# Pion production ratios at 180° in relativistic nuclear reactions

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An attempt is made to understand the unexpected features of the recent experimental data on pion production at 180° in proton–nucleus collisions in the framework of the two-component EP–ET model. It is shown that: a) although the characteristic features in the two regions $0.8 \leq \varepsilon \leq 4$ GeV and $\varepsilon > 4$ GeV of the initial energy $\varepsilon$ are very different, the reaction mechanism is the same; b) the $\varepsilon$ dependence of the slope parameter of the spectra, as well as the $\pi^-/\pi^+$ production ratio, has a kinematic origin; c) future experiments to study the correlations between particles emitted forward and backward will be helpful for understanding the mechanisms of excitation and decay of the effective pion-emitting target.

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High-energy pions emitted in the "backward" direction in relativistic nuclear reactions have been observed by various experimental groups.[1–4] One of the main aims of these experiments was to establish whether pions are produced with energies above the kinematic limit of free nucleon–nucleon collisions and, if they are, whether there are grounds for assuming collective effects of nucleons within the nuclear target. It was noted[5] that there is a connection between this problem and the remarkable new features of hadron–nucleus and nucleus–nucleus collisions at high energies. In fact, it was shown that the main features of the existing experimental data on nuclear reactions at high energies can be understood in the framework of the simple physical picture provided by the two-component EP–ET model (Ref. 6).[1)]

In systematic investigations of the energy dependence (the kinetic energy ε of the bombarding protons was varied from 0.80 to 4.89 GeV) of the cross section for production of pions with kinetic energy $E_{\rm lab}^{\rm kin} \geq 100$ MeV and emission angle $\theta_{\rm lab} = 180°$, Schroeder *et al.*[4] found the following characteristic features of this process at the given initial energies:

a) At all the above initial energies, the observed pions exceed the admissible limit for the free kinematics of nucleon–nucleon collisions.

b) The invariant inclusive cross section for pion production can be described by the simple exponential dependence

$$E\frac{d^3\sigma}{d^3p} = C\exp\{-E_{\rm lab}^{\rm kin}/T_0(\varepsilon)\}, \tag{1}$$

where the coefficient $C$ depends weakly on only the initial energy ε. The "slope parameter" $T_0(\varepsilon)$ depends strongly on the initial energy for $\varepsilon < 4$ GeV but becomes a constant for $\varepsilon \geq 4$ GeV.

c) The slope parameter depends weakly on the mass of the target nucleus.

d) The $\pi^-/\pi^+$ yield ratio exhibits a dependence on the initial energy similar to that of the slope parameter. At energy 0.8 GeV, this ratio is 0.25; at $\varepsilon \geq 4$ GeV, it is equal to unity.

The data of Schroeder *et al.*,[4] taken together with the early experimental data of Baldin *et al.*,[1] Hayashiro *et al.*,[2] and also the data of Papp *et al.*[7] obtained in a coordinate system in which the fragmenting nucleus moves rapidly and the pions are observed at an angle of about 0°, were used to solve the following problems:

1) Is there experimental proof of collective participation of nucleons within the nuclear target in this type of experiment?

2) Is there a connection between the measurements in the backward direction[1–4] and in the forward direction?

3) How large or small are the momentum transfers that correspond to the observed energetic pions at 180°? Do they correspond—and this applies especially to the production of protons, deuterons, and tritium nuclei—to "hard" or "soft" processes?

While some progress has been made in the solution of these problems, other problems remain open.[4]

1. Are there different reaction mechanisms at low and high ($\varepsilon > 4$ GeV) energies?

2. If there are, what is the connection between them? If the reaction mechanism is the same, how can one understand qualitatively the difference between the dependences of the parameters in these two energy intervals?

3. Why does the ratio $\pi^-/\pi^+$ decrease with decreasing ε? Why is this dependence similar to the energy dependence of the "slope parameter"?

