A nonlocal quark model

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A nonlocal quark model is described. Quarks are described by a virton field, existing as virtual states only. Hadrons are described by standard quantized fields and have experimentally observable masses. Hadrons are bound states of quarks. The model satisfies all axioms of relativistic quantum field theory and does not contain ultraviolet divergences. Hadron-quark interaction Lagrangians are introduced. The strong, electromagnetic, and weak decays of the pseudoscalar and vector mesons as well as the baryon octet and decuplet are considered. The model contains only two free parameters, which characterize the quark field. Good agreement with the experimental data is obtained.

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INTRODUCTION

The discovery of SU(3) symmetry introduced into elementary-particle physics the concept of quarks as particles which constitute hadrons.[1] The numerous experimental attempts to detect quarks have not been crowned with success.[2] However, the success of the phenomenological quark models in describing strong-interaction physics indicates that quarks represent a reality although they do not exist in the free state like electrons, protons, pions, etc. In addition, modern experiments convincingly demonstrate that hadrons are complex composite systems, and experiments on deep inelastic scattering of electrons by protons find their most natural interpretation in terms of the parton model, in which it is assumed that the proton consists of a collection of elementary constituents, or partons.[3] Further experiments made it possible to interpret the partons as quarks.

However, a definitive relativistic theory of the strong interactions does not yet exist. It is natural to assume (though earlier there were considerable doubts about this) that the strong interactions, like the electromagnetic and weak interactions, must be described in the framework of relativistic quantum field theory (QFT). Indeed, at the conceptual level QFT alone is, on the basis of the simplest assumptions about the form of the interaction, capable of describing all the observed mutual transformations of particles, which form the basis of hadron physics. The ideas and methods of quantum electrodynamics (QED), which describes the electromagnetic interactions of the elementary particles with remarkable accuracy, provided the basis of the modern development of QFT. The ideas and concepts of QED such as gauge invariance, charge conservation, and universality of the interaction also became the point of departure for the construction of quantum-field models of the strong and weak interactions. The idea of gauge invariance proved to be particularly fruitful. In conjunction with the concept of spontaneous symmetry breaking, it formed the basis of the construction of the unified theory of the weak and the electromagnetic interactions—the Weinberg–Salam model.[4] All the available experimental data relating to these interactions match the predictions of this model well.[5]

Further development of the idea of gauge invariance led to the creation of quantum chromodynamics (QCD), which describes the interaction of colored quarks with massless gauge fields—gluons.[6] By analogy with QED, the basic principle in the construction of QCD is the requirement of gauge invariance under local transformations of the color group $SU_c(3)$. Quantum chromodynamics has proved to be a fairly complicated theory for investigation. The main problem in QCD—the explanation of quark confinement, the formation of hadrons as bound states of quarks, and the absence of colored hadron states—is still far from its solution. This is due primarily to the fact that, quite generally, in all more or less realistic QFT models strong-coupling methods have not been developed at all. Nevertheless, QCD does have the remarkable property of asymptotic freedom, namely, the renormalized charge decreases with increasing energy. This means that in the limit of large momenta (or at short distances) quarks become almost free and one can use the methods of perturbation theory. Some processes, in particular processes with large momentum transfers, have been described in the framework of QCD.[7]

When we come to quark confinement, QCD loses its predictive power and becomes a philosophical point of view rather than a mathematical formalism for investigating low-energy hadron physics. One can say that in this energy region QCD is at the present time one of the phenomenological approaches in strong-interaction physics.

The absence of any description of strong interactions in the framework of QFT, especially at low energies, gave rise to the development of many phenomenological approaches. The ideas of current algebra[8] have proved to be the most fruitful in the description of low-energy hadron physics.

On the basis of the quark model, it was suggested that the vector and axial charges satisfy the algebra SU(3)×SU(3). It is assumed that the vector currents are conserved exactly (the CVC hypothesis) but the axial currents only partially (the PCAC hypothesis). Thus, ignorance of the dynamical structure of the hadronic currents could be compensated by the establishment of certain relations between the amplitudes of different physical processes, these amplitudes being the matrix elements of the hadronic currents. However, in this approach too it is necessary to make some additional hypotheses, for example, that of vector dominance, etc.

The development of the method of investigation in current algebra based on the use of nonlinear realizations of chiral symmetry led to the development of a direction associated with the description of low-energy hadron physics by chiral-invariant nonpolynomial interaction Lagrangians. In the framework of this direction, it was possible to describe virtually all the decays of the octet of pseudoscalar mesons.[9] However, this approach is not concerned at all with the quark structure of the hadrons.

The notion of a hadron as a composite system has been developed in numerous phenomenological quark models[10] based on the ideas of nonrelativistic quantum mechanics. In these models, quark confinement is ensured by the introduction of a potential which increases at large distances. By an appropriate choice of the potential, one can obtain more or less accurately the hadron spectrum, the magnetic moments, and the other static characteristics of the hadrons.

The quasiclassical theory of "bags"[11] is in this direction. In the corresponding models, it is assumed that the quark fields satisfy free relativistic equations within some restricted region of space (the bag), and the boundary conditions on the surface of the region are formulated to make the energy–momentum flux into the exterior space zero. This ensures confinement of the quarks. In this model, the hadron is a bag, and its quantum numbers and other characteristics are determined by the quark fields within the bag.

It should be noted that in bag theory the problem of quark confinement is, strictly speaking, still not solved, since one quark in a bag is a bag with the quantum numbers of the quark, i.e., a physically observable quark, just as two quarks in a bag is a boson, three a baryon, and so forth. Therefore, the absence of quarks in color states is essentially an additional requirement of the theory.

Basically, in the bag models one attempts to explain the static characteristics of the hadrons. In these models, the interaction of two hadrons is an extremely complicated problem.

Returning to QCD, which in accordance with the "official" opinion currently accepted is the pretender for the theory of the strong interactions, one can say with confidence that even in the case of successful solution of the confinement problem (which, of course, will be one of the significant achievements of the theory) the real mathematical language of the description of hadron physics in the confinement region which arises from QCD will be much simpler than the mathematical structures with which one currently operates in QCD.

Clearly, for hadron physics in the confinement region there must exist, within the framework of relativistic quantum field theory, a fairly simple scheme which solves the following problems:

1) quarks do not exist in the free state;

2) hadrons consist of quarks;

3) colored hadronic states are completely absent;

4) all the axioms of relativistic quantum field theory are satisfied;

5) there exist hadron–quark interaction Lagrangians which describe low-energy hadron physics (strong, weak, and electromagnetic decays and low-energy hadron scattering);

6) there is a minimal set of free parameters characterizing the quarks alone.

The nonlocal quark model of Ref. 12 makes claims to be such a scheme.

The usual approach to the explanation of quark confinement[10,11,13] is based on the assumption that quarks exist primordially as physical Dirac particles but cannot exist as free particles outside hadrons because of some dynamical interaction. The nonlocal quark model is based on an entirely different hypothesis, namely, that quarks do not exist at all as ordinary physical particles but exist only in the virtual state. This hypothesis was realized as follows.[12] In the framework of QFT, one introduces "particles" called virtons, which have the following properties. First, the field describing free virtons vanishes identically, i.e., the virtons do not exist in the free state. Second, the causal Green's function, i.e., the propagator of the virton field, is nonzero and a nontrivial function. In other words, virtons exist only in the virtual state.

The virton field is a good candidate for describing the quark field and does not require any additional fields (such as a gluon field) to ensure quark confinement.

Further, it is assumed that hadrons are described by ordinary local quantized fields but that they interact with one another not directly but by exchanging quanta of the virton–quark field. For example, the interaction of the meson, $\pi(x)$, and baryon, $B(x)$, fields with the virton field $q(x)$ can be described by a Lagrangian of the type

$$\mathcal{L}_I(x) = ig\pi(x)(\bar{q}(x)\gamma_5 q(x)) + if(\bar{B}(x)q(x))(\bar{q}^c(x)\gamma_5 q(x)) + \text{h.c.}$$

The assumption that hadrons are bound states of quarks is equivalent to the requirement that the renormalization constant Z_h of the hadron wave function ($h=\pi, B$) is zero.[14] This condition fixes the coupling constants g and f in the interaction Lagrangian.

In this approach, a finite unitary S matrix was constructed by the methods developed in quantum field theory with a nonlinear interaction.[15] Thus, this model satisfies all the requirements listed above.

The calculations showed that the nonlocal quark model is capable of describing successfully low-energy hadron physics.

In Sec. 1 we describe the nonlocal quark model, and in Sec. 2 we give the description in the framework of this model of low-energy hadron physics.

1. THE VIRTON FIELD

1. *Formulation of the problem.* We introduce a

quantized field which describes particles that do not exist in the free state in the way that electrons and protons do, but exist only in the virtual state. We call these nonexistent particles virtons and the field $q(x)$ which describes these particles the virton field.

The virton field can be constructed as follows. The fact that ordinary elementary particles are observable means in the quantum-field formalism that the fields which describe free particles are solutions of the corresponding equations (Dirac, Klein–Gordon, etc.) and these solutions are nonzero. It is natural to assume that the unobservability or the impossibility of the existence of the particles in the free state means that the field of the free virtons is identically zero.

In the framework of the standard Lagrangian formalism, this means the following. Suppose the Lagrangian of the free virton field $q(x)$, which is assumed to be a fermion field, is written in the form

$$\mathscr{L}_0(x)=\bar{q}(x)\,Z(\hat{p})\,q(x), \tag{1}$$

where $Z(\hat{p})$ is an operator which depends on $\hat{p}=i\hat{\partial}=i\gamma_\mu\partial/\partial x_\mu$.

Then the assumption of the impossibility of the existence of virtons in the free state means that in the equation of motion

$$Z(\hat{p})\,q(x)=0 \tag{2}$$

the operator $Z(\hat{p})$ must be chosen in such a way that the only solution of this equation is identically equal to zero, i.e.,

$$q(x)\equiv 0. \tag{3}$$

In addition, we require that the Green's function of the field $q(x)$, which satisfies the equation

$$Z(\hat{p})\,G(x-y)=i\delta(x-y), \tag{4}$$

is nontrivial:

$$G(x-y)=iZ^{-1}(\hat{p})\,\delta(x-y)\not\equiv 0. \tag{5}$$

In the framework of the standard methods of local classical or quantum field theory, it is impossible to satisfy simultaneously Eq. (2) with the solution (3) and Eq. (4) with the solution (5), since any Green's function of the free field is constructed from solutions of the free equation. However, in the framework of the nonlocal quantum theory developed in Ref. 15 this problem can be solved.

The idea is as follows. One constructs a regularized quantized field $q^\delta(x)$, which is defined on some Fock space $\mathscr{H}$, which satisfies the conditions

$$\lim_{\delta\to 0}\langle\Psi_1\,|:q^\delta(x_1)\cdot\ldots\cdot q^\delta(x_n):|\,\Psi_2\rangle=0 \tag{6}$$

for all $n\geqslant 1$ and $\Psi_1,\Psi_2\in\mathscr{H}$, i.e., in the weak sense there exists on $\mathscr{H}$ the limit

$$q(x)=w\lim_{\delta\to 0}q^\delta(x)=0, \tag{7}$$

$$\lim_{\delta\to 0}\langle 0\,|\,T(q^\delta(x)\,\bar{q}^\delta(y))\,|\,0\rangle=G(x-y)=iZ^{-1}(\hat{p})\,\delta(x-y)\not\equiv 0. \tag{8}$$

A field $q^\delta(x)$ satisfying all these conditions is a solution to the problem.

We now find the general form of the operators $Z(\hat{p})$ for which the unique solution of Eq. (2) is (3).

2. *Choice of the function $Z(\hat{p})$.* The function $Z(z)$ must satisfy the following conditions.

1. The function $Z(z)$ must be an entire analytic function in the complex z plane, and $[Z(z)]^*=Z(z^*)$. This follows from the requirement that the action functional $S=\int dx\mathscr{L}_0(x)$ must exist and be real on sufficiently smooth functions $q(x)$ in Minkowski space and Euclidean space.

2. The function $Z(z)$ must not have zeros. This follows from the requirement that the unique solution of Eq. (2) be identically zero: $q(x)\equiv 0$.

3. In the limit $z^2\to-\infty$, the function $Z^{-1}(z)=G(z)$ must decrease sufficiently rapidly: $G(z)\to 0$. This condition means that we shall construct the virton field in the framework of a nonlocal quantum field theory which requires that the Green's functions of the fields decrease in the Euclidean metric.

We write down the general form of a function satisfying the conditions listed above:

$$Z(z)=\exp\{W(z)\}. \tag{9}$$

Here, $W(z)$ is an entire real function which increases as $z^2\to-\infty$.

The functional arbitrariness in the definition of the function $Z(z)$ in (9) can be eliminated by introducing an additional requirement.

4. This is the requirement of minimality. What is the minimal degree of the function $W(z)$ which ensures that all the preceding conditions are satisfied? In this case,

$$Z(z)=\exp\{-a-bz-cz^2\},$$

where a,b,c are real, and $c>0$.

Thus, the functional arbitrariness reduces to the three independent parameters a, b, and c, but it turns out that the parameter a is not independent. We write the operator $Z(\hat{p})$ in the form

$$Z(\hat{p})=\frac{1}{L}\exp\left\{-l\hat{p}-\frac{L^2}{4}p^2\right\}, \tag{10}$$

where l and L are two independent parameters. Hence, for the Green's function we obtain

$$G(\hat{p})=Z^{-1}(\hat{p})=L\exp\left\{l\hat{p}+\frac{L^2}{4}p^2\right\}. \tag{11}$$

The constants l and L are fundamental in the approach and determine the dynamics of all the possible virton interactions.

There is a definite shortcoming in the use of the Green's function in the form (11) and, quite generally, in the form of an entire function of finite degree, i.e.,

$$G(\hat{p})=\exp\left\{\sum_{k=1}^{2n}a_k\hat{p}^k\right\}. \tag{12}$$

This shortcoming is that for any choice of the coefficients a_k and the degree n in (12) integrals of, for example, the form

$$F(p^2)=\int dk_E\,\mathrm{Tr}\,\{\gamma_5G(\hat{k}_E)\,\gamma_5G(\hat{k}_E-\hat{p}_E)\} \tag{13}$$

($p^2=-p_E^2$), which exist in Euclidean space, determine a function $F(p^2)$ which in Minkowski space increases as

$$F(p^2)=O(\exp\{a(p^2)^n\})\quad (a>0).$$

This means that at sufficiently high energies the terms of the perturbation series grow and, therefore, it is not possible to make a restriction to the lowest perturbation orders, i.e., we enter the strong-coupling regime.

One can pose the following problem: Do there exist functions $W(\hat{p})$ in (9) for which the function $F(p^2)$ in (13) is bounded as $p^2\to+\infty$? It can be shown that such functions do exist. We require that $W(\hat{p})$ have no zeros; then there exists a unique function satisfying the imposed condition:

$$W(\hat{p})=\exp\{-a-b\hat{p}-cp^2\},$$

where a, b, and c are real parameters, and $c>0$. For the Green's function, we have

$$G(\hat{p})=Z^{-1}(\hat{p})=A\exp\{-\exp\{-a-b\hat{p}-cp^2\}\}. \tag{14}$$

Thus, the operator $Z(\hat{p})$ and, therefore, the Green's function are determined up to the three real parameters a, b, and c [the parameter A in (14) is not independent].

In the considered quark model (see Sec. 2) we use the Green's function (11), since we consider low-energy hadron physics.

3. *Quantization of the virton field*. The next problem is that of quantizing the system described by the Lagrangian (1) with the operator $Z(\hat{p})$ (9) or (10). This is an unusual problem, since the classical solution of Eq. (2) vanishes identically. To solve this problem, we use the methods developed in quantum field theory with a nonlocal interaction.[15] The idea of our quantization method is to replace the operator $Z(\hat{p})$ in the Lagrangian (1) by a regularized operator $Z^\delta(\hat{p})$ such that, first, the function $Z^\delta(z)$ has an infinite number of zeros

$$Z^\delta(z)\sim\prod_{j=1}^{\infty}\left(1-\frac{z}{M_j(\delta)}\right)$$

at the points

$$z_j=M_j(\delta)>0,$$

which in the limit when the regularization is lifted ($\delta\to 0$) go to infinity,

$$M_j(\delta)\to+\infty,$$

and, second,

$$\lim_{\delta\to 0}Z^\delta(z)=Z(z)$$

uniformly in any bounded domain $\Gamma\subset C$.

