Problems of relativistic quark dynamics and the quark structure of the deuteron

A. P. Kobushkin

Institute of Theoretical Physics, Ukrainian Academy of Sciences, Kiev

V. P. Shelest

All-Union Scientific-Research Center for the Study of Surface Properties and Vacuum, Moscow

Fiz. Elem. Chastits At. Yadra **14**, 1146–1192 (September-October 1983)

Approaches to a relativistic description of quark systems and their application to the study of nuclear-physics effects at short distances are discussed. Detailed consideration is given to the problem of the quark structure of the simplest nucleus, i.e., the deuteron, and allowance for it in various problems (elastic *ed* scattering, electrodisintegration of the deuteron, and deuteron fragmentation at high energies). Further experimental investigations of deuteron structure at short distances are discussed.

PACS numbers: 21.40. + d, 12.35.Eq

INTRODUCTION

The deuteron has always been the object on which nuclear physics has tested its new methods. But even now, despite a history of its investigation that is long and rich in remarkable discoveries, the deuteron is still largely a mysterious object of nature capable of many surprises. It is still the ideal object for studying problems of the relativistic dynamics of bound systems. And although in this direction there are already great achievements, especially thanks to the contribution of Shirokov,[1-6] there is still no flagging of the bitter disputes about the part played by relativistic effects in the various processes of interaction of the deuteron with particles and nuclei.

A qualitative jump took place recently in our ideas about the nature of nuclear forces. It was found that quantum chromodynamics (QCD)—the theory of the interaction of the local fields of colored quarks and gluons—is the basis of strong-interaction physics. This immediately changed our views about the deuteron structure. Whereas at "large" distances, i.e., at distances greater than those at which confinement occurs ($r \gtrsim 1$ F), the deuteron, like every other nucleus, is still assumed to be constructed from nucleons, which now must be regarded as $3q$ bags, at distances less than 1 F the quarks and gluons themselves become the structural blocks. The quark–gluon structure of the nucleus must be manifested in a fundamentally new form—in the form of a quark–gluon plasma, multiquark ($6q, 9q, 12q, \ldots$) bags, etc. If this is the case, then it will be possible to study various properties of quarks and gluons that cannot, or can only with great difficulty, be studied in particle–particle collisions.

These ideas about the structure of nuclear matter find experimental confirmation in investigations of the so-called *cumulative effect*,[7-9] in the study of electron scattering by light nuclei at values of q^2 of the order of a few $(\text{GeV}/c)^2$,[10-14] in the fragmentation of relativistic deuterons into protons,[15-17] etc. For more complete acquaintance with this problem, we refer the reader to the reviews of Refs. 8, 9, 18, and 19.

Let us dwell briefly on the behavior of the form factors. At sufficiently large q^2, it follows from scaling arguments[20,21] that the decrease in the form factor of a bound system of n constituent quarks satisfies the law

$$F^{(nq)}(q^2) \sim (1/q^2)^{n-1} \quad (q^2 \equiv -t). \tag{1}$$

If a virtual photon with sufficiently large q^2 is to "see" a nucleus as an n-quark bag, the nuclear form factors must have the asymptotic behavior (1), in which $n = 3A$.

In Fig. 1 we have plotted $F^{(nq)}(q^2)(q^2)^{n-1}$ for systems with different quark contents: the pion ($n = 2$), nucleons ($n = 3$), the deuteron ($q = 6$), ^{3}He ($n = 9$), and ^{4}He ($n = 12$). The transition to the asymptotic behavior begins later, the larger is n: For the pion it begins at $q^2 \approx 1\ (\text{GeV}/c)^2$, for the nucleons at $q^2 \approx 2$–$3\ (\text{GeV}/c)^2$, and for the deuteron at $4\ (\text{GeV}/c)^2$. For ^{3}He and ^{4}He, the form factors have not yet reached their asymptotic behavior at $q^2 \approx 4\ (\text{GeV}/c)^2$. Nevertheless, one can clearly see a tendency for the curves corresponding to them to "flatten out."

The influence of many-quark configurations on nuclear structure at short distances can be studied not only in high-energy nuclear collisions. As was shown in Ref. 22 for the example of the circular polarization P_γ, it can have a strong influence on parity-violation effects in nuclear capture reactions. However, a consistent calculation of this mechanism of enhancement of P-odd correlations in nucleon–nucleon interactions has not yet been made.

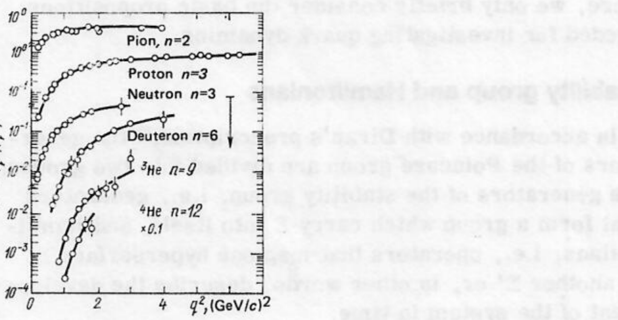

FIG. 1. Comparison with experiment of the quark-counting rules for the form factors of systems containing different numbers of constituent quarks.

 0090-4759/83/050483-21$04.40

It is to be expected that the quark structure of the nucleus can be described by the same methods that are traditionally employed in hadron physics, i.e., the methods in which a particle is regarded as a system of an unchanged number of components, the constituent quarks, which are bound by a confining potential.

A consistent description of quark systems can be based on various relativistic approaches: on the Hamiltonian formalism of relativistic systems with a constant number of interacting particles, developed in 1949 by Dirac,[23] on the Bethe–Salpeter equation,[24] and on the Logunov–Tavkhelidze quasipotential equation.[25] Therefore, in this paper we shall briefly review the methods of relativistic description of systems of a fixed number of interacting particles without particularizing the mechanism of their interaction. We present the methods of relativistic description of a system of constituent quarks. In particular, we consider the description of quark systems in the relativistic three-dimensional harmonic-oscillator model. We discuss the problems of quark description of the deuteron structure. We describe in detail the "hybrid" model of the deuteron, in which it is regarded as a quantum-mechanical mixture of a state in which the quarks cluster in two "bags" (a neutron and a proton) and the state in which they are in one bag. We give the approaches to the construction of many-quark systems for the simplest configurations s^6 and s^4p^2 with the quantum numbers of the deuteron. We consider the application of the hybrid model for the deuteron wave function to the description of the deuteron form factors, $d\to p$ fragmentation on nuclei at high energies, and other processes.

1. HAMILTONIAN FORMALISM OF RELATIVISTIC SYSTEMS WITH A FIXED NUMBER OF PARTICLES

The central question in the relativistic description of systems with a fixed number of particles is the question of how one should realize the ten generators of the Poincaré group in terms of a given set of dynamical variables (coordinates, momenta, spins, etc.), satisfying at the same time the causality requirement, i.e., the requirement that all variables are on a hypersurface Σ of Minkowski space that does not contain time-like directions.

A detailed review of the present state of the problem of the Hamiltonian description of relativistic systems with a fixed number of particles can be found in Ref. 26. Here, we only briefly consider the basic propositions needed for investigating quark dynamics.

Stability group and Hamiltonians

In accordance with Dirac's prescription,[23] the generators of the Poincaré group are divided into two groups: the generators of the stability group, i.e., generators that form a group which carry Σ into itself, and Hamiltonians, i.e., operators that map one hypersurface Σ to another Σ' or, in other words, describe the development of the system in time.

Because of a number of physical advantages, the hypersurface Σ is most often taken to be the so-called *null plane*, which is defined by

$$x^0 = x^3. \tag{2}$$

For an arbitrary vector a_μ, we shall use the components

$$a_+ = a^- = (a_0 + a_3)/2;\; a_- = a^+ = a_0 - a_3;\; \mathbf{a}_\perp = (a_1\ \ a_2). \tag{3}$$

In this notation, the generators of the Poincaré group have the components

$$\hat{P}_+;\; \hat{P}_r;\; \hat{P}_-;\; E_r = M_{+r};\; K_3 = M_{-+};\; F_r = M_{-r};\; J_3 = M_{12},\ r = 1,\ 2. \tag{4}$$

These generators are readily divided into the generators of the stability group of the null plane:

$$\hat{P}_+,\ \hat{\mathbf{P}}_\perp,\ \mathbf{E}_\perp,\ K_3,\ J_3 \tag{5}$$

and into the Hamiltonians:

$$\hat{P}_-,\ \mathbf{F}_\perp. \tag{6}$$

From the generators (5) of the stability group we can construct the operators of the helicity

$$J_3 \equiv W_+/\hat{P}_+ = J_3 + \hat{P}_+^{-1}(E_1\hat{P}_2 - E_2\hat{P}_1); \tag{7}$$

$$W_\mu = \varepsilon_{\mu\nu\alpha\beta}\hat{P}^\nu M^{\alpha\beta}/2 \tag{8}$$

and the mass

$$\hat{M}^2 = 2\hat{P}_+\hat{P}_- - \hat{\mathbf{P}}_\perp^2. \tag{9}$$

We can now choose a representation in which the generators $\hat{P} \equiv (\hat{P}_+, \hat{\mathrm{P}}_\perp)$, $\hat{M}^2$, and J_3 are diagonal. We shall denote the Hilbert state corresponding to this representation by $|p, m^2, \lambda\rangle$, where p, m^2, λ are the eigenvalues of the corresponding operators.

The action of the remaining generators on $|p, m^2, \lambda\rangle$ is determined by

$$\mathbf{E}_\perp|p, m^2, \lambda\rangle = \mathrm{i}p_+\frac{\partial}{\partial \mathbf{p}_\perp}|p, m^2, \lambda\rangle; \tag{10}$$

$$K_3|p, m^2, \lambda\rangle = \mathrm{i}p_+\frac{\partial}{\partial p_+}|p, m^2, \lambda\rangle; \tag{11}$$

$$J_3|p, m^2, \lambda\rangle = \left(\mathrm{i}\varepsilon_{rs}p_r\frac{\partial}{\partial p_s} + \lambda\right)|p, m^2, \lambda\rangle; \tag{12}$$

$$\hat{P}_- = \frac{1}{2\hat{P}_+}(M^2 + \hat{\mathbf{P}}_\perp^2); \tag{13}$$

$$F_r = \frac{1}{\hat{P}_+}[\hat{P}_rK_3 + \hat{P}_-E_r - \varepsilon_{rs}(\hat{P}_sJ_3 + \hat{M}J_3)], \tag{14}$$

where the perpendicular components of the spin are

$$\mathbf{J}_\perp = \hat{M}^{-1}(\mathbf{W}_\perp - \hat{P}_+^{-1}\hat{\mathbf{P}}_\perp W_+), \tag{15}$$

and $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$. Defined in this manner, the operators J_1, J_2, J_3 satisfy the commutation relations of angular momentum.

Basic principles of the relativistic quantum mechanics of systems of a fixed number of particles

We consider first the simplest composite system—two noninteracting particles of equal mass on the null plane. In this case, the representation of the Lorentz group acts on the space of vectors $|p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle$ which are eigenfunctions of the operators $\hat{p}^{(i)}$, $j_3^{(i)}$, and $\hat{M}^{(i)2}$ with eigenvalues $p^{(i)}$, $\lambda^{(i)}$, and m^2, respectively.

We define the operator of the "3-momenta" of the system as a whole by

$$\hat{P} = \hat{p}^{(1)} + \hat{p}^{(2)}. \tag{16}$$

Then the operators $\mathbf{E}_\perp$ and K_3 are expressed in terms of $\hat{P}$ in accordance with (10)–(12). For the operator J_3 additivity holds. Then

$$J_3 |p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle = \left(i\varepsilon_{rs} p_r^{(i)} \frac{\partial}{\partial p_s^{(i)}} + \lambda^{(i)} \right) |p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle. \quad (17)$$

We define the relative momenta of the particles. We usually obtain the perpendicular relative momenta as

$$\mathbf{k}_\perp = \mathbf{p}_\perp - \alpha \mathbf{P}_\perp, \quad (18)$$

where

$$\alpha \equiv \hat{P}_+^{-1} \hat{p}_+; \quad (19)$$

$$\hat{p} \equiv (\hat{p}^{(1)} - \hat{p}^{(2)})/2. \quad (20)$$

The role of the longitudinal component of the relative momentum is played by the variable α—the half-difference between the fractions of the momentum of the "bound" state carried by the first and second particle in the infinite-momentum frame.

It is important that the relative energy $\hat{p}_-$ is not an independent variable and is determined from the condition

$$\hat{P}_+ \hat{p}_- + \hat{P}_- \hat{p}_+ - \hat{\mathbf{P}}_\perp \hat{\mathbf{p}}_\perp = \hat{P}^\mu \hat{p}_\mu = 0 \quad (21)$$

through the operators $\hat{P}_+$, $\mathbf{k}_\perp$, α and the as yet unknown operator $\hat{M}^2$ (this last occurs in the definition of $\hat{\boldsymbol{P}}_-$).

The condition (21) has a simple physical meaning (see, for example, Ref. 27): The wave function of the system of two relativistic particles does not contain the relative time in the rest frame of the bound state. In what follows, we shall call (21) the *Markov–Yukawa condition.*[28,29]

We rewrite the equation

$$(2\hat{p}_+^{(1)}\hat{p}_-^{(1)} - \hat{\mathbf{p}}_\perp^{(1)2} + 2\hat{p}_+^{(2)}\hat{p}_-^{(2)} - \hat{\mathbf{p}}_\perp^{(2)2} - 2m^2) |p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle \equiv (\hat{p}_\mu^{(1)}\hat{p}^{(1)\mu} + \hat{p}_\mu^{(2)}\hat{p}^{(2)\mu} - 2m^2)|\ldots\rangle = 0 \quad (22)$$

in terms of the momentum $\hat{P}$ and "energy" $\hat{P}_-$ of the bound state and the relative momentum $\hat{p}$ and energy $\hat{p}_-$ of the particles. We must then express the Hamiltonians $\hat{P}_-$ and $\hat{p}_-$ in terms of the generators of the stability group in accordance with (13) and (21), respectively:

$$M^2 |p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle = \frac{\mathbf{k}_\perp^2 + \hat{m}^2}{1/4 - \alpha^2} |p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle. \quad (23)$$

Thus, the mass operator depends only on the relative variables $\mathbf{k}_\perp$ and α and the mass operator $\hat{m}^2$ of the subsystem.

It now remains to find the spin operator $\mathbf{J}_\perp$. This is readily done, since additivity holds for the generators $\mathbf{F}_\perp^{(i)}$. After simple transformations, we obtain from (14)

$$J_r = i\hat{M}^{-1}\varepsilon_{rs}\left[-k_s \frac{\partial}{\partial\alpha} + \frac{1}{2}\left(\frac{1}{4} - \alpha^2\right)\frac{\partial}{\partial\alpha}\hat{M}^2\frac{\partial}{\partial k_s}\right] + \frac{1}{1/2 + \alpha}[k_r j_3^{(1)} + \hat{m} j_r^{(1)}] + \frac{1}{1/2 + \alpha}[-k_r j_3^{(2)} + \hat{m} j_r^{(2)}]. \quad (24)$$

The relativistic wave functions, the solutions of Eq. (23), are functions normalized with the relativistically invariant measure $d\alpha d\mathbf{p}_\perp(\frac{1}{4} - \alpha^2)$. The usual probability interpretation holds for them.

It is possible to make a unitary transformation[30] that simplifies the expression for $J_\perp$,

$$\mathbf{J}_\perp \to V\mathbf{J}_\perp V^{-1} = \mathbf{L}_\perp + \mathbf{J}_\perp^{(1)} + \mathbf{J}_\perp^{(2)} \equiv \mathbf{L}_\perp + \mathbf{s}_\perp \quad (25)$$

and does not change $\mathcal{J}_3$. At the same time, the operators $\mathbf{L}$ and $\mathbf{s}$ are defined in such a way that the commutation relations of angular-momentum operators hold for each of them, and their third projections have the form of the nonrelativistic operators of the z projections of orbital angular momentum and spin. Thus, the wave functions obtained from the old functions by application of the unitary transformation $\boldsymbol{V}$ become eigenfunctions of the operators of the square of the angular momentum $\mathbf{L}^2$ and the spin $\mathbf{s}^2$. In other words, a $SU(2)_s \otimes O(3)_L$ classification becomes valid for them.

In the theory of quarks, such a transformation is called a *transformation from constituent to current quarks* (see, for example, Refs. 27 and 30–33).

For interacting particles, as for noninteracting, it is necessary to have a choice of the states $|p^{(1)}, \lambda^{(1)}; p^{(2)}, \lambda^{(2)}; m^2\rangle$ for which the condition (16) is satisfied and there is additivity for J_3, or such that the generators of the stability group act on these vectors as before. The Hamiltonians are changed. Thus, in contrast to the case of noninteracting particles, the mass operator of the composite particle will depend not only on the relative momenta α and $\mathbf{k}_\perp$ and on the masses of the constituents, m^2, but also on their relative coordinates η and $\mathbf{x}$, i.e., the quantities canonically conjugate to $\boldsymbol{\alpha}$ and $\mathbf{k}_\perp$ (see Ref. 26), and on the helicities $\lambda^{(1)}$ and $\lambda^{(2)}$.

A key to the solution of the bound-state problem for interacting particles is the condition of conservation of the commutation relations between the generators of the Poincaré group, i.e., the conservation of relativistic invariance. For this, it is sufficient to require the mass operator to satisfy the so-called *angular conditions*

$$[J_i, \hat{M}^2(\ldots)] = 0. \quad (26)$$

Relativistic description of bound states of constituent quarks

Almost all modern approaches to the description of bound states of quarks are based on a single physical idea formulated in 1965 by Bogolyubov and his students.[34–36] The essence of this idea is that the relativistic equations for a system of strongly bound quarks at distances less than the hadron dimensions go over into equations for quarks with a renormalized mass moving within a bounded region of space–time as almost free particles.

