A method of calculating a scattering amplitude at high energies using unitarity and analyticity

S. M. Troshin and N. E. Tyurin

Institute of High Energy Physics, Serpukhov
Fiz. Elem. Chastits At. Yadra **15**, 53–93 (January–February 1984)

A dynamical approach in the theory of strong interactions is developed directly on the basis of the general properties of analyticity and unitarity of the S matrix, which makes it possible to describe scattering through all angles in a unified manner. The method proposed for calculating the scattering amplitude uses its analytic properties in the impact-parameter representation. For the region of fixed $t \neq 0$ and in the case of asymptotically increasing total cross sections, the scattering amplitude is expanded with respect to a small parameter $\tau(\sqrt{-t})$. In the region of fixed scattering angles ($s \to \infty$, t/s fixed), the cross section decreases in accordance with a power law. This result is due to the analytic properties of the amplitude in the cosine of the scattering angle. The method provides a basis for analyzing the behavior of scattering cross sections and spin effects in the region of large angles, in particular the energy dependence of the spin-correlation parameters.

INTRODUCTION

We consider a dynamical approach in the theory of strong interactions based on an analysis of the singularities of the scattering amplitude in the complex plane of the impact parameter in the direct reaction channel. This approach is based directly on the use of general properties of the S matrix such as analyticity and unitarity,[1] and makes it possible to treat scattering in the complete region of momentum transfers from a unified point of view. It is well known that quantum chromodynamics, which possesses a computational formalism—albeit nonunique—for the region of large values of the square of the momentum transfer, encounters serious difficulties in calculations of the amplitudes of soft processes.

In the present paper, we proceed from a three-dimensional dynamical equation $F = F[U]$ for the scattering amplitude,[2] its kernel, the U matrix, being the relativistic analog of the reaction matrix of quantum mechanics. In this framework, the interaction dynamics of hadrons is determined by the equation $F = F[U]$, whose structure has a manifestly unitary nature, the kernel being chosen with regard to the consequences of the analytic properties of the amplitude in $\cos\theta$.

In the region of fixed t, we shall obtain an expansion of the scattering amplitude with respect to a parameter $\tau(\sqrt{-t})$ which decreases with increasing momentum transfer. In the region of scattering through fixed angles, the analytic properties of the amplitude together with the unitarity relation lead to a power-law of decrease of the scattering cross section. Moreover, there is no need to use perturbation theory; this is very important because of the large value of the strong-interaction coupling constant.

This large value of the coupling constant means that one cannot use the standard methods of perturbation theory to calculate, for example, the cross sections of two-particle processes. However, it is well known that in the framework of various dynamical schemes, which we wish to discuss here, the scattering amplitude can be represented as an expansion with respect to some parameter, which depends on the scattering variables,[3] and there exists a range of variation of the variables in which it is small. In contrast to a coupling constant in Lagrangian field theory, this parameter depends on the characteristics of the original dynamical quantity. Thus, if it is a smooth effective interaction quasipotential $V(s,r^2) = g(s,r^2)\exp[-\varphi(s,r^2)]$, the corresponding expansion parameter can be expressed in terms of the functions g and φ and their derivatives at the point $r^2 = 0$. The representation mentioned above, which is common to different models, has the form of the following iterative series:

$$F(s,\ t) = \sum_n c_n\,[\tau(s)]^n \exp\left[\frac{a(s)\,t}{n}\right]. \qquad (1)$$

Essentially, all that is needed to obtain the series (1) is smoothness of the effective interaction quasipotential (the Born term). It is readily seen that, irrespective of the concrete form of the coefficients c_n, the amplitude $F(s,t)$ decreases exponentially with increasing square of the momentum transfer at small t, whereas an estimate of the series (1) in the region beyond the diffraction peak leads to the dependence $F(s,t) \sim \exp[-b(s)\sqrt{-t}]$. Such behavior of the scattering amplitude correctly reflects the experimentally observed dependence of the differential cross sections on the square of the momentum transfer.[4] This circumstance justifies the efforts that have been made in seeking quantitative agreement with the experimental data in models in which the amplitude is determined by an iterative series of the type (1).

The parameter $\tau(s)$ in Eq. (1) can be expressed in terms of the quantities that determine the Born term in the chosen scheme. In the case of an asymptotically constant total interaction cross section, the use of the series (1) to calculate the slope parameter $B(s)$ of the angular distribution at $t = 0$ and the total interaction cross section $\sigma_{\rm tot}(s)$ makes it possible to obtain the energy dependence of the expansion parameter: $\tau(s) \sim \sigma_{\rm tot}^{(\infty)}/B^{(\infty)}(s)$. Thus, in this case the theory in the limit $s \to \infty$ contains a small parameter $\tau(s) \sim (\ln s)^{-1}$. This result justifies representation of the amplitude as an expansion in

 0090-4759/84/010025-17$04.40

an iterative series with respect to the number of successive rescatterings. All the experimentally measurable quantities can then be represented in the form of expansions with respect to the parameter $\tau(s)$.

It is well known that allowance for unbounded growth of the total interaction cross sections required changes in the values of the parameters that determine the behavior of the kernel of the dynamical equation (the Born term). As a result, the energy dependence[5] of the parameter τ is such that $\tau \to \infty$ as $s \to \infty$, and therefore an expansion of the type (1) for the amplitude becomes meaningless.

Naturally, in such a situation the question arises of finding an appropriate small parameter and constructing a new expansion of the scattering amplitude with respect to this parameter, valid in the case of increasing total interaction cross sections. Consideration of the singularities of the amplitude in the complex plane of the impact parameter makes it possible to obtain for the region of fixed t in the case of an increasing $\sigma_{\rm tot}(s)$ an expansion of the scattering amplitude with respect to a parameter that depends on $\sqrt{-t}$. As we have noted, this parameter decreases with increasing momentum transfer.

The expressions obtained in the present paper for the region of large scattering angles using the analytic properties of the amplitude will be compared with calculations made by perturbation theory in quantum chromodynamics.

For the scattering amplitude, we use a one-time dynamical equation[2] that relates it to the generalized reaction matrix:

$$F(\mathbf{p}, \mathbf{q}) = U(\mathbf{p}, \mathbf{q}) + \frac{i\pi\rho(s)}{8} \int d\Omega_{\hat{\mathbf{k}}} U(\mathbf{p}, \mathbf{k}) F(\mathbf{k}, \mathbf{q}). \quad (2)$$

The kernel of the equation (the U matrix) is the relativistic analog of the reaction matrix of quantum mechanics. The dynamical quantities that occur in Eq. (2) are on the energy shell, and, thus, do not contain ambiguities associated with the extrapolation procedure. The solution of Eq. (2) in partial waves has the form

$$f_l(s) = u_l(s)/[1 - i\rho(s)\, u_l(s)]. \quad (3)$$

Thus, Eq. (2) automatically leads to a scattering amplitude satisfying the unitarity relation $\operatorname{Im} f_l(s) \geqslant |f_l(s)|^2$, provided the anti-Hermitian part of the U matrix is non-negative. On the other hand, the representation (3) has, besides this remarkable property, a manifestly resonance nature.[6,7] By this we mean the fact that the very representation of the amplitude in the form (3) generates singularities in the l plane, different from the singularities of $u_1(s)$, whose positions are determined from the condition

$$1 - i\rho(s)\, u_l(s) = 0. \quad (4)$$

We shall show that the corresponding poles $\{l_k(s)\}$ are situated near the values $l_0(s)$, and $l_0(s)/p = R(s)$, where $R(s)$ is the range of the interaction. These poles are analogous to the Regge poles in the direct reaction channel, and they can be interpreted as the manifestation of s-channel peripheral resonances excited at the boundary of the interaction region.

In the impact-parameter representation as $s \to \infty$, the solution of Eq. (2) can be represented in the form of the ratio

$$f(s, \beta) = u(s, \beta)/[1 - iu(s, \beta)], \quad (5)$$

where $\beta = b^2$, which ensures for the analytic continuation of the scattering amplitude fulfillment of the condition $|f(s, re^{i\varphi})| \leqslant 1$ when $r \to \infty$. This consequence of the unitary nature of the representation of the scattering amplitude makes it possible to apply the expression (5) effectively for using the analytic structure of the amplitude in the β plane. It is natural to expect that the contribution to the amplitude of the singularities determined by Eq. (4) must decrease exponentially with increasing momentum transfer, since $R(s) \gg r_0$, where r_0 is the radius of the region of localization of the hadron constituents. At the same time, the presence of internal structure must also result in inhomogeneity in the region of impact parameters near zero. It can be shown that a consequence of the analytic properties of the amplitude with respect to the cosine of the scattering angle is a singularity of the function $u(s, \beta)$ at the point $\beta = 0$, from which there follows a power-law behavior of the amplitude in the region of large scattering angles.

In the first part of the paper, we consider the general properties of the singularities of the partial-wave amplitude $l_k(s)$ (see Sec. 1), formulate a scheme for calculating $F(s,t)$ and obtain an expansion in a series with respect to the small parameter (see Sec. 2), obtain a power law of decrease of the amplitude in the region of fixed angles (see Sec. 3), and propose a method for taking into account the composite structure of hadrons (the introduction of a number of valence constituents) in constructing the kernel of the basic equation, i.e., the U matrix (see Sec. 4).

In the second part of the paper, we discuss the phenomenological consequences of the expressions for the amplitude obtained in the first part. We consider the behaviors of the angular distributions, the polarization parameter (large angles), and the energy and angular dependence of the spin-correlation parameters.

1. THE U MATRIX AND GENERAL PROPERTIES OF THE POLES OF THE PARTIAL-WAVE AMPLITUDE IN THE l PLANE

As we have already noted in the Introduction, the poles of the partial-wave amplitude in the complex l plane are determined by the roots of Eq. (4), whose solutions depend, of course, on the particular form of the function $u_l(s)$. However, the general properties of the solutions $l_k(s)$ as $s \to \infty$ can be obtained from the analytic properties of the function $U(s,t)$. In Ref. 8, the analytic properties of the U matrix were considered, and it was shown that for the components $U^{\pm}(s,t)$ even and odd with respect to the cosine of the scattering angle the following spectral representation holds when $s \geqslant 4m^2$;

$$U^{\pm}(s, t) = \int_{t_0}^{\infty} dt' \frac{\rho^{\pm}(s, t)}{t' - t}, \quad (6)$$

provided an analogous spectral representation holds for the scattering amplitude. For the function $u_l(s)$, omitting for simplicity the parity symbol, we obtain from this

$$u_l(s) = \int_{t_0}^{\infty} \rho(s, t')\, Q_l\left(1 + \frac{2t'}{s - 4m^2}\right) dt'. \quad (7)$$

Since the partial waves with large l make the main contribution to the scattering at high energies, for the functions $Q_l(z)$

we can use the asymptotic expression

$$Q_l\left(1+\frac{2t'}{s-4m^2}\right)\cong\left(\frac{2l}{\sqrt{s}}\right)^{-1/2}\exp\left[-\frac{2l}{\sqrt{s}}\sqrt{t'}\right]. \quad (8)$$

Then for $u_l(s)$ we can readily obtain the expression

$$u_l(s)\cong i\varphi(s,\ l,\ \mu)\exp\left[-\mu\frac{2l}{\sqrt{s}}\right],\quad \mu=\sqrt{t_0}. \quad (9)$$

The function $\varphi(s,l,\mu)$ which occurs in the last expression can be represented in the case of large $l/\sqrt{s}$ by the asymptotic expansion

$$\varphi(s,\ l,\ \mu)=\sum_{n=0}^{N}\frac{\tilde{\rho}^{(n)}(s,\ \mu)}{(2l/\sqrt{s})^{n+3/2}}+O\left[\left(\frac{2l}{\sqrt{s}}\right)^{-N-5/2}\right],$$

where $\tilde{\rho}(s,x)=-i\sqrt{x}\,\rho(s,x)$.

The equation determining the positions of the poles now has the form

$$1+\varphi(s,\ l,\ \mu)\exp\left[-\mu\frac{2l}{\sqrt{s}}\right]=0. \quad (10)$$

Its solutions are the functions

$$l_k(s)=\frac{\sqrt{s}}{2\mu}[\ln\varphi(s,\ \mu)+i\pi k],\quad k=\pm 1,\ \pm 3\ \ldots \quad (11)$$

In obtaining $l_k(s)$, we ignore the terms $\sim\ln l$ compared with l. It is also readily seen that $R(s)=(1/2\mu)\ln\varphi(s,\mu)$ is the effective interaction range. Thus, the general properties of the poles of the amplitude in the l plane that follow from the manifestly unitary form of the representation (3) and from the analytic properties of the generalized reaction matrix are

$$\begin{aligned} &1)\quad |l_k(s)|\sim\sqrt{s};\\ &2)\quad \mathrm{Re}\ l_k(s)\simeq\sqrt{s}R(s);\\ &3)\quad \mathrm{Im}\ l_k(s)\simeq\sqrt{s}k. \end{aligned} \quad (12)$$

The first property is a reflection of the quasiclassical nature of the scattering at large angles, by virtue of which the orbital angular momentum enters all quantities through the impact parameter. It is therefore convenient to go over from expansion with respect to the partial waves to the impact-parameter representation, which makes it possible to obtain a perspicuous semiclassical picture of the interaction at high energies.

The functions $l_k(s)$ are asymptotic with respect to the Regge trajectories $\alpha_k(s)$. The Regge trajectories relate the asymptotic behavior to the behavior of the amplitude in the crossed channel in the region of small values of the square of the momentum transfer s. As will be shown, the values of these trajectories also determine the behavior of the scattering amplitude in the direct reaction channel in the region of large values of the square of the momentum transfer t.

