Complete breakup of light nuclei induced by elementary particles

R. I. Dzhibuti

Institute of Physics, Georgian Academy of Sciences
Fiz. Elem. Chastits At. Yadra 14, 741–772 (July–August 1983)

The review presents systematically the existing theoretical and experimental results on complete breakup of light nuclei induced by elementary particles. A restriction is made to the recent theoretical studies that advance the unified approach to investigation of the structure of light nuclei and nuclear reactions on light nuclei. Analysis of the example of three-particle scattering illustrates the difficulties of the modern theory of many-particle reactions associated with the complicated asymptotic behavior of the wave function in the coordinate representation of the singularities of the scattering amplitude in the momentum representation. Various methods in the theory of the complete breakup of light nuclei induced by elementary particles are discussed. Experimental data on some reactions of this kind are given and compared with theoretical results. The following reactions are considered: complete photodisintegration of three- and four-particle nuclei; three-particle photodisintegration of the ^{6}Li nucleus in the cluster model; three-α-particle photodisintegration of the ^{12}C nucleus; complete breakup of light nuclei by $\pi^{\pm}$ mesons; and breakup with the participation of muons and hyperons.

PACS numbers: 25.10. + s, 27.10. + h, 25.20.Lj, 25.80.Hp

INTRODUCTION

During the last two decades, intensive investigations have been made in the physics of few-nucleon systems. Scientists from many scientific centers in the world have made high-accuracy calculations of the ground states of the ^{3}H and ^{3}He nuclei using Faddeev's integral equations, hyperspherical functions, and functions of the translationally invariant shell model with the aim of obtaining unambiguous conclusions about the forms of the nucleon–nucleon potentials. However, the characteristics of the ground states of three-particle nuclei were found to be insufficiently sensitive to the shape of the nuclear potential and did not solve the problem.[1] Recent calculations[2] for the ground state of the ^{4}He nucleus made by means of hyperspherical functions showed that the problem remains unsolved in this case too.

In such a situation, it is natural to extend the investigation by considering not only the ground states of light nuclei but also nuclear processes on light nuclei with the participation of elementary particles. The many-body aspects of nuclear structure and a nuclear system in the continuous spectrum must be manifested most clearly in processes such as the complete breakup of nuclei. Recently, there has been a strong growth of interest in problems of this kind.

The present paper reviews some of the results that have been obtained, explains the difficulties in the theory and the modern methods used in it, presents the existing more or less reliable experimental results, and gives their theoretical explanations.

1. DIFFICULTIES OF THE THEORY

Among the multimode of nuclear processes, complete-breakup reactions have until recently been the least studied. The complete breakup of a nucleus into nucleons is a many-particle process, and a complicated experimental technqiue is required to identify such events. No less complicated must be their theoretical description. For example, in the case of complete breakup of a three-particle nucleus with allowance for the final-state interaction it is necessary to consider the complicated process of three-particle scattering. One then encounters the great difficulties discussed intensively in Refs. 3–12. Such discussions and especially the intensive investigations of Merkur'ev[3] have made us very aware that these difficulties are related to the boundary conditions for the three-particle wave functions. In the momentum representation, they are expressed by a singularity of the total T scattering matrix and, as is shown in Refs. 8, 10, and 11, force us to reconsider the employed definitions of scattering observables. The problem can be expected to become even more complicated in the case of systems of four or more particles, though the asymptotic behavior of, for example, four-particle wave functions has not yet been investigated.

Despite these problems in the study of many-particle processes, there has been a strong growth of interest recently in the complete breakup of light nuclei induced by elementary particles. In the first place, this is due to the significant advance in few-body theory achieved by the creation and further development of the method of exact integral Faddeev equations and the hyperspherical-basis method. This has led to the hope of a model-free description of not only bound few-body states but also nuclear reactions. In addition, the calculations show that the continuum wave functions describing the final state in a complete-breakup reaction of three- and four-particle nuclei are changed strongly by allowance for the interaction between all the reaction products, and the corresponding cross sections depend rather critically on the choice of the nucleon–nucleon interaction potential. This result is very attractive, offering a prospect of attacking the multimode of forms of nucleon–nucleon potentials in which modern nuclear physics is so rich.

Comparatively systematic investigations of, for example, complete breakup of three-particle nuclei by γ rays were initiated by the application of Faddeev's integral equations.[13] Although Delves[14] made a first attempt as early as 1959 to use the method of hyperspherical functions for this purpose, systematic investigation of complete-breakup reactions of light nuclei using expansion with respect to a basis of hyperspherical functions have been made recently by Levinger's group, Fabre de la Ripelle and Fang in the United States, and the group of Baz' and the Tbilisi group in the Soviet Union.

The possibilities of using hyperspherical functions in problems of the continuous spectrum of many-particle systems are more restricted than is the case for Faddeev's exact integral equations. The many-particle continuum wave function obtained using hyperspherical functions behaves asymptotically like a multidimensional spherical wave [for three bodies, it has the form $\exp(ik_6\rho_6)/\rho_6^{5/2}$, and for four bodies $\exp(ik_9\rho_9)\rho_9^{-4}$, where ρ and k are the length of the radius vector and the momentum in the corresponding multidimensional spaces]. On the other hand, the three-particle energy operator H, for example, has $1+\sum_{i=1}^{3} N_i$ types of wave function[9,13]; here, N_i is the number of two-particle states with negative energy ε_i^α (the number of bound states in the i-th two-particle system). We denote these functions by $\psi_{\varkappa_0}(\rho)$ and $\psi^\alpha_{p_i}(\rho)$, with $i=1, 2, 3$, and $\alpha=1, 2, 3, \ldots, N_i$, where $\rho\equiv\{\eta_i, \xi_i\}$ and $\varkappa_0\equiv\{q_i, p_i\}$ are the six-dimensional radius vector and momentum. The wave function $\psi_{\varkappa_0}(\rho)$ describes three-particle scattering when all three particles are initially free, and the wave function $\psi^\alpha_{p_i}(\rho)$ describes the process in which pair i is in the bound state α before scattering. These functions are solutions of the same Schrödinger equation ($\hbar=2M=1$):

$$[E+\Delta_\rho-\sum_{i=1}^{3} v_i(\eta_i)]\,\psi(\rho)=0, \tag{1}$$

where $E=q_i^2-\varepsilon_i^\alpha\equiv E_i^\alpha$ for the function $\psi^\alpha_{p_i}(\rho)$ and $E=\varkappa_0^2$ for $\psi_{\varkappa_0}(\rho)$.

However, the boundary conditions for these solutions are quite different.

For the $i=1$ channel, the wave function $\psi^\alpha_{p_i}(\rho)$ can be represented in the form

$$\psi^\alpha_{p_1}(\rho)=\chi^\alpha_{p_1}(\rho)+\sum_{i=1}^{3}[\sum_{\beta=1}^{n_i^\alpha}\psi_i^\beta(\eta_i)\,U^{\beta\alpha}_{ip_1}(\xi_i)+U^\beta_{ip_1}(\rho)], \tag{2}$$

where

$$\chi^\alpha_{p_1}(\rho)=\psi_1^\alpha(\eta_1)\exp(ip_1\xi_1); \tag{3}$$

$\psi_i^\beta(\eta_i)$ is the wave function of the bound state of pair i.

The asymptotic form of the function $U^\alpha_{ip_1}(\rho)$ is given by

$$U^\alpha_{ip_1}(\rho)=\frac{\exp[i(E)^{1/2}\rho]}{\rho^{5/2}}[A^\alpha_{ip_1}(\eta_i, \xi_i)+O(1)], \tag{4}$$

and that of $U^{\beta\alpha}_{ip_1}(\xi_i)$ by

$$U^{\beta\alpha}_{ip_1}(\xi_i)=\frac{\exp[i(E_i^\beta)^{1/2}\xi_i]}{\xi_i}[a^{\beta\alpha}_{ip_1}(\xi_i)+O(1)]. \tag{5}$$

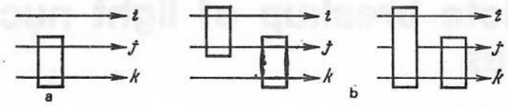

FIG. 1. Diagrams of the contributions of single (a) and double (b) scattering to the three-particle $3\to3$ process.

It can be seen from (2)–(5) that if bound subsystems can form in a three-particle system, the asymptotic behavior of the corresponding wave function is different from $\exp(ik_6\rho_6)/\rho_6^{5/2}$, so that the hyperspherical-basis method is ineffective for describing such states.

We now consider the wave function $\psi_{\varkappa_0}(\rho)$. It can be represented in the form

$$\psi_{\varkappa_0}(\rho)=\chi_{\varkappa_0}(\rho)+\sum_{i=1}^{3}[U^s_{i\varkappa_0}(\rho)+\sum_{\beta=1}^{N_i}\psi_i^\beta(\eta_i)\,U^\beta_{i\varkappa_0}(\xi_i)+U_{i\varkappa_0}(\rho)], \tag{6}$$

where $\chi_{\varkappa_0}(\rho)$ is a plane wave in the six-dimensional space; $U^s_{i\varkappa_0}(\rho)$ describe the contributions of single and double scattering shown in Fig. 1. The functions $U^\beta_{i\varkappa_0}(\xi_i)$ have the asymptotic behavior (5). If there are no two-particle bound states in the three-body system, the function (6) takes the form

$$\psi_{\varkappa_0}(\rho)=\chi_{\varkappa_0}(\rho)+\sum_{i=1}^{3}[U^s_{i\varkappa_0}(\rho)+U_{i\varkappa_0}(\rho)]. \tag{7}$$

In the asymptotic region, $U_{i\varkappa_0}(\rho)$ behaves as $\exp(ik_6\rho_6)/\rho_6^{5/2}$.

The entire difficulty in the problem of the continuous spectrum of a three-particle system is associated with the functions $U^s_{i\varkappa_0}(\rho)$. The point is that, because of the presence of the effects shown in Fig. 1b, the wave function exhibits different behavior in its dependence on the magnitude and direction of the six-dimensional radius vector ρ.

The double scatterings shown in Fig. 1 can occur both on and off the energy shell. Accordingly, the function $U^s_{i\varkappa_0}(\rho)$ contains two terms in the asymptotic region. One of them tends to zero as ρ^{-2}, the other as $\rho^{-5/2}$. The first corresponds to the case when particles i and j scatter, and then, after a macroscopic distance has been traversed, a different pair, for example, jk, scatters. For fixed values of the initial momenta, such double scattering is allowed for particular values of the final momenta determined by the momentum conservation law.

Following Newton and Shtokhamer,[8] we introduce the concept of a "ridge" of the double scattering in the six-dimensional space. The ridge is determined by the direction of the six-dimensional radius vector ρ corresponding to double scattering on the energy shell. If there is such a ridge, the overall picture becomes very complicated. Indeed, on the ridge itself, because there is in fact independent two-body scattering on the energy shell, the function $U^s_{i\varkappa_0}(\rho)$ will behave asymptotically as $\rho^{-2}\exp(iq_i\rho)$. The function takes its most complicated form when the scattering takes place near the ridge. As was shown in Refs. 3 and 5, a Fresnel function appears in this case. Then the vector q_i is no longer directed along the six-dimensional radius vector ρ, and the scattered wave ceases to be spherical. Physically, the origin of the nonspherical wave is the large distance between the two successive two-body scatterings. When this distance is large, the particles from the second scattering will not "originate" from the origin and, therefore, the corresponding wave will be nonspherical. Far from the ridge, the wave function $U^{s}_{i\varkappa_0}(\rho)$ behaves asymptotically as a six-dimensional spherical wave $\rho^{-5/2}\exp(i\varkappa_0\rho)$. Thus, if a ridge arises in a $3\to 3$ scattering process, fundamental difficulties are immediately encountered.

We now consider processes of complete breakup of light nuclei induced by elementary particles. Clearly, in the case of complete breakup of a three-particle nucleus there will be $3\to 3$ scattering processes at the end of the reaction. How probable is the appearance of a dangerous ridge in this case? Considering the inverse reaction, we can arrive at the conclusion that macroscopic distances in $3\to 3$ processes in the case of formation of a three-particle nucleus with emission of an elementary particle should not play a significant part. Therefore, the appearance of a ridge is improbable, and fundamental difficulties do not arise in the theory. Mathematically, this is due to the fact that the wave function of a bound state of a three-particle nucleus decreases rapidly with increasing ρ, with the consequence that the parts of the final-state wave function associated with processes at macroscopic distances do not contribute effectively to the transition matrix element.

