Coherent radiation of photons by fast particles in excited matter

M. I. Ryazanov

Engineering-Physics Institute, Moscow
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The theory of the coherent emission of photons by fast charged particles when they interact with atoms of matter excited by an external electromagnetic field is reviewed.

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INTRODUCTION

The development of sensitive light detectors—photomultipliers—stimulated the rapid development of the use of the radiation of fast particles in matter to obtain information about the properties of the particles and, in particular, to detect particles. Ten years after the discovery and explanation of Cherenkov radiation by Cherenkov, Tamm, and Frank,[1–3] photomultipliers were used to detect fast particles,[4] and the velocities of fast particles in a beam were directly measured[5,6] using the radiation. Somewhat later, total absorption Cherenkov counters were used to detect high-energy particles[7]; in this case, the Cherenkov radiation of the secondary charged particles of the shower initiated by the primary particle is detected.

From the microscopic point of view, it is obvious that the source of the Cherenkov radiation is not the particle itself but the atoms of the matter which it excites. It can therefore be said that the widely used scintillation counters are analogous to Cherenkov counters and differ from them only in that in scintillators there is incoherent de-excitation of excited impurity atoms, whereas in a Cherenkov counter there is coherent de-excitation of excited atoms of the main material.[8]

Searches for new methods of detecting high energy particles have led to the discovery of new possibilities for detecting particles from their radiation in matter. The attention of theoreticians was drawn to the radiation, predicted theoretically by Ginzburg and Frank,[9] of a uniformly moving charge when it intersects the interface of two media with different permittivities.

The theoretical investigation of this radiation, which Ginzburg and Frank called transition radiation, showed that in this case the radiated energy increases in proportion to the energy of the particle.[10] This was an important development in the theoretical investigations and stimulated study of the transition radiation of particles in layered and inhomogeneous media.[11] In particular, a method was proposed for the detection of particles on the basis of their radiation in layered media.[12,13] A study was also made of the possibility of detecting particles from their transition radiation in a single crystal,[13,14] including the cases when the emitted photons undergo Bragg scattering.[15–17] During the last decade, there has been a considerable growth of interest in transition radiation in inhomogeneous media in the x-ray range. In particular, this is due to the fact that in a streamer chamber one can observe visually the tracks of photoelectrons produced by transition-radiation x-rays.[18] It became clear that experimentally it is more convenient to use not layered media but a porous substance with a large number of randomly distributed walls as in styrofoam.[18,19] Subsequently, it was suggested that a material consisting of a large number of fine superconducting granules should be used as a detector based on the use of x-ray transition radiation. The photoelectrons produced by the transition x rays lead to the destruction of the superconducting state of the granules of a superheated colloid and thus strongly change the electromagnetic properties of the detector.[20] These questions were considered in detail at the International Symposium on Transition Radiation of High Energy Particles (Erevan, 1977) in the Proceedings of which[21] information about the present state of the art can be found.

Note that transition radiation can be regarded as the de-excitation of the atoms of the matter excited by the fast particle. In the case of transition radiation, some of the momentum is transmitted in the process of the radiation to inhomogeneities of the matter, whereas in the case of Cherenkov radiation the changes in the energy and momentum of the particle are, respectively, equal to the energy and momentum of the emitted photon. This is the difference between the two types of radiation. The wave properties of particles have the consequence that momentum cannot be transferred to matter at one point[22]: the effective length over which the momentum Δp is transferred to matter is of order $(\Delta p)^{-1} \times (\hbar = c = 1)$. It is over this length that the process of transition radiation effectively occurs, and the excited atoms are de-excited coherently. In transition radiation, not all the atoms along the path of the particle are de-excited coherently but only groups of atoms over the coherence length, i.e., over sections of the path of length $(\Delta p)^{-1}$; the radiation from neighboring sections is incoherent and in a homogeneous material is mutually suppressed, whereas in an inhomogeneous material this occurs only partly, which results in the transition radiation. In contrast, $\Delta p = 0$ in the case of Cherenkov radiation, and the coherence length increases unboundedly, i.e., the de-excitation of the atoms along the entire path of the particle is coherent. Since the intensity of coherent de-excitation is N times greater than the intensity of incoherent de-excitation (N is the number of emitting atoms), it is obvious why the Cherenkov radiation is readily detected while the incoherent de-excitation of the atoms in the same material is unnoticed.

The question arises of whether it is possible to have processes of coherent de-excitation of atoms of matter

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excited by a particle which are analogous but not identical to Cherenkov radiation. The advantages of such processes with regard to the intensity are obvious. However, if such processes are more complicated electromagnetic interactions, their intensity must contain extra powers of the fine-structure constant $e^2=1/137$. Therefore, the main question is whether the increase in the intensity due to the coherence can compensate the decrease in the intensity due to the greater complexity of the process.

As is shown in Ref. 23, there is such a possibility if one considers the coherent de-excitation of atoms in matter excited by a particle in a specially chosen external electromagnetic field. The consideration of such a situation reveals an entire class of new types of electromagnetic processes in matter, and these are not only of independent interest but also provide possibilities for new methods of obtaining information about the properties of charged high-energy particles.

1. THE SELF-FIELD OF A CHARGE MOVING IN MATTER AND THE OCCURRENCE OF RADIATION. THE COHERENCE LENGTH

Before we consider the occurrence of radiation in matter, it is helpful to find the field of a charge e moving uniformly with velocity v in matter with permittivity $\varepsilon(\omega)$. It is well known that in matter with frequency dispersion of the permittivity the Maxwell equations can, strictly speaking, be written only for the Fourier transforms $\mathbf{E}(\mathbf{R}, \omega)$ and $\mathbf{H}(\mathbf{R}, \omega)$ of the fields. These satisfy the equations ($c=1$)

$$\left.\begin{aligned}(\Delta+\omega^2\varepsilon(\omega))\,\mathbf{E}(\mathbf{R},\omega)&=-4\pi i\omega\mathbf{j}(\mathbf{R},\omega)+4\pi\nabla\rho(\mathbf{R},\omega);\\(\Delta+\omega^2\varepsilon(\omega))\,\mathbf{H}(\mathbf{R},\omega)&=-4\pi\,\mathrm{curl}\,\mathbf{j}(\mathbf{R},\omega).\end{aligned}\right\}\tag{1}$$

Seeking the solution of these equations in the form*

$$\left.\begin{aligned}\mathbf{E}(\mathbf{R},\omega)&=\int d^3q\mathbf{E}(\mathbf{q})\exp(i\mathbf{qR});\\\mathbf{H}(\mathbf{R},\omega)&=\int d^3q\mathbf{H}(\mathbf{q})\exp(i\mathbf{qR}),\end{aligned}\right\}\tag{2}$$

we readily obtain

$$\left.\begin{aligned}\mathbf{H}(\mathbf{R},\omega)&=4\pi i\int d^3q\exp(i\mathbf{qR})\frac{[\mathbf{q},\mathbf{j}(\mathbf{q},\omega)]}{q^2-\omega^2\varepsilon};\\\mathbf{E}(\mathbf{R},\omega)&=4\pi i\int\frac{d^3q}{\varepsilon}\exp(i\mathbf{qR})\frac{\omega\varepsilon\mathbf{j}(\mathbf{q},\omega)-\mathbf{q}\rho(\mathbf{q},\omega)}{q^2-\omega^2\varepsilon}.\end{aligned}\right\}\tag{3}$$

It follows that for a uniformly moving charge

$$\left.\begin{aligned}\mathbf{E}_u(\mathbf{R},t)&=\int d^3q\mathbf{E}_u(\mathbf{q})\exp(i\mathbf{qR}-i\mathbf{q}vt);\\\mathbf{E}_u(\mathbf{q})&=\frac{ie}{2\pi^2\varepsilon}\frac{\mathbf{v}(\mathbf{qv})\varepsilon-\mathbf{q}}{q^2-(\mathbf{qv})^2\varepsilon};\\\mathbf{H}_u(\mathbf{R},t)&=\int d^3q\mathbf{H}_u(\mathbf{q})\exp(i\mathbf{qR}-i\mathbf{q}vt);\\\mathbf{H}_u(\mathbf{q})&=\frac{ie}{2\pi^2}\frac{[\mathbf{q},\mathbf{v}]}{q^2-(\mathbf{qv})^2\varepsilon}.\end{aligned}\right\}\tag{4}$$

We not find $\mathbf{H}(\mathbf{R}, \omega)$ at large distances from the field source. For this, it is convenient to use the well-known asymptotic formula

$$\int d^3q\frac{f(\mathbf{q})\exp(i\mathbf{qR})}{(k+q+i\delta)(k-q+i\delta)}\approx-2\pi^2f(k\mathbf{n})\frac{\exp(ikR)}{R};\tag{5}$$

where $kR\gg1$; $f(q)$ does not have singularities, and we have introduced the notation $\mathbf{n}=\mathbf{R}/R$. Denoting $\mathbf{k}(\omega)=\mathbf{n}\omega\sqrt{\varepsilon(\omega)}$ and using (5), we can obtain from (4) the asymptotic behavior of the Fourier transform of the magnetic field at large distances:

$$\mathbf{H}(\mathbf{R},\omega)=-i(2\pi)^3[\mathbf{k}(\omega),\mathbf{j}(\mathbf{k}(\omega),\omega)]\exp(ik(\omega)R)/R.\tag{6}$$

The inverse proportionality of the field to the distance R from the system of charges means that (6) is a radiation field. It is well known that the energy radiated in the direction $\mathbf{n}$ in the element $d\Omega$ of solid angle in the frequency interval $d\omega$ has the form

$$d\mathcal{E}(\mathbf{n},\omega)=\frac{1}{\sqrt{\varepsilon(\omega)}}R^2\,d\Omega\,d\omega\,|\mathbf{H}(\mathbf{R},\omega)|^2.\tag{7}$$

Substitution of (6) in (7) gives

$$d\mathcal{E}(\mathbf{n},\omega)=\omega^2\sqrt{\varepsilon(\omega)}\,|(2\pi)^3[\mathbf{n},\mathbf{j}(\mathbf{k}(\omega),\omega)]|^2\,d\omega\,d\Omega.\tag{8}$$

We now assume that a particle with charge e moves with constant velocity $\mathbf{v}$ between the times $t=-\tau$ and $t=+\tau$ but is otherwise at rest. In this case,

$$\mathbf{j}(\mathbf{q},\omega)=\frac{e\mathbf{v}}{(2\pi)^4}\int_{-\tau}^{\tau}\exp[i(\omega-\mathbf{qv})t]\,dt=\frac{e\mathbf{v}}{(2\pi)^4}2\frac{\sin\{(\omega-\mathbf{qv})\tau\}}{(\omega-\mathbf{qv})},\tag{9}$$

so that the radiated energy has the form

$$d\mathcal{E}(\mathbf{n},\omega)=\frac{e^2}{\pi^2}\omega^2\sqrt{\varepsilon(\omega)}[\mathbf{nv}]^2\frac{\sin^2\{(\omega-\mathbf{k}(\omega)\mathbf{v})\tau\}}{(\omega-\mathbf{k}(\omega)\mathbf{v})^2}\,d\omega\,d\Omega.\tag{10}$$

First of all, we consider the limiting case $\tau\to\infty$, when

$$\mathbf{j}(\mathbf{k}(\omega),\omega)=\frac{e\mathbf{v}}{(2\pi)^3}\delta(\omega-\mathbf{k}(\omega)\mathbf{v})=\frac{e\mathbf{v}}{(2\pi)^3\omega}\delta(1-v\sqrt{\varepsilon}\cos\vartheta).\tag{11}$$

For $v^2\varepsilon<0$, the argument of the δ function never passes through zero, and therefore $\mathbf{j}(\mathbf{k}(\omega), \omega)$, and with it the radiated energy, vanishes. For $v^2\varepsilon>0$ substitution of (11) in (8) gives the Cherenkov radiation

$$d\mathcal{E}^{\mathrm{Ch}}=e^2\omega\,d\omega\,(T/2\pi)\,d\Omega\,[\mathbf{nv}]\sqrt{\varepsilon}\delta(1-v\sqrt{\varepsilon}\cos\vartheta),\tag{12}$$

for which the proton emission angle ϑ is determined by

$$\cos\vartheta=1/[v\sqrt{\varepsilon(\omega)}].\tag{13}$$

We now consider the change in the radiation intensity at finite τ. From zero and to $\tau_{\mathrm{coh}}=(\pi/2)(\omega-\mathbf{vk}(\omega))^{-1}$, the radiated energy increases monotonically in proportion to the square of τ, since the radiation emitted from different points of the path of the particle arrives at the point of observation with the same phase and is added coherently. The decrease in the intensity for $\tau>\tau_{\mathrm{coh}}$ can be attributed to the fact that some of the radiated waves arrive at the point of observation out of phase and there is partial suppression of the waves. For $\tau=2\tau_{\mathrm{coh}}$, the entire radiated energy is zero, which means that there has been mutual suppression of all the radiated waves at the point of observation. It follows that the maximal interval of the particle's path over which coherent radiation takes place is

$$l_{\mathrm{coh}}\approx 2v\tau_{\mathrm{coh}}=\pi v(\omega-\mathbf{vk}(\omega))^{-1}.$$

The qualitative picture of the radiation for arbitrary motion of the particle can be obtained by assuming that the entire path is divided into sections of length $\sim v(\omega-\mathbf{vk}(\omega))^{-1}$, the radiation from each such section being coherent, while that from neighboring sections is incoherent. The introduction of the *coherence length* $l_{\mathrm{coh}}\sim v(\omega-\mathbf{vk}(\omega))^{-1}$ is convenient for the qualitative analysis of the radiation of ultrarelativistic particles.[22,25] The absence of radiation in the case of a uniformly moving charge is due to the mutual suppression of the radiation

*Translation Editor's Note. The Russian notation for scalar and vector products is retained here and throughout the article.

from the different coherence lengths. Anything which changes the interference of the fields at the point of observation must therefore lead to radiation. For example, this could be a change in the velocity of the charge, when the fields from the sections near the turning point of the trajectory do not suppress each other and bremsstrahlung arises. In the case of uniform motion of a charge in matter with inhomogeneous permittivity properties, the field of the charge excites atoms of the matter along the path of the particle, and the inhomogeneity of the permittivity properties of the matter has the consequence that the radiation from neighboring coherence lengths is different in magnitude and does not cancel as a result of interference. The radiation due to the scattering of the self-field of the charge by the atoms of the material has become known as *transition radiation.*

Naturally, in the general case it is possible to have radiation associated with scattering of the charged particle in the matter and the scattering of its field in the matter. However, it is convenient to investigate the properties of these two types of radiation separately.