2. SINGULARITIES OF THE AMPLITUDE IN THE IMPACT-PARAMETER PLANE. EXPANSION OF THE AMPLITUDE IN THE REGION OF FIXED $t\neq 0$

To study the singularities of the scattering amplitude in the complex b plane, we use a spectral representation for $U(s,t)$ and go over to the impact-parameter representation:

$$u(s,\ b)=\frac{\pi^2}{s}\int_0^\infty\sqrt{-t}\,d\sqrt{-t}\,U(s,\ t)\,J_0(\sqrt{-tb^2}). \quad (13)$$

We introduce the variable $\beta=b^2$; then, using the integral

$$\int_0^\infty\sqrt{-t}\,d\sqrt{-t}\,\frac{J_0(\sqrt{-t\beta})}{t'-t}=K_0(\sqrt{t'\beta}) \quad (14)$$

we obtain for $u(s,\beta)$

$$u(s,\ \beta)=\frac{\pi^2}{s}\int_{t_0}^\infty\rho(s,\ t')\,K_0(\sqrt{t'\beta})\,dt'. \quad (15)$$

The representation (15) makes it possible to continue the function $u(s,\beta)$ analytically to the complex β plane. It follows from this representation that the function $u(s,\beta)$ is analytic in the complete β plane except for a cut along the negative real half-axis. Using for $K_0(z)$ the asymptotic expression $K_0(z)\cong\sqrt{\pi/2z}\,e^{-z}$, which gives a good approximation of the function $K_0(z)$ even at not too large z, we can obtain for $u(s,\beta)$ the approximate expression

$$u(s,\ \beta)\cong i\,g(s)\exp[-\mu\sqrt{\beta}]. \quad (16)$$

The general form of the function $u(s,\beta)$ that can be proposed on the basis of the analytic properties of the amplitude with respect to the variable t is[9]

$$u(s,\ \beta)=g(s,\ \beta)\,(\mu^2\beta)^{-\gamma}\ln^{\alpha}(\mu^2\beta)\exp[-\mu\sqrt{\beta}],\quad (0\leqslant\gamma<1,\ \alpha>-1), \quad (17)$$

where $g(s,\beta)$ is an entire function of the variable β.

In the impact-parameter representation, the scattering amplitude, which satisfies the basic equation (2), is determined by the integral

$$F(s,\ t)=\frac{s}{2\pi^2}\int_0^\infty d\beta\frac{u(s,\ \beta)}{1-iu(s,\ \beta)}J_0(\sqrt{-t\beta}). \quad (18)$$

For effective use of the analytic structure of the amplitude in the complex plane of the impact parameter, we transform the integral (18) along the positive half-axis $\beta\in[0,\infty)$ into an integral around a contour in the β plane (Fig. 1).

For this, we use the following relationship between the Bessel function $J_0(z)$ and the Macdonald function $K_0(z)$:

$$J_0(\sqrt{|z|})=\frac{i}{\pi}[K_0(\sqrt{-|z|+i0})-K_0(\sqrt{-|z|-i0})]. \quad (19)$$

Then the expression (18) can be rewritten in the form of the contour integral

$$F(s,\ t)=-\frac{is}{2\pi^3}\int_C d\beta f(s,\ \beta)\,K_0(\sqrt{t\beta}),\quad t<0, \quad (20)$$

where the contour C, which is shown in Fig. 1, surrounds the positive half-axis in the β plane.

Since Eq. (2) determines the amplitude $f(s,\beta)$ in the form of a ratio that ensures fulfillment of the condition for analytic continuation to the β plane, $|f(s,re^{i\varphi})|\leqslant 1$ as $r\to\infty$, and also

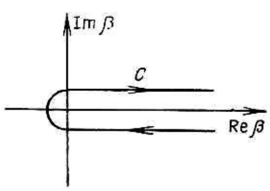

FIG. 1. Contour of integration C.

bearing in mind that $K_0(z) \sim \exp(-\mathrm{Re}\, z)$ as $|z| \to \infty$, we can close the contour C at infinity, passing around at the same time all the singularities of the amplitude $f(s, \beta)$ in the β plane.

Note that to calculate integrals of the function $f(s, \beta)$, which has a singularity at the point $\beta = 0$, it is necessary to go over to the function $f(s, \beta + \beta_0)$, displacing, thus, the beginning of the cut to the point $\beta = -\beta_0$, and consider integrals of the function $f(s, \beta + \beta_0)$, and then go to the limit $\beta_0 \to 0$ in the resulting expressions. Then the contour C in (20) will pass around the cut $\beta \in [-\beta_0, -\infty)$, and the value of the integral calculated as $\beta_0 \to 0$ will converge uniformly to the expression (18), which determines the amplitude $F(s,t)$.

In accordance with what we have said, the scattering amplitude $F(s,t)$ is determined by the sum of the contributions from the poles and the cuts in the β plane: $F(s,t) = F_p(s,t) + F_c(s,t)$, where F_p and F_c are the contributions to the amplitude from the poles and the cuts, respectively. The pole contribution will be determined by the corresponding sum of residues:

$$F_p(s, t) = \sum F^{[\beta_n(s)]}(s, t) = -\frac{s}{\pi^2} \sum_n \mathrm{Res}\, [f(s, \beta) K_0(\sqrt{t\beta})]_{\beta=\beta_n(s)}, \qquad (21)$$

where the function $\beta_n(s)$ determines the trajectory of the n-th pole in the β plane.

The expression (21) is valid for any form of the function $u(s, \beta)$ and, as we shall show, leads to an expansion of the scattering amplitude in a series with respect to a parameter that decreases with increasing momentum transfer.

For the function $u(s, \beta)$, we use the expression (16), which correctly takes into account the analytic properties in the plane of the impact parameter. We note immediately that the use in place of the representation (16) of the more general representation (17) does not affect the results relating to the behavior of the pole part $F_p(s,t)$. This is a consequence of the fact that the poles are generated by the very form of the representation of the amplitude $f(s, \beta)$ and depend weakly on the actual form of $u(s, \beta)$.

Thus, the positions of the poles are determined by the roots of the equation

$$1 + g(s) \exp[-\mu \sqrt{\beta}] = 0, \qquad (22)$$

and solving this equation we find

$$\beta_n(s) = \frac{1}{\mu^2} [\ln g(s) + i\pi n]^2, \quad n = \pm 1, \pm 3 \ldots . \qquad (23)$$

It is readily seen that $\mathrm{Im}\, \beta_n(s) \sim n$, and the real part $\mathrm{Re}\, \beta_n(s)$ is determined by the interaction range $R(s) = (1/\mu) \ln g(s)$.

Calculating the values of the residues at the points $\beta = \beta_n(s)$, for the pole part of the scattering amplitude we obtain

$$F_p(s, t) = -\frac{2is}{\mu\pi^2} \sum_n \sqrt{\beta_n(s)} K_0(\sqrt{t\beta_n(s)}). \qquad (24)$$

Using for the function $K_0(z)$ the asymptotic expression, we arrive at the expansion

$$F_p(s, t) \cong s \sum_{m=1}^{\infty} \left[\exp\left(-\frac{2\pi}{\mu}\sqrt{-t}\right)\right]^m \Phi_m(R(s), \sqrt{-t}), \qquad (25)$$

where

$$\Phi_m(R(s), \sqrt{-t}) \cong \frac{1}{\mu}\left(\frac{R(s)}{\sqrt{-t}}\right)^{1/2} \left\{\exp\left[\left(iR(s) + \frac{\pi}{\mu}\right)\sqrt{-t}\right] \times \left(-i + (m - 1/2)\frac{2\pi}{\mu R(s)}\right)^{1/2} - \exp\left[\left(-iR(s) + \frac{\pi}{\mu}\right)\sqrt{-t}\right] \times \left(i + (m - 1/2)\frac{2\pi}{\mu R(s)}\right)^{1/2}\right\}. \qquad (26)$$

It should be noted that in the case when the function $u(s, \beta)$ is not purely imaginary and has a phase $\alpha(s)$ different from $\pi/2$, i.e., $u(s, \beta) = g(s)\exp(-\mu\sqrt{\beta} + i\alpha(s))$, the expansion (25) will also hold, but the function Φ_m will then have a phase dependence of the form

$$\Phi_m[R(s), \alpha(s), \sqrt{-t}] \cong \frac{1}{\mu}\left(\frac{R(s)}{\sqrt{-t}}\right)^{1/2} \times \left\{\exp\left[\left(iR(s) + \frac{\pi}{\mu} - \frac{1}{\mu}\left(\alpha(s) - \frac{\pi}{2}\right)\right)\sqrt{-t}\right] \times \left[-i + \frac{1}{\mu R}\left(\alpha(s) - \frac{\pi}{2} + \pi(2m-1)\right)\right]^{1/2} - \exp\left[\left(-iR(s) + \frac{\pi}{\mu} + \frac{1}{\mu}\left(\alpha(s) - \frac{\pi}{2}\right)\right)\sqrt{-t}\right] \times \left[i + \frac{1}{\mu R}\left(-\alpha(s) + \frac{\pi}{2} + \pi(2m-1)\right)\right]^{1/2}\right\}. \qquad (27)$$

As we shall show, the presence of a phase of the function $u(s, \beta)$ different from $\pi/2$ leads to important physical consequences.

We now consider the energy dependence of the function $u(s, \beta)$, which we determine from the condition of unbounded growth of the total interaction cross sections as $s \to \infty$.

It is easy to show that this condition requires growth of $g(s)$ as $s \to \infty$. Indeed, in this case $\sigma_{\mathrm{tot}}(s) = (4\pi/\mu^2)\ln^2 g(s)$ and the power-law behavior of $g(s)$ ensures Froissart growth of the cross sections.

Therefore, taking into account as well the polynomial boundedness of the generalized reaction matrix, $|U(s,t)| < s^N$, we set $g(s) = gs^{\lambda}$. It is readily seen that the energy dependence of the functions Φ_m for fixed t and $s \to \infty$ is determined by the expression $\sqrt{\ln g(s)}$. We shall show that the contribution from the cut in this kinematic region decreases as a power with respect to the energy, i.e., for fixed t the pole contribution is the most important, since

$$F_p(s, t) = O(s\sqrt{\ln g(s)}),$$

and

$$F_c(s, t) = O(sg^{-1}(s)).$$

Thus, in the case of increasing total cross sections it is the pole contribution that is decisive for fixed values of t, and the scattering amplitude can be represented as an expansion with respect to the parameter $\tau(\sqrt{-t}) = \exp(-(2\pi/\mu)\sqrt{-t})$, which decreases with increasing momentum transfer:

$$F(s, t) \cong s \sum_{m=1}^{\infty} [\tau(\sqrt{-t})]^m \Phi_m(R(s), \alpha(s), \sqrt{-t}). \qquad (28)$$

For sufficiently large values of the momentum transfer, it is sufficient to retain only the first term in the expansion (28).

The experimental consequences of such a representation of the amplitude will be discussed in the second part of the review.

We note that an expansion of the amplitude of the type (28) will also hold for other forms of the function $u(s, \beta)$.

Thus, in the framework of the approach in the theory of strong interactions based on a three-dimensional dynamical equation, and in the case of asymptotically increasing total cross sections, $\sigma_{tot}^{\infty}(s) \to \infty$, we have obtained for large $|t|$ a representation of the scattering amplitude in the form of a series with respect to the parameter $\tau(-\sqrt{t})$, which decreases with increasing momentum transfer.[9] The expansion was obtained without recourse to perturbation theory with respect to the Born term (the kernel of the integral equation) on the basis of an analysis of the singularities of the partial-wave amplitude in the impact-parameter plane. This contribution to the amplitude is determined by poles in the plane of the impact parameter, generated by the very form of the representation of the amplitude in terms of the U matrix (the basic equation). The properties of the poles depend weakly on the particular form of the function $u(s, \beta)$.

The m-th term of the expansion (25) corresponds to a pole in the plane of the impact parameter with position determined by the function

$$b_m(s) = R(s) + icm. \tag{29}$$

Thus, $\operatorname{Re} b_m(s)$ is equal to the interaction range, and $\operatorname{Im} b_m(s)$ satisfies the relation $\operatorname{Im} b_{m+1}(s) = \text{const}$. The fact that $\operatorname{Im} b_m(s)$ does not depend on s as $s \to \infty$ leads to an Orear regime in the behavior of the scattering amplitude for the corresponding region of momentum transfers.

Note that an expansion analogous to (28) can also be obtained for the scattering amplitude parametrized in the eikonal form $f(s,b) = (1/2)(1 - e^{-\chi(s,b)})$. In this case, assuming that $\chi(s,0) \to \infty$ as $s \to \infty$, we must calculate the positions of the saddle points determined by the equation $(d/db)(-\chi(s,b) + ibq) = 0$.

The poles of the scattering amplitude in the plane of the impact parameter were considered in Ref. 10, and also in Ref. 11, in which hadron scattering is simulated by the scattering of a plane wave by a spheroidal surface.

3. ANALYTICITY OF THE AMPLITUDE IN cosθ AND POWER-LAW DECREASE OF THE FIXED-ANGLE SCATTERING AMPLITUDE

In the previous section, we considered the contribution $F_p(s,t)$ of the poles $b_m(s)$ to the scattering amplitude. Since these singularities are situated near values $b \sim R(s)$, it is natural that the corresponding contribution to the amplitude decreases exponentially with increasing momentum transfer, since, of course, $R(s)$ is much greater than the characteristic dimension of the central region, where one can assume the hadron constituents to be localized.

We note that the radius r_0 of the central region in the framework of the approach of Ref. 38, in which the hadron is represented as the result of the interaction of particles bound strongly ($g \gg 1$) to a quantized field, is determined by the expression

$$r_0 \sim 1/\sqrt{g}.$$

In scattering with fixed values of the momentum transfer when the ratio t/s is small, one actually considers impact parameters $b \gg r_0$, where the effects of the existence of a virtual cloud are manifested and the interacting hadrons are represented by extended objects. Allowance for their structure at such distances is equivalent to the introduction of a certain distribution of the hadronic matter, or to the description of the interaction by a smooth quasipotential. It is therefore natural that the contribution of the singularities in the plane of the impact parameter, whose positions are determined by Eq. (23), leads to a term $F_p(s,t)$ in the scattering amplitude that decreases exponentially with respect to the momentum transfer.

