Collective acceleration of ions in straight relativistic electron beams

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A review is given of experimental and theoretical investigations on collective acceleration of ions in straight relativistic electron beams, an area which was first studied by V. I. Veksler, G. I. Budker and B. Ya. Faĭnberg. The acceleration methods considered are compared in relation to their promise of resulting in viable accelerators in the range up to several hundred MeV per nucleon.

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INTRODUCTION

For more than twenty years the creation of accelerators with final particle energy up to hundreds of MeV or a few GeV per unit charge and accelerated-particle current at the level of tens of amperes or a few kilo-amperes has been associated with collective confinement and acceleration in the intrinsic fields of intense charged bunches, uncompensated beams, and plasma. The attractiveness of the idea of using the intrinsic collective fields is due to their substantially greater magnitude in comparison with the permissible values of the external microwave fields used in ordinary accelerator technology. Simple estimates show that with quite moderate densities of plasma or beams these fields can reach 10^6–10^7 V/cm, which is one or two orders of magnitude greater than the accelerating fields in resonators.

The early proposals of this type considered the possibility of acceleration of protons by electron bunches or by the focus of an electron flux while it was scanned,[1] which according to the estimates of the authors should provide an acceleration rate $E \approx 250$ keV/cm.

Collective methods of acceleration were placed on a firm basis as a new direction in accelerator physics for the first time in the works of Veksler,[2] Faĭnberg,[3] and Budker,[4] which were presented in 1956 at the International Symposium on High Energy Accelerators at Geneva.

Collective methods of acceleration and confinement of particles can be broken down into the following classes[2-5]:

1) acceleration as a whole of an intense electron bunch loaded with ions, by means of external fields, bunches, or beams;

2) acceleration of bunches of ions by slow waves in an electron beam or plasma.

One of the important aspects of acceleration of a bunch as a whole is its creation and confinement without destruction during an entire acceleration cycle. The solution of this problem involved the idea of a self-stabilized relativistic electron beam with partial neutralization, which was advanced by Budker.[4] In such a beam, if one assumes a small transverse energy of the electrons ($E_\perp/E_\parallel \ll 1$), a force-free equilibrium in the intrinsic electric and magnetic fields could be obtained by means of partial neutralization, $f_e = n_i/n_e = 1/\gamma_e^2$ (the Bennett–Budker condition[6]). For example, the acceleration as a whole of an electron–ion bunch in which $m_i N_i \lesssim m_e N_e$, where $m_{i,e}$ and $N_{i,e}$ are the masses and numbers of ions and electrons, respectively, will occur on fulfillment of the requirement[7]

$$E_{\max} > m_i E_1 / z_i m_e \gamma_e, \qquad (1)$$

where $E_{\max}$ is the maximum field strength in the bunch; E_1 is the strength of the external electric field which accelerates the bunch.

The essence of the collective-acceleration methods proposed by Veksler lay in creation of rotating ring-shaped electron bunches with a large value of the azimuthal relativistic factor γ_φ, loading them with ions, and acceleration of them in a direction perpendicular to the plane of the rings by means of external magnetic and electric fields or the fields of other bunches. The maximum field strength in such an electron ring is determined from the following relation[8]:

$$E \approx eN_e/\pi r_0 a, \qquad (2)$$

where N_e is the total number of electrons in the ring; r_0 and a are the mean radius and characteristic dimension of the ring cross section in centimeters.

The electron-ring acceleration method received extensive attention and development, first theoretically at the P. N. Lebedev Physics Institute of the USSR Academy of Sciences, the Joint Institute for Nuclear Research, the Radio-Technical Institute of the Academy of Sciences, the Moscow Engineering-Physics Institute, and the Institute of Theoretical and Experimental Physics,[2-5] and then experimentally in the Soviet Union at the Joint Institute for Nuclear Research[7,8] and abroad in the USA[9] and West Germany.[10] At the present time as the result of the intensive work of these scientific groups a complete and rigorous theory has been produced for the generation and acceleration of electron rings with ions, and acceleration of ions with mass number up to $A = 137$ has been accomplished. Taking into account that the electron-ring method of acceleration of ions has been described with exhaustive completeness by Sarantsev and Perel'shteĭn,[8] we direct the reader to this book.

Methods of collective acceleration of ion bunches in uncompensated beams or plasma in slow waves (space-charge, cyclotron, or waveguide-mode waves), which were first proposed by Faĭnberg,[3] differ from ion ac-

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celeration in individual electron bunches as discussed above in several fundamental respects, namely:

a) electron density waves in uncompensated beams, in the course of their propagation, produce oscillatory motions of the beam electrons, as a result of which the average repulsive force acting on the electrons of the beam is zero, which improves the transverse stability conditions;

b) the relativistic γ factor of the beam electrons remains at the moderate relativistic level in comparison with γ_φ of the electrons in an intense ring, which assures higher values of effective accelerating fields;

c) the limiting values of the accelerating fields and the variable density of charges of the beam in a slow wave are limited only by the requirement that there be no capture of beam electrons by the wave.

These methods of collective acceleration are based on the possibility of propagation of slow waves with phase velocity less than the velocity of light in a bounded plasma or in uncompensated beams[11,12] and on the existence, in addition to excitation from an external microwave source, of efficient mechanisms for excitation of such waves in beam–plasma, beam–beam, or beam–drift-tube interactions.[12-21] The references cited are only a small part of the work in this area.

A particular case of processes of the type considered is the collective acceleration of an ion bunch in the front of a relativistic electron beam (REB), which represents an isolated slow space-charge wave.[22] Recently the rapid development of high-current electron technology has stimulated the study of collective acceleration of ions in REB, and this work has been described in several reviews.[28,46,49]

In the present article we make a further attempt to review these studies. However, the limited size of the article and the definite range of the interests of the authors permit us to discuss only three basic types of such acceleration:

1) acceleration of ions in the front of an REB drifting in a neutral gas;

2) acceleration of ions in the front of an REB drifting in vacuum;

3) acceleration of ions in slow waves (space-charge or cyclotron waves) in an REB drifting in vacuum.

In regard to the acceleration of ions in an REB drifting in plasma—a subject which has been developed intensively for a number of years at the Physico-Technical Institute of the Ukrainian Academy of Sciences, we direct the readers to the corresponding publications (see, for example, Refs. 7, 14, 18, and 19).

1. MODEL OF COLLECTIVE ACCELERATION OF IONS IN RELATIVISTIC ELECTRON BEAMS DRIFTING IN A NEUTRAL GAS

The phenomenon of collective acceleration of ions in a relativistic electron beam drifting in a neutral gas,[22] first discovered more than ten years ago by Graybill *et al.*, has become the subject of extensive study. At the present time a large amount of experimental data have been accumulated which reflect numerous characteristics and features of this process.[22-39] At the same time a unified theory of the phenomenon, the physical essence of which is sufficiently clear in its general features, does not yet exist, although there are a number of models and numerical calculations which agree with varying degrees of completeness and accuracy with the experimental results.[40-52] This situation is due to the substantial nonstationarity and nonlinearity, and to the multiparametric dependence of the processes on the external conditions (pressure, type of gas, drift geometry) and on the characteristics of the REB (geometry, voltage, current, length of front, and so forth). The greatest acceptance has been given to frontal-ionization models which relate the nature of the accelerating fields to the space-charge wave at the front of the REB, i.e., models of a quasielectrostatic type. These are all various modifications of the Rostoker one-dimensional model.[41]

The essence of this model reduces to the following (Fig. 1). On injection of a one-dimensional monoenergetic electron beam into a drift half-space through a grounded anode plane, at its front there is a slowing down of the electrons with a corresponding rise in their density up to diverging values and formation of a virtual cathode, i.e., a potential step of height $(\gamma_e - 1)m_e c^2/e = \mathscr{E}_e/e$ at a distance c/ω_e from the anode (ω_e is the Langmuir frequency: $\omega_e^2 = 4\pi n_e e^2/m_e\gamma_e$). In a stationary discussion the charge density beyond the virtual cathode is equal to zero, i.e., the entire beam is reflected back to the anode. The presence of gas in the drift space leads to its ionization by the electron beam, the rate of occurrence of which can be determined from the formula[53]

$$dn_i/dt \approx 10^{-1} z_M n_e p, \tag{3}$$

where z_M is the average number of electrons per molecule, n_i is the density of ions, n_e is the density of electrons, and p is measured in millimeters Hg. The ions produced are accelerated on the slope of the potential step toward the virtual cathode, and on reaching it acquire an energy $\mathscr{E}_i = z_i \mathscr{E}_e$, where z_i is the charge of the ion.

In this case, as the space charge in the region near the anode is neutralized, the virtual cathode is dis-

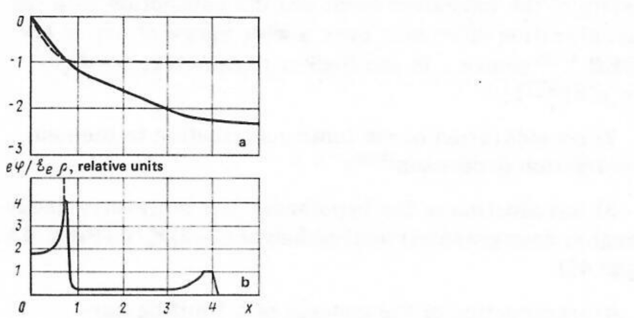

FIG. 1. Distribution of the potential $e\varphi/\mathscr{E}_e$ (a) and space-charge density ρ (b) in the Rostoker model (dashed curves[41]) and in the Rostoker–Poukey model (solid curves).[43]

placed with an average velocity $v \approx L/\tau_n$, where $L \approx c/\omega_e$; τ_n (the average time of neutralization) is determined from Eq. (3). As the result of the presence of pre-ionization in the drift region lying in front of the virtual cathode (as the result of photoionization, microwave radiation, and retreating electrons at the front of the REB), the rate of displacement of the virtual cathode (VC) will grow with time, so that when the cynchronization condition $\ddot{x}_i \gtrsim \ddot{x}_{VC}$ is satisfied, capture of ions into an acceleration regime will be observed up to energies significantly greater than the depth of the potential step $\mathscr{E}_e/e$. Here $\ddot{x}_i = (1/cm_i)z_i\mathscr{E}_e\omega_e$ is the acceleration of the ions; $\ddot{x}_{VC}$ is the acceleration of the motion of the virtual cathode due to previously existing ionization. Estimation of the maximum energy of the ions in the framework of the model leads to the expression $\mathscr{E}_{i,\max} \approx (n_e/n_e^*)z_i\mathscr{E}_e$, where n_e^* is the density of electrons in the region beyond the virtual cathode, which enters into the model as a free parameter, choice of which can provide agreement with experiment. In particular, for $n_e^*/n_e = 0.2$ we obtain values which agree quite well with the observed values, namely: the energy of the accelerated ions turns out to be 1–5 times greater than $z_i\mathscr{E}_e$, the acceleration length corresponds to a few centimeters or a few tens of centimeters, and the duration of the pulses of ions is from fractions of a nanosecond up to a few nanoseconds. The Rostoker model is extremely simple and does not take into account many factors, in particular, the following:

1) change of steepness of the virtual cathode in the course of its displacement;

2) influence of the ions on the neutralization processes;

3) the real geometry of the relativistic electron beam and the drift space;

4) the real departure from a monoenergetic beam, i.e., the existence of a voltage and current front of finite length in the REB.

Nevertheless the simplicity of the model and the correctly captured essence of the acceleration mechanism have permitted it to become the basis for all subsequent modified approaches to this problem. Among such modifications are the following:

1) inclusion of the distribution of electrons and ions in density and energy in the front of the REB; time variation of the electric fields and their dependence on the width of the ionization front and the connection with the acceleration efficiency over a wide range of ν/γ of the REB,[46-48] where ν is the Budker parameter, $\nu = J\gamma_e e/m_e c^3\sqrt{\gamma_e^2 - 1}$;

2) consideration of the ionic contribution to the neutralization processes[45-49];

3) introduction of the hypothesis of a stationary anode-region deep potential well of height $(2-3)\mathscr{E}_e/e$ (Refs. 43 and 49);

4) introduction of the concept of a limiting current[42,49];

5) inclusion of the expansion of the REB in the near-anode region, which leads to an initial increase of the limiting current of the injected beam[34,51];

6) introduction of the hypothesis of a nonstationary deep well with a small lifetime[34,50,52];

7) an energy approach to the motion of the front of an REB loaded with ions, for high ν/γ of the beam and high gas pressures.[36]

The first modification is actually a theoretical justification of the Rostoker model. V. I. Kucherov, using a thermalized distribution of oscillating electrons in the region between the anode and the virtual cathode,

$$n_e(x) = 2n_e(0)\exp[U(x)/\gamma_e^*], \quad (4)$$

where $U(x) = e\varphi(x)/m_e c^2$, obtained in the quasistatic approximation expressions for the electric field strength (the profile of the ionization front step) in the form

$$E(x) = 4[\pi n_e m_e c^2 \gamma_e^*]^{1/2}/\{1 + 2x[\pi n_e e^2/m_e c^2\gamma_e^*]^{1/2}\} \quad (5)$$

and for the density distribution of the ions and their velocity:

$$\left.\begin{aligned} n_i(x, t) &= 2n_e \exp[-\sqrt{m_i/m_e\gamma_e^*}(x/ct)]; \\ v_i(x, t) &= x/t + \sqrt{(m_e/m_i)\gamma_e^*}c. \end{aligned}\right\} \quad (6)$$

Here $\gamma_e^* m_e c^2$ is the effective temperature of the REB.

