Excited states of hypernuclei

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The present status of theoretical and experimental investigations of excited states of hypernuclei is reviewed. Topics considered are the mechanism of formation of hypernuclei in the (K^-, π^-) reaction, the influence of the kinematic conditions on the nature of the excitations, and the main features of the cross sections for production of hypernuclei at small momentum transfers. The hyperon–nucleus and residual ΛN interactions are analyzed in a model of particle–hole excitations and in the many-particle shell model. The status of the weak-coupling model is discussed. The properties of the low-lying levels of light Λ hypernuclei are considered. Various approaches to the description of the structure of strangeness analog states are presented. Strong decays of Λ hypernuclei and the associated nucleon–association effects are considered. The problem of $\Sigma\Lambda$ conversion of Σ hypernuclei is discussed.

INTRODUCTION

The investigation of hypernuclei—nuclear-like systems consisting of nucleons and hyperons—was begun by the pioneering work of Danysz and Pniewski,[1] who found experimental indications of the existence of a bound state of a Λ hyperon and a nucleus. For about 20 years, the study of individual hypernucleus production events following the capture of stopped K^- mesons by photoemulsion nuclei and their weak decays remained the most productive direction in hypernuclear physics. More than 20 ground states of light hypernuclei were identified (see Table I), their binding energies were determined, and in some cases also the quantum numbers and lifetimes,[2–4] and double Λ hypernuclei were discovered.[5] The experimental data on the ground states of hypernuclei, a unique source of information about the hyperon–nucleon interaction, were frequently subjected to theoretical analysis (this stage of the investigations has been reviewed in detail by Gal[3]). The main result that was obtained concerns the saturation property of the hyperon–nucleus forces (see Refs. 3 and 6). It finally became obvious that data on the ground states of hypernuclei are not sufficient to resolve such complex problems as the choice of the most flexible model of the hypernucleus and the determination of the best parametrization of the hyperon–nucleus interaction.[1)]

Thus, further progress in hypernuclear physics was inconceivable without systematic study of the excited states of hypernuclei. This question was already considered at the end of the fifties, but advance was delayed by the limited possibilities of the emulsion technique. Nevertheless, in 1962 experimental data were published[7] that indicated the possible existence of a long-lived ${}^7_\Lambda$He isomer, an excited state of the ${}^7_\Lambda$He hypernucleus with a very low radiative-transition rate to the ground state comparable with the weak-decay rate. At the end of the sixties, an excited state of the ${}^{12}_\Lambda$C hypernucleus was found in the (K^-, π^-) reaction on carbon using stopped kaons and the emulsion technique.[8]

The systematic study of hypernuclear excitations began with experiments on the gamma spectroscopy of hypernuclei, the first results of which were published at the beginning of the seventies.[9] Such experiments revealed excited states of the s-shell hypernuclei ${}^4_\Lambda$H and ${}^4_\Lambda$He, and γ transitions in p-shell hypernuclei were also observed.[10–14]

At the same time, detailed calculations of the spectra of the low-lying excitations of the light hypernuclei were made. The first detailed analysis of p-shell hypernuclei in the framework of the shell model with intermediate coupling was made by Gal, Soper, and Dalitz.[15,16]

A qualitatively new stage in hypernuclear physics commenced with the appearance of extensive experimental data on excited states of hypernuclei produced in the (K^-, π^-) reaction using beams of slow kaons. The production of hypernuclei using slow-kaon beams was proposed in 1963 by Podgoretskiĭ,[17] who noted that in the strangeness-exchange reaction

$$K^- + {}^A Z \to \pi^- + {}^A_\Lambda Z \quad (1)$$

it is possible to achieve conditions of so-called recoilless kinematics, when the momentum transferred from the kaon to the pion is small compared with the Fermi momentum $q_F = 250$ MeV/c of the nucleons in the nucleus. Coherent interaction must with high probability result in the production of a hypernucleus, which differs from the target nucleus by replacement of one of the neutrons by a Λ hyperon. The conditions favorable for coherent production of hypernuclei can be readily established from the kinematic relations for the elementary reaction

$$K^- + n \to \pi^- + \Lambda \quad (2)$$

on a neutron at rest. Figure 1 shows the dependence of the momentum transfer $q = |\mathbf{p}_K - \mathbf{p}_\pi|$ on the kaon momentum p_K for different pion emission angles θ in the laboratory system. There is a critical momentum $p_K = 530$ MeV/c at

[1)]When the cross sections of low-energy Λp scattering were measured for the first time in a bubble chamber, it was found that the parameters of the hyperon–nucleon interaction obtained earlier from hypernuclei were completely incorrect.[3]

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TABLE I. Binding energies of the ground states of Λ hypernuclei.[2]

${}^{A}_{\Lambda}Z$	B_Λ, MeV	${}^{A}_{\Lambda}Z$	B_Λ, MeV	${}^{A}_{\Lambda}Z$	B_Λ, MeV
${}^{3}_{\Lambda}H$	0,13±0,05	${}^{8}_{\Lambda}Li$	6,80±0,03	${}^{10}_{\Lambda}B$	8,89±0,12
${}^{4}_{\Lambda}H$	2,04±0,04	${}^{9}_{\Lambda}Li$	8,53±0,15	${}^{11}_{\Lambda}B$	10,24±0,05
${}^{4}_{\Lambda}He$	2,39±0,03	${}^{7}_{\Lambda}Be$	5,16±0,08	${}^{12}_{\Lambda}B$	11,37±0,06
${}^{5}_{\Lambda}He$	3,12±0,02	${}^{8}_{\Lambda}Be$	6,84±0,05	${}^{12}_{\Lambda}C$	10,76±0,19
${}^{6}_{\Lambda}He$	4,18±0,10	${}^{9}_{\Lambda}Be$	6,71±0,04	${}^{13}_{\Lambda}C$	11,69±0,12
${}^{8}_{\Lambda}He$	7,16±0,70	${}^{10}_{\Lambda}Be$	9,11±0,22	${}^{14}_{\Lambda}C$	12,17±0,33
${}^{7}_{\Lambda}Li$	5,58±0,03	${}^{9}_{\Lambda}B$	7,88±0,15	${}^{15}_{\Lambda}N$	13,59±0,15

which q vanishes for $\theta = 0°$ (for the reaction on a nucleus, the critical momentum has a similar value). If allowance is also made for the energy dependence of the charge-exchange cross section $d\sigma/d\Omega$ of the reaction (2) (see Fig. 2), it can be seen that the most favorable conditions for recoilless production of hypernuclei are realized for kaon beam momenta $p_K = 0.5$–1 GeV/c and pion emission angles $\theta = 0$–$15°$. The corresponding experimental possibilities were realized in the middle of the seventies at CERN[19–23] and somewhat later at BNL.[24]

Ideas about the structure of the hypernuclear states produced by the coherent K^-–nucleus interaction were strongly influenced by the ideas of Sakata $SU(3)$ symmetry. The fusion of these ideas with the nuclear phenomenon of analog states resulted in the concept of a strangeness analog state (SAS). This concept was introduced in 1971 by Kerman and Lipkin[25,26] on the basis of the following arguments. The low-lying states of the Λ hyperon and a nucleon in a nucleus have wave functions of similar shape, and if one assumes in a first approximation that the distances between the single-particle hyperon and nucleon levels are the same, then the wave functions of the nuclei and hypernuclei can be appropriately classified by means of the irreducible representations of Sakata $SU(3)$ symmetry. The hypernuclear states ${}^{A}_{\Lambda}Z$ belonging to the same $SU(3)$ representation as the ground state ${}^{A}Z_{gs}$ were called stangeness analog states. The SAS wave function is obtained from the ${}^{A}Z_{gs}$ wave function by application of the operator for lowering the total U spin:

$$|\,{}^{A}_{\Lambda}Z_{SAS}\rangle = (A - Z)^{-1/2}\hat{U}^-|\,{}^{A}Z_{gs}\rangle; \tag{3}$$

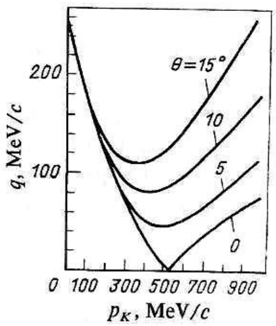

FIG. 1. The momentum transfer q to the Λ hyperon in the (K^-, π^-) reaction on a neutron at rest as a function of the kaon momentum p_K for different pion emission angles θ.

$$\hat{U}^- = \int d^3x\,\hat{\psi}^+_\Lambda(x)\,\hat{\psi}_n(x) = \sum_{nljm} \hat{b}^+_\Lambda(nljm)\,\hat{a}_n(nljm). \tag{4}$$

Here, $\hat{b}^+_\Lambda$, $\hat{\psi}^+_\Lambda(x)$ and $\hat{a}_n$, $\hat{\psi}_n(x)$ are the corresponding operators of creation of the Λ hyperon and annihilation of the neutron; the second equation in (4) presupposes identity of the hyperon and neutron single-particle wave functions for the shell states $(nljm)$.

As was noted by Feshbach and Kerman,[27] the transition realized by the operator $\hat{U}^-$ is realized physically in the (K^-, π^-) reaction at small momentum transfers, when the amplitudes of the elementary processes (2) are added coherently.[17] Thus arose the original SAS version, in accordance with which it was expected, with allowance for the lifting of the degeneracy with respect to the total isospin T, that in coherent production of hypernuclei on a target nucleus with $T \neq 0$ two strong peaks corresponding to strangeness analog states with $T = T \pm 1/2$ would be dominant.

However, such discussions encountered objections on account of the circumstance that the distances between the single-particle levels of the Λ hyperon and the nucleon in the nucleus are not equal.[28–30] Closer to the eigenstates of the hypernuclear Hamiltonian will be states of the form $\Sigma_m \hat{b}^+_\Lambda(nljm)\hat{a}_n(nljm)|{}^{A}Z_{gs}\rangle$ rather than the states (3). This interpretation of the strangeness analog states, according to which coherence of the $n \to \Lambda$ transitions is realized when hypernuclei are produced within one shell, was proposed by Hüfner *et al.*[29–31] and was subsequently confirmed experimentally.

Thus, questions of hypernuclear structure were shown to be intimately related to the mechanism of hypernuclear production in the (K^-, π^-) reaction, since without a solid understanding of the latter it is impossible to have any reli-

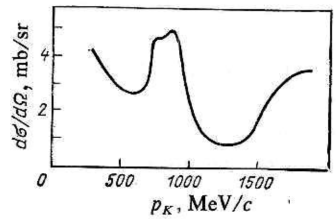

FIG. 2. Dependence of the differential cross section $d\sigma/d\Omega|_{\theta=0°}$ of the $K^- + n \to \pi^- + \Lambda$ reaction on the kaon momentum p_K (laboratory system).[18]

able interpretation of hypernuclear excitations. The mechanism of hypernucleus production in the (K^-, π^-) reaction was investigated in detail by Hüfner, Lee, and Weidenmüller,[29] who showed that the condition of small momentum transfer predetermines a predominance of single strangeness exchange, and, bearing in mind the strong absorption of mesons by nuclei, they proposed the use of the impulse approximation with distorted waves.

In the middle of the seventies, a Heidelberg–Saclay–Strasbourg collaboration at CERN obtained excitation spectra of the hypernuclei ${}^{12}_{\Lambda}C$, ${}^{16}_{\Lambda}O$, ${}^{32}_{\Lambda}S$ and ${}^{40}_{\Lambda}Ca$, confirming the existence of coherent transitions in individual shells.[32–34] At the same time, it was found that even for small momentum transfers ($q \simeq 70$ MeV/c) coherent transitions do not exhaust the complete spectrum of final states. As Dalitz and Gal showed, to describe this spectrum it is necessary to take into account the so-called quasifree transitions, in which the produced Λ hyperon occupies a state different from the one occupied by the initial neutron.[35] In contrast to the (e, ep) and $(p, 2p)$ reactions, quasifree transitions in the (K^-, π^-) reaction are important already at momenta significantly less than the Fermi momentum, since the prohibition associated with the Pauli principle did not extend to the final states of the Λ hyperon. With increasing q and increasing mass of the target nucleus, the importance of the quasifree transitions compared with the coherent (quasielastic) transitions increases.

Progress in the experimental field associated with perfection of the kaon beams and detectors made it possible to obtain in the (K^-, π^-) reaction spectra of hypernuclei with an energy resolution of order 1 MeV, at which one can distinguish the structure of individual levels excited not only in quasielastic transitions, and one can also investigate the dependence of the cross sections for production of different hypernuclear states on the momentum transfer q.[23,36] Thus, hypernuclear spectroscopy acquired a solid base, making it possible to establish with a certain degree of certainty the form of the hyperon–nucleus and hyperon–nucleon interactions.[37–39] At the same time, the development of the hypernuclear models reached a level at which one could pose the question of using hypernuclear data to study the structure of ordinary nuclei and the relationship between the effective-particle–nucleus interactions and the fundamental baryon–baryon interactions.[39–42]

In the present paper, we review theoretical and experimental investigations of excited states of hypernuclei. In recent years, a number of publications have been devoted to this theme, not only reviews[23,43–45] but also review papers at various conferences.[36–39,42] Nevertheless, the time now appears ripe for an exposition of the field in which the main accent is no longer on the novelty of the individual results and the technical details but rather on the internal integrity of the models and methods of hypernuclear physics.

In Sec. 1, we consider the mechanism of hypernucleus production in the (K^-, π^-) reaction at small momentum transfers. The main attention here is concentrated on the occurrence of the correlations characteristic for the given kinematic conditions between the hyperon and nuclear excitations. Section 2 contains an analysis of the structure of hypernuclei in the approximation of closed shells and the main features of the Λ-hyperon–nucleus interaction. In Sec. 3, in the framework of the many-particle shell model, we consider low-lying excitations of light Λ hypernuclei. In Sec. 4, we discuss the structure of the strangeness analog states produced in the (K^-, π^-) reaction on $1p$-shell nuclei. We also consider questions such as the use of the translationally invariant shell model to describe light hypernuclei, the supermultiplet structure of the strangeness analog states, the status of the weak-coupling approximation and the main features of the strong decays of hypernuclei, and also alternative approaches based on using Sakata $SU(3)$ symmetry and the model of nucleon associations. In Sec. 5, we give examples of the analysis of the residual ΛN interaction using detailed information about the excitation spectra of light hypernuclei. Section 6 is devoted to Σ hypernuclei and the problem of $\Sigma\Lambda$ conversion.

To conclude this introductory section, we consider the weak-coupling approximation, to which we shall frequently return in what follows. This approximation is based on the assumption that the influence of the Λ hyperon on the nucleon core is small and that the wave function of the hypernucleus ${}^{A}_{\Lambda}Z$ has the form of the direct product of the wave functions of the nuclear core ${}^{A-1}Z$ in a state with energy E, total angular momentum J_N, parity π, and isospin T and the Λ hyperon moving in the average field of the nucleus:

$$|{}^{A}_{\Lambda}Z,\ J\rangle = [|\ {}^{A-1}Z,\ E,\ J^{\pi}_{N},\ T\rangle \otimes |(nlj)_{\Lambda}\rangle]_{J=J_N+j}. \quad (5)$$

For the analysis of experimental data, this approximation is a very convenient initial approximation, since it requires minimal information about the structure of the nucleon core and frequently makes it possible to obtain not only a qualitative but also a quantitative description of the hypernuclear spectra. The deviations from the predictions of the weak-coupling approximation are of particular interest, since they are associated with mixing of the functions (5) of the zeroth approximation and are, therefore, an important source of information about the hyperon–nucleon interaction.

1. PRODUCTION OF HYPERNUCLEI IN THE (K^-, π^-) REACTION

The possibility of producing hypernuclei under controllable conditions is the great advantage of the (K^-, π^-) reaction: For a special choice of the kinematics (large momentum of the incident kaon and small momentum transfer from the K^- to the π^-), the single-step strangeness-exchange mechanism is dominant, in which one of the neutrons is replaced by a Λ hyperon with minimal disturbance in the motion of the remaining nucleons of the nucleus. With the accumulation of experimental information, it became clear that the excitation spectra of light hypernuclei in the ${}^{A}Z(K^-, \pi^-){}^{A}_{\Lambda}Z$ reaction admit in many cases a simple interpretation in the framework of the weak-coupling approximation, when the hyperon and nuclear degrees of freedom (the motion of the hyperon in the field of the undeformed nucleon core and the excitation of the nuclear core) in the wave function of the hypernucleus factorize. The single-step strangeness-exchange mechanism ensures a definite correlation between the hyperon and nuclear excitations, essential use of which is made in the interpretation of the experimental data.[23]

Amplitude of the (K^-, π^-) Reaction in the Distorted-Wave Born Approximation

Analyzing the mechanism of hypernucleus production in the (K^-, π^-) reaction at small momentum transfers, Hüfner *et al.*[29] gave a justification for using the distorted-wave Born approximation (DWBA). Following Ref. 46, we consider the standard derivation of the expression for the amplitude of the reaction (1).

For large momenta of the initial kaon and final pion and small pion emission angle, the Glauber approximation is valid,[47] and the amplitude for production of the hypernucleus ${}^A_\Lambda Z$ in the state $|f\rangle$ can be written in the form

$$M_{if} = \int d^2b e^{i\mathbf{q}\mathbf{b}} \langle f \mid \Gamma(\mathbf{b}) \mid i\rangle. \quad (6)$$

Here, $|i\rangle$ is the initial state of the target nucleus, and $\mathbf{q}=\mathbf{p}_\pi - \mathbf{p}_K$ is the momentum transfer. The profile function $\Gamma(\mathbf{b}, \mathbf{x}_1, \ldots, \mathbf{x}_A)$, which depends on the impact parameter $\mathbf{b}$ and on the coordinates $\mathbf{x}_1, \ldots, \mathbf{x}_A$ of the nucleons of the target nucleus, can be expressed in terms of the profile functions of the binary processes $K^-N \to K^-N$, $K^-n \to \pi^-\Lambda$ and $\pi^-N \to \pi^-N$:

$$\Gamma(\mathbf{b}, \mathbf{x}_1, \ldots, \mathbf{x}_A) = \sum_{n=1}^{A} \prod_{i=1}^{n-1} (1-\Gamma_K(\mathbf{b}-\mathbf{x}_i)) \times \Gamma_{K\pi}(\mathbf{b}-\mathbf{x}_n) \prod_{j=n+1}^{A} (1-\Gamma_\pi(\mathbf{b}-\mathbf{x}_j)).$$

The profile functions $\Gamma_K, \Gamma_{K\pi}, \Gamma_\pi$ are related to the corresponding amplitudes of the elementary reactions $f_K, f_{K\pi}, f_\pi$ by

$$\Gamma(\mathbf{b}) = \frac{1}{2\pi i k} \int d^2q e^{-i\mathbf{q}\mathbf{b}} f(\mathbf{q}),$$

where k is the c.m.s. momentum of the initial particles.

Since the considered values of q are small compared with the momentum that characterizes the scale of appreciable variation of the amplitudes f_π and f_K ($q_c \sim 1$ GeV/c), the dependence of the latter on q can be ignored:

$$f_{\pi(K)}(\mathbf{q}) = f_{\pi(K)}(0) = \frac{ik}{4\pi} \sigma_{\pi(K)} (1 + i\alpha_{\pi(K)}).$$

Here, σ_π and σ_K are the total cross sections of the π^-N and K^-N interactions, and $\alpha_{\pi(K)} = -\mathrm{Re} f_{\pi(K)}(0)/\mathrm{Im} f_{\pi(K)}(0)$. In the charge-exchange amplitude at small q there is no spin-flip term, since it vanishes for collinear momenta of the kaon and pion.

The next approximation is to go to the optical limit in the calculation of the matrix element $\langle f|\Gamma(\mathbf{b})|i\rangle$. After integration over the coordinates of all the nucleons except the one on which the charge exchange $K^- + n \to \pi^- + \Lambda$ occurs, the amplitude is reduced to the form

$$M_{if} = f_{K\pi} \int d^3x \langle f \mid \hat{u}(\mathbf{x}) \mid i\rangle \times \exp\left[i\mathbf{q}\mathbf{x} - \frac{\sigma_K}{2}(1-i\alpha_K) \int_{-\infty}^{z} \rho(\mathbf{b}, z')\, dz' - \frac{\sigma_\pi}{2}(1+i\alpha_\pi) \int_{z}^{+\infty} \rho(\mathbf{b}, z')\, dz' \right],$$

where $\hat{u}(\mathbf{x}) = \hat{\psi}^+_\Lambda(\mathbf{x})\psi_n(\mathbf{x})$ is the operator that lowers the U spin, i.e., transforms the neutron into the Λ hyperon; $\rho(\mathbf{b}, z) \equiv \rho(\mathbf{x})$ is the nucleon density. The result obtained is the amplitude of the reaction (1) in the DWBA[29]:

$$M_{if} = f_{K\pi} \int d^3x \chi^{(-)}_{\mathbf{p}_\pi}(\mathbf{x})\, \chi^{(+)}_{\mathbf{p}_K}(\mathbf{x}) \langle f \mid \hat{u}(\mathbf{x}) \mid i\rangle, \quad (7)$$

where the meson wave functions are determined by means of the eikonal formula

$$\chi^{(\pm)}_{\mathbf{p}}(\mathbf{x}) = \exp\left[\pm \left(i\mathbf{p}\mathbf{x} - \frac{1}{2}\sigma(1 \mp i\alpha) \int_{\mp\infty}^{z} \rho(\mathbf{b}, z')\, dz' \right) \right].$$

One can arrive at the expression (7) without recourse to the Glauber approximation if the meson wave functions are determined by the optical potentials of the meson–nucleus interaction.[29,48] Such an approach is used, in particular, to describe the (K^-, π^-) reaction "at rest."[29] The accuracy of the DWBA for the (K^-, π^-) reaction was discussed in Refs. 29, 48, and 49. According to Refs. 29 and 49, the corrections to the differential cross section $d\sigma/d\Omega dE$ calculated in the DWBA can change the result by several tens of percent. The experimental data agree with the calculations to within this error (see Secs. 2 and 5).