It is obvious that the presence of the internal structure must lead to an uncertainty of the interaction in the region of small impact parameters $b \sim 0$. As was shown in Sec. 2, a consequence of the analyticity of the amplitude in the cosine of the scattering angle is the presence of a singularity of the function $u(s, \beta)$ at the point $\beta = 0$. We shall in fact see that the contribution $F_c(s,t)$ to the scattering amplitude of the cut $\beta \in [0, -\infty)$ determines the behavior of the cross sections in the region of fixed scattering angles ($s \to \infty$, t/s fixed).

After these remarks, we turn to the calculations of the function $F_c(s,t)$, which is determined by the integral

$$F_c(s, t) = -\frac{s}{\pi^3} \int_{-\infty}^{0} d\beta \operatorname{disc} f(s, \beta) K_0(\sqrt{t\beta}). \tag{30}$$

It is readily seen that in the general case

$$\operatorname{disc} f(s, \beta) = \frac{\operatorname{disc} u(s, \beta)}{[1 - iu(s, \beta + i0)][1 - iu(s, \beta - i0)]}. \tag{31}$$

Using in the general case the representation (15) for the generalized reaction matrix, we obtain for the discontinuity of the function $u(s, \beta)$ the representation

$$\operatorname{disc} u(s, \beta) = -\frac{\pi^3}{2s} \int_{\mu^2}^{\infty} dx \rho(s, x) J_0(\sqrt{x|\beta|}). \tag{32}$$

Using for the function $u(s, \beta)$ the representation (16), which has the correct analytic properties in the β plane, we obtain for the discontinuity of the scattering amplitude

$$\operatorname{disc} f(s, \beta) = -\frac{ig(s) \sin(\mu\sqrt{|\beta|})}{1 + 2g(s)\cos(\mu\sqrt{|\beta|}) + g^2(s)}. \tag{33}$$

Bearing in mind that $g(s) \to \infty$ as $s \to \infty$, we represent the expression for $F_c(s,t)$ in the form

$$F_c(s, t) = -\frac{2is}{\pi^3 g(s)} \left\{ \int_0^{\infty} x \sin \mu x K_0(x\sqrt{-t})\, dx + O(g^{-1}(s)) \right\}. \tag{34}$$

From this we find the asymptotic (with respect to s) contribution of the singularity of the partial-wave amplitude $f(s, \beta)$ at the point $\beta = 0$:

$$F_c^{(\infty)}(s, t) = -\frac{is\mu}{\pi^2 g(s)} \frac{1}{(\mu^2 - t)^{3/2}}. \tag{35}$$

Comparing the expressions (25) and (35), we readily see that at $|t| > |t_0| = (\mu^2/\pi^2) \ln^2 g(s)$ the contribution of the function $F_c(s,t)$ to the scattering amplitude becomes the main one. Retaining in the sum in (25) the term linear in $\tau(\sqrt{-t})$, we obtain for the scattering amplitude the expression

$$F(s,t)=s\exp\left(-\frac{2\pi}{\mu}\sqrt{-t}\right)\Phi_1(R(s),\sqrt{-t})-\frac{is\mu}{\pi^2 g(s)}(\mu^2-t)^{-3/2}. \quad (36)$$

In the region of fixed angles when $s, t\to\infty$, with t/s fixed, the second term in (36) is decisive. Note that the separation of the kinematic regions in which the contributions $F_p(s,t)$ or $F_c(s,t)$, respectively, are decisive is due to the fact that $g(s)\to\infty$ as $s\to\infty$; in other words, it is due to the asymptotic growth of the total interaction cross section. Thus, in the region of scattering through fixed angles we obtain as a consequence of the analytic properties of the scattering amplitude in $\cos\theta$ the following expression for the differential cross section[12]:

$$\frac{d\sigma^{(\infty)}}{dt}=\frac{32\pi\mu^2}{g^2}\left(\frac{1}{s}\right)^{3+2\lambda}(1-\cos\theta)^{-3}. \quad (37)$$

At the same time, the unitarity relation is saturated and the total cross section increases by the maximal permissible amount:

$$\sigma_{\text{tot}}^{(\infty)}(s)=\frac{4\pi}{\mu^2}\ln^2 g(s)=\frac{4\pi\lambda^2}{\mu^2}\ln^2 s.$$

The asymptotic growth of the total interaction cross section and the power-law decrease of the cross section in the region of scattering through large angles are dynamically related.

As we have already noted, the power-law decrease of the fixed-angle scattering cross section is a consequence of the fact that the function $u(s,\beta)$ has a singularity at the point $\beta=0$. If $u(s,\beta)$ is an analytic function in the neighborhood of the point $\beta=0$, then the cross section decreases in the entire region of momentum transfers exponentially. Note that to deduce the existence of a singularity of the amplitude at $\beta=0$ it is sufficient to use analyticity with respect to the variable $\cos\theta$ in an ellipse[1] without having recourse to the Mandelstam analyticity on which the representation (15) is based. We note that a power-law decrease was obtained for the first time on the basis of the self-similarity principle and the quark counting rules.[13]

The expression obtained for the cross section is asymptotic. We give the expression that is obtained if in the calculation of the integral $F_c(s,t)$ we take into account not only the leading term in $1/g(s)$:

$$\frac{d\sigma}{dt}=\frac{32\pi\mu^2}{g^2}\left(\frac{1}{s}\right)^{3+2\lambda}(1-\cos\theta)^{-3}|f(s,0)|^4, \quad (38)$$

or

$$\frac{d\sigma}{dt}=\frac{d\sigma^{(\infty)}}{dt}|f(s,0)|^4.$$

The factor $|f(s,0)|^4$ in the expression for the cross section is determined by the density of the hadronic matter at the point $b=0$ or, in other words, characterizes the probability of a central collision of hadrons, for which large-angle scattering occurs.

The angular dependence of the cross sections is determined by the function $(1-\cos\theta)^{-3}$. This dependence agrees fairly well with the experimental data on large-angle scattering.[1)]

In the general case, the function $u(s,\beta)$ has the more general form (17), although the absence of information about the behavior of the spectral density $\rho(s,x)$ does not permit a more accurate calculation of the pre-exponential factor in the expression for $u(s,\beta)$.

Note that if the function $u(s,\beta)$ does not contain logarithmic factors $\ln^\alpha\beta$, the cross section satisfies asymptotically a simple power law: $d\sigma/dt\sim s^{-N}$. But if there are logarithmic factors in $u(s,\beta)$, the asymptotic expression for the differential cross section contains a factor of the form $(\ln|t|)^{-2\alpha}$. The most interesting case in our view is when

$$u(s,\beta)=ig(s)(\mu^2\beta)^{-\gamma}\ln^\alpha(\mu^2\beta)\exp(-\mu\sqrt{\beta}). \quad (39)$$

The choice of the phase is here dictated solely by considerations of convenience. For the large-angle scattering cross section we then have

$$\frac{d\sigma}{dt}=\frac{1}{\pi\mu^4 g^2(s)}\left(\frac{\mu^2}{|t|}\right)^{2+2\gamma}\frac{\varphi^2(\ln^{-1}|t|/\mu^2)}{\ln^{2\alpha}(|t|/\mu^2)},\quad \varphi(0)=1. \quad (40)$$

We note first of all that the expression (40) is asymptotic with respect to $1/s$, and that the function that depends on the logarithms has been taken into account fully. The expression (40) is derived in the Appendix.

The form of the expression (40) is the same as that of the expression for the cross section obtained when perturbation theory in quantum chromodynamics is used.[14] The exponent $2\lambda+2\gamma+2$ of the decrease of the large-angle scattering cross section in quantum chromodynamics is determined by the number of hadron constituents, and the power of the logarithm is determined, in addition, by the so-called anomalous dimensions.

The expression (40) can also be effectively represented as a power-law function with exponent that depends on the square of the momentum transfer:

$$\frac{d\sigma}{dt}=\frac{1}{\pi\mu^4 g^2(s)}\left(\frac{\mu^2}{|t|}\right)^{N(t)}\varphi^2[\ln^{-1}(|t|/\mu^2)], \quad (41)$$

where

$$N(t)=2(1+\gamma)+2\alpha[\ln\ln(|t|/\mu^2)]\ln^{-1}(|t|/\mu^2).$$

The nature of the singularity of the generalized reaction matrix $u(s,\beta)$ at the point $\beta=0$ is a reflection of the internal structure of the hadrons. We shall consider this question in more detail in the following section.

In the expressions given above, an important part is played by the parameter μ, which determines the momentum-transfer scale. In contrast to quantum chromodynamics, in which the analogous role is played by the parameter Λ, which does not have a clear physical meaning, the parameter μ can, as will be shown in the following section, be related to the masses of the quarks.

4. ALLOWANCE FOR THE COMPOSITE STRUCTURE OF PARTICLES IN THE CONSTRUCTION OF THE GENERALIZED REACTION MATRIX

To conclude the first part of the review, we discuss a way of taking into account the composite structure of interacting hadrons h_1 and h_2 in constructing the expression for the generalized reaction matrix.[15] We shall obtain the dependence of the characteristic parameters, in particular $\tau(\sqrt{-t})$ and the exponent N of the decrease of the large-angle scattering cross section on the number of constituent (valence)

[1)]Since we here do not take into account the exchange interaction, our expressions correspond to forward scattering: $\cos\theta>0$.

quarks n_1 and n_2. The method of introducing the numbers n_1 and n_2 corresponds to the quark model with the factorizability assumption.[16]

In this model, it is assumed that when the hadrons h_1 and h_2 interact the valence quarks are scattered independently by some effective potential $V_{\rm eff}$ and the amplitude of the process $h_1h_2 \to h_1h_2$ is represented accordingly in the form of the product

$$\tilde{f}_{h_1h_2} = \prod_{i=1}^{n_1} f_i \prod_{j=1}^{n_2} f_j, \tag{42}$$

where f_i is the amplitude for scattering of quark i by the effective field.

The potential $V_{\rm eff}$ can be regarded as arising as follows. When the hadrons h_1 and h_2, each of which consists of valence quarks concentrated in the center and a virtual cloud of other constituents ($q\bar{q}$ pairs, gluons), scatter off each other, the virtual clouds interact with one another. This gives rise to a certain effective field, whose potential $V_{\rm eff}$ can in this case be regarded as universal. The valence quarks are scattered by this potential independently.

We assume that a factorization of the form (42) holds in the impact-parameter representation. Of course, the function then obtained can be regarded only as a first approximation to the scattering amplitude. It is therefore natural to regard the function $\tilde{f}_{h_1h_2}$ as the kernel of our basic equation. The amplitude then obtained will satisfy the unitarity condition. Thus, the form we take for the generalized reaction matrix is

$$u_{h_1h_2}(s, b) = \prod_{i=1}^{n_1} f_i(s_i, b) \prod_{j=1}^{n_2} f_j(s_j, b). \tag{43}$$

For the amplitude of the scattering of quark i by the potential $V_{\rm eff}$ we choose the expression

$$f_i(s_i, b) = g_i(s_i)\, e^{-\mu b}. \tag{44}$$

We shall assume that the parameter μ does not depend on the quark species. Then the function $u(s,b)$ can be represented in the form

$$u(s, b) = \left[\prod_{i=1}^{n_1} g_i(s_i) \prod_{j=1}^{n_2} g_j(s_j)\right] \exp\left[-\mu(n_1+n_2)b\right]. \tag{45}$$

For simplicity, we shall assume that the momentum of the particle is distributed uniformly between the valence constituents and shall use the variable s as the argument of the quark amplitudes. In accordance with what we said above, we finally obtain for the generalized reaction matrix

$$u(s, b) = i\left[\prod_{i=1}^{n_1+n_2} g_i(s)\right] \exp\left[-\mu(n_1+n_2)b\right]. \tag{46}$$

Constructing now the scattering amplitude in accordance with Eq. (2), we describe the interaction picture considered above in accordance with the unitarity condition. Of course, the scattering amplitude $F(s,t)$ will then no longer correspond to independent scattering of the constituent quarks by the potential $V_{\rm eff}$.

The dependence on the impact parameter contained in the expression for the quark amplitude (44) is chosen to make the function $u(s, \beta)$ have the correct analytic properties in the complex β plane. We note further that the growth of the total interaction cross sections necessarily leads to the requirement of growth of the function $\Pi_{i=1}^{n_1+n_2} g_i(s)$ with the energy. Therefore, with allowance for the polynomial boundedness of $U(s,t)$, it is natural to choose for the functions $g_i(s)$ a dependence of the form $g_i(s) = c_i s^{\lambda}$. The exponent is taken to be the same for all quark species, while the constant c_i depends on the species of quark i, i.e., for $u(s,b)$ we have

$$u(s, b) = iC_{h_1h_2} s^{\lambda(n_1+n_2)} \exp\left[-\mu(n_1+n_2)b\right].$$

This expression takes into account in the simplest manner the composite structure of hadrons.

