Microscopic description of Gamow-Teller resonance and collective isobaric 1^+ states of spherical nuclei

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A theory of collective isobaric 1^+ states of spherical nuclei is constructed on the basis of the theory of finite Fermi systems. Particular attention is devoted to the description of a new type of giant isobaric resonance. This is the Gamow-Teller resonance, whose existence has recently been confirmed in charge-exchange reactions in experiments made at the Universities of Michigan and Indiana. The possibility of describing the Gamow-Teller resonance and the ground states of nuclei in a Wigner symmetry scheme based on the group SU(4) is analyzed. It is shown that such a description improves with increasing $N-Z$ in the region of heavy nuclei. The experimental investigations on the Gamow-Teller resonance and collective isobaric 1^+ states are reviewed. A method for calculating the properties of nuclei far from the β-stability line is developed on the basis of a theory of isobaric states, the use of which is illustrated by a number of examples.

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INTRODUCTION

The problem of discovering new nuclear resonances in complicated nuclei is currently attracting much attention. Besides the well-known E1 resonance, magnetic dipole, electric monopole, quadrupole, and a number of other resonances corresponding to different modes of collective excitation of the nucleus have been discovered and actively studied.[1] In a microscopic approach, all these resonances correspond, on the basis of the charge of particle–hole pairs, to the neutral ($\Delta Q = 0$) excitation branch of the core, the ground state of an even–even nucleus $A(N, Z)$. In the charged excitation branches, in which the charge of the particle–hole pairs above the core is nonzero ($\Delta Q = \pm 1$), only one type of collective state has so far been studied in detail; this is the isobar analog resonance. It is a special example of isobaric collective states of the proton–neutron–hole type ($p\bar{n}$ type). Whereas the neutral branch of the particle–hole excitations of the nucleus $A(N, Z)$ corresponds to states of the same nucleus and is manifested, for example, in γ transitions and electron or nucleon scattering reactions, the excitation branches with charges $\Delta Q = \pm 1$ of the particle–hole pair correspond to excitations of the neighboring isobars $A(N - \Delta Q, Z + \Delta Q)$ and are manifested in the corresponding charge-exchange reactions, for example, (p, n), $(\bar{\nu}_e, e^-)$, (n, p), $(\bar{\nu}_e, e^+)$, or in β transitions.

The states characteristic of the charged excitation branches are usually called isobaric states.

Not only analog resonances but also other collective isobaric states have been observed in charge-exchange reactions. In the first place, there are the analogs of the giant dipole 1^- resonance (for example, of $n\bar{p}$ type[2]) and various collective isobaric 1^+ states. The basic possibility of the existence of collective isobaric 1^+ states with nearly the same structure as analog states was first noted by Ikeda, Fujii, and Fujita.[3] They pointed out the importance of these states for understanding the reasons for the suppression of allowed β transitions in nuclei with $N > Z$. In the early studies,[4-6] it was assumed that the region in which the collective isobaric 1^+ states are concentrated lies below the analog resonance on the energy scale. Isobaric 1^+ states were in fact found in this energy region, but their degree of collectivization was comparatively small. At the beginning of the seventies, the present authors[7] showed on the basis of the theory of finite Fermi systems that for spherical nuclei the most strongly collectivized 1^+ resonance, the Gamow–Teller resonance, must lie above the analog resonance, these states approaching each other with increasing A and $N - Z$.[8,9] In deformed nuclei, the physics of the Gamow–Teller resonance was investigated theoretically in Refs. 10 and 11. In this case, the picture is complicated by the splitting of the Gamow–Teller resonance with respect to the projection of the total angular momentum onto the symmetry axis of the nucleus.

For a long time, the experimental situation with regard to the collective isobaric 1^+ states, and above all the Gamow–Teller resonance, remained unresolved. In 1975, Galonsky's group began intensive searches for the Gamow–Teller resonance above the analog resonance using the direct (p, n) reaction.[12-14] At the present time, these studies are being continued in the United States with the cyclotron of the University of Indiana using protons of energy 50–200 MeV, a new magnetic-deflection technique, and the time-of-flight method of neutron detection. In a large cycle of experimental studies[15-20] during 1979–1980, the existence of the Gamow–Teller resonance in the region above the analog resonance was finally proved for spherical nuclei from light to heavy nuclei (^{208}Pb), the decreasing distance between these states with increasing $N - Z$ in the Zr isotopes was confirmed, and it was shown that in ^{208}Pb the analog resonance and the Gamow–Teller resonance are practically degenerate. The investigations simul-

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taneously confirmed the existence of a less collective 1^+ state below the analog resonance and discovered a spin–dipole giant resonance of $p\bar{n}$ type and, apparently, an analog of the magnetic dipole resonance of the neutral excitation branch.

This strong progress in the experimental results is again attracting interest to the problem of collective isobaric 1^+ states. The present review is devoted to a detailed analysis of the modern theoretical and experimental investigations in the region of spherical nuclei. On the basis of the theory of finite Fermi systems,[21] we analyze the origin of the various types of collective isobaric 1^+ states of spherical nuclei.

In Sec. 1, the equations of finite Fermi systems are solved numerically for specific spherical nuclei in a wide range of A values, namely, from As to La. In Sec. 2, we describe an approximate method of solving the equations which makes it possible to understand the nature of the different collective isobaric 1^+ states and to formulate a number of qualitative conclusions. In Sec. 3, we consider the important question of the phenomenological description of the Gamow–Teller resonance and the ground states of nuclei in the Wigner supermultiplet scheme. In Sec. 4, we discuss the experimental investigation of the Gamow–Teller resonance and other collective isobaric states. Finally, in Sec. 5 we apply the theory of isobaric 1^+ states to the prediction of the properties of nuclei far from the stability line. The review summarizes a series of papers by the authors on this subject.

The isobaric 1^+ states considered in the present review belong to the spin–isospin excitation branch. In recent years, A. B. Migdal and his collaborators have shown that the behavior of this branch at momentum transfers $q \sim m_\pi c$ is essentially related to the soft pionic excitation mode of nuclear matter.[22] In this connection, the theory of finite Fermi systems was reformulated with allowance for this mode.[23] However, in the present paper we consider the region of spin–isospin phenomena in which the influence of the mode is weak. This is the region of collective isobaric states corresponding to small momentum transfer: $q \approx 0$ (small changes Δn in the principal quantum number). At the present stage of the investigations of the spin–isospin excitation branch, it appears natural to consider the phenomena of particle–hole and pionic type separately and to compare the properties of the former with other types of collective isobaric states.

1. ISOBARIC 1⁺ STATES OF SPHERICAL NUCLEI

In the theory of finite Fermi systems, the parameters of isobaric 1^+ states are found by solving the equation for the effective field of Gamow–Teller type:

$$\left.\begin{aligned} &V_{\lambda\lambda'} = V^{\omega}_{\lambda\lambda'} + \sum_{\lambda_1\lambda_2} \Gamma^{\omega}_{\lambda\lambda'\lambda_1\lambda_2} A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{\nu_1\nu_2} \Gamma^{\omega}_{\lambda\lambda'\nu_1\nu_2} A_{\nu_1\nu_2} V_{\nu_2\nu_1};\\ &V_{\nu\nu'} = \sum_{\lambda_1\lambda_2} \Gamma^{\omega}_{\nu\nu'\lambda_1\lambda_2} A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{\nu_1\nu_2} \Gamma^{\omega}_{\nu\nu'\nu_1\nu_2} A_{\nu_1\nu_2} V_{\nu_2\nu_1};\\ &V^{\omega} = e_q \boldsymbol{\sigma\tau}^+; \quad A^{(p\bar{n})}_{\lambda\lambda'} = \frac{n^n_\lambda (1 - n^p_{\lambda'})}{\varepsilon^n_\lambda - \varepsilon^p_{\lambda'} + \omega}; \quad A^{(n\bar{p})}_{\lambda\lambda'} = \frac{n^p_\lambda (1 - n^n_{\lambda'})}{\varepsilon^p_\lambda - \varepsilon^n_{\lambda'} - \omega}. \end{aligned}\right\} \tag{1}$$

Here, the matrix elements of the effective field $V(\mathbf{r}, \omega)$ and the bare field $V^{\omega}(\mathbf{r})$ are taken between the single-particle functions of the quasiparticles in the neutron (n) and proton (p) potential of the nucleus $A(N, Z)$; n_λ and ε_λ are the population numbers and energies of the states λ in these potentials. The indices ν distinguish the matrix elements of the l-forbidden part of the effective field with the selection rules $\Delta l = 2$ (not allowed). The expression for Γ^{ω} contains not only the angular parts and radial integrals but also the coupling constant g'_0 of the effective spin–isospin interaction. The effective charge of the Gamow–Teller field is $e_q = 0.9$.

Equations (1) do not contain terms associated with allowance for the soft pionic mode. Analysis shows that allowance for pions leads, first, to an effective renormalization of the phenomenological constant g'_0,

$$g'_{\rm eff} = g'_0 - e_q^2 f_\pi^2 \frac{dn}{d\mu} \varkappa^{-2} \left[\frac{q^2}{1+q^2} + \frac{4\varkappa}{\pi m_\pi R} (1+q^2)^{-2} \right], \tag{2}$$

where $q = \pi \Delta n / m_\pi R$ [the constant $\varkappa \approx 0.5$ takes into account the influence of the $\Delta(1236)$ isobar, Δn is the change in the principal quantum number in the β transition, f_π is the coupling constant of the πN interaction, and m_π is the effective pion mass with allowance for the Δ isobar], and, second, to a modification of the l-forbidden terms as a result of the tensor nature of the interaction. For transitions with $\Delta n = 0$ and $\pi \Delta n \ll m_\pi R$, the effective constant differs little from g'_0. The pions can have an important influence in the calculations of l-forbidden and single-particle β transitions due to the influence of reflected waves, but for collective isobaric states this factor is comparable with the errors of the theory. Thus, Eqs. (1) are valid for calculations of collective isobaric states $[\log(ft) \leqslant 5]$ and must be modified with allowance for pions for l-forbidden and single-particle states.

The position of the isobaric 1^+ states relative to the ground state of the even–even nucleus $A(N, Z)$ is determined by the poles of the system of equations (1). The system gives frequency solutions of two signs. Solutions with frequency

$$\omega > M(N-1, Z+1) - M(N, Z) \tag{3}$$

[M is the mass of the nucleus $A(N, Z)$] are interpreted as isobaric $\bar{p}n$ states of charge $\Delta Q = +1$ and are associated with the levels of the nucleus $A(N-1, Z+1)$. Solutions with frequency

$$\omega < M(N, Z) - M(N+1, Z-1)$$

are interpreted as isobaric np states of charge $\Delta Q = -1$ and are associated with levels of the nucleus $A(N+1, Z-1)$. The important parameter of the problem is the mean relative displacement of the neutron and proton potentials. It was found earlier in a calculation of isobaric 0^+ states[24] and depends on the coupling constant f'_0 of the effective isospin interaction of the quasiparticles. In Ref. 24, the relative displacement of the single-particle potentials and the constant f'_0 were chosen in a mutually consistent manner to make the position of the analog 0^+ states exactly equal to the difference between the Coulomb energies of the corresponding nuclei. This procedure gave $f'_0 = 1.35$. A recent independent verification based on the energy splitting of the analog and antianalog states confirms this value. It should be noted that although this self-consistency scheme is simple and convenient for application in many nuclei, it is approximate. In the meanwhile, more rigorous self-consistency schemes have been developed (see, for example, Refs. 25–27). The results of calculations of individual nuclei in accordance with a complete scheme for a number of collective 1^+ states and the comparison with the present calculation are discussed in Sec. 4.

Equations (1) also make it possible to determine the matrix elements of the Gamow–Teller β transitions between the isobaric 1^+ states and the 0^+ ground state of the even–even nucleus $A(N, Z)$ in terms of the residues $\chi\lambda_1\lambda_2(\omega)$ of the field $V\lambda_1\lambda_2(\omega)$ at the pole $\omega=\omega_k$ corresponding to this isobaric state:

$$M_{GT}^2=\sum_{\lambda_1\lambda_2}\chi_{\lambda_1\lambda_2}A_{\lambda_1\lambda_2}V^{\omega}_{\lambda_2\lambda_1}. \quad (4)$$

The residues are normalized by

$$\Big[\sum_{\lambda_1\lambda_2}\chi_{\lambda_1\lambda_2}A_{\lambda_1\lambda_2}V^{\omega}_{\lambda_2\lambda_1}+\sum_{\nu_1\nu_2}\chi_{\nu_1\nu_2}A_{\nu_1\nu_2}V^{\omega}_{\nu_2\nu_1}\Big]_{\omega=\omega_k}$$
$$=-\Big[\Big|\sum_{\lambda_1\lambda_2}\chi_{\lambda_1\lambda_2}\frac{dA_{\lambda_1\lambda_2}}{d\omega}\chi_{\lambda_2\lambda_1}+\sum_{\nu_1\nu_2}\chi_{\nu_1\nu_2}\frac{dA_{\nu_1\nu_2}}{d\omega}\chi_{\nu_2\nu_1}\Big|\Big]_{\omega=\omega_k} \quad (5)$$

and determine the contribution of the particle–hole configurations $\lambda_1\lambda_2(\nu_1\nu_2)$ to the isobaric state ω_k and thus the degree of its collectivization.

