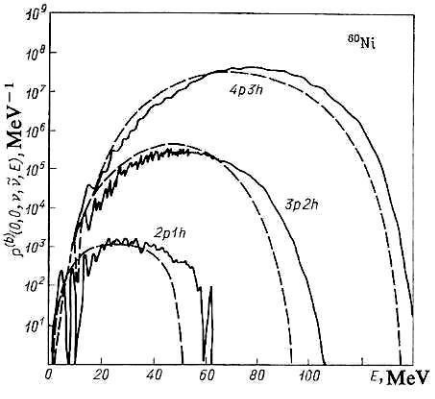


FIG. 3. Densities of quasibound states $\rho^{(b)}(0, 0, \nu, \tilde{\nu}, E)$ of the nucleus ^{60}Ni with separation of the particles and holes into protons and neutrons. The continuous curves are realistic densities, and the broken curves are the densities in the equidistant approximation.

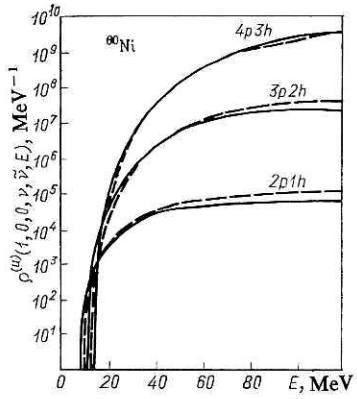


FIG. 4. Densities of unbound states $\rho^{(u)}(1, 0, 0, \nu, \tilde{\nu}, E)$ of the nucleus ^{60}Ni with one continuum quasiparticle, as functions of the excitation energy E (neutron excitation branch, $\pi = \tilde{\pi} = 0$). The notation is as in Fig. 3.

culated densities $r^{(b)}(\pi, \tilde{\pi}, \nu, \tilde{\nu}, E)$ allowance is made for the division of the particles (π, ν) and holes ($\tilde{\pi}$, $\tilde{\nu}$) into protons and neutrons, respectively. It can also be seen that there is an energy gap for the different configurations. For the example, the represented densities were calculated for the neutron excitation branch ($\pi = \tilde{\pi} = 0$), for which the number of particles (p_N) and holes (h_N) is determined solely by the neutrons (in the general case, $p_N = \pi + \nu$, $h_N = \tilde{\pi} + \tilde{\nu}$). Figure 4 shows calculated realistic densities of the unbound states of the ^{60}Ni nucleus as functions of the excitation energy when there is one quasiparticle in the continuum.

In the region of energies $\varepsilon_a \lesssim 100$ MeV, the nuclear reaction will most probably take place in two stages. The first is an intranuclear cascade (DP + SMDP), after which the sytem goes over to quasibound configurations of the intermediate nucleus; the second is the process of relaxation to equilibrium (SMCP). In the framework of the combined (SMDP + SMCP) model of pre-equilibrium reactions, the analysis of the multistage emission process requires the determination of the mean squares of the matrix elements, $\langle|I|^2\rangle_{uu}$, $\langle|I|^2\rangle_{ub}$, $\langle|I|^2\rangle_{bb}$, $\langle|I|^2\rangle_{bu}$, which do not depend on the number of excitons. In the microscopic model of pre-equilibrium decay, all these quantities can be determined from calculations based on data on the self-consistent potential and the residual interaction. It is also necessary to calculate the densities of the admissible final states corresponding to these types of transition:

$$\rho_+^{(u)}(m,\ p_N - m,\ h_N,\ E_a),\ \rho_f^{(b)}(p_N,\ h_N,\ U_C),$$
$$\rho_+^{(b)}(p_N,\ h_N,\ E_a),\ \rho_f^{(b)}(p_N,\ h_N,\ U_B);$$

these densities contain an explicit dependence on the number of excitons.

In the framework of the microscopic model of pre-equilibrium decay,[30,44] we investigate the properties of the contributions to the cross section from the various mechanisms of the pre-equilibrium nuclear reactions (DP, SMDP, SMCP, and SMDP + SMCP). If the incident particle has an energy of the order of tens of mega-electron-volts ($k_a R \gg 1$), the possible range of variation of the total spin and orbital angular momentum is fairly large. Summing in an expression of the type (24), which describes the statistical multistage compound processes, over all possible values of l, l', J, J_B' under the assumption of a weak spin dependence of the averaged matrix elements (this assumption is confirmed by numerical calculations with residual forces) with a corresponding averaging over the spins (such averaging leads to an isotropic angular distribution of the secondary particles), we obtain for the hard part [$\varepsilon_b \sim 0.5(E_a - B_b)$] of the spectrum

$$\frac{d^2\sigma^{\text{SMCP}}}{d\varepsilon_b\, d\Omega_b} = \sum_{n=1}^{\bar{n}} \frac{\Gamma_{nb}^{\uparrow}}{\Gamma_n}\left[\prod_{k=1}^{n-1}\frac{\Gamma_k^{\downarrow}}{\Gamma_k}\right]\sigma_a(\varepsilon_a), \tag{50}$$

$$\Gamma_{nb}^{\uparrow} = 2\pi\,\langle|I(\varepsilon_b)|^2\rangle_{bu}\rho(\varepsilon_b,\ \Omega_b)\,\rho_f^{(b)}(a+2n-b,\ U_B),$$
$$\Gamma_k^{\downarrow} = 2\pi\,\langle|I(E_a)|^2\rangle_{bb}\rho_+^{(b)}(k,\ E_a).$$

