Quantum symmetries in particle interactions[1)]

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The concept of a quantum symmetry is introduced as a symmetry in the formulation of which quantum representations and specific quantum notions are used essentially. Three quantum symmetry principles are discussed: the principle of renormalizability (possibly super-renormalizability), the principle of local gauge symmetry, and the principle of supersymmetry. It is shown that these principles play a deterministic role in the development of quantum field theory. Historically their use has led to ever stronger restrictions on the interaction mechanism of quantum fields.

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During the last 15 to 20 years the theory of the elementary interactions of particles has been influenced more and more clearly by symmetry principles of a special type whose formulation makes essential use of quantum representations and specific quantum concepts. By this is meant:

1) the principle of renormalizability (or even super-renormalizability, i.e., the absence of divergent renormalizations);

2) the principle of local gauge symmetry;

3) the principle of supersymmetry.

To demonstrate the part played by these principles in forming the modern picture of quantum field theory, we recall some notable results achieved by their use.

We note, first, that in contrast to quantum mechanics, in which the formulation of the problem of the interaction of a given set of particles admits an arbitrary function (a potential), in quantum field theory the arbitrariness in the interaction Lagrangian of a fixed system of fields is limited to a small number of coupling constants. Quantum field theory does not admit arbitrary functions,[2)] as a consequence of the requirement of renormalizability.

Connections between different coupling constants can be established by the principle of local gauge symmetry. This principle was given full formulation in 1954 by Yang and Mills, and on its basis they introduced a new class of self-interacting (i.e., satisfying nonlinear equations) relativistic vector fields—non-Abelian gauge fields, frequently also called Yang–Mills fields. These fields, besides their internal features associated with the nonlinearity of the free equations of motion, are remarkable from the point of view of their interaction with the remaining fields. The minimal interaction of a gauge field with matter fields is unique in form and is characterized by a single coupling constant. In the case of a non-Abelian field, the same constant occurs in the nonlinear terms of the free equation.

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[2)] This fact was noted more than 20 years ago by Feynman,[1] who, however, emphasized mainly in this connection relativistic invariance.

The simplest gauge field is the electromagnetic field $A_\mu(x)$, whose interaction with the matter fields $u(x)$ arises as a result of the operation of "extending" the derivatives with respect to the space–time coordinates in accordance with the rule

$$\partial u(x)/\partial x^\mu \equiv \partial_\mu u(x) \to D_\mu(A)\, u(x) \equiv [\partial_\mu + ieA_\mu(x)]\, u(x). \quad (1)$$

For spinor matter fields, for which the free Lagrangian is linear in the derivatives, this prescription leads to the appearance in the Lagrangian (or Hamiltonian) of contributions linear in the electromagnetic field of the form

$$(\text{potential}) \times (\text{current}), \text{ i.e., } eA_\mu(x)\, j^\mu(\psi),$$

where $j^\mu(\psi)$ is the current of the field ψ.

The essence of local gauge symmetry can be formulated by means of the concept of the phase of the matter field:

$$u(x) = |u(x)| \exp(i\varphi(x)).$$

It is well known from quantum mechanics that the phase φ of the wave function u is unobservable, with the consequence that a transformation involving a change of phase,

$$u \to u' = u \exp(i\alpha),\ \varphi \to \varphi + \alpha, \quad (2)$$

does not lead to any physical consequences. If the parameter α does not depend on the space–time coordinates $x = (\mathbf{x}, t)$, then (2) is called a local gauge transformation. It is obviously compatible with the equations of motion. In the more general case, a transformation of the form

$$u(x) \to u'(x) = u(x) \exp[i\alpha(x)] \quad (3)$$

is called a local gauge transformation. Form invariance, i.e., covariance of the equations of motion, can now be achieved with respect to simultaneous transformation of the complex wave function of the matter field,

$$u \to u \exp[i\alpha(x)],\quad \overset{*}{u} \to \overset{*}{u} \exp[-i\alpha(x)],$$

and a displacement-type transformation of the vector field:

