Problems of the theory of nuclear interactions in the relativistic approach

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A review is given of the main aspects of the relativistic approach in the theory of nuclear interactions and some of its applications. The approach is based on a scale-invariant relativization of the nuclear wave functions, and this leads to a number of predictions for observable quantities that are confirmed by analysis of the experimental data on the interaction of high-energy nuclei. The kinematics of deep inelastic lepton–hadron coincidence processes is considered, and the scaling properties of the structure functions (form factors) of such processes are predicted

I. INTRODUCTION

The experiments with beams of high-energy nuclei made since the beginning of the seventies at Dubna Berkeley, and a number of other laboratories have stimulated interest in the study of nuclei as relativistic composite systems and the processes of interaction between them (see, for example, Refs. 1–15 and the bibliography given there). So far, beams of nuclei with different atomic numbers (from the deuteron to iron) have been obtained in the range of energies 0.4–10 GeV/nucleon. We note that energies above 2 GeV/nucleon were achieved for the first time in the Dubna synchrophasotron. Experiments with colliding beams of deuterons and α particles with energies up to 16 GeV/nucleon were also made using the storage rings at CERN.[16] This range of energies corresponds to relativistic physics, and for the description of the collisions of the nuclei the nonrelativistic theory of nuclear reactions based on the Schrödinger formalism of quantum mechanics is no longer valid. At such energies, it is necessary to take into account relativistic effects, and a relativistic formalism must be used to achieve an adequate description of the interaction of high-energy nuclei. Accordingly, it is preferable to represent the experimental data in terms of variables that have a relativistic origin. A stimulus to the development of investigations in the field of relativistic nuclear physics was Baldin's prediction[17] of cumulative production of particles in nucleus–nucleus collisions.

In this review, we describe the main features of the relativistic approach in the theory of nuclear interactions at high energies, give some of its applications, and compare some theoretical calculations with experimental data. Aspects of the nonrelativistic theory of nuclear reactions are described in fair detail in, for example, Ref. 18.

In the majority of experiments with the participation of relativistic nuclei,[19–24] the single-particle inclusive spectra of secondary particles (pions, protons, light nuclei) were measured in a wide range of the kinematic variables and for different projectile nuclei and target nuclei. Originally, pion production in collisions of relativistic deuterons with a copper target in a kinematic region extending beyond the boundary of nucleon–nucleon kinematics was investigated.[19]

In subsequent experiments,[25] cumulative production of protons and light nuclei was studied. The energy of a secondary particle can be related to the cumulation number $N_{\min}$, the effective number of nucleons of the fragmenting nucleus that participate in the process. This variable is a complicated combination of the momentum and emission angle of the created particle and at high energies of the colliding nuclei has the form[4,7,8,26]

$$N_{\min}=\begin{cases}(E-p_z)/m - \text{ in the rest frame of the beam,}\\ (E+p_z)/(E^0+p_z^0) - \text{ in the rest frame of the target.}\end{cases}$$

Here, E and p_z are the energy and z component of the momentum of the cumulative particle, m is the proton mass, and E^0 and p_z^0 are the energy and momentum per nucleon of the incident nucleus. The variable $N_{\min}$ is analogous to the "light-front" variables that will be discussed below. Light-front variables for describing processes with the participation of high-energy nuclei were introduced for the first time in Ref. 27 in connection with relativization of nuclear wave functions.

A simple exponential dependence of the cross section for production of cumulative particles on the variable $N_{\min}$,

$$E\frac{d\sigma}{d\mathbf{p}}\sim C\exp(-aN_{\min})$$

(the parameters a and C do not depend on the properties of the target in the beam fragmentation region), describes satisfactorily not only the momentum but also the angular distributions. The region of cumulative production corresponds to $N_{\min}>1$. This means that the production occurs as a result of interaction with a group of nucleons of the fragmenting nucleus.

Since the production of cumulative particles must take place at short internucleon distances, the quark degrees of freedom in the nuclei must be taken into account. The need to take into account the quark structure of nuclei is also indicated by experiments on the elastic scattering of electrons by light nuclei,[28] which yielded indications of a change from exponential decrease of the form factors at small q^2 to a power-law decrease at large momentum transfers, in agreement with the theoretical predictions of the quark counting rules.[29] The quark counting rules were introduced for the first time by Matveev, Muradyan, and Tavkhelidze[30] and Brodsky and Farrar.[31] They successfully explain the power-law decrease of the cross sections of hadron scattering with large momentum transfer. In Ref. 32, Matveev and Sorba used the quark bag model to estimate the contribution of six-quark states to the deuteron wave function, and the probabil-

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ity of a tunneling transition of the deuteron from the two-nucleon to the six-quark state was found to be 7%. A number of studies have been made of the contribution of quark degrees of freedom in nuclei (see, for example, Ref. 33 and the bibliography given there).

To describe the experimental characteristics of processes with the participation of relativistic nuclei, various theoretical models have been developed, but as yet there does not exist a theoretical scheme which satisfactorily describes all the experimental data. In experiments with beams of high-energy nuclei, it is not uncommon to encounter situations in which short internucleon distances must be taken into account, this corresponding to large relative momenta of the constituent particles of the nucleus. The problem arises of giving a satisfactory description of nuclei for arbitrary momenta of the nucleons in them. Since high-energy nuclei are relativistic composite systems, to describe processes in which they participate it is more convenient to use wave functions that take into account the relativistic nature of the motion of the nucleons within a nucleus in place of the ordinary quantum-mechanical nuclear wave functions, which correspond to motion of the nucleons with small internal momenta. It is necessary to use the formalism of quantum field theory,[34] in the framework of which a composite system is described by means of a Fock column[8,26] defined at a certain instant of time.

One of the methods of describing relativistic bound states is to use various equations of quantum field theory (Bethe-Salpeter equation,[35] quasipotential equations[36]) for two- and many-particle relativistic composite systems. But the construction of the kernel of the corresponding integral equation for the relativistic wave function is a complicated problem, and it is not realistic to hope for a solution of this problem in the near future. Therefore, a heuristic way of introducing relativistic nuclear wave functions appears to be the way forward. One can construct relativistic wave functions that reproduce the experimental behavior well and have the correct nonrelativistic limit, i.e., that go over into the well-known nuclear wave functions in that limit.

In analyzing the consequences of theoretical models and comparing them with experiments, it is important to identify a class of experimentally observable phenomena in which various features characteristic of the relativistic treatment of bound states are presumed to be manifested. Among the relatively distinguished types of processes with relativistic incident nuclei are the processes with spectator fragments as the outcome. Processes of such kind were considered by the present authors in the framework of a many-particle[37] relativistic formalism in light-front variables[38]; when applied to processes with the participation of high-energy nuclei, this formalism gives a number of scale-invariant types of behavior for observable quantities. We mention that the possibility of constructing a relativistic dynamics with commutation relations on a light-front hypersurface was already pointed out by Dirac[39] in 1949.

In this approach, a relativistic nucleus of A particle with total 4-momentum P_A can be described by means of a relativistic wave function $\Phi_{P_A}^{(A)}([x_i^{(A)}, \mathbf{p}_{i,\perp}])$, in which the "longitudinal motion" of the constituents within the relativistic nuclei is parametrized by means of scale-invariant variables:

$$x_i^{(A)} = (p_{i,0} + p_{i,z})/(P_{A,0} + P_{A,z}), \tag{1}$$

where $p_{i,\mu}$ ($\mu = 0,1,2,3$ is the Lorentz index) and $P_{A,\mu}$ are the individual 4-momentum of particle i within the composite system and the total 4-momentum of the system, respectively. The quantities $x_i^{(A)}$ are ratios of the light-front variables. In terms of these variables, the wave function of the composite system reflects, in particular, the dependence of the internal motion of the constituents on the total momentum of the relativistic composite system. The "inequivalence" of the longitudinal and transverse degrees of freedom characteristic of high-energy physics[40] finds here a natural reflection.

Assuming that the nucleons are fairly good "quasiparticles" for describing the properties of nuclei, we can ascribe an index i to the individual nucleons in the nucleus. Here and in what follows, upper-case letters will denote the characteristics (momenta, masses, etc.) of the composite systems, and lower-case letters the characteristics of individual nucleons. Square brackets in the argument of the wave function $\Phi_{P_A}^{(A)}$ identify the set of the corresponding variables $x_i^{(A)}$ and $\mathbf{p}_{i,\perp}$, which satisfy the conditions

$$\sum_{i=1}^{A} x_i^{(A)} = 1; \quad 0 < x_i^{(A)} < 1; \quad \sum_{i=1}^{A} \mathbf{p}_{i,\perp} = \mathbf{P}_{A,\perp} \tag{2}$$

The superscript of the variable $x_i^{(A)}$ means that this variable is defined in a system of particles whose number is equal to this index.

The formalism can be directly generalized to take into account quark degrees of freedom in nuclei.

1. PROCESSES WITH THE PARTICIPATION OF HIGH-ENERGY NUCLEI AND THE PROBLEM OF RELATIVIZATION OF NUCLEAR WAVE FUNCTIONS

Scale-invariant parametrization of the relativistic wave function of the deuteron

We begin by considering the simplest case of a two-nucleon nucleus, i.e., the deuteron. To construct a relativistic wave function of the deuteron, we use the fact that in the framework of the relativistic formalism in light-front variables[38] the Lorentz-invariant combination $(\mathbf{p}_\perp^2 + m^2)/x(1-x)$ [where $\mathbf{p}_\perp$ is the relative transverse momentum of the constituents, $x = 1/2 + (p_0 + p_z)/(P_0 + P_z)$, p_μ and P_μ are the relative 4-momentum of the internal motion of the constituents and the total 4-momentum of the motion of the composite system as a whole, respectively, and m is the mass of a constituent] plays a part analogous to that of the rotationally invariant (in three-dimensional space) square $\mathbf{p}^2$ of the three-dimensional relative momentum in nonrelativistic theory. The relativistic wave functions can be obtained from the corresponding nonrelativistic expressions by the substitution

$$\mathbf{p}^2 \to \frac{\mathbf{p}_\perp^2 + m^2}{x(1-x)}$$

accompanied by the natural replacement of the nonrelativistic numerical parameters by the relativistic ones.

In this way one can obtain, for example, the relativistic analog of the Hulthén wave function[27]:

$$\Phi_R(x, \mathbf{p}_\perp) = C_R\left[\frac{\mathbf{p}_\perp^2+m^2}{x(1-x)} - \alpha_R\right]^{-1}\left[\frac{\mathbf{p}_\perp^2+m^2}{x(1-x)} - \beta_R\right]^{-1}, \quad (3)$$

which is written down in an arbitrary coordinate system for arbitrary momenta of the deuteron as a whole and arbitrary internal momenta of its constituent nucleons. The finding of relativistic analogs of more accurate deuteron wave functions does not lead to fundamental difficulties. In particular, in what follows we shall use the relativistic analog of the Gartenhaus–Moravcsik function. We shall not describe here other possible ways of relativizing the wave functions. Some of them can be found in Refs. 41–47.

In the rest frame of the deuteron the wave function (3) for momenta of the internal motion of the deuteron satisfying the condition $|\mathbf{p}|/m \ll 1$ goes over into Hulthén's well-known nonrelativistic wave function

$$\Phi_{NR}(\mathbf{p}) = C_{NR}(\mathbf{p}^2 + \alpha_{NR}^2)^{-1}(\mathbf{p}^2 + \beta_{NR}^2)^{-1}. \quad (4)$$

In (4) and (3), α_R, β_R and α_{NR}, β_{NR} are variable parameters of the relativistic and nonrelativistic wave functions, respectively, and C_R and C_{NR} are normalization coefficients.

Going to the nonrelativistic limit in the expression (3) and ignoring the deuteron binding energy in the numerator of the expression for the variable x, we obtain the following connection between the parameters of the wave functions Φ_R and Φ_{NR}:

$$\alpha_R = \frac{m_d}{m}(2m^2 - \alpha_{NR}^2), \quad \beta_R = \frac{m_d}{m}(2m^2 - \beta_{NR}^2). \quad (5a)$$

Here, m_d is the deuteron mass.

If we completely ignore the binding energy in the expression for x, then from the condition that the wave function Φ_R go over into Φ_{NR} we obtain the following connection between the parameters:

$$\alpha_R = 4(m^2 - \alpha_{NR}^2), \quad \beta_R = 4(m^2 - \beta_{NR}^2). \quad (5b)$$

Normalizing the wave function (4) by the condition

$$\int d\mathbf{p}\, |\Phi_{NR}(\mathbf{p})|^2 = 1, \quad (6)$$

we obtain the following expression for the normalization coefficient:

$$C_{NR} = \frac{1}{\pi}(\alpha_{NR} + \beta_{NR})^{3/2}\alpha_{NR}^{1/2}\beta_{NR}^{1/2}. \quad (7)$$

For the normalization of the wave function (3), it is in general necessary to know the form of all interactions within the two-particle composite system. Assuming, however, that the total quasipotential of the interaction does not depend on the 4-momentum of the deuteron as a whole, we obtain the normalization condition[1)]

$$\int_0^1 \frac{dx}{x(1-x)} \int d\mathbf{p}_\perp |\Phi_R(x, \mathbf{p}_\perp)|^2 = 8\pi. \quad (8)$$

Substituting here the wave function (3), we obtain for the normalization coefficient C_R the expression[2)]

$$C_R = 2^{3/2}(\alpha_R - \beta_R)\,[f(\alpha_R, \beta_R) + f(\beta_R, \alpha_R)]^{-1/2}, \quad (9)$$

where

$$f(\alpha_R, \beta_R) = \frac{4[m^2(\alpha_R - \beta_R) + \alpha_R(4m^2 - \alpha_R)]}{(\alpha_R - \beta_R)\alpha_R^{3/2}(4m^2 - \alpha_R)^{1/2}} \operatorname{arctg}\left(\frac{\alpha_R}{4m^2 - \alpha_R}\right)^{1/2} - \frac{1}{\alpha_R}.$$

We also give the connection in the nonrelativistic limit between the wave functions Φ_R and Φ_{NR} that is obtained using (5a) and (5b):

$$\Phi_R(x, \mathbf{p}_\perp) \to 2^{7/4}\pi^{1/2}m^{1/2}\Phi_{NR}(\mathbf{p});$$

$$\Phi_R(x, \mathbf{p}_\perp) \to 2\pi^{1/2}m^{1/2}\Phi_{NR}(\mathbf{p}).$$

Here, the relativistic wave function Φ_R is normalized by the condition (8), and the nonrelativistic wave function Φ_{NR} is normalized by the condition (6).

Breakup of the relativistic deuteron

For the study of the dynamics of the relativistic deuteron, experiments with hydrogen targets are the most convenient, since in this case there are no effects associated with disintegration of the target, and the selection of the spectator nucleons makes it possible to obtain direct information about the deuteron wave function.