Coherent and Quasifree Transitions

We consider the structure of the matrix element $\langle f|\hat{u}(\mathbf{x})|i\rangle$ in the shell model. The action of the neutron annihilation operator on the state vector of the target nucleus can be represented in the form

$$\hat{\psi}_n(\mathbf{x}) \mid i\rangle = \sum_{knlj} g^k_{nlj} \Psi_{nlj}(\mathbf{x}) \mid k\rangle, \quad (8)$$

where the sum is over all states k of the nucleus ${}^{A-1}Z$ and the neutron shells nlj (for simplicity, the coefficients of the vector addition of the angular momenta and isospins are omitted); $\Psi_{nlj}(\mathbf{x})$ are the wave functions of the neutrons in the state nlj; g^k_{nlj} are the fractional-parentage coefficients (the spectroscopic amplitudes of neutron separation in the state nlj). We take the state vector of the hypernucleus in the weak-coupling approximation (5):

$$| f\rangle = | k_0\rangle \otimes | (n_\Lambda l_\Lambda j_\Lambda)_\Lambda\rangle.$$

Thus, we have

$$\langle f \mid \hat{\psi}^+_\Lambda(\mathbf{x})\, \psi_n(\mathbf{x}) \mid i\rangle = \sum_{nlj} g^{k_0}_{nlj} \Psi_{nlj}(\mathbf{x})\, \Phi^*_{n_\Lambda l_\Lambda j_\Lambda}(\mathbf{x}), \quad (9)$$

where $\Phi_{n_\Lambda l_\Lambda j_\Lambda}(\mathbf{x})$ is the wave function of the Λ hyperon in the state $(n_\Lambda l_\Lambda j_\Lambda)$. To illustrate the part played by the various terms on the right-hand side of (9), we replace for simplicity the meson wave functions by plane waves. Then the amplitude M_{ij} determined by (7) takes the form

$$M_{if}(\mathbf{q}) = f_{K\pi} \sum_{LM} \sum_{nljm} g^{k_0}_{nlj} A_{if}(l, j, l_\Lambda, j_\Lambda, L) C^{J_f M_f}_{J_i M_i LM} Y_{LM}(\mathbf{q}/q) \times \int_0^\infty R^{(\Lambda)}_{n_\Lambda l_\Lambda j_\Lambda}(r)\, j_L(qr)\, R^{(n)}_{nlj}(r)\, dr, \quad (10)$$

where $R^{(n)}_{nlj}$ and $R_{n_\Lambda l_\Lambda j_\Lambda}$ are the radial wave functions of the neutron and the Λ hyperon; $A_{if}(l, j, l_\Lambda, j_\Lambda, L)$ is the coefficient that arises on the addition of the angular momenta and isospins, the explicit form of which is not important for what follows (for more details, see Refs. 29 and 50). For $q = 0$, only the terms with $L = 0$ make a nonvanishing contribution to the right-hand side of (10). In addition, the absence of spin-flip components in the amplitude of the elementary reaction leads to the equation $j_\Lambda = j$. Such $n \to \Lambda$ transitions ($l_\Lambda = l, j_\Lambda = j$) are said to be coherent, or recoilless, and also, by analogy with elastic scattering by nuclei, quasielastic; to indicate the shell in which the coherent transition takes place, we use the notation $(nlj, (nlj)^{-1})^{0+}_{\Lambda n}$. The contribution of the coherent transitions to the differential cross section for production of the hypernucleus ${}^A_\Lambda Z$ is proportional to the square of the form factor $F(q) = \int_0^\infty R^{\Lambda}_{nlj}(r) j_0(qr) R^{(n)}_{nlj}(r) dr$ and decreases with increasing momentum transfer.

The transitions as a result of which the Λ hyperon occupies a state with quantum numbers $n_\Lambda l_\Lambda j_\Lambda$ different from the quantum numbers nlj of the initial neutron are called quasifree transitions and are denoted by $(n_\Lambda l_\Lambda j_\Lambda, (nlj)^{-1})^{L}_{\Lambda n}$.[2)] The quantity $L = |\mathbf{l} - \mathbf{l}_\Lambda|$ determines the nature of the q dependence of the contribution of a quasifree transition to the differential cross section for production of the hypernucleus (see below). Quasifree transitions recall inelastic scattering by nuclei, but, in contrast to the nucleon, the Pauli principle does not extend to the final Λ hyperon, and therefore these transitions can compete with the coherent transitions at momentum transfers comparatively small compared with q_F.

Meson–Nucleus Interaction in the Initial and Final States. Sum Rule for Differential Cross Sections of the Reactions for Angle 0°

The total cross sections of the K^-N and π^-N interactions at momenta 0.5–1 GeV/c have the order of magnitude of several tens of millibarns, so that the mean free path of K^- and π^- in nuclear matter is about 2 F. Thus, only neutrons lying in a narrow annular layer near the surface of the nucleus participate effectively in the strangeness-exchange reaction. We consider the differential cross section of the reaction (K^-, π^-) at angle 0°, integrated over all final states of the hypernucleus:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=0^\circ} = N_{\text{eff}} \frac{d\sigma_{K^- n \to \pi^- \Lambda}}{d\Omega}. \tag{11}$$

Here, N_{eff}, the total effective number of neutrons, can be calculated theoretically by means of a sum rule.[35] This sum rule follows from Eq. (7) if we ignore the weak dependence of p_π on the energy of the final states $|f\rangle$ and use the completeness relation for the system of hypernuclear wave functions:

$$N_{\text{eff}} = \int d^3x \rho(\mathbf{x}) \,|\, \chi^{(-)}_{\mathbf{p}_\pi}(\mathbf{x}) \chi^{(+)}_{\mathbf{p}_K}(\mathbf{x}) \,|^2. \tag{12}$$

If the eikonal approximation is used for the meson wave functions, the expression (12) becomes

$$N_{\text{eff}} = \frac{N}{A}\int d^2b(\sigma_\pi - \sigma_K)^{-1}[\exp(-\sigma_K T(\mathbf{b})) - \exp(-\sigma_\pi T(\mathbf{b}))],$$

where $T(\mathbf{b}) = \int_{-\infty}^{+\infty} \rho(\mathbf{b}, z)dz$, $\int d^2bT(\mathbf{b}) = A$. In the absence of absorption, as one would expect, N_{eff} is equal to the total number $N = A - Z$ of neutrons in the target nucleus. If we set $\sigma_\pi = \sigma_K = \sigma$, then $N_{\text{eff}} = (N/A)\int d^2bT(\mathbf{b})e^{-\sigma T(\mathbf{b})}$. The value of N_{eff} was calculated in Refs. 35, 48, 51, and 52 and was measured for nuclei with $A = 6$–209 by the Heidelberg–Saclay–Strasbourg collaboration.[53] The experimental data are give in Table II together with the results of Bouyssy's calculations[52]; it should be noted that good agreement between the theory and experiment is obtained only when realistic functions of the nuclear density $\rho(r)$ are used.

Although allowance for the meson–nucleus interaction is important for calculating the absolute magnitude of the hypernucleus production cross section, the q dependences of the intensities of the coherent and quasifree transitions in the *outer shells* retain the qualitative behavior established in the plane-wave approximation for the meson wave functions (see Refs. 46 and 50).

Main Features of the Excitation Spectra of Λ Hypernuclei in the (K$^-$, π^-) Reaction at Small Momentum Transfers. The Relationship Between Quasielastic and Quasifree Transitions

The differential cross sections $d\sigma/d\Omega dE\,|_{\theta+\theta^\circ}$ of the reaction (1) on the nuclei ^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, ^{16}O, ^{27}Al, ^{32}S, ^{40}Ca, ^{51}V, ^{89}Y, ^{209}Bi were measured at CERN by the Heidelberg–Saclay–Strasbourg (HSS) collaboration (Refs. 33, 34, 41, and 53) and at BNL by a Carnegie-Mellon–MIT–Houston collaboration on the ^{12}C, ^{13}C, ^{14}N, ^{18}O nuclei[24] in a wide range of hypernuclear excitations E. Figure 3 shows the experimental data on the hypernuclei ${}^{12}_\Lambda$C, ${}^{27}_\Lambda$Al, ${}^{51}_\Lambda$V, ${}^{209}_\Lambda$Bi; for convenience of comparison, all the hypernuclear spectra are given in the common energy scale of the transition $M_{\text{nuc}} - M_\Lambda$, the mass difference of the hypernucleus ${}^A_\Lambda Z$ and the target nucleus $^A Z$, and for each hypernucleus we have given the energy of the Λ hyperon relative to the ground state of the nucleon core: $E_\Lambda = M_{\text{nuc}} - m_\Lambda - M_{A-1}$ (the scale of the binding energy $B_\Lambda = -E_\Lambda$ is frequently used). Depending on the momentum of the kaon beam and the mass of the produced hypernucleus, the momentum transfer q is in the interval 50–80 MeV/c.

A characteristic feature of the exhibited spectra is the presence in each of them of a peak at $M_{\text{nuc}} - M_\Lambda = 190$–196 MeV, this peak being particularly pronounced in light nuclei (${}^{12}_\Lambda$C). It follows from what was said above that these peaks must correspond to coherent transitions in the principal (closed or nearly closed) outer shell: $(1p_{3/2}, 1p^{-1}_{3/2})^{0+}_{\Lambda n}$ in the case of ${}^{12}_\Lambda$C, $(1d_{5/2}, 1d^{-1}_{5/2})^{0+}_{\Lambda n}$ for ${}^{27}_\Lambda$Al, $(1f_{7/2}, 1f^{-1}_{7/2})^{0+}_{\Lambda n}$ for ${}^{51}_\Lambda$V, and $(1i_{13/2}, 1i^{-1}_{13/2})^{0+}_{\Lambda n}$ for ${}^{209}_\Lambda$Bi. The fact that the corresponding transition energies exceed the Λ-hyperon–neutron mass difference $m_\Lambda - m_n = 176$ MeV by the amount $\Delta E = 14$–

[2)]The concept of quasifree interaction in the (K^-, π^-) reaction was proposed by Dalitz and Gal[35] as a simple interpretation of the hump in the energy dependence of the cross section $d\sigma/d\Omega dE$ at high excitation energies (above the region of coherent transitions). Using the Fermi-gas model to describe the nucleons of the target nucleus, they found a connection between the mean energy transfer in the (K^-, π^-) reaction to the depth of the hyperon–nucleus potential U_Λ and the momentum transfer q. Since at large q the quasifree interaction is dominant, the concept "quasifree" acquired the meaning of an "alternative to coherent," and in this meaning it began to be used not only for transitions to continuum final states but also to characterize transitions to discrete and quasidiscrete hypernuclear levels.

TABLE II. Differential cross sections $d\sigma/d\Omega$ of the (K^-, π^-) reactions on nuclei ${}^AZ$ for kaons with momentum p_K, total effective neutron numbers N_{eff}, and effective neutron numbers $N_{\text{eff}}^{\text{coh}}$ for coherent transitions (references are indicated by square brackets).

${}^AZ$	p_K, MeV/c	Experiment (Ref. 53)		Theory (Ref. 52)	
		$\frac{d\sigma}{d\Omega}\Big\vert_{\theta=0°}$	N_{eff}	N_{eff}	$N_{\text{eff}}^{\text{coh}}$
^{6}Li	790	3,6±1,0	1,5±0,4	1,50	1.16
^{7}Li	720	3,4±1,0	1,5±0,4	1,58	1,14
^{7}Li	790	4,4±1,2	1,8±0,5	1,58	1,14
^{9}Be	720	3,5±1,0	1,5±0,4	1,86	1,28
^{9}Be	790	3.8±0,9	1.6±0,4	1,86	1,28
^{12}C	720	3,8±1,0	1,7±0,4	2,0 [35] 1,67 2,2 [35]	1,08
^{16}O	720	3,9±1,3	1,7±0.6	1,80 [48] 1,97 2,3 [35]	1.21
^{27}Al	720	7,8±2,3	3,5±1,0	2,35	1,10
^{32}S	720	5.6±1,6	2.5±0.7	2.52 2,8 [35]	1.17
^{40}Ca	790	4,9±2,0	2,1±0,8	2,40 3,0 [35] 3,54 [48]	1.04
^{51}V	720	3,0±1,4	1.3±0.6	2,70	1.15
^{89}Y	720	4.8±3,9	2,1±1.7	3,91	1.08
^{209}Bi	640	10.5±3.7	5,0±1,8	7,00	1.01

20 MeV, which is close to the difference between the depths of the potentials of the nucleon–nucleus and hyperon–nucleus interactions ($U_N - U_\Lambda \simeq 20$ MeV), strengthens such an interpretation. As will be seen from what follows, the identification of these peaks with coherent transitions is directly confirmed by the q dependence of their excitation cross sections.

In accordance with what was said above, in medium and heavy nuclei coherent transitions in inner shells are suppressed by the strong absorption of the mesons by the nucleus. In addition, the hypernuclear states produced by coherent transitions in the inner shells have a strongly excited nucleon core. The width $\Gamma^{\downarrow}$ of their decay through rearrangement of the nuclear core is determined by the width of the corresponding neutron–hole state of the nucleus ${}^{A-1}Z$, and this width increases rapidly with increasing A. Therefore, coherent transitions in inner shells can be well observed only in light nuclei (see below); in particular, in the excitation spectrum of ${}^{12}_{\Lambda}$C at $E_\Lambda \simeq 10$ MeV the contribution of a $(1s_{1/2}, 1s_{1/2}^{-1})^{0+}_{\Lambda n}$ coherent transition can be noted.

The theoretically calculated values of the contribution of the coherent transitions to N_{eff} are given in Table II,[52] and in Fig. 3 they are shown by the broken curves. Thus, the effective number of neutrons that participate in the quasielastic transitions is close to 1 and depends weakly on A; at the same time, the importance of the quasifree transitions increases with increasing A, and in heavy nuclei they predominate over the quasielastic transitions even at comparatively small q.[51,52]

Determination of the Quantum Numbers of Hypernuclear States from Analysis of the q Dependences of their Excitation Cross Sections. Spectrum of the Hypernucleus ${}^{12}_{\Lambda}$C

The interpretation of the resonance structures in the energy dependence of the cross sections of the (K^-, π^-) reaction presupposes determination of the quantum numbers of the hypernuclear states, which is complicated by the fact that peaks may arise through the superposition of several transitions to different hypernuclear states. The dependence of the cross section $d\sigma/d\Omega dE$ on the momentum transfer q is the characteristic whose investigation permits solution of the posed problem. It is here in no way unimportant that such an analysis does not require detailed knowledge of the hyperon–nucleus and hyperon–nucleon interactions and, therefore, may precede their detailed study.

An example, the excitation spectrum of the hypernucleus ${}^{12}_{\Lambda}$C at $q = 70$ MeV/c, is shown in Fig. 3. The position of the peak at $E_\Lambda = -11$ MeV coincides in energy with the ${}^{12}_{\Lambda}$C ground state (the binding energy $B_\Lambda = 10.8$ MeV is known from emulsion experiments (Ref. 2).[3)] To the resonance with minimal excitation energy there must correspond the quasifree transition $(1s_{1/2}, 1p_{3/2}^{-1})^{1-}_{\Lambda n}$, and therefore the ${}^{12}_{\Lambda}$C ground state has the quantum numbers $J^\pi = 1^-$, since $J^\pi = 0^+$ for ${}^{12}_{\Lambda}C_{\text{gs}}$. In Fig. 4, the result of the theoretical calculation[54] of the cross section $d\sigma/d\Omega$ is compared with the experimental data obtained at BNL.[24] Apart from an overall normalization factor, the two q dependences are similar.

We now consider the peak at $E_\Lambda = 0$ MeV, at which, as we noted above, the quasielastic transition $(1p_{3/2}, 1p_{3/2}^{-1})^{0+}_{\Lambda n}$ is dominant at small q; the resulting hypernuclear state has quantum numbers $J^\pi = 0^+$. The experimental data on the dependence of the differential cross section $d\sigma/d\Omega$ on the pion emission angle in the region of the considered peak do indeed agree with the theoretical curve calculated for a coherent transition (see Fig. 4) for $q \lesssim 150$ MeV/c. With further increase in the momentum transfer, the sharp decrease in the cross section becomes less steep. Analysis of ths fact leads to

3)In the weak-coupling model, the structure of ${}^{12}_{\Lambda}C_{\text{gs}}$ is ${}^{11}C_{\text{gs}}(3/2^-) \otimes (1s_{1/2})_\Lambda$, i.e., for the total angular momentum and parity J^π one can expect the values of 1^- or 2^-.

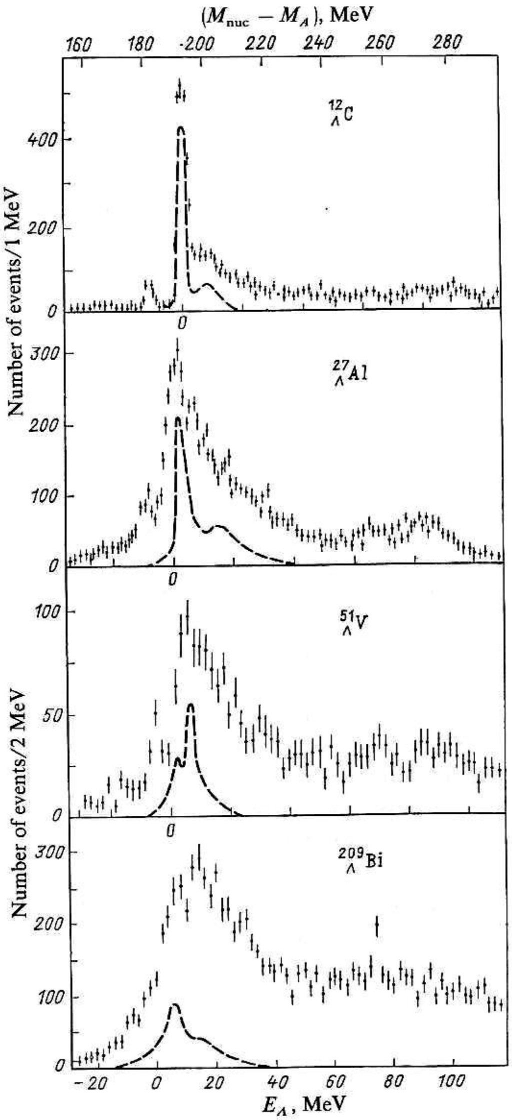

FIG. 3. Excitation spectra of Λ hypernuclei in the (K^-, π^-) reaction at $\theta = 0°$ and $p_K = 720$ MeV/c ($^{12}_{\Lambda}$C, $^{27}_{\Lambda}$Al, $^{51}_{\Lambda}$V) and $p_K = 690$ MeV/c (^{209}Bi).[53] The broken curves are the contribution of the coherent transitions.

the following conclusion[24]: The peak in the region of $E_\Lambda = 0$ MeV is the result of superposition of the coherent transition $(1p_{3/2}, 1p^{-1}_{3/2})^{0+}_{\Lambda n}$ and the quasifree transitions $(1p_{3/2}, 1p^{-1}_{3/2})^{2+}_{\Lambda n}$ and $(1p_{1/2}, 1p^{-1}_{3/2})^{2+}_{\Lambda n}$. The quasifree transitions lead to the production of two hypernuclear states with quantum numbers $J^\pi = 2^+$, separated by a distance less than the experimental resolution (2 MeV). (In the weak-coupling model, these states have the structure $[^{11}{\rm C}_{\rm gs} \otimes (1p_{3/2})_\Lambda]_{J\pi=2+}$ and $[^{11}{\rm C}_{\rm gs} \otimes (1p_{1/2})_\Lambda]_{J\pi=2+}$). The transition $0^+ \to 2^+$ requires a change in the angular momentum $L = 2$, and the maximum of its intensity is attained at $q \simeq 200$ MeV/c (see Fig. 4). Thus, by varying the momentum transfer, it is possible to change the relative intensities of the excitation of the various states. In the case of $^{12}_{\Lambda}$C, it is possible to obtain bounds on the splitting of the closely spaced levels:

$$| E(2^+) - E(0^+) | < 0.42 \text{ MeV};$$
$$| E(2^+_1) - E(2^+_2) | < 0.82 \text{ MeV}.$$

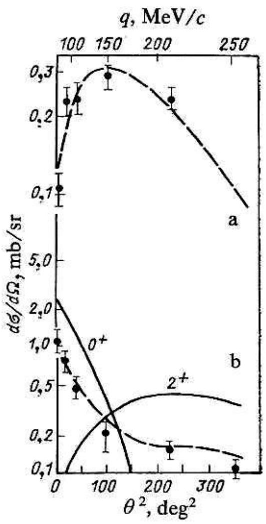

FIG. 4. Dependence of the differential cross section for production of the hypernucleus $^{12}_{\Lambda}$C on the pion emission angle θ (momentum transfer q) for the ground state (a) and for the peak with excitation energy $E = 11$ MeV (b).[24] The broken curves are the result of the theoretical calculation of Ref. 54 (normalization using the experimental data), and the continuous curves are the contributions of the $(1p, 1p^{-1})^{0+,2+}_{\Lambda n}$ transitions.

We note that because the spin-flip terms in the elementary amplitude are small (see above), the transitions with non-natural parity, $\pi \neq (-1)^L$, are suppressed, and therefore the transitions $(1p_{3/2}, 1p^{-1}_{3/2})^{1+,3+}_{\Lambda n}$ and $(1p_{1/2}, 1p^{-1}_{3/2})^{1+}_{\Lambda n}$ are not observed.

Excitation Spectra of Light Hypernuclei in the (K^-, π^-) Reaction at Small Momentum Transfers in the Weak-Coupling Limit

As follows from the results given above, coherent transitions are dominant in the excitation spectra of light nuclei at small momentum transfers q. In the weak-coupling limit, the produced hypernuclear states have the structure

$$[| ^{A-1}Z, J^\pi_N, \alpha\rangle \otimes | (nl)_\Lambda \rangle]_{J=J_N+l_\Lambda+s_\Lambda},$$

where the nucleon core $^{A-1}Z(J^\pi_N, \alpha)$ is a neutron–hole $(nlj)^{-1}_n$ state of the target nucleus. The ^{A-1}Z excitations are characterized structurally by the coefficients $g^{J_N,\alpha}_{nlj} = \langle ^{A-1}Z, J_N, \alpha \| \hat{a}_n(nlj) \| ^A Z_{\rm gs} \rangle$ (see Eq. (8)), which arise when the wave function of the target-nucleus ground state is expanded with respect to a complete system of functions of the daughter nucleus. The fractional-parentage properties of $1p$-shell nuclei have been well studied[55,56]; they were considered in great detail in connection with the quasielastic knockout reactions $(p, 2p)$, (p, d), (e, ep). The fractional-parentage coefficients calculated with realistic wave functions make it possible to find the probabilities for the production of a nucleon hole in different shells and to determine the fragmentation of holes belonging to definite shells. The degree of correlation of the spectrum of the neutron hole levels with the structure of the observed hypernuclear excitations can be used as a criterion for the applicability of the weak-coupling approximation. Thus, hypernuclear spectroscopy is intimately related to the parentage of the nuclear states. The importance of this was first pointed out by Balashov.[40]

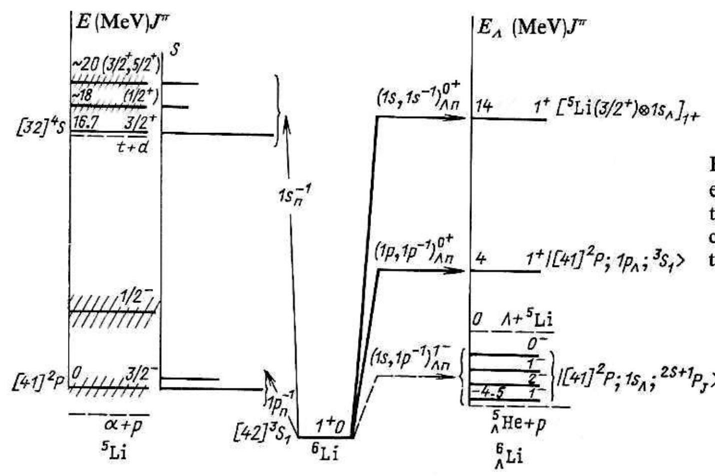

FIG. 5. Fractional-parentage schemes of the coupling $^6Li \to n + {}^5Li$ and excitation of the hypernucleus ${}^6_\Lambda Li$ in the (K^-, π^-) reaction at small q. For the nucleon states the Young tableau and the LS structure of the dominant configuration $|[f]^{2S+1}L_J\rangle$ are given; S are the neutron spectroscopic factors (rel. units).