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \quad (4)$$

The equations of motion obtained from the Lagrangian

$$L_0(u, \overset{*}{u}, D_\mu(A)u, \overset{*}{D}_\mu(A)\overset{*}{u}) + L_{tr}(A) = L(u, A), \quad (5)$$

which describes the system of the two fields u and A,

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are covariant. The first term on the left-hand side of (5) is obtained by modifying the derivatives in accordance with the rule (1) in the Lagrangian $L_0(u, \overset{*}{u}, \partial u, \partial\overset{*}{u})$ of the free matter field u, $\overset{*}{u}$, and the second term is the Lagrangian of the free field A, which is invariant with respect to the transformation (4). Thus, it is not the free equations for the field u that are covariant but the equations of motion for the coupled system of fields u and A.

Equations (1)–(5) correspond to the simplest case of the one parameter group of transformations (2), which is equivalent to the group of rotations $U(1)$ of the plane. This group is commutative, i.e., Abelian. The corresponding Abelian vector field—the electromagnetic field—satisfies linear equations of motion as an exception.

In the more general case, the matter field $u(x)$ has several components,

$$u \to u_a, \quad a = 1, 2, \ldots, A,$$

and forms an A-dimensional representation of some internal symmetry group G. The analog of the transformation (3) mixes the components u_a, i.e., has a matrix form:

$$u'_a = [\exp i\alpha]^{ab} u_b.$$

The number of independent parameters of the transformation α is equal to the dimension of the group G. In the general case, two such transformations do not commute, and G is non-Abelian. The corresponding non-Abelian gauge field B^{ab}_μ "carries" indices of the group G, i.e., it takes, as one says, values in the group G and realizes the so-called adjoint representation, transforming in accordance with a law analogous to (4).

A specific feature of any gauge field is its masslessness, which is due to the fact that a mass term in the Lagrangian and the equations of motion must directly contain the vector potential (and not its derivative) and is therefore manifestly noninvariant with respect to transformations of the form (4). The masslessness of the gauge fields also complicates the procedure of their quantization, since it is associated with a "redundancy" in the description of the corresponding physical system in terms of potentials (and not field intensities), and ultimately has the consequences that, formally, a system of fields that includes a gauge field is analogous to a mechanical system with constraints.

Because no massless particles with spin unity were observed experimentally, apart from photons, non-Abelian gauge fields were for a long time after their discovery regarded as an "elegant toy" with no bearing on the real world.

Nevertheless, at the beginning of the seventies, following the successful overcoming of the difficulties in the quantization of field systems with contraints and the discovery of a method for introducing mass that does not break the gauge symmetry (the method of spontaneous symmetry breaking and the Higgs mechanism), these fields came to occupy a central position in the formalism of the modern theory of interaction of particles. Besides the Abelian gauge field that represents the core of quantum electrodynamics, Yang–Mills fields form the foundation of the theory of electroweak interactions of Glashow, Weinberg, and Salam, and also quantum chromodynamics, the modern basis of the theory of strong interactions. The quanta of the non-Abelian gauge field of the color group $SU(3)$—gluons—have taken the place of Yukawa's mesons of the universal carriers of the strong interactions between quarks and the gluons themselves.

The property of renormalizability has a history no less tortuous. At the beginning of the more than 30-year period of existence of renormalizable perturbation theory, this property appeared to be the weak link in quantum field theory, severely restricting the class of field interactions for which it is possible to calculate uniquely the radiative corrections; it appeared as an imperfection of the available theoretical methods, in the first place perturbation theory and perturbative renormalizations. But at the beginning of the seventies the renormalizability property, imposed as a requirement, played an important part in the construction of a unified renormalizable model of the electromagnetic and weak interactions and ultimately led to the prediction of the existence of neutral currents and a fourth, so-called charmed quark.

There is now no doubt that the renormalizability property of perturbation theory is satisfied by the three fundamental interactions (electromagnetic, weak, and strong) constructed on the basis of the mechanism of local gauge symmetry. The question of the renormalizability of quantum gravity remains open. Here, however, hopes associated with supersymmetry and supergravity have made their appearance and gained strength greatly during recent years.