In Ref. 49, Glagolev *et al.* noted the convenience of studying the dp reaction in the kinematic scheme in which the deuteron is incident and the proton is at rest, since in this case the products of the deuteron fragmentation have velocities close to that of the incident nucleus and they can be clearly seen in a bubble chamber.

We consider the process $d + p \to p_{SP}\ (n_{SP}) + X_N$ in which a relativistic deuteron interacts with a hydrogen target and a spectator nucleon and the system X_N of hadrons are produced. One of the simplest processes of this type is direct deuteron disintegration, $dp \to ppn$.

Assuming that the spectator nucleon does not interact with the target, we can obtain for it the invariant distribution[27,50]

$$E_{SP}\frac{d\sigma}{d\mathbf{p}_{SP}} \sim \frac{\lambda^{1/2}(s_{NN}, m^2, m^2)}{\lambda^{1/2}(s, m^2, m_d^2)}\sigma_{\text{in}}(s_{NN})\left|\frac{\Phi_R(x, \mathbf{p}_\perp)}{1-x}\right|^2. \quad (10)$$

Here, s is the ordinary Mandelstam variable for the deuteron–proton system, and s_{NN} is the analogous variable for the subsystem consisting of the interacting nucleon of the deuteron and the target proton. Energy–momentum conservation leads to the following relation between these variables:

$$s_{NN} = s(1 - X_{SP}) + m^2 - \frac{\mathbf{p}_{SP,\perp}^2 + m^2}{X_{SP}},$$

where $\sigma_{\text{in}}(s_{NN})$ is the integrated inelastic cross section of the nucleon–nucleon interaction in the given channel $NN \to X_N$, m_d is the deuteron mass, and m is the nucleon mass. The flux factor is defined as $\lambda(x, y, z) = (x - y - z)^2 - 4yz$. The variable X_{SP} is defined as follows:

$$X_{SP} = \frac{(E_{SP} + p_{SP,z})}{(E_d + P_d) + (E_p + p_p)}. \quad (11)$$

Here, E_{SP}, E_d, E_p, and $p_{SP,z}$, $P_{d,z}$, $p_{p,z}$ are the energies and z components of the momenta of the spectator nucleon and the colliding deuteron and proton, respectively. Note that the variable X_{SP} is scale-invariant and Lorentz-invariant un-

[1)]General questions relating to the normalization of three-dimensional relativistic wave functions were considered by Faustov and Khelashvili.[48]

[2)]*Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic trigonometric functions, etc., is retained here and throughout the article in the displayed equations.

der transformations of the frame of reference along the collision axis (the z axis).

The arguments of the wave function $\Phi_R(x,\mathbf{p}_\perp)$ are related to the arguments X_{SP} and $\mathbf{p}_{SP,\perp}$ by

$$x=1-\left(1+\frac{E_p+p_{p,z}}{E_d+P_{d,z}}\right)X_{SP};$$

$\mathbf{p}_\perp = -\mathbf{p}_{SP,\perp}$ in the frame in which $\mathbf{P}_{d,\perp}=0$. It follows from these expressions that by observing experimentally the distributions of the spectator nucleon we can obtain information about the internal motion of the nucleons within the relativistic deuteron.

In the high-energy limit, the distribution of the spectator, summed over all possible hadron systems X_N, takes the form

$$\frac{1}{\sigma_{tot}(\infty)}\frac{d\sigma}{d\mathbf{p}_{SP}/E_{SP}}\bigg|_{s\to\infty}\sim\frac{|\Phi_R(X_{SP},p_{SP,\perp})|^2}{1-X_{SP}}.$$

Note the analogy between this expression and the predictions for the inclusive distributions obtained in the framework of the hypothesis of limiting fragmentation,[51] the parton model,[52] and the self-similarity principle for strong interactions.[53] Deviations from self-similar behavior may occur because of a possible weak dependence of the wave-function parameters on the energy. In this connection, there is undoubted interest in the experimental study of processes with beams of deuterons of different energies. We shall return once more to the discussion of this question when comparing our calculations with experimental data.

A characteristic feature of the distribution (10) for the spectator nucleons in the frame in which the target proton is at rest is the prediction of a maximum at the point.

$$\widetilde{X}_{SP}=\frac{1}{2\,[1+m/(E_d+P_{d,z})]},\qquad(12)$$

this approaching its limiting value $\widetilde{X}_{SP}=\frac{1}{2}$ as the energy increases. It will be seen from what follows that the positions of the maxima of the experimental X_{SP} distributions agree with the values $\widetilde{X}_{SP}$ predicted by Eq. (12).

To compare the relativistic parametrization (3) with the nonrelativistic Hulthén wave function (4), we consider the momentum distributions of the spectator nucleons in the deuteron rest frame.[54] The momentum distribution of the spectators is related to the invariant differential cross section (10) as follows:

$$\frac{d\sigma}{dp_{SP}}=\frac{2\pi p_{SP}^2}{(p_{SP}^2+m^2)^{1/2}}\int_{-1}^{1}d\cos\theta_{SP}\left(E_{SP}\frac{d\sigma}{d\mathbf{p}_{SP}}\right).$$

In Fig. 1, the theoretical calculations with the relativistic wave function (3) and with the nonrelativistic Hulthén wave function (4) are compared with the experimental distribution of spectator neutrons in the deuteron rest frame in the direct deuteron-disintegration process $d+p\to p+p+n_{SP}$. The experimental data were obtained in the 1-m hydrogen bubble chamber of the Laboratory of High Energies at the JINR, Dubna, bombarded by 3.3-GeV/c deuterons,[55] and they have been converted to the antilaboratory frame, in which the momentum of the incident proton is 1.65 GeV/c. Experimentally, the spectator was chosen as the particle having the least momentum in the deuteron rest frame. The small admixture of other nucleons selected with the spectators evidently cannot significantly influence our considerations (for more detail, see Ref. 55). The parameters α_R and β_R of the relativistic deuteron wave function were calculated in accordance with (5a) using the values $\alpha_{NR}=0.0456$ GeV/c and $\beta_{NR}=0.26$ GeV/c (Ref. 56) of the parameters of the nonrelativistic Hulthén wave function. The expressions (5b) give very similar values for the parameters α_R and β_R. The cross section of elastic proton–proton scattering at these energies is constant and equal to 24 mb.[57] The continuous curve in Fig. 1 corresponds to the theoretical calculation with the relativistic wave function (3) and the parameters $\alpha_R=3.521$

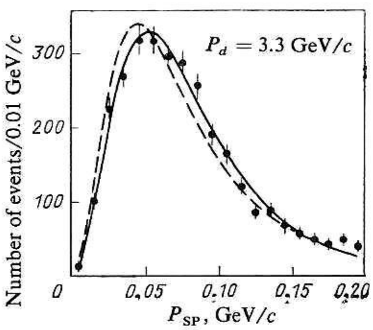

FIG. 1. Distribution with respect to the momentum of the spectator neutron in the deuteron rest frame in the reaction $dp\to ppn_{SP}$. The continuous and broken curves are the results of calculation with the relativistic and the nonrelativistic wave function, respectively.

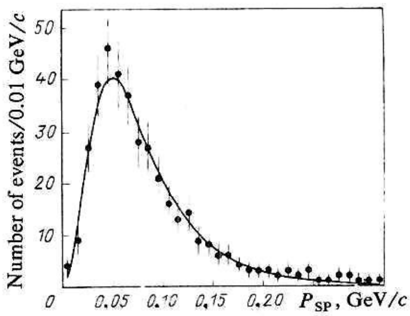

FIG. 2. Distribution with respect to the momentum of the spectator proton in the deuteron rest frame in the reaction $dp\to p(p\pi^-)p_{SP}$.

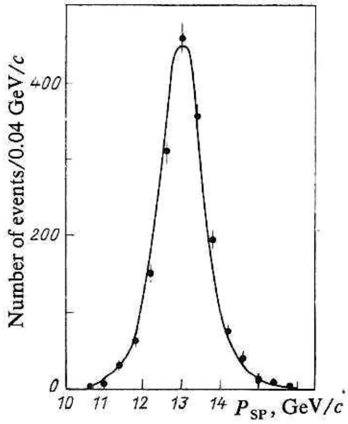

FIG. 3. Distribution with respect to the momentum of the spectator proton in the system of the colliding beams in the reaction $dp\to p(p\pi^-)p_{SP}$.

TABLE I. Parameters of the relativistic Hulthén wave function.

Reaction	α_R, $(\text{GeV}/c)^2$	β_R, $(\text{GeV}/c)^2$	χ^2/N_p
Calculation in accordance with (5)	3.521	3.390	—
$d+p \to p+(p\pi^-)+p_{SP}$ $\sqrt{s}=52$ GeV Deuteron rest frame	3,522±0.006	3,383±0.033	11/32
$d+p \to p+(p\pi^-)+p_{SP}$ $\sqrt{s}=52$ GeV Colliding-beam system	3.513±0.005	3,476±0.019	28/14
$d+\text{Al} \to n_{SP}+X$ $P_d=3{,}46$ GeV/c $P_d=4{,}46$ GeV/c $P_d=7{,}66$ GeV/c $P_d=10{,}2$ GeV/c	 3.5156, fixed 3.5156, fixed 3.5156, fixed 3.5156, fixed	 3.5240±0,0004 3.5222±0.0006 3,5203±0.0010 3,5170±0.0021	 1.9 1.2 2,9 1,1
$d+p \to p+p+n_{SP}$ $P_d=3.3$ GeV/c $d\sigma/dX_{SP}$ In three intervals of $p_{SP,\perp}$	3.5156, fixed	3,4572±0.0005	232/150
$d+p \to p+p+n_{SP}$ $P_d=3,3$ GeV/c $d\sigma/dp_{SP,\perp}$ In three intervals of X_{SP}	3.5156, fixed	3,4579±0,0005	311/150

$(\text{GeV}/c)^2$ and $\beta_R = 3.390$ $(\text{GeV}/c)^2$ ($\chi^2/N_p = 27/20$). The broken curve corresponds to the calculation with the nonrelativistic wave function (4) and the parameters $\alpha_{NR} = 0.0456\,\text{GeV}/c$ and $\beta_{NR} = 0.26\,\text{GeV}/c$ ($\chi^2/N_p = 65/20$). It can be seen that at momenta $P_{SP} < 0.2$ GeV/c the relativistic wave function gives a somewhat better description of the data than the nonrelativistic one.

To avoid cluttering the figures, in the remaining graphs we give only the curves obtained by means of the relativistic wave functions.

Figure 2 shows the momentum distribution of the spectator protons in the deuteron rest frame in the reaction $d + p \to p + (p\pi^-) + p_{SP}$ at energy $\sqrt{s} = 52$ GeV. The data were obtained in an experiment at the CERN ISR with colliding deuteron–proton beams.[16] Figure 3 shows the momentum distribution of the spectator nucleons in the frame of the colliding beams in the same reaction at the same energy. The curves in Figs. 2 and 3 correspond to the calculation with the wave function (3). The cross section $\sigma_{in}(s_{NN})$ of neutron diffraction dissociation $n + p \to p + (p\pi^-) + p$ at the considered energies was taken to be constant and equal to 185 μb.[16] The values of the parameters α_R and β_R obtained by fitting the experimental data and the corresponding values of χ^2/N_p are given in Table I. The agreement between the theoretical calculations and the experimental data confirms the validity of using the relativistic parametrization (3) of the wave function to describe the deuteron both at rest and when moving.

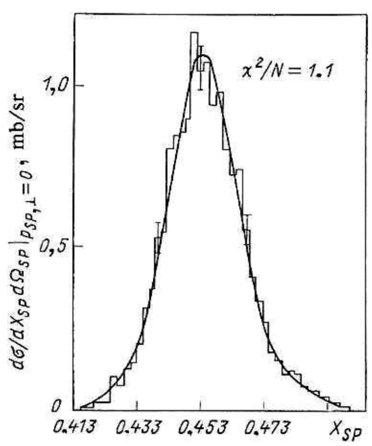

FIG. 4. The distribution of $d\sigma/dX_{SP}d\Omega_{SP}|_{p_{SP,\perp}=0}$ at momentum $P_d = 3.46$ GeV/c of the incident deuteron.

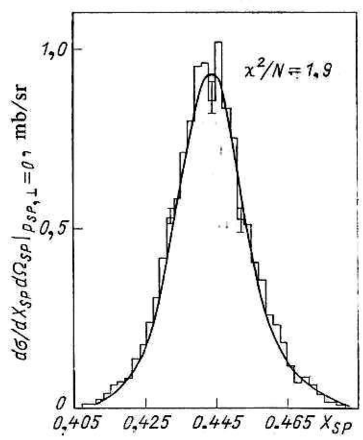

FIG. 5. The distribution of $d\sigma/dX_{SP}d\Omega_{SP}|_{p_{SP,\perp}=0}$ at momentum $P_d = 4.46$ GeV/c of the incident deuteron.

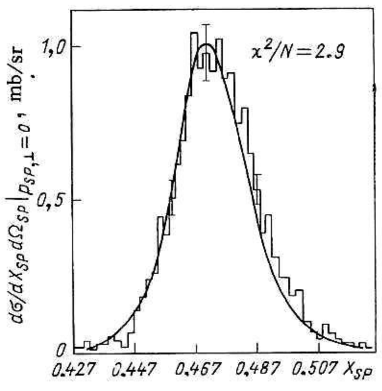

FIG. 6. The distribution of $d\sigma/dX_{SP}\,d\Omega_{SP}|_{p_{SP,\perp}=0}$ at momentum $P_d = 7.66$ GeV/c of the incident deuteron.

To verify the scaling properties inherent in the relativistic wave functions, a comparison was made of theoretical calculations with the experimental distributions of the spectator nucleons at other momenta of the incident deuteron as well.[58] The distribution of the spectator neutrons calculated using the wave function (3) was compared with the experimental distribution of neutrons from deuteron stripping.[59]

Figures 4–7 give the theoretical (continuous curves) and experimental distributions $(d\sigma/dX_{SP}\,d\Omega_{SP})|_{p_{SP,\perp}=0}$ for four energies of the incident deuteron; the distributions are normalized to unity at the points of the corresponding maxima. The relationship between the distribution given in the graph and the invariant distribution (10) at zero values of the transverse momentum of the spectator has the form[58]

$$\frac{d\sigma}{dX_{SP}\,d\Omega_{SP}}\Big|_{p_{SP,\perp}=0} = \frac{[(m+P_d+E_d)\,X_{SP}]^2 - m^2}{4\,(m+P_d+E_d)^2\,X_{SP}^3} \times \left(E_{SP}\frac{d\sigma}{d\mathbf{p}_{SP}}\right)\Big|_{SP,\perp=0}.$$

Analysis shows that in the considered region of momenta of the incident deuteron the parameters α_R and β_R depend weakly on the energy of the incident beam (Table I). This can be regarded as an indication that in a fairly wide range of energies the deuteron wave function Φ_R contains no other dependence on the energy apart from the dependence on the variable x, and it can be assumed that the relativistic deuteron is fairly well described by the wave function with

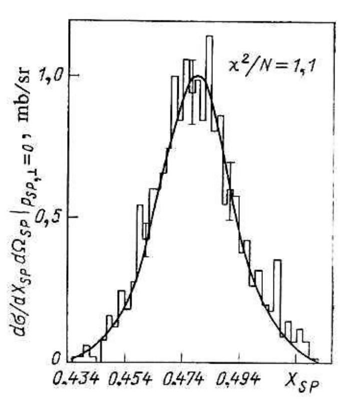

FIG. 7. The distribution of $d\sigma/dX_{SP}\,d\Omega_{SP}|_{p_{SP,\perp}=0}$ at momentum $P_d = 10.2$ GeV/c of the incident deuteron.