Kurilko and Kucharov[47,48] showed that the motion of the ionization front has a scaling nature, stationarity of which cannot be provided by electron impact ionization alone. Their analysis enabled them to obtain a time picture of the spreading of the space-charge wave in the front of the REB with its advance and with increase of the neutralization, the intensity of which increases with decrease of the ν/γ of the REB. Therefore to increase the efficiency of acceleration Kucharov proposed external control of the ionization wave.[48]

A second modification introduced by Alexander *et al.*[45] involves consideration of the contribution of the ions to the neutralization processes, which becomes important under the condition of cutoff of charge-exchange processes. The latter is written in the form $E/p > A \cdot 10^6$,[54] where $A \geq 1$; p is in millimeters Hg and E is in V/cm. In this case, in a mean free path the ions can be accelerated to energies $\mathscr{E}_i > (50-100)$ keV, at which the cross sections for ionic impact ionization are significantly greater than the cross sections for their charge exchange leading to loss of ions from the acceleration process. Thus, the fact that ions are accelerated to an energy $\mathscr{E}_i > (50-100)$ keV unambiguously indicates the occurrence of avalanche ionization of the gas under the influence of the ions, which is an order of magnitude more efficient than electron ionization at the same average energies. For example, for H_2 we have $\tau_i = 0.33/p$ and $\tau_e = 5/p$, where $\tau_{i,e}$ are the mean times for ionization by ions and electrons in nanoseconds if p is in millimeters Hg.

Alexander and Hintze analyzed the one-dimensional self-consistent stationary motion of the front of an REB loaded with ions, using the kinetic approach with inclusion of electron and ion impact ionization and charge-exchange processes.[46] The analysis showed that there are two regions of pressure in which the behavior of such characteristics as the velocity of the REB front, the number of accelerated ions, and so forth, differ qualitatively. For example, with increase of pressure above the limits of the first region the velocity of the front experiences a discontinuity. Correspondingly the number of ions captured by the front in this region drops to 2%.

The essence of the idea of a potential well and limiting current is as follows. A time analysis of the injection of a one-dimensional monoenergetic beam into a plane drift half-space carried out by Poukey and Rostoker[43] showed that in the process of its establishment the potential step formed in this case has a depth significantly greater (by 2–3 times) than $\mathscr{E}_e/e$. This result, which is so unexpected at first glance, is due to the geometry of the problem considered and to the monoenergetic nature of the beam. In this case the maximum height of the potential step is due to the head part of the REB—escaping electrons which in the one-dimensional case move in a force-free drift, traveling arbitrarily far from the injection plane. However, the main part of the beam electrons are slowed down to a complete stop, forming a virtual cathode of depth $\mathscr{E}_e/e$, located at a distance c/ω_e from the anode. Similar results can be obtained in consideration of a monoenergetic beam with a finite duration of the current rise τ_{crt}, and also in a relativistic approach. Here the depth of the step reaches values $\varphi_w \approx 2.25\mathscr{E}_e/e$. The results of the one-dimensional analysis were taken by Olson as the postulated basis of his two-dimensional model of ion acceleration.[49] In contrast to the results of Poukey and Rostoker,[43] Olson postulated the appearance of a deep potential well of this type only when the condition $J_{inj} > J_{lim}$ is satisfied, where

$$J_{lim} = \frac{m_e c^3}{e} \frac{(\gamma_e^{2/3} - 1)^{3/2}}{1 + 2\ln(R_{tu}/r_b)} \frac{1}{(1 - f_e)} \quad (7)$$

is the Rukhadze limiting vacuum current for a cylindrical beam of radius r_b in a cyclindrical tube of radius R_{tu}; f_e is the degree of charge neutralization. In the framework of this model, ion acceleration processes occur as follows.

With increase of the injection current there is a deepening of the potential well in the cavity of the anode with expansion of the beam and simultaneous occurrence of neutralization processes. The rate of the neutralization processes will depend on the gas pressure and also on the strength of the electric fields and can be due to both electron and ionic components.

On reaching the current value J_{lim} the beam stops in the near-anode region and expands to the walls, and a deep potential well $\varphi_w \approx (2-3)\mathscr{E}_e/e$ is formed, which has a stationary nature. The time of formation of this potential well amounts to a fraction of a nanosecond. The time of existence of the deep stationary potential well is determined by the time of neutralization of the region near the anode, with allowance for both electronic and ionic contributions. The solution of the equation of the charge state,

$$dn_i/dt \approx n_e/\tau_e + n_i/\tau_i, \quad (8)$$

leads to expressions for $f_e(t)$ in the following form:

$$f_e(t) = (\tau_i/\tau_e)\,[\tau_i \exp(t/\tau_i) - t - \tau_i]/t. \quad (9)$$

Simultaneously with processes of neutralization of the near-anode region there is acceleration of ions in the two-dimensional potential well to an energy $0 < \mathscr{E}_i < (2-3)\mathscr{E}_e z_i$. On reaching $f_e \lesssim 1$ there is a transition to the state of a moving potential well with smaller depth $\varphi_w \leqslant \mathscr{E}_e/e$. Part of the ions accelerated in the axial direction turn out to be captured into the acceleration mode. The condition of capture of an ion by the potential well is written in the form

$$\mathscr{E}_{i0}/(m_i v_w^2/2) \geqslant [1 - z_i e\varphi_w/(m_i v_w^2/2)]^{1/2}, \quad (10)$$

where v_w is the velocity of the motion of the well; $\mathscr{E}_{i0}$ is the initial energy of the ions.

The model considered, in spite of its postulative nature, has been developed very completely and permits comparison with the experimental results for many parameters. This applies first of all to verification of the very ideas of a limiting current and a deep potential well, to the dependences of the acceleration efficiency on the injection current, gas pressure, drift-space geometry, beam voltage and current-front duration, the external magnetic field, and its geometry.

Allowance for the geometry of an expanding electron beam injected into a drift space, in the approach of Bystritskiĭ *et al.*,[51] leads to a substantial change of the limiting current in the near-anode region and of the nature and sequence of the processes which occur here. As estimates and numerical calculations show, the limiting current in such a geometry in the first approximation turns out to be greater than follows from Eq. (7) by about $[1-2\ln(R_{tu}/r_b)]$ times.[34] As a result of this, the formation of the virtual cathode sets in on reaching a current of higher value than given by Eq. (7). The beginning of acceleration is associated only with satisfaction of the single condition of cutoff of the charge-exchange processes, $E/p > 10^6$ V/(cm · mm Hg), and not with a necessary exceeding of the limiting current.

Analysis of the acceleration of ions, for example, for hydrogen, at a low pressure ($p \lesssim 1/\tau_{vrt}$) for two ranges of injection current ($J_{inj} \approx J_{lim}$ and $J_{inj} \gg J_{lim}$) leads to the following approximate expressions for the minimum value of the ratio (J_{inj}/J_{lim}) at which acceleration of ions begins:

$$J_{inj}/J_{lim} \geqslant 10Lk\tau_{crt}/[\varphi_A(2\tau_e - \tau_{vrt} + \Delta t)(\tau_{vrt} - \Delta t)]; \quad (11)$$

$$J_{inj}/J_{lim} \geqslant 4 \cdot 10^2 r_b^2 \beta_e [\tau_{crt}/(\tau_{vrt} - \Delta t)]/[\varphi_A^2 J_{lim}(2\tau_e - \tau_{vrt} + \Delta t)^2], \quad (12)$$

where τ_{crt} and τ_{vrt} are the current and voltage rise times; Δt is the time interval of delay of the injection current relative to the voltage, in nanoseconds.

These expressions, which are functions of the complete set of parameters of the REB and the drift space, were obtained on the basis of the following assumptions. For $J_{inj} \approx J_{lim}$ the maximum sag of the potential in the region of the virtual cathode φ_{VC} is determined by the relation $\varphi_{VC} \approx (J_{inj}/kJ_{lim})\varphi_A(1 - f_e)$, where k is the coefficient of increase of the limiting current as the result of expansion of the beam; φ_A is the REB injection voltage, the extent of the slope of the virtual cathode is $L = \xi R_{tu}$, and $\xi \leqslant 1$. The beginning of acceleration of ions in the limiting case occurs at the moment of time t

$\approx \tau_{vrt}$. For $J_{inj} \gg J_{lim}$ the maximum depth of the sag in the near-anode region is $\varphi_A(1-f_e)$, and the extent of the slope of the virtual cathode is taken approximately equal to c/ω_e. In the approach, we have considered two pressure regimes which differ in the length of the beam voltage and current fronts and the time of occurrence of the neutralization process.

The complete sequence of processes leading to acceleration of ions is as follows: in the initial stage of injection of the beam, its dissipation occurs in a length equal approximately to R_{tu}, and slow neutralization by electrons $f_e \approx t/2\tau_e$ occurs over the entire region of dissipation of the REB, since charge-exchange processes are dominant. The ions do not take part in ionization of the gas. On reaching the necessary steepness in the near-anode slope of the potential well ($E/p > 10^6$), ionic avalanche ionization and ion acceleration begin, while on the flat front slope of the potential this condition is not satisfied and acceleration of ions toward the anode does not occur. As a result of the rapid neutralization of the rear slope of the potential well, it begins to depart from the anode in accompaniment of the accelerated ions with a velocity determined from the relation $v_w \approx L_c/\tau_n$, where L_c is the length of the rear slope. In its motion the rear slope enters the region with an increasing degree of initial neutralization, which leads to a decrease of the total time of neutralization of the front of the REB, and to a corresponding decrease of the steepness in depth of the potential well to zero. A feature of this approach is rejection of the postulative nature of the Olson model and prediction of the beginning of ion acceleration over a very wide range of J_{inj}/J_{lim}, including both the regions $J_{inj} > J_{lim}$ and $J_{inj} < J_{lim}$. The model does not make reference to the idea of a deep potential well, and in the framework of the model the acceleration of ions to maximum values $\mathscr{E}_i \gg z_i \mathscr{E}_e$ occurs in the stage of the moving potential well as in the approaches considered in Refs. 41, 42, 46, and 47. The model predicts a decrease of the minimum value of J_{inj}/J_{lim} with increase of the electron energy of the REB and a shift of the acceleration cutoff pressure and the optimal pressure to higher values with increase of J_{inj}/J_{lim}.

In use of the hypothesis of a deep near-anode potential well of a nonstationary nature with a small lifetime $t \lesssim (c/\omega_e)(2z_i\mathscr{E}_e/m_i)^{-1/2}$ in the studies by Bystritskiĭ *et al.*[50] and Kolomenskiĭ *et al.*[52] its formation is associated with satisfaction of the condition $J_{inj} > J_{lim}(t_1)$, where t_1 is the moment of reaching the degree of equalization f_e corresponding to the beginning of a pinch of the REB. For real, "hot" beams with a distribution in the angle θ after traversal of the anode foil the necessary value of f_e at which the transition from expansion of the beam to contraction begins is determined from the relation

$$f_e \geqslant 1-\beta_e^2\cos^2\overline{\theta} \text{ (i.e., } f_e > 1/\gamma_e^2). \tag{13}$$

In the model discussed in Ref. 50, on reaching this value of f_e the beam stops spreading and begins to contract toward the axis. The contraction is completed by formation of a virtual cathode and by reflection from it of part of the current to the anode: $\Delta J = J_{inj} - J_{lim}(1-f_e)^{-1}$, since the limiting current corresponding to a beam with constant radius is smaller. The resulting decrease of the total current in the region between the cathode and virtual cathode leads to generation of a rotational emf and to a corresponding acceleration of electrons to an energy greater than $e\varphi_A$:

$$\mathscr{E} \approx e\varphi_A + (\Delta J/c)\,[1+2\ln(R_{tu}/r_b)], \tag{14}$$

which deepens the sag of the potential at the virtual cathode. The model considered is to be preferred for REB with large γ_e and for a low gas pressure where $1/\gamma_e^2 \ll 1$, and the times for achievement of $(f_e > 1/\gamma_e^2)$, turn out to be greater than the time to reach the limiting current.

The formation of a nonstationary deep well in Ref. 52 is also due to contraction of the beam in the region of strong overcompensation ($f_e > 1/\gamma_e^2$), which leads to generation of axial electric fields which produce a rapid deepening and steepening of the potential well which is formed at the anode. Here in calculation of the longitudinal electric fields one uses the expression

$$E(x) = \frac{J}{J_A}\left(-\frac{J}{cr_b}\frac{\partial r_b}{\partial t} + \frac{J}{2n}\frac{\partial n}{\partial x} + \frac{k}{e}\frac{\partial J}{\partial t}\right). \tag{15}$$

This expression is similar to that obtained by Putnam in the model of a localized self-accelerating pinch,[44] which is distinct from electrostatic acceleration models. The occurrence of a pinch here is due to a local increase of the ion density, which leads to a rapid compression of the beam and to generation of a rotational emf, which accelerates the ions. The displacement of the ion trap in the field of the rotational emf is accompanied by further capture of ions into the pinch and by increase of the axial electric field to saturation at a level $E_x \approx 60J/r_b$. Cutoff of the acceleration sets in as the result of the rapid decrease of f_e in the region beyond the localized pinch, corresponding to breakup of the beam. In contrast to many electrostatic approaches, the model does not relate the motion of the ion bunch with the REB front, and in the framework of this model generation of several ion bunches is possible.

In concluding this brief review of models of ion acceleration in a neutral gas of the electrostatic type, we note the following:

1) all of these models involve the mechanism of acceleration in an isolated space-charge wave at the beam front;

2) in all of these models, consequently, a linear dependence must be observed between the charge state and the final energy of the ions;

3) each of these models predicts that the efficiency of the acceleration processes and the characteristics of the accelerated ions have definite relations to the various parameters of the beam and drift space, and some of them predict threshold values of injection current below which acceleration is not observed. As a result of the very large amount of experimental statistics, a comparative analysis of the theoretical predictions and the experimental and numerical results should preferably be carried out on the basis of groups of studies devoted to individual aspects of the total problem.