An important difference between transition radiation and bremsstrahlung is the different dependence of the radiation intensity on the particle's rest mass. Bremsstrahlung is inversely proportional to the square of the particle's mass, whereas transition radiation does not depend on the particle's mass. Therefore, transition radiation is basic in the study of particles with large mass, whereas bremsstrahlung is more important for light particles.

Note that in the collision of a fast particle with an atom recoil electrons are produced and there is radiation by them. In such a process, the transfer of relatively small portions of energy is more probable, and therefore the motion of the fast particle can be approximately assumed to be uniform. Therefore, the radiation of the recoil electrons must also be termed transition radiation.

It is necessary to point out specific features of the emission of photons by ultrarelativistic particles. If a charge is scattered elastically during bremsstrahlung by atoms of matter at rest or if the properties of the matter do not change in time, then exchange of energy with the medium does not occur during the radiation process, and only the momentum $\overline{\Delta p}$ is transferred to the matter. The conservation laws for the process have the form

$$E_0 - E - \omega = 0; \quad \mathbf{p}_0 - \mathbf{p} - \mathbf{k} = \Delta\mathbf{p}. \tag{14}$$

To find the coherence length, we determine the momentum $\Delta p_\parallel$ transferred to the matter in the longitudinal (along $\mathbf{p}_0$) direction. It is readily shown that for $p_0 \gg M$

$$\approx M^2\omega/(2E_0E) + p(1-\cos\vartheta) + \omega - k\cos\vartheta_\gamma$$

and for high frequencies, when $\varepsilon(\omega) = 1 - (\omega_p/\omega)^2$, $\omega^2 \gg \omega_p^2 = 4\pi ne^2/m$, and for small angles $\vartheta \ll 1$, $\vartheta_\gamma \ll 1$ one can obtain

$$\Delta p_\parallel = \frac{\omega}{2}\left(\frac{M^2}{E_0E} + \frac{\omega_p^2}{\omega^2} + \frac{p}{\omega}\vartheta^2 + \vartheta_\gamma^2\right). \tag{15}$$

It is readily seen from (15) that the longitudinal momentum transfer cannot be smaller than the minimal value

$$\Delta p_\parallel^{\min} = (\omega/2)(M^2/E_0E + \omega_p^2/\omega^2). \tag{16}$$

The coherence length is determined by the reciprocal of the longitudinal momentum transfer:

$$l_{\text{coh}} \sim v(\omega - \mathbf{k}(\omega)\mathbf{v})^{-1} \sim (1/\omega)(M^2/E^2 + \omega_p^2/\omega^2)^{-1}. \tag{17}$$

The most important thing for the radiation of ultrarelativistic particles is the circumstance that the coherence length (17) can significantly exceed atomic dimensions at superhigh energies. The formation of the quantum of radiation occurs over macroscopic lengths, and the influence of competing processes therefore becomes possible. For the bremsstrahlung of fast electrons, this circumstance was noted for the first time by Landau and Pomeranchuk,[26] and for pair production by Migdal.[27] Coherence effects in bremsstrahlung and the influence of competing processes are considered in Refs. 13, 22, and 28.

2. EXCITATION OF LONGITUDINAL ELECTRIC VIBRATIONS OF A FAST PARTICLE. WAKE CHARGE

The Maxwell equation

$$\operatorname{div}\mathbf{E}(\mathbf{R},\omega) = 4\pi\rho(\mathbf{R},\omega)/\varepsilon(\omega) = 4\pi\rho(\mathbf{R},\omega) + 4\pi\rho(\mathbf{R},\omega)(1/\varepsilon(\omega) - 1) \tag{18}$$

contains the permittivity $\varepsilon(\omega)$ in the denominator, and therefore the result of integration on the transition to time-dependent quantities depends on the position of the zeros of the permittivity. It is well known that the zeros of $\varepsilon(\omega)$ lie below the real axis of ω. Therefore, when the Fourier transformation (18) is inverted, we obtain

$$\operatorname{div}\mathbf{E}(\mathbf{R},t) = 4\pi\rho(\mathbf{R},t) + 4\pi\int_0^\infty d\tau\rho(\mathbf{R},t-\tau) \times \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}\exp(i\omega\tau), \tag{19}$$

where the lower limit of integration over τ is replaced by zero in the absence of zeros of $\varepsilon(\omega)$ in the upper half-plane. The integral over the frequencies in (19) is determined by the poles of the integrand, i.e., by the zeros of $\varepsilon(\omega)$, and reduces to the sum of the contributions from the different poles, which makes it possible to investigate the contribution of each pole separately. The poles situated far from the real axis of ω lead to the appearance in the integral over τ of a rapidly decreasing exponential, which makes it possible to take $\rho(\mathbf{R}, t-\tau)$ in front of the integral over τ at the point $\tau = 0$. Therefore, the contribution from such a pole leads to a term consisting of the product of $\rho(\mathbf{R}, t)$ and a factor small compared with unity. This term will be small compared with the first term on the right-hand side of (19), and therefore the poles far from the real axis of ω make an unimportant contribution to (19). Therefore, the main attention must be devoted to the poles near the real axis. But because the real part of $\varepsilon(\omega)$ is even, the zeros of $\varepsilon(\omega)$ along the real axis are situated pairwise at the points $\pm\omega_p - i\gamma_p$, $\gamma_p \ll \omega_p$. The contribution of such poles gives from (19)

$$\operatorname{div}\mathbf{E}(\mathbf{R},t) = 4\pi\rho(\mathbf{R},t) + 4\pi\rho_W(\mathbf{R},t), \tag{20}$$

where

$$\rho_W(\mathbf{R}, t) = -\int_0^\infty d\tau \rho(\mathbf{R}, t-\tau) \sum_p \frac{2\sin\omega_p\tau}{\left|\frac{\partial \operatorname{Re}\varepsilon(\omega)}{\partial\omega}\right|_{\omega=\omega_p}} \exp(-\gamma_p\tau). \quad (21)$$

For a charge Ze moving uniformly with velocity v, this gives

$$\rho_W(\mathbf{R}, t) = \frac{Ze}{v}\delta(y)\delta(z)\theta(vt-x)\sum_p 2\frac{\sin(\omega_p x/v-\omega_p t)}{|\partial \operatorname{Re}\varepsilon(\omega)/\partial\omega|_{\omega=\omega_p}}, \quad (22)$$

where $\Theta(x)$ is the unit step function: $\Theta(x)=0$ for $x<0$, $\Theta(x)=1$ for $x>0$. It can be seen from (22) that the behavior of $\rho_W(\mathbf{R}, t)$ differs from that of $\rho(\mathbf{R}, t)$. After the passage of the particle, longitudinal vibrations of the electric charge density arise in its wake, and these exist for a time of order γ_p^{-1}. These vibrations arise because the moving particle excites in the matter longitudinal vibrations–plasmons, longitudinal optical phonons, and longitudinal excitons. It is well known that the frequencies of such vibrations are determined by the condition of vanishing of $\varepsilon(\omega)$. The charge density (22) has become known as the *wake charge density*. Qualitative arguments suggesting the existence of such vibrations were put forward by Bohr,[29] and the wake charge density for a fast particle in a plasma was calculated in Ref. 30. Attention was drawn to the wake charge when a fast particle moves through a solid in Ref. 31, in which bound states in the wake potential of an ion were considered. This was followed by a large number of studies devoted to the influence of wake charge on the separation of ions produced in the breakup of fast molecules passing through thin films of matter.[32-40]

It should be emphasized that the above macroscopic treatment of the wake charge vibrations is valid only when the spatial period v/ω_p of these vibrations remains macroscopic, i.e., exceeds the interatomic distance. This imposes a lower bound on the velocity of the considered particles:

$$v > (\omega_p/me^2). \quad (23)$$

If this condition is violated, a special study is necessary. For a large class of solids, collective vibrations of electrons–plasmons–are possible, their frequency corresponding to vanishing of a permittivity of the form

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega^2 + 2i\gamma_p\omega), \quad (24)$$

where $\omega_p^2 = 4\pi ne^2/m$. For metals n is the number of conduction electrons per unit volume, and for semiconductors n is the number of electrons of the valence band per unit volume.[41] For the existence of plasmons, it is sufficient for (24) to hold in the region of frequencies $\omega \sim \omega_p$. But, as is well known, an expression of the form (24) is valid at high frequencies for any material. Deviations from (24) arise only at frequencies comparable with the eigenfrequencies. Therefore, if (24) is to hold in the region $\omega \sim \omega_p$ it is sufficient if the eigenfrequencies are less than ω_p. In semiconductors, the condition $\omega_p > E_g$, where E_g is the width of the forbidden band, is satisfied, and therefore plasmons can exist.[41] For spatial dispersion, i.e., a dependence of ε on not only the frequency but also the wave vector, the expression for the wake charge density is somewhat more complicated. Instead of (18), one must proceed from the equation

$$iq_i\varepsilon_{is}(\mathbf{q}, \omega) E_s(\mathbf{q}, \omega) = 4\pi\rho(\mathbf{q}, \omega). \quad (25)$$

Bearing in mind that the permittivity is given by

$$\varepsilon_{is}(\mathbf{q}, \omega) = (\delta_{is} - q_i q_s q^{-2})\varepsilon^t(q, \omega) + q_i q_s q^{-2}\varepsilon^l(q, \omega), \quad (26)$$

we can transform (25) to the form

$$iq\mathbf{E}(\mathbf{q}, \omega) = 4\pi\rho(\mathbf{q}, \omega) + 4\pi\rho(\mathbf{q}, \omega)(1/\varepsilon^l(q, \omega) - 1). \quad (27)$$

For the wake charge density, this gives the formula

$$\rho_W(\mathbf{R}, t) = \int d^3r \int d\tau \rho(\mathbf{R}-\mathbf{r}, t-\tau) \times \int\int \frac{d^3q\, d\omega}{(2\pi)^4}\left(\frac{1}{\varepsilon^l(q, \omega)} - 1\right)\exp(i\omega\tau - iqr). \quad (28)$$

The vibrations of the wake charge produce the wake potential, which acts on the original particle, decelerating it. From Eq. (28) and

$$\Delta\varphi_W(\mathbf{R}, t) = -4\pi\rho_W(\mathbf{R}, t) \quad (29)$$

we can find the wake potential, which at the point $\mathbf{r}_0(t)$ where the original particle is situated has the form

$$\varphi_W(\mathbf{r}_0(t), t) = -\frac{Ze}{4\pi^3}\int\frac{d^3q}{q^2}\left(\frac{1}{\varepsilon^l(q, \omega)} - 1\right) \times \int_0^\infty d\tau \exp\{i\omega\tau - iq(\mathbf{r}_0(t) - \mathbf{r}_0(t-\tau))\}. \quad (30)$$

Assuming that the change in the velocity is small, $\mathbf{r}_0(t-\tau) \approx \mathbf{r}_0(t) - \tau\mathbf{v}_0(t)$, we can obtain

$$\varphi_W(\mathbf{r}_0(t), t) = -\frac{Ze}{2\pi^2}\int\frac{d^3q}{q^2}\frac{1}{\varepsilon^l(q, \mathbf{q}\mathbf{v}_0(t))}. \quad (31)$$