As an example, we consider the $^6Li(K^-, \pi^-){}^6_\Lambda Li$ reaction. The $^6Li \to n + {}^5Li$ fractional-parentage scheme is shown in Fig. 5. Production of a neutron hole in the $1p$ shell of ^6Li leads with probability 2/3 to production of the ^5Li ground state $(3/2^-)$, and with probability 1/3 to the $^5Li(1/2^-)$ state. In shell models without allowance for coupling to the continuum, the distance between these levels is less than 1 MeV (Ref. 56).[4)]

The neutron hole excitations $(1s^{-1})_n$ of the nucleus ^6Li lead with the greatest probability to production of a narrow $^5Li(3/2^+)$ state with E^+ 16.7 MeV. In the weak-coupling approximation, the hypernuclear states have the form $^5Li(J^\pi_N) \otimes (nl)_\Lambda$. In Fig. 5, the continuous arrows show the coherent $(1p, 1p^{-1})^{0+}_{\Lambda n}$ and $(1s, 1s^{-1})^{0+}_{\Lambda n}$ transitions, which lead to SAS production, and the broken arrow shows the $(1s, 1p^{-1})^{1-}_{\Lambda n}$ transitions to the low-lying states with $J^\pi = 1^-, 2^-, 1^-, 0^-$.[5)] The distance between the $1s$ and $1p$ shells for the Λ hyperon in the nucleus is $E((1p)_\Lambda) - E((1s)_\Lambda) = 8$ MeV.[57] For other nuclei, the transition schemes are constructed similarly in the weak-coupling approximation.

We now turn to the comparison with the experimental data.

In Refs. 58 and 59, the correspondence between the excitation spectra of $1p$-shell hypernuclei and the spectra of the corresponding neutron-hole excitations was analyzed. Figure 6 shows the spectroscopic factors for separation of $1p$-shell neutrons[60] and the experimental spectra,[33,41,98] these being placed on top of each other in such a way that the strong low-lying peak coincides with the strongest hole excitation of the target nucleus. For all nuclei for which the (K^-, π^-) reaction was studied, one can trace a clear correlation both in the energy position and in the excitation intensity of the individual peaks or energy regions. This result confirms

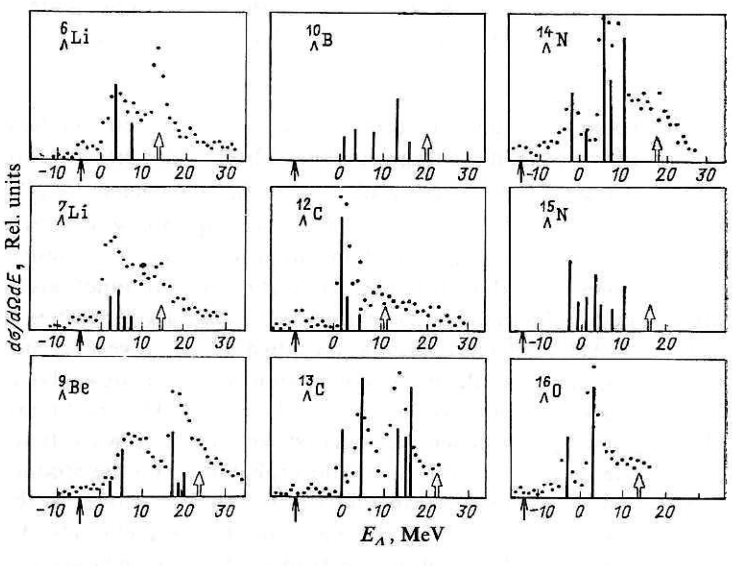

FIG. 6. Excitation spectra of hypernuclei of the $1p$ shell at small q. The straight segments are the spectroscopic factors for separation of $1p$-shell neutrons.

[4)]The appreciable excitation energy $E = 5$–10 MeV and the corresponding large width $\Gamma = 5 \pm 2$ MeV of the $1/2^-$ state are the result of strong coupling to the open $^4He + p$ channel.

[5)]The ${}^6_\Lambda Li$ ground state with $E_\Lambda = -4.5$ MeV (Ref. 41) is unstable with respect to the strong decay ${}^6_\Lambda Li \to {}^5_\Lambda He + p$ and has not been observed in emulsion experiments.

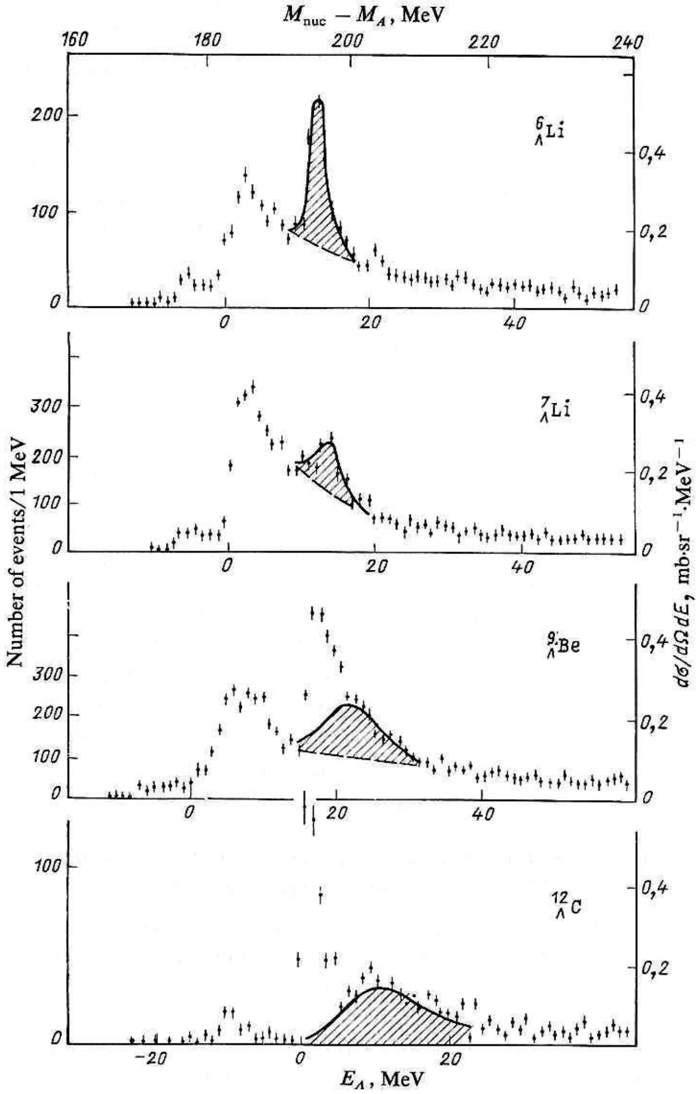

FIG. 7. Energy dependence of the differential cross sections $d\sigma/d\Omega dE$ of (K^-, π^-) reactions on light nuclei at $\theta = 0°$. The hatching indicates the contribution of the $(1s, 1s^{-1})^{0+}_{\Lambda n}$ transition.[41]

the correctness of the weak-coupling hypothesis, since otherwise a strong difference between the spectra would be observed. Use of the experimental spectroscopic factors extracted from the (p, d) reaction makes it possible to predict the excitation spectra of the hypernuclei $^{10}_{\Lambda}$B, $^{11}_{\Lambda}$B, $^{14}_{\Lambda}$C, $^{15}_{\Lambda}$N.[58]

On the other hand, the realization of the weak-coupling limit makes the use of the (K^-, π^-) reaction to study neutron hole states at small momentum transfers particularly attractive. Earlier, these states were studied in the (p, d) neutron-pickup reaction at comparatively large momentum transfers $q \gtrsim 150$ MeV/c, at which the search for high-lying ($1s$-hole) excitations is difficult on account of fragmentation of the residual nucleus. In Ref. 41, the excitation spectra of the hypernuclei $^6_\Lambda$Li, $^7_\Lambda$Li, $^9_\Lambda$Be, $^{12}_\Lambda$C (Fig. 7) were used to determine the $1s$-neutron binding energy $B_n(1s)$ and the width Γ_n of the neutron hole state. The resonance with the highest excitation energy is identified with the state $((1s^{-1})_n, (1s)_\Lambda)$. The standard procedure (with allowance for the experimental resolution) gave the position and width of the resonance, and the binding energy $B_n(1s)$ was found from the relation

$$M(^A_\Lambda Z) - M(^A Z) = B_n(1s) - B_\Lambda(1s) + m_\Lambda - m_n,$$

where $B_\Lambda(1s)$ are the known Λ-hyperon binding energies. The results agree well with the data on the (p, d) reaction, and also demonstrate a clear similarity with the parameters of the proton hole states[61] (Table III). As was shown in Ref.

TABLE III. Energies B_n and widths Γ_n of neutron hole states $(1s^{-1})_n$ determined from data on $^AZ(K^-, \pi^-)^A_\Lambda Z$ reactions,[41] compared with the parameters of the proton hole states $(1s^{-1})_p$.[61]

^{A}Z	$M_{nuc} - M_A$, MeV	B_n, MeV	Γ_n, MeV	B_p, MeV	Γ_p, MeV
^{6}Li	195,5±0,5	24,0±0,5	0,7±1,0	21,4	1,2
^{7}Li	197,9±0,5	26,6±0.5	1,6±1,5	25,5±0,4	5,9
^{9}Be	199,0±1,0	29,7±1,0	5,0±2,0	25,4±0,5	6,3
^{12}C	204.5±1,5	39,3±1,5	9,0±2,0	34,0±2,0	9.2

62, allowance for the dependence of B_Λ on the excitation of the nucleon core reduces the given values of $B_n(1s)$ by 2–3 MeV, so that at the end the difference $B_n(1s) - B_p(1s)$ is close to the Coulomb correction ΔB^c.

2. STRUCTURE OF HYPERNUCLEI IN THE CLOSED-SHELL APPROXIMATION

It is expedient to begin the analysis of hypernuclear excitations in the $^AZ(K^-, \pi^-)^A_\Lambda Z$ reaction with nuclei having doubly closed (sub)shells. In this case, the excitation spectrum of the nucleon core has the simplest form, so that the corresponding hypernuclear excitations can be described in terms of particle–hole configurations $|1p - 1h\rangle$.[29,51] The hyperon and nuclear degrees of freedom are described, respectively, by the hyperon–nucleus and nucleon–nucleus potentials, and a coupling between them is introduced by means of a residual-interaction potential. The small number of important configurations makes it possible to take into account their mixing and thus establish the limits of applicability of the weak-coupling approximation.

Closed-Shell Approximation. Particle–Hole Excitations

A model of hypernuclear excitations in the closed-shell approximation was proposed in Ref. 29 and was successfully used to describe experimental data on the excitation spectra of $^{12}_\Lambda$C, $^{16}_\Lambda$O, $^{32}_\Lambda$S, $^{40}_\Lambda$Ca,[37,46,51] which provided a good possibility for determining the parameters of the hyperon–nucleus potential and the residual ΛN interaction.

Following Refs. 37 and 51, we consider the main elements in the model of particle–hole hypernuclear excitations.

Basis Functions. The basis functions are direct products of the single-particle wave functions of the Λ hyperon and the nucleon in the hyperon-nucleus and nucleon-nucleus potentials. The hyperon-nucleus potential contains central and spin–orbit parts of Thomas type:

$$\left.\begin{aligned} &V_\Lambda = V_\Lambda^C + V_\Lambda^{LS}; \quad V_\Lambda^C = U_\Lambda \rho(r); \\ &V_\Lambda^{LS} = U_\Lambda^{LS} \frac{d}{m_\pi^2 r\, dr} \rho(r)\, \mathbf{ls}. \end{aligned}\right\} \tag{13}$$

The central part is the Woods–Saxon potential:

$$\left.\begin{aligned} &\rho(r) = [1 + \exp((r-a)/b)]^{-1}; \\ &a = r_0 A^{1/3}, \quad r_0 = 1.1 \ \mathrm{F}, \quad b = 0.6 \ \mathrm{F}. \end{aligned}\right\} \tag{14}$$

The parameters U_Λ and U_Λ^{LS} are free. The nucleon–nucleus potential is the standard shell-model potential (the energies of the neutron–hole states are taken from experiments).

Configuration Mixing. The residual interaction is diagonalized on the basis of the functions corresponding to the configurations $[(n'l'j')_\Lambda, (nlj)_n^{-1}]_J$

$$V_{\Lambda N} = v_0 \delta(\mathbf{x}_\Lambda - \mathbf{x}_N)(1 + \alpha_\Lambda \sigma_\Lambda \sigma_N), \tag{15}$$

where the value of v_0 is matched to U_Λ by means of the condition

$$U_\Lambda = v_0 \int_0^\infty \rho(r) r^2\, dr,$$

and the parameter α_Λ is free. Data on the binding energies of the ground states of light hypernuclei[63] and on low-energy Λp scattering[64] reveal a weak spin dependence of the residual interaction ($\alpha_\Lambda = -0.05$), and a result of a similar order of magnitude also follows from analysis of the splitting of the 0^+ and 1^+ levels of the hypernuclei $_\Lambda$H and $^4_\Lambda$He ($\alpha_\Lambda = -0.13$).[11]

Excitation Cross Sections. The resulting hypernuclear wave functions were used to calculate the cross sections of the (K^-, π^-) reaction on the ^{12}C, ^{16}O, ^{32}S, ^{40}Ca nuclei in the impulse approximation with distorted waves. From the condition of best agreement between the theoretical cross sections and the experimental data, the following values of the parameters were found:

$$\begin{aligned} &U_\Lambda = (32 \pm 2)\,\mathrm{MeV}; \quad U_\Lambda^{LS} = (4 \pm 2)\,\mathrm{MeV}; \quad \alpha_\Lambda = \\ &= -0.05 \pm 0.10 \ [37]. \end{aligned} \tag{16}$$

The corresponding parameters for a nucleon are

$$U_N \simeq 50\,\mathrm{MeV}; \quad U_N^{LS} = 20\,\mathrm{MeV}; \quad \alpha_N = 0.3.$$

As is shown in Fig. 8, the theoretical calculation agrees well with the experimental data of the Heidelberg–Saclay–Strasbourg collaboration.[33,34] The most important result of the analysis is the conclusion that the spin–orbit hyperon–nucleus interaction is weak compared to the central interaction, since it followed from early analyses of the binding energies of the ground states of p-shell hypernuclei that the spin–orbit potentials for the nucleon and the Λ hyperon in the nucleus must be similar in order of magnitude.[15]

Intensity of the Hyperon–Nucleus Spin–Orbit Interaction. Excitation Spectrum of the $^{16}_\Lambda$O Hypernucleus

One of the ways of determining the magnitude of the spin–orbit interaction is to study the energy splitting of coherent excitations in the shells nlj ($j = l \pm \frac{1}{2}$). It was used by the HSS collaboration to analyze data on the ^{16}O$(K^-, \pi^-)^{16}_\Lambda$O reactions at $p_K = 715$ MeV/c. Two strong peaks at $E_\Lambda = -3$ MeV and $E_\Lambda = 3$ MeV in the differential cross section $d\sigma/d\Omega dE$ (Fig. 9) are identified in the weak-coupling approximation with the quasielastic transitions $(1p_{3/2}, 1p_{3/2}^{-1})^{0+}_{\Lambda n}$ and $(1p_{1/2}, 1p_{1/2}^{-1})^{0+}_{\Lambda n}$. The distance $\Delta E = 6$ MeV between these peaks is equal to the energy splitting $E((1p_{3/2}^{-1})_n) - E((1p_{1/2}^{-1})_n) = 6$ MeV of the neutron holes. From this it may be concluded that for the Λ hyperon in the nucleus the spin–orbit potential is much weaker than for the nucleon.[33]

Note that the distance between the two peaks at $E_\Lambda = -14$ MeV and $E_\Lambda = -8$ MeV must also be equal to the splitting $E((1p_{3/2}^{-1})_n) - E((1p_{1/2}^{-1})_n) = 6$ MeV. The first of the peaks corresponds to a quasifree transition to the state $^{16}_\Lambda$O$(1^-) = {}^{16}$O $gs \otimes (1s_{1/2})_\Lambda$, and the second to the quasifree transition $(1s_{1/2}, 1p_{3/2}^{-1})^{1-}_{\Lambda n}$. Both states have quantum numbers $J^\pi = 1^-$, and their excitation cross section must, in contrast to the excitation cross sections of the coherent peaks, increase with increasing momentum transfer q, and this is confirmed by the experimental data.[33,34]

Thus, the main features of the excitation spectrum of the hypernucleus $^{16}_\Lambda$O (splitting of certain levels) find a simple explanation in the weak-coupling model. The accuracy of this model is limited by configuration mixing. In particular, mixing effects influence the relative intensity of the quasi-

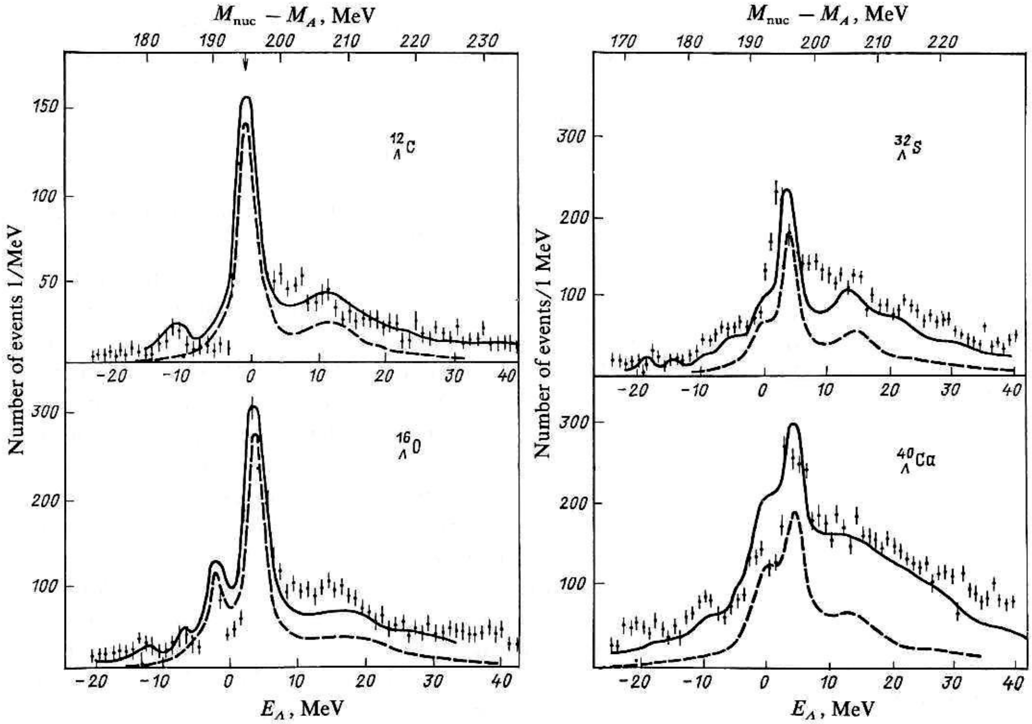

FIG. 8. Excitation spectra of the hypernuclei $^{12}_{\Lambda}C$, $^{16}_{\Lambda}O$, $^{32}_{\Lambda}S$, $^{40}_{\Lambda}Ca$ in the (K^-, π^-) reaction at $\theta = 0°$ and $p_K = 720$ MeV/c, calculated in the closed-shell approximation[37] (continuous curve) and compared with the experimental data.[33,34] The broken curve shows the contribution of the quasielastic transitions.

elastic transitions. The experimental ratio of the excitation cross sections of the 0^+ states with $E_\Lambda = 3$ MeV and $E_\Lambda = -3$ MeV is 3 ± 0.5, whereas if meson absorption and mixing of the $|(1p_{3/2})_\Lambda, (1p_{3/2}^{-1})_n, 0^+\rangle$ and $|(1p_{1/2})_\Lambda, (1p_{1/2}^{-1})_n, 0^+\rangle$ configurations is ignored, the ratio of the intensities of the quasielastic transitions in the $1p_{3/2}$ and $1p_{1/2}$ shells at $q = 0$ must be equal to the ratio of the numbers of neutrons in thse shells (2:1). If we limit ourselves to just these two configurations, the experimental ratio of the cross sections can be reproduced for the following choice of the exact

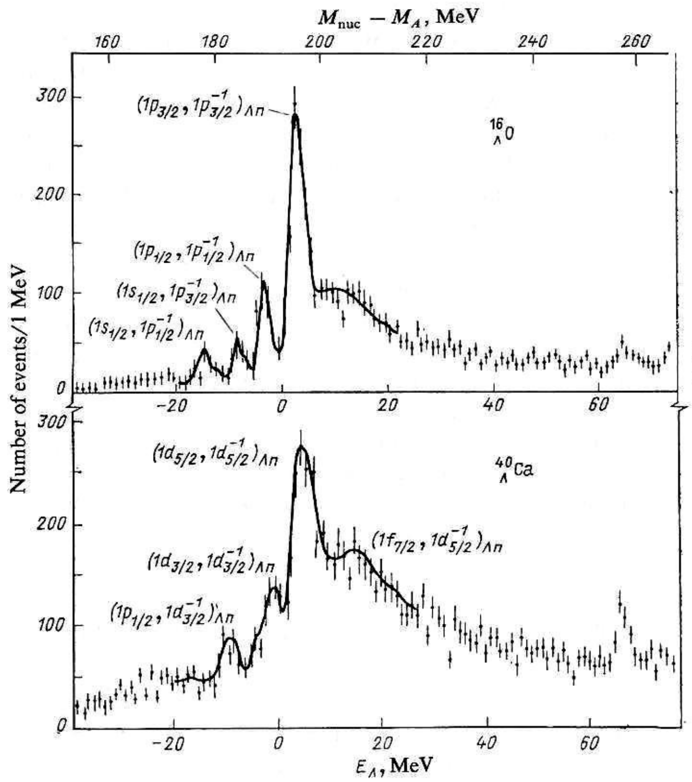

FIG. 9. Excitation spectra of the hypernuclei $^{16}_{\Lambda}O$ and $^{40}_{\Lambda}Ca$ in the (K^-, π^-) reaction at $\theta = 0°$ and $p_K = 715$ MeV/c.[33,34]

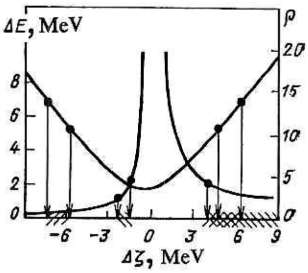

FIG. 10. Dependence of the energy splitting ΔE and the ratio ρ of the excitation intensities at $q=0$ of the $J^\pi = 0^+$ states of the hypernucleus ${}^{16}_{\Lambda}$O on the spin–orbit splitting $\Delta\zeta$ of the basis states $|(1p_{1/2})_\Lambda, (1p^{-1}_{1/2})_n, 0^+\rangle$ and $|(1p_{3/2})_\Lambda, (1p^{-1}_{3/2})_n, 0^+\rangle$.[65]

wave functions of the 0^+ states[64]:

$$|0_1^+\rangle = \cos\theta\,|(1p_{3/2})_\Lambda,\ (1p^{-1}_{3/2})_n,\ 0^+\rangle - \sin\theta\,|(1p_{1/2})_\Lambda,\ (1p^{-1}_{1/2})_n,\ 0^+\rangle;$$

$$|0_2^+\rangle = \sin\theta\,|(1p_{3/2})_\Lambda,\ (1p^{-1}_{3/2})_n,\ 0^+\rangle + \cos\theta\,|(1p_{1/2})_\Lambda,\ (1p^{-1}_{1/2})_n,\ 0^+\rangle,$$

where $\theta = 0.32$. Weak mixing is equivalent to a weak influence of the residual interaction on the level splitting. Therefore, in a first approximation, the latter is equal to the sum of the spin–orbit splittings of the corresponding neutron-hole and Λ-hyperon states.