2. EXPERIMENTAL AND NUMERICAL INVESTIGATIONS OF THE COLLECTIVE ACCELERATION OF IONS IN RELATIVISTIC ELECTRON BEAMS IN A NEUTRAL GAS. COMPARATIVE ANALYSIS

The general arrangement of experiments on acceleration of ions in relativistic electron beams has not undergone significant changes in more than ten years of investigations. It reduces to injection of an REB into a drift tube supplied with appropriate detectors of the ions and the electromagnetic fields of the beam (Fig. 2). The measurable parameters are the characteristics of the ion bunches (energy, number, shape, length, and rate of acceleration) and of the expanding electron beam (energy, velocity of propagation, depth of potential sag at the virtual cathode, and so forth). On departure of the front of the drifting beam loaded with ions from the drift tube, a magnetic field is used to separate the electrons from the ion bunch, which continues its motion to the detectors (time-of-flight detectors, nuclear activation or range spectrometers, and so forth).

Investigation of the Nature of the Accelerating Fields. The energy of the axially accelerated ions, its dependence on their charge composition and on the length of acceleration, and also the energy of the electrons and ions accelerated in the radial direction in the near-anode region were investigated in Refs. 22–37. Good statistics exist for experiments on accelerators with various ν/γ values and voltages, which confirm the approximate constancy of $\mathscr{E}_i/z_i$ in the range $(1-3)\mathscr{E}_e$. In experiments carried out with a mixture of gases (helium, hydrogen), in spite of the preferential acceleration of protons with the maximum value of z_i/m_i, accelerated doubly charged helium ions with corresponding energies were also observed.[26] As a rule, the lengths of acceleration in which collection of energy by the ions occurs are of the order of the drift-tube diameter and depend on the ratio ν/γ.[28-31] This may be due to the characteristic length of the slope of the virtual cathode (in the stage of acceleration in a stationary potential well) or to the length of the combined motion of the potential well and the ions. It should be noted that the very measurement of the acceleration length can introduce distortions into the accelerating fields (placement of a probe in the immediate vicinity of the anode). Location of a large probe with a grounded enclosure at a distance from the anode $x \lesssim R_{tu}$ leads to a significant decrease ($\gtrsim 20\%$) of the depth of the potential well which is formed and consequently to a decrease of the energy of the accelerated ions. Results of such measurements, which confirm the electrostatic nature of the near-anode fields, nevertheless can provide quantitative information on the acceleration length. The dependence of the yield of accelerated ions on the length of the drift tube is shown in Fig. 3.[29]

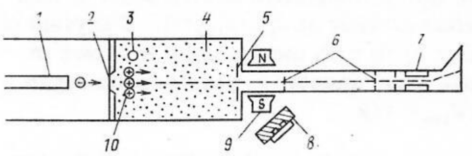

FIG. 2. Diagram of experimental apparatus for study of collective acceleration of ions in a relativistic electron beam[30]: 1—cathode, 2—foil anode, 3—Rogowski belt for measurement of the pure current of the REB, 4—working gas, 5—activated target, 6—time-of-flight grids, 7—mass spectrometer, 8—neutron detector, 9—deflecting magnet, 10—ion bunch.

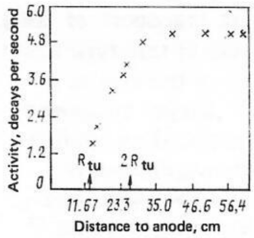

FIG. 3. Yield of accelerated ions from the reaction $^{12}C(d,n)^{13}N$ as a function of the drift-tube length.[29]

In almost all studies the ions are detected immediately after the arrival of the head electrons of the front, i.e., in complete correspondence with all frontal-ionization models. Here it must be pointed out that the large extent of the front, which is of the order of the drift-tube radius R_{tu}, may produce a separation of the ion bunch from the head electrons, which with increase of the drift-tube radius will become quite significant.

Experiments have confirmed the existence of an initial phase of "anticipation" of the beam near the anode (before the beginning of the motion of the potential well) which is due to the rate of occurrence of charge-neutralization processes, the duration of which is determined by the gas pressure. In Fig. 4 we have shown results obtained in time-of-flight measurements of the moment of arrival of the electron front and the ion bunches, which confirm the frontal nature of the ion motion.[25,61]

In regard to some experiments of the Physics International group and the group at the Tomsk Nuclear Physics Institute, in which second ion bunches were observed following the first bunches at the front of the REB with a time interval of a few nanoseconds to tens of nanoseconds,[25,27] the occurrence of a second ion bunch may be due to the nonstationary nature of the processes within the diode and to the rotational fields generated in contraction of the REB in the near-anode region.[49]

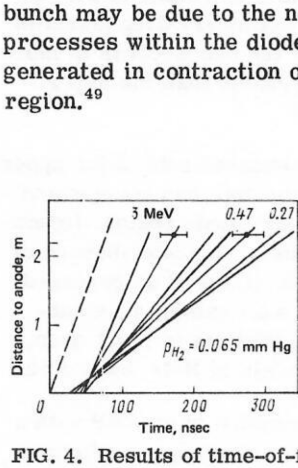

FIG. 4. Results of time-of-flight measurements of accelerated ions. The origin coincides with the moment of injection of the relativistic electron beam (on the basis of the signal from the Rogowski belt)[61]: dashed line—front velocity $\beta = 0.1c$, $J = 100$ kA, $\varphi_{max} = 2$ MV, $R_{tu} = 2.5$ cm, $r_e = 1$ cm.

Measurement of the depth of the potential well formed in the near-anode region in transport of REB. To obtain unique results, experiments of this type are undoubtedly decisive for verification of the idea of a deep potential well $\varphi_w \approx (2-3)\mathscr{E}_e/e$.[49] Direct measurement of the near-anode sag of the potential on application of an axial magnetic field which magnetizes the REB (r_b = const) was carried out in an NSU-600 accelerator with parameters $\varphi_A = 500$ kV and $J_{inj} \lesssim 10$ kA.[33] The values obtained for the depth of the potential well in the near-anode region did not exceed $(\gamma_e - 1)m_e c^2/e$, and far from the anode they did not exceed $(\gamma_e - \gamma_e^{1/3})m_e c^2/e$, which agrees with the formation of a virtual cathode and the passage of the limiting current through the drift tube. These results do not support the hypothesis of formation of a stationary deep well advanced by Olson,[49] since the latter should be observed also in the presence of a magnetic field. At the same time they are consistent with the picture of the appearance of a nonstationary deep potential well which is suppressed in the presence of a strong magnetic field.[50,52] In this connection particular significance is acquired by direct measurements of the depth of the near-anode potential well in an expanding REB without a guiding magnetic field.

Determination of the upper limit of the energy spectrum of electrons and accelerated ions ejected radially from the beam in the near-anode region permits an answer to be given to the question of the depth of the potential well. In a series of measurements carried out at the Nuclear Physics Institute at Tomsk[37] it was shown that the energy of the investigated electrons did not exceed $\mathscr{E}_e$, which also does not agree with the picture of a stationary deep potential well. Measurements of the fluxes of radially accelerated ions in the near-anode region $[x \lesssim (1/2)R_{tu}]$ showed that they are 3–4 orders of magnitude less intense than ions accelerated in the axial direction both as the result of dominance of charge-exchange processes in the periphery of the drift tube where $E/p < 10^6$ V/(cm · mm Hg) and as the result of outflow of ions from this region along the axis. In regard to the upper limit of the ion energy, we have $\mathscr{E}_i \lesssim z_i \mathscr{E}_e$. At the same time the expected number of ions with energy $\mathscr{E}_i > z_i \mathscr{E}_e$ in numerical calculations with assumption of the existence of a stationary deep potential well of duration of the order of ten nanoseconds is more than two orders of magnitude greater than the experimental detection threshold.[55,56]

Thus, the set of results of measurements of the upper limits of the spectra of electrons and ions accelerated in the radial direction in the near-anode region, together with the numerical calculations, indicate absence of a stationary stage of a deep potential well as proposed by Olson.[49] In such a potential well exists, it is substantially nonstationary and its lifetime is $\tau_w < 1$ nsec, which is consistent with the models of Refs. 50 and 52.

Numerical modeling of the injection of an REB with a current $J_{inj} > J_{lim}$ into drift tubes of various configurations showed that formation of a potential well with depth $\varphi_w \approx (2-3)\mathscr{E}_e/e$ in the near-anode region does not take place in the presence of a current front with duration comparable with the time of traversal by the beam of the characteristic lengths of the drift tube,[50,57] and also in the presence of a voltage front with duration even an order of magnitude smaller. Time dependences of the maximum depth of the potential well for several values of the current and voltage front durations, obtained for a one-dimensional drift-tube and beam geometry, are given in Fig. 5. The corresponding calculations were carried out in the quasistatic approximation where the only fields taken into account are the electrostatic fields which lead to formation of a deep potential well in the models of Refs. 43 and 49. In regard to the time of existence of the deep-potential-well stage (for its formation with a large delay of the current with respect to the voltage), it also does not exceed $L/\beta_f c$, where $L \approx R_{tu}$ for cylindrical geometry and $L = d$ for plane geometry, and amounts to small fractions of a nanosecond. This small lifetime of the deep potential well does not agree with the actual time of collection of the final energy of the ions in traversal of the slope of this potential well. A characteristic feature of the numerical solutions consists of time oscillations, maxima of the potential sag reaching $1.2\mathscr{E}_e/e$, and spatial oscillations of the virtual cathode. On injection of an REB into a drift tube such oscillations also exist, as is confirmed by the generation of microwaves in such systems.[58]

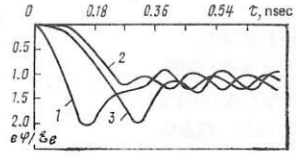

FIG. 5. Time pattern of potential formation in the front of an REB on injection of it into a planar drift space[57]: 1—$\tau_{vrt} = \tau_{crt} = 0$, 2—$\tau_{crt} = \tau_{vrt} = 0.3$ nsec; 3—$\tau_{crt} = 0.3$ nsec, $\tau_{vrt} = 0$; $d = 10$ cm, $\varphi_A = 1$ MV, $j_e = 1$ kA/cm^2, $\Delta t = 10^{-3}$ nsec.

Verification of the idea of limiting current. A series of experiments investigating the dependence of the acceleration efficiency on J_{inj}/J_{lim} was carried out in Refs. 29–31. They were supplemented by Refs. 34, 38, and 51. We note that in Refs. 29–31 the accelerators used had short rise times and significantly higher working voltages than others, which provided correspondingly large values of E/p in the usually used range of pressures. The main results of these studies reduce to the following.

1. In work with accelerators with voltages significantly greater than $m_e c^2/e$ (by five or more times) and short pulse rise times, collective acceleration of ions becomes important already at $J_{inj}/J_{lim} \approx 1$. The yield of the reaction $^{12}C(d,n)^{13}N$ with increasing p is shown in Fig. 6.[30] In these experiments ion acceleration began already at $J_{inj}/J_{lim} \gtrsim 1/2$.

2. In work with accelerators with voltages at the level $(1-2)m_e c^2$ and a large pulse rise time, collective acceleration of ions began with values of J_{inj}/J_{lim} significantly greater than unity. For example, in Ref. 38 $J_{inj}/J_{lim} \gtrsim 4$, in Ref. 32 $J_{inj}/J_{lim} \gtrsim 1.4$, and in Ref. 51 $J_{inj}/J_{lim} = 2$ for $\gamma_e = 3$ and $J_{inj}/J_{lim} = 5$ for $\gamma_e = 1.6$. These re-

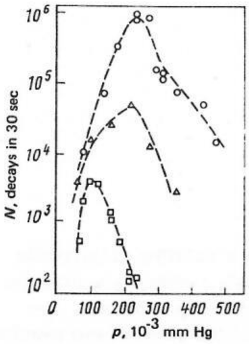

FIG. 6. Ion-acceleration efficiency on the basis of the activation yield in the reaction $^{12}C(d, n)^{13}N$ as a function of gas pressure for various values of J_{inj}/J_{lim} and R_{tu}/r_b from Ref. 30: ○—$R_{tu}/r_b=42$, $J/J_{lim}=2.2$; △—$R_{tu}/r_b=20$, $J/J_{lim}=1.6$; □—$R_{tu}/r_b=10$, $J/J_{lim}=1.3$.

sults agree better with the satisfaction of the requirement $E/p>10^6$ V/(cm·mm Hg) and not with the idea of a limiting current, exceeding which (as the single condition) is insufficient for beginning of ion acceleration.

Dependence of accelerated-ion energy on J_{inj}/J_{lim}. The set of results breaks up into two classes:

1. In experiments with a small value ($\nu/\gamma \lesssim 1$) of the ν/γ of the REB or at low pressures, a direct proportionality is observed between the REB current and the final ion energy.[22,30,34] For various REB parameters this dependence lies in the range $\mathscr{E}_i \sim J_{inj}^{(<2)}$. The corresponding experimental data are given in Fig. 7. These results, when considered in terms of a deep potential well of nonstationary nature in the near-anode region, indicate a dependence of the depth on the ratio J_{inj}/J_{lim}. The mechanism of acceleration in the stage of a moving potential well with depth $\varphi_w \lesssim \mathscr{E}_e/e$ is to be preferred; the efficiency of this mechanism depends on the length of joint motion of the rear slope of the potential well and the accelerated ions. With increase of ν/γ the extent of the rear slope of the potential well is smaller, and correspondingly the velocity of its departure from the anode, i.e., the motion in the initial stage, is less and the probability of capture of ions and synchronous motion of the ions with the potential well during its expansion and acceleration in a greater length is higher; see Eq. (10).