There are many ways of introducing such a regularization. For example,

$$Z^{-1}(z)=L\exp\left\{lz+\frac{L^2}{4}z^2\right\}$$

$$=L\exp\left(-\frac{1}{2}l\mu\right)\sum_{n=0}^{\infty}\frac{(z+\mu)^{2n}}{n!}\left(\frac{L^2}{4}\right)^n\to[Z^\delta(z)]^{-1}$$

$$=L\exp\left(-\frac{1}{2}l\mu\right)\sum_{n=0}^{\infty}\frac{(z+\mu)^{2n}}{n!}\left(\frac{L^2}{4}\right)^n\left[\prod_{j=1}^{2n+n_0}\left(1-\frac{\delta(z+\mu)L}{j^\sigma}\right)\right]^{-1}$$

$$=\sum_{j=1}^{\infty}\frac{(-1)^jA_j(\delta)}{M_j(\delta)-z}. \tag{15}$$

Here

$$M_j(\delta)=\frac{1}{L}\left(\frac{j^\sigma}{\delta}-\mu L\right)\quad (j=1,2,3\ldots),$$

$\mu=2l/L^2$, the parameter $\sigma>1$, $A_j(\delta)>0$, and the explicit form of these coefficients can be readily determined. The parameter n_0 can be chosen arbitrarily; it determines the rate of decrease of the regularized function $[Z^\delta(z)]^{-1}$ in the complex z plane:

$$[Z^\delta(z)]^{-1}=O\left(\frac{1}{|z|^{n_0}}\right)\quad (|z|\to\infty).$$

It follows that

$$\sum_{j=1}^{\infty}(-1)^jA_j(\delta)M_j^k(\delta)=0\quad (k=0,1,\ldots,n_0-1). \tag{16}$$

We introduce the system of fields

$$q_j^\delta(x)=\sqrt{A_j(\delta)}\frac{Z^\delta(\hat{p})}{\hat{p}-M_j(\delta)}q^\delta(x)\quad (j=1,2\ldots). \tag{17}$$

Then

$$q^\delta(x)=\sum_{j=1}^{\infty}(-1)^j\sqrt{A_j(\delta)}\,q_j^\delta(x)$$

and

$$\mathscr{L}_0(x)=\bar{q}(x)Z(\hat{p})q(x)\to\mathscr{L}_0^\delta(x)=\overline{q^\delta}(x)Z^\delta(\hat{p})q^\delta(x)$$

$$=\sum_{j=1}^{\infty}(-1)^j\overline{q_j^\delta}(x)(\hat{p}-M_j(\delta))q_j^\delta(x). \tag{18}$$

The fields $q_j^\delta(x)$ $(j=1,2\ldots)$ correspond to some fictitious unphysical quanta with mass $M_j(\delta)$ and do not describe any physical particles. They play an auxiliary part and vanish in the limit $\delta\to 0$.

The equation of motion

$$Z^\delta(\hat{p})q^\delta(x)=0$$

of infinite order can be expressed as an infinite system of Dirac equations:

$$(\hat{p}-M_j(\delta))q_j^\delta(x)=0\quad (j=1,2\ldots). \tag{19}$$

The solution of these equations has the standard form[16]

$$q_j^\delta(x)=\int\frac{d\mathbf{k}}{(2\pi)^{3/2}}[v_{j\mathbf{k}}^\delta d_{j\mathbf{k}}\exp(-ikx)+w_{j\mathbf{k}}^\delta h_{j\mathbf{k}}^+\exp(ikx)],$$

$$\overline{q_j^\delta}(x)=\int\frac{d\mathbf{k}}{(2\pi)^{3/2}}[\overline{v_{j\mathbf{k}}^\delta}d_{j\mathbf{k}}^+\exp(ikx)+\overline{w_{j\mathbf{k}}^\delta}h_{j\mathbf{k}}\exp(-ikx)], \tag{20}$$

where v_{jk}^δ and w_{jk}^δ are Dirac spinors and $k_{j0}=E_{jk}(\delta)=\sqrt{M_j^2(\delta)+\mathbf{k}^2}$.

The Hamiltonian which describes the regularized system (19) has the form

$$H_0^\delta=\sum_{j=1}^{\infty}(-1)^j\int d\mathbf{k}E_{j\mathbf{k}}(\delta)[d_{j\mathbf{k}}^+d_{j\mathbf{k}}-h_{j\mathbf{k}}h_{j\mathbf{k}}^+]. \tag{21}$$

Since the energy of the system must be positive, the spinor fields $q_j^\delta(x)$ must be quantized in accordance with the canonical procedure with the indefinite metric

$$\{d_{j\mathbf{k}},d_{j'\mathbf{k}'}^+\}=\{h_{j\mathbf{k}},h_{j'\mathbf{k}'}^+\}=(-1)^j\delta_{jj'}\delta(\mathbf{k}-\mathbf{k}'),$$

the remaining anticommutators being zero.

The state space $\mathscr{H}$, which contains all the fictitious particles, is a vector space with indefinite metric. It contains:

1) the vacuum state $|0\rangle$, which is unique and is determined by the conditions

$$d_{j\mathbf{k}}|0\rangle=h_{j\mathbf{k}}|0\rangle=0\quad (j=1,2\ldots),$$
$$\langle 0|0\rangle=1;$$

2) single- and many-particle states constructed using the basis vectors:

$$|jn, im\rangle = \frac{1}{\sqrt{n!m!}} d^{+}_{j_1 k_1}\cdot\ldots\cdot d^{+}_{j_n k_n} h^{+}_{i_1 p_1}\cdot\ldots\cdot h^{+}_{i_m p_m}\,|0\rangle.$$

The vacuum, single-particle, and many-particle states form a complete system of eigenstates in the vector space $\mathscr{H}$. It is important that the vacuum $|0\rangle$ and the operators d_{jk} and h_{jk} do not depend on the regularization parameter δ.

We define the state space $\mathscr{H}(E)$ as the space of normalized physical states $\Psi(E)$ with energy not exceeding the energy E:

$$\Psi(E) = \sum_{\{jn,\, im\}} \int d^n\mathbf{k} \int d^m\mathbf{p}\,\theta(E - \Sigma E_{j\mathbf{k}}(\delta) - \Sigma E_{i\mathbf{p}})(\delta)\,\tilde{f}_{\{jn,\, im\}}(\mathbf{k}, \mathbf{p})\,|jn, im\rangle,$$

where $d^n\mathbf{k} = d\mathbf{k}_1, \ldots, d\mathbf{k}_n$, $\sum E_{j\mathbf{k}}(\delta) = \sum_{\nu=1}^{n} E_{j_\nu \mathbf{k}_\nu}(\delta)$, and

$$\tilde{f}_{\{jn,\, im\}}(\mathbf{k}, \mathbf{p}) = \tilde{f}_{j_1, \ldots, j_n, i_1, \ldots, i_m}(\mathbf{k}_1, \ldots, \mathbf{k}_n, \mathbf{p}_1, \ldots, \mathbf{p}_n) \in \tilde{Z}_2.$$

The function space is $\tilde{Z}_2 = \cup \tilde{Z}_2(N)$, where $\tilde{Z}_2(N)$ consists of functions $\tilde{g}(\mathbf{k}_1, \ldots, \mathbf{k}_N)$ of the $3N$ variables $\mathbf{k}_1, \ldots, \mathbf{k}_N$ which are such that they are differentiable and for any $\varepsilon > 0$ there exists $C_\varepsilon > 0$ such that

$$|\tilde{g}(\mathbf{k}_1, \ldots, \mathbf{k}_N)| \leqslant C_\varepsilon \exp\left\{-\frac{1}{\varepsilon}\sum_{\nu=1}^{N} |\mathbf{k}_\nu|^2\right\}. \tag{22}$$

Then for any $\Psi(E) \in \mathscr{H}(E)$

$$\|\Psi\|^2 = (\Psi(E), \Psi(E))$$
$$= \sum_{\{jn,\, im\}} \int d^n\mathbf{k} \int d^n\mathbf{p}\,\theta(E - \Sigma E_{j\mathbf{k}}(\delta) - \Sigma E_{i\mathbf{p}}(\delta))\,(-1)^{\Sigma j + \Sigma i}\,|\tilde{f}_{\{jn,\, im\}}(\mathbf{k}, \mathbf{p})|^2 < \infty.$$

The space Z_2, which consists of the Fourier transforms of the functions in $\tilde{Z}_2$, contains entire analytic functions $g(\mathbf{z}_1, \ldots, \mathbf{z}_N)$ such that for any $\varepsilon > 0$ there exists $C_\varepsilon > 0$ such that

$$|g(\mathbf{z}_1, \ldots, \mathbf{z}_N)| \leqslant C_\varepsilon \exp\{\varepsilon \sum_{\nu=1}^{N} |\mathbf{z}_\nu|^2\};$$

$$\int d\mathbf{x}_1 \cdot\ldots\cdot \int d\mathbf{x}_N\,|g(\mathbf{x}_1 + i\mathbf{y}_1, \ldots, \mathbf{x}_N + i\mathbf{y}_N)| < \infty$$

for all real $\mathbf{y}_1, \ldots, \mathbf{y}_N$.

We shall regard the vector space $\mathscr{H} = \cup_E \mathscr{H}(E)$ as the inductive limit of the spaces $\mathscr{H}(E)$ with respect to the embedding $\mathscr{H}(E) \to \mathscr{H}$.

Thus, we have constructed the field operators $q_j^\delta(x)$ and $q^\delta(x)$ (20) and the vector space $\mathscr{H}$ on which these operators exist.

It should be noted that the operator $q^\delta(x)$ (17) for $\delta > 0$ is a local operator on the space $\mathscr{H}$, since for $(x-y)^2 < 0$

$$\{q^\delta(x), \overline{q^\delta}(y)\} = 0.$$

4. *Green's functions of the virton field*. We now consider what happens to the field $q^\delta(x)$ and the various Green's functions in the limit $\delta \to 0$ when the regularization is lifted. Physically, it is clear that in this limit any physical state characterized by a definite value of the energy cannot contain fictitious quanta, since their masses increase in accordance with (15). Mathematically, this can be readily proved. The convergence on the space $\mathscr{H}$ is defined as convergence on the spaces $\mathscr{H}(E)$ for all fixed $E > 0$. Then for all $\Psi_1(E)$, $\Psi_2(E) \in \mathscr{H}(E)$ and all $n \geqslant 1$ Eq. (6) is satisfied, since for any fixed $E > 0$ there exists $\delta(E) > 0$ such that $M_1(\delta) > E$ and for all $\delta < \delta(E)$

$$(\Psi_1(E),\ :q^\delta(x_1)\cdot\ldots\cdot q^\delta(x_n):\ \Psi_2(E)) = 0.$$

This means that in the weak sense on $\mathscr{H}$

$$q(x) = w\lim_{\delta\to 0} q^\delta(x) = 0. \tag{23}$$

Thus, the quantized free virton field $q(x)$ is zero.

In the limit $\delta \to 0$, the Green's functions are generalized functions defined on Z_2. Therefore, it is necessary to consider the improper limit

$$\lim_{\delta\to 0} \int dx\, G^\delta(x) f(x) = \lim_{\delta\to 0} \int dp\, G^\delta(p)\tilde{f}(p).$$

We introduce the standard Green's functions

$$G^\delta_{(+)}(x-y) = \{q^\delta(x), \overline{q^\delta}(y)\};$$
$$G^\delta_{(-)}(x-y) = \langle 0|\, q^\delta(x)\,\overline{q^\delta}(y)\,|0\rangle;$$
$$G^\delta(x-y) = \langle 0|\, T(q^\delta(x)\,\overline{q^\delta}(y))\,|0\rangle.$$

It can be shown that

$$\lim_{\delta\to 0} \int dx G^\delta_{(+)}(x) f(x) = \lim_{\delta\to 0} \int dx G^\delta_{(-)}(x) f(x) = 0.$$

Since the corresponding calculations are simple but lengthy, we shall not give them here but merely refer to Ref. 15, in which the analogous calculations are made for the scalar case.

We now consider the causal Green's function $G(x)$. We have

$$G^\delta(x-y) = \sum_{j=1}^{\infty} A_j(\delta)\,\langle 0|\, T(q_j^\delta(x)\,\overline{q_j^\delta}(y))\,|0\rangle$$

$$= \int \frac{dp}{(2\pi)^4 i} \exp[-ip(x-y)] \sum_{j=1}^{\infty} \frac{(-1)^j A_j(\delta)}{M_j(\delta) - \hat{p} - i0}. \tag{24}$$

In the limit $\delta \to 0$, we have in accordance with (15) and (22)

$$\lim_{\delta\to 0} \int dx G^\delta(x) f(x) = \lim_{\delta\to 0} \int \frac{dp}{(2\pi)^4 i} \tilde{f}(p) \sum_{j=1}^{\infty} \frac{(-1)^j A_j(\delta)}{M_j(\delta) - \hat{p} - i0}$$
$$= \int \frac{dp}{(2\pi)^4 i} \tilde{f}(p)\, L \exp\left\{l\hat{p} + \frac{L^2}{4} p^2\right\}.$$

This means that the causal function $G^\delta(x)$ in the limit $\delta \to 0$ is transformed into the nonlocal propagator

$$\lim_{\delta\to 0} G^\delta(\hat{p}) = G(\hat{p}) = L \exp\left\{l\hat{p} + \frac{L^2}{4} p^2\right\}. \tag{25}$$

Thus, we have satisfied the conditions (3) and (5) formulated above.

It should be noted that the explicit form of the regularization procedure is not important for studying the limit expressions. It is important only that the regularization procedure 1) exists and defines a local regularized field; 2) ensures the possibility of transition to the Euclidean metric; and 3) admits the passage to the limit $\delta \to 0$.

In what follows, we shall use the representation

$$G(\hat{p}) = A(-p^2) + \hat{p}B(-p^2), \tag{26}$$

where

$$A(-p^2) = L\,\mathrm{ch}\, l\sqrt{p^2}\exp\left(\frac{L^2p^2}{4}\right) = L\,\mathrm{ch}\,\xi\sqrt{\frac{L^2p^2}{4}}\exp\left(\frac{L^2p^2}{4}\right);$$

$$B(-p^2)=L\frac{\operatorname{sh} l\sqrt{p^2}}{\sqrt{p^2}}\exp\left(\frac{L^2p^2}{4}\right)=L\frac{L}{2}\frac{\operatorname{sh}\xi\sqrt{\frac{L^2p^2}{4}}}{\sqrt{\frac{L^2p^2}{4}}}\exp\left(\frac{L^2p^2}{4}\right).$$

On the transition to the Euclidean metric, $p^2\to -p_E^2$,

$$A(p_E^2)=L\cos\xi\sqrt{\frac{L^2p_E^2}{4}}\exp\left(-\frac{L^2p_E^2}{4}\right);$$
$$B(p_E^2)=L\frac{L}{2}\frac{\sin\xi\sqrt{\frac{L^2p_E^2}{4}}}{\sqrt{\frac{L^2p_E^2}{4}}}\exp\left(-\frac{L^2p_E^2}{4}\right). \tag{27}$$

The regularized Green's function satisfies

$$G^\delta(\hat{p})=A^\delta(-p^2)+\hat{p}B^\delta(-p^2)$$
$$=\sum_{j=1}^{\infty}\frac{(-1)^j A_j(\delta)}{M_j(\delta)-\hat{p}-\mathrm{i}0}=\sum_{j=1}^{\infty}(-1)^j A_j(\delta)\,S_j^\delta(\hat{p}); \tag{28}$$
$$A^\delta(-p^2)=\sum_{j=1}^{\infty}\frac{(-1)^j A_j(\delta)\,M_j(\delta)}{(M_j-\mathrm{i}0)^2-p^2}\xrightarrow[\delta\to 0]{}A(-p^2);$$
$$B^\delta(-p^2)=\sum_{j=1}^{\infty}\frac{(-1)^j A_j(\delta)}{(M_j-\mathrm{i}0)^2-p^2}\xrightarrow[\delta\to 0]{}B(-p^2).$$

Here

$$S_j^\delta(\hat{p})=\frac{1}{M_j(\delta)-\hat{p}-\mathrm{i}0}. \tag{29}$$

5. *Interactions of the virton field with the fields of the physical particles.* We now consider how we can use the constructed virton field in hadron physics. It is attractive to use the virton field to describe quarks, since the principal dynamical property of quarks—their unobservability—is already contained in the virton field. It should be emphasized that there is then no need to introduce any gluon fields, which in the standard approach are needed to ensure quark confinement.

