Investigation of nuclear reactions and decays by analysis of their duration

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The paper reviews theoretical studies of the duration of collision, scattering, reaction, and decay processes, and also experimental methods of measuring the durations of nuclear reactions in the interval 10^{-20}–10^{-14} sec. The connection between the duration of a nuclear reaction and the S matrix and cross section is analyzed. The evolution of the decay of compound nuclei is considered. Particular attention is devoted to study of the duration of collisions in the region of overlapping resonances and in the threshold region. It is shown how measurements of the lifetimes of compound nuclei can yield information about the densities and widths of unresolved resonances not observed in the cross sections.

INTRODUCTION

Besides the measurement of cross sections, correlations, polarizations, and other quantities not directly related to the effects of the duration and evolution of nuclear reactions and decays,[1)] another type of experiment has long been known in nuclear physics—direct measurement of the duration of nuclear processes (decays) and quantities that depend explicitly on the way in which the processes evolve in time. The simplest experiments of such type are the ones that establish the exponential form of a decay law and measurements of the lifetimes of radioactive nuclei that live fairly long (from 10^{-9} sec to 10^{14} years, and, in some cases, up to 10^{18} years and more).

Excited states of nuclei with energies below the threshold for emission of nucleons decay through electromagnetic transitions with characteristic times of 10^{-15}–10^{-9} sec. For time measurements in this range, various experimental methods have already been developed on the basis of Coulomb excitation, the Doppler effect, delayed coincidences, resonance γ scattering, etc.

With regard to the lifetime of nuclei in an excited state whose energy exceeds the dissociation energy of the nuclei (i.e., composite systems and compound nuclei), their characteristic values fill the interval from 10^{-23} to, at least, 10^{-15} sec. Methods of measuring durations on such scales began to be developed only about 20 years ago.

Although time measurements are actually as common in the experimental practice of nuclear research as measurements of other fundamental quantities (energy, momentum, etc.), the development of systematic theoretical investigations of the duration and evolution of nuclear reactions and decays was for a long time delayed by serious problems in the mathematical formalism of quantum theory. Since in a consistent theory of collisions it is impossible to get by without a picture of the evolution in time and each model mechanism of nuclear reactions is based essentially on some picture of the evolution of the processes that take place in the system of colliding particles, the theoretical investigations of nuclear reactions also confronted the problems involved in analyzing their duration and evolution. Gradually, as the duration of collision processes was studied and the properties of time as an observable in quantum mechanics were examined at the general theoretical level, it became clear that the duration of a collision can be of interest from several points of view:

1) as an observable directly related to other observables and characteristics of quantum systems, i.e., the S matrix, T matrix, scattering amplitude, cross section, statistical density of continuum states, and, indeed moreover, the number of bound states of a composite system;

2) as an observable that is a source of information about the properties of quantum systems that cannot always be extracted in practice from other observables;

3) as one of the most important characteristics of the mechanism of nuclear reactions;

4) as a quantity used in the causality condition that strongly influences the analytic (in particular, pole) structure of the S matrix.

1. TIME AS AN OBSERVABLE IN QUANTUM MECHANICS

It was already noted in the year when the creation of the foundations of quantum mechanics was completed that there is a difference between time and other fundamental physical quantities (energy, momentum, angular momentum, spatial quantities). In accordance with the generally accepted propositions of quantum mechanics, all physical quantities (observables) are associated with linear self-adjoint operators, for which real eigenvalues are defined and a complete set of orthogonal eigenfunctions exists. The uniqueness of the spectral function (orthogonal decomposition of unity) of such an operator guarantees the uniqueness (equivalence) of calculations of the mean values and variances of the corresponding quantities, the transition probabilities, and so forth, in all possible representations, and also the universality of the fundamental uncertainty relations for pairs of canonically conjugate quantities. However, the construction of a linear self-adjoint time operator proved to be impossible. The impossibility of constructing in nonrelativistic quantum mechanics a self-adjoint time operator canonically conjugate to the energy operator was pointed out for the first time by Pauli.[1] Ultimately, this impossibility is due to the fact that the energy spectrum is bounded below for any real physical system (for more detail on this, see, for example, Refs. 2–5).

[1)]The decay of nuclei can be regarded as the final stage of nuclear reactions.

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This circumstance had the consequence that in the literature on quantum mechanics, and on atomic and nuclear physics, since then and until very recently there was no clear approach to the definition of the duration of processes of collision, scattering, reaction, decay, and so forth. Moreover, in standard, orthodox treatments of quantum mechanics time was not even regarded as an observable, but was used only as an evolution parameter of systems. Its interpretation actually remained classical and was certainly not developed to the same level of clarity and rigor as in the case of the other fundamental quantities. Nevertheless, in the majority of studies there was little doubt with regard to the existence of an uncertainty relation for the energy and time, $\Delta E \times \Delta t \sim h$, or, more rigorously,

$$\Delta E \cdot \Delta t \geqslant \hbar/2, \tag{1}$$

which up to the seventies had nevertheless not been derived with such rigor and universality as the uncertainty relation for the coordinate and momentum[2)]:

$$\Delta p_x \Delta x \geqslant \hbar/2, \tag{2}$$

which follows directly from the relationship between the two linear self-adjoint operators of the coordinate $\hat{x}$ and the momentum $\hat{p}_x$:

$$[\hat{x},\ \hat{p}_x] = i\hbar. \tag{3}$$

Th relation (1) has frequently been discussed, illustrated, and confirmed by means of special examples, thought experiments, and arguments lacking sufficient rigor, clarity, and universality. The most general (indirect) justification of the relation (1) up to 1973 was given in Ref. 6. But it still lacked sufficient universality. The very meaning of time in quantum mechanics has not yet been elucidated. The derivation of Ref. 6 was taken in Ref. 7 as the basis of a fundamental interpretation of the relation (1).

The unsatisfactory state of the problem of the interpretation of time in quantum mechanics generated numerous attempts to solve this problem. Papers were published that discussed the part played by time in quantum mechanics and analyzed various suggestions about the properties of time[8,9] that in one way or another anticipated further investigations into the properties of time in quantum mechanics. Some papers considered various examples and properties of the forms of a time operator $\hat{T}$, which were obtained by postulating the operator relation

$$[\hat{T},\ \hat{H}] = i\hbar \tag{4}$$

($\hat{H}$ is the Hamiltonian operator of the system),[2,10–17] and some authors (for example, in Ref. 14) announced the impossibility of interpreting time as an observable in the framework of the traditional concept of a measurement, while others (for example, in Ref. 2) pointed out the possibility of extending the family of observables, including in it, in particular, time. However, even in the papers that claimed that time could be regarded as an observable (Refs. 2, 10–13, and 15–17), this assertion was not adequately justified and, in addition, there was no discussion or no sufficiently rigorous derivation of the uncertainty relation (1) for the energy and time. Moreover, these papers entirely ignored applications to real physical processes with the calculation of the expectation values of durations, variances of durations, and other time characteristics. Finally, all these studies[2,10–17] considered the operator

$$\hat{T} = i\hbar\partial/\partial E \ \ (0 < E < \infty), \tag{5}$$

which satisfies the relation (4) and, as was shown in Refs. 3–5, leads when applied to real quantum processes to the incorrect sign in the expression for the expectation value of a duration.

Such a state of the problem of time in quantum mechanics was naturally incapable of providing a basis for developing methods of calculation of the duration of quantum processes at the same level of rigor as in the cases of the expectation values of other observables. The absence of reliable theoretical basis explains the appearance of numerous devices, prescriptions, algorithms, procedures, and models for determining the times of scattering, collision, reactions, decay, and so forth—by no means always mutually consistent. For example, the studies of Refs. 18–20 from the very beginning eliminated the time that occurs in the formalism of time-dependent quantum mechanics (time-dependent Schrödinger equation) and in the framework of a stationary (timeless) approach, using artificial devices or quasiclassical parameters, postulated a speculative "time" suitable for a restrictive class of problems. Attempts to generalize such procedures for the duration of larger classes of collisions (with allowance, for example, for collision processes over finite distances within interaction regions, with the emission of more than two final particles, etc.) encounter serious difficulties. The elucidation of the very meaning of such a time gives rise to difficulties.[3)] Finally, in the framework of the stationary approach the problem of justifying the uncertainty relation (1) is completely unsolved.

The first estimates of the duration of the simplest scattering processes in the framework of the time-dependent Schrödinger equation were made in Refs. 21–23. In Refs. 21–23, and also in Ref. 24, the duration of the scattering of wave packets was calculated on the basis of the determination of the location of the centers of packets of converging and diverging waves (or an incident packet of plane waves and a final packet of scattered waves) in accordance with the conditon of the interference maximum. In Refs. 25–28 there was a somewhat more detailed but nevertheless clearly restricted study of the delay time of scattered packets.

In Refs. 29 and 30, and then in other papers (for example, in Refs. 31 and 32) the duration of the simplest processes of two-particle collisions was determined by averaging with respect to a variable time coordinate (regarded as a parameter). The effective time of passage of the initial and final

[2)]Despite what would appear to be a direct possibility of transition from (2) to (1) or the simple substitution $\Delta p_x = (\mu/p_x)\Delta E$, $\Delta x = (p_x/\mu)\Delta t$ for freely moving wave packets (μ is the particle mass).

[3)]In particular, the definition of the scattering time in Ref. 19 in terms of the ratio of the deflection angle θ of the spin of the scattered particle in a magnetic field, introduced speculatively into the region of the nuclear interaction, to the precession frequency is internally inconsistent,[5] since it contains the assumption of uniform increase of θ with the time, which is refuted by analysis of the same assumption even in the case of uniform motion of a free particle.

wave packets through an arbitrary point of space far from the region of interaction of the incident particle and the scatterer was determined by averaging the moment of time with respect to the duration of passage of the packet. The effective collision duration was defined as the difference between the effective moments of passage of the final and initial moments through a sphere of radius appreciably exceeding the radius of the region of interaction of the colliding particles.

In Refs. 3 and 4, in demonstrating the uniqueness of the definition of the collision duration, the present author introduced a time operator $\hat{t}$ that reduces in the time (t) representation to multiplication by t and has the form

$$\hat{t} = -i\hbar\partial/\partial E \quad (0 < E < \infty) \tag{6}$$

in the energy (E) representation. In particular,

$$\int_{-\infty}^{\infty} \varphi_1^*(x,\ t)\, t\varphi_2(x,\ t)\, dt =$$

$$= \int_0^{\infty} g_1^*(x,\ E)\left(-i\hbar\frac{\partial}{\partial E}\right) g_2(x,\ E)\, dE \equiv (g_1,\ \hat{t}g_2), \tag{7}$$

where

$$\varphi_{1,2}(x,\ t) = \int_0^{\infty} g_{1,2}(x,\ E)\exp(-iEt/\hbar)\, dE \equiv$$

$$\equiv \int_0^{\infty} G_{1,2}(E)\exp(ikx - iEt/\hbar)\, dE$$

is a square-integrable one-dimensional wave packet describing the motion of a particle in the direction of the x axis ($E = \hbar^2k^2/2\mu$, where μ is the particle mass). Obviously, the operator (6) is Hermitian, i.e., the relation

$$(g_1,\ \hat{t}g_2) = ((\hat{t}g_1),\ g_2)$$

holds, only if all square-integrable functions $G(E)$ in the space on which it is defined vanish together with their derivatives for $E = 0$. In other words, the Hermitian operator (6) is defined in a subspace $B \subset \mathscr{L}^2$, where $\mathscr{L}^2$ is the Hilbert space of all square-integrable functions. In accordance with the results of the mathematical theory of operators (see, for example, Ref. 33), it is a non-self-adjoint operator with equal deficiency indices, and a maximal Hermitian (symmetric) operator with deficiency indices (0, 1) and has a unique generalized (nonorthogonal) spectral function (generalized decomposition of unity). It follows from the last property that the results of calculating expressions of the type (7) are unique (including averaging over a wave-packet duration), i.e., such expressions are equivalent in the t and E representations. Using the properties of the operator (6), it is easy to justify the uncertainty relation (1) for the energy and time with the same degree of rigor and generality as was done for the relations (2); see, for example, Refs. 34 and 35 (and also Appendix A). The operator (6), which is defined in contrast to (5) by a procedure for calculating the duration of real quantum processes, satisfies the commutation relation

$$[\hat{H},\ \hat{t}] = i\hbar \tag{8}$$

instead of (4).

By means of simple calculations using the properties of the time operator (6) it is easy to show[4] that for motion of a wave packet Ehrenfest's theorem holds in the form $\langle \Delta t \rangle = \Delta x/\langle v \rangle$, where $\Delta x = x_2 - x_1$ is the distance between the points (or surfaces) through which the packet passes successively, $v = \hbar k/\mu$, and $\langle \Delta t \rangle = \langle t(x_2) \rangle - \langle t(x_1) \rangle$ is the interval between the moments of passage of the packet through the points (surfaces) x_2 and x_1 averaged with respect to the duration of its passage.

For the operator (6), as for every maximal Hermitian operator, exact eignevalues and orthogonal eigenfunctions are not defined. If the continuous spectrum of energies of physical systems were unbounded below, the operator (6) would be self-adjoint on the axis $-\infty < E < \infty$, and for it there would exist exact continuous eigenvalues and orthogonal eigenfunctions. If the energy spectrum were bounded not only below but also above ($0 < E < E_m$), the operator (6) would also be self-adjoint and, like the momentum operator in a bounded box with rigid walls, would have discrete eigenvalues (separated by $t_0 = 2\pi\hbar/E_m$). The generalized decomposition of unity of the operator (6) can be approximated with arbitrary accuracy by an orthogonal decomposition of unity.[36] Moreover, for it one can construct approximate eigenvalues and approximate orthogonal eigenfunctions with any degree of accuracy (see Appendix B).

The uniqueness of the spectral function of the operator (6) makes it possible to establish uniqueness of the calculation of not only the expectation values of durations and variances of durations of quantum processes in any possible representation (time, energy, momentum) but also the expectation values of any analytic function of the time expanded in a convergent series of the type $\Sigma_n c_n t^n$.

Thus, the introduction and use of the properties of the operator (6) makes it possible to regard time as an observable in quantum mechanics and to extend the probability interpretation of the wave function as follows: The quantity $|\varphi(x, t)|^2 dt$ characterizes the probability of finding the particle at the point (on the surface) x in the time interval from t to $t + dt$, and $j_x\, dt$, where

$$j_x(x, t) = \mathrm{Re}\varphi^*(x, t)\left(-\frac{i\hbar}{2\mu}\frac{\partial}{\partial x}\right)\varphi(x, t),$$

characterizes the probability that the particle passes through the surface $x = \text{const}$ in the direction of the x axis in the time interval from t to $t + dt$. In this light, we obtain, in particular, a natural explanation of the well-known empirical fact of the statistical distribution of the lifetimes of radioactive nuclei as a manifestation of the probabilistic distribution of the durations of quantum processes.

The results presented here[3,4] were used in the subsequent development of the mathematical formalism of the time operator in Refs. 37 and 38, and also in the reviews of Refs. 39 and 40. Independently, it was also established in Refs. 41 and 42 that time is an observable in quantum mechanics.

