Quasiclassical theory of rapid rotation

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Some aspects of the quasiclassical theory of nuclear rotation are considered. By a generalization of Kirzhnits's method, it is possible to study the smooth characteristics of a rotating nucleus, in particular the changes in the density distribution and the shape at high spins. Shell effects can also be taken into account by direct summation using Poisson's formula.

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INTRODUCTION

High-spin states are a new field of study in nuclear structure. Particularly interesting are the evolution of the shape with increasing angular momentum, the behavior of the smooth part of the deformation energy and the role of shell fluctuations, and the influence of rotation on the average field. As in other branches of nuclear physics, the quasiclassical approach adequately reflects the physical essence of these phenomena. The shell-correction method provides a typical example of a quasiclassical treatment in which the physical characteristics are divided into averaged and fluctuating quantities.

In recent years, there have been several studies on the quasiclassical theory of rotation. Brack and Jennings[1] studied the smooth part of the rotation energy in the framework of a generalized Thomas–Fermi method, and many aspects of the shell fluctuations were studied in Ref. 2 by the method of summation of classical trajectories. Some problems in the quasiclassical theory of rotation, including an analysis of the smooth part of the deformation energy and the angular momentum, and also the shell corrections to the angular momentum, are analyzed in Refs. 3 and 4. The main results of these investigations are included in the present review.

To describe the averaged part of the energy and the angular momentum, we generalize Kirzhnits's method,[4] which was originally developed to calculate the density in atomic physics. The smooth parts can be represented in the form of expansions in the quasiclassical parameter $\xi = \hbar/(P_F L) \ll 1$, where L is the characteristic distance over which the potential varies appreciably. In the zeroth order in ξ, we obtain the results of the Thomas–Fermi method; the following order (ξ^2) leads to quantum effects such as Landau diamagnetism and Pauli paramagnetism. These corrections are important for the correct description of the density in the surface region of a nucleus.

By means of a variational method for the density functional, we have studied in detail the effects of the redistribution of the density as a result of rotation. It is found that for nuclei with $A > 100$ the assumption of the drop model with additivity of the drop energy and the rotational energy[5] is well satisfied. However, for light nuclei ($40 \leq A \leq 80$), the effects of the redistribution of the density that are not included in the drop model begin to have an influence. Because of the compressibility of nuclear matter, the density at the center of the nucleus decreases, and the surface layer becomes radially inhomogeneous.

To consider the shell correction to the total angular momentum of a rotating nucleus, we generalize the method of Kirzhnits *et al.*[6] to the problem of the rotation of a spherical nucleus, i.e., to the problem of the alignment of the angular momenta of the nucleons. We use Poisson's formula to calculate the sum over the principal quantum number, and we carry out quasiclassical quantization. We have shown that this procedure leads to the exact quantum-mechanical result, and one can see clearly the origin of the smooth part and the shell fluctuation associated with level crossing.

1. QUASICLASSICAL HARTREE–FOCK APPROXIMATION

To construct our quasiclassical theory of the rotation of nuclei, we use Kirzhnits's method,[4] by means of which one can obtain a consistent quasiclassical approximation of the Hartree–Fock method. We give here the main relations of this method without allowance for rotation.

In Kirzhnits's approach, a central part is played by Wigner's quantum distribution function, which can be represented in the form

$$f(\mathbf{r}, \mathbf{p}) = \exp(-i\hat{\mathbf{p}}\mathbf{r})\,\hat{\rho}\,\exp(i\hat{\mathbf{p}}\mathbf{r}), \quad \hbar = 1, \tag{1}$$

where p is the classical momentum, $\hat{\mathbf{p}}$ is the momentum operator,

$$\hat{\rho} = \Theta(\varepsilon_F - \hat{h}) \tag{2}$$

is the density operator defined in accordance with the Hartree–Fock method, and ε_F is accordingly the Fermi energy and $\hat{h}$ the single-particle self-consistent Hamiltonian:

$$\hat{h} = \frac{\hat{p}^2}{2\mu} + \hat{V}_{HF}, \quad \hat{V}_{HF}(1) = \mathrm{tr}_2\,(\hat{\rho}^{(2)}\hat{V}(12)(1 - \hat{P}^{12})). \tag{3}$$