This method of calculations for a large group of spherical nuclei from ^{72}As to ^{140}Pr was used in a numerical computer calculation to find the positions of the isobaric $p\bar{n}$ 1^+ states and the matrix elements of β^+ decay of these states to the 0^+ ground state of the neighboring even–even nucleus $A(N, Z)$, which serves as the unexcited core for the formation of these states. In the scheme, the energies of the single-particle levels of the potentials of the nucleus $A(N, Z)$ were calculated by approximating the single-particle Woods–Saxon potentials of the nearest magic nuclei, and the oscillator approximation was used for the wave functions. In the system of equations (1), the summation was over a large basis of single-particle states from two shells (plus–minus one shell from the edge), i.e., allowance was made for transitions with $\Delta n=0$ and partly with $\Delta n=\pm1$. More distant single-particle transitions were taken into account by a renormalization of the constant f_0', as described in Ref. 24. For nonmagic nuclei, pairing was also taken into account by means of the usual substitution $\varepsilon_\lambda\to E_\lambda=\sqrt{\varepsilon_\lambda^2+\Delta_\lambda^2}$. The energy Δ_λ was calculated in accordance with the theory of finite Fermi systems. The effective coupling constant of the spin–isospin interaction was varied in the range $g_0'=1.0$–1.3.[1)]

The numerical calculations reveal the following features of the structure of the isobaric $p\bar{n}$ 1^+ states. Among the collective states, one is clearly distinguished with matrix element $M^2\sim N-Z$ and separated in energy from the remainder. The main contribution to the structure of the wave function of this state is made by the configurations $j_p=l-\frac{1}{2}$ and $j_{\bar{n}}=l+\frac{1}{2}$, which correspond to transition of a neutron into a proton with spin flip. Lower in energy, there is a group of collective isobaric $p\bar{n}$ states that are less collectivized. The matrix elements of the β decay of these states is $M_{GT}^2\approx0.1$–1.0, whereas the matrix elements of the β decay of the single-particle states correspond to $M_{GT}^2\approx10^{-2}$–10^{-3}. The main contribution to the structure of the wave functions for them are made by the configurations $j_p=l\pm\frac{1}{2}=j_{\bar{n}}$, corresponding to transitions of a neutron into a proton with reversal of the total angular momentum. In addition, configurations with spin flip ($j_p=l-\frac{1}{2}$, $j_{\bar{n}}=l+\frac{1}{2}$) may play a part. Finally, in the cases when this is permitted by the selection rules, the low-lying $p\bar{n}$ states may include collective 1^+ states corresponding to transitions of a neutron into a proton with the opposite spin flip: $j_p=l+\frac{1}{2}$, $j_{\bar{n}}=l-\frac{1}{2}$. All these features are clearly revealed in the quasiclassical model of isobaric states.

The results of the numerical calculations are presented in Tables I–III. Tables I, IIa, and IIb contain the main characteristics and structure of the most strongly collectivized of the investigated isobaric states, i.e., the Gamow–Teller resonance; Table III gives the characteristics of the other collective isobaric 1^+ states. In Table I, we give the calculated position of E, the energy of the Gamow–Teller resonance formed on the basis of the even–even nucleus $A(N, Z)$, relative to the ground state of the odd–odd nucleus $A(N-1, Z+1)$, in which it is actually situated, and the square of the matrix element M_{GTR}^2 of the β decay of this state to the 0^+ ground state of the nucleus $A(N, Z)$. The calculations were made for the two values $g_0'=1.0$ and 1.3 of the effective coupling constant of the spin–isospin interaction. The values of the frequencies for constants that are slightly different can be obtained by linear interpolation. The assumed error in the calculations of the energy of the resonance is 1.0–2.0 MeV, which is less than the assumed width of the Gamow–Teller resonance.

The structure of the wave function of the Gamow–Teller resonance is illustrated in Tables IIa and IIb. In them, we give the results of calculation of the contributions of the individual configurations $\lambda_1\lambda_2$ to the structure of the Gamow–Teller resonance. As can be seen from Table II, the Gamow-Teller resonance is basically formed through transitions of spin–orbit type ($l+\frac{1}{2}\to l-\frac{1}{2}$) with spin flip and partly by j–j transitions. For experimental investigations of isobaric 1^+ states, the parameters of the other collective states in the particular nuclei are also important. The results of numerical calculations for the most strongly collectivized of them in each of the investigated nuclei are given in Table III. The energies E of the isobaric states are given relative to the ground states of the $A(N-1, Z+1)$ nuclei. For each state, we also give the values of $\log(ft)$ of the β^+ decay to the ground state of the nucleus $A(N, Z)$, determined from the calculated values of M_{GT}^2. The frequencies and matrix elements were found with allowance for the quasiclassical corrections for $\Delta n>0$ transitions (Δn is the change of the principal quantum

1) The given value of g_0' has a normalization different from the constant in Refs. 26 and 27. They are related as follows: g_0' (Refs. 26 and 27) $=g_0'4/\pi$.

TABLE I. Positions and matrix elements of 1^+-0^+ β decays of Gamow–Teller resonances.

Nucleus $A\,(N-1,\,Z+1)$	$g_0' = 1.0$		$g_0' = 1.3$	
	E, MeV	M^2_{GTR}	E, MeV	M^2_{GTR}
^{72}As	8.93	5.14	10.23	5.42
^{74}As	11.11	6.59	12.65	6.70
^{76}As	12.17	7.89	13.98	8.35
^{78}As	13.97	9.23	16.06	9.86
^{74}Br	6.0	3.18	7.76	3.43
^{76}Br	9.16	4.52	10.17	4.92
^{75}Br	10.07	6.22	11.41	6.66
^{80}Br	10.78	7.66	13.42	8.25
^{82}Br	12.11	8.57	14.11	9.95
^{80}Rb	9.86	4.34	10.84	4.84
^{82}Rb	10.82	5.67	11.97	6.23
^{84}Rb	11.97	7.39	13.46	7.97
^{86}Rb	13.54	9.33	15.37	9.66
^{88}Rb	14.38	10.57	16.04	11.15
^{86}Y	10.42	6.09	11.60	6.59
^{88}Y	11.49	7.74	12.98	8.21
^{90}Y	13.62	8.61	14.35	9.19
^{92}Y	15.04	10.50	15.93	11.17
^{88}Nb	8.66	4.98	9.62	5.22
^{90}Nb	9.03	6.50	10.29	6.80
^{92}Nb	13.65	7.35	15.02	7.79
^{94}Nb	14.12	8.56	15.63	9.19
^{96}Nb	14.29	9.93	15.99	10.65
^{92}Tc	9.23	6.29	10.40	6.61
^{94}Tc	12.76	7.60	14.13	8.09
^{96}Tc	12.70	8.66	15.21	9.33
^{98}Tc	13.79	10.21	15.49	11.00
^{100}Tc	14.90	10.44	16.70	11.36
^{102}Tc	15.16	12.38	17.20	13.32
^{98}Rh	12.97	8.63	14.42	9.32
^{100}Rh	12.88	9.89	14.49	10.76
^{102}Rh	13.54	10.87	15.28	11.83
^{104}Rh	14.04	11.66	15.92	12.68
^{106}Rh	14.08	12.85	16.14	13.88
^{102}Ag	12.72	9.27	14.23	10.39
^{104}Ag	12.98	10.09	14.59	11.15
^{106}Ag	13.69	10.71	15.39	11.77
^{108}Ag	13.35	11.65	15.15	12.68
^{110}Ag	12.77	12.49	14.68	13.58
^{112}Ag	13.91	12.79	15.86	13.96
^{106}In	10.97	9.36	12.43	10.44
^{108}In	12.11	9.98	13.86	11.09
^{110}In	12.20	10.79	12.82	11.79
^{112}In	12.90	11.19	14.57	12.27
^{114}In	13.18	11.45	14.91	12.83
^{116}In	13.47	13.62	15.44	14.78
^{118}In	14.58	14.94	16.72	16.14
^{114}Sb	9.71	9.15	11.06	10.08
^{116}Sb	10.93	10.17	12.60	11.28
^{118}Sb	11.02	12.19	12.73	13.32
^{120}Sb	11.79	13.59	13.68	14.69
^{122}Sb	12.47	15.47	14.60	16.48
^{124}Sb	13.26	17.07	15.59	18.04
^{120}I	10.27	10.94	11.77	12.13
^{122}I	11.02	12.31	12.69	13 39
^{124}I	11.54	13.34	13.58	14.67
^{126}I	12.62	15.54	14.69	16.52
^{128}I	13.68	16.06	15.82	17.45
^{130}I	14.39	17.85	16.76	19.05
^{132}I	15.23	19.38	17.30	20.50
^{126}Cs	10.99	12.11	12.58	13.20
^{128}Cs	11.61	13.21	13.33	14.37
^{130}Cs	12.68	14.56	14.57	15.95
^{132}Cs	13.72	16.27	15.82	17.51
^{134}Cs	14.02	17.99	16.23	19.11
^{136}Cs	15.00	19.77	17.53	20.78
^{130}La	10.48	12.09	12.01	13.16
^{132}La	11.80	12.17	13.36	13.65
^{134}La	12.54	14.28	14.33	15.52
^{136}La	13.18	16.59	15.25	17.73
^{138}La	14.13	18.31	16.41	19.35
^{138}Pr	12.61	14.75	14.41	15.89
^{140}Pr	13.39	16.40	15.39	17.47

number in the configuration $\lambda_1\lambda_2$); these will be described below.

Analysis of the numerical results and comparison of them with the parameters of the analog states of the corresponding nuclei make it possible to draw a number of qualitative conclusions about the properties of the Gamow–Teller resonance and the collective isobaric states.

1. Among the isobaric 1^+ states of even–even nuclei, there always exists a high-lying $p\bar{n}$ state with maximal degree of collectivization; this is the Gamow–Teller isobaric 1^+ resonance. In the central group of spherical nuclei (As–La), the Gamow–Teller resonance is situated 3–6 MeV above the analog resonance.

2. There is a general tendency for the Gamow–Teller resonance and the analog resonance to get closer together when the mass number A and the difference $N-Z$ increase.

TABLE IIa. Contribution of single-particle $p\bar{n}$ transitions, %, to the structure of the Gamow–Teller resonance (f–g shell).

Nucleus $A\,(N-1,\,Z+1)$	$1f_{7/2}-1f_{5/2}$	$2p_{3/2}-2p_{1/2}$	$1g_{9/2}-1g_{7/2}$	$2d_{5/2}-2d_{3/2}$	$j-j$
^{72}As	90	6	0	—	4
^{74}As	61	5	27	—	6
^{76}As	50	5	36	—	8
^{78}As	33	5	50	—	10
^{74}Br	92	6	0	—	2
^{76}Br	55	5	36	—	4
^{78}Br	44	5	44	—	6
^{80}Br	27	4	60	—	8
^{82}Br	25	4	60	—	11
^{80}Rb	17	5	74	—	4
^{82}Rb	10	4	80	—	6
^{84}Rb	9	4	78	—	8
^{86}Rb	8	4	76	—	11
^{88}Rb	1	5	82	2	10
^{86}Y	3	4	87	0	6
^{88}Y	2	4	85	0	9
^{90}Y	0	4	82	3	11
^{92}Y	0	4	70	10	15
^{88}Nb	0	0	95	0	5
^{90}Nb	0	0	92	0	8
^{92}Nb	0	0	89	2	9
^{94}Nb	0	0	83	6	11
^{96}Nb	0	0	76	10	14

TABLE IIb. Contribution of single-particle $p\bar{n}$ transitions, %, to the structure of the Gamow–Teller resonance (g–h shell).

Nucleus $A\,(N-1,\,Z+1)$	$1g_{9/2}-1g_{7/2}$	$2d_{5/2}-2d_{3/2}$	$1h_{11/2}-1h_{9/2}$	$j-j$	$l-1/2 \to l+1/2$
^{92}Tc	93	0	0	6	1
^{94}Tc	88	2	1	8	1
^{96}Tc	82	6	2	8	1
^{98}Tc	74	10	2	11	2
^{100}Tc	72	7	5	13	3
^{102}Tc	60	11	6	16	6
^{98}Rh	81	7	2	7	2
^{100}Rh	74	10	3	7	5
^{102}Rh	68	11	5	11	5
^{104}Rh	63	11	7	12	6
^{106}Rh	57	11	8	16	7
^{102}Ag	70	9	11	7	2
^{104}Ag	70	11	7	8	3
^{106}Ag	66	11	8	9	5
^{108}Ag	62	12	9	12	5
^{110}Ag	57	12	11	14	5
^{112}Ag	57	14	8	16	5
^{106}In	72	11	8	6	4
^{108}In	68	12	9	9	2
^{110}In	63	13	10	11	2
^{112}In	60	13	12	12	3
^{114}In	59	14	9	15	3
^{116}In	51	14	12	19	4
^{118}In	46	12	16	20	6
^{114}Sb	70	15	5	10	0
^{116}Sb	62	14	10	13	1
^{118}Sb	51	16	13	18	2
^{120}Sb	46	15	17	18	4
^{122}Sb	41	15	20	19	5
^{124}Sb	38	14	23	19	6
^{120}I	52	16	15	15	2
^{122}I	48	15	17	16	4
^{124}I	43	13	27	13	4
^{126}I	37	14	25	19	5
^{128}I	28	12	37	17	5
^{130}I	27	10	39	18	5
^{132}I	25	10	41	19	5
^{126}Cs	40	16	23	17	4
^{128}Cs	37	12	30	17	4
^{130}Cs	28	11	40	17	4
^{132}Cs	25	10	43	18	4
^{134}Cs	23	10	44	19	4
^{136}Cs	21	9	45	20	5
^{130}La	34	15	30	16	4
^{132}La	23	10	50	14	3
^{134}La	21	9	57	11	2
^{136}La	20	10	49	17	4
^{138}La	19	10	49	18	4
^{138}Pr	17	10	54	15	3
^{140}Pr	16	9	56	16	3

TABLE III. Positions and log(ft) of the 1^+-0^+ β^+ decays of isobaric configuration 1^+ states.

