Quasiparticle–phonon model of the nucleus. III. Single-phonon states in spherical nuclei

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The basic physical ideas and the general structure of the Hamiltonian of the quasiparticle–phonon nuclear model are discussed in relation to spherical nuclei. Equations are obtained for the energy and structure of the single-phonon states with allowance for the various components of the effective internuclear forces (separable multipole, spin–multipole, spin–isospin–multipole, etc.). The properties of single-phonon excitations and some general properties of nuclear operators are discussed in detail. The results of calculations made with different radial dependences of the effective separable forces are compared. The accuracy of the random-phase approximation in spherical nuclei is investigated.

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INTRODUCTION

One of the obvious achievements of the intensive study of nuclear structure during the last two decades has been the confirmation of nuclear-structure models based essentially on the concept of a nucleus as a system of interacting nucleons moving in a self-consistent average potential common to all of them. Such models—in particular variants and with particular methods for constructing the average potential itself and the corresponding effective internuclear forces in the nucleus—are at present very widely used to analyze and interpret experimental data. In fact, we have to thank the development of this direction in the theory of nuclear structure for the fact that detailed quantitative description of the great variety of nuclear spectra has now become a reality, at least up to excitation energies 2–3 MeV. An important step in the confirmation of such nuclear models[1)] has become their successful use to describe nuclear properties not only at these excitations but also at excitation energies in the continuum—from the neutron binding energy and above to several tens of MeV. This point is important because semimicroscopic models have been developed in the process of theoretical elucidation of the properties of low-lying nuclear excitations, and these have been used in an initial stage to analyze excited states of nuclei lying in the so-called *spectroscopic region.* The significant extension of the region of application of the semimicroscopic models has demonstrated the strength of the physical principles on which they are based and also—not unimportant in quantitative calculations—their practicality.

The quasiparticle–phonon model is one of the semimicroscopic models. It is a "descendant" of the superfluid model, which in the sixties and beginning of the seventies was widely used to analyze the low-lying part of spectra, mainly of deformed nuclei.[1,2] In principle, the quasiparticle–phonon model makes it possible to analyze many properties of the excited states of nuclei. However, its main region of application hitherto has been the region of intermediate and high excitation energies—from B_n to 20–30 MeV in the continuum.[3–6]

All the characteristics of excited nuclear states studied in the framework of the quasiparticle–phonon model are united by an important feature. They are associated with the distribution over the spectrum of nuclear excitations of the strength of very simple configurations (one-, two-, or three-quasiparticle configurations). The distribution of the strength of these simple configurations, i.e., their fragmentation, determines many observable effects—the spreading width of giant resonances and deep hole states, the neutron and radiative strength functions, and so forth.

In principle, the reason for the fragmentation is clear—it is the interaction of the simple configurations with more complicated configurations. But from the simple recognition of this fact to quantitative calculations that take into account the interaction of simple configurations at excitation energies 10–20 MeV with the sea of more complicated states surrounding them is a long and laborious path. Technically, the problem is extremely complicated, and a straightforward solution by direct diagonalization of matrices of huge dimension is simply unrealistic. It is necessary to separate the most important configurations from the great profusion of complicated configurations. The selected configurations must include collective states, since it is they that have the strongest interaction with the other configurations. This distinctive feature of collective vibrational excitations was pointed out long ago.[7] However, one cannot completely ignore the influence of a large number of weakly interacting states, since it is precisely the interaction with them that smooths and spreads the gross structure of the distribution reflected by the interaction with the collective excitations.

In the quasiparticle–phonon model, this problem is solved by transition to the phonon basis; i.e., phonon excitations, which are superpositions of two-quasiparticle excitations, are used as the elementary blocks from which nuclear excitations of more complicated structure are constructed. The expediency of using phonons as elementary bricks to analyze nuclear spectra is now recognized by many. Quadrupole, octupole, and pairing phonons are traditional elements in the description of low-lying nuclear excitations. Phonon excitations are widely used in nuclear field theory,[8] and in

[1)] These models are frequently said to be *microscopic* or, less often, *semimicroscopic*. The latter term is more precise, and we shall use it in the paper.

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some cases they are introduced phenomenologically in microscopic schemes to study high-lying excitations (see, for example, Ref. 9). The quasiparticle–phonon model goes further in this direction than all other models, including in the phonon basis not only the quadrupole and octupole vibrations of lowest energies but also phonons of all possible moments and parities (in practice, with λ^π from $1^\pm$ to $7^\pm$) and, in addition, phonons from a very wide range, $E_x \leq 25$–30 MeV, of excitation energies. The structure of the phonons is calculated microscopically, in the random-phase approximation, and the interaction of the phonons with one another or with the odd quasiparticle in an odd nucleus is already determined by the structure of the phonons. In even–even nuclei, not only the simplest single-phonon excitations but also two- and three-phonon excitations and so forth are considered; in odd nuclei, there is the following hierarchy of states: single-quasiparticle, quasiparticle + phonon, quasiparticle + two phonons, etc. Why is the phonon basis convenient? Its main advantage is in enabling one to take into account automatically both collective and noncollective nuclear excitations. Further, the phonon basis includes collective states of intermediate and high excitation energies corresponding to electric and magnetic resonances of various types. And the significant simplification of numerical calculations is not the least advantage of the phonon basis.

However, it is readily seen that the advantages which accrue on the transition to the phonon basis are accompanied by a number of shortcomings. The most obvious is that in taking phonon excitations to satisfy Bose statistics we ignore their fermion structure and violate the Pauli principle in the many-phonon states. Correct allowance for the Pauli principle significantly complicates the problem already at the level of two-phonon or the quasiparticle + phonon states.[10] A second problem is that the "many-phonon" basis is overdetermined compared with the equally complicated and correctly symmetrized "many-quasiparticle" basis, i.e., the total number of quasiparticle + phonon states with given spin is, for example, greater than the total number of three-quasiparticle states with the same spin. In part, this is due to the violation of the Pauli principle that we have already mentioned, but there is a further reason—the regrouping of the quasiparticles that occur in the phonons that form the many-phonon states. The combined effect of these factors means that direct extension of the phonon-basis idea to ever more complicated configurations necessarily leads either to large errors or to serious complications in attempts to avoid them.

The present paper is one of a series of publications giving a systematic exposition of the formalism of the quasiparticle–phonon model and the results obtained in its framework. In this sense, it continues Refs. 4 and 5, in which deformed nuclei were considered. Here, we consider single-phonon states of spherical nuclei, their structure, and their properties, since they largely determine the characteristic features of the spectra of both even–even and odd nuclei.

1. GENERAL CHARACTERIZATION OF THE HAMILTONIAN OF THE QUASIPARTICLE–PHONON MODEL

Summarizing the results of the various studies in the framework of the quasiparticle–phonon model, we can write the most general form of the model Hamiltonian as

$$H = H_{\rm SP} + H_{\rm PAIR} + H_M^{ph} + H_{SM}^{ph} + H_M^{pp}, \tag{1}$$

where $H_{\rm SP} = H_{\rm SP}^p + H_{\rm SP}^n$ describes the independent motion of the protons and neutrons in their average potentials; $H_{\rm PAIR} = H_{\rm PAIR}^p + H_{\rm PAIR}^n$ are the monopole pairing forces acting only between the neutrons and between the protons; H_M^{ph} is the sum of the isoscalar and isovector separable multipole interactions in the particle–hole channel; H_{SM}^{ph} is the sum of the isoscalar and isovector separable spin–multipole interactions in the particle–hole channel; and H_M^{pp} is the sum of the separable multipole interactions in the particle–particle channel (multipole pairing).

In the second-quantization representation, these terms can be written as

$$H_{\rm SP}^n + H_{\rm PAIR}^n = \sum_{jm} E_j a_{jm}^+ a_{jm} - \frac{G_n}{4} \sum_{jj'}^{n} \sum_{mm'} (-1)^{j-m} (-1)^{j'-m'} \times a_{jm}^+ a_{j-m}^+ a_{j'-m'} a_{j'm'}; \tag{2}$$

$$\left.\begin{aligned} H_M^{ph} &= -\frac{1}{2} \sum_{\lambda} (\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)} \tau_1 \tau_2) \sum_{\mu} M_{\lambda\mu}^+ M_{\lambda\mu}; \\ M_{\lambda\mu}^+ &= \sum_{jj'} \sum_{mm'} \langle jm \mid i^\lambda R_\lambda(r) Y_{\lambda\mu} \mid j'm' \rangle a_{jm}^+ a_{j'm'}; \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} H_{SM}^{ph} &= -\frac{1}{2} \sum_{\lambda} \sum_{L=\lambda,\ \lambda\pm1} (\varkappa_0^{(\lambda L)} + \varkappa_1^{(\lambda L)} \tau_1 \tau_2) \sum_{M} (S_{LM}^\lambda)^+ S_{LM}^\lambda; \\ (S_{LM}^\lambda)^+ &= \sum_{jj'} \sum_{mm'} \langle jm \mid i^\lambda R_\lambda(r) [\sigma Y_{\lambda\mu}(\Omega)]_{LM} \mid j'm' \rangle a_{jm}^+ a_{j'm'}; \\ [\sigma Y_{\lambda\mu}(\Omega)]_{LM} &= \sum_{\nu\mu} \langle 1\nu\lambda\mu \mid LM \rangle \sigma_\nu Y_{\lambda\mu}(\Omega); \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} H_M^{pp} &= -\frac{1}{2} \sum_{\lambda\mu} [G_n^{(\lambda)} P_{\lambda\mu}^+(n) P_{\lambda\mu}(n) + G_p^{(\lambda)} P_{\lambda\mu}^+(p) P_{\lambda\mu}(p) \\ &\quad + G_{np}^{(\lambda)} (P_{\lambda\mu}^+(n) P_{\lambda\mu}(p) + P_{\lambda\mu}^+(p) P_{\lambda\mu}(n))]; \\ P_{\lambda\mu}^+(n) &= \sum_{jj'}^{n} \sum_{mm'} \langle jm \mid i^\lambda R_\lambda(r) Y_{\lambda\mu} \mid j'm' \rangle (-1)^{j'-m'} a_{jm}^+ a_{j'-m'}^+. \end{aligned}\right\} \tag{5}$$

In Eqs. (2)–(5), a_{jm}^+ and a_{jm} are the operators of creation and annihilation of a nucleon in the level of the average field with quantum numbers $(n, l, j) \equiv j$ and projection m of the total angular momentum j (unless specially stated, summation over j simultaneously means summation over the isotopic index $\tau = n, p$); E_j are the single-particle energies; G_n and G_p are the constants of the monopole pairing; $G_n^{(\lambda)}$, $G_p^{(\lambda)}$, and $G_{np}^{(\lambda)}$ are the constants of the multipole pairing ($\lambda \neq 0$); $\varkappa_0^{(\lambda)}$ and $\varkappa_1^{(\lambda)}$ are the isoscalar and isovector constants of the multipole p–h interaction; and $\varkappa_0^{(\lambda L)}$ and $\varkappa_1^{(\lambda L)}$ are the isoscalar and isovector constants of the spin–multipole interaction.

The Hamiltonian (1)–(5) contains many parameters. It is no simple matter to determine them, and in fact this problem has not yet been completely solved. The reason for this is that H is an effective Hamiltonian. By *effective* we mean that H is not constructed on the basis of any "first principles" by a rigorously justified procedure. And although there are weighty—both pure-

ly theoretical and experimental—grounds for hoping that many important features of nuclear spectra can be described by means of such a Hamiltonian, it must be clearly recognized that the actual parametrization of the average field, the residual forces, etc., is not rigorously derived and can be justified only by the analysis of experimental data (in a broad sense).

The effective nature of the Hamiltonian means that the parameters also depend on the truncation of the employed basis. By the *basis* we shall understand not only the basis of single-particle states but also the space of model basis states in which the eigenvectors and eigenvalues of H are sought. For example, it could consist solely of 1p–1h configurations (as in the random-phase approximation), or 1p–1h and 2p–2h, etc. Any change in the basis or the lifting of physical or even computational restrictions can—strictly speaking, must—lead to a readjustment of the parameters of the Hamiltonian, the functional form of the forces, etc. Thus, the parameters of the Hamiltonian (1)–(5) have meaning only in the framework of a given set of approximations and are justified by the extent to which they can describe a large number of phenomena in these approximations.

We begin the discussion with the parameters of the single-particle potential. At the present time, the Woods–Saxon potential is the one most widely used:

$$V(r)+V_{ls}(r)=\frac{V_0^{n,p}}{1+\exp[\alpha(r-R)]}$$
$$-\frac{1}{r}\frac{d}{dr}\frac{V_{ls}^{n,p}}{1+\exp[\alpha_{ls}(r-R_{ls})]}(\mathbf{ls}), \qquad (6)$$

where $V_0^{n,p}(V_{ls}^{n,p})$, $\alpha(\alpha_{ls})$, and $R=r_0A^{1/3}$ $(R_{ls}=r_{ls}A^{1/3})$ are the depth, diffuseness, and radius parameters of the central (respectively, spin-orbit) part of the potential, respectively. To the potential for the protons there is also added the Coulomb term, which is the Coulomb potential of a charged sphere of radius R. The advantages of the Woods–Saxon potential associated with its finite depth and correct asymptotic behavior of the single-particle wave functions are well known. However, it cannot be claimed that there exists a generally accepted set of parameters of this potential. Each paper has its own parametrization. The expression (6) allows the possibility of different radius and diffuseness parameters for the central and spin–orbit parts. This was the case for the parametrization of the Woods–Saxon potential by the Jülich group in Ref. 11. In Soviet literature, the Chepurnov parametrization[12] is, as a rule, used; in it, $\alpha=\alpha_{ls}, r_0=r_{ls}$, and

$$\varkappa=V_{ls}^{n,p}/V_0^{n,p}=0.263[1-2(n-p)/A].$$

In the parametrization proposed in Ref. 13 for the neutron potential, V_0^n, V_{ls}^n, and $\alpha=\alpha_{ls}$ do not depend on A, n, and p but the dependence on A of the radius is changed, $R=r_0A^{1/3}+r_1$, which effectively means an increase in r_0 for light nuclei. The last two parametrizations are used in studies in the framework of the quasiparticle–phonon model for spherical nuclei.

A further important point related to the single-particle part of the Hamiltonian is the question of the completeness of the single-particle basis. It is particularly important for highly excited states, since there exists a continuous part of the spectrum in a potential of finite depth. A method for taking into account correctly the continuous part of the single-particle basis has hitherto been developed only in the framework of the random-phase approximation (RPA) for nuclei in which there is no pairing.[14] The problem of taking into account the complete single-particle basis in approximations that go beyond the RPA has not yet been solved.

In the existing variant, the quasiparticle–phonon model has as its aim the study of the influence of the interaction of the simple with the complicated configurations on the properties of states of intermediate and high excitation energy, and it makes essential use of the discrete nature of the single-particle spectrum. At the same time, the single-particle basis includes discrete and quasidiscrete states, whose existence is due to the centrifugal and Coulomb barriers. The numerical method of solving the Schrödinger equation with a spherically symmetric potential[16] in the REDMEL program[15] makes it possible to find the energies and wave functions of quasibound states with small width $\Gamma \ll E_{nlj}$. Since the height of the centrifugal barrier is proportional to l^2, states with large angular momenta are predominant among the quasidiscrete states calculated in this manner, while the influence of the Coulomb barrier means that there are more proton than neutron quasibound states. Different forms of the single-particle level scheme for the mass number $A=90$ are given in Fig. 1.[11,17,18]

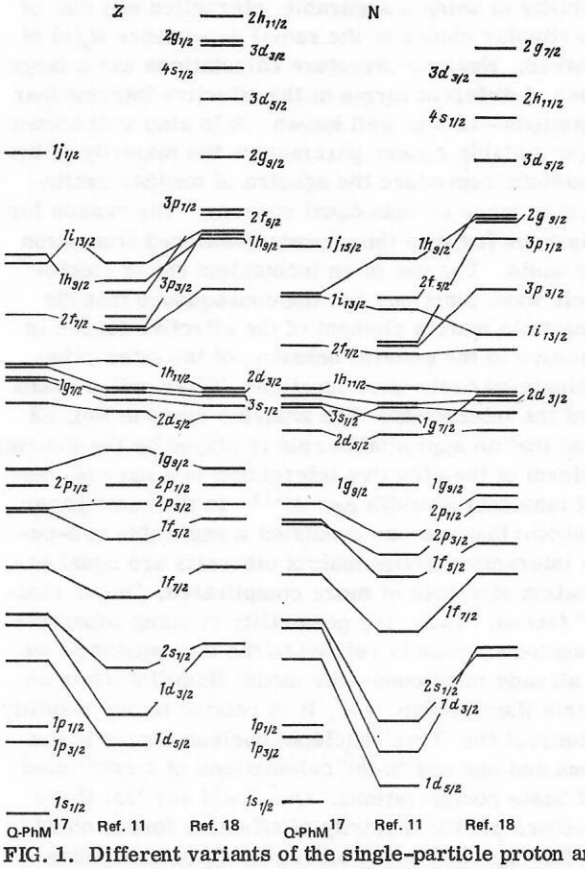

FIG. 1. Different variants of the single-particle proton and neutron schemes for mass number $A=90$ (Refs. 11, 17, and 18).