The reactions we are considering can be divided into two large groups. One of them contains the breakup reactions in the final states of which bound subsystems cannot be formed. These reactions are the most favorable from the theoretical point of view, since the corresponding channels are unique. In addition, if the appearance of a dangerous ridge is improbable on account of the decay nature of the process, a single asymptotic behavior of the continuum wave function in the form of a multidimensional spherical wave is sufficient. The method of hyperspherical functions is very promising for the study of decay processes of this type, which include, for example, the reactions

$$\pi^+ + {}^3\mathrm{He}\to p+p+p;\quad \pi^- + {}^3\mathrm{H}\to n+n+n;$$
$$\gamma + {}^{12}\mathrm{C}\to \alpha+\alpha+\alpha;$$
$$\pi^- + {}^4\mathrm{He}\to \pi^+ + n+n+n+n;$$
$$\pi^+ + {}^4\mathrm{He}\to \pi^- + p+p+p+p;$$
$$\pi^- + {}^6\mathrm{Li}\to \alpha+n+n;\quad \pi^+ + {}^6\mathrm{Li}\to \alpha+p+p;$$
$$\mu^- + {}^3\mathrm{H}\to n+n+n+\nu_\mu.$$

The computational scheme and a review of the results for these reactions will be given in the following sections of the present paper.

The second group of decay processes is comparatively large and combines all complete-breakup reactions of light nuclei for which the final state contains subsystems capable of forming bound states. Of course, in this case "democracy" is violated, the situation may be complicated, and in the general case it is necessary to take into account channel coupling. However, there are cases when the violation of democracy can be assumed to be not too serious and such decay reactions can be investigated by means of hyperspherical functions. In this group of reactions, comparatively detailed studies have so far been made of the reactions

$$\gamma + {}^3\mathrm{He}\to p+p+n;\quad \gamma + {}^4\mathrm{He}\to p+p+n+n;$$
$$e + {}^3\mathrm{He}\to e' + p+p+n.$$

The corresponding results are reviewed below.

2. THE METHOD OF FADDEEV'S INTEGRAL EQUATIONS IN THE MOMENTUM REPRESENTATION FOR STUDY OF THE NUCLEAR PROCESS OF DIRECT COMPLETE BREAKUP OF THREE-PARTICLE NUCLEI

We shall restrict ourselves here to the problem of complete breakup of three-particle nuclei, since the method of Faddeev's integral equations has not yet been fully developed for the continuous spectra of four or more particles.

Faddeev's equations were first used to study the problem of transition from a bound state of three bodies to a continuum state (complete breakup of a nucleus) by Aaron, Amado, and Yam.[15] The scheme was then intensively used by other authors in calculations of the complete photodisintegration of the ^{3}He nucleus,[16-18] and also in investigations of the breakup of three-particle nuclei induced by other elementary particles.[19,20] In this direction, an especially large contribution on the photodisintegration of ^{3}He was made by Barbour and Phillips[16] and Gibson and Lehman.[17] We briefly describe the computational scheme proposed in Ref. 17.

We consider a three-particle system with Hamiltonian H. We denote by H^1 the Hamiltonian of the interaction of the external agent with the nucleons of the nucleus that causes the complete breakup. Further, let $|\psi_{\mathrm{bd}}\rangle$ and $|\psi_{\alpha n\mathbf{pk}}\rangle$ be the wave functions of the bound state and the continuum state of the three-particle system. These functions will satisfy the equations

$$H\,|\,\psi_{\mathrm{bd}}\rangle = -E_{\mathrm{bd}}\,|\,\psi_{\mathrm{bd}}\rangle,\quad E_{\mathrm{bd}}>0; \tag{8}$$

$$H\,|\,\psi_{\alpha n\mathbf{pk}}\rangle = E^{(3)}_{\alpha n}\,|\,\psi_{\alpha n\mathbf{pk}}\rangle;\quad E^{(3)}_{\alpha n} = \frac{p_\alpha^2}{2m_\alpha} + \frac{k^2_{\beta\gamma}}{2\mu_{\beta\gamma}}, \tag{9}$$

where $\mathbf{p}_\alpha$ is the momentum of particle α relative to the center of mass of particles β and γ, and $k_{\beta\gamma}$ is the relative momentum of particles β and γ. The reduced masses m_α and $\mu_{\beta\gamma}$ can be expressed in terms of the particle masses M_α by

$$m_\alpha = M_\alpha\,(M_\beta+M_\gamma)/(M_\alpha+M_\beta+M_\gamma);\quad \mu_{\beta\gamma} = M_\beta M_\gamma/(M_\beta+M_\gamma),\quad \alpha\neq\beta\neq\gamma=(1,\,2,\,3). \tag{10}$$

The amplitude of three-particle breakup can be represented in the form

$$A_3(\alpha,\,n,\,\mathbf{p},\,\mathbf{k}) = \langle\psi^{(-)}_{\alpha n\mathbf{pk}}\,|\,H'\,|\,\psi_{\mathrm{bd}}\rangle. \tag{11}$$

The wave function of three-particle scattering can be obtained by solving the equation

$$|\psi^{(-)}_{\alpha n\mathbf{pk}}\rangle = [1 - G\,(E^{(3)}_{\alpha n} - i\eta)\,V]\,|\,\Phi^0_{\alpha n\mathbf{pk}}\rangle,\quad \eta>0, \tag{12}$$

where

$$V=\sum_{\alpha=1}^{3} V_\alpha \equiv \sum_{i<j} V_{ij} \quad (13)$$

(V_{ij} is the two-nucleon potential),

$$G(z)=(H-z)^{-1} \quad (14)$$

is the resolvent operator, and $|\Phi^0_{\alpha n p \mathbf{k}}\rangle$ is a six-dimensional plane wave, an eigenfunction of the three-body kinetic-energy operator (H_0). Following Ref. 17, we introduce the operator

$$\Omega_0^{(+)}(z)=\Omega_0(z+i\eta)=1-VG(z+i\eta). \quad (15)$$

Then the amplitude of three-particle breakup can be written in the form

$$A_3(\alpha, n, \mathbf{p}, \mathbf{k})=\langle\Phi^0_{\alpha n p \mathbf{k}} \mid \Omega_0^{(+)}(z) H' \mid \psi_{\text{bd}}\rangle. \quad (16)$$

We introduce the operators $X_{\alpha\beta}(z)$, which couple the particle–correlated-pair states. For these operators, Gibson and Lehman[17] obtained the equations

$$X_{\alpha\beta}(z)=G_0(z)(1-\delta_{\alpha\beta})-G_0(z)\sum_\gamma(1-\delta_{\alpha\gamma})T_\gamma(z)X_{\gamma\beta}(z), \quad (17)$$

where

$$G_0(z)=(H_0-z)^{-1}; \quad (18)$$

$T_\gamma(z)$ is the two-particle T operator determined by the condition

$$T_\gamma(z)G_0(z)=V_\gamma G_\gamma(z); \quad (19)$$

$$G_\gamma(z)=(H_0+V_\gamma-z)^{-1}. \quad (20)$$

For the amplitude of three-particle breakup, we finally have

$$A_3(\alpha, n, \mathbf{p}, \mathbf{k})=\langle\Phi^0_{\alpha n p \mathbf{k}} \mid H' \mid \psi_{\text{bd}}\rangle - \sum_{\beta=1}^{3}\langle\Phi^0_{\alpha n p \mathbf{k}} \mid T_\beta(z)G_0(z)H' \mid \psi_{\text{bd}}\rangle + \sum_{\beta,\gamma=1}^{3}\langle\Phi^0_{\alpha n p \mathbf{k}} \mid T_\gamma(z)X_{\gamma\beta}(z)T_\beta(z)G_0(z)H' \mid \psi_{\text{bd}}\rangle, \quad z=E^{(3)}_{\alpha n}+i\eta. \quad (21)$$

The first term on the right-hand side of (21) is the plane-wave Born term, the second corresponds to single rescatterings, and the third to all the remaining rescattering effects.

On the basis of the arguments in Sec. 1, the second and third terms of the amplitude A_3 will not contain dangerous singularities if the single and all remaining rescatterings take place off the energy shell. Then the continuum wave function of the free three-particle system formed by the breakup of the three-particle nucleus will behave asymptotically as $\rho^{-5/2}$. In Ref. 17, this scheme was used to calculate the reaction $\gamma + {}^3\text{He} \to p+p+n$. The nucleon–nucleon potential was taken to be a nonlocal potential with separable variables parametrized by means of the low-energy nucleon–nucleon scattering data. The results agree reasonably well with the experimental data.[21] The calculations showed that in the case of complete ^{3}He photodisintegration at low photon energies the three nucleons are formed predominantly in a final state with isospin $T_f=3/2$, the cross section of the $T_f=\frac{1}{2}$ channel being approximately an order of magnitude smaller.

Other decay reactions have been investigated by the method of Faddeev's integral equations in the momentum representation. Radiative capture of pions by the ^{3}He nucleus with complete breakup was investigated by Phillips and Roig[22] in the framework of Amado's model,[23] which had been used earlier by Barbour and Phillips[16] to study complete photodisintegration of three-particle nuclei. In Amado's model, the probability of radiative capture of pions by the ^{3}He nucleus with complete breakup is proportional to the square of the three-nucleon scattering amplitude. This amplitude is represented as a sum of the Born term and a term that takes into account the formation, propagation and decay of nucleon–deuteron and nucleon–singlet-pair configurations.

These computational schemes for three-particle decay reactions are attractive in that they are based on Faddeev's integral equations in the momentum representation, which provide the only completely correct way of treating three-particle quantum mechanics. On the other hand, the possibilities of using this scheme are limited. First, using this scheme, it is hardly possible at present to attack the problem posed above of establishing the status of realistic potentials within the nucleus and choosing between the multimode of existing forms of the nucleon–nucleon potentials—in the scheme, one is in fact restricted to nonlocal potentials with separable variables. Second, because of the extreme complexity, no step has yet been made in the direction of four-particle systems, and for three-body systems the number of decay reactions is small. One of them is, for example, the reaction $\mu^- + {}^3\text{H} \to n+n+n+\nu_\mu$, which was initially investigated by means of the above scheme by Phillips, Roig, and Ros[19] and Torre, Gignoux, and Goulard,[20] and later by Torre and Goulard,[24] who proposed for this reaction a new scheme for studying the transition from a three-body bound state to a continuum state. This scheme is also based on Faddeev's equations, but uses the coordinate representation.

3. FADDEEV'S EQUATIONS IN THE COORDINATE REPRESENTATION FOR INVESTIGATION OF DECAY REACTIONS

We consider the complete breakup of the ^{3}H nucleus by a μ^- meson. The transition probability can be represented in the form[24]

$$\lambda_\mu({}^3\text{H} \to 3n) = \frac{2\pi}{\hbar}\int \frac{d^3\nu}{(2\pi)^3} \mid \langle \nu \otimes \psi^{(-)}_{\mathbf{pq}} \mid H' \mid \varphi_{\text{bd}} \otimes \varphi_\mu \rangle \mid^2 \delta(E_f-E_i)\frac{d^3p\, d^3q}{(2\pi)^6}, \quad (22)$$

where $|\nu\rangle$ is the plane wave describing the motion of the outgoing neutrino, $\psi^{(-)}_{\mathbf{pq}}$ is the scattering wave function of the three neutrons, φ_{bd} is the wavefunction of the ^{3}H bound state, $E=m_\mu c^2-E_{\text{bd}}$, where E_{bd} is the ^{3}H binding energy, and φ_μ is the muon wave function. As was pointed out in Sec. 1, the function $|\psi^{(-)}_{\mathbf{pq}}\rangle$ has a complicated asymptotic behavior, and the solution of the corresponding differential Faddeev equation with realistic nucleon–nucleon potentials is a very difficult task. Torre and Goulard[24] suggested that this difficulty should be avoided by going over to a new function $|\chi_\nu\rangle$ having the asymptotic behavior

$$\langle\rho \mid \chi_\nu\rangle_{|\rho|\to\infty,\,|\eta|/|\xi|=\text{const}}=A_\nu(\eta, \xi, \eta/\xi)\frac{\exp(ik_6\rho)}{\rho^{5/2}}, \quad (23)$$

where $A_\nu(\eta, \xi, \eta/\xi)$ is the three-particle scattering amplitude, η and ξ are Jacobi coordinates, and $\rho = \sqrt{\xi^2+\eta^2}$. The function $|\chi_\nu\rangle$ is obtained by solving the inhomogeneous equation

$$(E - E_\nu - H_\varkappa) \mid \chi_\nu\rangle = \langle \nu \mid H' \mid \varphi_{bd} \otimes \varphi_\mu\rangle, \tag{24}$$

where E_ν is the neutrino energy and $H_\varkappa$ is the nuclear Hamiltonian.