2. In experiments with a large value of ν/γ and a high gas pressure ($\tau_n \approx 2L/\beta_i c$) an inverse dependence

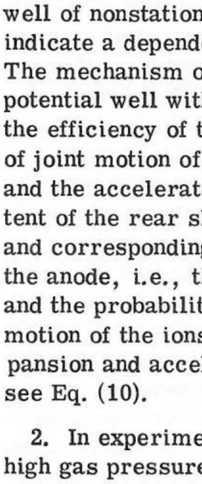

FIG. 7. Accelerated-ion energy as a function of injection current for $\nu/\gamma<1$ from Ref. 22 for $p=0.1$–0.3 mm Hg, $\varphi_A=1.7$ MV, $\tau=30$ nsec.

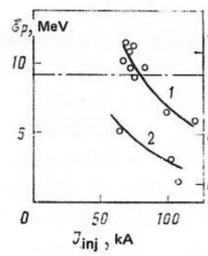

FIG. 8. Accelerated proton energy $\mathscr{E}_p$ and velocity β_p as a function of injection current for $\nu/\gamma>1$ at high pressure ($J_{inj}=J_{trans}$). Solid lines—calculated velocity of REB front in the Putnam model[36]; dot–dash line—calculated REB front velocity in the model of Ref. 49; the gas is hydrogen at $p=0.55$ mm Hg; 1—electron beam energy 0.89 MeV, 2—electron beam energy 0.48 MeV.

of the ion energy on the REB current is observed,[36] which is shown in Fig. 8. These results, in spite of the apparent inconsistency with Olson's model,[49] can be explained by considering the energy balance of the transported beam. At a high gas pressure the maximum rate of motion of the front and of the ion synchronous with it is limited already not by the rate of occurrence neutralization processes or by the length of joint motion of the ions, but by the power of the REB expended in generation of electromagnetic waves in the drift tube. In fact, as a result of the fact that the magnetic energy stored in it is proportional to $\mathscr{L}J^2/2$, while the energy injected into the drift tube is proportional to $J_{inj}\varphi_A$, further rise of the REB current beginning with definite values of J will lead to a decrease of its kinetic energy, so that there is a preservation of the energy-balance condition

$$J\varphi_A = (J^2/4c)\,\beta_f\,(1+4\ln R_{tu}/r_b) + \varphi_A J\beta_f/\beta_e + k_1\varphi_A J\beta_f/\beta_e;\quad J \approx J_{inj}, \tag{16}$$

where the first term corresponds to the magnetic energy of the beam, the second to the kinetic energy, and the third to the electrostatic energy expended by the beam in ejection of secondary electrons from the beam channel and in loss of electrons at its front with average energy $k_1\mathscr{E}_e$. From this it follows that there is an inverse dependence of the maximum front velocity on the REB current

$$\beta_f = \varphi_A/[\varphi_A(k_1+1)/\beta_e + J(1+4\ln R_{tu}/r_b)/4c]. \tag{17}$$

The quantity k_1 is an empirical constant determined from experiment.[36]

Here we should note the analysis carried out by Alexander and Hintze in Ref. 46, where it was shown that for most of the experimental data considered by them, an inverse dependence of the maximum ion energy on the REB current density is observed. This dependence agrees with the kinetic model of collective acceleration advanced by them.[46] At the same time in the experiments of Bystritskiĭ *et al.*,[51] where only the REB current density was varied, a direct dependence of the ion energy on the REB density was observed, which has the form $\mathscr{E}_i \sim (J_{inj}/J_{lim})^{<1}$. The existing ambiguity of the experimental results for the dependence of the ion energy on the value (or density) of the REB current indicates the necessity of further more careful studies.

Dependence of the ranges of gas pressure for ion acceleration on the parameters of the relativistic electron beam. In all experiments ion acceleration has been observed in a rather narrow region of pressures $p \lesssim 1$ mm Hg, and this region shifts to lower pressures for REB with lower current and longer rise time.[30,49,51] A characteristic of this dependence is the proportionality of the optimal pressure at which the maximum acceleration efficiency is observed to the ratio J_{inj}/J_{lim}. In ionization-front models the shift of the pressure optimum finds a natural explanation, namely that with increase of J_{inj}/J_{lim} the length of the rear slope of the virtual cathode decreases approximately as $(J_{inj}/J_{lim})^{-1}$ and correspondingly the time of traversal of it by an accelerated ion decreases. Obviously for onset of synchronism between the motion of the ions and the rear slope, the time of neutralization of the REB front must decrease correspondingly, i.e., with other parameters of the beam remaining unchanged $J_{inj}/J_{lim} \sim p$. This also applies to the upper pressure of cutoff of acceleration processes, the analytic dependence between which and J_{inj}/J_{lim} is given in Eqs. (11) and (12). The condition $2\tau_e - \tau_{vrt} + \Delta t > 0$ gives an estimate for the upper pressure of acceleration cutoff $p \lesssim 2A/(\tau_{vrt} - \Delta t)$, where A is an empirical constant; the lower pressure of cutoff of ion acceleration is due to the substantial lag of neutralization processes behind the motion of the accelerated ions, as a result of which instead of synchronous motion of the ions with the beam front one observes slow oscillations of ions in the potential well and consequently there is no accelerated ion bunch. The minimum pressure, for example, for hydrogen, can be estimated as $p \lesssim 1/\tau_b$, where τ_b is the beam duration in nanoseconds and p is in millimeters Hg.

Influence of an external magnetic field on ion acceleration in a relativistic electron beam. In all experiments on application of an external magnetic field of the same direction to the entire drift tube $B \gtrsim 0.05$ T, strong suppression of collective acceleration of ions has been observed (by two orders of magnitude), and the ions detected had an energy not exceeding $\mathscr{E}_i \lesssim z_i \mathscr{E}_e$.[25,34] These results find various explanations in the framework of the enumerated models, but the essence of the phenomenon which is responsible for them is the same: on application of a guiding magnetic field the beam is transported along the entire drift tube in the course of a few nanoseconds and the potential well formed has an extent equal to the drift length. The experimentally established substantial slowing down of neutralization processes on application of a strong field as the result of magnetization of the secondary electrons[33] leads to the result that the potential well does not leave the anode together with the ions, and the latter do not acquire a final energy greater than $z_i \mathscr{E}_e$ (for a sufficient duration of the pulse they can execute oscillations in the axial direction).

The absence of synchronism in the motion of the potential well and the ions leads to a rapid decrease of the number of ions arriving at the detector. The resulting neutralization pattern is due not to the motion of the potential well with a steep rear edge from the anode to the other end of the tube, but to neutralization of the entire drift space, i.e., to the slow disappearance of the long potential well. Use of an opposing-magnetic-field configuration, a so-called cusp, of the necessary strength in the near-anode region and of a uniform field in the remaining portion leads to stopping of the beam immediately beyond the cusp (or to a strong retardation of it so that $v_f \ll \beta_e c$) and to formation of a steep potential well similar to that discussed above in the absence of an external field. The slow propagation of the steep front of the rotating electron beam and the neutralization wave leads to efficient collective acceleration (Fig. 9), as has been observed in Ref. 35.

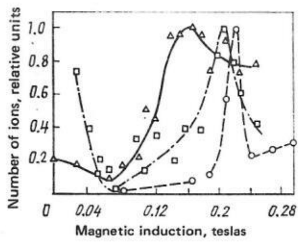

FIG. 9. Ion-acceleration efficiency in a rotating relativistic electron beam which has passed through a cusp, as a function of the magnetic induction[35]: ○—Faraday cup readings, △—results of measurement on the basis of the yield of the reaction $^{12}C(d,n)^{13}N$; □—results of measurements on the basis of the yield of the reaction $^{7}LI(p,n)^{7}Be$.

Similar results were obtained also in a uniform magnetic field for the condition of strong heating of the REB as a consequence of its passage through scattering foils so that the front velocity β_f was also small. This means of obtaining an REB with low velocity of the front motion can be extended also to rotating beams without an external magnetic field. A configuration of this type is realized on injection of a rotating relativistic electron beam into a drift tube with conducting walls which is filled with a neutral gas, which provides charge neutralization. In this case an equilibrium state of the rotating electron layer is achieved as the result of the intrinsic axial and azimuthal fields, which have a closed configuration with a reversal, so that the zero total magnetic flux through the drift tube is preserved.[39] Typical front velocities lie in the range $\beta_f \approx 0.1$ and are estimated from the expression $Z/\mathscr{L} f_M^2$, where Z is the impedance of the diode; $\mathscr{L}$ is the inductance per unit length of the beam + tube system; f_M is the degree of current neutralization. In such a rotating electron layer the main energy of the REB is stored in the azimuthal kinetic energy and ν/γ_e of the beam reaches 20–30.

Experiments[39] have shown a high ion-acceleration efficiency in this configuration. The average number of accelerated ions in a pulse reached 10^{14} at an optimal pressure 75×10^{-3} mm Hg of hydrogen. As in all other experiments, the accelerated ion bunch was associated with the REB front for a maximum ion energy $\mathscr{E}_i \approx \mathscr{E}_e z_i$.

The methods considered for acceleration of ions in a rotating REB in an external field and without it have several earmarks of the electron-ring methods, where the ions are accelerated in the entire length by a single ring-shaped electron bunch, in contrast to acceleration in direct beams (at current levels $J_{\text{inj}} \gtrsim J_{\text{lim}}$), where it occurs at the REB front in a flux of electrons which continuously pass through the virtual cathode, expanding to the walls or departing along the drift tube.

Control of collective acceleration of ions in the front of a relativistic electron beam in a neutral gas. The promise of use of the phenomenon considered of collective acceleration of ions depends on the efficiency of conversion of the REB energy into the energy of the ion bunch at its front.

The expression for the relative efficiency of the ion acceleration considered above in a single bunch at the REB front in the general case has the form[36]

$$\eta \leqslant \frac{1}{(\gamma_e - 1)} \frac{m_i}{2m_e} \beta_i^2 \frac{1}{c} \frac{r_i^2}{r_{\text{b}}^2} \frac{L_i}{\beta_e \tau_{\text{b}}} \frac{n_i}{n_e}, \tag{18}$$

where r_i is the mean radius of the ion bunch; L_i is the length of the ion bunch. The usual estimates give $\eta < 1\%$.

As follows from Sec. 1, the efficiency of collective acceleration of ions and their average energy can be increased substantially if synchronism is provided between the motion of the ion bunch and the frontal space-charge wave of the REB for a large length. We shall set forth briefly some suggestions in this direction, observing a definite chronological sequence, and the first results achieved. Among them are the following: 1) use of a gas gradient or variable drift-tube cross section without an external magnetic field; 2) the same but in an external magnetic field with a specified time profile of the injection current; 3) externally or internally controllable motion of the REB crossover; 4) control of the rate of neutralization of the front by means of an external ionization source.

Suggestions of the first type are the simplest. In fact, on creation of a positive (or negative) gradient of the gas along the drift tube it is possible in this way to control the rate of occurrence of charge-neutralization processes, since $\tau_{ie} \sim A/p$, and correspondingly to control the velocity of propagation of the neutralization wave. Naturally, the upper limit of this velocity cannot exceed the value determined from the energy balance of the beam.[36] However, in experiments carried out in Refs. 26 and 61 it was not possible to obtain unambiguous results indicating an increase of the ion energy on drifting in a gas with a positive gradient, and the predicted rate of rise of the velocity of the REB front was not observed. This ambiguity is due to the substantial role of pre-ionization in regions lying along the path of the REB which completely change the pattern of neutralization-wave propagation. As a result of this, the wave exists in an extremely limited length, regardless of the further change of the gas gradient. The influence of pre-ionization increases with increase of the working pressure. Everything that we have said is valid for acceleration of ions in an REB during its transport in a diverging drift tube with constant gas pressure, where the velocity of the neutralization wave at the initial stage is proportional to R_{tu}.

Suggestions of the second type have been oriented toward control of the motion of the virtual cathode of the REB for a given rate of rise of the injection current and a definite change of the drift-tube radius along its length. Here, to provide equally accelerated motion of the virtual cathode toward the anode for a linear rise of the current, the dependence $R_{\text{tu}}(x)$ can be expressed by the equation[62]

$$\left.\begin{aligned} &h_{\text{tu}}(x) = h(x_0) + (\alpha v_e \tau_{\text{crt}})^{-1} \{[2v_e^2 (x_0 - x)/a]^{1/2} + (x_0 - x)\};\\ &\alpha = (\gamma_e^{2/3} - 1)^{3/2} m_e c^3 / eJ_{\text{inj}}\ ;\ v_e = \beta_e c;\\ &h_{\text{tu}} = \{1 + 2\ln[R_{\text{tu}}(x)/r_{\text{b}}]\}^{-1}, \end{aligned}\right\} \tag{19}$$

where x_0 is the initial position of the virtual cathode; t_0 is the initial moment of formation of the virtual cathode, associated with the condition

$$x_{\text{vc}} = v_e t_0 - \alpha v_e \tau_{\text{crt}} h(x_0); \tag{20}$$

here a is the acceleration of the virtual cathode.