The restriction of the single-particle basis to discrete and quasidiscrete levels is evidently a satisfactory approximation for the description of the characteristics of states that depend significantly on the completeness of the basis (the probability of $E\lambda$ transitions, the contribution to the sum rule) up to energies 15–20 MeV in heavy nuclei. This at least is confirmed by a comparison of the results of RPA calculations made for the ^{208}Pb states with a complete basis[19] and with allowance for only bound and quasibound levels,[20] which shows that similar results are obtained not only for low-lying states ($2_1^+, 3_1^-, 4_1^+, 5_1^-, 6_1^+$) but also for isovector $E1$ and isoscalar $E2$ and $E3$ resonances. The influence of the single-particle continuum will increase for resonances of high multipolarity and in nuclei with $A<100$. At the same time, of course, it must be borne in mind that there are characteristics of highly excited states (for example, the escape widths) whose description simply cannot be attempted without allowance for the continuum.

There is no point in dwelling in detail on the parametrization of the monopole pairing interaction, since it has been studied in detail.[21] When bound and quasibound states are included in the single-particle basis, the values of G_n and G_p determined using the even–odd nuclear mass differences fluctuate in the ranges $G_nA = 15.5–16.5$ MeV and $G_pA = 16.5–17.5$ MeV.

The terms H_M^{ph} and H_{SM}^{ph} warrant a more detailed discussion. Here, there are two problems—that of the possibility of using a separable interaction and that of the particular choice of the radial dependence $R_\lambda(r)$ of the forces. Nuclear-structure calculations use a large number of different forms of the effective internuclear interactions—this is well known. It is also well known that for suitably chosen parameters the majority of the interactions reproduce the spectra of nuclear excitations with more or less equal success. The reason for this is to be found in the already mentioned truncation of the basis. The use of an incomplete set of single-particle wave functions has the consequence that the two-particle matrix element of the effective forces is insensitive to the general behavior of the interaction. The single-particle wave functions "filter out" a small part of the interaction. The analysis made in Ref. 22 showed that an appreciable role is played by the Fourier transform of the effective interaction in a narrow interval of momenta of width $\Delta q \sim A^{-1/3}$. In the same paper it is shown that one can construct a separable two-nucleon interaction whose matrix elements are equal to the matrix elements of more complicated, "more realistic" forces. Thus, the possibility of using separable internucleon forces is related to the circumstance we have already mentioned—the model Hamiltonian is an effective Hamiltonian, i.e., it is related to our inability to construct the "true" nucleon–nucleon forces in the nucleus and our use in the calculations of a restricted set of basis configurations. One could say that these two factors put the majority of effective forces on an equal footing. But under otherwise equal conditions preference is given to separable forces—the calculations with them are much simpler.

We now discuss the radial dependence of the forces. The use of $R_\lambda(r) = r^\lambda$ has a long historical tradition.[23,24] True, it is generally only multipolarities $\lambda \leq 3$ that are considered, this being due to the well-studied state of the low-lying vibrational 2_1^+ and 3_1^- states, the giant isovector $E1$ resonance, and the isoscalar $E2$ and $E3$ resonances. The forces $r_1^\lambda Y_{\lambda\mu}(\Omega_1) r_2^\lambda Y_{\lambda\mu}(\Omega_2)$ automatically ensure coherent enhancement of the $E\lambda$ transition from the collective state with $J, \pi = \lambda, (-1)^\lambda$ to the ground state of the nucleus. But the same enhancement can be obtained with all forces that have a maximum on the surface of the nucleus. Frequent use is made of a λ-independent radial dependence: $R_\lambda(r) \sim \partial V/\partial r$, where V is the central part of the single-particle potential (6). Some characteristics of nuclear excitations calculated with these two types of forces were compared in Ref. 25. The results were found to be very similar. This question will be considered in more detail below.

In the framework of the quasiparticle–phonon model the choice of the constants of the effective multipole forces is based on experimental data (in the first place, on low-lying states) and qualitative estimates. Estimates for the multipole constants $\varkappa_0^{(\lambda)}$ and $\varkappa_1^{(\lambda)}$ of forces with $R_\lambda(r) = r^\lambda$ in the harmonic-oscillator potential are given in Ref. 24. They are based on the condition of proportionality between small oscillations of the single-particle density and the single-particle potential of the nucleus (see also Ref. 26):

$$\varkappa_0^{(\lambda)} = -\frac{4\pi}{2\lambda+1}\frac{m\omega_0^2}{A\langle r^{2\lambda-2}\rangle}, \quad q = \varkappa_1^{(\lambda)}/\varkappa_0^{(\lambda)} = -0.5\,(2\lambda+3). \tag{7}$$

Of course, these are rough estimates and are unsuitable for determining the energies of the 2_1^+ and 3_1^- levels of spherical nuclei, which are very sensitive to the value of $\varkappa_0^{(\lambda)}$. The value of q is also overestimated, which, for example, makes it impossible to reproduce the position of the isovector $E2$ resonance. In determining the constants $\varkappa_0^{(2)}$ and $\varkappa_0^{(3)}$, it is best to rely on the experimental energies of the 2_1^+ and 3_1^- levels, and for the constant $\varkappa_1^{(1)}$ on the position of the maximum of the $E1$ resonance. To eliminate an admixture of the ghost 1^- state associated with the breaking of translational invariance, it is simplest to use the method of Ref. 27, i.e., to choose the constant $\varkappa_0^{(1)}$ such that the energy of the lowest 1^- state in the RPA vanishes. Admittedly, this method is not entirely correct when the dipole forces have the radial dependence $R_\lambda(r) = r$ and the average field is described by the Woods–Saxon potential (the resulting errors are 10–15% for the 1^- states below the resonance at energies $E_x \lesssim 7–10$ MeV; see Ref. 28). It is difficult to determine the constant $\varkappa_0^{(\lambda)}$ for higher multipolarities from experimental data, since (except, perhaps, for ^{208}Pb) there are virtually no data on excitations with $\lambda > 3$ having a simple single-phonon (or particle–hole) structure. In spherical nuclei with developed pairing the lowest $4_1^+, 5_1^-, 6_1^+$ states contain a significant admixture of complicated (two- and three-phonon) components. Therefore, the values of $\varkappa_0^{(\lambda)}$ for $\lambda > 3$ must be chosen on the basis of an analysis of the experimental situation in the considered nucleus in an approximation that goes beyond the RPA. At the same time, the estimates (7) are taken into account.

It appears that the use of $R_\lambda(r)=\partial V/\partial r$ is preferable. With these forces, the ghost 1^- state is correctly eliminated. The single-particle matrix elements $\langle j|\partial V/\partial r|j'\rangle$ between quasistationary states do not depend on the upper limit of the integration, in contrast to the matrix elements $\langle j|r^\lambda|j'\rangle$ at large λ. Calculations made in the RPA with a complete single-particle basis for the ^{208}Pb nucleus[19] show that the constants $\varkappa_{0,1}^{(\lambda)}$ determined from the self-consistency conditions for $E1$ excitations also give a reasonable description of the single-phonon states of other multipolarities, both low-lying and resonances. Of course, for a truncated single-particle basis the constants $\varkappa_{0,1}^{(\lambda)}$ will depend on λ, but their values for different λ's are found to differ comparatively little.

Separable forces have not been wisely used to study spin and spin–isospin excitations. Apart from the studies made in the quasiparticle–phonon model, we can mention but few, and these have mainly been made by the Copenhagen school.[18,29,30] In Ref. 29, an expression is given for the constants $\varkappa_{0,1}^{(\lambda L)}$ when $R_\lambda(r)=r^\lambda$:

$$\varkappa_0^{(\lambda L)}=\varkappa_1^{(\lambda L)}=-\frac{4\pi\cdot 25}{A\langle r^{2\lambda}\rangle}, \tag{8}$$

where $\varkappa_0^{(\lambda L)}$ is measured in MeV/F$^{2\lambda}$.

The coefficient of proportionality in front of the dependence on A and $\langle r^{2\lambda}\rangle$ was chosen on the basis of data on the $M1$ resonance in ^{208}Pb. Of course, it depends on the potential used to calculate the single-particle wave functions, the actual scheme of single-particle levels, and so forth. In addition, it is well known that uncertainty has developed in recent years with regard to the $M1$ resonance; the old experimental data have been reviewed and partly rejected.[31] However, the $\varkappa_1^{(\lambda L)}$ values obtained by analyzing a large set of data in the framework of the quasiparticle–phonon model[32,33] are close to (8).

If we now turn to the expressions (3) and (4) for the multipole and spin–multipole interactions in the particle–hole channel, we easily see that these interactions must also determine the properties of the charge-exchange interactions, and the constants $\varkappa_1^{(\lambda)}$ and $\varkappa_1^{(\lambda L)}$ must have the same values that we discussed above. This will serve as an additional verification of them. We note that in recent years our ideas about the properties and structure of magnetic and charge-exchange excitations have been deepened. In the light of the recent investigations, we cannot rule out the possibility that it is in the spin and spin–isospin channels that we come up against the fundamental limitation of a separable effective interaction.

Below, we shall discuss the term H_M^{pp}. It seems that the possibility of including in the effective nuclear Hamiltonian forces in the particle–particle channel with nonvanishing moment was first posed in Ref. 34. The requirement of gauge invariance of the internucleon interaction was advanced as justification. On its basis, and under the assumption of separability of the new forces with radial dependence $R_\lambda(r)=r^\lambda$, estimates were obtained for their constants:

$$G_{n,p}^{(\lambda)}\approx G_{n,p}/2\langle Q_\lambda^2\rangle. \tag{9}$$

Later, the influence of multipole pairing in spherical nuclei was investigated in Refs. 35–40, the investigations of Refs. 35–40, being in the framework of the quasiparticle–phonon model. In Refs. 35–40, this pairing was included in the model Hamiltonian independently of the monopole pairing. The constants $G_{n,p}$ and $G_{n,p}^{(\lambda)}$ were in no way related to each other; in addition, the existence of neutron–proton multipole pairing forces ($G_{nn}^{(\lambda)}=G_{pp}^{(\lambda)}=G_{np}^{(\lambda)}$) was assumed. Multipole forces in the particle–particle channel strongly influence the two-nucleon transfer reaction cross sections, and in Ref. 36 it was actually proposed that their constants should be chosen on the basis of correct description of the (p,t) reaction cross sections.

In conclusion, it should be pointed out that the theory contains other parameters, of the type of effective charges, which arise in calculations of specific nuclear processes. We shall discuss them when we come to describe the particular nuclear characteristics associated with them.

We now turn to a discussion of the part played by the various components of the Hamiltonian and their influence on the properties of the single-phonon excitations of spherical nuclei.

2. SINGLE-PHONON STATES OF ELECTRIC TYPE

The structure of single-phonon states of electric type of multipolarity λ is determined by the Hamiltonian

$$\begin{aligned}\mathcal{H}_I=H_{\rm SP}+H_{\rm PAIR}&-\frac{1}{2}(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)}\tau_1\tau_2)\sum_\mu M_{\lambda\mu}^+M_{\lambda\mu}\\&-\frac{1}{2}(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)}\tau_1\tau_2)\sum_\mu (S_{\lambda\mu}^\lambda)^+S_{\lambda\mu}^\lambda\\&-\frac{1}{2}\sum_\mu[G_n^{(\lambda)}P_{\lambda\mu}^+(n)P_{\lambda\mu}(n)+G_p^{(\lambda)}P_{\lambda\mu}^+(p)P_{\lambda\mu}(p)\\&+G_{np}^{(\lambda)}(P_{\lambda\mu}^+(n)P_{\lambda\mu}(p)+P_{\lambda\mu}^+(p)P_{\lambda\mu}(n))].\end{aligned} \tag{10}$$

In this section, we shall not consider multipole pairing, and in (10) we retain only the first four terms. After a Bogolyubov transformation to the quasiparticle creation and annihilation operators α_{jm}^+ and α_{jm},

$$a_{jm}=u_j\alpha_{jm}+(-1)^{j-m}\alpha_{j-m}^+v_j,$$

where u_j and v_j are chosen to minimize the ground-state energy for the Hamiltonians $H_{\rm SP}^n+H_{\rm PAIR}^n$ and $H_{\rm SP}^p+H_{\rm PAIR}^p$, we obtain for $H_{\rm SP}+H_{\rm PAIR}$ the expression

$$\begin{aligned}H_{\rm SP}+H_{\rm PAIR}=&\sum_{jm}\varepsilon_j\alpha_{jm}^+\alpha_{jm}-\frac{1}{4}\sum_\tau G_\tau\sum_{jj'}\hat j\hat j'\{u_j^3A^+(j)-v_j^3A(j)\}\\&\times\{u_{j'}^3A(j')-v_{j'}^3A^+(j')\}\\&+\frac{1}{2}\sum_\tau G_\tau\sum_{jj'}\hat j u_jv_j\{[u_j^3A^+(j)-v_j^3A(j)]B(j')+\text{h. c.}\},\end{aligned} \tag{11}$$

where

$$\begin{aligned}&A(j)=\frac{1}{\sqrt2}\sum_m\frac{(-1)^{j-m}}{\hat j}\alpha_{j-m}\alpha_{jm};\quad B(j)=\sum_m\alpha_{jm}^+\alpha_{jm};\quad \hat j=\sqrt{2j+1};\\&H_M^{ph}(\lambda)+H_{SM}^{ph}(\lambda)=-\frac{1}{2}\sum_\mu(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})[M_{\lambda\mu}^+(n)M_{\lambda\mu}(n)\\&+M_{\lambda\mu}^+(p)M_{\lambda\mu}(p)]+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})[M_{\lambda\mu}^+(n)M_{\lambda\mu}(p)+M_{\lambda\mu}^+(p)M_{\lambda\mu}(n)]\\&-\frac{1}{2}\sum_\mu(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})[(S_{\lambda\mu}^\lambda(n))^+S_{\lambda\mu}^\lambda(n)+(S_{\lambda\mu}^\lambda(p))^+S_{\lambda\mu}^\lambda(p)]\\&+(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})[(S_{\lambda\mu}^\lambda(n))^+S_{\lambda\mu}^\lambda(p)+(S_{\lambda\mu}^\lambda(p))^+S_{\lambda\mu}^\lambda(n)],\end{aligned} \tag{12}$$

where $M^+_{\lambda\mu}=M^+_{\lambda\mu}(n)+M^+_{\lambda\mu}(p)$ [similarly, $(S^\lambda_{\lambda\mu})^+$]. The operators $M^+_{\lambda\mu}(n)$ and $[S^\lambda_{\lambda\mu}(n)]^+$ expressed in terms of α^+_{jm} and α_{jm} have the following form [for $M^+_{\lambda\mu}(p), (S^\lambda_{\lambda\mu}(p))^+$ similarly]:

$$\left.\begin{aligned} M^+_{\lambda\mu}(n)&=\frac{1}{\hat\lambda}\sum_{jj'}^{n} f^{(\lambda)}_{jj'}\Big\{\frac{1}{2}u^{(+)}_{jj'}[A^+(jj'\lambda\mu)\\ &+(-)^{\lambda-\mu}A(jj'\lambda-\mu)]+v^{(-)}_{jj'}B(jj'\lambda\mu)\Big\};\\ (S^\lambda_{LM}(n))^+&=\frac{1}{\hat\lambda}\sum_{jj'}^{n} f^{(\lambda L)}_{jj'}\Big\{\frac{1}{2}u^{(-)}_{jj'}[A^+(jj'LM)\\ &-(-)^{L-M}A(jj'L-M)]+v^{(+)}_{jj'}B(jj'LM)\Big\}. \end{aligned}\right\} \quad (13)$$

In (13), we have introduced the notation

$$A^+(jj'\lambda\mu)=\sum_{mm'}\langle jmj'm'\mid\lambda\mu\rangle\,\alpha^+_{jm}\alpha^+_{j'm'};$$

$$B(jj'\lambda\mu)=\sum_{mm'}(-1)^{j'+m'}\langle jmj'm'\mid\lambda\mu\rangle\,\alpha^+_{jm}\alpha_{j'-m'};$$

$$u^{(\pm)}_{jj'}=u_jv_{j'}\pm u_{j'}v_j;\quad v^{(\pm)}_{jj'}=u_ju_{j'}\pm v_jv_{j'};$$

$$f^{(\lambda)}_{jj'}=\langle j\parallel R_\lambda(r)\,i^\lambda Y_{\lambda\mu}\parallel j'\rangle;\quad f^{(\lambda L)}_{jj'}=\langle j\parallel R_\lambda(r)\,i^\lambda\{\sigma Y_{\lambda\mu}\}_{LM}\parallel j'\rangle.$$

In what follows, the second term in Eq. (11) will be omitted, since it contributes only to the 0^+ states. We write down the commutator of the operators $A(jj'\lambda\mu)$ and $A^+(jj'\lambda\mu)$:

$$\begin{aligned}[A(j_1j_2\lambda\mu),\ A^+(j_1'j_2'\lambda'\mu')]&=\delta_{\lambda\lambda'}\delta_{\mu\mu'}(\delta_{j_1j_1'}\delta_{j_2j_2'}+(-)^{j_2-j_1+\lambda}\delta_{j_1j_2'}\delta_{j_2j_1'})\\ &-\sum_{\substack{m_1m_2\\ m_1'm_2'}}\langle j_1m_1j_2m_2\mid\lambda\mu\rangle\langle j_1'm_1'j_2'm_2'\mid\lambda'\mu'\rangle[\delta_{j_1'j_1}\delta_{m_1'm_1}\alpha^+_{j_2'm_2'}\alpha_{j_2m_2}\\ &+\delta_{j_2'j_2}\delta_{m_2'm_2}\alpha^+_{j_1'm_1'}\alpha_{j_1m_1}-\delta_{j_2j_1'}\delta_{m_2m_1'}\alpha^+_{j_2'm_2'}\alpha_{j_1m_1}-\delta_{j_1j_2'}\delta_{m_1m_2'}\alpha^+_{j_1'm_1'}\alpha_{j_2m_2}]. \end{aligned} \quad (14)$$

To terms $\sim\alpha^+\alpha$, the operators $A(j_1j_2\lambda\mu)$ and $A^+(j_1'j_2'\lambda'\mu')$ commute as boson operators. We now make a linear transformation from the operators $A^+(jj'\lambda\mu)$ and $A(jj'\lambda\mu)$ to the phonon creation and annihilation operators:

$$Q^+_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}[\psi^{\lambda i}_{jj'}A^+(jj'\lambda\mu)-(-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}A(jj'\lambda-\mu)]. \quad (15)$$

The commutator of the phonon operators has the form

$$\begin{aligned}[Q_{\lambda'\mu'i'},\ Q^+_{\lambda\mu i}]&=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\sum_{jj'}(\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'}-\varphi^{\lambda i}_{jj'}\varphi^{\lambda i'}_{jj'})\\ &-\sum_{jj'j_2}\sum_{\substack{mm'\\ m_2}}\{\psi^{\lambda i}_{j'j_2}\psi^{\lambda'i'}_{jj_2}\langle j'm'j_2m_2\mid\lambda\mu\rangle\\ &\times\langle jmj_2m_2\mid\lambda'\mu'\rangle-(-1)^{\lambda+\lambda'-\mu-\mu'}\varphi^{\lambda i}_{jj_2}\varphi^{\lambda'i'}_{j'j_2}\\ &\times\langle jmj_2m_2\mid\lambda-\mu\rangle\langle j'm'j_2m_2\mid\lambda'-\mu'\rangle\}\alpha^+_{jm}\alpha_{j'm'}. \end{aligned} \quad (16)$$

Assuming now that the ground state Ψ_0 of the even–even nucleus is the phonon vacuum, and that the excited states are obtained by applying to Ψ_0 the phonon creation operators ($\Psi_{\lambda\mu i}=Q^+_{\lambda\mu i}\Psi_0$), we consider the expression (16) from the point of view of the restrictions imposed on the coefficients of the transformation (15). The different single-phonon excited states must be orthogonal and normalized. This means that $\langle\Psi_0\alpha^+_{jm}\alpha_{jm}\Psi_0\rangle\approx 0$ and

$$\sum_{jj'}\psi^{\lambda i}_{jj'}\psi^{\lambda'i'}_{jj'}-\varphi^{\lambda i}_{jj'}\varphi^{\lambda'i'}_{jj'}=2\delta_{ii'}. \quad (17)$$

The requirement of a small number of quasiparticles in the nuclear ground state is the main assumption in the random-phase method. It is in this approximation, namely, that the structure of the phonons—the basis states of the quasiparticle–phonon model—is calculated.