4. What can one understand about the reaction mechanism on the basis of the existing ε dependence of the $\pi^-/\pi^+$ ratio? Will further investigations of the $\pi^-/\pi^+$ ratio be helpful?

In the present paper, we attempt to give answers to questions 1–4, and we show the following. First, the reaction mechanisms at high and low energies are the same! Second, the ε dependence of the slope parameter and of the $\pi^-/\pi^+$ ratio has a kinematic origin. Third, measurements of the $\pi^-/\pi^+$ ratio are helpful for under-

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[1)] In this model the collision of nuclei is regarded as the interaction of an effective projectile (EP) with an effective target (ET), a "tube" (Ed.).

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standing the reaction mechanism at low and intermediate energies.

In Refs. 5 and 6, the two-component EP–ET model was proposed; in it, the production of pions at 180° in proton–nucleus collisions occurs as a result of soft (with small momentum transfer[2)]) processes between the initial proton and the effective target (ET), and the observed remarkable features of the high-energy data of nuclear reactions become a direct consequence of trivial kinematics.[3)]

We recall that the effective target is a group of nucleons arranged along the trajectory of the initial hadron within the nuclear matter. In a first approximation, this group of nucleons can be regarded as a nuclear hadronic object with mass $\nu_{ET}M$, where $M$ is the nucleon mass and $\nu_{ET}$ is the number of nucleons of the effective target. Such an assumption is possible, since the formation time of the final many-particle states in high-energy hadron collisions is *much longer* than the time of flight of the initial particle through the nucleus. In this sense, the nucleons along the trajectory of the initial hadron can be regarded as a single object.

In the dynamical picture of *projectile fragmentation* the effective target moves with kinetic energy $\nu_{ET}\varepsilon$ before the interaction event if the projectile has energy $\varepsilon$ per nucleon. After the collision, a particle of the effective target (in the general case, excited) has, losing relatively small portions of energy and momentum (only *soft* collisions), a velocity near the initial velocity. It was shown earlier[5] that $E_p^{\mathrm{kin}}/\nu_{ET}\varepsilon$ is a good scaled variable. For a nuclear target (with mass number $A_T$) the inclusive cross section for the production of pions at different initial energies is described by the dependence

$$Ed^3\sigma/d^3p = C\exp\{-E_p^{\mathrm{kin}}/(\alpha\nu_{ET}\varepsilon)\}, \tag{2}$$

where both the constants $C$ and $\alpha$ are independent of $\varepsilon$ and, in addition, do not depend on $A_T$. In Ref. 5, the relation (2) was interpreted in terms of a statistical model, in which $\alpha\nu_{ET}\varepsilon$ is the mean kinetic energy of the emitting system. Thus, by analogy with a free Boltzmann gas, the temperature is proportional to the total kinetic energy of the system. (We note that the temperature is fairly high in terms of absolute values, so that the use of the Boltzmann distribution is justified.)

In the considered case, since the observed pions for $\theta_{\mathrm{lab}} = 180°$ were fairly energetic ($E_{\mathrm{lab}}^{\mathrm{kin}} \geqslant 100$ MeV), their energy and momentum are nearly the same not only in the projectile system but also in the laboratory system, i.e., in a first approximation the pions can be regarded as massless and

$$E_{\mathrm{lab}}^{\mathrm{kin}} = \sqrt{(1-\beta)/(1+\beta)}\,E_p^{\mathrm{kin}}, \tag{3}$$

where $\beta = \sqrt{\varepsilon(\varepsilon+2M)}/(\varepsilon+M)$ is the velocity of the projectile relative to the target, and $M$ is the nucleon mass.

From the relations (1)–(3), we readily obtain

$$T_0(\varepsilon) = \alpha\nu_{ET}(\varepsilon M)\,[E+M-\sqrt{\varepsilon(\varepsilon+2M)}\,]. \tag{4}$$

The function (4) gives a very good[4)] approximation to the curve in Fig. 1a in Ref. 4. It should be noted that: a) the form does not depend on the constant $\alpha\nu_{ET}$; b) the constant $\alpha\nu_{ET}$ can be directly determined from the experimental data of Ref. 1.