Supersymmetry was discovered at the end of the sixties and the beginning of the seventies, initially in algebraic form (as an extension of the algebra of the Poincaré group including anticommuting elements) and then in a quantum-field formulation. Supersymmetry can be defined as a symmetry between Bose and Fermi fields, i.e., between quantum fields, some of which satisfy Bose–Einstein commutation relations and others Fermi–Dirac statistics and the Pauli exclusion principle.

A manifestly invariant formulation of a theory with symmetry between field functions of different statistics involves the use of a special algebraic object—a Grassmann algebra. The simplest Grassmann algebra with an involution (the analog of the operation of conjugation) is based on two generators θ and $\bar{\theta}$, which anticommute and each have vanishing square: $\theta\bar{\theta} + \bar{\theta}\theta = \theta^2 = \bar{\theta}^2 = 0$. Thus, this algebra is nilpotent. More complicated Grassmann algebras contain an even number of generators $\theta_\alpha, \bar{\theta}_\beta, 1 \leqslant \alpha, \beta \leqslant n$ such that

$$\theta_\alpha\theta_\beta + \theta_\beta\theta_\alpha = \bar{\theta}_\alpha\theta_\beta + \theta_\beta\bar{\theta}_\alpha = \bar{\theta}_\alpha\bar{\theta}_\beta + \bar{\theta}_\beta\bar{\theta}_\alpha = \theta^2_\alpha = \bar{\theta}^2_\beta = 0.$$

We now consider an arbitrary function of the generators θ, $\bar{\theta}$. As is readily seen, it can be expressed by a power-law expansion with a small number of terms. For example, for $n=1$

$$\Phi(\theta, \bar{\theta}) = \varphi + \bar{\theta}\psi + \bar{\psi}\theta + \bar{\theta}\theta u.$$

Terms of higher powers are absent because of the nilpotency. If it is assumed further that the generators θ, $\bar{\theta}$ are spinors of the Poincaré group (form a real Majorana spinor), i. e., have a spinor index that takes two values, it is possible to exploit the properties of the coefficients φ, ψ, $\bar{\psi}$, and u in such a way as to obtain simple properties for Φ. Namely, assuming that φ and u are Hermitian scalar Bose fields and that ψ, $\bar{\psi}$ is a spinor Fermi field, we find that the commutator of two Hermitian forms

$$\Phi(x, \theta, \bar{\theta}) = \varphi(x) + \bar{\theta}\psi(x) + \bar{\psi}(x)\theta + \bar{\theta}\theta u(x) \qquad (6)$$

can be expressed linearly,

$$[\Phi(x, \theta, \bar{\theta}); \quad \Phi(y, \theta, \theta)] = \Delta(x-y, \theta, \bar{\theta}),$$

in terms of the commutator of the scalar field φ and the anticommutator of the spinor field and does not contain operator quantum fields.

Thus, a combination of the form (6), which is a scalar of the inhomogeneous Lorentz group, satisfies Bose–Einstein commutation relations. It may be called a scalar boson superfield or, briefly, a scalar superfield. One can construct analogous linear forms corresponding to spinor representations of the supersymmetry group, for example, the superspinor

$$\Psi_\alpha(x, \theta) = \psi_\alpha(x) + \varphi(x)\theta_\alpha,$$

which satisfies anticommutation relations of the form

$$\{\Psi_\alpha(x, \theta), \bar{\Psi}_\beta(x, \bar{\theta})\}_+ = S_{\alpha\beta}(x-y, \theta, \bar{\theta}),$$

i. e., is a super-Fermi field, etc. The theory of representations provides regular rules for constructing superfields.