This scheme is rather exotic. In the first place we are discussing acceleration of positive ions in the backward direction to the anode. Acceleration on the fall of the current pulse obviously is inefficient as a result of the occurrence of neutralization processes. Second, the acceleration on the forward extended slope of the virtual cathode with a small value of E/p cannot have high efficiency, as a result of dominance of charge-exchange processes, i.e., operation in the range of very low pressures and correspondingly small n_i is assumed. Nevertheless we note that the scheme has survived experimental verification, which confirms in principle the possibility of acceleration of ions by this means.

Among the exotic schemes of control of the velocity of an isolated wave of space charge we can include also those presented in the third group. In one of them the REB rotates in an external electromagnetic field perpendicular to the direction of motion of the electrons as a whole together with the ions confined by the Coulomb field. Here the ion bunch is under the influence of centrifugal forces which eject it along the path of motion of the electrons in the beam.[63]

A shift of the leading crest (or crossover) of an REB can be obtained by means of a gas layer located in the path of the beam if we made use of the fact that the rise in neutralization of the gas leads to focusing of the REB beyond the gas layer and to a shift of the focus with acceleration determined by the rate of pinching of the REB.[64]

The most promising and most viable scheme, obviously, is that of active control of the front velocity by means of external sources of gas ionization, which is realized in the ionization-front accelerator (IFA) of Olson.[65,66] The essence of the scheme lies in creation of a narrow ionization wave in the front under the action of external laser illumination, sweeping of which along the drift tube of the REB is accomplished by means of fiber optics. The general scheme of the IFA is shown in Fig. 10. As a result of the low ionization potential and the large ionization cross sections, cesium vapor was chosen as the working gas in the first IFA model, and in the second model N-N-dimethylaniline; these gases are ionized in a two-step process by

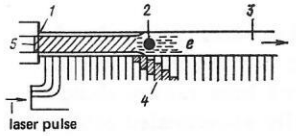

FIG. 10. Diagram of ionization-front accelerator (IFA)[65]: 1—anode, 2—ion bunch, 3—drift tube, 4—light-pipe system, 5—cathode.

two (or one) laser pulses.

As the result of choice of definite lengths of fiber optics cables connecting the light sources with the drift chamber, the two-step light pulse moves along the chamber with a programmed acceleration. In the IFA-1 accelerator the first pulse accomplishes excitation of the cesium vapor and the second produces ionization of it. Measurements of the propagation of the ionization wave by means of electron-optical streak cameras have shown the possibility of controlling the velocity of the wave over a wide range of β.[66] By means of a different set of light pipes in a separate series of experiments it was possible to establish the effective slowing down and the absence of beam transport at the working pressures of Cs vapor and the residual gas without laser illumination and the high efficiency of transport of the REB in the presence of illumination. Combined triggering in the necessary sequence of the REB and the two-step laser pulse have confirmed the motion of the REB front with the calculated acceleration along the entire length of the drift chamber. The first measurements of ions accelerated in the front of the IFA showed agreement with the expected parameters. The accelerated-ion energies of 5 MeV corresponded to the selected mode of scanning of the ionization wave.[66] At the same time the large statistical spread in the time sequence of the different phases of acceleration in IFA-1 showed the necessity of conversion to a single laser pulse: a XeCl* laser and the new working gas N-N-dimethylaniline.

In discussing the possibility of use of IFA to obtain accelerated ions with much higher energies, we note that the main limitation, which is due to the energy balance in the electromagnetic energy, permits ions with energy up to 1 GeV to be obtained with efficiency up to 10% with use of REB with parameters at the level $\mathscr{E}_e = 3$ MeV, $J_{inj} = 30$ kA, and $\tau_b = 100$ nsec.

3. COLLECTIVE ACCELERATION OF IONS IN A RELATIVISTIC ELECTRON BEAM DRIFTING IN VACUUM

The first studies of collective acceleration of ions in vacuum were carried out in the early 1960s.[67,68] In experiments carried out in vacuum and plasma-filled diodes, Plyutto *et al.* studied the acceleration of ion bunches in the anode–cathode gap of a diode on formation in it of electron beams. The collective ion acceleration considered here differs from the approach of Plyutto in that it is observed mainly in an evacuated equipotential drift space of the REB outside the anode–cathode gap. This approach is now surviving its second introduction as the result of progress which has been achieved in generation of high-current relativistic electron beams. The experimental information which has been accumulated up to the present time is still much inferior to the statistics on acceleration in a neutral gas, but it is already clear that the corresponding processes which occur in the general case cannot be discussed just as the propagation of a solitary charge wave in the front of an REB with the ions being accelerated in it. The scheme which most closely approaches the ionization-front idea is the collective acceleration of ions in an REB drifting in vacuum in a strong magnetic field through a localized boundary of dense plasma at the anode, $(10^{18}–10^{19})$ cm^{-3}, which was discussed by Ryutov and Stupakov.[69,70] In this case on injection into the drift space of an REB with a current significantly greater than J_{lim} beyond the plasma boundary there is formed a virtual cathode, and in the cathode-anode–virtual cathode region oscillating fluxes of electrons and a flow of ions from the plasma boundary toward the virtual cathode arise. The flux of ions away from the anode is charge-limited. Here the accumulation of space charge of the oscillating electrons, which have a distribution in the axial component of the energy, provides ion currents significantly (an order of magnitude or more) greater than the well known Child–Langmuir value. The motion of the ions leads to expansion of the cloud of oscillating electrons and a displacement of the virtual cathode, which carried behind it the ions. The pattern has the scaling nature of expansion of a gas into vacuum.

Since the motion of the ions is rather slow in comparison with the electrons, in the electron–ion flows there is established a quasistationary potential distribution which is shown in Fig. 11. Its characteristic feature is the presence of a narrow near-anode Debye layer $r_d \approx (\varphi_A/4\pi n_e e)^{1/2}$, in which a substantial drop of potential is observed $(\gtrsim 0.2–0.6)\varphi_A$, which provides a lower threshold value of the accelerated-ion energy. The corresponding equations of continuity and ion motion, for example, in the one-dimensional case have the form

$$dn_i/dt + (\partial/\partial x)\, n_i v_i = 0; \tag{21}$$

$$\partial v_i/\partial t + v_i \partial v_i/\partial x = -(z_i e/m_i)\, \partial\varphi/\partial x. \tag{22}$$

Here we introduce the condition of quasineutrality of the electron–ion flux $n_i = n_e(\varphi)$ in the main part of the drift. Thus, in the acceleration method considered the "freezing" of the expanding cloud of oscillating electrons is accompanied by transfer of energy to the ions, which permitted the authors of Refs. 69 and 70 to name it the gas-dynamical method. In fact, in spite of the collective mechanism of ion acceleration, this method is extremely close to the direct schemes of production of intense ion beams in reflecting systems,[71] and differs from them in the presence of a moving virtual cathode

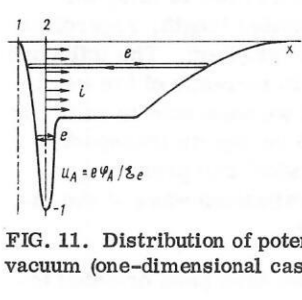

FIG. 11. Distribution of potential in an electron–ion flux in vacuum (one-dimensional case)[70]: 1—cathode, 2—anode.

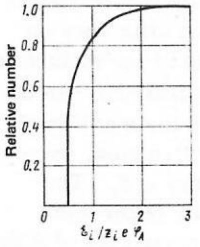

FIG. 12. Ion distribution function in energy $F(\mathscr{E}_i)$.[70]

during the entire period of acceleration. Scaling solutions of Eqs. (21) and (22) lead to final values for the maximum ion energies $\mathscr{E}_i = 3\mathscr{E}_e z_i$ and $\mathscr{E}_i = 5\mathscr{E}_e z_i$ in the nonrelativistic and relativistic cases, respectively.[70] The calculated distribution function of the accelerated ions in energy is given in Fig. 12.

Analysis shows that the maximum efficiency of the gas-dynamical method of ion acceleration is given by the expression

$$\eta \approx 0.77/[1 + 6.44\,(\delta W/e\varphi_{\rm A})\,(m_i/m_e)^{1/2}], \qquad (23)$$

where δW is the average energy loss by an oscillating electron in one crossing of the anode.

Thus, to obtain acceptable values of efficiency, especially for heavy ions $[(m_i/m_e)^{1/2} \gg 1]$, a significant number of oscillations of the electrons through the anode is necessary before they are absorbed. This requirement, on the other hand, must be reconciled with the absence of blocking of the diode by the space charge of the oscillating electrons. In this case it is preferable to work with heavy anodes ($Z_{\rm A} \gg 1$) which strongly scatter the electrons, or to use neutralization of the anode–cathode gap of the diode by ions.[69]

Experiments carried out in the Crab, Water 1–10, and Aquagen accelerators on the basis of this scheme have confirmed the acceleration of ions with an efficiency exceeding ordinary schemes of acceleration in a gas (tens of percent). The detected ion fluxes have exceeded 10^{13} at an energy $(2-3)z_i\varepsilon_e$ and with use of an axial magnetic field $B \approx 1.5$ T.[72-74] As recent experiments carried out in the Aquagen accelerator have shown, application of a falling magnetic field in the drift region beyond the anode leads to a rise of the maximum accelerated-ion energy, which is evidently due to the more efficient transfer of the momentum of the oscillating electron fluxes to the accelerated ions in the region of the virtual cathode.[73]

The pattern considered of ion acceleration in the front of an expanding cloud of oscillating electrons obviously exists in the collective acceleration of ions in reflecting systems, where groups of collectively accelerated ions are observed with energy significantly above the applied voltage.[71,79] In addition, as the result of the substantial nonstationarity of the processes in a triode, potential oscillations and generation of a rotational emf occur with appearance of fast electrons with energy $\mathscr{E}_e > e\varphi_{\rm A}$, which form a virtual cathode with a sag below the potential of the walls of the surrounding cavity. The situation becomes similar to that discussed above, and the neutralizing ion flux extracted from the anode provides a displacement of the virtual cathode, which leads to pre-acceleration of the head part of the ion bunch. The efficiency of such acceleration depends in many ways on the moment of appearance of "fast" electrons, their number, and on the geometry of the diode space. The experimentally measured energies of the fast fraction of the ions lie in the range up to $15z_i\mathscr{E}_e$ and that of the fast fraction of the electrons in the range $\lesssim 1.5\mathscr{E}_e$.

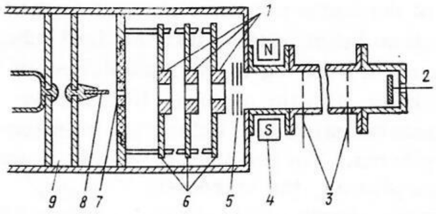

FIG. 13. Diagram of diode system with insulated electrodes[78]: 1—electrodes beyond the anode, 2—Faraday cup, 3—time-of-flight grids, 4—deflecting magnet, 5—activated foils, 6—anode supports, 7—dielectric anode, 8—point cathode, 9—gas-filled predischarge gap.

The picture of ion acceleration in systems with an isolated anode and floating electrodes beyond the anode (or without them), which was first proposed by Luce,[75] is less clear. The arrangement usually used for such a system is shown in Fig. 13. Its characteristic features are as follows: 1) use of a cathode of a point or pencil type (modifications are possible); 2) use of a dielectric anode with a central opening, and also of insulated or grounded electrode tubes beyond the anode.

The sequence of physical processes which lead to acceleration of ions in the system is evidently as follows: on arrival of a high-voltage pulse at the point cathode in the initial stage of electron emission, charging of the dielectric anode begins with its subsequent breakdown along the surface and formation of a dense plasma around the central opening. Before onset of breakdown the beam turns out to be locked in the anode–cathode gap by the negative potential of the anode. During this phase in the anode–cathode gap and the region beyond the anode the degree of neutralization increases, and this plays an important role in all subsequent stages. After the occurrence of breakdown, the beam penetrates into the region beyond the anode, passing through the expanding anode plasma with a falling density gradient. As a result of the significant degree of neutralization in the near-anode region and beyond the anode, the beam turns out to be strongly compressed by its own field and the injection current significantly exceeds the limiting value corresponding to the drift-tube geometry. The virtual cathode formed in the region beyond the anode (its steepness and depth) also will depend on the state of the anode–cathode gap, on the symmetry and density of the anode plasma, its gradient and temperature, and the amplitude of the injected current.

Ions extracted by the electric field of the virtual cathode are accelerated in its direction, producing simultaneous displacement and spreading of the REB front, as in gas-dynamical acceleration. Breakdown of the charged anode produces oscillations of its potential.

The oscillations of the anode potential can lead to modulation of the electron beam passing into the drift tube, which produces a rapid pumping of the instabilities of interaction of the beam with the plasma. In this case the pattern is characteristic for acceleration in space-charge waves in plasma. For the REB parameters used and a collisionless plasma, the increment $\delta \sim \omega_{pl}(n_e/n_{pl})^{1/3}/\gamma_e$ can provide growth of instability in a time 10^{-8} sec, and the spatial increment $\varkappa \sim (n_e\beta_e c/n_{pl}v_{th})^{1/3}\omega_{pl}/\beta_e c\gamma_e$ provides pumping of the wave already in a length of centimeters or fractions of a centimeter. Here v_{th} is the thermal velocity of the plasma electrons; n_{pl} is the plasma density and ω_{pl} is the plasma frequency. In contrast to the gas-dynamical acceleration scheme, in the geometry considered the REB has a small diameter (a beam of the pencil type) and the ion fluxes are clearly two-dimensional. This means that in the near-anode region a significant radial acceleration of ions should be observed.