Nucleus $A(N-1, Z+1)$	E_1, MeV	lg (ft_1)	E_2, MeV	lg (ft_2)	E_3, MeV	lg (ft_3)	E_4, MeV	lg (ft_4)
^{72}As	—	—	3.20	3.6	0.87	4.5	—	—
^{74}As	7.41	4.6	4.58	3.6	3.85	5.3	1.81	4.5
^{76}As	7.21	4.9	5.00	3.6	5.85	5.4	1.53	4.6
^{78}As	9.87	5.3	7.62	3.9	6.71	4.6	5.80	3.8
^{74}Br	2.78	3.5	2.05	5.1	0.18	4.5	—	—
^{76}Br	6.54	5.0	3.96	3.5	2.67	5.0	1.45	4.6
^{78}Br	6.28	5.6	3.90	3.5	2.99	4.9	1.62	4.6
^{80}Br	6.48	4.1	4.44	3.5	3.98	4.6	1.50	4.7
^{82}Br	5.84	4.3	4.19	3.6	3.66	4.3	0.87	4.3
^{80}Rb	6.24	3.9	4.83	3.8	2.23	4.0	1.21	4.4
^{82}Rb	5.14	3.6	3.80	3.8	2.18	3.9	0.28	5.0
^{84}Rb	5.52	3.6	4.44	4.0	2.99	4.0	0.76	4.9
^{86}Rb	6.43	3.6	5.65	4.3	4.17	3.9	1.86	4.8
^{88}Rb	7.76	3.6	7.49	4.6	6.27	4.0	4.29	4.1
^{86}Y	4.41	4.0	3.70	3.8	2.92	3.9	1.31	4.1
^{88}Y	4.99	4.1	4.46	4.0	3.88	3.9	1.98	4.0
^{90}Y	8.41	3.6	7.02	4.0	4.97	4.0	—	—
^{92}Y	8.82	3.7	7.11	4.0	5.96	4.6	4.82	4.0
^{88}Nb	4.08	6.3	1.50	3.5	1.42	5.6	—	—
^{90}Nb	1.69	3.5	0.47	6.0	—	—	—	—
^{92}Nb	6.97	3.7	5.05	3.9	3.37	5.3	3.54	5.1
^{94}Nb	8.04	3.5	6.24	4.1	5.37	3.7	3.64	5.8
^{96}Nb	9.04	3.6	7.14	4.4	5.96	3.7	4.65	4.6
^{92}Tc	3.48	5.3	1.51	3.9	0.87	3.7	—	—
^{94}Tc	6.32	3.7	5.21	5.2	4.71	4.0	4.26	3.9
^{96}Tc	8.75	3.6	6.66	4.6	5.61	3.8	3.87	4.3
^{98}Tc	11.51	4.8	8.54	3.6	6.81	4.4	5.49	3.7
^{100}Tc	12.40	4.6	7.74	3.3	6.55	4.4	5.24	4.5
^{102}Tc	11.91	4.5	9.39	3.6	7.92	4.2	6.34	3.8
^{98}Rh	11.37	5.0	6.72	3.7	6.21	4.6	5.02	3.6
^{100}Rh	10.91	4.7	7.84	3.6	6.01	4.5	4.75	3.7
^{102}Rh	11.41	4.5	8.60	3.6	6.89	4.3	5.58	3.8
^{104}Rh	11.25	4.4	8.69	3.6	7.07	4.2	5.72	3.8
^{106}Rh	10.75	4.3	8.42	3.6	7.55	4.4	5.78	3.8
^{102}Ag	10.64	4.2	7.66	3.6	5.77	4.7	4.56	3.8
^{104}Ag	10.95	4.4	8.14	3.6	6.19	4.5	4.66	3.8
^{106}Ag	11.23	4.3	8.69	3.6	6.89	4.4	5.64	3.8
^{108}Ag	10.49	4.3	8.23	3.6	7.17	4.3	5.47	3.9
^{110}Ag	9.56	4.1	7.52	3.6	6.59	4.3	4.75	4.0
^{112}Ag	10.96	4.3	8.88	3.6	8.19	4.6	6.10	4.0
^{106}In	9.14	4.3	6.33	3.6	4.40	4.6	1.26	4.4
^{108}In	10.17	4.2	7.59	3.6	5.72	4.4	4.56	3.9
^{110}In	8.67	4.2	6.57	3.6	5.29	4.2	3.73	3.9
^{112}In	10.19	4.1	9.12	3.6	7.12	4.3	5.34	4.0
^{114}In	10.66	4.1	8.55	3.6	7.75	4.4	5.81	4.0
^{116}In	10.08	4.0	8.56	3.7	7.76	4.5	4.63	4.4
^{118}In	10.46	3.7	9.10	3.8	4.99	4.5	4.10	4.7
^{114}Sb	8.31	4.4	5.77	3.6	5.72	4.4	2.90	3.9
^{116}Sb	9.00	4.1	5.78	3.6	5.90	4.4	4.10	4.0
^{118}Sb	8.13	3.9	6.10	3.8	5.66	4.3	2.79	4.4
^{120}Sb	8.12	3.7	6.90	4.1	6.16	4.3	1.08	4.5
^{122}Sb	8.01	3.6	6.98	4.6	6.56	4.6	2.84	4.5
^{124}Sb	8.10	3.6	6.19	4.6	5.37	4.3	3.00	4.5
^{120}I	7.61	3.9	7.07	3.7	5.97	4.1	2.46	4.3
^{122}I	7.64	3.8	6.59	3.8	5.61	4.3	2.05	4.5
^{124}I	7.21	3.7	6.00	3.8	5.12	4.3	2.14	4.4
^{126}I	7.93	3.6	6.48	4.4	3.12	4.6	0.94	4.3
^{128}I	8.56	3.4	7.24	4.7	6.91	4.5	3.10	4.4
^{130}I	8.45	3.4	7.67	4.7	3.27	4.5	1.85	4.5
^{132}I	8.15	3.4	7.37	5.0	3.01	4.6	1.62	4.5
^{126}Cs	7.47	3.7	6.78	4.1	5.77	4.2	0.69	4.3
^{128}Cs	7.54	3.6	6.81	3.9	5.83	4.7	0.97	4.3
^{130}Cs	7.96	3.4	6.87	4.2	6.19	4.5	3.07	4.2
^{132}Cs	8.37	3.4	7.35	4.5	6.81	4.6	2.00	4.4
^{134}Cs	7.96	3.4	6.54	4.7	3.19	4.6	1.77	4.3
^{136}Cs	8.35	3.4	7.34	4.7	3.69	4.5	2.09	4.3
^{130}La	6.86	3.6	6.40	4.6	5.35	4.2	2.52	4.2
^{132}La	7.87	3.4	6.78	3.9	5.88	4.3	2.96	4.6
^{134}La	7.65	3.4	6.98	4.3	6.46	4.3	1.79	4.2
^{136}La	7.64	3.4	7.01	4.7	6.39	4.5	1.71	4.2
^{138}La	7.98	3.4	6.73	4.6	3.49	4.6	2.06	4.2
^{138}Pr	7.63	3.4	7.04	4.2	6.16	4.3	1.97	4.0
^{140}Pr	7.71	3.4	7.24	4.5	6.44	4.4	2.19	4.0

3. The square of the matrix element of the β^+ decay of the Gamow–Teller resonance to the ground state of the nucleus $A(N, Z)$ is near $N-Z$.

4. The structure of the Gamow–Teller resonance is basically determined by transitions of neutrons to proton states with spin flip.

5. As a rule, the group of collective isobaric 1^+ states is situated below the analog state and includes three or four states with $\log(ft)\approx 3.5-4.5$.

2. QUASICLASSICAL DESCRIPTION OF COLLECTIVE ISOBARIC 1⁺ STATES (BETA MODEL)

To investigate the physical picture of the formation of collective isobaric states and elucidate the principal quantitative results obtained in the numerical solution of the equations of the theory, we construct an approximate analytic method for solving Eqs. (1) (the so-called beta model). We developed it earlier[24] for isobaric 0^+ states. Application of the model to 1^+ states required a modification associated with the average spin properties of the system. Since a quasiclassical averaging of the single-particle characteristics is used in the method, it can be called a quasiclassical method.

In Eqs. (1), we ignore the l-forbidden terms (as a rule, the error resulting from this approximation is of the order of a few percent) and go over to the r representation:

$$V(r)=V^{\omega}(r)+\frac{g_0'}{4\pi\, d\rho/d\mu}\sum_{\lambda_1\lambda_2} R^{+}_{\lambda_1}(r)\, R_{\lambda_2}(r)\, \beta_{\lambda_1\lambda_2} A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1}(r). \qquad (6)$$

Here, $R_\lambda(r)$ are the radial parts of the single-particle wave functions $\varphi_\lambda(r)$; $\beta_{\lambda_1\lambda_2}$ are the coefficients associated with the angular parts:

$\beta_{\lambda_1\lambda_2}$	$j_1=l-1/2$	$j_1=l+1/2$
$j_2=l-\frac{1}{2}$	$\frac{2j_1+1}{3}\left(1-\frac{1}{l+1/2}\right)$	$\frac{2j_1+1}{3}\left(2-\frac{1}{l+1/2}\right)$
$j_2=l+\frac{1}{2}$	$\frac{2j_1+1}{3}\left(2+\frac{1}{l+1/2}\right)$	$\frac{2j_1+1}{3}\left(1+\frac{1}{l+1/2}\right)$.

We introduce average energy parameters of the nucleus $A(N, Z)$: E_{ls} is the average energy of the spin–orbit splitting, ΔE is the energy width of the layer of excess neutrons, and $\mu_\pm$ is the chemical potential determined by the mass differences of the nuclei $A(N, Z)$ and $A(N\pm 1, Z\pm 1)$. We have

$$\left.\begin{aligned} &E_{ls}=\sum_{\lambda_1\lambda_2} n_{\lambda_1}(1-n_{\lambda_2})\,\varepsilon^{ls}_{\lambda_1\lambda_2}\Big/\sum_{\lambda_1\lambda_2} n_{\lambda_1}(1-n_{\lambda_2});\\ &\Delta E=(4/3)\,\varepsilon_F\,(N-Z)/A\,(\varepsilon_F\approx 40\ \text{MeV}). \end{aligned}\right\} \qquad (7)$$

We split the sum in Eq. (6) into three parts in accordance with the three types of particle–hole configurations $p\bar{n}$ (or $n\bar{p}$) coupled by the selection rules in accordance with the angular momenta j_p and $j_{\bar{n}}$ (j_n and $j_{\bar{p}}$). In the first group, we put the $p\bar{n}$ transitions with $\Delta j=j_p-j_{\bar{n}}=0$. If the principal quantum number does not change in the transition ($\Delta n=0$), this group corresponds to transitions of the $N-Z$ excess neutrons with reversal of the total angular momentum. In the second group, we put the $p\bar{n}$ transitions with $j_{\bar{n}}-j_p=1$, corresponding to transitions of neutrons into protons with spin flip, and in the third group we put the $n\bar{p}$ transitions with $j_n-j_{\bar{p}}=-1$ or the reverse $p\bar{n}$ transitions $j_{\bar{n}}-j_p=-1$ with opposite spin flip, which arise at large $N-Z$ ($\Delta E > E_{ls}$). Note that in the last group different signs of the frequency correspond to the $p\bar{n}$ and $n\bar{p}$ transitions in our equation.