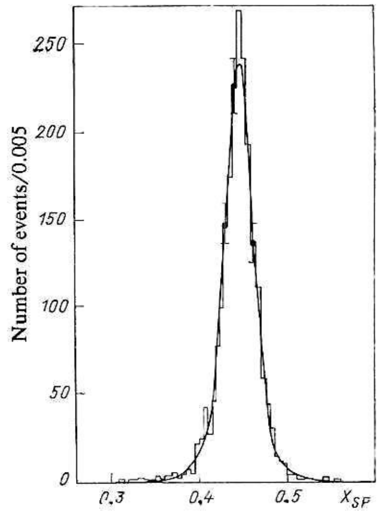

FIG. 8. The $d\sigma/dX_{SP}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.01 < p_{SP,\perp} < 0.04$ GeV/c.

the scale-invariant parametrization of the "longitudinal motion" in the light-front variables.

To study the distributions of the spectators with respect to the transverse momentum, data on the interaction of 3.3-GeV/c deuterons with a hydrogen chamber were used.[60] From the experimental point of view, there are fairly complete data for the direct disintegration channel $dp \to ppn$. In this case, $\sigma_{in}(s_{NN})$ in Eq. (10) must be replaced by the total elastic cross section $\sigma_{el}(s_{NN})$ of the nucleon–nucleon interaction. We restricted ourselves to the case of a neutron spectator, since there are for the cross section σ_{el}^{pp} far fewer experimental data than for σ_{el}^{pn} (see, for example, Ref. 57). In our case, s_{pp} varies in an interval in which σ_{el}^{pp} is effectively constant at 24 mb.

The experimental distributions $d\sigma/dX_{SP}$ and $d\sigma/dp_{SP,\perp}$ in the rest frame of the target proton were analyzed. They are related to the invariant differential cross section as follows:

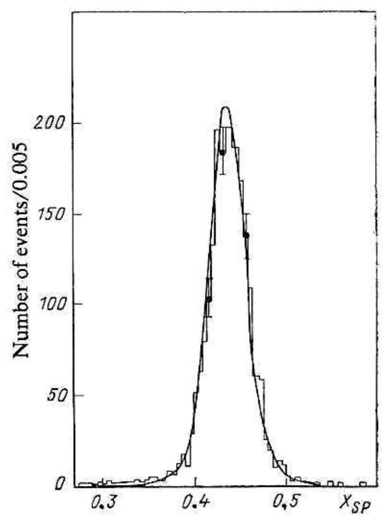

FIG. 9. The $d\sigma/dX_{SP}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.04 < p_{SP,\perp} < 0.07$ GeV/c.

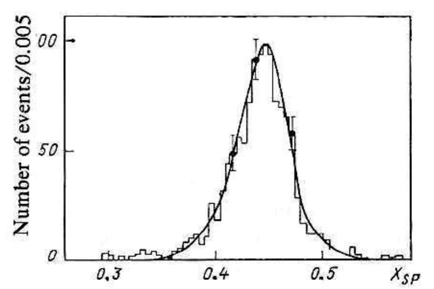

FIG. 10. The $d\sigma/dX_{SP}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.07 < p_{SP,\perp} < 0.1$ GeV/c.

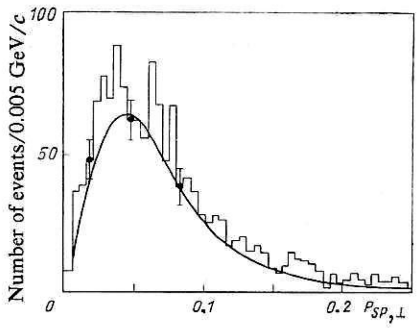

FIG. 12. The $d\sigma/dp_{SP,\perp}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.43 < X_{SP} < 0.46$.

$$\frac{d\sigma}{dX_{SP}} = \int_{p_{SP,\perp\,\min}}^{p_{SP,\perp\,\max}} \frac{d\sigma}{dX_{SP}\, dp_{SP,\perp}}\, dp_{SP,\perp}; \tag{13a}$$

$$\frac{d\sigma}{dp_{SP,\perp}} = \int_{X_{SP,\,\min}}^{X_{SP,\,\max}} \frac{d\sigma}{dX_{SP}\, dp_{SP,\perp}}\, dX_{SP}; \tag{13b}$$

$$\frac{d\sigma}{dX_{SP}\, dp_{SP,\perp}} = 2\pi \frac{p_{SP,\perp}}{X_{SP}} \left(E_{SP} \frac{d\sigma}{dp_{SP}} \right). \tag{13c}$$

The experimental distributions $d\sigma/dX_{SP}$ $(d\sigma/dp_{SP,\perp})$, integrated in three different intervals of $p_{SP,\perp}$ (X_{SP}), were compared with the results of the foregoing theoretical scheme.

The results of the analysis are given in Figs. 8–13. The theoretical curves in these figures correspond to the values of the parameters of the relativistic wave function (3) given in Table I. The values of χ^2/N_p (where N_p is the number of experimental points) indicate that the model agrees satisfactorily with the experimental data. In accordance with the prediction (12), the maximum in the X_{SP} distribution is at $\tilde{X}_{SP} \approx 0.44$.

More complicated nuclei

It is of undoubted interest to attempt to relativize the wave functions of more complicated nuclei, to compare the corresponding results with experimental data, and thus to verify the universality of the scale-invariant properties of the relativistic wave functions in the developed formalism.

In the framework of the impulse approximation we consider the process of knockout of a nucleon from a relativistic nucleus A in a collision with a hydrogen target. Assuming that the target interacts only with the nucleon that is knocked out and that the remaining A-1 nucleons of the initial nucleus continue to exist in the form of a fragment nucleus (we shall call it the spectator fragment), we can calculate the distributions of these fragments.

We shall describe the incident nucleus and the spectator nucleus by means of the wave functions of the many-particle relativistic formalism in light-front variables. We shall describe the nucleus of A nucleons with total 4-momentum P_A by means of a wave function $\Phi_{P_A}^{(A)}([x_i^{(A)}, \mathbf{p}_{i,\perp}])$, in which the "longitudinal motion" of the constituents is parametrized by means of scale-invariant variables $x_i^{(A)}$, which are defined as the ratio of the + components of the 4-momenta of nucleon i and the nucleus A [see Eq. (1)]. The arguments of the wave function $\Phi_{P_A}^{(A)}$ satisfy the conditions (2).

The distribution of the spectator fragments in the process of nucleon knockout from the nucleus in the laboratory frame (nucleus A is incident along the z axis and the target proton is at rest) has the form[61,62]

$$E_{SP} \frac{d\sigma}{d\mathbf{P}_{SP}} \sim \frac{\lambda^{1/2}(s_{NN}, m^2, m^2)}{\lambda^{1/2}(s, m^2, M_A^2)} \sigma_{NN}^{\rm el}(s_{NN}) \left| \frac{I(X_{SP}, \mathbf{P}_{SP,\perp})}{1-\alpha X_{SP}} \right|^2. \tag{14}$$

Here, s is the usual Mandelstam variable for the system con-

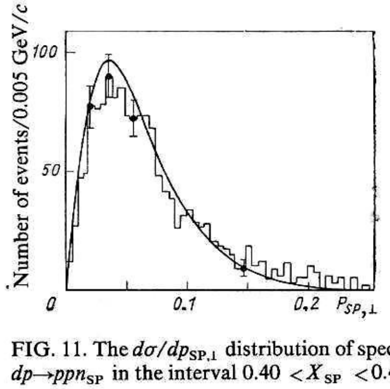

FIG. 11. The $d\sigma/dp_{SP,\perp}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.40 < X_{SP} < 0.43$.

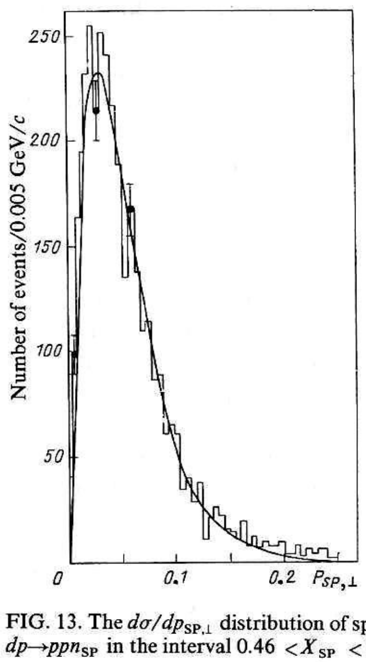

FIG. 13. The $d\sigma/dp_{SP,\perp}$ distribution of spectator neutrons in the reaction $dp \to ppn_{SP}$ in the interval $0.46 < X_{SP} < 0.49$.

sisting of the incident nucleus A and the target nucleon, s_{NN} is the analogous variable for the subsystem consisting of the nucleon that interacts and the target nucleon,

$$s_{NN}=s(1-X_{SP})+M_{SP}^2-\frac{\mathbf{P}_{SP,\perp}^2+M_{SP}^2}{X_{SP}};$$

$\sigma_{NN}^{el}(s_{NN})$ is the total elastic cross section for interaction of the knocked-out nucleon with the target, $\lambda(x,y,z)$ is, as before, the flux factor, m is the nucleon mass, M_A is the mass of the incident nucleus, M_{SP} is the mass of the spectator fragment, α is given by

$$\alpha=1+m/(E_A+P_{A,z}),$$

the variable X_{SP} by

$$X_{SP}=\frac{E_{SP}+P_{SP,z}}{m+E_A+P_{A,z}},\qquad(15)$$

and $P_{A,z}$, E_A and $P_{SP,z}$, E_{SP} are the z components of the momentum and the energy of the incident nucleus A and the spectator fragment $(A-1)$, respectively.

The overlap integral $I(X_{SP}, \mathbf{P}_{SP,z\perp})$ of the relativistic wave functions of the incident nucleus and the spectator fragment is given by

$$I(X_{SP},\mathbf{P}_{SP,\perp})=\int_0^1\prod_{i=1}^{A-1}\frac{dy_i^{(A-1)}}{y_i^{(A-1)}}\delta\left(1-\sum_{i=1}^{A-1}y_i^{(A-1)}\right)\times\int\prod_{i=1}^{A-1}d\mathbf{q}_{i,\perp}\delta^{(2)}\left(\mathbf{P}_{SP,\perp}-\sum_{i=1}^{A-1}\mathbf{q}_{i,\perp}\right)\times\Phi^{+(A-1)}_{f,\mathbf{P}_{SP,\perp}=0}([y_i^{(A-1)},\mathbf{q}_{i,\perp}-y_i^{(A-1)}\mathbf{P}_{SP,\perp}])\,\Phi_i^{(A)}([x_i^{(A)},\mathbf{p}_{i,\perp}]).\qquad(16)$$

The variables $x_i^{(A)}$ and $y_i^{(A-1)}$ are defined as follows:

$$x_i^{(A)}=\frac{p_{i,0}+p_{i,z}}{P_{A,0}+P_{A,z}},\quad 0<x_i^{(A)}<1,\quad\sum_{i=1}^{A}x_i^{(A)}=1;$$

$$y_i^{(A-1)}=\frac{q_{i,0}+q_{i,z}}{P_{SP,0}+P_{SP,z}},\quad 0<y_i^{(A-1)}<1,\quad\sum_{i=1}^{A-1}y_i^{(A-1)}=1.$$

The wave-function overlap integral has a direct analog in the nonrelativistic theory of nuclear reactions (see, for example, Ref. 18 and the bibliography given there).

The arguments of the wave function $\Phi_i^{(A)}$ of the incident nucleus are related to the variables of integration and the observable quantities X_{SP} and $P_{SP,\perp}$ as follows:

$$\left.\begin{aligned}&x_i^{(A)}=\alpha X_{SP}y_i^{(A-1)},\ \mathbf{p}_{i,\perp}=\mathbf{q}_{i,\perp},\ i=1,2,\ldots,A-1,\\&x_A^{(A)}=1-\alpha X_{SP},\ \mathbf{p}_{A,\perp}=-\mathbf{P}_{SP,\perp}.\end{aligned}\right\}\qquad(17)$$

Thus, observation of the spectator fragment makes it possible to obtain information about the nature of the "longitudinal" and transverse momentum distributions of the nucleons in the incident nucleus. A. Equation (16) contains the relativistic wave functions $\Phi_i^{(A)}$ and $\Phi_f^{(A-1)}$ of the initial and final nuclei in frames in which their total transverse momenta are zero. They are related to the wave functions with arbitrary total 4-momentum P by[37]

$$\Phi^{(A)}_{\mathbf{P}_\perp=0}([x_i^{(A)},\mathbf{p}_{i,\perp}-x_i^{(A)}\mathbf{P}_\perp])=\Phi_P^{(A)}([x_i^{(A)},\mathbf{p}_{i,\perp}]).\qquad(18)$$

As an example, we consider the following very simple parametrization of the relativistic wave functions of the initial nucleus and the target nucleus:

$$\Phi^{(A)}([x_i^{(A)},\mathbf{p}_{i,\perp}])=C_A\exp\left(-a_A^R\sum_{i=1}^{A}\frac{\mathbf{p}_{i,\perp}^2+m_i^2}{x_i^{(A)}}\right)\qquad(19)$$

and similarly for $\Phi^{(A-1)}$ with the substitution $A\to(A-1)$.

Such a parameterization appears reasonable for light nuclei ($A\leqslant4$). In Eq. (19), C_A is a normalization coefficient, and a_A^R is a numerical parameter that can be varied. If the scale-invariant parametrization put into the wave function $\Phi^{(A)}$ is valid, then when theory is compared with experiment the parameter a_A^R must be found to be about the same when data at different energies of the incident nucleus are approximated.