This approach made it possible to calculate some characteristics of hadrons—the magnetic moments, the axial constant of β decay, and so forth.[37–39] Further development of the approach of Ref. 34 led to the so-called MIT bag,[40] which has been widely used in different problems in the physics of particles and nuclei.

We consider the simplest system—a bound state of a quark and antiquark. In this case, the wave function depends on two coordinates—the quark's $x^{(1)}$ and the antiquark's $x^{(2)}$. In what follows, the subscript of the wave functions will denote the spin and unitary index of the quark, the superscript the antiquark's. As the equation for the bound states of the quark–antiquark system $(q\bar{q})$ we can take[1)]

$$(M_1^2 - p^{(1)^2})(M_2^2 - p^{(2)^2})\Psi_A^B(x^{(1)}, x^{(2)}) = \int dy^{(1)} dy^{(2)} U_{AB'}^{BA'}(x^{(1)}, x^{(2)}; y^{(1)}, y^{(2)}) \delta(x^{(1)} + x^{(2)} - y^{(1)} - y^{(2)}) \Psi_{A'}^{B'}(y^{(1)}, y^{(2)}). \quad (27)$$

Unfortunately, the problem of finding the operator U on the basis of field-theoretical ideas (for example, in the framework of QCD) has not yet been solved. It is therefore necessary to choose U on the basis of model considerations.

It is natural to attempt to expand U with respect to a small parameter. In Ref. 34, this was taken to be the inverse of the quark mass M:

$$U = M_1^2 M_2^2 + M^2 W + M^2 O(M^{-1}). \quad (28)$$

Suppose at the same time $M_1^2 = M^2 + \mu_1^2$, $M_2^2 = M^2 + \mu_2^2$, $\mu_1^2/M^2 \sim \mu_2^2/M^2 \to 0$, $W/M^2 \to 0$. Then

$$(p^{(1)^2} + p^{(2)^2} - \mu_1^2 - \mu_2^2 + W)\Psi_B^A(x^{(1)}, x^{(2)}) = 0. \quad (29)$$

In this equation, the center-of-mass variables and the relative variables separate:

$$(m^2 + 4p^2 - 2\mu_1^2 - 2\mu_2^2 + 2W)\varphi(x) = 0; \quad (30)$$

$$(P^2 - m^2)\Phi_B^A(X) = 0; \; P \equiv i\partial/\partial X; \; p \equiv i\partial/\partial x; \quad (31)$$

$$\Psi_B^A(x^{(1)}, x^{(2)}) \Phi_B^A(X)\varphi(x), \quad (32)$$

where $X = (x^{(1)} + x^{(2)})/2$, $x = x^{(1)} - x^{(2)}$.

Equations (30) and (31) do not describe the spinor structure of the meson wave function. To describe it, (31) can be replaced by the Bargmann–Wigner equation[2)]:

$$(P\gamma^{(1)} - m)\Phi_B^A(X) = 0; \; (P\gamma^{(2)} - m)\Phi_B^A(X) = 0. \quad (33)$$

It is obvious that the wave equation will then also satisfy (31).

Following the general principles, it is necessary to go over in (30) to the variables of the stability group. We restrict the treatment to the case of equal masses $\mu_1 = \mu_2 = \mu$. On the operator of the relative momentum we impose the Markov–Yukawa condition. At the same time, the wave function in the configuration space is given by the Fourier transform (for more details, see Ref. 26)

$$\varphi(\eta, \mathbf{x}_\perp) = (2\pi)^{-3/2} \int d\mathbf{k}_\perp \int_{-1/2}^{1/2} \frac{d\alpha}{1 - 4\alpha^2} \exp(-i\mathbf{k}_\perp \mathbf{x}_\perp - i\alpha\eta)\varphi(\alpha, \mathbf{k}_\perp). \quad (34)$$

Similarly, one can make a Fourier transformation for the potential:

$$W(\eta, \mathbf{x}_\perp) = (2\pi)^{-3/2} \int d\mathbf{k}_\perp \int_{-1/2}^{1/2} \frac{d\alpha}{1 - 4\alpha^2} \exp(-i\mathbf{k}_\perp \mathbf{x}_\perp - i\alpha\eta) w(\eta, \mathbf{k}_\perp). \quad (35)$$

Then (30) becomes the eigenvalue equation for the operator of the square of the mass of the bound state,

$$\hat{m}^2 = 4(\mu^2 + \mathbf{k}_\perp^2)/(1 - 4\alpha^2) - V, \quad (36)$$

where V is an integral operator:

$$V\varphi(\alpha, \mathbf{k}_\perp) = (2\pi)^{-3/2} \frac{2}{1 - 4\alpha^2} \times \int d\mathbf{p}_\perp \int_{-1/2}^{1/2} \frac{d\beta}{1 - 4\beta^2} w(\alpha - \beta, \mathbf{k}_\perp - \mathbf{p}_\perp)\varphi(\beta, \mathbf{p}_\perp). \quad (37)$$

As for a free particle, we shall assume that $-\frac{1}{2} \leqslant \alpha \leqslant \frac{1}{2}$.

It is possible to construct a relativistically invariant three-dimensional vector

$$\mathbf{k} = (\mathbf{k}_\perp, k_3); \; k_3 = 2\alpha\sqrt{(\mu^2 + \mathbf{k}_\perp^2)/(1 - 4\alpha^2)}. \quad (38)$$

Then Eq. (30) takes the form of the ordinary nonrelativistic Schrödinger equation

$$m^2\varphi(\mathbf{k}) = (4\mu^2 + 4\mathbf{k}^2 - V)\varphi(\mathbf{k}). \quad (39)$$

Thus, we obtain a simple connection between the relativistic wave function $\varphi(\mathbf{k})$ and the nonrelativistic $\varphi_{\mathrm{nonrel}}(\mathbf{q}^2)$. For this, it is necessary to replace the nonrelativistic relative momentum by the relativistic (38):

$$\varphi_{\mathrm{non\,rel}}(\mathbf{q}^2) \to \varphi(\alpha, \mathbf{k}_\perp) = \varphi_{\mathrm{non\,rel}}(\mathbf{k}^2) \equiv \varphi_{\mathrm{non\,rel}}[(\mu^2 + \mathbf{k}_\perp^2)/(1 - 4\alpha^2) - \mu^2]. \quad (40)$$

The normalization condition now takes the form

$$\int d\mathbf{k}\,|\varphi(\mathbf{k}^2)|^2 = 2\int d\mathbf{k}_\perp \int_{-1/2}^{1/2} \frac{d\alpha}{1 - 4\alpha^2} \times \sqrt{\frac{\mu^2 + \mathbf{k}_\perp^2}{1 - 4\alpha^2}} \left|\varphi\left(\frac{\mu^2 + \mathbf{k}_\perp^2}{1 - 4\alpha^2} - \mu^2\right)\right|^2 = 1. \quad (41)$$

For the relativistic wave function (40), the usual probability interpretation holds. Such a relativistic wave function is widely used in the most varied problems.

We construct the operators of the angular momentum and the spin. We begin with the latter. The spin operator is the sum of the spin operators W of the quark and antiquark:

$$\mathbf{S} = \mathbf{W}^{(1)} + \mathbf{W}^{(2)}. \quad (42)$$

The relativistic operator of the angular momentum is readily constructed from the momenta $\mathbf{k}$:

$$\mathbf{L} = -i\mathbf{k} \times \frac{\partial}{\partial \mathbf{k}}. \quad (43)$$

It is natural to require the operator V to commute with the operators $\mathbf{S}$ and $\mathbf{L}$. Then, of course, the conditions $[\mathbf{J}, \hat{m}^2] = 0$ will also hold for the operator $\mathbf{J} = \mathbf{S} + \mathbf{L}$ of the total angular momentum.

Thus, for a meson we have succeeded in constructing an operator of the total angular momentum in which the spin operator $\mathbf{S}$ and angular-momentum operator $\mathbf{L}$ are separated. In other words, a $SU(2)_s \otimes O(3)_L$ classification is valid for the solutions of our equations.

Similar results hold for more complicated $3q$, $6q$, etc., systems. For this it is necessary, as for mesons, to make an expansion with respect to the reciprocal mass of the quark, impose the Markov–Yukawa conditions on the relative momenta of the quarks, and go over to the rest frame of the bound state.

"Minimal boosting" scheme

In most studies (see, for example, Refs. 31, 32, and 34–36), the spinor parts of the wave functions of bound quarks are determined from the Bargmann–Wigner equations (33). However, there is an alternative,[42]

[1)]It is, however, possible to proceed from a separable Bethe–Salpeter equation, since there exists a transformation[43] of the Bethe–Salpeter wave function into a wave function that satisfies the condition (27).

[2)]There is another possibility,[42] which we shall discuss below.

called by its proponents the *"minimal boosting" scheme*. In this case, the boost generators K_i are taken to be the operators

$$K_i = \frac{i}{2} \rho_1 \otimes \sigma_i. \tag{44}$$

where for a system consisting of n quarks and m antiquarks

$$\sigma_i = \sum_{r=1}^{n} \sigma_i^{(r)} - \sum_{s=1}^{m} \sigma_i^{T(s)}; \tag{45}$$

here, T denotes the transpose.

The matrices ρ_i (two-component Pauli matrices) refer to the $(nq + m\bar{q})$ system as a whole. In the approach based on the Bargmann–Wigner equations, matrices $\rho_i^{(r)}$ for each quark and antiquark are introduced, i.e., in this case

$$K_i = \frac{i}{2} \Sigma \rho_1^{(r)} \otimes \sigma_i^{(r)}!$$

In contrast to the scheme based on the Bargmann–Wigner equations, the equations that determine the wave function of the center-of-mass motion $\Phi(X)$ of the bound state are not linear in the derivative $\partial/\partial X^\mu$ in the case of the minimal boosting scheme. For example, in the case of baryons the equation will contain derivatives of third order, and in the case of mesons the second order. Therefore, such equations will permit not only the ordinary solutions but also solutions with complex and purely imaginary masses.

Despite this shortcoming, such an approach has a number of advantages over the approach based on the Bargmann–Wigner equations. Thus, in the minimal boosting scheme the wave functions have a minimal number of components. In Ref. 43, it was also noted that there is a difference between the behavior of the particle form factors in the schemes based on the Bargmann–Wigner equations and on the minimal boosting equations.

Internal symmetries of quark Hamiltonians

We now turn to the model of quasi-independent quarks,[37–39] in which equations linear in the quark momentum are considered. As for the quadratic equations, it is assumed that the potential that binds the quarks can be expanded in the reciprocal of the quark mass. At distances of the order of or less than the hadron diameter one studies, instead of the equation for a bound state of the quarks, a system of equations for the individual quarks moving in an external confining potential:

$$(\hat{\partial}^{(i)} - \mu_i - W^{(i)})\,\psi = 0. \tag{46}$$

The possibility of such an approximation for the example of a quark moving in a deep potential well was first pointed out in Ref. 37, where, in particular, it was demonstrated that such a treatment leads to an important effect—enhancement of the quark magnetic moment (see also Ref. 38).

It should be noted that in the model of quasi-independent quarks, as in the MIT bag, there is a serious difficulty (see, for example, Ref. 44) associated with the separation of the center-of-mass variables. Therefore, the wave function in these models can, strictly speaking, be constructed only in the hadron rest frame.

There is a connection between the equations of quasi-independent quarks and the equations considered above, namely, by means of a unitary transformation one can go over from wave functions that are a solution of (46) to wave functions that are a solution of the quadratic equations.

We consider one very instructive example—that of a weak potential $W \sim v^2/c^2$ (see Refs. 27, 30–32, and 45). In this case, Eq. (46) can be solved in the lowest expansion order in v/c. Omitting the details, which the reader can find in Ref. 32, we give only the final result: In the rest frame of the bound state, the wave function of a quasi-independent quark has the form (for brevity, we omit the quark index i)

$$\psi_{\mathbf{P}=0}(\mathring{\mathbf{p}}) = \frac{m/N + \mu - \mathring{\mathbf{p}}\boldsymbol{\gamma}}{[(m/N + \mu + \mathring{p}_3)^2 - \mathring{\mathbf{p}}_\perp^2]^{1/2}}\, \phi_{\mathbf{P}=0}(\mathring{\mathbf{p}}) \equiv U\phi_{\mathbf{P}=0}(\mathring{\mathbf{p}}), \tag{47}$$

where $\phi_{\mathbf{p}=0}$ satisfies the Bargmann–Wigner conditions in the rest frame of the hadron, $\varphi(\dot{\mathbf{p}})$ is a scalar function satisfying the equation

$$[m^2/N^2 - \mu^2 - \mathring{\mathbf{p}}^2 - (m/N + \mu)\,W]\,\varphi(\mathring{\mathbf{p}}) = 0, \tag{48}$$

and λ are ordinary Pauli spinors. Apart from the notation, Eq. (48) is identical to the equation (39) on the null plane.

Thus, in the Hamiltonian of the quasi-independent quark there are no terms containing the spin operators **S** or, therefore, breaking the $SU(2)_S$ symmetry of the Hamiltonian up to second order in v/c [Eq. (47)]. In higher orders, this is no longer so. Thus, the terms that break the $SU(2)_S$ symmetry have order $(v/c)^3$. However, if the Hamiltonian of the quasi-independent quark is transformed by means of the transformation U, the resulting Hamiltonian will contain spin operators only in terms of order $(v/c)^4$ and higher.[45] Therefore, one can construct a new operator **S'**, which will generate the group $SU(2)_{S'}$, breaking the symmetry of the Hamiltonian only from the fourth order in v/c. If we require the transformed Hamiltonian to contain terms that break the symmetry only from the fifth order in v/c, such a transformation will depend on the interaction W.

There are potentials that lead to Hamiltonians with exact higher symmetry and satisfying the requirement of quark confinement, for example, a potential of the type (46).

The bag model

As we have already said, the model of quasi-independent quarks, or, as it is also called, the Dubna bag model, was developed further in Ref. 40. An important difference between the MIT bag and the Dubna bag is the introduction of a phenomenological parameter, the external pressure B. The shape and the size of the bag are determined by the condition of equilibrium on the surface of the bag between the pressure B and the internal quark–gluon pressure. The physical meaning of the pressure B is that it represents the difference between the energy densities of the perturbation-theory vacuum and the true QCD vacuum. The degeneracy with respect to the total spin of the hadron in the bag model can be lifted by introducing chromomagnetic corrections[47] to the quark–quark interaction. As a result, the mass of the Δ isobar becomes greater than the nucleon mass. In the model of quasi-independent quarks with confining potentials of the type (46), the single-gluon corrections to the hadron masses were calculated in Ref. 48.

In Refs. 49 and 50, chiral generalizations of the bag models were considered. Rather promising in this direction is the large-bag model,[50] in which the hadron is a bag with a dimension of about 1 F surrounded by a pion field. The pion field does not penetrate into the bag, but interacts only on its surface. The model makes it possible to describe many properties of the lightest baryons in a unified manner.

Three-dimensional relativistic harmonic oscillator

For many physical problems, a useful approximation is the one in which the quark–quark interaction is approximated by the potential of a relativistic harmonic oscillator.[51–57]

We shall consider mesons. The generalization to the case of the more complicated systems $3q$, $6q$, $4q\bar{q}$, etc., is trivial. The equation describing this system has the form

$$(\Box^{(1)}+\Box^{(2)}-\Omega\,(x^{(1)}-x^{(2)})^2+2m_0^2)\,\Psi^{A}_{B}(x^{(1)},x^{(2)})=0 \qquad (49)$$

(we again restrict ourselves to the case when the masses of the quarks are equal). To describe the spinor structure of the bound state, we also make the wave function satisfy the Bargmann–Wigner conditions (33) or the equations of the minimal boosting scheme (see above).

In (49), we separate the center-of-mass variables (32). Then the wave function of the relative motion of the quark and the antiquark satisfies the equation

$$[\Box^{(x)}-(\Omega^2/2)\,x^2+m_0^2]\,\varphi(x)=M^2\varphi(x)/4. \qquad (50)$$

In accordance with the general method, it is necessary to impose on the wave function $\varphi(x)$ the Markov–Yukawa condition:

$$P_\mu\partial/\partial x_\mu\varphi(x)=0, \qquad (51)$$

and also the Feynman–Kislinger–Ravndal condition[53]

$$P_\mu\,[\partial/\partial x_\mu+(\Omega/\sqrt{2})\,x^\mu]\,\varphi(x)=0. \qquad (52)$$

Obviously, by virtue of the Markov–Yukawa condition the condition (52) reduces to

$$P_\mu x^\mu=0. \qquad (53)$$

Because of the conditions (51) and (53), Eq. (50) in the meson rest frame reduces to the Schrödinger equation

$$4\,[-\partial^2/\partial\mathring{\mathbf{x}}^2+(\Omega^2/2)\,\mathring{\mathbf{x}}^2+m_0^2]\,\varphi_{\mathbf{P}=0}(\mathring{\mathbf{x}})=M^2\varphi_{\mathbf{P}=0}(\mathring{\mathbf{x}}). \qquad (54)$$

The solutions of this equation are

$$\varphi_{\mathbf{P}=0;\,n_1,\,n_2,\,n_3}(\mathring{\mathbf{x}})=N_{n_1n_2n_3}H_{n_1}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_1)\times H_{n_2}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_2)H_{n_3}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_3)\exp[(-\Omega/2\sqrt{2}\,\mathring{x}^2]. \qquad (55)$$

The mass spectrum has the form

$$M^2_{n_1n_2n_3}=2\sqrt{2}\,\Omega\,(n_1+n_2+n_3+3)+4m_0^2. \qquad (56)$$

The wave functions are normalized by the condition

$$\int d^4x\,\delta\left(\frac{P_\mu}{M}x^\mu\right)|\varphi_{\mathbf{P};\,n_1,\,n_2,\,n_3}(x)|^2=0, \qquad (57)$$

whence

$$N_{n_1n_2n_3}=2^{n_1+n_2+n_3}n_1!n_2!n_3! \qquad (58)$$

To determine the angular-momentum operators **L**, it is necessary to construct the Pauli–Lubański vector

$$W_\mu=-i\varepsilon_{\mu\nu\sigma\rho}P^\nu x^\sigma\partial/\partial x_\rho, \qquad (59)$$

impose the restrictions (51) and (53) on the variables x_μ and $\partial/\partial x_\mu$, and write down W_μ in the meson rest frame ($\mathbf{P}=0$):

$$\mathbf{W}=M\mathbf{L};\quad \mathbf{L}=i\left[\frac{\partial}{\partial\mathring{\mathbf{x}}}\times\mathring{\mathbf{x}}\right]. \qquad (60)$$

Then the spin operator of the meson in its rest frame takes the form

$$\mathbf{J}=\mathbf{L}+(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})/2. \qquad (61)$$

Note that the spin operators (61) commute with the operator of the square of the meson mass, M^2, which is determined by Eq. (54):

$$\hat{M}^2=4\,(-\partial^2/\partial\mathring{\mathbf{x}}^2+(\Omega^2/2)\,\mathring{\mathbf{x}}^2+m_0^2). \qquad (62)$$

This is sufficient to construct all the generators of the Poincaré group in accordance with the appropriate formulas.