The energy lost by the particle per unit time is determined by the deceleration force exerted on the particle by the wake charge:

$$d\mathscr{E}/dt = Ze\mathbf{v}_0(t)\mathbf{E}_W(\mathbf{r}_0(t), t) = i\frac{Z^2e^2}{\pi}\int\omega\, d\omega\int\frac{q_\perp\, dq_\perp}{q_\perp^2v^2+\omega^2}\frac{1}{\varepsilon(\sqrt{q_\perp^2+\omega^2v_0^2}(t), \omega)}, \quad (32)$$

which agrees with the usual expression.[42]

The spatial oscillations of the wake charge have the consequence that when one particle follows another the vibrations of the wake charge can be mutually enhanced or suppressed, depending on the distance between the particles. Accordingly, the wake potential may be enhanced or suppressed and, with it, the energy losses of the two particles. Thus, the wake charge vibrations lead to interference between the energy losses of the two particles. This circumstance was investigated theoretically in Refs. 34, 43, and 44 and has been confirmed experimentally.[44,45]

3. RADIATION OF A UNIFORMLY MOVING CHARGE IN EXCITED MATTER

The energy and momentum conservation laws associated with the radiation of a photon in matter by a charged particle were considered above without allowance for energy transfer to the matter. We now take into account the possibility that some energy is transferred to the matter when the radiation occurs. Then instead of (14), we can write[46]

$$E_0 - E - \omega = \Delta E;\ \mathbf{p}_0 - \mathbf{p} - \mathbf{k} = \Delta\mathbf{p}. \quad (33)$$

Repeating the arguments in the deviation of (16), we can

readily see that the minimal value of the longitudinal momentum transferred during the radiation process takes the form

$$(\Delta p_{\parallel})_{\min} = \Delta E + (\omega/2)\,(m^2/E^2 + \omega_p^2/\omega^2). \tag{34}$$

If energy is transferred to the matter, the minimal momentum transfer increases, which means that the coherence length decreases and, therefore, so does the probability of the process. Opposite results are obtained if energy is transferred from the matter during the radiation process, i.e., if prior to the radiation the matter is in an excited state and after the radiation the energy of the matter is reduced. In this case,

$$(\Delta p_{\parallel})_{\min} = (\omega/2)\,(m^2/E^2 + \omega_p^2/\omega^2) - |\,\Delta E\,|. \tag{35}$$

At high energies of the particles and high frequencies of the emitted photons, energy transfer has an influence even at small $|\Delta E|$. Of particular interest is the case when the momentum transfer to the matter is zero. In this case, emission of a photon is made possible solely by energy transfer. Such a process can be realized in a uniformly excited material. We shall assume that the permittivity $\varepsilon(\omega)$ of the undisturbed material is known. Let us consider what happens when matter containing excited atoms is polarized by an electromagnetic field $\mathbf{E}(\mathbf{R}, t) = \int d\omega \mathbf{E}(\mathbf{R}, \omega)\exp(-i\omega t)$. Before the field is applied, the wave function of an excited atom is a superposition of stationary states of the atom: $\Psi(\mathbf{r}, t) = \sum_n a_n \Psi_n^0(\mathbf{r}, t)$, where the expansion coefficients can be assumed to be slow functions of the time. This assumes that all the relaxation processes are slow compared with the considered radiation process. Application of the field changes the wave function by the small amount $\Phi(\mathbf{r}, t) = \sum_n C_n(t)\Psi_n^0(\mathbf{r}, t)$, in which the expansion coefficients $C_n(t)$ change in time much more rapidly than the a_n. In the approximation linear in the field, we can use the Schrödinger equation with operator of the interaction with the field of the form $-\mathbf{d}\cdot\mathbf{E}$ ($\mathbf{d}$ is the operator of the dipole moment of the atom) to express C_n in terms of a_n ($\mathbf{R}$ is the coordinate of the center of mass of the atom):

$$C_n(\mathbf{R}, t) = \sum_s a_s \int \frac{d\omega}{\omega_{ns} - \omega - i0}\, \mathbf{d}_{ns}\, \mathbf{E}(\mathbf{R}, \omega)\exp\{i\,(\omega_{ns} - \omega)\,t\}, \tag{36}$$

where it is assumed that the field is applied at the time $t = -\infty$. The dipole moment induced by the field per unit volume of the matter can be conveniently split into the part $\mathbf{P}_0(\mathbf{r}, t)$ corresponding to the undisturbed matter and the correction $\mathbf{P}_1(\mathbf{r}, t)$ to it due to the presence of the excitations:

$$4\pi \mathbf{P}_1(\mathbf{r}, t) = \sum_{n,s}{}' \int d\omega Q_{ns}(\omega)\,\mathbf{E}(\mathbf{r}, \omega)\exp\{i\,(\omega_{ns} - \omega)\,t\}, \tag{37}$$

where

$$\mathbf{E} = \mathbf{e}\,|\,E\,|;\ Q_{ns}(\omega) = 4\pi n_0 a_n^* a_s \sum_m (\mathrm{ed}_{nm})\,(\mathrm{ed}_{ms})\,[(\omega_{ms} - \omega)^{-1} + (\omega_{mn} + \omega)^{-1}];\ n \neq s.$$

It follows that the displacement $\mathbf{D}$ is related to the field $\mathbf{E}$ by

$$\mathbf{D}(\mathbf{r}, \omega) = \varepsilon(\omega)\,\mathbf{E}(\mathbf{r}, \omega) + \sum_{n,s}{}' Q_{ns}(\omega + \omega_{ns})\,\mathbf{E}(\mathbf{r}, \omega + \omega_{ns}). \tag{38}$$

Thus, if in the unexcited matter the displacement oscillates with the same frequency as the field, then in the excited matter there is a component of the displacement which oscillates with the combination frequencies $\omega + \omega_{ns}$. This proves the equivalence of matter with excited atoms to matter with nonstationary properties, i.e., with time-dependent permittivity.[47] The radiation of a particle in a medium with phenomenologically specified dependence $\varepsilon = \varepsilon_0 + \varepsilon_1\cos(\mathbf{k}\cdot\mathbf{r} - \Omega t)$ was studied in Ref. 48.

We now turn directly to the problem of studying a uniformly moving charge in a uniformly excited material. The presence of the corrections due to the excitation in the polarization and the displacement (37) has the consequence that the field $\mathbf{E}$, $\mathbf{H}$ of a uniformly moving charge in excited matter will differ from the field $\mathbf{E}_u$, $\mathbf{H}_u$ (4) in unexcited matter by $\mathbf{E}_1$ and $\mathbf{H}_1$. Subtracting from the Maxwell equation for excited matter the same equations for the unexcited matter, we can, for example, obtain an equation for $\mathbf{H}_1(\mathbf{r}, \omega)$ in the form

$$(\Delta + \omega^2\varepsilon)\,\mathbf{H}_1(\mathbf{r}, \omega) = -\omega\varepsilon \sum_{n,s}{}' (\omega + \omega_{ns})\,Q_{ns}(\omega + \omega_{ns})\,\mathbf{H}_0(\mathbf{r}, \omega + \omega_{ns}). \tag{39}$$

Adding this equation to (1), we readily see that the right-hand side plays the part of the curl of the current density, and therefore the solution (39) at large distances can be obtained in the same way as (10). The energy radiated in a solid angle $d\Omega$ in the direction $\mathbf{n} = \mathbf{r}/r$ in the frequency interval $d\omega$ can be obtained[47] in the same way as (12) and ($\mathbf{k}(\omega) = \mathbf{n}\omega\sqrt{\varepsilon(\omega)}$)

$$d\mathscr{E} = \frac{T}{2\pi}\,\frac{\omega^2\varepsilon^{5/2}(\omega)\,(\mathbf{kv})^2\,[\mathbf{kv}]^2}{k^2 - (\mathbf{kv})^2\,\varepsilon(\mathbf{kv})} \times \sum_{n,s}{}' |\,Q_{ns}(\mathbf{kv})\,|^2\,\delta(\omega + \omega_{ns} - \mathbf{kv})\,d\omega\,d\Omega, \tag{40}$$

where T is the total time of flight of the charge through the matter. It follows from (40) that to each value of ω_{ns} there corresponds the emission angle θ_{ns} of the radiation. This angle is related to the velocity of the particle and the frequency of the radiation by[46,47]

$$\cos\theta_{ns} = [1/v\sqrt{\varepsilon(\omega)}]\,(1 - |\,\omega_{ns}\,|/\omega), \tag{41}$$

which goes over in the limit $\omega_{ns} \to 0$ into the Cherenkov radiation condition.

We emphasize that radiation for given ω_{ns} is possible only if

$$|\,\omega_{ns}\,| > \omega\,(1 - v\sqrt{\varepsilon(\omega)}). \tag{42}$$

4. COHERENT DE-EXCITATION OF ATOMS EXCITED BY A PARTICLE IN A MATERIAL IN AN EXTERNAL FIELD

As was noted above, when a photon is emitted by a fast particle in matter, all the atoms within a certain effective region of space participate in the process coherently. The longitudinal length of this region (along the initial momentum of the particle)–the coherence length–increases with the energy and at high energies may reach macroscopic dimensions. Can one have a situation when the atoms along the entire path of the particle participate coherently in the radiation process? The widely known Cherenkov radiation is the simplest example of such a process. Indeed, from the macroscopic point of view one can say that the source of the Cherenkov radiation is not the charged particle itself but the atoms of the material excited by it. Since there is no energy or momentum transfer to the matter in the

case of Cherenkov radiation, the longitudinal momentum transfer to the matter tends to zero, so that the coherence length increases unboundedly, i.e., the excited atoms lying along the entire path of the particle are de-excited coherently. When the particle's velocity changes, $v^2\varepsilon$ varies and may pass through the value unity. Above this value, when $v^2\varepsilon > 1$, the Cherenkov radiation exists; below it, it does not. This means that coherent de-excitation of the excited atoms is possible (at given frequency) at a single angle, which changes with the velocity, and for $v^2\varepsilon < 1$ the direction of coherent de-excitation disappears. Naturally, the direction of the coherent de-excitation depends on the space-time distribution of the excited atoms. It is this distribution that changes with the particle velocity. It follows that the application to matter of an electromagnetic field can change the distribution of the excited atoms and, therefore, change the direction of the coherent de-excitation or lead to the appearance of new directions of coherent de-excitation.

The electromagnetic field must be such that the radiation process takes place without energy or momentum transfer to the matter. This is completely ensured by the fact that an atom which participates in the radiation process remains after the interaction with the particle and with the field and the emission of the photon in the same state as before the interaction. The action of the field of an electromagnetic wave on an atom can be reduced to the absorption of photons of the wave by the atom. Assuming that the fast particle which excites the atom changes its momentum in the process from $\mathbf{p}$ to $\mathbf{p}-\mathbf{q}$ and that, in addition, the atom absorbs photons with momenta $\mathbf{k}_1$, $\mathbf{k}_2$, ... and emits a photon with momentum $\mathbf{k}$, we can write the energy and momentum conservation laws

$$\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2 \ldots = \mathbf{k}; \tag{43}$$

$$(p^2+m^2)^{1/2}-[(p-q)^2+m^2]^{1/2}+\omega_1+\omega_2 \ldots = \omega; \tag{44}$$

for $q \ll p$, (44) takes the form

$$\mathbf{qv}+\omega_1+\omega_2 \ldots = \omega. \tag{45}$$

Substitution of (43) in (45) leads to an expression for the emission angle θ of the emitted photon[53]:

$$\cos\theta = [1/v\sqrt{\varepsilon(\omega)}]\{1-(1/\omega)(\omega_1-\mathbf{k}_1\mathbf{v}+\omega_2-\mathbf{k}_2\mathbf{v} \ldots)\}. \tag{46}$$

If the frequency of the absorbed photons tends to zero, (46) goes over into the usual expression for the emission angle of Cherenkov radiation.[3]

This effect can be described as the result of the action of three fields the field of the particle, the field of the wave with frequency ω_1, and the field of the wave with frequency ω_2, on the atom, i.e., as the result of nonlinear interaction of the electromagnetic field with the matter. Phenomena of this kind are treated in nonlinear optics by the methods of nonlinear macroscopic electrodynamics.[49,50] The nonlinear properties of the matter are described in this case phenomenologically by the introduction of nonlinear susceptibilities of various orders. For not too strong fields, the dependence of the nonlinear polarization of the medium on the total field $\mathbf{E}^t(\mathbf{r}, t)$ in initially homogeneous, isotropic, and stationary matter has the form

$$P_i^{NL}(\mathbf{r}, \omega) = \int\int d\omega'\, d\omega'' \chi(\omega, \omega', \omega'') E_i^t(\mathbf{r}, \omega-\omega') E_j^t(\mathbf{r}, \omega'-\omega'') E_j^t(\mathbf{r}, \omega''), \tag{47}$$

where $\chi(\omega, \omega', \omega'')$ is the nonlinear susceptibility of third order (for a number of materials, the values of χ were measured in Refs. 51 and 52). We now choose the exciting electromagnetic field in the form[23]

$$\mathbf{E}_0(\mathbf{r}, t) = \mathbf{e}\{E_{01}\cos(k_1 r-\omega_1 t-\varphi_1)+E_{02}\cos(k_2 r-\omega_2 t-\varphi_2)\}, \tag{48}$$

so that the total field is

$$\mathbf{E}^t(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}, t)+\mathbf{E}(\mathbf{r}, t), \tag{49}$$

where $\mathbf{E}(\mathbf{r}, t)$ includes the particle's self-field and the field of the coherent de-excitation of the excited atoms. If we consider frequencies not equal to the frequencies of the exciting field, then the linear part of the polarization of the matter depends only on the field $\mathbf{E}(\mathbf{r}, t)$ and can be taken into account by the introduction of a permittivity of the matter.