In Refs. 43 and 44, and then in Ref. 45, an alternative approach to the definition of collision durations was developed on the basis of averaging of coordinates with respect to the spatial dimensions of wave packets. Such a possibility was also discussed in Refs. 46–48 and follows essentially from Ehrenfest's theorem in the form $\langle \Delta x \rangle = \langle v \rangle \Delta t$, where

$\Delta t = t_2 - t_1$ is a time interval, and $\langle \Delta x \rangle = \langle x(t_2) \rangle - \langle x(t_1) \rangle$ is the distance between the two positions of the spatial center of the wave packet at the times t_2 and t_1 averaged over the extension of the packet in the direction of the average velocity of its motion. The effective collision duration in this approach was defined as the difference between the moment of separation of the spatial centers of the separating packets of the final particles and the moment of encounter of the centers of the colliding packets of the initial particles (in the directions of the average velocities of their relative motion) to a certain distance exceeding the radius of the interaction region. At large distances and small spreads of the momenta, the results obtained in Refs. 43–48 are almost identical to the results of calculation of durations by means of averaging over the time. However, such an approach is more restricted, since there are difficulties in applying it at short distances (near the boundary of and, especially, within the interaction region).

2. GENERAL EXPRESSIONS FOR THE COLLISION DURATION

In the case of short-range interactions that vanish completely outside a region of radius R, and if the spins of the particles are ignored, the effective (average) duration of the reaction $i \to j$ is defined in accordance with Refs. 3–5, 29, and 49 by

$$\begin{aligned}\langle \tau_{j^{(n)}(i)} \rangle &= \langle t_{j^{(n)}} \rangle_{j^{(n)}} - \langle t_i \rangle_i \\ &\equiv \int_{-\infty}^{\infty} t J_{j^{(n)}}(z_{j^{(n)}}, t)\, dt \Big/ \int_{-\infty}^{\infty} J_{j^{(n)}}(z_{j(n)}, t)\, dt \\ &\quad - \int_{-\infty}^{\infty} t J_i(z_i, t)\, dt \Big/ \int_{-\infty}^{\infty} J_i(z_i, t)\, dt \\ &\equiv \langle |T_{ji}(E, \Omega_j)|^2 [\hbar \partial \arg G_{j^{(n)}} T_{ji} / \partial \varepsilon_{j^{(n)}} \\ &\quad + z_{j^{(n)}} / v_{j^{(n)}}] \rangle_{j^{(n)}} / \langle |T_{ji}(E, \Omega_j)|^2 \rangle_{j^{(n)}} \\ &\quad - \langle [\hbar \partial \arg G_i / \partial \varepsilon_i + z_i / v_i] \rangle_i, \end{aligned} \tag{9}$$

where

$$J_\nu(z_\nu, t) = \mathrm{Re}\left(\varphi_\nu, \left(-i\hbar/2\mu_\nu \frac{\partial}{\partial z_\nu}\varphi_\nu\right)_\nu\right)$$

is the z_ν-th component of the probability flux; the brackets $(\ldots)_\nu$ denote integration over all the coordinates occurring in the wave functions φ_ν except z_ν; the z_ν axis is taken along the average velocity $\langle \mathbf{v}_\nu \rangle$ of the relative motion of the pair of particles with reduced mass μ_ν, radius factor $\mathbf{r}_\nu$ of the relative motion, and wave function $|\nu\rangle$ and energy e_ν of the internal motion: $v_\nu = \hbar k_\nu / \mu_\nu$, $\varepsilon_\nu = \hbar^2 k_\nu^2 / 2\mu_\nu = E - e_\nu$; the brackets $\langle \ldots \rangle_\nu$ denote averaging of the form

$$\begin{aligned}\langle A \rangle_\nu &= \int |G_\nu|^2 A dE\, d\omega_\nu \Big/ \int |G_\nu|^2 dE\, d\omega_\nu, \quad \nu = i,\ j^{(n)}; \\ \varphi_i &= |i\rangle (2\pi)^{-3/2} \int G_i(E, \omega_i) \exp(i\mathbf{k}_i \mathbf{r}_i - iEt/\hbar)\, dE\, d\omega_i; \end{aligned} \tag{10}$$

$$\begin{aligned}\varphi_{j^{(n)}} = |j^{(n)}\rangle (2\pi)^{-3/2} &\int G_{j^{(n)}}(E, \omega_{j^{(n)}}) T_{ji}(E, \Omega_j) \\ &\times \exp(i\mathbf{k}_{j^{(n)}} \mathbf{r}_{j^{(n)}} - iEt/\hbar)\, dE\, d\omega_{j^{(n)}} \end{aligned} \tag{11}$$

(for simplicity, we have omitted the index i of $\varphi_{j(n)}$ and $J_{j(n)}$); n is an index introduced in the presence of more than two particles in the exit channel, and it labels different pairs of final particles, $G_{j^{(n)}} = \Lambda_{j^{(n)}}(\varepsilon_{j^{(n)}}, \omega_{j^{(n)}}) G_i(E, \omega_i)$, where $G_i(E, \omega_i)$ is the weight function of the initial wave packet of the colliding particles; the function $\Lambda_{j^{(n)}}(\varepsilon_{j^{(n)}}, \omega_{j^{(n)}})$, which without loss of generality can be taken to be real, describes the resolution properties of the detectors of the final particles; ω_ν are the angular variables that describe the spreads of the directions of the velocities v_ν about the average values $\langle \mathbf{v}_\nu \rangle$; $T_{ji}(E, \Omega_j)$ is an element of the T matrix, E is the total energy of the system, Ω_j is the set of angular coordinates that characterize the directions of emission of the products of the reaction $i \to j$ (in the center-of-mass system), and $z_{j^{(n)}} \geqslant R$, $z_i \leqslant -R$. As usual, we shall assume that the spread of energies ΔE in the packets of the colliding and separating particles satisfies the condition $\Delta E = \Delta \varepsilon_\nu \ll \varepsilon_\nu$, and that the spreads $\Delta \omega_i$ and $\Delta \omega_{j^{(n)}}$ are so small that the variables ω_i and $\omega_{j^{(n)}}$ in the functions G_i and $G_{j^{(n)}}$, and also the integration over $d\omega_i$ and $d\omega_{j^{(n)}}$, can be omitted. Then the index ν in $\langle \ldots \rangle_\nu$ can also be omitted, and henceforth we shall use the symbol $\langle \ldots \rangle$. Instead of (9), one frequently uses the concept of the effective (average) time of delay (or advance) in the reaction $i \to j$, which is determined by the expression

$$\langle \Delta \tau_{j^{(n)}i} \rangle = + \langle \tau_{j^{(n)}i} \rangle - \langle \tau^{(0)}_{j^{(n)}i} \rangle, \tag{12}$$

where $\langle \tau^{(0)}_{j^{(n)}i} \rangle$ differs from $\langle \tau_{j^{(n)}i} \rangle$ by the substitution of $|T_{ji}|$ in place of T_{ji} in the integrands and describes the total time of free motion of a particle with mass μ_i and velocity $\langle \mathbf{v}_i \rangle$ along the distance $|z_i|$ and of a particle with mass $\mu_{j^{(n)}}$ and velocity $\mathbf{v}_{j^{(n)}}$ along the distance $z_{j^{(n)}}$. For

$$|\langle \partial \arg G_i / \partial \varepsilon_i \rangle| \ll |\partial \arg T_{ji} / \partial \varepsilon_{j^{(n)}} \rangle| \tag{13}$$

the quantity $\langle \Delta \tau_{j^{(n)}i} \rangle$ determined by the relation (12) is equal to

$$\langle \Delta \tau_{j^{(n)}i}(E, \Omega_j) \rangle = \langle |T_{ji}(E, \Omega_j)|^2 \hbar \partial \arg T_{ji} / \partial \varepsilon_{j^{(n)}} \rangle / \langle |T_{ji}(E, \Omega_j)|^2 \rangle. \tag{14}$$

The condition (13) means physically that when $\langle \Delta \tau_{j^{(n)}i} \rangle$ is appreciably greater than $\langle \tau^{(0)}_{j^{(n)}i} \rangle$ the influence of the distortion in the shape of the colliding packets in the process of the reaction on $\langle \Delta \tau_{j^{(n)}i} \rangle$ can be ignored.

In the cases when the interactions do not vanish strictly outside a region of finite radius and, moreover, the charged particles have a Coulomb potential, (10) and (11) must be replaced by more general expressions containing distorted rather than plane waves, or in place of the T matrix one must introduce the concept of an evolution operator.[5,30,49]

In such a case, the right-hand sides of (9) and (14) become much more complicated.[4)] But if we go to the limit $z_j \to \infty$, $z_i \to -\infty$ in accordance with the boundary conditions in experimental determinations of cross sections and durations of nuclear reactions, we obtain the previous result

[4)] A convenient method of estimating scattering durations at short distances is Calogero's variable-phase method.[50] The use of this method made it possible to obtain an analytic expression for the delay time of a simple process of potential scattering, the expression being valid for all values of r.[4]

(14). Thus, in the general case it is not (9) but the delay time (14) that has a real physical meaning in characterizing a reaction process as a whole.

In reality, it must also be borne in mind that in the experimental study of collisions one is generally dealing, not with a single collision event between two particles, but with a large number of collisions resulting from the bombardment of a large number of target nuclei by a beam of incident particles. To take into account this and also particle spins the density-matrix formalism can be used.[5] In the case of a macroscopic ensemble of pairs of unpolarized colliding particles whose average energies are distributed in the interval $\Delta_m E$, and whose initial phases are uncorrelated, the expression (14) keeps the same form, provided the symbol $\langle \ldots \rangle$ denotes averaging over the spread of E and $\omega_{j^{(n)}}$ both in an individual packet, and in the incoherent mixture of packets, and also averaging over the initial and summation over the final spin states.

In the case of two-particle reactions (the index n is omitted) and under the usual simplifications (central interactions, neglect of the spins) one frequently uses the concepts of the partial and integrated delay times $\langle \Delta\tau_{ji}^{(J)}(E)\rangle$ and $\langle \Delta\tau_{ji}^{\mathrm{int}}(E)\rangle$, which are determined by means of the expressions

$$\langle \Delta\tau_{ji}^{(J)}(E)\rangle = \hbar \langle |T_{ji}^{(J)}(E)|^2 \, d \arg T_{ji}^{(J)}(E)/dE\rangle / \langle |T_{ji}^{(J)}(E)|^2\rangle \quad (15)$$

and

$$\langle \Delta\tau_{ji}^{\mathrm{int}}(E)\rangle = \hbar \frac{\int |G_i T_{ji}(E,\ \vartheta_j)|^2 (\partial \arg T_{ji}(E,\ \vartheta_j)/\partial\varepsilon_j)\, dE\, d\Omega_j}{\int |G_i T_{ji}(E,\ \vartheta_j)|^2\, dE\, d\Omega_j} = \hbar \frac{\sum\limits_J \langle \sigma_{ji}^J(E)\, d \arg T_{ji}^{(J)}/dE\rangle}{\sum\limits_J \langle \sigma_{ji}^{(J)}(E)\rangle}. \quad (16)$$

where $T_{ji}^{(J)}$ and $\sigma_{ji}^{(J)}$ are defined by the expansions

$$T_{ji}(E,\ \vartheta_j) = -\frac{\hbar^2}{4\pi\mu_j} f_{ji}(E,\ \vartheta_j) = \sum_J T_{ji}^{(J)}(E) \sqrt{4\pi(2J+1)}\, Y_{J0}(\vartheta_j) \equiv -\frac{\hbar^2}{8\pi i \mu_j (k_j k_i)^{1/2}} \sum_J [S_{ji}^{(J)} - \delta_{ji}] \sqrt{4\pi(2J+1)}\, Y_{J0}(\vartheta_j), \quad (17)$$

$$\vartheta_j = \arccos \mathbf{k}_j \mathbf{k}_i / k_j k_i,$$

$$\sigma_{ji} = \sum_J \sigma_{ji}^{(J)} \equiv \frac{\pi}{k_i^2} \sum_J (2J+1) |S_{ji}^{(J)} - \delta_{ji}|^2. \quad (18)$$

The average delay time in channel i is defined as[51]

$$\langle \Delta\tau_i^{(\mathrm{int})}\rangle = \frac{\sum\limits_j \int d\Omega_j \langle \Delta\tau_{ji}(E,\ \vartheta_j)\rangle \langle |T_{ji}(E,\ \vartheta_j)|^2\rangle}{\sum\limits_j \int d\Omega_j \langle |T_{ji}(E,\ \vartheta_j)|^2\rangle} = \hbar \frac{\sum\limits_j \sum\limits_J \langle \sigma_{ji}^{(J)}(E)\, d \arg T_{ji}^{(J)}(E)/dE\rangle}{\sum\limits_j \sum\limits_J \langle \sigma_{ji}^{(J)}(E)\rangle} \equiv \frac{\sum\limits_J \langle \Delta\tau_i^{(J)}(E)\rangle \sum\limits_j \langle \sigma_{ji}^{(J)}(E)\rangle}{\sum\limits_J \sum\limits_j \langle \sigma_{ji}^{(J)}(E)\rangle}. \quad (19)$$

3. VARIANCE OF THE DISTRIBUTIONS OF THE DURATIONS

To describe the evolution of collision processes, one uses not only the expectation value of the duration but also the variance of the distributions of the durations. Defining the variance of the duration as

$$\mathscr{D}\tau_{ji} = \langle (t_j - \langle t_i\rangle)^2\rangle - (\langle t_j\rangle - \langle t_i\rangle)^2 = \langle t_j^2\rangle - \langle t_j\rangle^2, \quad (20)$$

and applying the technique of the time operator (6), we obtain the expression

$$\langle t_j^2\rangle = \langle [(\hbar\partial \arg G_j T_{ji}/\partial\varepsilon_j + z_j/v_j)^2 + \hbar^2 (\partial |G_j T_{ji}|/\partial E)^2]\rangle / \langle |T_{ji}|^2\rangle.$$

For sufficiently large collision duration, when $|\partial \arg G_j/\partial\varepsilon_j| \ll |\partial \arg T_{ji}/\partial\varepsilon_j|$, $z_j/v_j \ll \hbar\partial \arg T_{ji}/\partial\varepsilon_j$ and $|\partial |G_j|/\partial\varepsilon_j| \ll \partial \arg T_{ji}/\partial\varepsilon_j$, the variance of the duration takes the form

$$\mathscr{D}\tau_{ji} \approx \hbar^2 \{ \langle [|T_{ji}|^2 (\partial \arg T_{ji}/\partial\varepsilon_j)^2 + (\partial |T_{ji}|/\partial\varepsilon_j)^2]\rangle / \langle |T_{ji}|^2\rangle - |\langle |T_{ji}|^2\, \partial \arg T_{ji}/\partial\varepsilon_j\rangle / \langle |T_{ji}|^2\rangle|^2 \}. \quad (21)$$