We use the following approximations: 1) $\Gamma_k^{\downarrow} \gg \Gamma_k^{\uparrow}$, which is justified for intermediate and heavy nuclei[8]; 2) $\rho_+^{(b)}(k, E_a)$ depends weakly on k, and $\langle|I(\varepsilon_b)|^2\rangle_{bu}$ does not depend on ε_b [$\varepsilon_b \sim 0.5(E_a - B_b)$, $k_b R \gg 1$], this being due to the particular way in which we have defined the density $\rho(\varepsilon_b)$ [see (47) – (49)], which contains almost the entire dependence on ε_b. Then from (50) we finally obtain

$$\frac{d^2\sigma^{\text{SMCP}}}{d\varepsilon_b\, d\Omega_b} = \sum_{n=1}^{\bar{n}} C_n^{\text{SMCP}}(\varepsilon_a,\ E_a)\,\varepsilon_b^{1/2}\exp\left[-\varepsilon_b/\varkappa_n(\varepsilon_b)\right], \tag{51}$$

where $C_n^{\text{SMCP}}(\varepsilon_a, E_a)$ is a function that depends weakly on ε_b, and $\varkappa_n(\varepsilon_b)$ is the nuclear quasitemperature, defined in the complete range of variation of $\varepsilon_b (0 < \varepsilon_b \leqslant E_a - B_b)$ by means of the exact relation

$$\ln\rho_f(a+2n-b,\ E_a - B_b - \varepsilon_b) = S_0(E_a - B_b) - \varepsilon_b/\varkappa_n(\varepsilon_b). \tag{52}$$

For the densities ρ_f calculated in the equidistant appproximation, we have

$$\rho_f(a+2n-b,\ U_B) = g(gE_a)\left(\frac{U_B}{E_a}\right)^{a+2n-b-1}, \tag{53}$$

where g is the single-particle density. It follows from (52) and (53) that

$$\varkappa_n(\varepsilon_b) = (E_a - B_b)/a_1(\varepsilon_b)\,[a+2n-b-1], \tag{54}$$

where the factor $a_1(\varepsilon_b)$ is determined by

$$a_1(\varepsilon_b) = -\ln\left(1 - \frac{\varepsilon_b}{E_a - B_b}\right)\Big/\left(\frac{\varepsilon_b}{E_a - B_b}\right). \tag{55}$$

The factor $a_1(\varepsilon_b)$, calculated in accordance with (55), depends weakly on ε_b in the range of values (0.3–0.6)$(E_a - B_b)$.

As in the SMCP scheme of analysis described above, for $\varepsilon_b \sim 0.5(E_a - B_b)$ we obtain

$$\frac{d^2\sigma^{\text{SMDP}}(\mathbf{k}_b,\ \mathbf{k}_a)}{d\varepsilon_b d\Omega_b} = \sum_{n=1}^{N_{ub}} C_n^{\text{SMDP}}(\mathbf{k}_b,\ \mathbf{k}_a,\ E_a)\,\varepsilon_b^{1/2} \times \exp\left[-\varepsilon_b/\varkappa_n(\varepsilon_b)\right], \tag{56}$$

where $C_n^{\text{SMDP}}(\mathbf{k}_b, \mathbf{k}_a, E_a)$ is a function that depends weakly on ε_b, determines the shape of the angular distribution of the secondary particles, and is determined by an expression of the type (29). Analysis of the structure of the functions $C_n^{\text{SMDP}}(\mathbf{k}_b, \mathbf{k}_a, E_a)$ in the framework of the theory of direct

reactions shows that at small angles the main contribution is made by the first stage of the process ($\varkappa_i$); with increasing scattering angle ϑ_b the contribution of the second stage increases, and for a definite range of angles ϑ_b the contribution of the second stage becomes the main one or comparable with the contribution of the first stage, etc.; the contributions of the following stages increase with increasing angle ϑ_b. This behavior of the functions $C_n^{\mathrm{SMDP}}(\mathbf{k}_b, \mathbf{k}_a, E_a)$ for the direct processes is also confirmed by direct SMDP calculations for reactions with nucleons.[9,18] Thus, analysis of the dependences of $\ln[(d^2\sigma/d\varepsilon_b d\Omega_b)\varepsilon_b^{-1/2}$ on ε_b at different angles ϑ_b for the individual SMDP stages ($n = 1$–6) made using the DWUCK program for the reaction ^{120}SN(p, n) at $\varepsilon_p = 45$ MeV (Ref. 9; see Fig. 2) confirms the validity of the SMDP expression (56) in the region $\varepsilon_p \sim 0.5(E_a - B_b)$. The values of $\varkappa_n$, in MeV, obtained from these calculations for $n = 1$–6 are: $\varkappa_1 = 25.0$, $\varkappa_2 = 8.5$, $\varkappa_3 = 5.0$, $\varkappa_4 = 3.9$, $\varkappa_5 = 3.2$, and $\varkappa_6 = 2.9$. The corresponding estimates of $\varkappa_i$ in accordance with (54) are $\varkappa_1 = 25.2$, $\varkappa_2 = 8.4$, $\varkappa_3 = 5.1$, $\varkappa_4 = 3.8$, $\varkappa_5 = 3.3$, and $\varkappa_6 = 2.9$. It can be seen from Fig. 2 that as ϑ_b increases the contributions of the second, third, and following stages increase. Therefore, the conclusion[24–26] established by analysis of experimental data that the "quasitemperature" $\varkappa(\vartheta_b)$ depends on the scattering angle ϑ_b is a reflection of such behavior of C_n^{SMDP}, namely, with increasing angle ϑ_b the dominant contributions of the successive SMDP stages increase. Thus, the microscopic theory of pre-equilibrium nuclear reactions makes it possible in principle to determine the type of mechanism of a pre-equilibrium nuclear reaction by analyzing the linear sections in dependences of the form $\ln[(d^2\sigma/d\varepsilon_b d\Omega_b)\varepsilon_b^{-1/2}$ on ε_b for given angle ϑ_b in the region $\varepsilon_b \sim 0.5(E_a - B_b)$, where C_n^{SMCP}, C_n^{SMDP}, and $\varkappa_n(\varepsilon_b)$ depend weakly on ε_b.