At the first glance, such results contradict the result obtained on the null plane.[26] Indeed, instead of the Markov–Yukawa and Feynman–Kislinger–Ravndal [or the equivalent (53)] conditions, one has in that case a pair of restrictions: the restriction to the null plane $x_+=(x_0+x_3)/2$ and the Feynman–Kislinger–Ravndal condition. It is obvious that the condition (53) is not identical to the condition $x_+=0$ (in the first case we have a condition of the type $\omega_\mu x^\mu$ with $\omega^2>0$, and in the second case one with $\omega^2=0$!). Nevertheless, as we shall now show, the Hamiltonian obtained in the light-front approach and the Hamiltonian found from the Markov–Yukawa condition are related by a gauge transformation, i.e., these approaches are in fact equivalent.

In the approach of Ref. 26, the operator of the square of the mass in the meson rest frame has the form

$$\hat{M}^2_{\mathrm{LC}}=4\,[-\partial^2/\partial\mathring{\mathbf{x}}^2_\perp-\partial^2/\partial\mathring{x}_3^2+(\Omega/2)\,\mathring{\mathbf{x}}^2_\perp+\sqrt{2}\,\Omega\,(\partial/\partial\mathring{x}_3)\,\mathring{x}_3+\tilde{m}_0^2], \qquad (63)$$

(LC = Light Cone).

which differs from the operator $\overline{M^2}$ obtained in the approach with the Markov–Yukawa conditions. The eigenfunctions of the Hamiltonian (61) are

$$\varphi^{\mathrm{LC}}_{n_1n_2n_3}(\mathring{\mathbf{x}}_\perp\mathring{x}_3)=N_{n_1n_2n_3}H_{n_1}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_1)\times H_{n_2}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_2)\,H_{n_3}(\sqrt[4]{\Omega^2/2}\,\mathring{x}_3)\exp[(-\Omega/2\sqrt{2})\,\mathring{\mathbf{x}}^2_\perp]. \qquad (64)$$

The wave functions (62) are normalized with the measure $\exp[(-\Omega/\sqrt{2})x_3^2]$. The angular-momentum operators are

$$\left.\begin{aligned} L_r^{\mathrm{LC}} &= -\mathrm{i}\varepsilon_{rs}\left[\mathring{x}_s\partial/\partial\mathring{x}_3-\mathring{x}_3\partial/\partial\mathring{x}_s-(\Omega/\sqrt{2})\,\mathring{x}_s\mathring{x}_3\right]; \\ L_3^{\mathrm{LC}} &= -\mathrm{i}\,(\mathring{x}_1\partial/\partial\mathring{x}_2-\mathring{x}_2\partial/\partial\mathring{x}_1). \end{aligned}\right\} \quad (65)$$

We make the gauge transformation[56]

$$\varphi(\mathring{\mathbf{x}}) \to \varphi^{\mathrm{LC}}(\mathring{\mathbf{x}}_1\quad \mathring{x}_3) = V(\mathring{x})\,\varphi(\mathring{\mathbf{x}});\qquad V(\mathring{x}_3)=\exp(\Omega\mathring{x}_3^2/2\sqrt{2}). \quad (66)$$

Then the operator $\hat{M}^2$ is transformed into $\hat{M}^2_{\mathrm{LC}}$, and the angular-momentum operators $\mathbf{L}$ defined in (60) are transformed into the operators $\mathbf{L}^{\mathrm{LC}}$ (65).

As we have already said, the wave functions in the approach on the light front[26] are normalized with measure $\exp(-\Omega\mathring{x}_3^2/\sqrt{2})$. The gauge transformation (64) restores this measure:

$$d^4x\,\delta(Px/M) = d\mathring{x} \to \exp(-\Omega\mathring{x}_3^2/\sqrt{2})\,d\mathring{x}_\perp\,d\mathring{x}_3, \quad (67)$$

which proves the equivalence of the descriptions of the harmonic-oscillator equation on the null plane and by means of the Markov–Yukawa conditions.

It is important to emphasize that the gauge transformation conserves the commutation relations between the angular-momentum operators and the condition of commutation of the mass operator with the angular-momentum operators.

Matrix elements of the currents

We consider the matrix element of some operator J in the nonrelativistic limit. We shall assume that additivity holds (to be specific, we consider a $3q$ system):

$$J = J^{(1)} + J^{(2)} + J^{(3)}. \quad (68)$$

In the momentum representation the matrix element of J is given by

$$\begin{aligned} (\mathbf{P}'|J(0,\mathbf{k})|\mathbf{P}) &= 3\delta(\mathbf{P}-\mathbf{P}'+\mathbf{k})\int d\mathbf{l}^{(1)}\,d\mathbf{l}^{(2)}\,d\mathbf{l}^{(3)}\,d\mathbf{l}'^{(1)} \\ &\times d\mathbf{l}'^{(2)}\,d\mathbf{l}'^{(3)}\,\delta\Big(\sum_{i=1}^{3}\mathbf{l}^{(i)}-\mathbf{P}\Big)\,\delta\Big(\sum_{i=1}^{3}\mathbf{l}'^{(i)}-\mathbf{P}'\Big)\,\delta(\mathbf{l}^{(2)}-\mathbf{l}'^{(2)}) \\ &\times \delta(\mathbf{l}^{(3)}-\mathbf{l}'^{(3)})\,\overline{\Psi}(\mathbf{l}'^{(1)},\mathbf{l}'^{(2)},\mathbf{l}'^{(3)})\,J^{(1)}\Psi(\mathbf{l}^{(1)},\mathbf{l}^{(2)},\mathbf{l}^{(3)}). \end{aligned} \quad (69)$$

For the relativistic generalization of (67) we must:

1) make the substitution $d\mathbf{l}^{(i)} \to d^4l^{(i)}$, $d\mathbf{l}'^{(i)} \to d^4l'^{(i)}$;

2) make the substitution $\delta(\sum_{i=1}^{3}\mathbf{l}^{(i)}-\mathbf{P}) \to \delta^4(\sum_{i=1}^{3}l^{(i)}-P)$ and similarly for the final state;

3) on the zeroth components of the 4-momenta of the quarks impose the Markov–Yukawa condition;

4) find the relativistic generalization of the three-dimensional δ function $\delta(\mathbf{l}^{(i)}-\mathbf{l}'^{(i)})$. It is obvious that this three-dimensional δ function cannot be replaced by the ordinary four-dimensional $\delta^4(l^{(i)}-l'^{(i)})$, since the correct nonrelativistic limit for the matrix elements of the currents is then not obtained. The required three-dimensional δ function is[27,55,57]

$$\frac{1}{L_0}\,\delta\Big(\lambda^{(i)}-\frac{\lambda_0^{(i)}}{L_0}\,\mathbf{L}\Big), \quad (70)$$

where

$$\lambda^{(i)} = l^{(i)} - l'^{(i)};\ L = (P'/m' - P/m)/2, \quad (71)$$

and m and m' are the masses of the hadrons in the initial and final states.

The relativistic matrix element of the current now takes the form

$$\begin{aligned} \langle\mathbf{P}'|J(0)|\mathbf{P}\rangle &= \frac{3}{L_0^2}\int\delta\Big(\sum_{i=1}^{3}l^{(i)}-P\Big)\,\delta\Big(\sum_{i=1}^{3}l'^{(i)}-P'\Big) \\ &\times\prod_{j=2}^{3}\delta\Big[\Big(l^{(j)}-\frac{P}{3}\Big)\frac{P}{M}\Big]\,\delta\Big[\Big(l'^{(j)}-\frac{P'}{3}\Big)\frac{P'}{M}\Big]\,\delta\Big(\lambda^{(j)}-\frac{\lambda_0^j}{L_0}\,\mathbf{L}\Big) \\ &\times\overline{\Psi}(l'^{(1)},l'^{(2)},l'^{(3)})\,J^{(1)}\Psi(l^{(1)},l^{(2)},l^{(3)}) \\ &\times d^4l^{(1)}\,d^4l^{(2)}\,d^4l^{(3)}\,d^4l'^{(1)}\,d^4l'^{(2)}\,d^4l'^{(3)}. \end{aligned} \quad (72)$$

The explicit form of $J^{(1)}$ depends on the equation solved to find the wave functions. If (29) is taken as the equation and the electromagnetic interaction is included in it by minimal coupling, we obtain

$$J_\mu^{(i)} = \frac{3}{2}\,\frac{\hat{Q}^{(i)}}{M}\,(\hat{l}'^{(i)}\gamma_\mu^{(i)}+\gamma_\mu^{(i)}\hat{l}^{(i)}), \quad (73)$$

where $\hat{Q}^{(i)}$ is the charge operator of quark i. If we consider the equations of the model of quasi-independent quarks, then it follows from minimal coupling of the electromagnetic current that

$$J_\mu^{(i)} = \hat{Q}^{(i)}\gamma_\mu^{(i)}. \quad (74)$$

In both cases, conservation of the electromagnetic current can be proved.[32,55]

2. THE DEUTERON IN THE LANGUAGE OF CONSTITUENT QUARKS

As we emphasized in the Introduction, nuclear structure must be described at two levels—large and small distances. At the first, the nucleus is a bound system of several nucleons; in the second, a configuration in which individual nucleons are combined into multiquark systems.

Hybrid model for the deuteron wave function

We restrict ourselves in what follows to the simplest nucleus—the deuteron. Its example will show us the changes that result from taking into account the six-quark ($6q$) state at short distances.

The model that we shall discuss below is based on the assumption[58-60] that the deuteron wave function can be represented as a weakly bound state of two three-quark clusters (called the proton and neutron; in what follows, the np state) with a small admixture of a $6q$ state, in which all six quarks are in one bag:

$$|\,\mathrm{deuteron}\rangle = \alpha\,|\,np\rangle + \beta\,|\,6q\rangle. \quad (75)$$

The quark degrees of freedom can be taken into account in the two-nucleon system in the framework of the *quark compound-bag model*.[61] Analysis of the S-wave phase shifts of NN scattering shows that the $6q$ states can play a dominant role when the kinetic energy of the incident nucleon is 300 MeV and more. The masses of the so-called *primitives*, i.e., the masses of the $6q$ states, which are obtained from the equations without the potential of the transition of the many-quark state to the nucleon state, correspond to the poles of the P matrix.[62] However, in the S matrix, because of the interaction between the quark channel and the nucleon channel, the poles are shifted relative to the poles of the P matrix along both the real and the imaginary axis. For a definite choice of the potential of the transition from the many-quark state to the nucleon state this shift may be so strong that it is impossible to observe the $6q$-bound states experimentally.

However, for the calculation of the deuteron form factors and in some other problems it is not necessary to know the $6q \leftrightarrow 2N$ transition potential explicitly. One can proceed from the phenomenological hybrid wave function (75) with parameters determined by fitting the results of the calculation to the experimental data.

We now turn to the correct description of each of the components of the hybrid wave function of the deuteron.

The *np* wave function

We begin by introducing convenient relative quark coordinates and momenta. Let $x^{(i)}$ be the coordinate of quark i, and X be the center-of-mass coordinate:

$$X = (x^{(1)} + x^{(2)} + x^{(3)} + x^{(4)} + x^{(5)} + x^{(6)})/6. \quad (76)$$

We determine the six relative coordinates

$$\left.\begin{array}{ll} x^{(1)} = X + r/2 - 2\xi; & x^{(4)} = X - r/2 - 2\mu; \\ x^{(2)} = X + r/2 + \xi - \sqrt{3}\eta; & x^{(5)} = X - r/2 + \mu - \sqrt{3}\nu; \\ x^{(3)} = X + r/2 + \xi + \sqrt{3}\eta; & x^{(6)} = X - r/2 + \mu + \sqrt{3}\nu. \end{array}\right\} \quad (77)$$

In the deuteron rest frame, the np wave function can be represented in the form

$$|np\rangle = \phi(\mathbf{r})\,\Psi_{3q}(\xi, \eta)\,\Psi_{3q}(\mu, \nu) \times |3q + 3q; j = 1, j_3; T = 0\rangle, \quad (78)$$

where $\phi(\mathbf{r})$ is the wave function of the relative motion of the two $3q$ clusters. It can be expressed in the usual manner in terms of the u and w wave functions:

$$\phi(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\frac{1}{r}\left[u(r) + \frac{S_{pn}(\hat{r})}{\sqrt{8}}w(r)\right], \quad (79)$$

where $S_{pn}(\hat{r})$ is a known tensor operator. For $\phi(\mathbf{r})$, we can take one of the phenomenological wave functions (for example, from Refs. 63–66). The function $|3q + 3q; j = 1; j_3; T = 0\rangle$ is the spin–isospin color wave function with spin $j(=1)$, third projection j_3, and isospin 0. The function $3q + 3q$ means that it is constructed from products of two $3q$ states with the quantum numbers of the proton and neutron. The explicit form of the spin–isospin color wave functions of the proton and neutron can be found, for example, in Appendix 1 of Ref. 53.

The wave functions $\Psi_{3q}(\xi,\eta)$, $\Psi_{3q}(\mu,\nu)$ describe the relative motion of the quarks within the first and second quark triplets, respectively. We shall determine them as the eigenfunctions of the Hamiltonian of the relativistic oscillator quark model on the null plane:

$$\Psi_{3q}(\xi, \eta) = N_3 \exp\left\{-\frac{3\omega}{\sqrt{2}}(\xi^2 + \eta^2)\right\}; \; N_3 = (6\omega/\sqrt{2}\pi)^{3/2}. \quad (80)$$

These wave functions are normalized by the condition

$$\int d\xi\, d\eta\, |\Psi_{3q}(\xi, \eta)|^2 = 1. \quad (81)$$

The 6*q* wave function

The construction of a $6q$ wave function with the quantum numbers of the deuteron was considered in Ref. 60. The deuteron is a color singlet, and therefore the $6q$ state must also be one. This will be the case if and only if the color part has the symmetry of the Young tableau $[2^3]$. However, the total wave function must be antisymmetric with respect to permutations of all quark pairs. If the ground state is taken as the orbital part of the $6q$ state, the color spin–flavor part must be completely antisymmetric with respect to quark permutations. The color spin–flavor wave function will be completely antisymmetric if and only if the spin–flavor part has symmetry conjugate to the symmetry of the color part, i.e., the symmetry $[3^2]$.

As follows from its symmetry, the color part is characterized by one of the five Y symbols (Yamanouchi symbol)[3)] of the symmetry $[2^3]$. The spin–flavor part is also characterized by one of the five Y symbols, but of the symmetry $[3^2]$, and also by spin $j = 1$, its z component, by isospin T, and by its z component T_3. Since only the constituent quarks u and d occur in the $6q$ state of the deuteron, it is sufficient to take the isospin group $SU(2)_T$ as the flavor group. Below, we shall denote the color and spin–isospin wave functions by $|6q; C_2 = 0, Y\rangle$ and $|6q; j = 1, j_3; T = 0; Y\rangle$, respectively; here C_2 is the eigenvalue of the square of the Casimir operator. The completely antisymmetric color spin-isospin wave function is

$$|6q;\ C_2=0;\ j=1,\ j_3;\ T=0\rangle = \sqrt{\frac{1}{5}}\sum_Y \Lambda_Y|6q;\ C_2=0;\ Y\rangle|6q;\ j=1,\ j_3;\ T=0;\ \tilde{Y}\rangle, \quad (82)$$

where $\Lambda_Y/\sqrt{5}$ is the Clebsch–Gordan coefficient of the symmetric group S_6 for the completely antisymmetric state in the inner product of the representations $[3^2] \times [2^3]$; Λ_Y is a phase factor, equal to -1 if Y is an even and $+1$ if Y is an odd permutation of the numbers in $Y = [332211]$; $\tilde{Y}$ is the symbol conjugate to the Y symbol Y, i.e., $\tilde{Y}$ is obtained from Y if in the corresponding partitioning of the Young tableau the positions of the rows and columns are interchanged (for example, $Y = [323121]$, $\tilde{Y} = [312211]$).