The exceptionally good agreement between the experiment and the theory, together with the explanation of other characteristic features of nuclear reactions at high energies, gives confidence that we are on the correct path. In fact, it can be hoped that the present model gives a natural explanation of other properties (see items 1–4 listed above) just as well.

We now calculate the ratio $\pi^-/\pi^+$. At high energies ($\varepsilon > 4$), this ratio is equal to unity, which raises the following question: What effects must be introduced into consideration on the transition to the low-energy region?

The most important and also the most trivial effect is that of considering the influence of the kinematics! Indeed, when the initial energy $\varepsilon$ increases there is an increase in not only the relative velocity between the effective target and the projectile but also the maximal value of the kinetic energy available for production of particles. Thus, the pion multiplicity at low bombarding energies is small. In fact, simple estimates show that at $\varepsilon = 0.80$ GeV not more than two energetic ($E_{\mathrm{lab}}^{\mathrm{kin}} \geqslant 100$ MeV) pions are produced at the angle of observation 180°. This means that the pions observed at the lowest bombarding energies in the experiment of Schroeder *et al.* were produced in reactions with production of one or two pions. In contrast to the high-energy case, states with definite isospin play an important part in few-hadron systems.

The importance of these effects can be demonstrated by the following simple calculation. First, we recall the following:

a) In the two-component EP–ET model the effective target is treated in a first approximation as a single hadron with large mass.

b) The excitation of the colliding objects in a proton-proton interaction, i.e., EP–ET-proton, is due to the exchange of vacuum quantum numbers at high energies and one-pion exchange at low energies.[8]

c) The mean number of nucleons in the effective target, namely, $\bar{\nu}_{ET}$, for the copper nucleus, for example, is approximately two. According to Ref. 9, it is easy to

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2) This conclusion is true for the transverse-momentum transfer in a collision of particles, but, on the other hand, the assumption of the EP–ET model to the effect that there exists an effective target (ET) consisting of several nucleons presupposes large transfers to the inclusive particle of longitudinal momentum, the longitudinal momentum of all the nucleons forming the effective target (Ed.).

3) See the references in Refs. 1–4 and 7; additional references can be found in Refs. 5 and 6.

4) Predictions for the experiment of Schroeder *et al.*[4] were made in Ref. 5, in which the inclusive cross section was calculated for different targets and different initial energies using the universal curve found at 180° (Ref. 1) and 2.5° (Ref. 7) in terms of our scaled variable. An approximate expression for $T_0(\varepsilon)$ was found.

obtain for the mean number of nucleons $\overline{\nu}_p$ encountered by the initial hadron as it passes through a target nucleus with mass number $A$ the relation $\overline{\nu}_p = A\sigma_{pp}^{in}/\sigma_{pA}^{in}$, where $\sigma_{pp}^{in}$ and $\sigma_{pA}^{in}$ are the inelastic cross sections of $pp$ and $pA$ interactions, respectively. At the same time, we have assumed that the soft collisions are predominantly peripheral.[5)] Thus, $\overline{\nu}_{ET} < 2.5 < \overline{\nu}_p$ for copper. On the other hand, because the nucleons within the effective target behave as a single entity, energetic pions emitted backward (such as were observed in the experiment of Schroeder *et al.*) are fragments of an effective target with a large number of nucleons, i.e., $\overline{\nu}_{ET} > 1$.

The facts noted above make it possible to conclude that the mechanism of one-pion exchange is a reasonable approximation to the dynamics of soft $p$–ET processes at initial energies for which this approximation gives a good description of the $pp$ interaction.