Under such assumptions, the Hamiltonian (12) becomes

$$\begin{aligned}\mathcal{H}_I=\sum_{jm}\varepsilon_j\alpha^+_{jm}\alpha_{jm}&-\frac{1}{8}\frac{1}{\hat\lambda^2}\sum_{\mu}\sum_{ii'}\{(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})[D_n^{\lambda i}D_n^{\lambda i'}+D_p^{\lambda i}D_p^{\lambda i'}]\\ &+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})[D_n^{\lambda i}D_p^{\lambda i'}+D_n^{\lambda i'}D_p^{\lambda i}]+(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})\\ &\times[\mathscr{D}_n^{(\lambda,\lambda)i}\mathscr{D}_n^{(\lambda,\lambda)i'}+\mathscr{D}_p^{(\lambda,\lambda)i}\mathscr{D}_p^{(\lambda,\lambda)i'}]\\ &+(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})(\mathscr{D}_n^{(\lambda,\lambda)i}\mathscr{D}_p^{(\lambda,\lambda)i'}+\mathscr{D}_p^{(\lambda,\lambda)i}\mathscr{D}_n^{(\lambda,\lambda)i'})\}\\ &\times(Q^+_{\lambda\mu i}+(-)^{\lambda-\mu}Q_{\lambda-\mu i})(Q^+_{\lambda-\mu i'}+(-)^{\lambda-\mu}Q_{\lambda\mu i}), \end{aligned} \quad (18)$$

where

$$\left.\begin{aligned} D^{\lambda i}_{n(p)}&=\sum_{jj'}^{n(p)}u^{(+)}_{jj'}f^{(\lambda)}_{jj'}(\psi^{\lambda i}_{jj'}+\varphi^{\lambda i}_{jj'})=\sum_{jj'}^{n(p)}u^{(+)}_{jj'}f^{(\lambda)}_{jj'}g^{\lambda i}_{jj'};\\ \mathscr{D}^{(\lambda L)i}_{n(p)}&=\sum_{jj'}^{n(p)}u^{(-)}_{jj'}f^{(\lambda L)}_{jj'}(\psi^{Li}_{jj'}-\varphi^{Li}_{jj'})=\sum_{jj'}^{n(p)}u^{(-)}_{jj'}f^{(\lambda L)}_{jj'}w^{Li}_{jj'}. \end{aligned}\right\} \quad (19)$$

The energy $\omega_{\lambda i}$ of the single-phonon state $Q^+_{\lambda\mu i}\Psi_0$ can be found by minimizing the expectation value of the Hamiltonian (18) with respect to this state subject to the condition (17), which can be rewritten as

$$\sum_{jj'}g^{\lambda i}_{jj'}w^{\lambda j}_{jj'}=2\delta_{ii'}. \quad (20)$$

Thus,

$$\begin{aligned}\langle\Psi_0Q_{\lambda\mu i}\mathcal{H}_IQ^+_{\lambda\mu i}\Psi_0\rangle&=\frac{1}{4}\sum_{jj'}(\varepsilon_j+\varepsilon_{j'})[(g^{\lambda i}_{jj'})^2+(w^{\lambda i}_{jj'})^2]\\ &-\frac{1}{4}\frac{1}{\hat\lambda^2}\{(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})[(D_n^{\lambda i})^2+(D_p^{\lambda i})^2]\\ &+(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})[(\mathscr{D}_n^{(\lambda\lambda)i})^2+(\mathscr{D}_p^{(\lambda\lambda)i})^2]+2(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})D_n^{\lambda i}D_p^{\lambda i}\\ &+2(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})\mathscr{D}_n^{(\lambda\lambda)i}\mathscr{D}_p^{(\lambda\lambda)i}\}. \end{aligned} \quad (21)$$

Varying (21) with respect to $g^{\lambda i}_{jj'}$ and $w^{\lambda i}_{jj'}$ subject to the constraint (20), we obtain a system of linear homogeneous equations for the functions $D^{\lambda i}_{n(p)}$ and $\mathscr{D}^{(\lambda\lambda)i}_{n(p)}$, this having a nontrivial solution when the following conditions are satisfied:

$$0=\begin{vmatrix} X_M(n)(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})-1 & X_M(n)(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}) & X_{MS}(n)(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)}) & X_{MS}(n)(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})\\ X_M(p)(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}) & X_M(p)(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})-1 & X_{MS}(p)(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)}) & X_{MS}(p)(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})\\ X_{MS}(n)(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)}) & X_{MS}(n)(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}) & X_S(n)(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})-1 & X_S(n)(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})\\ X_{MS}(p)(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}) & X_{MS}(p)(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)}) & X_S(p)(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)}) & X_S(p)(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})-1 \end{vmatrix}, \quad (22)$$

where

$$\left.\begin{aligned} X_M(n,\ p)&=\frac{1}{\hat\lambda^2}\sum_{jj'}^{n,\ p}\frac{(u^{(+)}_{jj'}f^{(\lambda)}_{jj'})^2\varepsilon_{jj'}}{\varepsilon^2_{jj'}-\omega^2_{\lambda i}};\\ X_S(n,\ p)&=\frac{1}{\hat\lambda^2}\sum_{j,j'}^{n,\ p}\frac{(u^{(-)}_{jj'}f^{(\lambda\lambda)}_{jj'})^2\varepsilon_{jj'}}{\varepsilon^2_{jj'}-\omega^2_{\lambda i}};\\ X_{MS}(n,\ p)&=\frac{1}{\hat\lambda^2}\sum_{jj'}^{n,\ p}\frac{u^{(+)}_{jj'}u^{(-)}_{jj'}f^{(\lambda)}_{jj'}f^{(\lambda\lambda)}_{jj'}\omega_{\lambda i}}{\varepsilon^2_{jj'}-\omega^2_{\lambda i}}. \end{aligned}\right\} \quad (23)$$

For the neutron amplitudes $\psi^{\lambda i}_{j_nj_n'}$ and $\varphi^{\lambda i}_{j_nj_n'}$, we have the expressions

$$\left.\begin{aligned} \psi^{\lambda i}_{j_nj_n'}&=\frac{\mathscr{N}}{\varepsilon_{j_nj_n'}-\omega_{\lambda i}}\{f^{(\lambda)}_{j_nj_n'}u^{(+)}_{j_nj_n'}[(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})A_{41}\\ &+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})A_{42}]+f^{(\lambda\lambda)}_{j_nj_n'}u^{(-)}_{j_nj_n'}[(\varkappa_0^{(\lambda\lambda)}+\varkappa_1^{(\lambda\lambda)})A_{43}\\ &+(\varkappa_0^{(\lambda\lambda)}-\varkappa_1^{(\lambda\lambda)})A_{44}]\};\\ \varphi^{\lambda i}_{j_nj_n'}&=\frac{\varepsilon_{j_nj_n'}-\omega_{\lambda i}}{\varepsilon_{j_nj_n'}+\omega_{\lambda i}}\psi^{\lambda i}_{j_nj_n'}; \end{aligned}\right\} \quad (24)$$

where A_{ik} are the minors of the matrix that occurs in Eq. (22); the normalization coefficient $\mathscr{N}$ can be found

from the condition (20). Equation (22) was obtained in Ref. 41.

If we do not take into account the spin–multipole interaction ($\varkappa_0^{(\lambda\lambda)} = \varkappa_1^{(\lambda\lambda)} = 0$), then from (22) and (24) we obtain the well-known expressions for the energies of single-phonon states of electric type generated by the separable multipole forces with constants $\varkappa_{nn}^{(\lambda)} = \varkappa_{pp}^{(\lambda)} = \varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)}$, $\varkappa_{np}^{(\lambda)} = \varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)}$ (see Refs. 1 and 4):

$$
\begin{aligned}
&[X_M(n) + X_M(p)]\,(\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)}) - 4\varkappa_0^{(\lambda)}\varkappa_1^{(\lambda)} X_M(n)\, X_M(p) = 1;\\
&\psi_{j_n j_n'}^{\lambda i} = \frac{1}{\sqrt{2y_n^{\lambda i}}} \frac{u_{j_n j_n}^{(+)} f_{j_n j_n'}^{(\lambda)}}{\varepsilon_{j_n j_n'} - \omega_{\lambda i}};\\
&y_n^{\lambda i} = Y_n^{\lambda i} + Y_p^{\lambda i} \left\{ \frac{1 - (\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)})\, X_M(n)}{(\varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)})\, X_M(p)} \right\}^2;\\
&Y_n^{\lambda i} = \frac{1}{2} \frac{\partial}{\partial \omega} X_M(n)\Big|_{\omega=\omega_{\lambda i}}\\
&= -\frac{1}{\hat{\lambda}^2} \sum_{jj'}^{n} \frac{(u_{jj'}^{(+)} f_{jj'}^{(\lambda)})^2\, \varepsilon_{jj'}\, \omega_{\lambda i}}{(\varepsilon_{jj'}^2 - \omega_{\lambda i}^2)^2}.
\end{aligned}
\qquad (25)
$$

The expressions for the proton components are similar.

The single-phonon states of electric type are the ones that have been studied theoretically most fully. There are also numerous calculations with a separable interaction (Refs. 1, 2, 5, 19, 22–26, and 28). Thus, their properties are relatively well known. On the other hand, we do not propose to make comparisons with experiment in this paper. Therefore, in discussing the properties of states of electric type, we shall restrict ourselves to a single example, which combines the most typical features of the spectrum of single-phonon excitations. More detailed discussions of the properties of $E1$, $E2$, and $E3$ resonances can be found in Refs. 17, 20, 42, and 43.

We consider octupole single-phonon excitations of the nucleus ^{140}Ce. Calculations were made with forces $R_3(r) = r^3$; the constants $\varkappa_0^{(3)}$ and $\varkappa_1^{(3)}$ were determined from the experimental energy of the 3_1^- level and the ratio[32]

$$
\varkappa_1^{(\lambda)}/\varkappa_0^{(\lambda)} = q = -0.2\,(2\lambda + 3), \qquad (26)
$$

which for $\lambda = 3$ gives $q = -1.8$. Figure 2 shows a histogram of the density of the two-quasiparticle states with $\lambda^\pi = 3^-$ and the single-phonon states with the largest $B(E3, 0_{g.s.}^+ \to 3_i^-)$ values. The single-particle basis included all bound states and the quasibound states with relatively small width of the proton system and the neutron system. The total number of two-quasiparticle (and, hence, single-phonon) states with $\lambda^\pi = 3^-$ up to the energy $E_x = 35$ MeV is about 130. In Fig. 2a, one can clearly see two peaks in the density of the two-quasiparticle states, these corresponding to transitions through one and through three shells. Comparison of Figs. 2a and 2b shows how the interaction redistributes the strength of the $E3$ transitions, pushing it into the region of low density of the two-quasiparticle states. At the same time, the isoscalar part of the interaction shifts the strength of the $E3$ transitions to lower excitation energies, while the isovector part shifts it to higher energies lying above the region of concentration of the single-particle transitions.[44] However, the isovector component of the multipole interaction also has an appreciable influence on the low-lying states, especially in half-magic nuclei.

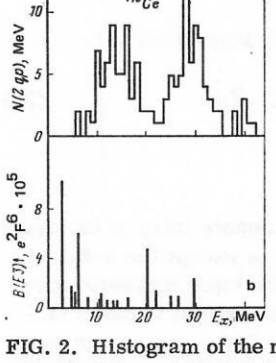

FIG. 2. Histogram of the number of two-quasiparticle poles with $\lambda^\pi = 3^-$ per 1-MeV interval in ^{140}Ce and distribution of the probabilities $B(E_3, 0_{g.s.}^+ \to 3_i^-)$ over the single-phonon $\lambda_i^\pi = 3_i^-$ states in the ^{140}Ce spectrum.

This can be most clearly demonstrated by the example of the 2_1^+ states of spherical nuclei. Experimental and calculated values of $B(E2, 0_{g.s.}^+ \to 2_1^+)$ for a large number of spherical nuclei are given in Fig. 3. The calculations were made with an effective charge having vacuum value (unity for protons and zero for neutrons) with a purely isoscalar quadrupole interaction and with allowance for an isovector component. In both cases, the condition $\omega(2_1^+) = E(2_1^+)_{exp}$ was satisfied. The $B(E2)$ values calculated with $\varkappa_1^{(2)} \neq 0$ significantly exceed the values calculated only with the isoscalar interaction; as a rule, they are also larger than the experimental values. In nuclei with a closed proton shell ($Z = 50$ and 82), calculations with $\varkappa_1^{(2)} = 0$ gave $B(E2)$ values below the experimental values. The isovector component led to a sharp increase in the contribution of the proton components to the structure of the 2_1^+ states of these nuclei, and as a result the $B(E2)$ values increased.

We now discuss the distribution of $B(E3)$ in ^{140}Ce. The residual multipole forces lead to a concentration of the strength of the $E3$ transitions on a small number of collective states. Using the expression *collective*, we understand here single-phonon states for which a large number of two-quasiparticle components makes a contribution to their structure. Fairly often, this word is encountered in the literature with a different meaning—by collective states, states with high probabilities of $E\lambda$ or $M\lambda$ transitions are meant. Sometimes, these two meanings coincide, but this is by no means always so. On the one hand, an enhancement of, for example, the probability of an $E\lambda$ transition may be due, not to a coherent contribution of many components, but simply to a large value of the single-particle matrix element. On the other hand, "structural" collectivization of a state may be manifested in some processes but not in others.