Estimates of the expected number of ions in the pulse can be obtained in the one-dimensional approximation from the well known expression for the Child–Langmuir ion current for a space-charge limited ion flow in the bipolar regime[77]:

$$n_i \approx (1/9e\pi)\,(j_e/j_{CL})\,(\varphi_{vc}/d^2)\,(1+e\varphi_A/2m_ec^2)^{1/2}, \tag{24}$$

where j_e/j_{CL} is the ratio of the electron current density at the anode to the nonrelativistic Child–Langmuir current density; φ_{VC} is the potential at the virtual cathode.

Equation (24) gives values for n_i in the range 10^{12}–10^{13} cm^{-3}, depending on the parameters of the pulse, which are sufficient for rapid neutralization of the potential well and its departure from the anode. As a result of this the emission from the plasma surface drops sharply, which determines the short duration of the ion bunch moving on the slope of the virtual cathode. In Refs. 76, 78, and 81 in study of accelerated ions by the time-of-flight method they usually observed two or three ion bunches with a current amplitude equal to a few kiloamperes and a duration of a few nanoseconds. Here the Faraday cup which measured the beam current recorded large oscillations of the amplitude at the peak, and the shunt installed in the anode circuit recorded anode-current oscillations with a frequency of 1 GHz, while the spectrometer recorded the presence of an axial component of electrons with energy greater than $e\varphi_A$. Simultaneously with this, generation of microwaves was observed over a wide range of wavelengths as short as $\lambda \gtrsim 5$ cm.

In the experiments carried out by Boyer *et al.*[76] and by Adamski *et al.*[77] the acceleration corresponded to the pattern of a moving isolated virtual cathode. Characteristic ion pulses of duration 5–10 nsec with rise time 3 nsec reached current amplitudes 4 kA, and the maximum energies of the ions were 30 MeV with use of the FX-75 accelerator with a voltage 3 MV and current 90 kA. These results are in good agreement with the picture of outflow of a space-charge-limited ion flux toward the virtual cathode. Measurements of the velocity of motion of the front of the REB indicate that there is no "stationary" phase, which is observed in injection of an REB into a gas. In this connection we must mention the experiments of Heberling and Payton,[80] who established a dependence of the acceleration efficiency on J_{inj}/J_{lim} extremely similar to the case of the ionization-front acceleration mechanism in a neutral gas. An interesting feature of this experiment is the significant yield of accelerated ions in the axial direction, even at $J_{inj}/J_{lim} \lesssim 1/2$ with the maximum energy $z_i\mathscr{E}_e$. The authors associate the occurrence of the latter with processes inside the diode. In the experiments of Adler *et al.*[81] in the immediate vicinity of the anode radially accelerated ions were also detected with energy $>z_i\mathscr{E}_e$, the origin of which was not associated with the anode-cathode gap. Here there were no accompanying radial electrons with such energies. This result agrees with those of Bystritskiĭ *et al.*[37] and can be considered as not confirming the idea of a deep stationary well $|\varphi_w| > |\varphi_A|$. Experiments showed that the most energetic component of the ions is not accelerated at the beginning of the pulse and, as a rule, its appearance is correlated with the development in the near-anode region of instabilities of the REB leading to periodic pinching and expansion of it to the walls of the drift tube.[78,81]

Measurement of the acceleration lengths in various studies[78-81] have given values which vary within very wide limits from $R_{tu}/2$ up to $2R_{tu}$, with corresponding field strengths reaching ≈ 2 MV/cm. The results of measurement of the divergence of the electron beam in the region beyond the anode are also ambiguous, and some studies[75] have recorded a strong divergence of the beam and a substantial radial acceleration of ions, while others have found a very small divergence of the beam $\alpha/2 \approx 4°$ (Refs. 78 and 81) and good transport of the beam in vacuum without a magnetic field. These results indicate a large influence of various kinds of instabilities arising in the drift-tube–plasma–beam system and leading to various patterns of ion acceleration.

As is clear from the above, the efficiency and even the possibility itself of acceleration of ions in the front of an REB are sensitive to the injection current and the state of the anode–cathode gap at the moment of breakdown of the anode and of formation of the virtual cathode. In the presence of a large charge prepulse the initial ionization of the anode–cathode gap and of the region beyond the anode, produced by the prespulse, leads to formation of a flatter potential well, a rapid drop of the diode impedance, a corresponding decrease of the potential sag at the virtual cathode, and a resulting low efficiency of ion acceleration. In regard to acceleration in slow waves generated in pumping of instabilities, long pulse durations are preferable for this, and also axial symmetry and a falling profile of the density of the plasma which is formed.

As a result, for such diodes it is customary to use prepulse spark gaps which suppress the long (hundreds of nanoseconds) charge prepulse, which is especially large in operation with double shaping lines. Experiments carried out in Ref. 78 have shown that introduction of a prepulse spark gap (gas or electrical), which suppresses the prepulse and shortens the rise of the main pulse, leads to a significant increase of the average energy of the ions (from $2z_i\mathscr{E}_e$ to $4z_i\mathscr{E}_e$), an increase of the integrated number of ions, and better repeatability of the results. Similar results were obtained in Refs. 81 and 82.

A general feature of the acceleration scheme under discussion is the use as an ion source of a surface anode plasma generated in the field of the beam and by the beam itself. As a result of a certain spread in the beam parameters and the characteristics of the breakdown of the dielectric from pulse to pulse, the reproducibility is poor and improvement of it will require empirical modification of all elements of the system. It must be acknowledged that by this means it has been possible to obtain unique results in which the energies of the accelerated ions reach $(20-25)z_i\mathscr{E}_e$, the neutron flux yields are approximately 10^{11}, and the efficiency is several tens of percent.[80]

Furthermore, use of additional electrodes beyond the anode, either insulated, grounded, or in a mixed modification, further increase the values achieved. The record result can be considered the 50-fold increase of the proton-beam energy obtained in a system which included three pairs of electrodes beyond the anode, grounded and floating.[75] There is no clear idea of the role of the electrodes beyond the anode, and the processes which occur in this case reduce not only to the successive charging of the electrodes to the beam potential and discharging, i.e., to an advance of the virtual cathode, but also to a focusing action of the electrodes on the plasma and correspondingly on the ion-electron fluxes. Optical and physical studies of the plasma luminescence in the interelectrode gaps and time measurements of the electrode breakdowns confirm this.

In this connection we point out that in the experiments of Bystritskiĭ *et al.* with use of grounded electrodes in arrangements of the diaphragmed-waveguide type[78] a significant energy increase was also observed, which indicates possible acceleration in slow waves excited in the interelectrode plasma on passage of the REB through it. As these studies have shown, use of a larger number of electrodes ($n>3$) is not very effective and the average increment of energy per electrode drops rapidly, as a result of the finite length of acceleration and the synchronous motion of the beam front with the ions. In this connection it is convenient to distinguish the processes of anode and electrode plasma formation and the injection of the beam itself (as was achieved in ionization front acceleration[66]) as the result of use of an external plasma source, for example: in illumination of the anode surface by a laser, in controlled breakdown of the anode, and in pulsed admission of gas into the near-anode region.[82,83]

In contrast to the method discussed, the motion of a neutralization wave behind the front of the REB can be produced by ions extracted from the walls of an evacuated dielectric drift tube. Acceleration of ions in this arrangement was observed previously in study of transport of an REB in evacuated dielectric channels with a current several times greater than the value $J_{\rm lim}$, which indicated a significant neutralization of the beam by ions.[84-86]

A physical picture of such transport can be represented as the succession of the following processes: formation of the virtual cathode and corresponding expansion of the REB to the surface of the dielectric tube, leading to its charging to breakdown potential; formation of a surface plasma primarily as the result of the breakdown; extraction of ions by the space-charge field of the REB and acceleration of them in the radial direction; neutralization of the REB front by ions and advance of the front along the drift tube; and acceleration of ions captured into the potential well at the REB front, in the axial direction. In the framework of this representation we can expect that the velocity of the REB front should be proportional to the rate of formation of the plasma and the arrival from it of ions into the expanding frontal part of the beam, i.e.,

$$\beta_b \sim JR_{\rm tu}^{-1}. \tag{25}$$

It follows from Eq. (25) that equally accelerated motion of the REB front can be achieved in a cylindrical drift tube with use of an REB which increases linearly with time. Estimation of the rate of acceleration in the REB front, on the basis of the expression for the field strength at its boundary, $E_{\rm bd}\sim(4/3)J/(r_b\beta_e c)$, gives a value approximately equal to $5\cdot10^4$ V/cm.[87]

As a whole the experimental results correspond to the model constructions. The studies carried out by Agafonov *et al.*,[88] Greenwald *et al.*,[89] and Pasour *et al.*[90] have shown that the energy of collectively accelerated ions rises rather linearly with increase of the REB current and drift-tube length and decreases with increase of its radius. For illustration of this we have shown in Fig. 14 the experimental dependence of the energy of ions accelerated in the axial direction on the injection current density of the REB. In regard to radially accelerated ions, the experiments of Ref. 90 have confirmed the existence of preferential radial acceleration along the entire drift of the REB, characterized by the absence of azimuthal symmetry. On the average in all experiments the energy of ions accelerated in the axial direction has exceeded the electron energy by 2–10 times with a yield at the level 10^{13}, and the ion spectrum has the form $N_i(E)\sim\exp(-E_i)$. The form of accelerated ions, as a rule, is protons, which are contained in excess in any dielectric. In the first approximation one can state that there is satisfactory agreement of the experimental results with the quasistatic idea of a moving solitary space-charge wave at the front of the REB, which accelerates the ions.

In conclusion we note that in comparison with acceleration in a gas (without controlled motion of the REB front) collective acceleration in vacuum is more effi-

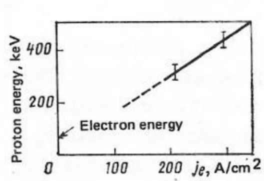

FIG. 14. Ion energy as a function of injection-current density in collective acceleration in vacuum with a dielectric liner.[85]

cient in the ion-current amplitudes and the energies achieved but requires significant modification directed toward increase of the degree of synchronism of the motion of the REB front with the ions. In regard to systems with an insulated anode, they are still awaiting a theoretical description.

4. COLLECTIVE ACCELERATION OF IONS BY SLOW WAVES IN A RELATIVISTIC ELECTRON BEAM

Slow waves, as we have mentioned in the Introduction, are characteristic of high-current charged-particle beams. Their appearance is due to Doppler effects and is related to the Langmuir oscillations of the density of charges in the beam and to cyclotron rotation of the electrons, the frequencies of which depend on the parameters of the relativistic electron beam and the drift space. Both Langmuir and cyclotron oscillations propagate in REB with a group velocity which corresponds to the beam velocity and a phase velocity determined from the dispersion relations. Interactions of the wave modes of the drift tube and the REB are responsible for various instabilities observed in beams. The strong concentrations and rarefactions of the charges of the REB observed here (space-charge density waves—SCDW) under certain conditions can be used for collective acceleration of ions.[11] Here we must have the conditions for ion capture by the wave and a definite rate of change of its phase velocity. Analysis of these possibilities can be carried out on the basis of dispersion equations for these waves.

As an example we shall give the dispersion equations for two simple geometries (plane and cylindrical) of the REB and the drift space with assumption of a monochromatic electron beam, following Refs. 91 and 92. In spite of the simplifications used, they reflect the main regularities and features of the propagation of waves in beam. These equations, which establish the dependence $\omega(k)$, where ω is the frequency of the wave and k is its wave vector in the linear approximation, can be obtained by simultaneous solution for ω of the equations of continuity, conservation of momentum, and Maxwell's equations. In the one-dimensional case of a monoenergetic beam moving in the x direction, the dispersion equation has the form[91]:

$$(\omega-kv_e)^2=\omega_e^2, \tag{26}$$

where ω_e is the Langmuir frequency of the beam and v_e is the velocity of the beam electrons.

Equation (26) has two types of wave solutions:

$$\omega = kv_e \pm \omega_e \tag{27}$$

with corresponding phase velocities

$$v_{sw} = v_e/(1 + \omega_e/\omega); \quad v_{fw} = v_e/(1 - \omega_e/\omega), \tag{28}$$

of which the first characterizes slow Langmuir waves and the second-fast waves. For acceleration of ions we are interested in slow waves, the phase velocities of which for $\omega_e/\omega \gg 1$ can be significantly less than v_e, i.e., suitable for capture of ions. The characteristic feature of these waves is that their energy is negative, which corresponds to transfer of the beam kinetic energy into excitation of the electromagnetic fields of the wave and a flow of its kinetic energy, which is transferred to ions.

By the energy of the wave we mean the difference between the total energy of the system (beam + field) before excitation of a wave in it and its total energy after excitation. The fact that the energy of the wave is negative follows from conservation of energy and momentum of the wave:

$$\mathrm{div}\,(\mathbf{P}_{EM}+\mathbf{P}_K)+\frac{\partial}{\partial t}(\mathscr{E}_E+\mathscr{E}_M+\mathscr{E}_K)=0, \tag{29}$$

where $\mathbf{P}_{EM}$ is the electromagnetic energy flux of the wave, $\mathbf{P}_K$ is the kinetic energy flux of the wave, $\mathbf{P}_W = \mathbf{P}_{EM}+\mathbf{P}_K$ is the total energy flux carried by the wave, $\mathscr{E}_E$ and $\mathscr{E}_M$ are the energies associated with the magnetic and electric fields of the wave, and $\mathscr{E}_K$ is the kinetic energy of the wave. In the presence of ions moving with the wave, conservation of the total energy flux through any cross section of the drift space requires fulfillment of the equality

$$\mathbf{P}_i + \mathbf{P}_W = \mathrm{const}, \tag{30}$$

where $\mathbf{P}_i$ is the energy flux carried by ions.