4. DURATION OF SINGLE-CHANNEL SCATTERING AND DECAY PROCESSES

In the simplest case of single-channel scattering, when one partial J wave is predominant [for example, for slow particles ($J = 0$) and in the region of J resonances], we have in accordance with (15) for $\Delta E \ll \tilde{\Delta}E \ll E$ ($\tilde{\Delta}E$ is the interval of significant variation of $T_{ii}^{(J)}(E)$)

$$\langle \Delta\tau_{ii}^{(J)}\rangle \approx \Delta\tau_{ii}^{(J)}(E) = \hbar d\delta_J(E)/dE, \quad (22)$$

where δ_J is a real phase shift determined by $S_{ii}^{(J)}(k_i) \equiv S_J(k) = \exp(2i\delta_J)$. For the large class of interactions that either vanish completely or decrease sufficiently rapidly outside a finite region $r \leqslant R$, $S_{ii}^{(J)}$ can be represented in the form[52,53]

$$S_J(k) = e^{-2i\alpha k} \prod_\lambda \frac{1 + k/k_{\lambda J}}{1 - k/k_{\lambda J}} \prod_\mu \frac{1 + k/\kappa_{\nu J}}{1 - k/k_{\mu J}} \prod_\nu \frac{(1 + k/k_{\nu J})(1 - k/k_{\nu J}^*)}{(1 - k/k_{\nu J})(1 + k/k_{\nu J}^*)}, \quad (23)$$

where $\alpha \leqslant R$; $k_{\lambda J} = i\chi_{\lambda J}$ ($\chi_{\lambda J} > 0$); $k_{\mu J} = i\chi_{\mu J}$ ($\chi_{\mu J} < 0$); $k_{\nu J}$ ($\mathrm{Re}\, k_{\nu J} > 0$, $\mathrm{Im}\, k_{\nu J} < 0$) are poles corresponding to bound virtual and resonance states, respectively, or, sufficiently far from the point $E = 0$, in the simplified form

$$S_J(k) \simeq \tilde{S}_J(k) \prod_\nu \frac{E - E_\nu^{(J)} - i\Gamma_\nu^{(J)}/2}{E - E_\nu^{(J)} + i\Gamma_\nu^{(J)}/2}, \quad (24)$$

where $\tilde{S}_J = \exp(2i\delta_J)$ is the "background" (nonresonance) S matrix; $E_\nu^{(J)} = \hbar^2 |k_{\nu J}|^2/2\mu$; $\Gamma_\nu^{(J)} = -2k\, \mathrm{Im}\, k_{\nu J} \hbar^2/2\mu$ (for $J = 0$).

With allowance for the representation (23), we can rewrite the expression (22) in the form

$$\Delta\tau_{ii}^{(J)}(E) = \Big[-\alpha - \sum_\lambda \frac{\chi_{\lambda J}}{k^2 + \chi_{\lambda J}^2} - \sum_\mu \frac{\chi_{\mu J}}{k^2 + \chi_{\mu J}^2} - \sum_\nu \frac{2\, \mathrm{Im}\, k_{\nu J}(k^2 + |k_{\nu J}|^2)}{(k^2 - |k_{\nu J}|^2)^2 + 4k^2 (\mathrm{Im}\, k_{\nu J})^2} \Big] \Big/ v; \quad v = \hbar k/\mu. \quad (25)$$

From (25), taking into account the results of Refs. 52 and 53,

it can be seen that for values of r exceeding some minimal value $\tilde{R} \geqslant R/2 + \chi_{\lambda_0 J}/4$, where $\chi_{\lambda_0 J}$ corresponds to the bound state of the system consisting of the scatterer and the incident particle with binding energy of the smallest absolute magnitude, a causality condition is satisfied in the form

$$\Delta\tau_{ii}^{(J)}(E) + 2r/v \geqslant 0. \tag{26}$$

Further, it follows directly from (25) that for only one bound or virtual state of the system as $E \to 0$

$$\Delta\tau_{ii}^{(J)}(E) \xrightarrow[E\to 0]{} -a/v\,[1 + k^2a^2], \tag{27}$$

where a, the scattering length, is equal to $(\chi_{\lambda J})^{-1} > 0$ or $(\chi_{\mu J})^{-1} < 0$.

In the region of one isolated resonance,

$$\langle\Delta\tau_{ii}^{(J)}(E)\rangle = \begin{cases} \hbar/\Gamma_r & \text{for } \Gamma_r \ll \Delta E, \\ \hbar\Gamma_r/2\,[(E-E_r)^2 + \Gamma_r^2/4] & \text{for } \Delta E \ll \Gamma_r. \end{cases} \tag{28}$$

At the same time, calculation in accordance with Eq. (21) shows that in the case $\Gamma_r \ll \Delta E$ the variance is $\mathscr{D}\tau_{ii}^{(J)} \approx \langle\Delta\tau_{ii}^{(J)}\rangle^2 = (\hbar/\Gamma_r)^2$. Going over to the time representation, we obtain in the approximations in which the interval of integration in $\langle\Delta\tau_{ii}^{(J)}\rangle$ is extended to the entire real axis ($-\infty < E < \infty$) and $\Gamma_r = \text{const}$ an exponential form of the decay law of such a resonance state in the form $\exp(-\Gamma_r t/\hbar)$. A more detailed study (see, for example, Refs. 54–57) shows that even for an isolated resonance the exponential form of the decay law is an approximation satisfied in a time interval bounded below by $t_1 \sim t_0\,\Gamma_r/E_r$ and above by $t_2 \sim t_0 \ln(E_r/\Gamma_r)$, where $t_0 \sim \hbar/\Gamma_r$, the accuracy of the description by the exponential form being better, the smaller the ratio Γ_r/E_r. At times less than t_1 and greater than t_2 appreciable deviations from the exponential law can be observed. Ultimately, these are due, first, to the fact that the energy continuum is bounded below by zero, second, to the presence of poles of the S matrix corresponding to other resonances, and other singularities of the S matrix, and, third, to the dependence $\Gamma_r(E)$ different from $\sqrt{E}$.

It is well known that at energies below the Coulomb barrier long-lived states can be associated with the interaction of protons, α particles, and heavy ions with nuclei. This makes it possible to use data on the lifetimes of the corresponding radioactive compound nuclei to establish the hyperfine structure of the cross sections. Experimental study of the deviations from the exponential decay law of such nuclei for $t \gg t_0$ can in principle be a source of information about the density of the energy distribution in the wave packets describing the decaying nuclei and, moreover, the initial stage of the corresponding reactions.

If the interval ΔE contains a set of resonances and

$$\Delta E \gg \Gamma_J \gg D_J \tag{29}$$

(Γ_J and D_J are the average width and average distance between the J resonances), then calculation of $\langle\Delta\tau_{ii}^{(J)}(E)\rangle$ on the basis of (15) with allowance for (24) gives

$$\langle\Delta\tau_{ii}^{(J)}(E)\rangle = \hbar \sum_{\nu} \frac{\Gamma_\nu^{(J)}/2}{(E-E_\nu^{(J)})^2 + (\Gamma_\nu^{(J)}/2)^2} \approx \pi\hbar\rho_J; \tag{30}$$

where $\rho_J = D_J^{-1}$. Thus, the average delay time in elastic scattering in the region (29) is determined by the density of the resonances and not by their width. Note that the expression (30) is obtained in the continuum approximation:

$$\left(\sum_\nu \to \int dE_\nu^{(J)}\,\rho_J\right).$$

When the interval ΔE contains a set of well-separated resonances, i.e., $\Delta E \gg D_J \gg \Gamma_J$, it is readily seen that if the widths $\Gamma_\nu^{(J)} = \Gamma_J$ of all the levels are equal

$$\langle\Delta\tau_{ii}^{(J)}(E)\rangle \simeq \hbar/\Gamma_J, \tag{31}$$

as in the case of an isolated resonance of the same width.

In the case of very strong overlapping of the resonances, when

$$D_J \ll \Delta E < \Gamma_J, \tag{32}$$

$$\langle\Delta\tau_{ii}^{(J)}(E)\rangle \sim \hbar \int_{E_{\min}}^{\infty} dE'\rho_J E' \frac{\Gamma_J(E')/2}{(E-E')^2 + [\Gamma_J(E')/2]^2}, \tag{33}$$

where the lower limit must satisfy the obvious conditions $E_{\min} \lesssim E - \Gamma_J$ and $\rho_J(E_{\min}) < (\rho_J)_{\min}$.[58] It is obvious that in the case of an equidistant distribution of resonances ($\rho_J = \text{const}$) and energy-independent widths ($\Gamma_J = \text{const}$) for all possible values of the energy $\langle\Delta\tau_{ii}^{(J)}(E)\rangle \sim \pi\hbar\rho_J$, i.e., we again obtain Eqs. (30).

The duration of scattering is sometimes related to the density of the continuum states.[59] In essence, this last, being infinite, is devoid of real physical meaning. However, considering the scattering of particles in a large but finite volume and imposing the condition of vanishing of the wave functions on the boundaries of the volume, we can readily obtain the following expression for the density of states dn_J/dE: $dn_J/dE = [R/\hbar v + d\delta_J/dE]/\pi$.[60] In the limit $R \to \infty$, this quantity diverges. Nevertheless, since $\delta_J = 0$ for free motion, the change in the density of continuum states of a system of particles due to their interaction remains finite and has the form $\Delta(dn_J/dE) = (d\delta_J/dE)/\pi = \Delta\tau_{ii}^{(J)}/\pi\hbar$. But for all that the density of the continuum states does not have a physical meaning and it is evidently preferable to use the concept of the density ρ_J of resonance states, which is also related to the delay time by Eq. (30) or (33).

Finally, by means of the delay time in elastic scattering one can give a direct physical meaning to Levinson's theorem by relating the delay time to the number N_J of bound states of the system[59]:

$$\int_0^\infty \Delta\tau_{ii}^{(J)}(\varepsilon_i)\,d\varepsilon_i = \hbar\,[\delta_J(\infty) - \delta_J(0)] = -\pi\hbar N_J. \tag{34}$$

It follows in particular from this formulation of the theorem that if in some region of energies $\mathscr{D}E$ there is a very narrow resonance for a very dense group of strongly overlapping resonances, where $\Delta\tau_{ii}^{(J)}$ is very large and positive, then, since the right-hand side of Eq. (34) is nonpositive, in other energy regions far from $\mathscr{D}E$ there must be a corresponding increase in the time of advance (with allowance, of course, for the restrictions imposed by the causality condition).

5. DURATIONS OF MULTICHANNEL SCATTERING PROCESSES

Duration of Scattering in Threshold Regions

In the ideal case of strictly defined levels of excited states of the target nucleus near the threshold of inelastic

scattering (for example, in the region $k_1 \approx 0$) of neutrons[5,56]

$$S_{ii}^{(J)}(k_0, k_1) = \tilde{S_J}(k_0)\,[1 + 2\alpha(k_0)\,k_1^{2J+1}],$$

where k_0 and k_1 are related by $E = e_0 + \hbar^2 k_0^2/2\mu = e_1 + \hbar^2 k_1^2/2\mu$, e_0 and e_1 are the levels of the ground state and the first excited state of the target nucleus, $\tilde{S}_J(k_0)$ has the same analytic properties as the single-channel function $S_{ii}^{(J)}(k_0)$ in the approximation of one channel, $\alpha(k_0)$ is a function of k regular on the real axis and real for real potentials, and

$$\Delta\tau_{ii}^{(J)} \xrightarrow[k_1 \to 0]{} \begin{cases} \Delta\tilde{\tau}_{ii}^{(J)} - \hbar(-1)^J \alpha d \mid k_1 \mid^{2J+1}/dE \\ \qquad \text{for} \quad k_0^2 < 2\mu(e_1 - e_0)/\hbar^2, \\ \Delta\tilde{\tau}_{ii}^{(J)} \quad \text{for} \quad k_0^2 > 2\mu(e_1 - e_0)/\hbar^2, \end{cases} \tag{35}$$

where $\Delta\tilde{\tau}_{ii}^{(J)} = (\hbar/2) d \arg \tilde{S}_{ii}^{(J)}/dE$. In the case of a smooth dependence of $\arg \tilde{S}_i^{(J)}(E)$, i.e., a small value of $\Delta\tilde{\tau}_{ii}^{(J)}$, then for $J = 0$ the quantity $\Delta\tau_{ii}^{(0)}$ as the scattering threshold is approached from below behaves as $\Delta\tau_{ii}^{(0)} \sim -\alpha\mu/\hbar|k_1|$, i.e., for $\alpha \neq 0$ at the point $k_1 = 0$ the lifetime of such a "threshold state" increases unboundedly as the threshold point is approached from below (α cannot be positive by virtue of the causality condition).

In reality, however, the excited states of the target nucleus are metastable. If the levels of such states are represented in the form of complex quantities $\tilde{e}_n = e_n - i\gamma_n/2$, where $\gamma_n > 0$ is the width corresponding to the decay of the state (usually radiative decay), then instead of the threshold point one must introduce the concept of a threshold region of width $\sim \gamma_n$ and, following Ref. 61, replace e in (35) by $\tilde{e}_n$, with

$$k_1 \to [2\mu\,(E - e_1 + i\gamma_0/2)/\hbar^2]^{1/2}$$

in the threshold region. Then in the threshold region the function $\Delta\tau_{ii}^{(J)}(E)$ remains finite and continuous and, in particular, for s-wave neutrons we obtain

$$\Delta\tau_{ii}^{(0)}(E) \simeq \hbar b_0 \frac{1 - (E - E_{\text{th}})/d_{\text{th}}}{[d_{\text{th}} - (E - E_{\text{th}})]^{1/2}}, \tag{36}$$

where

$$b_0 = (1 - \operatorname{Re}\tilde{S}_J)^{-1} \operatorname{Re}\tilde{S}_J \alpha \sqrt{\mu/\hbar^2};$$

$E_{\text{th}} = e_1 - e_0$, and $d_{\text{th}} = [(E - E_{\text{th}})^2 + \gamma_1^2/4]^{1/2}$ under the assumption that $|1 - \operatorname{Re}\tilde{S}_J| \geqslant |\alpha\sqrt{\mu/\hbar^2}\,|E - E_{\text{th}}|^{1/2}$ and $\gamma_1 \to 0$. Obviously, $0 < \Delta\tau_{ii}^{(0)}(E_{\text{th}}) < \hbar/\gamma_1$ for $(2\gamma_1)^{-1/2} > b_0 > 0$.