In Fig. 2b, we can clearly identify single-phonon states with large $B(E3)$, these corresponding to a 3_1^- level, a low-lying ($1\hbar\omega$) isoscalar $E3$ resonance, a high-lying ($3\hbar\omega$) isoscalar $E3$ resonance, and a high-lying ($3\hbar\omega$) isovector $E3$ resonance. As was noted in Ref. 45, there could also be a low-lying ($1\hbar\omega$) isovector $E3$ resonance. In Fig. 2b, it could be associated with several single-phonon states with not too large but never-

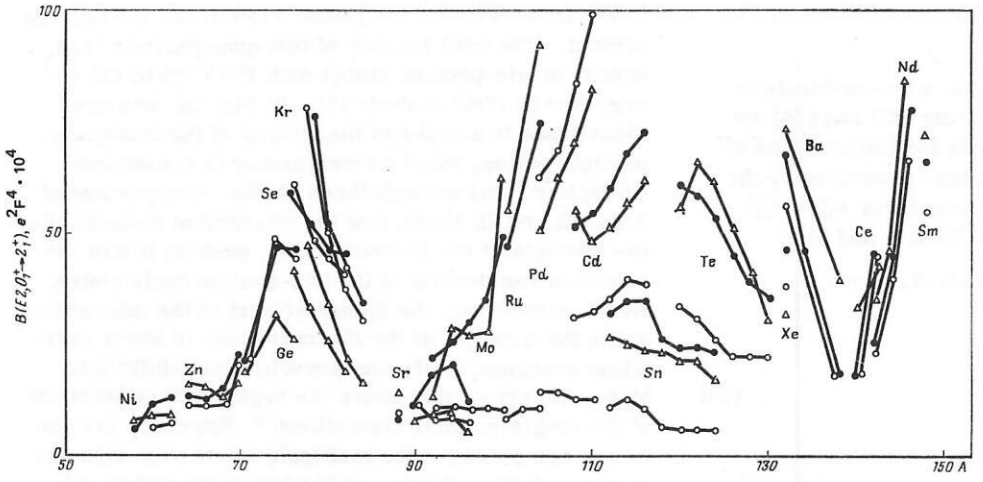

FIG. 3. The probabilities $B(E2, 0^+_{g.s.} \to 2^+_1)$ in spherical nuclei with $60 < A < 150$. The open triangles are the experimental values, the open circles the results of calculations with isoscalar multiple–multipole interaction, and the black circles the results of calculations with isoscalar and isovector multipole–multipole interactions. In both cases, $\omega(2^+_1) = E(2^+_1)_{exp}$.

theless significant $B(E3)$ in the interval 10–15 MeV. In this connection, let us consider the isotopic structure of the single-phonon $E3$ states. Although the multipole interaction is isotopically invariant, this is not true for the model Hamiltonian as a whole. Besides the Coulomb term in the average potential, we use different parameters of the Woods–Saxon potential for the neutrons and protons, and also an isotopically noninvariant monopole pairing. Nevertheless, isotopic symmetry is preserved to a considerable degree in the structure of the single-phonon states. This is shown by Fig. 4, in which we have plotted as functions of E_x, summed over an interval of 1 MeV, the reduced probabilities of electromagnetic $E3$ transitions, isoscalar $E3$ transitions, and isovector $E3$ transitions. In other words, we have plotted the squares of the reduced matrix elements between the ground state and the single-phonon 3^- states of the following operators (summed over an interval of 1 MeV):

$$\left.\begin{aligned} &\mathfrak{M}(E3) = e\sum_i^p r_i^3 Y_{3\mu}(\Omega_i);\\ &\mathfrak{M}(IS,3) = \frac{e}{2}\left\{\sum_i^p r_i^3 Y_{3\mu}(\Omega_i) + \sum_i^n r_i^3 Y_{3\mu}(\Omega_i)\right\};\\ &\mathfrak{M}(IV,3) = \frac{e}{2}\left\{\sum_i^p r_i^3 Y_{3\mu}(\Omega_i) - \sum_i^n r_i^3 Y_{3\mu}(\Omega_i)\right\}. \end{aligned}\right\} \qquad (27)$$

The maxima in the distribution of the isoscalar $E3$ transitions are situated just where the 3^-_1 level and the $1\hbar\omega$ and $3\hbar\omega$ isoscalar $E3$ resonances are situated. And just for these states the strength of the isovector transitions is almost equal to zero. Therefore, the protons and neutrons make a coherent contribution to their structure. For the $3\hbar\omega$ isovector $E3$ resonance the opposite situation is realized. At the same time, the collective states in the region $10 < E_x < 15$ MeV have probabilities of the isoscalar and isovector transitions of the same order of magnitude.

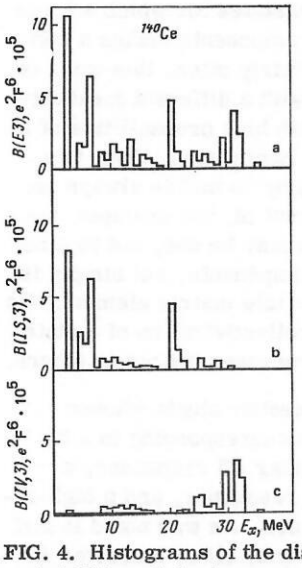

FIG. 4. Histograms of the distributions of the reduced probabilities of electromagnetic (a), isoscalar (b), and isovector (c) $E3$ transitions in ^{140}Ce [see Eq. (27)]. The interval of summation is 1 MeV.

The isotopic structure of the single-phonon states can be seen in the behavior of their transition densities. The transition densities are important characteristics of nuclear states; they determine the probabilities of their excitation in different nuclear reactions. The charge, $\rho(\mathbf{r})$, and current, $\mathbf{J}(\mathbf{r})$, transition densities of the nucleus between the initial state Ψ_i and the final state Ψ_f are defined as follows:

$$\left.\begin{aligned} &\rho(\mathbf{r}) = e\sum_k \delta(\mathbf{r}-\mathbf{r}_k)\{\Psi_f^*\Psi_i\}\, g_l^k;\ \mathbf{J}(\mathbf{r}) = \mathbf{J}^c(\mathbf{r}) + \mathbf{J}^m(\mathbf{r})\\ &= \frac{e\hbar}{2m}\Big[\sum_k \delta(\mathbf{r}-\mathbf{r}_k)\, g_l^k\{\Psi_f^*\nabla_k\Psi_i - \Psi_i\nabla_k\Psi_f\}\\ &+ \sum_k \delta(\mathbf{r}-\mathbf{r}_k)\,\mu_s^k\nabla_k\{\Psi_f^*\sigma_k\Psi_i\}\Big]. \end{aligned}\right\} \qquad (28)$$

The curly brackets in Eqs. (28) denote integration over the coordinates of all the nucleons except the k-th; m is the nucleon mass; μ_s^k is the intrinsic magnetic moment of the nucleon; and g_l^k is the orbital nucleon factor, zero for a neutron and unity for a proton. In the expression for $\mathbf{J}(\mathbf{r})$, we have omitted the exchange component of the nuclear current, which presupposes the necessity of using effective values of the nucleon mag-

netic moments. The functions $\mathbf{J}(\mathbf{r})$ and $\rho(\mathbf{r})$ can be conveniently expressed as expansions with respect to the partial densities $\rho_\lambda(r)$ and $\rho_{\lambda L}(r)$:

$$\left.\begin{aligned}\rho(\mathbf{r}) &= e\sum_{\lambda\mu}(-1)^\lambda\langle J_iM_i\lambda\mu|J_fM_f\rangle\,\rho_\lambda(r)\,Y^*_{\lambda\mu}(\Omega);\\ \mathbf{J}^{c,m}(\mathbf{r}) &= ec\sum_{\lambda L\mu}(-\mathrm{i})^L\langle J_iM_i\lambda\mu|J_fM_f\rangle\,\rho^{c,m}_{\lambda L}(r)\,\mathbf{Y}^\mu_{L}(\Omega).\end{aligned}\right\}\qquad(29)$$

At present, we are interested in only the charge transition density of the single-phonon states[46]:

$$\left.\begin{aligned}&\rho_\lambda(r)=\sum_{j>j'}^{n,p}\frac{u^{(+)}_{jj'}}{1+\delta_{jj'}}\,g^{\lambda i}_{jj'}\rho^{(\lambda)}_{jj'}(r);\\ &\rho^{(\lambda)}_{jj'}(r)=-\{1+(-)^{l+l'+\lambda}\}(-)^{j+\lambda+1/2}\\ &\times\frac{\hat j\,\hat j'}{\hat\lambda\sqrt{4\pi}}(j^1/_2j'-{}^1/_2|\lambda0)\,u^*_j(r)\,u_{j'}(r)\,(\mathrm{i})^{l'-l+\lambda}.\end{aligned}\right\}\qquad(30)$$

Here, $j>j'$ means that in the sum (j,j') states are not encountered twice; $u_j(r)$ is the radial part of the single-particle wave function of the nlj state.

The charge transition densities of some collective 3^- states are shown in Fig. 5. All of them except for the one for the state with $E_x=12.4$ MeV have a clearly expressed surface peak. We also show the transition densities corresponding to the neutron component of these states (only the proton components contribute to the charge transition density, since the effective charge for the neutrons is zero). For the states that earlier were called isovector states, the proton and neutron transition densities as a function of the radius r vary in phase on, at least, the surface of the nucleus. For the state with $E_x=29.8$ MeV (an isovector $3\hbar\omega$ $E3$ resonance) the proton and neutron transition densities vary in counterphase. Among the states in the interval $\Delta E_x=10$–15 MeV, there are ones with both coherent and incoherent variation of $\rho_n(r)$ and $\rho_p(r)$. Thus, they can hardly be regarded as states of the isovector $1\hbar\omega$ resonance.

How are the properties of the single-phonon states of electric type changed if the contribution to their structure of the spin–multipole interaction is taken into account? It can be seen from the expressions (24) for $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ that this influence is manifested in two ways: First, because of the different angular-momentum selection rules in the single-particle matrix elements $f^{(\lambda)}_{jj'}$ and $f^{(\lambda\lambda)}_{jj'}$, the number of poles of Eq. (22) may be increased after inclusion of the spin–multipole forces; second, there is a renormalization of $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ due to the appearance of a term proportional to $f^{(\lambda\lambda)}_{jj'}$. The calculations made in Ref. 41 show that the influence of the first factor is slight and basically the

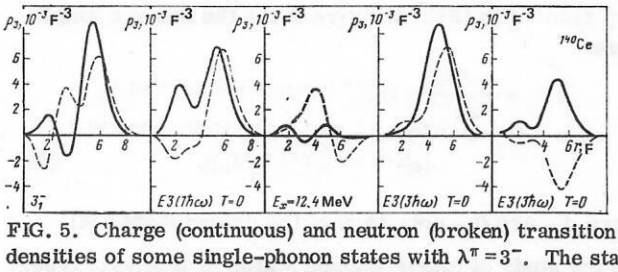

FIG. 5. Charge (continuous) and neutron (broken) transition densities of some single-phonon states with $\lambda^\pi=3^-$. The state with energy $E_x=12.4$ MeV is the region of the isovector $1\hbar\omega$ $E3$ resonance.

amplitudes $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ are renormalized. A more detailed analysis made for several spherical nuclei with $R_\lambda(r)=\partial V/\partial r$ forces and with allowance for only the isovector component of the spin–multipole interaction showed that the properties of the low-lying states and the isoscalar resonance $E\lambda$ states change very little. More significant are the changes in the regions of the isovector resonances, especially the $E1$ resonance. The extent of the redistribution of the strength of the $E1$ transitions that arises in the region of localization of the resonance when the spin–dipole interaction is included can be gauged from Fig. 6, in which we give the results of calculations for ^{208}Pb and ^{90}Zr. The spin–multipole forces also change the transition densities, mainly the current densities. We give expressions for the partial current convection, $\rho^c_{\lambda L}(r)$, and magnetic, $\rho^m_{\lambda L}(r)$, transition densities of single-phonon states[47]:

$$\left.\begin{aligned}&\rho^{c,m}_{\lambda L}(r)=\sum_{j>j'}^{n,p}\frac{u^{(-)}_{jj'}}{1+\delta_{jj'}}\,w^{\lambda i}_{jj'}\rho^{c,m}_{jj',\lambda L}(r);\\ &\rho^{c}_{jj',\lambda L}(r)=\frac{\hbar}{mc}\hat L\mathrm{i}^{l'-l+L+1}(-)^{j'-l+1}\\ &\times\hat j\,\hat j'\begin{Bmatrix}l\,l'&\lambda\\ j\,j'&\frac12\end{Bmatrix}g_l\tfrac12\{\hat l\,(l\|Y_{\lambda L1}\nabla\|l')\\ &-(-1)^\lambda\hat l'\,(l'^*\|Y_{\lambda L1}\nabla\|l^*)\};\\ &(l\|Y_{\lambda L_1}\nabla\|l')=(-1)^{\lambda+1}\frac{1}{\sqrt{4\pi}}\\ &\times\sum_{l''=l'-1}^{l'+1}(l''-l')\,(l0L0|l'0)\begin{Bmatrix}l'&1&l''\\ l&L&\lambda\end{Bmatrix}u^*_{nlj}(r)\\ &\times D_{l',\,l''-l'}(r)\,u_{n'l'j'}(r);\\ &D_{l',\,\pm1}(r)=\Big[l'+\tfrac12(1\pm1)\Big]^{1/2}\Big[\frac{d}{dr}\mp\frac{l'+(1\mp1)/2}{r}\Big];\\ &\rho^{m}_{jj',\lambda L}(r)=-\frac{\hbar}{2mc}\,i^{l'-l+1+L}(-1)^{j+\lambda+1/2}\\ &\times g_s\frac{\hat j\,\hat j'}{\hat\lambda^2\sqrt{4\pi}}(j^1/_2j'-1/2|\lambda0)\,F^{jj'}_{\lambda L}(r)\\ &\times u^*_{nlj}(r)\,u_{n'l'j'}(r);\\ &F^{jj'}_{\lambda\lambda}(r)=-\frac{\hat\lambda}{\sqrt{\lambda(\lambda+1)}}\Big[\frac{\lambda(\lambda+1)}{r}+[(l-j)\,\hat j^2\\ &+(l'-j')\,\hat j'^2]\Big(\frac{d}{dr}+\frac1r\Big)\Big];\\ &F^{jj'}_{\lambda,\lambda\pm1}(r)=\frac{1}{\lambda+1/2\pm1/2}\,[(l-j)\,\hat j^2-(l'-j')\,\hat j'^2]\,D_{\lambda,\,\pm1}(r).\end{aligned}\right\}\qquad(31)$$

The excitation cross section of states of normal parity is determined by the current transition densities $\rho^{c,m}_{\lambda,\lambda+1}(r)$ and $\rho^{c,m}_{\lambda,\lambda-1}(r)$ as well as by the charge transition density. All three transition densities for the two 1^-

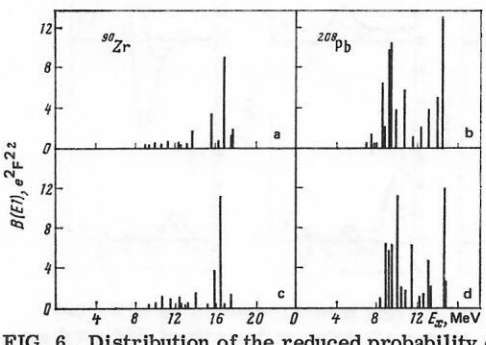

FIG. 6. Distribution of the reduced probability of $E1$ transitions in the spectra of ^{90}Zr (a, c) and ^{208}Pb (b, d): a) and b) are the results of calculation with only dipole–dipole forces, and c) and d) are the results of calculation with allowance for the spin–dipole interaction.

states in ^{208}Pb with maximal $B(E1)$ are shown in Fig. 7. The current transition densities change very appreciably. Since they determine the behavior of the electric form factor $F_\lambda^E(q^2)$, which plays the main part in electron scattering through $\theta \approx 180°$, the spin forces will have a strong influence on the probability of excitation of the $E1$ resonance in backward scattering.

This does not exhaust the role of the spin–multipole forces of "normal" parity. They lead to the appearance of a new type of collective excitation—$E\lambda$ states formed by two-quasiparticle components, which correspond to single-particle transitions with spin flip.[41] Such 1^- states arise at $E_x \approx 20$ MeV and are distinguished by the small value of $B(E1)$. For example, in ^{208}Pb such a state has $E_x = 19.4$ MeV and $B(E1)\uparrow = 0.5\ e^2F^2$, which is 20 times less than the $B(E1)\uparrow$ of the stronger 1^- states in the region of the ordinary $E1$ resonance in this nucleus. The collective nature of the spin 1^- state is manifested in another way—it is strongly excited in (e, e') backward scattering. At energies $E_0 = 40$–60 MeV of the incident electrons and large scattering angles, such an $E1$ state must be excited an order of magnitude more strongly than the 1^- states of the ordinary $E1$ resonance. The transition densities for the collective "spin" 1^- state of the ^{58}Ni nucleus are shown in Fig. 8. With the example of these excitations we encounter states of a collective structure that can be seen in some processes but not in others.

3. SINGLE-PHONON STATES OF MAGNETIC TYPE

To calculate the energies and structure of single-phonon ML states in the RPA, it is necessary to find the eigenvalues of the Hamiltonian

$$\mathscr{H}_{\rm II} = H_{\rm SP} + H_{\rm PAIR} - \frac{1}{2}(\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)}\tau_1\tau_2) \times \sum_M (S_{LM}^{L-1})^+ S_{LM}^{L-1} - \frac{1}{2}(\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)}\tau_1\tau_2) \sum_M (S_{LM}^{L+1})^+ S_{LM}^{L+1}. \quad (32)$$

Going over to quasiparticle operators, and then to

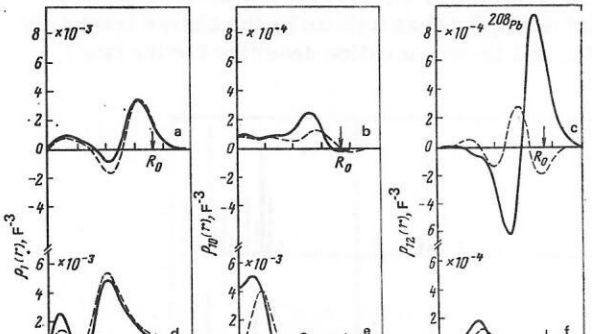

FIG. 7. Charge and current transition densities of the two collective single-phonon 1^- states with maximal $B(E1)$ values of the nucleus ^{208}Pb (see Fig. 6d): a), b), and c) refer to the state $E_x = 14.7$ MeV; d), e), and f) to the state with $E_x = 9.6$ MeV. The continuous curves were calculated with the dipole–dipole interaction, and the broken curves with allowance for the spin–dipole forces.