If it is assumed that the quarks are described by the virton field, then to describe hadron physics, two approaches are possible. In the one, it is possible to introduce a Lagrangian of the virton field of the type

$$\mathscr{L}=(\bar{q}Zq)+\lambda(\bar{q}\Gamma q)(\bar{q}\Gamma q) \tag{30}$$

and seek bound states in the virton–quark system and identify these bound states with the known hadrons. This idea warrants special investigation and, technically, is not simple, since the problem of finding bound states has not yet been solved in quantum field theory.

In the other approach it is assumed that the hadrons are elementary particles and are described by standard quantized fields satisfying the ordinary Dirac, Klein-Gordon fields, etc. However, the hadron fields [for example, of the pseudoscalar mesons $P(x)$, the vector mesons $V_\mu(x)$, the baryons $B(x)$, etc.] interact with one another not directly but through the intermediate virton–quark field $q(x)$. In this sense, the interaction Lagrangian can be chosen, for example, in the form

$$\mathscr{L}_I(x)=\mathrm{i}g_P P(x)\,(\bar{q}(x)\gamma_5 q(x))+g_V V_\mu(x)\,(\bar{q}(x)\gamma_\mu q(x))$$
$$+\mathrm{i}g_B(\bar{B}(x)\,q(x))\,(\bar{q}^c(x)\gamma_5 q(x))+\text{h.c.} \tag{31}$$

The S matrix which describes the strong interactions of the hadrons with the Lagrangian (31) can be constructed by the methods of nonlocal quantum field theory.[15] Instead of $\mathscr{L}_I(x)$, it is necessary to introduce in (31) the regularized interaction Lagrangian

$$\mathscr{L}_I^\delta(x)=\mathrm{i}g_P P\,(\bar{q^\delta}\gamma_5 q^\delta)+g_V V_\mu\,(\bar{q^\delta}\gamma_\mu q^\delta)+\mathrm{i}g_B\,(\bar{B}q^\delta)\,(\bar{q}^{\delta c}\gamma_5 q^\delta)+\text{h.c.} \tag{32}$$

The regularized S^δ matrix is determined in the usual manner:

$$S^\delta=T\exp\left\{\mathrm{i}\int dx\mathscr{L}_I^\delta(x)\right\}, \tag{33}$$

since the field $q^\delta(x)$ is local. The limit

$$\lim_{\delta\to 0}S^\delta=S \tag{34}$$

exists, as is readily proved by using the methods developed in Ref. 15, and it determines a finite unitary causal S matrix in each order of perturbation theory. The obtained S matrix describes only the interaction of the hadrons, since there are no quarks in the initial or final states.

Naturally, all the calculations of the various physical effects are made in perturbation theory. We must therefore consider what is the effective expansion constant of the perturbation series for interaction Lagrangians of the type (31).

We consider first the interaction of bosons with the virtons. Since this interaction is of Yukawa type, it is to be expected that any matrix element in the n-th order of perturbation theory will be of the order of any diagram of n-th order. The amplitude T corresponding to some process can be written formally as

$$T=\sum_n g_P^{2n+n_0}\int\prod_{i=1}^{s}\frac{dk_i}{(2\pi)^4}\prod_{j=1}^{R}L\exp\left\{\mathrm{i}l(\hat{k}_j+\hat{p})-\frac{L^2}{4}(k_j+p)^2\right\}\prod_{l=1}^{Q}\frac{1}{m^2+(k_l+p)^2}.$$

The integrals are already expressed in the Euclidean metric. Here, n_0 depends on the investigated process, R is the number of internal virton lines, s is the number of independent integrations, and E and Q are, respectively, the numbers of external and internal meson lines. These numbers are related by

$$2n+n_0=R=E+2Q;$$
$$s=Q+R-2n-n_0+1.$$

Introducing dimensionless momenta of integration $q_i=Lk_i/2$ and separating the dimensional factors from the virton and meson propagators, we obtain

$$T=\frac{(4\pi)^E}{(2\pi)^2}\left(\frac{g_P}{4\pi}\right)^{n_0}\sum_{n=0}^{\infty}\left[\frac{g_P^2}{(4\pi)^2}\right]^n I_n;$$
$$I_n=\int\prod_{i=1}^{s}\left(\frac{4}{\pi^2}dq_i\right)\prod_{j=1}^{R}\exp\left\{\mathrm{i}\xi\left(\hat{q}_i+\frac{L\hat{p}}{2}\right)-\left(q_i+\frac{Lp}{2}\right)^2\right\}$$
$$\times\prod_{t=1}^{Q}\frac{1}{\left(\frac{mL}{2}\right)^2+\left(q_t+\frac{Lp}{2}\right)^2}.$$

Since $(mL/2)^2\lesssim 1$ for mesons and the integration over q_j is performed with a Gaussian exponential, we have in order of magnitude in the limit $n\to\infty(R\sim 2n, Q\sim n, s\sim n)$

$$I_n\approx\prod_{i=1}^{n}\int\left(\frac{4}{\pi^2}dq_j\exp(-2q_j^2)\right)\prod_{t=1}^{n}\frac{1}{q_t^2}\approx 1,$$

since q_t is a sum of the momenta of integration. Therefore, the parameter

$$\lambda_P=g_P^2/(4\pi)^2 \tag{35}$$

can be taken as the effective expansion constant of the perturbation series.

In the case of the Lagrangian of the four-fermion interaction (31), we readily show by making similar estimates[17] that the effective expansion constant is the parameter

$$\lambda_B = (1/2)\,(g_B/L^2)^2\,(2/3\pi)^4. \tag{36}$$

6. *Physical meaning of the parameters l and L.* Let us elucidate the physical meaning of the constants l and L in (11). For this, we investigate the interaction potential between two point sources exchanging quanta of the virton field, as in the derivation of the Yukawa potential. Suppose that there are two fermion sources $\psi_1(x)$ and $\psi_2(x)$ and that the interaction between them and the virton field $q(x)$ is described by the Lagrangian

$$\mathcal{L}_I(x) = g\,[(\bar{\psi}_1(x)\,q(x)) + (\bar{\psi}_2(x)\,q(x))] + \text{h.c.} \tag{37}$$

The energy of the interaction between them in the second order of perturbation theory in g is

$$W = g^2 \int\int dx_1\, dx_2\, [\bar{\psi}_1(x_1)\, G(x_1 - x_2)\, \psi_2(x_2) + \text{h.c.}\,].$$

We assume that these sources are at rest and

$$\psi_j(x) = \psi_j(\mathbf{x}, t) = u\delta^{(3)}(\mathbf{x} - \mathbf{r}_j) \qquad (j = 1, 2),$$

where u is the Dirac spinor describing the sources at rest and normalized such that

$$\bar{u}u = 1, \quad \bar{u}\gamma_\mu u = 0.$$

Then, denoting $r = |\mathbf{r}_1 - \mathbf{r}_2|$, we obtain

$$W(r) = g^2 \bar{u} G(r)\, u$$
$$= \text{const}\left[\left(1 + \frac{l}{r}\right)\exp\left[-\frac{(r+l)^2}{L^2}\right] + \left(1 - \frac{l}{r}\right)\exp\left[-\frac{(r-l)^2}{L^2}\right]\right], \tag{38}$$

where $G(\mathbf{r}) = \int d\mathbf{p}\exp(i\mathbf{p}\mathbf{r})\tilde{G}(\mathbf{p})$ and

$$\tilde{G}(\mathbf{p}) = L\left[\cos l\sqrt{\mathbf{p}^2} - \gamma\mathbf{p}\,\frac{\sin l\sqrt{\mathbf{p}^2}}{\sqrt{\mathbf{p}^2}}\right]\exp\left(-\frac{L^2\mathbf{p}^2}{4}\right).$$

The potential $W(r)$ decreases as $r \to \infty$ as $W(r) \sim \exp\{-(r/L)^2\}$ and is bounded as $r \to 0$. In this, it differs from the Yukawa potential $\exp\{-mr\}/r$.

Note that the potential (38) does not have any relation to an increasing, quark-confining potential. Its physical meaning is that the interaction of the sources due to the exchange of virtons decreases with increasing distance much more rapidly than in the case of the exchange of particles with mass m.

We now calculate the mean value $\langle \mathbf{r}^2\rangle_W$ determined by the distribution $W(r)$:

$$\langle \mathbf{r}^2\rangle_W = \frac{\int d\mathbf{r}\,(\mathbf{r}^2)\, W(r)}{\int d\mathbf{r} W(r)} = \frac{3}{2}\,(2l^2 + L^2). \tag{39}$$

For the Yukawa potential,

$$\langle \mathbf{r}^2\rangle_Y = \frac{\int dr r^2 \frac{1}{r}\exp(-mr)}{\int dr \frac{1}{r}\exp(-mr)} = \frac{6}{m^2}. \tag{40}$$

From the relations (39) and (40), we can define a formal virton mass:

$$m_q = \sqrt{\frac{6}{\langle \mathbf{r}^2\rangle_W}} = \frac{2}{\sqrt{L^2 + 2l^2}} = \frac{2}{L}\,\frac{1}{\sqrt{1 + \frac{1}{2}\xi^2}}. \tag{41}$$

7. *The bound-state condition.* Thus, we assume that all hadrons are bound states of quarks. If the hadron and quark fields are included independent in the original Lagrangian, as in (31), then in S-matrix theory the fact that the hadron consists of quarks means that the wave-function renormalization constant of the hadron h must vanish:

$$Z_h(g,\, m_h L,\, \xi) = 0. \tag{42}$$

Indeed, the constant Z_h is the probability that the physical particle is in the "bare" state:

$$Z_h = |\langle \mathbf{h} \mid h\rangle|^2,$$

where $|\mathbf{h}\rangle$ and $|h\rangle$ are the "dressed" and "bare" state of the hadron h, respectively. Therefore, the condition (42) means that a physical particle is always dressed and never in a state described by the free Hamiltonian (see Ref. 14 for more details about the conditions which determine a bound state in quantum field theory). It should be noted that in the considered model all the renormalization constants are finite.

Thus, the coupling constant g can be determined from the bound-state condition (42) as a function of the hadron mass and the quark parameters L and ξ.

In the present paper, we shall take into account the condition (42) up to the second order of perturbation theory:

$$Z_h = 1 + g^2 \frac{d}{dp^2}\,\tilde{\Sigma}_2^h(p^2)\,|_{p^2 = m^2} = 0, \tag{43}$$

where $\tilde{\Sigma}_2^h(p^2)$ is the hadron mass operator in the second order of perturbation theory.

Allowance for the higher orders of perturbation theory requires complete renormalization of the theory. We shall not do that here, since the effective expansion constant is found to be less than unity, and therefore the matrix elements of all the considered processes will be calculated in the lowest orders of perturbation theory.

We now calculate the mass operator in the second order of perturbation theory in the case of the pion field. We represent the term of the S matrix corresponding to the diagram in Fig. 1 in the form

$$S_2^\delta = -\frac{i}{2}\int\int dx\, dy P(x)\, g_P^2 \Sigma_2^\delta (x - y)\, P(y),$$

where

$$g_P^2 \Sigma_2^\delta(x - y) = -ig_P^2\, \mathrm{Tr}\,\{\gamma_5 G^\delta(x - y)\,\gamma_5 G^\delta(y - x)\}$$
$$= g_P^2 \int \frac{dp}{(2\pi)^4}\exp[ip(x - y)]\,\tilde{\Sigma}_2^\delta(p^2).$$

Going over to the Euclidean metric and lifting the regularization, we obtain

$$g_P^2 \tilde{\Sigma}_2(p^2) = \lim_{\delta\to 0} g_P^2 \tilde{\Sigma}_2^\delta(p^2) = -4\int \frac{dk_E}{(2\pi)^4}\,[A(k_E^2)\, A((k-p)_E^2)$$
$$+ k_E(k_E - p_E)\, B(k_E^2)\, B((k-p)_E^2)]$$
$$= -4\left(\frac{2}{L}\right)^2 \frac{g_P^2}{(4\pi)^2}\, Y\left(\frac{p^2L^2}{4},\ \xi\right),$$

where

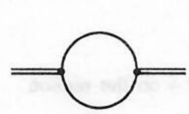


FIG. 1. Meson self-energy diagram.

$$Y(q^2, \xi)=16\int_0^\infty dt\,\frac{I_1(\sqrt{q^2}t)}{\sqrt{q^2}}\Big\{\Big[\int_0^\infty dss^2\cos\xi s\exp(-s^2)J_1(st)\Big]^2+\Big[\int_0^\infty dss^2\sin\xi s\exp(-s^2)J_2(ts)\Big]^2\Big\},$$

and $I_\nu(z)$ and $J_\nu(z)$ are Bessel functions.

The renormalizations of the mass of the pion and its wave function are determined in the standard manner:

$$g_P^2\widetilde{\Sigma}_2(p^2)=\delta m^2+(Z-1)(p^2-m^2)+\widetilde{\Sigma}_{2R}(p^2);$$
$$\delta m^2=g_P^2\widetilde{\Sigma}_2(m^2);\quad Z=1+g_P^2\widetilde{\Sigma}_2'(m^2).$$

Setting $Z=0$, we determine the effective expansion constant $\lambda_P=g_P^2/(4\pi)^2$ as a function of the parameters $mL/2$ and ξ. The corresponding family of graphs is shown in Fig. 2.

8. *Virtrons and the electromagnetic field.* Since we intend to describe quarks by the virton field, and quarks are charged particles, we consider the electromagnetic interactions of the virton field. The Lagrangian of the classical free quark–virton field has the form (1). The Lagrangian which describes the interaction of the electromagnetic field with the virtons is obtained by the standard "minimal" substitution

$$i\frac{\partial}{\partial x_\mu}\to i\frac{\partial}{\partial x_\mu}+e_qA_\mu(x).$$

Then the gauge-invariant Lagrangian can be written in the form

$$\mathcal{L}_{em}(x)=-\frac{1}{4}F_{\mu\nu}(x)F_{\mu\nu}(x)+\overline{q}(x)Z(\hat{p}+e_q\hat{A}(x))q(x). \qquad (44)$$

When the electromagnetic interaction is introduced, it is necessary to preserve gauge invariance at all stages of the calculations. This means that the electromagnetic field must be introduced in such a way that the regularized Lagrangian is also gauge invariant, i.e.,

$$\mathcal{L}\to\mathcal{L}^\delta=\overline{q}^\delta(x)Z^\delta(\hat{p}+e_q\hat{A}(x))q(x). \qquad (45)$$

We now introduce the system of fields

$$q_j^\delta(x)=\sqrt{A_j(\delta)}\frac{Z^\delta(\hat{p}+e_q\hat{A})}{\hat{p}+e_q\hat{A}-M_j(\delta)}q^\delta(x)\quad(j=1,2\ldots); \qquad (46)$$
$$q^\delta(x)=\sum_{j=1}^\infty(-1)^j\sqrt{A_j(\delta)}\,q_j^\delta(x).$$

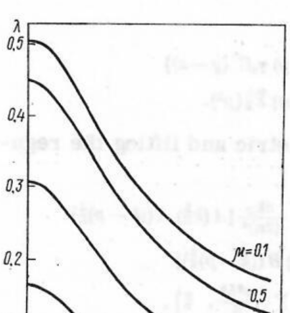


FIG. 2. Dependence of the coupling constant λ on the meson mass and on the parameter ξ.

Then under the gauge transformations

$$q^\delta(x)\to\exp[ie_qf(x)]\,q^\delta(x)$$

the fields $q_j^\delta(x)$ transform as follows:

$$q_j^\delta(x)\to\exp[ie_qf(x)]\,q_j^\delta(x).$$

The Lagrangian (45) can be represented in the form

$$\mathcal{L}^\delta(x)=\sum_{j=1}^\infty(-1)^j\,\overline{q}_j^\delta(x)(\hat{p}+e_q\hat{A}(x)-M_j(\delta))\,q_j^\delta(x)=\mathcal{L}_0^\delta+\mathcal{L}_{I em}^\delta, \qquad (47)$$

where $\mathcal{L}_{I em}^\delta=e_qJ_\mu^\delta(x)A_\mu(x)$, in which

$$J_\mu^\delta(x)=\sum_{j=1}^\infty(-1)^j\,\overline{q}_j^\delta(x)\gamma_\mu q_j^\delta(x). \qquad (48)$$

Thus, in the given model there is a conserved vector current J_μ and in regularized form it is represented by (48). It should be emphasized that the vector current

$$I_\mu^\delta(x)=\overline{q}^\delta(x)\gamma_\mu q^\delta(x) \qquad (49)$$

is not conserved. The existence of the two vector expressions (48) and (49), one of which is a conserved current but the other simply a vector, is a feature of the model.