Duration of Resonance Multichannel Scattering Processes

At energies far from the threshold regions, one can use the following representation of the multichannel S matrix[62]:

$$\hat{S}^{(J)}(E) = \hat{U}^{(J)} \prod_{\nu=1}^{N} \left(1 - \frac{i\Gamma_\nu^{(J)} \hat{P}_\nu^{(J)}}{E - E_\nu^{(J)} + i\Gamma_\nu^{(J)}/2}\right) \hat{U}^{(J)T}, \tag{37}$$

where the unitary matrix $\hat{U}^{(J)}$ and the projection matrices $P_\nu^{(J)}$ ($\hat{P}_\nu^{(J)+} = \hat{P}_\nu^{(J)} = \hat{P}_\nu^{(J)2}$, $\operatorname{Tr}\hat{P}_\nu^{(J)} = 1$) are almost independent of the energy $\hat{U}^{(J)T})_{ij} = U_{ij}^{(J)}$ ($i, j = 1, 2, \ldots, n$), n is the number of open channels, and $\tilde{S}^{(J)} = \hat{U}^{(J)}\hat{U}^{(J)T}$ is the symmetric unitary background (nonresonance) S matrix, the relations $\tilde{S}_{ij}^{(J)} = \tilde{S}_{ji}^{(J)T}$, $\Sigma_{k=1}^n \tilde{S}_{ik}^{(J)}\tilde{S}_{jk}^{(J)*} = \delta_{ij}$ holding. The representation (37) is, strictly speaking, valid only for binary (two-particle) reactions. It has the advantage that in it, in contrast to representations with additive sets of resonance terms, the unitarity property is taken into account explicitly.

$$\sum_{k=1}^{n} S_{ik}^{(J)} S_{jk}^{(J)*} = \delta_{ij}, \tag{38}$$

and, in contrast to representations of the form $\hat{S}^{(J)} = (1 + i\hat{K}^{(J)})(1 - i\hat{K}^{(J)})^{-1}$, there are no inverse matrices, the operations with which greatly complicate the analysis.

Using the Hermitian matrix

$$\hat{Q}^{(J)}(E) = i\hbar\hat{S}^{(J)}(E)\frac{d\hat{S}^{(J)+}(E)}{dE},$$

which was introduced in Ref. 18, it is easy to show in conjunction with (37) that

$$\operatorname{Tr}\hat{Q}^{(J)}(E) = \sum_{\nu=1}^{N} \frac{\Gamma_\nu^{(J)}}{(E - E_\nu^{(J)})^2 + (\Gamma_\nu^{(J)}/2)^2}, \tag{39}$$

this relation being independent of the matrices $\hat{U}^{(J)}$ and $\hat{P}_\nu^{(J)}$, i.e., independent of the smooth (effectively constant) background associated with the potential scattering and the direct reactions.[51] Averaging Eq. (39) within the region in which the resonances are situated, we obtain in accordance with the $\langle\ \rangle$ procedure and under the assumption of the conditions (29)

$$\operatorname{Tr}\langle\hat{Q}^{(J)}(E)\rangle = \mathcal{T}_J, \tag{40}$$

where $\mathcal{T}_J = 2\pi\hbar/D_J$ is the Poincaré cycle time.[5)] Using the transformations

$$i\hbar\langle S_{ij}^{(J)}\, dS_{ij}^{(J)*}/dE\rangle = \hbar\langle\mid S_{ij}^{(J)}\mid^2\rangle\langle\Delta\tau_{ij}^{(J)}\rangle + i\hbar\langle d\mid S_{ij}^{(J)}\mid^2/dE\rangle/2, \quad i \neq j,$$

$$i\hbar\langle S_{ii}^{(J)}(dS_{ii}^{(J)*}/dE)\rangle = \hbar\langle\mid 1 - S_{ii}^{(J)}\mid^2\rangle\langle\Delta\tau_{ii}^{(J)}\rangle + \hbar\,\mathrm{Im}\langle dS_{ii}^{(J)}/dE\rangle + i\hbar\langle d\mid S_{ii}^{(J)}\mid^2/dE\rangle/2,$$

where $\mathrm{Im}\langle dS_{ii}^{(J)}/dE\rangle$ for sufficiently large ΔE can be ignored in the random-phase approximation, and the equation

$$\sum_j \langle d\mid S_{ji}^{(J)}\mid^2/dE\rangle = 0,$$

which follows directly from the unitarity condition (38), we obtain from (40) the following sum rules for $\langle\Delta\tau_{ji}^{(J)}\rangle$ and $\langle\Delta\tau_i^{(J)}\rangle$:

$$\left.\begin{aligned} \sum_{i,j}\langle\Delta\tau_{ij}^{(J)}\rangle\langle\mid S_{ij}^{(J)} - \delta_{ij}\mid^2\rangle &= \mathcal{T}_J, \\ \sum_i \langle\Delta\tau_i^{(J)}\rangle[1 - \operatorname{Re}\langle S_{ii}^{(J)}\rangle] &= \mathcal{T}_J/2, \end{aligned}\right\} \tag{41}$$

which were first found in Ref. 51. With allowance for the definitions (15) and (16), the sum rules (41) can be rewritten in the form

$$\sum_{i,j}\int d\Omega_j\langle\Delta\tau_{ji}(E, \vartheta_j)\rangle k_j k_i\langle\sigma_{ji}(E, \vartheta_j)\rangle = \sum_J \pi(2J+1)\mathcal{T}_J.$$

If we assume equal durations in all channels, i.e., $\langle\Delta\tau_i^{(J)}\rangle = \langle\Delta\tau_{ji}^{(J)}\rangle = \langle\Delta\tau^{(J)}\rangle$ ($i, j = 1, 2, \ldots, n$, then in the

[5)] More precisely, the Poincaré cycle time for a system with equidistant purely discrete levels for which $|E_{\nu+1}^{(J)} - E_\nu^{(J)}| = D_J$ and $\Gamma_\nu^{(J)} = 0$.

approximation $\langle S_{ii}^{(J)}\rangle \approx 0$ (which, as will be seen below, can hold for $\Gamma_J \gg nD_J/2\pi$ in the so-called approximation of equivalent incoming channels[51]) we obtain[51]

$$\langle \Delta\tau^{(J)}\rangle \simeq \mathcal{T}_J/2n. \tag{42}$$

But if $\Sigma_i \operatorname{Re}\langle S_{ii}^{(J)}\rangle = n-1$, which is possible in the so-called Newton case of complete correlation between the decay amplitudes of all resonances in the approximation of equal projection matrices $\hat{P}_\nu^{(J)} = \hat{P}_J$ ($\nu = 1, 2, \ldots, N$) and $\hat{U}^{(J)} = 1$, when the unitary S matrix has the form $\hat{S}^{(J)} = \hat{1} + [\exp(2i\delta_J) - 1]\hat{P}_J$, then

$$\langle \Delta\tau^{(J)}\rangle = \mathcal{T}_J/2. \tag{43}$$

The result (43) was obtained in Ref. 63.

If $\operatorname{Re}\langle S_{ii}^{(J)}\rangle = 1 - \pi\Gamma_J/nD_J$, which, as will be seen below, can hold when $\Gamma_J \ll nD_J/2\pi$, then $\langle \Delta\tau^{(J)}\rangle = \hbar/\Gamma_J$, as in the case of one isolated resonance with total width $\Gamma_J \ll \Delta E$.

If we proceed from the direct definition (15) for $\langle \Delta\tau_{ij}^{(J)}\rangle$, then in the approximation $\hat{P}_\nu^{(J)} = \hat{P}_J$ ($\nu = 1, 2, \ldots, N$) it is easy to show that under the condition (29)

$$\hat{S}^{(J)} = \tilde{S}^{(J)} - \hat{\alpha}^{(J)} + \hat{\alpha}^{(J)} \prod_\nu \frac{E - E_\nu^{(J)} - i\Gamma_\nu^{(J)}/2}{E - E_\nu^{(J)} + i\Gamma_\nu^{(J)}/2}$$

and

$$\left.\begin{aligned} \langle \Delta\tau_{ij}^{(J)}\rangle &= |\,\alpha_{ij}^{(J)}\,|^2 \mathcal{T}_J/[\,|\,\tilde{S}_{ij}^{(J)} - \alpha_{ij}^{(J)}\,|^2], \quad i \neq j; \\ \langle \Delta\tau_{ii}^{(J)}\rangle &= |\,\alpha_{ii}^{(J)}\,|^2 \mathcal{T}_J/[\,|\,1 - \tilde{S}_{ii}^{(J)} + \alpha_{ii}^{(J)}\,|^2], \end{aligned}\right\} \tag{44}$$

where $\alpha_{ij}^{(J)} = (\hat{U}^{(J)}\hat{P}_J\hat{U}^{(J)T})_{ij}$, and $\Sigma_{ij}|\alpha_{ij}^{(J)}|^2 = 1$. This follows directly from the unitarity of $\hat{U}^{(J)}$ and the property $\operatorname{Tr}\hat{P}_J = 1$. Since $|\alpha_{ij}^{(J)}| = (1/n)$ in the approximation of equal $|\alpha_{ij}^{(J)}|$, and $|\tilde{S}_{ij}^{(J)}| = (1/\sqrt{n})$ in the approximation of equal $|\tilde{S}_{ij}^{(J)}|$,

$$\left.\begin{aligned} \langle \Delta\tau_{ij}^{(J)}\rangle &\approx \mathcal{T}_J/n, \quad i \neq j; \\ \langle \Delta\tau_{ii}^{(J)}\rangle &\approx \mathcal{T}_J/n^2. \end{aligned}\right\} \tag{44a}$$

The result (44a) was obtained for the first time in Ref. 58.

But if we set

$$|\,\alpha_{ij}^{(J)}\,| = \frac{1}{\sqrt{2n}}\,\delta_{ij} + \frac{1}{\sqrt{2n(n-1)}}\,(1 - \delta_{ij})$$

and $|\tilde{S}_{ij}^{(J)}| = (1/\sqrt{n})$, then

$$\left.\begin{aligned} \langle \Delta\tau_{ij}^{(J)}\rangle &= \mathcal{T}_J/2n, \quad i \neq j; \\ \langle \Delta\tau_{ii}^{(J)}\rangle &\approx \mathcal{T}_J/2n^2. \end{aligned}\right\} \tag{44b}$$

In both the last cases $\langle \Delta\tau_{ij}^{(J)}\rangle$ for $i \neq j$ are equal to each other, and $\langle \Delta\tau_{ii}^{(J)}\rangle$ is n times smaller than each of them. This last result can be explained by the unavoidable presence of effectively instantaneous diffraction scattering in each entrance channel, the fraction of which in the total probability (total cross section) of all reaction processes (due to the term δ_{ij} in $T_{ij}^{(J)}$) is equal to the total fraction of all the direct processes

$$\Big(\sum_j |\,\tilde{S}_{ji}\,|^2 = \sum_j \delta_{ji} = 1\Big).$$

In the region (32),

$$\mathcal{T}_J = \operatorname{Tr}\langle \hat{Q}^{(J)}(E)\rangle \sim \hbar \int_{E_{\min}}^{\infty} \rho_J(E') \frac{\Gamma_J(E')}{(E-E')^2 + [\Gamma_J(E')/2]^2}\,dE', \tag{45}$$

and the expressions (41)–(44) retain their form.

We now consider which of the results (42), (43), (44a), and (44b) do not conflict with experimental data on the behavior of the total reaction cross sections

$$\langle \sigma_i\rangle = \sum_j \langle \sigma_{ji}\rangle = \frac{2\pi}{k_i^2} \sum_J (2J+1)\,[1 - \operatorname{Re}\langle S_{ii}^J\rangle].$$

In accordance with experiment with a large number of channels ($n \gg 1$), with increasing energy $\sigma_i \sim 2\pi R^2$, where in the given case R is the range of the interaction of the incident particle and the target nucleus in the i-th entrance channel.[64] Since the main contribution to the cross section is made by waves with $J \sim k_i R \gg 1$, the result $2\pi R^2$ is obtained only under the condition $|\operatorname{Re} S_{ii}^{(J)}| \ll 1$, which is satisfied when the results (42), (44a), and (44b) hold, and not in the case (43), when, as was noted in Ref. 65, only one of the eigenvalues of the matrix $\langle S^{(J)}\rangle$ is zero, the remaining $n-1$ being equal to unity. Thus, because of the contradiction with the experimental data, the approximations leading to the result (43) must be rejected.

6. COLLISION DURATION AND THE MECHANISM OF NUCLEAR REACTIONS

Direct and Compound Processes

In accordance with ideas established long ago (see, for example, Refs. 66 and 67), one considers in the theory of nuclear reactions two limiting types of mechanism—direct processes, which take place almost instantaneously, and compound processes, corresponding to the delayed decay of compound nuclear states. More precisely, direct processes must be characterized by times of the order of the time of free flight of the projectile particle through the region of the nucleus with diameter of about 10^{-13}–10^{-12} cm. If the incident nucleons have energy $\varepsilon_0 \geqslant 1$ MeV, then

$$\langle \tau_{ij}^d\rangle \leqslant 10^{-23} \div 10^{-21} \text{ sec}. \tag{46}$$

Compound processes are characterized by times

$$\langle \tau_{ij}^c\rangle \gg \langle \tau_{ij}^d\rangle. \tag{47}$$

To estimate the time $\langle \tau^c\rangle$ in the region of isolated compound resonances ($\Gamma_J \ll D_J$) one uses in accordance with the uncertainty relation (1) the expression $\langle \tau_{ij}^{c(J)}\rangle \sim \hbar/\Gamma_J$. In the region of overlapping resonances, it was already proposed in Refs. 68 and 69 that one should use an expression that depends essentially on the density ρ_J of the resonance levels. The following was proposed. If it is assumed that the state of the compound nucleus can be described by a linear combination of the excited states of some Hamiltonian with equal distances D_J between the levels, then the Poincaré cycle time (the interval of time over which the system returns to the initial state) is $\mathcal{T}_J = 2\pi\hbar/D_J$. When a compound nucleus is formed in a reaction process with some entrance channel i, the system can return after a time approximately equal to $\mathcal{T}_J$ to the initial state and then decay through the same channel i, so that the lifetime of the compound nucleus under the condition that only one channel i is open can be determined by means of the relation $\tau_i^{(J)} \sim \mathcal{T}_J/t_i^{(J)}$, where $t_i^{(J)}$ is the penetrability of the centrifugal and Coulomb barriers on

the surface of the nucleus for escape of the final particle to the exterior region. Under the assumption of an exponential law of decay of the compound nucleus, the probability of decay through channel i per unit time is

$$P_i^{(J)}(t) = \frac{1}{\tau_i^{(J)}} \exp\left[-\left(\frac{t}{\tau^{(J)}}\right)\right],$$

where $\tau^{(J)}$ is the lifetime of the compound nucleus for its decay through all the open channels. Since

$$\mathcal{P}_i^{(J)} \equiv \int_0^\infty P_i^{(J)}(t)\, dt = \frac{\tau^{(J)}}{\tau_i^{(J)}}$$

and $\Sigma_i \mathcal{P}_i^{(J)} = 1$, it follows that $(1/\tau^{(J)}) = \Sigma_i (1/\tau_i^{(J)})$ and, therefore,

$$\tau^{(J)} = \mathcal{T}_J / \sum_i t_i^{(J)}. \tag{48}$$

In later studies (see, for example, Ref. 70) it was suggested that as $t_i^{(J)}$ in (48) one should use the expression $1 - |S_{ii}^{(J)\mathrm{om}}|$, where $S_{ii}^{(J)\mathrm{om}}$ is the element of the elastic S matrix obtained in the framework of the optical model.

It was usually assumed that compound processes, in contrast to direct processes, are due to a number of successive collisions of nucleons within the compound system (see, for example, Ref. 69). The development of the theoretical studies led to a number of refinements and appreciable particularization of the original simple ideas.