It follows from (47)–(49) that in the indicated range of frequencies the displacement is related to the field by[23]

$$D_i(\mathbf{r}, \omega) = \varepsilon_{ij}E_j(\mathbf{r}, \omega) + \sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1} Q_{ij}(\omega, \alpha, \beta, \xi, \eta) \times \exp(i\xi\mathbf{k}_\alpha\mathbf{r}-i\eta\mathbf{k}_\beta\mathbf{r})E_i(\mathbf{r}, \omega-\xi\omega_\alpha+\eta\omega_\beta), \tag{50}$$

where

$$Q_{ij}(\omega, \alpha, \beta, \xi, \eta) = \pi(1-\delta_{\alpha\beta}\delta_{\xi\eta})\{\delta_{ij}\chi(\omega, -\xi\omega_\alpha+\eta\omega_\beta, \eta\omega_\beta) + e_i e_j(\chi(\omega, \omega-\xi\omega_\alpha, \eta\omega_\beta) + \chi(\omega, \omega-\xi\omega_\alpha, \omega-\xi\omega_\alpha+\eta\omega_\beta))E_{0\alpha}E_{0\beta}\exp(i\xi\varphi_\alpha-i\eta\varphi_\beta); \tag{51}$$

$$\varepsilon_{ij} \equiv (\varepsilon+\varkappa)\delta_{ij}+e_i e_j b = \varepsilon\delta_{ij} + \pi\sum_{\alpha=1,2}\sum_{\xi=\pm1} E_{0\alpha}^2\{\delta_{ij}\chi(\omega, 0, \xi\omega_\alpha) + e_i e_j(\chi(\omega, \omega-\xi\omega_\alpha, \xi\omega_\alpha)+\chi(\omega, \omega-\xi\omega_\alpha, \omega))\}. \tag{52}$$

It follows from (50) that action of the exciting field makes the matter anisotropic, inhomogeneous, and nonstationary.

For the Fourier transform

$$\mathbf{E}(\mathbf{q}, \omega) = (2\pi)^{-3}\int d^3r\mathbf{E}(\mathbf{r}, \omega)\exp(-i\mathbf{qr})$$

of the field, Maxwell's equations lead to the equation

$$(q^2\delta_{ij}-q_i q_j-\omega^2\varepsilon_{ij})E_j(\mathbf{q}, \omega) = 4\pi i\omega e v_i(2\pi)^{-3}\delta(\omega-\mathbf{qv}) + \omega^2\sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1} Q_{ij}(\omega, \alpha, \beta, \xi, \eta) \times E_j(\mathbf{q}-\xi\mathbf{k}_\alpha+\eta\mathbf{k}_\beta, \omega-\xi\omega_\alpha+\eta\omega_\beta). \tag{53}$$

If we ignore the anisotropy of the originally homogeneous and isotropic matter induced by the field, we can obtain an expression for the energy radiated by the coherent de-excitation in the solid angle $d\Omega$ and frequency interval $d\omega$:

$$d\mathcal{E}(\mathbf{n}, \omega) = \omega^4\sqrt{\varepsilon(\omega)}\, d\omega\, d\Omega\, T(2\pi^3) \times\{Q_{ij}(\omega)(\delta_{ie}-n_i n_e)Q_{se}^*(\omega)E_{ps}^*(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2) \times E_{pj}(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)\}\delta(\omega-\omega_1-\omega_2-\mathbf{v}(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)), \tag{54}$$

where

$$\mathbf{k}(\omega) = \mathbf{n}\omega\sqrt{\varepsilon(\omega)};$$

$$Q_{ij}(\omega) = Q_{ij}(\omega, 1, 2, 1, -1)+Q_{ij}(\omega, 2, 1; 1, -1).$$

The presence in (44) of the δ function ensures fulfill-

ment of the conservation laws (43) and (44) in the case of coherent de-excitation. The angle between the direction of the coherent de-excitation and the velocity of the particle is determined in (46).

Since the nonlinear susceptibilities are small (of the order of the ratio of the external fields to the atomic fields), the quantities Q_{ij}, which are proportional to them, are small. Therefore, the intensity of the coherent de-excitation is fairly low. One must seek ways of increasing the intensity of the coherent de-excitation. One such possibility is to use resonance pumping fields. But then the treatment used above becomes incorrect and a new treatment is required.

5. COHERENT DE-EXCITATION OF ATOMS IN A MATERIAL EXCITED BY A PARTICLE IN A RESONANCE FIELD

As we have already noted, the change in the space-time distribution of excitations under the influence of a field may lead to the occurrence of directions of coherent de-excitation of atoms excited by a particle in a material.

However, the largest change in the populations of excited states of the atoms arises under the influence of a resonance field with frequency equal to the transition frequency of an electron in an atom.[57] In this case, when the field is in resonance with the transitions from the ground to an excited state, the matter is excited by the field more strongly than by the particle. If the main excitation of the matter is to be by the particle, one can choose the field in resonance with transitions between excited states. Then the field hardly affects an unexcited atom, but in an excited atom the resonance field significantly changes the population distribution of the excited levels. We therefore choose the frequencies of the field (48) such that[23]

$$\omega_1 = E_2 - E_1 + \varepsilon_1 = \omega_{21} + \varepsilon_1; \quad \omega_2 = E_3 - E_2 + \varepsilon_2 = \omega_{32} + \varepsilon_2, \tag{55}$$

where $E_3 > E_2 > E_1$ are the energies of the excited atomic states.

The absence of sufficiently detailed experimental information about the behavior of the nonlinear susceptibility $\chi(\omega, \omega', \omega'')$ in the region of the resonance frequencies makes it impossible to estimate the effect by the phenomenological method considered above. Therefore, to estimate the effect, we can use a microscopic treatment, calculating directly the nonlinear part of the polarization of the matter:

$$\mathbf{P}^{NL}(\mathbf{q}, \omega) = (2\pi)^{-3} \int d^3r \mathbf{P}^{NL}(\mathbf{r}, \omega) \exp(-i\mathbf{qr}). \tag{56}$$

This is related to the induced dipole moment at the point $\mathbf{r}$ of the atom,

$$\mathbf{d}(\mathbf{r}, t) = \int d^3q \int d\omega \mathbf{d}(\mathbf{q}, \omega) \exp(i\mathbf{qr} - i\omega t), \tag{57}$$

by

$$\mathbf{P}^{NL}(\mathbf{q}, \omega) = n_0 \zeta \mathbf{d}^{NL}(\mathbf{q}, \omega). \tag{58}$$

The coefficient ζ, which takes into account the deviation from the mean of the field acting on the atom, has the form[50]

$$\zeta = \prod_{i=1}^{n} \frac{1}{3}(\varepsilon(\omega_i) + 2). \tag{59}$$

The nonlinear part $\mathbf{d}^{NL}(\mathbf{q}, \omega)$ of the dipole moment can be expressed in terms of the population amplitudes $C_s(\mathbf{R}, t)$ of the excited states of the atom in the field:

$$\mathbf{d}^{NL}(\mathbf{q}, \omega) = \sum_s \{\mathbf{d}_{0s} C_s^{NL}(\mathbf{q}, \omega - \omega_{s0}) + \mathbf{d}_{s0} C_s^{NL*}(-\mathbf{q}, -\omega - \omega_{s0})\}, \tag{60}$$

where

$$C_s(\mathbf{q}, \omega) = (2\pi)^{-4} \int d^3R \int dt\, C_s(\mathbf{R}, t) \exp(i\omega t - i\mathbf{qR}), \tag{61}$$

and it is assumed that the excitation of the atom is weak, $C_s \ll C_0 \approx 1$. It is well known that in a long-wavelength field the $C_s(\mathbf{R}, t)$ satisfy the equation

$$i \frac{\partial C_s(\mathbf{R}, t)}{\partial t} = -\sum_n \mathbf{d}_{sn} \mathbf{E}(\mathbf{R}, t) C_n(\mathbf{R}, t) \exp(i\omega_{sn} t). \tag{62}$$

In solving the system of equations (62), we assume that the transitions $3 \to 2$ and $2 \to 1$ are due to the resonance field (48), the transitions $0 \to 2$ are forbidden, and the higher levels E_n ($n \geq 4$) do not influence the population of the levels E_1, E_2, E_3. We also assume that the resonance field can transfer an electron from one level to another during the lifetime of the excited state, i.e., that

$$|V_{21}| \gg \gamma_1, \gamma_2; \ |V_{32}| \gg \gamma_1\gamma_3; \ V_{21} = -\mathbf{d}_{21}\mathbf{E}_{01}/2; \quad V_{32} = -\mathbf{d}_{32}\mathbf{E}_{02}/2. \tag{63}$$

Under the assumptions we have made, we obtain from (62) the system of equations

$$\begin{aligned} (\omega + i\gamma_3) C_3(\mathbf{q}, \omega) &= V_{32} C_2(\mathbf{q} - \mathbf{k}_2, \omega - \varepsilon_2) + U_{30}(\mathbf{q}, \omega); \\ (\omega + i\gamma_2) C_2(\mathbf{q}, \omega) &= V_{21} C_1(\mathbf{q} - \mathbf{k}_1, \omega - \varepsilon_1) + V_{23} C_3(\mathbf{q} + \mathbf{k}_2, \omega + \varepsilon_2); \\ (\omega + i\gamma_1) C_1(\mathbf{q}, \omega) &= V_{12} C_2(\mathbf{q} + \mathbf{k}_1, \omega + \varepsilon_1) + U_{10}(\mathbf{q}, \omega), \end{aligned} \tag{64}$$

where $U_{n0}(\mathbf{q}, \omega) = -\mathbf{d}_{n0}\mathbf{E}_p(q, \omega)$ and it is assumed that the excitation of the atom from the ground state is caused by the field of the particle. Ignoring the width of the levels, we can obtain from (64) the expression[23]

$$C_3(\mathbf{q}, \omega) = \frac{1}{\omega} U_{30}(\mathbf{q}, \omega) + \frac{(\omega - \varepsilon_1 - \varepsilon_2)|V_{32}|^2 U_{30}(\mathbf{q}, \omega) + \omega V_{32} V_{21} U_{10}(\mathbf{q} - \mathbf{k}_1 - \mathbf{k}_2, \omega - \varepsilon_1 - \varepsilon_2)}{\omega\{\omega(\omega - \varepsilon_2)(\omega - \varepsilon_1 - \varepsilon_2) - |V_{21}|^2 \omega - (\omega - \varepsilon_1 - \varepsilon_2)|V_{32}|^2}. \tag{65}$$

For frequencies near ω_{30}: $|\omega - \omega_{30}| \ll \omega_{30}$, the dipole moment induced in an atom in the presence of the resonance field differs from the dipole moment induced by the particle in the absence of the resonance field by the amount

$$\mathbf{d}^{NL}(\mathbf{q}, \omega) = \mathbf{d}_{03} C_3^{NL}(\mathbf{q}, \omega - \omega_{30}) = \frac{-\mathbf{d}_{03} V_{32} V_{21} (\mathbf{d}_{10}\mathbf{E}(\mathbf{q} - \mathbf{k}_1 - \mathbf{k}_2, \omega - \omega_1 - \omega_2))}{(\omega - \omega_{30})(\omega - \omega_{30} - \varepsilon_2)(\omega - \omega_{30} - \varepsilon_1 - \varepsilon_2) - |V_{21}|^2(\omega - \omega_{30}) - |V_{32}|^2 \times (\omega - \omega_{30} - \varepsilon_1 - \varepsilon_2)}. \tag{66}$$

Note that the dependence of d^{NL} on the amplitude of the electromagnetic field (48) is much more complicated than in the nonresonance case considered in the previous section; the important thing is the dependence on the field in the denominator of (66). We now give the final expression for the polarization of the matter[23]:

$$\mathbf{P}(\mathbf{q}, \omega) = \varepsilon(\omega) E(\mathbf{q}, \omega) - \frac{\mathbf{d}_{03} V_{32} V_{21} (\mathbf{d}_{10}\mathbf{E}(\mathbf{q} - \mathbf{k}_1 - \mathbf{k}_2, \omega - \omega_1 - \omega_2))\zeta}{(\omega - \omega_{30})(\omega - \omega_{30} - \varepsilon_2)(\omega - \omega_{30} - \varepsilon_1 - \varepsilon_2) - |V_{21}|^2(\omega - \omega_{30}) - |V_{32}|^2(\omega - \omega_{30} - \varepsilon_1 - \varepsilon_2)}. \tag{67}$$