One can decompose the spin–isospin wave functions into inner products of spin and isospin wave functions. The spin and isospin wave functions satisfy separately the symmetries $[4, 2]$ and $[3^2]$, which ensures the required values of the spin and isospin. Thus, the spin part of the wave function is characterized by spin $j = 1$, its z projection, and one of the nine Y symbols of the Young tableau $[4, 2]$, and the isospin part is characterized by isospin $T = 0$, $T_3 = 0$, and one of the five Y symbols of the tableau $[3^2]$. The total spin–isospin wave function has the form

$$|j=1,\ j_3;\ T=0;\ Y\rangle = \sum_{Y_j,\, Y_t} S^{[3^2][3^2][4,2]}_{Y\ \ Y_t\ \ Y_j}\,|T=0;\ Y_t\rangle\,|j=1,\ j_3;\ Y_j\rangle, \quad (83)$$

where $S^{\alpha\ \beta\ \gamma}_{Y_\alpha Y_\beta Y_\gamma}$ are Clebsch–Gordan coefficients of the symmetric group. For the spin and isospin wave functions we introduce the notation $|j = 1, j_3; Y_j\rangle$ and $|T = 0; Y_t\rangle$. Tables of the corresponding Clebsch–Gordan coefficients can be found, for example, in Ref. 60.

The spin and isospin wave functions can be found from the rules of addition of angular momenta (for more details, see Appendix 1 in Ref. (60).

---

3)We refer the reader to the book of Ref. 67, in which all the group-theroetical concepts of this section are given.

The total $6q$ wave function has the form

$$|6q\rangle = |6q;\ C_2=0;\ j=1,\ j_3;\ T=0 > \phi_{6q}(\mathbf{r}, \xi, \eta, \mu, \nu). \quad (84)$$

Note that in such an approach the np and $6q$ states are, strictly speaking, nonorthogonal. However, a numerical calculation shows that the contribution of the interference term to the various quantities is negligibly small. For example, it makes a contribution of less than 0.1% to the deuteron form factors.

The orbital wave function $\phi_{6q}(\mathbf{r}, \xi, \eta, \mu, \nu)$ is the ground state of the relativistic oscillator equation in light-front variables:

$$\left.\begin{aligned}\phi_{6q}(\mathbf{r}, \xi, \eta, \mu, \nu) &= N_6 \exp\{-3\omega(\mathbf{r}^2/4+\xi^2+\eta^2+\mu^2+\nu^2)\};\\ N_6 &= (1/6)\sqrt{1/2}\,(6\omega/\pi)^{15/4}.\end{aligned}\right\} \quad (85)$$

The normalization condition is

$$\int d\mathbf{r}\, d\xi\, d\eta\, d\mu\, d\nu\, |\phi_{6q}(\mathbf{r}, \xi, \eta, \mu, \nu)|^2 = 1. \quad (86)$$

The "composition" of the 6q component of the deuteron

The spin–isospin color part of the wave function of the $6q$ state can be decomposed[59] into a sum of different $3q$ pairs. It can be shown that such a decomposition contains not only true, i.e., colorless, baryons but also colored $3q$ systems (hidden color). The probability of finding such states in the $6q$ state can be readily found by using the decomposition (82). Indeed, of the five $6q$ color wave functions $|6q; C_2=0; Y\rangle$ in (82) distinguished by the Yamanouchi number, four will be orthogonal to the wave function $|3q; C_2'=0; Y=[321]\rangle\,|3q; C_2''=0; Y=[321]\rangle$ and only for one will the overlap be nonzero:

$$\begin{aligned}&\langle 3q;\ C_2'=0;\ Y'=[321]\ |\ \langle 3q;\ C_2''=0;\\ &Y=[321]\ |\ 6q;\ C_2=0,\ Y=[321\ 341]\rangle = 1.\end{aligned} \quad (87)$$

Thus, the probability of detecting in the $6q$ state states with hidden color is 4/5.

The spin–isospin color part of the wave function of the $6q$ component of the deuteron contains not only "nucleons" but also "isobars." As a result, we have the following set of $3q$ pairs in the $6q$ state with the quantum numbers of the deuteron: 11% for np, 9% for $\Delta\Delta$, and 80% for hidden color. The estimates obtained[68] in the $6q$ model for the probability of finding $\Delta\Delta$ pairs in the deuteron do not contradict the experimental upper limit obtained for it.

The presence of hidden color in the deuteron $6q$ component leads to additional production of Δ isobars in inclusive reactions compared with quasi-two-particle breakup of the deuteron,[69] namely, in quasi-two-particle reactions Δ isobars previously "prepared" in the deuteron are produced. In inclusive reactions, an additional number of isobars from the following process may be produced. A color octet with isospin 3/2 (Δ^8) is knocked out of the part of the $6q$ component with hidden color. Then, emitting a soft gluon, the Δ^8 becomes a real isobar. Thus, the number of Δ isobars produced in the inclusive reactions must exceed by more than two times their number in quasi-two-particle knockout reactions on the deuteron. This prediction is in qualitative agreement with experiment.[70]

Orbitally excited 6q systems

So far, we have only considered a $6q$ system in which all six quarks are in the s state (s^6). However, as was shown in Ref. 71, another configuration, in which four quarks are in the s state and two in the p state (s^4p^2), can determine the deuteron structure at short distances on an equal footing with the s^6 configuration.

The mass of the $6q$ state is determined as the sum

$$M(6q) = M_0(6q) + M_{\text{UM}}(6q), \quad (88)$$

where M_0 is the mass determined by one of the dynamical models: the oscillator quark model, the bag model, etc. This mass depends only on the radial excitation modes. Thus, for the nonrelativistic harmonic-oscillator model[4)]

$$M_0 = N\Omega + \text{const}, \quad (89)$$

where $N=0$ for the s^6 configuration and $N=2$ for the s^4p^2; M_{cm} is the correction for the chromomagnetic qq interaction[47]:

$$M_{\text{cm}} = -3\alpha_s \sum_{i<j} \sum_{a=1}^{b} \langle 6q\ |\ \lambda_a^{(i)}\lambda_a^{(j)}\sigma^{(i)}\sigma^{(j)} M_{ij}, \quad (90)$$

where $\alpha_s = g_s^2/4\pi$, g_s is the quark–gluon coupling constant, $\lambda_a^{(i)}$ are the generators of the color group $SU(3)_C$, and $|6q\rangle$ are the unperturbed wave functions of the $6q$ system, i.e., wave functions that are solutions of the dynamical equations with the eigenvalues $M_0(6q)$.

The presence in the mass formula (88) of the chromomagnetic term leads to a splitting of the levels corresponding to one value of N.

The wave function $|6q\rangle$ for the s^6 configuration is constructed in Ref. 60. For more complicated configurations, the $|6q\rangle$ are constructed similarly.[73] The completely antisymmetric wave function of the $6q$ system is

$$\begin{aligned}|6q\rangle = &\sum_{Y=1}^{n_f} \frac{\Lambda_Y}{\sqrt{n_f}}\, \phi_{6q;\,[f]Y\omega_X}(\mathbf{r}, \xi, \eta, \mu, \nu)\\ &\times |6q\,[\tilde f];\ C_2=0;\ s,\ s_3;\ T,\ T_3;\ \omega_{CST};\ \tilde Y\rangle,\end{aligned} \quad (91)$$

where the orbital wave function $\phi_{6q;[f]Y\omega X}(\cdots)$ is determined by the symmetry type $[f]$ (Young tableau) with respect to permutation of the particles, by the Yamanouchi symbol Y, and the repetition index ω_X; the spin–isospin color wave function is determined by the symmetry $[\tilde f]$ conjugate to the symmetry $[f]$ of the orbital wave function, by the Yamanouchi symbol $\tilde Y$ (conjugate to Y), by the values of the spin s and its projection s_3, and by the isospin T and its projection T_3, and it must be a color singlet, i.e., its color part must have the symmetry $[2^3]$; ω_{CST} are additional quantum numbers that characterize the spin–isospin color function; n_f is the dimension of the representation of the symmetric group S_6 corresponding to the Young tableau $[f]$; $\Lambda_Y/\sqrt{n_f}$ are the Clebsch–Gordan coefficients of the symmetric group S_6 for the completely symmetric state in the inner product of the Young tableaux $[f]\times[\tilde f]$,

---

4) In the relativistic three-dimensional oscillator model (see Sec. 1) there is an analogous expression for the square of the mass. In this case, Eq. (88) is obtained when the spin–orbit and tensor interactions are ignored.[72]

$\Lambda_Y = \pm 1$.

For the s^6 configuration $[f] = [6]$, $n_6 = 1$, the repetition indices ω_X and ω_{CST} have only one value, and the expression (91) goes over into (84).

The spin–isospin color wave function $|6q; [f]; C_2 = 0; s, s_3; T, T_3; \omega_{CST}; Y\rangle$ is constructed by means of the Clebsch–Gordan coefficients of the group S_6 from the spin, isospin, and color wave functions, which have the symmetries $[(s+3),(s-3)]$, $[(T+3),(T-3)]$, and $[2^3]$, respectively. Tables of the corresponding Clebsch–Gordan coefficients for the configuration s^4p^2 can be found in Ref. 73.

Because the chromomagnetic term M_{cm} for the s^4p^2 configurations with the quantum numbers of the deuteron is negative, and for the s^6 it is positive, it may partly or even completely compensate the difference $M_0(s^4p^2) - M_0(s^6)$ of the color-unperturbed terms (88) for these states. Thus, the calculation of Ref. 71 in the framework of the MIT bag led to

$$M(s_{1/2}^6) = 2165\ \text{MeV}/c^2; \quad M(s_{1/2}^4, p_{1/2}^2) = 2379\ \text{MeV}/c^2, \tag{92}$$

whereas $\Delta M_0 = 538\ \text{MeV}/c^2$. An analogous calculation in the framework of the nonrelativistic oscillator quark model shows that the s^4p^2 state may even be lighter than the ground s^6:

$$M_{\text{osc}}(s^6) = 2380\ \text{MeV}/c^2; \quad M_{\text{osc}}(s^4p^2) = 2160\ \text{MeV}/c^2. \tag{93}$$

Such a phenomenon was also considered in Ref. 74 for systems consisting of an arbitrary number n of quarks and it was shown that from a certain value of n the phenomenon of chromomagnetic collapse can occur.

3. ELECTRON–DEUTERON SCATTERING AND PROBLEMS OF THE QUARK STRUCTURE OF THE DEUTERON

We now discuss the part played by the $6q$ component of the deuteron in electron–deuteron scattering and in deuteron fragmentation on nuclei. It should be noted that in nuclear physics there are approaches—the introduction of exchange currents, introduction of a soft core in the NN potentials, etc.—that may take into account partly the contributions from the many-quark phase of the nucleus. Therefore, we shall compare such approaches with the quark approaches and attempt to find the effects that cannot be explained by the traditional methods of nuclear physics.

We begin with a discussion of high-energy electron–deuteron scattering. The study of elastic and inelastic scattering of electrons by particles and nuclei has always been one of the most reliable methods of investigating their structure, and therefore the attempt to investigate the structure of the deuteron by the same method is very natural.

Kinematics of elastic *ed* scattering and the nonrelativistic impulse approximation

Elastic *ed* scattering is completely described by three form factors—the electric $F_C(q^2)$, the quadrupole $F_Q(q^2)$, and the magnetic $F_M(q^2)$. These are combinations of the form factors $F_1(q^2)$, $F_2(q^2)$, and $G(q^2)$, which in the Breit system (Fig. 2) are determined as follows in terms of the matrix elements of the electromagnetic current[75]:

$$\langle \mathbf{q}/2, \lambda' = 0 | J_1(0) | -\mathbf{q}/2, \lambda = 1 \rangle = (q/\sqrt{2}E)\, G(q^2); \tag{94}$$

$$\langle \mathbf{q}/2, \lambda' = -1 | J_0(0) | -\mathbf{q}/2, \lambda = 1 \rangle = F_1(q^2); \tag{95}$$

$$\begin{aligned} \langle \mathbf{q}/2, \lambda' = 0 | J_0(0) | -\mathbf{q}/2, \lambda = 0 \rangle &= F_1(q^2) + (q^2/2M^2)\,[F_1(q^2) \\ &+ (1 + q^2/4M^2)\, F_2(q^2) \\ &- 2(1 + q^2/4M^2)\, G(q^2)], \end{aligned} \tag{96}$$

where λ and λ' are the polarizations of the deuteron, m is the nucleon mass, and M is the deuteron mass. The charge $F_C(q^2)$, quadrupole $F_Q(q^2)$, and magnetic $F_M(q^2)$ form factors can be expressed in terms of the form factors $F_1(q^2)$, $F_2(q^2)$, and $G(q^2)$:

$$F_C = F_1 + \frac{q^2}{6M^2}[F_1 + (1 + q^2/4M^2)\,F_2 - 2(1 + q^2/4M^2)\,G]; \tag{97}$$

$$F_Q = F_1 + (1 + q^2/4M^2)\,F_2 - 2(1 + q^2/4M^2)\,G; \tag{98}$$

$$F_M = G. \tag{99}$$

If the electrons and deuterons are not polarized, then the differential cross section of elastic *ed* scattering can be expressed as follows in terms of the squares of the form factors F_C, F_Q, and F_M:

$$(d\sigma/d\Omega_{\text{lab}}) = (d\sigma_{\text{Mott}}/d\Omega_{\text{lab}})\,\{A(q^2) + B(q^2)\tan^2\theta/2\}, \tag{100}$$

where

$$A(q^2) = F_C^2 + \frac{q^4}{18M^4}F_Q^2 + \frac{q^2}{6m^2}(1 + q^2/4M^2)\,F_M^2; \tag{101}$$

$$B(q^2) = \frac{q^2}{3m^2}(1 + q^2/4M^2)^2\,F_M^2 \tag{102}$$

and θ is the scattering angle of the electron in the laboratory system.

In the nonrelativistic impulse approximation (Fig. 3)

$$F_C(q^2) = F(q^2)\,[L(q^2) + 8K(q^2)]; \tag{103}$$

$$F_Q(q^2) = 12F(q^2)\,(M^2/q^2)\,[M(q^2) - R(q^2)]; \tag{104}$$

$$\begin{aligned} F_M(q^2) = F(q^2)\,\{(\mu_p + \mu_n)\,[L(q^2) - 4K(q^2) + \\ + 2M(q^2) + R(q^2)] + R(q^2) + 6K(q^2) + 3M(q^2)\}, \end{aligned} \tag{105}$$

where $F(q^2)$ is the proton electric form factor. It is assumed here that for the electromagnetic form factors of the nucleons the following relations hold:

$$G_E^p(q^2) \equiv F(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n}; \; G_E^n(q^2) = 0. \tag{106}$$

For the proton electric form factor one can use either the empirical dipole fit

$$F(q^2) = (1 + q^2/0{,}71\,[(\text{GeV}/c)^2])^{-2}, \tag{107}$$

or the expression that follows from the relativistic os-

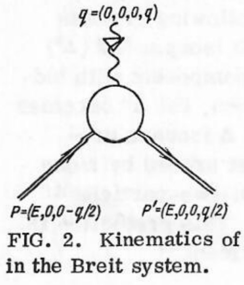

FIG. 2. Kinematics of elastic electron–deuteron scattering in the Breit system.

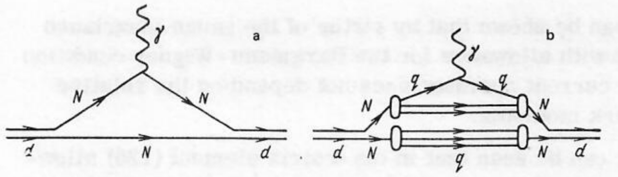

FIG. 3. Impulse approximation in nucleon language (a) and quark language (b).

cillator quark model[52, 55]:

$$F(q^2)=(1+q^2/m_\rho^2)^{-1}(1+q^2/4m^2)^{-1}\exp\{-q^2/[3\sqrt{2}\,\omega(1+q^2/4m^2)]\}. \tag{108}$$

In (103)–(105), the functions $L(q^2)$, $K(q^2)$, $M(q^2)$, and $R(q^2)$ can be expressed as follows in terms of the radial parts of the S and D deuteron wave functions:

$$L(q^2)=\int_0^\infty dr\, j_0(rq)\, u^2(r); \tag{109}$$

$$K(q^2)=\frac{1}{8}\int_0^\infty dr\, j_0(rq)\, w^2(r); \tag{110}$$

$$M(q^2)=\frac{1}{4}\int_0^\infty dr\, j_2(rq)\, w^2(r); \tag{111}$$

$$R(q^2)=\sqrt{\frac{1}{2}}\int_0^\infty dr\, j_2(rq)\, w(r)\, u(r). \tag{112}$$

At the present time, there are no data on measurement of all three electromagnetic form factors of the deuteron. The problem is that in accordance with (100) experiments with unpolarized electrons and deuterons measure only the combination $A(q^2)$ and the magnetic form factor, which is uniquely determined by the function $B(q^2)$.

The combination $A(q^2)$ has been measured[10] up to values $q^2 = 4\ (\mathrm{GeV}/c)^2$. There are also preliminary data[14] on an "indirect" measurement of it at $q^2 = 6$ and $8\ (\mathrm{GeV}/c)^2$. The behavior of the function $A(q^2)$ in the region $q^2 < 1.5\ (\mathrm{GeV}/c)^2$ can be well described by the nonrelativistic impulse approximation (103)–(105). At large values of q^2, the theoretical estimates depend strongly both on the choice of the NN potential and on the parametrization of the electromagnetic form factors of the nucleons.[76]

Various relativistic effects have been taken into account on occasion.[77, 78] It is found that the data on $A(q^2)$ up to $q^2 \approx 4\ (\mathrm{GeV}/c)^2$ can be described only with a very specific choice of the nucleon form factors.