In particular, since each collision event is accompanied by a nonvanishing probability for decay of the compound system, there must be a wide interval of times satisfying the condition (47) and, moreover, characterizing continuous transition from (46) to (47). Such ideas formed the basis of a number of models of so-called precompound processes and pre-equilibrium decay of compound systems (see, for example, Refs. 71–73). Moreover, the development of semiempirical models of multistep direct and compound processes (see Refs. 74–76) indicates the need to review the part played by characteristics of the reaction mechanism such as the number of degrees of freedom and the duration of the process.

Indeed, whereas earlier (see, for example, Ref. 77) it was assumed that for direct processes it is characteristic for a small number of degrees of freedom to participate, in the framework of modern ideas as many degrees of freedom may participate as in the compound processes. A minimal number of degrees of freedom participates only in single-step direct reactions. Multistep direct processes differ from multistep compound processes in the nature of the spectrum of states of the participating particles, the former taking place in a nonresonance region (or in a region of giant resonances), the latter in a resonance (quasidiscrete) region of the continuum.

Here, we shall restrict the analysis to the evolution of nuclear reactions in the framework of a simpler model. In accordance with the the well-known treatment of Friedman and Weisskopf,[66] which was generalized in Ref. 72 to the case of an arbitrary number of open channels and an incoherent ensemble of pairs of colliding initial particles with spread ΔE of the average energies of individual pairs, the interaction of an incident particle with a nucleus when $\Delta E \gg \Gamma_J$ and $\Delta E \gg D_J$ (for all values of J actually taken into account) can be separated in time into the essentially instantaneous stage of diffraction (potential) scattering and direct processes and the stage of formation of the compound nucleus, and the reaction cross section $\langle \sigma_{ji}(E, \vartheta_j) \rangle$ can be represented in the form of the incoherent sum

$$\langle \sigma_{ji}(E, \vartheta_j) \rangle \simeq \sigma_{ji}^{\mathrm{d}}(E, \vartheta_j) + \langle \sigma_{ji}^{\mathrm{c}}(E, \vartheta_j) \rangle, \tag{49}$$

where $\sigma_{ji}^{\mathrm{d}}(E, \vartheta_j) = |\langle f_{ji}(E, \vartheta_j) \rangle|^2$ and $\langle \sigma_{ji}^{\mathrm{c}}(E, \vartheta_i) \rangle = \langle |f_{ji}^{\mathrm{c}}(E, \vartheta_j)|^2 \rangle \equiv \langle |f_{ji}(E, \vartheta_j)|^2 \rangle - |\langle f_{ji}(E, \vartheta_j) \rangle|^2$ are the cross sections of the direct and compound processes, respectively. In the approximation of random phases of the fluctuation amplitudes $f_{ji}^{\mathrm{c}(J)} = f_{ji}^{(J)} - \langle f_{ji}^{(J)} \rangle$ we can set

$$\langle \sigma_{j_2 i_2}^{\mathrm{c}}(E, \vartheta_j) \rangle \simeq \sum_J \langle \sigma_{ji}^{\mathrm{c}(J)}(E, \vartheta_j) \rangle \equiv \sum_J \langle |f_{ji}^{\mathrm{c}(J)}(E, \vartheta_j)|^2 \rangle.$$

After simple manipulations, setting $\tau_{ji}^{\mathrm{d}(J)} = \hbar \partial \arg \langle f_{ji}^{(J)}(E, \vartheta_j) \rangle / \partial E = 0$, we can readily show that

$$\left.\begin{aligned}
&\langle \Delta\tau_{ji}^{(J)}(E) \rangle = \langle \Delta\tau_{ji}^{\mathrm{c}(J)}(E) \rangle \langle |S_{ji}^{\mathrm{c}(J)}(E)|^2 \rangle / \langle |S_{ji}^{(J)} - \delta_{ji}|^2 \rangle, \\
&\langle \Delta\tau_{ji}(E, \vartheta_j) \rangle = \sum_j \langle \Delta\tau_{ji}^{\mathrm{c}(J)}(E) \rangle \langle \sigma_{ji}^{\mathrm{c}(J)}(E, \vartheta_j) \rangle / \langle \sigma_{ji}(E, \vartheta_j) \rangle, \\
&\langle \Delta\tau_{ji}^{\mathrm{c}}(E, \vartheta_j) \rangle = \sum_J \langle \Delta\tau_{ji}^{\mathrm{c}(J)}(E) \rangle \langle \sigma_{ji}^{\mathrm{c}(J)}(E, \vartheta_j) \rangle / \sum_J \langle \sigma_{ji}^{\mathrm{c}(J)}(E, \vartheta_j) \rangle,
\end{aligned}\right\} \tag{50}$$

where the lifetimes $\langle \Delta\tau_{ji}^{\mathrm{c}} \rangle$ and $\langle \Delta\tau_{ji}^{\mathrm{c}(J)} \rangle$ of the compound states are determined on the basis of the relation (14) with T_{ji} replaced by f_{ji}^{c} and $f_{ji}^{\mathrm{c}(J)}$ or $S_{ji}^{\mathrm{c}(J)} = S_{ji}^{(J)} - \langle S_{ji}^{(J)} \rangle$, respectively.

Average Lifetime and Evolution of the Decay of a Compound Nucleus

Taking into account the first equation of (50), we can rewrite the first of the sum rules (41) in the form

$$\sum_{i,j=1}^{n} \langle \Delta\tau_{ji}^{\mathrm{c}(J)} \rangle \langle |S_{ji}^{\mathrm{c}(J)}|^2 \rangle = \mathcal{T}_J, \tag{51}$$

from which it follows directly that in the approximation of equal $\langle \Delta\tau_{ji}^{\mathrm{c}(J)} \rangle = \langle \Delta\tau^{\mathrm{c}(J)} \rangle$ $(i, j = 1, 2, \ldots, n)$ their values are determined by

$$\langle \Delta\tau^{\mathrm{c}(J)} \rangle = \mathcal{T}_J / \sum_j T_j^{(J)}, \tag{52}$$

where

$$T_j^{(J)} = 1 - \sum_{j'} |\langle S_{j'j}^{(J)} \rangle|^2$$

are the transmission coefficients.[65,78]

In accordance with Ref. 79, we can represent $\langle S_{ji}^{(J)} \rangle$ in the form $\langle \hat{S}^{(J)}(E) \rangle = \hat{U}_0^{(J)} \hat{S}_0^{(J)} \hat{U}_0^{(J)T}$, where the unitary matrix $\hat{U}_0^{(J)}$ is equal to $\hat{U}^{(J)}$ under the assumption that in the absence of a constant background the resonance S matrix does not describe direct reactions, and $\hat{S}_0^{(J)}$ is a diagonal matrix whose elements satisfy

$$\prod_{j=1}^{n} | S_{0,jj}^{(J)} | = \exp(-\pi \Gamma_J / D_J).$$

In the approximation of equivalent incoming channels,[51]

$$|\langle S_{ii}^{(J)}(E) \rangle| = \delta_{ji} \exp(-\pi \Gamma_J / n D_J);$$

$$T_j^{(J)} = 1 - \exp(-2\pi \Gamma_J / n D_J)$$

and

$$\langle \Delta \tau^{c(J)} \rangle = \mathcal{T}_J / n \, [1 - \exp(-2\pi \Gamma_J / n D_J)].$$

If, in addition, $\Gamma_J \gg n D_J / 2\pi$, i.e., $\langle S_{ii}^{(J)} \rangle \simeq 0$ $(i = 1, 2, \ldots, n)$, then[6)]

$$\langle \Delta \tau^{c(J)} \rangle = \mathcal{T}_J / n. \tag{53}$$

In the approximation of equal projection matrices $\hat{P}_\nu^{(J)}$ in (37) $(\hat{P}_\nu^{(J)} = \hat{P}_J, \nu = 1, 2, \ldots, N)$,[58] when

$$S_{ji}^{c(J)} = \alpha_{ji}^{(J)} \left[\prod_\nu \frac{E - E_\nu^{(J)} - i\Gamma_\nu^{(J)}/2}{E - E_\nu^{(J)} + i\Gamma_\nu^{(J)}/2} - \exp(-\pi \Gamma_J / D_J) \right], \tag{54}$$

it is readily seen that for $\Gamma_J \gg D_J$

$$\sum_{i,j=1}^{n} \langle |S_{ji}^{c(J)}|^2 \rangle \simeq \sum_{i,j=1}^{n} |\alpha_{ji}^{(J)}|^2 = 1$$

and

$$\langle \Delta \tau^{c(J)} \rangle = \mathcal{T}_J. \tag{55}$$

But if for $\Gamma_J \gg D_J / 2\pi$ it is found that $\Gamma_J \ll n D_J / 2\pi$, then under the condition that there is no constant background one can use the Bethe pole representation with real residues and decay amplitudes of the different levels uncorrelated in sign (see, for example, Ref. 64). Then in the approximation of equal total widths of all levels (i.e., $\Gamma_\nu^{(J)} = \Gamma_J$, $\nu = 1,2,\ldots,N$) $|\langle S_{ii}^{(J)} \rangle| = \mathrm{Re} \langle S_{ii}^{(J)} \rangle \simeq 1 - \pi \Gamma_J / n D_J$, and $\langle \Delta \tau^{c(J)} \rangle \simeq \langle \Delta \tau^{(J)} \rangle = \hbar / \Gamma_J$, in accordance with (52), i.e., we have exactly the same result as when $\Gamma_J \ll D_J$.

In the region (32), the results (51)–(55) retain their form when allowance is made for the relation (45) instead of $2\pi\hbar / D_J$. For a rough estimate of the integral if (45), we can restrict ourselves to the simplest approximations of the type $\Gamma_\nu^{(J)}(E) \neq \Gamma_{0,J} [(E - e_i)/(E_\nu^{(J)} - e_i)]^{J+1/2}$ or even Γ_J = const when $E > E_{\min}$. In this last case, the integral in (45) is largely determined by the function $\rho_J(E)$. In various versions of the compound-nucleus model with single-nucleon decay frequent use is made of the expression

$$\rho_J(E) \sim \rho_{J,min} \exp(\sqrt{\alpha_J E^*}), \tag{56}$$

where $E^* = E - e_i + B_n$ is the excitation energy of the compound nucleus; B_n is the nucleon binding energy in the groundstate of the compound nucleus. However, (56) is valid only for not too high excitations (near the Fermi surface). In any realistic model, the discrete region of the spectrum must be bounded above by the continuum. In this connection, the growth with the energy of the density $\rho_J(E)$ of the compound-nucleus states formed, for example, from single-particle states in the average field of the nucleus must slow down and, at a certain energy $\mathscr{E}_J$, go over into a decrease[81,82] in accordance with the law

$$\rho_J(E) \sim \rho_{max,J} \exp(-\sqrt{\alpha_J' E^*}), \quad E > \mathscr{E}_J. \tag{57}$$

In accordance with Refs. 81 and 82, for two-particle reactions of elastic and inelastic scattering of nucleons by nuclei and reactions of the type (n, p) and (p, n) the difference $\mathscr{E}_J - e_0$ is in the interval between zero and the energy for nucleon knockout from the lowest single-particle level of the nucleus. The elementary estimate (45) leads in conjunction with (56) and (57) to the result

$$\mathcal{T}_J \xrightarrow[E \to \infty]{} \hbar \Gamma_J \left[\frac{2\rho_{J,\min} x}{(E - \mathscr{E}_J)^2 + \Gamma_J^2/4} + \frac{\rho_{J,\max}}{E^2 (\alpha_J')} \right]; \tag{58}$$

$$x \sim |\sqrt{\mathscr{E}_J} \exp \sqrt{\alpha_J \mathscr{E}_J} - \sqrt{E_{J,\min}} \exp \sqrt{\alpha_J E_{J,\min}}| / \sqrt{\alpha_J}$$
$$- [\exp \sqrt{\alpha_J \mathscr{E}_J} - \exp \sqrt{\alpha_J E_{J,\min}}] / \alpha_J$$

for $E \gg \mathscr{E}_J$.

Thus, in the limit $E \to \infty$ the lifetime $\langle \Delta \tau_{ji}^{(c)} \rangle$ must decrease asymptotically in accordance with the law E^{-2} in the approximation of equal $\hat{P}_\nu^{(J)}$ $(\nu = 1, 2, \ldots, N)$ or $[E^2 n(E)]^{-1}$ in the approximation of equivalent entrance channels.

When for different values different relationships between the quantities ΔE, Γ_J, D_J and $n D_J / 2\pi$ are observed, the delay times $\langle \Delta \tau_{ji}^{c(J)} \rangle$ take different values in the interval from $\hbar / \Gamma_J$ to $2\pi\hbar / D_J$ or up to the values determined by (58), taking the values $2\pi\hbar / n D_J$ for equivalent entrance channels for $\Gamma_J \gg n D_J / 2\pi$.

We consider the evolution of the decay of a compound nucleus using the definition (21). In the approximation of a fairly large lifetime and random phases of $f_{ji}^{c(J)}$ when $\langle \sin (\cos) \arg f_{ji}^{c(J)} \rangle \approx 0$, we obtain

$$\langle (\Delta \tau_{ji}^{c(J)})^2 \rangle \approx \hbar^2 \langle [|f_{ji}^{c(J)}|^2 (\partial \arg f_{ji}^{c(J)} / \partial E)^2$$
$$+ (\partial |f_{ji}^{c(J)}| / \partial E)^2] \rangle / \langle |f_{ji}^{c(J)}|^2 \rangle.$$

If

$$|\partial |f_{ji}^{c(J)}| / \partial E| \ll |f_{ji}^{c(J)}| \, |\partial \arg f_{ji}^{c(J)} / \partial E|, \tag{59}$$

then

$$\langle (\Delta \tau_{ji}^{c(J)})^2 \rangle \approx \hbar^2 \langle |f_{ji}^{c(J)}|^2 (\partial \arg f_{ji}^{c(J)} / \partial E)^2 \rangle / \langle |f_{ji}^{c(J)}| \rangle.$$

The approximation (59) is automatically satisfied in the region (29) under the condition of equal $\hat{P}_\nu^{(J)} = \hat{P}_J$ $(\nu = 1, 2, \ldots, N)$, when $|S_{ji}^{c(J)}| \simeq |\alpha_{ji}^{(J)}| = \text{const}$. Since

$$\hbar \partial \arg f_{ji}^{c(J)} / \partial E = \sum_\nu \frac{\Gamma_\nu^{(J)}}{(E - E_\nu^{(J)})^2 + (\Gamma_\nu^{(J)})^2/4} \approx \mathcal{T}_J,$$

by virtue of (54),

[6)] See also Refs. 65 and 80.

$$\langle(\Delta\tau_{ji}^{c}{}^{(J)})^2\rangle \approx \langle\mathscr{T}_J\rangle^2, \quad (60)$$

$$\mathscr{D}\tau_{ji}^{c}{}^{(J)} \approx 0. \quad (61)$$

As is shown in Ref. 65, in the case of n equivalent entrance channels when $\Gamma_J \gg nD_J/2\pi$ Eq. (59) holds and the variance of the duration is also near zero (being a quantity of order D_J/Γ_J compared with $\langle\Delta\tau_{ji}^{c(J)}\rangle^2$). Thus, the compound nucleus then decays not in accordance with an exponential law but during a very short time interval $[\mathscr{D}\tau_{ij}^{c(J)}]^{1/2}$ after the time $\langle\Delta\tau_{ij}^{c(J)}\rangle$ following the first, effectively instantaneous stage of the direct processes. Since all $\langle S_{ij}^{(J)}(E)\rangle \approx 0$, $i.j = 1, 2, \ldots, n$ (as in the case of diffraction by a black sphere) and

$$\sum_j \sigma_{ji}^{d} = \sigma_{ii}^{d} = \sum_j \langle\sigma_{ji}^{c}\rangle,$$

in any entrance channel i half of all the interaction events belong to the stage of instantaneous elastic scattering, and the other half to the delayed stage of the fluctuation processes associated with the decay of the compound nucleus. It is easy to show that the same result with (45) substituted for $\mathscr{T}_J$ in place of $2\pi\hbar/D_J$ also holds at higher energies in the region (32).