We express $A\lambda_1\lambda_2$ in terms of the average energy parameters and expand the effective field $V(r)$ in the eigenfunctions of the integral equation that are associated with the different types of radial symmetry of the effective field. Then for the symmetry of each type we obtain a secular equation to determine the eigenfrequencies corresponding to this symmetry. Thus, for the fields $V(r)=\text{const}$ the secular equation has the form

$$\frac{1}{g_0'}=\frac{\Delta E}{\Omega}a+\frac{\Delta E+E_{ls}}{\Omega-E_{ls}}b_+ +\frac{\Delta E-E_{ls}}{\Omega+E_{ls}}b_-,$$
$$\Omega=\omega-\mu;\quad a=\sum_{\lambda}R_{\lambda}^2(r)\,\beta_{\lambda\lambda}\delta_{l_1l_2}; \tag{8}$$
$$b_\pm=\sum_{\lambda_1\lambda_2}R_{\lambda_1}(r)\,R_{\lambda_2}(r)\,\beta_{\lambda_1\lambda_2}^{(\pm)}\delta_{l_1l_2}\,(j_1-j_2=\pm1).$$

In the approximation of completeness of the system of functions R_λ, which corresponds to constancy of the density of the excess neutrons and is approximately satisfied for sufficiently heavy nuclei ($A\gtrsim 120$) and large $N-Z\gtrsim 20$,

$$a\approx 1/3;\ b_\pm\approx(1/3)\,[1\mp(2A)^{-1/3}];\ b=b_+ + b_-\approx 2/3. \tag{9}$$

The solutions of Eq. (8) describe the frequencies of three nominal collective 1^+ states of isobaric type. The frequency ω_+ corresponds to the Gamow–Teller resonance (state of spin-flip type); the frequency ω_0, to a collective state of $j-j$ type (state of core-polarization type) formed mainly by quasiparticle $p\bar{n}$ transitions with $\Delta j=0$; and the frequency ω_-, to a collective state associated with $j_{\bar{n}}-j_p=-1$ transitions (of $p\bar{n}$ type, a state of reverse spin-flip type) or, alternatively, $j_{\bar{p}}-j_n=-1$ ($n\bar{p}$ type, states of spin-flip type with reversed sign of the charge). Analytic expressions for the solutions can be conveniently represented relative to the analog resonance:

$$\frac{\omega_+-\omega_{AP}}{E_{ls}}\approx\frac{b\,(1+bg_0')\,g_0'\Delta E/E_{ls}}{(a+b)\,(g_0'\Delta E/E_{ls})^2+[1+2\,(a+b)\,g_0']/3A^{1/3}} \tag{10}$$
$$(\text{for } \Delta E>E_{ls},\ g_0'=f_0');$$
$$\frac{\omega_+-\omega_{AP}}{E_{ls}}\approx b\,\frac{1+bg_0'}{(a+b)\,g_0'}\,\frac{E_{ls}}{\Delta E}\ (\text{for } \Delta E>2E_{ls}); \tag{11}$$
$$\frac{\omega_{AP}-\omega_0}{\Delta E}=f_0'-\frac{ag_0'}{1+bg_0'}\approx bf_0'\ (\text{for } g_0'=f_0'); \tag{12}$$
$$\frac{\omega_0-\omega_-}{E_{ls}}\approx\frac{1+bg_0'}{(a+b)\,g_0'}\left(\frac{E_{ls}}{\Delta E}+ag_0'\right)\ (\text{for } \Delta E>E_{ls}). \tag{13}$$

In Fig. 1, in the relative units $\Delta E/E_{ls}$, we show the solution of Eq. (8), and also the most collective configuration state $\omega_+^{(1)}$ [see the relation (17)] for $f_0'=g_0'=1.35$. For $\Delta E=0$ ($N=Z$), the Gamow–Teller resonance goes over into the $p\bar{n}$ state symmetric with respect to the $n\bar{p}$ state ω_- and forms with it an isotriplet (the Coulomb shift is included in $\mu_\pm$). In the limit $\Delta E\to 0$, the collective $j-j$ state ω_0 tends to zero together with the analog resonance. With increasing ΔE [$(N-Z)/A$ increases], the Gamow–Teller resonance approaches the analog resonance asymptotically, which is illustrated in Fig. 2 (the analytic solutions are given for $f_0'=1.35, g_0'=1.3$). As can be seen from the figure, the quasiclassical method satisfactorily describes the

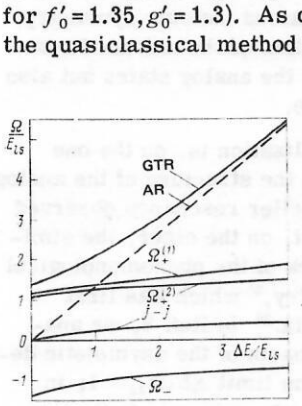

FIG. 1. Energies of the most collective isobaric 1^+ states as functions of the dimensionless quantity $\Delta E/E_{ls}\sim(N-Z)/A$ [solutions of Eq. (8)].

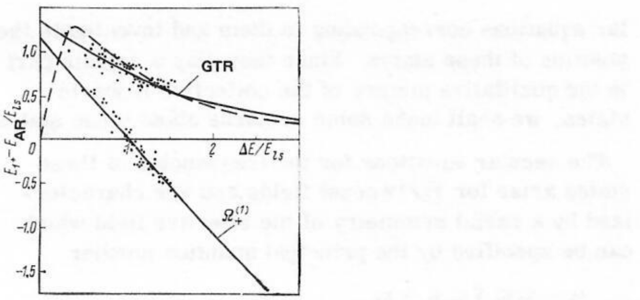

FIG. 2. Quasiclassical (curves) and numerical (points) values of the energies of the Gamow–Teller resonance and the most intense isobaric configuration 1^+ state relative to the energy of the analog resonance. The broken curve is the approximation (10).

exact solutions of the equations of the theory of finite Fermi systems. In the same figure, we show the solution for one of the most intense collective isobaric configuration states near the analog resonance for nuclei with $E_{ls}\approx\Delta E$.

The quasiclassical approximation can also be used to calculate the matrix elements of the β decay of isobaric states in accordance with Eqs. (4) and (5). For the Gamow–Teller resonance when $\Delta E>E_{ls}$,

$$M_{GTR}^2=e_q^2(N-Z)\,(1-\delta),\quad \delta\approx\frac{2aE_{ls}^2}{g_0'\,(E_{ls}^2+\Delta E^2)}, \tag{14}$$

where δ characterizes the admixture of transitions of other types to the $j_{\bar{n}}-j_p=1$ transitions, which basically form the Gamow–Teller resonance as a collective state. The purity of the Gamow–Teller resonance as a state of spin-flip type increases with increasing $N-Z$ (see Fig. 3). The parameter δ is evidently associated with the width of the Gamow–Teller resonance.

The numerical calculations show that besides the collective isobaric states with the radial symmetry of the effective field $V(r)=\text{const}$ one can have other less collective isobaric states with a different (higher in Δn) radial symmetry of the effective field: $V(r)=\alpha+\beta r^2$, $V(r)=\alpha+\beta r^2+\gamma r^4$, etc. The appearance of such isobaric states (we call them collective isobaric configuration states) depends on the particular population of the shells and need not occur in the general case. In the quasiclassical approximation, one can obtain the secu-

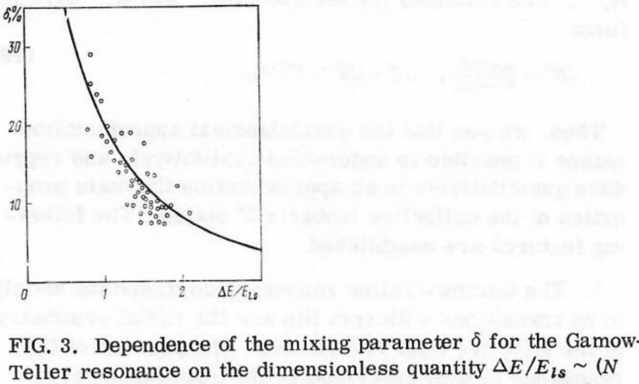

FIG. 3. Dependence of the mixing parameter δ for the Gamow–Teller resonance on the dimensionless quantity $\Delta E/E_{ls}\sim(N-Z)/A$. The open circles are numerical solutions, and the curve is based on Eq. (14).

lar equations corresponding to them and investigate the position of these states. Since they play a certain part in the qualitative picture of the collective isobaric states, we shall make some remarks about these states.

The secular equations for the frequencies of these states arise for $V(r) \neq$ const fields and are characterized by a radial symmetry of the effective field which can be specified by the principal quantum number

$$n_1 - n_2 \leqslant N \leqslant n_1 + n_2,$$

where n_1 and n_2 are the principal quantum numbers for the particles and holes which participate in the transitions. The secular equations for $\Delta N \neq 0$ have the form

$$1 = \frac{g_0'}{2} \sum_{m=0}^{N} \Big[\Big(\frac{\Delta E + E_m}{\Omega - E_m} + \frac{\Delta E - E_m}{\Omega + E_m} \Big) a_m + \Big(\frac{\Delta E + E_m + E_{ls}}{\Omega - E_m - E_{ls}} + \frac{\Delta E - E_m + E_{ls}}{\Omega + E_m - E_{ls}} \Big) b_+^{(m)} + \Big(\frac{\Delta E + E_m - E_{ls}}{\Omega + E_m + E_{ls}} + \frac{\Delta E - E_m - E_{ls}}{\Omega - E_m + E_{ls}} \Big) b_-^{(m)} \Big], \tag{15}$$

where E_m is the mean distance between the shells with $\Delta n = k$. The quantities a_m and b_m can be described approximately by constants:

$$a_m \approx ap_m;\ b_\pm^{(m)} \approx b_\pm p_m;\ b_m \approx bp_m;\ p_m \approx (2m+1)^{-1}. \tag{16}$$

Analysis shows that among the solutions of these equations the most collective states are $\Omega_+^{(N)}$, for which one can obtain approximate analytic expressions (here $k = N$):

$$\Omega_+^{(k)} = \omega_+^{(k)} - \mu = X_+^{(k)} E_{ls} - \frac{g_k a_k (1 + b_k g_k)\,\Delta E}{X_+^{(k)} (a_k + b_k) g_k (\Delta E/E_{ls}) - (1 + b_k g_k)}, \tag{17}$$

$$2X_\pm^{(k)} = (a_k + b_k) g_k \Delta E/E_{ls} \pm [(a_k + b_k)^2 g_k^2 (\Delta E/E_{ls})^2 + 4(1 + b_k g_k)]^{1/2},$$

where g_k are the renormalized values of the constant of the effective interaction:

$$g_k = g_0' (1 + \alpha_k g_0')^{-1};\ \alpha_k = 2p_k \beta_k (1 + 2g_0' \beta_k)^{-1};\ \beta_k = \sum_{m=1}^{k} p_m. \tag{18}$$

As for the main isobaric branch with $k = 0$, the quasiclassical formulas satisfactorily describe the numerical solutions of Eq. (1). In Fig. 2 we compare the exact and quasiclassical values for the frequencies of the state $\Omega_+^{(1)}$. The solutions for the states $\Omega_0^{(k)}$ and $\Omega_-^{(k)}$ have the form

$$\Omega_0^{(k)} = \frac{a_k g_k \Delta E}{1 + b_k g_k};\quad \Omega_-^{(k)} = \Omega_0^{(k)} - X_-^{(k)} E_{ls}. \tag{19}$$

Thus, we see that the quasiclassical approximation makes it possible to understand qualitatively and reproduce quantitatively in an approximation the main properties of the collective isobaric 1^+ states. The following features are established.

1. The Gamow–Teller resonance corresponds mainly to $p\bar{n}$ transitions with spin flip and the radial symmetry of the effective field $V(r) =$ const. The position of the resonance is well described by the quasiclassical approximation, which shows that it must tend asymptotically from above to the analog resonance with increasing value of the parameter $\Delta E/E_{ls}$. The matrix element of the β^+ decay of the Gamow–Teller resonance is close to the value $e_q\sqrt{N-Z} = 0.9\sqrt{N-Z}$.

2. Besides the Gamow–Teller resonance, there are two collective isobaric 1^+ states with the same radial symmetry:

a) A collective $j-j$ state (of core polarization type). It corresponds to $p\bar{n}$ transitions with reversal of the total angular momentum ($\Delta j = 0$), is situated below the Gamow–Teller resonance, and has $M_{\mathrm{GT}}^2 \lesssim 1$. For it, the approximate rule $(E_{\mathrm{AR}} = E_{j-j})/\Delta E \approx$ const must hold.

b) A collective state corresponding to reverse spin flip. In nuclei with $N = Z$, it forms an isotriplet with the Gamow–Teller resonance and describes $n\bar{p}$ transitions in the isobaric branch $\Delta Q = -1$. With increasing $N - Z$ (or rather, $\Delta E/E_{ls}$) it sinks to the ground state of the nucleus $A(N+1, Z-1)$, and then for $\Delta E \gtrsim E_{ls}$ is transformed into a $p\bar{n}$ state of the branch $\Delta Q = +1$. Since it is characterized by a change in the sign of the charge with increasing $N - Z$, it can also be called a charge-variable state.

3. Collective isobaric configuration states corresponding to a higher radial symmetry of the effective field can exist. The most collective among them are the spin-flip states $\omega_+^{(1)}$. For light and medium nuclei ($\Delta E \gtrsim E_{ls}$), they may lie near the analog state and even above it.

3. PHENOMENOLOGICAL DESCRIPTION OF THE GROUND STATES AND THE MOST COLLECTIVE ISOBARIC STATES OF NUCLEI IN THE WIGNER SCHEME BASED ON SU(4) SYMMETRY

In modern nuclear theory, one uses not only microscopic theories but also a phenomenological description of states of a general nature in the framework of approximate symmetry schemes. The discovery of isobar analog states of medium and heavy nuclei made it possible to describe phenomenologically the ground states and a number of low-lying states in terms of the isotopic spin. The investigation of the Gamow–Teller resonance and the collective isobaric 1^+ states raises the question of an extension of the phenomenological description. It is possible to extend isotopic symmetry in such a way that the multiplets of the extended symmetry group contain not only the analog states but also the Gamow–Teller resonance.