An axial current J_μ^A can be introduced similarly. The regularized form of this current is

$$J_\mu^{A\delta}(x)=\sum_{j=1}^\infty(-1)^j\,\overline{q}_j^\delta(x)\gamma_\mu\gamma_5q_j^\delta(x). \qquad (50)$$

The current J_μ^A can be used in weak interactions.

The regularized S matrix is defined in the usual manner:

$$S^\delta=T\exp\Big\{ie_q\int dx\,J_\mu^\delta(x)A_\mu(x)\Big\}. \qquad (51)$$

All physical matrix elements are obtained in the limit $\delta\to0$. Since the virton field $q^\delta(x)$ vanishes in this limit, we consider the Feynman diagrams containing only closed virton loops.

We analyze the interaction between photons due to the virtons. First, we consider the vacuum-polarization diagram (Fig. 3). The term of the S matrix corresponding to this diagram can be represented in the form

$$-i:A_\mu(x)\,\Pi_{\mu\nu}^\delta(x-y)\,A_\nu(y):,$$

where

$$\Pi_{\mu\nu}^\delta(x-y)=-ie_q^2\,\mathrm{Sp}\sum_{j=1}^\infty\{\gamma_\mu S_j^\delta(x-y)\gamma_\nu S_j^\delta(y-x)\}.$$

The vacuum-polarization diagram contains ultraviolet divergences. To eliminate them, we use the gauge-invariant Pauli–Villars regularization with subsidiary conditions.[15,16] We then obtain

$$\widetilde{\Pi}_{\mu\nu}^\delta(p)=\int dx\exp(-ipx)\,\Pi_{\mu\nu}^\delta(x)=(g_{\mu\nu}p^2-p_\mu p_\nu)\,\Pi^\delta(p^2);$$

$$\Pi^\delta(p^2)=\frac{e_q^2}{12\pi^2}\sum_{j=1}^\infty\frac{p^2}{4M_j^2(\delta)}\int_0^1\frac{du\,\sqrt{1-u}\left(1+\frac{1}{2}u\right)}{1-\frac{p^2}{4M_j^2(\delta)}u-i0}.$$

This series converges well, and therefore $M_j(\delta)\approx(j^\sigma/$

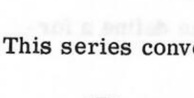


FIG. 3. Vacuum-polarization diagram.

$\delta)(1/L)$ $(\sigma > 1)$ as $\delta \to 0$, and in this limit

$$\Pi^{\delta}(p^2) \approx \frac{e_q^2}{60\pi^2}(p^2L^2)\sum_{j=1}^{\infty}\frac{\delta^2}{j^{2\sigma}}.$$

The function $\Pi^{\delta}(p^2)$ tends to zero in the limit $\delta \to 0$, as δ^2, since the series converges.

The virton loops containing more than two photon lines (Fig. 4) can be represented in the form

$$\Pi^{\delta}_{\mu_1, \dots, \mu_{2n}}(p_1, \dots, p_{2n}) \sim \sum_{j=1}^{\infty}\int dk \sum_{(1, \dots, 2n)} \mathrm{Tr}\Big\{\gamma_{\mu_{2n}} \times S_j^{\delta}\Big(k + \sum_{i=1}^{2n-1} p_i\Big)\gamma_{\mu_{2n-1}} \cdot \ldots \cdot \gamma_{\mu_2} S_j^{\delta}(k+p_1)\gamma_{\mu_1} S_j^{\delta}(k)\Big\}. \quad (52)$$

Here, $\sum_{(1, \dots, 2n)}$ denotes the sum over all permutations of the photon vertices $\gamma_{\mu_1}, \dots, \gamma_{\mu_{2n}}$. The integral in (52) does not contain ultraviolet divergences, and the series over j converges well. In the limit $\delta \to 0$, using $M_j(\delta) \to \infty$, we obtain

$$\lim_{\delta \to 0} \Pi^{\delta}_{\mu_1, \dots, \mu_{2n}}(p_1, \dots, p_{2n}) = 0.$$

Thus, any element of the S matrix which describes the photon–photon interaction through virton carriers is zero.

However, if a virton loop contains even one hadron vertex, then the matrix element corresponding to such a diagram is nonzero. Below, we consider examples of such diagrams.

It should also be noted that the virton diagrams with electromagnetic radiative corrections of the type shown in Fig. 5 contain ultraviolet divergences. This is due to the local nature of the electromagnetic current (48). At the same time, the theory is renormalizable, like ordinary spinor electrodynamics.

9. *The technique for calculating matrix elements*. As an example, we obtain the matrix element of the transition $V \to 2P$. The Feynman diagram describing this process in the lowest order of perturbation theory is shown in Fig. 6. The invariant amplitude can be written in the form

$$M_{V \to 2P} = g_P^2 g_V \varepsilon_\mu \lim_{\delta \to 0}\int \frac{dk}{(2\pi)^4 i} \times \mathrm{Tr}\{\gamma_\mu G^{\delta}(\hat{k} - \hat{p}_2)\gamma_5 G^{\delta}(\hat{k})\gamma_5 G^{\delta}(\hat{k} + \hat{p}_1)\},$$

where ε_μ is the polarization vector of the vector meson.

In the calculation of such integrals in the case of small ($\lesssim 1$ GeV) particle masses, we can use an expansion in the external momenta of the particles, which corresponds to an expansion in the parameter $\mu = mL/2 \lesssim 1$, where m is the mass of the physical particle.

In the given case, the first approximation is the linear approximation in p_1 and p_2. It is readily seen that

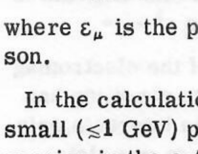


FIG. 4. Virton loop describing the photon–photon interaction.

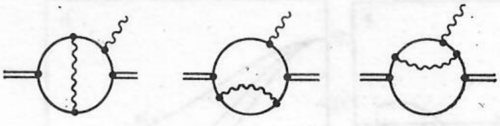


FIG. 5. Virton loops with radiative corrections containing ultraviolet divergences.

the above integral changes sign under the transposition $p_1 \rightleftharpoons p_2$, and it is therefore sufficient to make the calculation for $p_2 = 0$. Going over to the Euclidean metric and using the relation (26), we obtain

$$\lim_{\delta \to 0}\int \frac{dk}{(2\pi)^4 i}\frac{1}{4}\mathrm{Tr}\{G^{\delta}(\hat{k})\gamma_5 G^{\delta}(\hat{k})\gamma_5 G^{\delta}(\hat{k} + \hat{p}_1)\gamma_\mu\}$$
$$= \lim_{\delta \to 0}\int \frac{dk}{(2\pi)^4 i}[(A^{\delta}(-k^2))^2 - k^2(B^{\delta}(-k^2))^2] \times B^{\delta}(-(k+p_1)^2)(k+p_1)_\mu$$
$$\approx p_{1\mu}\frac{\pi^2}{(2\pi)^4}\int_0^{\infty} du\,u\,[A^2(u) + uB^2(u)][B(u) + \frac{u}{2}B'(u)]$$
$$= p_{1\mu}\frac{L^2}{(4\pi)^2}\int_0^{\infty} du\,u\,\exp\left(-\frac{uL^2}{2}\right)[B(u) + \frac{u}{2}B'(u)]$$
$$= p_{1\mu}\frac{L^2}{(4\pi)^2}\frac{L^2}{4}\int_0^{\infty} du\,u^2\exp\left(-\frac{uL^2}{2}\right)B(u)$$
$$= p_{1\mu}\frac{1}{\pi^2}\int_0^{\infty} dt\,t^5\exp(-3t^2)\frac{\sin\xi t}{t} = \frac{p_{1\mu}}{2\pi^2}\frac{2\xi}{27}S_2\left(\frac{\xi}{\sqrt{3}}\right).$$

Here and in what follows, the structure integrals are, as a rule, expressed in terms of the functions

$$C_n(\xi) = \frac{2}{n!}\int_0^{\infty} dt\,t^{2n+1}\cos\xi t\exp(-t^2);$$
$$S_n(\xi) = \frac{2}{n!}\int_0^{\infty} dt\,t^{2n+1}\frac{\sin\xi t}{\xi t}\exp(-t^2). \quad (53)$$

The graphs of these functions for $n = 0, 1, 2$ are shown in Fig. 7.

For the matrix element $M_{V \to 2P}$, we finally obtain

$$M_{V \to 2P} = g_P^2 g_V \frac{4\xi}{27\pi^2}S_2\left(\frac{\xi}{\sqrt{3}}\right)((p_1 - p_2)\varepsilon).$$

We now consider examples of processes with the participation of hadrons and the electromagnetic field. The integrals corresponding to the diagrams shown in Fig. 8 can be written in the form

$$T^{\delta}_{\mu\nu}(P \to \gamma\gamma) = \int \frac{dk}{(2\pi)^4 i}\sum_{j=1}^{\infty}(-1)^j A_j(\delta) \times \mathrm{Tr}\{S_j^{\delta}(\hat{k})\gamma_\mu S_j^{\delta}(\hat{k} + \hat{k}_1)\gamma_\nu S_j^{\delta}(\hat{k} + \hat{k}_1 + \hat{k}_2)\gamma_5\};$$
$$T^{\delta}_{\mu\nu}(V \to \gamma) = \int \frac{dk}{(2\pi)^4 i}\sum_{j=1}^{\infty}(-1)^j A_j(\delta)\,\mathrm{Tr}\{S_j^{\delta}(\hat{k})\gamma_\nu S_j^{\delta}(\hat{k} + \hat{q})\gamma_\mu\};$$
$$T^{\delta}_{\mu\nu}(V \to P\gamma) = \int \frac{dk}{(2\pi)^4 i}\sum_{j=1}^{\infty}(-1)^j A_j(\delta) \times \mathrm{Tr}\{S_j^{\delta}(\hat{k})\gamma_\mu S_j^{\delta}(\hat{k} + \hat{k}_1)\gamma_\nu G^{\delta}(\hat{k} + \hat{k}_1 + \hat{k}_2)\gamma_5\}.$$

Making the standard calculations (taking the trace, Feyman's α parametrization, transition to the Euclidean metric, and integration over the spherical angles), we obtain

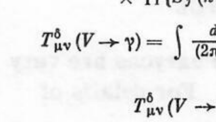


FIG. 6. Diagram describing the decay $V \to 2P$.

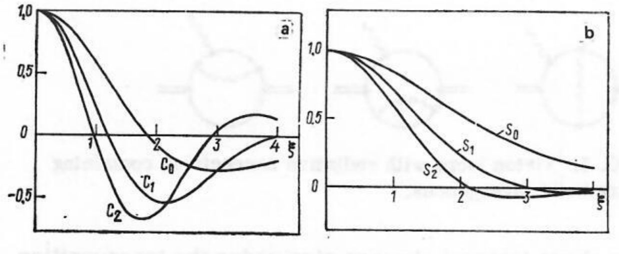


FIG. 7. Dependence of the functions C_n (a) and S_n (b) on ξ.

$$T^{\delta}_{\mu\nu}(P\to\gamma\gamma)=\frac{1}{8\pi^2 i}\varepsilon_{\mu\rho\nu\sigma}k_{1\rho}k_{2\sigma}F^{\delta}_P(k_1^2,k_2^2,p^2)\quad(p=k_1+k_2);$$

$$F^{\delta}_P(k_1^2,k_2^2,p^2)=2\int\int\int_0^1 d\alpha_1\,d\alpha_2\,d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$$

$$\times\sum_{j=1}^{\infty}\frac{(1-)^j A_j(\delta)M_j(\delta)}{[M_j^2(\delta)-\alpha_1\alpha_2k_1^2-\alpha_2\alpha_3k_2^2-\alpha_1\alpha_3p^2-i0]};$$

$$T^{\delta}_{\mu\nu}(V\to\gamma)=\frac{\pi^2}{6}[g_{\mu\nu}q^2-q_\mu q_\nu]\Pi^{\delta}(q^2),$$

$$\Pi^{\delta}(q^2)=6\int_0^1 d\alpha\alpha(1-\alpha)\int_0^{\infty}du\sum_{j=1}^{\infty}\frac{(-1)^j A_j(\delta)}{M_j^2(\delta)+u-\alpha(1-\alpha)q^2-i0}.$$

In the first nonvanishing approximation in the external momenta,

$$T^{\delta}_{\mu\nu}(V\to P\gamma)=\frac{1}{4\pi^2 i}\varepsilon_{\mu\rho\nu\sigma}k_{1\rho}k_{2\sigma}K^{\delta};$$

$$K^{\delta}=\int_0^{\infty}du\,u\sum_{j=1}^{\infty}(-1)^jA_j\Big(\delta\Big\{-\frac{u}{2}[A^{\delta}(u)]'\frac{1}{(M_j^2+u)^2}$$

$$+\frac{u}{2}[B^{\delta}(u)]'\frac{M_j}{(M_j^2+u)^2}+B^{\delta}(u)\frac{M_j}{(M_j^2+u)^2}\Big\}.$$

Using Eqs. (27) and (28) and then lifting the regularization, $\delta\to 0$, we finally obtain

$$F_P=\lim_{\delta\to 0}F^{\delta}_P=2\int\int\int_0^1 d\alpha_1\,d\alpha_2\,d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$$

$$\times\cos\xi\sqrt{\alpha_1\alpha_2\left(\frac{Lk_1}{2}\right)^2+\alpha_2\alpha_3\left(\frac{Lk_2}{2}\right)^2+\alpha_1\alpha_3\left(\frac{Lp}{2}\right)^2}$$

$$\times\exp\left\{\alpha_1\alpha_2\left(\frac{Lk_1}{2}\right)^2+\alpha_2\alpha_3\left(\frac{Lk_2}{2}\right)^2+\alpha_1\alpha_3\left(\frac{Lp}{2}\right)^2\right\}$$

$$\approx 1+\frac{1}{12}\left(1+\frac{\xi^2}{2}\right)(\mu_1+\mu_2+\mu).$$

Here

$$\mu_i=(Lk_i/2)^2;\quad \mu=(Lp/2)^2.$$

$$\Pi(q^2)=\lim_{\delta\to 0}\Pi^{\delta}(q^2)$$

$$=12\xi\int_0^1 d\alpha\alpha(1-\alpha)\int_0^{\infty}dtt\frac{\sin\xi\sqrt{t^2-\alpha(1-\alpha)Q^2}}{\xi\sqrt{t^2-\alpha(1-\alpha)Q^2}}$$

$$\times\exp\{-t^2+\alpha(1-\alpha)Q^2\};$$

$$Q^2=(Lq/2)^2;\quad \Pi(0)=\xi S_0(\xi);$$

$$K=\lim_{\delta\to 0}K^{\delta}=\xi[1+2S_1(\sqrt{2}\xi)-C_0(\sqrt{2}\xi)].$$

The calculations associated with the baryons are very cumbersome and require a computer. For details of the calculations, see Refs. 17 and 18.

10. *Electromagnetic interactions of hadrons and virtons and the bound-state condition.* As an example, we consider a system consisting of π mesons, charged quarks, virtons, and photons. We write the Lagrangian describing the electromagnetic and strong interactions of this system in the form

$$\mathcal{L}_I=ig_\pi(\bar q\gamma_5M(\pi)q)+\delta m_\pi^2\left(\pi^+\pi^-+\frac{1}{2}\pi^{02}\right)$$

$$-ie(\pi^+\partial_\mu\pi^--\partial_\mu\pi^+\pi^-)A_\mu+e^2A_\mu A_\mu\pi^+\pi^-$$

$$+e\left(\frac{2}{3}J^1_\mu-\frac{1}{3}J^2_\mu\right)A_\mu.$$

Here

$$M(\pi)=\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0 & \pi^-\\ \pi^+ & -\frac{1}{\sqrt{2}}\pi^0\end{pmatrix};\quad q=\begin{pmatrix}q_1\\ q_2\end{pmatrix};$$

δm_π^2 is the counterterm responsible for the renormalization of the pion mass; J^a_μ $(a=1,2)$ is the local quark-virton current (28); and the quarks q_1 and q_2 have, respectively, the charges $e_1=2e/3$ and $e_2=-e/3$.

We consider the electromagnetic form factor of a charged pion in the second order of perturbation theory in the constant g_π. The corresponding Feynman diagrams are shown in Figs. 9a and 9b. The sum of the matrix elements corresponding to the diagrams in Fig. 9a is

$$-ie(\pi^+\partial_\mu\pi^--\partial_\mu\pi^+\pi^-)[1+g_\pi^2\tilde\Sigma_2'(m^2)]=0$$

by virtue of the bound-state condition (42) and (43). Therefore, the electromagnetic form factor is determined solely by the diagram in Fig. 9b. The form factor $F_\pi(q^2)$ calculated in accordance with this diagram is normalized, $F_\pi(0)=1$, and decreases as $q^2\to-\infty$.