But if $\Gamma_J \ll nD_J/2\pi$ when the condition (29) is satisfied, the total widths of all levels are equal, and there is no constant background, then the decay of the compound nucleus will be described by an exponential in the approximation of equivalent entrance channels.[51,65]

For completeness, we mention Ref. 83, in which an explicit analytic expression describing the evolution of the decay of a compound system is obtained in the framework of a single-channel model with an equidistant distribution of poles with equal widths and residues of the S matrix. It is interesting that for this special model there is not one delayed stage but a whole series of brief "pulses" of decay of the compound nucleus, these being separated by time intervals equal to the Poincaré cycle time; the intensity of each successive decay "pulse" is less than the preceding one by the factor $\exp(-2\pi\Gamma_J/D_J)$.

7. BRIEF ANALYSIS OF METHODS OF DETERMINATION AND MEASUREMENT OF THE DURATIONS OF NUCLEAR REACTIONS

Determination of the Durations of Reactions on the Basis of the Energy Dependence of the Cross Sections

In Refs. 5 and 84 a method of practical determination of durations was considered. It is based on data on the measured energy dependences of the reaction cross sections in the region $\Gamma_J < D_J$. In the cases when the main contribution to the amplitude and the cross section of a two-particle reaction is made by a term with one value of J (for example, in the region $k_j R \ll 1$ ($J = 0$) or J resonances), the expressions (17) and (18) in conjunction with the unitary relation (38) are sufficient for one, using the definition (15) of the duration $\langle\Delta\tau_{ii}^{(J)}\rangle \approx \Delta\tau_{ji}^{(J)}$ for $\Delta E \ll \Gamma_J < D_J$, to express $|\Delta\tau_{ji}^{(J)}|$ in terms of $k_i^2\sigma_{ji}^{(J)}$ and $dk_i^2\sigma_{ji}^{(J)})/dE$. For example, in the region of energies below the threshold of inelastic processes, when $\sigma_{ji}^{(J)} = 0$ for $j \neq i$,

$$|\Delta\tau_{ii}^{(J)}| = (\hbar/2)\,(dB_i^{(J)}/dE)\,[B_i^{(J)}\,(4 - B_i^{(J)})]^{1/2},$$

where $B_i^{(J)} = k_i^2\sigma_{ii}^{(J)}/4\pi(2J+1)$, and $0 \leqslant B_i^{(J)} \leqslant 4$..

In Refs. 5 and 84 the results are given of calculations of the durations of neutron scattering by the nuclei ^{12}C in the range of energies 1–500 keV, ^{19}F (55–130 keV), ^{24}Mg (150–500 keV), ^{27}Al (10–60 keV), and ^{32}S (10–160 keV) on the basis of experimental data on the cross sections taken from Refs. 85–89. In the nonresonances region $\Delta\tau_{ii}^{(J)} \approx 10^{-21}$–$10^{-20}$ sec, while in the region of resonance peaks $\Delta\tau_{ii}^{(J)} = 0.5\times10^{-19}$–$10^{-18}$ sec. A detailed analysis of the $\Delta\tau_{ii}^{(L)}(E)$ curves is given in Ref. 5. It should be noted that in such calculations of $\Delta\tau_{ii}^{(J)}(E)$ the requirements on the accuracy of the original experimental data on $\sigma_{ji}(E)$ are higher, since the calculations include the operation of differentiation. If the experimental conditions made it possible to measure with resolution $\Delta E < D_J$, then in principle the method could also be used in the region of overlapping resonances, when $\Gamma_J > D_J$.

The basic possibility of determining a duration on the basis of differential cross sections of reactions with two or three particles in the exit channel is considered in Refs. 5, 49, and 84.

Determination of the Duration of a Nuclear Reaction by Studying the Fluctuations of the Cross Section

In the region $\Gamma_J \gg D_J$, one can use analysis of the fluctuations in the reaction cross sections due to the random distribution of the values of the resonance parameters.

An important feature of fluctuations is the existence of a correlation between the values f of the amplitude for two energies separated by a fairly small interval ε (when the same group of levels makes an appreciable contribution to f). The correlation is characterized by the autocorrelation coefficients of the amplitudes and the cross sections[7]:

$$r(\varepsilon) = [\langle f(E) f^*(E+\varepsilon)\rangle - \langle f(E)\rangle\langle f^*(E+\varepsilon)\rangle]/[\langle|f(E)|^2\rangle - |\langle f(E)\rangle|^2]$$

and

$$R(\varepsilon) = [\langle\sigma(E)\,\sigma(E+\varepsilon)\rangle - \langle\sigma(E)\rangle\langle\sigma(E+\varepsilon)\rangle]/[\langle\sigma^2(E)\rangle - \langle\sigma(E)\rangle^2].$$

In Ericson's theory $r(\varepsilon) = [1 + i\varepsilon/\Gamma_{\text{corr}}]^{-1}$, $R(\varepsilon) = [1 + (\varepsilon/\Gamma_{\text{corr}})^2]^{-1}$,[90] and the correlation interval Γ_{corr}, which in Ref. 90 is taken equal to the total width Γ, is usually related by means of (1) to the characteristic lifetime of the compound nucleus: $\tau^c \sim \hbar/\Gamma_{\text{corr}}$.

In the simulation in Refs. 71 and 91 of the nuclear matrix elements describing compound transitions by the method of random matrices with allowance for the unitarity condition of the S matrix, represented in the form

$$\hat{S} = (1 + i\hat{K})\,(1 - i\hat{K})^{-1},$$

$$K_{ji} = K_{ji}^{(0)} + \sum_k \gamma_{kj}\gamma_{ki}\,(E_k - i\Gamma_k/2 - E)^{-1}$$

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[7]For simplicity, we restrict ourselves to reactions with a single value of J, and we omit the index J.

[$\hat{K}^{(0)}$ is the "background" K matrix, $\Sigma_j(\gamma_{kj})^2=\Gamma_k$], the following expression was obtained under the condition $\Gamma \gg D$ for the correlation coefficient $R(\varepsilon)$:

$$R_{ji}(\varepsilon)=\frac{\langle\sigma_{ji}(E)\,\sigma_{ji}(E+\varepsilon)\rangle-\langle\sigma_{ji}(E)\rangle^2}{\langle\sigma^c_{ji}(E)\rangle\,[\langle\sigma^c_{ji}(E)\rangle+2\sigma^d_{ji}(E)]}=\frac{(\mathrm{Tr}\,\hat{T})^2}{(\mathrm{Tr}\,\hat{T})^2+\varepsilon^2(2\pi/D)^2}, \tag{62}$$

where

$$\langle\sigma^{cji}(E)\rangle\sim\langle|S^c_{ji}(E)|^2\rangle;\ \sigma^d_{ji}\sim|\langle S_{ji}\rangle|^2;\ T_{ji}=\delta_{ji}-\sum_k\langle S_{jk}\rangle\langle S^*_{ki}\rangle.$$

In accordance with (62), the correlation interval is of order $\Gamma_{\mathrm{corr}}\approx D\,\mathrm{Tr}\,\hat{T}/2\pi$, and the characteristic time of the compound nucleus is $\tau^c\approx 2\pi\hbar\rho/\mathrm{Tr}\,\hat{T}$, which agrees with (52). In Ref. 92, in the framework of a generalization of the Weisskopf–Wigner K-matrix model and the eikonal model using the approximation of equivalent channels (i.e., $\langle|S_{ji}|^2\rangle=1/n$, $i,j=1,2,\ldots,n$), it was concluded that $\Gamma_{\mathrm{corr}}\approx nD/2\pi$, from which we obtain the estimate $\tau^c\approx 2\pi\hbar/nD$, which agrees with (54).

In Ref. 93 for single-channel reactions and in Refs. 51 for multichannel collisions the following expression was obtained for $R(\varepsilon)$ in the region $\Gamma\gg nD/2\pi$ using the unitary representation (37) of the S matrix under the assumption of statistical independence of the levels E_ν and the widths Γ_ν:

$$R_i(\varepsilon)=\frac{\langle\sigma_i(E)\,\sigma_i(E+\varepsilon)\rangle-\langle\sigma_i(E)\rangle^2}{\langle\sigma_i^2(E)\rangle-\langle\sigma_i(E)\rangle^2}=\cos\left(-\frac{2\pi\Gamma\varepsilon^2}{nD(\Gamma^2+\varepsilon^2)}\right)\exp\left(-\frac{2\pi\Gamma\varepsilon^2}{nD(\Gamma^2+\varepsilon^2)}\right),$$

from which it also follows that $\Gamma_{\mathrm{corr}}\simeq nD/2\pi$ and $\tau^c\simeq 2\pi\hbar/nD$.

The difficulties in determining the durations of compound processes by analyzing the fluctuations in the cross sections are aggravated by the circumstance that Γ_{corr} can be determined under the condition that the experimental resolution $\Delta_{\mathrm{exp}}E$ of the projectile particle beams does not exceed in order of magnitude Γ_{corr} (for more detail on this question, see Ref. 94). Moreover, direct measurements of the level density ρ in the cross sections are so far possible only under the restriction $\Delta_{\mathrm{exp}}E\lesssim D$.

Measurement of Lifetimes of Compound Nuclei by Means of the Blocking Effect

Since this method has already been described in detail in the review literature (see, in particular, Refs. 94 and 95), we shall here merely discuss its basic idea and the features required in the light of the results (50), (51), and (60) in the region of strongly overlapping resonances.

The idea of using such an effect to measure lifetimes of compound nuclei was first proposed by Tulinov[96] and also Gemmell and Holland,[97] and is as follows. In the reaction process, the compound nucleus is displaced from the lattice site by the pulse imparted by the incident particle. The small displacements of the source of the final particles affect the shape of the shadows in the angular distributions. But if before decaying the compound nucleus succeeds in traveling a distance appreciably greater than the amplitude of the vibrations of the nuclei, the effect is rearranged and the shadow pattern effectively disappears.

It is clear that the range of measured lifetimes depends strongly on the velocity acquired by the compound nucleus, i.e., it depends ultimately on the energy of the initial particles and on the masses of the target nuclei. The estimates made in Ref. 94 for various initial particles from protons to ^{40}Ar ions with energy of the order of the Coulomb fusion barrier and nuclei with medium and large mass numbers, showed that such a method enables one in principle to measure lifetimes in the range 10^{-15}–10^{-18} sec.

In accordance with Refs. 98 and 99, the relative intensity χ_{ji} of the emitted particles in the region of the shadows is described in the case of small displacements of the nuclei by

$$\chi_{ji}=(C/\langle\Delta\tau^c_{ji}\rangle)\int_0^\infty P_{ji}(\tau/\langle\Delta\tau^c_{ji}\rangle)\ln[1-(V^{(i)}_\perp)^2\tau^2/r_0^2]^{-1}\,d\tau\approx 2C\pi Nd\,(V^{(i)}_\perp\langle\Delta\tau^c_{ji}\rangle)^2, \tag{63}$$

where the factor C is related to the approximation of the continuous potential of the chain of atoms (for displacements of the compound nuclei small compared with the distance between the atomic chains, $C\approx 2$–3; with increasing displacement of the nuclei, the value of C decreases to 1); $P(\tau/\langle\Delta\tau^c_{ji}\rangle)$ is the probability of decay of the compound nucleus through a definite reaction channel ($i\to j$) at the time τ after its formation; $V^{(i)}_\perp$ is the component of the displacement velocity of the compound nucleus normal to the crystallographic axis; $r_0=[\pi Nd]^{-1/2}$; N is the density of the atoms in the crystal; and d is the distance between the atoms along the crystallographic axis. On the transition to the right-hand side of (63) we use the exponential decay law $P(\tau\langle\Delta\tau^c_{ji}\rangle)=\exp(-\tau/\langle\Delta\tau^c_{ji}\rangle)$, when $\langle(\Delta\tau^c_{ji})^2\rangle=2\langle\Delta\tau^c_{ji}\rangle^2$, and expansion of the logarithmic function in powers of $(V^{(i)}_\perp\langle\Delta\tau^c_{ji}\rangle)^2/r_0^2$.

Note that determination of the duration $\langle\Delta\tau^c_{ji}\rangle$ on the basis of the expression (70) or more complicated expressions in the shadow region involves, in addition to everything else, use of Ehrenfest's theorem in the form $\langle\Delta x\rangle=\langle v^{(i)}_\perp\rangle\tau$.

According to the results of measurements published in Refs. 94 and 100–113, the reaction times for fission of the nuclei ^{235}U and ^{238}U induced by neutrons, protons, and light ions (^{2}H, ^{3}He, ^{4}He) with energies in the region 0.2–50 MeV and for scattering of 5–7 MeV protons by Ni and Ge ions have values 10^{-17}–10^{-15} sec, and for fission reactions of the Ta, W, and U nuclei induced by heavy ions (^{12}C, ^{16}O, ^{20}Ne, ^{22}Ne) with energy 80–200 MeV are in the interval 10^{-18}–10^{-17} sec. It is shown in Ref. 114 that the nonexponential form of the decay law in the region (29) makes it necessary to correct (63) and, therefore, the results obtained for $\langle\Delta\tau^c_{ji}\rangle$. Indeed, since in this case the relation (61) holds, the factor 2 on the right-hand side of (63) must be deleted. For the same reason, the same correction must be introduced in the region (32).