The expression for the total interaction cross section has the form

$$\sigma_{\rm tot}^{h_1h_2}(s) = 4\pi\left[\frac{\lambda^2}{\mu^2}\ln^2 s + \frac{2\lambda \ln C_{h_1h_2}}{\mu^2(n_1+n_2)}\ln s + \frac{\ln^2 C_{h_1h_2}}{\mu^2(n_1+n_2)^2}\right] \equiv 4\pi R^2_{h_1h_2}(s), \tag{47}$$

where the interaction range is related to the numbers n_1 and n_2 by

$$R_{h_1h_2}(s) = \frac{\lambda}{\mu}\ln s + \frac{1}{\mu(n_1+n_2)}\ln C_{h_1h_2}. \tag{48}$$

Thus, in the considered approach the doubly logarithmic growth of the total cross sections has a universal nature, whereas the terms that increase logarithmically with the energy or are constant with respect to the energy depend on the species of the hadrons h_1 and h_2. In the asymptotic region, $\sigma_{\rm tot}^{h_1h_2}(s)/\sigma_{\rm tot}^{\tilde{h}_1\tilde{h}_2}(s) \to 1$ as $s \to \infty$ for any two pairs of hadrons h_1, h_2 and $\tilde{h}_1$, $\tilde{h}_2$. We note that in the additive quark model this ratio depends on the total number of valence quarks in the systems (h_1,h_2) and $(\tilde{h}_1, \tilde{h}_2)$. The expression for the cross section $\sigma_{\rm inel}^{h_1h_2}(s)$ of inelastic interactions has the form

$$\sigma_{\rm inel}^{h_1h_2}(s) = \frac{16\pi\lambda}{\mu^2(n_1+n_2)}\ln s + \frac{16\pi \ln C_{h_1h_2}}{\mu^2(n_1+n_2)^2}. \tag{49}$$

In contrast to the total cross sections, the asymptotic term is here not universal, the coefficient in front of $\ln s$ in the expression for $\sigma_{\rm inel}^{h_1h_2}(s)$ depending on the sum $(n_1 + n_2)$.

Using the results of Sec. 2, we can readily show that for scattering with fixed momentum transfer when $s \to \infty$ and t/s is small the dependence of the scattering amplitude on the numbers n_1 and n_2 of valence quarks will be determined by the expression

$$F_{h_1h_2}(s, t) = s\sum_{m=1}^{\infty} \tau^m(\sqrt{-t})\, \Phi_m(R_{h_1h_2}(s), \sqrt{-t}), \tag{50}$$

where

$$\tau(\sqrt{-t}) = \exp\left[-2\pi\sqrt{-t}/\mu(n_1+n_2)\right].$$

The function $\Phi_m(R_{h_1h_2}(s), \sqrt{-t})$ is obtained from the expression (26) by replacing μ by $\mu(n_1 + n_2)$. At sufficiently large values of the momentum transfer, only one term need to be retained in the expansion (50). In this case, the scattering amplitude decreases in accordance with the Orear law, the slope parameter depending on the total number $(n_1 + n_2)$ of valence quarks.

We now consider large-angle scattering ($s \to \infty$, t/s fixed). The scattering amplitude in this kinematic region is determined by the region of small impact parameters $b \sim 0$. As was shown in Sec. 3, the existence of a singularity of the function $u(s, \beta)$ at $\beta = 0$ leads to a power-law decrease of the scattering amplitude.

Naturally, in the region of large angles the scattering amplitude depends strongly on the number $(n_1 + n_2)$ of valence quarks.

Using the expressions of Sec. 3, we obtain for the large-angle scattering amplitude

$$F_{h_1h_2}(s,\ t) = -\frac{is}{\pi^2}\frac{\mu(n_1+n_2)}{C_{h_1h_2}s^{\lambda(n_1+n_2)}|t|^{3/2}}|f_{h_1h_2}s,\ 0|^2. \quad (51)$$

For the differential scattering cross section in the region of large angles we obtain

$$\frac{d\sigma_{h_1h_2}}{dt} = \frac{32\pi\mu^2(n_1+n_2)^2}{C^2_{h_1h_2}}\left(\frac{1}{s}\right)^{2\lambda(n_1+n_2)+3} \times |f_{h_1h_2}(s,\ 0)|^4(1-\cos\theta)^{-3}. \quad (52)$$

The phenomenological consequences of the resulting expressions will be discussed in the second part of the review.

In this section, we consider a way of taking into account the composite structure of the hadrons in the choice of the kernel of the one-time equation. The interaction dynamics of the constituent quarks is taken into account by the function $U(s,t)$. In particular, the composition law (43) for the kernel of the one-time equation is determined by the interaction at the quark level in the form of the quasi-independent scattering of the valence quarks by the field $V_{\rm eff}$ produced by the virtual clouds. The interaction dynamics of the hadrons h_1 and h_2 is determined by the equation $F = F[U]$. As we shall show, the obtained dependence on the number of valence constituents has nontrivial and experimentally verifiable consequences.

The considered way of constructing an expression for the U matrix can be used for the kernel of any dynamical equation and, in particular, in the choice of the interaction quasipotential $V(s,t)$.

Finally, we note once more that in choosing the quark scattering amplitude $f_i(s,b)$ we assumed that the range of their interaction with the field $V_{\rm eff}$ does not depend on the quark species. A consequence of this, in particular, was the universality of the doubly logarithmic term in the expression for the total cross sections. An alternative is to introduce for each quark species a corresponding range $1/\mu_i$ and to identify the parameters μ_i with the masses m_i of the corresponding quarks. Then in the expressions of Secs. 2 and 3 it is necessary to make the substitutions $\mu \to M$ and $\lambda \to \lambda N$, where

$$M \equiv \sum_i m_i n_i;\quad N \equiv n_1+n_2 = \sum_i n_i$$

and n_i is the total number of valence quarks of species i in the hadrons h_1 and h_2.

5. ANALYSIS OF THE BEHAVIOR OF THE CROSS SECTIONS. EXPERIMENTAL CONSEQUENCES

Smooth behavior of the angular distributions of *pp* scattering in the region beyond the second diffraction peak

It is well known that the absence of diffraction dips in the angular distribution of elastic *pp* scattering in the region of squares of the momentum transfer from 2 to 10 $({\rm GeV}/c)^2$ was a difficult point for numerous models of high-energy scattering.[17] In the present section, we shall consider this question from the point of view of the representation of the amplitude in a series in the parameter $\tau(\sqrt{-t})$, which is valid in the region of fixed t (t/s small).

Suppose t is sufficiently large for us to be able to retain in the expansion (25) only the leading term in $\tau(\sqrt{-t})$. Then in the case of a purely imaginary generalized reaction matrix we obtain for the differential cross section the expression

$$\frac{d\sigma}{dt} \cong \frac{32\pi^2}{\mu^2}\frac{R(s)}{\sqrt{-t}}e^{-\frac{2\pi}{\mu}\sqrt{-t}}\cos^2\left(R(s)\sqrt{-t}+\frac{\pi}{4}\right), \quad (53)$$

which has an oscillating nature.

However, if the generalized reaction matrix is not purely imaginary and has a phase $\alpha(s)$ different from $\pi/2$, then, using the expression (27), we obtain for $d\sigma/dt$ an Orear behavior without oscillating factors:

$$\frac{d\sigma}{dt} \cong \frac{8\pi^2}{\mu^2}\frac{R(s)}{\sqrt{-t}}e^{-b(s)\sqrt{-t}}, \quad (54)$$

where

$$b(s) = \frac{2\pi}{\mu}\left[1 \pm \left(\frac{\alpha(s)}{2\pi} - 1\right)\right], \quad (55)$$

the plus sign being taken in the case when $\alpha(s) < \pi/2$ and the minus when $\alpha(s) > \pi/2$. The expression (54) leads to a smooth behavior, without alternation of maxima and minima, of the angular distributions in the region beyond the second diffraction peak and agrees well with the experimental data (Fig. 2).[3,18] The energy dependence of the slope parameter $b(s)$ is determined by the energy dependence of the phase $\alpha(s)$. Note that the experimental data indicate that the energy dependence of the parameter $b(s)$ at ISR energies is very weak.

Thus, allowance for the phase of the function $U(s,t)$ makes it possible to explain in the framework of the considered approach the absence of a second minimum in the angular distributions of elastic *pp* scattering. Of course, there could be other explanations for the absence of a second minimum at ISR energies associated, for example, with spin degrees of freedom.[19]

Relationship between the exponents of decrease of the elastic and inclusive cross sections in the region of large-angle scattering

In this section, we shall use the method presented in the first part to analyze the behavior of the inclusive spectra.

We note that the quark counting rules[13] and calculations based on the model of hard collisions[20] for inclusive processes $a + b \to c + \ldots$ at large $p_\perp$ lead to the law of decrease $E(d\sigma/d\mathbf{p}_c) \sim p_\perp^{-4}$ of the cross section. Such behavior is actually due to allowance for the simplest subprocesses at the level of the constituent quarks.

However, decrease of the cross sections in accordance with the law $p_\perp^{-4}$ is in disagreement with the existing data.[21] In a number of papers,[22] this disagreement was attributed to screening of the behavior $p_\perp^{-4}$ in the pre-asymptotic region by various particle production mechanisms. Logarithmic corrections were also calculated in the framework of QCD.[23]

We shall obtain a simple relation between the exponent of decrease $N_{\rm incl}$ of the inclusive cross section and the exponent of decrease $N_{\rm el}$ of the elastic cross section. It follows from the obtained relation, which is in agreement with the experimental data, that $N_{\rm incl}$ is appreciably larger than the value predicted by the behavior $p_\perp^{-4}$.

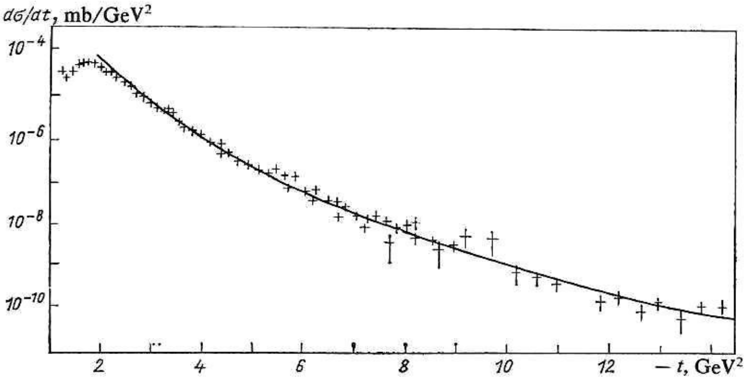

FIG. 2. Differential cross section for elastic pp scattering. The experimental points correspond to $p = 400$ GeV/c (FNAL) and $\sqrt{s} = 52.8$ GeV, $\sqrt{s} = 62$ GeV (ISR).

For the inclusive cross section, we shall use a simple relation connecting it to the contribution of the inelastic channels to the unitary relation for the functions $\eta(s,b)$:

$$\frac{d\sigma_{\text{incl}}}{dq^2} = \left[\int_0^\infty \sqrt{\eta(s,b)}\, J_0(b\sqrt{q^2})\, b\, db\right]^2, \quad q^2 = -t. \tag{56}$$

Equation (56) was obtained in Ref. 24, which considered the relations between the characteristics of elastic scattering and the single-particle inclusive spectrum. The relation (56) makes it possible to calculate the inclusive cross section if the elastic scattering amplitude is known. We shall use it to obtain the cross section $d\sigma_{\text{incl}}/dq^2$ at large momentum transfers ($p_\perp$, $q \sim \sqrt{s}$). Note that the exponents of the decrease of the cross section $E\,(d\sigma/d\mathbf{p}_c)$ with respect to the variable s (or $p_\perp$) and of an integrated inclusive cross section, such as $d\sigma_{\text{incl}}/dq^2$, are equal in this kinematic region.

To calculate the integral (56), we use the method developed in Secs. 2 and 3. The contribution of the inelastic channels to the unitarity relation can be expressed in terms of the function $u(s,\beta)$ as follows:

$$\eta(s,\beta) = \frac{\operatorname{Im} u(s,\beta)}{|1 - iu(s,\beta)|^2}. \tag{57}$$

For simplicity, we shall assume that the generalized reaction matrix is purely imaginary. Allowance for the phase is unimportant in the case. Going over to a contour integral, we obtain for $d\sigma_{\text{incl}}/dq^2$

$$\sqrt{\frac{d\sigma_{\text{incl}}}{dq^2}} = -\frac{i}{2\pi}\int_C \frac{u^{1/2}(s,\beta)}{1+u(s,\beta)} K_0(\sqrt{-\beta q^2})\, d\beta. \tag{58}$$

Using for the function $u(s,\beta)$ a general expression of the form (17), which takes into account correctly the analytic properties of the scattering amplitude, and making calculations like those in Sec. 3, we obtain for the cross section $d\sigma_{\text{incl}}/dq^2$

$$\frac{d\sigma_{\text{incl}}}{dq^2} \simeq \frac{\sin^2\frac{\pi\gamma}{2}}{\pi^3\mu^4 g(s)} \left(\frac{1}{1+\gamma/2}\right)^4 \left(\frac{\mu^2}{q^2}\right)^{2+\gamma} \frac{1}{\ln^\alpha \frac{q^2}{\mu^2}}\, \varphi^2\left(\ln^{-1}\frac{q^2}{\mu^2}\right),$$

$$\varphi(0) = 1. \tag{59}$$

The approximate expression can be represented in the form

$$\frac{d\sigma_{\text{incl}}}{dq^2} = \frac{1}{\pi\mu^4 g(s)} \left(\frac{\mu^2}{q^2}\right)^{N(q^2)} \varphi^2\left[\ln^{-1}\left(\frac{q^2}{\mu^2}\right)\right],$$

where

$$N(q^2) = 2 + \gamma + \alpha\frac{\ln\ln(q^2/\mu^2)}{\ln(q^2/\mu^2)}. \tag{60}$$

Thus, with allowance for the power-law growth of the function $g(s) = gs^\lambda$ we obtain for the inclusive cross section for production of particles with large transverse momenta ($\sim\sqrt{s}$) a power-law decrease of the form $(1/s)^{\lambda + N(q^2)}$. Comparing this with the behavior $d\sigma_{\text{el}}/dq^2 \sim (1/s)^{2\lambda - 2 + 2N(q^2)}$ of the elastic scattering cross section (see Sec. 3), we obtain the desired relation between the exponents:

$$N_{\text{incl}} = \frac{1}{2}N_{\text{el}} + 1. \tag{61}$$

It follows from (60) and (61) that $2 < N_{\text{incl}} < N_{\text{el}}$.