Electromagnetic form factors of the deuteron in the hybrid model

In the hybrid model, the electromagnetic form factors of the deuteron were calculated in Refs. 60 and 78–83. In Ref. 60, the calculation was made in the framework of the relativistic three-dimensional oscillator quark model. For this, relativistic matrix elements of the electromagnetic current of the type (71) were used; in the Breit system, they take the form

$$\langle -\mathbf{P}|J_\mu(0)|\mathbf{P}\rangle = 6\left(\frac{M}{E}\right)^5\int \delta\left(\frac{Pp}{M}\right)\delta\left(\frac{P'p'}{M}\right)\delta\left(\frac{P\tilde{\xi}}{M}\right)\delta\left(\frac{P'\tilde{\xi}'}{M}\right)$$
$$\times\,\delta\left(\frac{P\tilde{\eta}}{M}\right)\delta\left(\frac{P'\tilde{\eta}'}{M}\right)\delta\left(\frac{P\tilde{\mu}}{M}\right)\delta\left(\frac{P'\tilde{\mu}'}{M}\right)\delta\left(\frac{P\tilde{\nu}}{M}\right)\delta\left(\frac{P'\nu'}{M}\right)$$
$$\times\prod_{j=2}^{6}\delta(\mathbf{p}^{(j)}-\mathbf{p}'^{(j)})\,\overline{\Psi}_D(p',\tilde{\xi}',\tilde{\eta}',\tilde{\mu}',\tilde{\nu}')\,\Gamma_\mu^{(1)}\,\Psi_D(p,\tilde{\xi},\tilde{\eta},\tilde{\mu},\tilde{\nu})$$
$$\times\, d^4p\, d^4p'\, d^4\tilde{\xi}\, d^4\tilde{\xi}'\, d^4\tilde{\eta}\, d^4\tilde{\eta}'\, d^4\tilde{\mu}\, d^4\tilde{\mu}'\, d^4\tilde{\nu}\, d^4\tilde{\nu}',$$
$$\mathbf{P}=(0,\ 0,\ -q/2);\ E=\sqrt{M^2+P^2};\ p,\ \tilde{\xi},\ \tilde{\eta},\ \tilde{\mu},\ \tilde{\nu}\ \text{и}\ p',\ \tilde{\xi}',\ \tilde{\eta}',\ \tilde{\mu}', \tag{113}$$

where $\mathbf{P} = (0, 0, -q/2)$; $E = \sqrt{M^2 + P^2}$; $p, \tilde{\xi}, \tilde{\eta}, \tilde{\mu}, \tilde{\nu}$ and $p', \tilde{\xi}', \tilde{\eta}', \tilde{\mu}', \tilde{\nu}'$ are the quantities canonically conjugate to the relative coordinates; Ψ_D is the wave function (75) of the hybrid model in the momentum representation.

It follows from (113) that

$$\langle -\mathbf{P}|J_\mu(0)|\mathbf{P}\rangle = \alpha^2\langle -\mathbf{P};\ np|J_\mu(0)|\mathbf{P};\ np\rangle + \beta^2\langle -\mathbf{P};\ 6q|J_\mu(0)|\mathbf{P};\ 6q\rangle + \text{interference term}. \tag{114}$$

The first term in (114) is the contribution to the matrix element from the np component of the wave function and corresponds to the diagram in Fig. 3b. In the region up to $q^2 \approx 1.5\ (\mathrm{GeV}/c)^2$ it is effectively equal to the nonrelativistic matrix element given above with electromagnetic form factors of the nucleons in the parametrization (106)–(108); the difference between the nonrelativistic and relativistic matrix element is proportional to terms of order $q^2/4M^2$ (see below). The second term in (114) is the contribution to the matrix element of the electromagnetic current of the deuteron from the $6q$ component of the deuteron wave function and corresponds to the diagram in Fig. 4. In what follows, it is this contribution in which we shall be interested. The interference term depends strongly on the choice of the np wave function. For wave functions with a hard[63–65] and even with a soft core,[65, 68] it makes a negligibly small contribution to the deuteron form factors[60, 82] because of the approximate orthogonality of the np and $6q$ wave functions. Therefore, in what follows we shall ingore it.

Thus, we consider the contribution of the $6q$ component of the wave function to the matrix element. As a result of integration, we have

$$\langle -\mathbf{P};\ 6q|J_\mu(0)|\mathbf{P};\ 6q\rangle = 6\left(\frac{M}{E}\right)^{15}\int \bar{\phi}_{6q}(\mathbf{p},\ \tilde{\xi}',\ \tilde{\eta}',\ \tilde{\mu}',\ \tilde{\nu}')\times\Gamma_\mu^{(1)}(\mathbf{q},\ \mathbf{p},\ \tilde{\xi})\,\phi_{6q}(\mathbf{p},\ \tilde{\xi},\ \tilde{\eta},\ \tilde{\mu},\ \tilde{\nu})\,d\mathbf{p}\,d\tilde{\xi}\,d\tilde{\eta}\,d\tilde{\mu}\,d\tilde{\nu}, \tag{115}$$

where $\phi_{6q}(\mathbf{p},\ldots,\tilde{\nu})$ and $\phi_{6q}(\mathbf{p}',\ldots,\tilde{\nu}')$ are obtained from the relativistic wave functions $\Psi_{6q}(p,\ldots,\tilde{\nu})$ and $\Psi_{6q}(p',\ldots,\tilde{\nu}')$ by imposing on the zeroth components of the relative 4-momenta the Markov–Yukawa conditions

$$p_0 = P_3 p_3/E;\ \tilde{\xi}_0 = P_3\tilde{\xi}_3/E,\ \ldots;\ \tilde{\nu}_0 = P_3\tilde{\nu}_3/E; \tag{116}$$

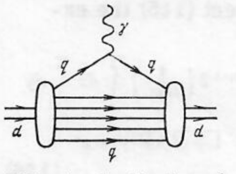

FIG. 4. Scattering of a virtual photon by the $6q$ component of the deuteron.

$$p'_0 = -P_3 p'_3/E;\ \tilde{\xi}'_0 = -P_3\tilde{\xi}'_3/E,\ \ldots;\ \tilde{v}'_0 = -P_3\tilde{v}'_3/E. \quad (117)$$

At the same time,

$$\mathbf{p}' = \mathbf{p} + \mathbf{P}; \quad (118)$$

$$\tilde{\xi}' = \tilde{\xi} + 4\mathbf{P}; \quad (119)$$

$$\tilde{\mu}' = \tilde{\mu};\ \tilde{\eta}' = \tilde{\eta};\ \tilde{v}' = \tilde{v}. \quad (120)$$

The current operator $\Gamma^{(1)}_\mu(\mathbf{q}, \mathbf{p}, \xi)$ is obtained by imposing the restrictions (116)–(119) on the current operator $\Gamma^{(1)}_\mu(p^{(1)}, p'^{(1)})$, which follows from minimal electromagnetic coupling in the harmonic-oscillator equation[53,55]:

$$\Gamma^{(1)}_\mu(p^{(1)}, p'^{(1)}) = (3/M)\, Q^{(1)}(\hat{p}'^{(1)}\gamma^{(1)}_\mu + \gamma^{(1)}_\mu \hat{p}^{(1)}). \quad (121)$$

Here, the factor $3/M$ is chosen from the condition which ensures that at $q^2 = 0$ the zeroth component of the electromagnetic current (115) is equal to the charge of the deuteron; in other words, the charge form factor determined by (94) and (95) is equal to the charge of the deuteron at $q^2 = 0$.

Indeed, by virtue of the Bargmann–Wigner conditions [in contrast to (33), there will be six and not two of them in the present case]

$$(P\gamma^{(1)} - M)\ \varphi_{6q}(\ldots) = (P\gamma^{(2)} - M)\ \varphi_{6q}(\ldots) \ldots = (P\gamma^{(6)} - M)\ \varphi_{6q}(\ldots) = 0 \quad (122)$$

the spinors at $q^2 = 0$, i.e., for $\mathbf{P} = \mathbf{P}' = 0$, have only upper components. Then the expectation value of the operator $\Gamma^{(1)}_0(0, \mathbf{p}, \tilde{\xi})$ between the $6q$ wave functions will be

$$\begin{aligned}&\bar{\varphi}_{6q}(\mathbf{p}, \tilde{\xi}, \ldots, \tilde{v})\, \Gamma^{(1)}_0(0, \mathbf{p}, \tilde{\xi})\, \varphi_{6q}(p, \tilde{\xi}, \ldots, \tilde{v})\\ &= (3/M)\, \varphi^*_{6q}(\mathbf{p}, \tilde{\xi}, \ldots, \tilde{v})\, \varphi_{6q}(\mathbf{p}, \tilde{\xi}, \ldots, \tilde{v})\, (p^{(1)}_0 + p'^{(1)}_0)\\ &\times \langle 6q;\ C_2 = 0,\ j = 1,\ j_3;\ T = 0 \mid \hat{Q}^{(1)} \mid 6q;\ C_2 = 0,\ j = 1,\ j_3;\ T = 0\rangle,\end{aligned} \quad (123)$$

where $\varphi_{6q}(\ldots)$ is the orbital and $|6q; C_2 = 0; j = 1; j_3; T = 0\rangle$ the spin–isospin–color parts of the $6q$ wave function. As a result of (118) and the Markov–Yukawa conditions (116) and (117), $p^{(1)}_0 + p'^{(2)}_0 = M/3$ for $q^2 = 0$. The expectation value of the operator $\hat{Q}^{(1)}$ with respect to the spin–isospin–color wave function is 1/6 of the deuteron charge by virtue of the complete antisymmetry of this wave function with respect to the interchange of any pair of quarks. Substituting (123) in (117), we obtain the required equation as a result of normalization of the orbital wave functions.

In view of the restrictions (116)–(120) for the initial and final states, we have

$$\phi_{6q}(\mathbf{p}, \tilde{\xi}, \ldots, \tilde{v}) = \tilde{N}_6 \exp\left\{-\frac{1}{12\omega}\left[4\frac{M^2}{E^2}p_3^2 + 4\mathbf{p}^2_\perp + \frac{M^2}{E^2}\tilde{\xi}^2_3 + \tilde{\xi}^2_\perp + \ldots + \frac{M^2}{E^2}\tilde{v}^2_3 + \tilde{v}^2_\perp\right]\right\}; \quad (124)$$

$$\phi_{6q}(\mathbf{p}', \tilde{\xi}', \ldots, \tilde{v}') = \tilde{N}_6 \exp\left\{-\frac{1}{12\omega}\left[4\frac{M^2}{E^2}(p_3 - q/2)^2 + 4\mathbf{p}^2_\perp + \frac{M^2}{E^2}(\tilde{\xi}_3 - 2q)^2 + \tilde{\xi}^2_\perp + \frac{M^2}{E^2}\tilde{\eta}^2_3 + \tilde{\eta}^2_\perp + \ldots + \frac{M^2}{E^2}\tilde{v}^2_3 + \tilde{v}^2_\perp\right]\right\}, \quad (125)$$

where $\tilde{N}_6 = 2\sqrt{2}(1/6\omega\pi)^{15/4}$.

Integrating over all variables except p_3 and ξ_3 and making the change of variables $x = (M/E)p_3$ and $y = (M/E)\tilde{\xi}_3$, we obtain for the matrix element (115) the expression

$$\begin{aligned}\langle \mathbf{q}/2;\ 6q| J_\mu(0) |-\mathbf{q}/2;\ 6q\rangle &= 6\,(1 + q^2/4M^2)^{-5}\, 2\left(\frac{1}{6\omega\pi}\right)\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\\ &\times \exp\left\{-\frac{1}{12\omega}\left[4x^2 + 4\left(x - q/2\sqrt{1 + q^2/4M^2}\right)^2 + y^2 + \left(y^2 - 2q/\sqrt{1 + q^2/4M^2}\right)^2\right]\right\}\langle 6q;\ C_2 = 0;\ j = 1,\ j'_3;\\ &T = 0|\Gamma_\mu(0)|6q;\ C_2 = 0;\ j = 1,\ j_3;\ T = 0\rangle.\end{aligned} \quad (126)$$

It can be shown that by virtue of the gauge invariance and with allowance for the Bargmann–Wigner condition the current operator does not depend on the relative quark momenta.

It can be seen that in the matrix element (126) allowance for the relativistic effects has led, first, to the appearance of the factor $(1 + q^2/4M^2)^{-5}$ multiplying the matrix element, this ensuring decrease of the form factors in agreement with the quark-counting rules,[20,21] and, second, to a Lorentz contraction of the argument of the nonrelativistic form factor ($q \to q/\sqrt{1 + q^2/4M^2}$). One can also show[60] that this holds for a larger class of wave functions.

Calculating the matrix elements (126) for the transitions (94)–(96) and using the definitions (97)–(99), we obtain for the charge, quadrupole, and magnetic form factors of the $6q$ system

$$F^{(6q)}_C(q^2) = (1 + q^2/4M^2)^{-5}\exp\left[-\frac{5}{24\omega}\frac{q^2}{1 + q^2/4M^2}\right]; \quad (127)$$

$$F^{(6q)}_Q(q^2) = 0; \quad (128)$$

$$F^{(6q)}_M(q^2) = \mu_{6q}F^{(6q)}_C(q^2); \quad (129)$$

$$\mu_{6q} = 1 \text{ — the nuclear magneton.} \quad (130)$$

It was shown in Refs. 52 and 55 that the description of the data on the electromagnetic and weak form factors of the nucleons and the data on the cross sections of electroproduction and weak production of resonances is significantly improved if one uses "vector dominance." In this case, it reduces to making the following substitution for one of the factors in front of the form factors (127) and (130):

$$(1 + q^2/4M^2)^{-1} \to (1 + q^2/m^2_\rho)^{-1}. \quad (131)$$

It is found[60] that this procedure also significantly improves the agreement between the hybrid model and experiment.

Comparison of the hybrid model with data on the electromagnetic form factors of the deuteron

In accordance with (115), the deuteron form factors have the decomposition

$$F_i(q^2) = (1 - \beta^2)\, F^{(np)}_i(q^2) + \beta^2 F^{(6q)}_i(q^2), \quad (132)$$

where $i = C, Q, M$. In (132), the contributions of the interference form factors are ignored. To calculate the form factors $F^{(np)}_i(q^2)$ of the np system, it is necessary to use the impulse approximation given above. The form factors $F^{(6q)}_i(q^2)$ of the $6q$ system are determined in Eqs. (127)–(131). For obvious physical reasons, the parameter β, which determines the probability of finding a $6q$ component in the deuteron, must be appreciably smaller than unity. Hence, at comparatively small q^2 the behavior of the form factors must be entirely determined by the impulse approximation. As we have already said, it does indeed give a good description of the behavior of the function $A(q^2)$ in the region $q^2 < 1.5$ (GeV/c)^2. It also satisfactorily describes the behavior of the magnetic form factor in the region up to $q^2 \approx 0.6$ (GeV/c)^2. At larger q^2, the contribution of the $6q$ component is important.

The model contains two unknown parameters: the parameter ω of the oscillator potential and the probability β^2 of finding the $6q$ state in the deuteron. Their values are determined by fitting the data at large q^2: $\omega = 8.5\ \mathrm{F}^{-2}$, $\beta^2 = 0.025$. The value of ω is close to the value obtained by fitting[52] the electromagnetic form factors of the nucleons: $\omega = 7.9\ \mathrm{F}^{-2}$.

The contributions of the different mechanisms to the charge form factor of the deuteron are shown in Fig. 5. In all nonquark approaches (impulse approximation, exchange currents,[84] etc.) the presence of the core of the NN potential causes the charge form factor to change sign in the region $q^2 \approx 0.6\ (\mathrm{GeV}/c)^2$. In the hybrid model, the region of the core is "occupied" by the $6q$ phase. As a result, $F_C(q^2)$ does not change sign in this model.[62] Experimental investigation of this must be an important qualitative test of the hypothesis of the $6q$ nature of the deuteron at short distances.

In Fig. 6, the results of calculating the function $A(q^2)$ in accordance with the hybrid model are compared with experiment. Relativization[77] of the Reid potential with a hard core (chain curve in Fig. 6) cannot explain the experimentally observed behavior of $A(q^2)$ at $q^2 > 1.5\ (\mathrm{GeV}/c)^2$. It has already been noted that by means of such calculations one can achieve agreement with experiment at $q^2 \approx 4\ (\mathrm{GeV}/c)^2$ only by using a specific behavior of the electric form factor of the neutron.[77,78] At the same time, the hybrid model gives a fairly good description of the behavior of $A(q^2)$ in the complete measured region. In addition, it should be noted that relativistic calculations without allowance for the $6q$ component of the deuteron contradict the "independent" measurements of $A(q^2)$ (broken curve in Fig. 6),[14] whereas calculations using the hybrid model agree well with them. The predictions of the hybrid model are fairly stable with respect to the choice of not only the nuclear potential but also the nucleon form factors. This is so because different potentials and fits for the nucleon form factors lead to appreciable differences in the behavior of the np-state form factors only at comparatively large q^2, i.e., in the region in which the behavior of the deuteron form factors is basically determined by the $6q$ component.

The results of calculations of the function $B(q^2)$ are compared with experiment[12] in Fig. 7. In the region $q^2 \leqslant 1\ (\mathrm{GeV}/c)^2$, the results hardly differ for any choice of fit for the nucleon form factors—the one that follows

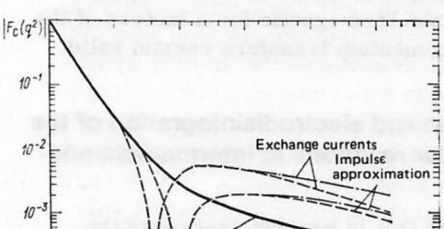

FIG. 5. Charge form factor of the deuteron in the hybrid model,[60] in the impulse approximation, and in the model with exchange currents.[84] The broken curves correspond to the fit of Ref. 85 for the nucleon form factors; the chain curves correspond to the dipole fit.

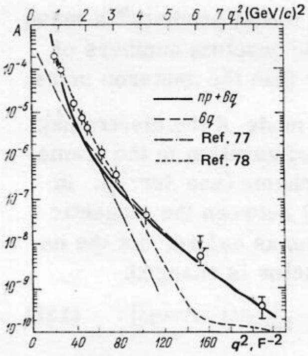

FIG. 6. The function $A(q^2)$ in the hybrid model[60] (continuous curve) and in the impulse approximation with relativized Reid wave function[77] (broken curve). The open-circle data are from Ref. 14.

from the relativistic oscillator quark model or the ordinary dipole fit. It should be noted especially that the different models used to describe the electromagnetic form factors of the deuteron lead to very different results for the magnetic form factor already at $q^2 \approx 1\ (\mathrm{GeV}/c)^2$.