In the general case, the slope of the linear section is determined in the hard part of the emission spectrum by the contributions of several stages (as a rule, not more than three SMDP stages). In this situation, it is possible to introduce a generalized nuclear quasitemperature $\varkappa_{i,i+1}(\bar{\varepsilon}_b, \vartheta_b)$ (the contribution of two successive stages i and $i+1$) and $\varkappa_{i,i+1,i+2}(\bar{\varepsilon}_b, \vartheta_b)$ (the contribution of three successive stages) by means of the relation

$$\varkappa_{i,i+1}^{-1}(\bar{\varepsilon}_b, \vartheta_b) = [\varkappa_i^{-1}(\bar{\varepsilon}_b)\, p_i + \varkappa_{i+1}^{-1}(\bar{\varepsilon}_b)\, p_{i+1}]/(p_i + p_{i+1}). \quad (57)$$

A similar relation holds for $\varkappa_{i,i+1,i+2}^{-1}$. In (57), $p_i = d^2\sigma_i(\bar{\varepsilon}_b, \vartheta_b)/d\varepsilon_b d\Omega_b$ is the partial contribution of stage i to the emission spectrum at $\varepsilon_b \approx 0.5(E_a - B_b)$; more precisely, $\bar{\varepsilon}_b$ corresponds to the middle of the observed linear section in the hard part of the emission spectrum. As follows from the calculations, the relation (57) holds with satisfactory accuracy. The above analysis of the linear sections in the dependences of $\ln[(d^2\sigma/d\varepsilon_b d\Omega_b)\varepsilon_b^{-1/2}]$ on ε_b in the framework of the microscopic theory of pre-equilibrium reactions differs from the formalism of a locally heated nucleus,[27] which is valid only for sufficiently high energies ($\varepsilon_a > 100$ MeV).

On the basis of Eqs. (54), (56), and (57), we analyze the experimental dependences of the quasitemperature $\varkappa(\vartheta_b)$ on the scattering angle ϑ_b.

Example 1. The reactions ^{62}Ni(^{3}He, n), $\varepsilon(^3\mathrm{He}) = 40.9 \pm 0.8$ MeV and ^{61}Ni(α, p), $\varepsilon_\alpha = 52.6 \pm 0.8$ MeV.[24] Figure 5 shows the experimental dependence of $(d^2\sigma/d\varepsilon_n\, d\Omega_n)\varepsilon_n^{-1}\sigma_{inv}^{-1}(E_n)$ on the neutron energy ε_n for different detection angles ϑ_n and the dependence of the nuclear quasitemperature $\varkappa(\vartheta_n)$ on ϑ_n for the reactions ^{62}Ni(^{3}He, n) (black circles) and ^{61}Ni(α, p) (open circles).

Example 2. The reactions (^{3}He, p) and (^{3}He, d) on the nuclei ^{27}Al, ^{59}Co, ^{90}Zr, and ^{112}Sn at $\varepsilon(^3\mathrm{He}) = 50.5$ MeV.[28] Figure 6 shows the experimental dependences $\varkappa(\vartheta)$ for these reactions. The broken lines in Fig. 6 show the estimates of $T_{\mathrm{eq}} = (E_a - B_b/a')^{1/2}$ and $a' = \pi^2 g/6$. In Table I we give estimates of $\varkappa$ in accordance with the expressions (54) and (57) for the reactions considered above (Examples 1 and 2), obtained in the approximation of equally large contributions of successive stages. For an odd target nucleus, the initial number of excitons a must be replaced by $a + 1$. Comparison of $\varkappa(\vartheta_b)$ (see Figs. 5 and 6) and $\varkappa_{ik}$ (see Table I) shows that for the considered reactions the slopes of the linear sections in the dependences on ϑ_b in the region $\varepsilon_b \sim 0.5(E_a - B_b)$ are determined by the contributions of the first to the third SMDP stages for $\vartheta_b \lesssim 150°$ and by the contribution of the third SMDP and SMCP stages for $150° < \vartheta_b < 180°$. Summarizing the results of the above analysis, it can be said that the

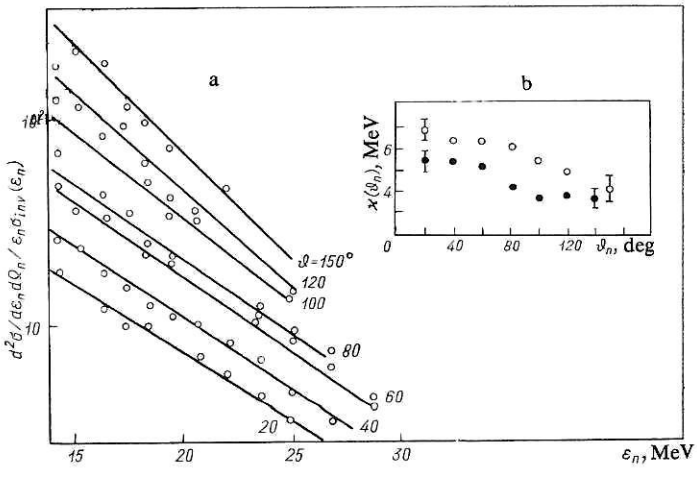


FIG. 5. The dependences of $(d^2\sigma/d\varepsilon_n\, d\Omega_n)/\varepsilon_n\, \sigma_{\mathrm{inv}}$ on the neutron energy for different detection angles (a) and of the nuclear temperature $\varkappa(\vartheta)$ on the neutron emission angle for the reactions ^{62}Ni(^{3}He, n) (black circles) and ^{61}Ni(α, n) (open circles) (b).

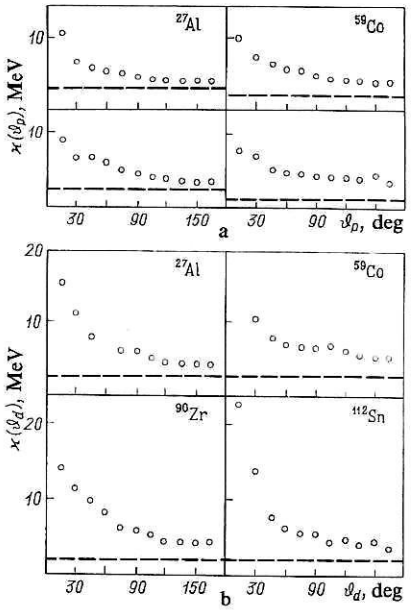


FIG. 6. Experimental dependences $\varkappa(\vartheta)$ at $\varepsilon(^3\text{He}) = 50.5$ MeV: a) for the reaction (^{3}He, p); b) for the reaction (^{3}He, d). The broken lines show the estimates of T_{eq}. the open circles are the estimates of $\varkappa$ with a correction for the ^{3}He disintegration contribution.

main part in the formation of the differential cross section is played by the first to the third SMDP + SMCP stages, i.e., there is a combined SMDP + SMCP mechanism of multistage emission.