The basis of such a generalization is, on the one hand, the similarity between the structure of the analog resonance and the Gamow–Teller resonance observed in the microscopic model and, on the other, the similarity known in the framework of the phenomenological Wigner spin–isospin symmetry,[28] which was first pointed out by Ikeda and Fujita.[29] In Ref. 8, we analyzed this possibility on the basis of the asymptotic degeneracy of these states in the limit $\Delta E/E_{ls} \gg 1$; in particular, we brought forward arguments indicating that the 0^+ analog resonance and the 1^+ Gamow–Teller resonance may occur in a single Wigner $(T_0, 0, 0)$ multiplet, where $T_0 = (N-Z)/2$. We briefly describe these and a number of new facts which support the reality of the Wigner multiplets.

Since the phenomenological scheme is essentially an approximate scheme, one can obtain the phenomenological characteristics by comparing microscopic results with the parameters of the phenomenological model. Let us consider the question of the isospin of the Gamow–Teller resonance. We use the fact that the matrix element of the β^+ transition from the Gamow–Teller resonance to the 0^+ ground state of the even–even nucleus $A(N, Z)$ is near $N-Z$. We compare it with the matrix element of the isovector $Y_+ = \sum_n \sigma^{(n)} \tau_+^{(n)}$:

$$M = | \langle T_0, T_z = T_0 | Y_+ | T', T'_z = T_0 - 1 \rangle |;$$
$$\frac{M^2}{N-Z} = \begin{cases} (2T_0-1)/(2T_0+1) & \text{for } T' = T_0 - 1; \\ 0 & \text{for } T' = T_0; \\ 2/(2T_0+1)(2T_0+3) & \text{for } T' = T_0 + 1. \end{cases} \quad (20)$$

The comparison shows that $T_{\rm GTR} = T' = T_0 - 1$. The additions of the states with $T' = T_0 + 1$ are small, $\sim T_0^{-2}$, which corresponds to the microscopic estimates (14).

In the isoinvariant scheme, the 0^+ ground state of the even–even nucleus $A(N, Z)$ and its analog resonance 0^+ in the nucleus $A(N-1, Z+1)$ belong to a single isomultiplet $(T, S^\pi) = (T_0, 0^+)$, together with all the 0^+ states (there are $2T_0 + 1$ of them) in the isobars with $T_0 \geqslant (N-Z)/2 \geqslant -T_0$. Taking the isospin of the Gamow–Teller resonance to be $T_0 - 1$, we can assert that it belongs to the $(T_0 - 1, 1^+)$ isomultiplet, which includes the 1^+ states $(2T_0 - 1$ in number) of the same isobars with

$$T_0 - 1 \geqslant (N-Z)/2 \geqslant -T_0 + 1.$$

Using the asymptotic degeneracy of the Gamow–Teller resonance and the analog resonance and the nearly equal radial symmetry of their effective field $[V(r) = \text{const}]$, we can suppose that the two isomultiplets belong to a single Wigner supermultiplet $(P, P', P'') = (T_0, 0, 0)$. To this supermultiplet there must belong the following states: in the nucleus $A(N, Z)$, one 0^+ state (degenerate); in the nucleus $A(N-2, Z+2)$, two 0^+, one 1^+, and one 2^+ states (degenerate), etc. The first of them is the ground state of the nucleus $A(N, Z)$, and the second and the third are the analog resonance and the Gamow–Teller resonance in the nucleus $A(N-1, Z+1)$. From our assumption there follows the possible existence of four (of which one is a 2^+!) doubly analog resonances in the nucleus at the height of two Coulomb energies [if we measure from the nucleus $A(N, Z)$].

This hypothesis of a single supermultiplet $(T_0, 0, 0)$ based on the 0^+ ground state of the even–even nucleus $A(N, Z)$ is in fact just one special case of Wigner's general hypothesis concerning the structure of the ground states of nuclei in the scheme of spin–isospin symmetry based on SU(4) (formulated by Wigner[28] in 1937). According to this hypothesis, the ground states of the nuclei are the highest weights of the following supermultiplets specified by the triplet of quantum numbers (P, P', P''):

$$(P, P', P'') = \begin{cases} (T_0, 0, 0) & \text{for even-even nuclei,} \\ (T_0, 1, 0) & \text{for odd-odd nuclei,} \\ (T_0, 1/2, \pm 1/2) & \text{for } A\text{-odd nuclei,} \end{cases}$$

where P is the largest value of the component T_z of the isotopic spin in the supermultiplet; P' is the largest value of the spin component S_3 in the state in which $T_z = P$ (or, conversely, $S_3 = P$ and $T_z = P'$); and the number P'' denotes the largest value of the sum $\sum_{i=1}^{r} S_3^{(i)} t_z^{(i)}$ for the states with $T_z = P$ and $S_3 = P'$ (or $S_3 = P$ and $T_z = P'$).

Although Wigner's hypothesis in its original form aimed only at a description of the ground states of nuclei, we have seen in the example of the $(T_0, 0, 0)$ multiplet that it must actually also contain information about a series of many-particle isobaric states (the analog resonance and Gamow–Teller resonance states with $Q = +1$) of $p\bar{n}$ type, double analog and Gamow–Teller states with $Q = +2$ of $pp\bar{n}\bar{n}$ type, etc. In accordance with the Wigner symmetry, all these states occur in a single supermultiplet and are degenerate in the case of exact symmetry. The Coulomb and other interactions which break the symmetry lift the degeneracy. Conversely, the experimental discovery of many-particle states of such supermultiplets would indicate approximate realization of Wigner symmetry, and the detection of approximate degeneracy of $Q = +1$ and $Q = +2$ states of similar structure would be even stronger evidence for the existence of symmetry. We note that an interesting example of reactions for studying double analog states is, for example, the (π^+, π^-) reaction.

In Ref. 28, Wigner obtained a mass formula for nuclei with given number of nucleons A on the basis of spin–isospin symmetry:

$$M(A) = a(A) + b(A)(1/2)(P^2 + 4P + P'^2 + 2P' + P''^2). \quad (21)$$

As a test of the realization of Wigner symmetry, Franzini and Radicatti[30] proposed to test whether the ratio

$$R = \frac{M(A, T_0) - M(A, T_0-2)}{M(A, T_0-1) - M(A, T_0-2)} \quad (22)$$

is independent of $a(A)$ and $b(A)$.

Their analysis of the masses of nuclei with $A \lesssim 130$ showed that the ground states of the nuclei satisfy the propositions of Wigner symmetry, the agreement improving with increasing A and $N-Z$.

Since the Franzini–Radicatti test is an independent test of the supermultiplet structure of the Gamow–Teller resonance, we made a similar test for all known nuclei ($A < 300$), making it more precise in one respect. In Wigner's mass formula, we introduced phenomenologically the usual pairing correction:

$$\Delta M = \begin{cases} \Delta & \text{for even-even nuclei,} \\ 0 & \text{for } A\text{-odd nuclei,} \\ -\Delta & \text{for odd-odd nuclei,} \end{cases}$$

where $\Delta = 12/\sqrt{A}$ MeV, and we took into account the Coulomb energy $E_{\rm Coul} \approx 0.584 Z^2 M^{-1/3}$.

Figure 4 shows the data of such an evaluation in groups of nuclei; for nuclei with the same T_0, the mean experimental value of the parameter R is obtained with a mean relative error of 2%. The curves show the theoretical, Wigner values of R. We evaluated separately the nuclear masses corresponding to the even–even, odd–odd, and odd-A groups, these masses corresponding to T_0. As can be seen from the data, in no case does the Franzini–Radicatti test contradict the

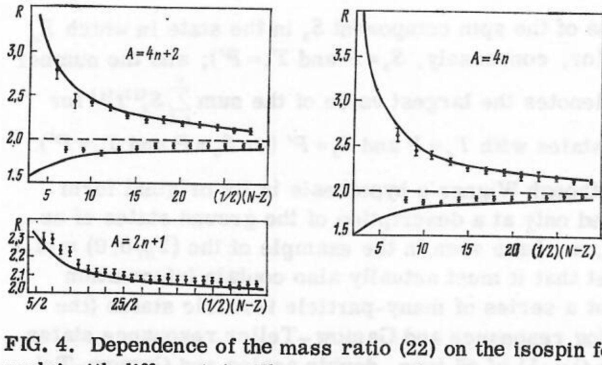

FIG. 4. Dependence of the mass ratio (22) on the isospin for nuclei with different A. The curves are based on the SU(4) theory, and the points are the experimental data.

experiments, and the agreement improves with increasing T_0 or $N-Z$. The most important thing is the good agreement between the theoretical and experimental values for the odd–odd nuclei. At the least, the results confirm the assumption that the nuclear ground states occur in Wigner supermultiplets: $(T_0, 0, 0)$ for even–even nuclei, $(T_0, 1, 0)$ for odd–odd nuclei, and $(T_0, \frac{1}{2}, +\frac{1}{2})$ for odd-A nuclei.

A natural question arises: What are the reasons for the restoration of Wigner symmetry in the charged excitation branch in the heavy nuclei and in what connection do they stand with the well-known fact of a large spin–orbit energy of nucleons in nuclei, which leads to $j-j$ coupling? For the example of the even–even nuclei $A(N, Z)$, we construct a qualitative model of this phenomenon, combining the main hypotheses introduced on the basis of the theoretical and experimental analysis, namely, the description of the nuclear ground states by Wigner multiplets, the asymptotic degeneracy of the analog resonance and the Gamow–Teller resonance in the limit of large $N-Z$, and the restoration of the symmetry with increasing A and $N-Z$ for real nuclei.[31]

As we have shown, the Wigner multiplet $(T_0, 0, 0)$ contains the ground state of the nucleus $A(N, Z)$, the analog resonance and the Gamow–Teller resonance in the nucleus $A(N-1, Z+1)$, etc. In the case of exact Wigner symmetry, these states are degenerate, the degeneracy being lifted when allowance is made for the broken symmetry. The breaking is associated with four factors: the spin–orbit energy of the nucleons, the difference between the constants f_0' and g_0' of the residual interaction of the quasiparticles, the Coulomb energy, and pairing. The Coulomb energy determines the relative splitting of the components of the multiplet in neighboring nuclei and, as was shown by Lane,[32] hardly breaks the isotopic, and with it the Wigner symmetry. With allowance for the residual average spin–orbit interaction, the distance between the Gamow–Teller resonance and the analog resonance is

$$\delta\bar{E} \approx (g_0'-f_0')\,\Delta E + \frac{2}{3}\,\frac{1+(2/3)\,g_0}{g_0'}\,\frac{\bar{E}_{ls}^2}{\Delta E} \approx c\,(g_0')\,\frac{\bar{E}_{ls}^2}{\Delta E}, \quad f_0' \approx g_0. \tag{23}$$

In the theory of finite Fermi systems, one does indeed observe proximity of f_0' and g_0' (see Table IV). The contribution of pairing in nuclei with $A > 100$ is less than 1 MeV. The ground state of the nucleus $A(N-1, Z+1)$ belongs to a different multiplet, $(T-1, 1, 0)$, though its nature may be different from that of the $(T, 0, 0)$ states. It appears natural to define the distance between these multiplets as the distance between states of nearly the same nature. Among the $(T-1, 1, 0)$ states, such as, for example, the antianalog 0^+ state, which in accordance with the microscopic approach is constructed from the same particle–hole configurations as the analog resonance, differing from it in the value of the isospin and the radial symmetry of the effective field.[24] The mean distance between the analog resonance and the antianalog is

TABLE IV. Comparison of parameters of the Gamow–Teller resonances obtained experimentally and calculated in accordance with the theory of finite Fermi systems in different methods.

Final nucleus	Oscillator basis			Quasiclassical beta model			Self-consistency with respect to the constant f_0'			Experiment		
	E, MeV	$M^2_{1^+-0^+}$	g_0'/f_0'	E, MeV	$M^2_{1^+-0^+}$	g_0'/f_0'	E, MeV	$M^2_{1^+-0^+}$	g_0'/f_0'	E, MeV	M^2_{GT}	Literature (references in square brackets)
^{90}Nb	9.4	6.6	0.815	10.3	7.0	0.926	8.2	7.6	0.938	8.7	8.3	[12, 42, 44]
^{92}Nb	14.1	7.5	0.815	13.5	8.2	0.926	—	—	—	12.4	—	[44]
^{94}Nb	14.6	8.8	0.815	13.4	9.5	0.926	—	—	—	12.3	—	[44]
^{112}Sb	—	—	—	10.4	8.3	0.926	10.1	7.9	0.938	10.0	—	[64]
^{120}Sb	12.4	14.0	0.815	11.9	14.3	0.926	12.1	12.6	0.938	12.3	—	[12–15]
^{124}Sb	14.0	17.4	0.815	13.7	17.2	0.926	14.3	14.9	0.938	13.1	—	[64]
^{203}Pb	—	—	—	15.7	33	0.926	15.3	28	0.938	15.6	—	[19]

$$\Delta E_{SU} \approx b\,(A)\,T \approx (1+c_1)\,\frac{1+f_0'}{1+c_1 f_0'}\,f_0'\Delta E, \quad c_1 \approx 4/9. \tag{24}$$

The condition of restoration of Wigner symmetry is that the splitting of the individual components of the $(T, 0, 0)$ and $(T-1, 1, 0)$ multiplets must be small compared with the distance between them: $\Delta E_{SU} > \delta\bar{E}$. Bearing in mind that on the stability line $\bar{E}_{ls} \approx 30A^{-1/3}$ MeV, $T_{st} \approx 3\times10^{-3}A^{5/3}$, and $b(A) \approx 100\,A^{-1}$ MeV, we obtain an estimate for the boundary in A of restoration of Wigner symmetry: $A > 120$.[31] At such values of A, one can at least assert that the mean distance between the multiplets is greater than the splitting of the levels of one multiplet due to the influence of the spin–orbit forces and pairing, so that the Wigner symmetry must be restored in this region of nuclei. Overall, it can be seen that the experimental data on the nuclear masses indicate a restoration of the Wigner symmetry in heavy nuclei and the possibility of describing the ground states and their analog resonance and Gamow–Teller resonance in the framework of single Wigner multiplets.