Supersymmetric models of quantum field theory can be expressed either in terms of superfields.

$$\Phi(x, \theta, \bar{\theta}); \quad \Psi(x, \theta); \quad \bar{\Psi}(x, \bar{\theta}); \quad B_\mu(x, \theta, \bar{\theta}); \ldots$$

or in terms of the constituent fields

$$\varphi(x), \ \psi(x), \bar{\psi}(x), \ u(x), \ b_\mu(x), \ldots,$$

these being representations of the Poincaré group. It is then found that a fairly simple supermodel with one coupling constant, for example, $g\Phi^3$, is, when written in Poincaré components, equivalent to the Lagrangian of the system of corresponding constituent fields, consisting of a sum of products of different ordinary fields, each of the terms of the Lagrangian containing the same coupling constant g. As a result, supersymmetry establishes simple connections between the coupling constants of a set of Bose and Fermi fields, the set of which is also fixed.

Investigation of the structure of the divergences of a number of supersymmetric models already in the middle of the seventies revealed a number of interesting properties. In particular, it was found that in such models there is frequently a tendency for the divergences to cancel. Models were found for which the interaction Lagrangian, when represented in terms of the constituent fields, is expressed by a sum of terms each of which taken separately requires the introduction of a large number of infinite counterterms. However, when the sum of the contributions of a given order in g from all the terms of the total interaction Lagrangian is calculated, a small number of counterterms, which do not break the supersymmetry, is sufficient. The majority of the counterterms, expressed in terms of the constituent fields, compensate each other. It is here convenient to formulate the Feynman rules directly in terms of the superfields. The corresponding superpropagators

$$\langle T\Phi(x, \theta, \bar{\theta})\,\Phi(y, \theta', \bar{\theta}')\rangle_0 = \Delta^c(x-y, \theta, \bar{\theta}, \theta', \bar{\theta}')$$

are functions of the difference $x-y$ and the Grassmann generators $\theta, \bar{\theta}$. If one now calculates the perturbation-theory contributions described above using the Feynmann super-rules, the mutually compensating divergences do not arise at all.

This fact has a well-known historical analogy. After the creation at the end of the forties of covariant perturbation theory, in which the particles and antiparticles in virtual states were combined into the unified Stückelberg–Feynman propagators, it was found that the degree of some of the divergences changed. The divergence of the electron self-energy was reduced from linear to logarithmic.

It may be that, from the physical point of view, the most important examples are those of complete compensation of divergences in some specific models. We are referring to the so-called models of extended supersymmetry, which depend on $4N$ Grassmann generators (i. e., on $2N$ two-component Grassmann spinors $\theta_1^\alpha, \bar{\theta}_1^\alpha, \ldots, \theta_N^\beta, \bar{\theta}_N^\beta$; $\alpha, \beta = 1, 2$). Among them there are models with minimal values of the spins of the constituent fields, in which the values of these spins do not exceed some maximal value J, which is related to the number N by the simple relation $N = 4J$. Three such models are known[2]:

a) the extended $N=2$ model with maximal spin $J=\frac{1}{2}$;

b) the extended $N=4$ model with maximal spin $J=1$, which corresponds to a gauge vector field (supergauge model);

c) the extended $N=8$ model with maximal spin $J=2$, which corresponds to the graviton (extended supergravity).

In these three models, which on the basis of a formal count of the degrees of divergence in terms of the constituent fields (i. e., in calculation by the ordinary Feynman rules) are logarithmically divergent, the cancellation of the divergence leads to complete compensation of the divergent contributions to the renormalization of the coupling constant (i. e., to vanishing contributions to the so-called renormalization-group beta function). So far, this has been established:

1) for the $N=2$ extended supersymmetric, so-called sigma model in an arbitrary order of perturbation theory;

2) for the $N=4$ supergauge model in the one-, two-, and three-loop approximations.

It is found that when supersymmetry is taken into account systematically in the intermediate stages of the calculations, i. e., in the calculations in accordance with the Feynman super-rules, the logarithmic divergences of the renormalizations of the propagators and vertex functions, which compensate each other in the renormalization of the coupling constant in calculations made component by component, do not arise at all.

Moreover, in supergravity the absence of divergences was established without any calculations on the basis of the impossibility of constructing a supersymmetric operator form corresponding to the structure of one- and two-loop Feynman diagrams.

Thus, there have been found serious indications of the possible existence of a small number of "exceptional" models of quantum field theory in four-dimensional space–time that do not contain ultraviolet divergences in the renormalization constants of the operator field functions. Of course, for complete liberation from divergences it is still necessary to solve the problem of the self-masses of the particles of the matter fields.