In practical calculations, we shall proceed on the basis of complete analogy between $p$–ET and the corresponding $pp$ processes.[10] Thus, we adopt the following picture: 1. After the interaction, the projectile $p$ and the effective target are in the general case excited (denoted by $p^*$ and ET*, respectively, in Fig. 1) and can emit pions through the decay of isobars.[10] 2. In this energy interval, only the lowest pion–nucleon resonances ($\Delta$) play a dominant part. This means, for example, that for the case when ET is a two-nucleon system its decay into $F_1^*$ and $F_2^*$, i.e., into a nucleon or a $\Delta$ particle, is possible. In a concrete case, the number of $\Delta$ particles depends on the initial energy. For a fixed number of $\Delta$ particles, the relative probability of the different configurations is determined by Clebsch–Gordan coefficients. The result of the calculations is summarized in Tables I and II.

At initial energies below the production threshold for two pions only one $\Delta$ particle is produced,[11] and then the $\pi^-/\pi^+$ ratio for pions observed at 180° is 0.19 if the effective target consists of two nucleons. (We note that $\pi^-/\pi^+$ is 0.09 if the effective target is a single nucleon and also that in peripheral collisions the effective target may also consist of more than two nucleons.) We introduce $g$, the probability that the effective target is a single nucleon, and $1-g$, the probability that it is a two-nucleon system. Then the number of $\pi^-$ is equal to

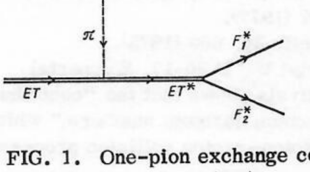


FIG. 1. One-pion exchange collision between projectile (P) and effective target (ET).

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5) The mean value mentioned above is the mean over all possible processes, both *soft* (mainly peripheral) and *hard* (mainly central). For *soft* collisions responsible for backward particle production processes this mean value is somewhat smaller. Such an effect is discussed in Ref. 5, where it is also pointed out that it becomes stronger with increasing $A$. Thus, the $A$ dependence of $\overline{\nu}_{ET}$ for soft processes is weaker than for $\overline{\nu}_p$.

TABLE I. Probability of pion emission from an excited two-nucleon effective target for $\pi^+$ and $\pi^-$ exchange.