4. EXPERIMENTAL INVESTIGATION OF ISOBARIC 1^+ STATES

The above analysis of the nature of the various types of collective isobaric 1^+ states of spherical nuclei makes it possible to examine critically the experimental data relating to these states. While the analog states and the associated isobaric configuration 0^+ states have been studied for a comparatively long time and the basic structure of this, the simplest of the isobaric branches is now essentially understood, the study of the isobaric 1^+ states is only beginning. Until recently, the main difficulty of the experimental investigation of these states was the complexity of the separation from the background of the other nuclear states. According to the theory, the main feature of the collective 1^+ states is the large value of the matrix elements of their β decay to the ground state of the nucleus $A(N,Z)$, so that a direct method of separation from the background would involve excitation of these states in the inverse β-decay reaction in neutrino beams. However, this method is as yet very problematic. Now, after the discovery of the Gamow–Teller resonance in the (p,n) reaction, it has become clear that an alternative is to excite the collective 1^+ states in charge-exchange reactions of the type (p,n), $(^3He,T)$, and $(^6Li,^6He)$ at angles near 0° in the energy range 100–300 MeV. Under such conditions, the excitation channel of the 0^+ states is suppressed compared with that of the 1^+ states. Such a method makes it possible to begin the systematic study of all types of collective 1^+ states, whose spectrum is much richer than the 0^+ spectrum.

As we have shown, the collective isobaric 1^+ states can be divided into four classes: the Gamow–Teller resonance, as the state with maximal degree of collectivization; the $j-j$ state; the charge-variable state associated with reverse spin flip (of $n\bar{p}$ type in proton-rich nuclei and of $p\bar{n}$ type in neutron-rich nuclei), and the class of isobaric configuration states. The collective states of the different classes are manifested in different phenomena: in charge-exchange reactions, in the β decay of proton- and neutron-rich nuclei, and in the systematics of β transitions. The features of the 1^+ spectrum lead to new phenomena that have no analogs in the case of 0^+ states. We present successively the experimental facts associated with these groups of isobaric states, emphasizing the specific features of each group.

The Gamow–Teller resonance. As we have already mentioned, the question of the existence of the Gamow–Teller resonance remained open for a long time. Up to 1975, it had been observed directly only in the β^+ decay of ^{33}Ar into ^{33}Cl.[33] The ^{33}Ar nucleus is the extreme right-hand member of a $T=3/2$ multiplet and has $Z>N$ and $T_z=-3/2$, so that there remained the question of whether the Gamow–Teller resonance exists in multiplets with larger T in medium and heavy nuclei. The attempts to discover it in the γ decay of the analog resonance made in Refs. 34 and 35 did not lead to success, though a group of collective 1^+ states of $j-j$ type was discovered. Since 1975, Galonsky's group has made systematic investigations of the (p,n) reaction at proton energies 25, 35, and 45 MeV using the cyclotron of the University of Michigan. In the series of investigations of Refs. 12–14, they studied in detail the resonances in the neutron spectrum in the nuclei ^{48}Ca, ^{90}Zr, ^{120}Sn, and ^{208}Pb in a wide range of angles (0–160°) with an energy resolution of about 1.5 MeV and background conditions of the reaction.[14] The preliminary conclusion of the authors was that they had observed the Gamow–Teller resonance in ^{90}Zr and ^{120}Sn, but for ^{208}Pb an individual peak separate from the analog resonance was not observed. A detailed analysis of the results was not made. At the same time, attempts to investigate the $(^6Li,^6He)$ reaction by a method developed earlier were begun[36] As a rule, light nuclei from ^6Li to ^{48}Ca were investigated,[37,38] since the background conditions for $E_{Li}\approx 30$ MeV did not permit study of states above the analog resonance in heavier nuclei.

A summary of this period was provided by the results of Sterrenburg *et al.*,[15] presented at the conference at Dresden in 1980. They reported observation of a broad (3–5 MeV) peak in the (p,n) reaction at $E_p=45$ MeV in 17 nuclei, $^{90,91,92,94,96}Zr$, ^{93}Nb, $^{94,96,97,98,100}Mo$, $^{112,116,120,122,124}Sn$, ^{208}Pb, near and somewhat above (by 0.3–3.9 MeV) the analog resonance. The observations were made at the angle 7.5°. Simultaneously, a 1^- giant spin-dipole resonance was discovered even higher (9–15 MeV).

In 1979, a facility for magnetic beam deflection was put into operation at the cyclotron of the University of Indiana; with a base of length 60–70 m, this made it possible to achieve a resolution in the neutron spectrum of 200–300 keV at proton energy $E_p=50$–200 MeV. The facility was used for a large number of searches for the Gamow–Teller resonance in different nuclei, and these were crowned with success[16] (see also Refs. 17 and 18). The following experiments were made.

1. The reaction $^7Li(p,n)^7Be$ at $E_p=80$ MeV (Ref. 39) and the reaction $^{13}C(p,n)^{13}N$ at $E_p=80$, 120, 160 MeV (Ref. 40) were investigated. Low-lying resonances corresponding to charge exchange of a neutron with spin flip, which are equivalent to the Gamow–Teller resonance in light nuclei, were found.

2. The (p,n) reaction on ^{27}Al, ^{40}Ca, and ^{48}Ca was investigated at $E_p=160$ MeV and small angles.[41] The study of the reaction $^{48}Ca(p,n)^{48}Sc$ revealed a broad group corresponding to a split Gamow–Teller resonance, and a narrow peak, which is interpreted as the analog state of the M1 resonance in ^{48}Ca.

3. The reaction $^{90}Zr(p,n)^{90}Nb$ was investigated at $E_p=45$, 120, and 160 MeV and angles 0–10°. A spectrum of 1^+ resonances was observed, among them the Gamow–Teller resonance and the analog of the M1 resonance of ^{90}Zr. It was shown that with increasing proton energy and decreasing emission angle of the neutrons the Gamow–Teller resonance begins to play the dominant role in the resonance spectrum.[42]

4. The connection between the cross section of the (p,n) reaction at 0° and the matrix elements of the β decay of the 1^+ states to the ground state of the target nucleus was investigated. For some nuclei from ^7Li to ^{90}Zr, the effective values of M^2_{GT} were estimated from the known cross sections of the reaction at 0° (Ref. 43), and it was shown (see also Refs. 39, 64, and 65) that with increasing E_p the ratio of the contribution $\sigma\tau$ to the τ component of the cross section increases.

5. In the nuclei $^{90,92,94}Zr$, the behavior of the Gamow–Teller resonance with varying $N-Z$ was investigated at $E_p=120$ MeV. It was shown that with increasing $N-Z$ the Gamow–Teller resonance is shifted toward the analog resonance.[44]

6. The reaction $^{208}Pb(p,n)^{208}Bi$ was investigated at $E_p=120$ MeV and angle 0°. The Gamow–Teller reso-

nance and a number of collective 1^+ resonances were found. It was shown that the Gamow–Teller resonance and the analog resonance are practically degenerate with respect to the energy.[19]

7. There have been reports of experiments at $E_p = 200$ MeV on ^{169}Tm (Ref. 45), ^{206,208}Pb (Ref. 46), and ^{89}Y (Ref. 20). For the last, a qualitative conclusion is formulated: the experiments on many nuclei from Li to Pb reveal the presence of two 1^+ resonances: one below the analog resonance and the other above it; in light nuclei the former plays the dominant role; in medium and heavy nuclei, the latter.

Thus, when taken together the new experimental data lead to the following conclusions.

1. In a large class of nuclei there is a giant isobaric 1^+ resonance; it is manifested in the charge-exchange reaction (p,n) and in the ^{90}Zr–^{208}Pb region lies 4–0.5 MeV above the analog resonance. The resonance has a width of the order of several mega-electron-volts and is excited predominantly at proton energies $E_p > 40$ MeV at an angle near 0°. It is interpreted as a 1^+ state associated with spin flip of a nucleon.

2. The differential cross section of the reaction at the angle 0° shows that the effective matrix element of the β decay of this state is large ($M^2_{GT} \gtrsim N-Z$), and this confirms the interpretation. The theoretical estimates show[43] that the differential cross section must increase to $E_p \approx 200$–400 MeV, and the square of the matrix element, M^2_{GT}, must reach a maximal value of about $3(N-Z)$.

3. Investigations on isotopes of a given element show that this resonance approaches the analog resonance with increasing $N-Z$ (Refs. 44 and 49) and $(N-Z)/A$ for a large class of nuclei.[66] In the nuclei ^{206,208}Pb and ^{169}Tm the resonance is virtually degenerate in energy with the analog resonance. This confirms the most important qualitative property of the Gamow–Teller resonance, which leads to realization of the hypothesis of restoration of the Wigner symmetry in heavy nuclei.

4. Besides this resonance, a further 1^+ resonance is observed at an energy lower than the analog resonance. In light nuclei, its excitation predominates over the other resonance, but in medium and heavy nuclei the cross section for its excitation decreases. Comparing the qualitative results with the theoretical conclusions of Sec. 1, we arrive at the unambiguous conclusion that the newly discovered giant resonance must be identified with the Gamow–Teller resonance. It is interesting to compare the experimental results on the position of this resonance with the theoretical results obtained in Secs. 1 and 2 on the basis of numerical and quasiclassical calculations. The free parameter of the models is the coupling constant g'_0 of the spin–isospin interaction, which is chosen to give the best match of the theoretical and experimental values (Table IV). In Table IV we also give preliminary data obtained on the basis of new calculations with more rigorous allowance for the conditions of self-consistency of the two-particle isovector interaction of the quasiparticles and the single-particle potentials. The calculations were made in collaboration with V. G. Aleksankin using the method of Ref. 26 with an additional control based on the analog–antianalog splitting. As can be seen from the table, the theory satisfactorily describes the experimental data, the model with oscillator wave functions used in Sec. 1 giving somewhat overestimated (by 1–2 MeV) values, whereas the more rigorous model with self-consistency puts the position of the Gamow–Teller resonance too low at the same ratio g'_0/f'_0. The ratio of the constants is $g'_0/f'_0 \approx 0.9$, which also indicates proximity of the calculated model to the Wigner symmetry scheme, in which the rigorous equality $g'_0 = f'_0 = g_0$ holds (g_0 is the coupling constant of the spin–spin interaction of the quasiparticles).

There is one further possible independent experimental verification of the existence of the Gamow–Teller resonance, namely, its excitation in other charge-exchange reactions, for example, (^{3}He, T) and (^{6}Li, ^{6}He). First data on the (^{3}He, T) reaction in the nuclei ^{90}Zr (Ref. 47) and ^{40}Ca (Ref. 48) have already appeared and confirm the existence of the resonance.

Experiments on the reaction (^{6}Li, ^{6}He) at energy $E_{Li} = 90$ MeV have been made for the ^{90}Zr nucleus at the I. V. Kurchatov Institute of Atomic Energy[50] by a method developed and tested earlier.[36,37] The Gamow–Teller resonance is clearly observed on the background of the other states. An important feature of the (^{6}Li, ^{6}He) reaction is that the background of analog states present in the (p,n) and (^{3}He, T) reactions is absent because of the specific selection rules ($\Delta J = 1, \Delta T = 1$).

The j–j state. Collective isobaric 1^+ states forming a narrow group below the analog resonance were discovered for the first time for some nuclei in the region of ^{48}Ca (^{51}V, ^{49}Ca) in experiments on the γ decay of the analog resonance.[34,35] These states are clearly distinguished by the large matrix elements of the M1 transitions and are situated 4–5 MeV below the analog resonance. Initially, attempts were made to associate them with the Gamow–Teller resonance, but it is now clear that they must be associated with a split $j-j$ state, which according to the quasiclassical estimate lies in this energy region. They were found in the first experiments on the (^{6}Li, ^{6}He) reaction on ^{48}Ca.[38] It follows from the new data on the (p,n) reaction that the $j-j$ state must be associated with the 1^+ resonance that lies below the analog resonance and is observed together with the Gamow–Teller resonance in light and medium nuclei. According to the calculations in Sec. 1, the collective 1^+ states with such excitation energy have $M^2_{GT} \approx 1$, while the Gamow–Teller resonance has $M^2_{GT} \approx N-Z$, so that naturally these states can compete with the Gamow–Teller resonance as regards the excitation cross section only in the region of light nuclei. The tendency in the behavior of the experimentally observed resonance agrees with that expected for the $j-j$ state; in particular, an experiment[44] with ^{90,92,94}Zr nuclei reveals an increase in the distance of the resonance from the analog resonance with increasing $N-Z$, in agreement with the quasiclassical conclusions of Sec. 2.