Of course, the asymptotic behavior of $|\chi_\nu\rangle$ is much simpler than that of $\psi_{pq}^{(-)}$. This greatly simplifies the problem. It is much harder to solve the equation for the function $\psi_{pq}^{(-)}$ than to find the function $|\chi_\nu\rangle$ from the inhomogeneous differential equation.[24]

It was shown in Refs. 8 and 9 that for the amplitude A_ν there exists the integral representation

$$A_\nu(\eta, \xi, \eta/\xi) = \frac{M}{\hbar^2} \frac{\exp(i\pi/4)}{2(2\pi)^{5/2}} \varkappa^{3/2} \langle \psi^{(-)}_{p\hat{\xi},\, q\hat{\eta}} \otimes \nu \mid H' \mid \varphi_{bd} \otimes \varphi_\mu\rangle \tag{25}$$

(with the condition $p/q = \xi/\eta$), where q and p are the Jacobi momenta conjugate to the Jacobi coordinates η and ξ, and $\varkappa^2 = p^2+q^2$.

Further, bearing in mind that the detector is a long way from the origin, we can assume $\hat{\mathbf{p}} = \hat{\xi}$ and $\hat{\mathbf{q}} = \hat{\eta}$, and from Eqs. (22) and (25) we finally obtain for the probability of the $\mu^- + {}^3H \to 3n + \nu_\mu$ process

$$\lambda_\mu({}^3H \to 3n) = \frac{2\hbar}{M} \int \frac{d^3\nu}{(2\pi)^3} d^3p\, d^3q \frac{\varphi_\mu}{\varkappa^3} \mid A_\nu(\mathbf{p}, \mathbf{q}, p/q) \mid^2. \tag{26}$$

Equation (24) is obtained under the assumption that the function $|\chi_\nu\rangle$, which is a pure outgoing six-dimensional spherical wave, is generated by the action of the perturbation operator H^1 on the ground-state wave function. Under this assumption, we can write

$$\mid \chi_\nu\rangle = G_n^+(E - E_\nu) \langle \nu \mid H' \mid \varphi_{bd} \otimes \varphi_\mu\rangle, \tag{27}$$

where G_n^+ is the three-nucleon Green's function. Equation (27) is equivalent to Eq. (24).

Using Faddeev's equation in the coordinate representation, Merkur'ev and Pozdneev[25] considered the scattering of a charged particle by a bound pair of charged particles (elastic scattering, breakup). A bound state of three charged particles was investigated by the same method. Attractive in this approach is the possibility of unified description (in the framework of the same formalism) of the bound state of the three-particle system and the scattering state of three bodies with Coulomb interaction. The theory based on the unified approach to the investigation of nuclear structure and nuclear reactions is called the unified theory of the nucleus.[26] Recent investigations show that in such a unified theory one can successfully use the method of expansion with respect to basis hyperspherical functions. These questions will be considered below.

4. THE METHOD OF HYPERSPHERICAL FUNCTIONS IN THE MOMENTUM REPRESENTATION AND DECAY REACTIONS

We consider the capture of stopped π^- mesons by the ^3H nucleus. In this case, there is only a single open channel— complete breakup of the nucleus. Such a reaction with allowance for the interaction simultaneously between all the neutrons in the final state was investigated for the first time in Ref. 27, in which the present author, Krupennikova, and Tomchinskii developed the method of hyperspherical functions in the momentum representation for the continuous spectrum. For the discrete spectrum, the method was proposed and developed earlier in Ref. 28.

The probability of the reaction $\pi^- + {}^3H \to n+n+n$ can be written in the form

$$W = \frac{2\pi}{\hbar} \int \sum \mid \langle f \mid H' \mid i\rangle \mid^2 \delta\left(\frac{\hbar^2\varkappa_0^2}{2m} + E_{bd} - \mu c^2\right) \rho_f, \tag{28}$$

where E_{bd} is the binding energy of the ^3H nucleus, $\varkappa_0^2 = p_0^2 + q_0^2 = 2mE/\hbar^2$, E is the energy of the three neutrons in the center-of-mass system, and μ is the mass of the π^- meson.

The density of the final states is

$$\rho_f = p_0^2\, dp_0\, q_0^2\, dq_0\, d\Omega_{p_0}\, d\Omega_{q_0}. \tag{29}$$

We assume that the π^- meson is absorbed from the S orbit of the mesic atom by a nucleon pair,[29,30] and we take the Hamiltonian of the $\pi - NN$ interaction in the nonrelativistic approximation:

$$H'_{12} = \frac{f}{k} \frac{\mu}{m} \Phi_S (T^- \sigma \cdot \nabla + \tau^- \mathbf{S} \cdot \nabla), \tag{30}$$

where f is the $\pi - N$ coupling constant, k is the π^- Compton wavelength, and Φ_S is the wave function of the S orbit of the mesic atom. Since the meson wave function changes little in the region of the nucleus, following Refs. 29 and 30, we take $\Phi_S = \pi^{-1/2} a_0^{-3/2}$, where a_0 is the Bohr radius of the orbit of the mesic atom. In (30), we have introduced the notation

$$\left.\begin{array}{ll} \sigma = \sigma(1) - \sigma(2); & \mathbf{S} = [\sigma(1) + \sigma(2)]/2; \\ \tau^- = \tau^-(1) - \tau^-(2); & T^- = [\tau^-(1) + \tau^-(2)]/2; \\ \tau^-(i) = [\tau_x(i) - i\tau_y(i)]/\sqrt{2}; & \nabla = [\nabla(1) - \Delta(2)]/2. \end{array}\right\} \tag{31}$$

For the transition matrix element we have

$$\langle f \mid H' \mid i\rangle = -\frac{3i}{2\sqrt{2}} \frac{f}{k} \frac{\mu}{m} \Phi_S \int \langle \psi^f_{p_0 q_0}\left(\mathbf{p}, \mathbf{q}; \{\sigma, \tau\}\right) \times \mid T^- \sigma \cdot \mathbf{q} + \tau^- \mathbf{S} \cdot \mathbf{q} \mid \psi^i(\mathbf{p}, \mathbf{q}; \{\sigma, \tau\}) > d\mathbf{p}\, d\mathbf{q}, \tag{32}$$

where $\langle \| \rangle$ is the matrix element in the spin–isospin space, $\psi^i(\mathbf{p}, \mathbf{q}; \{\sigma, \tau\})$ is the total wave function of the triton in the momentum representation (p and q are Jacobi momenta), $\psi^f_{p_0 q_0}(\mathbf{p}, \mathbf{q}; \{\sigma, \tau\})$ is the three-neutron continuum wave function, and σ and τ are the spin and the isospin variables.

We expand the antisymmetrized triton function with respect to a complete basis of hyperspherical functions in the momentum representation[28]:

$$\psi^i(\mathbf{p}, \mathbf{q}; \{\sigma, \tau\}) = \sum_{K[f]\mu[f]\nu} \varphi^{[f]\nu}_{KO}(\varkappa) \frac{1}{\sqrt{h_{[f]}}} \Phi^{[f]\mu[f]\nu}_{KO}(\Omega)\, V_1^{[\tilde{f}]\tilde{\mu}[\tilde{f}]}(\sigma, \tau), \tag{33}$$

where $\Phi^{[\tilde{f}]\tilde{\mu}[\tilde{f}]\nu}_{KO}(\Omega)$ are hyperspherical functions on a hypersphere of unit radius in the six-dimensional space of the momenta p and q, $\varkappa^2 = p^2 + q^2$, Ω is a set of five angles, four of which determine the direction of the vectors p and q, and the fifth is introduced by the conditions $p=\varkappa\sin\alpha$ and $q=\varkappa\cos\alpha$, $V_1^{[f]\tilde{\mu}[f]}(\sigma,\tau)$ are spin–isospin functions, $h_{[f]}$ is the dimension of the representation $[f]$, and $\mu_{[f]}$ is the Yamanouchi symbol. In (32), $L_i=0$ and $L_f=1$.

For the hyper-radial functions $\varphi_{KO}(\varkappa)$ in the momentum representation, we obtain a system of coupled one-dimensional integral equations ($\bar{\varkappa}_0^2=2m\varepsilon/\hbar^2$, where ε is the triton binding energy):

$$(\varkappa^2+\bar{\varkappa}_0^2)\,\varphi_{KO}^{[f]\nu}(\varkappa)=-\frac{6m}{\hbar^2}\frac{1}{\varkappa^2}\sum_{K'[f]'\nu' l_q l_p}\sum_{\mu_{[f]}\mu_{[f']}}C_{KO}^{[f]\mu[f]\nu}(l_q l_p)$$
$$\times C_{K'O}^{[f']\mu[f']\nu'}(l_q l_p)\frac{1}{\sqrt{h_{[f]}h_{[f']}}}\,i^{K-K'}\int J_{K+2}(\varkappa\rho)\,J_{K'+2}(\varkappa'\rho)$$
$$\times\tilde{\Phi}_K^{l_q l_p}(\alpha_\rho)\,\tilde{\Phi}_{K'}^{l_q l_p}(\alpha_\rho)\,\langle V_1^{[f']\tilde{\mu}'[\tilde{f}']}(\sigma,\tau)\,|\,U(\eta)\,|$$
$$\times V_1^{[f]\tilde{\mu}[\tilde{f}]}(\sigma,\tau)\rangle\,\varphi_{K'O}^{[f']\nu'}(\varkappa')\,d\tilde{\Omega}_\rho\rho\,d\rho\varkappa'^3\,d\varkappa', \qquad (34)$$

where

$$\left.\begin{aligned}&\tilde{\Phi}_K^{l_q l_p}(\alpha_\rho)=\sqrt{\frac{2n!\,(K+2)\,(n+l_p+l_q+1)!}{\Gamma(n+l_q+3/2)\,\Gamma(n+l_p+3/2)}}(\cos\alpha_\rho)^{l_q}(\sin\alpha_\rho)^{l_p}\\&\qquad\times P_n^{l_p+1/2,\,l_q+1/2}(\cos 2\alpha_\rho);\\&d\tilde{\Omega}=\cos^2\alpha\sin^2\alpha\,d\alpha;\qquad 2n=K-l_q-l_p;\end{aligned}\right\}\qquad (35)$$

$P_n^{l_1 l_2}$ are Jacobi polynomials, and l_q and l_p are the orbital angular momenta. In (34), $C_{KO}^{[f]\mu[f]\nu}(l_q l_p)$ are the coefficients of the orthogonal transformations from the functions with definite $l_q l_p K$ to functions of the given symmetry. As is shown in Ref. 27, these coefficients can be expressed in terms of the Raynal–Revai coefficients,[31] which are the coefficients of the unitary transformations of hyperspherical functions defined on different sets of Jacobi coordinates. For the Raynal–Revai coefficients simple recursion relations are obtained in Ref. 32.

Bearing in mind that the final-state wave function corresponds to $T=3/2$ and $S=\frac{1}{2}$, it can be represented in the form of the expansion

$$\psi^f_{\mathbf{p}_0\mathbf{q}_0}(\mathbf{p},\mathbf{q},\{\sigma,\tau\})$$
$$=\frac{1}{2}\sum_{K\nu}\varphi_{K1}(\varkappa,\varkappa_0)[\Phi_{K1}^{m2\nu}(\Omega_0)-\Phi_{K1}^{m1\nu}(\Omega_0)]$$
$$\times[V_2^{m2}(\sigma,\tau)\Phi_{K1}^{m1\nu*}(\Omega)-V_2^{m1}(\sigma,\tau)\Phi_{K1}^{m2\nu*}(\Omega)], \qquad (36)$$

where V_2 are spin–isospin functions.