Since in our treatment we do not distinguish protons from neutrons, the wave function $\Phi^{(A)}([x_i^{(A)},\mathbf{p}_{i,\perp}])$ is a symmetric function of its arguments $x_i^{(A)}$ and $\mathbf{p}_{i,\perp}$. Solving the problem of a conditional extremum under the conditions (2), we find that the wave function (19) has a maximum at zero values of the transverse momenta of the constituent nucleons and at values of the variables $x_i^{(A)}$ equal to

$$x_i^{(A)}=\frac{m_i}{\sum_{i=1}^{A}m_i}=\frac{1}{A}.$$

Taking into account the connection between the variables $x_i^{(A)}$ and X_{SP} [see Eqs. (17)], we find that the distribution of the spectator fragments with respect to X_{SP} must have a maximum at

$$\widetilde{X}_{SP}=\frac{A-1}{A\left(1+\frac{m}{E_A+P_{A,z}}\right)}.\qquad(20)$$

We note that properties such as the scale invariance of the wave functions and the position of the maximum in the X_{SP} distribution of the spectator fragments remain valid for an arbitrary parametrization of the relativistic wave functions.

We normalize the relativistic wave functions as in the two-particle case under the assumption that the total quasipotential of the interaction does not depend on the total 4-momentum of the bound system. In this case,[62]

$$\int_0^1\prod_{i=1}^{A}\frac{dx_i^{(A)}}{x_i^{(A)}}\delta\left(1-\sum_{i=1}^{A}x_i^{(A)}\right)\int\prod_{i=1}^{A}d\mathbf{p}_{i,\perp}\delta^{(2)}\left(\mathbf{P}_{A,\perp}-\sum_{i=1}^{A}\mathbf{p}_{i,\perp}\right)\times|\Phi^{(A)}(x_i^{(A)},\mathbf{p}_{i,\perp})|^2=2(4\pi)^{A-1}.\qquad(21)$$

Substituting in the normalization condition (21) the wave function $\Phi^{(A)}([x_i^{(A)},\mathbf{p}_{i,\perp}])$ in the form (19), we obtain an approximate expression for the normalization coefficient C_A:

$$C_A\simeq2^{1/2}(4\pi)^{\frac{A-1}{2}}\left(\sum_{i=1}^{A}m_i\right)^{\frac{3(A-1)+1}{4}}\left(\prod_{i=1}^{A}m_i\right)^{-\frac{1}{4}}\left(\frac{2a_A^R}{\pi}\right)^{\frac{3(A-1)}{4}}\times\exp\left[a_A^R\left(\sum_{i=1}^{A}m_i\right)^2\right].\qquad(22)$$

One of the arguments used in choosing the relativistic wave functions is that in the nonrelativistic limit they should go over into the well-known nonrelativistic nuclear wave functions. The nonrelativistic wave function $\Phi_{NR}^{(A)}([\mathbf{p}_i])$, whose relativistic analog is the function (19) has Gaussian form:

$$\Phi_{NR}^{(A)}([\mathbf{p}_i]) = \left(\frac{4a_A^{NR}}{\pi}\right)^{\frac{3(A-1)}{4}} \exp\left(-a_A^{NR}\sum_{i=1}^{A}\mathbf{p}_i^2\right) \tag{23}$$

and is normalized by the condition

$$\int \prod_{i=1}^{A} d\mathbf{p}_i\, \delta^{(3)}\left(\sum_{i=1}^{A}\mathbf{p}_i\right) |\Phi_{NR}^{(A)}([\mathbf{p}_i])|^2 = 1. \tag{24}$$

From the condition of agreement in the nonrelativistic limit between the wave function (19) and the nonrelativistic wave function (23) we obtain the connection between the parameters of the relativistic and nonrelativistic functions:

$$a_A^R = \frac{m_i}{\sum_{i=1}^{A} m_i} a_A^{NR} = \frac{1}{A} a_A^{NR}. \tag{25}$$

For the normalized wave functions, the nonrelativistic limit is

$$\Phi^{(A)}([x_i^{(A)}, \mathbf{p}_{i,\perp}]) \to 2^{1/2}(2\pi^2 m^2)^{\frac{A-1}{4}} A^{1/4}\Phi_{NR}^{(A)}([\mathbf{p}_i]).$$

Substituting now the relativistic wave functions $\Phi^{(A)}$ and $\Phi^{(A-1)}$ in the expression (16) for the overlap integral, taking into account (18), and integrating over the transverse momenta, we obtain

$$I(X_{SP}, \mathbf{P}_{SP,\perp}) = C_A C_{A-1}\left(\frac{\pi}{a_{A-1}^R + \frac{a_A^R}{\alpha X_{SP}}}\right)^{A-2} \exp\left(-\frac{a_A^R m_A^2}{1-\alpha X_{SP}}\right)$$
$$\times \exp\left[-\frac{a_A^R \mathbf{P}_{SP,\perp}^2}{\alpha X_{SP}(1-\alpha X_{SP})}\right] J(X_{SP}), \tag{26}$$

where

$$J(X_{SP}) = \int_0^1 \prod_{i=1}^{A-1} dx_i^{(A-1)} \delta\left(1 - \sum_{i=1}^{A-1} x_i^{(A-1)}\right)$$
$$\times \exp\left[-\left(a_{A-1}^R + \frac{a_A^R}{\alpha X_{SP}}\right)\left(\sum_{i=1}^{A-1}\frac{m_i^2}{x_i^{(A-1)}}\right)\right]. \tag{27}$$

The integral over the variable $x_i^{(A-1)}$ can be calculated approximately by means of the multidimensional method of steepest descent[63] in the form of an asymptotic expansion in inverse powers of the large parameter $(a_{A-1}^R + a_A^R X_{SP})$. The leading term of the expansion has the form

$$J_0(X_{SP}) = \left(\prod_{i=1}^{A-1} m_i\right)^{1/2}\left(\sum_{i=1}^{A-1} m_i\right)^{-\frac{3(A-2)+1}{2}}$$
$$\times\left(\frac{\pi}{a_{A-1}^R + \frac{a_A^R}{\alpha X_{SP}}}\right)^{\frac{A-2}{2}} \exp\left[-\left(a_{A-1}^R + \frac{a_A^R}{\alpha X_{SP}}\right)\left(\sum_{i=1}^{A-1} m_i\right)^2\right]. \tag{28}$$

Substituting in (26) the expressions for the normalization coefficients C_A and C_{A-1} from (22) and the expression for the integral $J_0(X_{SP})$, we obtain finally for the overlap integral

$$I(X_{SP}, \mathbf{P}_{SP,\perp}) = 2(4\pi)^{\frac{2A-3}{2}}\left(\frac{2a_A^R}{\pi}\right)^{\frac{3(A-1)}{4}}\left(\frac{2a_{A-1}^R}{\pi}\right)^{\frac{3(A-2)}{4}}$$
$$\times\left(\sum_{i=1}^{A} m_i\right)^{\frac{3(A-1)-1}{4}}\left(\sum_{i=1}^{A-1} m_i\right)^{-\frac{3(A-2)+1}{4}}$$
$$\times\left(\prod_{i=1}^{A} m_i\right)^{-\frac{1}{4}}\left(\prod_{i=1}^{A-1} m_i\right)^{\frac{1}{4}}$$
$$\times\left(\frac{\pi}{a_{A-1}^R + \frac{a_A^R}{\alpha X_{SP}}}\right)^{\frac{3(A-2)}{2}} \exp\left[-\frac{a_A^R \mathbf{P}_{SP,\perp}^2}{\alpha X_{SP}(1-\alpha X_{SP})}\right]$$
$$\times \exp\left\{-a_A^R \frac{\left[\left(\sum_{i=1}^{A} m_i\right)\alpha X_{SP} - \left(\sum_{i=1}^{A-1} m_i\right)\right]^2}{\alpha X_{SP}(1-\alpha X_{SP})}\right\}. \tag{29}$$

It can be seen from the expression (29) that the distribution with respect to the transverse momentum $\mathbf{P}_{SP,\perp}$ of the spectator fragment must have a Gaussian form, and the X_{SP} distribution will have a maximum at the point predicted by Eq. (20).

The expression (14) for the differential cross section with the overlap integral (29) can now be used for a comparison with experimental data.

To verify the scaling properties of the relativistic wave functions and extract information about the values of their parameters, we used experimental data on ^{4}Hep interactions with an incident ^{4}He nucleus having momenta 8.56 GeV/c and 13.5 GeV/c and ^{3}Hep interactions at momentum 13.5 GeV/c of the ^{3}He nucleus; these data were obtained using the 1-m hydrogen bubble chamber of the Laboratory of High Energies at Dubna.[64,65] We also used data on the momentum distribution of the ^{3}He nuclei emitted at angle 0.65° in the ^{4}He$p\to{}^2$HeX reaction with an incident ^{4}He nucleus having momentum 6.85 GeV/c.[66]

The experimental $d\sigma/dX_{SP}$ and $d\sigma/dP_{SP,\perp}$ distributions of the ^{3}H and ^{3}He spectator fragments in the ^{4}He$p\to{}^3$Hpp and He$p\to{}^3$Hepn reactions were analyzed in the rest frame of the target proton. (The spectators were assumed to be the fragments having the smallest momentum among the reaction products in the ^{4}He rest frame.)

These distributions are related to the invariant differential cross section by equations of the type (13), in which $E_{SP}\, d\sigma/d\mathbf{P}_{SP}$ is given by the expression (14) with the overlap integral (29) with $A = 4$.

The limits of integration $X_{SP,\min}$ and $X_{SP,\max}$ were taken from the corresponding experimental distributions of the spectator fragments with respect to the variable X_{SP}. The limit $P_{SP,\perp\max}$ is the kinematic limit determined by the condition of positivity of the factor $\lambda(s_{NN}, m^2, m^2)$ in Eq. (14). It has the form

$$(P_{SP,\perp\text{макс}})^2 = (sX_{SP} - M_{SP}^2)(1 - X_{SP}) - 4m^2 X_{SP}. \tag{30}$$

The total elastic NN cross section in Eq. (14) can be assumed to be effectively constant in the considered range of energies and approximately equal to 24 mb.[57]

The overlap integral (29) depends weakly on the values of the parameter of the spectator nucleus. Therefore, when the parameters of the relativistic wave functions of the final

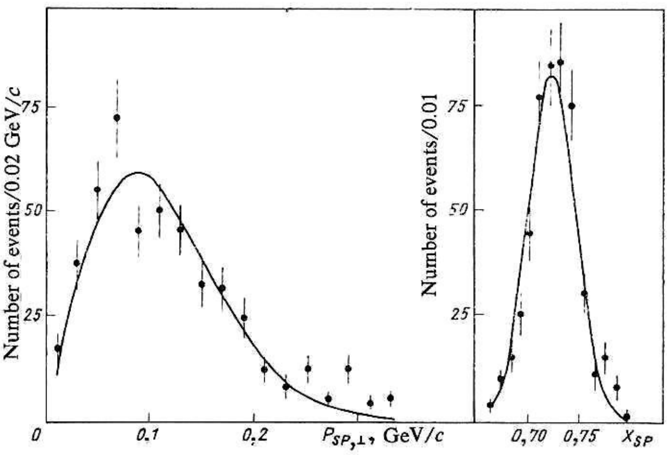

FIG. 14. The $P_{SP,\perp}$ and X_{SP} distributions for the spectator fragment ^3H in the reaction $^4He p \to {}^3H pp$ at ^4He momentum 8.56 GeV/c.

nuclei ^3H and ^3He are determined by fitting the data, there are very large errors. For this reason, the fitting was done for a fixed value of the parameter of the wave function of the three-nucleon nucleus. The value $a_3^R = 8$ (GeV/c)$^{-2}$ obtained by means of Eq. (25) from the value $a_3^{NR} \approx 24$ (GeV/c)$^{-2}$ of the parameter of the nonrelativistic wave function (23) was chosen. It should be mentioned that the parameters of even the nonrelativistic Gaussian wave functions of the nuclei ^4He, ^3He, and ^3H are determined insufficiently well, and we do not have reliable information about them. The values of the parameters a_4^{NR} and a_3^{NR} given in the literature vary in a fairly wide range (see, for example, Ref. 67). The greatest number of data are available for the parameters of the nonrelativistic ^4He wave function, but the values of the parameter a_4^{NR} of the Gaussian parametrization (23) vary in the range $a_4^{NR} = 20$–28 (GeV/c)$^{-2}$.

Figure 14 shows the experimental and theoretical $P_{SP,\perp}$ and X_{SP} distributions of the spectator nucleus ^3H in the $^4He p \to {}^3H pp$ reaction at momentum 8.56 GeV/c of the incident 4He nucleus. Figure 15 shows the same distributions for the spectator fragment ^3He in the $^4He p \to {}^3He pn$ reaction at the same momentum. The theoretical curves in Figs. 14 and 15 correspond to the parameter values of the relativistic nuclear wave functions given in Table II.

The position of the maximum in the X_{SP} distributions of the spectator fragments, $\widetilde{X}_{SP} = 0.715$, agrees well with the

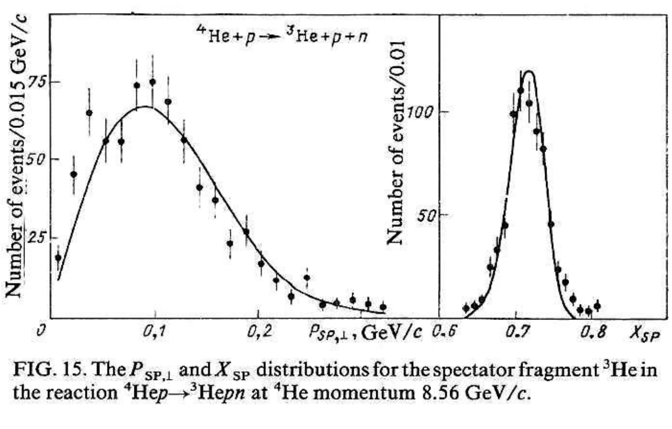

FIG. 15. The $P_{SP,\perp}$ and X_{SP} distributions for the spectator fragment ^3He in the reaction $^4He p \to {}^3He pn$ at ^4He momentum 8.56 GeV/c.

value of $\widetilde{X}_{SP}$ predicted by Eq. (20).

Figure 16 gives the spectrum of the ^3He nuclei emitted at angle 0.65° in the laboratory system in the $^4He p \to {}^3He X$ reaction at momentum 6.85 GeV/c of the incident ^4He nucleus, and Fig. 17 shows the $P_{SP,\perp}$ and X_{SP} distributions of the spectator ^3H fragment in the $^4He p \to 3H pp$ reaction at momentum 13.5 GeV/c of the incident nucleus. The curves in Figs. 16 and 17 correspond to the parameter values obtained by fitting and given in Table II.

It can be seen from Table II that the values of the parameter of the relativistic ^4He wave function obtained by fitting the experimental data at different momenta of the incident nucleus are close to each other and agree satisfactorily with the values $a_4^R \approx 5$–7 (GeV/c)$^{-2}$ predicted by Eq. (25). The approximate energy-independence of the values of the parameters of the relativistic wave function in the considered region of energies of the incident ^4He nucleus suggests that in the wave function (19) there is no other energy dependence apart from the dependence on the scale-invariant variables $x_i^{(A)}$.