Allowance for the composite structure of hadrons in the construction of the generalized reaction matrix makes it possible to relate N_{el} to the number of valence quarks of the colliding hadrons (see Sec. 4). Using values for N_{el} that agree with the experimental data, we arrive at values for N_{incl} that also agree with the corresponding experimental values obtained by analyzing the inclusive cross sections, which even in the asymptotic region must decrease faster than $p_\perp^{-4}$.[25]

6. ALLOWANCE FOR THE EXCHANGE INTERACTION AND DESCRIPTION OF THE ANGULAR DEPENDENCE IN THE REGION OF LARGE-ANGLE SCATTERING

To describe the angular dependence of the large-angle scattering cross section in the region of angles near 90° and backward scattering, it is necessary to take into account the effects of the exchange interaction.

For this, we represent the kernel of the integral equation—the U matrix—in the form of the sum

$$U(s,t) = U_1(s,t) + U_2(s,u). \tag{62}$$

We represent the scattering amplitude determined by Eq. (2) in the form[2)]

$$F(s,t) = F_1(s,t) + F_2(s,u), \tag{63}$$

where the functions $F_1(s,t)$ and $F_2(s,u)$ are determined by the integral equations

$$F_1(\mathbf{p},\mathbf{q}) = U_1(\mathbf{p},\mathbf{q}) + i\frac{\pi}{8}\rho(s)\int d\Omega_{\hat{\mathbf{k}}}[U_1(\mathbf{p},\mathbf{k})F_1(\mathbf{k},\mathbf{q}) + U_2(\mathbf{p},\mathbf{k})F_2(\mathbf{k},\mathbf{q})];$$

$$F_2(\mathbf{p},\mathbf{q}) = U_2(\mathbf{p},\mathbf{q}) + i\frac{\pi}{8}\rho(s)\int d\Omega_{\hat{\mathbf{k}}}[U_1(\mathbf{p},\mathbf{k})F_2(\mathbf{k},\mathbf{q}) + U_2(\mathbf{p},\mathbf{k})F_1(\mathbf{k},\mathbf{q})]. \tag{64}$$

---

2) The functions $F_{1(2)}(s,t(u))$ are analytic in the topological product of the s and $t(u)$ planes with corresponding cuts.

The representation of the kernel of the dynamical equation as a sum corresponds to the assumption that there is a dynamical distinction between the processes of forward and backward scattering.

In partial waves, Eqs. (64) reduce to algebraic equations and can be solved. Going over to the impact-parameter representation, we finally obtain

$$F_1(s,t) = \frac{s}{\pi^2}\int_0^\infty b\,db\,\frac{u_1(s,b)[1-iu_1(s,b)]+iu_2^2(s,b)}{[1-iu_1(s,b)]^2+u_2^2(s,b)}J_0(b\sqrt{-t});$$

$$F_2(s,u) = \frac{s}{\pi^2}\int_0^\infty b\,db\,\frac{u_2(s,b)}{[1-iu_1(s,b)]^2+u_2^2(s,b)}J_0(b\sqrt{-u}). \quad (65)$$

Our assumption concerning the analytic properties of $F_1(s,t)$ and $F_2(s,u)$ with respect to the momentum transfer enables us to write down for these functions dispersion relations with respect to the variables t and u, respectively, which together with Eqs. (64) enable us to obtain for the functions $U_1(s,t)$ and $U_2(s,u)$ the spectral representations $[s \geqslant (m_1+m_2)^2]$

$$U_{1(2)}(s,t(u)) = \int_{t_0(u_0)}^\infty \frac{\rho_{1(2)}(s,x)}{x-t(u)}dx,$$

from which we obtain

$$u_{1(2)}(s,\beta) = \int_{\mu_{1(2)}^2}^\infty \rho_{1(2)}(s,x)K_0(\sqrt{x\beta})\,dx. \quad (66)$$

The representation (66) enables us to obtain for the functions $u_{1(2)}(s,\beta)$ the expressions

$$u_{1(2)}(s,\beta) = g_{1(2)}(s,\beta)\exp(-\mu_{1(2)}\sqrt{\beta}), \quad (67)$$

where $u_{1(2)} = \sqrt{t_0(u_0)}$.

The parameters μ_1 and μ_2 are related to the masses of the intermediate states in the t and u channels. For πN scattering, the smallest masses of the intermediate states are, respectively, $2m_\pi$ and $m_\pi + m_N$, and we shall therefore assume that $\mu_1 < \mu_2$ and consider in what follows nonidentical particles. For identical particles, allowance for the exchange interaction reduces to symmetrization of the amplitude with respect to the variables t and u. The actual form of the dependence of $g(s,\beta)$ on β at $\beta \sim 0$ is determined by the form of the spectral functions $\rho_{1(2)}(s,x)$. We shall discuss various possibilities.

First, we consider the simplest case, when

$$u_{1(2)}(s,\beta) = ig_{1(2)}(s)\exp(-\mu_{1(2)}\sqrt{\beta}). \quad (68)$$

As we have already noted, growth of the total cross section requires growth of the function $g_1(s)$ as $s\to\infty$: $g_1(s)\sim s^{\lambda_1}$. For $g_2(s)$, we assume the same form of the energy dependence: $g_2(s)\sim s^{\lambda_2}$. Experimental data on backward scattering indicate that $d\sigma/du|_{u=0}$ decreases with increasing energy in a power-law fashion.[22] It is easy to show that for this fulfillment of the inequality $\lambda_2<\lambda_1$ is required.

Therefore, bearing in mind the smallness of the exchange interaction, for the functions $F_1(s,t)$ and $F_2(s,u)$ we can write

$$F_1(s,t) = \frac{s}{\pi^2}\int_0^\infty b\,db\,\frac{u_1(s,b)}{1-iu_1(s,b)}J_0(b\sqrt{-t});$$

$$F_2(s,u) = \frac{s}{\pi^2}\int_0^\infty b\,db\,\frac{u_2(s,b)}{[1-iu_1(s,b)]^2}J_0(b\sqrt{-u}). \quad (69)$$

The integrals in (69) can be calculated in accordance with the scheme set forth in Secs. 2 and 3. As a result, for the contributions of the poles in the plane of the impact parameter to the scattering amplitude we have, respectively,

$$F_{1p}(s,t) = s\sum_{m=1}^\infty\left[\exp\left(-\frac{2\pi}{\mu_1}\sqrt{-t}\right)\right]^m\Phi_m^{(1)}(R_1(s),\sqrt{-t});$$

$$F_{2p}(s,t) = s\exp[-\mu_2[R_1(s)-R_2(s)]]\times\sum_{m=1}^\infty\left[\exp\left(-\frac{2\pi}{\mu_1}\sqrt{-u}\right)\right]^m\Phi_m^{(2)}(R_1(s),\sqrt{-t});$$

$$\Phi_m^{(1)}(R_1(s),\sqrt{-t}) = \frac{1}{\pi\mu_1}\left[\frac{2R_1(s)}{\pi\sqrt{-t}}\right]^{1/2}\times\left\{\left[-i+(m-1/2)\frac{2\pi}{\mu_1R_1(s)}\right]^{1/2}\exp\left[\left(iR_1(s)+\frac{\pi}{\mu_1}\right)\sqrt{-t}\right]\right.$$
$$\left.\left[i+(m-1/2)\frac{2\pi}{\mu_1R_1(s)}\right]^{1/2}\exp\left[\left(-iR_1(s)+\frac{\pi}{\mu_1}\right)\sqrt{-t}\right]\right\};$$

$$\Phi_m^{(2)}(R_1(s),\sqrt{-u}) = \frac{i}{\pi\mu_1}\left[\frac{2R_1(s)\sqrt{-u}}{\pi}\right]^{1/2}\times\left\{\exp\left[i\pi\frac{\mu_2}{\mu_1}(2m-1)+\sqrt{-u}\left(iR_1(s)+\frac{\pi}{\mu_1}\right)\right]\right.\times\sqrt{-i+(m-1/2)\frac{2\pi}{\mu_1R_1(s)}}$$
$$\left.+\exp\left[-i\pi\frac{\mu_2}{\mu_1}(2m-1)+\sqrt{-u}\left(-iR_1(s)+\frac{\pi}{\mu_1}\right)\right]\times\sqrt{i+(m-1/2)\frac{2\pi}{\mu_1R_1(s)}}\right\}. \quad (70)$$

Thus, the pole part of the scattering amplitude can be represented as a sum of two series in the parameters $\tau(\sqrt{-t})$ and $\tau(\sqrt{-u})$, respectively, which decrease exponentially with increasing $\sqrt{-t}$ and $\sqrt{-u}$. The contribution of the exchange interaction decreases with the energy in accordance with a power law.

It follows from the representation (66) that the amplitudes $f_{1(2)}(s,\beta)$ have in the β plane a cut $\beta\in[0,-\infty)$, its contribution to the large-angle scattering amplitude $F_{1(2)}(s,t)$ being decisive. The corresponding integrals have the form

$$F_{1c}(s,t) = \frac{s}{\pi^3}\int_{-\infty}^0 d\beta K_0(\sqrt{t\beta})\,\mathrm{disc}\,f_1(s,\beta);$$

$$F_{2c}(s,u) = \frac{s}{\pi^3}\int_{-\infty}^0 d\beta K_0(\sqrt{u\beta})\,\mathrm{disc}\,f_2(s,\beta). \quad (71)$$

Calculating the discontinuities of the amplitudes $f_1(s,\beta)$ and $f_2(s,\beta)$ across the cut, we have

$$\mathrm{disc}\,f_1(s,\beta) = -\frac{-i\sin\mu_1\sqrt{|\beta|}}{g_1(s)}+O\left(\frac{1}{g_1^2(s)}\right);$$

$$\mathrm{disc}\,f_2(s,\beta) = \frac{ig_2(s)\sin(2\mu_1-\mu_2)\sqrt{|\beta|}}{g_1^2(s)}+O\left(\frac{g_2(s)}{g_1^3(s)}\right). \quad (72)$$

For the contribution of the cut $\beta\in[0,-\infty)$ to the amplitude we find

$$F_{1c}(s,t) = \frac{-is\mu_1}{\pi^2 g_1(s)}\frac{1}{|t|^{3/2}}+O\left(\frac{1}{g_1^2(s)}\right);$$

$$F_{2c}(s,t) = \frac{-isg_2(s)(\mu_2-2\mu_1)}{\pi^2 g_1^2(s)}\frac{1}{|u|^{3/2}}+O\left(\frac{g_2(s)}{g_1^3(s)}\right). \quad (73)$$

Comparing the expressions $F_{2p}(s,u)$ and $F_{2c}(s,u)$, we see that when we consider scattering into the backward hemisphere (fixed u) it is necessary to take into account the contribution from not only the poles but also the cuts. The pole contribution need not be dominant if the ratio μ_2/μ_1 is sufficiently large: $\mu_2/\mu_1 > 2$. We note that for πN scattering precisely this case is realized.

In contrast, for forward scattering in the region of fixed t the pole contribution is dominant, while the cut contribution $F_{1c}(s,t)$ is suppressed with respect to the energy as a power.

For scattering in the region of large angles, the differential cross section has the form

$$\frac{d\sigma}{dt} \simeq \frac{32\pi\mu^2}{g^2}\left(\frac{1}{s}\right)^{\lambda_1+3}\left\{(1-\cos\theta)^{-3/2} + \left[\frac{g_2(s)}{\mu_1 g_1(s)}(\mu_2-2\mu_1)+O\left(\frac{g_2(s)}{g_1^2(s)}\right)\right](1+\cos\theta)^{-3/2}\right\}. \tag{74}$$

The expression (74) makes it possible to describe the angular distribution of the large-angle scattering cross sections. Figures 3 and 4 show the results of such description for the case of $\pi^{\pm}p$ scattering. The overall normalization and the coefficient in the square brackets were assumed to be free parameters. The agreement obtained with the experimental data is fairly good.

Consideration of more general expressions for $u_{1(2)}(s,\beta)$ does not change the main results. In the general case,

$$u_{1(2)}(s,\beta) = i g_{1(2)}(s)\,(\mu_{1(2)}^2\beta)^{-\gamma_{1(2)}} \times \ln^{\alpha_{1(2)}}(\mu_{1(2)}^2\beta)\exp(-\mu_{1(2)}\sqrt{\beta}). \tag{75}$$

Then the differential cross section in the region of large angles has the form

$$\frac{d\sigma}{dt} \simeq \frac{1}{\pi\mu^4}\frac{1}{g_1^2(s)}\left[\frac{1}{(1+\gamma_1)^2}\left(\frac{\mu^2}{|t|}\right)^{1+\gamma_1}\frac{1}{\ln^{\alpha_1}\frac{|t|}{\mu_1^2}}\varphi_1\left(\ln^{-1}\frac{|t|}{\mu_1^2}\right) + (-1)^{\alpha_1-\alpha_2}\frac{g_2(s)}{g_1(s)}\left(\frac{\mu_1^2}{\mu_2^2}\right)^{2\gamma_1+1}\frac{1}{(1+2\gamma_1-\gamma_2)^2}\left(\frac{\mu_2^2}{|u|}\right)^{2\gamma_1-\gamma_2+1} \times \frac{1}{\ln^{2\alpha_1-\alpha_2}\frac{|u|}{\mu_2^2}}\varphi_2\left(\ln^{-1}\frac{|u|}{\mu_2^2}\right)\right]^2, \quad \varphi_{1(2)}(0)=1, \tag{76}$$

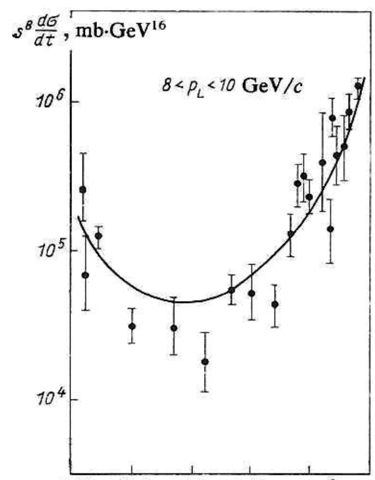

FIG. 3. Angular dependence of elastic π^+p scattering.