Thus, in the case of the interaction of the electromagnetic field with a hadron the bound-state condition has the consequence that effectively a proton interacts only with the quarks which form the hadron, in complete agreement with the ideas about the composite structure of hadrons.

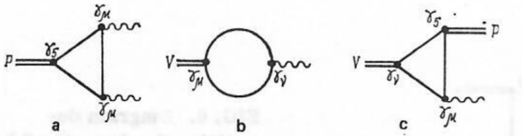


FIG. 8. Diagrams describing the processes $P\to 2\gamma$ (a), $V\to\gamma$ (b), and $V\to P\gamma$ (c).

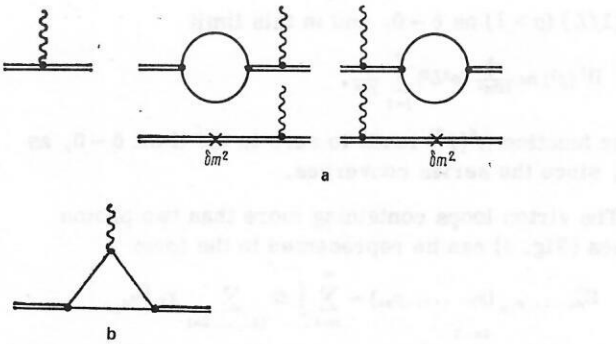


FIG. 9. Diagrams describing the electromagnetic radius of the meson.

2. LOW-ENERGY HADRON PHYSICS IN THE NONLOCAL QUARK MODEL

11. *The nonlocal quark model.* The diagram scheme developed above is taken as the basis for the construction of a model which permits the description of physical effects. The model is based on the following assumptions.

1. Hadrons satisfy the SU(3) classification and are described by standard quantized fields satisfying the

ordinary Dirac, Klein–Gordon, etc., equations. It is assumed that the SU(3) symmetry is broken by the hadrons having physical masses.

2. In the group SU(3)×SU$_c$(3), quarks are described by the fields

$$q_a(x)=(q_a^m(x)) =\begin{pmatrix} q_a^1(x)\\ q_a^2(x)\\ q_a^3(x)\end{pmatrix}=\begin{pmatrix} p_a(x)\\ n_a(x)\\ \lambda_a(x)\end{pmatrix}, \qquad (54)$$

where m ($m=1,2,3$) and a ($a=1,2,3$) are, respectively, the SU(3) and color indices. The quantized quark fields $q_a^m(x)$ are virton fields, so that for the free field $q_a^m(x)\equiv 0$, and the propagator of the quark field is

$$\overline{q_a^m(0)\,\bar{q}_{a'}^{m'}}(p)=\delta_{mm'}\delta_{aa'}L_m\exp\left\{l_m\hat{p}+\frac{L_m^2}{4}p^2\right\}. \qquad (55)$$

Note that the sign of the parameter l_m in (55) is not determined.

Since the SU(3) symmetry is a broken symmetry, it is natural to assume that the parameters in the propagators of the p and n quarks are equal, $L_1=L_2$ and $\xi_1=\xi_2$, and that the parameters L_3 and ξ_3 in the propagator of the λ quark are such that the "mass" (41) of the λ quark is greater than the "mass" of the p quark:

$$\frac{2}{L_3}\frac{1}{\sqrt{1+\frac{1}{2}\xi_3^2}}>\frac{2}{L_1}\frac{1}{\sqrt{1+\frac{1}{2}\xi_1^2}}.$$

However, the calculations showed that in the considered model the parameters of the nonstrange and strange quarks differ by not more than 10–20%. Therefore, in what follows, we shall assume that the parameters of all three quarks are equal:

$$L_1=L_2=L_3=L;\ \xi_1=\xi_2=\xi_3=\xi.$$

As a result of fitting of the experimental data, the following parameter values were determined:

$$\xi=\frac{2l}{L}=1.45\pm 0.05;\quad L=3.12\ \mathrm{GeV}^{-1}=\frac{1}{320\ \mathrm{MeV}}. \qquad (56)$$

3. It is assumed that the hadrons are bound states of quarks. This assumption is equivalent to the requirement that the hadron wave-function renormalization constant be zero [the bound-state condition (42) and (43)].

Thus, the strong interactions are determined by the two parameters L and ξ, and the hadron–quark coupling constants are determined from the condition (43). There is an additional breaking of the SU(3) symmetry, since the coupling constants depend on the physical masses of the hadrons. This dependence can be ignored in the case of the mesons, but for the baryons it must be taken into account.

4. It is assumed that the hadrons interact not directly with each other but by the exchange of quarks. We choose the Lagrangians that couple the hadrons to the quarks in the simplest form without derivatives. In practice this means that in the nonrelativistic limit the quarks are in the states with the smallest orbital angular momentum.

The nonet of pseudoscalar mesons $P(\pi,K,\eta,\eta')$:

$$\mathcal{L}_P=\frac{ig_P}{\sqrt{2}}\left\{\varphi_i(\bar{q}_a\gamma_5\lambda_i q_a)+\sqrt{\frac{2}{3}}\eta_1(\bar{q}_a\gamma_5 q_a)\right\}, \qquad (57)$$

where λ_i are the Gell-Mann matrices, and φ_i are the meson fields associated with the fields π, K, and η_8 by the well-known relations[19]

$$\eta_8=\eta\cos\theta_P+\eta'\sin\theta_P;\quad \eta_1=-\eta\sin\theta_P+\eta'\cos\theta_P.$$

The mixing angle is $\theta_P=-11°$, as follows from the quadratic mass formulas.[19] The effective expansion parameter calculated from the condition (43) is $\lambda_P=(g_P/4\pi)^2=0.08$. However, as was found in Ref. 12, better agreement with the data on the various decays is obtained for

$$\lambda_P=(g_P/4\pi)^2=0.13. \qquad (58)$$

We regard the differences between these values as in reasonable limits. Below, we use the coupling constant (58).

It should be noted that for neither the pseudoscalar nor the vector mesons does the bound-state condition depend on the sign of ξ, i.e., the sign of l_m in (55).

The nonet of vector mesons $V(\rho,K^*,\omega,\varphi)$:

$$\mathcal{L}_V=\frac{1}{\sqrt{2}}g_V\left\{v_{i\mu}(\bar{q}_a\gamma_\mu\lambda_i q_a)+\sqrt{\frac{2}{3}}\omega_{1\mu}(\bar{q}_a\gamma_\mu q_a)\right\}; \qquad (59)$$

$$\omega_{8\mu}=\omega_\mu\cos\theta_V+\varphi_\mu\sin\theta_V;$$

$$\omega_{1\mu}=-\omega_\mu\sin\theta_V+\varphi_\mu\cos\theta_V;\ \tan\theta_V=\frac{1}{\sqrt{2}}.$$

In this case, the bound-state condition gives for the coupling constant

$$\lambda_V=(g_V/4\pi)^2=0.13. \qquad (60)$$

The octet of baryons $B(p,n,\Lambda,\Sigma,\Xi)$. For the baryon octet, three types of interactions are possible:

$$\mathcal{L}_B=i\{g_{BS}(\bar{B}^{kr}q_a^r)(\overline{q_b^{mc}}\gamma_5 q_c^n) + g_{BA}(\bar{B}^{kr}\gamma_\mu\gamma_5 q_a^r)(\overline{q_b^{mc}}\gamma_\mu q_c^n) + g_{BP}(\bar{B}^{kr}\gamma_5 q_a)(\overline{q_b^{mc}}q_c^n)\}\,\varepsilon_{kmn}\varepsilon^{abc}+\mathrm{h.c.} \qquad (61)$$

Here, B^{kr} is the octet matrix of the baryons.

In the nonrelativistic limit, the interactions in the Lagrangian (61) correspond to the following quark, q_a^r, and diquark (qq) states in $(q_a^{mc}\Gamma q_b^n)$:

S variant: diquark 0^+, s state,

A variant: diquark 1^+, s state,

P variant: diquark 1^-, p state.

The renormalization constant of the baryon wave function is determined by the expression[17]

$$Z_j=1+\lambda_j F\left(\xi,\frac{ML}{2}\right)\quad (j=S,\ A,\ P);$$

$$\lambda_j F_j\left(\xi,\frac{ML}{2}\right)=\frac{\partial}{\partial\hat{p}}\tilde{\Sigma}_j(\hat{p})\big|_{\hat{p}=M},$$

where $\tilde{\Sigma}_j(\hat{p})$ is the baryon mass operator determined by the Feynman diagram (Fig. 10). The dependence of the functions $F_j(\xi)$ on ξ for the nucleon is shown in Fig. 11.

It is important that $F_j(\xi)=-F_j(-\xi)$. Therefore, the bound-state condition $Z_j=0$ can be satisfied for the following cases:

The S variant for $\xi>0$;

FIG. 10. Baryon self-energy diagram.

The A and P variants for $\xi < 0$.

For the nucleon, we have[17]

$$\lambda_S = 0.0017 \pm 0.003 \qquad (\xi = 1{,}4 \pm 0{,}1);$$
$$\left.\begin{array}{l}\lambda_A = 0.003 \pm 0.002\,,\\ \lambda_P = 0{,}011 \pm 0{,}005\end{array}\right\} (\xi = -1{,}4 \pm 0{,}1). \tag{62}$$

Comparing the coupling constant λ_S in (62) with the constant $G = 10^{-5}/m_p^2$ of the weak interactions, we obtain $g_S/G \approx 10^7$. Thus, the constant of the strong interactions is 10^7 times greater than that of the weak interactions. Nevertheless, in the framework of the nonlocal quark model perturbation theory is valid for the description of the strong interactions. This is due to the circumstance that in Euclidean metric the quark propagator decreases very rapidly.

The decuplet of baryons $\mathscr{D}(\Delta, \Xi^*, Y^*, \Omega)$:

$$\mathscr{L}_{\mathscr{D}} = \mathrm{i}g_{\mathscr{D}}\{(\overline{\mathscr{D}_\mu^{kmn}} q_a^k)(\overline{q_b^{m^c}}\gamma_\mu q_c^n) + \frac{1}{2}(\overline{\mathscr{D}_\mu^{kmn}}\gamma_\nu q_a^k)(\overline{q_b^{m^c}}\sigma_{\nu\mu} q_c^n)\}\,\varepsilon^{abc} + \mathrm{h.c.}; \tag{63}$$
$$\sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu);$$

$\mathscr{D}_\mu^{kmn}$, the decuplet matrix of the baryons, is symmetric with respect to permutation of the indices k, m, n. For the free fields of the decuplet, which are Rarita–Schwinger spinors, the following conditions are satisfied:

$$\partial_\mu \mathscr{D}_\mu^{kmn}(x) = 0; \quad \gamma_\mu \mathscr{D}_\mu^{kmn}(x) = 0.$$

The Lagrangian (63) is the unique Lagrangian without derivatives invariant under C, P, and T transformations and SU(3) permutations.

In the case of the decuplet, the coupling constants determined by the bound-state condition depend on the masses of the decuplet particles. The calculation gives the values[18]

$$\lambda_\Delta = \frac{1}{2}\left(\frac{g_\Delta}{L^2}\right)^2\left(\frac{3}{2\pi}\right)^4 = 2.3\cdot 10^{-4};$$
$$\lambda_{Y^*} = 1.1\cdot 10^{-4};\ \lambda_{\Xi^*} = 0.5\cdot 10^{-4}. \tag{64}$$

However, the interaction Lagrangians listed above do not take into account the approximate chiral invariance of low-energy physics. Therefore, processes with the participation of four or more hadrons (for example, $\pi\pi$ and $K\pi$ scattering, the decay $K \to 3\pi$, etc.) cannot be correctly described in the framework of the Lagrangian chosen above. To correct the situation, we introduce a nonet of nonproduced σ particles, which leads to the appearance of an additional third parameter in the model. This question will be considered in more detail below.

Thus, in the nonlocal quark model there are actually

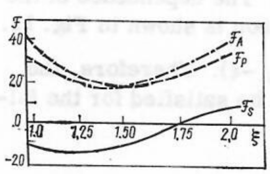
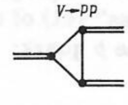
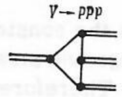
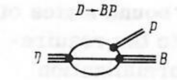
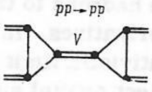


FIG. 11. Dependence of the functions F_j $(j = S, A, P)$ on ξ.

two free parameters ξ and L, while the third parameter associated with the introduction of the σ particles will be determined from a condition.

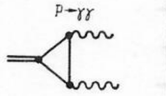
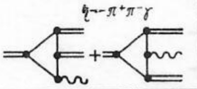
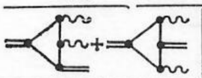
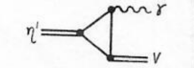
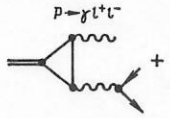
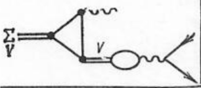
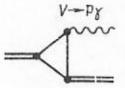
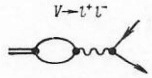
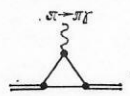
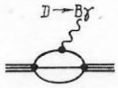
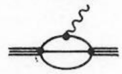
In the constructed model, the various physical processes are calculated by means of the technique developed in the first section. For the sake of a more compact and clearer exposition, the obtained results will be presented as follows. In Tables I–III, we give the Feynman diagrams corresponding to the process, the experimental value of the calculated characteristic, and its theoretical prediction in the nonlocal quark model. In the text, we give expressions for the invariant amplitude and, if necessary, for the decay width. Details of the calculations can be found in the original

TABLE I.

Process, diagram	Mode	Measured quantity, units	Experiment*	Theory (nonlocal quark model)*
$V \to pp$ 	$\rho \to \pi\pi$	Γ, MeV	1.52±4 [29]	145 [12]
	$K^* \to K\pi$		49.4±1.8	43
	$\varphi \to K^+K^-$		1.91±0.18	1.97
	$\varphi \to K^0\overline{K}^0$		1.44±0.15	1,29
$V \to ppp$ 	$\omega \to 3\pi$	Γ, MeV	8.99±0.45 [29]	5.5 [12]
	$\Delta^{++} \to p\pi^+$	Γ, MeV	111.5±0.67 [29]	100 [18]
	$\Delta^+ \to p\pi^0$		76.1±0.46	68
	$\Delta^+ \to n\pi^+$		36.8±0.22	33
	$\Delta^0 \to p\pi^-$		38.3±0.23	34
	$\Delta^0 \to n\pi^0$		76.7±0.46	71
	$\Delta^- \to n\pi^-$		116±0.69	106
$D \to BP$ 	$\Xi^{*0} \to \Xi^0\pi^0$		3.26±0.2	3.3
	$\Xi^{*0} \to \Xi^-\pi^+$		5.54±0.25	5.3
	$\Xi^{*-} \to \Xi^-\pi^0$		3.09±0.2	3.1
	$\Xi^{*-} \to \Xi^0\pi^-$		6.56±0.4	6.6
	$Y^{*+} \to \Lambda\pi^+$		30.8±0.61	21
	$Y^{*+} \to \Sigma^+\pi^0$		2.3±0.046	1.5
	$Y^{*+} \to \Sigma^0\pi^+$		1.9±0.04	1.2
	$Y^{*0} \to \Lambda\pi^0$		30.8±0.61	23
	$Y^{*0} \to \Sigma^-\pi^+$		1,86±0.037	1.2
	$Y^{*0} \to \Sigma^+\pi^-$		2.3±0.046	1.5
	$Y^{*-} \to \Lambda\pi^-$		35.2±0.71	23
	$Y^{*-} \to \Sigma^0\pi^-$		2.4±0.048	1.5
$pp \to pp$ 	$\pi\pi \to \pi\pi$	a_0^0	[0.10; 0.60] [32]	0.18 [20]
		a_0^2	[−0.10; −0.03]	−0.9
		a_1^1	[0,042; 0.04]	0.045
	$K\pi \to K\pi$	$a_0^{1/2}$	[−0.1; 0.4] [33]	0.147 [20]
		$a_0^{3/2}$	[−0.2; 0]	−0.074

*References to the literature are indicated by square brackets.

TABLE II.