Quasiparticle–phonon model of pre-equilibrium decay

In the microscopic theories of pre-equilibrium nuclear reactions described above it was assumed that the intermediate states in the multistage relaxation process are classified only by the number of excitons, and admissible transitions with $\Delta N = 0, \pm 2$ were considered. Despite the successes of this description, it is necessary to extend the microscopic approaches by taking into account explicitly not only the excitons but also the collective degrees of freedom of the nucleus (phonons). The intermediate stages will then be classified by both the number of excitons N and the number of phonons, N_{ph}, i.e., we consider configurations of the type $|N, N_{ph}\rangle$. We define accordingly the densities of the intermediate states: $\rho(N, N_{ph}, E)$. To describe the multistage relaxation process, we introduce two types of intranuclear transitions with corresponding transition probabilities λ_+:

a) with a change in the number of excitons by $\Delta N = +2$,

$$\lambda_+^e(N, N_{ph}, E) = \frac{2\pi}{\hbar}\langle|I|^2\rangle_e \rho_f^e(N, N_{ph}, E);$$

b) with a change in the number of phonons by $\Delta N_{ph} = +1$,

$$\lambda_+^{ph}(N, N_{ph}, E) = \frac{2\pi}{\hbar}\langle|I|^2\rangle_{ph}\rho_f^{ph}(N, N_{ph}, E),$$

where $\langle|I|^2\rangle_e$ and $\langle|I|^2\rangle_{ph}$ are the mean squares of the matrix elements of the transitions with $\Delta N = +2$ and $\Delta N_{ph} = +1$, respectively, and $\rho_f^{(e)}(N, N_{ph}, E)$ and $\rho_f^{ph}(N, N_{ph}, E)$ are the densities of the admissible final states in the corresponding transitions. The values of $\langle|I|^2\rangle_e$ and $\langle|I|^2\rangle_{ph}$ can be extracted from estimates of the corresponding transition probabilities in the framework of the microscopic approach:

$$\lambda_+(1, 0, E) = \lambda_+^e(1, 0, E) + \lambda_+^{ph}(1, 0, E),$$

$$\lambda_+^e(1, 0, E) = \frac{2\pi}{\hbar}\left\langle\sum_{j_1, j_2, j_3}|\langle j|I|j_1, j_2, j_3\rangle|^2 \times\left\{\frac{1}{\pi}\frac{\Delta E/2}{[E-(e_{j_1}+e_{j_2}-e_{j_3})]^2+(\Delta E/2)^2}\right\}\right.,$$

$$\lambda_+^{ph}(1, 0E) = \frac{2\pi}{\hbar}\left\langle\sum_{\lambda, j'}\frac{\beta_\lambda^2(E_\lambda)}{(2\lambda+1)(2j'+1)}|\langle j||T_\lambda||j'\rangle|^2 \times\left\{\frac{1}{\pi}\frac{\Delta E/2}{(E-(e_{j'}+E_\lambda)]^2+(\Delta E/2)^2}\right\}\right.,$$

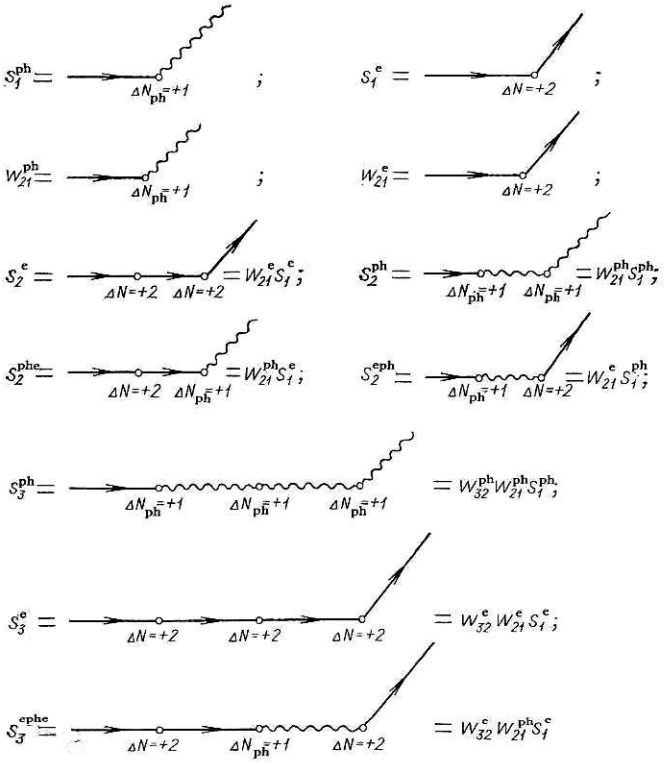
where $T_{\lambda\mu} = R(\partial U/\partial r)Y_{\lambda\mu}$, and $\beta_\lambda(E_\lambda)$ is the deformation parameter for a phonon of multipolarity λ and energy E_λ, the above quantities being taken from RPA calculations; j is the set of quantum numbers that characterize the single-particle state with energy e_j in the average field U of the nucleus; ΔE is the interval of averaging. It follows from numerical calculations of λ_+, λ_+^e, and λ_+^{ph} on the basis of the above expressions [29,30] that the contribution of the direct transition with $\Delta N_{ph} = +1$ and the excitation of low-lying collective modes (2_1^+, $T=0$; 3_1^-, $T=0$, giant quadrupole

TABLE I. Calculation of the nuclear quasitemperature, MeV.