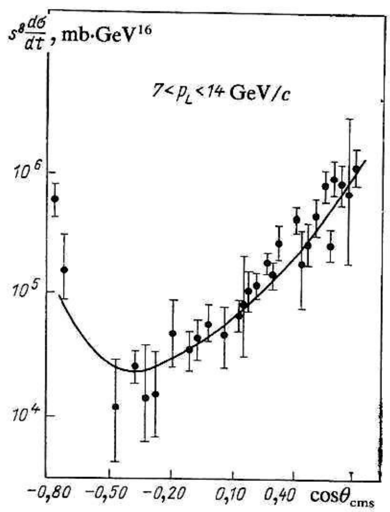

FIG. 4. Angular dependence of elastic π^-p scattering.

and has the same form as the expression for the cross section obtained in QCD.[14]

To conclude this section, we note that the function $F_{2p}(s,u)$, which determines the behavior of the scattering amplitude in the region of angles near 180°, contains, compared with the function $F_{1p}(s,t)$, an additional small factor $\exp\{-\mu_2[R_1(s)-R_2(s)]\}$, which has a clear geometrical meaning, namely, the poles in the plane of the impact parameter are situated near values $b \sim R_1(s)$, whereas the region of impact parameters associated with backward scattering has dimensions $b \lesssim R_2(s)$. The ranges of the direct and exchange interactions control the scattering in the region of small t and u, respectively.[26] In contrast, in the region of fixed scattering angles, in which the region of impact parameters $b \sim 0$ is important, the relative contribution of the direct and exchange interactions is determined, not by the ranges $R_i(s)$, but by the ratio of the intensities of these interactions[27]: $g_2(s)/g_1(s)$.

7. SPIN EFFECTS IN ELASTIC SCATTERING

Polarization effects in elastic scattering

In this section, we shall consider the application of the method to the analysis of spin effects in elastic scattering.

Data on measurement of the spin-correlation parameter A_{nn} in elastic pp scattering, and also a number of other experiments, lead to the conclusion that it is important to take into account the spin in a study of hadron processes in the region of large momentum transfers and large scattering angles.

We consider first the behavior of the polarization parameter in large-angle elastic scattering. Calculations made in the framework of QCD and based on perturbation theory with respect to the coupling constant $\alpha_s(Q^2)$ predict a polarization parameter in large-angle scattering processes equal to zero, as a consequence of conservation of the s-channel helicity in a gauge theory with vector gluons.[28] The polarization parameter is also predicted to be zero in the quasipotential approach when the requirement of γ_5 invariance is imposed.[29]

Bearing in mind the ambiguity of the conclusions that can be drawn from the analysis of experimental data, there is undoubted interest in approaches that predict a nonzero value of the polarization in the region of large scattering angles.[30]

To analyze the behavior of the polarization parameter, we use the method developed in the first part of the review. For simplicity, we shall consider the scattering of a spinless particle by a spin-$\frac{1}{2}$ particle, and we shall ignore the exchange interaction. Allowance for it and consideration of the scattering of two spinor particles do not influence the results of the present section.

In the considered case, there are two independent amplitudes—the amplitude $F_+(s,t)$ without change in the helicity and the amplitude $F_-(s,t)$ with one:

$$F_\pm(s,t)=\frac{s}{2\pi^2}\int_0^\infty d\beta f_\pm(s,\beta)J_{\substack{0\\1}}(\sqrt{-\beta t}),\quad \beta=b^2, \tag{77}$$

these being related to the generalized matrix by the relations[39]

$$\begin{aligned} f_+(s,\beta)&=\frac{u_+(s,\beta)[1-iu_+(s,\beta)]-i[u_-(s,\beta)]^2}{[1-iu_+(s,\beta)]^2-[u_-(s,\beta)]^2},\\ f_-(s,\beta)&=\frac{u_-(s,\beta)}{[1-iu_+(s,\beta)]^2-[u_-(s,\beta)]^2}, \end{aligned} \tag{78}$$

where

$$u_\pm(s,\beta)=\frac{\pi^2}{s}\int_0^\infty \sqrt{-t}\,d\sqrt{-t}\,U_\pm(s,t)J_{\substack{0\\1}}(\sqrt{-\beta t}).$$

From the representation (78), we obtain the following equation for finding the poles in the complex β plane:

$$[1-iu_+(s,\beta)]^2-[u_-(s,\beta)]^2=0. \tag{79}$$

For the functions $u_\pm(s,\beta)$, we shall use the expression

$$u_\pm(s,\beta)=ig_\pm(s,\beta)\,e^{-\mu\sqrt{\beta}}, \tag{80}$$

which correctly takes into account the analytic properties of the scattering amplitude. Here, $g_\pm(s,\beta)$ are certain functions of β that vary slowly compared with the exponential function. The parameter μ is taken to be the same for the functions $u_\pm(s,\beta)$, and the factor i is separated for convenience. Using the relations

$$J_1(z)=\frac{z}{2}[J_0(z)+J_2(z)];$$

$$J_0(\sqrt{|z|})=\frac{i}{\pi}[K_0(\sqrt{-|z|+i0})-K_0(\sqrt{-|z|-i0})];$$

$$J_2(\sqrt{|z|})=-\frac{i}{\pi}[K_2(\sqrt{-|z|+i0})-K_2(\sqrt{-|z|-i0})], \tag{81}$$

we obtain for the amplitudes $F_\pm(s,t)$

$$F_+(s,t)=-\frac{is}{2\pi^3}\int_C d\beta f_+(s,\beta)K_0(\sqrt{t\beta}),\quad t<0;$$

$$F_-(s,t)=-\frac{is\sqrt{-t}}{4\pi^3}\Big\{\int_C d\beta\sqrt{\beta}f_-(s,\beta)\times[K_0(\sqrt{t\beta})-K_2(\sqrt{t\beta})]\Big\}. \tag{82}$$

The contour of integration C is shown in Fig. 1. The method of calculating the integrals in (82) is similar to that already described in Secs. 2 and 3. As a result, for the pole contribution to the amplitude we obtain the expansions

$$F_{\pm,p}(s,t)=s\sum_{m=1}^\infty[\tau(\sqrt{-t})]^m\{\Phi_m(G_+(s),\sqrt{-t})\pm\Phi_m(G_-(s),\sqrt{-t})\},$$

where $G_\pm(s)\equiv(1/\mu)\ln[g_+(s)\pm ig_-(s)]=R_\pm(s)+iI_\pm(s)$ and the functions $\Phi_m(G_\pm(s),\sqrt{-t})$ have the form

$$\begin{aligned}\Phi_m(G_\pm(s),\sqrt{-t})&=\frac{1}{2\pi\mu}\Big[\frac{2R_\pm(s)}{\pi\sqrt{-t}}\Big]^{1/2}\\ &\times\Big\{\Big[-i+\frac{1}{R_\pm(s)}\Big(I_\pm(s)+\frac{\pi(2m-1)}{\mu}\Big)\Big]^{1/2}\\ &\times\exp\Big[\Big(iR_\pm(s)-I_\pm(s)+\frac{\pi}{\mu}\Big)\sqrt{-t}\Big]\\ &-\Big[i+\frac{1}{R_\pm(s)}\Big(-I_\pm(s)+\frac{\pi(2m-1)}{\mu}\Big)\Big]^{1/2}\\ &\times\exp\Big[\Big(-iR_\pm(s)+I_\pm(s)+\frac{\pi}{\mu}\Big)\sqrt{-t}\Big]\Big\}.\end{aligned} \tag{83}$$

For fixed values of the momentum transfer, the contribution $F_{\pm,p}(s,t)$ is dominant.

As we have already noted, the function $u(s,\beta)$ has in the β plane the cut $\beta\in[0,-\infty)$. The presence of the singularity at the point $\beta=0$ agrees with the idea that the internal structure of the particles must be manifested in the interaction at short distances.

The cut contribution to the amplitude is determined by the integrals

$$\begin{aligned}F_{+,c}(s,t)&=\frac{s}{\pi^3}\int_{-\infty}^0 d\beta\,\mathrm{disc}\,f_+(s,\beta)K_0(\sqrt{t\beta});\\ F_{-,c}(s,t)&=-\frac{s\sqrt{-t}}{\pi^3}\int_{-\infty}^0 d\beta\,\mathrm{disc}(\sqrt{\beta}f_-(s,\beta))\frac{K_1(\sqrt{t\beta})}{\sqrt{t\beta}}.\end{aligned} \tag{84}$$

Making the calculations, we obtain for the amplitude of scattering through fixed angles

$$\begin{aligned}F_+(s,t)&=-\frac{is\mu}{\pi^2}\frac{g_+(s)}{g_+^2(s)+g_-^2(s)}\frac{1}{(\mu^2-t)^{3/2}};\\ F_-(s,t)&=-\frac{is}{\pi^2}\frac{g_-(s)}{g_+^2(s)+g_-^2(s)}\frac{\sqrt{-t}}{(\mu^2-t)^{3/2}}.\end{aligned} \tag{85}$$

For simplicity, we have here assumed that the functions $g_\pm(s,\beta)$ do not depend on β.

For the cross section of scattering through large angles we have

$$\frac{d\sigma}{dt}=\frac{4\pi}{|g_+^2(s)+g_-^2(s)|^2}\Big\{\frac{\mu^2|g_+(s)|^2}{|t|^3}+\frac{|g_-(s)|^2}{|t|^2}\Big\}. \tag{86}$$

It follows from the expression (86) that scattering in which the helicity changes makes an important contribution to the large-angle differential scattering cross section and leads to a weaker angular dependence than scattering without change in the helicity.

In the more general case when $g_\pm(s,\beta)=g_\pm(s)(\mu^2\beta)^{-\nu_\pm}\ln^{\alpha_\pm}(\mu^2\beta)$, we obtain for the cut contributions

$$\begin{aligned}F_{+,c}(s,t)&=\frac{is(-1)^{\alpha_+}}{2\pi^3\mu^2}\frac{g_+(s)}{g_+^2(s)+g_-^2(s)}\frac{1}{(1+\gamma_+)^2}\\ &\times\Big(\frac{\mu^2}{|t|}\Big)^{1+\gamma_+}\frac{1}{\Big(\ln\frac{|t|}{\mu^2}\Big)^{\alpha_+}}\varphi_+\Big(\ln^{-1}\frac{|t|}{\mu^2}\Big),\\ F_{-,c}(s,t)&=\frac{is(-1)^{\alpha_-}}{\pi^3\mu^2}\frac{g_-(s)}{g_+^2(s)+g_-^2(s)}\frac{1}{\gamma_-}\\ &\times\Big(\frac{\mu^2}{|t|}\Big)^{\gamma_-+1/2}\frac{1}{\Big(\ln\frac{|t|}{\mu^2}\Big)^{\alpha_-}}\varphi_-\Big(\ln^{-1}\frac{|t|}{\mu^2}\Big);\end{aligned} \tag{87}$$

where $\varphi_{\pm}(0) = 1$.

At the same time, it follows from (87) that the asymptotic form of the angular dependence is $(1-\cos\theta)^{-4+\delta}$, $\delta > 0$.

As we noted, the functions $g_{\pm}(s)$ must increase as powers as $s \to \infty$. We assume that $g_+(s) = (C_+/\sqrt{2})s^{\lambda_+}$, $g_-(s) = C_- e^{-i\varphi(s)s_\lambda}$. Here, the function $\varphi(s)$ is the relative phase, which can be related to the behavior of the polarization parameter at small values of the momentum transfer. Taking into account the power-law behavior of the functions $g_{\pm}(s)$, we obtain for the polarization parameter the expression

$$P(s, z)|_{\substack{s,\, t\to\infty \\ t/s-\text{fixed}}} = -2\sin\varphi(s)\Big[\frac{C_-}{\mu C_+} s^{\lambda_- - \lambda_+ + \frac{1}{2}}(1-z)^{1/2} + \frac{\mu C_+}{C_-} s^{\lambda_+ - \lambda_- - \frac{1}{2}}(1-z)^{-1/2}\Big]^{-1}, \tag{88}$$

$$z = \cos\theta.$$

It follows from the expression (88) that the polarization parameter $P(s,z)$ decreases at fixed z as a power, provided $\lambda_+ \neq \lambda_- + \frac{1}{2}$. The case when $\lambda_+ = \lambda_- + \frac{1}{2}$ has the greatest interest, since the polarization in the region of large-angle scattering does not vanish as $s \to \infty$. Then between the intensities $g_+(s)$ and $g_-(s)$ the relation $g_-(s) \simeq (1/\sqrt{s})\, g_+(s)$ holds. It also follows from (88) that the dependence of the polarization on the scattering angle is weak. The use of the more general expressions (87) leads to the following expression for the polarization parameter:

$$P(s, t) = \frac{\operatorname{Im} g_-^*(s)}{2g_+(s)}\Big[\chi\Big(\frac{|t|}{\mu^2}\Big) + \frac{|g_-(s)|^2}{g_+^2(s)}\chi^{-1}\Big(\frac{|t|}{\mu^2}\Big)\Big]^{-1},$$

where

$$\chi\Big(\frac{|t|}{\mu^2}\Big) = \frac{\gamma_-(-1)^{\Delta_\alpha}}{2(1+\gamma_+)^2}\Big(\frac{\mu^2}{|t|}\Big)^{\Delta_\gamma}\frac{1}{\Big(\ln\frac{|t|}{\mu^2}\Big)^{\Delta_\alpha}}\Big[\frac{\varphi_+(\ln^{-1}|t|/\mu^2)}{\varphi_-(\ln^{-1}|t|/\mu^2)}\Big]$$

and $\Delta_\nu \equiv \gamma_+ - \gamma_- + 1/2$, $\Delta_\alpha \equiv \alpha_+ - \alpha_-$. It is natural that the introduction of the additional singularity in the functions $g_+(s,\beta)$ and $g_-(s,\beta)$ changes the power of decrease of the large-angle scattering cross section and leads to the appearance of additional logarithmic factors in the expressions for the cross section and the polarization parameter.