For applications, the most interesting possibility is extended $N=8$ supergravity, which offers the hope of the existence of a mechanism of complete unification of all four (electromagnetic, weak, strong, and gravitational) interactions, free not only of unrenormalizable divergences, which usually accompany the procedure of quantization of the gravitational field, but of divergences altogether.[3]

If these hopes are fulfilled, we have a chance of obtaining a picture of complete unification (i.e., superunification), in which in the region of superhigh energies at

$$E \gg M_{\text{п}},\ M_{\text{п}} \sim (10^{14} \div 10^{18})\ \text{GeV}$$

the completely unified running coupling constant $\bar{\alpha}_{\text{cu}}$ ceases to "run," i.e., to change with a further increase in the energy, and is equal to some constant α_0, which corresponds to the coupling constant in the Lagrangian. In practical low-energy calculations, the complete-unification mass M_{cu} plays the part of an effective momentum cutoff. From the point of view of such calculations, the contribution of diagrams with superheavy virtual particles, which compensate the divergences of the "light" diagrams, can be regarded as a kind of "materialization of Pauli–Villars ghosts."

The ultraviolet divergences appear in such a case as parasitic effects that do not correspond to the physics of the situation but as an artificial consequence of the imperfection of the traditional method of investigation that split up the unified supersymmetric Lagrangian into nonsymmetric terms considered independently of each other in the first stage of the investigation.

We can liken the world of elementary interactions of particles to a beautiful vase with smooth shapes of a large statue, and a physicist to a pygmy that chips from the vase a piece and carries it to his laboratory for analysis. After a time he knows everything about the chip and nothing about the vase as a whole. However, besides the traditional laboratory means and equipment he needed iodine, cotton wool, and bandage to heal the cuts in his hands received from the sharp edges of the fragment of the vase. It may be that divergences, counterterms, and regularization are as unnecessary for understanding the picture of the interaction as a whole as medicine is for investigation of the vase as a whole.

Successive allowance for the three quantum symmetries corresponding to the chronological order of their use can be represented in the form of the following rather striking progressions.

1. The transition from quantum mechanics to renormalizable quantum field theory (allowance for the renormalizability principle) led to the replacement of the functional arbitrariness characterized by a potential function by only a set of arbitrary constants—the coupling constants.

2. The principle of local gauge symmetry, i.e., the introduction of gauge fields, led to the establishment of connections (in the simplest cases, equations) between the constants of the coupling of a given gauge field with matter fields.

3. The use of the supersymmetry principle introduces quite definite sets consisting of both Bose and Fermi fields (supermultiplets) whose interactions are described by a single coupling constant.

Finally, the possibilities of the fourth stage appear very exciting:

4. The imposition of the requirement of super-renormalizability (understood as the condition of absence of ultraviolet divergences in models that contain divergences in accordance with the formal rules for counting the powers of the fields and their derivatives in the Lagrangian) for a given value of the spin of the participating particles fixes the supersymmetric model. Taking this spin equal to two, we obtain the chance of achieving a unique description of the world of interactions of quantum fields in the region of sufficiently high energies.

Thus, there may begin to appear before our eyes Einstein's dream of more than half a century ago: "...we wish to know not only how nature is constructed (how natural phenomena occur) but also, if possible, achieve the aim, which may appear Utopian and audacious, of understanding why nature is as it is and not otherwise."[4]

As we have already noted, all four principles—renormalizability, local gauge symmetry, supersymmetry, and super-renormalizability—are quantum in nature, since purely quantum concepts are needed for their formulation:

1) the concept of the phase of the wave function, observable quantities being independent of it;

2) the concept of different quantum statistics and of Bose and Fermi fields;

3) the concept of the origin and structure of ultraviolet divergences in quantum field theory and the renormalization procedure.

As we have seen, the successive use of these quantum principles introduces a powerful deterministic flow in the world of quantum phenomena, which in its basis is subject to statistical laws.

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