Process, diagram	Mode	Measured quantity, units	Experiment*	Theory (nonlocal quark model)*
P→γγ 	$\pi^0 \to \gamma\gamma$	Γ, eV	7.92±0.42 [29]	6.8 [12.22]
	$\eta \to \gamma\gamma$	Γ, eV	323±54 [29]	342 [22]
	$\eta' \to \gamma\gamma$	Γ, keV	5.4±2.1 [21]	5.3 [22]
η→π+π−γ 	$\eta \to \pi^+\pi^-\gamma$	Γ, eV	41.6±7.0 [29]	41 [22]
	$\eta \to \pi^0\gamma\gamma$	Γ, eV	26±14 [29]	0.45 [22]
	$\eta' \to \rho^0\gamma$	Γ, keV	83±52 [21]	106 [22]
	$\eta' \to \omega\gamma$	Γ, keV	5.9±2.4 [34]	11
P→γl+l− 	$\pi^0 \to \gamma e^+e^-$	M_π^{-2}, GeV^{-2}	5.5±1.6 [21]	2.3 [22.23]
	$\eta \to \gamma\mu^+\mu^-$	M_η^{-2}, GeV^{-2}	3±1 [21]	2.6
	$\eta' \to \gamma\mu^+\mu^-$	M_η^{-2}, GeV^{-2}		1.4
V→Pγ 	$\omega \to \pi^0\gamma$	Γ, keV	888±55 [29]	920 [12]
	$\omega \to \eta\gamma$		$3^{+2.5}_{-1.6}$ [35]	7.7
	$\rho^- \to \pi^-\gamma$		67±7 [36]	98
	$\rho^0 \to \eta\gamma$		50±13 [35]	62
	$K^{*0} \to K^0\gamma$		75±35 [29]	216
	$K^{*-} \to K^-\gamma$		40±15 [36]	68
	$\varphi \to \eta\gamma$		55±12 [35]	170
	$\varphi \to \eta'\gamma$			0.85
V→l+l− 	$\rho^0 \to e^+e^-$	Γ, keV	6.44±0.89 [29]	4.7 [12]
	$\omega \to e^+e^-$		0.76±0.47	0.53
	$\varphi \to e^+e^-$		1.31±0.15	1.30
π→πγ 		r_π, F	0.56±0.04 [30]	0.46 [24]
D→Bγ 		μ_p	1.79 [29]	1.66 [17]
		μ_n	−1.91	−1.88
		g_w	3.70	3.54
B→Bγ 	$\Delta^+ \to p\gamma$	c	1.25±0.2 [31]	0.82 [18]
	$\Delta^0 \to n\gamma$		1.25±0.2	0.82
	$Y^{*+} \to \Sigma^+\gamma$		−(1.25±0.2)	−0.99
	$Y^{*0} \to \Sigma^0\gamma$		0.63±0.1	0.5
	$Y^{*0} \to \Lambda\gamma$		1.08±0.17	0.72
	$\Xi^{*0} \to \Xi^0\gamma$		1.25±0.2	0.91

*References to the literature are indicated by square brackets.

papers.

12. *Strong interactions*. The effective constants of the perturbation expansions (58), (60), (62), and (64) are found to be less than unity, i.e., in the description of the strong interactions in the nonlocal quark model perturbation theory can be used.

We begin by analyzing the processes $V \to PP$, $\omega \to 3\pi$, $\mathscr{D} \to BP$, and also the strong interactions in the octet of pseudoscalar mesons, i.e., $\pi\pi$ and $K\pi$ scattering. These are the basic processes in the considered energy region, and therefore the good agreement with experiment (Table I) indicates that the constructed scheme with a small expansion constant is capable of describing correctly the strong-interaction dynamics.

The decay $V \to PP$ (Ref. 12):

$$M(\rho^0 \to \pi^+\pi^-) = G_{\rho^0\pi^+\pi^-}\varepsilon^\mu (p_+ - p_-)_\mu,$$

where ε is the polarization of the ρ meson, and

$$G_{\rho^0\pi^+\pi^-} = \lambda^{3/2}\frac{2^9\pi}{9\sqrt{2}} S_2\left(\frac{\xi}{\sqrt{3}}\right);$$

$$\Gamma(\rho^0 \to \pi^+\pi^-) = \frac{2p^3}{3m_\rho^2}\frac{G^2_{\rho\pi\pi}}{4\pi};$$

p is the c.m.s. momentum of the ρ meson.

The decay $\omega \to 3\pi$ (Ref. 12):

$$M(\omega \to 3\pi) = \varepsilon_{\mu\nu\alpha\beta}\varepsilon^\mu k_1^\nu k_2^\alpha k_3^\beta G_{\omega\to 3\pi},$$

where ε is the polarization vector of the ω meson, k_1, k_2, k_3 are the 4-momenta of the pions, and

$$G_{\omega\to 3\pi} = 72\pi^2\lambda^2 L^3 \frac{\xi}{2}\{S_0(\xi) - S_0(2\xi) + 3C_0(\xi)\};$$

$$\Gamma(\omega \to 3\pi) = G^2_{\omega\to 3\pi}\frac{196{,}5 m_\pi^{10}}{3\pi^3 M_\omega^8}.$$

The decay $\mathscr{D} \to BP$ (Ref. 18):

$$M = 8\pi\left(\frac{3}{4}\right)^4 \sqrt{\lambda_D\lambda_B\lambda_P}\,\frac{L}{2} C_{ks,\,tri,\,ml} \times \bar{B}^{ks}(q)\,k_\mu R\mathscr{D}_\mu^{tri}(p)\,M^{ml}(k);$$

$$C_{ks,\,tri,\,ml} = -6\varepsilon^{km'n}\delta_{st}[\delta_{mt}(\delta_{nl}\delta_{m'r} - \delta_{m'l}\delta_{nr}) + \delta_{mr}(\delta_{nl}\delta_{m't} - \delta_{nt}\delta_{m'l})];$$

$$M^{ks} = \frac{1}{\sqrt{2}}\sum_m \varphi_m\lambda_m^{ks}.$$

The explicit expression for R is excessively cumbersome and is not given here.[18]

Pion–pion and kaon–pion scattering.[20] The simplest Lagrangian (57) is invariant under transformations of the group SU(3) and does not take into account the approximate chiral invariance, which holds for the considered energy region. In particular, for such a choice of the Lagrangian $\pi\pi$ scattering is described by the diagram in Fig. 12. In this case, the ratio of the $\pi\pi$ scattering lengths is $a_0^0/a_0^2 = 5/2$, in complete agreement with the prediction of SU(3) symmetry. However, this result is in strong contradiction with the experimental value $(a_0^0/a_0^2)_{\exp} = -(2-4)$. Chiral theory[8] predicts $a_0^0/a_0^2 = -7/2$, which agrees well with the experimental data.

In the framework of the model, we attempt to take into account chiral invariance. For this, by analogy with the linear σ model, we introduce a nonet of hypothetical unobservable σ particles in such a way as to achieve a correct description of the $\pi\pi$ and $K\pi$ scattering lengths.

TABLE III.

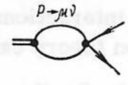
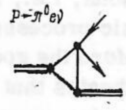
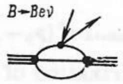
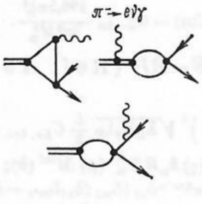
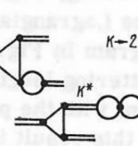
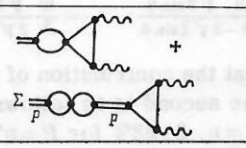
Process, diagram	Mode	Measured quantity, units	Experiment*	Theory (nonlocal quark model)*	
$P\to\mu\nu$ 	$\pi\to\mu\nu$	Γ, MeV	$(2.528\pm0.002)\cdot10^{-14}$	$2.6\cdot10^{-14}$ [24]	
	$K\to\mu\nu$		$(3.383\pm0.016)\cdot10^{-14}$	$3.1\cdot10^{-14}$	
$P\to\pi^0 e\nu$ 	$\pi^-\to\pi^0 e\nu$	Γ, GeV	$(2.58\pm0.18)\cdot10^{-25}$	$2.8\cdot10^{-25}$ [24]	
	$K^-\to\pi^0 e\nu$	Γ, GeV	$(2.56\pm0.03)\cdot10^{-18}$	$2.78\cdot10^{-18}$	
		ξ (0)	-0.35 ± 0.14 [29]	-0.2 [24]	
		λ_+	0.026 ± 0.08	0.016	
		λ_-		-0.009	
		λ_0	-0.003 ± 0.001	0	
$B\to Be\nu$ 	$\Sigma^-\to ne^-\nu$	Γ, MeV	$(1.08\pm0.04)\cdot10^{-3}$	$1.12\cdot10^{-3}$ [17]	
	$\Sigma^-\to n\mu^-\nu$		$(4.5\pm0.4)\cdot10^{-4}$	$5.15\cdot10^{-4}$	
	$\Lambda\to pe\nu$		$(8.07\pm0.28)\cdot10^{-4}$	$7.32\cdot10^{-4}$	
	$\Lambda\to p\mu\nu$		$(1.57\pm0.35)\cdot10^{-4}$	$1.21\cdot10^{-4}$	
	$\Sigma^-\to\Lambda e^-\nu$		$(0.60\pm0.06)\cdot10^{-4}$	$0.61\cdot10^{-4}$	
	$\Sigma^+\to\Lambda e^+\nu$		$(0.20\pm0.04)\cdot10^{-4}$	$0.2\cdot10^{-4}$	
	$\Xi^-\to\Lambda e^-\nu$		$(0.69\pm0.18)\cdot10^{-3}$	$0.25\cdot10^{-3}$	
	$\Xi^-\to\Sigma^0 e^-\nu$		$<0.5\cdot10^{-3}$	$0.078\cdot10^{-3}$	
	$\Xi^0\to\Sigma^+ e^-\nu$		$<1.1\cdot10^{-3}$	$0.25\cdot10^{-3}$	
	$\Xi^-\to\Xi^0 e^-\nu$		$<2.3\cdot10^{-4}$	$0.09\cdot10^{-10}$	
		$\beta=G_A/G_V$	1.25 ± 0.01	1.19	
		$\alpha=\mathscr{D}/(F+\mathscr{D})$	0.66 ± 0.07	0.68	
$\pi^-\to e\nu\gamma$ 	$\pi^-\to e\nu\gamma$	$\gamma=F_A/F_V$	0.15 ± 0.11 [29]	1 [24]	
				A [20]	B [20]
$K_L^0\to P$ 	$K_L^0\to\pi^0$	$M(K\to\pi)$ MeV²		$2.8\cdot10^{-2}$	$2.4\cdot10^{-2}$
	$K_L^0\to\eta$			$1.3\cdot10^{-2}$	$1.7\cdot10^{-2}$
	$K_L^0\to\eta'$			$-1.9\cdot10^{-2}$	$1.6\cdot10^{-2}$
$K\to 2\pi$ 	$K_S^0\to\pi^0\pi^0$	Γ, MeV	$(2.32\pm0.02)\cdot10^{-12}$	$2.4\cdot10^{-12}$	$1.8\cdot10^{-12}$
	$K_S^0\to\pi^+\pi^-$		$(5.06\pm0.03)\cdot10^{-12}$	$4.9\cdot10^{-12}$	$3.5\cdot10^{-12}$
	$K^+\to\pi^+\pi^0$		$(1.13\pm0.01)\cdot10^{-14}$	0	0

TABLE III. (*continued*).

Process, diagram	Mode	Measured quantity, units	Experiment*	Theory (nonlocal quark model)*	
$K\to 3\pi$ (see Fig. 15)	$K_L^0\to\pi^+\pi^-\pi^0$	Γ, MeV	$(1.57\pm0.03)\cdot10^{-15}$	$1.1\cdot10^{-15}$	$0.9\cdot10^{-15}$
		σ_{+-0}	$-(0.33\pm0.73)$	-0.48	-0.48
	$K_L^0\to\pi^0\pi^0\pi^0$	Γ, MeV	$(2.73\pm0.11)\cdot10^{-15}$	$2.0\cdot10^{-15}$	$1.5\cdot10^{-15}$
	$K^+\to\pi^+\pi^+\pi^-$	Γ, MeV	$(2.97\pm0.02)\cdot10^{-15}$	$1.8\cdot10^{-15}$	$1.3\cdot10^{-15}$
		σ_{++-}	0.11 ± 0.02	0.24	0.24
	$K^+\to\pi^+\pi^0\pi^0$	Γ, MeV	$(0.92\pm0.03)\cdot10^{-15}$	$0.6\cdot10^{-15}$	$0.4\cdot10^{-15}$
		σ_{+00}	$-(0.28\pm0{,}01)$	-0.48	-0.48
	$K_L^0\to\gamma\gamma$	Γ, MeV	$(6.22\pm0.64)\cdot10^{-18}$	$0.72\cdot10^{-18}$	$10\cdot10^{-18}$
	$K_L^0\to\pi^+\pi^-\gamma$	Γ, MeV	$(7.62\pm2.54)\cdot10^{-19}$	$3.1\cdot10^{-19}$	$13\cdot10^{-19}$

*References to the literature are indicated by square brackets.

The Lagrangian of the interaction of the σ particles with the quarks is written in the form

$$\mathscr{L}_{\sigma qq}=\frac{g_P}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}\,\sigma^0(\bar{q}_a I q_a)+\sigma^i(\bar{q}_a\lambda^i q_a)\right].$$

It is assumed that:

1) The σ particles are unobservable, i.e.,

$$\overline{\sigma^i(x)}\,\sigma^j(y)=\delta_{ij}\frac{1}{i}\frac{1}{m_\sigma^2}\,\delta(x-y).$$

The field of the σ particles is quantized in exactly the same way as the virton field.

2) The "mass" m_σ is determined by the condition that the contributions of the diagrams in Figs. 12 and 13 cancel.

If it is assumed that the masses of all the σ particles are the same, then by virtue of the obvious equation

$$\mathrm{Tr}\,(\lambda^{i_1}\lambda^{i_2}\lambda^{i_3}\lambda^{i_4})=\frac{1}{2}\,\mathrm{Tr}\,(\lambda^{i_1}\lambda^{i_2}\lambda^{j})\times\mathrm{Tr}\,(\lambda^{j}\lambda^{i_3}\lambda^{i_4})+\frac{1}{3}\,\mathrm{Tr}\,(\lambda^{i_1}\lambda^{i_2})\,\mathrm{Tr}\,(\lambda^{i_3}\lambda^{i_4})$$

we

$$\lambda\frac{3}{2}\frac{W^2}{(Lm_\sigma)^2}=1;$$
$$W(\xi)=\xi\,[1+3S_1(\xi)].$$

For $L=1/320$ MeV and $\xi=1.45$, we find that $m_\sigma=446$ MeV.

Thus, the main contribution to the amplitudes of $\pi\pi$ and $K\pi$ scattering are made by the resonance diagrams (see Table I). The numerical values of the scattering lengths are given in Table I. It can be seen that they are in good agreement with experiment.

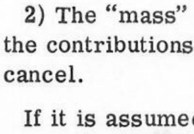


FIG. 12. Quark loop describing $\pi\pi$ scattering.

It is readily seen that the σ particles do not contribute to the amplitudes of the decays $V\to PP$ and $\omega\to3\pi$ in the lowest orders of perturbation theory. The only decay for which the σ particles have an important influence is $K\to3\pi$ (see below).

It must be emphasized that the introduction of the σ particles is to be regarded as merely an attempt to take into account chiral invariance phenomenologically and not as a final solution to the problem.