A detailed analysis of the experimental data on the excitation of 0^+ states of the hypernucleus ${}^{16}_{\Lambda}$O with allowance for configuration mixing was made in Ref. 65. The energy splitting ΔE and the ratio ρ of the excitation intensities of the 0^+ states obtained after diagonalization of the residual interaction on the basis of the states $|(1p_{1/2})_\Lambda, (1p^{-1}_{1/2})_n, 0^+\rangle$ and $|(1p_{3/2})_\Lambda, (1p^{-1}_{3/2})_n, 0^+\rangle$ were calculated as functions of the spin–orbit splitting $\Delta\zeta$ of the basis states for different forms of the residual ΛN interaction. In Fig. 10, the experimental data are compared with the results of the calculation for single-particle oscillator wave functions ($\omega_N = 1.44\omega_\Lambda$) and the residual interaction

$$V_{\Lambda N} = v_0\delta(\mathrm{x}_\Lambda - \mathrm{x}_N)(1 - \varepsilon_x + \varepsilon_x P_x), \quad \varepsilon_x = 0.25. \quad (17)$$

The experimental splitting ΔE agrees with the calculated value in two $\Delta\zeta$ intervals. The interval $4.7 < \Delta\zeta < 6.1$ corresponds to the case when the hyperon–nucleus spin–orbit interaction is much weaker than the nucleon–nucleus interaction. If $-8.3 < \Delta\zeta < -5.9$, then the spin–orbit potential for the Λ hyperon is approximately twice as intense as the nucleon–nucleus potential and differs from it in sign. Comparison with the admissible $\Delta\zeta$ values for which the observed ratio of the excitation cross sections is ensured rules out the second possibility; therefore, $\Delta\zeta = 5.4 \pm 0.7$ MeV. Hence, the intensity of the spin–orbit interaction is found to be

$$U_\Lambda^{ls} = (2 \pm 2)\ \text{MeV}.$$

This conclusion is unchanged by variation of the form of the residual interaction (see also Ref. 36).

A similar analysis of the excitation spectrum of the hypernucleus ${}^{40}_{\Lambda}$Ca confirms the results (see Fig. 9). The energy splitting of the 0^+ states produced as a result of the quasi-elastic transitions $(1d_{3/2}, 1d^{-1}_{3/2})^{0+}_{\Lambda n}$ ($E_\Lambda = 0$ MeV) and $(1d_{5/2}, 1d^{-1}_{5/2})^{0+}_{\Lambda n}$ ($E = 5$ MeV) agrees with the energy difference of the neutron-hole states $(1d^{-1}_{3/2})_n$ and $(1d^{-1}_{5/2})_n$ in the ^{40}Ca nucleus: $\Delta E = 5$ MeV.[34]

Energies of the Λ-Hyperon Shells and the Central Part of the Hyperon–Nucleus Potential

An estimate of the intensity of the potential of the central Λ-hyperon–nucleus interaction using the binding energies of the ground states of light hypernuclei and the upper limits for the binding energies of heavy hypernuclei gives the value $U_\Lambda \simeq 30$ MeV,[6] i.e., about 3/5 of the corresponding quantity for the nucleon–nucleus potential. If the motion of the Λ hyperon is indeed determined by an effective field produced by the nucleons of the nucleus, then the distance between the Λ-hyperon shells must correspond to the value of U_Λ given above. As was noted by Wilkinson (see Ref. 66), the question of the existence of an effective field for the Λ hyperon is important for understanding the nature of the quasiparticle excitations in the nucleus (in the shell model, the existence of an effective field for the nucleons is to a large degree ensured by the Pauli principle).

The shell structure of Λ hypernuclei was considered in the weak-coupling approximation in Ref. 34. The distance between the 1s and 1p shells for the Λ hyperon in the hypernucleus ${}^{16}_{\Lambda}$O (see Fig. 9) is determined from the energy differences of the transitions $(1s_{1/2}, 1p^{-1}_{3/2})^{1-}_{\Lambda n}$ and $(1p_{3/2}, 1p^{-1}_{3/2})^{+}_{\Lambda n}$ or $(1s_{1/2}, 1p^{-1}_{1/2})^{1-}_{\Lambda n}$ and $(1s_{1/2}, 1p^{-1}_{1/2})^{0+}_{\Lambda n}$ and is 11 MeV, approximately half the corresponding value for the nucleon. Another example is the distance between the 1p, 1d, and 1f shells in the ^{40}Ca hypernucleus identified from the transitions $(1p_{1/2}, 1d^{-1}_{3/2})^{1-}_{\Lambda n}$, $(1d_{3/2}, 1d^{-1}_{3/2})^{0+}_{\Lambda n}$, $(1d_{5/2}, 1d^{-1}_{5/2})^{0+}_{\Lambda n}$ and $(1f_{7/2}, 1d^{-1}_{5/2})^{1-}_{\Lambda n}$; it is approximately 9 MeV, which is again close to half the distance between the nucleon shells.

Detailed calculations of the excitation spectra of hypernuclei in the (K^-, π^-) reaction[37] confirm the possibility of using a local single-particle Λ-hyperon–nucleus interaction potential to describe the hypernuclear excitations with the Λ hyperon in different shells. The accuracy of such a description, in which the single-particle levels and the wave functions are generated by a potential that does not depend on the states of the Λ hyperon, is limited, in particular, by the effects of polarization of the nucleon core.[62,67] In connection with the possible existence of an effective field for the Λ hyperon, it is of great interest to investigate the structure of hypernuclei with a deformed nucleon core.[67] The connection between the hyperon–nucleus interaction and the elementary ΛN interaction was considered in Refs. 3, 68, and 69. The energies of the Λ-hyperon shells were calculated for different single-particle potentials in Refs. 49 and 70.

3. LOW-LYING EXCITATIONS OF LIGHT HYPERNUCLEI

The hypernuclear states with the Λ hyperon in the 1s shell are the simplest from the point of view of describing the hypernuclear degrees of freedom, and therefore the ΛN interaction can be investigated in this case in some detail. In the weak-coupling approximation, the wave functions of such states have the form[6)]

$$|{}^{A}_{\Lambda}Z,\ J,\ \alpha\rangle=[|{}^{A-1}Z,\ \alpha\rangle\otimes|(1s_{1/2})_{\Lambda}\rangle]_{J}. \tag{18}$$

Here, α denotes the quantum numbers of the nucleon core: $\alpha=(E, J^{\pi}_{N}, T)$. If the residual ΛN interaction is taken into account in the first order of perturbation theory, the corrections to (18) must above all include additional excitations of the nucleon core with $\alpha'\neq\alpha$, since the energy intervals between them are much less than the distance between the single-particle 1s and 1p levels for the Λ hyperon in the nucleus:

$$|{}^{A}_{\Lambda}Z,\ J,\ \alpha\rangle=[(|{}^{A-1}Z,\ \alpha\rangle + \sum_{\alpha'\neq\alpha}\frac{\langle {}^{A-1}Z,\ \alpha'|V_{\text{tot}}|{}^{A-1}Z,\ \alpha\rangle}{E_{\alpha}-E_{\alpha'}}|{}^{A-1}Z,\ \alpha'\rangle)\otimes|(1s_{1/2})_{\Lambda}\rangle]_{J}. \tag{19}$$

Here, $V_{\text{tot}}=\langle(1s_{1/2})_{\Lambda}|V_{\Lambda N}|(1s_{1/2})_{\Lambda}\rangle$ is the residual ΛN interaction averaged over the Λ-hyperon density. The effects of the polarization of the nucleon core due to V_{tot} play the largest part for the low-lying collective ${}^{A-1}Z$ states. It is well known[55,56,73] that the low-lying excitations of light nuclei (of the 1p shell) can be successfully described in the many-particle shell model with an intermediate type of coupling of the angular momenta. Therefore, the extension of this model to light hypernuclei is of great importance for the study of hypernuclei outside the framework of the weak-coupling approximation.

A many-particle model of 1p-shell hypernuclei was developed very actively by Gal, Soper, and Dalitz in Refs. 15, 16, 74, and 75 (see also Refs. 76 and 77), who investigated in detail the parts played by the various components of the residual ΛN interaction (spin–spin, spin–orbit, tensor) in forming the spectra of individual hypernuclei. Much attention was paid to the theoretical study of γ transitions between the hypernuclear levels and of the $(K^-, \pi^-\gamma)$ reaction.

It is known that a reliable set of parameters of a residual interaction cannot be obtained without detailed experimental information on the low-lying excitations; the same applies to hypernuclei. Therefore, the existing calculations of hypernuclear spectra must be treated with caution, since it is difficult to gauge the degree of arbitrariness in them.[50] For example, on the basis of the calculations of Refs. 15 and 16 a residual ΛN interaction with a very complicated structure was proposed, its parametrization being done with allowance for only the binding energies of the ground states of the 1p-shell hypernuclei; moreover, the results of the parametrization varied appreciably when B_{Λ} was varied within the experimental errors. Since the phenomenological parameters serve more or less to correct the imperfection of the model, it is of interest to have different principles for choosing the residual interaction. In particular, a ΛN potential with a comparatively simple form satisfactorily describing the available data on low-energy Λp scattering and the binding energies of the ground states of hypernuclei was used in Ref. 77.

6)The simplest model of hypernuclear excitations, the model of excitations of the nuclear core, derives from the work of De Shalit.[71,72]

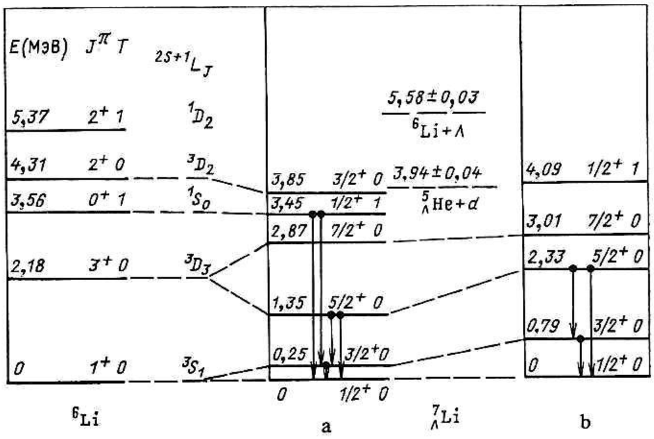

FIG. 11. Spectrum of the levels of the ^{6}Li nucleus and theoretical spectra of the low-lying excitations of the hypernucleus $^7_\Lambda$Li in accordance with Ref. 74 (a) and Ref. 77 (b). The arrows indicate the strongest γ transitions.

We consider as an example of a theoretical calculation the spectrum of the hypernucleus $^7_\Lambda$Li, which is the first of the 1p-shell hypernuclei having excited states stable with respect to nuclear decays. It is well known that the wave functions of the low-lying states of the nuclei at the beginning of the 1p shell can be well reproduced by a single LS configuration: $|(1s)^4_N(1p)^{A-4}_N[f]\ LSJT\rangle$.[73] To classify the corresponding hypernuclear levels, it is expedient to use the LS coupling scheme. Moreover, this scheme is convenient for analyzing the (K^-, π^-) reaction at small momentum transfers, when the amplitudes of the spin-flip transitions are negligibly small. To the low-lying configurations there corresponds the transition $(1s, 1p^{-1})^{1-}_{\Lambda n}$ with selection rules $|\mathbf{L}_i-\mathbf{L}_f|=1$, $\pi_f=-\pi_i$, $S_j=S_i$, where L, π, and S are, respectively, the orbital angular momentum, the parity, and the spin of the target nucleus (i) and the hypernucleus (f).

The spectrum of low-lying excitations of the hypernucleus $^7_\Lambda$Li, obtained in accordance with the calculations of Refs. 74 and 77, is shown in Fig. 11 together with the level scheme of the nucleon core ^{6}Li. Table IV gives the LS structure of the various $^7_\Lambda$Li states, and also the relative probability of their production in the (K^-, π^-) reaction at $q=0$, calculated in the limit of LS coupling and using the exact wave functions in the intermediate-coupling scheme.[74] The target nucleus has $^2P_{3/2}$ as a dominant configuration, and only ^2S and ^2D states can be excited in the limit of LS coupling. As can be seen from Table IV, the excitation probabilities calculated for this case differ little from the results of the exact calculation. As regards the energies of the hypernuclear levels, their values vary appreciably from model to model (cf. Figs. 11a and 11b). It may, however, be expected that the levels $J^{\pi}=1/2^+, 3/2^+, 5/2^+, 7/2^+$ $(T=0)$ will be stable with respect to strong decays.

The doublet of hypernuclear levels $(J\pi T)=(1/2^+0)$, $(3/2^+0)$ has a nucleon core with the structure $^6\text{Li}_{\text{gs}}$ $(L_N=0, S_N=1)$, and therefore the doublet splitting $E(3/2^+0)E(1/2^+0)$ is determined by the spin–spin ΛN interaction and is not very sensitive to the two-particle spin–orbit forces. The latter may play an important part in the splitting of the levels $(5/2^+0)$ and $(7/2^+0)$, since here the main component of the

TABLE IV. Low-lying $J^{\pi}T$ levels of the hypernucleus ${}^{7}_{\Lambda}$Li. E is the excitation energy, ${}^{2S+1}L$ is the dominant configuration, w are the relative probabilities of production in the (K^-, π^-) reaction at $q = 0$ in the limit of LS coupling (LS) and in the intermediate-coupling scheme (IC).[74]

J^{π} T	E, MeV	${}^{2S+1}L$	w (LS)	w (IC)
$1/2^+$ 0	0	${}^{2}S$	5/12	0,360
$3/2^+$ 0	0,25	${}^{4}S$	—	0,001
$5/2^+$ 0	1,35	$-\frac{\sqrt{7}}{3}\,{}^{2}D - \frac{\sqrt{2}}{3}\,{}^{4}D$	7/30	0,277
$7/2^+$ 0	2,87	${}^{4}D$	—	—
$1/2^+$ 1	3.45	${}^{2}S$	5/36	0,148
$3/2^+$ 0		$\sqrt{\frac{1}{2}}\,({}^{2}D + {}^{4}D)$	1/60	0,022
$3/2^+$ 1		${}^{2}D$	1/90	0,013
$5/2^+$ 1		${}^{2}D$	1/10	0,087

nucleon core has the nonvanishing orbital angular momentum $L_N = 2$. Observation of γ transitions in the hypernucleus ${}^{7}_{\Lambda}$Li would be of great value for determining the parameters of the ΛN interaction.

Recently, data were obtained at BNL on the $(K^-, \pi^-\gamma)$ reaction on the nucleus ^{7}Li that indicated the existence of a γ transition in the hypernucleus ${}^{7}_{\Lambda}$Li with energy $E_{\gamma} = 2$ MeV.[14] The intensity of the γ line agrees well with the theoretical value for the transition $5/2^+ \to 1/2^+$, and the value of E_{γ} is close to the splitting of the corresponding levels of the nucleon core: $\Delta E(1^+ - 3^+) = 2.18$ MeV. Earlier, there was a report[12] of observation of a γ transition with energy $E_{\gamma} = 0.789 \pm 0.004$ MeV accompanying the (K^-, π^-) reaction on the nucleus ^{7}Li ($p_K = 1.7$ GeV/c), but the theoretical interpretation of this result is not entirely unambiguous (see Refs. 74 and 75).

Other examples of spectra of light hypernuclei of the $1p$ shell can be found in the review of Ref. 75, in which γ transitions between hypernuclear levels are also considered in detail.[7)] The hypernuclei of the $1s$ shell with $A = 4$ have been investigated in the framework of an exact solution of the 4-body problem with potentials constructed on the basis of the one-boson exchange potential from the studies of Ref. 78. The Coulomb corrections to the binding energies and the effects of the violation of isotopic invariance in the hypernuclear isodoublet with $A = 4$ were estimated in Refs. 79–81.

It follows from the analysis of the Λ hypernuclei with $A = 4$ in the framework of $SU(6)$ symmetry[82] that the double hypernuclei ${}^{4}_{\Lambda\Lambda}$H and ${}^{4}_{\Lambda\Lambda}$He apparently do not form bound states. The possible existence of a ${}^{7}_{\Lambda}$He hypernuclear isomer was discussed in Refs. 7 and 83.

4. STRANGENESS ANALOG STATES

We already encountered the simplest examples of strangeness analog states in Sec. 1 when considering the coherent production of hypernuclei in the approximation of closed nucleon shells. In this case, the hypernuclear excitations have the structure $|1p - 1h\rangle$ and the strangeness analog states are states of the type $|(nlj)_{\Lambda}, (nlj)_n^{-1}, 0^+\rangle$ (in the jj-coupling scheme). The present section is devoted to the description of the SAS structure beyond the approximation of closed shells. We shall concentrate our main attention on light $1p$-shell nuclei for the following reasons.

1. Strangeness analog states are dominant in the excitation spectra of light hypernuclei at small momentum transfers q (see Table II).

2. In light hypernuclei, the number of particles is not yet so large as to make many-particle shell calculations a hopeless undertaking, and, therefore, there is still the possibility of making a detailed analysis of the hypernuclear structure.

3. The possibility of studying the fractional-parentage connection between the target nucleus and the various states of the nucleon core of the hypernucleus in the strangeness-exchange reaction at small q is of practical interest for the study of neutron hole states.

4. For reliable identification of γ transitions in the planned γ-spectroscopic experiments with light hypernuclei it is, in particular, necessary to know the spectrum of the lowly excited secondary hypernuclei produced by the strong decays of strangeness analog states.

The strangeness analog states are hypernuclear configurations obtained from the ground state of the nucleus, ${}^{A}Z_{gs}(J_i)$, as a result of coherent transitions in individual shells:

$$|{}^{A}_{\Lambda}Z_{SAS}\rangle = N^{-1/2}\int d^3x\,\hat{\psi}_{\Lambda}(\mathbf{x})\,\psi_n(\mathbf{x})|{}^{A}Z_{gs}\rangle \simeq$$
$$\simeq N^{-1/2}\sum_{nljm}\hat{b}^{+}_{\Lambda}(nljm)\hat{a}_n(nljm)|{}^{A}Z_{gs}\rangle = N^{-1/2}\sum_{nl}|{}^{A}_{\Lambda}Z_{SAS\,(nl)}\rangle \tag{21}$$

$$|{}^{A}_{\Lambda}Z_{SAS\,(nl)}\rangle = \sum_{jm}\hat{b}^{+}_{\Lambda}(nljm)\,\hat{a}_n(nljm)\,|{}^{A}Z_{gs}\rangle =$$
$$= \sum_{J\alpha,\,j} g^{J\alpha}_{nlj}\,[\,|{}^{A-1}Z;\,J,\,\alpha\rangle \otimes |(nlj)_{\Lambda}\rangle]_{J_i}. \tag{22}$$

[*Translation Editor's Note*. There is no Eq. (20) in the Russian article.]

Here, $\hat{a}_n(nljm)$ and $\hat{b}^{+}_{\Lambda}(nljm)$ are the operators of annihilation of a neutron and creation of a Λ hyperon in the single-particle shell states with quantum numbers $nljm$, and $g^{J\alpha}_{nlj}$ are the coefficients of fractional parentage:

[7)] As was shown in Ref. 75, the distribution of the γ rays over the angle of emission θ_{γ} with respect to the collinear momenta of the K^- and π^- can be used to identify γ transitions in the $(K^-, \pi^-\gamma)$ reaction.

$$g_{nlj}^{J\alpha} = \langle {}^{A-1}Z;\ J,\ \alpha \,||\, \hat{a}_n\,(nlj)\,||\, {}^{A}Z_{gs}\rangle.$$

In more detail, suppose we have chosen a suitable basis for the nucleon states, for example, the basis formed by the configurations $|[f],\ LSTJ\rangle$. Then the coherent transitions $n \rightarrow \Lambda$ ($L_f = L$, $S_f = S$, $T_f = T \pm 1/2$, $[f]_f = [f]$), generate for each configuration of A nucleons analog configurations of the system of $A = 1$ nucleons and the Λ. Note that, because of the change in the symmetry properties of the wave function on the appearance of the particle not identical to the remainder, there exist hypernuclear configurations that are not analogs of any nucleon configuration. In general, the basis nucleon functions and analog configurations are not identical to the eigenfunctions of the Hamiltonians of the nucleus and the hypernucleus. If some nucleon configuration is dominant in the wave function of the target nucleus, then the corresponding analog configurations play the part of doorway states[84] in the (K^-, π^-) reaction at small momentum transfers. When hypernuclear states containing these analog configurations with significant probability are localized in relatively narrow energy intervals, the presence of the strangeness analog states leads to intermediate structures in the energy dependence of the excitation cross sections.

Supermultiplet Structure of the Strangeness Analog States

To go beyond the closed-shell approximation even in the weak-coupling limit, we require a very detailed description of the state of the nucleon core, this being so primarily because of the nontrivial symmetry properties of the many-particle wave function. It is known that the excitation spectra of the light nuclei at the start of the $1p$ shell have a supermultiplet structure associated with the important part played by the Majorana term in the residual NN interaction.[85] The monopole part of the Majorana forces is the Casimir operator of the group of permutations of the orbital coordinates (within the shell), and this explains the appearance of the Young tableau $[f]$ of the orbital symmetry as a good quantum number. States with different $[f]$ differ appreciably in energy and are weakly mixed (this is manifested particularly clearly in the well-known "fouring" effect[85]). Since the weak-coupling limit is a good initial approximation for describing the hypernuclei, the supermultiplet structure must also be manifested in the properties of the $1p$-shell hypernuclei.