Thus, the helicity-changing interaction makes an important contribution to the large-angle scattering cross section. The polarization parameter in this kinematic region is nonzero and does not decrease as $s \to \infty$ if $g_-(s) \simeq s^{-1/2} g_+(s)$. The polarization parameter has a weak angular dependence. Note that allowance for the exchange interaction does not affect these conclusions about the behavior of the polarization parameter. We have considered the scattered of particles with spins 0 and $\frac{1}{2}$. For the case of the scattering of two spin-$\frac{1}{2}$ particles, all the main results obtained in the present section remain unchanged.

Energy dependence of the spin-correlation parameters in large-angle *pp* scattering

Study of large-angle scattering processes is important from the point of view of manifestation of the interaction dynamics of the hadron constituents. The spin properties of the hadron–hadron scattering amplitude are sensitive to this dynamics. For example, variants of quark and QCD models using perturbation theory agree with the main results on the elastic and inclusive distributions at large angles but encounter difficulties in attempting to reconcile the polarization experiments in this region,[32] which, thus, reveal details of the quark models that appear to be unimportant from the point of view of describing the angular distributions.

In the present section, we consider the application of the method we have developed for calculating the amplitudes to the analysis of the behavior of the spin-correlation parameters in elastic *pp* scattering. The interest in this problem is due to the experimental discovery of a large value of the parameter A_{nn} for 90° scattering. It has been found[31] that in the interval of energies from 6 to 12 GeV the parameter A_{nn} increases its value reaching 0.59 ± 0.09. The spin-correlation parameter A_{nn} is directly related to the ratio of the scattering cross sections with parallel, σ_p, and antiparallel, σ_a, spins:

$$\frac{\sigma_p}{\sigma_a} \equiv \frac{\sigma_{\uparrow\uparrow}+\sigma_{\downarrow\downarrow}}{\sigma_{\uparrow\downarrow}+\sigma_{\downarrow\uparrow}} = \frac{1+A_{nn}}{1-A_{nn}}.$$

It follows from the value given above for the parameter $A_{nn}(\pi/2)$ that the cross section for scattering with parallel spins exceeds the antiparallel cross section by four times. The large value of this ratio and its rapid growth with the energy on the transition from 8 GeV to 12 GeV do not agree with the predictions of the majority of models of large-angle scattering.[32,33]

Thus, a simple model taking into account quark exchange[32] leads to a constant value of this ratio equal to 2, so that to obtain the value 4 one must invoke further arguments that go beyond the scope of perturbation theory in QCD and are based, for example, on allowance for instanton effects or quark-confinement effects.[32] Despite the introduction of additional assumptions, these models do not explain the growth of the ratio σ_p/σ_a with the energy, though they do make it possible to obtain a value of σ_p/σ_a in the interval from 3 to 4. In the quark–parton model,[33] the ratio is predicted to be 1.25. The studies of Refs. 32 and 33 are devoted to explanation of the angular dependence of the parameter $A_{nn}(\theta)$ on the transition to the value $\theta = 90°$ and for a fixed value of the energy equal to 11.75 GeV. The given dependence is the result of comparison with the data of a function that depends only on $\cos\theta$.

In the present section, we discuss the energy dependence of the parameter $A_{nn}(s, \cos\theta)$ found in the region of ZGS energies. Expressions are obtained from which there follows the possibility of an oscillating (with respect to s) behavior of the spin-correlation parameters $A_{nn}(90°)$ and $A_{ll}(90°)$. This makes it possible to explain the growth of the parameter A_{nn} in the region of energies and to reconcile the existing data. Asymptotically, $\sigma_p/\sigma_a = 2$, i.e., the value is predicted to be the same as in the quark model. The angular dependence of the spin-correlation parameters for fixed s also agrees well with the data.

The process of elastic nucleon–nucleon scattering can be described by means of five independent helicity amplitudes $F_i(s,t)$ ($i = 1,...,5$): two amplitudes $F_{1,3}(s,t)$ without change in helicity, one amplitude $F_5(s,t)$ with one change, and two amplitudes $F_{2,4}(s,t)$ with a double change. For $F_i(s,t)$, we use a one-time dynamical equation relating the amplitude to the U matrix; in the case of the scattering of two spin-$\frac{1}{2}$ particles, this equation has the following form in the helicity basis and the center-of-mass system:

$$F_{\lambda_3\lambda_4\lambda_1\lambda_2}(\mathbf{p},\ \mathbf{q}) = U_{\lambda_3\lambda_4\lambda_1\lambda_2}(\mathbf{p},\ \mathbf{q}) + i\frac{\pi}{8}\rho(s)\sum_{\nu_1\nu_2}\int d\Omega_{\hat{\mathbf{k}}}\,U_{\lambda_3\lambda_4\nu_1\nu_2}(\mathbf{p},\mathbf{k})\,F_{\nu_1\nu_2\lambda_1\lambda_2}(\mathbf{k},\mathbf{q}), \quad (89)$$

where

$$F_1 = F_{1/2\ 1/2\ 1/2\ 1/2}; \quad F_2 \equiv F_{-1/2\ -1/2\ 1/2\ 1/2};$$

$$F_3 \equiv F_{1/2\ -1/2\ 1/2\ -1/2}; \quad F_4 \equiv F_{1/2\ -1/2\ -1/2\ 1/2};$$

$$F_5 \equiv F_{1/2\ 1/2\ 1/2\ -1/2}.$$

In constructing the expression for the generalized reaction matrix, we use the quark model with the factorizability assumption (see Sec. 4). For generalization to the case of the scattering of particles with spin, we use the fact that the hadron helicity λ_h is equal to the sum of the helicities of the valence constituents: $\lambda_h = \Sigma_{i=1}^{n_h} s_i$.

We do not assume conservation of the quark helicities in an interaction and, thus, we introduce two amplitudes f_+ and f_-. For the quark scattering amplitudes with a change in the helicity, $f_-(s,b)$, and without, $f_+(s,b)$, we use the expressions

$$f_\pm(s,\ b) = g_\pm(s)\exp[-\mu_\pm b + i\varphi_\pm(s)]. \quad (90)$$

The amplitudes $f_\pm$ are the amplitudes for scattering of a valence quark by some field $V_{\rm eff}$. Although we do not need to know the concrete form of the potential $V_{\rm eff}$, it is interesting to use an analogy with electron scattering in a central nuclear field, when the optical potential is chosen in the form[34]

$$V_0(r) + \frac{1}{r}\frac{dV_0(r)}{dr}(\sigma\mathbf{L}). \quad (91)$$

The allowance for relativistic effects is expressed, above all, in the energy dependence of the potential. In addition, a potential with different interaction ranges is more realistic, namely, with spin-dependent and spin-independent parts. Of course, this potential must have an imaginary part that increases with the energy. Therefore, the expression given above must be at least generalized to the form

$$V_{\rm eff} = V_0(s,\ r^2) + iW_0(s,\ r^2) + \frac{1}{r}\left[V(s)\frac{\partial V_0}{\partial r} + iW(s)\frac{\partial W_0}{\partial r}\right](\sigma\mathbf{L}). \quad (92)$$

It is obvious that such a potential must lead to an energy dependence of the phases of the quark amplitudes and to a difference between them: $\varphi_+(s) \neq \varphi_-(s)$. We also make the usual assumption that $g_-(s)/g_+(s) \to 0$ as $s \to \infty$. We shall use the relation

$$g_-(s) = \frac{m}{\sqrt{s}}g_+(s), \quad (93)$$

where the parameter m has the dimensions of mass. This relation is approximate in nature. For example, it can be assumed that, in contrast to the valence-quark interaction leading to spin flip, the scattering without change in helicity has a shadow nature, a contribution to it being made by every possible intermediate state, and in this case we can estimate the ratio $g_+(s)/g_-(s)$ by the number of such states. The relation (93) is not necessary for the conclusions of the present section, but it is important from the point of view of the analysis of the behavior of the polarization parameter in the region of large scattering angles. As before, we choose the energy dependence in the form $g_+(s) = g_+ s^\Delta$.

Thus, we arrive at the following expressions for the functions $u_i(s,b)$:

$$\begin{aligned} u_{1,3}(s,\ b) &= g_0(s)\exp[-\mu_0 b + i\varphi_0(s)]; \\ u_5(s,\ b) &= g_1(s)\exp[-\mu_1 b + i\varphi_1(s)]; \\ u_{2,4}(s,\ b) &= g_2(s)\exp[-\mu_2 b + i\varphi_2(s)], \end{aligned} \quad (94)$$

where

$$\begin{aligned} g_k(s) &= [g_+(s)]^{n_1+n_2-k}[g_-(s)]^k; \\ \mu_k &= (n_1+n_2-k)\mu_+ + k\mu_-; \\ \varphi_k(s) &= (n_1+n_2-k)\varphi_+(s) + k\varphi_-(s), \quad k=0,\ 1,\ 2. \end{aligned} \quad (95)$$

Using the method developed in the first part of the review and taking into account the identity of the particles and the expressions (94) and (95), we obtain the following expressions for the five helicity amplitudes[37]:

$$\begin{aligned} F_1(s,\ t) &= (n_1+n_2)\mu_+\omega(s)(|t|^{-3/2} + |u|^{-3/2}); \\ F_2(s,\ t) &= [(n_1+n_2)\mu_+ + 2(\mu_+ - \mu_-)]\,\omega(s) \times \left(\frac{g_-(s)}{g_+(s)}\right)^2 e^{-i\Delta(s)}(|t|^{-3/2} + |u|^{-3/2}); \\ F_3(s,\ t) &= \omega(s)\left[\frac{(n_1+n_2)\mu_+}{|t|^{3/2}} - \left(\frac{g_-(s)}{g_+(s)}\right)^2 \times \frac{3[(n_1+n_2)\mu_+ + 2(\mu_+-\mu_-)]}{|u|^{3/2}}e^{-i\Delta(s)}\right]; \\ F_4(s,\ t) &= \omega(s)\left[\left(\frac{g_-(s)}{g_+(s)}\right)^2\frac{3[(n_1+n_2)\mu_+ + 2(\mu_+-\mu_-)]}{|t|^{3/2}} \times e^{-i\Delta(s)} - \frac{(n_1+n_2)\mu_+}{|u|^{3/2}}\right]; \\ F_5(s,t) &= \omega(s)\frac{g_-(s)}{g_+(s)}e^{-i\frac{\Delta(s)}{2}}(|t|^{-1} - |u|^{-1}), \end{aligned} \quad (96)$$

where

$$\omega(s) = \frac{s e^{-i(n_1+n_2)\varphi_+(s)}}{\pi^2[g_+(s)]^{n_1+n_2}}, \quad \frac{1}{2}\Delta(s) = \varphi_+(s) - \varphi_-(s).$$

Knowing the expressions for the amplitudes $F_i(s,t)$, we can readily obtain expressions for the spin-correlation parameters. For 90° scattering, the quantities in which we are interested are determined by the amplitude combinations

$$\begin{aligned} \sigma A_{nn} &= {\rm Re}[F_1F_2^* + |F_3|^2]; \\ \sigma A_{ll} &= -\frac{1}{2}[|F_1|^2 + |F_2|^2 - 2|F_3|^2]; \\ \sigma A_{ss} &= {\rm Re}[F_1F_2^* - |F_3|^2], \end{aligned} \quad (97)$$

where $\sigma = (1/2)[|F_1|^2 + |F_2|^2 + 2|F_3|^2]$ and, as is readily seen,

$$A_{nn} - A_{ll} - A_{ss} = 1.$$

Using the expressions (96) and the relation (93), we obtain the following expansions for the spin-correlation parameters:

$$\begin{aligned} A_{nn} &= \frac{1}{3}\left[1 - \frac{\varkappa}{s}\cos\Delta(s) + O\left(\frac{1}{s^2}\right)\right]; \\ A_{ll} &= -\frac{1}{3}\left[1 + \frac{\varkappa}{s}\cos\Delta(s) + O\left(\frac{1}{s^2}\right)\right]; \\ A_{ss} &= -\frac{1}{3} + O\left(\frac{1}{s^2}\right), \end{aligned} \quad (98)$$

where

$$\varkappa = 8m^2\left[1+\frac{2(\mu_+-\mu_-)}{(n_1+n_2)\mu_+}\right].$$

The expressions (98) determine the energy dependence of the spin-correlation parameters at $\theta = 90°$. It follows from them, in particular, that there can be oscillations of the parameters $A_{nn}(\pi/2)$ and $A_{ll}(\pi/2)$ as s varies.

The nature of the oscillations is determined by the behavior of the phase difference $\varphi_+(s) - \varphi_-(s)$ and of the valence-quark scattering amplitudes without and with spin flip, respectively. Therefore, the possible oscillating behavior of the spin-correlation parameters in the pre-asymptotic region has a dynamical origin, in contrast to the variations of the parameter A_{nn} for $\theta \to \pi/2$ and a fixed value of s.

The presence in the pre-asymptotic term of the factor $\cos\Delta(s)$ makes it possible to describe the growth of the parameter $A_{nn}(\pi/2)$ in the region of energies from 8 to 12 GeV if it is assumed that in this interval the phase difference $\varphi_+(s) - \varphi_-(s)$ has an increment of $\pi/2$. Figure 5 shows the results of the reconciliation of the expression (98) with the experimental data in this case.

The possibility of subsequent oscillations in the behavior of the spin-correlation parameters at $p_L > 12$ GeV/c depends on the behavior of the function $\Delta(s)$. We note that the asymptotic region commences at an energy of ~ 1000 GeV.