The continuum hyper-radial functions $\varphi_{K1}(\varkappa,\varkappa_0)$ are solutions of the system of coupled one-dimensional inhomogeneous integral equations

$$\varphi_{K1}^{m\nu}(\varkappa,\varkappa_0)=\frac{\delta(\varkappa-\varkappa_0)}{\varkappa_0^5}-\frac{3m}{\hbar^2}\varkappa^{-2}(\varkappa^2-\varkappa_0^2-i\varepsilon)^{-1}$$
$$\times\sum_{K'\nu' l_q l_p\mu_m}C_{K1}^{\mu_m\nu}(l_q l_p)\,C_{K'1}^{\mu_m\nu'}(l_q l_p)\,i^{K-K'}$$
$$\times\int J_{K+2}(\varkappa\rho)\,J_{K'+2}(\varkappa'\rho)\tilde{\Phi}_K^{l_q l_p}(\alpha_\rho)\,\tilde{\Phi}_{K'}^{l_q l_p}(\alpha_\rho)$$
$$\times\langle V_2^{\tilde{\mu}_m}(\sigma,\tau)\,|\,U(\eta)\,|\,V_2^{\tilde{\mu}_m}(\sigma,\tau)\rangle$$
$$\times\varphi_{K'1}^{m\nu'}(\varkappa',\varkappa_0)\,d\tilde{\Omega}_\rho\rho\,d\rho\,\varkappa'^3\,d\varkappa'. \qquad (37)$$

In (34) and (37), $U(\eta)$ is the potential of the two-nucleon interaction. Specifying the form of this potential, we obtain from the integral equations (34) the binding energy and establish the structure of the initial nucleus. Solving then the system of integral equations (37), we obtain for the same nucleon–nucleon potential the continuum wave functions, and Eqs. (28)–(32) make it possible to investigate by the same method the nuclear reaction $\pi^-+{}^3\mathrm{H}\to n+n+n$. Thus, one obtains a unified approach to nuclear structure and nuclear reactions of complete-breakup type without using model assumptions about the nuclear structure or additional assumptions about the reaction mechanism. In solving the system of equations (37), it is more convenient to go over to the amplitude functions

$$\varphi_{K1}^{m\nu}(\varkappa,\varkappa_0)=\frac{\delta(\varkappa-\varkappa_0)}{\varkappa_0^5}+(\varkappa^2-\varkappa_0^2-i\varepsilon)^{-1}F_{K1}^{m\nu}(\varkappa,\varkappa_0). \qquad (38)$$

Then for the functions F we obtain the system

$$F_{K1}^{m\nu}(\varkappa,\varkappa_0)=\sum_{K'}f_{KK'1}^{m\nu}(\varkappa,\varkappa_0)$$
$$+\sum_{K'}\int\frac{\varkappa'^5\,d\varkappa'}{\varkappa'^2-\varkappa_0^2-i\varepsilon}f_{KK'1}^{m\nu}(\varkappa,\varkappa')\,F_{K'1}^{m\nu}(\varkappa',\varkappa_0), \qquad (39)$$

where

$$f_{KK'1}^{m\nu}(\varkappa,\varkappa_0)=-\frac{2m}{\hbar^2}\frac{1}{(\varkappa\varkappa_0)^2}\sum_{l_q l_p\mu_m}C_{K1}^{\mu_m\nu}(l_q l_p)$$
$$\times C_{K'1}^{\mu_m\nu}(l_q l_p)\,i^{K-K'}\int J_{K+2}(\varkappa\rho)\,J_{K'+2}(\varkappa_0\rho)$$
$$\times\tilde{\Phi}_K^{l_q l_p}(\alpha_\rho)\tilde{\Phi}_{K'}^{l_q l_p}(\alpha_\rho)$$
$$\times\langle V_2^{\tilde{\mu}_m}(\sigma,\tau)\,|\,U(\eta)\,|\,V_2^{\tilde{\mu}_m}(\sigma,\tau)\rangle\,d\tilde{\Omega}_\rho\rho\,d\rho. \qquad (40)$$

We can study similarly the reaction $\pi^++{}^3\mathrm{He}\to p+p+p$ with the difference that it is now necessary to calculate the cross section of the process:

$$\sigma=\frac{2\pi}{v_\pi}\int\sum|\langle f\,|\,H\,|\,i\rangle|^2\,\delta(E_i-E_f)\,d\rho_f, \qquad (41)$$

where $v_\pi=k_\pi/\mu$ is the velocity of the π^+ meson incident on the nucleus, $E_i=k_\pi^2/2\mu+\mu c^2$, $E_f=h^2k_0^2/2m+E_0$, and the matrix element is calculated in accordance with

$$\langle f\,|\,H\,|\,i\rangle$$
$$=-\frac{3i}{2\sqrt{2}}\Phi_{\pi^+}(O)\frac{f}{k}\frac{\mu}{m}\int\langle\psi^f_{\mathbf{p}_0\mathbf{q}_0}(\mathbf{p},\mathbf{q};\{\sigma,\tau\})$$
$$\times|\,T^+\sigma\cdot\mathbf{q}+\tau^+\mathrm{S}\cdot\mathbf{q}\,|\,\psi^i(\mathbf{p},\mathbf{q};\{\sigma,\tau\})\rangle\,d\mathbf{p}\,d\mathbf{q}. \qquad (42)$$

The formalism described above was used in Ref. 33 to consider the three-particle cluster structure of the ^{12}C nucleus and the decay reaction $\gamma+{}^{12}\mathrm{C}\to\alpha+\alpha+\alpha$. In Ref. 34, the three-particle cluster structure of the ^{6}Li nucleus and the three-particle reactions $\pi^-+{}^6\mathrm{Li}\to\alpha+n+n$ and $\pi^++{}^6\mathrm{Li}\to\alpha+p+p$ were investigated. In Ref. 35, the structure of hypertritium and the hypertritium complete-breakup reaction were studied. In Ref. 36, the three-particle photodisintegration of the ^{6}Li nucleus was studied by the same method. In Sec. 6, we shall give the results of some of the quoted papers with the corresponding experimental data.

It should be noted that the variant of the unified approach to the structure of the three-particle nucleus and three-particle decay reactions proposed in Ref. 27 admits a natural generalization to the case of four bodies. The corresponding theory is given in Ref. 37.

5. INVESTIGATION OF DECAY REACTIONS BY THE METHOD OF HYPERSPHERICAL FUNCTIONS IN THE COORDINATE REPRESENTATION

We consider specifically the double-charge-exhange reaction of $\pi^\pm$ mesons on the ^4He nucleus:

$$\pi^- + {}^4\mathrm{He} \rightarrow \pi^+ + 4n; \tag{43}$$

$$\pi^+ + {}^4\mathrm{He} \rightarrow \pi^- + 4p. \tag{44}$$

These reactions were investigated for the first time by the method of hyperspherical functions in the coordinate representation in Refs. 38 and 39. The differential cross section of these reactions can be written in the form

$$d\sigma = 2\pi \frac{P_f E_i E_f}{P_i} \mid M_{fi} \mid^2 \delta\left(E_i - \frac{\mathbf{q}_1^2+\mathbf{q}_2^2+\mathbf{q}_3^2}{2m} - Q - E_f\right) \frac{d\mathbf{q}_1\, d\mathbf{q}_2\, d\mathbf{q}_3}{(2\pi)^9} \frac{d\mathbf{p}_f}{(2\pi)^3}, \tag{45}$$

where $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ are the Jacobi momenta for the four neutrons or protons, and they are determined by the nucleon momenta by means of the relations

$$\mathbf{q}_1 = (\mathbf{k}_1 - \mathbf{k}_2)/\sqrt{2};\quad \mathbf{q}_2 = (\mathbf{k}_3 + \mathbf{k}_4 - \mathbf{k}_1 - \mathbf{k}_2)/2;\quad \mathbf{q} = (\mathbf{k}_3 - \mathbf{k}_4)/\sqrt{2}; \tag{46}$$

$P_{i\mu} \equiv (\mathbf{p}_i, E_i)$ and $P_{f\mu} \equiv (\mathbf{p}_f, E_f)$ are the 4-momenta of the pions in the initial and final states, respectively, E_i is the binding energy of the ^{4}He nucleus, and M_{fi} is the transition matrix element,

$$M_{fi} = \int \psi_f^* (\xi_1, \xi_2, \xi_3) F(\mathbf{p}_i, \mathbf{p}_f, \xi_1, \xi_2) \times \psi_i(\xi_1, \xi_2, \xi_3)\, d\xi_1\, d\xi_2\, d\xi_3, \tag{47}$$

where ξ_i are the Jacobi coordinates conjugate to the momenta $\mathbf{q}_i$. In (47), $F(\mathbf{p}_i, \mathbf{p}_f, \xi_1, \xi_2)$ is the double-charge-exchange amplitude, and $\psi_i(\xi_1, \xi_2, \xi_3)$ and $\psi_f(\xi_1, \xi_2, \xi_3)$ are the wave functions of the ^{4}He nucleus and the four-nucleon system in the final state, respectively.

We go over from the Jacobi momenta $\mathbf{q}_i$ to hyperspherical coordinates in the momentum space. The hyperspherical radius vector in the nine-dimensional momentum space is determined by $\varkappa = (q_1^2 + q_2^2 + q_3^2)^{1/2}$, and the hyperspherical angles β_1 and β_2 are introduced as follows:

$$q_1 = \varkappa \cos\beta_1 \sin\beta_2;\quad q_2 = \varkappa \sin\beta_1 \sin\beta_2;\quad q_3 = \varkappa \cos\beta_2. \tag{48}$$

The remaining six angles determine the directions of the vectors $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3$.

Using the hyperspherical variables and noting that $d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 = \varkappa^8 d\varkappa d\Omega_\varkappa$, we obtain from (45) for the differential double-charge-exchange cross section

$$\frac{d^2\sigma}{dE_f\, d\Omega} = 2\pi \frac{E_i}{P_i} \rho_f \int \mid M_{fi} \mid^2 d\Omega_\varkappa, \tag{49}$$

where

$$d\Omega_\varkappa = \sin^2\beta_1 \cos^2\beta_1 \sin^5\beta_2 \cos^2\beta_2\, d\beta_1\, d\beta_2\, d\Omega_{q_1}\, d\Omega_{q_2}\, d\Omega_{q_3}; \tag{50}$$

ρ_f is the five-particle density of the final states,

$$\rho_f = (2\pi)^{-12} P_f E_f [(2m)^{9/2}/2]\, (E_i - E_f - Q)^{7/2}. \tag{51}$$

We expand the wave function of the ^{4}He nucleus with respect to four-particle symmetrized hyperspherical functions,

$$\psi_i(\xi_1, \xi_2, \xi_3) = \sum_{\mu [f] \lambda L M} \chi^{\mu [f] L \lambda}(\rho)\, \Gamma^{LMST}_{\mu [f] \lambda}(\Omega_\rho, \sigma, \tau), \tag{52}$$

where μ is the generalized angular momentum in the nine-dimensional space; λ is the Yamanouchi symbol; L and M are the total orbital angular momentum and its projection, and S and T are the total spin and isospin, respectively; $\rho = (\xi_1^2 + \xi_2^2 + \xi_3^2)^{1/2}$ is the length of the radius vector in the nine-dimensional coordinate space; $\Omega_\rho \equiv (\alpha_1, \alpha_2, \hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3)$ is a set of eight angles, six of which, $\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3$, determine the directions of the unit vectors corresponding to the Jacobi coordinates, while the angles α_1 and α_2 are introduced by analogy with the relations (48) using the hyper-radius ρ. The symmetrized four-particle functions $\Gamma^{LMST}_{\mu [f] \lambda}(\Omega_\rho, \sigma, \tau)$ are constructed from the spin–isospin and hyperspherical functions[37]:

$$\Gamma^{LMST}_{\mu [f] \lambda}(\Omega_\rho, \sigma, \tau) = \frac{1}{\sqrt{h_{[f]}}} \sum_\nu \Phi^{[f]\nu\lambda}_{\mu LM}(\Omega_\rho)\, \Phi^{ST}_{[\tilde{f}]\tilde{\nu}}(\sigma, \tau), \tag{53}$$

where $h_{[f]}$ is the dimension of the representation of the group S_4, ν labels the rows of this representation, $\Phi^{ST}_{[\tilde{f}]\tilde{\nu}}(\sigma, \tau)$ are four-particle spin–isospin functions, $\Phi^{f\nu\lambda}_{\mu LM}(\Omega_\rho)$ are symmetrized hyperspherical functions introduced in accordance with

$$\Phi^{[f]\nu\lambda}_{\mu LM}(\Omega_\rho) = \sum_{l_1 l_2 l_{12} l_3 K} C^{[f]\nu\lambda}_{\mu L}(l_1 l_2 l_{12} l_3 K)\, \Phi^{l_1 l_2 l_{12} l_3}_{\mu L K M}(\Omega_\rho); \tag{54}$$

l_1, l_2, and l_3 are the orbital angular momenta corresponding to the Jacobi coordinates ξ_1, ξ_2, and ξ_3, and here

$$\Phi^{l_1 l_2 l_{12} l_3}_{\mu L K M}(\Omega_\rho) = \sum_{m_1 m_2 m_{12} m_3} \langle l_1 l_2 m_1 m_2 \mid l_{12} m_{12}\rangle \langle l_{12} l_3 m_{12} m_3 \mid LM\rangle \times \psi^{l_1 l_2 l_3 m_1 m_2 m_3}_{\mu K}(\Omega_\rho); \tag{55}$$