Reaction	$\varkappa_1$	$\varkappa_{12}$	$\varkappa_2$	$\varkappa_{23}$	$\varkappa_3$	$\varkappa_{3n}$	T_{eq}
^{62}Ni(^{3}He, n)	10.5	7,9	6,3	5,3	4,5	3.4	2.4
^{61}Ni(α, n)	8.5	6,8	5,7	4,8	4,3	3,4	2,5
^{27}Al(^{3}He, p)	10.8	8,4	7.2	6,1	5,4	4,7	4.1
^{59}Co(^{3}He, p)	9,9	7,9	6,6	5,5	5,0	3.6	2.8
^{90}Zr(^{3}He, p)	11,5	9,1	7,5	6,4	5,6	3.7	2,2
^{112}Sn(^{3}He, p)	12,0	9,1	7,3	6,2	5,2	3.5	1,9
^{27}Al(^{3}He, d)	9,0	7,0	5,4	4,6	4,0	3,9	3,8
^{59}Co(^{3}He, d)	7.0	5,1	4,2	3,4	3,0	2,8	2,6
^{90}Zr(^{3}He, d)	11.2	8,6	6,9	6,3	5,8	3,9	2,1
^{112}Sn(^{3}He, d)	10,8	8,7	7,3	6,0	5,1	3,4	1.8

resonance) is about 60% of λ_+; accordingly, the contribution of the direct transition with excitation of a particle and a hole ($\Delta N = +2$) and weakly collective phonons ($\Delta N_{\rm ph} = +1$) is ~40%. In what follows, we shall describe this group of transitions in a unified manner as transitions with $\Delta N = +2$.

The main pre-equilibrium reaction mechanism at energies of the incident particle up to ~100 MeV is a combined multistage process involving the first to the third SMDP + SMCP stages (see Sec. 2). The hard part of the emission spectrum is determined predominantly by the contributions of the first to the third SMDP stages. When the two types of transition are particularized ($\Delta N = +2$ and $\Delta N_{\rm ph} = +1$), the following reaction schemes are possible (first to the third SMDP stages, i.e., 1SDP, 2SDP, 3SDP):



+ corresponding diagrams for S_3 with the possible alternations of transitions with $\Delta N = +2$ and $\Delta N_{\rm ph} = +1$. In the above diagrams, we have introduced the notation (see Sec. 1)

$$S_1 = \frac{d^2\sigma^{1\,\mathrm{SDP}}(\mathbf{k}_1, \mathbf{k}_i)}{d\varepsilon_1 d\Omega_1}, \quad W_{21} = \frac{d^2 W_{21}(\mathbf{k}_2, \mathbf{k}_1)}{d\varepsilon_2 d\Omega_2},$$

$$\Delta N = +2, \ \Delta N_{\rm ph} = +1.$$

To obtain formulas to be used in calculations, we make the following approximations:

1) under the assumption that the angular distributions of the secondary particles are determined solely by the direction of motion of the incident particle, we shall use a factorized expression for the matrix elements[15]:

$$\langle |I_{m,\,m+1}(\mathbf{k}_m, \mathbf{k}_{m+1})|^2 \rangle = \langle |I(\varepsilon_m, \varepsilon_{m+1})|^2 \rangle W(\Omega_m \to \Omega_{m+1}),$$

where

$$W(\Omega_m \to \Omega_{m+1}) = \pi^{-1} \cos(\vartheta_{m+1} - \vartheta_m)\, \Theta\left[\frac{\pi}{2} - (\vartheta_{m+1} - \vartheta_m)\right]$$

with the normalization condition

$$\int d\Omega_{m+1} W(\Omega_m \to \Omega_{m+1}) = 1;$$

2) for $k_{m+1}R \gg 1$, the matrix elements $\langle |I(\varepsilon_m, \varepsilon_{m+1})|^2 \rangle$ are determined solely by the properties of the wave functions for $r \lesssim R$, and therefore $\langle |I(\varepsilon_m, \varepsilon_{m+1})|^2 \rangle$ is almost independent of ε_{m+1} if the density $\rho(\varepsilon_{m+1})$ within the sphere of radius R is defined in accordance with (47)–(49);

3) the total transition probability $\lambda_+(1, 0, \varepsilon)$ is determined by the real mean free path $\Lambda(\varepsilon)$ of a nucleon in the nucleus, $\lambda_+(1, 0, \varepsilon) = v/\Lambda(\varepsilon)$;

4) the densities $\rho(N, N_{\rm ph}, E)$ can be calculated on the basis of the single-particle level scheme and the spectrum of phonons in the nucleus;

5) the probabilities of transitions with $\Delta N = +2$ and $\Delta N_{\rm ph} = +1$ are determined by the relations of Refs. 29 and 30: $\lambda_+^{\rm ph}/\lambda_+ \simeq 0.6$ and $\lambda_+^{\rm e}/\lambda_+ \simeq 0.4$.

Using these approximations, we calculated the contributions $d^2\sigma^{1\mathrm{SDP}}/d\varepsilon_f d\Omega_f$, $d^2\sigma^{2\mathrm{SDP}}/d\varepsilon_f d\Omega_f$, $d^2\sigma^{3\mathrm{SDP}}/d\varepsilon_f d\Omega_f$, SMCP, and 3SDP + SMCP to the total cross section for the reactions ^{60}Ni(p, p') and ^{120}Sn(p, p') at $\varepsilon_p = 61.7$ MeV with allowance for excitation of the phonons 2_1^+, 2_2^+, 3_1^-, 4_1^- and a giant quadrupole resonance (2^+). The results of the calculation[31] of the reaction ^{120}Sn(p, p') are shown in Fig. 7.

The analysis of the (p, p') reactions in the framework of the quasiparticle–phonon model of pre-equilibrium decay in the SMDP + SMCP formalism demonstrates the importance of taking into account the contributions with excitation of collective modes to the total cross section, this also making it possible to explain the intermediate structure in the hard part of the spectrum and to estimate the partial contributions of the individual types and stages of the multistage reaction mechanism (the first to the third SMDP stage, 3SDP + SMCP, SMCP). The resulting formulas obtained in the framework of the SMDP + SMCP formalism are valid for reactions with emission of composite particles ($b \leqslant 4$) under the assumption that the mechanism of formation of the composite particle is described by the coalescence model.[31] In this case, the expressions contain an additional factor γ_b, which determines the probability of formation in the final state f of the composite particle b from b excited quasiparticles.