Measurement of the Lifetimes of Compound Nuclei by Means of x-Ray Spectroscopy

In Ref. 115, Gugelo already suggested that the x-ray emission accompanying the rearrangement of the atomic

electron shell should be used as a "clock" for measuring the lifetime of a compound nucleus formed in a collision of a charged particle with an atom. The basis of the method and the first experiments of this kind were described in Ref. 116. The lifetime τ^K of the vacancy formed in the K shell can be determined by measuring the natural line width of the x rays. In accordance with the x-ray spectroscopic data given in Ref. 116, the lifetime τ^K varies from 5×10^{-16} to $\sim5\times10^{-18}$ sec as the atomic number Z is varied from 20 to 90. From this it is clear that the time of population of the electron vacancy is comparable with the lifetime τ^c of the compound nucleus. If $\tau^K \ll \tau^c$, the vacancy is already filled in the electron shell of the compound nucleus. If $\tau^K \gg \tau^c$, the vacancy is filled in the shell of the final nucleus. If $\tau^K \sim \tau^c$, the vacancy is filled simultaneously with the decay of the compound nucleus. If charged final particles are emitted, the compound and final nuclei differ in their atomic numbers, and therefore the x-ray lines accompanying the filling of the vacancy have different energies. This makes it possible to determine the lifetime of the compound nucleus by measuring the x-ray spectrum in coincidence with the final particles of the nuclear reaction.

In Refs. 116 and 117, τ^c was determined under the assumption of an exponential form of the decay law of the atomic and nuclear states and $\tau^c \lesssim \tau^K$ on the basis of an expression of the type $\tau^K/\tau^c = (J_p J^t_{r,\mathrm{el}}/J^c_r J_{p,\mathrm{el}})r^K_{\mathrm{in}} - 1$, where J_p and $J_{p,\mathrm{el}}$ are the intensities of the detected final particles for the investigated reaction and for elastic scattering, respectively; J^c_r and $J^t_{r,\mathrm{el}}$ are the intensities of the x-ray emission of the K shell of the compound atom in coincidence with the products of the nuclear reaction and of the target nucleus coincidence with the elastically scattered particles, respectively; $r^K_{\mathrm{in}} = p^K_{\mathrm{in}}/p^K$ (p^K_{in} and p^K are the probabilities of K-shell ionization in the process of elastic scattering and in the investigated reaction, respectively).

Although the basic principle of the method is fairly simple, in practical measurements there are great experimental difficulties, and the accuracy of the measurements is not great.

In Ref. 116, Chemin *et al.* determined the lifetimes of the states of the compound nucleus ^{107}In with excitation energies 13.6 and 15.6 MeV formed in the reaction ^{106}Cd(p,p') with protons of energies 10 and 12 MeV. The lifetimes were found to be $(6.5\pm5)\times10^{-17}$ sec and $(5\pm2.5)\times10^{-17}$ sec, respectively. In Ref. 117, a similar method was used to measure the average widths Γ (or rather, Γ_{corr}) of the states of the compound nucleus ^{113}Sb with excitation energies 10.1, 13, and 15 MeV formed in the reactions ^{112}Sn(p,p) and ^{112}Sn(p,p') at proton energies 7.138, 10, and 12 MeV, respectively. The values obtained were $\Gamma \geqslant 3.9$ eV, 16.5 ± 15.1 eV, and 19.1 ± 11.1 eV, respectively, to which there correspond τ^c values of order $\lesssim 1.8\times10^{-16}$, 4.2×10^{-17}, and 3.6×10^{-17} sec. It was also pointed out in Ref. 117 that in a region of strongly overlapping resonances, as is the case for the data on ^{107}In and ^{113}Sb (in these cases, estiamtes based on the nuclear Fermi-gas model give $\Gamma/D\sim10^4$), the quantity Γ must in accordance with Refs. 117 and 118 actually be the correlation interval of the Ericson fluctuations.

The method of x-ray spectroscopy was also used to measure the lifetimes of the ^{69}As nucleus[119] and ^{73}Br nucleus,[120] which are unstable with respect to proton decay and are formed by electron capture by the nuclei ^{69}Se and ^{73}Kr, respectively. The same method can in principle be used to determine the durations of heavy-ion–nucleus reactions.

In Ref. 118, McVoy *et al.* give a quantum-mechanical analysis of the interaction of an incident proton with a nucleus and the electron shell of the target atom. Calculation of the ratio R of the intensities of the x-ray emission before and after the decay of the compound nucleus in the cases of an isolated compound resonance as well as strongly overlapping resonances leads to the result

$$R \approx \Gamma^K_{Z+1}/(\Gamma^K_{Z+1} + 2\Gamma_{\mathrm{corr}}),$$

where Γ^K_{Z+1} is the line width of the x rays emitted following population of the K-shell vacancy of the atom with atomic number $Z+1$.

Measurements of the Durations of Nuclear Reactions by Means of the Interference Between the Bremsstrahlung of Charged Particles

The idea of this method is explained in Refs. 121 and 122. When a charged particle is scattered by a nucleus, it emits bremsstrahlung. One of the components in the bremsstrahlung-emission amplitude arises in connection with the discontinuation of the current corresponding to the incident particle, the other to the appearance of the current corresponding to the emitted particle. This amplitude depends on the charge and velocity of the particles, on the scattering angle, on the emission angle of the bremsstrahlung, and on the duration τ of the reaction. The dependence on τ leads to an interference term of the two components with the characteristic factor $\exp(i\omega\tau)$ (ω is the frequency of the radiation). At small values of $\omega\tau$, the two components interact coherently (they amplify or suppress each other, depending on the scattering angle of the particle and on the emission angle of the bremsstrahlung), giving as a result the ordinary bremsstrahlung spectrum. With increasing $\omega\tau$, the interaction between the component changes. Thus, the effect can be observed for the part of the spectrum in which $\omega\tau\sim1$. Such a picture is observed when either the incident or the emitted particle is not charged or both are not charged, since the recoil nucleus is charged.

Figure 1 shows the bremsstrahlung spectrum for the case when the scattering and bremsstrahlung-emission angles are such that both components enhance each other without a time delay. It is clear from this that measurement of the

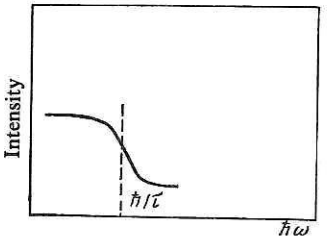

FIG. 1. Qualitative form of bremsstrahlung spectrum.

shape of the bremsstrahlung spectrum in the region $\omega\tau \sim 1$ can give information about the time τ.

In Ref. 123, the following expression for the differential cross section of bremsstrahlung in proton elastic scattering was obtained in the framework of a stationary quantum-mechanical description of the bremsstrahlung process:

$$\frac{\partial^3\sigma}{\partial\Omega_\gamma\partial\Omega_f\partial\varepsilon_\gamma} \simeq \frac{\alpha}{(2\pi)^2}(1-Z/A)^2\frac{p_f^2}{\varepsilon_\gamma}[(\hat{k}_\gamma\times\hat{p}_f)^2|f(\varepsilon_f)|^2 + (\hat{k}_\gamma\times p_i)^2|f(\varepsilon_i)|^2 - 2(\hat{k}_\gamma\times\hat{p}_f)(\hat{k}_\gamma\times\hat{p}_i)\,\mathrm{Re}\,f^*(\varepsilon_i)f(\varepsilon_f)], \quad (64)$$

where α is the fine-structure constant, $\mathbf{p}_{f,i} = \hat{\mathbf{p}}_{f,i}p$ is the momentum vector of the relative motion of the proton and the target nucleus, ε_γ is the energy of the emitted photon, ε_i is the energy of the incident proton, $\varepsilon_f = \varepsilon_i - \varepsilon_\gamma$, $\hat{\mathbf{k}}_\gamma = c\mathbf{k}_\gamma/\varepsilon_\gamma$ ($\mathbf{k}_\gamma$ is the photon momentum), $f(\varepsilon)$ is the proton elastic-scattering amplitude, and $\partial\Omega_\gamma$ and $\partial\Omega_f$ are the elements of the solid angles characterizing the direction of emission of the photon and the scattered proton, respectively. The expression (64) is obtained in the approximation $k_\gamma R \ll 1$ (R is the radius of the target nucleus). In the case of neutron scattering, it is necessary to replace the factor $(1-Z/A)^2$ in (64) by $(Z/A)^2$. The difference between the phases of the amplitudes $f(\varepsilon_i)$ and $f(\varepsilon_f)$ in the final, interference term in (64) depends on the duration time of the nuclear reaction.

Various aspects of the theoretical description of the bremsstrahlung spectrum and its dependence on the correlation coefficient in the presence of Ericson fluctuations and on the duration time of the nuclear reaction were also considered in Refs. 124–127. The experimental possibilities of the method and the first results of measurements of the duration time of nuclear reactions are considered in Refs. 128–131. In accordance with Refs. 129–131, the delay time in the process of elastic scattering of ~1.7-MeV protons by ^{12}C nuclei, measured by this method, is $\sim 10^{-20}$ sec.

Measurements of the Lifetimes of Compound Nuclei by Means of the "Proximity Scattering" Effect

In Ref. 132, Fox proposed a method of measuring lifetimes $\gtrsim 10^{-20}$ sec of compound nuclei based on allowance for the effect of "proximity scattering" (rescattering) of two emitted particles in a region far from the final nucleus. The essence of the method can be readily understood in the framework of the following simple quasiclassical treatment,[133] which agrees in its basic features with Ref. 132.

Consider the reaction

$$a + X \to Y + b + c \quad \{d + {}^{12}\mathrm{C} \to {}^{12}\mathrm{C} + n + p\} \quad (65)$$

(in the curly brackets we shall write down the actual data of the measurements described in Ref. 133). Suppose that the reaction (65) proceeds through two such stages:

$$a + X \to Y + b, \quad Q = Q_Y \{d + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N}^* + n,\ Q_Y = -3{,}82\ \mathrm{MeV}\}; \quad (66)$$

$$Y \to Z + c,\ Q = Q_Z \{{}^{13}\mathrm{N}^* \to {}^{12}\mathrm{C} + p,\ Q_Z = 1.59\ \mathrm{MeV}\}, \quad (67)$$

where Y is an intermediate compound nucleus that decays during a time τ^Y long compared with the time required for each of the particles to pass through its region of interaction with any other particle; Q is the reaction energy.

If the particles b and c are emitted in one direction, and the velocity v_c of particle c is greater than the velocity v_b of particle b, then particle c can overtake particle b at the distance D_{bc} from the nucleus Z, which leads to their "proximity scattering" of each other. Denoting by τ_0 the time interval between the emission of particle b and the beginning of the process of proximity scattering, and by τ the interval of time between the emission of paricles b and c, we readily see that $D_{bc} = v_b\tau_0 = v_c(\tau_0 - \tau)$ and hence, eliminating τ_0, we find

$$D_{bc} = \frac{v_b v_c}{v_b - v_c}\tau\,\{340\ \mathrm{F}\}. \quad (68)$$

Assuming that the distribution of the emitted particle c with respect to $\mathbf{v}_b$ is isotropic, we can describe the probability W of occurrence of the process of proximity scattering by the expression

$$W \approx \frac{\sigma(\varepsilon_{bc}^{(Y)})}{4\pi D_{bc}^2} = \frac{\sigma(\varepsilon_{bc}^{(Y)})}{4\pi}\left(\frac{v_b - v_c}{v_b v_c}\right)^2\frac{1}{\tau^2}\{9\cdot 10^{-4}\}, \quad (69)$$

where $\sigma(\varepsilon_{bc}^{(Y)})$ is the integrated cross section for scattering of particles b and c on each other in the rest frame of the nucleus Y;

$$\varepsilon_{bc}^{(Y)} = \frac{1}{2}\frac{m_b m_c}{m_b + m_c}(v_b - v_c)^2\{0.04\,\mathrm{MeV}\}.$$

Of course, such a quasiclassical description is certainly oversimplified and it was used only as an initial approximation to the quantum-mechanical picture of the rescattering process. On the basis of this approach the following scheme for measuring τ was proposed and tested in Ref. 133. The energy spectra of particles b and c are measured by means of two detectors working in coincidence. From these spectra, the velocities v_b and v_c are calculated. The fraction W of particles that undergo "proximity scattering" is also measured (see below). If the cross section $\sigma(\varepsilon_{bc})$ is known from other experiments, it is possible on the basis of (69) to determine τ (or, rather, $\langle\tau^Y\rangle$).

The proximity scattering changes the kinetic energies of particles b and c, which take values ε_b' and ε_c' in the intervals $\varepsilon_b^{(Y)} < \varepsilon_b^{(Y)} < (\varepsilon_b^{(Y)})_{\max}$ and $(\varepsilon_c^{(Y)})_{\min} < \varepsilon_c^{(Y)} < \varepsilon_c^{(Y)}$, where

$$(\varepsilon_b^{(Y)})_{\max} = \frac{1}{2}m_b\left[\frac{(m_b - m_c)v_b + 2m_c v_c}{m_b + m_c}\right]^2\{0.85\ \ldots\ 1.47\,\mathrm{MeV}\};$$

$$(\varepsilon_c^{(Y)})_{\min} = \frac{1}{2}m_c\left[\frac{(m_c - m_b)v_c + 2m_b v_b}{m_b + m_c}\right]^2\{1.47\ \ldots\ 0.85\,\mathrm{MeV}\}.$$

If the finite dimensions of the particles (packets) are ignored, the following expression can be obtained for the maximal angle (in the Y system) between the directions of emission of particles b and c after the process of proximity scattering:

$$(\theta_{bc}^{(Y)})_{\max} = \arcsin\frac{(v_c - v_b)(m_b v_b + m_c v_c)}{2(\varepsilon_b^{(Y)} + \varepsilon_c^{(Y)})}\{15°\}.$$

The experiment described in Ref. 133 measured the spectrum of the particles b and c in the directions θ_b, Φ_b, and θ_c, Φ_c, by means of two detectors in coincidence. In the spectrum it was necessary to find a comparatively narrow line of intensity I^0 at energy corresponding to $\varepsilon_b^{(Y)}$ or $\varepsilon_c^{(Y)}$. The

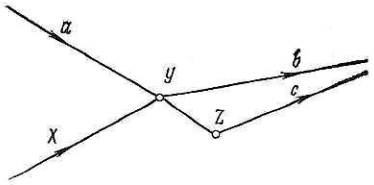

FIG. 2. Two-stage decay without proximity scattering.

width of this line is determined by $\Gamma = \hbar/\langle\tau^Y\rangle$ and the experimental resolution. For $\theta_{bc}^{(Y)} < (\theta_{bc}^{(Y)})_{\max}$ one expects a continuous broadening of this line up to energies $(\varepsilon_b^{(Y)})_{\max}$ or $\varepsilon_c^{(Y)})_{\max}$ (see Fig. 4 below). The intensity I^N of this continuous broadening must satisfy

$$\int\limits_{[\theta_{bc}<(\theta_{bc})_{\text{макс}}]} I^N d\Omega = 4\pi W I^0.$$

The total cross section of the reaction (65) was calculated in Ref. 133 approximately with allowance for the interference of the amplitudes corresponding to the diagrams shown in Figs. 2 and 3.

Figure 4 shows the results of such calculations in a comparison with experimental data for 5.39-MeV deuterons at one chosen set of angles θ_n, θ_p, and $\varphi_{np} = \phi_n - \phi_p$, the two-stage nature of the decay being taken into account: $^{12}C + d \rightarrow {}^{13}N^*$ (3.5 MeV) $+ n$, $^{13}N^* \rightarrow {}^{12}C + p$. The excited nucleus $^{13}N^*$ has two levels with energies 3.51 MeV ($\Gamma = 55$ keV) and 3.56 MeV ($\Gamma = 61$ keV), which in the experiment were not sufficiently well resolved and were regarded as a single resonance.