The part played by the supermultiplet structure in the production and decay of strangeness analog states for $1p$-shell hypernuclei was investigated in Refs. 86 and 87. In this case, the coherent transitions lead to excitations of the type $1\hbar\omega$ in terms of the oscillator shell model. Since it is necessary to take care to eliminate the "spurious" states when one is working with $1\hbar\omega$ excitations, the translationally invariant shell model is used.[85]

Following Ref. 87, we consider first the qualitative results that are obtained in the approximation of doorway states (residual interaction switched off).

Formation of Strangeness Analog States. The ground-state wave function of the target nucleus can be approximated by a single configuration:

$$|\,i\rangle = |\,(1s)_N^4,\ \ (1p)_N^k,\ [f]\,L_iS_iT_iJ_i\rangle.$$

To hypernuclear $1\hbar\omega$ excitations there correspond configurations of two types:

$$\left.\begin{array}{l} |\,(1s)_N^4\,(1p)_N^{k-1}\,(1p)_\Lambda\rangle \\ |\,(1s)_N^3\,(1p)_N^k\,(1s)_\Lambda\rangle \end{array}\right\} = |\,(1s)_N^{3+l_\Lambda}\,(1p)_N^{k-l_\Lambda}\,[f_N]\,L_NS_NT_N;$$

$$l_\Lambda,\ L_iS_iJ_i\rangle.$$

The amplitude of coherent transition is proportional to the coefficient of fractional parentage

$$\langle (1s)^4\,(1p)^k\,[f_i]\,L_iS_iT_i\,||\,l_\Lambda;\,(1s)^{3+l_\Lambda}\,(1p)^{k-l_\Lambda}\,[f_N]\,L_NS_NT_N\rangle.$$

Thus, fragmentation of the strangeness analog states over the supermultiplets already occurs in the doorway-state approximation. For $q = 0$ (without allowance for absorption) the effective number of neutrons is determined by

$$N_{\text{эф}}\,([f_N]\,L_NS_NT_N,\ l_\Lambda) = A\,|\,C_{T_NT_N^3\,1/2\,-1/2}^{T_iT_i^3}\,|^2$$

$$\times\,|\,\langle (1s)^4\,(1p)^k\,[f_i]\,L_iS_iT_i\,||\,l_\Lambda;$$

$$(1s)^{3+l_\Lambda}\,(1p)^{k-l_\Lambda}\,[f_N]L_NS_NT_N\rangle\,|^2. \qquad (23)$$

For the transition to different supermultiplets characterized by Young tableau $[f_N]$ of the orbital symmetry of the nucleon core and orbital state l_Λ of the Λ hyperon, the effective numbers of neutrons calculated in accordance with (23) for a number of $1p$-shell nuclei are given in Table V.

Experimental Observation of Strangeness Analog States in the (K^-, π^-) Reaction on Light Nuclei

The excitation cross sections of hypernuclei at the beginning of the $1p$ shell, ${}^6_\Lambda$Li, ${}^7_\Lambda$Li, ${}^9_\Lambda$Be, ${}^{12}_\Lambda$C, were measured in the (K^-, π^-) reaction at kaon beam momenta $p_K = 720$–790 MeV/c and zero pion emission angle by the HSS group at CERN[41] (see Fig. 7). A preliminary DWBA analysis[57] showed that the weak-coupling model makes it possible to obtain satisfactory quantitative agreement between the theoretical calculations and the experimental excitation spectra of the light hypernuclei. The supermultiplet structure of the strangeness analog states is here manifested fairly clearly.

${}^6_\Lambda$Li. Two strong peaks in the ${}^6_\Lambda$Li excitation spectrum at $E_\Lambda = 4$ MeV and $E_\Lambda = 14$ MeV are interpreted as coherent transitions in the $1p$ and $1s$ shells. In accordance with Table V, the resulting states have the supermultiplet structure $[41]p_\Lambda$ and $[32]s_\Lambda$ (the probability of transition to the supermultiplet $[41]s_\Lambda$ is small). The distance beween the $1s$ and $1p$ shells for the Λ hyperon is $E((1p)_\Lambda) - E((1s)_\Lambda) = 8$ MeV, and the distance between the $1p$- and $1s$-neutron holes for ^{6}Li is approximately 16 MeV, so that the weak-coupling model predicts for the difference between the excitation energies of the considered states the value 8 MeV, in satisfactory agreement with the experimental value $\Delta E = 10$ MeV. The intensity ratio of the peaks is also close to the calculated ratio 1.20:1.67 (the naive estimate 1:2 is the ratio of the numbers of neutrons in the $1p$ and $1s$ shells in ^{6}Li).

${}^7_\Lambda$Li. Here again two strong peaks are observed, corresponding to coherent transitions in the $1p$ and $1s$ shells to the

TABLE V. Fragmentation of strangeness analog states in the approximation of doorway states.[87] $[f]$ is the Young tableau for the parent nucleus, $[f_N]l_\Lambda$ is the SAS structure, N_{eff} is the effective number of neutrons that participate in the formation of the strangeness analog state (without allowance for the meson–nucleus interaction, $q = 0$), E is the excitation energy of the hypernucleus (from the experiment of Ref. 41), and E_c is the energy release in the channel with emission of particle c.

${}^A_\Lambda Z$	$[f]$	$[f_N]\,l_\Lambda$	N_{eff}	E, MeV	E_c, MeV
${}^6_\Lambda Li$	[42]	[41] p_Λ	1,20	8,5	$E_\Lambda=4$; $E_p=9$
		[32] s_Λ	1,67	18,5	$E_d=0,2$
		[41] s_Λ	0,13	—	$E_p=19$
${}^7_\Lambda Li$	[43]	[42] p_Λ	2,33	9,1	$E_\Lambda=3,5$; $E_p=3,1$
		[42] s_Λ	0,67	21,1	$E_p=15,1$
		[33] s_Λ	1,00	—	$E_t=2,1$
${}^9_\Lambda Be$	[441]	[44] p_Λ	1,41	12,7	$E_\Lambda=6$
		[44] s_Λ	0,09	—	—
		[431] p_Λ	1.97	23.7	$E_p=6,5$
		[431] s_Λ	1.53	29.7	$E_p=12.5$; $E_t=5,9$
${}^{12}_\Lambda C$	[444]	[443] p_Λ	4.64	11,0	$E_\Lambda=0,2$; $E_p=1.2$
		[443] s_Λ	1.36	21,0	$E_p=11.2$

multiplets $[42]p_\Lambda$ ($E'_\Lambda = 3.5$ MeV) and $[33]s_\Lambda$ ($E_\Lambda = 15.5$ MeV). The distance between them agrees with the estimate in the weak-coupling model: $\Delta E = 12$ MeV. The lower peak is noticeably stronger than the upper one, in complete agreement with the theoretical calculations and in contrast to the naive estimate 1:1. The supermultiplet $[42]s_\Lambda$ does not appear to have a clear structure and merges with the background.

${}^9_\Lambda$Be. This hypernucleus is of particular interest, since it clearly demonstrates the part played by the supermultiplet structure for coherent transitions in one shell. In the given case, the coherent transitions in the $1p$ shell lead with comparable probabilities to the states $[44]p_\Lambda$ and $[431]p_\Lambda$ (see Table V). The latter is distinguished from the former by a broken Young tableau and, therefore, an appreciable excitation energy of the nucleon core (more than 10 MeV). Coherent transition in the $1s$ shell makes an appreciable contribution only to the excitation of the states $[431]s_\Lambda$. Experimentally, two strong peaks are observed (Fig. 7); the peak at $E_\Lambda = 6$ MeV corresponds to the supermultiplet $[44]p_\Lambda$, while the peak at $E_\Lambda = 17$ MeV must be the result of superposition of transitions to the supermultiplets $[431]p_\Lambda$ and $[431]s_\Lambda$.

${}^{12}_\Lambda$C. The case of closed subshells was considered above. Note that the effective numbers of neutrons (without allowance for absorption) for coherent transitions in different shells, calculated in the translationally invariant shell model, are not equal to the numbers of neutrons in these shells (see Table V).

Clustering Effects and SAS Decays

The majority of the strangeness analog states are situated in the region of the continuum and are unstable with respect to strong decays. The calculation of the widths of such states is, on the one hand, of spectroscopic interest, since comparison of the theoretical and experimental values of the widths is important in deciphering the hypernuclear structure. On the other hand, the decay products may again be hypernuclei, and this is important for the γ spectroscopy of hypernuclei. Spectroscopic factors are widely used[58,86] to describe the decay properties of hypernuclei. The corresponding technique has been well developed[85,88] in nuclear shell models (in particular, in the translationally invariant shell model). The partial width of decay through the two-particle channel is determined by

$$\Gamma_c = 2\,kP\gamma_c^2,$$

where k is the relative momentum of the particles in channel c, P is the penetrability, and γ_c^2 is the reduced width. The spectroscopic factor S_c relates γ_c^2 to the single-particle reduced width $\gamma_0^2 = \hbar^2/M_c a_c$:

$$\gamma_c^2 = S_c\gamma_0^2,$$

where M_c is the reduced mass of the particles in channel c, and a_c is the channel radius. We shall consider different types of decays and illustrate them by examples for $1p$-shell hypernuclei.

${}^A_\Lambda Z \to {}^{A-1}Z + \Lambda$. This decay (if it is allowed energetically) is characteristic for strangeness analog states formed by coherent transitions in the outer shell ($[f]p_\Lambda$). If the decay threshold is not too close (as, for example, in the case of ${}^{12}_\Lambda$C), the corresponding decay width $\Gamma^\uparrow$ is of the order of several mega-electron-volts. Quite different is the situation for analog states constructed on the $1s$ shell, namely, the spectroscopic amplitude for Λ-hyperon emission is zero, and such states can decay only by virtue of the admixture of p_Λ configurations or through rearrangement processes (emission of nucleons and nucleon clusters). An example is the strangeness analog state in ${}^{6}_{\Lambda}$Li—the peak at $E_\Lambda = 14$ MeV corresponds to the state $[32]s_\Lambda$ and has width $\Gamma \simeq 1$ MeV.

${}^{A}_{\Lambda}Z \to {}^{A_1}_{\Lambda}Z_1 + {}^{A_2}Z_2$. The spectroscopic amplitude of such a decay can be expressed in the translationally invariant shell model in terms of the spectroscopic amplitude of the corresponding decay ${}^{A-1}Z \to {}^{A_1-1}Z_1 + {}^{A_2}Z_2$ of the nucleon core. In particular, for the strangeness analog state with the $1\hbar\omega$ s_Λ configuration[87]

$$\frac{SA\left({}^{A}_{\Lambda}Z \to {}^{A_1}_{\Lambda}Z_1 + {}^{A_2}Z_2\right)}{SA\left({}^{A-1}Z \to {}^{A_1-1}Z_1 + {}^{A_2}Z_2\right)} = \left(1 - \frac{A_2}{(A-1)\,A_1}\right)^{k/2}$$

($k = A - 4$ is the number of nucleons in the outer $1p$ shell).

In the analysis of the strong decays of hypernuclei, the Young-tableau selection rules are helpful.[58,86,87] Let $[f]$, $[f_1]$, and $[f_2]$ be the Young tableaux for the orbital part of the wave functions of the nucleons for ${}^{A}_{\Lambda}Z$, ${}^{A_1}_{\Lambda}Z_1$, and ${}^{A_2}Z_2$. Then a necessary condition for the decay ${}^{A}_{\Lambda}Z \to {}^{A_1}_{\Lambda}Z_1 + {}^{A_2}Z_2$ is the presence of $[f]$ in the decomposition of the outer product $[f_1] \otimes [f_2]$ into irreducible representations. For example, for the strangeness analog states considered above with the structure $[32]s_\Lambda$ and $[33]s_\Lambda$ the energetically advantageous decays ${}^{6}_{\Lambda}\mathrm{Li} \to {}^{5}_{\Lambda}\mathrm{He} + p$ and ${}^{7}_{\Lambda}\mathrm{Li} \to {}^{6}_{\Lambda}\mathrm{He} + p$ are suppressed by the selection rules $[32] \not\to [14] \otimes [1]$ and $[33] \not\to [41] \otimes [1]$. The cluster decays $[32] \to [3] \otimes [2]$ and $[33] \to [3] \otimes [3]$ are allowed, but the thresholds of the corresponding channels are situated much higher. Thus, the state $[32]s_\Lambda$ of the hypernucleus ${}^{6}_{\Lambda}$Li lies only 0.2 MeV above the threshold of the first open cluster channel ${}^{4}_{\Lambda}\mathrm{He} + d$ and as a result is fairly narrow ($\Gamma = 0.7^{+1}_{-0.7}$ MeV (Ref. 41)).

Since the effects of the coupling to the continuum may lead to an appreciable broadening and shift of the hypernuclear levels, particular interest attaches to methods of calculation in which the presence of open channels is taken into account explicitly in the consideration of the structure of the hypernuclear states. In this respect, it is promising to apply the ideas of the unified theory of the excitation of the nuclei in the region of the continuum[40] to the description of the (K^-, π^-) reaction. Continuum effects in the excitation of light hypernuclei were considered in Refs. 89 and 90.

Following Ref. 89, we consider the application of the Green's-function method to take into account the continuum in the $\Lambda + {}^{A-1}Z$ channel. In the closed-shell approximation, the hypernuclear system is described by the Green's function $G(E)$ of the configuration (Λ, N^{-1}); it satisfies the equation

$$G(E) = G_0(E) + G_0(E)\,V_{\mathrm{ph}}\,G(E). \qquad (24)$$

Here

$$G_0(E) = \sum_h \varphi_h^*(\mathbf{x}'_N)\,\langle \mathbf{x}'_\Lambda \,|\, (E - H_0 - \varepsilon_h)^{-1} \,|\, \mathbf{x}_\Lambda \rangle\, \varphi_h(\mathbf{x}_N)$$

is the Green's function for the system consisting of the nucleon hole and the Λ hyperon without allowance for the residual ΛN interaction, and $\varphi_h(\mathbf{x})$ are the wave functions of the single-particle nucleon states. The matrix element of the Green's function $(E - H_0)^{-1}$ can be expressed in closed form in terms of the regular and irregular solutions of the Schrödinger equation for the Λ hyperon in the hyperon–nucleus potential. Equation (24) with the residual interaction $V_{\mathrm{ph}} = v_0\delta(\mathbf{x}_N - \mathbf{x}_\Lambda)$ was solved numerically in the configuration space. The poles of $G(E)$ lying on the second sheet of the complex E plane determine the positions and widths $\Gamma^\uparrow$ of the hypernuclear resonances. Calculations made for ${}^{6}_{\Lambda}$Li, ${}^{7}_{\Lambda}$Li, and ${}^{9}_{\Lambda}$Be give the following values for the widths of the decays through the channel $\Lambda + {}^{A-1}Z$: $\Gamma^\uparrow = 7$–10 MeV for states of the type $((1p)_\Lambda, (1p^{-1})_n)$ and $\Gamma \lesssim 1$ MeV for states of the type $((1s)_\Lambda, (1s^{-1})_n)$.

The extension of the configuration space of the traditional shell model by taking into account the continuum of single-particle states is an important step towards achieving a unified description of the resonance and direct processes in the (K^-, π^-) reaction. It is of greatest interest to use such models (continuum shell models) in the case of large momentum transfers—in the investigation of the angular dependences of the differential cross sections $d\sigma/d\Omega dE$,[90,91] and also in the analysis of alternative hypernucleus-generation reactions of the type (π^+, K^+) and (γ, K^+).

Detailed Calculations of SAS Structure in the Many-Particle Shell Model

A very detailed analysis of SAS structure for the example of the hypernucleus ${}^{6}_{\Lambda}$Li was made in Ref. 86. In this case, the number of configurations is not yet too large, and complete allowance for them makes it possible to demonstrate the difference between the many-particle approach and the simplest approaches in which the total fragmentation of the neutron holes is not considered. Since the experimentally observed excitations encompass a wide energy interval ($E = 0$–40 MeV), all $0\hbar\omega$ and $1\hbar\omega$ states were included in the basis ($\omega_N = \omega_\Lambda = 17$ MeV). The states belonging to the ground-state configuration have the form

$$|\,(1s)^4_N\,(1p)_N,\ (1s_\Lambda);\ J^\pi\,\rangle,\quad J^\pi = 1^-, 0^-, 1^-, 2^-.$$

The basis for the $1\hbar\omega$ excitations contains states of two types:

$$1)\ \left(\frac{5m_N}{5m_N + m_\Lambda}\right)^{1/2} |\,\varphi_1\,[41]\ {}^{22}P,\ \ p_\Lambda;\ \ {}^{2S+1}L\rangle -$$

$$-\left(\frac{m_\Lambda}{5m_N + m_\Lambda}\right)^{1/2} |\,\tilde{\varphi}_2\,[41]\ {}^{22}L;\ \ s_\Lambda;\ \ {}^{2S+1}L\rangle,$$

$$2)\ |\,\varphi_2\,[f]^{(2T_N+1)(2S_N+1)}L;\ \ s_\Lambda;\ \ {}^{2S+1}L\rangle.$$

Here, φ_1 is the ground-state configuration of the nucleon core, $\tilde{\varphi}$ is a "ghost" state of the nucleon core ($L = 0, 1, 2$), and φ_2 is a $1\hbar\omega$ excitation of the nucleon core in the translationally invariant shell model with quantum numbers $[41]^{22}S$, ${}^{22}D$, $[32]^{22}S$, ${}^{22}D$, ${}^{24}S$, ${}^{24}D$, $[311]^{22}P$, ${}^{24}P$. The residual interaction was diagonalized on the complete basis. The NN interaction contained a central part with exchange term of Rozenfeld type and spin–orbit and tensor parts and was fitted to the well-know $1/2^+$ and $3/2^+$ excitations of ^{5}Li. The residual ΛN interaction was chosen in the form

$$V_{\Lambda N} = v_0\,(1 - a_S + a_S P_\sigma)\left[(1 - a_x + a_x P_x) + \right.$$

$$\left. + \frac{v_T}{v_0}\,(1 - a_x^T + a_x^T P_x)\,\hat{O}_T\right] e^{-r^2/\mu^2},$$

where the Majorana exchange component a_x and the parameters v_T and a_x^T of the tensor interaction $\hat{O}_T$ were varied, while the parameters v_0 and a_S were fitted using the binding

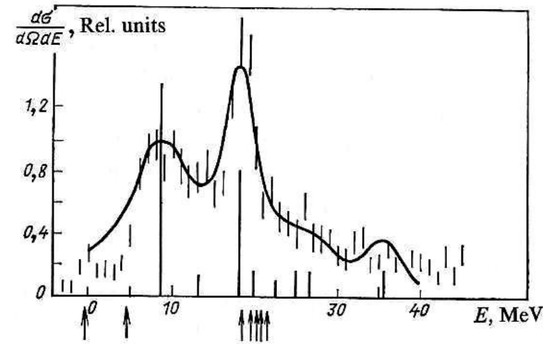

FIG. 12. Theoretical spectrum of excitations of the hypernucleus ${}^6_\Lambda$Li in the ^{6}Li(K^-, π^-)${}^6_\Lambda$Li reaction at $p_K = 720$ MeV/c and $\theta = 0°$.[86] The arrows indicate the thresholds of the channels (from left to right) ${}^5_\Lambda$He + p, ^{5}Li + Λ, ${}^4_\Lambda$He + d, ${}^4_\Lambda$He* + d, ${}^4_\Lambda$H + 2p, ^{3}He + ${}^3_\Lambda$H, + ${}^4_\Lambda$H* + 2p. The experimental data are taken from Ref. 57.

energies of the ${}^6_\Lambda$He and ${}^4_\Lambda$He ground states. The wave functions obtained for the hypernuclear states were used to calculate the cross section of the (K^-, π^-) reaction in the DWBA, a realistic wave function being taken for ^{6}Li. The results of the calculations for one of the parameter sets ($v_0 = -32.9$ MeV, $a_x = 0.6$, $a_S = -0.15$, $v_T/v_0 = 0.3$, $a_x^T = 0.5$, $\mu = 1.044$ F) are shown in Fig. 12. As one would expect, in the regime of recoilless production the main contribution to the cross section is made by 1^+ states, among which two states with $E = 8.4$ MeV and $E = 18$ MeV are dominant. Table VI gives the probabilities with which the different configurations occur in the wave functions of the 1^+ states.

The state with the excitation energy $E = 8.4$ MeV can be well described by the single configuration $|[41]^{22}P, p_\Lambda; {}^3S_1\rangle$, which corresponds to the main intensity of the transition to the supermultiplet $[41]p_\Lambda$. The total width of this state is determined by the decay through the channel $\Lambda + {}^5$Li. An appreciable fraction of the transition intensity to the supermultiplet [32] is concentrated in the state with $E = 18$ MeV (with principal component $|[32]^{24}S, s_\Lambda, {}^3S_1\rangle$). Its structure can also be represented in the form $[{}^5\text{Li}(3/2^+) \otimes (1s_{1/2})_\Lambda]_{1^+}$. The component $[32]s_\Lambda$ admits a cluster decay through the channel ${}^4_\Lambda$He + d [the corresponding spectroscopic factor is $S({}^4_\Lambda\text{He} + d) = 0.306$], but in the considered variant of the calculation the level is below the threshold of the ${}^4_\Lambda$He + d channel, which is at $E = 18.3$ MeV. When the parameters are varied, this channel may become open (the experimental value is $E = 18.5$ MeV), but the decay width will contain a small factor due to the proximity of the threshold. Decay of this state through the $p + {}^5_\Lambda$He channel is possible only by virtue of a small admixture (about 4%) of the component $|[41]^{22}D, s_\Lambda, {}^3D_1\rangle$.

To the supermultiplet $[32]s_\Lambda$ there also belong states with $E = 24.8$ MeV and $E = 26.4$ MeV, which decay predominantly through the ${}^4_\Lambda$He + d and ${}^4_\Lambda$He* + d channels. The decays through the ${}^4_\Lambda$He* + d channel are accompanied by a subsequent $1^+ \to 0^+$ gamma transition in the hypernucleus ${}^4_\Lambda$He. Such gamma transitions were observed in Refs. 9–11.

A state with $E = 35.6$ MeV has the configuration $|(1s)^3_N(1p)^2_N[311]^{22}P_{3/2}, s_\Lambda, {}^1P_1\rangle$, and a coherent transition to it is made possible by a P-wave admixture in the ^{6}Li ground-state wave function.

Thus, detailed calculations in the framework of the translationally invariant shell model make it possible to obtain a good description of the excitation spectra of hypernuclei in the (K^-, π^-) reaction. In addition, they confirm the validity of the qualitative results obtained in the approximation of doorway states (weak coupling) and demonstrate the further fragmentation of the strangeness analog states. For quantitative comparison of the theory with experiment, it is important to take into account the details of the structure of both the target nucleus and the produced hypernucleus. This last circumstance makes it possible to use hypernuclear spectroscopy to investigate the fractional parentage of nuclear states.