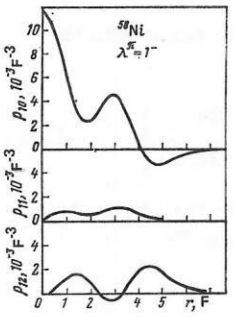

FIG. 8. Charge and current transition densities of the "spin" collective $E1$ state of the nucleus ^{58}Ni ($E_x = 24$ MeV).

phonons, as we did at the beginning of the previous section, and using the expression (13) for the operator S_{LM}^{λ}, we obtain the following expression for $H_{\rm II}$:

$$\mathscr{H}_{\rm II} = \sum_{jm} \varepsilon_j \alpha_{jm}^+ \alpha_{jm} - \frac{1}{8}\frac{1}{\hat{L}^2}\sum_M \sum_{ii'} \{(\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)}) \times (\mathscr{D}_n^{(L-1,\,L)i}\mathscr{D}_n^{(L-1,\,L)i'} + \mathscr{D}_p^{(L-1,\,L)i}\mathscr{D}_p^{(L-1,\,L)i'}) + (\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})(\mathscr{D}_n^{(L+1,\,L)i}\mathscr{D}_n^{(L+1,\,L)i'} + \mathscr{D}_p^{(L+1,\,L)i}\mathscr{D}_p^{(L+1,\,L)i'}) + (\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})(\mathscr{D}_n^{(L-1,\,L)i}\mathscr{D}_p^{(L-1,\,L)i'} + \mathscr{D}_n^{(L-1,\,L)i'}\mathscr{D}_p^{(L-1,\,L)i}) + (\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})(\mathscr{D}_n^{(L+1,\,L)i}\mathscr{D}_p^{(L+1,\,L)i'} + \mathscr{D}_n^{(L+1,\,L)i'}\mathscr{D}_p^{(L+1,\,L)i})\} \times (Q_{LMi}^+ + (-1)^{L-M}Q_{L-Mi})((-)^{L-M}Q_{L-Mi'}^+ + Q_{LMi'}), \quad (33)$$

where the functions $\mathscr{D}_{n,p}^{(\lambda L)i}$ are determined by the expression (19). After calculation of the expectation value of the Hamiltonian (33) in the single-phonon state $Q_{LMi}^+\Psi_0$ and variation of it with respect to the functions $g_{jj'}^{Li}, w_{jj'}^{Li}$ subject to the constraint (20), we obtain a system of linear homogeneous equations for the functions $\mathscr{D}_{n,p}^{(\lambda L)i}$ and as a condition for the existence of a nontrivial solution of it an equation for the state energy ω_{Li} (see Ref. 48):

$$\begin{vmatrix}
(\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(n) - 1 & (\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(n) \\
(\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(p) & (\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(p) - 1 \\
(\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)})\,X_S^{(L-1,\,L+1)i}(n) & (\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})\,X_S^{(L-1,\,L+1)i}(n) \\
(\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})\,X_S^{(L-1,\,L+1)i}(p) & (\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)})\,X_S^{(L-1,\,L+1)i}(p) \\
(\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})\,X_S^{(L-1,\,L+1)i}(n) & (\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})\,X_S^{(L-1,\,L+1)i}(n) \\
(\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})\,X_S^{(L-1,\,L+1)i}(p) & (\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})\,X_S^{(L-1,\,L+1)i}(p) \\
(\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})\,X_S^{L+1,\,i}(n) - 1 & (\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})\,X_S^{L+1,\,i}(n) \\
(\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})\,X_S^{L+1,\,i}(p) & (\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})\,X_S^{L+1,\,i}(p) - 1
\end{vmatrix} = 0. \quad (34)$$

The functions $X_S^{L-1,\,i}(n,p)$ and $X_S^{L+1,\,i}(n,p)$ are defined in (23), and the form of the functions $X_S^{(L-1,\,L+1)i}(n,p)$ is

$$X_S^{(L-1,\,L+1)i}(n,\,p) = \frac{1}{\hat{L}^2}\sum_{jj'}^{n,\,p} \frac{f_{jj'}^{(L-1,\,L)} f_{jj'}^{(L+1,\,L)} (u_{jj'}^-)^2}{\varepsilon_{jj'}^2 - \omega_{Li}^2}\varepsilon_{jj'}. \quad (35)$$

The expression for the amplitudes $\psi_{jj'}^{Li}$ and $\varphi_{jj'}^{Li}$ is also very similar to (24). We give it for the neutron amplitudes:

$$\psi_{j_n^{-}j'_n}^{Li} = \frac{\mathscr{N} u_{j_n^{-}j'_n}^-}{\varepsilon_{j_n j'_n} - \omega_{Li}} \{f_{j_n^{-}j'_n}^{(L-1,\,L)}[(\varkappa_0^{(L-1,\,L)} + \varkappa_1^{(L-1,\,L)})\,A_{41} + (\varkappa_0^{(L-1,\,L)} - \varkappa_1^{(L-1,\,L)})\,A_{42}] + f_{j_n^{+}j'_n}^{(L+1,\,L)}[(\varkappa_0^{(L+1,\,L)} + \varkappa_1^{(L+1,\,L)})\,A_{43} + (\varkappa_0^{(L+1,\,L)} - \varkappa_1^{(L+1,\,L)})\,A_{44}]\}, \quad (36)$$

where A_{ik} are the cofactors of the determinant (34).

In the study of single-phonon states of magnetic type, the terms proportional to $(S_{LM}^{L+1})^* S_{LM}^{L-1}$ are often ignored. Then the expressions (34)–(36) simplify to

$$
\left.\begin{aligned}
&[X_S^{L-1,\,i}(n)+X_S^{L-1,\,i}(p)]\,(\varkappa_0^{(L-1,\,L)}+\varkappa_1^{(L-1,\,L)})\\
&-4\varkappa_0^{(L-1,\,L)}\varkappa_1^{(L-1,\,L)}X_S^{L-1,\,i}(n)\,X_S^{L-1,\,i}(p)=1;\\
&\psi_{j_n'j_n'}^{Li}=\frac{1}{\sqrt{2\mathcal{Y}_n^{L-1,\,i}}}\,\frac{f_{j_n'j_n'}^{(L-1,\,L)}u_{j_n'j_n'}^{(-)}}{\varepsilon_{j_n'j_n'}-\omega_{Li}};\\
&\mathcal{Y}_n^{L-1,\,i}=Y_n^{L-1,\,i}+Y_p^{L-1,\,i}\left\{\frac{1-(\varkappa_0^{(L-1,\,L)}+\varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(n)}{(\varkappa_0^{(L-1,\,L)}-\varkappa_1^{(L-1,\,L)})\,X_S^{L-1,\,i}(p)}\right\}^2;\\
&Y_n^{L-1,\,i}=\frac{1}{2}\,\frac{\partial}{\partial\omega}\,X_S^{L-1,\,i}(n)|_{\omega=\omega_{Li}}\\
&=-\frac{1}{\hat{L}^2}\sum_{jj'}^{n}\frac{(j_{jj'}^{(L-1,\,L)}u_{jj'}^{(-)})^2\,\varepsilon_{jj',\omega_{Li}}}{(\varepsilon_{jj'}^2-\omega_{Li}^2)^2}.
\end{aligned}\right\}
\tag{37}
$$

Use of an interaction proportional to $(S_{LM}^{L-1})^+S_{LM}^{L-1}$ automatically ensures coherent enhancement of the ML transition from a collective magnetic state of multipolarity L, since the operator S_{LM}^{L-1} is identical to the spin part of the magnetic transition operator $\mathfrak{M}(ML)$. An interaction of this type was used in several investigations of the $M1$ resonance.[29,33,49]

Typical distributions of the strength of the $M1$ and $M2$ transitions obtained in calculations with simple spin and spin–dipole forces are shown in Fig. 9.[32,50] The calculations were made with forces having radial dependence $R_L(r)=r^{L-1}$ and effective gyromagnetic factors $g_s^{\text{eff}}=0.8g_s^{\text{free}}$ and $g_l^{\text{eff}}=g_l^{\text{free}}$. Equation (8) for the constants $\varkappa_{0,1}^{(\lambda L)}$ presupposes the absence of a neutron–proton spin–multipole interaction. In this case, the single-phonon states of magnetic type will have either purely neutron or purely proton structure. In Fig. 9, we show the results of calculations with different constants $\varkappa_{0,1}^{(\lambda L)}$. The absolute values of the constants differ slightly from (8):

$$
\varkappa_1^{(\lambda L)}=-4\pi 28/A\,\langle r^{2\lambda}\rangle, \tag{38}
$$

where $\varkappa_1^{(\lambda L)}$ is measured in MeV/F$^{2\lambda}$.

Since the isoscalar and the isovector components of the spin–multipole interaction have the same signs, the isoscalar component does not significantly influence the distribution of $B(ML)$. Its role reduces to a redistribution of the strength of the ML transitions between the resonance single-phonon states.

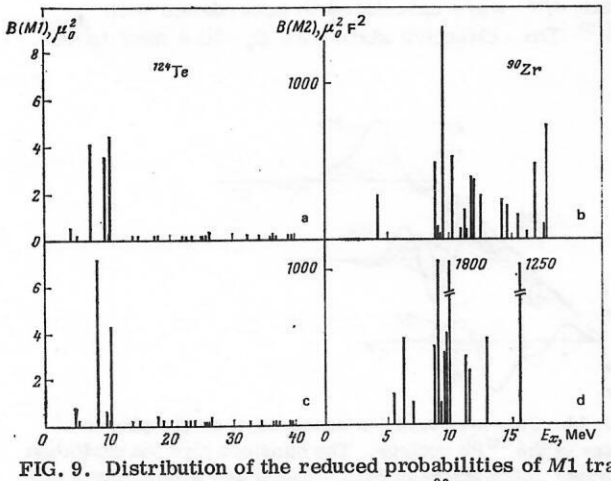

FIG. 9. Distribution of the reduced probabilities of $M1$ transitions in ^{124}Te (a, c) and $M2$ transitions in ^{90}Zr (b, d): a) and b) are the calculation with $\varkappa_0^{(\lambda L)}=0$; c) and d) are with $\varkappa_0^{(\lambda L)}=\varkappa_1^{(\lambda L)}$.

The structure of the 1^+ phonons is well known from numerous calculations. The single-phonon states forming the $M1$ resonance are a typical example of resonance but weakly collective states. Their wave functions have a contribution from two or three two-quasiparticle components, which are formed by quasiparticles in levels of a spin–orbit doublet. The large values of $B(M1)$ are explained by the large single-particle matrix elements of the magnetic dipole operator $\mathfrak{M}(M1)$ between the states of the doublet. The current transition densities of the single-phonon resonance 1^+ states have a maximum near the surface of the nucleus[51] (Fig. 10). Spin–quadrupole forces, which will be considered below, have a strong influence on the structure and properties of 1^+ states with higher excitation energies.

The distribution of the $B(M2)$ probability in the spectra of spherical nuclei differs strongly from the $B(M1)$ distribution that we have just discussed. States with large $B(M2)$ occupy a wide range of excitation energies, the range increasing with increasing mass number from 4 MeV (^{58}Ni) to 10 MeV (^{208}Pb). In nuclei with $A<100$–120, an appreciable strength of the $M2$ transitions is concentrated, in addition, on one state with energy $E_x\approx 20$ MeV. It can be clearly seen in Fig. 9 in ^{90}Zr. In heavier nuclei, this state sinks and adjoins the low-lying region of $B(M2)$ concentration. Such a distribution of the $B(M2)$ probabilities is shown in Fig. 11 (^{140}Ce). From this figure it can also be concluded that such a state exists because the residual interaction forces the transition strength into a region of low density of the two-quasiparticle states.[2)] The resonance 2^- states have a collective structure, and their current transition densities behave quite differently from the 1^+ states (see Fig. 10). The current transition densities of the 2^- states are concentrated within the volume of the nucleus and decrease toward its surface.

The relative importance of the terms $(S_{LM}^{L-1})^+S_{LM}^{L-1}$ and $(S_{LM}^{L+1})^+S_{LM}^{L+1}$ of the spin–multipole interaction is forming

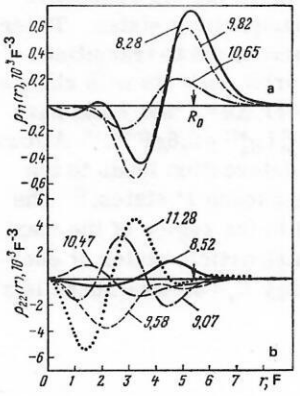

FIG. 10. Current transition densities of single-phonon 1^+ states (a) and 2^- states (b) of the nucleus ^{90}Zr. The numbers are the excitation energies of the states in MeV. The arrow indicates the position of the radius of the nucleus. The transition densities of the various states are shown by different types of curve.

[2)] It was shown in Refs. 32 and 52 that the high-lying collective 2^- state is almost completely broken up as a result of interaction with more complicated states.

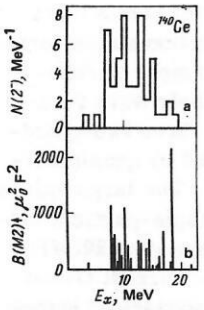

FIG. 11. Histogram of the density of two-quasiparticle states with $L^\pi = 2^-$ in ^{140}Ce (a) and the distribution of $B(M2)$ in the ^{140}Ce spectrum (b).

the structure of the single-phonon ML states is determined by the same factors as we mentioned above in connection with the contribution of the spin–multipole forces to the structure of the $E\lambda$ states. Here, however, we can give an example of a situation that did not arise for the $E\lambda$ states, namely, allowance for the spin–quadrupole component of the interaction strongly increases the number of two-quasiparticle components that participate in the formation of the 1^+ phonons. Since the selection rules for simple spin forces admit a contribution by only two-quasiparticle states with $\Delta l = 0$, while states with $\Delta l = 2$ are allowed for the spin–quadrupole forces, the number of poles in (34) greatly exceeds the number (see Fig. 12) in Eq. (37). However, the new components do not contribute to the probability of the $M1$ transitions unless the polarization term $g_s r^2 Y_{20}$ is included in the $\mathfrak{M}(M1)$ operator. There is also little change in the properties of the 1^+ states with energy $E_x < 10$–15 MeV and, therefore, the same is true of the properties of the resonance $M1$ states. Allowance for the spin–quadrupole interaction leads to other changes, in their own way striking, in the structure and properties of high-lying $M1$ excitations.

In calculations with simple $\sigma\sigma$ forces, all the single-phonon 1^+ states with energy above the $M1$ resonance are found to be purely two-quasiparticle states. These states (see Figs. 9 and 13) correspond to transitions between components of spin–orbit doublets with change in the principal quantum number $\Delta n = 1$ and 2 and have very small $B(M1) \lesssim (0.1$–$0.2)\mu_0^2$ ($g_s^{\text{eff}} = 0.8 g_s^{\text{free}}$).[49] Allowance for the spin–quadrupole interaction leads to the formation of collective single-phonon 1^+ states.[53] The majority of them are situated in the region of the maximum of the density of two-quasiparticle states at energies $15 < E_x < 22$ MeV. At energy $E_x \approx 30$ MeV there is a

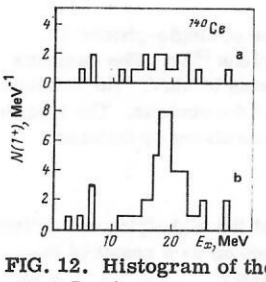

FIG. 12. Histogram of the density of two-quasiparticle poles with $L^\pi = 1^+$ of Eqs. (37) (a) and (34) (b) in the ^{140}Ce nucleus.[48]

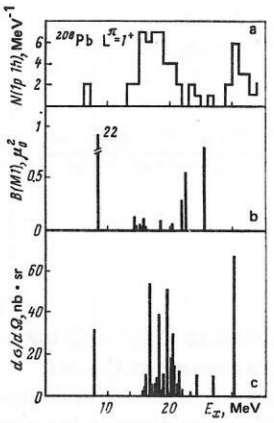

FIG. 13. Histogram of the density of two-quasiparticle states with $L^\pi = 1^+$ for an interval of 1 MeV in ^{208}Pb (a), the distribution of $B(M1)$ in the ^{208}Pb spectrum (b), and the excitation probabilities of 1^+ states of the ^{208}Pb nucleus in (e, e') backward scattering at $E_0 = 60$ MeV (c).

strongly collective state formed by two-quasiparticle components corresponding to transitions with spin flip. Its position is very sensitive to the constant $\varkappa_1^{(21)}$. Typical current transition densities of the high-lying 1^+ excitations, including two collective states with $E_x = 19.2$ MeV and $E_x = 30.4$ MeV,[53] are shown in Fig. 14. In contrast to the densities of the $M1$ resonance, the transition densities shown in Fig. 14 have a volume nature. The transition density of the state with $E_x = 19.2$ MeV oscillates strongly, which is due to the superposition of the current transition densities of the two-quasiparticle components in its structure in counterphase. For the collective state with $E_x = 30.4$ MeV, the situation is the opposite. Therefore, the probability of excitation in (e, e') scattering of the first state is much less than for the second.

If we now consider the probabilities of excitation of 1^+ states in inelastic backward scattering of electrons, we obtain the picture shown in Fig. 13c. This calculation was made with allowance for only the isovector components of the spin and spin–quadrupole forces. The radial dependence of the latter was $R_2(r) = r^2$, and the constants $\varkappa_1^{(\lambda 2)}$ were calculated in accordance with Eq. (38).[53] The collective state with $E_x = 30.4$ MeV is ex-

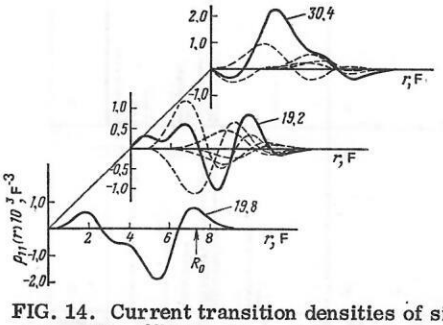

FIG. 14. Current transition densities of single-phonon 1^+ states of the ^{208}Pb nucleus. The numbers give the excitation energies of the states in MeV. The states with $E_x = 19.2$ and 30.4 MeV are collective; the broken curves show the current transition densities of the main particle–hole components that contribute to their structure.

cited most strongly. It is interesting that in the group of states with relatively large excitation probabilities ($15 < E_x < 22$ MeV) it is the two-quasiparticle, and not the collective, states that are most strongly excited. Figure 14 shows the current transition density of one such state ($E_x = 19.8$ MeV). Like the density of the state with $E_x = 30.4$ MeV, it does not fluctuate strongly but has a fairly broad minimum near $r \approx 5$ F. It is interesting to note that again, as we have already seen in the example of the collective "spin" $E1$ state and the high-lying collective $M2$ state, the collective high-lying $M1$ state excited strongly in (e, e') scattering is "pushed" into the region of low density of two-quasiparticle states.