A calculation of the magnetic form factor based on the hybrid wave function of the deuteron in the framework of the MIT bag model is made in Ref. 83. There is a discussion of the predictions of the hybrid model and the model with exchange currents for the deuteron magnetic form factor.

In Ref. 79, the behavior of $A(q^2)$ in the region $q^2 > 0.8\ (\mathrm{GeV}/c)^2$ is described by means of the expressions

$$A(q^2) = F_D^2(q^2); \tag{133}$$

$$F_D(q^2) = |\beta|^2 (1 + q^2/2M_D^2)^{-5} \exp\left\{-\frac{5}{4\alpha_D}\frac{q^2}{1+q^2/2M_D^2}\right\}, \tag{134}$$

which follow from the relativistic four-dimensional oscillator quark model with scalar quarks. The value of α_D, i.e., the parameter that determines the size of the $6q$ state, is found from data on the elastic electromagnetic form factors of the nucleons $[\alpha_D = 1.4\ (\mathrm{GeV}/c)^2]$, and $|\beta|^2$ and the "mass of a possible deuteron multiplet" M_D are assumed to be fitted: $\beta^2 = 0.07$ and $M_D = 1.2\ \mathrm{GeV}/c^2$. Thus, M_D is significantly smaller than the deuteron mass. It seems to us that such a value of M_D cannot be regarded as physically well justified. We

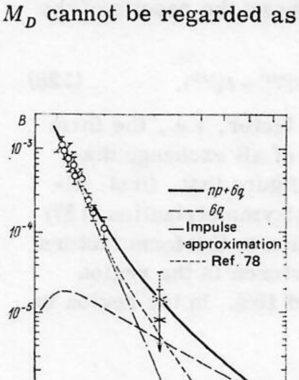

FIG. 7. The function $B(q^2)$ in the hybrid model. Experimental points: open circles from Ref. 86, crosses from Ref. 12 (the latter with allowance for the result of Refs. 86 and 87).

note, for example, that the bag model predicts[59] a mass of the s-wave $6q$ system with the quantum numbers of the deuteron 270 MeV/c^2 higher than the deuteron mass.

In Ref. 80, calculations were made of the electromagnetic form factors for the s^6 configuration in the framework of the minimal boosting scheme (see Sec. 1). In this case, the relationship (129) between the magnetic and charge form factors remains as before, but the expression for the charge form factor is changed:

$$F_C^{(6q)}(q^2)=(1+7q^2/4M_D^2)(1+q^2/2M_D^2)^{-6}\exp\left\{-\frac{5}{24\omega}q^2/(1+q^2/2M_D^2)\right\}. \qquad (135)$$

Since the mass M_D of the possible deuteron multiplet differs from the mass M of the real deuteron, the value of the magnetic moment of the $6q$ system is no longer equal to one nuclear magneton, as is obtained in the approach based on the Bargmann–Wigner equations, but is increased by a factor M/M_D. Because only the s-wave $6q$ state is considered, the quadrupole moment of the $6q$ system remains equal to zero. The values of the parameters of the model are obtained by fitting to experimental data: $|\beta|^2 = 0.05$, $M_D = 1.3$ GeV/c^2, $\omega = 4.58$ F^{-2}.

Exchange form factors of the deuteron

Strictly speaking, the np wave function considered above must be antisymmetrized with respect to the interchange of all pairs of quarks in different nucleons:

$$|np\rangle \rightarrow \hat{A}\,|np\rangle, \qquad (136)$$

where

$$\hat{A}=\sqrt{\frac{1}{10}}\Big(1-\sum_{i=1}^{3}\sum_{j=4}^{6}P_{ij}\Big); \qquad (137)$$

here, P_{ij} is the operator that interchanges quarks i and j.

Allowance for the antisymmetrization of the $|np\rangle$ wave function must lead to additional terms in the decomposition (132) of the deuteron form factors. Because of the action of $\hat{A}$ on the np wave function, the so-called *exchange diagrams*[82,88] shown in Fig. 8 are added to the diagrams in Figs. 3b and 4.

The contribution of the exchange diagrams to the deuteron form factors was estimated in Ref. 82 in nonrelativistic calculations. Figure 9 shows the results of the calculations of the ratio

$$R=[(F_C^{np}+F_C^{\rm exc})^2+(F_Q^{(np)}+F_Q^{\rm exc})^2]/(F_C^{(np)^2}+F_Q^{(np)^2}), \qquad (138)$$

where $F_i^{\rm exc}$ is the exchange form factor, i.e., the form factor corresponding to the sum of all exchange diagrams. It can be seen from the figure that, first, allowance for the effects of the antisymmetrization (137) leads to changes in the contribution to the form factors from the np component of the deuteron in the region $q^2 \lesssim 6$ (GeV/c)^2 that do not exceed 10%. In the region in

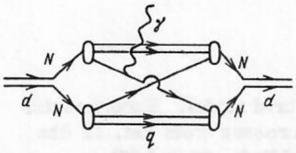

FIG. 8. Exchange quark diagram.

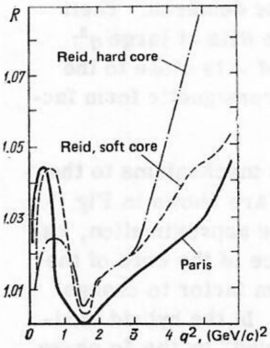

FIG. 9. Contribution of exchange form factors in the nonrelativistic approximation to the cross section of elastic ed scattering without allowance for the $6q$ component.[82]

which the contribution of the np component is dominant [$q^2 \lesssim 1.5$ (GeV/c)^2], the contribution of the exchange form factor is only a few percent. Second, the contribution from the exchange form factor depends strongly on the choice of the np wave function.

In contrast to the calculations of the deuteron form factors in the framework of the hybrid model, the deuteron wave function was not decomposed into two components in Ref. 88; instead, only the electromagnetic current of the neutron was split into the direct and exchange parts in accordance with the action of the antisymmetrizer (137). An expansion of the exchange contribution with respect to the $6q$ states was then considered. The contribution corresponding to the impulse approximation with respect to the nucleons was subtracted from the expansions with respect to the $6q$ configurations. The wave functions of the translationally invariant shell model[89] were used. To describe the exchange current in this case one can restrict the consideration to only two levels, corresponding to the principal quantum number $n = 0$ and 2. The calculations made in Ref. 88 show that the contributions of the exchange form factors corresponding to the s^6 and s^4p^2 configurations cancel each other, which leads to a negligibly small contribution of the exchange form factors to the function $A(q^2)$. However, in the presence of a small resonance enhancement of one of the $6q$ states the part played by the uncompensated contributions must increase strongly. Thus, the main conclusions of the hybrid model for the deuteron wave function to the effect that the $6q$ component plays a dominant part in the behavior of the electromagnetic form factors of the deuteron at large momentum transfers remain valid.

Photodisintegration and electrodisintegration of the deuteron and similar reactions at intermediate and low energies

It is quite possible that in various processes the many-quark degrees of freedom of a nucleus may be manifested at intermediate or even low energies. Photo- and electrodisintegration of the deuteron are processes of this kind. Thus, for the example of the similar reaction of radiative capture of protons by thermal neutrons it has been noted[22] that the $6q$ configurations must lead to a significant enhancement of the P-odd contributions to the NN interaction. Besides the s^6 configuration considered in this paper, such a phenomenon was discussed in Ref. 90 for the deuteron with allowance for two configurations: s^6 and s^4p^2.

Analysis of the data on the total cross sections of photodisintegration of the deuteron at $E_\gamma \leqslant 100$ MeV led to the conclusion in Ref. 91 that it is necessary to introduce a new non-nucleon state whose $E1$ contribution must be added incoherently to the impulse-approximation contribution. However, it should be noted that there are very serious objections[92] against the arguments brought forward in Ref. 91 for such a conclusion.

The process of backward deuteron electrodisintegration was studied in Ref. 83. The $6q$ component of the deuteron was described in the framework of the shell model. The MIT bag model was used to determine the quark wave functions. The three configurations s^6, s^4p^2, and s^5d^1 in the deuteron were taken into account. It was shown that in electrodisintegration at 180° in the q^2 region from 0.4 $(\text{GeV}/c)^2$ and above one must expect an appreciable influence of multiquark contributions to the cross section compared with the impulse approximation and the contributions of the pion and two-baryon currents.

In Refs. 93 and 94, a study is made of the influence of the $6q$ component of the deuteron on the cross section of the inclusive process

$$e + d \to p + X \tag{139}$$

with a proton emitted in the backward hemisphere in the region of quasielastic disintegration. In other words, a study is made of the cross section of the deuteron-electrodisintegration reaction integrated over the angles of the scattered electron.

The results of the nonrelativistic calculations of the cross section of this process are given in Fig. 10. It can be seen that without allowance for the $6q$ component the results of the calculation depend strongly on the choice of the phenomenological np wave function. At the same time, allowance for a $6q$ admixture of only 2% strongly "smooths" the discrepancy between the results of the calculations with different np wave functions. It can also be seen that the contribution of the $6q$ component is dominant at proton momenta $p \gtrsim 2.5\ \text{F}^{-1}$

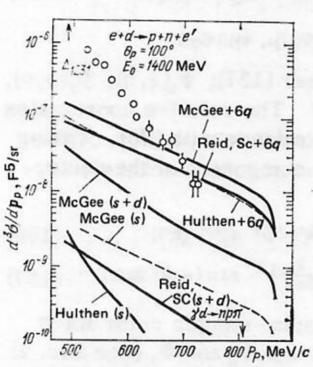

FIG. 10. Cross section for deuteron electrodisintegration in the hybrid model. The $6q$ contribution to the deuteron wave function has been taken to be 2%.

(the energy of the initial electron was chosen to be 1.4 GeV, and $\theta_p = 100°$). The experimental points in Fig. 10 are the result of conversion from the process of proton photoknockout from deuterons. With increasing proton momentum, they lie on the theoretical curve obtained with allowance for the $6q$ component of the deuteron. The region of the reaction (139) from the threshold of the process $\gamma d \to pn\pi$ (indicated by the arrow in Fig. 10) to the kinematic limit has not been investigated experimentally at all. But in fact this is the most interesting region, since in it the calculation can be made absolutely "cleanly." Indeed, the reaction is due solely to the process of deuteron electrodisintegration and in it the fragmentation nucleons have relative momentum of order 1.5 GeV/c, i.e., in it the effects of final-state interaction are unimportant.

4. STUDY OF THE QUARK STRUCTURE OF THE DEUTERON IN EXPERIMENTS ON THE FRAGMENTATION OF RELATIVISTIC DEUTERONS

Investigation of fragmentation processes of relativistic deuterons on nuclei,

$$d + A \to p + X, \tag{140}$$

are important, as the study of ed scattering at large momentum transfers, for establishing the nature of the high-momentum component of the deuteron.[15,78,95] In this section, we shall show that data on this process may be not only an independent but also a complementary source of information, in addition to elastic ed scattering, on the quark structure of the deuteron at short distances.

Fragmentation of unpolarized deuterons into protons and the hybrid model of the deuteron wave function

Questions related to the process of deuteron fragmentation on nuclei have been considered in various studies.[78,95–97] Bertocchi and Treleani[96] developed an approach to the description of this reaction in the framework of Sitenko–Glauber theory without allowance for multiquark degrees of freedom. It was found that in the framework of such a model one can successfully describe the data in the region of small (of order 1–2°) emission angles of the protons and for values $p_\parallel^* \lesssim 200$ MeV/c of the longitudinal proton momentum in the deuteron rest frame. They assumed that the $d \to p$ fragmentation cross section is a sum of the cross sections for disintegration of the deuteron on the nucleus ($d + A \to p + n + A'$) and the cross section for absorption of the neutron by the nucleus $[d + A \to p + (n + X) + A']$. They assumed that the wave function of the broken deuteron d' is a plane wave renormalized by a coefficient C_d, which effectively takes into account the nonorthogonality of the wave functions d and d'. It is determined in such a way as to separate from all the processes that result from the deuteron–nucleus collision those that lead to deuteron breakup:

$$C_d \approx (\sigma_{dA}^T - \sigma_{dA}^{\text{el}})/\sigma_{dA}^T = \sigma_{dA}^{\text{in}}/\sigma_{dA}^T, \tag{141}$$

where σ_{dA}^T is the total cross section of deuteron–nucleus scattering, and σ_{dA}^{el} is the sum of the coherent and incoherent elastic cross sections.[5)]

The invariant cross section of proton yield in the reaction (140) in the Bertocchi–Treleani model has the form

$$E_p \frac{d^3\sigma}{d\mathbf{p}} = C_d \Big[\phi^2 (p_d;\ \mathbf{p}_T,\ p_\parallel)\, \sigma^T_{NA} - 2\phi (p_d;\ \mathbf{p}_T,\ p_\parallel) \times \int d\mathbf{q}_T \phi (p_d;\ \mathbf{q}_T,\ p_\parallel)\, \sigma_{NA} (\mathbf{p}_T - \mathbf{q}_T) + \int d\mathbf{q}_T \phi^2 (p_d;\ \mathbf{q}_T,\ p_\parallel)\, \sigma_{NA} (\mathbf{p}_T - \mathbf{q}_T) \Big], \tag{142}$$

where $\mathbf{p}_T$ and $p_\parallel$ are the transverse and longitudinal components of the proton momentum, σ^T_{NA} is the total cross section of nucleon–nucleus scattering, and $\sigma_{NA}(\mathbf{q}_T)$ is the differential cross section of elastic nucleon–nucleus scattering. The wave function $\phi(p_d; \mathbf{p}_T, p_\parallel)$ is normalized by the condition

$$\int \frac{d\mathbf{p}}{E_p} \left| \phi (p_d;\ \mathbf{p}_T,\ p_\parallel) \right|^2 = 1;\ E_p = \sqrt{m^2 + p_T^2 + p_\parallel^2}, \tag{143}$$

where m is the nucleon mass.

It is evidently most consistent to describe the deuteron wave function in the light-front variables $\mathbf{k}_\perp$ and α defined in Sec. 1. If the coordinate system is chosen such that $\mathbf{p}_d = (0, 0, p_d)$, the variables $\mathbf{k}_\perp$ and α can be expressed as follows in terms of the proton and deuteron momenta[6)]:

$$\mathbf{k}_\perp = \mathbf{p}_T; \tag{144}$$

$$\alpha = (E_p + p_\parallel)/(E_d + p_d). \tag{145}$$

Then in accordance with the general principles of relativistic quantum mechanics with a fixed number of particles, the relativistic wave function is[99]

$$\varphi^2 (\alpha,\ \mathbf{k}_\perp) = \sqrt{\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)}}\ \varphi^2_{\text{non-rel}} \left(\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)} - m^2 \right); \tag{146}$$

$$\varphi^2_{\text{non-rel}} (\mathbf{k}^2) = u^2 (\mathbf{k}^2) + w^2 (\mathbf{k}^2). \tag{147}$$

The normalization condition is

$$\frac{1}{2} \int_0^1 \frac{d\alpha}{\alpha (1-\alpha)} \int d\mathbf{k}_\perp \varphi^2 (\alpha,\ \mathbf{k}_\perp) = 1. \tag{148}$$

Substituting (146) in (148) and going over in it from the variable α to the variable $p_\parallel$ and comparing the obtained normalization condition with the normalization condition (143) for the function $\phi(p_d; \mathbf{p}_T, p_\parallel)$, we obtain

$$\phi (p_d;\ \mathbf{p}_T,\ p_\parallel) = \frac{1}{2} (1-\alpha)^{-1} \left(\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)} \right)^{1/2} \varphi^2_{\text{non-rel}} \left(\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)} - m^2 \right). \tag{149}$$

One can introduce a "nonrelativistic" momentum $\mathbf{k} = (\mathbf{k}_\perp, k_3)$ on the basis of the condition that its square be equal to the argument of the wave function φ_{nonrel} in (149):

$$k_3 = (2\alpha - 1) \sqrt{(m^2 + \mathbf{k}_\perp^2)/[4\alpha (1-\alpha)]}. \tag{150}$$

This momentum is not equal to the longitudinal proton momentum in the deuteron rest frame:

$$p^*_\parallel = (\alpha^2 M^2 - m^2 - \mathbf{k}_\perp^2)/2M\alpha. \tag{151}$$

The momenta k_3 and $p^*_\parallel$ are equal only in the nonrelativistic region, i.e., when $p^{*2}_\parallel + \mathbf{p}_T^2 \ll m^2$.

For infinite momentum of the incident deuteron, the maximal value $(p^*_\parallel)_{\max} \approx (3m^2 - \mathbf{k}_\perp^2)/4m$ corresponds to infinite momentum k_3. At the same time, $\varphi_{\text{nonrel}}(\mathbf{k}^2) \to 0$, which ensures dynamic cutoff of the cross section at the boundary of the phase space. For more details, see Ref. 78.