To determine from experiments the parameter a_3^R of the relativistic wave function of the three-nucleon nucleus, we consider the $^3He p \to dpp$ reaction. In this case, the overlap integral has the form[68]

$$I(X_{SP}, \mathbf{P}_{SP,\perp}) = 4\pi C_d 3^{7/4} m (a_3^R)^{3/2} \exp\left[-\frac{a_3^R \mathbf{P}^2_{SP,\perp}}{\alpha X_{SP}(1-\alpha X_{SP})}\right]$$
$$\times \exp\left[-a_3^R \frac{m^2(3\alpha X_{SP}-2)^2}{\alpha X_{SP}(1-\alpha X_{SP})}\right]$$
$$\times \sum_{i=1}^{5} A_i^R \exp(-4m^2\alpha_i^R)\left(\alpha_i^R+\frac{a_3^R}{\alpha X_{SP}}\right)^{-3/2}. \qquad (31)$$

Here, C_d is a normalization coefficient, and A_i^R and α_i^R are the parameters of the relativistic deuteron wave function which we chose as the relativistic analog of the Gartenhaus–Moravcsik wave function:

$$\Phi_d(x, \mathbf{p}_\perp) = C_d \sum_{i=1}^{5} A_i^R \exp\left[-\alpha_i^R \frac{\mathbf{p}_\perp^2+m^2}{x(1-x)}\right]. \qquad (32)$$

This wave function is normalized by the condition (8), and the normalization coefficient is

$$C_d = 2^{5/2}\pi^{-1/4} m^{1/2}\left\{\sum_{i=1}^{5}\sum_{j=1}^{5}\frac{A_i^R A_j^R}{(\alpha_i^R+\alpha_j^R)^{3/2}} \times \exp\left[-4m^2(\alpha_i^R+\alpha_j^R)\right]\right\}^{-1/2}.$$

The parameters A_i^R and α_i^R of the relativistic wave function (32) are related to the parameters of the nonrelativistic wave function

$$\Phi_d^{NR}(\mathbf{p}) = C_d^{NR}\sum_{i=1}^{5} A_i^{NR}\exp(-\alpha_i^{NR}\mathbf{p}^2) \qquad (33)$$

by

$$A_i^R = A_i^{NR}\exp(m^2\alpha_i^{NR}),\quad \alpha_i^R = \frac{1}{4}\alpha_i^{NR}. \qquad (34)$$

In the comparison of the theoretical calculations with the experimental data on the distributions of the spectator deuterons in the $^3He p \to dpp$ reaction, the parameters of the deuteron wave function (32) were fixed by the values obtained in accordance with the expressions (34) from the val-

TABLE II. Parameters of relativistic wave functions of three- and four-nucleon nuclei.

Reaction	a_3^R, $(\mathrm{GeV}/c)^{-2}$	a_4^R, $(\mathrm{GeV}/c)^{-2}$	χ^2/N_p	
$^4\mathrm{He}\, p \to {}^3\mathrm{H}\, pp$ $P_{^4\mathrm{He}} = 8{,}56$ GeV/c	8, fixed	7,39±0,26	$d\sigma/dX_{SP}$ 31,12/18	$d\sigma/dP_{SP,\perp}$ 27,65/22
$^4\mathrm{He}\, p \to {}^3\mathrm{He}\, pn$ $P_{^4\mathrm{He}} = 8{,}56$ GeV/c	8, fixed	5,86±0,21	$d\sigma/dX_{SP}$ 54,63/18	$d\sigma/dP_{SP,\perp}$ 43,15/22
$^4\mathrm{He}\, p \to {}^3\mathrm{He}\, X$ $P_{^4\mathrm{He}} = 6{,}85$ GeV/c	8, fixed	7,21±0,34	$d\sigma/dP_{SP}d\Omega$ 34,93/20	
$^4\mathrm{He}\, p \to {}^3\mathrm{H}\, pp$ $P_{^4\mathrm{He}} = 13{,}5$ GeV/c	8, fixed	6,23±0,25	$d\sigma/dX_{SP}$ 29,01/14	$d\sigma/dP_{SP,\perp}$ 37,41/17

ues of the nonrelativistic wave function (33) given in Ref. 69.

In Fig. 18, the theoretical curve corresponds to calculation with the overlap integral (31) with the parameter value $a_3^R = 8.28 \pm 0.71\ (\mathrm{GeV}/c)^{-2}$ obtained by fitting the experimental data ($\chi^2/N_p = 13.6/11$). This value of the parameter a_3^R agrees well with the value $a_3^R = 8\ (\mathrm{GeV}/c)^{-2}$ predicted by (25) and used by us in the earlier calculations.

The position of the maximum in the X_{SP} distribution of the spectator deuterons also agrees well with the prediction $\tilde{X}_{SP} = 0.644$ of Eq. (20).

It is of interest to compare the theoretical calculations on the distributions of the spectator deuterons with experimental data at other energies of the incident ^{3}He nucleus.

It is also of interest to consider wave functions of a more complicated form. However, our simplest parametrizations of the relativistic nuclear wave functions make it possible to establish a number of features that do not depend on their specific parametrization (in particular, the scaling properties with respect to $x_i^{(A)}$ and the nonrelativistic limit of the wave functions).

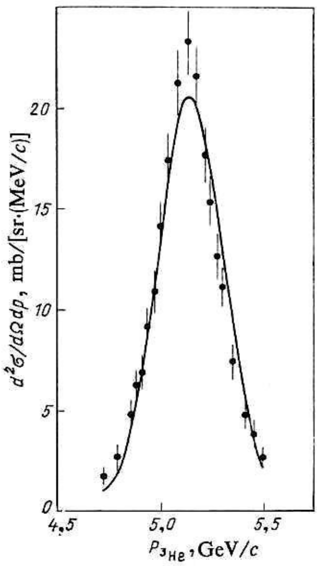

FIG. 16. Momentum spectrum of ^{3}He fragments emitted at angle 0.65° in the reaction $^4\mathrm{He}p \to {}^3\mathrm{He}X$ at ^{4}He momentum 6.85 GeV/c.

2. DEEP INELASTIC LEPTON–NUCLEUS COINCIDENCE PROCESSES

The study of deep inelastic processes with the participation of leptons plays an important part in our understanding of the nature of elementary particles and the forces that act between them. If the leptons (electron, muon, neutrino) are regarded as point particles and the interactions between them as known, they can be used as good test bodies to study strongly interacting hadrons, whose internal structure is described by the introduction of structure functions (form factors). A scaling property of these structure functions was discovered in experiments[70] on deep inelastic lepton–hadron interactions. This stimulated a large number of theoretical studies aimed at describing the experimentally observed behavior. However, later experiments with high-energy beams of electrons, muons,[71–73] and neutrinos[74] revealed deviations from scaling behavior with respect to the Bjorken variable x_B.

Among the attempts to explain the deviation from scaling in deep inelastic processes we mention the search for new scaling variables in the framework of the parton model (see, for example, the review of Ref. 75 and the bibliography given there) and the calculation of deep inelastic processes in the framework of quantum chromodynamics.[76] Without going into the successes and shortcomings of the various ap-

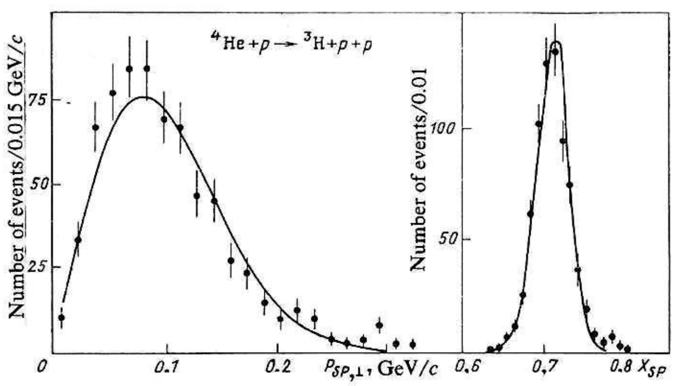

FIG. 17. Distributions with respect to $P_{SP,\perp}$ and X_{SP} of the spectator fragment ^{3}H in the reaction $^4\mathrm{He}p \to {}^3\mathrm{H}pp$ at ^{4}He momentum 13.5 GeV/c.

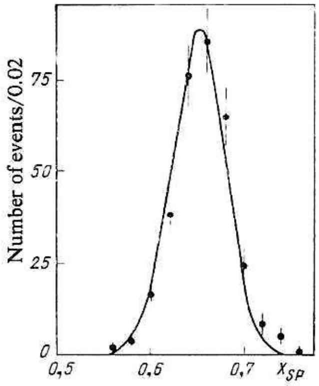

FIG. 18. Distribution with respect to X_{SP} of the spectator deuterons in the reaction ^{3}He$p \to dpp$ at ^{3}He momentum 13.5 GeV/c.

proaches, we mention that deep inelastic lepton–nucleus processes can help to clarify some of the problems that arise. Besides the independent interest associated with the possibility of studying the structure of the nuclei themselves, lepton–nucleus interactions can in a certain sense simulate lepton–hadron interaction processes (see, for example, Refs. 4 and 77). Nuclei, treated relativistically, can serve as models for the study of hadrons. In both cases we are dealing with composite systems, though with a different nature of the constituents. This circumstance gives rise to interest in the theoretical study of deep inelastic lepton–nucleus processes. The analogies in the hadronic and nuclear interactions become particularly clear in the framework of many-particle relativistic dynamics in light-front variables.[37]

We present some results of study of lepton–nucleus interactions obtained using the self-similarity principle and the many-particle relativistic formalism in light-front variables.[78,79] We shall investigate deep inelastic lepton–nucleus processes with the participation of a spectator fragment nucleus in the final state.

Deep inelastic scattering of charged leptons by nuclei. The process $lA \to l'(A-1)X$

We consider the deep inelastic scattering of an electron (or muon) with 4-momentum k by a nucleus A with 4-momentum P_A when a lepton with 4-momentum k' and a spectator $(A-1)$ fragment nucleus with 4-momentum P^{SP}_{A-1} are detected in coincidence in the final state. All the arguments remain valid when the fragment of the nucleus consists of an arbitrary number A' of nucleons. We define the kinematic invariants of the studied $lA \to l'(A-1)X$ process as follows:

$$\left.\begin{array}{l} q^2=(k-k')^2,\ \nu_A=P_Aq,\ \nu_{SP}=P^{SP}_{A-1}q, \\ \varkappa=P_AP^{SP}_{A-1},\ s_{lA}=(k+P_A)^2. \end{array}\right\} \quad (35)$$

Note that unless otherwise stated we use the frame in which the virtual photon and the initial nucleus A move along the z axis:

$$q=(q_0,\ 0,\ 0,\ q_z),\ P_A=(E_A,\ 0,\ 0,\ P_{A,z});$$
$$P^{SP}_{A-1}=(E^{SP}_{A-1},\ P^{SP}_{A-1,\perp}\cos\varphi_{SP},\ P^{SP}_{A-1,\perp}\sin\varphi_{SP},\ P^{SP}_{A-1,z}).$$

The differential cross section of this process in the single-photon approximation is

$$\frac{d\sigma\,(lA\to l'(A-1)\,X)}{dq^2\,d\nu_A\,d\nu_{SP}\,d\varkappa\,d\varphi_{SP}}=$$
$$=\frac{1}{8\,(2\pi)^5\,\lambda\,(s_{lA},\ m_l^2,\ M_A^2)}\,(\nu_A^2-M_A^2q^2)^{-1/2}\left(\frac{4\pi\alpha}{q^2}\right)^2 l_{\mu\nu}\hat{W}^{lA}_{\mu\nu}. \quad (36)$$

The tensor $l_{\mu\nu}$, which describes the lepton part of the process, has the form

$$l_{\mu\nu}=\sum_{\text{spin}} j_\mu j^*_\nu=2\,[k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}\,(kk'-m_l^2)]. \quad (37)$$

All the information about the strongly interacting part of the process is contained in the tensor $\hat{W}^{lA}_{\mu\nu}$:

$$\hat{W}^{lA}_{\mu\nu}=\sum_{\text{spin}}\sum_X (2\pi)^4\,\delta^{(4)}\,(P_A+q-P^{SP}_{A-1}-p_X)$$
$$\times\langle P_A|J_\nu(0)|P^{SP}_{A-1},\ X\rangle\,\langle P^{SP}_{A-1},\ X|J_\mu(0)|P_A\rangle. \quad (38)$$

In this expression there is a summation over all the final states with a distinguished spectator fragment, and for each such state there is a summation over the spins and an integration over the momenta (over the phase spaces) of the undetected hadrons.

Using the momenta $P_{A,\mu}$, q_μ and $P^{SP}_{A-1,\mu}$, we can construct a tensor $\hat{W}^{lA}_{\mu\nu}$ satisfying the conditions of gauge invariance and symmetry as follows:

$$\begin{aligned}\hat{W}^{lA}_{\mu\nu}=&\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\hat{W}^{lA}_1\\ &+\left(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\right)\left(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\right)\hat{W}^{lA}_2\\ &+\left(P^{SP}_{A-1,\mu}-\frac{\nu_{SP}}{q^2}q_\mu\right)\left(P^{SP}_{A-1,\nu}-\frac{\nu_{SP}}{q^2}q_\nu\right)\hat{W}^{lA}_3\\ &+\frac{1}{2}\Big[\left(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\right)\left(P^{SP}_{A-1,\nu}-\frac{\nu_{SP}}{q^2}q_\nu\right)\\ &\quad+\left(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\right)\left(P^{SP}_{A-1,\mu}-\frac{\nu_{SP}}{q^2}q_\mu\right)\Big]\hat{W}^{lA}_4.\end{aligned} \quad (39)$$

It follows from the Herniticity of the hadronic electromagnetic current J_μ that the structure functions $\hat{W}^{lA}_i$ are real; the latter are functions of the invariant kinematic variables q^2, ν_A, ν_{SP}, $\varkappa$:

$$\hat{W}^{lA}_i=\hat{W}^{lA}_i\,(q^2,\ \nu_A,\ \nu_{SP},\ \varkappa).$$

When the cross section (36) is integrated over the azimuthal angle φ_{SP} of the spectator fragment, the integration affects only the tensor $W^{lA}_{\mu\nu}$. This integration over the angle φ_{SP} makes it possible to define new structure functions $\hat{W}^{lA}_i$ as follows:

$$\begin{aligned}\tilde{W}^{lA}_{\mu\nu}=&\int_0^{2\pi} d\varphi_{SP}\,\hat{W}^{lA}_{\mu\nu}\\ =&\int d\varphi_{SP}\,d\nu_{SP}\,d\varkappa\,\delta\,(\nu_{SP}-P^{SP}_{A-1}q)\,\delta\,(\varkappa-P_AP^{SP}_{A-1})\,\hat{W}^{lA}_{\mu\nu}\\ =&\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\hat{W}^{lA}_1\\ &+\left(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\right)\left(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\right)\tilde{W}^{lA}_2.\end{aligned} \quad (40)$$

Using the new structure functions $\hat{W}^{lA}_i$, we write the cross section in the form

$$\frac{d\sigma(lA\to l'(A-1)X)}{dq^2\,d\nu_A\,d\nu_{SP}\,d\varkappa}=\frac{1}{8(2\pi)^5\lambda(s_{lA},m_l^2,M_A^2)(\nu_A^2-M_A^2q^2)^{1/2}}\left(\frac{4\pi\alpha}{q^2}\right)^2$$

$$\times\Big\{-2(q^2+2m_l^2)\widetilde{W}_1^{lA}(q^2,\nu_A,\nu_{SP},\varkappa)+\Big[(s_{lA}-\nu_A-M_A^2-m_l^2)^2$$

$$+q^2\Big(M_A^2-\frac{\nu_A^2}{q^2}\Big)\Big]\widetilde{W}_2^{lA}(q^2,\nu_A,\nu_{SP},\varkappa)\Big\}. \qquad (41)$$

A similar treatment can be given for deep inelastic lepton–hadron scattering $lN\to l'hX$ with one detected hadron in the final state. This process was studied in the framework of the parton model in Ref. 80. It was studied in the framework of quantum field theory in Ref. 81.