The best agreement between the calculations and the experimental data was obtained for $\langle\tau^{(Y)}\rangle = (0.7 \pm 0.1) \times 10^{-20}$ sec. In Fig. 4 one can note the appearance of a step, which is characteristic for the process of proximity scattering and is determined by the admissible kinematic momentum transfer.[133,134]

However, the analysis made in Ref. 133 is oversimplified and must be made much more refined and more complicated in the case of a large number of closely spaced compound resonances. Therefore, it is of interest to consider another kind of experiment—measurement of the dependence of the angular correlation of particles b and c in the region of small angles $\theta_{bc} = \theta_b - \theta_c$ on $\langle\tau^Y\rangle$. This effect was considered for the first time in a simplified form in Ref. 132. It is given a more complete quantum-mechanical analysis in Refs. 5 and 49.

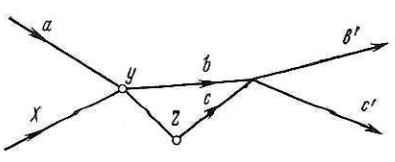

FIG. 3. Two-stage decay with proximity scattering.

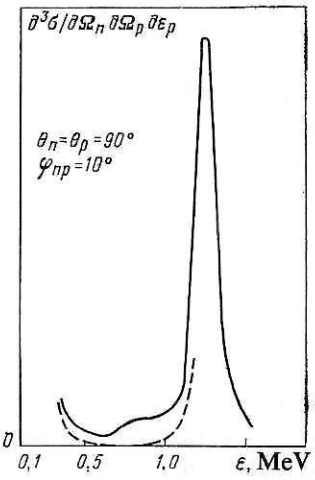

FIG. 4. Differential cross section $\partial^3\sigma/\partial\Omega_n\,\partial\Omega_p\,\partial\varepsilon_p$ (in relative units). The continuous curve describes the theoretical values for $\langle r^Y\rangle = 0.7 \times 10^{-20}$ sec; the broken curve is constructed with neglect of the process of proximity scattering.

8. INVESTIGATION OF THE DURATIONS OF NUCLEAR REACTIONS IN THE REGION OF STRONGLY OVERLAPPING RESONANCES—A FUNDAMENTALLY NEW SOURCE OF INFORMATION ABOUT THE PROPERTIES OF COMPOUND STATES

In Ref. 135, Malaguti *et al.* already pointed out that analysis of the evolution of the decay of a compound nucleus on the basis of an experiment using the blocking effect in a region of weak overlapping of resonances ($\Gamma \ll D$) could give information about the average partial widths, and also, with allowance for data on the strength functions, about the level density of the compound nucleus. In Refs. 105, 106, and 136, the results of an investigation of the decay of compound nuclei produced in proton scattering by Mo nuclei in the region of energies 5.89, 6.09, and 6.55 MeV, by ^{58}Ni in the region of energies 5.65 and 6.50 MeV, and by ^{70,72,74}Ge in the region of energies 5.035 and 5.110 MeV were used to obtain information about the average width of compound resonances.

The third of the relations (50) and the sum rule (51) give a basically simple possibility of extracting information about the density of resonances that cannot be obtained using data only on the cross sections in the region of strongly overlapping resonances.[137]

Indeed, the averaged cross sections $\langle\sigma_{ji}^{c(J)}(E,\vartheta_j)\rangle$ can be calculated in the framework of an optical–statistical analysis. For this, in particular, one can use the statistical Hauser–Feshbach model of compound processes in conjunction with the optical model of elastic scattering or the coupled-channel method (generalized optical model) for elastic and inelastic scattering to calculate $T_j^{(J)}$. Determining in this way $\langle\sigma_{ji}^{c(J)}(E,\vartheta_j)\rangle$, one can on the basis of measurements of $\langle\Delta\tau_{ij}^c(E,\vartheta_j)\rangle$ and the third of the relations (53) calculate $\langle\Delta\tau_{ji}^{c(J)}(E)\rangle$ on the basis of simple devices for solving systems of linear algebraic equations. Then, using the sum rule (51), one can directly calculate D_J, i.e., the density of the compound resonances. For this one requires, of course, a sufficiently complete set of $\langle\Delta\tau_{ji}^c(E,\vartheta_j)\rangle$ for all open channels and

different values of the angles ϑ_j. It is sufficient to take the number of the latter equal to the final number of J values actually taken into account. This method of determining the density of strongly overlapping resonances can, in its turn, serve as an orientation for measurements of $\langle \tau_{ji}^{c}(E,\vartheta_j)\rangle$ with the aim of ensuring the required completeness of the set of data.

Moreover, using the optical model (or the coupled-channel method) one can use the Moldauer–Simonius theorem[79] to calculate

$$|\det \langle \hat{S}^{(J)} \rangle| = \exp(-\pi\Gamma_J/D_J)$$

or

$$\prod_{j=1}^{n} |\langle S_{jj}^{(J)} \rangle| \simeq \exp(-\pi\Gamma_J/D_J)$$

and the average width Γ_J of strongly overlapping resonances.

Such a method of determining D_J and Γ_J applies not only in the region (29) but also at higher energies in the region (32). Since in this last case $\langle \Delta\tau_{ji}^{c(J)} \rangle$ are determined by means of integral relations of the type (33) and (45) by the functions $D_J(E)$ and $\Gamma_J(E)$, it is necessary to use in the calculations models that describe such functions in a fairly wide range of energies.

9. CONCLUSIONS

This review makes it possible to draw the following conclusions:

1. Time is an observable in quantum mechanics. The Hermitian time operator of the evolution of a quantum system is canonically conjugate to the self-adjoint energy operator of the system.

2. General expressions have been obtained for the expectation values of the durations and the variances of the durations for effectively all types of collisions and nuclear rections.

3. The present state of the investigtion of the durations and evolution of nuclear reactions and decays has a perfectly reliable theoretical foundation and unambiguously indicates the possibility of obtaining new information about the properties of quantum systems.

We end by formulating some problems that are important and promising for the further development of investigations of the duration and evolution of collision processes and also for extracting new information about the properties of nuclear compound systems by analyzing the durations and evolution of nuclear reactions.

1. The investigation of the correlations between the dependences $\Delta\tau_{ji}^{(J)}(E)$, $\tilde{S}_{ji}^{(J)}(E)$ and $D_J(E)$ in different regions of energies on the basis of the generalization of Levinson's theorem for multichannel scattering.

2. The generalization of the expressions (50) and (51) for the durations in the region of overlapping resonances in the case of channels with three or more particles in the continuum. For channels with N particles in the continuum ($N>2$), the channel index j must characterize not only the corresponding set of discrete quantum numbers (j_0) but also the continuous components of the $N-2$ momentum vectors of the relative motion of the particles. Denoting these last by the symbol $\mathbf{Q}$, we can write $j=\{j_0,\mathbf{Q}\}$. If we begin by considering the simplest problem and propose the same structure of the representation (37), taking the elements $\hat{U}_{ji}^{(J)}$, $(\hat{P}_{\nu}^{(J)})_{ji}$ and $S_{ji}^{(J)}$ to be the same for the different admissible values of $\mathbf{Q}$ (for the same sets of the discrete quantum numbers), then the results (50)–(61) obtained for the two-particle reactions retain their form and meaning for channels with an arbitrary number of particles in the continuum. It is only necessary in counting the number of channels n to understand by a "channel" a set of colliding or emitted particles with definite energy levels of the Hamiltonians of their internal motion.

3. Further investigations of the variances of the durations and the evolution of the decay of compound nuclei in a region of overlapping resonances. In particular, it would be interesting to establish what are the actually encountered cases in which the approximation $|\partial|S_{ji}^{c}|/\partial E| \ll \partial \arg S_{ji}^{c}/\partial E$ is invalid. Clearly, investigations outside the framework of the approximations considered in Sec. 6 are of interest.

4. Searches for and development of new methods to measure the durations and evolution of nuclear reactions. For example, it was suggested in Ref. 138 that the oscillations in the energy spectrum of δ electrons accompanying the nuclear reaction of deep inelastic interaction of complex nuclei should be exploited. Such an effect makes it possible in principle to measure durations $\gtrsim 10^{-21}$ sec of nuclear rections.[8)] An interesting variant of the use of the rescattering effect in the Coulomb fields of interacting fission fragments to determine the lifetime of a fissioning reaction product is given in Ref. 139. Moreover, the possibilities of using the effect of resonance transition of nucleons and clusters between equivalent states of nuclei, which was already proposed in Ref. 140, have not yet been studied.

5. Searches for experimental possibilities of measuring the durations of threshold states and neutron scattering and, possibly, γ rays.

6. Experimental investigations of the deviations from the exponential form of the decay law of nuclear states. A start was made on such investigations in Refs. 141 and 142.

APPENDIX A: DERIVATION OF THE ENERGY–TIME UNCERTAINTY RELATION

We define the variances

$$\mathscr{D}t = \int_{-\infty}^{\infty} (\hat{t} - \langle t\rangle)^2 \varphi^* \varphi \, dt = \int_{0}^{\infty} g^* (\hat{t} - \langle t\rangle)^2 g \, dE \equiv (g, (\hat{t} - \langle t\rangle)^2 g)$$
$$\equiv \| (t - \langle t\rangle) g \|^2 \text{ и } \mathscr{D}E = \| (\hat{E} - \langle E\rangle) g \|^2$$

[8)]The recently published paper of Ref. 143 gives the first results of determination of the durations τ^c of deep inelastic U + U and U + Cm collisions with energy in the region 5.9–8.4 MeV by analysis of the experimental spectra of δ electrons and positrons created in these processes. Preliminary estimates give $\tau^c \sim 10^{-21}$ sec. It was also found that the expression (14) for the delay time is valid under a condition less restrictive than (13):

$$|\langle |T_{ji}|^2 \hat{\partial} \arg G_i | \partial\varepsilon_i \rangle / \langle |T_{ji}|^2 \rangle - \langle \hat{\partial} \arg G_i / \partial\varepsilon_i \rangle| \ll |\langle \Delta\Gamma_j n_i(E,\Omega_j)\rangle| .$$

The expression (21) for the variance of the distribution of the collision durations is also valid for a larger class of collisions when the distortion of the shape of the packet in the collision process can be ignored.

and $\mathscr{D}E = \|(\hat{E} - \langle E\rangle)g\|^2$ and choose as measures of the uncertainties of t and E the rms fluctuations $\Delta t = \sqrt{\mathscr{D}t}$ and $\Delta E = \sqrt{\mathscr{D}E}$. In accordance wth the Cauchy–Schwarz inequality $|(f,g)| \leqslant \|f\|\,\|g\|$, where the equality sign holds only when the vector f is parallel to the vector g, i.e., $f = cg$, we have

$$\Delta E \Delta t \geqslant |([\hat{\hat{E}} - \langle E\rangle]\, g,\ [\hat{\hat{t}} - \langle t\rangle]\, g)|.$$

Bearing in mind that $|x| \geqslant |\operatorname{Im} x|$ and $\operatorname{Im}(f,g) = \{(f,g) - (g, f)\}/2i$, we obtain

$$\Delta E \Delta t \geqslant |\operatorname{Im}([\hat{E} - \langle E\rangle]\, g,\ [\hat{t} - \langle t\rangle]\, g)| = \frac{1}{2}|\langle [\hat{E},\ \hat{t}]\rangle| = \hbar/2$$

etc.

For strictly stationary bound states, when the energy levels are well defined and $\Delta E = 0$, we have $\Delta t = \infty$ in accordance with (1), i.e., the lifetime of the state becomes completely indefinite.

APPENDIX B: APPROXIMATE EIGENVALUES AND APPROXIMATE ORTHONORMALIZED EIGENFUNCTIONS OF THE TIME OPERATOR

Using the example given in the third part of Ref. 2, we can specify the following approximate eigenvalues and eigenfunctions of the operators $\hat{t}$ and $\hat{t}^2$:

$$\hat{t}\varphi_t^{\delta,\eta}(E) \approx t\varphi_t^{\delta,\eta}(E); \tag{B1}$$

$$\hat{t}^2\varphi_t^{\delta,\eta}(E) \approx t^2\varphi_t^{\delta,\eta}(E), \tag{B2}$$

where

$$\varphi_t^{\delta,\eta}(E) = C \exp(\mathrm{i}\, Et/\hbar)\, f_\delta(E)\, g_\eta(E): \tag{B3}$$

C is an arbitrary constant; t is the continuous real eigenvalue of the operator $\hat{t}$;

$$f_\delta(E) = 2\frac{\sin \delta E/\hbar}{E/\hbar},$$

$$g_\eta(E) = \begin{cases} 3(E/\eta)^2 - 2(E/\eta)^3 & \text{for } 0 \leqslant E \leqslant \eta; \\ 1 & \text{for } \eta \leqslant E, \end{cases}$$

δ is a positive parameter that describes the width of the wave packet formed from the functions $\exp(iEt/\hbar)$; and the limit of the sequence of functions $g_\eta(E)$ as $\eta \to 0$ is a generalized function $\theta(E)$. As is readily seen by direct calculation of the left-hand sides of Eqs. (B1) and (B2), the functions $\varphi_t^{\delta,\eta}(E)$ approximate the eigenfunctions of the operators $\hat{t}$ and $\hat{t}^2$ more accurately and are more nearly orthogonal for different t, the better the relations

$$\delta/t \ll (\delta\eta/\hbar)^{1/2} \ll 1 \tag{B4}$$

and

$$(\delta/t)^2 \ll (\delta\eta/\hbar)^{3/2} \tag{B5}$$

hold as $\delta \to 0$ and $\eta \to 0$. Further, for $\delta \to 0$ and $\eta \to 0$ and under fulfillment of the condition (B4) the variance

$$\mathscr{D}t = \langle \varphi_t^{\delta,\eta}|\hat{t}^2|\varphi_t^{\delta,\eta}\rangle - |\langle \varphi_t^{\delta,\eta}|\hat{t}|\varphi_t^{\delta,\eta}\rangle|^2$$

in the state $\varphi_t^{\delta,\eta}$ tends to 0. The constant C can be chosen to make the norm $\|\varphi_t^{\delta,\eta}(E)\|$ equal to unity.

Note that the function (B3) differs from simple wave packets of the form

$$\varphi_t^\delta(E) = 2C \exp(\mathrm{i}\, Et/\hbar)\, f_\delta(E),$$

which are typical for "eigendifferentials" in the continuous spectrum of lineary self-adjoint operators,[34] only by the presence of the factor

$$g_\hbar(E) \to \theta(E).$$

Similarly, one can construct approximate eigenvalues and eigenfunctions of other maximal symmetric operators: the momentum operator $-i\hbar\partial/\partial x$ in a half-space with a rigid wall at $x = 0$ $(0 < x < \infty)$, the radial momentum operator $-i\hbar(\partial/\partial r + 1/r)$ $(0 < r < \infty)$, etc.

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