We give the expression for the energy $d\mathscr{E}$ acquired by the particle during the complete time of flight T in the

solid angle $d\Omega$ (in the direction $\mathbf{n}$) in the frequency interval $d\omega$ for a field in exact resonance, $\varepsilon_1=\varepsilon_2=0$. In this case, it is necessary to take into account the widths of the levels[23] and the radiated energy takes the form

$$d\mathcal{E}=\frac{2\pi^3}{9}n_0^2T\omega^4\sqrt{\varepsilon}\,d\omega\,d\Omega\,\zeta^2[ne]^2|d_{03}d_{31}|^2$$
$$\times|e\mathbf{E}_p(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)|^2$$
$$\times\frac{|V_{21}^0|^2|V_{22}^0|^2}{G^2\{(\omega-\omega_{30})^2+\Gamma^2\}}\delta(\omega-\omega_1-\omega_2-\mathbf{vk}(\omega)+\mathbf{vk}_1+\mathbf{vk}_2), \quad (68)$$

where n_0 in the number of atoms per unit volume, $E_p(\mathbf{k})$ is the Fourier transform of the particle's self-field (6), and

$$G=(\omega-\omega_{30})^2-\Omega^2;\ \Omega^2=|V_{21}^0|^2+|V_{32}^0|^2;$$
$$V_{32}^0=-\frac{1}{2\sqrt{3}}d_{32}E_{02};\quad V_{21}^0=-\frac{1}{2\sqrt{3}}d_{21}E_{01};$$
$$\Gamma G=(\omega-\omega_{30})^2(\gamma_1+\gamma_2+\gamma_3)-\gamma_3|V_{21}^0|^2-\gamma_1|V_{32}^0|^2.$$

It is convenient to compare the intensity of the radiation (68) with the intensity of Cherenkov radiation in matter with $\varepsilon-1\sim1$:

$$(d\mathcal{E}/d\mathcal{E}_{\text{Ch}})\sim(n_0a^3)^2(\omega_{aT}/\Delta\omega)^2, \quad (69)$$

where a is the dimension of the atom, ω_{aT} is the atomic frequency, and $\Delta\omega$ is the smallest of the quantities $(\omega-\omega_{30})$, G, γ_1, γ_2, γ_3. It follows from (69) that for a gas ($n_0\approx10^{19}$ cm^{-3}) the intensity of coherent de-excitation in the field will be comparable with the intensity of the ordinary Cherenkov radiation.

Let us now consider the limits of applicability of the obtained expression. First, in considering the polarization of the matter, we ignored the interaction of the a atoms of the matter with each other and the dipole–dipole transfer of excitation to neighboring atoms. Allowance for this circumstance leads to the appearance of large levels widths, which significantly lowers the radiation intensity. Therefore, from the point of view of the radiation intensity the favorable case is still the one in which one can speak of atomic energy levels rather than bands, i.e., in a gas or when one is considering impurity atoms in a solid.

Another restriction is the following circumstance. In considering radiation at frequency $\omega\sim\omega_{30}$, we assumed above that the particle excites the transition $0\to1$, the field excites the transitions $1\to2$ and $2\to3$, and then de-excitation takes place. However, in some cases the opposite process takes place at the same time, namely, excitation by the particle of the transition $0\to3$, excitation by the field of the transitions $3\to2$ and $2\to1$, and de-excitation at frequency $\omega\sim\omega_{10}$. When such processes are taking place simultaneously, there is a nonlinear interaction of the modes with the frequencies ω_{30} and ω_{10}, and the mutual influence of the direct and inverse processes reduces the radiation intensity compared with (68) and (69). (This case is considered in Ref. 54.) Therefore, the cases of interest are those when for some reason the direct process takes place but the reverse process is suppressed. Then the above treatment is valid.

6. COHERENT RADIATION OF IMPURITY ATOMS EXCITED RESONANTLY BY WAKE CHARGE VIBRATIONS

One of the cases when only one of the atomic levels is excited by the particle is the case of resonance between the vibration frequency ω_p of the wake charge and the excitation frequency ω_{10} of the atom. Such a situation can arise in matter with impurity atoms when the vibration frequency of the wake charge is determined by the main matter; then, for example, by varying the density of the main matter one can achieve the equation $\omega_p=\omega_{10}$. Then for the impurity atom the situation considered in the previous section is realized. Indeed, if we study the excitation of the matter by the vibrations of the wake charge, we can ignore the excitation of the transitions $0\to3$, since for them the vibrations of the wake charge are not resonant, and we need take into account only the excitation of the transitions $0\to1$. Therefore, the intensity of the coherent de-excitation of the impurity atoms at the frequency $\omega\sim\omega_{30}$ will be appreciably higher than at the frequency $\omega\sim\omega_{10}$. It is important to note that even if one takes into account the circumstance that the particle excites the transitions $0\to3$ and $0\to1$ about equally, the resonance excitation of the transitions $0\to1$ by the wake charge vibrations ensures predominance of excitation of the transitions $0\to1$. This means that the treatment developed above can be applied directly to the given case.

The only difference which must be taken into account is the replacement of the general expression for the self-field of the particle

$$\mathbf{E}_p(\mathbf{r},t)=\int d^3q\mathbf{E}_p(\mathbf{q})\exp(i\mathbf{qr}-i\mathbf{qv}t); \quad (70)$$
$$\mathbf{E}_p(\mathbf{q})=\frac{ie}{2\pi^2\varepsilon}\frac{\mathbf{v}(\mathbf{qv})\varepsilon-\mathbf{q}}{q^2-(qv)^2\varepsilon} \quad (71)$$

by the field of the wake charge. Since the wake charge density ρ_W is related to the charge density ρ by

$$\rho_W(\mathbf{r},\omega)=\rho(\mathbf{r},\omega)(1/\varepsilon(\omega)-1), \quad (72)$$

the field of the wake charge is

$$\mathbf{E}_W(\mathbf{r},t)=\int d^3q\mathbf{E}_W(\mathbf{q})\exp(i\mathbf{qr}-i\mathbf{qv}t), \quad (73)$$

where

$$\mathbf{E}_W(\mathbf{q})=\frac{ie}{2\pi^2}\frac{\mathbf{q}}{q^2}\frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}. \quad (74)$$

In the special case when $\varepsilon(\omega)$ has the form (24), we can write

$$\mathbf{E}_W(\mathbf{q})=\frac{ie}{2\pi^2}\frac{\mathbf{q}}{q^2}\frac{\omega_p^2}{(\mathbf{qv})^2-\omega_p^2+2i\gamma_p(qv)} \quad (75)$$

and the expressions for the intensity of coherent de-excitation are obtained from (68) by replacing $\mathbf{E}_p(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)$ by $\mathbf{E}_W(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)$:

$$d\mathcal{E}=\frac{e^2}{18\pi}T\omega^4\sqrt{\varepsilon}\,d\omega\,d\Omega\,n_0^2\zeta^2[\mathbf{ne}]^2|d_{03}d_{31}|^2$$
$$\times|V_{21}^0|^2|V_{32}^0|^2\frac{[\mathbf{e},\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2]^2}{(\mathbf{k}(\omega)-\mathbf{k}_1-\mathbf{k}_2)^4}$$
$$\times\frac{\omega_p^4\delta(\omega-\omega_1-\omega_2-\mathbf{vk}(\omega)+\mathbf{vk}_1+\mathbf{vk}_2)}{G^2[(\omega-\omega_{30})^2+\Gamma^2]\{(\omega-\omega_1-\omega_2)^2-\omega_p^2+2i\gamma_p(\omega-\omega_1-\omega_2)\}}. \quad (76)$$

7. COHERENT SCINTILLATOR

In the widely used scintillation counters for particle detection, the incoherent de-excitation of impurity atoms excited by the passage of a particle is used. The processes which take place can be described as follows.

The original particle excites the atoms of the material, and this excitation migrates through the material in, for example, the form of an exciton. Prolonged migration

of excitations occurs because in dense matter the excited atom (or molecule) transmits the excitation energy with overwhelming probability to a neighboring atom (or molecule) and does not radiate it in the form of a photon.

The excitation energy ω_{10} of an impurity atom is taken lower than the excitation energy of the host material but in its transparency region, i.e., $\mathrm{Im}\varepsilon(\omega_{10}) \to 0$. If an impurity atom is in a region in which the matter is excited, the excitation energy is transmitted to the impurity atom. The presence of an energy difference requires the participation of phonons, which carry away the excess energy. The opposite transfer of excitation from the impurity atom to the matter requires the absorption of phonons and is therefore improbable. Transfer of the excitation to the nearest impurity atom can also be ignored because of the large distance between them. To ensure this, the concentration of the impurity atoms is taken low. The only possibility of de-excitation of the impurity atom is through emission of a photon. The emission of phonons in the excitation transfer process has the consequence that the radiation from the different impurity atoms is incoherent, and the process itself takes place slowly. The creation of conditions for coherent de-excitation of the impurity atoms would make it possible not only to increase the radiation intensity but also to decrease the de-excitation time. To obtain a coherent scintillator, one can use the methods discussed above for creating the conditions for coherent de-excitation of atoms of not very dense matter.[55]

As a very simple example, we consider dense matter with impurity atoms, choosing atoms such that the excitation energy of the first excited level of an atom (or molecule) of the host material is equal to the energy E_3 of the third excited level of the impurity atom. Suppose also that in the impurity atom it is possible to have transitions between the levels E_3 and E_1, E_3 and E_0, E_2 and E_1, E_2 and E_0 ($E_3 > E_2 > E_1 > E_0$), but the transitions $3 \to 2$ and $1 \to 0$ are forbidden. The transition $3 \to 0$ is a resonance transition with respect to the atoms of the host material and leads to transfer of excitation to a neighboring atom of the host material; the transition $3 \to 1$ leads to emission of a photon. Since the state E_1 of the impurity atom is orthogonal to the original ground state E_0, the radiation of the impurity atoms at the frequency $\omega_{31} = E_3 - E_1$ will be incoherent. Transitions from the state E_1 to the state E_0 proceed through the states E_2 or E_3, and are therefore improbable.

To ensure coherent radiation of photons by different impurity atoms, we impose on the matter the additional electromagnetic field

$$\begin{aligned}\mathbf{E}(\mathbf{R}, t) = \mathbf{E}_0\{\exp(i\mathbf{k}_0\mathbf{R} - i\omega_0 t) + \text{c.c.}\} \\ + \mathbf{E}_1\{\exp(i\mathbf{k}, \mathbf{R} - i\omega_1 t) + \text{c.c.}\},\end{aligned} \tag{77}$$

choosing the frequencies ω_0 and ω_1 in the form

$$\omega_0 = \omega_{21} \equiv E_2 - E_1; \quad \omega_1 = \omega_{31} \equiv E_3 - E_1. \tag{78}$$

The resonance field E_1 gives rise to the transition $3 \to 1$ of the impurity atom, and the resonance field E_0 gives rise to the transition $1 \to 2$. After this, the impurity atom is in the state 2 and can radiate a photon with frequency $\omega \approx \omega_{20} \equiv E_2 - E_0$.

Thus, after the interaction with the particle and with the fields E_0 and E_1 and the radiation of the photon, the impurity atom returns to the ground state E_0. Therefore, the energy ω and momentum k of the emitted photon have arisen through a change in the momentum of the particle, $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{q}$, and in its energy $E_1 - E_2 \approx \mathbf{q}\cdot\mathbf{v}$ ($q \ll p_1$) and a change in the energy and momentum of the fields E_0 and E_1. Therefore, the energy and momentum conservation laws for the complete process can be written in the form

$$\mathbf{q} + \mathbf{k}_0 - \mathbf{k}_1 = \mathbf{k}; \quad \mathbf{q}\mathbf{v} + \omega_0 - \omega_1 = \omega, \tag{79}$$

from which there follows the relation

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon}}[1 - (\omega_0 - \mathbf{k}_0\mathbf{v} - \omega_1 + \mathbf{k}_1\mathbf{v})/\omega], \tag{80}$$

which is the normal relation for processes of coherent de-excitation of atoms excited by a particle in matter in a field.

To estimate the intensity of the radiation of the impurity atoms, we assume first that the dipole moments of the transition $0 \to 1$ in an atom of the host material and the transition $0 \to 3$ in the impurity atom are equal in magnitude. Bearing in mind that in accordance with the assumption above the excitation energies for these transitions are also equal, one can say that in such a case the impurity atoms participate in the resonance transfer of excitation and the migration of the excitations on an equal footing with the atoms of the host material. But in this case one can say that the emission of a photon by an impurity atom hardly changes the population amplitude $a_3(\mathbf{R}, t)$ of the level E_3 of the impurity atom, since the population is maintained by the resonance transfer of excitation from the neighboring atoms of the host material. Therefore, in writing the equations for the population amplitudes $a_2(\mathbf{R}, t)$ and $a_1(\mathbf{R}, t)$ of the levels E_2 and E_1 of the impurity atom, we can assume that the amplitude $a_3(\mathbf{R}, t)$ is given.