The terms $\sim (S^{L+1}_{LM})^+ S^{L+1}_{LM}$ do not strongly influence the structure of the states with higher spins. Essentially, they renormalize the amplitudes $\psi^{Li}_{jj'}$ and $\varphi^{Li}_{jj'}$. The resulting changes in the distribution of the probabilities $B(ML)$ are noticeable only at energies $E_x > 15$ MeV. For example, when allowance is made for the spin–octupole term for the states with $L^\pi = 2^-$ in some nuclei (for example, in ^{90}Zr) the strength of the $M2$ transition to the high-lying collective 2^- state is distributed over two or three neighboring one-phonon states.[48]

The structure of the single-phonon states of magnetic type and the distribution of the probabilities $B(ML)$ in the spectra of spherical nuclei are strongly different from what we saw for states of electric type and the probabilities $B(EL)$. The distributions $B(ML)$ and the probabilities of (e, e') excitation of 1^+ states differ appreciably. It is interesting that there exist collective 1^+ excitations that do not have resonance properties. However, their strong interaction with the neighboring states, being a consequence of their collective nature, can significantly, although indirectly, influence the characteristics of the nuclear spectra.

4. CHARGE-EXCHANGE PHONONS AND SINGLE-PHONON STATES WITH $T_>$

Discussing the model Hamiltonian as a whole in Sec. 1, we pointed out that the isotopically invariant expression for the multipole and spin–multipole interactions presupposes the possibility of investigating neutron–proton (or charge-exchange) excitations. These excitations are generated by the action of the charge-exchange phonon operator

$$\Omega^+_{\lambda\mu i} = \frac{1}{\sqrt{2}} \sum_{j_n j_p} \psi^{\lambda i}_{j_p j_n} A^+ (j_p j_n \lambda\mu) - (-)^{\lambda-\mu} A (j_p j_n \lambda - \mu)\, \varphi^{\lambda i}_{j_p j_n}, \qquad (39)$$

where

$$A^+ (j_p j_n \lambda\mu) = \sum_{m_n m_p} \langle j_p m_p j_n m_n | \lambda\mu \rangle \alpha^+_{j_p m_p} \alpha^+_{j_n m_n},$$

on the wave function of the ground state of the neighboring even–even nucleus. To find the excitation energy of the state $\Omega^+_{\lambda\mu i}\Psi_0$, we must diagonalize in the random-phase approximation the part of the Hamiltonian that includes the terms proportional to $\tau_1^{(-)}\tau_2^{(+)} + \tau_1^{(+)}\tau_2^{(-)}$. In this section, we consider the case of charge-exchange states of electric type:

$$\mathscr{H}_{\rm III} = H_{\rm SP} + H_{\rm PAIR} - 2\varkappa_1^{(\lambda)} \sum_\mu M^+_{\lambda\mu}(np) M_{\lambda\mu}(np) - 2\varkappa_1^{(\lambda\lambda)} \sum_\mu [S^\lambda_{\lambda\mu}(np)]^+ S^\lambda_{\lambda\mu}(np), \qquad (40)$$

where

$$M^+_{\lambda\mu}(np) = \sum_{\substack{j_p m_p \\ j_n m_n}} \langle j_p m_p | i^\lambda R_\lambda(r)\, Y_{\lambda\mu} \tau^{(-)} | j_n m_n \rangle\, a^+_{j_p m_p} a_{j_n m_n};$$

$$[S^\lambda_{\lambda\mu}(np)]^+ = \sum_{\substack{j_p m_p \\ j_n m_n}} \langle j_p m_p | i^\lambda R_\lambda(r) \{\sigma Y_{\lambda\mu}\}_{\lambda\mu} \tau^{(-)} | j_n m_n \rangle\, a^+_{j_p m_p} a_{j_n m_n}.$$

The expressions for the operators $M^+_{\lambda\mu}(np)$ and $[S^\lambda_{\lambda\mu}(np)]^+$ in terms of quasiparticle operators have the same form except for the replacement $f^{(\lambda)}_{j_p j_n} \to f^{(\lambda\lambda)}_{j_p j_n}$, where $f^{(\lambda),(\lambda\lambda)}_{j_p j_n}$ is the reduced single-particle matrix element of the multipole or spin–multipole operator between the proton and neutron states:

$$M^+_{\lambda\mu}(np) = \frac{1}{\hat\lambda} \sum_{j_n j_p} f^{(\lambda)}_{j_p j_n} \{u_{j_p} v_{j_n} A^+ (j_p j_n \lambda\mu) + (-)^{\lambda-\mu} v_{j_p} u_{j_n} A (j_p j_n \lambda - \mu) + u_{j_p} u_{j_n} B (j_p j_n \lambda\mu) + (-)^{j_p + j_n + \lambda} v_{j_p} v_{j_n} B (j_n j_p \lambda\mu)\}. \qquad (41)$$

The operator $A^+(j_p j_n \lambda\mu)$ is defined in (39), and the operator $B(j_p j_n \lambda\mu)$ has the form

$$B (j_p j_n \lambda\mu) = \sum_{m_p m_n} \langle j_p m_p j_n m_n | \lambda\mu \rangle (-)^{j_n + m_n} \alpha^+_{j_p m_p} \alpha_{j_n - m_n}.$$

Going over to the phonon operators (39), we obtain in the random-phase approximation the expression for the Hamiltonian $\mathscr{H}_{\rm III}$:

$$\begin{aligned}\mathscr{H}_{\rm III} = \sum_{jm}^{n,p} \varepsilon_j \alpha^+_{jm} \alpha_{jm} - \frac{1}{4} \sum_\mu \frac{1}{\hat\lambda^2} \sum_{ii'} \varkappa_1^{(\lambda)} \{(D^{\lambda i}_+ + D^{\lambda i}_-)\, \Omega^+_{\lambda\mu i} \\ + (D^\lambda_+ - D^{\lambda i}_-)\, \Omega_{\lambda-\mu i} (-)^{\lambda-\mu}\} \{(D^{\lambda i'}_+ + D^{\lambda i'}_-)\, \Omega_{\lambda\mu i'} + (D^{\lambda i'}_+ - D^{\lambda i'}_-) \\ \times \Omega^+_{\lambda-\mu i'} (-)^{\lambda-\mu}\} + \varkappa_1^{(\lambda\lambda)} \{(\mathscr{D}^{\lambda i}_+ + \mathscr{D}^{\lambda i}_-)\, \Omega^+_{\lambda\mu i} \\ + (\mathscr{D}^{\lambda i}_+ - \mathscr{D}^{\lambda i}_-)\, \Omega_{\lambda-\mu i} (-)^{\lambda-\mu}\} \{(\mathscr{D}^{\lambda i'}_+ + \mathscr{D}^{\lambda i'}_-)\, \Omega_{\lambda\mu i'} \\ + (\mathscr{D}^{\lambda i'}_+ - \mathscr{D}^{\lambda i'}_-)\, \Omega^+_{\lambda-\mu i'} (-)^{\lambda-\mu}\},\end{aligned} \qquad (42)$$

where

$$\left.\begin{aligned} D^{\lambda i}_+ &= \sum_{j_p j_n} u^{(+)}_{j_p j_n} f^{(\lambda)}_{j_p j_n} g^{\lambda i}_{j_p j_n}; \quad D^{\lambda i}_- = \sum_{j_p j_n} u^{(-)}_{j_p j_n} f^{(\lambda)}_{j_p j_n} w^{\lambda i}_{j_p j_n}; \\ \mathscr{D}^{\lambda i}_+ &= \sum_{j_p j_n} u^{(+)}_{j_p j_n} f^{(\lambda\lambda)}_{j_p j_n} g^{\lambda i}_{j_p j_n}; \quad \mathscr{D}^{\lambda i}_- = \sum_{j_p j_n} u^{(-)}_{j_p j_n} f^{(\lambda\lambda)}_{j_p j_n} w^{\lambda i}_{j_p j_n}. \end{aligned}\right\} \qquad (43)$$

Calculating the expectation value of the Hamiltonian (42) in the single-phonon state, we obtain

$$\langle \Psi_0 \Omega_{\lambda\mu i} \mathscr{H}_{\rm III} \Omega^+_{\lambda\mu i} \Psi_0 \rangle = \frac{1}{4} \sum_{j_p j_n} \varepsilon_{j_p j_n} [(g^{\lambda i}_{j_p j_n})^2 + (w^{\lambda i}_{j_p j_n})^2] - \frac{1}{2\hat\lambda^2} \{\varkappa_1^{(\lambda)} [(D^{\lambda i}_+)^2 + (D^{\lambda j}_-)^2] + \varkappa_1^{(\lambda\lambda)} [(D^{\lambda i}_+)^2 + (\mathscr{D}^{\lambda i}_-)^2]\}$$

and after variation with respect to $g^{\lambda i}_{j_p j_n}$ and $w^{\lambda i}_{j_p j_n}$ we finally arrive at the following secular equation for the state energy $\omega_{\lambda i}$:

$$\begin{vmatrix} \varkappa_1^{(\lambda)}X_M^{(+)}-1 & \varkappa_1^{(\lambda)}X_M^{(+-)} & \varkappa_1^{(\lambda\lambda)}X_{SM}^{(+)} & \varkappa_1^{(\lambda\lambda)}X_{SM}^{(+-)} \\ \varkappa_1^{(\lambda)}X_M^{(+-)} & \varkappa_1^{(\lambda)}X_M^{(-)}-1 & \varkappa_1^{(\lambda\lambda)}X_{SM}^{(+-)} & \varkappa_1^{(\lambda\lambda)}X_{SM}^{(-)} \\ \varkappa_1^{(\lambda)}X_{SM}^{(+)} & \varkappa_1^{(\lambda)}X_{SM}^{(+-)} & \varkappa_1^{(\lambda\lambda)}X_S^{(+)}-1 & \varkappa_1^{(\lambda\lambda)}X_S^{(+-)} \\ \varkappa_1^{(\lambda)}X_{SM}^{(+-)} & \varkappa_1^{(\lambda)}X_{SM}^{(-)} & \varkappa_1^{(\lambda\lambda)}X_S^{(+-)} & \varkappa_1^{(\lambda\lambda)}X_S^{(-)}-1 \end{vmatrix} = 0;$$

$$X_M^{(+)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(+)}f_{j_pj_n}^{(\lambda)})^2\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};\quad X_S^{(+)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(+)}f_{j_pj_n}^{(\lambda\lambda)})^2\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_M^{(-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(-)}f_{j_pj_n}^{(\lambda)})^2\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};\quad X_S^{(-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(-)}f_{j_pj_n}^{(\lambda\lambda)})^2\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_M^{(+-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{u_{j_pj_n}^{(-)}u_{j_pj_n}^{(+)}(f_{j_pj_n}^{(\lambda)})^2\omega_{\lambda i}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_S^{(+-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{u_{j_pj_n}^{(-)}u_{j_pj_n}^{(+)}(f_{j_pj_n}^{(\lambda\lambda)})^2\omega_{\lambda i}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_{SM}^{(+)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(+)})^2f_{j_pj_n}^{(\lambda)}f_{j_pj_n}^{(\lambda\lambda)}\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_{SM}^{(-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{(u_{j_pj_n}^{(-)})^2f_{j_pj_n}^{(\lambda)}f_{j_pj_n}^{(\lambda\lambda)}\varepsilon_{j_pj_n}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2};$$

$$X_{SM}^{(+-)} = \frac{2}{\hat{\lambda}^2}\sum_{j_pj_n}\frac{u_{j_pj_n}^{(+)}u_{j_pj_n}^{(-)}f_{j_pj_n}^{(\lambda)}f_{j_pj_n}^{(\lambda\lambda)}\omega_{\lambda i}}{\varepsilon_{j_pj_n}^2-\omega_{\lambda i}^2}. \qquad (44)$$

Equations (22), (34), and (44) are very similar. So are the expressions for the amplitudes $\psi_{j_pj_n}^{\lambda i}$ are $\varphi_{j_pj_n}^{\lambda i}$:

$$\psi_{j_pj_n}^{\lambda i} = \frac{\mathcal{N}}{\varepsilon_{j_pj_n}-\omega_{\lambda i}}\{\varkappa_1^{(\lambda)}f_{j_pj_n}^{(\lambda)}[u_{j_pj_n}^{(+)}A_{41}+u_{j_pj_n}^{(-)}A_{42}] + \varkappa_1^{(\lambda\lambda)}f_{j_pj_n}^{(\lambda\lambda)}[u_{j_pj_n}^{(+)}A_{43}+u_{j_pj_n}^{(-)}A_{44}]\};$$
$$\varphi_{j_pj_n}^{\lambda i} = \frac{\varepsilon_{j_pj_n}-\omega_{\lambda i}}{\varepsilon_{j_pj_n}+\omega_{\lambda i}}\psi_{j_pj_n}^{\lambda i}, \qquad (45)$$

where A_{ik} are the cofactors of the determinant (44); $\mathcal{N}$ is determined from the normalization condition.

In Ref. 54, equations were obtained for the energies and structure of charge-exchange $E\lambda$ states with allowance for only the multipole interaction:

$$(\varkappa_1^{(\lambda)}X_M^{(+)}-1)(\varkappa_1^{(\lambda)}X_M^{(-)}-1)-(\varkappa_1^{(\lambda)})^2(X_M^{(+-)})^2=0;$$

$$\psi_{j_pj_n}^{\lambda i} = 2\sqrt{\frac{\varkappa_1^{(\lambda)}}{1-\varkappa_1^{(\lambda)}X_M^{(+)}}}\frac{1}{\sqrt{\mathcal{Y}_{\lambda i}}}\frac{f_{j_pj_n}^{(\lambda)}}{\hat{\lambda}}\frac{u_{j_p}v_{j_n}\varkappa_1^{(\lambda)}X_M^{(+-)}+v_{j_p}u_{j_n}(1-\varkappa_1^{(\lambda)}X_M^{(+)})}{\varepsilon_{j_pj_n}-\omega_{\lambda i}},$$

where $\mathcal{Y}_{\lambda i}$ is the value of the derivative with respect to ω of the left-hand side of the secular equation at $\omega = \omega_{\lambda i}$. Spin–isospin charge-exchange excitations were studied in Ref. 55 in connection with the problem of the Gamow–Teller resonance.

The charge-exchange phonon wave function can be used to contruct the wave function of a single-phonon state of an even–even nucleus having isospin larger by unity than the isospin of the ground state (we shall denote it by T_0).[54] The wave function $\Omega_{\lambda\mu i}^{+}\Psi_0$ corresponds to a state of the odd–odd nucleus with isospin and isospin projection $|T, T_z\rangle = |T_0+1, T_0+1\rangle$. Applying to it the operator $T^{(-)}=\sum_{k=1}^{A}\tau_k^{(-)}$,[56] we obtain the state $\langle T_0+1, T_0\rangle$, which corresponds to the state $T_>$ of the initial even–even nucleus. Thus,

$$|T_0+1, T_0\rangle = \frac{1}{\sqrt{2T_0+2}}T^{(-)}\Omega_{\lambda\mu i}^{+}\Psi_0. \qquad (46)$$

The energy of the state (46) is determined by

$$\frac{1}{2T_0+2}\langle\Psi_0\Omega_{\lambda\mu i}T^{(+)}\mathcal{H}_{\mathrm{III}}T^{(-)}\Omega_{\lambda\mu i}^{+}\Psi_0\rangle = \omega_{\lambda i}+\Delta E_c, \qquad (47)$$

where ΔE_c is the Coulomb energy.

In Ref. 54 the energies and transition probabilities for dipole resonances $T_>$ in the $N=50$ isotopes and Sn isotopes were calculated. The Coulomb energy was not calculated microscopically but was estimated in accordance with the phenomenological formula $\Delta E_c = 1.444(Z-1/2)/A^{1/3}-1.131$, where ΔE_c is measured in MeV. Typical results of this calculation are compared with $B(E1)$ for the ordinary dipole resonance in Fig. 15.

5. INFLUENCE OF THE RADIAL DEPENDENCE OF SEPARABLE MULTIPOLE AND SPIN-MULTIPOLE FORCES ON THE STRUCTURE AND PROPERTIES OF THE SINGLE-PHONON STATES

The results of the concrete numerical calculations discussed above were obtained with different radial dependences of the forces $R_\lambda(r)$. It is natural to consider to what extent they depend on the actual choice of the radial dependence. This question was considered for states of electric type in Ref. 25. Here, we shall consider it more systematically.

Comparison of results obtained with different radial dependence of the forces requires the enunciation of some principle for consistent determination of the constants of effective forces. In Ref. 25, they were found on the basis of Bohr–Mottelson consistency.[24] Since the constants found in this way lead to large errors in the description of the energies of low-lying states,[25] we shall follow our traditional path, normalizing the constants by means of the known experimental values.

For the $E\lambda$ states, such normalization does not encounter great difficulties. We determine the isoscalar constants for the forces $R_\lambda(r)=r^\lambda$ and $R_\lambda(r)=\partial V/\partial r$ to make the energies of the lowest single-phonon states with given λ^π coincide. We shall calculate the isovector constant for r^λ forces by using the ratio q (26), which reproduces the position of the $E2$ and $T1$ resonances. For the $\partial V/\partial r$ forces, we shall assume that q does not depend on the multipolarity and choose them empirically using the ratio $\varkappa_1^{(1)}/\varkappa_0^{(1)}$ of the dipole constants; we discussed the principles of this determination in Sec. 1 (see also Ref. 19).