Isobaric $n\bar{p}$ states. The group of V. A. Karnaukhov investigated the strength functions of the β^+ transitions of proton-rich nuclei of medium A (≈120) produced by the proton decay of proton-rich nuclides.[51,52] For the nuclei ^{109}Te, ^{115}Xe, and ^{117}Xe at $E<E_{AR}$, a broad peak of the β^+-decay strength function is observed, which corresponds to $n\bar{p}$ configurations with spin flip. The value of log(ft) for these β^+ transitions is of order 4.5. With increasing $N-Z$, the energy of the states decreases relative to the ground state of the nucleus. These facts show that in these experiments a collective isobaric $n\bar{p}$ state with spin flip is observed. A rigorous calculation with allowance for the deformation of the nuclei[11] confirms this conclusion.

Low-lying isobaric 1^+ *states*. Data on the systematics of allowed Gamow–Teller transitions of nuclei with $N>Z$ indicate that these β transitions are suppressed by a strong factor compared with the single-particle estimates. A measure of the suppression is $F=M^2_{\rm exp}/M^2_{\rm sp}$. Figure 5 shows the values of log F for the known β transitions of Gamow–Teller type. The suppression is clearly visible, and it increases with increasing $N-Z$.

A qualitative explanation of this picture in the scheme of collective isobaric states was proposed by Ikeda and Fujita.[3] The sharp decrease in the matrix elements of the β decay of the ground states of nuclei with $N>Z$ is due to the fact that the collective isobaric 1^+ states through which the β transitions pass virtually are situated in the region of high excitation energies. The energy of these excitations increases with increasing $N-Z$. There is also an increase in the total number of collective configuration states corresponding to $\Delta n>0$. By virtue of the existence of the approximate sum rules, the part corresponding to β transitions from the low-lying states decreases, and F^{-1} increases. This conclusion can be confirmed qualitatively by plotting in Fig. 5 the quasiclassical value of the parameter

$$\lg F_N=\lg\Big[1-\sum_{i=\Delta n=0}^{N} M_{GT}^{(i)2}(N-Z)^{-1}\Big], \qquad (25)$$

where N is the number of collective $\omega_+^{(i)}$ states, including the Gamow–Teller resonance. This expression gives an upper bound for the values of the suppression factor as a function of $N-Z$.

Exact calculations confirm this picture well. Figure

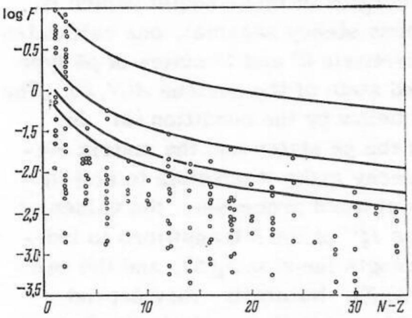

FIG. 5. Suppression factor for Gamow–Teller β transitions. The curves are the estimates (25) with allowance for the high-lying collective isobaric states; the open circles are experimental values.

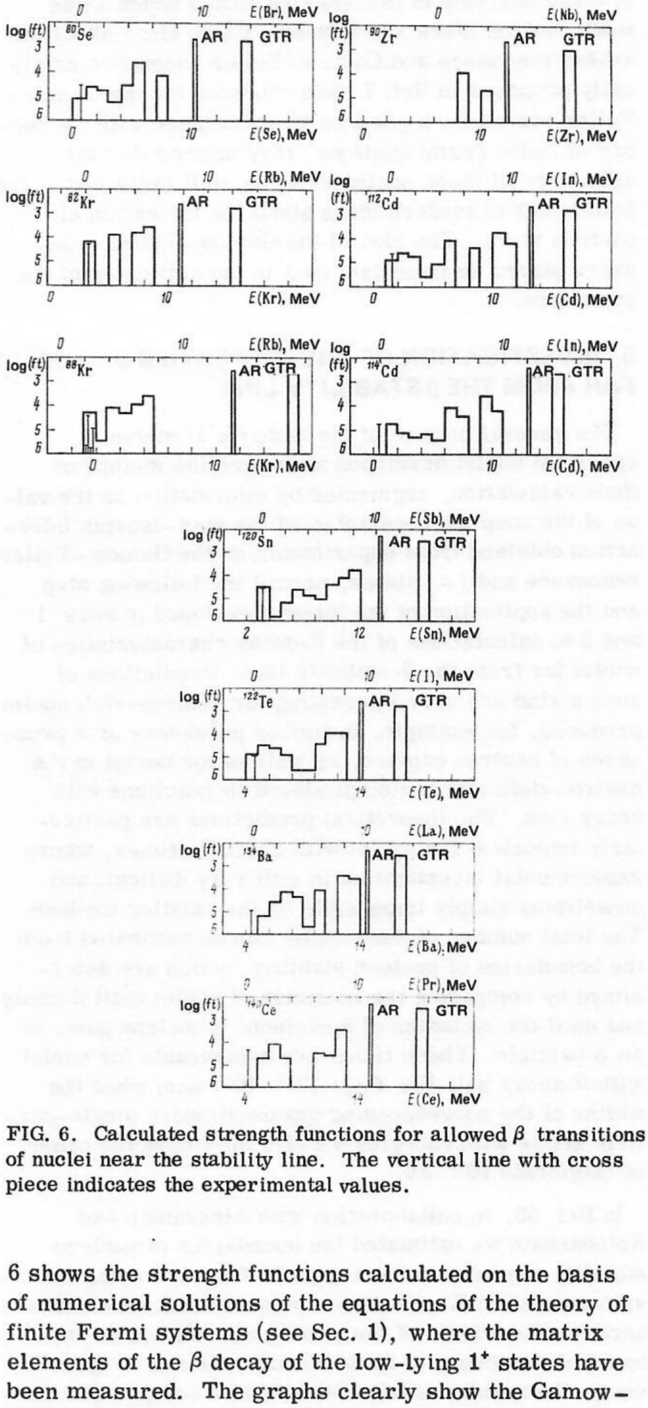

FIG. 6. Calculated strength functions for allowed β transitions of nuclei near the stability line. The vertical line with cross piece indicates the experimental values.

6 shows the strength functions calculated on the basis of numerical solutions of the equations of the theory of finite Fermi systems (see Sec. 1), where the matrix elements of the β decay of the low-lying 1^+ states have been measured. The graphs clearly show the Gamow–Teller resonance, the analog resonance, the collective $j-j$ states and the configuration states, and the region of the low-lying β transitions, to which the usually measured β transition, indicated by the vertical line with cross piece, belongs. The numerical value of log(ft) is well explained in all cases.

Thus, the general picture of the collective isobaric 1^+ states at present agrees well with the theory.

Interesting indirect arguments in favor of the existence of the Gamow–Teller resonance were recently obtained from astrophysical investigations into the question of the formation of the group of the so-called bypassed nuclides with $80\leqslant A\leqslant 140$. Domogatsky and Nadyozhin[53,54] investigated the mechanism of formation of these nuclides in the neutrino fluxes which arise when neutron stars are formed. Using ideas about the analog resonance and Gamow–Teller resonance originally proposed in Ref. 7, and choosing for the Gamow–Teller resonance a position in accordance with the theory of finite Fermi systems, they showed that the abundance of these nuclides can be well explained in the framework of modern ideas about the formation of neutron stars. The idea of the Gamow–Teller resonance played an important part in the estimates of the abundance.

5. INVESTIGATION OF THE PROPERTIES OF NUCLEI FAR FROM THE β-STABILITY LINE

The general picture of the isobaric 1^+ states of spherical nuclei developed above and the method of their calculation, augmented by information on the value of the coupling constant g_0' of the spin–isospin interaction obtained from experiments on the Gamow–Teller resonance and $j-j$ states, permit the following step and the application of the theory developed in Secs. 1 and 2 to calculations of the β-decay characteristics of nuclei far from the β-stability line. Predictions of such a kind are very interesting for neutron-rich nuclei produced, for example, in fission processes or r processes of neutron capture, as well as for nuclei in the neutron-deficient region produced in reactions with heavy ions. The theoretical predictions are particularly important for nuclei with short lifetimes, whose experimental investigation is still very difficult and sometimes simply impossible by the existing methods. The total number of such nuclei can be estimated from the boundaries of nucleon stability, which are determined by comparing the lifetimes of nuclei until β decay and until the emission of a nucleon, a nucleon pair, or an α particle. These times are comparable for nuclei with β-decay half-life $T_{1/2}=10^{-2}-10^{-4}$ sec, when the widths of the corresponding quasistationary single-particle states with energies 0.5–1.5 MeV have the order of magnitude 10^{-13} eV.

In Ref. 55, in collaboration with Aleksankin and Kolobashkin we estimated the boundaries of nucleon stability of nuclei in the region $70 \leqslant A \leqslant 170$ using the self-consistent Woods–Saxon potential and with allowance for the effects of the pairing correlation on the basis of the theory of finite Fermi systems. Figure 7 shows the results of this estimate in a comparison with the results of estimates based on Hartree–Fock theory[56] and the known experimental boundaries of the investigated nuclei. It can be seen from Fig. 7 that the number of uninvestigated nuclei is very large. Thus, in the region $70 \leqslant A \leqslant 170$ the number of neutron-rich nuclei with unknown decay schemes reaches 940, of which about 520 must be emitters of delayed neutrons. The number of unknown neutron-deficient nuclei is about 530. In processes in which such nuclei participate, an important part is played by the β-decay chains of these nuclei to the ground and excited states of the neighboring nuclei, these chains including delayed emission of neutrons (for neutron-rich nuclei) or protons and α particles (for neutron-deficient nuclei). These estimates emphasize the exceptional importance

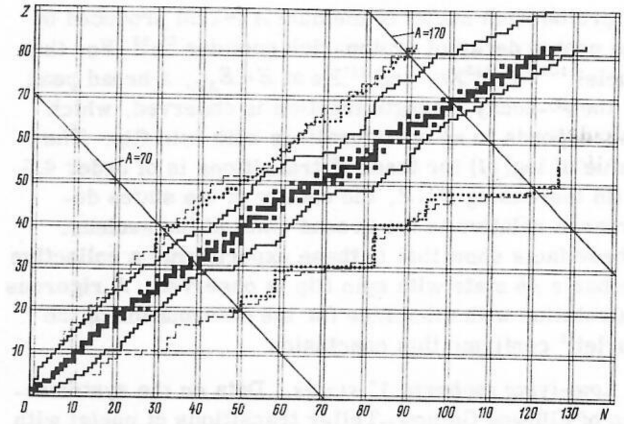

FIG. 7. Chart of the nuclei. The continuous line shows the boundaries of the experimentally studied nuclei (nuclei with known β-decay schemes). The boundaries of nucleon stability are calculated using the self-consistent Woods-Saxon potential with allowance for pairing in accordance with the theory of finite Fermi systems (Ref. 55; points) and in accordance with Hartree-Fock theory (Ref. 56; broken line).

of creating methods for predicting the β-decay characteristics of nuclei far from the β-stability line.

The method of calculating isobaric 1^+ states of spherical nuclei presented in the present paper and augmented by the analogous method for calculating the isobaric 0^+ states (see Ref. 24) provides a basis for such prediction which is much more accurate than the predictions based on gross theory,[57,58] phenomenological theory,[59,60] or non-self-consistent microscopic calculations (see, for example, Ref. 61). The main difference between our method and the others is in the correct allowance for all types of collective isobaric states, in particular of $j-j$ type and collective states with reverse spin flip, and also in the method of making the coupling constant f_0' of the isospin interaction self-consistent with the difference between the proton and neutron potentials (see Sec. 1). We present the general scheme of the calculation and its basic features for neutron-rich and neutron-deficient even–even spherical nuclei.

For a given difference between the ground-state energies of the even–even nucleus $A(N,Z)$ and the neighboring odd–odd nucleus $A(N-1,Z+1)$ (determined from independent mass relations[62]) and the difference between the Coulomb energies of these nuclei (which is needed for the self-consistency scheme), one calculates the spectrum of the isobaric 0^+ and 1^+ states of $p\bar{n}$ type based on the 0^+ ground state of the nucleus $A(N,Z)$. The spectrum is bounded below by the condition (3). In turn, the energies of the $p\bar{n}$ states and the matrix elements M^2 of their β decay make it possible to find the main characteristics of the β processes: the values of $\log(ft)$, the intensities $I_\beta^{(i)}$ of the β transitions to individual levels, the strength function $S_\beta(E)$, and the half-life of the nucleus $A(N,Z)$. Naturally, they depend strongly on the extent to which the states in the β-decay "window" are collective.