The elastic cross section of NA scattering can be well approximated by the dependence

$$\sigma_{NA} (\mathbf{q}_T) = (A/\pi) \exp (-B\mathbf{q}_T^2), \tag{152}$$

where B for nuclei of the type of carbon is approximately 65 $(\text{GeV}/c)^{-2}$. As follows from numerical calculations with different phenomenological wave functions,[62–66] the deuteron wave function (149) in the region $\alpha \gtrsim 0.6$ (we shall call this the hard region) does not change appreciably if the change in the transverse momentum is $|\Delta \mathbf{k}_\perp| \sim B^{-1/2}$. In this case, the Gaussian function (152) in the integrands of (142) can be replaced by a δ function:

$$\sigma_{NA} (\mathbf{q}_T) \approx \sigma^{el}_{NA} \delta (\mathbf{q}_T);\quad \sigma^{el}_{NA} = A/B. \tag{153}$$

Then the invariant cross section takes the extremely simple form[95]

$$E_p \frac{d^3\sigma}{d\mathbf{p}} \approx \frac{1}{2} C_d \sigma^{\text{in}}_{NA} (1-\alpha)^{-1} \times \sqrt{\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)}}\ \varphi^2_{\text{non-rel}} \left(\frac{m^2 + \mathbf{k}_\perp^2}{4\alpha (1-\alpha)} - m^2 \right). \tag{154}$$

Thus, in the hard region the corrections for rescattering reduce to renormalization of the cross section $(\sigma^{\mathrm{T}}_{NA} \to \sigma^{\text{in}}_{NA})$. We obtain a possibility of measuring the deuteron wave function.

The approximate expression (154) of the Bertocchi–Treleani model, reformulated in light-front variables, is very similar to the expression (11) of Ref. 78.

In the hybrid model, the deuteron contains other states besides two nucleons. Then the wave function $\varphi_{\text{nonrel}}(\mathbf{k}^2)$, which occurs in (141) through (146), must be replaced by the overlap integral of the hybrid wave function (74) and the wave function of the unbound state:

$$\varphi_{\text{non-rel}} (\mathbf{k}^2) \to \varphi_{np} (\mathbf{k}^2) = (1-\beta^2)^{1/2} \varphi_{\text{non-rel}} (\mathbf{k}^2) + \beta \left(\frac{6!}{3!3!} \right)^{1/2} \times \int d\mathbf{r}\, d\boldsymbol{\mu}\, d\boldsymbol{\nu}\, d\boldsymbol{\eta}\, d\boldsymbol{\xi}\, (2\pi)^{-3/2} \exp (-i\mathbf{k}\mathbf{r}) \times \hat{A} (\Psi^*_n (\boldsymbol{\xi},\ \boldsymbol{\eta})\, \Psi^*_p (\boldsymbol{\mu},\ \boldsymbol{\nu}))\, |6q\rangle, \tag{155}$$

where $\hat{A}$ is the antisymmetrizer (137); $\Psi_n(\xi, \eta)$, $\Psi_p(\rho, \nu)$, and $|6q\rangle$ are defined in Sec. 2. The relative coordinates $\mathbf{r}, \mu, \nu, \eta, \xi$ are defined in accordance with (76). Taking the s^6 configuration as the $6q$ component of the deuteron, we obtain

$$\varphi_{np} (\mathbf{k}^2) = (1-\beta^2)^{1/2}\ \varphi_{\text{non-rel}}\ (\mathbf{k}^2) + \beta I\ \varphi^{(np)}_{6q}\ (\mathbf{k}^2); \tag{156}$$

$$\varphi^{np}_{6q} (\mathbf{k}^2) = \left(\frac{6!}{3!3!} \right)^{1/2} 2^{3/2} \left(\frac{2}{1+\sqrt{2}} \right)^6 \left(\frac{2}{3\pi\omega} \right)^{3/4} \exp (-\mathbf{k}^2/3\omega). \tag{157}$$

Here, I is the overlap of the spin–isospin color wave functions of the $6q$ component and Ψ_n and Ψ_p (see Sec. 2). In other words, in (147) we must make the substitution

$$u (\mathbf{k}^2) \to u_{np} (\mathbf{k}^2) = (1-\beta^2)^{1/2}\, u (\mathbf{k}^2) + \beta I \varphi^{np}_{6q} (\mathbf{k}^2); \tag{158}$$

$$w (\mathbf{k}^2) \to (1-\beta^2)^{1/2}\, w (\mathbf{k}^2). \tag{159}$$

[5)]In Ref. 97, an attempt is made to calculate the cross section of the process (140) with orthogonal d and d' wave functions. However, success is achieved only in the coherent cross section. As a result, the calculation in accordance with the expression of Ref. 97 already exceeds the experimental cross section[98] by 30% in the "trivial" region $p_\parallel \approx p_d/2$.

[6)]When α is defined in this way, it differs from α defined in Sec. 1 by 1/2: $\alpha^{\text{new}} = \alpha^{\text{old}} + 1/2$. Therefore, the new α varies in the range from 0 to 1.

The fact that more detailed information about the quark structure of the deuteron can be obtained from fragmentation of deuterons into protons than from elastic *ed* scattering experiments can already be seen from the nonrelativistic treatment of these processes. In elastic *ed* scattering the Fourier transform of the square of the wave function (the form factor) is measured; in fragmentation experiments, the square of the Fourier transform. Therefore, if the np and $6q$ components are orthogonal in the coordinate space, interference between the np and $6q$ contributions will not enter into the form factors. Indeed, as we have already noted in Sec. 3, this is confirmed by numerical calculations with realistic wave functions of the np system. In the case of $d \to p$ fragmentation, this is not so—the Fourier transformation "mixes" the region $r > r_0$, in which the np component exists, and the region $r < r_0$, where the $6q$ component exists. Thus, the contributions from the fragmentation of the np and $6q$ components can in principle be added coherently even if the np and $6q$ wave functions are orthogonal. To establish whether these contributions are added coherently or incoherently, we can introduce the incoherence angle $\varkappa$:

$$\varphi^2_{\text{non-rel}}(\mathbf{k}^2) \to (1-\beta^2)\varphi^2_{\text{non-rel}}(\mathbf{k}^2) + \beta^2 I^2 [\varphi^{np}_{6q}(\mathbf{k}^2)]^2 + 2\cos\varkappa\beta(1-\beta^2)^{1/2} \times I\varphi_{\text{non-rel}}(\mathbf{k}^2)\varphi^{np}_{6q}(\mathbf{k}^2). \quad (160)$$

One can regard as physical only the values of $\varkappa$ equal to 0 and π (constructive and destructive interference) and $\pi/2$, corresponding to incoherent addition of the contributions of the np and $6q$ components. The angle $\varkappa$ must be chosen by comparison with experimental data.

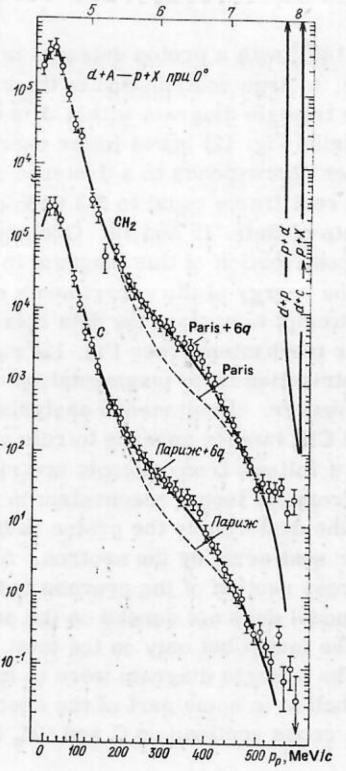

FIG. 11. Spectra of the fragmentation of unpolarized deuterons into protons at 8.9 GeV/c at zero emission angle of the protons on C and CH_2 targets.[15,16] The curves correspond to calculation by the hybrid model (continuous curves) and calculation without allowance for the $6q$ component of the deuteron (broken curves). The wave function for the Paris potential is chosen for the np component.

Comparison of the predictions of the hybrid model with experiments on $d \to p$ fragmentation. Measurement of the parameters of the hybrid model

Experimental investigations of inclusive reactions of the type (140) are reported in Refs. 15–17, 100, and 101. In Ref. 100, measurements were made of the cross section for proton production at 180° for collisions of 8.9-GeV/c protons with deuterons in the region $p_{\parallel}^* > 250$ MeV/c. In Ref. 101, the cross section of the reaction (140) was determined for a number of targets at momenta 3.5 and 5.8 GeV/c of the incident deuteron at proton emission angle 2.5° in the laboratory system and in the region of longitudinal momenta $p_{\parallel}^* < 200$ MeV/c in the deuteron rest frame.

However, to separate the contributions to the $d \to p$ fragmentation spectrum from the different mechanisms (Fermi motion of the nucleons in the deuteron, fragmentation of the $6q$ system, etc.) it is desirable to have detailed data on a maximally wide $p_{\parallel}^*$ region. Such data were obtained in Refs. 15 and 16, in which the reaction (140) was studied for C and CH_2 targets at momentum $p_d = 8.9$ GeV/c of the incident deuteron. The momentum $p_{\parallel}^*$ is varied in the region from 0 to 580 MeV/c (Fig. 11) and $\theta_p \leq 0.4°$.

As we said above, the wave function (149) takes into account the boundary of the phase space for a deuteron with infinite momentum. To take into account the boundary of the phase space at finite deuteron momentum, the expression for the cross section (141) is multiplied in Ref. 15 by the factor $(\alpha_{\max} - \alpha)/[(2\alpha_{\max} - 1)(1 - \alpha)]$.

The experimental data of Refs. 15 and 16 were compared with hybrid-model calculations. Three parameters were regarded as adjustable: the mean-square radius of the $6q$ system, which is related to the parameter ω,

$$r^2_{6q} = 5/4\omega, \quad (161)$$

the probability β^2 of the $6q$ admixture, and the phase $\varkappa$. The results of the fit are given in Table I.

The following conclusions can be drawn from the analysis of the data in Refs. 15 and 16:

1) the parameters of the hybrid model estimated from the data hardly depend on the choice of the np wave function;

2) the estimated parameters do not depend on the target nucleus;

3) the value of $\varkappa$ is found to be close to 90°, which indicates incoherent addition of the contributions from the fragmentation of the np and $6q$ components of the deuteron.

There are grounds for assuming (for more details, see Ref. 102) that in the exclusive process

$$p + d \to p + n + p \tag{162}$$

[which is similar to (140)] with a proton detected in the backward hemisphere, a large contribution to the hard region is made by the triangle diagram with a Δ isobar in the intermediate state (Fig. 12) but at lower energies ($p_p = 1.65$ GeV/c, which corresponds to a deuteron momentum in the proton rest frame equal to 3.3 GeV/c) than in the experiments of Refs. 15 and 16. One could therefore expect the contribution of this diagram to remain appreciable at the energy of the experiments of Refs. 15 and 16 and attempt to explain the data obtained in them by the "isobar mechanism" (see Fig. 12) rather than as due to the contribution from fragmentation of the 6q component. However, simultaneous analysis of the data for the C and CH_2 targets appears to rule out this possibility. For it follows from isotopic invariance that the contribution from the isobar mechanism in the case of scattering of the deuteron by the proton is five times greater than for scattering by the neutron. At the same time, the cross section of the process in the Bertocchi–Treleani model does not depend on the neutron–proton ratio of the target but only on the total number A. Thus, if the triangle diagram were to make an appreciable contribution in some part of the spectrum, the ratio of the cross sections on C and CH_2 targets in this region,

$$R = d\sigma\,(\mathrm{C})/d\sigma\,(\mathrm{CH_2}) \tag{163}$$

would have to differ from the value in the remaining part of the spectrum. However, to within the errors of the experimentally measured R no dependence on the momentum of the detected proton is observed.[17]

In the region of $p_{\parallel}^*$ values from 300 to 400 MeV/c a peak is observed in the difference between the experimentally measured and theoretically calculated cross sections (Fig. 13). In all probability, it can be interpreted[16] as the production of a dibaryon resonance in the process

$$d + N \to d^* + N \\ \quad\;\; \hookrightarrow p\ (\text{detected}) + N. \tag{164}$$

One could attempt to interpret this peak as the contribution of the triangle diagram in Fig. 12. However, such an interpretation contradicts the consequences of isotopic invariance. We introduce

$$R_{\text{res}} = \sum\left[(d\sigma_{\exp})_i - (d\sigma_{\text{theor}})_i\right] / \sum (d\sigma_{\text{theor}})_i, \tag{165}$$

where the summations are over all the experimental points in the interval from 300 to 400 MeV/c. The ratio of R_{res} on the CH_2 and C targets is

$$R_{\text{res}}\,(\mathrm{CH_2})/R_{\text{res}}\,(\mathrm{C}) = 1.00 \pm 0.07. \tag{166}$$

At the same time, it follows from the isotopic invariance for the mechanism of Fig. 12 that this ratio must

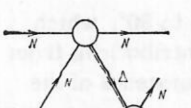

FIG. 12. Triangle diagram with Δ isobar in the intermediate state.

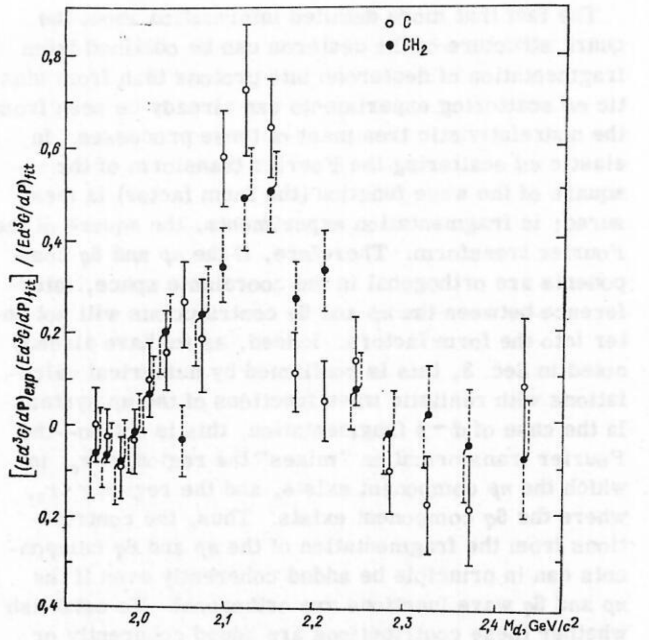

FIG. 13. Peak in the difference between the experimentally measured and theoretically calculated cross sections in the region 300 MeV/$c \leqslant p_{\parallel}^* \leqslant 400$ MeV/c.

be equal to two if we assume that the effective number of nucleons in carbon is equal to four. Even if we assume that the effective number of nucleons in carbon is 12, the ratio (166) for the isobar mechanism is 1.3. If the observed peak is interpreted as a contribution to the cross section from (164), then it follows from (166) that the isospin of the observed dibaryon must be equal to zero. The estimates $M = 2.13$ GeV/c^2 and $\Gamma < 80$ MeV/c^2 have also been obtained[16] for the dibaryon resonance.

Fragmentation of polarized deuterons in the hybrid model

One of the predictions of the hybrid model for the deuteron wave function is the absence of a dip in the modulus of the charge form factor $|F_C(q^2)|$ [in the region $q^2 \approx 0.6$ (GeV/c)^2][60] (see also Sec. 3 of the present review). However, experiments to measure the charge form factor of the deuteron are very complicated and probably will not be made in the near future. However, it is possible to study a phenomenon somewhat similar to the phenomenon of the filling of the dip in $|F_C(q^2)|$ in experiments with beams of relativistic polarized deuterons.[95] The importance of making experiments on the fragmentation of relativistic polarized deuterons in order to investigate the dynamics of few-nucleon correlations in nuclei has also been emphasized in Ref. 103.

If the deuteron is polarized in such a way that it has z projection of the spin ±1, then, to calculate the cross section for its fragmentation on nucleus A into an unpolarized proton emitted at zero angle, the nonrelativistic function $u^2(\mathbf{k}^2) + w^2(\mathbf{k}^2)$ in (142), which is related to the relativistic function by (149), must be replaced by

$$[u\,(\mathbf{k}^2) - \sqrt{1/2}\,w\,(\mathbf{k}^2)]^2. \tag{167}$$

As is shown by calculations with realistic wave functions,[63–66] this combination has a dip in the region $|\mathbf{k}| \approx 300–350$ MeV/c. This dip lies close to the value k_0 at which $u(\mathbf{k}^2)$ changes sign (Fig. 14, case $\beta = 0$). In the hybrid model, the behavior of the function $u_{np}(k^2)$ determined by Eq. (158) is very sensitive to the parameters of the $6q$ admixture for $|\mathbf{k}| \gtrsim k_0$. Figure 14 shows how the behavior of $|u_{np}(\mathbf{k}^2)|$ depends on the value of β for the cases when the $6q$ component makes a contribution that interferes constructively (see Fig. 14a) and destructively (see Fig. 14b) with the contribution of the np component. The parameter ω was taken to be 7.9 F^{-2}. If there is constructive interference, then at values $\beta^2 \gtrsim 0.02$ the function $u_{np}(\mathbf{k}^2)$ does not change sign. This phenomenon has the same physical nature as the filling of the dip of $|F_C(q^2)|$.

This qualitative change in the behavior of the function $u_{np}(\mathbf{k}^2)$ is reflected qualitatively in the change in the fragmentation spectrum of polarized deuterons. Thus, it leads to the appearance of a second dip in the cross section $d^3\sigma^{\pm}(p, \theta = 0°)/d\mathbf{p}$ for fragmentation of a deuteron with z projection of the spin ± 1 into a proton at zero angle. The position of this dip depends very strongly on the value of β. The reader can find graphs of the cross sections for different values of β in Ref. 95.