Knowing the distribution function $f(\mathbf{r},\mathbf{p})$, we find the ground-state expectation value of the single-particle operator $\hat{a} = \hat{a}(\mathbf{r}, \hat{\mathbf{p}})$, which is

$$\langle \hat{a} \rangle = \mathrm{tr}\,(\hat{\rho}\,\hat{a}) = \int d^3r \int \frac{d^3p}{(2\pi)^3}\,\hat{a}\,(\mathbf{r}, \mathbf{p}, \hat{\mathbf{p}})\,f(\mathbf{r}, \mathbf{p}). \tag{4}$$

We can also calculate directly the mixed density

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_V \Theta(\varepsilon_F - \varepsilon_V)\,\varphi_V^*(\mathbf{r})\,\varphi_V(\mathbf{r}')$$
$$= 2\int \frac{d^3p}{(2\pi)^3} \exp(i\mathbf{p}\,|\,\mathbf{r} - \mathbf{r}'\,|)\,f(\mathbf{r}, \mathbf{p}), \tag{5}$$

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where the factor 2 takes into account the spin and we consider only one species of particle. In particular, for the density we obtain, setting $r=r'$ in (5),

$$\rho(\mathbf{r})=\rho(\mathbf{r},\ \mathbf{r}')\,|_{\mathbf{r}=\mathbf{r}'}=2\int\frac{d^3p}{(2\pi)^3}f(\mathbf{r},\ \mathbf{p}). \tag{6}$$

The quasiclassical approximation is introduced by expanding the function $f(\mathbf{r},\mathbf{p})$ with respect to the parameter ξ (see the Introduction). In accordance with (1), it is necessary to know the action of the operator function $\hat{\rho}=\theta(\varepsilon_F-\hat{h})$ on the plane wave $\exp(i\mathbf{p}\cdot\mathbf{r})$. It is expedient to represent the density operator in the form

$$\hat{\rho}=\Theta(p_0^2-\hat{p}^2), \tag{7}$$

where

$$p_0^2=2\mu(\varepsilon_F-\hat{V}_{HF}) \tag{8}$$

is the square of the local Fermi momentum. Since the potential $\hat{V}_{HF}$ itself depends functionally on $\hat{p}_0$ or, equivalently, on the densities $\rho(r)$ and $\rho(r,r')$, the relation (8) is an integral equation for the self-consistent calculation of p_0.

To determine the action of $\theta(p_0^2-\hat{p}^2)$ on $\exp(i\mathbf{p}\cdot\mathbf{r})$, we shall use an operator Taylor expansion applied to an eigenfunction of the operator $\hat{a}$ (Ref. 4):

$$\begin{aligned}\varphi(\hat{a}+\hat{b})=\varphi(a+\hat{b})-\varphi''(a+\hat{b})\,[\hat{b},\ \hat{a}]/2\\+\varphi'''(a+\hat{b})\,\{[\hat{b},\ [\hat{b},\ \hat{a}]]-[\hat{a},\ [\hat{b},\ \hat{a}]]\}/6\\+\varphi''''(a+\hat{b})\,[\hat{b},\ \hat{a}]^2/8+O(\xi^4),\end{aligned} \tag{9}$$

where a is an eigenvalue of $\hat{a}$.

Setting $\hat{a}=-\hat{p}^2$, $\hat{b}=p_0^2$, we see that (9) really does give an expansion in powers, the first term being the zeroth degree and the following the second degree in ξ. By means of the definition (1), we obtain from (9)

$$\begin{aligned}f(\mathbf{r},\mathbf{p})=\Theta(p_0^2-p^2)+(\Delta p_0^2+2ip\nabla p_0^2)\,\delta'(p_0^2-p^2)/2\\+[(\nabla p_0^2)^2-2(p\nabla)^2p_0^2]\delta''(p_0^2-p^2)/3\\-(p\nabla p_0^2)^2\delta'''(p_0^2-p^2)/2.\end{aligned} \tag{10}$$

The first term of Eq. (10) gives the smooth, quasiclassical part of the expectation values (4) and, in particular, the density (6). The terms of order $\hbar^2,\hbar^4,\ldots$ take into account the smooth quantum corrections associated with the density inhomogeneity at the surface of the nucleus. Besides the smooth quantum corrections, we must also include the fluctuating shell effects. These are considered in Sec. 5.