| ET | | ET* | | $F_1^* + F_2^*$ | | $m$ | Relative probabilities for $\pi^+:\pi^0:\pi^-$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $\pi^+$-exchange | | | | | | |
| 1/4 | $\begin{bmatrix}1\\1\end{bmatrix}$ | 1/1 | $\begin{bmatrix}2\\2\end{bmatrix}$ | 1/1 | $\Delta^{++}\Delta^+$ | 2 | 1/4 | (4/3 | 2/3 | 0) |
| | | | | 1/1 | $\Delta^{++}p$ | 1 | 1/1 | (1 | 0 | 0) |
| 1/4 | $\begin{bmatrix}1\\0\end{bmatrix}$ | 1/2 | $\begin{bmatrix}2\\1\end{bmatrix}$ | 1/1 | $\Delta^{++}\Delta^0$ | 2 | 1/8 | (1 | 2/3 | 1/3) |
| | | | | 1/4 | $\Delta^{++}n$ | 1 | 1/32 | (1 | 0 | 0) |
| | | | | 3/4 | $\Delta^+p$ | 1 | 3/32 | (1/3 | 2/3 | 0) |
| | | 1/2 | $\begin{bmatrix}1\\1\end{bmatrix}$ | 3/5 | $\Delta^{++}\Delta^0$ | 2 | 3/40 | (1 | 2/3 | 1/3) |
| | | | | 2/5 | $\Delta^+\Delta^+$ | 2 | 1/20 | (2/3 | 4/3 | 0) |
| | | | | 3/4 | $\Delta^{++}n$ | 1 | 3/32 | (1 | 0 | 0) |
| | | | | 1/4 | $\Delta^+p$ | 1 | 1/32 | (1/3 | 2/3 | 0) |
| 1/4 | $\begin{bmatrix}1\\-1\end{bmatrix}$ | 1/6 | $\begin{bmatrix}2\\0\end{bmatrix}$ | 1/2 | $\Delta^{++}\Delta^-$ | 2 | 1/48 | (1 | 0 | 1) |
| | | | | 1/2 | $\Delta^+\Delta^0$ | 2 | 1/48 | (1/3 | 4/3 | 1/3) |
| | | | | 1/2 | $\Delta^+n$ | 1 | 1/48 | (1/3 | 2/3 | 0) |
| | | | | 1/2 | $\Delta^0p$ | 1 | 1/48 | (0 | 2/3 | 1/3) |
| | | 1/2 | $\begin{bmatrix}1\\0\end{bmatrix}$ | 9/10 | $\Delta^{++}\Delta^-$ | 2 | 9/80 | (1 | 0 | 1) |
| | | | | 1/10 | $\Delta^+\Delta^0$ | 2 | 1/80 | (1/3 | 4/3 | 1/3) |
| | | | | 1/2 | $\Delta^+n$ | 1 | 1/16 | (1/3 | 2/3 | 0) |
| | | | | 1/2 | $\Delta^0p$ | 1 | 1/16 | (0 | 2/3 | 1/3) |
| | | 1/3 | $\begin{bmatrix}0\\0\end{bmatrix}$ | 1/2 | $\Delta^{++}\Delta^-$ | 2 | 1/24 | (1 | 0 | 1) |
| | | | | 1/2 | $\Delta^+\Delta^0$ | 2 | 1/24 | (1/3 | 4/3 | 1/3) |
| 1/4 | $\begin{bmatrix}0\\0\end{bmatrix}$ | 1/1 | $\begin{bmatrix}1\\1\end{bmatrix}$ | 3/5 | $\Delta^{++}\Delta^0$ | 2 | 3/20 | (1 | 2/3 | 1/3) |
| | | | | 2/5 | $\Delta^+\Delta^+$ | 2 | 1/10 | (2/3 | 4/3 | 0) |
| | | | | 3/4 | $\Delta^{++}n$ | 1 | 3/16 | (1 | 0 | 0) |
| | | | | 1/4 | $\Delta^+p$ | 1 | 1/16 | (1/3 | 2/3 | 0) |
| | | | | $\pi$-exchange | | | | | | |
| 1/4 | $\begin{bmatrix}1\\1\end{bmatrix}$ | 1/2 | $\begin{bmatrix}2\\1\end{bmatrix}$ | 1/1 | $\Delta^{++}\Delta^0$ | 2 | 1/8 | (1 | 2/3 | 1/3) |
| | | | | 1/4 | $\Delta^{++}n$ | 1 | 1/32 | (1 | 0 | 0) |
| | | | | 3/4 | $\Delta^+p$ | 1 | 3/32 | (1/3 | 2/3 | 0) |
| | | 1/2 | $\begin{bmatrix}1\\1\end{bmatrix}$ | 3/5 | $\Delta^{++}\Delta^0$ | 2 | 3/40 | (1 | 2/3 | 1/3) |
| | | | | 2/5 | $\Delta^+\Delta^+$ | 2 | 1/20 | (2/3 | 4/3 | 0) |
| | | | | 3/4 | $\Delta^{++}n$ | 1 | 3/32 | (1 | 0 | 0) |
| | | | | 1/4 | $\Delta^+p$ | 1 | 1/32 | (1/3 | 2/3 | 0) |
| 1/4 | $\begin{bmatrix}1\\0\end{bmatrix}$ | 2/3 | $\begin{bmatrix}2\\0\end{bmatrix}$ | 1/2 | $\Delta^{++}\Delta^-$ | 2 | 1/12 | (1 | 0 | 1) |
| | | | | 1/2 | $\Delta^+\Delta^0$ | 2 | 1/12 | (1/3 | 4/3 | 1/3) |
| | | | | 1/2 | $\Delta^+n$ | 1 | 1/12 | (1/3 | 2/3 | 0) |
| | | | | 1/2 | $\Delta^0p$ | 1 | 1/12 | (0 | 2/3 | 1/3) |
| | | 1/3 | $\begin{bmatrix}0\\0\end{bmatrix}$ | 1/2 | $\Delta^{++}\Delta^-$ | 2 | 1/24 | (1 | 0 | 1) |
| | | | | 1/2 | $\Delta^+\Delta^0$ | 2 | 1/24 | (1/3 | 4/3 | 1/3) |
| 1/4 | $\begin{bmatrix}1\\-1\end{bmatrix}$ | 1/2 | $\begin{bmatrix}2\\-1\end{bmatrix}$ | 1/1 | $\Delta^+\Delta^-$ | 2 | 1/8 | (1/3 | 2/3 | 1) |
| | | | | 3/4 | $\Delta^0n$ | 1 | 3/32 | (0 | 2/3 | 1/3) |
| | | | | 1/4 | $\Delta^-p$ | 1 | 1/32 | (0 | 0 | 1) |
| | | 1/2 | $\begin{bmatrix}1\\-1\end{bmatrix}$ | 3/5 | $\Delta^+\Delta^-$ | 2 | 3/40 | (1/3 | 2/3 | 1) |
| | | | | 2/5 | $\Delta^0\Delta^0$ | 2 | 1/20 | (0 | 4/3 | 2/3) |
| | | | | 1/4 | $\Delta^0n$ | 1 | 1/32 | (0 | 2/3 | 1/3) |
| | | | | 3/4 | $\Delta^-p$ | 1 | 3/32 | (0 | 0 | 1) |
| 1/4 | $\begin{bmatrix}0\\0\end{bmatrix}$ | 1/1 | $\begin{bmatrix}1\\0\end{bmatrix}$ | 9/10 | $\Delta^{++}\Delta^-$ | 2 | 9/40 | (1 | 0 | 1) |
| | | | | 1/10 | $\Delta^+\Delta^0$ | 2 | 1/40 | (1/3 | 4/3 | 1/3) |
| | | | | 1/2 | $\Delta^+n$ | 1 | 1/8 | (1/3 | 2/3 | 0) |
| | | | | 1/2 | $\Delta^0p$ | 1 | 1/8 | (0 | 2/3 | 1/3) |