Thus, when synchronism is provided between the motion of the accelerated ions and the slow wave, the removal by the ions of the energy of the wave will provide continuous amplification of the wave (as a consequence of the negative value of its energy–momentum) and consequently acceleration of ions up to saturation in the nonlinear stage of capture of electrons by the wave (see below), i.e., the process has an autoresonant nature and accelerators based on this principle are called autoresonant accelerators (ARA).

The dispersion equation for a cylindrical cold monochromatic relativistic electron beam (a hollow beam or a beam of the pencil type) drifting in a strong magnetic field in a cylindrical drift tube has the form

$$(\omega - kv_e)^2 = (k^2c^2 - \omega^2)\, 2J \ln (R_{tu}/r_b)/J_A\gamma_e^3\beta_e. \tag{31}$$

The expression for the phase velocity of the slow space-charge wave obtained from Eq. (31) depends explicitly on J_{inj}/J_{lim}:

$$v_{ps} \approx c\,(\gamma_e^{2/3}-1)^{1/2}\,(1-J_{inj}/J_{lim})^{1/2}. \tag{32}$$

A more general analysis carried out by Sloan and Drummond[93] and which includes perturbation of the beam in the direction transverse to its motion, leads to a dispersion equation the solutions of which are not only Langmuir waves, but also cyclotron and electromagnetic waveguide modes:

$$0=(\omega-k_xv_e)^2(\omega^2-\omega_e^2-k^2c^2)^2[(\omega-k_xv_e)^2-\omega_e^2/\gamma_e^2] - \omega_H^2(\omega^2-k^2c^2)[(\omega-k_xv_e)^2(\omega^2-k^2c^2)-(\omega_e^2/\gamma_e^2)(\omega^2-k_x^2v_e^2)], \tag{33}$$

where $\omega_H = eB/m_e\gamma_e c$.

This equation was obtained on the assumption of a continuous uniform beam drifting in a tube in a magnetic field and satisfying the conditions of equilibrium and stability: $\omega_e^2<\gamma_e\omega_H c/r_b$; $2\omega_e^2<\gamma_e^2\omega_H^2$; $\omega_e^2 r_b^2<4c^2$; $\omega_e^2 <\gamma_e\omega_H c/R_{tu}$. The corresponding real branches of the solution of this equation are given in Fig. 15. As can be seen from this figure, to a rather good approximation

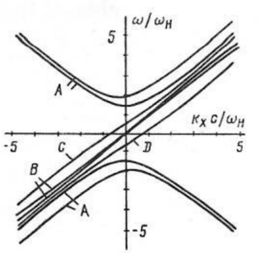

FIG. 15. Solution of the dispersion equation[93]: A—intrinsic electromagnetic modes of the cavity; B—Langmuir modes with Doppler shift; C—fast branch of the cyclotron mode with Doppler shift; D—slow branch of the cyclotron mode with Doppler shift.

the expression for the phase velocity of the slow cyclotron wave can be written in a form similar to Eq. (28):

$$v_{sw} = v_e/(1 + \omega_H/\omega). \quad (34)$$

In regard to the Langmuir mode, in the region $\omega > \omega_e$ the linear dispersion relation $\omega = kv_e - \omega_e$ also is a good approximation, and the phase velocity is expressed by Eq. (28). While the Langmuir waves are longitudinal waves of charge density propagating in a relativistic electron beam, on the other hand the slow cyclotron waves are transverse waves in which the beam electrons execute oscillations in the r directions and rotation in the θ direction, with a radius which pulsates with a frequency ω_H. As a whole this appears as a corrugation traveling along the surface of the REB.

Analysis of the curves of Fig. 15 leads to the conclusion that of the two forms of slow wave, the cyclotron mode has an unquestioned advantage over the Langmuir mode for the purpose of ion capture, since it provides a smooth control of v_{sw}, beginning with a value $v_{sw} \ll c$, which is extremely important in the initial stage of ion acceleration if injection of ions with low initial energies is used. Indeed, it follows from Eq. (32) that with increase of the current and on approach of the current to its limiting value, $v_{sw} \to 0$; however, this dependence does not have a smooth nature, since the quantity dv_{ps}/dJ_{inj} diverges as $J_{inj} \to J_{lim}$. As a result of the impossibility of sufficiently smooth regulation of J_{inj} in real high-current pulsed accelerators, capture of ions into the Langmuir slow wave can be accomplished with a final value $v_{sw} \approx (0.05 + 0.1)c$ and a corresponding initial energy of the ions.

Use of these waves for regular acceleration of ions in autoresonant accelerators is a complicated problem, which includes the following aspects:

1) excitation and amplification of a slow wave of a certain type which provides a Doppler resonance with the beam $\omega = kv_e - \omega_e$ (or ω_H) and a Čerenkov resonance with the ions ($v_{sw} = v_i = \omega/k$) with suppression of all parasitic frequencies which do not satisfy these conditions;

2) efficient loading by ions in the initial stage or under the conditions of the developed slow wave;

3) modulation of the phase velocity of the wave in the ion acceleration process, providing synchronism of their combined motion in the linear and nonlinear stages of development of the wave.

Initial excitation of the required slow wave in an REB can be accomplished by an external source of microwaves, and its further growth can be accomplished in retarding structures of which the intrinsic modes have phase velocities close to the required values (0.03 −0.1)c. In this regard the best systems are structures of the helical or diaphragmed type which represent a chain of weakly coupled resonators. The former have been studied in the development of autoresonant accelerators in cyclotron waves,[94,95,97,98] and the latter have been studied for accelerators using Langmuir waves.[96]

With use of a helix, the electron beam propagating along its axis excites an electromagnetic wave with frequency $\omega = kc \sin\psi$, where ψ is the winding pitch of the helix. Interaction of the slow cyclotron wave with the main eigenmode of the helix leads to instability of the convective type with a group velocity near $c/2$. For a strongly relativistic REB ($\gamma_e \gg 1$) of cylindrical shape and uniform over its cross section, for $kr_b \gg 1$ the expression for the increment of this instability has the form[95]

$$\Gamma = \frac{\omega_e}{2\gamma_e}\left(\omega_H \frac{r_b}{c} \sin 2\psi\right)^{1/2} \{\exp[-2k(r_b - r_{he})] - \exp[-2k(R_{tu} - r_b)]\}, \quad (35)$$

where r_{he} is the radius of the helix.

Numerical calculations have confirmed the validity of Eq. (35) with a good degree of approximation. The parasitic mode, appearance of which leads to deterioration of the efficiency of amplification of the necessary cyclotron mode in an autoresonant accelerator, is a cyclotron wave with zero frequency which satisfies the condition $\omega_{HO} = kv_e$. The appearance of this wave is due to the presence of static perturbations in the beam and in particular to the grounded anode plane through which the REB is injected into the drift space, which screens the radial electric field at a distance approximately up to $R_{tu}/2$ from it. The local weak pinch effect which arises leads to appearance of a stationary corrugation wave in the REB, which results in loss of the outer layers of the beam to the walls (or to the helix).[94,95] Suppression of this mode turned out to be possible with use of a definite configuration of axial magnetic field (attenuated in the injection plane by about 1.3 times) and an anode–cathode gap which provides injection of electrons into the drift space at definite angles to the axis such that $P_r = P_0 r \sin\alpha / r_b$, where P_r is the radial momentum of the electron; P_0 is the initial momentum of the electron (axial); α is the divergence angle. The optimum value of α for the REB and drift-space parameters given above is 9°.

With use of Langmuir slow waves in an autoresonant accelerator, amplification of the waves can be accomplished in a series of weakly coupled resonators, where the amplitudes of all higher harmonics in the spectrum of excited harmonics are small in comparison with the principal harmonic.[96] It is also possible to use a diaphragmmed coaxial resonator with a smooth inner wall, into which a ring-shaped REB is injected. In this case the interaction of the space-charge density wave with the principal mode of such a resonator leads to growth of the principal and first harmonics of the space-charge wave.[99] An analysis carried out by Sprangle *et al.*[99] showed that for moderate values of the phase velocity of the first harmonic, $v_{1sw}=(0.05-0.1)c$, its amplitude amounts to approximately $E_{x1}/E_{x0}=0.28$ of the principal harmonic, i.e., it can be used for acceleration of ions.

The development of a slow wave of any of the types indicated continues until saturation sets in if suppression of the parasitic modes is provided, and the discussed interaction with waveguide modes takes place. The level of saturation of the wave is determined by the occurrence of capture of the electrons of the relativistic electron beam by the field of the wave. The maximum field of the wave can be determined from the following relation on conversion to the system of reference attached to the wave:

$$E_x \approx (\alpha k/\gamma_{sw})(\gamma_{ew}-1)\, m_e c^2/e, \tag{36}$$

where γ_{sw} is the γ factor of the slow wave, γ_{ew} is the γ factor of the wave–beam relative motion, $\gamma_{ew}=\gamma_e\gamma_{sw}(1-\beta_e\beta_{sw})$, and α is a numerical coefficient which takes into account that the growth of the wave is not cut off immediately on capture of the first electron but can continue up to substantial values ($1<\alpha<10$).[100] Estimates on the basis of this formula gives values which, depending on α, lie in the range $\leq 3\cdot 10^5$ V/cm, which is less than the field strength in the front of an REB in acceleration of electrons by an isolated wave of charge. Nevertheless the possibility of regular acceleration of several ion bunches over a substantial length, i.e., quasicontinuous acceleration, and the corresponding high efficiency of the process compensate this deficiency. In particular, if synchronism is provided between the motion of the wave and the ions up to the stage of saturation E_{max} the upper limit of the ion-acceleration efficiency can be obtained from the relation $\eta=P_i/P_b$, where $P_b=\pi r_b^2\beta_e(L)n_e(L)[\gamma_e(L)-1]m_ec^3$ is the energy flux of the beam measured at the end of the accelerator section;

$$P_i = 2\pi \int_0^{r_b} n_i(L)\,\beta_i(L)\,[\gamma_i(L)-1]\,m_ic^3 r\,dr \tag{37}$$

is the energy flux of ions at the same cross section.

The corresponding value of γ_i for the condition of synchronism of the motion of the wave with the ions before saturation can be determined from v_{sw}. For an upper-limit estimate $P_i=P_w$, η turns out to be a quadratic function of α and lies in the range up to 10%,[100] which is an order of magnitude or more greater than the acceleration efficiency in a neutral gas and approaches the acceleration efficiency in the front of a relativistic electron beam in vacuum and in systems with an insulated anode.

When the slow wave has reached the necessary amplitude, it is loaded with ions. The simplest method is to pass the REB through a narrow region containing ionized gas at low pressure. However, in use of Langmuir slow waves the ions must have a significant initial energy at a level 10–15 MeV and in this scheme capture of ions by the wave will be observed on injection of an ion beam with appropriate energy along the REB. A possible injection scheme of this type is to use a two-

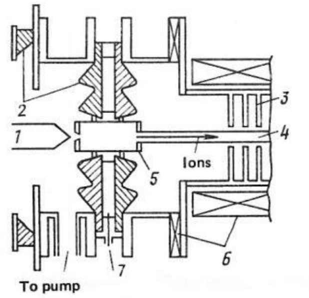

FIG. 16. Diagram of two-stage Luce diode for injection of ions into a relativistic electron beam[10]: 1—cathode, 2—insulator, 3—wave-slowing structure, 4—relativistic electron beam, 5—anode, 6—magnet system, 7—resistive voltage divider.

step Luce diode, which is illustrated in Fig. 16.[101] In this arrangement ions generated by the Luce diode pass inside a hollow tube which is a second cathode, which injects into the amplifier section a hollow electron beam. The γ factor of the second electron beam is, accordingly, below that of the first (by about 20%) and is determined by a resistive potential divider. This scheme is inefficient ($\leq$0.1%) as a result of the high level of electron loss ($\approx$90%), but it demonstrates the possibility in principle of axial injection of ions along the REB. Analysis of the behavior of the ion beam after its injection into an REB with a developed slow wave shows that the ion beam breaks up into bunches which are attracted to equilibrium phases. The capture coefficient here can exceed 80%.[102] Further motion of the ions together with the REB can be accompanied by modulation of the phase velocity. In the absence of modulation, the acceleration of the ions is saturated at a level determined by the "capture" of the ions by the wave (by analogy with the capture of beam electrons which has been discussed).

Analysis of the expressions for the phase velocity of slow waves leads to several possible means of regulating it: spatial variation of the external magnetic field as adopted in the autoresonant accelerator in the cyclotron wave, and variation of the drift-tube radius in a uniform magnetic field as proposed in the first autoresonant accelerator arrangement using Langmuir waves by Sprangle *et al.*[100] The slow falloff of the field in the first scheme with conservation of the total flux B through the REB leads to expansion of the beam according to a law

$$r_b = r_{0b}(B_0/B)^{1/2}, \tag{38}$$

where r_{0b} and B_0 are the initial values of the beam radius and the magnetic field strength.