Strangeness Analog States of the Hypernucleus ${}^9_\Lambda$Be

The structure of the strangeness analog states of the hypernucleus ${}^9_\Lambda$Be was considered in detail in Ref. 50. This case is particularly interesting, since we have here for the first time "fouring" in the 1p shell, and at the same time the wave functions have a single dominant LS configuration (for nuclei with $N_{1p} \geqslant 6$ the jj coupling begins to operate). The target nucleus ^{9}Be($3/2^-$, 1/2) has the structure $|[441]^2P_{3/2}\rangle$ (the admixture of other configurations is about 30% (Ref. 73)).

TABLE VI. Strangeness analog states of ${}^6_\Lambda$Li(1^+).[86] The excitation energy is E; $d\sigma/d\Omega$ is the production cross section in the ^{6}Li(K^-, π^-)${}^6_\Lambda$Li reaction ($p_K = 800$ MeV/c, $\theta = 0°$); $w([f_N]l_\Lambda)$ are the probabilities of the configurations $[f_N]l_\Lambda$; $S(c)$ are the spectroscopic factors for decay through channel c.

E, MeV	$d\sigma/d\Omega$, mb/sr	$w(p_\Lambda)$	$w([41]s_\Lambda)$	$w([32]s_\Lambda)$	$w([311]s_\Lambda)$	$S({}^4_\Lambda\text{He}+d)$	$S({}^4_\Lambda\text{He}^*+d)$
8.4	1,36	0,97	0,03	0	0	0,001	0
13,0	0,15	0.99	0,01	0	0	0	0
18,0	0.84	0	0,04	0,92	0.04	0.306	0.213
19,7	0.18	0,04	0,82	0,13	0,01	0,037	0.014
22.4	0,11	0,01	0,33	0,27	0,39	0.040	0.033
24.8	0,18	0	0,17	0,82	0.01	0,142	0.299
26,4	0,18	0	0,03	0,93	0,04	0.108	0.158
35.6	0,19	0	0	0,03	0.97	0,005	0.004

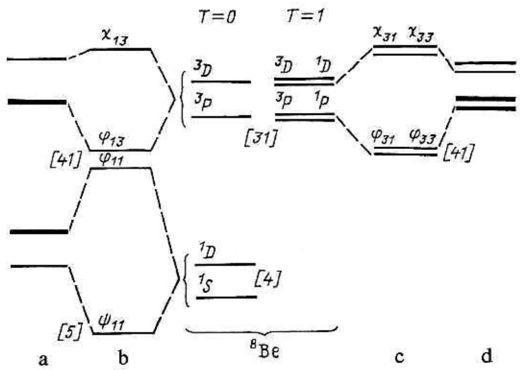

FIG. 13. Scheme of $^9_\Lambda$Be levels corresponding to the configurations $\{(1s)^4_N(1p)^4_N(1p)_\Lambda\}$: a), d) in the physical case; b), c) in the limit of $SU(6)$ symmetry. The Young tableaux $[f]$ of the orbital symmetry of the baryons in the $1p$ shell and the LS structure of the dominant configurations are indicated.[50]

The foregoing discussion of the supermultiplet structure of the strangeness analog states was restricted to classification in accordance with the Young tableaux $[f_N]$. For more detailed specification, we introduce in addition to $[f_N]$ the orbital angular momentum L_N of the nucleon system. The level scheme of the ^{8}Be nucleus without allowance for the spin–isospin dependence of the residual NN interaction is shown in Fig. 13. Following Refs. 31 and 50, we first determine the structure of the $^9_\Lambda$Be levels corresponding to the configurations $\{(1s)^4_N(1p)^4_N(1p)_\Lambda\}$ that would arise in the limit of Sakata $SU(3)$ symmetry. The aim of the approach is to identify the qualitative effects that could be important in $^9_\Lambda$Be excitations. This is particularly important in the absence of information about the ΛN interaction, when any particularization leads to uncontrollable uncertainties.

Thus, if we ignore the spin-dependent forces, we can speak of $SU(6)$ symmetry. There here arises in a natural manner a classification of the hypernuclear states in accordance with the Young tableau of orbital symmetry $[f]$ common to all particles of the $1p$ shell. Figure 13 shows the states with total orbital angular momentum $L = L_N(^9\mathrm{Be}_{gs}) = 1$ which arise on the decomposition of the direct product $([f_N]L_N)\otimes([1]p)$ with respect to functions of the type $([f]L)$. The strangeness analog states belong to the supermultiplet $[41]P$, which contains functions $^{2S+1}\varphi_{(2T+1)(2S_N+1)}$ with different values of the isospin T, nucleon spin S_N, and total spin S. The supermultiplet $[5]P$ contains the so-called supersymmetric state $\psi_{11}(T=0, S_N=0)$. The states $\chi_{(2T+1)(2S_N+1)}$ belong to the supermultiplet $[32]P$. The breaking of $SU(6)$ symmetry due to the difference between the NN and ΛN interactions even when allowance is made for the forces that depend on the spins of the particles gives rise to a splitting of the strangeness analog states, and through the fouring effect there are formed two groups of states differing in the Young tableaux of the orbital symmetry of the dominant nucleon configurations (see Fig. 13). The result of diagonalization of the phenomenological residual interaction on the basis of the functions $^{2S+1}\varphi_{(2T+1)(2S_N+1)}$ is given in Table VII.

The main contribution to the cross section of the (K^-, π^-) reaction at small q is made by three states—a "lower" state with dominant configuration $^2\varphi_{11}$ and two isospin-degenerate "upper" states with configurations $^2\varphi_{13}$ and $^2\varphi_{33}$. The experimental value $\Delta E = 11$ MeV of the energy splitting of the two peaks agrees with the theoretical value $\Delta E = 11.8$ MeV. The theoretical ratio of the cross sections for excitation of the upper states and the lower state is 1.5, in good agreement with the experimental value 1.4.

In accordance with the calculations of Ref. 50, the supersymmetric state $^9_\Lambda\mathrm{Be}(3/2^-)$ is shifted by about -5 MeV relative to the lower strangeness analog state. The structure $|(1s)^4_N(1p)^4_N[44], (1p)_\Lambda\rangle$ of this state permits it to decay readily through the channel $^8\mathrm{Be}+\Lambda$ (the width of the peak corresponding to it is $\Gamma\simeq 5$ MeV). For the upper strangeness analog states the transition to all the open channels is forbidden in the approximation of the principal components by the selection rule with respect to the Young tableau.[58,87] Because of this, the peak with $E_\Lambda = 17$ MeV in the spectrum of $^9_\Lambda$Be (see Fig. 7) has only the moderate width $\Gamma\simeq 6$ MeV despite the appreciable excitation energy.

An approach to the study of hypernuclear structure based on the use of $SU(3)$ symmetry is an alternative to the weak-coupling approximation. Therefore, its use is of the greatest interest in the cases when there is observed experimentally a large deviation from the predictions of the weak-coupling model and the probable cause of these deviations can be the partly preserved consequences of the $SU(3)$ symmetry. Such an example for the hypernucleus ^{13}C was considered in Sec. 5.

TABLE VII. The $3/2^-$ state of $^9_\Lambda$Be with isospin T in the region of excitation of the "upper" strangeness analog states.[50] The energy ΔE is measured from the lowest strangeness analog state; ρ is the relative probability of production in the (K^-, π^-) reaction at $q=0$ (for the lowest strangeness analog state, $\rho=1$); a is the amplitude of the dominant configuration indicated in the brackets.

T	ΔE, MeV	ρ	a
0	11,8 13,6	0,69 0,06	0,96 ($^2\varphi_{13}$) 0,96 ($^4\varphi_{13}$)
1	11,8 13,3 14,2	0,78 0,15 0,07	0,96 ($^2\varphi_{33}$) 0,88 ($^4\varphi_{33}$) 0,89 ($^2\varphi_{31}$)

Structure of Light Hypernuclei in the Nucleon-Association Model

Although the fouring effects can be explained qualitatively in the shell model, the large magnitude of these effects means that the shell wave functions for the nuclei at the start of the $1p$ shell must be strongly distorted. To describe the α association in ordinary nuclei, the nucleon-association model is successfully used[85,92]; it derives from Wheeler's resonating-group method.[93] In Ref. 94, the nucleon-association model was used to calculate the spectrum of the hypernucleus ${}^{9}_{\Lambda}$Be. The wave function of the hypernucleus was written in the form

$$\psi_L = \sum_{l\lambda} w_{l\lambda} \left[\varphi_{\alpha\alpha}^{(\lambda)}(l) \otimes \chi_{\lambda}^{(l)}(R)\, Y_{\lambda}(\mathbf{R}/R)\right]_L.$$

Here, $\varphi_{\alpha\alpha}^{(\lambda)}(l)$ is the wave function of an $\alpha\alpha$ system with orbital angular momentum l of the relative motion, constructed by the method of generator coordinates,[92] the function $\chi_{\lambda}^{(l)}$ characterizes the motion of the Λ hyperon in the field of the α clusters, λ is the orbital angular momentum of the Λ with respect to the center of mass of the $\alpha\alpha$ system, and $\mathbf{L} = \mathbf{I} + \boldsymbol{\lambda}$ (the Λ hyperon is regarded as a spinless particle). The nucleon–nucleon interaction was described by the Volkov potential, which reproduces the properties of ^{4}He and low-energy $\alpha\alpha$ scattering. A ΛN interaction potential of Gaussian type was fitted to the experimental value of the binding energy of the hypernucleus ${}^{5}_{\Lambda}$He. The system of equations for the functions $\varphi_{\alpha\alpha}^{(\lambda)}(l)$ and $\chi_{\lambda}^{(l)}(R)$ was solved iteratively (the initial approximation was the free $\alpha\alpha$ system). The results of the calculation are given in Table VIII.

For states belonging to the ground-state rotational band with $K = 0^+$, the weak-coupling approximation $[\alpha\alpha]_L \otimes \Lambda_{0^+}$ reproduces well the structure of the main term in the wave function, although a small admixture of configurations with $\lambda \neq L$ is important in the calculation of the energy levels (without allowance for the mixing of the configurations the binding energy of the ground state is $B_\Lambda = 5.1$ MeV, while the result of the exact calculation is $B_\Lambda = B_\Lambda^{\text{exp}} = 6.7$ MeV). The α-clustering effects lead to a significant enhancement of the transitions within a band; for example, for the probability of the $E2$ transition $2^+ \to 0^+$ the nucleon-association model gives the value $B(E2, 2^+ \to 0^+) = 12.3 e^2 \cdot \mathrm{F}^4$, which exceeds by six times the prediction of the shell model.

Coherent $(1p, 1p^{-1})^{0^+}_{\Lambda n}$ transitions in the ^{9}Be (K^-, π^-) ${}^{9}_{\Lambda}$Be reaction lead to the formation of states with $L^\pi = 1^-$. In the given case, there are two such states at $E_\Lambda = 0.46$ MeV ($K = 0^-$) and $E_\Lambda = 6.66$ MeV ($K = 1^-$), and both are characterized by strong mixing of the configurations $(l\lambda) = (01, 21)$ through the strong deformation of the nucleon core. Since the $K = 1^-$ band has a structure analogous to that of ^{9}Be, and the $K = 0^-$ band is not manifested in the ordinary nuclei because of the Pauli principle, the dominant contribution to the coherent production of ${}^{9}_{\Lambda}$Be is made by the upper 1_2^- state. Its energy agrees well with the experimentally measured position $E_\Lambda = 6$ MeV of the coherent peak. In the shell-model limit, the lower 1_1^- state corresponds to the supersymmetric state ($[f] = [54]$). The splitting $\Delta E = 6.2$ MeV of the 1^- states in the nucleon-association model is close to the theoretical splitting of the supersymmetric state and the lower strangeness analog state: $\Delta E = 5$ MeV.[50] The structural differences between the 1_1^- and 1_2^- states are also manifested in their decay properties—for the first of them, the main decay channel is $\alpha + {}^{5}_{\Lambda}$He, while for the second it is $\Lambda + {}^{8}$Be.

The use of the nucleon-association model for describing hypernuclei was developed further in Ref. 95, in which a microscopic three-cluster $\alpha + x + \Lambda$ $(x = n, d, t, \alpha)$ model was used to describe the excitations of ${}^{6}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Li, ${}^{9}_{\Lambda}$Be. The calculations reproduce well the position of the ground state and the lowest coherent peak in the (K^-, π^-) reaction. For some of the low-lying levels (for example, of ${}^{8}_{\Lambda}$Li) the predictions of the model differ strongly from the predictions of the shell model,[16] and therefore the study of γ transitions in hypernuclei is also of interest for clarifying the part played by clustering effects. Excitations of ${}^{9}_{\Lambda}$Be were also considered in the framework of a three-particle molecular approach in Ref. 96, and as a three-body problem on the basis of the Faddeev equations in Ref. 97.

TABLE VIII. Spectrum of the hypernucleus ${}^{9}_{\Lambda}$Be in the nucleon-association model (Ref. 94). J^π is the total orbital angular momentum and parity of the level with energy E_Λ belonging to the rotational band K; $w^2(L\lambda)$ are the probabilities of the configurations with relative orbital angular momentum L of the α clusters and orbital angular momentum λ of the Λ hyperon relative to the $\alpha + \alpha$ center-of-mass system; S is the spectroscopic factor for decay through the channel $\alpha + {}^{5}_{\Lambda}$He.

K	J^π	E_Λ, MeV	$w^2(L\lambda)$						S
0^+	0^+	−6,71	0,944(00)	0,054(22)	0,001(44)				0.150
	2^+	−3.73	0,018(02)	0,944(20)	0,019(22)	0,001(24)	0,018(42)	0,000(44)	0.116
	4^+	3.93	0,000(04)	0,032(22)	0,000(24)	0,954(40)	0,013(42)	0,000(44)	0,075
0^-	1^-	0.46	0,545(01)	0,424(21)	0,019(23)	0,012(43)	0,001(45)		0,21
	3^-	4.50	0,015(03)	0,829(21)	0,013(23)	0,001(25)	0,135(41)	0,006(43)	0,214
	5^-	13,12	0,001(05)	0,051(23)	0,001(25)	0,938(41)	0,010(43)	0.000(45)	0,181
1^-	1^-	6,66	0,483(01)	0,511(21)	0,001(23)	0,006(43)	0,000(45)		0,013
	2^-	8,43	0,994(21)	0,003(23)	0,003(43)	0,000(45)			0
	3^-	13,20	0,002(03)	0,337(21)	0,002(23)	0,000(25)	0,657(41)	0,003(43)	0.018
	4^-	15,93	0,006(23)	0,000(25)	0,991(41)	0,003(43)	0,000(45)		0

5. THE ΛN INTERACTION STUDIED THROUGH SPECTROSCOPY OF HYPERNUCLEI

Study of the hyperon–nucleon interaction on the basis of hypernuclear data under conditions when detailed experimental results are only beginning to be accumulated requires a physically motivated system for choosing the most important terms in the ΛN interaction in the construction of the hypernuclear model. It is therefore important to elucidate the part played by forces of different types informing the hypernuclear spectra. For example, weakness of the spin–orbit Λ-nucleus interaction compared with the central interaction is established by analyzing the energy splitting and intensities of the coherent peaks in the (K^-, π^-) reaction on nuclei with doubly closed (sub)shells. The appearance of experimental data on the excitation spectra of light hypernuclei in the (K^-, π^-) reaction in a wide range of momentum transfers with good energy resolution makes it possible to turn to the study of the structure of the residual ΛN interaction.

Residual ΛN Interaction from Analysis of the $^{13}C(K^-, \pi^-)^{13}_{\Lambda}C$ Reaction

The cross section of the (K^-, π^-) reaction on the ^{13}C nucleus, measured at BNL with beam momentum $p_K = 800$ MeV/c and pion emission angles in the range $\theta = 0$–$25°$,[98] exhibits a very rich spectrum of levels of the hypernucleus $^{13}_{\Lambda}C$ (Fig. 14), five peaks being distinguishable at $\theta = 4°$. The $^{13}_{\Lambda}C$ excitation scheme in the weak-coupling limit is shown in Fig. 15. The two lowest peaks (1 and 2) correspond to $(1s, 1p^{-1})^{1^-}_{\Lambda n}$ transitions to the ground state $^{13}_{\Lambda}C_{gs} = [^{12}C_{gs} \otimes (1s_{1/2})_{\Lambda}]_{1/2+} (E=0)$ and the state $[^{12}C(2^+0) \otimes (1s_{1/2})_{\Lambda}]_{3/2+}$ with an excited nucleon core ($E = 4.4$ MeV). Such an interpretation is confirmed by the

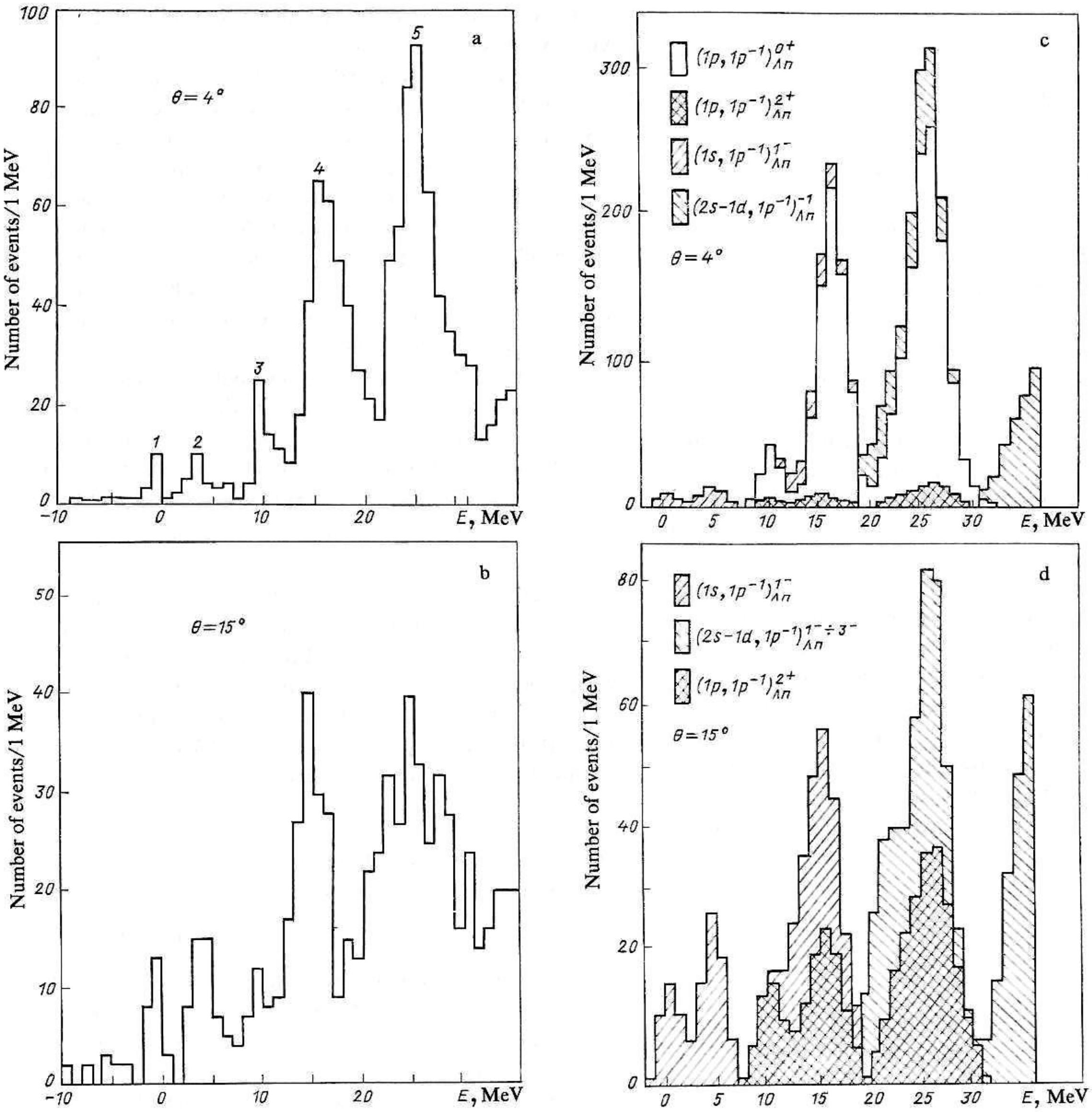

FIG. 14. Excitation spectra of the hypernucleus $^{13}_{\Lambda}C$ in the $^{13}C(K^-, \pi^-)^{13}_{\Lambda}C$ reaction at $p_K = 800$ MeV/c and θ equal to 4 and 15°: a), b) the experiment of Ref. 98; c), d) the theoretical calculation of Ref. 99. The contributions of the different transitions $(n'l', (nl)^{-1})^{L}_{\Lambda n}$ are indicated separately.

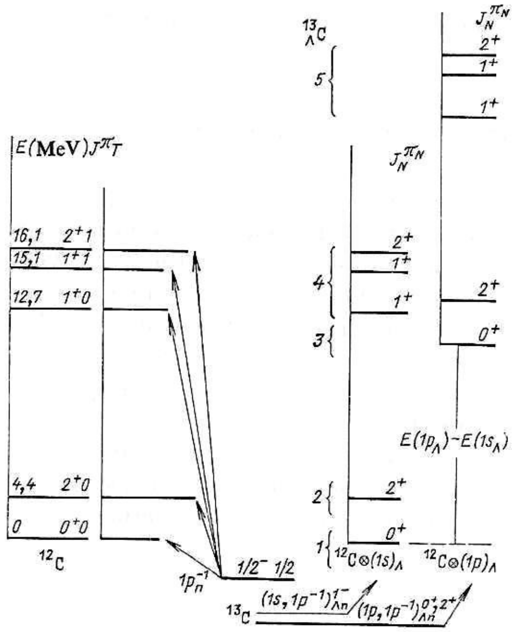

FIG. 15. The $^{13}C_{gs} \to n + {}^{12}C(J^{\pi}T)$ fractional-parentage scheme and excitation spectrum of the hypernucleus $^{13}_{\Lambda}C$ in the weak-coupling limit. The levels identified by the numbers 1–5 correspond to the five experimentally observed peaks (see Fig. 14a) at $E = 0$, 4.4, 10.4, 16.4, and 25.7 MeV.

angular dependence of the excitation cross section of these peaks, which is characteristic of $(1s, 1p^{-1})^{1-}_{\Lambda n}$ transitions (Fig. 16). The third peak at the excitation energy $E = 10.4$ MeV is due to a coherent $(1p_{1/2}, 1p^{-1}_{1/2})^{0+}_{\Lambda n}$ transition to the state $^{13}_{\Lambda}C(1/2^-_{(1)}) = [{}^{12}C(0^+0) \otimes (1p_{1/2})_\Lambda]_{1/2-}$. With increasing angle, the peak is shifted downward $[\Delta E_3(4-15°) = 0.36 \pm 0.3$ MeV], the dominant contribution in this case being made by a $(1p_{3/2}, 1p^{-1}_{1/2})^{2+}_{\Lambda n}$ transition to the state $[{}^{12}C(0^+0) \otimes (1p_{3/2})_\Lambda]_{3/2-}$. Then, assuming the validity of the weak-coupling approximation, it may be concluded that the spin–orbit splitting of the hypernuclear levels of the $1p$ shell is small: $\varepsilon_p = E((1p_{1/2})_\Lambda) - E((1p_{3/2})_\Lambda) = 0.36 \pm 0.3$ MeV.