*Note.* The numbers in the brackets in columns 2 and 4 characterize the isospin states (total spin and its third projection). The probabilitites of these states are given in columns 1 and 3. The fragments of the excited ET (ET*) and the corresponding relative widths are given in columns 6 and 5. The relative probabilities for $\pi^+$ and $\pi^-$ are normalized to $m$, the number of produced mesons. Note that the factor in front of each bracket in the last column does not include the corresponding weights, which are given in Table II.

$0.09g + 0.10(1-g)$ and the number of $\pi^+$ to $0.41g + 0.51(1-g)$.

Therefore,

TABLE II. Relative weights for the exchange of different pions for emission of pions by an excited projectile P*.

| Projectile | | | Pion from P* $(\pi^+ : \pi^0 : \pi^-)$ | | | Pion from exciation of ET |
|---|---|---|---|---|---|---|
| $P$ | $P*$ | | | | | |
| $p$ | 1/3 | $p$ | 0 | 0 | 0 | $\pi^0$ |
| | 2/3 | $n$ | 0 | 0 | 0 | $\pi^+$ |
| | 1/2 | $\Delta^{++}$ | 1 | 0 | 0 | $\pi^-$ |
| | 1/3 | $\Delta^+$ | 1/3 | 2/3 | 0 | $\pi^0$ |
| | 1/6 | $\Delta^0$ | 0 | 2/3 | 1/3 | $\pi^+$ |
| $n$ | 2/3 | $p$ | 0 | 0 | 0 | $\pi^-$ |
| | 1/3 | $n$ | 0 | 0 | 0 | $\pi_0$ |
| | 1/6 | $\Delta^+$ | 1/3 | 2/3 | 0 | $\pi^-$ |
| | 1/3 | $\Delta^0$ | 0 | 2/3 | 1/3 | $\pi^0$ |
| | 1/2 | $\Delta^-$ | 0 | 0 | 1 | $\pi^+$ |