$\psi^{l_1 l_2 l_3 m_1 m_2 m_3}_{\mu K}(\Omega_\rho)$ are hyperspherical functions in the nine-dimensional coordinate space,[40] eigenfunctions of the square of the nine-dimensional angular momentum:

$$\hat{M}^2 \psi^{l_1 l_2 l_3 m_1 m_2 m_3}_{\mu K}(\Omega_\rho) = \mu(\mu + 7)\, \psi^{l_1 l_2 l_3 m_1 m_2 m_3}_{\mu K}(\Omega_\rho). \tag{56}$$

A simple method of calculation is given in Ref. 37 for the four-particle symmetrization coefficients $C^{f\nu\lambda}_{\mu L}(l_1 l_2 l_{12} l_3 K)$. These coefficients are related in simple ways to the four-particle Raynal–Revai coefficients introduced in Ref. 40. These last have a simple connection with the three- and four-particle Talmi–Moshinsky coefficients.[41] In Ref. 37, recursion relations are found for the four-particle Raynal–Revai coefficients.

Substituting the expansion (52) in the four-particle Schrödinger equation, we obtain for the hyper-radial functions $\chi^{\mu [f] L\lambda}(\rho)$ an infinite system of coupled differential equations.[42]

We represent the continuum wave function of the four-particle system in the form of the expansion[37]

$$\psi^{[f]\beta\beta'}_{\bar{f}\bar{\beta}\bar{\beta}'}(\xi_1, \xi_2, \xi_3, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = \sum_{\mu L M} \frac{\varphi_{\mu L}(\varkappa\rho)}{\rho^4} \hat{A} \psi^{[f]\beta\beta'}_{\mu LM}(\Omega_\rho, \sigma, \tau)\, \psi^{*[f]\bar{\beta}\bar{\beta}'}_{\mu LM}(\Omega_\varkappa), \tag{57}$$

where β, β' and $\bar{\beta}, \bar{\beta}'$ correspond to the symbols s and

a in Ref. 43 and label in pairs the functions of the five irreducible representations of the group S_4: the symmetric [4], the antisymmetric [1111], the two three-dimensional [31] and [211], and the one two-dimensional [22]. In particular, for the three-dimensional representations we have the functions ([31], ss), ([31], aa), ([211], aa), ([211], as). The functions $\psi_{\mu LM}^{[f]\beta\beta'}(\Omega\rho, \sigma, \tau)$ are composed of four-particle spin–isospin and hyperspherical functions.

Substituting the expansion (57) in the Schrödinger equation for the four particles in the continuum, we obtain an infinite system of coupled differential equations for the hyper-radial functions:

$$\frac{d^2\varphi_{\mu L}^{[f]}(\varkappa, \rho)}{d\rho^2}+\left[\varkappa^2-\frac{(\mu+3)(\mu+4)}{\rho^2}\right]\varphi_{\mu L}^{[f]}(\varkappa, \rho) = \sum_{[f']\mu'} V_{\mu\mu'}^{[f][f']}(\rho)\,\varphi_{\mu' L}^{[f']}(\varkappa, \rho), \tag{58}$$

where $V_{\mu\mu'}^{[f][f']}(\rho)$ is the effective four-particle potential energy, multiplied by $2m/\hbar^2$, formed from realistic nucleon–nucleon potentials.

In Ref. 44, the variable-phase method was generalized for the quantum mechanics of three and four bodies under the restriction of "truly" many-particle scatterings (the asymptotic behavior of the wave function of many-particle scattering is represented in the form of a multidimensional spherical wave, and the dangerous singularities of the scattering amplitude discussed in Sec. 1 do not arise). The method proposed in Ref. 44 is very productive in the solution of the system of differential equations (58). We write this system in the matrix form

$$\frac{d^2\varphi(\varkappa, \rho)}{d\rho^2}+\left[\varkappa^2-\frac{(\mu+3)(\mu+4)}{\rho^2}\right]\varphi(\varkappa, \rho)=V(\rho)\,\varphi(\varkappa, \rho), \tag{59}$$

where

$$\varphi(\varkappa, \rho)=\begin{pmatrix}\varphi_{\alpha_1}(\varkappa, \rho)\\ \varphi_{\alpha_2}(\varkappa, \rho)\\ \vdots\end{pmatrix}; \tag{60}$$

$$V(\rho)=\begin{pmatrix}V_{\alpha_1\alpha_1}(\rho) & V_{\alpha_1\alpha_2}(\rho) & \dots\\ V_{\alpha_2\alpha_1}(\rho) & V_{\alpha_2\alpha_2}(\rho) & \dots\\ \dots & \dots & \dots\end{pmatrix}; \tag{61}$$

$\alpha\equiv(\mu[f]\lambda)$ is the set of quantum numbers. We seek the solution of the matrix equation (59) in the form

$$\varphi(\varkappa, \rho)=[j(\varkappa, \rho)-n(\varkappa, \rho)\,K(\varkappa, \rho)]\,A(\varkappa, \rho), \tag{62}$$

where

$$j(\varkappa, \rho)=\begin{pmatrix}j_{\mu+3}(\varkappa, \rho) & 0 & 0\\ 0 & j_{\mu+5}(\varkappa, \rho) & 0\dots\\ 0 & 0 & j_{\mu+7}(\varkappa, \rho)\\ \dots & \dots & \dots\end{pmatrix} \tag{63}$$

and

$$n(\varkappa, \rho)=\begin{pmatrix}n_{\mu+3}(\varkappa, \rho) & 0 & 0\dots\\ 0 & n_{\mu+5}(\varkappa, \rho) & 0\dots\\ 0 & 0 & n_{\mu+7}(\varkappa, \rho)\ 0\dots\\ \dots & \dots & \dots\end{pmatrix} \tag{64}$$

are matrices composed, respectively, of Riccati–Bessel and Riccati–Neumann functions; $A(\varkappa, \rho)$ is a column matrix; $K(\varkappa, \rho)$ is a diagonal matrix with elements $K_{\alpha_1}(\varkappa, \rho), K_{\alpha}, (\varkappa, \rho), K_{\alpha_3}(\varkappa, \rho)\dots$. For $K(\varkappa, \rho)$ a nonlinear first-order matrix equation is obtained[44]:

$$dK(\varkappa, \rho)/d\rho=-[j(\varkappa, \rho) - K(\varkappa, \rho)\,n(\varkappa, \rho)]\,V(\rho)\,[j(\varkappa, \rho)-n(\varkappa, \rho)\,K(\varkappa, \rho)], \tag{65}$$

and the equation for the amplitude matrix has the form

$$dA(\varkappa, \rho)/d\rho = n(\varkappa, \rho)\,V(\rho)\,[j(\varkappa, \rho)-n(\varkappa, \rho)\,K(\varkappa, \rho)]\,A(\varkappa, \rho). \tag{66}$$

In Refs. 38 and 39, in which various forms of modern realistic nucleon–nucleon potentials are used, the above approach is used to investigate simultaneously the ground-state structure of the ^{4}He nucleus and $\pi^{\pm}$ double-charge-exchange nuclear reactions at low energies. It is assumed that these reactions take place through two successive charge-exchange πN scatterings with production on one nucleon and subsequent capture of the π^0 meson by another nucleon. The corresponding amplitude of the elementary event was constructed in Ref. 45. It is clear that the contribution of this mechanism will be predominant at low pion exchanges. We shall discuss the results together with the experimental data below.

In Refs. 46 and 47, the same method was used to investigate the complete photodisintegration of the ^{4}He nucleus, and in Ref. 48 a quasi-α-particle mechanism of double $\pi^{\pm}$ charge exchange on the ^{12}C nucleus was introduced and calculated.

A basis of three-particle hyperspherical functions in the coordinate representation was intensively used to investigate complete photodisintegration of the ^{3}He nucleus by Levinger, Fabre de la Ripelle, Fang, and Fitzgibbon in Refs. 49–54. Also working in this direction are Vostrikov and Zhukov,[55,56] who have investigated in detail not only complete photodisintegration but also the channel of two-particle photodisintegration of the ^{3}H nucleus using the interpolation approach of Baz' and Zhukov.[57] The same approach was used by Tartakovskiĭ and collaborators[58] in an investigation of the electrodisintegration of the ^{3}H and ^{3}He nuclei.

6. EXPERIMENTAL DATA ON SOME COMPLETE-BREAKUP REACTIONS OF LIGHT NUCLEI AND COMPARISON WITH THEORETICAL RESULTS

Complete photodisintegration of three- and four-particle nuclei.

Among the complete-breakup reactions of light nuclei induced by elementary particles the experimentally most studied reaction is

$$\gamma + {}^3\mathrm{He} \to p + p + n. \tag{67}$$

The experimental data and theoretical curves for the cross section of the reaction (67) are shown in Fig. 2. The theoretical results were obtained by the method of Faddeev's integral equations (see Sec. 2), a separable potential being taken as the nucleon–nucleon potential. It was shown in Refs. 16 and 17 that allowance for the final-state interaction significantly changes the reaction cross section (the cross section is reduced to al-

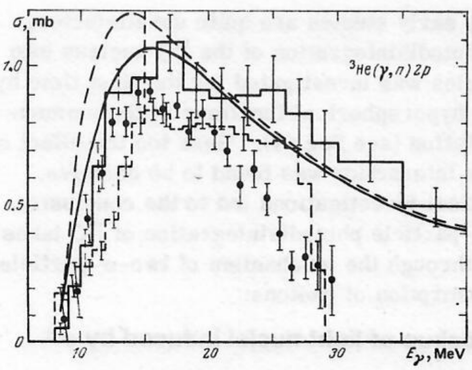

FIG. 2. Cross sections of the reaction $\gamma + {}^3\text{He} \to p + p + n$. The broken curve is taken from Ref. 16, and the continuous curve from Ref. 17. Experimental data: the continuous histogram is from Ref. 21, the broken histogram from Ref. 59, and the black circles are from Ref. 60.

most half, and the position of the maximum is shifted to lower energies). Experimental data from the same papers are given in Fig. 3, but the theoretical curves are taken from studies made by the method of hyperspherical functions in the coordinate representation (see Sec. 5). The continuous and broken curve are the results of Fabre de la Ripelle and Levinger[49] for the potentials V^X and $G2$, respectively, and the broken curve is the result of Vostrikov and Zhukov[55] for the Eikemaier–Harkenbroich potential. It can be seen that the results depend strongly on the form of the nucleon–nucleon potential.