Deep inelastic scattering of neutrinos (or antineutrinos) by nuclei. The process $\nu A\to l(A-1)X$

We consider the deep inelastic scattering of neutrinos (antineutrinos) by nuclei due to charged currents in the framework of the V-A theory of weak interactions. We define the kinematic invariants of this process in the same way as in the case of deep inelastic electroproduction [see Eq. (35)], except that now k and k' are the 4-momenta of the initial neutrino (or antineutrino) and the final lepton, respectively.

The differential cross section of the deep inelastic interaction of a neutrino (or antineutrino) with nucleus A when a lepton and $(A-1)$ spectator nucleus are detected in the final state has the form

$$\frac{d\sigma(\nu A\to l(A-1)X)}{dq^2\,d\nu_A\,d\nu_{SP}\,d\varkappa\,d\varphi_{SP}}=\frac{G^2m_{\mu\nu}^{\nu,\bar\nu}\hat{W}_{\mu\nu}^{\nu,\bar\nu}}{16(2\pi)^5(s_{\nu A}-M_A^2)^2(\nu_A^2-M_A^2q^2)^{1/2}}. \qquad (42)$$

The tensor $m_{\mu\nu}^{\nu,\bar\nu}$ describes the leptonic part of the process and has the form

$$m_{\mu\nu}^{\nu,\bar\nu}=\sum_{\text{spin}}(l_\mu^{\mp})^*l_\nu^{\mp}=8[k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}(kk')\pm i\varepsilon_{\mu\nu\lambda\rho}k_\lambda k'_\rho]. \qquad (43)$$

The strong-interaction part of the process is described by the tensor $\hat{W}_{\mu\nu}^{\nu,\bar\nu}$:

$$\hat{W}_{\mu\nu}^{\nu,\bar\nu}=\sum_{\text{spin}}\sum_X(2\pi)^4\delta^{(4)}(P_A+q-P_{A-1}^{SP}-p_X)$$
$$\times\langle P_A|(J_\mu^{\pm}(0))^*|P_{A-1}^{SP},X\rangle\langle P_{A-1}^{SP},X|J_\nu^{\pm}(0)|P_A\rangle. \qquad (44)$$

In contrast to the case of deep inelastic electroproduction on nuclei, the requirements of symmetry and gauge invariance are not imposed on the tensor $\hat{W}_{\mu\nu}^{\nu,\bar\nu}$.

The most general form of a second-rank tensor composed of the 4-momenta $P_{A,\mu}$ q_μ and $P_{A-1,\mu}^{SP}$ is

$$\hat{W}_{\mu\nu}^{\nu,\bar\nu}=\Big(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\Big)\hat{W}_1^{\nu,\bar\nu}+\Big(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\Big)\Big(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\Big)\hat{W}_2^{\nu,\bar\nu}$$
$$-\frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}P_{A,\lambda}q_\rho\hat{W}_3^{\nu,\bar\nu}+q_\mu q_\nu\hat{W}_4^{\nu,\bar\nu}+(P_{A,\mu}q_\nu+P_{A,\nu}q_\mu)\hat{W}_5^{\nu,\bar\nu}$$
$$+i(P_{A,\mu}q_\nu-P_{A,\nu}q_\mu)\hat{W}_6^{\nu,\bar\nu}+\Big(P_{A-1,\mu}^{SP}-\frac{\nu_{SP}}{q^2}q_\mu\Big)\Big(P_{A-1,\nu}^{SP}-\frac{\nu_{SP}}{q^2}q_\nu\Big)$$
$$\times\hat{W}_7^{\nu,\bar\nu}+\frac{1}{2}\Big[\Big(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\Big)\Big(P_{A-1,\nu}^{SP}-\frac{\nu_{SP}}{q^2}q_\nu\Big)$$
$$+\Big(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\Big)\Big(P_{A-1,\mu}^{SP}-\frac{\nu_{SP}}{q^2}q_\mu\Big)\Big]\hat{W}_8^{\nu,\bar\nu}-\frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}P_{A-1,\lambda}^{SP}q_\rho\hat{W}_9^{\nu,\bar\nu}+(P_{A-1,\mu}^{SP}q_\nu+P_{A-1,\nu}^{SP}q_\mu)\hat{W}_{10}^{\nu,\bar\nu}$$
$$+i(P_{A-1,\mu}^{SP}q_\nu-P_{A-1,\nu}^{SP}q_\mu)\hat{W}_{11}^{\nu,\bar\nu}-\frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}P_{A,\lambda}P_{A-1,\rho}^{SP}\hat{W}_{12}^{\nu,\bar\nu}+i(P_{A,\mu}P_{A-1,\nu}^{SP}-P_{A,\nu}P_{A-1,\mu}^{SP})\hat{W}_{13}^{\nu,\bar\nu}. \qquad (45)$$

It follows from the condition of Hermiticity of the weak hadronic currents and the invariance with respect to time reversal that certain structure functions vanish. In particular,

$$\hat{W}_6^{\nu,\bar\nu}=\hat{W}_{11}^{\nu,\bar\nu}=\hat{W}_{13}^{\nu,\bar\nu}=0.$$

When the cross section (42) is integrated over the azimuthal angle φ_{SP} of the spectator fragment, the integration affects only the tensor $\hat{W}_{\mu\nu}^{\nu,\bar\nu}$. This integration over the angle φ_{SP} makes it possible to define new structure functions $\widetilde{W}_i^{\nu,\bar\nu}$ as follows:

$$\widetilde{W}_{\mu\nu}^{\nu,\bar\nu}=\int_0^{2\pi}d\varphi_{SP}\hat{W}_{\mu\nu}^{\nu,\bar\nu}=\int d\varphi_{SP}\,d\nu_{SP}\,d\varkappa\,\delta(\nu_{SP}-P_{A-1}^{SP}q)\,\delta$$
$$\times(\varkappa-P_AP_{A-1}^{SP})\hat{W}_{\mu\nu}^{\nu,\bar\nu}=\Big(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\Big)\widetilde{W}_1^{\nu,\bar\nu}$$
$$+\Big(P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu\Big)\Big(P_{A,\nu}-\frac{\nu_A}{q^2}q_\nu\Big)\widetilde{W}_2^{\nu,\bar\nu}-$$
$$-\frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}P_{A,\lambda}q_\rho\widetilde{W}_3^{\nu,\bar\nu}+q_\mu q_\nu\widetilde{W}_4^{\nu,\bar\nu}+(P_{A,\mu}q_\nu+P_{A,\nu}q_\mu)\widetilde{W}_5^{\nu,\bar\nu}. \qquad (46)$$

Using the new structure functions, we obtain the following expression for the cross section of the process:

$$\frac{d\sigma(\nu A\to l(A-1)X)}{dq^2\,d\nu_A\,d\nu_{SP}\,d\varkappa}=\frac{G^2}{4(2\pi)^5(s_{\nu A}-M_A^2)^2(\nu_A^2-M_A^2q^2)^{1/2}}$$
$$\times\Big\{-\frac{1}{q^2}(2q^2+m_l^2)(q^2-m_l^2)\widetilde{W}_1^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)$$
$$+\Big[\Big(s_{\nu A}-\nu_A-M_A^2+m_l^2\frac{\nu_A}{q^2}\Big)^2+(q^2-m_l^2)\Big(M_A^2-\frac{\nu_A^2}{q^2}\Big)\Big]$$
$$\times\widetilde{W}_2^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)\pm[(s_{\nu A}-\nu_A-M_A^2)q^2+\nu_AM_A^2]$$
$$\times\widetilde{W}_3^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)-m_l^2(q^2-m_l^2)\widetilde{W}_4^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)$$
$$-2m_l^2(s_{\nu A}-M_A^2)\widetilde{W}_5^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)\Big\}. \qquad (47)$$

The $+$ sign in front of the third term corresponds to neutrino scattering; the $-$ sign, to antineutrino scattering.

It can be seen from (47) that when the lepton mass m_l is ignored the structure functions $\widetilde{W}_4^{\nu,\bar\nu}$ and $\widetilde{W}_5^{\nu,\bar\nu}$ do not contribute to the cross section of the process.

If we assume that the interaction between the neutrino (or antineutrino) and the nucleus takes place through the exchange of a virtual $W^{\pm}$ boson, then in this case the cross section of the process will differ from (47) when the lepton mass is ignored by the factor $(1-q^2/m_W^2)^{-2}$, which tends to unity when $m_W^2 \gg q^2$ (m_W is the mass of the intermediate W boson).

Production of lepton pairs in hadron–nucleus interactions. The process $aA \to l^+l^-(A-1)X$

We consider the process of hadron–nucleus interaction when a lepton pair and a spectator fragment are detected in the final state. The kinematics of this process generalizes naturally the kinematics of the well-studied process of production of a lepton pair in pion–nucleon interactions when a nucleon is also detected in the final state together with the lepton pair. A detailed analysis of such a process was made in Ref. 82.

Let p_a and P_A be the 4-momenta of the colliding hadron and nucleus, respectively, k and k' be the 4-momenta of the created leptons, and $P^{\rm SP}_{A-1}$ be the 4-momentum of the $(A-1)$ spectator fragment. We define the kinematic invariants of the studied process $aA \to l^+l^-(A-1)X$ as follows:

$$\left.\begin{array}{l} q^2=(k+k')^2, \quad \nu_A=P_A q, \quad \nu_a=p_a q, \quad \nu_{SP}=P^{SP}_{A-1}q; \\ \varkappa_A=P_A P^{SP}_{A-1}, \quad \varkappa_a=p_a P^{SP}_{A-1}, \quad s_{aA}=(p_a+P_A)^2. \end{array}\right\} \quad (48)$$

The cross section of the process, summed over the spin polarizations of the lepton pair, can be written in the form

$$d\sigma(aA\to l^+l^-(A-1)X)=\frac{1}{4(2\pi)^3\lambda^{1/2}(s_{aA},\ m_a^2,\ M_A^2)} \times\left(\frac{4\pi\alpha}{q^2}\right)^2\pi(q^2)(-g_{\mu\nu}q^2+q_\mu q_\nu)\hat\rho^{aA}_{\mu\nu}\frac{d^4q}{(2\pi)^4}\frac{d\mathbf{P}^{SP}_{A-1}}{E^{SP}_{A-1}}, \quad (49)$$

where the leptonic part of the process is described by the expression

$$\pi(q^2)(-g_{\mu\nu}q^2+q_\mu q_\nu)=\sum_{\rm spin}\int\frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6\,2E_k 2E_{k'}}(2\pi)^4\,\delta^{(4)}(q-k-k')\,\varepsilon^*_\mu\varepsilon_\nu$$
$$=\left(\frac{q^2-4m_l^2}{q^2}\right)^{1/2}\int\frac{d\Omega}{8\pi^2}\left[k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}\frac{1}{2}(k+k')^2\right]. \quad (50)$$

Here, m_l is the lepton mass, and $d\Omega$ is the element of solid angle associated with the direction of the momentum of one of the leptons in the center-of-mass system of the lepton pair ($\mathbf{q}=\mathbf{k}+\mathbf{k}'=0$) with

$$\mathbf{k}=-\mathbf{k}'=|\mathbf{k}|(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta);$$
$$E_k=E_{k'}=\sqrt{k^2+m_l^2}=\frac{1}{2}\sqrt{q^2}.$$

Calculating the integral over the angles in this system, we obtain

$$\pi(q^2)=\frac{1}{6\pi}\left(1+\frac{2m_l^2}{q^2}\right)\left(\frac{q^2-4m_l^2}{q^2}\right)^{1/2}\Bigg|_{m_l=0}=\frac{1}{6\pi}.$$

In Eq. (49), the tensor $\hat\rho^{aA}_{\mu\nu}$ describes the strong-interaction part of the process and is defined by

$$\hat\rho^{aA}_{\mu\nu}=\overline{\sum_{\rm spin}}\sum_X(2\pi)^4\delta^{(4)}(P_A+p_a-q-P^{SP}_{A-1}-p_X) \times\langle P_A,\ p_a;\ {\rm in}\,|\,J_\mu(0)\,|\,P^{SP}_{A-1},\ X;\ {\rm out}\rangle \times\langle P^{SP}_{A-1},\ X;\ {\rm out}\,|\,J_\nu(0)\,|\times P_A,\ p_a;\ {\rm in}\rangle. \quad (51)$$

The general form of the tensor $\hat\rho^{aA}_{\mu\nu}$, constructed from the 4-vectors $p_{a,\mu}$, $P^{\rm SP}_{A-1,\mu}$, will be

$$\hat\rho^{aA}_{\mu\nu}=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\hat\rho^{aA}_1+\mathcal{P}_{A,\mu}\mathcal{P}_{A,\nu}\hat\rho^{aA}_2+\mathcal{P}_{a,\mu}\mathcal{P}_{a,\nu}\hat\rho^{aA}_3$$
$$+(\mathcal{P}_{A,\mu}\mathcal{P}_{a,\nu}+\mathcal{P}_{A,\nu}\mathcal{P}_{a,\mu})\hat\rho^{aA}_4$$
$$+i(\mathcal{P}_{A,\mu}\mathcal{P}_{a,\nu}-\mathcal{P}_{A,\nu}\mathcal{P}_{a,\mu})\hat\rho^{aA}_5$$
$$+\mathcal{P}^{SP}_{A-1,\mu}\mathcal{P}^{SP}_{A-1,\nu}\hat\rho^{aA}_6+(\mathcal{P}_{A,\mu}\mathcal{P}^{SP}_{A-1,\nu}+\mathcal{P}_{A,\nu}\mathcal{P}^{SP}_{A-1,\mu})\hat\rho^{aA}_7$$
$$+i(\mathcal{P}_{A,\mu}\mathcal{P}^{SP}_{A-1,\nu}-\mathcal{P}_{A,\nu}\mathcal{P}^{SP}_{A-1,\mu})\hat\rho^{aA}_8$$
$$+(\mathcal{P}_{a,\mu}\mathcal{P}^{SP}_{A-1,\nu}+\mathcal{P}_{a,\nu}\mathcal{P}^{SP}_{A-1,\mu})\hat\rho^{aA}_9$$
$$+i(\mathcal{P}_{a,\mu}\mathcal{P}^{SP}_{A-1,\nu}-\mathcal{P}_{a,\nu}\mathcal{P}^{SP}_{A-1,\mu})\hat\rho^{aA}_{10}. \quad (52)$$

Here

$$\mathcal{P}_{A,\mu}=P_{A,\mu}-\frac{\nu_A}{q^2}q_\mu;\quad \mathcal{P}_{a,\mu}=p_{a,\mu}-\frac{\nu_a}{q^2}q_\mu;$$
$$\mathcal{P}^{SP}_{A-1,\mu}=P^{SP}_{A-1,\mu}-\frac{\nu_{SP}}{q^2}q_\mu.$$

Because of the symmetry of the lepton part [see Eq. (50)] only the symmetric part of the tensor $\hat\rho^{aA}_{\mu\nu}$ will contribute to the cross section of the process $aA \to l^+l^-(A-1)X$.