Thus, if we adopt the explanation of the observed growth of the parameter $A_{nn}(\pi/2)$ as due to growth of the phase difference $\Delta(s)$, then we must evidently conclude that resonance effects are present in the scattering of the valence quarks. Such effects are determined by the structure of the hadrons and their constituents, and in the framework of the present model they must be taken into account by the form of the potential $V_{\rm eff}$. Continuing the analogy with nucleon–nucleus scattering, we conclude that the picture of valence-quark scattering and the presence of resonance effects are analogous to the formation of a compound nucleus.[35] The possibility of formation of a compound nucleus is taken into account in nuclear physics by adding a negative imaginary part to a real potential. We note that oscillations of the parameters $A_{nn}(\pi/2)$ and $A_{ll}(\pi/2)$ must lead to the appearance of structure in the pp scattering cross section at 90° in the pre-asymptotic region. The existence of such structure in the experimental data was pointed out in Ref. 36.

An alternative to an oscillating behavior of the spin-correlation parameters is possible if it is assumed that the phase difference $\Delta(s)$ is close to some constant value. In this case, we can reconcile the existing experimental data at the maximal energies attainable in the ZGS region, and the parameters A_{nn} and A_{ll} tend to their asymptotic values fairly rapidly. In both cases, the angular dependence of the parameter A_{nn} in the region of angles near 90° agrees well with the experimental data. The expression for the parameter A_{nn}, valid also when $\theta \neq 90°$, has the form[37]

$$A_{nn}(s,\cos\theta) = \left(1+\left|\frac{t}{n}\right|^{3/2}+\left|\frac{u}{t}\right|^{3/2}\right)^{-1}$$
$$\times\left\{1+6G^2(s)\cos\Delta(s)\left[1+\frac{2(\mu_+-\mu_-)}{(n_1+n_2)\mu_+}\right]\right.$$
$$\times\left[\left(1+\left|\frac{t}{u}\right|^{3/2}+\left|\frac{u}{t}\right|^{3/2}\right)^{-1}-\frac{1}{3}\left(1+2\left|\frac{t}{u}\right|^{3/2}\right.\right.$$
$$\left.\left.+2\left|\frac{u}{t}\right|^{3/2}\right)\right]+\frac{2G^2(s)}{(n_1+n_2)^2\mu_+^2}\,|t|^{-1/2}|u|^{-1/2}$$
$$\left.\times(|t|-|u|)^2\left[1-\frac{2}{1+\left|\frac{t}{u}\right|^{3/2}+\left|\frac{u}{t}\right|^{3/2}}\right]\right\}, \qquad (99)$$

where $G(s) \equiv g_-(s)/g_+(s)$. In the limit $s\to\infty$, assuming that $G(s)\to 0$, we obtain the following behavior of the parameter A_{nn}:

$$A_{nn}^{(\infty)}(\cos\theta) = \left[1+\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{3/2}+\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{3/2}\right]^{-1}, \qquad (100)$$

which clearly reveals the kinematic nature of the growth of $A_{nn}(\cos\theta)$ as $\theta\to 90°$.

The asymptotic values obtained for the spin-correlation parameters agree with the result of models in which hadron–hadron scattering is associated with exchange of valence quarks. We note that in models like the constituent-interchange model it follows from the condition of conservation of the quark helicities in the interaction that $F_2(s,t) \equiv F_5(s,t) \equiv 0$. To reconcile these models with the experimental data on the dependence of the parameter A_{nn} on the scattering angle, more complicated contributions are now being taken into account; in particular, arguments are now used that go beyond the framework of perturbation theory in QCD, this leading to transition to a situation in which $F_2(s,t) \neq 0$. However, as before, $F_5(s,t) = 0$ and, therefore, the polarization parameter vanishes in the region of large scattering angles.

Allowance for the contributions that have order $O(m^2/s)$ relative to the amplitude $F_1(s,t)$ without spin flip makes it possible to describe the energy dependence of the spin-correlation parameters. The assumption (93) to the effect that $g_-(s) = (m/\sqrt{s})\,g_+(s)$ leads to the following relationship between the amplitudes (it is valid both for hadron scattering and the qq scattering amplitudes):

$$F_2(s,\ t\sim s) \sim \frac{m^2}{s}F_1(s,\ t\sim s).$$

Transition to the limit $s\to\infty$ corresponds to the transition to the case when $F_2(s,t\sim s)/F_1(s,t\sim s) = 0$ and, therefore, values of the parameters A_{nn} and A_{ll} asymptotic with respect to s. At the same time, however, the polarization $P(s,t\sim s)$ is nonzero ($\theta \neq 90°$).

It should be noted that the relation (93) is not necessary for conclusions about an oscillating behavior of the spin-correlation parameters and their asymptotic values. It is sufficient to assume that the ratio $G(s)$ decreases as $s\to\infty$. The expressions for the spin-correlation parameters have in this

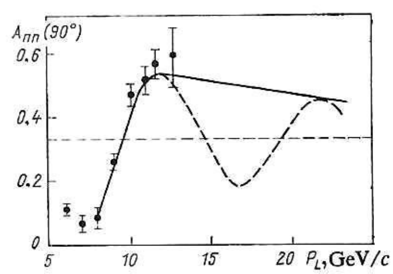

FIG. 5. Spin-correlation parameter $A_{nn}(90°)$ in elastic pp scattering.

case the form

$$A_{\substack{nn\\ ll}} = \pm \frac{1}{3}\left[1 \mp \frac{\varkappa}{m^2} G^2(s)\cos\Delta(s) + O(G^4(s))\right], \qquad (101)$$

where the upper sign refers to A_{nn}, and the lower to A_{ll}.

The curve in the figure corresponds to the value $\sigma_p/\sigma_a = 3.4$ of the ratio of the cross sections with parallel and antiparallel spins. It should be noted that if at the maximal ZGS energies $\sigma_p/\sigma_a > 4$, which does not contradict the data, then to reconcile the corresponding values of the parameter A_{nn} in the complete range $p_L = 6$–12 GeV/c it is necessary to introduce a weaker suppression of the quark scattering amplitude with spin flip than is given by the relation (93), or to assume that the expressions obtained for the spin-correlation parameters correspond to higher energies.

Note that in the framework of the considered model the value of the polarization parameter need not decrease at high energies in the region of fixed angles only if the condition (93) is satisfied. If it is not, the polarization parameter decreases in the region of fixed-angle scattering as a power with respect to s.

Study of the spin-correlation parameters at energies 100–1000 GeV is exceptionally important for investigating the mechanism of hadron interaction at the level of their constituents, since the parameters A_{nn} and A_{ll} carry nontrivial information about the quark-spin dynamics in hadron–hadron scattering. As in the diffraction region, study of the spin characteristics makes it possible to differentiate between different approaches from the point of view of the construction of the hadron–hadron scattering amplitude and the interaction mechanisms of the hadronic constituents.

I. CONCLUSIONS

In the present paper, we have considered a method for calculating the scattering amplitude, based on analysis of the singularities in the complex plane of the impact parameter. This method uses general properties of the S matrix such as analyticity and unitarity. For the scattering amplitude, we have used the three-dimensional dynamical equation $F = F[U]$ of quantum field theory,[2] whose solution automatically satisfies unitarity. In constructing the kernel of the equation—the generalized reaction matrix—we have made essential use of the analytic properties in the cosine of the scattering angle. We have shown that in the framework of the considered method the main features observed in hadron scattering can be reproduced qualitatively. In the region of fixed values of the momentum transfer (when t/s is small) we have obtained a representation of the scattering amplitude in the case of asymptotically increasing total cross sections in the form of a series with respect to a parameter $\tau(\sqrt{-t})$ that decreases with increasing momentum transfer. This expansion is obtained without the use of perturbation theory and is essentially a consequence of the manifestly unitary nature of the representation of the scattering amplitude in terms of the generalized reaction matrix. This part of the amplitude is determined by the contribution of the poles $\{b_m(s)\}$ in the plane of the impact parameter b, these poles being generated by the very form of the representation of the amplitude in terms of the U matrix. The properties of the poles depend weakly on the form of the generalized reaction matrix. In the region of large-angle scattering (when the ratio t/s is fixed), a power-law decrease of the cross sections in a consequence of the analytic properties of the amplitude in the cosine of the scattering angle. The corresponding part of the amplitude is determined by the contribution of the singularity at the point $b^2 = 0$ and is dominant in the region of fixed scattering angles. We have also considered the simplest way of taking into account the composite structure of hadrons in constructing the kernel of the equation.

We have used the method to analyze spin effects in elastic scattering and to establish a connection between the exponents of decrease of the elastic and the inclusive cross sections. We have also described the angular dependence of the large-angle scattering cross section, and we have proposed an explanation for the absence of a second dip in the angular distributions of pp elastic scattering, for which there is Orear behavior.

APPENDIX

We here calculate the contribution of the cut $\beta\in[0, -\infty)$ to the scattering amplitude $F_c(s,t)$ in the general case when the function $u(s,\beta)$ is represented by the expression (39).

Calculating the discontinuity of the function $u(s,\beta)$ across the cut $\beta\in[0, -\infty)$, for the corresponding contribution to $F(s,t)$ we find

$$F_c(s,t) = -\frac{is}{\pi^3\mu^2}\int_0^\infty dx K_0\left(\sqrt{x\frac{|t|}{\mu^2}}\right)\tilde{f}(s,x), \qquad \textbf{(A.1)}$$

where for $|\pi/\ln x| < 1$

$$\tilde{f}(s,x) = g(s)x^{-\gamma}\{[-\ln^\alpha x + O(\ln^{\alpha-2}x)]\sin(\sqrt{x}+\pi\gamma)$$
$$+[\alpha\pi\ln^{\alpha-1}x + O(\ln^{\alpha-3}x)]\cos(\sqrt{x}+\pi\gamma)\}$$
$$\times\{1+g^2(s)[\ln^2 x+\pi^2]x^{-2\gamma}+2g(s)x^{-\gamma}[\ln^\alpha x+O(\ln^{\alpha-2}x)]$$
$$\times\cos(\sqrt{x}+\pi\gamma)+2g(s)x^{-\gamma}[\alpha\pi\ln^{\alpha-1}x$$
$$+O(\ln^{\alpha-5}x)]\sin(\sqrt{x}+\pi\gamma)\}^{-1}.$$

If α is an integer, then the expressions in the square brackets are transformed into finite sums. As $x\to 0$, we obtain for $\tilde{f}(s,x)$

$$\tilde{f}(s,x) \simeq -\frac{1}{g(s)}x^\gamma(\ln x)^{-\alpha}. \qquad \textbf{(A.2)}$$

As $|t|\to\infty$, the main contribution to the integral (A.1) is given by the region of small values of x, since

$$\int_a^\infty dx K_0\left(\sqrt{x\frac{|t|}{\mu^2}}\right)\tilde{f}(s,x) \sim \exp\left(-\sqrt{a\frac{|t|}{\mu^2}}\right).$$

Therefore, taking into account the expression (A.2), we obtain

$$F_c(s,t) \simeq -\frac{is}{2\pi^3\mu^2 g(s)}\int_0^{\mu^2/|t|} dx \ln\left(x\frac{|t|}{\mu^2}\right)x^\gamma\ln^{-\alpha}x, \qquad \textbf{(A.3)}$$

where we have used Eq. (A.2) and the representation $K_0(z)\sim -\ln z$ as $z\to 0$. Making now the change of variables $x = \exp(-y)$ and introducing the incomplete Γ function

$$\Gamma(\xi, x) = \int_x^\infty e^{-z}z^{\xi-1}dz,$$

we find

$$F_c(s,t)=-\frac{is}{2\pi^3\mu^2 g(s)}\frac{(-1)^{1-\alpha}}{(1+\gamma)^{2-\alpha}}\left\{\Gamma\left(2-\alpha,\ \ln\left(\frac{|t|}{\mu^2}\right)^{1+\gamma}\right)\right.$$
$$\left.-\ln\left(\frac{|t|}{\mu^2}\right)^{1+\gamma}\Gamma\left(1-\alpha,\ \ln\left(\frac{|t|}{\mu^2}\right)^{1+\gamma}\right)\right\}. \qquad (A4)$$

Using the following asymptotic representation at large $|x|$,

$$\Gamma(\xi,x)=x^{\xi-1}e^{-x}\left[\sum_{m=0}^{M-1}\frac{(-1)^m\Gamma(1-\xi+m)}{x^m\Gamma(1-\xi)}+O(|x|^{-M})\right],$$
$$\xi\neq 2,3\ldots, \qquad (A.5)$$

we obtain

$$F_c(s,t)=\frac{is(-1)^{\alpha}}{2\pi^3\mu^2 g(s)}\frac{1}{(1+\gamma)^2}\left(\frac{\mu^2}{|t|}\right)^{1+\gamma}\frac{1}{\ln^{\alpha}\frac{|t|}{\mu^2}}\varphi\left(\ln^{-1}\frac{|t|}{\mu^2}\right). \qquad (A.6)$$

An expansion of the function $\varphi(\ln^{-1}|t|/\mu^2)$ in powers of the logarithm can be readily obtained by using the expressions (A.4) and (A.5), the relation $\varphi(0)=1$ holding. The last expression is valid for $\alpha\neq 1$; if $\alpha=1$, we have

$$F_c(s,t)=\frac{is}{2\pi^3\mu^2 g(s)}\frac{1}{(1+\gamma)^2}\left(\frac{\mu^2}{|t|}\right)^{1+\gamma}\frac{1}{\ln^2\frac{|t|}{\mu^2}}\varphi_1\left(\ln^{-1}\frac{|t|}{\mu^2}\right), \qquad (A.7)$$

where $\varphi_1(0)=1$. The expression for $\varphi_1(z)$ can be obtained using (A.5).

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Translated by Julian B. Barbour