At the present time, experimental data on the complete photodisintegration of the ^{4}He nucleus,

$$\gamma + {}^4\text{He} \to p + p + n + n, \qquad (68)$$

are known with much greater errors than for the data on the reaction considered above. This reaction was studied theoretically by the hyperspherical-basis method in the coordinate representation (see Sec. 5) in Refs. 46 and 47. The theoretical curves in Fig. 4 are taken from Ref. 47 and correspond to different nucleon–nucleon potentials. The chain curves are the cross sections obtained without allowance for the final-state interaction (the cross section is multiplied by 0.5). It

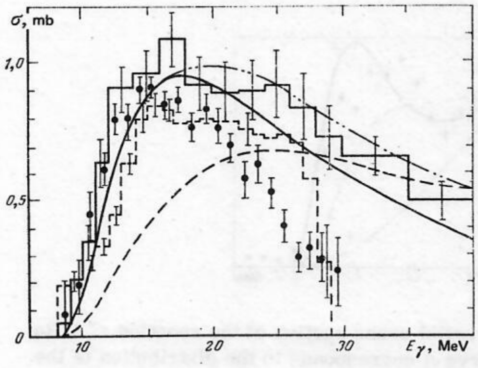

FIG. 3. Cross sections of the reaction $\gamma + {}^3\text{He} \to p + p + n$. The continuous and broken curves are taken from Ref. 49, and the chain curve from Ref. 55; the experimental data are the same as in Fig. 2.

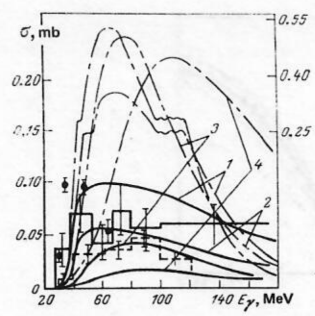

FIG. 4. Cross sections of the reaction $\gamma + {}^4\text{He} \to p + p + n + n$. The theoretical results are taken from Ref. 47: 1) for the Gogny–Pires–de Tourreil potential[64]; 2) for the Volkov potential[65]; 3) for a potential well[66]; 4) for the Baker potential.[67] Experimental data: the continuous histogram is from Ref. 61, the broken histogram from Ref. 62, and the black circles are from Ref. 63.

can be seen that the effect of interaction simultaneously between all the nucleons in the final state is manifested more strongly for the complete photodisintegration of ^{4}He (reducing the cross section by almost an order of magnitude) than it is for ^{3}He photodisintegration. Particularly important among the theoretical results in Refs. 46 and 47 is the strong dependence of the cross section on the chosen form of the nucleon–nucleon potential, and also the fact that the interaction between the nucleons in the final state suppresses the mechanism of single-particle photon absorption by the nucleus and two-particle absorption becomes the main mechanism.

In the process

$$\gamma + {}^6\text{Li} \to \alpha + n + p \qquad (69)$$

two mechanisms can contribute: 1) direct breakup into three interacting particles; 2) two-step breakup with the formation of a subsystem in an excited state in the first step and breakup of the subsystem in the second. The regions in which these two mechanisms act are more or less separated. Indeed, if we assume that the three-particle photodisintegration of the ^{6}Li nucleus in the cluster model in the region of energies $E_\gamma < 20$ MeV occurs in two steps, then in the first an excited state of ^{5}He or ^{5}Li is formed. For these nuclei, bound ground states are absent, and the first excited states with more or less narrow widths appear only at $E_x^{^5\text{Li}} = 16.81$ MeV and $E_x^{^5\text{He}} = 16.68$ MeV. Below $E_\gamma = 16$ MeV, the main contribution must be made by the mechanism of direct breakup into three particles. The two-step mechanism of the (γ, n) reaction on the ^{6}Li nucleus was investigated in detail in Refs. 68 and 69. In Ref. 36, the mechanism of direct three-particle breakup of the ^{6}Li nucleus was investigated by the method of hyperspherical functions in the momentum representation (see Sec. 4).

Figure 5 gives experimental data on the cross section of the (γ, n) reaction on the ^{6}Li nucleus.[70] The nucleon-nucleon potential used in the calculations was the Gogny–Pires–de Tourreil potential,[64] and the nucleon–

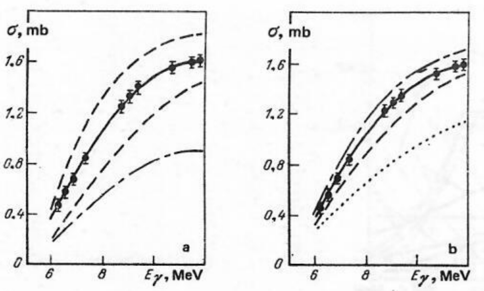

FIG. 5. Cross sections of the reaction $\gamma + {}^6\text{Li} \rightarrow \alpha + n + p$ without allowance for the contributions of the exchange currents (a) and with implicit allowance for these contributions (b) (theoretical results from Ref. 36). The dotted curve is the cross section in the plane-wave approximation with allowance for one harmonic in the final state; the broken curves are the cross section in the plane-wave approximation with allowance for $K=1$ and 3 harmonics at the end; the chain curves are the cross section with allowance for the interaction between all the particles and two harmonics in the final state. The experimental data are taken from Ref. 70.

α-particle potential was the Pearce–Swan potential.[71] It was shown that the main contribution to the norm of the wave function of the ^{6}Li nucleus is made by the harmonic $K=2$ (96.7%). The binding energy of the ^{6}Li nucleus then obtained (4.70 MeV) is close to the experimental value (4.53 MeV). It can be seen from comparison of the curves in Figs. 5a and 5b that without allowance for the final-state interaction the wave function of the $\alpha + n + p$ system is very inaccurate. Namely, as a result of this inaccuracy Siegert's theorem ceases to hold, and therefore in the plane-wave approximation with allowance for exchange currents (Siegert's theorem) the curves of the cross section (dotted and broken curves) in Fig. 5b pass below the corresponding curves of Fig. 5a (without allowance for exchange currents). The inclusion of the final-state interaction significantly corrects the wave function, and the chain curve in Fig. 5b, which takes into account the contributions of the exchange currents (in accordance with Siegert's theorem), passes appreciably above the corresponding curve in Fig. 5a, which does not take into account the contributions of the exchange currents. Such a result indicates that Seigert's theorem is very sensitive to inaccuracies of the wave functions and its use in even the long-wave approximation (at low energies) can lead to incorrect results.

The reaction

$$\gamma + {}^{12}\text{C} \rightarrow \alpha + \alpha + \alpha \tag{70}$$

has been studied little experimentally. The data of Maǐkov[72] are based on very sparse statistics, and the corresponding errors are not indicated on the experimental curve. A Khar'kov group[73,74] has begun a systematic study of this reaction. Until recently, there did not exist an acceptable theory of the reaction $\gamma + {}^{12}\text{C} \rightarrow 3\alpha$. Since the studies of Verde *et al.*,[75,76] nuclear physics and especially the three-body problem in nuclear physics have advanced significantly, and, of course, the early studies are quite unsatisfactory. In Ref. 33, photodisintegration of the ^{12}C nucleus into three α particles was investigated for the first time by the method of hyperspherical functions in the momentum representation (see Sec. 4). Here too the effect of the final-state interaction was found to be decisive. These theoretical investigations led to the conclusion that the three-particle photodisintegration of ^{12}C takes place mainly through the mechanism of two-α-particle quadrupole absorption of photons.

Complete breakup of light nuclei induced by $\pi^\pm$ mesons

The reactions

$$\pi^- + {}^3\text{H} \rightarrow n + n + n \quad \text{и} \quad \pi^+ + {}^3\text{He} \rightarrow p + p + p \tag{71}$$

have not yet been studied experimentally. A theory based on the use of a basis of hyperspherical functions in the momentum representation (see Sec. 4) is developed in Ref. 27. Among the obtained theoretical results, the most important is the establishment of a significant role of the interaction between all the nucleons in the final state. It is to be hoped that corresponding experiments will be made, so that it will be possible to verify the theoretical results and, in particular, the basic assumption about the mechanism of two-body absorption of $\pi^\pm$ mesons by three-particle nuclei.

For the reaction

$$\pi^- + {}^3\text{He} \rightarrow \pi^+ + 3n \tag{72}$$

there is the experimental study of Ref. 77, which gives the differential cross section as a function of the kinetic energy of the three-neutron system. The positive pions were detected in the interval of angles from 15 to 40°. A theory of the π^- double-charge-exchange reaction on the ^{3}He nucleus was developed by Phillips[78] using the method of Faddeev's integral equations in the momentum representation (see Sec. 2). However, these equations were used only to describe the final state of the three neutrons; the initial state of the nucleus was described by a Gaussian function. The theory[78] and experiment[77] are compared in Fig. 6. Curve C is obtained from curve B by replacing the Born amplitude by the amplitude calculated with al-

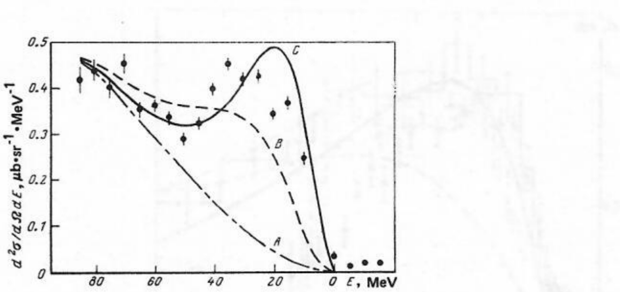

FIG. 6. Differential cross section of the reaction $\pi^- + {}^3\text{He} \rightarrow \pi^+ + 3n$. Curve A corresponds to the distribution of the four-particle phase space when the transition amplitude is assumed to be constant; curve B is obtained from A by incoherent addition of the Born amplitude; curve C is obtained with allowance for the final-state interaction. The experimental data are taken from Ref. 77.

lowance for the final-state interaction (the first iteration of the Faddeev equation is used). Despite the very approximate nature of the calculations, the theory shows that the interaction between the neutrons in the final state plays a decisive part. It was concluded that the experiment of Ref. 77 did not reveal a three-neutron resonance and that the distinctive structure of the distribution curve in Fig. 6 is the result of a strong effect of the final-state interaction.

We now consider the reaction

$$\pi^- + {}^3H \to \gamma + n + n + n. \qquad (73)$$

Experimentally, π^- radiative capture by the ^{3}He nucleus was first investigated in Ref. 79. Bistirlich *et al.*[80] then made a detailed experimental investigation of the reaction $\pi^- + {}^3H \to \gamma + 3n$. The experimental histogram of the spectrum of photons from the reaction $\pi^- + {}^3H \to \gamma + n + n + n$ is shown in Fig. 7, in which we have also plotted the theoretical curve obtained by Phillips and Roig.[22] They investigated π^- radiative capture by the ^{3}He nucleus by the method of Faddeev's integral equations in the momentum representation, using a nucleon–nucleon potential of separable form. The final-state interaction was taken into account in Amado's model (see Sec. 2). It can be seen from the figure that there is good agreement between the theory and the experiment, which, of course, can in no way be obtained without allowance for the interaction between the neutrons in the final state. The energy distribution of the photons has a clear peak at high energies, which corresponds to a low relative energy of the neutrons; however, this is a consequence of the interaction between the neutrons and does not indicate the formation of a three-neutron resonance state.

We now consider the reaction

$$\pi^- + {}^4He \to \pi^+ + 4n \text{ and } \pi^+ + {}^4He \to \pi^- + 4p. \qquad (74)$$

It is well known that the process of double charge exchange of pions on nucleons was first discovered and studied in experiments of a Dubna group,[81] and was subsequently studied in experiments made in other scientific centers.[82-84] On the basis of a unified treatment of the structure of the ^{4}He nucleus and the nuclear reactions (74), Kezerashvili, Sigua, and the present author[38,39] first studied double-charge-exchange processes by the hyperspherical-basis method

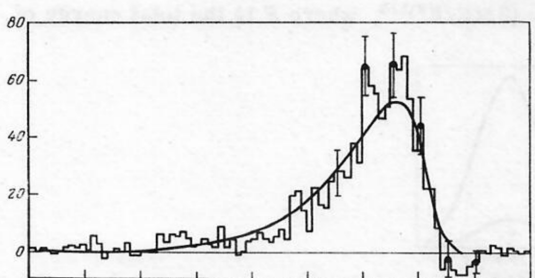

FIG. 7. Energy spectrum of photons in the reaction $\pi^- + {}^3H \to \gamma + 3n$. The theoretical curve is taken from Ref. 22. The experimental histogram is from Ref. 80.