The structure functions $\hat\rho^{aA}_i$ are real scalar functions of the kinematic invariants formed from the vectors q_μ, $P_{A,\mu}$, $p_{a,\mu}$, $P^{\rm SP}_{A-1,\mu}$:

$$\hat\rho^{aA}_i=\hat\rho^{aA}_i(s_{aA},\ q^2,\ \nu_A,\ \nu_a,\ \nu_{SP},\ \varkappa_A,\ \varkappa_a).$$

Rewriting the cross section of the process $aA \to l^+l^-(A-1)X$ by means of the variables (48), we obtain

$$\frac{d\sigma(aA\to l^+l^-(A-1)X)}{dq^2\,d\nu_A\,d\nu_a\,d\varphi_q\,d\varkappa_A\,d\varkappa_a\,d\varphi_{SP}}=\frac{1}{2(2\pi)^7\lambda^{3/2}(s_{aA},\ m_a^2,\ M_A^2)} \times\left(\frac{4\pi\alpha}{q^2}\right)^2\pi(q^2)(-g_{\mu\nu}q^2+q_\mu q_\nu)\hat\rho^{aA}_{\mu\nu}. \quad (53)$$

The arguments of the structure functions $\hat\rho^{aA}_i$ depend on the azimuthal angles of the virtual photon and the spectator fragment. Therefore, in the general case it is impossible to integrate the cross section (53) over the azimuthal angles, since the explicit form of the structure functions is not known. To perform such an integration, we consider the special case in which the colliding particles, hadron a and nucleus A, move along the z axis and the total transverse momentum of the lepton pair is zero, i.e., the transverse momentum of the virtual photon is zero, $q_\perp=0$. Then the dependence of the arguments of the structure functions $\hat\rho^{aA}_i$ on the azimuthal angles is eliminated, and the entire dependence on the azimuthal angle $\varphi_{\rm SP}$ will be contained in the coefficients of the expansion (52). As in the case of deep inelastic scattering of leptons by nuclei, integration of the tensor $\hat\rho^{aA}_{\mu\nu}$ over the azimuthal angle of the spectator fragment makes it possible to define new structure functions $\tilde\rho^{aA}_i$ by the relation

$$\widetilde{\rho}^{aA}_{\mu\nu}=\int\limits_0^{2\pi}d\varphi_{SP}\hat{\rho}^{aA}_{\mu\nu}=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\widetilde{\rho}^{aA}_1+\mathcal{P}_{A,\mu}\mathcal{P}_{A,\nu}\widetilde{\rho}^{aA}_2$$
$$+\mathcal{P}_{a,\mu}\mathcal{P}_{a,\nu}\widetilde{\rho}^{aA}_3+(\mathcal{P}_{A,\mu}\mathcal{P}_{a,\nu}+\mathcal{P}_{A,\nu}\mathcal{P}_{a,\mu})\,\widetilde{\rho}^{aA}_4$$
$$+\mathrm{i}\,(\mathcal{P}_{A,\mu}\mathcal{P}_{a,\nu}-\mathcal{P}_{A,\nu}\mathcal{P}_{a,\mu})\,\widetilde{\rho}^{aA}_5. \tag{54}$$

Then for the cross section of the process $aA\to l^+l^-(A-1)X$ we obtain

$$\frac{d\sigma\,(aA\to l^+l^-(A-1)X)}{dq^2\,d\nu_A\,d\nu_a\,d\varkappa_A\,d\varkappa_a}=\frac{1}{2\,(2\pi)^6\,\lambda^{3/2}\,(s_{aA},\,m_a^2,M_A^2)}$$
$$\times\left(\frac{4\pi\alpha}{q^2}\right)^2\pi\,(q^2)\,q^2\left\{3\widetilde{\rho}^{aA}_1-\left(M_A^2-\frac{\nu_A^2}{q^2}\right)\widetilde{\rho}^{aA}_2\right.$$
$$\left.-\left(m_a^2-\frac{\nu_a^2}{q^2}\right)\widetilde{\rho}^{aA}_3-\left(s_{aA}-M_A^2-m_a^2-\frac{2\nu_A\nu_a}{q^2}\right)\widetilde{\rho}^{aA}_4\right\}. \tag{55}$$

Dimensional analysis and self-similarity principle

We make a dimensional analysis of the structure functions introduced in the previous sections. For the case of deep inelastic scattering of neutrinos (or antineutrinos) by nuclei, we shall work in the center-of-mass system of nucleus A and the virtual intermediate particle in the limit $P_{A,z}\to\infty$:

$$P_A=(E_A,\,0,\,0,\,P_{A,z}),\quad q=(q_0,\,0,\,0,\,-P_{A,z}),$$

and for the case of production of a lepton pair in hadron–nucleus collisions we shall consider the center-of-mass system of the colliding particles in the high-energy limit.

It follows from the expressions (41), (47), and (55) that the structure functions of the considered processes have the dimensions

$$[\widetilde{W}^{lA}_1]=[m^{-2}],\quad[\widetilde{W}^{lA}_2]=[m^{-4}]; \tag{56a}$$
$$[\widetilde{W}^{\nu,\bar\nu}_1]=[m^{-2}],\quad[\widetilde{W}^{\nu,\bar\nu}_2]=[\widetilde{W}^{\nu,\bar\nu}_3]=[m^{-4}]; \tag{56b}$$
$$[\widetilde{\rho}^{aA}_1]=[m^{-4}],\quad[\widetilde{\rho}^{aA}_i]=[m^{-6}],\quad i=2,\,3,\,4. \tag{56c}$$

We define the Bjorken limit by the following values of the kinematic invariants: s_{lA}, q^2, $\nu_A\gg M_A^2$ with q^2/ν_A finite. We shall call the region of values of the kinematic invariants in which $\nu_{SP}\to\infty$ for a fixed ratio ν_{SP}/ν_A and finite $\varkappa$ the target (nucleus A) fragmentation region.

We introduce two scales for measuring momenta, longitudinal and transverse, and we formulate a self-similarity principle:

In the Bjorken limit in the target fragmentation region the structure functions $\widetilde{W}^{lA}_i$, $\widetilde{W}^{\nu,\bar\nu}_i$, and $\hat{\rho}^{aA}_i$ of the deep inelastic processes transform under scale transformations

$$p_z\to\lambda p_z,\quad\mathbf{p}_\perp\to\mathbf{p}_\perp \tag{57}$$

of the longitudinal momenta as homogeneous functions of the appropriate dimension. Applied to the interaction of elementary particles at high energies, this principle was formulated by Matveev, Muradyan, and Tavkhelidze[83] on the basis of the analogy, pointed out by Bogolyubov, between deep inelastic interaction processes and point explosions and two-dimensional explosions in hydrodynamics. For more details about the principles and its applications in elementary-particle physics, see Refs. 40 and 84.

For deep inelastic processes in the center-of-mass system of nucleus A and the virtual intermediate particle, we consider the case when $P_{A,z}\to\infty$ and $P^{SP}_{A-1,z}\to\infty$; we can then assume that

$$E^{SP}_{A-1}\approx P^{SP}_{A-1,\,z}+\frac{(P^{SP}_{A-1,\,\perp})^2+M^2_{SP}}{2P^{SP}_{A-1,\,z}}$$

and for the kinematic invariants of the process we obtain

$$\nu_{SP}\approx 2P_{A,\,z}P^{SP}_{A-1,\,z}\to\infty;$$
$$\varkappa\approx\frac{P_{A,\,z}}{2P^{SP}_{A-1,\,z}}[(P^{SP}_{A-1,\,\perp})^2+M^2_{SP}]-\text{finite}.$$

In accordance with the definition, this case belongs to the target fragmentation region. Under the scale transformations (57) of the momenta the kinematic invariants transform as follows:

$$q^2\to\lambda^2q^2,\quad\nu_A\to\lambda^2\nu_A,\quad\nu_{SP}\to\lambda^2\nu_{SP},\quad\varkappa\to\varkappa. \tag{58}$$

It follows from the self-similarity principle that in the Bjorken limit in the target fragmentation region the structure functions $\widetilde{W}^{lA}_i$ behave as follows:

$$\lim M_A^2\widetilde{W}^{lA}_1(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=f^{lA}_1\left(\frac{q^2}{\nu_A},\,\frac{\nu_{SP}}{\nu_A},\,\varkappa\right); \tag{59a}$$
$$\lim M_A^2\nu_A\widetilde{W}^{lA}_2(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=f^{lA}_2\left(\frac{q^2}{\nu_A},\,\frac{\nu_{SP}}{\nu_A},\,\varkappa\right); \tag{59b}$$
$$\lim M_A^2\widetilde{W}^{\nu,\bar\nu}_1(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=f^{\nu,\bar\nu}_1\left(\frac{q^2}{\nu_A},\,\frac{\nu_{SP}}{\nu_A},\,\varkappa\right); \tag{60a}$$
$$\lim M_A^2\nu_A\widetilde{W}^{\nu,\bar\nu}_2(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=f^{\nu,\bar\nu}_2\left(\frac{q^2}{\nu_A},\,\frac{\nu_{SP}}{\nu_A},\,\varkappa\right); \tag{60b}$$
$$\lim M_A^2\nu_A\widetilde{W}^{\nu,\bar\nu}_3(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=f^{\nu,\bar\nu}_3\left(\frac{q^2}{\nu_A},\,\frac{\nu_{SP}}{\nu_A},\,\varkappa\right), \tag{60c}$$

The ratio q^2/ν_A is proportional to the well-known Bjorken variable $x^B_A=-q^2/2\nu_A$. For the vatio ν_{SP}/ν_A, it is possible to obtain the expression $\nu_{SP}/\nu_A=(1-x^B_A)\,x^F_A$, where

$$x^F_A=2P^{SP}_{A-1\,,z}/\sqrt{s_{qA}},\quad s_{qA}=(q+P_A)^2.$$

In the considered case, the invariant $\varkappa$ can be expressed in the form

$$\varkappa=\frac{(P^{SP}_{A-1,\,\perp})^2+M^2_{SP}}{2x^F_A}.$$

Then in the Bjorken limit in the target fragmentation region the structure functions of deep inelastic electron scattering by nuclei can be expressed as functions of the three variables x^B_A, x^F_A, $P^{SP}_{A-1,\perp}$:

$$\lim M_A^2\widetilde{W}^{lA}_1(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=F^{lA}_1(x^B_A,\,x^F_A,\,P^{SP}_{A-1,\,\perp}); \tag{61a}$$
$$\lim M_A^2\nu_A\widetilde{W}^{lA}_2(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=F^{lA}_2(x^B_A,\,x^F_A,\,P^{SP}_{A-1,\,\perp}); \tag{61b}$$
$$\lim M_A^2\widetilde{W}^{\nu,\bar\nu}_1(q^2,\,\nu_A,\,\nu_{SP},\,\varkappa)=F^{\nu,\bar\nu}_1(x^B_A,\,x^F_A,\,P^{SP}_{A-1,\,\perp}); \tag{62a}$$

$$\lim M_A^2 \nu_A \widetilde{W}_2^{\nu,\bar\nu}(q^2, \nu_A, \nu_{SP}, \varkappa) = F_2^{\nu,\bar\nu}(x_A^B, x_A^F, P_{A-1,\perp}^{SP}); \tag{62b}$$

$$\lim M_A^2 \nu_A \widetilde{W}_3^{\nu,\bar\nu}(q^2, \nu_A, \nu_{SP}, \varkappa) = F_3^{\nu,\bar\nu}(x_A^B, x_A^F, P_{A-1,\perp}^{SP}). \tag{62c}$$

These results agree with the results obtained in the framework of Regge analysis.[85]

We introduce the variable $y = 2\nu_A/s_{qA}$ and, integrating the cross section over it from 0 to 1, we obtain

$$x_A^F \frac{d\sigma(lA\to l'(A-1)X)}{dx_A^B\, dx_A^F\, d(P_{A-1,\perp}^{SP})^2} = \frac{1}{8(2\pi)^5}\frac{s_{lA}}{M_A^2}\left(\frac{4\pi\alpha}{q^2}\right)^2 \times\left\{\frac{x_A^B}{3}F_1^{lA}(x_A^B, x_A^F, P_{A-1,\perp}^{SP}) + \frac{1}{2}F_2^{lA}(x_A^B, x_A^F, P_{A-1,\perp}^{SP})\right\}; \tag{63a}$$

$$x_A^F \frac{d\sigma(\nu A\to l(A-1)X)}{dx_A^B\, dx_A^F\, d(P_{A-1,\perp}^{SP})^2} = \frac{G^2}{4(2\pi)^5}\frac{s_{\nu A}}{M_A^2} \times\left\{\frac{x_A^B}{3}F_1^{\nu,\bar\nu}(x_A^B, x_A^F, P_{A-1,\perp}^{SP}) + \frac{1}{2}F_2^{\nu,\bar\nu}(x_A^B, x_A^F, P_{A-1,\perp}^{SP}) \mp \frac{x_A^B}{3}F_3^{\nu,\bar\nu}(x_A^B, x_A^F, P_{A-1,\perp}^{SP})\right\}. \tag{63b}$$