GENERALIZED MODEL OF THE DECAY OF GIANT MULTIPOLE RESONANCES

Microscopic theory of pre-equilibrium nuclear reactions and excitation of giant multipole resonances

The experimental discovery of giant multipole resonances (GMR) for a large class of nuclear reactions such as (γ, N), (e, e'), μ^- and π^- capture, and inelastic scattering of particles (N, d, t, h, α) was a major step forward in the interpretation of the excitation spectra of nuclei up to 40 MeV. The next task in the theoretical description of giant multipole resonances is the correct and complete allowance for complicated configurations ($2p2h$, $3p3h$, and more complicated ones). In recent years there have been numerous at-

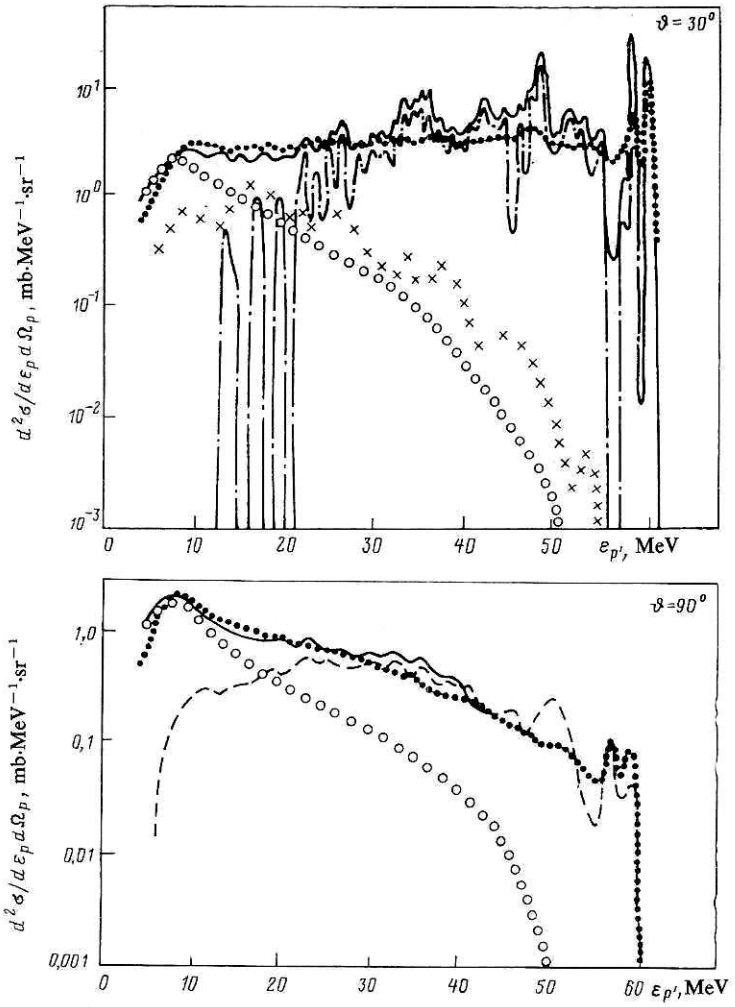


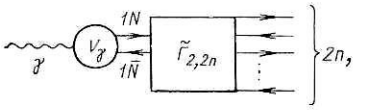
FIG. 7. Double differential cross sections for scattering of protons by the ^{120}Sn nucleus. The points represent the experiment of Ref. 45; the continuous curve, the calculated total cross section; the chain curve, the contribution of the first stage; the open circles, the SMCP contribution; the crosses, the 3SDP + SMCP contribution; and the chain curve in the second figure,the total contribution of the three SMDP stages.

tempts to take into account directly only the $2p2h$ configurations.[32–34] Allowance for all possible $2p2h$ configurations within a definite number of shells usually reduces to the diagonalization of a matrix of high order (with an effective $1p1h$ interaction). For intermediate and heavy nuclei such an approach, taking into account explicitly $2p2h$ configurations, is difficult on account of the large number of such states (for example, for the $E1$ resonance in the ^{208}Pb nucleus in the interval 5–20 MeV the number of $2p2h$ configurations that must be taken into account is $(3.5–5.5)\times 10^4$). The difficulties increase even more if one takes into account explicitly $3p3h$, $4p4h$, and more complicated configurations. One therefore usually separates the most "dangerous" $2p2h$ configurations (by introducing phonons–collectivized $1p1h$ states), which determine the fragmentation of the giant multipole resonances (intermediate structure of the resonances). In the framework of the quasiparticle–phonon nuclear model formulated in Refs. 33 and 35, numerous calculations have recently been made on the fragmentation of giant resonances.

Despite the successes in the study of such fragmentation, there remains much material on nuclear structure that has been little studied, namely, the many-quasiparticle and many-phonon components of the wave functions of the excited states of complex nuclei. Moreover, the traditional approaches based on the solution of secular equations with allowance for a complete basis of states ($2p2h$, $3p3h$, $4p4h$, etc.) encounters insuperable difficulties in the investigation of highly excited states because of the large number of complicated configurations. It is therefore expedient and of interest to consider approaches that do not use just the formalism of secular equations. Of particular interest in this connection is the formulation of a generalized model of the decay of giant resonances that takes into account the direct, pre-equilibrium, and equilibrium particle-emission mechanisms.[36–39] In such a formulation, a giant resonance is described in the framework of the shell model. The doorway wave functions ψ_d of the giant multipole resonances for nuclei with closed or almost closed shells can be found by diagonalizing the residual interaction on a $1p1h$ basis or on an extended basis that takes into account the dangerous $2p2h$ configurations.[32] The wave functions $\psi_d(J\pi, E)$ describe the initial stage in the process of production of a $1p1h$ pair (in the general case, it is necessary to take into account the fragmentation over more complicated dangerous configurations). In what follows, to simplify the treatment, we shall assume that $\psi_d(J\pi, E)$ is a collective $1p1h$ state. The process of creation of a collective state as a result of the reaction [GMR with