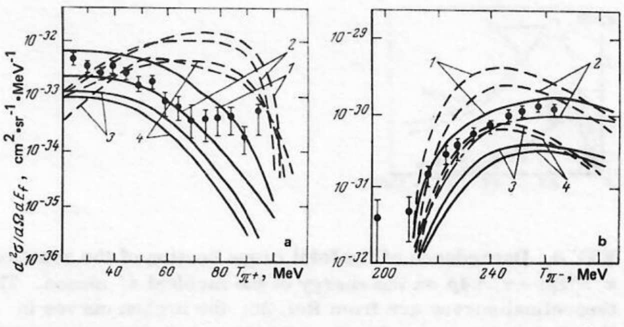

FIG. 8. Differential cross section of the reaction $^4He(\pi^-, \pi^+)4n$: a) the experimental data are taken from Ref. 85 (angle of observation of the π^+ meson at the end $\theta = 20°$ and $T_{\pi^-} = 140$ MeV; b) data taken from Ref. 83 (angle of observation $\theta = 0°$, $T_{\pi^+} = 176$ MeV); the theoretical curves are from Ref. 38: the broken curves are the results of the plane-wave approximation, and the continuous curves are with allowance for the interaction between all the neutrons in the final state. 1) For the Gogny–Pires–de Tourreil potential; 2) for the Volkov potential; 3) for a well potential; 4) for the Baker potential.

in the coordinate representation (see Sec. 5). Experimental data on the differential cross section of the reaction $^4He(\pi^-, \pi^+)4n$ are given in Fig. 8. The data of Fig. 8a are taken from Ref. 85 and give the dependence of the differential cross section on the kinetic energy (T_{π^+}) of the π^+ meson. It can be seen from the figure that without allowance for the final-state interaction the curves are far from the experimental data, and in the majority of cases do not even qualitatively reproduce the behavior of the cross section. The inclusion of the interaction simultaneously between all the nucleons in the final state changes the results of the plane-wave calculations qualitatively and quantitatively and leads to new results that correctly reproduce the behavior of the experimental curves of the cross sections. This is in fact achieved for all forms of the employed *NN* potentials. However, the results for the different potentials differ quantitatively. In addition, the theoretical results show that the contribution of the final-state interaction depends strongly on the energy transferred to the four-nucleon system in the double-charge-exchange process. The total cross section of the reaction $\pi^+ + {}^4He \to \pi^- + 4p$ is shown as a function of the energy of the incident π^+ in Fig. 9. The large effect of a final-state interaction is also demonstrated in this figure.

The reaction

$$\pi^- + {}^6Li \to \alpha + n + n \qquad (75)$$

is distinguished by its simplicity among the processes of capture of stopped π^- mesons by nuclei with emission of nucleon pairs, and it was investigated intensively in early studies, both theoretical[90,91] and experimental.[93,94] However, in the early theoretical studies the final-state interaction was taken into account only between two neutrons. In Ref. 34, the interaction between all the particles in the final state was taken into account for the first time, and the reaction (75) was investigated by the method of hyperspherical functions in the momentum representation (see Sec. 4). It was shown that elimination of the $N\alpha$ interaction from the effective

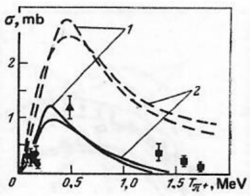

FIG. 9. Dependence of the total cross section of the reaction $\pi^+ + {}^4\mathrm{He} \rightarrow \pi^- + 4p$ on the energy of the incident π^+ meson. The theoretical curves are from Ref. 38: the broken curves in the plane-wave approximation, and the continuous curves with allowance for the interaction in the final state; 1) for the Gogny–Pires–de Tourreil potential; 2) for the Volkov potential. The experimental data are as follows: the black circles are from Refs. 86 and 87, the black triangles from Ref. 88, and the black squares from Ref. 89.

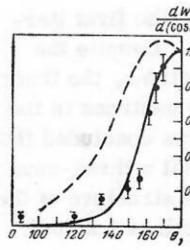

FIG. 11. Distribution of neutrons with respect to the angles between them for the reaction $\pi^- + {}^6\mathrm{Li} \rightarrow \alpha + n + n$. The theoretical results are taken from Ref. 34: the broken curve is without allowance for the internal structure of the α particle, and the continuous curve is with allowance for it. The experimental data are taken from Ref. 94.

three-particle interaction in the final state leads to results that are appreciably too large (the capture probability is increased by almost an order of magnitude). The distribution of the neutrons with respect to the recoil momenta of the α particle from the reaction (75) is given in Fig. 10. The distribution of the neutrons with respect to the angles between them for the reaction (75) is shown in Fig. 11. As can be seen from Figs. 10 and 11, one can achieve a satisfactory description of the experimental data in the investigation of the distribution with respect to the recoil momenta and the angles between the emitted nucleons by assuming that the α particle "takes up" the recoil momentum, not as a structureless "ball," but as a nuclear system with a definite internal structure. And for the total probability of the process allowance for the internal structure of the α particle leads to a satisfactory explanation of the experimental value.

Breakup reactions involving μ^- mesons and hyperons

The results of recent theoretical investigations[24] of the reaction

$$\mu^- + {}^3\mathrm{H} \rightarrow n + n + n + \nu_\mu \quad (76)$$

by the method of Faddeev's equations in the coordinate representation (see Sec. 3) increase the interest in corresponding experiments, which have not yet been made. In particular, the neutron energy spectrum is found to be rather sensitive to the effect of the final-state interaction of the neutrons. This result offers hope that when it becomes possible to test the conclusions of the theory it will be possible to draw conclusions about the acceptability of the employed nucleon–nucleon potentials.

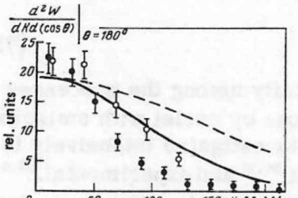

FIG. 10. Distribution of neutrons with respect to the recoil momenta from the reaction $\pi^- + {}^6\mathrm{Li} \rightarrow \alpha + n + n$. The theoretical results are from Ref. 34: the broken curve is the result of calculation without allowance for the internal structure of the α particle, while the continuous curve is with allowance for this structure; the black circles are the experimental data of Ref. 93, and the open circles the data of Ref. 96.

Let us consider the reactions

$${}^3_\Lambda\mathrm{H} \rightarrow \pi^- + p + p + n \text{ and } {}^3_\Lambda\mathrm{H} \rightarrow n + n + p. \quad (77)$$

The structure of hypertritium was investigated for the first time by the method of Fadeev's integral equations in the momentum representation in Refs. 97 and 98. Recently, this formalism has been used for intensive investigation of the structure of three- and four-particle hypernuclei.[99-101] The binding energy of three-particle hypernuclei was calculated in Ref. 102 using a hyperspherical basis in the coordinate representation. With regard to the reactions (77), the method of hyperspherical functions in the momentum representation (see Sec. 4) was used for the study of them for the first time in Ref. 35. It was shown that the interaction between all the nucleons in the final state significantly influences the probability of hypertritium decay with and without pion emission. The meson decay mechanism makes the main contribution to the complete breakup of hypertritium. Mesonless decay takes place with negligibly small probability because of the instability of hypertritium. Figure 12 shows the theoretical curves of $dW/d\varkappa_0$ as a function of $\varkappa_0$, where W is the probability of the partial decay ${}^3_\Lambda\mathrm{H} \rightarrow \pi^- + p + p + n$, and $\varkappa_0 = (2ME/\hbar^2)^{1/2}$, where E is the total energy of

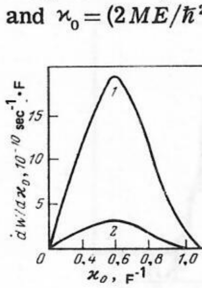

FIG. 12. The functions $dW/d\varkappa_0$ for the reaction ${}^3_\Lambda\mathrm{H} \rightarrow \pi^- + p + p + n$ from Ref. 35: 1) the result without allowance for interaction between the nucleons in the final state; 2) with allowance for the interaction.

the three-nucleon system at the end of the reaction (in the center-of-mass system). From curve 2 in Fig. 12 the lifetime is $\tau = 0.7\times10^{-10}$ sec. This value is in reasonable agreement with the experimental value of the total lifetime of hypertritium.[103] But if we use curve 1, the lifetime is found to be 10^{-11} sec, which is in strong contradiction with the experimental total lifetime. Of course, for more definite conclusions about the acceptability of the chosen forms of the nucleon–λ and nucleon–nucleon potentials we need more complete experimental data than those we have at our disposal at present. The fact that in Ref. 35 the binding energy of hypertritium was found to be $\varepsilon^{\rm theor}_{^3_\lambda H} = -2.30$ MeV, which is in good agreement with the experimental value $\varepsilon^{\rm exp}_{^3_\lambda H} = -2.35$ MeV, indicates the acceptability of the employed potentials; however, as we pointed out in the Introduction, this is not sufficient. We need to make a comparison with the experimental data of, for example, curve 2, and at the present time such data do not exist. The recent increase in the interest in hypernuclear physics offers hope that in the near future such experiments will be made.

CONCLUSIONS

The present review is a first attempt to systematize the existing theoretical and experimental results on the complete breakup of light nuclei induced by elementary particles. Of course, it is far from complete and is restricted to only the recent studies. For example, the comparatively early theoretical studies made using the variational formalism and without allowance for the interaction between all the particles in the final state have been outside the scope of the review, though it is well known that the first studies marked a very important step in the investigation of nuclear reactions of decay type.

The main aim of this paper was to show how interesting is the problem of direct complete breakup of light nuclei induced by elementary particles and how rich it is in possibilities of obtaining more or less reliable conclusions simultaneously about the structure of light nuclei and the mechanism of the nuclear reactions. The most interesting result appears to be the discovery of appreciable sensitivity of the cross sections of decay-type reactions to the chosen form of the nucleon–nucleon potential.

Of course, comparatively little has been done so far in investigating this problem. In the future, great efforts will be needed on the part of both the experimentalists and the theoreticians to achieve a deep and comprehensive study of this very interesting problem.

I am very grateful to N. B. Krupennikova for very valuable discussions.