For production of a lepton pair in hadron–nucleus collisions together with a spectator fragment, we consider in the center-of-mass system of the colliding particles ($P_A = (E_A, 0, 0, P_{A,z})$, $p_a = (E_a, 0, 0, -\mathrm{P}_{A,z})$) the case when $P_{A,z}\to\infty$ and $P_{A-1,z}^{\mathrm{RP}}\to\infty$, then

$$E_{A-1}^{SP} \approx P_{A-1,z}^{SP} + \frac{(P_{A-1,\perp}^{SP})^2 + M_{SP}^2}{2P_{A-1,z}^{SP}}.$$

Under the scale transformations (57), the kinematic invariants transform as follows:

$$\left.\begin{aligned} &s_{aA}\to\lambda^2 s_{aA},\ q^2\to\lambda^2 q^2,\ \nu_A\to\lambda^2\nu_A;\\ &\nu_a\to\lambda^2\nu_a,\ \varkappa_A\to\varkappa_A,\ \varkappa_a\to\lambda^2\varkappa_a.\end{aligned}\right\} \tag{64}$$

Applying the principle of self-similarity to the functions $\tilde\rho_i^{aA}$, we obtain

$$\left.\begin{aligned} &\lim M_A^2 q^2 \widetilde{\rho}_1^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = f_1^{\mathrm{I}}\left(\frac{q^2}{\nu_A}, \frac{\nu_a}{s_{aA}}, \frac{q^2}{\nu_a}, \frac{\varkappa_a}{s_{aA}}, \varkappa_A\right);\\ &\lim M_A^4 q^2 \widetilde{\rho}_i^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = f_i^{\mathrm{I}}\left(\frac{q^2}{\nu_A}, \frac{\nu_A}{s_{aA}}, \frac{q^2}{\nu_a}, \frac{\varkappa_a}{s_{aA}}, \varkappa_A\right),\end{aligned}\right| \tag{65a}$$

$i = 2, 3, 4$, in the region where $q_z\to\infty$, $q_0\approx q_z + (q^2/2q_z)$

$$\left.\begin{aligned} &\lim M_A^2 q^2 \widetilde{\rho}_1^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = f_1^{\mathrm{II}}\left(\frac{q^2}{\nu_A}, \frac{q^2}{\nu_a}, \frac{\nu_A}{s_{aA}}, \frac{\varkappa_a}{s_{aA}}, \varkappa_A\right);\\ &\lim M_A^4 q^2 \widetilde{\rho}_i^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = f_1^{\mathrm{II}}\left(\frac{q^2}{\nu_A}, \frac{q^2}{\nu_a}, \frac{\nu_A}{s_{aA}}, \frac{\varkappa_a}{s_{aA}}, \varkappa_A\right),\\ &i = 2, 3, 4\end{aligned}\right| \tag{65b}$$

in the region where $q_z\to-\infty$, $q_0\approx|q_z| + (q^2/2|q_z|)$.

We define the scale-invariant variables

$$x_A^B = -\frac{q^2}{2\nu_A},\ x_a^B = -\frac{q^2}{2\nu_a},\ x_q^F = \frac{2q_z}{\sqrt{s_{aA}}},\ x_{SP}^F = \frac{2P_{A-1,z}^{SP}}{\sqrt{s_{aA}}}. \tag{66}$$

In the high-energy limit and when $P_{A-1,z}^{\mathrm{SP}}\to\infty$ and $q_z\to\pm\infty$, the ratios of the kinematic invariants in Eqs. (65a) and (65b) can be expressed in terms of the variables (66). We finally obtain

$$\lim M_A^2 q^2 \widetilde{\rho}_1^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = F_1(x_A^B, x_a^B, x_q^F, x_{SP}^F, P_{A-1,\perp}^{SP}); \tag{67a}$$

$$\begin{aligned}&\lim M_A^4 q^2 \widetilde{\rho}_i^{aA}(s_{aA}, q^2, \nu_A, \nu_a, \varkappa_A, \varkappa_a) = F_i(x_A^B, x_a^B, x_q^F, x_{SP}^F, P_{A-1,\perp}^{SP}).\\ &i = 2, 3, 4\end{aligned} \tag{67b}$$

In connection with the observation in inclusive hadron–nucleus processes of the effects of cumulative particle production (see Refs. 4, 7, 19, 25, and 26) there is undoubted interest in an experimental search for cumulative lepton pairs in hadron–nucleus and nucleus–nucleus collisions (in this connection, see Ref. 86).

Connection between the structure functions of scattering by nuclei and by nucleons

We now establish the connection between the structure functions which we have introduced for deep inelastic lepton–nucleus scattering and for the production of lepton pairs in hadron–nucleus collisions and the structure functions for lepton–nucleon scattering and for the production of lepton pairs in hadron–nucleon collisions. We shall assume that in the case of lepton scattering by nucleus A the initial lepton interacts with only one nucleon of the nucleus, the remaining $A-1$ nucleons continuing their existence in the form of a residual fragment nucleus. We shall describe the initial nucleus A and the spectator fragment in terms of many-particle relativistic wave functions $\Phi_P^{(A)}([x_i^{(A)}, \mathbf{p}_{i,\perp}])$.

In the considered approximation, the cross section of the process $lA\to l'(A-1)X$ takes the form

$$\frac{d\sigma(lA\to l'(A-1)X)}{dq^2\, d\nu_A\, d\nu_{SP}\, d\varkappa} = \frac{1}{2(2\pi)^2(4\pi)^{2A-2}}\frac{\lambda(s_{lN}, m_l^2, m^2)}{\lambda(s_{lA}, m_l^2, M_A^2)} \times(\nu_A^2 - M_A^2 q^2)^{-1/2}\left|\frac{I(X^{SP}, \mathbf{P}_{A-1,\perp}^{SP})}{1-X^{SP}}\right|^2\frac{d\sigma(lN\to l'X)}{dq^2\, d\nu_N}. \tag{68}$$

Here, $d\sigma(lN\to l'X)/dq^2 d\nu_N$ is the cross section of deep inelastic lepton–nucleon scattering, and m is the nucleon mass.

The kinematic invariants

$$s_{lN} = (k + p_N)^2,\quad \nu_N = p_N q$$

of the deep inelastic lepton–nucleon interaction are related to the invariants of the lepton–nucleus interaction by

$$s_{lN} = s_{lA} + M_{SP}^2 - 2(kP_{A-1}^{SP}) - 2\varkappa; \tag{69a}$$

$$\nu_N = \nu_A - \nu_{SP}, \tag{69b}$$

where p_N is the 4-momentum of the nucleon within nucleus A that interacts with the lepton. In Eq. (68), $I(X^{\mathrm{SP}}, \mathbf{P}_{A-1,\perp}^{\mathrm{SP}})$ is the overlap integral of the wave functions of the initial nucleus and the fragment nucleus [see Eq. (16)]. In contrast to Sec. 1, we define the variable X^{SP} somewhat differently:

$$X^{SP} = (P_{A-1,0}^{SP} + P_{A-1,z}^{SP})/(P_{A,0} + P_{A,z}),$$

and the connection between the variables $x_i^{(A)}$ and $\mathbf{p}_{i,\perp}$, on the one hand, and the variables of integration $y_i^{(A-1)}$ and $\mathbf{q}_{i,\perp}$, on the other, takes the form

$$x_i^{(A)}=X^{SP}y_i^{(A-1)};\quad \mathbf{p}_{i,\perp}=\mathbf{q}_{i,\perp}\quad i=1,2,\ldots,A-1;$$
$$x_A^{(A)}=1-X^{SP},\quad \mathbf{p}_{A,\perp}=\mathbf{P}_{A,\perp}-\mathbf{P}^{SP}_{A-1,\perp}.$$

Using Eqs. (41) and (68) and the well-known expressions for the cross sections of deep inelastic electron scattering by nucleons[87] and nuclei,[79] we obtain the connection between the structure functions $\widetilde{W}_i^{lA}(q^2,\nu_A,\nu_{SP},\varkappa)$ and the structure functions $\widetilde{W}_i^{lN}(q^2,\nu_N)$ in the form

$$\widetilde{W}_1^{lA}(q^2,\nu_A,\nu_{SP},\varkappa)=\frac{1}{2(2\pi)^2(4\pi)^{2A-2}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2 W_1^{lN}(q^2,\nu_N); \tag{70a}$$

$$\widetilde{W}_2^{lA}(q^2,\nu_A,\nu_{SP},\varkappa)=\frac{1}{2(2\pi)^2(4\pi)^{2A-2}}\times\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\frac{(\nu_N^2-m^2q^2)}{(\nu_A^2-M_A^2q^2)}W_2^{lN}(q^2,\nu_N). \tag{70b}$$

It is easy to express the variables X^{SP} and $P^{SP}_{A-1,\perp}$ in terms of the invariants q^2, ν_A, ν_{SP}, and $\varkappa$:

$$X^{SP}=\frac{1}{M_A^2}\left[\varkappa+\frac{\varkappa\nu_A-M_A^2\nu_{SP}}{(\nu_A^2-M_A^2q^2)^{1/2}}\right];$$

$$(\mathbf{P}^{SP}_{A-1,\perp})^2=\frac{1}{q^2}\left[\nu_{SP}^2-\frac{(\nu_A\nu_{SP}-q^2\varkappa)^2}{(\nu_A^2-M_A^2q^2)}\right]-M_{SP}^2.$$

As in the case of electron scattering, it is possible to obtain a connection between the cross sections of deep inelastic neutrino (antineutrino) scattering by nuclei and by nucleons. The connection is

$$\frac{d\sigma(\nu A\to l(A-1)X)}{dq^2\,d\nu_A\,d\nu_{SP}\,d\varkappa}=\frac{1}{2(2\pi)^2(4\pi)^{2A-2}}\frac{(s_{\nu N}-m^2)^2}{(s_{\nu A}-M_A^2)^2}\times(\nu_A^2-M_A^2q^2)^{-1/2}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\frac{d\sigma(\nu N\to lX)}{dq^2\,d\nu_N}. \tag{71}$$

Here, $d\sigma(\nu N\to lX)/dq^2d\nu_N$ is the differential cross section for deep inelastic neutrino (antineutrino) scattering by nucleons.

Using Eqs. (47) and (71) and the well-known expressions for the cross sections of deep inelastic neutrino (antineutrino) scattering by nucleons[87] and nuclei,[79] we obtain the following connection between the structure functions $\widetilde{W}_i^{\nu,\bar\nu}$ and $W_i^{\nu,\bar\nu}$:

$$\widetilde{W}_1^{\bar\nu,\nu}(q^2,\nu_A,\nu_{SP},\varkappa)=\frac{1}{2(4\pi)^{2A-4}}\times\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2 W_1^{\nu,\bar\nu}(q^2,\nu_N); \tag{72a}$$

$$\widetilde{W}_2^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)=\frac{1}{2(4\pi)^{2A-4}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\times\frac{(\nu_N^2-m^2q^2)}{(\nu_A^2-M_A^2q^2)}W_2^{\nu,\bar\nu}(q^2,\nu_N); \tag{72b}$$

$$\widetilde{W}_3^{\nu,\bar\nu}(q^2,\nu_A,\nu_{SP},\varkappa)=\frac{1}{2(4\pi)^{2A-4}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\times\frac{(\nu_N^2-m^2q^2)^{1/2}}{(\nu_A^2-M_A^2q^2)^{1/2}}W_3^{\nu,\bar\nu}(q^2,\nu_N). \tag{72c}$$

We now consider the process of production of a lepton pair in hadron–nucleus interactions. We shall assume that the lepton pair is produced as a result of interaction of the incident hadron with one nucleon of the nucleus. In this case, it is possible to relate the structure functions of the processes of production of lepton pairs in hadron–nucleus and hadron–nucleon interactions.

In this approximation,

$$\tilde\rho_1^{aA}=\frac{2\pi}{(4\pi)^{2A-2}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\rho_1^{aN}; \tag{73a}$$

$$\tilde\rho_2^{aA}=\frac{2\pi}{(4\pi)^{2A-2}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\frac{(\nu_N^2-m^2q^2)}{(\nu_A^2-M_A^2q^2)}\rho_2^{aN}; \tag{73b}$$

$$\tilde\rho_3^{aA}=\frac{2\pi}{(4\pi)^{2A-2}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\rho_3^{aN}; \tag{73c}$$

$$\tilde\rho_4^{aA}=\frac{2\pi}{(4\pi)^{2A-2}}\left|\frac{I(X^{SP},\mathbf{P}^{SP}_{A-1,\perp})}{1-X^{SP}}\right|^2\times\frac{\left(s_{aN}-m^2-m_a^2-\frac{2\nu_N\nu_a}{q^2}\right)}{\left(s_{aA}-M_A^2-m_a^2-\frac{2\nu_A\nu_a}{q^2}\right)}\rho_4^{aN}. \tag{73d}$$

Here, the structure functions $\tilde\rho_i^{aA}$ are functions of the kinematic invariants s_{aA}, q^2, ν_A, ν_a, $\varkappa_A$, $\varkappa_a$, and the structure functions $\tilde\rho_i^{aA}$ depend on the invariants $s_{aN}=s_{aA}-2\varkappa_a-2\varkappa_A+M_{SP}^2$, q^2, $\nu_N=\nu_A-\nu_{SP}$, ν_a. Assuming that the structure functions ρ_i^{aN} at large values of their arguments are functions of their ratios, i.e., exhibit self-similar behavior, we find that in the high-energy limit the structure functions $\tilde\rho_i^{aA}$ are functions of the scale-invariant variables (66), in agreement with the results of the self-similarity principle.

The expressions obtained here make it possible to study the structure of nuclei in a relativistic manner. For example, by measuring experimentally the structure functions $\widetilde{W}_i^{lA}$, $\widetilde{W}_i^{\nu,\bar\nu}$ and having information about the structure functions W_i^{lN}, $W_i^{\nu,\bar\nu}$ (fairly detailed information about the structure functions of deep inelastic lepton scattering by nucleons is given, for example, in Ref. 88), one can study the overlap integral $I(X_{SP},P_{SP,\perp})$, which carries information about the relativistic wave functions of the initial and final nuclei.

CONCLUSIONS

In the review, we have given some results of the relativistic approach developed by the authors in the theory of nuclear interactions. The approach is based on a treatment of nuclei as relativistic composite systems; the nature of the constituents can be arbitrary. Although relativistically described nucleons have appeared here as the structural units of the nuclei, the developed formalism also makes it possible to consider quark degrees of freedom in nuclei.

In elucidating some general features in the interactions of relativistic nuclei (scaling properties of the cross sections, general properties of the relativistic wave functions, etc.), we have also left out of account the spin degrees of freedom of the nucleons. The employed method also contains the possibility of taking into account these characteristics in both the nucleons themselves and the quarks that form them. (Questions relating to particle spins in the light-front formalism can be found, for example, in Refs. 89 and 90).

At the present stage, the choice of the form of the relativistic wave functions was dictated largely by heuristic arguments, but in principle these (or other) wave functions could be obtained by solving appropriate dynamical equations with field-theoretical or phenomenological interaction kernels. The consideration of these questions goes beyond the framework of the present review.

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