For neutron-rich nuclei, the lower part of the spectrum of $p\bar{n}$ type, which is situated below the ground state of the nucleus $A(N, Z)$, characterizes the β-decay spectrum of a short-lived nucleus $A(N, Z)$. The major part of the spectrum is determined by the allowed β transitions (β transitions of the forbidden type can be calculated by the same method, but their contribution is slight); in the β^--decay window there are usually collective configuration 1^+ states and several single-particle states with small $M^2 \sim 10^{-3}-10^{-4}$. With increasing excess $N-Z$, the $j-j$ state is shifted upward into the β^--decay window, and at large $N-Z$ the charge-variable state with reverse spin flip appears from below. Since the half-life is proportional to E_β^{-5}, the appearance of the latter greatly decreases the lifetime of the neutron-rich nuclei with larger $N-Z$. With further increase in the neutron excess, $T_{1/2}$ decreases smoothly in accordance with the increase in the symmetry energy. For neutron-rich nuclei, the neutron separation energy S_n above which delayed neutrons are emitted after β^- decay from states of the quasicontinuous spectrum is of great importance. Naturally, the spectrum of the delayed neutrons carries information about the degree of collectivization of the isobaric states with energy higher than the separation energy. One must expect the manifestation in this spectrum of the most collective states, and at large $N-Z$ the collective $j-j$ state.

For neutron-deficient nuclei with $N \geq Z$, the β^+-decay spectrum is determined by the $n\bar{p}$ branch with states lying between the ground state of the nucleus $A(N, Z)$ and the ground state of the daughter nucleus $A(N+1, Z-1)$. Among the β transitions of this type, the principal part is played by the $n\bar{p}$ transitions of the charge-variable state ω_-, whose degree of collectivization increases with decreasing $N-Z$. On the transition through the line $N=Z$, we enter an essentially new region, in which the $n\bar{p}$ states are analogous to the $p\bar{n}$ branch of nuclei with $N>Z$. For the nuclei in this region, entirely new decay channels are opened, namely,

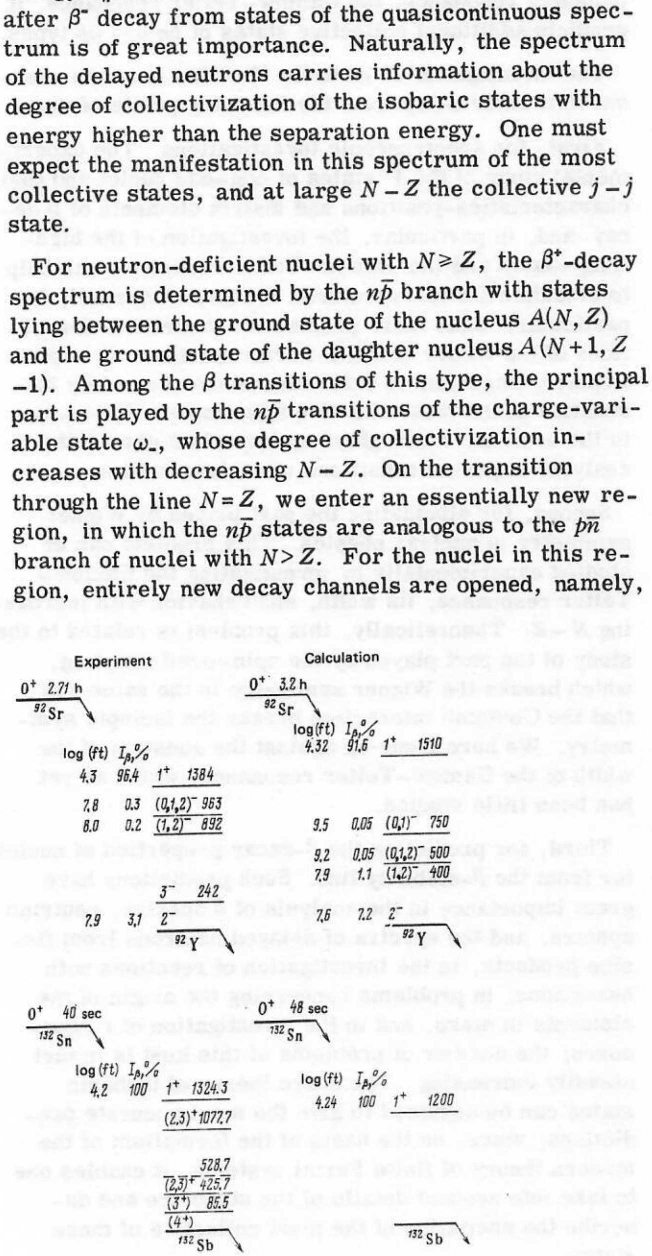

FIG. 8. Comparison of the β-decay schemes of the nuclei ^{92}Sr and ^{132}Sn obtained by calculation and experimentally.

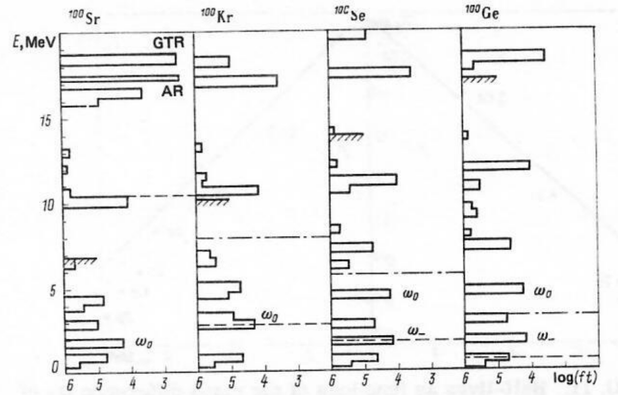

FIG. 9. Calculated β^--decay strength functions of even–even neutron-rich nuclei with $A = 100$ far from the β-stability line. The broken line is the neutron separation energy S_n; the chain line is the separation energy of two neutrons in the daughter nucleus.

decay to the analog resonance and the $j-j$ state and even to the Gamow–Teller resonance (at large Z). This must lead to a sharp decrease in the lifetime $T_{1/2}$ of nuclei with $Z>N$ and to the appearance of new lines in the spectra of delayed protons and α particles. For the neutron-deficient nuclei, the proton separation energy S_p plays the same part as for the neutron-rich nuclei, though for protons and α particles there is the additional Coulomb barrier, which hinders the emission of delayed particles after β^+ decay.

The results of the calculation of the characteristics of the β transitions in accordance with this method are shown in Fig. 8. First, to verify the parameters and estimate the accuracy of the calculations, the known β-decay schemes of several nuclei, including ^{92}Sr and ^{132}Sn were calculated (Fig. 8). It follows from comparison of the theoretical and experimental decay schemes that the error in the calculation of the energy E_β of low-lying states in the self-consistent scheme is 0.3–0.5 MeV, and the error in the calculation of $\log(ft)$ is of order 10%, which gives a total error of order 10% in the determination of the mean energy of the β transitions. The half-lives are calculated with a larger error, about 50%; however, the accuracy of the calculation exceeds the accuracy of the other predictions by several times. The accuracy could be increased by using more rigorous self-consistency schemes.

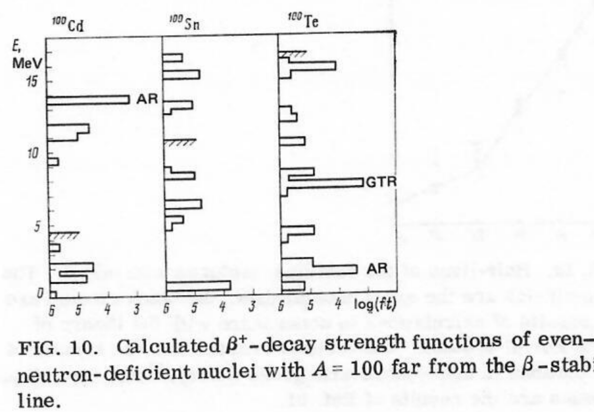

FIG. 10. Calculated β^+-decay strength functions of even–even neutron-deficient nuclei with $A = 100$ far from the β-stability line.

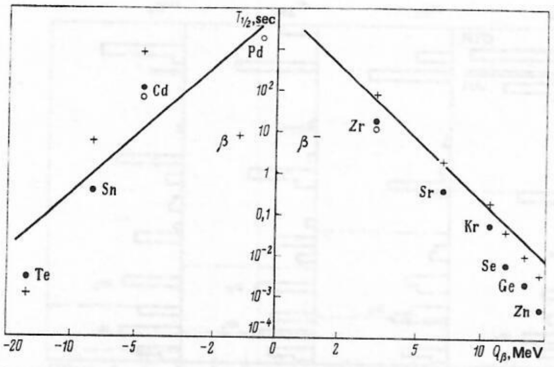

FIG. 11. Half-lives as functions of the mass difference Q_β of the nuclei for even–even nuclei with $A=100$. The open circles are the experimental data, the black circles are the results of calculation in accordance with the theory of finite Fermi systems, the crosses are the results of calculation in accordance with gross theory,[58] and the continuous line is the result of the phenomenological approach of Ref. 59.

As an example of the prediction of the properties of nuclei far from the β-stability line, we give calculations for neutron-rich (Fig. 9) and neutron-deficient (Fig. 10) nuclei with mass number $A=100$. In the schemes, we show the β-decay strength functions obtained by averaging the calculated values with a 0.5-MeV step, the β-decay windows, the neutron separation energies S_n, and also the principal collective states: the analog resonance, the Gamow–Teller resonance, ω_0, and ω_-. In Figs. 11 and 12, we compare the experimental data and the predictions of the various models for the values $T_{1/2}$ for the nuclei of the isobar $A=100$ and the rubidium isotopes.[63] It can be seen from these data that the model gives satisfactory agreement with experiment in all the known cases and that its predictions often differ appreciably from the predictions of other models. At the present time, the method of calculating the properties of nuclei far from the β-stability line is being used to predict the properties of a large number of nuclei and is being developed with a view to achieving a more accurate description of nuclear excitations with energy exceeding the nucleon separation energy.

In conclusion, we list the advantages of the method:

1) it enables one to predict the β-decay schemes, the

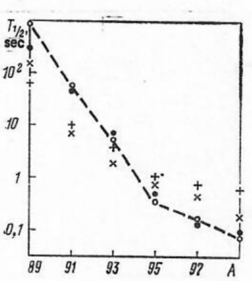

FIG. 12. Half-lives of the rubidium isotopes with odd A. The open circles are the experimental data, the black circles are the results of calculations in accordance with the theory of finite Fermi systems, the vertical crosses are the results of calculations in accordance with gross theory,[58] and the oblique crosses are the results of Ref. 61.

β spectra, and the neutrino spectra of nuclei far from the β-stability line for both neutron-rich, $N>Z$, and neutron-deficient, $Z\geqslant N$, nuclei;

2) it enables one to calculate the γ spectra accompanying β^- decays of nuclei and the probability of emission of delayed nucleons[67];

3) it enables one to determine the boundaries of nucleon stability with respect to neutron and proton decays and to investigate the structure and decays of nuclei near these boundaries.

CONCLUSIONS

The above theory of collective isobaric 1^+ states has a number of new consequences compared with the well-known theory of collective isobaric 0^+ states of the analog-resonance type. Besides the existence of the principal resonance, the Gamow–Teller resonance, it predicts additional collective states of $\bar{p}n$ and $\bar{n}p$ types.

The investigation of isobaric 1^+ states is at the present time interesting from the following points of view.

First, for spectroscopic investigations. The experimental study of the 1^+ states of odd–odd nuclei and their characteristics-positions and matrix elements of β decay–and, in particular, the investigation of the high-lying states and the Gamow–Teller resonance can help to elucidate the main features of these states and, in particular, make more precise the theoretical foundations of the theory of finite Fermi systems. It is particularly important to determine more accurately the constant of the local effective spin–isospin interaction in the nucleus, which plays an important part in the analysis of phase transitions in nuclear matter.

Second, for elucidating the part played by Wigner symmetry in nuclear physics. This problem can be studied experimentally by investigating the Gamow–Teller resonance, its width, and behavior with increasing $N-Z$. Theoretically, this problem is related to the study of the part played by the spin–orbit coupling, which breaks the Wigner symmetry in the same way that the Coulomb interaction breaks the isotopic symmetry. We here come up against the question of the width of the Gamow–Teller resonance, which as yet has been little studied.

Third, for predicting the β-decay properties of nuclei far from the β-stability line. Such predictions have great importance in the analysis of β spectra, neutrino spectra, and the spectra of delayed neutrons from fission products, in the investigation of reactions with heavy ions, in problems concerning the origin of the elements in stars, and in the investigation of r processes; the number of problems of this kind is in fact steadily increasing. The above theory of isobaric states can be assumed to give the most accurate predictions, since, on the basis of the formalism of the modern theory of finite Fermi systems, it enables one to take into account details of the structure and describe the excitation of the most collective of these states.

It is also interesting to follow the connection between the theory of isobaric states and the theory of charge-exchange reactions of the type (p,n), (^{3}He, T), or (^{6}Li, ^{6}He). First results in this direction have already been published (see, for example, Ref. 43).

Finally, it is tempting to use the data on the collective isobaric 1$^+$ states in the planning of neutrino experiments. These processes must be preferentially excited in inverse β-decay processes in neutrino beams, the neutrinos exciting the charge +1 branch, the antineutrinos the charge −1 branch. It should also be emphasized once more that the discovery of the Gamow–Teller resonance for heavy nuclei would create good neutrino detectors with gain of about $3(N-Z)\approx 150$.

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