$$\pi^-/\pi^+ = [0.11\,g + 0.29\,(1-g)]/[1.22g + 1.54\,(1-g)]. \quad (5)$$

Note that $0.09 \leqslant \pi^-/\pi^+ \leqslant 0.19$ in the considered energy interval agrees with the tendency exhibited by the data.[4] Further, since $\theta_{1ab} = 180°$, energetic pions ($\geqslant$100 MeV) are formed with greater probability from an effective target consisting of two nucleons than from single-nucleon effective targets, so that we expect[6)] $g \approx 0$. Therefore, $\pi^-/\pi^+ = 0.19$.

At initial energies at which the production of only two pions is possible, both pions can be imitated by an excited effective target (see Table I). Note that in this case the observed pion at $\theta_{1ab} = 180°$ is the only particle produced in the process of the collision. In this case, only $\pi^+$ and $\pi^0$ exchanges are possible for the excitation of the effective target (see Fig. 1 and Table II). Thus, if the initial particle is a neutron, only $\pi^-$ and $\pi^0$ exchanges are possible. The $\pi^-/\pi^+$ ratio changes in accordance with the variant in which the projectile is a proton, i.e., we obtain an interesting test of this point of view in an experiment by using a neutron or heavy-ion beam. Note also that in effective targets (or effective projectiles, if a beam of heavy ions is used) the probability of finding nucleons in the proton or neutron form is determined in general not only by the mass and charge number of the target (projectile) nucleus, but also by the nuclear structure. For simplicity, we shall take these probabilities in our calculations to be equal to each other (see column 1 in Table I).

However, it is more probable that in the collision process both, the projectile and the effective-target group, will be excited and each will emit its own pion. This last case has interesting consequences. Suppose we set up an experimental arrangement with a positively charged pion near $\theta_{1ab} = 0°$ and measure the relative yield of negative and positive charged energetic pions around $\theta_{1ab} = 180°$. In this case, the $\pi^-/\pi^+$ ratio differs quantitatively from what we observe in a single-pion inclusive experiment. In fact, the $\pi^-/\pi^+$ ratio will be of order 8.8, rather than 0.2, for $g$ factor equal to zero. This is an important prediction for "forward–backward" coincidence experiments.

The $\pi^-/\pi^+$ ratios in the limiting cases of high and low energies, and also the energy dependence of $\pi^-/\pi^+$ in the transition region, can be obtained by the statistical method of Chao and Yang.[11] Since 1) the pion multiplicity increases with increasing initial energy, 2) the isospin effects become relatively weak at large pion multiplicity, the $\pi^-/\pi^+$ ratio increases monotonically with increasing initial energy, reaching the limiting value (unity) at high initial energies. In practice, application of the Chao–Yang method to the considered problem consists in replacing a conglomerate (formed by the projectile and a target hadron in the Chao–Yang case) by an excited ET system.

After this note had been written, two preprints were published (L. S. Schroeder *et al.*, LBL-10899 and LBL-11102) and a Communication from the Joint Institute for Nuclear Research at Dubna (A. M. Baldin *et al.*, Soobshchenie 1-82-28). In the Berkeley publications, the $\pi^-/\pi^+$ ratio was obtained for beams of heavy ions (C and A) with nearly equal energies. These measurements agree with the predictions. The Dubna results on the production of $\pi^-$ and $\pi^+$ mesons, obtained by bombarding the lightest nuclear targets with a high-energy proton beam (8.9 GeV/$c$), contain the angular and momentum dependences, the dependences on the transverse component of the momentum for the pions, and the dependences on the mass number of the target. These data agree with the predictions of the two-component EP–ET model in general and of the present paper in particular (details of the EP–ET model relating to the production of pions in hard collisions with large momentum transfers and/or associative multiplicity can be found in Phys. Rev. Lett. **42**, 1331 (1979)).

We thank Gross for helpful discussions.

[6)] This is a special case, when the influence of the Fermi motion is assumed to be unimportant.

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