Bearing in mind what we have said, we can obtain from the wave equation equations for the population amplitudes of an impurity atom at the point R by retaining only the resonance terms in the form

$$\left.\begin{aligned} i\partial a_2(\mathbf{R}, t)/\partial t = -\mathbf{d}_{21}\mathbf{E}_0 \exp(i\mathbf{k}_0\mathbf{R})\, a_1(R_1 t); \\ i\partial a_1(\mathbf{R}, t)/\partial t = -\mathbf{d}_{12}\mathbf{E}_0 \exp(-i\mathbf{k}_0\mathbf{R}) a_2(\mathbf{R}, t) \\ -\mathbf{d}_{13}\mathbf{E}_1 \exp(-i\mathbf{k}_1\mathbf{R})\, a_3(\mathbf{R}, t). \end{aligned}\right\} \tag{81}$$

It follows that

$$\begin{aligned}(\partial^2/\partial t^2 + |\mathbf{d}_{21}\mathbf{E}_0|^2)\, a_2(\mathbf{R}, t) \\ = -(\mathbf{d}_{21}\mathbf{E}_0)(\mathbf{d}_{13}\mathbf{E}_1)\exp[-i(\mathbf{k}_0 - \mathbf{k}_1)R]\, a_3(\mathbf{R}, t),\end{aligned} \tag{82}$$

so that

$$a_2(\mathbf{R}, t) = \int d\omega a_2(\mathbf{R}, \omega)\exp(-i\omega t); \tag{83}$$

$$a_2(\mathbf{R}, \omega) = \frac{(\mathbf{d}_{21}\mathbf{E}_0)(\mathbf{d}_{13}\mathbf{E}_1)}{\omega^2 - |\mathbf{d}_{21}\mathbf{E}_0|^2}\, a_3(\mathbf{R}, \omega)\exp\{i(\mathbf{k}_0 - \mathbf{k}_1)\mathbf{R}\}. \tag{84}$$

The existence of populated excited states leads to the occurrence of a dipole moment of the impurity atom. Assuming that the populations of the excited states are low, we can obtain in the region of the frequencies $\omega \approx \omega_{20}$ the Fourier component of the dipole moment of the impurity atom in the form

$$\mathbf{d}(\mathbf{R}, \omega) = \frac{\mathbf{d}_{02}(\mathbf{d}_{21}\mathbf{E}_0)(\mathbf{d}_{13}\mathbf{E}_1)}{(\omega - \omega_{20})^2 - |\mathbf{d}_{21}\mathbf{E}_0|^2}\, a_3(\mathbf{R}, \omega - \omega_{20})\exp\{i(\mathbf{k}_0 - \mathbf{k}_1)\mathbf{R}\}. \tag{85}$$

The dipole moment (85) is the source of the radiation. The energy of such radiation from impurity atoms at the points $\mathbf{R}_a$ in the solid angle $d\Omega$ in the direction $\mathbf{n}$ in the frequency interval $d\omega$ near ω_{20} has the form

$$d\mathcal{E} = \omega^2 \sqrt{\varepsilon} \left| \frac{[\mathbf{n}\mathbf{d}_{02}]\,(\mathbf{d}_{21}\mathbf{E}_0)\,(\mathbf{d}_{13}\mathbf{E})}{(\omega-\omega_{20})^2 - |\,\mathbf{d}_{21}\mathbf{E}_0\,|^2} \times \sum_a \exp\left[\mathrm{i}\,(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}\,(\omega))\,\mathbf{R}_a\right] a_3\,(\mathbf{R}_a,\ \omega-\omega_{20}) \right|^2 d\omega\, d\Omega. \tag{86}$$

Therefore, to find the radiation intensity it remains to determine the excitation amplitude of the level E_3 of the impurity atom, and in accordance with the assumptions made above in the considered special case this is equal to the excitation amplitude of level 1 of a host atom.

The fields E_0 and E_1 are resonant fields only for the impurity atoms and have little influence on the host atoms. Therefore, the excitation of the host atoms occurs through the effect of the field of the particle:

$$\mathbf{E}_p\,(\mathbf{R},\ t) = \int d^3q \mathbf{E}_p\,(\mathbf{q}) \exp\,(\mathrm{i}\mathbf{q}\mathbf{R} - \mathrm{i}\mathbf{q}\mathbf{v}t); \tag{87}$$

$$\mathbf{E}_p\,(\mathbf{q}) = \frac{ie}{2\pi^2\varepsilon} \frac{\mathbf{v}\,(\mathbf{q}\mathbf{v})\,\varepsilon - \mathbf{q}}{q^2 - (\mathbf{q}\mathbf{v})^2\,\varepsilon}. \tag{88}$$

An equation for $a_3(\mathbf{R}, t)$ can be written down as for the population amplitude of level 1 of the host atoms:

$$\mathrm{i}\frac{\partial a_3\,(\mathbf{R}_a,\ t)}{\partial t} = -\,\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{R}_a,\ t) \exp\,(\mathrm{i}\omega_{30}t) + \sum_b V_3\,(\mathbf{R}_a - \mathbf{R}_b)\,a_3\,(\mathbf{R}_b,\ t), \tag{89}$$

where

$$V_3\,(\mathbf{R}) = \int d^3q V_3\,(\mathbf{q}) \exp\,(\mathrm{i}\mathbf{q}\mathbf{R}); \quad \mathbf{R}_{ab} = n_{ab}\,|\,\mathbf{R}_{ab}\,| = \mathbf{R}_a - \mathbf{R}_b \tag{90}$$

is the potential of the resonance dipole–dipole interaction, which at large distances has the form

$$V_3\,(\mathbf{R}_{ab}) = -\,\mathbf{R}_{ab}^{-3}\,\{\mathbf{d}_{30}^a\mathbf{d}_{03}^b - (\mathbf{n}_{ab}\mathbf{d}_{30}^a)\,(\mathbf{n}_{ab}\mathbf{d}_{03}^b)\}. \tag{91}$$

In homogeneous matter, summation over the atoms in (86) can be replaced by integration, after which (89) is reduced by a Fourier transformation to the algebraic equation

$$a_3\,(\mathbf{q},\ \omega) = -\frac{\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{q})}{\omega - n_0\,(2\pi)^3\,V_3\,(q)}\,\delta\,(\omega + \omega_{30} - \mathbf{q}\mathbf{v}), \tag{92}$$

where n_0 is the number of atoms of the material per unit volume, and

$$a_3\,(\mathbf{R},\ \omega) = \int d^3q a_3\,(\mathbf{q},\ \omega) \exp\,(\mathrm{i}\mathbf{q}\mathbf{R}). \tag{93}$$

Substituting (77) in (86), we can obtain the total radiation energy, which contains coherent and incoherent parts. The coherent part of the radiation is due to the average field, and therefore to obtain the coherent part of the radiation it is necessary to average the sum in (86) over the distribution of the impurity atoms. Assuming that they are distributed in the material uniformly with density n_1 atoms per unit volume, we obtain

$$\left\langle \sum_a a_3\,(\mathbf{R}_a,\ \omega-\omega_{20}) \exp\,(\mathrm{i}\Delta\mathbf{k}\mathbf{R}_a) \right\rangle = n_1 \int d^3R a_3\,(\mathbf{R},\ \omega-\omega_{20}) \exp\,(\mathrm{i}\Delta\mathbf{k}\mathbf{R}) \tag{A}$$

or

$$\left\langle \sum_a a_3\,(\mathbf{R}_a,\ \omega-\omega_{20}) \exp\left[\mathrm{i}\,(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}\,(\omega))\,\mathbf{R}_a\right] \right\rangle = n_1\,(2\pi)^3\,a_3\,(\mathbf{k}\,(\omega) - \mathbf{k}_0 + \mathbf{k},\ \omega - \omega_0 + \omega_1). \tag{94}$$

Before we substitute (94) in (86), we lift the restriction associated with the assumption that the dipole moment of the transition $0 \to 1$ in the host atom is equal to that of the transition $0 \to 3$ in the impurity atom, retaining through the assumption that the frequencies of these transitions are equal. Suppose the population amplitude of state 1 of the host atom is $b_1(\mathbf{R}, t)$. Obviously, the equation for $b_1(\mathbf{R}, t)$ is identical to (89), and the equation for $a_3(\mathbf{R}, t)$ has the form

$$\mathrm{i}\frac{\partial a_3\,(\mathbf{R}_a,\ t)}{\partial t} = -\,\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{R}_a,\ t) \exp\,(\mathrm{i}\omega_{30}t) + \sum_b V_3\,(\mathbf{R}_a - \mathbf{R}_b)\,\xi b_1\,(\mathbf{R}_b,\ t), \tag{95}$$

where ξ is the ratio of the moduli of the dipole moments of the transitions $0 \to 1$ in the host atom and $0 \to 3$ in the impurity atom. In homogeneous matter, we can obtain instead of (92)

$$\omega a_3\,(\mathbf{q},\ \omega) = -\,\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{q})\,\delta\,(\omega + \omega_{30} - \mathbf{q}\mathbf{v}) + n_0\,(2\pi)^3\,V\,(\mathbf{q})\,\xi b_1\,(\mathbf{q},\ \omega) \tag{96}$$

or, replacing $b_1(\mathbf{q}, \omega)$ by the expression (92),

$$a_3\,(\mathbf{q},\ \omega) = -\,\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{q})\,\frac{1}{\omega} \times \left\{ \frac{\omega - (1-\xi)\,n_0\,(2\pi)^3\,V\,(\mathbf{q})}{\omega - n_0\,(2\pi)^3\,V\,(\mathbf{q})} \right\} \delta\,(\omega + \omega_{30} - \mathbf{q}\mathbf{v}). \tag{97}$$

Substitution of this expression in (94) and of the expression (94) in (86) gives for the energy of the coherent de-excitation of the impurity atoms[55]

$$d\mathcal{E}^{\mathrm{coh}} = n_1^2\omega^4 \sqrt{\varepsilon}\,(2\pi)^5\,[\mathbf{n}\mathbf{d}_{02}]^2\,\{(\omega-\omega_{20})^2 + \gamma_2^2\}^{-1} \times [\mathbf{d}_{30}\mathbf{E}\,(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}\,(\omega))]^2 \left| \frac{(\mathbf{d}_{21}\mathbf{E}_0)\,(\mathbf{d}_{13}\mathbf{E}_1)}{(\omega-\omega_{20})^2 - |\,\mathbf{d}_{21}\mathbf{E}_0\,|^2} \right|^2 \times \left| \frac{\omega - \omega_{20} - (1-\xi)\,n_0\,(2\pi)^3\,V_3\,(k_0 - k_1 - k\,(\omega))}{\omega - \omega_{20} - n_0\,(2\pi)^3\,V_3\,(k_0 - k_1 - k\,(\omega))} \right|^2 \times T\delta\,(\omega - \omega_0 + \omega_1 - \mathbf{v}\mathbf{k}\,(\omega) + \mathbf{v}\mathbf{k}_0 - \mathbf{v}\mathbf{k}_1). \tag{98}$$

It is readily seen that in dense materials, when $\omega - \omega_{20}$ already exceeds the width of the line ω_{20} of the impurity atom but still remains small compared with $n_0(2\pi)^3 \times V(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k})$, i.e., for $\gamma_2 \ll \omega - \omega_{20} \ll n_0(2\pi)^3 V(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}(\omega))$, Eq. (98) simplifies considerably:

$$\frac{1}{T}\frac{d^2\mathcal{E}}{d\omega\,d\Omega} = \frac{n_1^2\,\omega^2\,\sqrt{\varepsilon}\,|\,[\mathbf{n}\mathbf{d}_{02}]\,(\mathbf{d}_{21}\mathbf{E}_0)\,(\mathbf{d}_{13}\mathbf{E}_1)\,(\mathbf{d}_{30}\mathbf{E}_p\,(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}))\,|^2}{\{(\omega-\omega_{20})^2 + \gamma_2^2\}\,\{(\omega-\omega_{20})^2 - |\,\mathbf{d}_{21}\mathbf{E}_0\,|^2\}} \times (2\pi)^5\delta\,(\omega - \omega_0 + \omega_1 - \mathbf{v}\,(\mathbf{k} - \mathbf{k}_0 + \mathbf{k}_1)). \tag{99}$$

In this case, the coherent de-excitation of the impurity atoms excited by a fast particle occurs with the greatest intensity near the frequencies

$$\omega \approx \omega_{20};\ \ \omega \approx \omega_{20} \pm |\,d_{21}E_0\,|. \tag{100}$$

Thus, we have shown above that the creation of a coherent scintillator is in principle possible. Naturally, the above model of a coherent scintillator is not the only one possible and other modifications may be proposed. It would appear that one thing must be common to all models–a specially chosen electromagnetic field must participate in the transfer of excitation from the host material to the impurity atoms.