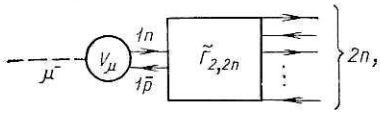
$\psi_d(J\pi, E)$] and the process of emission of nucleons (and, in the general case, composite particles) is described by diagrams of the following form:

a) For the photonuclear reaction (γ, N)



where $\overline{\Gamma}_{2,2n}$ is the total vertex part (total interaction amplitude) that takes the particle and hole that interact, $(N, \bar{N})$, to the final *npnh* state ($2n$ excitons); V_γ is the vertex of the noninteracting nucleons with respect to the γ field. In accordance with the selection rules for the operator V_γ, the predominantly excited states (in the long-wavelength approximation) are $J\pi = 1^-, 2^+$ with $T_>$ and $T_<$.

b) For μ^- capture



where V_μ is the effective interaction Hamiltonian that leads to μ^- capture by the nucleus with transformation of a proton into a neutron $(n\bar{p})$ and emission of a neutrino $(\tilde{\nu})$. In the reaction, isobar analogs of the isospin or spin–isospin resonances of the final nucleus are excited. There are similar diagrams for (e, e'), (π^-, γ), and inelastic scattering of light ions $(a \leqslant 4)$. In the diagrams, the total vertex $\tilde{\Gamma}_{2,2n}$ is the symmetrized sum of the Feynman diagrams that begin and end with an interaction event, and it is determined by the system of equations that are considered in the microscopic theory of pre-equilibrium nuclear reactions. [see (40) and (42)]. It follows from (23) that the integrated secondary-particle emission spectrum determined by statistical multistage compound processes in reactions with excitation of giant multipole resonances is described by the expression

$$\frac{d^2\sigma_{ba}^{\text{SMCP}}}{d\varepsilon_b} = \left\{ \sum_{n=2}^{\bar{n}} \frac{\Gamma_{nb}^{\uparrow}(J\pi, E_a, \varepsilon_b)}{\Gamma_n(J\pi, E_a)} \left[\prod_{k=2}^{n-1} \frac{\Gamma_{bx}^{\downarrow}(J\pi, E_a)}{\Gamma_{bx}(J\pi, E_a)} \times \frac{\Gamma_k^{\downarrow}(J\pi, E_a)}{\Gamma_k(J\pi, E_a)} \right] + \frac{\Gamma_{bx}^{\uparrow}(J\pi, E_a, \varepsilon_b)}{\Gamma_{bx}(J\pi, E_a)} \right\} \sigma_a^{\text{GMR}}(J\pi, E_a),$$

where $\sigma_a^{\text{GMR}}(J\pi, E_a)$ is the cross section for the production of the compound nucleus as a result of the given reaction in the doorway collective state $\psi_d(J\pi, E_a)$, and E_a is the excitation energy. For the μ^- and π^- capture reactions, σ_a^{GMR} must be replaced, respectively, by the μ^- capture probability $\Lambda_{0\to s}$ and by the probability of radiative π^- capture: $\Delta_\gamma(J_i \to J_f)$. The relaxation width of the doorway state ψ_d is determined by the expression

$$\Gamma_d^{\downarrow}(J\pi, E_a) = 2\pi \langle | \langle \psi_d(J\pi, E_a) | I_{2,4} | \tilde{\varphi}_{2p2h} \times (J\pi, E_a) \rangle |^2 \rangle \rho_+(2p2h, E_a).$$

The width of the decay of the doorway state ψ_d to the continuum with emission of a species-b particle, $\Gamma_d^{\uparrow}(J\pi, E_a, \varepsilon_b)$, can be calculated in accordance with the usual expressions of R-matrix theory, and $\Gamma_{nb}^{\uparrow}(J\pi, E_a, \varepsilon_b)$, $\Gamma_k^{\downarrow}(J\pi, E_a)$, and $\Gamma_k(J\pi, E_a)$ are determined by the usual expressions of the microscopic theory of pre-equilibrium decay. Thus, in the generalized GMR model based on the simultaneous use of the quasiparticle–phonon model of the nucleus (calculation of ψ_d on a restricted basis of "dangerous" configurations) and the microscopic model of pre-equilibrium decay (statistical allowance for the more complicated configurations $2p2h$, $3p3h$, etc.) one of the possibilities of taking into account the complete space of admissible complicated states is realized.

In the generalized GMR model, all possible complicated configurations are divided into two groups[36]: the group of dangerous configurations and the group of "statistical" configurations. The statistical group of complicated configurations is characterized by the following features: 1) the total density of states is high, $\rho_N(E) \gg 1$; 2) there is strong overlapping between the states, i.e., $\Gamma_n \gg D_{n-1} > D_n$; 3) the coupling matrix elements $\langle |I_{N,N'}|^2 \rangle$ are effectively small; 4) $\rho_{N+2}(E) \gg \rho_N(E)$, $N < \sqrt{2gE_a}$. It is this group of complicated configurations that determines the irreversible nature of the multistage relaxation process (pre-equilibrium decay of the doorway state ψ_d). The dangerous configurations do not satisfy the conditions of the statistical approach, and they require a special treatment in the framework of the shell model with allowance for the residual interaction. The spectrum of the states $\{\psi_d\}$ is found by diagonalizing the matrix of the residual interaction in the configuration space of the "dangerous" configurations (of the types $1p1h$, $1p1h$ + phonon, phonon + phonon, etc.). The structure of the spectrum of doorway states $\{\psi_d\}$ determines the intermediate structure of the giant multipole resonance (the fragmentation of the collective $1p1h$ state over the dangerous complicated configurations). The generalized GMR model makes it possible to take into account the specific nature of the collective doorway states in intranuclear transformations of the compound system, and it is possible to describe all the most important stages in the decay of giant resonances—the direct, pre-equilibrium, and equilibrium mechanisms of secondary-particle emission—in the framework of a unified approach.