13. *Electromagnetic interactions*. The minimal introduction of the electromagnetic field (see Sec. 2.8) leads to the interaction Lagrangian

$$\mathscr{L}_I=eA_\mu J_{q\mu}^{em}.$$

Here, $J_{q\mu}^{em}$ is the electromagnetic quark current, which in regularized form becomes

$$(J_{q\mu}^{em})^\delta=\sum_{j=1}(-)^j\,\bar{q}_{ja}^\delta\gamma_\mu Q q_{ja}^\delta;$$
$$Q=\frac{1}{2}\left(\lambda^3+\frac{1}{\sqrt{3}}\lambda^8\right).$$

The Lagrangians of the interaction of the electromagnetic field with the hadrons has the standard form.[19]

Below, we consider the basic radiative decays of the hadrons ($P\to\gamma\gamma$, $V\to P\gamma$, $\eta\to\pi\pi\gamma$, $V\to l^+l^-$), the pion mean-square radius, the magnetic moments of the baryons, the magnetic moment of the $\mathscr{D}$–B transition, and also a number of rare decays of the pseudoscalar mesons, $P\to\gamma l^+l^-$, $\eta\to\pi^0\gamma\gamma$, $\eta'\to V\gamma$ in which interest has grown recently. In the first place, this is due to the measurement of the characteristics of the form factors in the decays $\pi^0\to\gamma e^+e^-$, $\eta\to\mu^+\mu^-\gamma$, $\eta'\to\mu^+\mu^-\gamma$, $\eta'\to V\gamma$.[21] It should be noted that the calculation of the character-

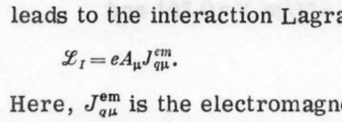


FIG. 13. Contribution of the σ particles to $\pi\pi$ scattering.

istics of the decays $P\to\gamma l^+l^-$ depends on the choice of the model. There are doubts concerning the experimental measurement of the width of the decay $\eta\to\pi^0\gamma\gamma$, since the existing theoretical estimates are much lower than the experimental value.[9]

The decay $P\to\gamma\gamma(P=\pi^0,\eta,\eta')$ (Refs. 12 and 22):

$$M(P\to\gamma\gamma)=e^2g_{P\gamma\gamma}\varepsilon_{\mu\nu\alpha\beta}p_1^\mu\varepsilon_1^\nu p_2^\alpha\varepsilon_2^\beta;$$

$$g^2_{\pi^0\gamma\gamma}=L^2\frac{\lambda}{2\pi^2};$$

$$g^2_{\eta\gamma\gamma}=g^2_{\pi^0\gamma\gamma}\cdot\frac{1}{3}(\cos\theta-2\sqrt{2}\sin\theta)^2;$$

$$g^2_{\eta'\gamma\gamma}=g^2_{\pi^0\gamma\gamma}\cdot\frac{1}{3}(2\sqrt{2}\cos\theta+\sin\theta)^2;$$

$$\Gamma(P\to\gamma\gamma)=\frac{1}{4}\pi\alpha^2m_P^3g^2_{P\gamma\gamma}.$$

The decay $\eta\to\pi^+\pi^-\gamma$ (Ref. 22):

$$M(\eta\to\pi^+\pi^-\gamma)=\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\mu q^\nu p_-^\alpha p_+^\beta C_\eta;$$

$$C_\eta=-e\frac{48\pi}{\sqrt{6}}\lambda^{3/2}L^3(\cos\theta-\sqrt{2}\sin\theta)R_\eta(\xi);$$

here, $R_\eta(\xi)$ is the structure integral

$$R_\eta(\xi)=\frac{1}{12}\Big\{C_0(\sqrt{3}\xi)-C_0\left(\frac{\xi}{\sqrt{3}}\right)+\frac{3\xi^2}{2}\Big[S_0(\sqrt{3}\xi)-S_0\left(\frac{\xi}{\sqrt{3}}\right)\Big]\Big\};$$

$$\Gamma(\eta\to\pi^+\pi^-\gamma)=m_\eta\cdot16\lambda^3\alpha(m_\pi L)^6R_\eta^2(\xi)\times(\cos\theta-\sqrt{2}\sin\theta)^2J;$$

$$J=\int_0^{3/2}dt\,t^3(3-2t)\sqrt{\frac{3-2t}{2(2-t)}}\approx0.52.$$

The decay $\eta\to\pi^0\gamma\gamma$ (Ref. 22):

$$M(\eta\to\pi^0\gamma\gamma)=-e^2\lambda L^2\frac{4}{3\sqrt{3}}[\cos\theta-\sqrt{2}\sin\theta]\times\varepsilon_\mu(k_1)\varepsilon_\nu(k_2)[(k_1k_2)g_{\mu\nu}-k_2^\mu k_1^\nu];$$

$$\Gamma(\eta\to\pi^0\gamma\gamma)=m_\eta\alpha^2\lambda^2(m_\pi L)^4\frac{4I}{27\pi}\times(\cos\theta-\sqrt{2}\sin\theta)^2,$$

where $I=\int_1^{M/m_\pi}du\sqrt{u^2-1}(u-M/m_\pi)^2\approx0.391$ and

$$M=(m_\eta^2+m_\pi^2)/2m_\eta.$$

The theoretical value (see Table II) is approximately 50 times lower than the experimental value, although above the predictions of chiral theory.[9] As we said above, here there are doubts concerning the correctness of the experimental data.

The decay $\eta'\to V\gamma$ (Ref. 22):

$$M(\eta'\to V\gamma)=eg_{\eta'V\gamma}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_\gamma^\mu\varepsilon_V^\nu p_\gamma^\alpha p_V^\beta.$$

Accordingly, the decay width is

$$\Gamma(\eta'\to V\gamma)=\frac{\alpha}{8}m_\eta^3\left[1-\frac{m_V^2}{m_\eta^2}\right]^3g^2_{\eta'V\gamma}.$$

Here

$$g^2_{\eta'\rho^0\gamma}=\lambda^2L^26\left(\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)^2[K_{PV}(\xi)]^2;$$

$$g^2_{\eta'\omega\gamma}=\frac{1}{9}g^2_{\eta'\rho^0\gamma},$$

$$K_{PV}(\xi)=\xi[1+2S_1(\sqrt{2}\xi)-C_0(\sqrt{2}\xi)].$$

The decay $P\to\gamma l^+l^-(P=\pi^0,\eta,\eta')$ (Refs. 22 and 23):

$$M(P\to\gamma l^+l^-)=-e^3\Phi_P(k_2^2)\frac{1}{k_2^2}\varepsilon_{\mu\rho\nu\sigma}\varepsilon^\mu(k_1)k_1^\rho k_2^\sigma j^\nu(k_2).$$

Here

$$k_1^2=0,\ p^2=(k_1+k_2)^2=m_P^2;$$

$$j^\nu(k_2)=\bar{l}(q_1)\gamma^\nu l(q_2)|_{q_1+q_2=k_2},\quad q_1^2=q_2^2=m_l^2;$$

$$\Phi_P(k_2^2)=g_{P\gamma\gamma}(k_2^2)+k_2^2\sum_V\frac{g_{PV\gamma}}{f_V}\frac{1}{m_V^2-k_2^2}.$$

For sufficiently small k_2^2, we have the parametrization

$$\Phi_P(k_2^2)=g_{P\gamma\gamma}(0)\left\{1+\frac{k_2^2}{M_P^2}\right\}.$$

Here

$$\frac{1}{M_P^2}=\frac{L^2}{4}\frac{1}{1+a(\xi)\mu_P^2}\{a(\xi)+F(\xi)r_P\};$$

$$\mu_P^2=\left(\frac{m_PL}{2}\right)^2;\ a(\xi)=\frac{1}{12}\left(1+\frac{1}{2}\xi^2\right);$$

$$F(\xi)=\frac{8\lambda}{m_\rho^2L^2}\xi^2S_0(\xi)[1+2S_1(\sqrt{2}\xi)-C_0(\sqrt{2}\xi)];$$

$$r_{\pi^0}=2;$$

$$r_\eta=\frac{10}{3}\frac{\cos\theta-\sqrt{2}\sin\theta}{\cos\theta-2\sqrt{2}\sin\theta};\ r_{\eta'}=\frac{10}{3}\frac{\sqrt{2}\cos\theta+\sin\theta}{2\sqrt{2}\cos\theta+\sin\theta}.$$

It is found that the contribution of the first diagram relative to the second is as follows: 1) 22% for $P=\pi^0$, 2) 16% for $P=\eta$, 3) 28% for $P=\eta'$. The numerical values for M_P^{-2} are given in Table II. In the case $P=\pi^0$, our result is about half the experimental value and comparable with the prediction of the vector-dominance model.[21] For the η meson, the result is in complete agreement with the recent experiment.[21] For the η' meson, there is as yet no experimental prediction.

The decay $V\to P\gamma$ (Ref. 12):

$$M(V\to P\gamma)=eg_{VP\gamma}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_\gamma^\mu k_\gamma^\nu\varepsilon_V^\alpha k_V^\beta;$$

$$g_{VP\gamma}=3\lambda LK_{PV}(\xi)\frac{1}{2}\mathrm{Tr}(\{\lambda^P,Q\}\lambda^V);$$

$$\Gamma(V\to P\gamma)=\frac{\alpha}{24}m_V^3\left[1-\frac{m_P^2}{m_V^2}\right]^3g^2_{VP\gamma}.$$

The decay $V\to l^+l^-$ (Ref. 12):

$$M(V\to l^+l^-)=\frac{e^2}{f_V}\bar{u}(k_1)\hat{\varepsilon}_Vu(-k_2);$$

$$\frac{1}{f_V}=\frac{\sqrt{2\lambda}}{\pi}\xi S_0(\xi)\mathrm{Tr}(\lambda^VQ);$$

$$\Gamma(V\to l^+l^-)=\frac{\alpha^2}{3}m_V\frac{4\pi}{f_V^2}.$$

The pion mean-square radius[24]:

$$M(\pi^-\to\pi^-\gamma)=ea_\mu(q)(p_1+p_2)^\mu F_+(q^2);$$

$$\langle r_\pi^2\rangle=6\frac{dF_+(t)}{dt}\Big|_{t=0}=\frac{3}{2}\lambda L\mathscr{D}(\xi);$$

for $\mathscr{D}(\xi)$, see Ref. 24.

Magnetic moments of the $\frac{1}{2}^+$ baryons[17]:

$$M(B\to B\gamma)=ef_0(k^2)\mathrm{Tr}(\bar{B}\gamma_\mu[Q,B])-\frac{e}{2m_N}f_1(k^2)\mathrm{Tr}(\bar{B}\sigma_{\mu\nu}k^\nu[Q,B])-\frac{e}{2m_N}f_2(k^2)\mathrm{Tr}(\bar{B}\sigma_{\mu\nu}k^\nu\{Q,B\}),$$

where

$$\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2;\quad k=p-p',$$

m_N is the nucleon mass, $f_0(k^2)$ is the form factor of the electric charge, which satisfies the condition $f_0(0)=1$, and $f_1(k^2)$ and $f_2(k^2)$ are the magnetic form factors, $f_1(0)$ and $f_2(0)$ determining the anomalous magnetic moments of the baryon octet. These last quantities are functions of the mass of the particle and the parameter ξ. We denote $f_j(0)=f_j(m,\xi)$ $(j=1,2)$, where m is the mass of the given particle. The anomalous magnetic moments of the baryon octet can be expressed in terms of $\tilde{f}_1(m,\xi)$ and $\tilde{f}_2(m,\xi)$ as follows:

$$\mu_j = -\frac{2}{3}\tilde{f}_2(m_j, \xi), \qquad j = n, \ \Xi^0;$$

$$\mu_j = \frac{1}{3}\tilde{f}_2(m_j, \xi) - \tilde{f}_1(m_j, \xi), \quad j = \Sigma^-, \ \Xi^-;$$

$$\mu_j = \frac{1}{3}\tilde{f}_2(m_j, \xi) + \tilde{f}_1(m_j, \xi), \quad j = p, \ \Sigma^+;$$

$$\mu_\Lambda = -\frac{1}{3}\tilde{f}_2(m_\Lambda, \xi); \quad \mu_{\Sigma^0} = \frac{1}{3}\tilde{f}_2(m_{\Sigma^0}, \xi).$$

The anomalous magnetic moments can be expressed in units of the nuclear magneton $\mu_N = e/2m_p$.

Magnetic moment of the $\mathscr{D}$–B transition.[18] The magnetic moment μ^* of the $\mathscr{D}$–B transition determines the nondiagonal matrix element of the operator of the electromagnetic current of the hadrons between the states of the baryon B and the baryon resonance $\mathscr{D}$. The magnetic moment μ^* can be found from the probability of the radiative decay $\mathscr{D} \to B + \gamma$. Since the amplitude for emission in the decay $\mathscr{D} \to B + \gamma$ of an electric quadrupole γ ray is appreciably less than the amplitude for emission of a magnetic dipole γ ray, it can be assumed with good accuracy that the width of this decay is determined solely by the magnetic moment μ^* of the transition:

$$M = \frac{e}{m_p}\bar{B}^{hs}(q)\gamma_5[V_1\gamma_\nu + V_2 q_\nu]\mathscr{D}^{itr}_\mu(p)\tilde{F}_{\mu\nu}(k),$$

where

$$\tilde{F}_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x).$$

In the considered model, V_1 and V_2 are invariant integrals (see Ref. 18). For μ^*, we have

$$\mu^* = \frac{1}{3}\sqrt{\frac{m_\mathscr{D}}{m_B}}\left[-V_1\left(3 + \frac{m_B}{m_\mathscr{D}}\right) + 2V_2 m_\mathscr{D}\left(1 - \frac{m_B}{m_\mathscr{D}}\right)\right],$$

where μ^* is expressed in units of the nuclear magneton. It is convenient to represent μ^* in the form

$$\mu^* = c\mu_N 2\sqrt{2}/3.$$

The value of c is given in Table II.

14. *Semileptonic weak interactions*. The weak leptonic and semileptonic interactions of the pseudoscalar mesons and baryons are described by Cabibbo's current–current theory:

$$\mathscr{L}^w_I = \frac{G}{\sqrt{2}}\{(I^P_\mu + I^B_\mu + J^Q_\mu)\, l_\mu + \text{h.c.}\},$$

where

$$I^P_\mu = -i\,\mathrm{Tr}([J, M], \partial_\mu M);$$

$$I^B_\mu = \mathrm{Tr}(\bar{B}, O_\mu[J, B]);$$

$$(J^Q_\mu)^\delta = \sum_{j=1}^{\infty}(-)^j \bar{q}^\delta_{ja} O_\mu J q^\delta_{ja};$$

$$J = \begin{pmatrix} 0 & 0 & 0 \\ \cos\theta & 0 & 0 \\ \sin\theta & 0 & 0 \end{pmatrix}; \quad O_\mu = \gamma_\mu(1 - \gamma_5).$$

Note that for the baryon weak current we choose only F coupling. A $\mathscr{D}$ coupling appears as a result of the interaction of the leptons with the quarks as an effect of the strong interactions. The following basic characteristics of the semileptonic decays of the mesons and baryons were calculated:

1) the widths of the decay $P \to \mu\nu (P = \pi, K)$, i.e., the decay constant f_π, which is the basic parameter of nonlinear chiral theory;

2) the form factors of the K_{l_3} decay;

3) the renormalization of the axial coupling constant due to the strong interaction, and the intensity of the contributions of the F and $\mathscr{D}$ couplings to the matrix elements of the semileptonic decays of the baryons;

4) the ratio of the contributions of the axial and vector parts of the amplitude of the decay $\pi^- \to e\nu\gamma$.

The decay $P \to \mu\nu (P = \pi, K)$ (Ref. 24):

$$\langle\mu\nu\,|\,M\,|\,P\rangle = s_P\frac{G}{\sqrt{2}}f_P m_\mu \bar{u}(p)(1 - \gamma_5)u(-p'),$$

where p and p' are the muon and neutrino momenta, respectively, and

$$s_P\colon\ s_\pi = \cos\theta; \quad s_K = \sin\theta;$$

$$f_\pi = f_K = \frac{12\sqrt{2}}{L\pi}C_0(\xi);$$

$$\Gamma(P \to \mu\nu) = \frac{G^2 f_P^2 s_P^2}{8\pi}m_P m_\mu^2\left[1 - \frac{m_\mu^2}{m_P^2}\right]^2;$$

The K_{l_3} decay[24]:

$$M(K^+ \to l^+\pi^0\nu) = -\frac{1}{\sqrt{2}}\sin\theta\, l^{(-)}_\mu T_\mu(p_1, p_2)\Big|_{p_1^2 = m_K^2,\ p_2^2 = m_\pi^2},$$

where $l^{(-)}_\mu$ is the leptonic weak current:

$$T_\mu(p_1, p_2) = F_+(t)(p_1 + p_2)_\mu + F_-(t)(p_1 - p_2)_\mu;$$
$$t = (p_1 - p_2)^2;$$

$$F_+(t) = 1 - \lambda(p_1^2 - p_2^2)C(\xi) + t\frac{L^2}{4}\lambda\mathscr{D}(\xi);$$

$$F_-(t) = (p_1^2 - p_2^2)\lambda\left[-\mathscr{D}(\xi) + t\frac{L^2}{4}E(\xi)\right]$$

The explicit form of the structure integrals $C(\xi)$, $\mathscr{D}(\xi)$, and $E(\xi)$ is given in Ref. 24. Using the standard parametrization of the form factors of the K_{l_3} decays, we obtain

$$F_\pm(t) = F_\pm(0)\left[1 + \lambda_\pm\frac{t}{m_\pi^2}\right];$$

$$F_+(0) = 1 - \lambda(\mu_K^2 - \mu_\pi^2)C(\xi);$$
$$\mu = mL/2;$$

$$F_-(0) = -\lambda(\mu_K^2 - \mu_\pi^2)\mathscr{D}(\xi);$$

$$\lambda_+ = \frac{\lambda\mu_\pi^2\mathscr{D}(\xi)}{1 - \lambda(\mu_K^2 - \mu_\pi^2)C(\xi)}; \quad \lambda_- = -\mu_\pi^2\frac{E(\xi)}{\mathscr{D}(\xi)};$$

$$\xi(0) = \frac{F_-(0)}{F_+(0)}; \quad \lambda_0 = \lambda_+ + \frac{m_\pi^2}{m_K^2 - m_\pi^2}\xi(0) = 0.$$

The numerical values for $\lambda_\pm$, λ_0, and $\xi(0)$ are given in Table III.