For the constants normalized in this way, we obtain very similar distributions of the strengths of the best

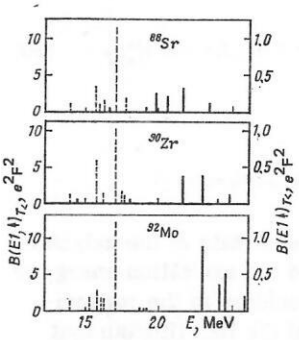

FIG. 15. Reduced probabilities of $E1$ transitions to single-phonon states of the giant dipole resonance $T_<$ (broken lines, left-hand scale) and $T_>$ (continuous lines, right-hand scale) in the isotopes $N=50$.

studied $E1$, $E2$, and $E3$ transitions in calculations using forces with different radial dependences. Typical results are given in Tables I and II for some low-lying and resonance single-phonon states in ^{58}Ni. For the $E2$ and $E3$ states, the differences are small and are within the accuracy of the model. The charge transition densities of some of these states (for convenience of comparison, they are normalized to unity) are given in Fig. 16. Their differences are also slight. For states of high multipolarity, the differences are greater. The structure of the 6_1^+ state of the same nucleus ^{58}Ni differs appreciably on the transition from $R_\lambda(r)=r^6$ to $R_\lambda(r)=\partial V/\partial r$, while the probability of the $E6$ transition $B(E6, 0^+_{g.s.} \to 6_1^+)$ is reduced by a factor 2. However, the behavior of the charge transition density changes little. The differences for the states of high multipolarity can be attributed to the truncation of the single-particle basis, which affects differently the states of different multipolarity and requires renormalization of q with increasing λ for the $\partial V/\partial r$ forces, which we have not done. With increasing mass number A, this factor will become weaker for given λ, which can be seen by the example of the distribution of the probability $B(E6)$ in ^{140}Ce (Fig. 17).

The situation is somewhat more complicated with regard to the normalization of the constants for the spin-multipole forces, for which the experimental data are mainly for highly excited states. Here, we have used the following convention. The constants $\varkappa_1^{(\lambda L)}$ for the forces with $R_\lambda(r)=r^\lambda$ were determined using the expression (38). The constant $\varkappa_1^{(01)}$ for $\partial V/\partial r$ forces was chosen to make the energy of the 1^+ state with maximal $B(M1)$ agree with the energy obtained with $\sigma\sigma$ forces and constant $\tilde{\varkappa}_1^{(01)}=-4\pi\times 28/A$ MeV. This value of the constant was then used for all multipolarities.

For the 1^+ states used to normalize the constants the resulting differences in the structure and $B(M1, 0^+_{g.s.} \to 1_i^+)$ are slight (Table III). However, they are accompanied by significant changes of the current transition densities (Fig. 18). These changes are stronger in the interior region of the nucleus, but, for example, in ^{140}Ce also affect the surface region. In Fig. 18, we also show the current transition densities (normalized to unity) of 2^- states of the nuclei ^{58}Ni and ^{140}Ce. The change in the function $R_\lambda(r)$ has not influenced them strongly. At the same time, the distribution of the strength of the $M2$ transitions has been changed (Fig. 19). In calculations with $R_\lambda(r)=\partial V/\partial r$ forces, it appears as if the transition strength had been shifted to higher excitation energies. By a renormalization of the constants it is possible to obtain closer distributions of $B(M2)$ than in Fig. 19. With increasing L, the differences in the $B(ML)$ distribution also increase. For $L \geqslant 4$ and the adopted convention about the constants, the interaction with $R_\lambda(r)=r^\lambda$ is effectively stronger than the interaction with $R_\lambda(r)=\partial V/\partial r$. One of the reasons for this difference is unphysical in nature. In the calculation of the constants (38), it was assumed that the distribution of the nucleon density in the nucleus has a rectangular shape and a sharp boundary. Allowance for the diffuseness of the nuclear surface strongly increases the value of $\langle r^{2\lambda}\rangle$ for $\lambda>3$, which must reduce the constants.

Overall, if λ and L are not large, the change in the radial dependence of the forces, accompanied by a corresponding renormalization of the constants, does not lead to significant changes in the results (except for the current transition densities for the 1^+ levels). At larger λ and L, when the truncation of the single-particle basis and the uncertainty in the constants begin to have more and more influence, there is simultaneously a greater sensitivity of the structure and properties of

TABLE I. Structure and $B(E\lambda)\uparrow$ values of the 2_1^+, 3_1^-, 6_1^+ states of the ^{58}Ni nucleus calculated with different functions $R_\lambda(r)$.

λ_i^π	Structure, %			$B(E\lambda)\uparrow$, $e^2F^{2\lambda}$	
	Indices	$R_\lambda(r)=r^\lambda$	$R_\lambda(r)=\partial V/\partial r$	$R_\lambda(r)=r^\lambda$	$R_\lambda(r)=\partial V/\partial r$
2_1^+	$2p_{3/2}-2p_{3/2}{}^n$	33	28	1140	1130
	$1f_{7/2}-2p_{3/2}{}^n$	7.0	9.8		
	$2p_{3/2}-2p_{1/2}{}^n$	8.8	7.4		
	$2p_{3/2}-1f_{5/2}{}^n$	2.5	3.4		
	$1f_{7/2}-2p_{3/2}{}^p$	38	39		
3_1^-	$2p_{3/2}-1g_{9/2}{}^n$	39	41	23900	20400
	$1f_{7/2}-1g_{9/2}{}^n$	9.0	14		
	$1f_{7/2}-1g_{9/2}{}^p$	26	21		
	$2s_{1/2}-1f_{5/2}{}^p$	3.9	3.7		
	$1d_{5/2}-2p_{3/2}{}^p$	4.5	3.6		
6_1^+	$1f_{7/2}-1f_{5/2}{}^n$	<1	29	$4.2\cdot10^8$	$2.4\cdot10^8$
	$1f_{7/2}-1f_{5/2}{}^p$	97	68		

TABLE II. Energies and $B(E\lambda)\uparrow$ probabilities for single-phonon phonon 2^+ and 3^- states of the ^{58}Ni nucleus calculated with different $R_\lambda(r)$ functions.

λ_i^π	E_x, MeV		$B(E\lambda)\uparrow$, $e^2F^{2\lambda}$	
	$R_\lambda(r)=r^\lambda$	$R_\lambda(r)=\partial V/\partial r$	$R_\lambda(r)=r^\lambda$	$R_\lambda(r)=\partial V/\partial r$
$E2$, $T=0$	13.5	13.6	218	154
	14.2	14.2	320	343
$E3$, $T=0$ $(1\hbar\omega)$	7.2	7.2	4140	3620
$E3$, $T=0$ $(3\hbar\omega)$	25.4	25.1	1930	2120
	26.0	26.0	1310	680
	27.1	27.1	430	304

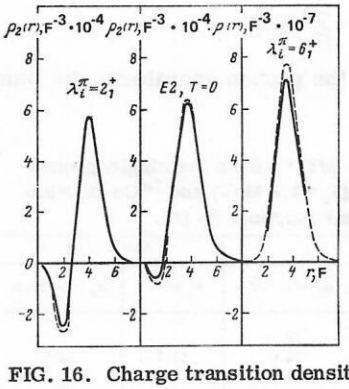

FIG. 16. Charge transition densities of states with $\lambda^\pi=2_1^+$ and 6_1^+ of the isoscalar $E2$ resonance of the ^{58}Ni nucleus. The continuous curves are calculated with forces $R_\lambda(r)=r^\lambda$, the broken curves with $R_\lambda(r)=\partial V/\partial r$.

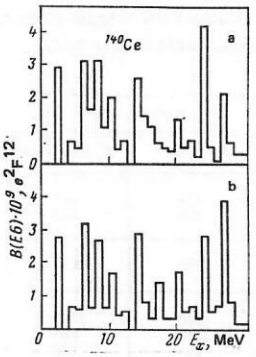

FIG. 17. Histogram of the distribution of $B(E6)\uparrow$ in the ^{140}Ce spectrum: a) calculation with $R_6(r)=r^6$; b) calculation with $R_6(r)=\partial V/\partial r$. The summation interval is 1 MeV.

the single-phonon states to the radial dependence of the forces.

6. MULTIPOLE INTERACTION IN THE PARTICLE–PARTICLE CHANNEL AND STRUCTURE OF PHONONS OF ELECTRIC TYPE

At the beginning of Sec. 3, when considering single-phonon $E\lambda$ states, we omitted the multipole interaction in the particle–particle channel from the Hamiltonian H_I. We now consider its influence on the structure and properties of the single-phonon states.

To write the corresponding part of the Hamiltonian (10) in terms of phonon operators, we first express the operator $P^+_{\lambda\mu}(n)$ (5) in terms of them:

$$P^+_{\lambda\mu}(n)=\frac{1}{\hat\lambda}\sum_{jj'}^{n} f^{(\lambda)}_{jj'}\{u_ju_{j'}A^+(jj'\lambda\mu) - (-)^{\lambda-\mu}v_jv_{j'}A(jj'\lambda-\mu)-2u_jv_{j'}B(jj'\lambda\mu)\}. \quad (48)$$

Omitting the term proportional to $B(jj'\lambda\mu)$ and going over from the operators $A^+(jj'\lambda\mu)$, $A(jj'\lambda\mu)$ to the operators $Q^+_{\lambda\mu i}$, $Q_{\lambda\mu i}$, we obtain

$$P^+_{\lambda\mu}(n)=\frac{1}{2\hat\lambda}\sum_i\sum_{jj'} f^{(\lambda)}_{jj'}[(v^{(-)}_{jj'}g^{\lambda i}_{jj'}+v^{(+)}_{jj'}w^{\lambda i}_{jj'})Q^+_{\lambda\mu i} + (-)^{\lambda-\mu}(v^{(-)}_{jj'}g^{\lambda i}_{jj'}-v^{(+)}_{jj'}w^{\lambda i}_{jj'})Q_{\lambda-\mu i}] = \frac{1}{2\hat\lambda}\sum_i (L^{\lambda i}_n+M^{\lambda i}_n)Q^+_{\lambda\mu i}+(-)^{\lambda-\mu}(L^{\lambda i}_n-M^{\lambda i}_n)Q_{\lambda-\mu i}, \quad (49)$$

where

$$L^{\lambda i}_n=\sum_i^n v^{(-)}_{jj'}g^{\lambda i}_{jj'}f^{(\lambda)}_{jj'};\quad M^{\lambda i}_n=\sum_{jj'}^n v^{(+)}_{jj'}f^{(\lambda)}_{jj'}w^{\lambda i}_{jj'}.$$

Expressed in terms of the phonon operators, the Hamiltonian (10) becomes (we omit the spin–multipole interaction)

$$\mathcal{H}'_1=\sum_{jm}\varepsilon_j\alpha^+_{jm}\alpha_{jm}-\frac{1}{8}\frac{1}{\hat\lambda^2}\sum_\mu\sum_{ii'}\{(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})[D^{\lambda i}_nD^{\lambda i'}_n+D^{\lambda i}_pD^{\lambda i'}_p] + (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})[D^{\lambda i}_nD^{\lambda i'}_p+D^{\lambda i'}_nD^{\lambda i}_p]\}(Q^+_{\lambda\mu i}+(-)^{\lambda-\mu}Q_{\lambda-\mu i}) \times(Q^+_{\lambda-\mu i'}+(-)^{\lambda-\mu}Q_{\lambda\mu i'})+G^{(\lambda)}_n[(L^{\lambda i}_n+M^{\lambda i}_n)Q^+_{\lambda\mu i} + (-)^{\lambda-\mu}(L^{\lambda i}_n-M^{\lambda i}_n)Q_{\lambda-\mu i}][(L^{\lambda i'}_n+M^{\lambda i'}_n)Q_{\lambda\mu i'} + (L^{\lambda i'}_n-M^{\lambda i'}_n)Q^+_{\lambda-\mu i'}(-)^{\lambda-\mu}]+G^{(\lambda)}_p[(L^{\lambda i}_p+M^{\lambda i}_p)Q^+_{\lambda\mu i} + (L^{\lambda i}_p-M^{\lambda i}_p)(-)^{\lambda-\mu}Q_{\lambda-\mu i}][(L^{\lambda i'}_p+M^{\lambda i'}_p)Q_{\lambda\mu i'} + (L^{\lambda i'}_p-M^{\lambda i'}_p)Q^+_{\lambda-\mu i'}(-)^{\lambda-\mu}]+G^{(\lambda)}_{np}\{[(L^{\lambda i}_n+M^{\lambda i}_n)Q^+_{\lambda\mu i} + (L^{\lambda i}_n-M^{\lambda i}_n)Q_{\lambda-\mu i}(-)^{\lambda-\mu}][(L^{\lambda i'}_p+M^{\lambda i'}_p)Q_{\lambda\mu i'} + (L^{\lambda i'}_p-M^{\lambda i'}_p)Q^+_{\lambda-\mu i'}(-)^{\lambda-\mu}]+[(L^{\lambda i}_p+M^{\lambda i}_p)Q^+_{\lambda\mu i} + (L^{\lambda i}_p-M^{\lambda i}_p)Q_{\lambda-\mu i}(-)^{\lambda-\mu}][(L^{\lambda i'}_n+M^{\lambda i'}_n)Q_{\lambda\mu i'} + (L^{\lambda i'}_n-M^{\lambda i'}_n)Q^+_{\lambda-\mu i'}(-)^{\lambda-\mu}]\}. \quad (50)$$

For the expectation value of $\mathcal{H}'_1$ in the single-phonon state, we obtain

$$\langle\Psi_0Q_{\lambda\mu i}\mathcal{H}'_1Q^+_{\lambda\mu i}\Psi_0\rangle=\frac{1}{4}\sum_{jj'}\varepsilon_{jj'}[(g^{\lambda i}_{jj'})^2+(w^{\lambda i}_{jj'})^2]-\frac{1}{4}\frac{1}{\hat\lambda^2}\{(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)}) \times[(D^{\lambda i}_n)^2+(D^{\lambda i}_p)^2]+2(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})D^{\lambda i}_nD^{\lambda i}_p+G^{(\lambda)}_n[(L^{\lambda i}_n)^2 + (M^{\lambda i}_n)^2]+G^{(\lambda)}_p[(L^{\lambda i}_p)^2+(M^{\lambda i}_p)^2]+2G^{(\lambda)}_{np}[L^{\lambda i}_nL^{\lambda i}_p+M^{\lambda i}_nM^{\lambda i}_p]\}. \quad (51)$$

The secular equation for the state energy $\omega_{\lambda i}$ is expressed by the vanishing of a determinant of sixth order:

TABLE III. Structure and $B(M1)\uparrow$ values for single-phonon resonance 1^+ states of ^{58}Ni (E_x=9.9 MeV) and ^{140}Ce (E_x=8.5 MeV) calculated with different functions $R_\lambda(r)$.

Nucleus	Structure, %			$B(M1)\uparrow$, μ_0^2	
	Indices	$R_\lambda(r)=1$	$R_\lambda(r)=\partial V/\partial r$	$R_\lambda(r)=1$	$R_\lambda(r)=\partial V/\partial r$
^{140}Ce	$1h_{11/2}-1h_{9/2}n$	79.3	75.1	12.4	12.5
	$1g_{9/2}-1g_{7/2}p$	18.7	20.0		
	$1h_{11/2}-1h_{9/2}p$	<1	1.3		
^{58}Ni	$1f_{7/2}-1f_{5/2}n$	57.5	46.4	10.4	10.5
	$1f_{7/2}-1f_{5/2}p$	41.5	51.2		

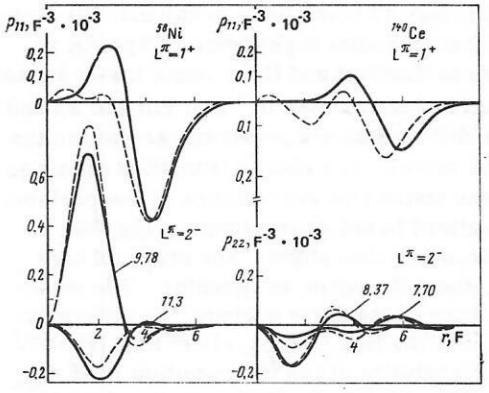

FIG. 18. Current transition densities of single-phonon 1^+ and 2^- states of the ^{58}Ni and ^{140}Ce nuclei. The continuous curves are calculated with $R_\lambda(r)=r^\mu$, the broken curves with $R_\lambda(r)=\partial V/\partial r$. The numbers give the excitation energies of the 2^- states in MeV.