[1]A. C. Phillips, Rep. Prog. Phys. **40**, 905 (1977).
[2]B. A. Fomin and V. D. Éfros, Yad. Fiz. **31**, 1441 (1980) [Sov. J. Nucl. Phys. **31**, 748 (1980)].
[3]S. P. Merkur'ev, Teor. Mat. Fiz. **8**, 235 (1971); Nucl. Phys. **A223**, 395 (1974).
[4]E. Gerjuoy, J. Phys. B **3**, 92 (1970); Philos. Trans. R. Soc. London **A270**, 197 (1971).
[5]J. Nuttall, J. Math. Phys. **12**, 1896 (1971).
[6]R. Aaron and R. D. Amado, Phys. Rev. Lett. **31**, 1157 (1973); R. D. Amado, Phys. Rev. Lett. **33**, 333 (1974); Phys. Rev. C **11**, 719 (1975); **12**, 1354 (1975).
[7]A. I. Badalyan and Yu. A. Simonov, Fiz. Elem. Chastits At. Yadra **6**, 299 (1975) [Sov. J. Part. Nucl. **6**, 119 (1975)].
[8]R. G. Newton, Ann. Phys. (N. Y.) **74**, 324 (1972); **78**, 561 (1973); R. G. Newton and R. Shtokhamer, Phys. Rev. A **14**, 642 (1976).
[9]S. P. Merkuriev, C. Gignoux, and A. Laverne, Ann. Phys. (N. Y.) **99**, 30 (1976).
[10]V. S. Potapov and J. R. Taylor, Phys. Rev. A **16**, 2264 (1977).
[11]M. Yamazaki, Prog. Theor. Phys. **60**, 112 (1978).
[12]R. I. Dzhibuti, "Method of *K* harmonics in the momentum representation and the continuum problem in the theory of many-particle reactions," Preprint YaF-1 [in Russian], Institute of Physics, Georgian Academy of Sciences (1980).
[13]L. D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960) [Sov. Phys. JETP **12**, 1014 (1961)]; Tr. Mat. Inst. Akad. Nauk SSSR **69** (1963).
[14]L. M. Delves, Nucl. Phys. **9**, 391 (1959); **20**, 275 (1960); **29**, 268 (1962).
[15]R. Aaron, R. Amado, and Y. Y. Yam, Phys. Rev. B **140**, 1291 (1965).
[16]I. M. Barbour and A. C. Phillips, Phys. Rev. C **1**, 165 (1970).
[17]B. F. Gibson and D. R. Lehman, Phys. Rev. C **13**, 477 (1976).
[18]C. R. Heimbach, D. R. Lehman, and J. J. O'Connell, Phys. Rev. C **16**, 2135 (1977).
[19]A. C. Phillips, F. Roig, and J. Ros, Nucl. Phys. **A237**, 493 (1975).
[20]J. Torre, C. Gignoux, and B. Goulard, Phys. Rev. Lett. **40**, 511 (1978).
[21]A. N. Gorbunov, "Photonucleon and photomeson processes." Tr. Fiz. Inst. Akad. Nauk SSSR **71**, 1 (1974).
[22]A. C. Phillips and F. Roig, Nucl. Phys. **A234**, 378 (1974).
[23]R. D. Amado, Phys. Rev. **132**, 548 (1963).
[24]J. Torre and B. Goulard, Phys. Rev. Lett. **43**, 1222 (1979).
[25]S. P. Merkur'ev and S. A. Pozdneev, Yad. Fiz. **30**, 941 (1979) [Sov. J. Nucl. Phys. **30**, 489 (1979)].
[26]K. Wildermuth and Y. C. Tang, A Unified Theory of the Nucleus, Academic Press, New York (1977).
[27]R. I. Jibuti (Dzhibuti), N. B. Krupennikova, and V. Yu. Tomchinsky, Nucl. Phys. **A276**, 421 (1977).
[28]R. I. Dzhibuti, N. B. Krupennikova, and V. Yu. Tomchinskiĭ, Yad. Fiz. **23**, 539 (1976) [Sov. J. Nucl. Phys. **23**, 285 (1976)].
[29]R. I. Jibuti (Dzhibuti) and T. I. Kopaleishvili, Nucl. Phys. **55**, 337 (1964).
[30]H. Garcilazo and T. U. Eisenberg, Nucl. Phys. **A220**, 13 (1974).
[31]J. Raynal and J. Revai, Nuovo Cimento **A68**, 612 (1970).
[32]R. I. Dzhibuti, N. B. Krupennikova, and V. Yu. Tomchinskiĭ, Soobshch. Akad. Nauk Gruz. SSR **81**, 69 (1976).
[33]R. I. Dzhibuti, N. B. Krupennikova, and V. Yu. Tomchinskiĭ, Yad. Fiz. **28**, 30 (1978) [Sov. J. Nucl. Phys. **28**, 15 (1978)].
[34]R. I. Dzhibuti, N. B. Krupennikova, and V. Yu. Tomchinskiĭ, Izv. Akad. Nauk SSSR, Ser. Fiz. **43**, 2343 (1979).
[35]R. I. Dzhibuti and Sh. M. Tsiklauri, Yad. Fiz. **34**, 58 (1981) [Sov. J. Nucl. Phys. **34**, 33 (1981)].
[36]R. I. Dzhibuti, N. B. Krupennikova, and V. Yu. Tomchinskiĭ, Yad. Fiz. **34**, 1185 (1981) [Sov. J. Nucl. Phys. **34**, 658 (1981)].
[37]R. I. Dzhibuti, N. B. Krupennikova, and N. I. Shubitidze, Teor. Mat. Fiz. **32**, 223 (1977).
[38]R. I. Dzhibuti, R. Ya. Kezerashvili, and K. I. Sigua, Yad. Fiz. **32**, 1536 (1980) [Sov. J. Nucl. Phys. **32**, 795 (1980)].
[39]R. I. Jibuti (Dzhibuti), R. Ya. Kezerashvili, and K. I. Sigua, Phys. Lett. **B102**, 381 (1981).

[40]R. I. Dzhibuti, V. Yu. Tomchinskiĭ, and N. I. Shubitidze, Yad. Fiz. 18, 1184 (1973) [Sov. J. Nucl. Phys. 18, 607 (1973)].
[41]R. I. Dzhibuti and N. I. Shubitidze, Yad. Fiz. 20, 666 (1974) [Sov. J. Nucl. Phys. 20, 357 (1975)].
[42]A. M. Badalyan *et al*., Yad. Fiz. 6, 473 (1967) [Sov. J. Nucl. Phys. 6, 345 (1967)].
[43]H. A. Jahn, Proc. R. Soc. 205, 192 (1951).
[44]R. I. Dzhibuti and K. I. Sigua, "Variable phase method in the quantum mechanics of three and four bodies," Preprint 31-YaF [in Russian], Institute of Physics, Georgian Academy of Sciences (1977).
[45]W. R. Gibbs *et al*., Phys. Rev. C 15, 1384 (1977).
[46]R. I. Dzhibuti, R. Ya. Kezerashvili, and K. I. Sigua, Yad. Fiz. 29, 65 (1979) [Sov. J. Nucl. Phys. 29, 31 (1979)].
[47]R. I. Jibuti (Dzhibuti), R. Ya. Kezerashvili, and K. I. Sigua, Phys. Lett. B81, 115 (1979).
[48]R. I. Dzhibuti and R. Ya. Kezerashvili, Yad. Fiz. 34, 1464 (1981) [Sov. J. Nucl. Phys. 34, 810 (1981)].
[49]M. Fabre de la Ripelle and J. S. Levinger, Nuovo Cimento A25, 555 (1975); Lett. Nuovo Cimento 16, 413 (1976).
[50]K. J. Myers, K. K. Fang, and J. S. Levinger, Phys. Rev. C 15, 1215 (1977).
[51]K. K. Fang, J. S. Levinger, and M. Fabre de la Ripelle, Phys. Rev. C 17, 24 (1978).
[52]J. S. Levinger and R. Fitzgibbon, Phys. Rev. C 18, 56 (1978).
[53]J. S. Levinger, Phys. Rev. C 19, 1136 (1979).
[54]J. S. Levinger, Phys. Rev. C 19, 2083 (1979).
[55]A. N. Vostrikov and M. V. Zhukov, Yad. Fiz. 26, 716 (1977) [Sov. J. Nucl. Phys. 26, 377 (1977)].
[56]A. N. Vostrikov and M. V. Zhukov, Yad. Fiz. 34, 344 (1981) [Sov. J. Nucl. Phys. 34, 195 (1981)].
[57]A. I. Baz' and M. V. Zhukov, Yad. Fiz. 16, 60, 958 (1972) [Sov. J. Nucl. Phys. 16, 31, 529 (1973)].
[58]V. K. Tartakovskiĭ *et al*., Yad. Fiz. 23, 727 (1976) [Sov. J. Nucl. Phys. 23, 383 (1976)].
[59]H. M. Getenberg and J. S. O'Connell, Phys. Rev. 144, 834 (1966).
[60]B. L. Berman, S. C. Fultz, and P. F. Yergin, Phys. Rev. C 10, 2221 (1974).
[61]A. N. Gorbunov, Yad. Fiz. 10, 469 (1969) [Sov. J. Nucl. Phys. 10, 268 (1970)].
[62]Yu. M. Arkatov *et al*., Yad. Fiz. 10, 1123 (1969) [Sov. J. Nucl. Phys. 10, 639 (1970)].
[63]F. Balestra *et al*., Nuovo Cimento A38, 145 (1977).
[64]D. Gogny, P. Pires, and R. de Tourreil, Phys. Lett. B32, 591 (1970).
[65]A. Volkov, Nucl. Phys. 74, 33 (1965).
[66]L. Hulthén and M. Sheawara, in: Handbuch der Physik, Vol. 39 (1957) [Russian translation published in: Stroenie atomnogo yadra (Structure of the Atomic Nucleus), Part 1, Izd. Inostr. Lit., Moscow (1959)].
[67]G. A. Baker *et al*., Phys. Rev. 125, 1754 (1962).
[68]C. U. Shakin and M. S. Weiss, Phys. Rev. C 7, 1820 (1973).
[69]C. U. Shakin and M. S. Weiss, Phys. Rev. C 9, 1679 (1974).
[70]B. L. Berman *et al*., Phys. Rev. Lett. 15, 727 (1965).
[71]W. A. Pearce and P. Swan, Nucl. Phys. 78, 433 (1966).
[72]V. N. Maĭkov, Yadernye reaktsii pri malykh i srednikh énergiyakh (Nuclear Reactions at Low and Medium Energies), Moscow (1958), p. 414.
[73]I. V. Dogyust *et al*., Voprosy atomnoĭ nauki i tekhniki. Ser. Fizika vysokikh énergiĭ i atomnogo yadra (Questions of Atomic Science and Technology. Ser. High Energy Physics and Nuclear Physics), No. 3 (1974).
[74]I. V. Dogyust *et al*., Tezisy dokl. XXVI soveshchaniya po yadernoĭ spektroskopii i strukture atomnogo yadra (Abstracts of Papers at the 27th Symposium on Nuclear Spectroscopy and Nuclear Structure), Nauka, Leningrad (1977), p. 228.
[75]V. L. Telegdi and M. Verde, Helv. Phys. Acta 22, 380 (1949).
[76]V. L. Telegdi and W. Zünti, Helv. Phys. Acta 23, 745 (1950).
[77]T. Sperinde *et al*., Phys. Lett. B32, 185 (1970).
[78]A. C. Phillips, Phys. Lett. B33, 260 (1970).
[79]P. Truöl *et al*., Phys. Rev. Lett. 32, 1286 (1974).
[80]J. A. Bistirlich *et al*., Phys. Rev. Lett. 36, 942 (1976).
[81]Yu. A. Batusov *et al*., Preprint R-1474 [in Russian], JINR, Dubna (1963); Zh. Eksp. Teor. Fiz. 46, 817 (1964) [Sov. Phys. JETP 19, 557 (1964)].
[82]T. P. Massue *et al*., in: Compte Rendus du Congress Intern. de Physique Nucleaire, Tuillet, Paris (1964), p. 326.
[83]L. Gilly *et al*., Phys. Lett. 19, 335 (1965).
[84]R. E. P. Davis *et al*., Bull. Am. Phys. Soc. 2, 627 (1964).
[85]L. Kaufman, V. Perez-Mendes, and T. Sperinde, Phys. Rev. 175, 1358 (1968).
[86]V. Falomkin *et al*., Nuovo Cimento 22, 333 (1974).
[87]V. Falomkin *et al*., Lett. Nuovo Cimento 16, 525 (1976).
[88]N. Carayannopoulos *et al*., Phys. Rev. Lett. 20, 1215 (1968).
[89]F. Gaille *et al*., Nuovo Cimento A40, 31 (1977).
[90]Y. Sakamoto, Nuovo Cimento 37, 774 (1965).
[91]T. I. Kopaleĭshvili and I. Z. Machabeli, Yad. Fiz. 4, 37 (1966) [Sov. J. Nucl. Phys. 4, 27 (1967)].
[92]D. S. Koltun and A. Reitan, Phys. Rev. 155, 1139 (1966).
[93]H. Davies, H. Muirhead, and T. U. Woulds, Nucl. Phys. 78, 663 (1966).
[94]U. E. Norberg, K. F. Kinsey, and R. L. Burman, Phys. Rev. 165, 1096 (1968).
[95]R. I. Dzhibuti and R. Ya. Kezerashvili, Yad. Fiz. 20, 340 (1974) [Sov. J. Nucl. Phys. 20, 181 (1975)].
[96]C. Il-Tong, KG Report, I, June (1968).
[97]I. Sh. Vashakidze and G. A. Chilashvili, Dokl. Akad. Nauk SSSR 157, 557 (1964).
[98]T. H. Hetherington and L. H. Schik, Phys. Rev. 139, B1169 (1965).
[99]B. F. Gibson, Phys. Rev. C 10, 888 (1974).
[100]B. F. Gibson and D. R. Lehman, Phys. Lett. B83, 289 (1979).
[101]H. Narumi, K. Ogawa, and Y. Sanami, Lett. Nuovo Cimento 26, 294 (1979).
[102]S. P. Verma and D. P. Sural, Phys. Rev. C 20, 781 (1979).
[103]M. M. Block *et al*., in: Proc. of the Sienna Conf. on Elementary Particles and High Energy Physics (1963), p. 63.

Translated by Julian B. Barbour