Concrete examples of calculations of emission spectra in the framework of the model of pre-equilibrium decay

Photonuclear Reactions. a) In the method of phenomenological kinetic master equations the photoneutron spectra were calculated at bremsstrahlung energy $E_\gamma^{\max} = 20$ MeV and $E_\gamma^{\max} = 14$ MeV using the expressions of the equidistant model for the densities of the exciton states in Ref. 40. The specific features of the photonuclear reactions due to the collectivization of the nuclear dipole-excitation levels were taken into account by the introduction of a modified density of $1p1h$ states. For all the considered nuclei and energies in Ref. 40, the contribution of the pre-equilibrium processes in the spectrum of the emitted neutrons reaches about 20%. Moreover, only allowance for the coherent nature of the photoabsorption reaction makes it possible to explain quantitatively the hard part of the photoneutron spectrum (the structural features).

b) In the framework of the combined description of the

photonucleon spectra based on simultaneous use of the shell model and the phenomenological exciton model of pre-equilibrium decay the predictions of the model were tested for the example of the ^{40}Ca nucleus.[37] The wave function of the state of the giant dipole resonance was chosen in the form

$$\psi\text{GDR} = \hat{D}\psi_0 \, (\langle \hat{D}\psi_0 \mid \hat{D}\psi_0 \rangle)^{-1/2},$$

where ψ_0 is the shell wave function of the nuclear ground state, and $\hat{D}$ is the operator of the electric dipole moment. The process of nuclear relaxation was described by means of phenomenological kinetic equations (balance equations), and the densities of the exciton states were calculated in accordance with the expressions obtained in the equidistant model. The mean-square matrix element $\langle |I|^2 \rangle_{bb}$ was extracted from data on the GDR widths. Overall, the agreement between the theory and experiment is satisfactory, which indicates that the combined computational scheme of the model of pre-equilibrium decay is correct. The discrepancies are due mainly to the use of unrealistic densities and a ψ_{GDR} without allowance for fragmentation. In the examples given above, the pre-equilibrium emission was analyzed in the framework of the exciton model, which is strictly valid only for $\Delta E > W_{opt}$.

c) In Ref. 38, the energy spectra of the photonucleons were investigated for two giant-resonance branches ($T_< = T_0$ and $T_> = T_0 + 1$, where T_0 is the ground-state isospin of the initial nucleus) in the framework of the generalized GMR model on the basis of the microscopic theory of pre-equilibrium reactions. The first stage of the reaction was described by the wave function ψ_d obtained by diagonalizing the matrix on the basis of $1p1h$ configurations and the dangerous configurations ($1p1h$ + phonons).

Figure 8 shows as an example the results of calculations of the photoproton spectrum in the $1p1h$ approximation and the spectrum of pre-equilibrium emission of photoprotons with allowance for $2p2h$ configurations and the evaporation stage ($\bar{n}$) for the ^{90}Zr nucleus.

Thus, the actual numerical calculations made for the giant dipole resonance $E1$ ($T_<$ and $T_>$) and the comparison of them with experiment confirm the validity of the generalized GMR model, which makes it possible to explain in a unified microscopic approach not only the intermediate structure of the $E1$ resonance but also the energy spectra of photonucleon emission.

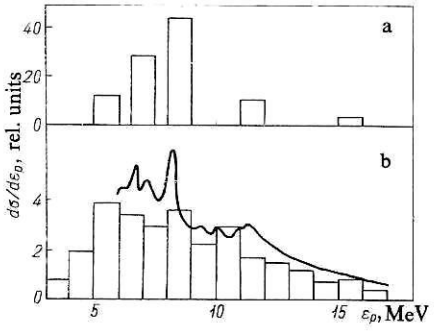


FIG. 8. Spectrum of photoprotons from ^{90}Zr: a) $1p1h$ approximation ($T_<$ and $T_>$); b) with allowance for $2p2h$ states ($T_<$ and $T_>$) and the evaporation stage ($\bar{n}$). The continuous curve represents the experiment.

μ^- and π^- Capture Reactions. The specific feature of these reactions is that the transition takes place predominantly with production of the daughter nucleus in, not the ground, but excited states, which are isospin analogs of the giant resonances of the target nucleus. The actual investigation of the emission spectra of the secondary particles for π^- capture[41] demonstrates the importance of taking into account the multistage emission process. It follows from the calculations that the main contribution is made by the equilibrium emission mechanism. When the nucleon emission for π^- capture is treated in the framework of the model of an intranuclear cascade, the spectra are described better in the high-energy region.[42]

The current application of the model of pre-equilibrium decay (in various modifications) to the description of a large class of nuclear reactions of the type (γ, N), (e, e'), and μ^- and π^- capture indicates the importance of the nonequilibrium approach, which makes it possible to obtain interesting quantitative information about the structure of the intermediate states and the multistage secondary-particle emission mechanism. At the same time, it should be noted that a consistent and quantitative description of the intermediate structure and decay of giant multipole resonances is possible only in the framework of the unified microscopic approach based on the quantum theory of pre-equilibrium nuclear reactions; this is particularly important in the analysis of experimental data with energy resolution $\Delta E < 100$ keV.

CONCLUSIONS

The microscopic models of pre-equilibrium processes considered in the present review on the basis of the unified theory of nuclear reactions (SMDP + SMCP formalism) make it possible to describe different reaction mechanisms for a wide range of energies, nuclei, and produced particles when the number of open channels and intermediate states is large. The practical possibility of calculations in the analysis of statistical multistage processes can serve as a criterion for the actual applicability of the SMDP + SMCP formalism for concrete calculations in, for example, the framework of the DWUCK program aimed at studying individual aspects of the physics of pre-equilibrium processes. Particularly attractive are the variants of the unified theory of nuclear reactions in the framework of which it is possible to find felicitous simplifications permitting quantitative description of varied experimental material. The most difficult and important problem that arises in the description of pre-equilibrium processes with the participation of composite particle clusters is the analysis of the mechanisms of formation of the composite particle in the intermediate states and in the exit channel, and also the correct choice of the densities of the bound and unbound states of the intermediate and final nucleus, the mean-square interaction matrix element coupling the different quasiparticle–phonon configurations, the optical potential that describes the relative motion of the fragments, etc. At the present time it is clearly necessary to improve further the computational formalism in order to achieve a more complete and systematic description of the mechanisms of pre-equilibrium nuclear reactions.

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