However, further examination reveals strong deviations from the predictions of the weak-coupling model.[99] A fourth peak at $E = 16.4$ MeV ($\theta = 4°$) corresponds to a coherent $1p_{3/2}, 1p^{-1}_{3/2})^{0+}_{\Lambda n}$ transition to the state $^{13}_{\Lambda}C(1/2^-_{(2)}) = [{}^{12}C(2^+0) \otimes (1p_{3/2})_\Lambda]_{1/2-}$. With increasing angle, the contribution of the $(1p, 1p^{-1})^{2+}_{\Lambda n}$ transition to the state $[{}^{12}C(2^+0) \otimes (1p)_\Lambda]_{5/2-}$ increases, and the peak is shifted appreciably downward $[\Delta E_4(4–15°) = 1.7 \pm 0.4$ MeV]. The attempt to explain this shift in the weak-coupling approximation leads to a contradiction with the previous estimate of ε_p, since it requires a negative value of ε_p of the order of several mega-electron-volts. The distance between the two coherent peaks corresponding to the transitions to the states $^{13}_{\Lambda}C(1/2^-_{(1)})$ and $^{13}_{\Lambda}C(1/2^-_{(2)})$, $\Delta E_{34} = (6.0 \pm 0.4$ MeV, is also significantly different from the difference between the energies of the nucleon cores (4.4 MeV). Neglect of the residual interaction leads here to a similar contradiction $[\varepsilon_p = -1.6 \pm 0.4 \text{ MeV} < 0]$. Finally, the most pronounced deviation from the predictions of the weak-coupling model is observed in the excitation cross sections of the two coherent peaks at $\theta = 4°$, namely experiment gives $\rho = (\sigma(1/2^-_{(2)})/\sigma(1/2^-_{(1)}) \simeq 5$, while the theoretical value, determined by the ratio of the corresponding spectroscopic factors, is 1.83.[56] All these data indicate that the residual ΛN interaction is important.

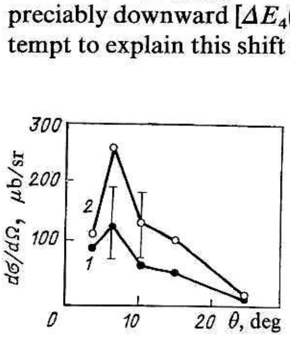

FIG. 16. Dependence of the differential cross section for excitation of the hypernucleus $^{13}_{\Lambda}C$ in the region of the peaks 1($E = 0$ MeV) and 2($E = 4.4$ MeV) on the pion emission angle θ.[98]

The $^{13}_{\Lambda}C$ excitations in the (K^-, π^-) reaction were analyzed in detail in Ref. 99. A theoretical spectrum of the $^{13}_{\Lambda}C$ levels was calculated in the many-particle shell model. Allowance was made for all states of the configurations $\{(1s)^4_N(1p)^8_N(1s)_\Lambda\}$ and $\{(1s)^4_N(1p)^8_N(1p)_\Lambda\}$, and at excitation energies $E > 18$ MeV the most important states of the configurations $\{(1s)^3_N(1p)^9_N(1s)_\Lambda\}$ and $\{(1s)^4_N(1p)^8_N(2s-1d)_\Lambda\}$ were included. The single-particle wave functions of the nucleons corresponded to Woods–Saxon potentials with parameters fitted to the energies of the single-particle levels and the charge density. The hyperon–nucleus potential had an identical geometry [$r_0 = 1.15$ F, $b = 0.63$ F in the notation of Eq. (14)], and its intensity was fitted to the given values of the binding energy of the $1p$ levels: $B_\Lambda(1p_{3/2}, 1p_{1/2}) = 0.6$ and 0.1 MeV. The residual NN interaction was taken into account in accordance with Ref. 56 (an original form of the residual interaction was also considered; it gave similar results). The ΛN interaction contained a central component and a two-particle spin–orbit component:

$$V_{\Lambda N} = -v(r)(1 - \varepsilon_x + \varepsilon_x P_x)(1 + \alpha_\Lambda \sigma_\Lambda \sigma_N) + v_\pm(r)(\sigma_\Lambda \pm \sigma_N)\mathbf{l}_{N\Lambda}.$$

Since the experimental data do not permit a choice to be made between the various versions of the calculations for $(1s)_\Lambda$ configurations, the following treatment concerns mainly the excitations in the region of the peaks 3, 4, and 5 [the description of the $(1s)_\Lambda$ configurations followed from Ref. 16]. For $l_N = l_\Lambda = 1$, the central ΛN interaction is determined by two Slater integrals:

$$F^{(k)} = \int dr_N\, dr_\Lambda R^2_\Lambda(r_\Lambda) R^2_N(r_N) V_k(r_N, r_\Lambda), \quad k = 0, 2,$$

where $R_{\Lambda,N}(r)$ are the radial functions of the Λ hyperon and nucleon in the $1p$ shell, and $V_k(r_N, r_\Lambda)$ are determined by

$$V(|\mathbf{r}_N - \mathbf{r}_\Lambda|) = \sum_{k=0}^{\infty} V_k(r_N, r_\Lambda) P_k(\mathbf{r}_N\mathbf{r}_\Lambda / r_N r_\Lambda).$$

The following parameters were fixed: $F^{(0)} = -1.16$ MeV (Ref. 50), $\alpha_\Lambda = 0.1$ (Refs. 10 and 37), and $\varepsilon_x = 0$; the parameters ε_p and $F^{(2)}$ were varied. The resulting wave functions of the hypernuclear levels were used in a DWBA calculation of the cross section of the $^{13}C(K^-, \pi^-)^{13}_{\Lambda}C$ reaction; optical potentials fitted to the available data on the elastic scattering of K^- and π^- by ^{12}C were used to describe the meson–nucleus interaction.

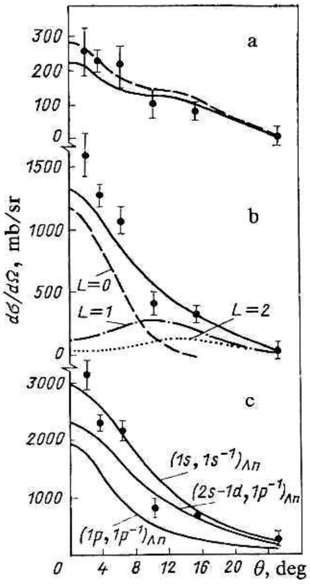

FIG. 17. Dependence of the differential cross section $d\sigma/d\Omega$ for production of the hypernucleus ${}^{13}_{\Lambda}C$ in the (K^-, π^-) reaction on the pion emission angle θ for peaks 3, 4, and 5 (a, b, and c).[99] The continuous and broken curves in case (*a*) correspond to calculations with two variants of the residual *NN* interaction (Refs. 99 and 57). In case (b), the continuous curve shows the total contribution of the transitions with $L = 0, 1, 2$. The experimental data are taken from Ref. 98.

The theoretical spectra for $\theta = 4$ and 15° are shown in Figs. 14c and 14d ($F^{(2)} = -3$ MeV, $\varepsilon_p = 0.5$ MeV). The contributions of the various transitions $(n'l', (nl)^{-1})^L_{\Lambda n}$ shown separately demonstrate the nature of the hypernuclear excitations at small and large q. Figure 17 shows the angular dependence of the differential cross sections $d\sigma/d\Omega$ for three intervals of excitation energy in the region of the peaks 3, 4, and 5. The good agreement between the theoretical spectra and angular dependences, on the one hand, and the experimental data on the other, both in the shape and in the absolute magnitude, makes it possible to use the developed model to determine the parameters of the residual ΛN interaction. Analysis of the hypernuclear structure in the region of the peaks 3 and 4 ($E = 10$–16 MeV) is found to be the most informative.

The hypernuclear states lying in the region of excitation energies $E = 10$–16 MeV have the structure $[{}^{12}C(0^+, 2^+) \otimes (1p)_\Lambda]_{J^-}$. The states $(J^\pi T) = (0^+\ 0), (2^+\ 0)$ of the ${}^{12}C$ nucleus can be approximated with good accuracy by a singlet wave function ($S_N = 0$), and therefore it is sufficient to take into account only the central component when diagonalizing the residual interaction. Suppose $\varepsilon_p = 0$; then $\mathbf{I} = \mathbf{J}_N + \mathbf{I}_\Lambda$ is a "good" quantum number, and the hypernuclear levels are doubly degenerate ($\mathbf{J} = \mathbf{I} + \mathbf{s}_\Lambda$). The spectrum of states $[{}^{12}C(0^+, 2^+) \otimes (1p)_\Lambda]_J$ is determined by the value of $F^{(2)}$ and for $F^{(2)} = -3$ MeV is shown in Fig. 18; the states that make the main contribution to the peaks 3 and 4 at $\theta = 4$ and 15° are indicated by an asterisk. Although the residual ΛN interaction is significantly weaker than the NN interaction (the corresponding Slater integral for the $1p$ shell is $F^{(2)}_N = 10$ MeV), the resulting wave functions demonstrate a tendency to the establishment of symmetry with respect to

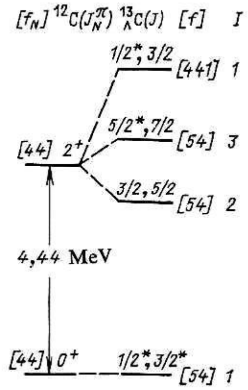

FIG. 18. Scheme of the ${}^{13}_{\Lambda}C = {}^{(12)}C(0^+, 2^+) \otimes (1p)_\Lambda$ levels for an interaction that does not depend on the spin of the Λ hyperon.

permutation of the baryons in the $1p$ shell, so that the different hypernuclear states can be characterized by the Young tableau $[f]$ of the orbital symmetry of the dominant configuration, as is shown in Fig. 18. Thus, the state $([f] = {}^{13}_{\Lambda}C(1/2^-_{(1)})$ has a structure close to that of the supersymmetric state ($[f] = [54]$), and, therefore, is not excited well at small momentum transfers, whereas the symmetry of the state ${}^{13}_{\Lambda}C(1/2^-_{(2)})$ is the same as the ${}^{13}C_{gs}$ symmetry [441], and this ensures coherence of its production at small q. This picture still holds when there is a weak spin dependence of the ΛN interaction. For example, for $F^{(2)} = -3.0$ MeV and $\varepsilon_p = 0.5$ MeV the wave function of the lowest $1/2^-$ state has the form

$$|{}^{13}_{\Lambda}C(1/2^-_{(1)})\rangle = \cos\theta_m [|{}^{12}C(0^+)\rangle \otimes |(1p_{1/2})_\Lambda\rangle]_{1/2^-} + \sin\theta_m [|{}^{12}C(2^+)\rangle \otimes |(1p_{3/2})_\Lambda\rangle]_{1/2^-},$$

where $\theta_m = 0.287$ and the supersymmetric component [54] is contained in it with amplitude 0.88. As a result, there is a strong deviation of the quantity $\rho = \sigma(1/2^-_{(2)})/\sigma(1/2^-_{(1)}) = [\Theta(3/2) - \Theta(1/2)\tan\theta_m]^2/[\Theta(1/2) - \Theta(3/2)\tan\theta_m]^2$, [$\Theta(j)$ of the spectroscopic amplitudes of the ${}^{13}C_{gs} \to {}^{12}C + n(1p_j)$] neutron pickup from the predictions of the weak-coupling model. For $-3.0 < F^{(2)} < -3.6$ MeV, $\rho = 6$–7, in good agreement with experiment. In the presence of exchange forces in the ΛN interaction ($\varepsilon_x \neq 0$, the tendency for the supersymmetric state to be isolated is enhanced. In particular, for Serber forces ($\varepsilon_x = 1/2$), $\rho_{th} \gg \rho_{exp}$, and therefore a choice with $\varepsilon_x \simeq 0$ appears preferable.

An interaction dependent on the spin of the Λ hyperon leads to a doublet splitting of the levels with given I. The available experimental data make it possible to determine only the splitting of the lowest doublet $(1/2^-, 3/2^-)$ ($I = 1$, $[f] = [54]$): The dominant contribution to peak 3 at $\theta = 4°$ is made by the $1/2^-$ state, and at $\theta = 15°$ by the $3/2^-$ state. As a result, it is possible to express ε_p in terms of $\Delta E_3(4\text{–}15°)$, and the result depends weakly on $F^{(2)}$, since in the expression relating $\Delta E_3(4\text{–}15°)$ to ε_p it is possible to ignore the terms of first order in $F^{(2)}$ (for not too large ε_p, $\varepsilon_p \lesssim 3$ MeV). If the interval of $F^{(2)}$ values is fixed by the value of ρ and $v_\pm = 0$, then $\varepsilon_p = c\Delta E_3$, where $c = 1.15 \pm 0.03$. It follows that $\varepsilon_p = 0.4 \pm 0.3$ MeV. In the case $v_\pm \neq 0$, the small value of ΔE_3 means that the single- and two-particle spin–orbit forces together give a weak effect (Fig. 19).

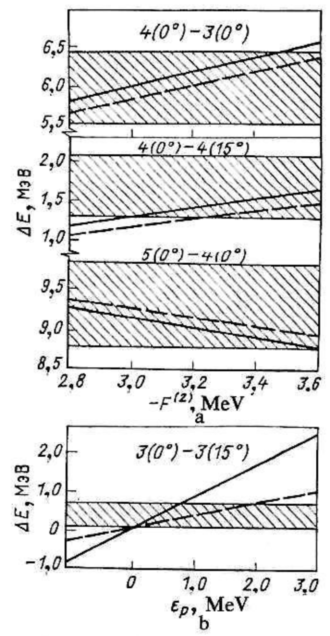

FIG. 19. Dependence of the energy splitting ΔE of individual pairs of groups of levels of ${}^{13}_{\Lambda}$C: (a) on $F^{(2)}$ for $\varepsilon_p = 0.5$ MeV and (b) on ε_p.[99] The hatched regions correspond to the experimental splittings and shifts of the peaks $\Delta E = E_4(0°) - E_3(0°)$, $\Delta E = E_4(0°) - E_4(15°)$, $\Delta E = E_5(0°) - E_4(0°)$, $\Delta E = E_3(0°) - E_3(15°)$.[98] The calculation for two variants of the residual NN interaction (Refs. 99 and 54) is shown in case (a) by the continuous and broken curves, respectively. In case (b) the continuous and broken lines represent the calculations with allowance for only the single-particle and two-particle spin–orbit interactions, respectively.

For fixed ε_p the restrictions on $F^{(2)}$ that arise from consideration of the energy splitting of peaks 3 and 4 at $\theta = 4°$, $\Delta E_{3,4}$, and the shifts of peaks 4 and 5 as the angle is varied from 4° to 15°, $\Delta E_{4,5}$(4–15°), are shown in Fig. 19. For $\varepsilon_p = 0.5$ MeV, all the deviations from the weak-coupling limit mentioned above can be explained if $-3.4 < F^{(2)} < -3.0$ MeV. However, the situation remains very far from the strong-coupling limit $V_{\Lambda N} \simeq V_{NN}$, which if realized would, for example, give instead of the two strong coherent peaks at $E = 16.4$ MeV and $E = 25.7$ MeV a single peak consisting of two closely spaced $T = 0$, 1 components.

Coherence in the (K^-, π^-) Reaction and Spin–Spin Interaction

In Sec. 1 we considered in the weak-coupling model the excitation spectrum of the hypernucleus ${}^{12}_{\Lambda}$C. The strong narrow peak at $E = 0$ (see Fig. 3) was identified with the coherent $(1p_{3/2}, 1p_{3/2}^{-1})^{0+}_{\Lambda n}$ transition to the state $[{}^{11}C_{gs} \otimes (1p_{3/2})_{\Lambda}]_{0+}$. In the jj-coupling limit, coherent transition in the $1p$ shell does indeed lead to the formation of a single ${}^{12}_{\Lambda}C(0^+)$ state. However, it follows from realistic calculations of the coefficients of fractional parentage for the ^{12}C nucleus[56] that the contribution of the ^{11}C ground state to the sum rule for the spectroscopic factors for separation of a $1p$ neutron is only 71%. The question therefore arises of how the other ^{11}C states needed to saturate the sum rule ($J^{\pi} = 1/2^-$, $E = 2.0$ MeV and $J^{\pi} = 3/2^-$, $E = 4.8$ MeV) appear in the ${}^{12}C(K^-, \pi^-){}^{12}_{\Lambda}C$ reaction. The answer to this question was given in Ref. 38, in which the spectrum of states

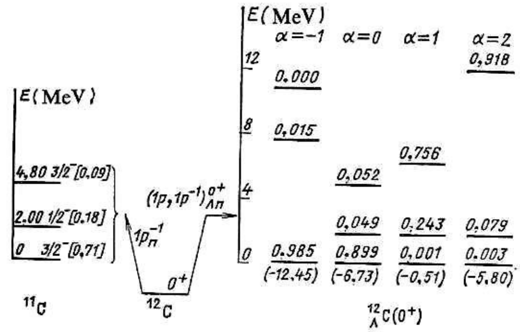

FIG. 20 The ${}^{12}C_{gs} \to {}^{11}C + n(1p_j)$ fractional-parentage scheme and spectrum of the ${}^{12}_{\Lambda}C(0^+) = [{}^{11}C(J^{\pi}E) \otimes (1p)_{\Lambda}]_{0+}$ levels for different values of the spin–spin interaction parameter α_{Λ}.[38] The neutron spectroscopic factors are given in the square brackets. For the hypernuclear levels, the relative probabilities of excitation in the reaction ${}^{12}C(K^-, \pi^-){}^{12}_{\Lambda}C$ for $a = 0$, are given; the absolute values of the interaction energy of the Λ hyperon in the $1p$ shell for the lowest level are given in the round brackets (MeV).

$|{}^{12}_{\Lambda}C(0^+)\rangle = [|{}^{11}C(J^{\pi}, E)\rangle \otimes (1p)_{\Lambda}\rangle]_{0+}$ was calculated with allowance for the excitations of the nucleon core. The residual interaction was chosen in the form $(1 + \alpha_{\Lambda} \boldsymbol{\sigma}_{\Lambda} \boldsymbol{\sigma}_N) v(r_{\Lambda N})$, the central part being characterized by Slater integrals $F^{(0)} = -1$ MeV and $F^{(2)} = -3$ MeV, and the parameter α_{Λ} was varied in the interval $[-1, 2]$. The calculated energies of the 0^+ levels and the relative probabilities of their excitation in the (K^-, π^-) reaction at $q = 0$ for different values of α_{Λ} are shown in Fig. 20. With increasing α_{Λ}, the maximum in the intensity of the coherent $(1p, 1p^{-1})^{0+}_{\Lambda n}$ transition is shifted to higher excitation energies. For $\alpha_{\Lambda} = 0$, which is close to the physical case ($\alpha_{\Lambda} \simeq -0.1$ according to Refs. 10 and 37), the lowest state has the structure $|{}^{12}_{\Lambda}C(0_1^+)\rangle \simeq 0.857 [|{}^{11}C(3/2^-; 0)\rangle \otimes |(1p_{3/2})_{\Lambda}\rangle]_{0+} + 0.515[|{}^{11}C(1/2^-; 2.00)\rangle \otimes |(1p_{1/2})_{\Lambda}\rangle]_{0+}$ and is responsible for 90% of the intensity instead of the 71% in the weak-coupling limit. This increase in the coherence is due to the constructive interference of the transitions to two lower states of the nuclear core. With decreasing α_{Λ}, the degree of coherence for the lowest state increases, reaching 98.5% for $\alpha_{\Lambda} = -1$. But if the parameter α_{Λ} is increased and takes positive values, coherent production becomes characteristic for the uppermost 0^+ state. For $\alpha_{\Lambda} = 1$, the probability of production of the lowest state is negligibly small, and the total intensity is distributed in the ratio 3:1 between the uppermost and central 0^+ levels, which are split by 4.5 MeV. This result means that the experimentally observed high degree of coherence in the production of Λ hypernuclei in the (K^-, $\pi^{\pm}$) reaction at small q imposes definite requirements on the hypernuclear structure and, as a consequence, on the ΛN interaction. Of great interest is the question of the degree of coherence in the production of Σ hypernuclei in the (K^-, $\pi^{\pm}$) reaction, since the absence of a significant spin dependence of the ΣN forces[100] means that the total intensity of the coherent transition can, in contrast to the case of Λ hypernuclei, be distributed over several Σ-hypernuclear states.[38]

6. Σ HYPERNUCLEI

A feature of Σ hypernuclei is the possibility of $\Sigma\Lambda$ conversion through the strong ΣN–ΛN channel coupling. Using the experimental cross section of the $\Sigma^- p \to \Lambda n$ reaction at low energies, $\sigma_{\Sigma\Lambda}$,[8)] one can estimate the conversion length of a Σ hyperon in a nucleus: $\lambda = 1/\rho\sigma_{\Sigma\Lambda} \sim 1$ F. Since λ does not exceed the nuclear diameter, it would appear that one cannot expect the existence of quasidiscrete levels in the Σ-nuclear system. More formally, suppose the interaction of the Σ hyperon with the nucleus is described by a local energy-independent optical potential

$$V_{\text{opt}}(r) = -\frac{2\pi}{M}\left(1+\frac{m_\Sigma}{m_N}\right)\bar{a}_{\Sigma N}\rho(r). \quad (25)$$

Here, M is the reduced mass, $\rho(r)$ is the nuclear density, and $\bar{a}_{\Sigma N}$ is the effective ΣN scattering length, which can be determined, for example, using data on Σ atoms[101]:

$$\bar{a}_{\Sigma N} = (0.35 \pm 0.04) + \mathrm{i}\,(0.19 \pm 0.03)\ \text{F}.$$

At the same time, $\operatorname{Re} V_{\text{opt}}(0) = -26$ MeV, $\operatorname{Im} V_{\text{opt}}(0) = -14$ MeV; a similar value $\operatorname{Im} V_{\text{opt}}(0) = -11$ MeV is obtained if $\operatorname{Im} \bar{a}_{\Sigma N}$ is expressed in terms of the low-energy cross section of the $\Sigma^- p \to \Lambda n$ reaction.[102] In the first order in $\operatorname{Im} V_{\text{opt}}$, the $\Gamma_{\Sigma\Lambda}$ conversion width is given by

$$\Gamma_{\Sigma\Lambda} = \frac{4\pi}{M}\left(1+\frac{m_\Sigma}{m_N}\right)\operatorname{Im}\bar{a}_{\Sigma N}\int |\psi(r)|^2\rho(r)\,d^3r, \quad (26)$$

where $\psi(r)$ is the wave function of the Σ hyperon. Thus, for states localized within the nucleus, $\Gamma_{\Sigma\Lambda} \simeq -2\operatorname{Im} V_{\text{opt}}(0) \simeq 20$–$30$ MeV.[103] This estimate is close to the results of numerical solution of the eigenvalue problem for the Hamiltonian of the Σ–nucleus system with the interaction (25) (Ref. 104): for the hypernucleus ${}^{12}_{\Sigma}$C, $\Gamma_{\Sigma\Lambda}(1s) = 23$ MeV $[B_\Sigma(1s) = 13$ MeV$]$, $\Gamma_{\Sigma\Lambda}(1p) = 13$ MeV $[B_\Sigma(1p) = 0$ MeV$]$; for the hypernucleus ${}^{40}_{\Sigma}$Ca, $\Gamma_{\Sigma\Lambda}(1s) = 28$ MeV, $\Gamma_{\Sigma\Lambda}(1p) = 23$ MeV, $\Gamma_{\Sigma\Lambda}(1d) \doteq 18$ MeV, $\Gamma_{\Sigma\Lambda}(2s) = 16$ MeV. Very unexpected therefore was the observation of narrow structures ($\Gamma \lesssim 10$ MeV) near the threshold of the $(\Sigma + {}^{A-1}Z')$ channel in the cross sections $d\sigma/d\Omega dE$ of the ${}^{A}Z(K^-, \pi^\pm)X$ reactions on light nuclei.