8. COHERENT COMBINATION RADIATION OF PHOTONS BY WAKE CHARGE IN RESONANTLY EXCITED MATTER

The participation of the wake charge of a particle in the generation of coherent electromagnetic waves is made difficult by two circumstances. First, the vibration frequencies ω_p of the wake charge correspond to the zeros of the permittivity, $\varepsilon(\omega_p)=0$, and at such frequencies transverse electromagnetic waves do not prop-

agate. Second, the vibrations of the wake charge are longitudinal, and this also hinders the generation of transverse waves.

To eliminate the first difficulty, it is necessary to consider a substance in which the frequency is not conserved, i.e., one with time-dependent dielectric properties, in particular, excited matter. The second difficulty can be eliminated if one takes matter in which the direction of propagation of the waves is not conserved, i.e., spatially inhomogeneous matter. In nonstationary and inhomogeneous matter the transformation of longitudinal waves into transverse electromagnetic waves with a frequency shift becomes possible. The resulting radiation is a special case of transition (in a wide meaning of the word) radiation in an inhomogeneous and nonstationary medium.

We consider matter to which a resonance pumping field is applied:

$$\mathbf{E}_0(r,t)=\int\int d^3k d\omega \mathbf{E}_0(\mathbf{k},\omega)\exp(i\mathbf{k}_0\mathbf{r}-i\omega_0 t). \tag{101}$$

Suppose that twice the frequency of the pumping field, $2\omega_0$, is close to the transition frequency ω_{10} of an atom from the ground state 0 to the excited state 1:

$$|2\omega_0-\omega_{10}|\ll\omega_{10}. \tag{102}$$

The polarization of the matter for such a pumping field has been calculated exactly in a number of papers.[56] For the Fourier transform

$$\mathbf{P}(q,\omega)=(2\pi)^{-4}\int\int d^2r\,dt\mathbf{P}(\mathbf{r},t)\exp(-i\mathbf{qr}+i\omega t) \tag{103}$$

of the polarization, we have the relation

$$\mathbf{P}_i(\mathbf{q},\omega)=W_0\chi_{ij}^{(0\cdot 0)}E_j(\mathbf{q},\omega)+W_1\chi_{ij}^{(1\cdot 1)}E_j(\mathbf{q},\omega)$$
$$+V\chi_{ij}^{01}E_j(\mathbf{q}-2\mathbf{k}_0,\omega-2\omega_0)+V^*\chi_{ij}^{(10)}E_j(\mathbf{q}+2\mathbf{k}_{01},\omega+2\omega_0), \tag{104}$$

where we have used the notation

$$W_{0(1)}=\frac{1}{2}(1\pm|\Delta|/2\Omega);\ \Delta=\omega_{10}-\omega_{20}-V_{11}^{(2)}-V_{00}^{(2)}; \tag{105}$$

$$V_{mm}^{(2)}=\sum_n|\mathbf{d}_{mn}\mathbf{E}_0|^2\left\{\frac{1}{\omega_{n0}-\omega_0}+\frac{1}{\omega_{n0}+\omega_0}\right\};\ \Omega=\sqrt{\frac{1}{4}\Delta^2+|Q|^2}; \tag{106}$$

$$Q=\sum_n\frac{(\mathbf{d}_{1n}\mathbf{E}_0)(\mathbf{d}_{n0}\mathbf{E}_1)}{\omega_{n0}-\omega};\ V=\frac{1}{2}\frac{Q}{|Q|}\sqrt{1-\left(\frac{\Delta^2}{4\Omega^2}\right)}; \tag{107}$$

W_0 and W_1 are the populations of the ground and excited states, and $\mathbf{d}_{ns}$ is the transition matrix element of the atom between the states n and s. The polarizabilities χ_{ij}^{ns} are determined by the formulas

$$\chi_{ij}^{ns}=n_0\sum_k\left\{\frac{d_{nk}^i d_{ks}^j}{\omega_{k0}-\omega+\Omega}+\frac{d_{nk}^j d_{ks}^i}{\omega_{k0}+\omega+\Omega}\right\}. \tag{108}$$

In the special case when the ground and excited states have angular momentum $l=0$ and projection $m=0$,

$$\chi_{ij}^{ns}=\delta_{ij}\chi^{(ns)}. \tag{109}$$

We now find the self-field of a charge Ze moving uniformly with velocity v in pumped matter whose polarization is described by (103)–(109). Substituting the explicit expression for the polarization in Maxwell's equations, we obtain for the considered case the equation

$$\{q^2\delta_{ij}-q_iq_j-\omega^2\varepsilon_i(\omega)\}E_j(\mathbf{q},\omega)=\frac{iZe}{2\pi^2}\omega v_i\delta(\omega-\mathbf{qv})$$
$$+\omega^2V\chi_{ij}^{(01)}E_j(\mathbf{q}-2\mathbf{k}_0,\omega-2\omega_0)+\omega^2V^*\chi_{ij}^{(10)}E_j(\mathbf{q}+2\mathbf{k}_0,\omega+2\omega_0). \tag{110}$$

Here, we have taken into account the circumstance that the pumping field contributes to the permittivity $\varepsilon_{ij}(\omega)$ of the pumped matter, which differs from the permittivity $\varepsilon_0(\omega)\delta_{ij}$ of the unexcited matter:

$$\varepsilon_{ij}(\omega)=\varepsilon_0(\omega)\delta_{ij}+4\pi W_0\chi_{ij}^{00}+4\pi W_1\chi_{ij}^{11}. \tag{111}$$

To avoid introducing irrelevant complications, we consider the special case of spherically symmetric states 0 and 1 of the atom ($l=0, m=0$), when $\chi_{ij}^{ns}=\delta_{ij}\chi^{(ns)}$.

Equation (110) couples three Fourier components, the fields with frequencies ω, $\omega-2\omega_0$, and $\omega+2\omega_0$. The nonlinear susceptibilities are small (of the order of the ratio of the pumping field to the atomic field), and the contribution of the terms with the combination frequencies $\omega\pm2\omega_0$ is small. Therefore, we can usually regard the terms proportional to χ_{ij} as small corrections.

However, in some cases one of these terms is not small. This will be the case when

$$\omega-2\omega_0=\omega_p. \tag{112}$$

The field at the combination frequency contains in the denominator the small quantity $\varepsilon(\omega_p)$, which compensates the smallness of the nonlinear susceptibility. The second term, which is proportional to the field with the frequency $\omega+2\omega_0$, is small and can be omitted. Thus, if the condition (112) is satisfied, Eq. (110) couples the two Fourier components of the field with frequencies ω and $\omega-2\omega_0$ by the system of the two equations

$$\left.\begin{aligned}&(q^2\delta_{ij}-q_iq_j-\omega^2\varepsilon(\omega)\delta_{ij})E_j(\mathbf{q},\omega)=\frac{iZe}{2\pi^2}\omega v_i\delta(\omega-\mathbf{qv})\\&+V\omega^2\chi_{ij}^{(01)}(\omega)E_j(\mathbf{q}-2\mathbf{k}_0,\omega-2\omega_0);\\&\{(\mathbf{q}-2\mathbf{k}_0)^2\delta_{ij}-(q_i-2k_{0i})(q_j-2k_{0j})\\&-(\omega-2\omega_0)^2\varepsilon(\omega-2\omega_0)\delta_{ij}\}E_j(\mathbf{q}-2\mathbf{k}_0,\omega-2\omega_0)\\&=\frac{iZe}{2\pi^2}(\omega-2\omega_0)v_i\delta(\omega-2\omega_0-\mathbf{v}(\mathbf{q}-2\mathbf{k}_0))\\&+V^*(\omega-2\omega_0)^2\chi_{ij}^{(10)}(\omega-2\omega_0)E_j(\mathbf{q},\omega).\end{aligned}\right\} \tag{113}$$

The solution of this system of equations has the form

$$E_i(\mathbf{q},\omega)=\{\delta_{ij}+(1+\varkappa)^{-1}\Phi_{ij}\}E_{pj}(\mathbf{q},\omega)$$
$$+\{\delta_{ij}+(1+\varkappa)^{-1}\Phi_{ij}\}\varphi_{jk}E_{pk}(\mathbf{q}-2\mathbf{k}_0,\omega-2\omega_0), \tag{114}$$

where $\varkappa=-\Phi_{ii}$

$$E_{pi}(\mathbf{q},\omega)=\frac{iZe}{2\pi^2\varepsilon}\frac{\omega v_i\varepsilon-q_i}{q^2-\omega^2\varepsilon}\delta(\omega-\mathbf{qv}) \tag{115}$$

is the self-field of the uniformly moving charge in the homogeneous and stationary medium with permittivity that takes into account the influence of the pumping, and

$$\varphi_{jk}=4\pi\chi^{01}(\omega)\frac{1}{q^2-\omega\varepsilon(\omega)}(\omega^2\delta_{jk}-q_jq_k\varepsilon^{-1}(\omega)); \tag{116}$$

$$\Phi_{ij}=-(4\pi^2|V|^2$$
$$\times\frac{\chi^{(10)}(\omega-2\omega_0)\chi^{(01)}(\omega)(q_s-2k_{0s})(q_i-2k_{0i})(\omega^2\delta_{sj}-\varepsilon^{-1}q_sq_j)}{(\mathbf{q}-2\mathbf{k}_0)^2\varepsilon(\omega-2\omega_0)\{q^2-\omega^2\varepsilon(\omega)\}}. \tag{117}$$

When the region of considered frequencies is determined, not by (112), but by the equation

$$\omega+2\omega_0=\omega_p, \tag{118}$$

the treatment is similar and the results can be obtained from (114)–(117) by means of the substitutions $\omega\to-\omega_0$, $\mathbf{k}_0\to-\mathbf{k}_0$, $\chi^{(01)}(\omega)\to\chi^{(10)}(\omega)$.

We now analyze the expression (114) obtained for the field $E(\mathbf{q},\omega)$, which consists of two terms. The first of these terms is proportional to the field of a uniformly moving charge in the medium with permittivity changed

by the pumping. If at the same time $v^2\varepsilon > 1$, this term describes the Cherenkov radiation adjusted in the field of the pumping wave. But if $v^2\varepsilon < 1$, then the first term does not describe radiation at all.

In accordance with the condition (112), the second term contains the field of the uniformly moving charge at the frequency ω_p, i.e., the field of the wake charge. Indeed, in the limit $\varepsilon(\omega_p) \to 0$ we can obtain from (115) for $\mathbf{E}(\mathbf{q}-2\mathbf{k}_{01},\ \omega-2\omega_0)$ the wake-charge field:

$$\mathbf{E}_W(\mathbf{q}-2\mathbf{k}_0,\ \omega-2\omega_0) = -\frac{iZe}{2\pi^2}\frac{\mathbf{q}-2\mathbf{k}_0}{(\mathbf{q}-2\mathbf{k}_0)^2}\frac{1}{(\omega-\omega_p)\,\partial\varepsilon/\partial\omega_p}. \qquad (119)$$

The field at large distances $\mathbf{E}(\mathbf{R}, \omega)$ can be found by using (5). In the case $v^2\varepsilon < 1$, the field at large distances is

$$E_i(\mathbf{R},\ \omega) = -\frac{\exp(i\omega\sqrt{\varepsilon}R)}{R}\frac{4\pi i Ze\chi^{(01)}(\omega)}{(\partial\varepsilon/\partial\omega_p)}\frac{\omega^2\delta_{is}-k_i(\omega)\,k_s(\omega)\,\varepsilon^{-1}(\omega)}{\omega-2\omega_0-\omega_p}$$
$$\times(\delta_{ij}-\psi_{ij}(\psi_{ss})^{-1})\frac{k_s(\omega)-2k_{0s}}{(\mathbf{k}(\omega)-2\mathbf{k}_0)^2}\delta(\omega-2\omega_0-\mathbf{vk}(\omega)+2\mathbf{k}_0\mathbf{v}), \qquad (120)$$

where $\mathbf{k}(\omega)=\mathbf{n}\omega\sqrt{\varepsilon(\omega)}$; and

$$\psi_{ij} = (k_i(\omega)-2k_{0i})(k_s(\omega)-2k_{0s})[\omega^2\delta_{js}-k_j(\omega)\,k_s(\omega)\,\varepsilon(\omega)^{-1}]. \qquad (121)$$

The energy radiated in the frequency interval $d\omega$ in the solid angle $d\Omega$ is[58]

$$d\mathscr{E} = \frac{T}{2\pi}\frac{(Ze)^2}{v}|Q|^2\delta(\omega-2\omega_0-\mathbf{vk}(\omega)+2\mathbf{vk}_0), \qquad (122)$$

where

$$Q_i = 4\pi^2\chi^{(01)}(\omega)(\delta_{ij}-\psi_{ij}(\psi_{ss})^{-1})$$
$$\times\frac{(k_n(\omega)-2k_{0n})[\omega^2\delta_{jn}-k_j(\omega)\,k_n(\omega)\,\varepsilon^{-1}(\omega)]}{(\mathbf{k}(\omega)-2\mathbf{k}_0)^2(\omega-2\omega_0-\omega_p)(\partial\varepsilon(\omega)/\partial\omega_p)}. \qquad (123)$$

The radiation is due to the coherent combination scattering of the wake-charge field in the inhomogeneous and nonstationary matter with transformation into a transverse wave. The ratio of the intensity of such radiation to the intensity of Cherenkov radiation in a medium with $(\varepsilon-1)\sim 1$ is in order of magnitude

$$(d\mathscr{E}/d\mathscr{E}_{Ch}) \sim n_0 a^3(\omega_p/\gamma_p)^2,$$

where a is a scale of the atomic order, and n_0 is the number of atoms per unit volume. For long-wavelength longitudinal electric oscillations, $\omega_p/\gamma_p \sim \omega^4$ in accordance with the estimates of Ref. 41. In this case, the intensity of the radiation is comparable with the intensity of Cherenkov radiation for $n_0 \approx 10^{20}$ cm^{-3}. This radiation can be used for the experimental measurement of the frequencies of the longitudinal vibrations [the zeros of $\varepsilon(\omega)$] of the investigated material.