If, as is indicated by experiments on the fragmentation of unpolarized deuterons, the $6q$ contribution to the fragmentation cross section is added incoherently to the np contribution, then, to calculate the cross section $E_p d^3\sigma^{\pm}(p, \theta = 0°)/d\mathbf{p}$ it is necessary to replace $u^2 + w^2$ in (142) by

$$(1-\beta^2)\,(u(\mathbf{k}^2) - \sqrt{1/2}\,w(\mathbf{k}^2))^2 + \beta^2 I^2 \varphi_{np}^{(6q)}(\mathbf{k}^2). \qquad (168)$$

Calculations[7)] with parameters of the $6q$ admixture given in Table I show that in this case $d^3\sigma^{\pm}(p, \theta = 0°)/d\mathbf{p}$

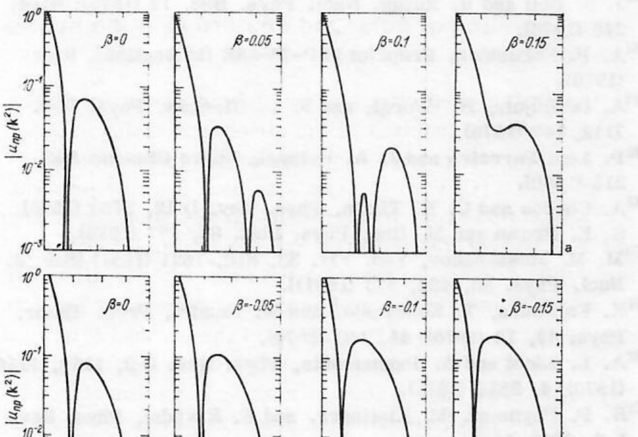

FIG. 14. Value of $|u_{np}(k^2)|$ for different values of β; $k_3 = (2\alpha - 1)\sqrt{m^2/[4\alpha(1-\alpha)]}$. [*Translator's note*: a footnote in the Russian, presumably added in proof, states that the values of β in this figure should be increased by $\sqrt{2}$ times.]

7) We are grateful to D. K. Nikitin for numerical computer calculations of the fragmentation cross sections of polarized deuterons.

TABLE I. Parameters of the hybrid model obtained from data on $d \to p$ fragmentation.

Deuteron wave function	Target	β^2, %	r_{6q}, F	$\varkappa$, deg	χ^2/N
Reid, soft core	C	0	—	—	12.7
Reid, soft core + $6q$	C	(6.2±0.4)	0.87±0.10	61±11	2.54
Paris	C	0	—	--	15.7
Paris + $6q$	C	(8.6±0.8)	0.95±0.05	82±6	1.95
Paris + $6q$	CH_2	(10.8±1.2)	0.99±0.04	95±7	1.6

does not have dips.

The results of calculation of the ratio of the cross section for fragmentation of a deuteron with 100% polarization to the cross section for fragmentation of an unpolarized deuteron into a proton at zero angle are shown in Fig. 15 and have the form

$$P^{(\lambda)} = \frac{d^3\sigma^{(\lambda)}(p,\ \theta = 0°)}{d\mathbf{p}} \Big/ \frac{d\sigma(p,\ \theta = 0°)}{d\mathbf{p}}, \qquad (169)$$

where λ is the z projection of the deuteron spin. Carbon was chosen as the target nucleus. For the np wave function, the wave function for the Paris potential was used.[66] The parameters of the $6q$ admixture were taken to be $\beta^2 = 0.086$, $r_{6q} = \sqrt{5/4\omega} = 0.95$ F. It can be seen that the fragmentation cross section of a polarized deuteron in the region $0.5 < \alpha \lesssim 0.56$ differs strongly from an unpolarized deuteron's, but is determined almost entirely by the np component. In the region $0.56 < \alpha < 0.85$, the nature of the fragmentation cross section for a polarized deuteron depends strongly on the presence or absence of a $6q$ component of the deuteron. Therefore, measurements of $P^{(\lambda)}$ must be an important step in the verification of the hybrid model for the deuteron wave function.

CONCLUSIONS

Thus, the quark methods initially used to describe the structure of hadrons and their interactions can also be used to describe at distances less than 1 F the structure of the simplest nucleus—the deuteron. It is natural to attempt to apply these methods to the description of the structure of more complicated nuclei. Here, the most important problem is evidently matching the results of calculations to the experimentally measured binding energies of tritium and helium.

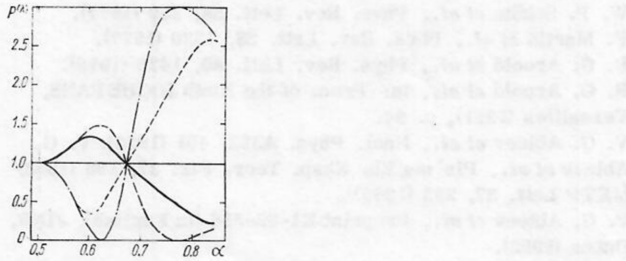

FIG. 15. The ratios $P^{(\lambda)}$ for deuteron with 100% polarization. The continuous and broken curves correspond to calculation with parameters of the $6q$ admixture $\beta^2 = 0.043$, $r_{6q} = 0.95$ F for $P^{(0)}$ and $P^{(\pm)}$, respectively; the broken and chain curves correspond to calculation without allowance for a $6q$ component for $P^{(0)}$ and $P^{(\pm)}$. The wave function for the Paris potential was taken as the np wave function.

With regard to further investigations of the quark structure of the deuteron, it seems to us that the most topical problem is the measurement of the fragmentation spectrum of relativistic polarized deuterons. Among the other topical problems we mention the following:

1) establishment of whether there is agreement between the results obtained in different experiments, for example, in the SLAC experiments on *ed* scattering and in the Dubna experiments on $d \to p$ fragmentation; this will permit a reliable answer to the question of whether these experiments are correctly interpreted as manifestation of a $6q$ component of the deuteron or whether they can be explained by "orthodox" mechanisms (exchange currents, isobar contributions, etc.);

2) measurements of the magnetic form factor of the deuteron at $q^2 \gtrsim 1\ (\mathrm{GeV}/c)^2$;

3) detailed investigation of P-odd correlations in NN interactions, in particular circular polarization in radiative capture of protons by thermal neutrons;

4) detailed study (both experimental and theoretical) of deuteron photodisintegration at energies of order 100 MeV and higher.

We are grateful to G. M. Zinov'ev, V. K. Luk'yanov, M. M. Musakhanov, M. Namiki, V. G. Neudachin, and L. N. Strunov for helpful discussions and comments.

[1]Yu. M. Shirokov, Nucl. Phys. B6, 159 (1968).
[2]V. E. Troitskiĭ and Yu. M. Shirokov, Teor. Mat. Fiz. **1**, 213 (1969).
[3]V. P. Kozhevnikov, V. E. Troitskiĭ, S. V. Trubnikov, and Yu. M. Shirokov, Teor. Mat. Fiz. 10, 47 (1972).
[4]V. E. Troitskiĭ, S. V. Trunbnikov, and Yu. M. Shirokov, Teor. Mat. Fiz. 10, 209, 349 (1972).
[5]V. I. Kukulin, V. E. Troitski, Yu. M. Shirokov, and S. V. Trubnikov, Phys. Lett. B39, 319 (1972).
[6]A. I. Kirillov, V. E. Troitskiĭ, S. V. Trubnikov, and Yu. M. Shirokov, Fiz. Elem. Chastits At. Yadra **6**, 3 (1975) [Sov. J. Part. Nucl. **6**, 1 (1975)].
[7]A. M. Baldin et al., Preprint R15-819 [in Russian], JINR Dubna (1971); Preprint E1-8054 [in English], JINR, Dubna (1974); Yad. Fiz. **21**, 1008 (1975) [Sov. J. Nucl. Phys. **21**, 517 (1975)].
[8]A. M. Baldin, Fiz. Elem. Chastits At. Yadra 8, 429 (1977) [Sov. J. Part. Nucl. 8, 175 (1977)].
[9]A. M. Baldin, Prog. Part. Nucl. Phys. **4**, 95 (1980).
[10]R. G. Arnold *et al.*, Phys. Rev. Lett. **35**, 776 (1975).
[11]W. P. Schütz *et al.*, Phys. Rev. Lett. **38**, 259 (1977).
[12]F. Martin *et al.*, Phys. Rev. Lett. **38**, 1320 (1977).
[13]R. G. Arnold *et al.*, Phys. Rev. Lett. **40**, 1429 (1978).
[14]R. G. Arnold *et al.*, in: Proc. of the Ninth ICOHEPANS, Versailles (1981), p. 94.
[15]V. G. Ableev *et al.*, Nucl. Phys. **A393**, 491 (1983); V. G. Ableev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 37, 196 (1983) [JETP Lett. 37, 233 (1983)].
[16]V. G. Ableev *et al.*, Preprint E1-82-516 [in English], JINR, Dubna (1982).
[17]V. G. Ableev *et al.*, in: Tr. Mezhdunarodnogo seminara po perspektivam issledovaniĭ v relyativistskoĭ yadernoĭ fizike (Proc. of the Intern. Seminar on Prospects for Investigations in Relativistic Nuclear Physics), Dubna (1982).
[18]V. K. Luk'yanov and A. I. Titov, Fiz. Elem. Chastits At. Yadra 10, 815 (1979) [Sov. J. Part. Nucl. **10**, 321 (1979)].
[19]V. S. Stavinskiĭ, Fiz. Elem. Chastits At. Yadra **10**, 949 (1979) [Sov. J. Part. Nucl. **10**, 373 (1979)].
[20]V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Lett. Nuovo Cimento **5**, 907 (1972).
[21]S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
[22]V. M. Dubovik and A. P. Kobushkin, Preprint ITP-78-85, Kiev (1978).
[23]P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[24]E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).
[25]A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
[26]H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) **112**, 94 (1978).
[27]A. P. Kobushkin and V. P. Shelest, Fiz. Elem. Chastits At. Yadra 3, 571 (1972) [Sov. J. Part. Nucl. **3**, 287 (1972)].
[28]M. A. Markov, J. Phys. **3**, 452 (1940).
[29]H. Yukawa, Phys. Rev. **77**, 219 (1950).
[30]H. J. Melosh, Phys. Rev. D **9**, 1095 (1974).
[31]A. P. Kobushkin and V. P. Shelest, Preprint E2-8594 [in English], JINR, Dubna (1975).
[32]A. P. Kobushkin and V. P. Shelest, Teor. Mat. Fiz. **31**, 156 (1977); A. P. Kobushkin, Teor. Mat. Fiz. **30**, 231 (1977).
[33]M. V. Terent'ev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)].
[34]N. N. Bogolyubov *et al.*, Preprint D-2075 [in Russian], JINR, Dubna (1965).
[35]N. N. Bogolyubov *et al.*, Preprint R-2141 [in Russian], JINR, Dubna (1965).
[36]A. N. Tavkhelidze, in: High Energy Physics and Elementary Particles, IAEA, Vienna (1965), p. 763.
[37]N. N. Bogolyubov, B. V. Struminskiĭ, and A. N. Tavkhelidze, Preprint D-1968 [in Russian], JINR, Dubna (1965).
[38]H. Lipkin and A. N. Tavkhelidze, Phys. Lett. **17**, 331 (1965).
[39]P. N. Bogolubov, Ann. Inst. Henri Poincaré 8, 163 (1968).
[40]A. Chodos *et al.*, Phys. Rev. D 9, 3471 (1974).
[41]M. Namiki and S. Saito, Prog. Theor. Phys. **53**, 1465 (1975).
[42]S. Ishida, A. Matsuda, and M. Namiki, Prog. Theor. Phys. 57, 210 (1977).
[43]S. Saito, Prog. Theor. Phys. 58, 1802 (1977).
[44]J. E. Donoghue and K. Johnson, Phys. Rev. D 21, 1975 (1980).
[45]J. S. Bell and H. Ruegg, Nucl. Phys. B98, 12 (1975); B104, 245 (1976).
[46]A. P. Kobushkin, Preprint ITP-76-58E [in English], Kiev (1976).
[47]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D12, 147 (1975).
[48]P. Leal Ferreira and J. A. Helayel, Nuovo Cimento A55, 215 (1980).
[49]A. Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975); G. E. Brown and M. Rho, Phys. Lett. 82, 177 (1979).
[50]M. M. Musakhanov, Yad. Fiz. 33, 810, 1621 (1981) [Sov. J. Nucl. Phys. 33, 422, 873 (1981)].
[51]K. Fujimura, T. Kobayashi, and M. Namiki, Prog. Theor. Phys. 43, 73 (1970); 44, 193 (1970).
[52]A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150, 1156 (1970); 4, 2810 (1971).
[53]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).
[54]R. G. Lipes, Phys. Rev. D 5, 2849 (1972).
[55]V. F. Dushenko, A. P. Kobushkin, and Yu. M. Shirokov, Yad. Fiz. **24**, 434, 655 (1976) [Sov. J. Nucl. Phys. **24**, 225, 344 (1976)].
[56]B. L. Bakker, L. A. Kondratyuk, and M. V. Terent'ev, Nucl. Phys. B158, 497 (1979).
[57]Yu. M. Sinjukov, Preprint ITP-75-14E [in English], Kiev (1975).
[58]A. P. Kobushkin, Preprint ITP-76-145E [in English], Kiev (1976).
[59]V. A. Matveev and P. Sorba, Lett. Nuovo Cimento **20**, 435 (1977).

[60]A. P. Kobushkin, Yad. Fiz. **28**, 495 (1978) [Sov. J. Nucl. Phys. **28**, 252 (1978)].
[61]Yu. A. Simonov, Phys. Lett. **B107**, 1 (1981); Preprint, ITEP, Moscow (1982).
[62]R. L. Jaffe and F. E. Low, Phys. Rev. D **19**, 2105 (1979).
[63]T. Hamada and I. D. Johnson, Nucl. Phys. 34, 382 (1962).
[64]I. J. McGee, Phys. Rev. **151**, 772 (1966).
[65]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[66]M. Lecombe *et al.*, Phys. Lett. **B101**, 139 (1981).
[67]M. Hamermesh, Group Theory and its Application to Physical Problems, Addison-Wesley, Reading, Mass. (1962) (Russian translation published by Mir, Moscow (1966)).
[68]Yu. F. Smirnov and Yu. M. Tchuvil'sky, J. Phys. G **4**, L1 (1978).
[69]H. Högaasen, P. Sorba, and R. Viollier, Z. Phys. C **4**, 131 (1980).
[70]P. Benz and P. Söding, Phys. Lett. **B53**, 367 (1978).
[71]I. T. Obukhovsky *et al.*, Phys. Lett. **B88**, 231 (1979).
[72]A. P. Kobushkin, Preprint ITP-82-59E [in English], Kiev (1982).
[73]I. T. Obukhovsky, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, J. Phys. A 15, 7 (1982).
[74]C. W. Wong, Phys. Lett. **B108**, 385 (1982).
[75]M. Gourdin, Nuovo Cimento **28**, 533 (1963).
[76]V. K. Mitrjushkin, P. K. Rashidov, and S. V. Trubnikov, Communication E2-82-156, JINR, Dubna (1982).
[77]R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. Lett. 38, 1516 (1977); Phys. Rev. C **21**, 1426 (1980).
[78]M. I. Strikman and L. L. Frankfurt, Fiz. Elem. Chastits At. Yadra 11, 571 (1980) [Sov. J. Part. Nucl. **11**, 221 (1980)].
[79]Y. Kizukuri, M. Namiki, and K. Okano, Prog. Theor. Phys. **61**, 559 (1979).
[80]Y. Kizukuri *et al.*, Prog. Theor. Phys. **64**, 1478 (1980).
[81]A. N. Mitra, Phys. Rev. D **17**, 729 (1978).
[82]V. V. Burov *et al.*, Preprint R2-81-621 [in Russian], JINR, Dubna (1981).
[83]L. S. Kisslinger, Phys. Lett. **B112**, 307 (1982).
[84]M. Gari and H. Hyuga, Nucl. Phys. **A264**, 409 (1976).
[85]F. Iachello, A. Jackson, and A. Landé, Phys. Lett. B43, 191 (1973).
[86]R. E. Rand *et al.*, Phys. Rev. Lett. **18**, 467 (1967); Phys. Rev. D 8, 3229 (1973).
[87]C. D. Buchanan *et al.*, Phys. Rev. Lett. **15**, 303 (1965); J. E. Elias *et al.*, Phys. Rev. **177**, 2075 (1969).
[88]I. T. Obukhovskiĭ and E. V. Tkalya, Yad. Fiz. 35, 288 (1982) [Sov. J. Nucl. Phys. **35**, 164 (1982)].
[89]Yu. Kudeuarov *et al.*, Nucl. Phys. A245, 429 (1975); V. G. Neudatchin, Yu. F. Smirnov, and F. F. Golovanova, Adv. Nucl. Phys. **11**, 1 (1979).
[90]V. M. Dubovik and I. T. Obukhovsky, Z. Phys. C 10, 123 (1981).
[91]E. Hadjimichael and D. P. Saylor, Phys. Rev. Lett. **45**, 1776 (1980).
[92]H. Arenhövel, Phys. Rev. Lett. C47, 749 (1981).
[93]S. I. Nagornyĭ and E. V. Inopin, VANT, Ser. Obshch. Yad. Fiz. No. 4(18), 26 (1981).
[94]E. V. Inopin, S. M. Nagornyĭ, and Yu. I. Titov, VANT, Ser. Obshch. Yad. Fiz. No. 1(19), 77 (1982).
[95]A. P. Kobushkin and L. Vizireva, J. Phys. G **8**, 893 (1982).
[96]L. Bertocchi and D. Treleani, Nuovo Cimento **A36**, 1 (1976).
[97]S. Nissen-Meyer, Nucl. Phys. A306, 499 (1978).
[98]V. G. Ableev *et al.*, Yad. Fiz. **37**, 132 (1983) [Sov. J. Nucl. Phys. 37, 75 (1983)].
[99]V. R. Garsevanishvili and V. A. Matveev, Teor. Mat. Fiz. **24**, 3 (1975).
[100]A. M. Baldin *et al.*, Communication R1-11168 [in Russian], JINR, Dubna (1977).
[101]J. Papp, Preprint LBL-3633, Berkeley (1975).
[102]N. S. Amelin, V. V. Glagolev, and G. I. Lykasov, Fiz. Elem. Chastits At. Yadra **13**, 130 (1982) [Sov. J. Part. Nucl. 13, 55 (1982)].
[103]L. L. Frankfurt and M. I. Strikman, Phys. Lett. B75, 257 (1978).

Translated by Julian B. Barbour