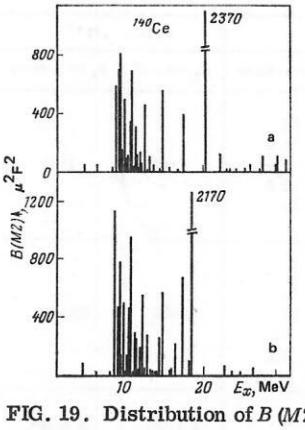

FIG. 19. Distribution of $B(M2)\uparrow$ in the ^{140}Ce spectrum: a) calculation with forces $R_1(r)=\partial V/\partial r$; b) calculation with $R_1(r)=r$.

$$\begin{vmatrix}
(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_M(n)-1 & (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_M(n) & G_n^{(\lambda)}X_{uv}^{(-)}(n) \\
(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_M(p) & (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_M(p)-1 & G_{np}^{(\lambda)}X_{uv}^{(-)}(p) \\
(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_{uv}^{(-)}(n) & (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_{uv}^{(-)}(n) & G_n^{(\lambda)}X_{vv}^{(-)}(n)-1 \\
(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_{uv}^{(-)}(p) & (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_{uv}^{(-)}(p) & G_{np}^{(\lambda)}X_{vv}^{(-)}(p) \\
(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_{uv}^{(+)}(n) & (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_{uv}^{(+)}(n) & G_n^{(\lambda)}X_{vv}^{(+-)}(n) \\
(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X_{uv}^{(+)}(p) & (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X_{uv}^{(+)}(p) & G_{np}^{(\lambda)}X_{vv}^{(+-)}(p)
\end{vmatrix}$$

$$\begin{vmatrix}
G_{np}^{(\lambda)}X_{uv}^{(-)}(n) & G_n^{(\lambda)}X_{uv}^{(+)}(n) & G_{np}^{(\lambda)}X_{uv}^{(+)}(n) \\
G_p^{(\lambda)}X_{uv}^{(-)}(p) & G_{np}^{(\lambda)}X_{uv}^{(+)}(p) & G_p^{(\lambda)}X_{uv}^{(+)}(p) \\
G_{np}^{(\lambda)}X_{vv}^{(-)}(n) & G_n^{(\lambda)}X_{vv}^{(+-)}(n) & G_{np}^{(\lambda)}X_{vv}^{(+-)}(n) \\
G_p^{(\lambda)}X_{vv}^{(-)}(p)-1 & G_{np}^{(\lambda)}X_{vv}^{(+-)}(p) & G_p^{(\lambda)}X_{vv}^{(+-)}(p) \\
G_{np}^{(\lambda)}X_{vv}^{(+-)}(n) & G_n^{(\lambda)}X_{vv}^{(+)}(n)-1 & G_{np}^{(\lambda)}X_{vv}^{(+)}(n) \\
G_p^{(\lambda)}X_{vv}^{(+-)}(p) & G_{pn}^{(\lambda)}X_{vv}^{(+)}(p) & G_p^{(\lambda)}X_{vv}^{(+)}(p)-1
\end{vmatrix}=0. \tag{52}$$

We write down the expressions for the functions $X(n,p)$:

$$X_{uv}^{(+)}(n,p)=\frac{1}{\hat{\lambda}^2}\sum_{jj'}^{n,p}\frac{(f_{jj'}^{(\lambda)})^2u_{jj'}^{(+)}v_{jj'}^{(+)}}{\varepsilon_{jj'}^2-\omega_{\lambda i}^2}\omega_{\lambda i};$$

$$X_{uv}^{(-)}(n,p)=\frac{1}{\hat{\lambda}^2}\sum_{jj'}^{n,p}\frac{(f_{jj'}^{(\lambda)})^2u_{jj'}^{(+)}v_{jj'}^{(-)}\varepsilon_{jj'}}{\varepsilon_{jj'}^2-\omega_{\lambda i}^2};$$

$$X_{vv}^{(\pm)}(n,p)=\frac{1}{\hat{\lambda}^2}\sum_{jj'}\frac{(f_{jj'}^{(\lambda)})^2v_{jj'}^{(\pm)2}\varepsilon_{jj'}}{\varepsilon_{jj'}^2-\omega_{\lambda i}^2};$$

$$X_{vv}^{(+-)}(n,p)=\frac{1}{\hat{\lambda}^2}\sum_{jj'}\frac{(f_{jj'}^{(\lambda)})^2v_{jj'}^{(+)}v_{jj'}^{(-)}}{\varepsilon_{jj'}^2-\omega_{\lambda i}^2}\omega_{\lambda i};$$

$X_M(n,p)$ is defined in (23). The expressions for the neutron amplitudes $\psi_{j_nj'_n}^{\lambda i}$, $\varphi_{j_nj'_n}^{\lambda i}$ are

$$\left.\begin{aligned}
&\psi_{j_nj'_n}^{\lambda i}=\frac{\mathcal{M}f_{j_nj_n}^{(\lambda)}}{\varepsilon_{j_nj'_n}-\omega_{\lambda i}}\{u_{j_nj'_n}^{(+)}[(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})A_{11}\\
&+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})A_{12}]+v_{j_nj'_n}^{(-)}[G_n^{(\lambda)}A_{13}+G_{np}^{(\lambda)}A_{14}]+v_{j_nj'_n}^{(+)}[G_n^{(\lambda)}A_{15}+G_{np}^{(\lambda)}A_{16}]\};\\
&\varphi_{j_nj'_n}^{\lambda i}=\frac{\mathcal{M}f_{j_nj'_n}^{(\lambda)}}{\varepsilon_{j_nj'_n}+\omega_{\lambda i}}\{u_{j_nj'_n}^{(+)}(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})A_{11}+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})A_{12}]\\
&+v_{j_nj'_n}^{(-)}[G_n^{(\lambda)}A_{13}+G_{np}^{(\lambda)}A_{14}]-v_{j_nj'_n}^{(+)}[G_n^{(\lambda)}A_{15}+G_{np}^{(\lambda)}A_{16}]\},
\end{aligned}\right\} \tag{53}$$

where A_{ik} are the cofactors of the determinant (52).

The example of Eq. (52) shows how the problem of finding the structure and energies of the single-phonon states becomes more complicated when more and more components of the residual interaction are taken into account. If, for example, we now also take into account the spin–multipole component of the residual forces, the secular equation is obtained by equating to zero a determinant of eighth order.

In our view, the part played by multipole pairing has not, despite many studies, been investigated with exhaustive depth, and quadrupole pairing has acquired a secure position in the formalism of so-called nuclear field theory.[8] An absolute majority of the papers has been devoted to studying the influence of multipole pairing on the lowest nuclear excitations. Forces with $G_n^{(\lambda)}=G_p^{(\lambda)}=G_{np}^{(\lambda)}$ have generally been used in concrete calculations. In this case, Eq. (52) simplifies greatly, reducing to the vanishing of a determinant of fourth order. Calculations made under such an assumption showed that the interaction in the particle–particle channel reduces the probability of transition from the ground state to the lowest vibrational levels of spherical nuclei.[36,38] If, for example, the quadrupole interaction in the particle–particle channel is strengthened and simultaneously the constant $\varkappa_0^{(2)}$ is changed to make the energy of the 2_1^+ level equal at all times to the experimental value, we obtain the variation of $B(E2,0_{g.s.}^+\to 2_1^+)$ shown in Fig. 20. If we recall that the $B(E2,0_{g.s.}^+\to 2_1^+)$ values are frequently found to be too large when forces in only the particle–hole channel are used (see Sec. 2 and Fig. 3), we see that there is hope of simultaneously describing the energy and excitation probability of 2_1^+ levels by means of quadrupole pairing. But it must be borne in mind that the probability of transition from the ground to the 2_1^+ state is reduced appreciably when anharmonic corrections are taken into account. In addition, a strong reduction in the $B(E2)$ values does not occur for any choice of the constants $G^{(2)}$. The main role in reducing $B(E2)$ is played by the neutron–proton quadrupole interaction in the particle–particle channel,[40] and when the constants $G_{n,p}^{(2)}$ are chosen using the condition of gauge invariance ($G_{np}^{(2)}=0$) the values of $B(E2)$ change little.

The influence of multipole pairing on higher-lying levels has hardly been studied. The calculations of Ref. 57 for the $^{144-150}$Sm isotopes, made under the assumption $G_n^{(2)}=G_p^{(2)}=G_{np}^{(2)}$, showed that there is a slight "deconcentration" of the strength of the $E2$ transitions in the resonance regions. It is most appreciable in half-magic nuclei. It is to be expected that the interaction in the particle–particle channel will be important for the correct description of the excitation probability of the so-called *deep two-hole states* in two-nucleon transfer reactions.[58] We hope that study of this question will make possible more definite conclusions about the interaction itself.

7. REGION OF APPLICABILITY OF THE RANDOM-PHASE APPROXIMATION IN SPHERICAL NUCLEI

The structure of our basis states is calculated in the random-phase approximation. It is therefore clear that the "quality" of our basis and, ultimately, the possibility of using it depend on the extent to which this approximation works well. The condition of applicability of the RPA is that there should be a small number of quasiparticles in the ground (no-phonon) state of the nucleus, i.e., one must calculate in different nuclei. $\langle\Psi_0|\sum_m\alpha_{jm}^+\alpha_{jm}|\Psi_0\rangle$ or, if it is normalized to unity,

$$n_j=\hat{j}^{-1}\langle\Psi_0\,|\,B(jj00)\,|\,\Psi_0\rangle. \tag{54}$$

An expression for n_j can be readily obtained from (54) either by expressing the function Ψ_0 in terms of the quasiparticle vacuum Ψ_{00} and the operators $A^+(jj'\lambda\mu)$, $A(jj'\lambda\mu)$,[59] or, conversely, by expressing the operator

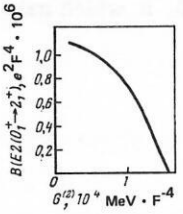

FIG. 20. Dependence of $B(E2,0_{g.s.}^+\to 2_1^+)$ on the constant $G^{(2)}$ in ^{126}Te. The constant $\varkappa_0^{(2)}$ is varied simultaneously with $G^{(2)}$ to ensure $\omega_{2_1^+}(\varkappa_0^{(2)},G^{(2)})=E(2_1^+)_{exp}$.

$B(jj'\lambda\mu)$ in terms of the phonon operators and averaging over Ψ_0. The result is

$$n_j = \frac{1}{\hat{j}^2} \sum_{\lambda} \hat{\lambda}^2 \sum_{j'i} (\varphi^{\lambda i}_{jj'})^2. \qquad (55)$$

From general considerations one can expect that the largest contribution to n_j will be made by low-lying vibrational excitations, since for them the inverse amplitudes $\varphi^{\lambda i}_{jj'}$ are greatest. Since $\varphi^{\lambda i}_{jj'} \sim (\varepsilon_{jj'} + \omega_{\lambda i})^{-1}$, it follows that with increasing i, i.e., with increasing excitation energy, the phonons will make a smaller and smaller contribution to n_j. The contribution of the spin–multipole phonons will be smaller than that of the multipole phonons, since the collective spin–multipole phonons have relatively high excitation energies. We note that phonons with energies $\omega_{\lambda i} \geq 10$ MeV have such small amplitudes $\varphi^{\lambda i}_{jj'}$ that they are essentially Tamm–Dancoff phonons.

These considerations are completely confirmed by calculations made in Ref. 59 for the series of $^{144-150}$Sm even isotopes. The constants $\varkappa_0^{(2)}$ and $\varkappa_0^{(3)}$ were determined in that paper from the transition probabilities $B(E2)$ and $B(E3)$ (the energies of the 2_1^+ and 3_1^- levels were too high), but the general conclusions are valid for a different choice of the constants. The 2_1^+ and 3_1^- states make a more than 50% contribution to n_j, which is well demonstrated in Fig. 21, which gives the values of n_j for the subshells $(2f_{7/2})_n$ and $(2d_{5/2})_p$. The contribution of the collective states from the resonance regions is negligible—it does not exceed tenths of a percent. It can be seen from Fig. 21 that the values of n_j increase in the sequence of Sm isotopes with increasing A. We can understand this, since, in moving from ^{144}Sm to ^{150}Sm, we go over from spherical nuclei to transitional nuclei, to nuclei in which the energies of the first vibrational states are small and the probabilities $B(E\lambda)\uparrow$ for them are large. In the structure of the single-phonon 2_1^+ and 3_1^- states of these nuclei the values of $\varphi^{\lambda i}_{jj'}$ reach 0.2–0.6. The values of n_j for shells near the Fermi surface (Table IV) in ^{150}Sm are fairly large, and in $^{148-150}$Sm the RPA no longer works. If $\varkappa_0^{(2)}$ and $\varkappa_0^{(3)}$ are determined using the energies of the 2_1^+ and 3_1^- levels, then the value of n_j increases.

As we saw in Sec. 6, multipole pairing forces with a sufficiently large constant $G^{(\lambda)}_{np}$ cause a significant decrease of the probabilities $B(E\lambda, 0^+_{g.s.} \to \lambda^\pi_i)$. As a result, by an appropriate choice of $G^{(\lambda)}$ and $\varkappa_0^{(\lambda)}$ one can achieve simultaneous agreement with the experimental values of the energies and the probabilities of $E\lambda$ excitation of the 2_1^+ and 3_1^- levels. It is natural to consider what happens in this case to the values of n_j. The answer to this question was given in Ref. 60, in which calculations were again made for the Sm isotopes as an example. The n_j values from states near the Fermi surface are given in Table V. Also given are the n_j calculated with $G^{(\lambda)} = 0$ and $\varkappa_0^{(\lambda)}$ chosen in two ways: by requiring the experimental and theoretical energies of the levels to be equal, and by requiring the experimental and theoretical probabilities $B(E\lambda)\uparrow$ to be equal.[3)] Comparison of all three values of n_j shows that allowance for pairing forces with $\lambda \neq 0$ leads to effectively the same consequences as a decrease in $\varkappa_0^{(2)}$. Of course, one can speak of a certain extension of the region of applicability of the RPA, but one must bear in mind all the reservations and doubts expressed with regard to the choice of the constants in Sec. 6.

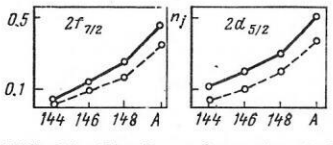

FIG. 21. Number of quasiparticles n_j in the shells $(2f_{7/2})_n$ and $(2d_{5/2})_p$ in the ground states of the Sm isotopes. The broken line is the contribution of only the 2_1^+ and 3_1^- phonons; the continuous line corresponds to the sum over all phonons.

TABLE IV. Number of quasiparticles n_j for subshells near the Fermi surface in the ground states of Sm isotopes.

	nlj	^{144}Sm	^{146}Sm	^{148}Sm	^{150}Sm		nlj	^{144}Sm	^{146}Sm	^{148}Sm	^{150}Sm
n	$2d_{3/2}$	0.05	0.06	0.06	0.05	p	$1g_{7/2}$	0.05	0.09	0.13	0.24
	$2f_{7/2}$	0.04	0.13	0.24	0.44		$2d_{5/2}$	0.11	0.20	0.29	0.49
	$1h_{9/2}$	0.02	0.03	0.04	0.06		$1h_{11/2}$	0.06	0.10	0.14	0.21
	$3p_{3/2}$	0.03	0.06	0.12	0.26		$3s_{1/2}$	0.09	0.10	0.14	0.27
	$1i_{13/2}$	0.02	0.04	0.05	0.08						

Thus, if we regard the results for the Sm isotopes as typical, the RPA in pure form is valid in a rather small number of spherical nuclei. Essentially, the n_j values in ^{148}Sm already reach 0.3, which can hardly be regarded as much smaller than unity, and this nucleus is separated from the half-magic ^{144}Sm by only four nucleons. A good qualtitative indicator of the applicability of the RPA is the energy of a 2_1^+ level. The example of the Sm isotopes shows that in nuclei with $E(2_1^+) \lesssim 0.5$ MeV the corrections to the RPA will be large, and the various boson (or phonon) expansions will converge badly. Other approaches must be sought.

CONCLUSIONS

In this review, we have not attempted to describe any definite experimental data. We have deliberately con-

TABLE V. Numbers of quasiparticles n_j for some subshells calculated with allowance for multipole pairing.

	^{144}Sm			^{146}Sm			^{148}Sm			^{150}Sm		
	$G^{(\lambda)}=0$			$G^{(\lambda)}=0$			$G^{(\lambda)}=0$			$G^{(\lambda)}=0$		
nlj	$\omega_{\lambda 1} = \omega_{exp}$	$B(E\lambda) = B(E\lambda)_{exp}$	$G^{(\lambda)} \neq 0$	$\omega_{\lambda 1} = \omega_{exp}$	$B(E\lambda) = B(E\lambda)_{exp}$	$G^{(\lambda)} \neq 0$	$\omega_{\lambda 1} = \omega_{exp}$	$B(E\lambda) = B(E\lambda)_{exp}$	$G^{(\lambda)} \neq 0$	$\omega_{\lambda 1} = \omega_{exp}$	$B(E\lambda) = B(E\lambda)_{exp}$	$G^{(\lambda)} \neq 0$
$n\ 2f_{7/2}$	0.04	0.016	0.027	0.25	0.09	0.10	0.47	0.17	0.18	0.70	0.35	0.4
$n\ 1h_{9/2}$	0.014	0.005	0.01	0.03	0.01	0.01	0.05	0.02	0.02	0.09	0.04	0.0
$n\ 3p_{1/2}$	0.02	0.006	0.013	0.09	0.03	0.04	0.19	0.07	0.09	0.39	0.21	0.2
$p\ 1g_{7/2}$	0.04	0.01	0.008	0.12	0.05	0.04	0.21	0.08	0.07	0.36	0.18	0.2
$p\ 2d_{5/2}$	0.12	0.03	0.02	0.33	0.12	0.11	0.51	0.18	0.19	0.78	0.37	0.4
$p\ 1h_{11/2}$	0.06	0.02	0.01	0.15	0.05	0.06	0.23	0.18	0.10	0.33	0.15	0.2

[3)] Since the contribution to n_j of only 2_1^+ and 3_1^- states was taken into account in Ref. 60, it is difficult to make a comparison with the data of Ref. 59 (see Table IV).

centrated our attention on formal and methodological subjects in order to elucidate what is most important—the nature of the phonon basis of the quasiparticle–phonon model, the structure and properties of the phonons in it, and the manner in which they are calculated. We shall consider later how the properties of the single-phonon states that we have considered are manifested in nuclear spectra.

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