In the initial stage for $\beta_{e0}\gg\beta_i$ the variation of the magnetic field should follow a law $B\sim\beta_{sw}^{-1}$. Here radial and phase stability is observed in the region of accelerating phases. On further development of the wave the nature of the motion of the resonance ions can be determined by simultaneous solution of the equations for the wave potential and the ion motion in the field of the wave for the condition of preservation of a constant accelerating phase.[102]

Decrease of the drift-tube radius in the second scheme of ion acceleration leads to a decrease of the potential sag of the REB with advance along the drift tube, i.e., to increase of its γ factor and correspondingly to decrease of the density n_e of the REB. In this case the expression for the γ_e factor which grows with the tube length has the form

$$\gamma_e(x) = \gamma_e(0) + \omega_c^2(0)(r_b^2/c^2)/4\,\{1 + 2\ln R_{tu}(0)/r_b - [v_{e0}/v_e(x)]\,[1 + 2\ln R_{tu}(x)/r_b]\}. \quad (39)$$

It was obtained from the condition of conservation of the total energy flux of the REB and the wave.[100] As a consequence of the stationarity of the system discussed with time, ω is a constant quantity, while the rise of $\gamma_e(x)$ corresponds to increase of β_e and decrease of ω_e, which leads to a rise of β_{sw}. A similar change of ω_H can be obtained on increase of r_b in a falling magnetic field, which also leads to a decrease of n_e for a constant drift-tube radius or by a combined variation of r_b and R_{tu}.[103]

An alternative approach to the problem of changing the phase velocity along the length is the controlled variation with time of the γ_e factor of the beam, so that the density n_e becomes a function of x and t. In this case with increase of γ_e with time there is a decrease of the corresponding characteristic frequencies ω_e and ω_H and a rise of β_{sw}. For example, for the cyclotron wave the phase velocity is given by the expression[104]

$$v_{ps} = \omega_0 - \omega_{H0}\frac{x}{v_{e0}}\frac{\partial}{\partial\eta}\left(\frac{\gamma_{e0}}{\gamma(\eta)}\right) \times \left\{k_0 - \frac{\omega_{H0}}{v_{e0}\gamma_{e0}}\left[1 - \frac{\gamma_{e0}}{\gamma(\eta)} + \frac{x}{v_{e0}}\frac{\partial}{\partial\eta}\left(\frac{\gamma_{e0}}{\gamma(\eta)}\right)\right]\right\}^{-1}, \quad (40)$$

where $\omega_0 = kv_{e0} - \omega_{H0}$; v_{e0}, γ_{e0}, k_0 are the initial values of the velocity and γ factor of the REB and of the wave vector of the cyclotron wave;

$$\eta = t - x/v_{e0}.$$

In addition to the methods discussed above for generation of slow waves and controlling their phase velocity in the ion-acceleration region, several other methods have been proposed:

1) injection of an REB into a cusp as suggested by Bonch-Osmolovskiĭ[105] in order to decrease the initial value v_{e0} and accordingly v_{sw} as the result of pumping of the axial energy of the REB into azimuthal energy. Further variation of the phase velocity of a slow wave can be carried out on the basis of any of the schemes discussed above;

2) use of the quasistatic fields of the REB as suggested by Belikov *et al.*,[106] which reduces essentially to transport of the REB in a corrugated drift tube or in a magnetic field which is periodic along the axis with a definite time modulation of the beam current. The idea is to produce a beam-current variation periodic in time and synchronized with the ion motion along the density-modulated electron beam such that passage by an ion bunch through an accelerating and decelerating modulation period of the beam is accompanied by a pure growth of the energy. The situation is in some sense analogous to periodic motion along the stationary field of a particle with periodically varying charge.

The efficiency of ion acceleration in this method will depend on several important questions, namely: a) on achievement of the necessary synchronization of the current change and the ion motion; b) on achieving the necessary depth of modulation of the charge density.

The suggestion under discussion was investigated theoretically by Lebedev *et al.*,[107] who showed that high accelerating fields require comparatively low values of the REB current. Here the maximum permissible field strength can be estimated from the expression

$$E_{x\,\max} \approx J\pi(\gamma_e - 1)/LcI_0(r_b/L), \quad (41)$$

where L is the length of the drift-tube period and I_0 is a modified Bessel function. Numerical estimates with this formula give $E_{x\,\max} \approx (10\text{–}40)$ MV/m for $J = 8$ kA, $\gamma_e = 10$, $L = 31$ cm, $r_b = 0.5$ cm, and the depth of corrugation of the drift tube equal to about 6 cm, depending on the degree of modulation of the REB density. The viability of this scheme of ion acceleration depends to a significant degree on the possibility of providing controlled high-frequency modulation of the electron current at levels of a few koloamperes to tens of kiloamperes. Nevertheless the first experiments at low energy and current levels have confirmed the basic postulates of this method.[107] For an electron beam energy 3 keV and a current 0.1–0.5 A the maximum energy increment of the ions was 5 keV with 50% modulation of the beam current at a frequency 3.1 MHz;

3) the third scheme is to use Langmuir waves of zero frequency (by analogy with the cyclotron stationary mode discussed above) as proposed by Lebedev and Pazin.[108] The idea reduces essentially to the possibility of excitation of such waves by an ion bunch in a system of reference fixed to the beam. Analysis shows that in the laboratory system of coordinates a force proportional to $q^2k_0^2$ acts on the ion bunch, where the quadratic dependence on the charge of the bunch indicates the coherent nature of the interaction. Estimates of the electric fields which develop from the wave give a value equal to about 100 MV/m for a beam-current density ≤ 10 kA/cm^2.

At the present time two schemes of autoresonant accelerators are being actively developed in their theoretical and experimental aspects—in the cyclotron and Langmuir modes as suggested by V. D. Shapiro, V. I. Shevchenko, *et al.*,[102] by Sloan and Drummond,[93] and by Sprangle *et al.*[100] In programs carried out in the USSR (at the Khar'kov Physico-technical Institute[97,98]) and in the USA (at Cornell University and by the Austin Company[101,109]), the experimental aspects of all of the problems considered above of generation and amplification of slow waves are being analyzed, together with their stable propagation, control of the phase velocity, and injection of ions. Among these studies an important position is occupied by the numerical calculations of Godfrey, Faehl, *et al.*,[94,95] which permit one to obtain solutions of the problems without simplifying assumptions or with a minimum number of them. The purpose of these programs is to put into operation a prototype autoresonant accelerator. For a prototype using cyclotron waves the following parameters have been chosen[109]: electron energy 3 MeV, electron beam current

30 kA, pulse length 200 nsec, frequency of excited waves 250 MHz, maximum magnetic field 2.5 T, maximum ion energy 30 MeV, ion current 30 A, amplitude of waves 250 kV, and accelerator length 4 m.

In the stage of development of a pulsed high-current electron accelerator and injection of an REB into a drift tube, special attention was devoted to suppression of the zero-frequency standing wave which we mentioned earlier. Excitation of a cyclotron wave with $m=0$ of amplitude 250 kV in a region with a moderate value of magnetic induction (<0.4 T) is accomplished by means of a microwave generator with a power of 100 kW operating in a pulsed mode with $T=10$ μsec. Further rapid growth of the cyclotron wave occurs in the drift-tube region. After the calculated wave amplitude is reached, the beam is compressed to a small radius in a strong magnetic field (2.5 T). This scheme of excitation of a cyclotron wave was chosen as a result of the inverse dependence of its increment on the magnetic induction. Then an REB with a developed wave enters an ion acceleration region with a falling magnetic field, which acts as an autoresonant accelerator. Injection of ions occurs after completion of the growth of the slow-wave amplitude by means of pulsed gas admission with subsequent stripping of the atoms. Dumping of the accompanying electron beam at the end of acceleration is accomplished by means of a bending magnet.

The experimental and numerical studies of a prototype autoresonant accelerator using Langmuir slow waves[101] are oriented toward an REB with energy in the range 300–700 keV. The source of ions is a Luce diode which uses an REB with energy 700 kV and $J_{inj}=60$ kA and which provides a proton flux of about 10^{10} with energy 10–15 MeV. As an amplifying section a series of coupled resonators is used in which it has been possible to achieve a minimum value $v_{sw}\approx 0.10c$ for corresponding values $E_x\approx 60$ kV/cm. Calculated curves of phase velocity for a thin beam for two different injection energies and the corresponding experimental values of v_{sw} are shown in Fig. 17. The experimental studies of the rate of growth of the slow wave give values 5 dB per period with a bandwidth 40 MHz.

The models and ion-acceleration schemes considered which employ slow waves are oriented mainly toward linear modes of space-charge density waves with calculated and strictly controlled growth rates of the wave amplitude and its phase velocity. At the same time considerable interest is presented by nonlinear modes of

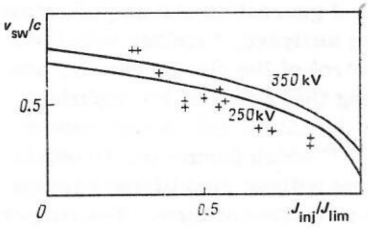

FIG. 17. Phase velocity of the slow wave as a function of J_{inj}/J_{lim} for two values of the relativistic electron beam energy: +++—experimental results; solid curves—calculation of Ref. 113.

space-charge density waves of the solution and shock-wave type and also self-accelerating highly nonlinear space-charge density waves. As has been shown by Khodataev and Tsytovich,[110] for the latter case a suitable arrangement is the so-called cyclotron-focusing instability associated with a space-charge density wave, in which electrons, executing cyclotron rotation in the transverse direction, create regions of constriction—foci, which follow ion concentrations. Analysis of the general dispersion equation obtained for a drift-space geometry and REB parameters similar to those of Ref. 93 shows a transition of the cyclotron-focusing instability in two limiting cases ($\omega_H \gg \gamma_e \omega_l$ and $\omega_H=0$) to pure cyclotron[93] or pure focusing[111] instability, respectively. Study of various regimes in ion current, when the nonlinearity of the interaction of the ions with the space-charge density wave may or may not be neglected (the weak-current and strong-current regimes, respectively) has shown that:

1) at the moment of formation of the ion bunches the depth of the space-charge modulation in the high-current regime reaches unity values of $\Delta=(\rho_{max}-\rho_{min})/(\rho_{max}+\rho_{min})$, in contrast to $\Delta=0.1$ which is realized in the weak-current regime of linear space-charge density waves characteristic of the acceleration schemes of Refs. 93 and 100;

2) the rate of acceleration in the high-current regime rises substantially in comparison with the linear regime, and the characteristic length of growth of the amplitude of the space-charge density wave turns out to be much less than the length of the acceleration region. Estimates of the rate of acceleration of ions by means of a highly nonlinear space-charge density wave give a final ion energy (in an acceleration length of 10–15 m) of the order 100–200 MV for a current of about 100 A and an electron beam with parameters $\varphi_A=1.5$–2 MV, $J_{inj}=20$–40 kA.

For the limiting case of development of space-charge density waves, the appearance of solitons is characteristic—local perturbations of the medium, i.e., of the density of the REB into which under certain conditions an arbitrary perturbation of the medium can break up in the general case.[112] The soliton field strength has the form

$$E(x)=E_0\exp\left[-i\Omega t+(iv_s/2)(x-v_st)(\omega_e/c)\right]^{-1} \times \cosh^{-1}(x-v_st)/x_0, \tag{42}$$

where E_0 is the soliton amplitude, Ω is the frequency shift, x_0 is the soliton width, and v_s is its velocity; $x_0=(c/\omega_e)\sqrt{2}\alpha/E_0\beta^{1/2}$; $\Omega=-\omega_e(v_s^2/4c^2+E_0^2\beta_-^2/2)$. Analysis of the motion of solitons in REB for current values close to J_{lim} has shown the existence of feedback between their amplitude and the velocity of the motion, so that when the maximum field strength is reached, the soliton can change the sign of its velocity. With increase of the amplitude of the soliton, it becomes narrower in the axial direction and $x_0<R_{tu}$. Breakup of solitons occurs at a φ level of the order $(0.4)\mathscr{E}_e/e$.[113] The possibility of existence of such solitons in relativistic electron beams as the final nonlinear stage of development of space-charge density waves is extremely attractive in the theory of collective acceleration.

CONCLUSION

The review which we have presented of the current state of collective methods of acceleration of straight electron beams permits one to make a comparative analysis of the possibilities and promise of these schemes. The last twenty years of study have shown that, in spite of the apparent simplicity and attractiveness of the idea of using collective fields of relativistic electron beams and plasma, its realization has required creation of a large set of pulsed systems of new types for obtaining intense electron beams and bunches and controlling them. The complexity and large scale of the problems which are being solved here at the various steps, as aptly expressed by Ya. B. Faĭnberg, have resulted in "... periods of cheerful and not always well founded hopes and of still less well founded feelings of hopelessness... ."[13] At the present time, of the entire set of collective-acceleration schemes one can single out as the most promising for production of high-energy ion bunches, the ionization-front accelerator of Olson[65] and the autoresonant accelerators employing cyclotron waves[93,98] and Langmuir waves (linear and nonlinear).[100] In spite of the modest initial results in comparison with Luce diodes and similar devices (accelerated ions have already been obtained in all three schemes), they have the main feature of classical accelerators, namely, regular and controlled accelerating fields, in contrast to the weakly controlled processes of acceleration in the front of a relativistic electron beam in its various modifications in a neutral gas or vacuum. Ionization-front accelerators and autoresonant accelerators can be considered as the prototypes of future high-current collective accelerators of the intermediate energy range (hundreds of MeV per unit charge) and intermediate current range (tens to hundreds of amperes). Such accelerators will permit solution of a number of urgent problems of nuclear physics and technology (production of new transuranium elements, creation of efficient neutron sources, new types of injectors for super-accelerators), and with use of heavy ions they will also permit competition with existing plans for thermonuclear reactors based on classical accelerator–storage-ring complexes.[114] Such accelerators present special interest for creation of high-current meson factories as part of the achievement of cold controlled thermonuclear fusion on the basis of muonic deuterium–tritium catalysis.[115]

The very extensive investigative work on collective methods of acceleration which has been carried out during the past two decades permits one to hope that these accelerators will appear as a working prototype during the next few years.

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