The parameters $\lambda_\pm$ and $\xi(0)$ are determined experimentally in three ways: by analysis of the Dalitz plots (study of the spectrum of the π mesons), by measuring the polarization of the μ mesons, and by measuring the ratio $\Gamma_{K\mu 3}/\Gamma_{K l 3}$. In Table III, we give the mean values of $\lambda_\pm$ and $\xi(0)$ taken from the polarization experiments.

The decay $B \to Be\nu$.[17] With allowance for gauge invariance and the bound-state condition $Z_2 = 0$, we obtain the following expression for the matrix element of the semileptonic decay of the baryon:

$$\bar{B}(p')\Lambda_\mu(p', k)B(p) = F^V_\mu - \beta[(1 - \alpha)F^A_\mu + \alpha\mathscr{D}^A_\mu],$$

where

$$F^V_\mu = \mathrm{Tr}(\bar{B}\gamma_\mu[J, B]);$$
$$F^A_\mu = \mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5[J, B]);$$
$$\mathscr{D}^A_\mu = \mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5\{J, B\}).$$

The dependence of β and α on the parameter $|\xi|$ for the variants S, A, and P of the strong coupling in (61) is shown in Fig. 14. As can be seen from the graphs, in the neighborhood of the point $|\xi| = 1.45 \pm 0.05$ only the S variant gives correct agreement with the experimental data (see Table III).

The decay $\pi^- \to e\nu\gamma$.[12,24] In the decay $\pi^- \to e\nu\gamma$, the most interesting quantity is the ratio of the contributions of the axial and vector parts of the amplitude. Usually, the structure-dependent part of the amplitude is parametrized by

$$M^{\mu\nu}_{S\mathscr{D}} = \varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma b(t) - \mathrm{i}\,(k^\mu p^\nu - g^{\mu\nu}(kp))\, a(t),$$

where k and p are the photon and pion momenta. In the considered approach, the amplitude is determined by a set of diagrams (see Table III). It is found that

$$a(0) = b(0) = \sqrt{\lambda}\, L/2\pi,$$

i.e., the experimentally determined quantity is

$$\gamma = a(0)/b(0) = 1,$$

as in the ordinary quark models.[25]

15. *Nonleptonic weak interactions*. Nonleptonic weak interactions are of interest in the first place because their study makes possible a deeper understanding of the structure of the weak interactions, and also the connection between the weak and strong interactions. The experimental data on the nonleptonic decays of hadrons indicate that the selection rule $\Delta T = \frac{1}{2}$ holds. So far, a convincing theoretical explanation of this fact has not been found, although a considerable number of studies have been devoted to this problem.[26] One of the commonly adopted ways of obtaining the selection rule $\Delta T = \frac{1}{2}$ is to assume that the Lagrangian of the nonleptonic interaction transforms as the sixth component of an octet[26]:

$$\mathscr{L}_I^{\Delta T=1/2} = \frac{G}{\sqrt{2}}\, 2d_{6mn} J^m_\mu J^n_\mu,$$

where J^m_μ is the octet of the weak hadronic currents (in the quark models, $J^m_\mu = \bar{q}\frac{1}{2}\lambda^m O_\mu q$). This corresponds to the introduction of neutral currents.

In color quark models[27] there is a different and original way of obtaining the selection rule $\Delta T = \frac{1}{2}$:

$$\mathscr{L}_I^{\Delta T=1/2} = \frac{G}{\sqrt{2}} \Big[\Big(\bar{q}_a O_\mu \frac{\lambda^1 - \mathrm{i}\lambda^2}{2} q_{a'} \Big) \times \Big(\bar{q}_b O_\mu \frac{\lambda^4 + \mathrm{i}\lambda^5}{2} q_{b'} \Big) + \mathrm{h.c.} \Big] \varepsilon_{abc} \varepsilon_{a'b'c}.$$

Here, a, b, c are color indices.

In the nonlocal quark model, the local weak currents

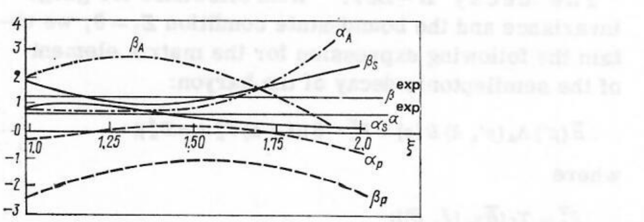


FIG. 14. Dependences of α and β on the interaction variant (S, A, P) and on the parameter ξ.

(50) can be used to consider these interactions, but then ultraviolet divergences appear in perturbation theory already in the lowest orders and special efforts are therefore needed[24] to avoid them.

However, in the nonlocal quark model there exists the possibility of using the following vectors instead of the local quark weak currents:

$$I^m_\mu = \bar{q}_a O_\mu \frac{\lambda^m}{2} q_a.$$

In this case, any product of these vectors leads to a finite result.

Thus, we consider the following two variants of interaction Lagrangians which ensure fulfillment of the selection rule $\Delta T = \frac{1}{2}$:

$$\mathscr{L}_I^{\Delta T=1/2} = \frac{G}{\sqrt{2}}\, 2d_{6mn} I^m_\alpha I^n_\alpha \quad \text{(variant A)};$$

$$\mathscr{L}_I^{\Delta T=1/2} = \frac{G}{\sqrt{2}} \Big[\Big(\bar{q}_a O_\mu \frac{\lambda^1 - \mathrm{i}\lambda^2}{2} q_{a'} \Big) \times \Big(q_b O_\mu \frac{\lambda^4 + \mathrm{i}\lambda^5}{2} q_{b'} \Big) + \mathrm{h.c.} \Big] \varepsilon_{abc} \varepsilon_{a'b'c} \quad \text{(variant B)}. \qquad (65)$$

It should be noted that these two variants hardly differ in the description of the decays $K \to 2\pi$ and $K \to 3\pi$, whereas the matrix element for the decay $K^0_L \to \gamma\gamma$ depends strongly on the choice of the variant. In variant A, the theoretical width of the decay $K^0_L \to \gamma\gamma$ is approximately an order of magnitude lower than the experimental width, which agrees with the conclusions of Ref. 28. In variant B, good agreement with the experiment is obtained, but as yet it is difficult to give a clear physical interpretation of such an interaction.

The transition $K^0_L \to P (P = \pi^0, \eta, \eta')$[20]:

$$M_A(K^0_L \to P) = \frac{\lambda}{L^2} \frac{G m_K^2}{\sqrt{2}} \frac{7\xi^2}{4\pi^2} (1 + 3S_1(\sqrt{2}\,\xi))^2 C^A_P;$$

$$C^A_{\pi^0} = 1;$$

$$C^A_\eta = \frac{1}{\sqrt{3}} \Big(\cos\theta + \frac{5\sqrt{2}}{7} \sin\theta \Big);$$

$$C^A_{\eta'} = \frac{1}{\sqrt{3}} \Big(\sin\theta - \frac{5\sqrt{2}}{7} \cos\theta \Big);$$

$$M_B(K^0_L \to P) = \frac{\lambda}{L^2} \frac{G m_K^2}{\sqrt{2}} \frac{3\xi^2}{2\pi^2} (1 + 3S_1(\sqrt{2}\,\xi))^2 C^B_P;$$

$$C^B_{\pi^0} = 1;$$

$$C^B_\eta = \frac{1}{\sqrt{3}} (\cos\theta - \sqrt{2} \sin\theta);$$

$$C^B_{\eta'} = \frac{1}{\sqrt{3}} (\sin\theta + \sqrt{2} \cos\theta).$$

It can be seen from Table III that the matrix elements M_A and M_B differ little for the transition $K^0_L \to \pi^0$ but that there is a fairly strong difference in the case of the transitions $K^0_L \to \eta, \eta'$.

The decay $K \to 2\pi$ (Ref. 20):

$$M(K^0_S \to \pi^+\pi^-) = \mathrm{i}\,\frac{1}{L} (G(m_K^2 - m_\pi^2)) \Big(\frac{4\sqrt{\lambda}}{3} \Big)^3 \frac{\xi^2}{2\pi} \times S_2\Big(\frac{\xi}{\sqrt{3}} \Big) (1 + 3S_1(\sqrt{2}\,\xi))\, C \Big\{ 1 + \frac{12\lambda}{L^2 (m^2_{K^*} - m^2_\pi)} (3 + C_1(\sqrt{2}\,\xi)) \Big\};$$

$$C = \begin{cases} 7 & \text{(variant A)}, \\ 6 & \text{(variant B)}, \end{cases}$$

i.e., variants A and B differ little in this case, and

$$\Gamma(K^0_S \to \pi^+\pi^-) = \frac{1}{16\pi m_K} \sqrt{1 - \frac{4m_\pi^2}{m_K^2}}\, |M(K^0_S \to \pi^+\pi^-)|^2.$$

The decay $K^+ \to \pi^+\pi^0$ is forbidden, since the Lagrangians (65) ensure exact fulfillment of the selection rule ΔT

$=\frac{1}{2}$.

The decay $K\to 3\pi$.[20] The corresponding diagrams are given in Fig. 15, and

$$M(K\to 3\pi)=a_{K\to 3\pi}[1-\sigma_{K\to 3\pi}y],$$

where $y=(s_3-s_0)/m_\pi^2, s_i=(k-p_i)^2$, k is the momentum of the K meson, p_i is the momentum of pion i, $s_0=m_\pi^2+m_K^2/3$, and

$$a_{+-0}=\lambda^2\frac{G}{\sqrt{2}}m_K^2Ca;\quad \sigma_{+-0}=b/a.$$

Here

$$a=a_0+a_\sigma+a_\rho+a_{K*}+a_{\pi\rho}+a_{K*K}+a_{K*\rho}$$
$$b=b_0+b_\sigma+b_\rho+b_{K*}+b_{\pi\rho}+b_{K*K}+b_{K*\rho};$$

a_j and b_j are the contributions of the various diagrams. The characteristics of the other modes of the decays $K\to 3\pi$ are related to a_{+-0} and σ_{+-0} by means of the isotopic relations.[8] We have

$$a=\begin{cases}-0.996 & \text{(variant A)},\\ -0.854 & \text{(variant B)},\end{cases}$$
$$b=0.474;$$
$$\Gamma(K\to 3\pi)=m_K\left(1-3\frac{m_\pi}{m_K}\right)^2\frac{|a_{K\to 3\pi}|^2}{2^7\pi^3(\sqrt{3})^3}.$$

The decay $K\to\gamma\gamma$ (Ref. 20):

$$M(K_L^0\to\gamma\gamma)=eg_{K\gamma\gamma}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_1^\alpha q_1^\beta\varepsilon_2^\mu q_2^\nu,$$

where ε_i and q_i are the polarization and momentum of photon i. It can be shown that the contact diagrams make a comparatively small contribution:

$$g^{(a)}_{K\to\gamma\gamma}=\begin{cases}L(Gm_K^2)\sqrt{\lambda}\frac{\xi^2}{(6\pi)^3}(1+3S_1(\sqrt{2}\xi))\\ \quad =0.016\cdot 10^{-10}\,\text{MeV}^{-1}\ \text{(A)}\\ 6L(Gm_K^2)\sqrt{\lambda}\frac{\xi^2}{(6\pi)^3}(1+3S_1(\sqrt{2}\xi))\\ \quad =0.096\cdot 10^{-10}\,\text{MeV}^{-1},\ \text{(B)}\end{cases}$$

and the main contribution is made by the resonance diagrams:

$$g^{(b)}_{K\to\gamma\gamma}=\sum_{P=\pi^0,\eta,\eta'}M(K_L^0\to P)\frac{1}{m_P^2-m_K^2}M(P\to\gamma\gamma)$$
$$=\begin{cases}-0.12\cdot 10^{-10}\,\text{MeV}^{-1}\ \text{(A)};\\ -0{,}55\cdot 10^{-10}\,\text{MeV}^{-1}\ \text{(B)};\end{cases}$$
$$\Gamma(K_L^0\to\gamma\gamma)=\frac{1}{4}\pi\alpha^2m_K^3g^2_{K\gamma\gamma}.$$

Since the main contribution is made by the resonance diagrams, the result depends strongly on the choice of the Lagrangian of the nonleptonic interaction (variants A and B). In the case of variant A, the result is about an order of magnitude lower than the experimental result, whereas variant B leads to good agreement with experiment.

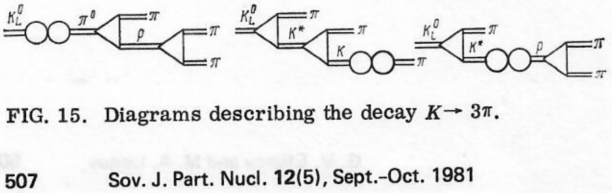


FIG. 15. Diagrams describing the decay $K\to 3\pi$.

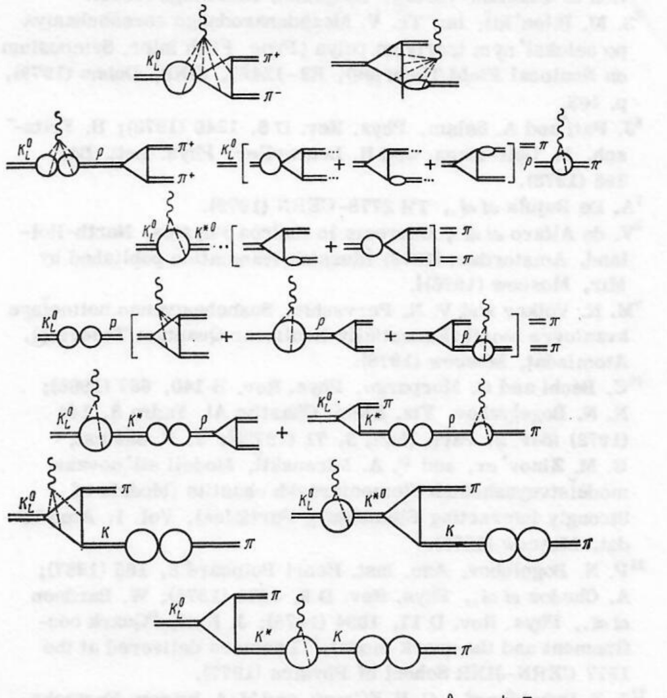


FIG. 16. Diagrams describing the decay $K_L^0\to\pi^+\pi^-\gamma$.

The decay $K_L^0\to\pi^+\pi^-\gamma$.[20] The corresponding diagrams are shown in Fig. 16, and

$$M(K_L^0\to\pi^+\pi^-\gamma)=eg_{K\to\pi\pi\gamma}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\mu q^\nu q_-^\alpha q_+^\beta,$$

where ε and q are the polarization and momentum of the photon, q_- and q_+ are the pion momenta,

$$g_{K\to\pi\pi\gamma}=g_0+g_\rho+g_{K*}+g_{K*K}+g_{K*\rho},$$

g_i are the contributions of the various diagrams, and

$$\Gamma(K_L^0\to\pi^+\pi^-\gamma)=\frac{0.13\pi^2}{(2\pi)^5}\left(\frac{17}{75}\right)^{11/2}m_K^7g^2_{K\to\pi\pi\gamma}.$$

CONCLUSIONS

The constructed nonlocal quark model is a quantum-field dynamical scheme of a relativistic bag. The model has just two free parameters, which characterize the unobservable quark field. The fact that the effective expansion constants are found to be less than unity makes it possible to use only the lowest orders of perturbation theory in the calculations. The obtained results show that the model correctly describes elementary-particle physics in the energy region corresponding to quark confinement.

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