9. COHERENT DEPOLARIZATION OF A LIGHT WAVE IN THE TRACK OF A FAST PARTICLE

We consider the propagation in matter of an electromagnetic wave that is linearly polarized along the z axis:

$$\mathbf{E}(\mathbf{R},\ t) = \mathbf{E}_0\cos(\mathbf{k}_0\mathbf{R}-\omega_0 t). \qquad (124)$$

The polarization of such a wave is conserved because of the dipole selection rules, which allow in an atom or a molecule under the influence of such a wave only transitions without change in the projection of the angular momentum onto the z axis. The excited state of an atom (or molecule) resulting from the absorption of a photon in the wave preserves information about the polarization of the absorbed photon and emits a photon of the same polarization. However, an additional field, for example, the field a fast charged particle, can influence the atom excited by the wave and lead to transitions in which the projection of the angular momentum changes, so that photons with the opposite polarization can be emitted.

We consider the case when the frequency ω_0 of the plane wave is equal to the energy difference of two excited states of the atom, $\omega_0 \approx E_2 - E_1$, the first excited state E_1 having angular momentum $l=1$, while the ground state and second excited state have $l=0$. Under normal conditions, the states E_1 and E_2 are not populated and the wave interacts with the atoms (or molecules) weakly, propagating in the matter without absorption and depolarization. The field of a fast charged particle moving through the matter is determined by Eq. (4). It excites an atom (or molecule), and the state E_1 becomes populated. The plane wave interacts resonantly with the electrons in the state E_1, mixing the populations of the levels E_1 and E_2. Important here is the circumstance that from the state E_2, $l=0$, $m=0$ one can have not only a transition to the state E_1, $l=1$, $m=0$ under the influence of the resonance wave field but also the spontaneous emission of a photon with different polarization as a result of transition to the states E_1, $l=1$, $m=\pm 1$. It is this process that is the source of the depolarized photons.

The photons depolarized at different atoms will be added coherently at the point of observation only when neither momentum nor energy is transferred to the matter in the depolarization process. After all the interactions with the wave and the particle and emission of the depolarized photon, the atom must return to the original ground state. After emission of the depolarized photon, the atom must go over from the state E_1, $l=1$, $m=\pm 1$ to the ground state under the influence of the particle's field (4).

We denote the population amplitudes of the states $(E_2, 0, 0)$, $(E_1, 1, 0)$, and $(E_1, 1, \pm 1)$, respectively, by $C_2(\mathbf{R}, t)$, $C_1^0(\mathbf{R}, t)$, and $C_1^{\pm}(\mathbf{R}, t)$, where $\mathbf{R}$ is the radius vector of the atom (or molecule). Introducing the notation

$$V_{21} = \mathbf{d}_{21}\mathbf{E}_0(\mathbf{R},\ t);\ V_{10}^{0(\pm1)} = \mathbf{d}_{10}^{0(\pm1)}\mathbf{E}_1(\mathbf{R},\ t);$$
$$\mathbf{d}_{10}^{0(\pm1)} = (E_1,\ 0,\ 0(\pm1)\,|\,\mathbf{d}\,|\,E_0,\ 0,\ 0)$$

[$\mathbf{d}$ is the operator of the dipole moment of the atom (or molecule)], we can obtain a system of equations for the Fourier components of the population amplitudes. In the resonance approximation for the wave field, this system of equations has the form

$$\left.\begin{aligned}
&\omega C_2(\mathbf{R},\ \omega) = \eta C_1^0(\mathbf{R}_1\omega+\Delta)\exp(i\mathbf{k}_0\mathbf{R});\\
&\omega C_1^0(\mathbf{R},\ \omega) = \eta^* C_2(\mathbf{R},\ \omega-\Delta)\exp(-i\mathbf{k}_0\mathbf{R})\\
&+V_{10}^0(\mathbf{R},\ \omega+\omega_{10});\\
&\omega C_1^{\pm}(\mathbf{R},\ \omega) = V_{10}^{\pm}(\mathbf{R},\ \omega+\omega_{10}),
\end{aligned}\right\} \qquad (125)$$

where $2\eta = \mathbf{d}_{21}\cdot\mathbf{E}_0$, $\Delta = \omega_{21}-\omega_0$.

The solution of the system (125) has the form

$$\left.\begin{aligned}
&C_2(\mathbf{R},\ \omega) = \eta\frac{V_{10}^0(\mathbf{R},\ \omega+\omega_{10}+\Delta)}{(\omega+\Delta/2)^2-\Omega^2}\exp(i\mathbf{k}_0\mathbf{R});\\
&C_1^0(\mathbf{R},\ \omega) = \frac{\omega-\Delta}{(\omega+\Delta/2)^2-\Omega^2}V_{10}^0(\mathbf{R},\ \omega+\omega_{10});\\
&C_1^{\pm}(\mathbf{R},\ \omega) = \frac{1}{\omega}V_{10}^{\pm}(\mathbf{R},\ \omega+\omega_{10}).
\end{aligned}\right\} \qquad (126)$$

In the absence of a particle, the polarization of the matter is directed along the field of the wave:

$$\mathbf{P}^0(\mathbf{R},\ \omega)=\frac{\varepsilon(\omega)-1}{4\pi}\mathbf{E}_0(\mathbf{R},\ \omega). \tag{127}$$

Depolarization of the wave in the particle's track means that under the influence of the particle additional polarization arises:

$$\mathbf{P}^{\pm}(\mathbf{R},\ \omega)=n_0\int\int d\omega_1\,d\omega_2\{\mathbf{d}_{21}^{\pm}C_2(\mathbf{R},\ \omega_2)\,C_1^{\pm *}(\mathbf{R},\ \omega_2)$$
$$+\mathbf{d}_{12}^{\pm}C_2^{*}(\mathbf{R},\ \omega_2)\,C_1^{\pm}(\mathbf{R},\ \omega_1)\}\,\delta(\omega_{21}+\omega_2-\omega_1-\omega). \tag{128}$$

Substitution of (126) in (128) gives

$$\mathbf{P}^{\pm}(\mathbf{R},\ \omega)=n_0\pi\mathbf{d}_{21}^{\pm}\exp(i\mathbf{k}_0\mathbf{R})\frac{V_{10}^{0}(\mathbf{R},\ \omega_{10}-\omega_0+\omega)\,V_{10}^{\pm *}(\mathbf{R},\ \omega_{10})}{(\omega_{21}-\omega-\Delta/2)^2-\Omega^2}$$
$$+n_0\pi\mathbf{d}_{12}^{\pm}\exp(-i\mathbf{k}_0\mathbf{R})\frac{V_{10}^{0*}(\mathbf{R},\ \omega_{10}-\omega_0-\omega)\,V_{10}^{\pm}(\mathbf{R},\ \omega_{10})}{(\omega_{21}+\omega-\Delta/2)^2-\Omega^2}. \tag{129}$$

Writing the Maxwell equation

$$\operatorname{curl}\mathbf{H}(\mathbf{R},\ \omega)=4\pi\mathbf{j}(\mathbf{R},\ \omega)-i\omega\varepsilon(\omega)\,\mathbf{E}(\mathbf{R},\ \omega)-i\omega\mathbf{P}^{\pm}(\mathbf{R},\ \omega)\,4\pi,$$

we can find the field of the depolarized waves as the radiation field produced by the additional polarization $\mathbf{P}^{\pm}(\mathbf{R},\omega)$. Assuming $P^{\pm}\ll P_0$, in finding $\mathbf{P}^{\pm}$ we can ignore the influence of the depolarization on the wave field. Then $-i\omega\mathbf{P}^{\pm}(\mathbf{R},\omega)$ plays the part of the current density which generates the depolarized component of the field. The energy of the depolarized component of the field propagating in the direction $\mathbf{n}$ in the solid angle $d\Omega$ in the frequency interval $d\omega$ over a long interaction time T has the form

$$d\mathcal{E}=T2\pi^5n_0^2\omega^4\sqrt{\varepsilon}\,v^2\,|\,(\mathbf{d}_{10}^{0})_i\,(\mathbf{d}_{01}^{\pm})_j\,\sigma_{ij}\,|^2$$
$$\times\frac{|\,\mathbf{d}_{12}\mathbf{E}_0\,|^2\,|\,\mathbf{e}^{*}\mathbf{d}_{12}^{\pm}\,|^2}{|\,(\omega_{21}-\omega-\Delta/2)^2-\Omega^2\,|^2}\,\delta(\omega-\omega_0-\mathbf{vk}(\omega)+\mathbf{vk}_0)\,d\omega\,d\Omega, \tag{130}$$

where

$$\sigma_{ij}=\frac{v_iv_j}{v^2}A_1+\frac{v_iq_j-q_iv_j}{qv}A_2+\frac{q_iq_j}{q^2}A_3;\ \mathbf{q}\equiv\mathbf{k}(\omega)-\mathbf{k}_0-\frac{\mathbf{v}}{v}(\omega-\omega_0);$$
$$A_1=\Lambda\left\{\frac{e}{\pi}\left(1-\frac{1}{v^2\varepsilon}\right)\frac{\omega_{10}}{s}\right\};\quad A_2=\Lambda\,\frac{e^2\omega_{10}}{\pi^2sv\varepsilon}\left(1-\frac{1}{v^2\varepsilon}\right);$$
$$A_3=\frac{1}{2}\left(\frac{e}{\pi v\varepsilon}\right)^2\left\{K_0(sa)-\frac{1}{2}-\frac{\xi}{2}\Lambda\right\};\quad \xi=\left(\frac{q}{s}\right);$$
$$s^2=\omega_{10}^2(v^{-2}-\varepsilon);\quad \Lambda=\frac{1}{\sqrt{\xi^2+4}}\ln\left(\frac{\sqrt{\xi^2+4}+4}{\sqrt{\xi^2+4}-4}\right);$$

a is a distance of the order of the atomic scale.

It follows from (130) that the frequency of the depolarized photons can change, but the greatest probability of depolarization in the case of exact resonance for the field ($\omega_0=\omega_{21}$) corresponds to frequencies $\omega=\omega_0\pm\Omega$ of the depolarized photons, for which it is necessary to take into account the finite energy width of the excited states. As in all processes of coherent de-excitation of atoms of matter excited by a particle, the emission angle of the depolarized photon is strictly related to its frequency:

$$\cos\vartheta=\frac{1}{v\sqrt{\varepsilon}}\left(1-\frac{\omega_0-\mathbf{vk}_0}{\omega}\right). \tag{131}$$

It follows from (132) in particular that the process of coherent depolarization has a threshold with respect to the energy of the particle at a given frequency of the depolarized photon. If coherent depolarization is to occur, the velocity v of the particle must be greater than

$$v_{\text{thr}}=(\omega\sqrt{\varepsilon}-k_0\cos\vartheta_0)/(\omega-\omega_0). \tag{132}$$

CONCLUSIONS

The processes considered above have a common feature–the coherent de-excitation of different atoms excited by the combined influence of a fast particle and the pumping field. The study of only coherent processes imposes similar conditions on the kinematics of the processes. It follows from the conservation laws that for such processes the photon emission angle is strictly related to its frequency forgiven characteristics of the fast particle and the pumping field. A change in the characteristics of the pumping field can significantly change the conditions of emission of the photon. There is an energy threshold of the process, which is possible at high particle velocities but impossible at low ones.

Although the considered processes have similar kinematics, their dynamics is different, and they may have different intensities. The intensity of the process can be raised by using resonance pumping fields, levels with small width, and the wake-charge field of the fast particle. In this case, the intensity is of the order of, or may exceed, the Cherenkov intensity. The analogy with Cherenkov radiation is clear, since these processes of coherent de-excitation can be regarded as a generalization of Cherenkov radiation to the case of matter excited by a field.

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