Σ Hypernuclei in the (K^-, π^-) Reaction

The first data indicating the existence of Σ hypernuclei were obtained by the Heidelberg–Saclay–Strasbourg collaboration at CERN.[105] Figure 21 shows the missing-mass spectrum for the ^{9}Be$(K^-, \pi^-)X$ reaction at $p_K = 720$ MeV/c and $\theta = 0°$. The two strong peaks in the left-hand part of the spectrum (E_Λ equal to 6 and 17 MeV) correspond to coherent production of ${}^9_\Lambda$Be (momentum transfer $q = 57$ MeV/c). About 80 MeV higher there is a structure consisting of two peaks ($E_{\Sigma^0} = E_\Lambda - 77$ MeV $= 9$ MeV and 20 MeV). These peaks correspond to states of a Σ hypernucleus produced by coherent transitions in the $1p$ shell. The momentum transfer to the Σ hypernucleus is $q = 129$ MeV/c, and the ratio of the cross sections for ${}^9_\Sigma$Be and ${}^9_\Lambda$Be production is approximately 1:4. For comparison, the insert shows the ${}^9_\Lambda$Be spectrum at $q = 67$ MeV/c ($p_K = 900$ MeV/c). The same collaboration obtained indications of the existence of a narrow ${}^{12}_{\Sigma}$C state at $E_{\Sigma^0} = 4$ MeV, and also broad structures near the $\Sigma + {}^{A-1}Z'$ threshold in the (K^-, π^-) reaction on ^{7}Li and the (K^-, π^+) reaction on ^{9}Be.[106]

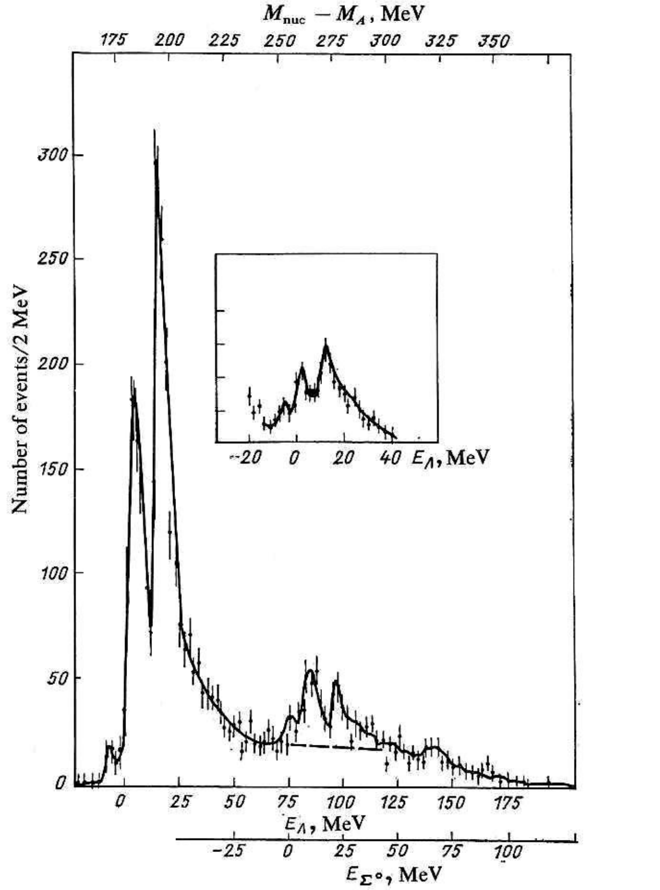

FIG. 21. Excitation spectra of Λ and Σ hypernuclei in the (K^-, π^-) reaction on the ^{9}Be nucleus at $p_K = 720$ MeV/c and $\theta = 0°$.[105] The energies E_Λ and E_Σ are measured from the thresholds of the $\Lambda + {}^8\text{Be}_{\text{gs}}$ and $\Sigma^0 + {}^8\text{Be}_{\text{gs}}$ channels. The insert shows the excitation spectrum of the ${}^9_\Lambda$Be hypernucleus at $p_K = 900$ MeV/c.

Experiments made at BNL[107] provided new proofs for the existence of narrow states of Σ hypernuclei. Figure 22

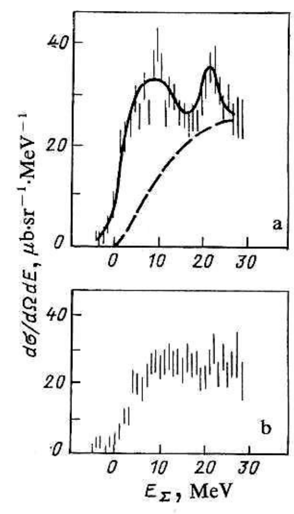

FIG. 22. Energy dependence of the differential cross section $d\sigma/d\Omega dE$ of the ^{6}Li$(K^-, \pi^-){}^6_\Sigma$H reaction at $p_K = 713$ MeV/c and $\theta = 3.7°$ (a) and 9° (b).[107]

[8)]The energy dependence of $\sigma_{\Sigma\Lambda}$ can be approximately described by $\sigma_{\Sigma\Lambda} = 65$ mb$/(v + 20v^2)$, where v is the relative velocity of the Σ^- and p.[100]

shows the excitation spectrum of the hypernucleus ${}^{6}_{\Sigma}\mathrm{H}$ in the ${}^{6}\mathrm{Li}(K^-, \pi^+){}^{6}_{\Sigma}\mathrm{H}$ reaction at $p_K = 713$ MeV/c. At $\theta = 3.7°$ there are two clearly defined peaks with energies E_Σ equal to 9.1 and 22 MeV and total widths (with allowance for the experimental resolution $\delta E = 3$ MeV) Γ equal to 12 and 4.6 MeV, respectively.[9)] Both peaks vanish with increasing angle θ, and this makes it possible to identify them with the coherent transitions $(1p, 1p^{-1})^{0+}_{\Sigma^-p}$ and $(1s, 1s^{-1})^{0+}$. In addition, it follows from the BNL data[107] that in the ${}^{16}\mathrm{O}(K^-, \pi^+){}^{16}_{\Sigma}\mathrm{C}$ reaction there is a broad structure at $E_\Sigma = 15$ MeV ($\Gamma \simeq 19$ MeV), which may be a superposition of several coherent transitions in the $1p$ shell.

The discovery of narrow states of Σ hypernuclei stimulated study of the $(K^-, \pi^\mp)$ reaction at lower momenta of the kaon beam under conditions of recoilless production of the Σ hypernuclei. The cross sections $d\sigma/d\Omega dE\,|_{\theta=0°}$ of the reactions ${}^{12}\mathrm{C}(K^-, \pi^-)X$, ${}^{12}\mathrm{C}(K^-, \pi^+)X$ and ${}^{16}\mathrm{O}(K^-, \pi^+)X$ measured by the Heidelberg–Saclay collaboration at CERN using a K^- beam with momentum $p_K = 400$–450 MeV/c are characterized by the presence of well-defined peaks corresponding to coherent transitions in the $1p$ shell.[108]

$\Sigma\Lambda$ Conversion and Structure of Σ Hypernuclei

One would expect Σ hypernuclei to have a more complicated structure than Λ hypernuclei. One of the fundamental questions that arises here concerns the validity of the average-field conception for describing the Σ hyperon in a nucleus. The point is that a many-channel treatment of the Σ hypernucleus with allowance for $\Sigma\Lambda$ conversion leads to an effective single-channel problem with a generalized optical potential that is essentially nonlocal and depends on the energy. Approximation of it by a local potential gives satisfactory results in the scattering problem and in the study of Σ atoms,[101] for which the form of the potential within the nucleus is of little importance on account of the strong absorption on the surface. However, if one is interested in Σ-nuclear states, for which localization in the intranuclear region is characteristic, such an approximation no longer appears unexceptionable. Another question relates to the applicability of the weak-coupling approximation for the description of Σ hypernuclei. Investigations of propagation of a Σ hyperon in nuclear matter show that an appreciable fraction of the energy release $(M_\Sigma - M_\Lambda)$ can be expended on excitation of the nuclear matter.[109] Applied to finite nuclei, this means that it is necessary to take into account the excited states of the nucleon core. Thus, the resulting picture of an unstable quasiparticle carrying strangeness is more complicated than the simple scheme of a "Σ hyperon in an undeformed nucleon core."

In pointing out the difference from Λ hypernuclei, we must also mention the more complicated isospin structure of the Σ hypernuclei and the appreciable—on nuclear scales—violation of isotopic invariance in the isotriplet $)\Sigma^-, \Sigma^0, \Sigma^+)$: $M_{\Sigma^-} - M_{\Sigma^+} = 8$ MeV. In addition, the spin–orbit forces for the Σ hyperon in the nucleus are expected to be as important as for the nucleon,[110,111] and the residual ΣN interaction has a strong spin–isospin dependence.[100] The degree of coherence of the production of Σ hypernuclei in the $(K^-, \pi^\mp)$ reaction at small q must also be established experimentally.[38]

Of particular interest at the present time is the problem of $\Sigma\Lambda$ conversion. There have been many theoretical calculations (see Refs. 104, 109, and 112–121) of the widths of Σ hypernuclei; we shall dwell on only some of them in order to demonstrate the variety of the approaches that are employed (see Ref. 120).

In the framework of the traditional approach, which uses a local energy-independent potential, one of the possible reasons for suppression of $\Sigma\Lambda$ conversion was pointed out in Ref. 104. Because of the dominance of one-pion exchange in the $\Sigma N \to \Lambda N$ process and the strong spin dependence of the ΣN interaction, the main contribution to the $\Sigma N \to \Lambda N$ conversion cross section at low energies ($\sim 90\%$) is made by the ${}^{13}S_1$ wave, in which there is attraction in the initial state (in the ${}^{11}S_0$ wave there is repulsion). In heavy nuclei, in which the spins and isospins are near saturation, such selectivity is of little importance, but in light nuclei, $A \leqslant 16$, it may lead to appreciable effects. Consider, for example, the Σ-hypernuclear state produced in the ${}^{6}\mathrm{Li}(K^-, \pi^+){}^{6}_{\Sigma}\mathrm{H}$ reaction as a result of the coherent $(1s, 1s^{-1})^{0+}_{\Sigma^-p}$ transition.[118] The structure of this state is such that it contains with high probability the cluster configuration $[{}^{4}_{\Sigma}n(T = 3/2, J = 0) \otimes d\,]_{1+}$.[10)] Since the $\Sigma^- p$ pair is in the cluster ${}^{4}_{\Sigma}n$ in the singlet state, strong $\Sigma^- p \to \Lambda n$ conversion is possible only on a proton of a deuteron cluster, and the width $\Gamma_{\Sigma\Lambda}$ acquires a certain smallness. The selectivity of $\Sigma\Lambda$ conversion can be taken into account by replacing in the formula

$$\Gamma_{\Sigma\Lambda} = \overline{(v\sigma_{\Sigma\Lambda})} \int d^3x_1 \ldots$$

$$\ldots d^3x_A\, d^3x_\Sigma \sum_{i=1}^{A} \delta(\mathbf{x}_i - \mathbf{x}_\Sigma)\, |\psi(\mathbf{x}_1, \ldots, \mathbf{x}_A, \mathbf{x}_\Sigma)|^2 \qquad (27)$$

the expression $\Sigma_{i=1}^{A}\delta(\mathbf{x}_i - \mathbf{x}_\Sigma)$ by $1/12\Sigma_i\delta(\mathbf{x}_i - \mathbf{x}_\Sigma)(3 + \sigma_i\sigma_\Sigma)(1 - \boldsymbol{\tau}_i\mathbf{t}_\Sigma)$ using $v\sigma_{\Sigma\Lambda}({}^{13}S_1)$ instead of the quantity $\overline{(v\sigma_{\Sigma\Lambda})}$, which is averaged over all possible initial ΣN states. Calculations made for light Σ hypernuclei show that for certain states the conversion on nucleons of the unfilled $1p$ shell is strongly suppressed, and the smallest width $\Gamma_{\Sigma\Lambda}$ is expected for states that are produced as a result of coherent $(1p, 1p^{-1})^{0+}_{\Sigma N}$ transitions and have the greatest possible isospin. For example, for the ${}^{12}_{\Sigma}\mathrm{C}$ strangeness analog state constructed on the $1p$ shell with isospin $T = 3/2$, conversion takes place only on nucleons of the $1s$ shell ($\Gamma_{\Sigma\Lambda} = 6$ MeV instead of 15 MeV). Other examples and analysis of the available experimental data from this point of view can be found in Refs. 36, 104, 118, and 120.

Possible reasons for the suppression of $\Sigma\Lambda$ conversion in the framework of the theory of infinite nuclear matter were sought in the effects of density dependence (Refs. 109, 115–117, and 121), off-shell effects, the Pauli principle,[109,115] and disperison effects.[109,121] In particular, in the presence of saturation in the ρ dependence of the mass opertor of the Σ

[9)]The values of E and Γ may vary somewhat, depending on the method used to subtract the background.[107]

[10)]A similar situation was considered for the hypernucleus ${}^{6}_{\Lambda}\mathrm{Li}$ in Ref. 86.

hyperon in nuclear matter[117] one can construct a local optical potential that agrees with the Σ-atomic data and gives a moderate width for the $\Sigma\Lambda$ conversion of Σ hypernuclei ($\Gamma \lesssim 5$ MeV). A suppression of $\Sigma\Lambda$ conversion by the effect of the Pauli principle arises if allowance is made for excitation of the nuclear matter in the $\Sigma N \to \Lambda N$ process when the final nucleon is emitted with momentum comparable with the Fermi momentum.[109,115]

The standard method of calculating the width $\Gamma_{\Sigma\Lambda}$ by means of Eq. (26), which does not take into account pairing correlations, was criticized by Kisslinger,[112] who proposed the estimate

$$\Gamma_{\Sigma\Lambda} = \tilde{\Gamma}\tilde{N}_{\rm eff} .$$

Here, $\tilde{\Gamma}$ is the width of the ΣN state that could be produced in the (K^-, π^-) reaction on an NN cluster in the nucleus.

$$\tilde{\Gamma} = -2\,{\rm Im}\,\langle \Sigma N \,|\, V^{+}_{\Lambda\Sigma}(E - H^{0}_{\Lambda N} + i\varepsilon)^{-1} V_{\Lambda\Sigma} \,|\, \Sigma N\rangle,$$

and $\tilde{N}_{\rm eff}$ is the effective number of pairs that participate in the conversion; it is comparable with the $N_{\rm eff}$ defined in Sec. 2. For the operator $V_{\Lambda\Sigma}$ of $\Sigma\Lambda$ conversion, a one-pion exchange potential was taken; the ΣN cluster was described by a wave function of Hulthén type. For a cluster with binding energy $B = 4$ MeV, $\tilde{\Gamma} \sim 1$ MeV, and the width must be of the order of several mega-electron-volts.

An alternative to the standard approach to the problem of the $\Sigma\Lambda$-conversion widths of Σ hypernuclei was proposed in Ref. 113, in which it was shown that if allowance is made for the interaction of the hyperons with the self-consistent field of the nucleons in the nucleus in the coupled-channel problem, the widths of $\Sigma\Lambda$ conversion must be small ($\Gamma_{\Sigma\Lambda} \lesssim 10$ MeV). Qualitatively, the suppression effect is due to the approximate orthogonality of the wave functions of the Σ hyperon in the bound state and of the Λ hyperon in the continuum, and to the slow variation of the $\Sigma \to \Lambda$ transition operator in the nucleus. Indeed, if the Σ–nucleus and Λ–nucleus forces are nearly equal, the wave functions of the hyperon–nucleus system corresponding to different energies are orthogonal. The $\Sigma\Lambda$ conversion potential, like the potentials of the interaction of the hyperons with the self-consistent field of the nucleus, is proportional to the nuclear density, since the range of the hyperon–nucleon forces is of the order of the distance between the nucleons in the nucleus. Then in the calculation of the $\Sigma \to \Lambda$ transition probability amplitude we are dealing with the matrix element of a slowly varying operator between orthogonal states, and the result acquires a certain smallness. If these arguments are correct, a small width $\Gamma_{\Sigma\Lambda}$ must be characteristic of all states of medium and light Σ hypernuclei. For example, for the ground state of the hypernucleus $^{12}_{\Sigma}$C, estimates of the widths give $\Gamma = 5$–10 MeV. The considered model predicts strong suppression of the high-energy component of the spectrum of Λ hyperons produced by the decay of Σ hypernuclei. An important element of such an approach is the assumption of a coherent nature of the conversion. In contrast to the estimate (26), it is the amplitudes for the different final channels that are added, and not the probabilities of $\Sigma N \to \Lambda N$ transitions on an individual nucleon. The part played by the greater energy release in the elementary $\Sigma N \to \Lambda N$ process was also noted in Ref. 119, in which it was pointed out that an appreciable fraction of the phase space for the final nucleon does not make a significant contribution to $\Gamma_{\Sigma\Lambda}$ on account of the suppression of the $\Sigma \to \Lambda$ transitions by structure factors due to the finite size of the nucleus.

In Ref. 114 it was conjectured that the observed narrow states of Σ hypernuclei are unstable bound states in the continuum. Such states, having a normalizable wave function, arise in the solution of the eigenvalue problem with the non-Hermitian Hamiltonian of the Σ–nucleus system. The poles of the S matrix corresponding to them lie in the lower half-plane of the first sheet of the Riemann surface $S(E_\Sigma)$. In contrast to the ordinary resonances lying on the second sheet near the physical region, the unstable bound states with ${\rm Re}\,E_\Sigma > 0$ are not manifested in Σ-hyperon scattering on a nucleus, but they may give a resonance structure in Λ–nucleus scattering near the threshold of the Σ–nucleus channel. For testing the hypothetical existence of unstable bound states, great interest attaches to study of the ^{4}He$(K^-, \pi^+)^4_\Sigma n$ reaction. As is shown by the estimates,[114] the $^4_\Sigma n$ state produced by the coherent $(1s, 1s^{-1})^{0+}_{\Sigma^- p}$ transition must lie in the continuum. If in the region $E_\Sigma > 0$ there is found to be a narrow state (in this case it cannot be an ordinary single-particle resonance because of the absence of a centrifugal barrier in the s wave), this will be a direct proof of the existence of an unstable bound state.

The variety of the approaches to the problem of $\Sigma\Lambda$ conversion in Σ hypernuclei indicates that its complete solution has not yet been found. Here we deal with the case in which the key to the description of the structure of a nuclear system is given by investigation of its decays. In this respect, great interest attaches to further experiments to study Σ hypernuclei in $(K^-, \pi^\mp)$ reactions on light nuclei, particularly at small momenta of the kaon beam ($p_K \lesssim 0.5$ GeV/c) under conditions when the coherent transitions are dominant.

CONCLUSIONS

There are several reasons for the interest in hypernuclei. One of them is the possibility of studying the low-energy interaction of hyperons with nucleons. Another, intimately related to the first, is the extension of the traditional methods and models of nuclear physics to the new region characterized by the presence in the nucleus of a strongly interacting particle not identical to the remaining constituents of the nucleus or, in other words, excitation in the nucleus of non-nucleon degrees of freedom associated with an additional quantum number—strangeness. The appearance of extensive experimental information on excited states of hypernuclei stimulated a rapid development of the theory. Two important circumstances simplifying the situation favored progress in this field, namely, the quasiparticle nature of the hyperon excitations and the presence of a large number of examples of weak coupling of the hyperon and nucleon degrees of freedom. It was established that the use of shell wave functions to describe the Λ hyperon in the nucleus makes it possible to obtain satisfactory agreement between theoretical calculations of the spectra and the cross sections for excitation of hypernuclei in the (K^-, π^-) reaction and the available experimental data. As a result, the shell model became the basis both for describing hypernuclear structure and for comparative analysis of the Λ–nucleus and N–nucleus interaction. The latter is of great importance for clarifying the nature of the various types of forces in the baryon–nucleus interaction, in particular the spin–orbit forces. The problem of LS forces and their relation to the elementary NN and ΛN interactions have recently been intensively investigated (see Refs. 110, 111, and 122), and it is possible that the study of hypernuclei will provide an answer to the old question of the nature of the nuclear spin–orbit forces.

Of considerable interest is the investigation of the interaction of hyperon and nucleon degrees of freedom, the most pronounced effects of which are expressed in deviations from the naive weak-coupling approximation. Such investigations have been begun for light Λ hypernuclei and provide material for determining the effective ΛN interction. Promising in this respect is the study of low-lying excitations in $(K^-, \pi^-\gamma)$ reactions. The high resolution is an important advantage of gamma-spectroscopic experiments, but their realization has hitherto been held back by the quality of the existing kaon beams. The possibilities of such experiments can be fully realized in kaon factories (Ref. 123).[11)]

The possibilities of investigating hypernuclei in the (K^-, π^-) reaction are far from exhausted. Study of the q dependences of the excitation cross sections of hypernuclei makes it possible not only to identify the individual states but also to consider more subtle questions of hypernuclear structure (mixing of configurations, splitting of levels). Strong decays of Λ hypernuclei are interesting in that their properties reflect the structure of the nucleon core and the presence of association effects. The $(K^-, \pi^{\mp})$ reaction is also the main source of Σ hypernuclei, whose structure attracts particular attention in connection with the importance of the $\Sigma N \to \Lambda N$ conversion processes, which do not have a close analog in traditional nuclear physics.

As alternative ways of producing hypernuclei, consideration has been given to the (π^+, K^+) reaction[49] and the (γ, K^+) reaction.[125] Compared with the (K^-, π^-) reaction, they have a number of advantages in the study of medium and heavy hypernuclei with high spin. It is to be expected that in heavy nuclei the Λ hyperon has a strong influence on the collective parameters (radius of the nucleus, moment of inertia),[67] and therefore study of heavy hypernuclei, about which very little is currently known, must significantly enrich our ideas about the interaction of hyperon and nucleon degrees of freedom.

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