2. QUASICLASSICAL DESCRIPTION OF ROTATION WITH INCLUSION OF THE SMOOTH QUANTUM CORRECTIONS

We consider here the problem of the rotation of a spherical nucleus around the z axis with angular frequency ω. Quantum mechanically we are dealing with the alignment of the angular momenta of the individual nucleons, but after averaging over the single-particle levels we obtain a classical picture of the rotation (see also Sec. 5). To find the smooth part of the total angular momentum M, we generalize Kirzhnits's method, though for simplicity we shall not consider questions of the self-consistency, and we shall replace the Hartree–Fock potential by a local potential V of the average field. Some aspects of the self-consistency will be discussed in Sec. 3.

To include rotation, we use the Hamiltonian

$$\hat{h}'=\hat{h}-\omega\hat{l}_z, \tag{11}$$

where

$$\hat{l}_z=-i[\mathbf{r}\times\nabla]_z.$$

The corresponding density operator is

$$\hat{\rho}=\Theta(\varepsilon_F-\hat{h}'), \tag{12}$$

and to determine the distribution function (1) we set $\hat{a}=-\hat{p}^2$ and $\hat{b}=2\mu(\varepsilon_F-V+\omega\hat{l}_z)=\hat{b}_0+2\mu\omega\hat{l}_z$ in (9), since $[\hat{l}_z,V]=0$ and $[\hat{l}_z,\hat{a}]=0$. In all the commutators $[\hat{b},\hat{a}]$, etc., in (9), the operator $\hat{b}$ can be replaced by $\hat{b}_0$, which corresponds to the case without rotation. Bearing in mind that in the order linear in ω

$$\begin{aligned}\delta'(b'+a)\exp(i\mathbf{pr})\,i\mathbf{p}\nabla p_0^2=\delta'(b+a)\exp(i\mathbf{pr})\,i\mathbf{p}\nabla p_0^2\\+\exp(i\mathbf{pr})\,\delta''(b+a)\,2\mu\omega\hat{l}_z i\mathbf{p}\nabla p_0^2,\end{aligned}$$

and noting that $\hat{l}_z\exp(i\mathbf{p}\cdot\mathbf{r})=l_z\exp(i\mathbf{p}\cdot\mathbf{r})$, where $l_z=xp_y-yp_x$ is the classical projection of the angular momentum onto the z axis, we obtain for the distribution function in the leading order in ω

$$\begin{aligned}f_\omega(\mathbf{r},\ \mathbf{p})=\Theta(\tilde{p}_0^2-p^2)+(\Delta p_0^2+2i\mathbf{p}\nabla p_0^2)\,\delta'(\tilde{p}_0^2-p^2)/2\\+[(\nabla p_0^2)^2-2(\mathbf{p}\nabla)^2p_0^2]\,\delta''(\tilde{p}_0^2-p^2)/3\\-(\mathbf{p}\nabla p_0^2)^2\delta'''(\tilde{p}_0^2-p^2)/2+i\delta''(p_0^2-p^2)\,2\mu\omega\hat{l}_z\mathbf{p}\nabla p_0^2.\end{aligned} \tag{13}$$

This expression contains the local Fermi momentum shifted because of the rotation:

$$\tilde{p}_0^2=2\mu(\varepsilon_F(\omega)-V(r)+\omega l_z). \tag{14}$$

Knowing the distribution function $f_\omega(\mathbf{r},\mathbf{p})$, we can calculate the total angular momentum M using the relation (4):

$$M=\mathrm{tr}(\hat{\rho}\,\hat{l}_z)=2\int d^3r\int\frac{d^3p}{(2\pi)^3}(l_z+\hat{l}_z)\,f_\omega(\mathbf{rp}). \tag{15}$$

Obviously, it is only when $\omega\neq0$ that there is a nonvanishing angular momentum $M\neq0$; for a spherically symmetric distribution function

$$\int d^3p\,l_z f_\omega=0$$

and

$$l_z\int f(\mathbf{r},\ \mathbf{p})\,d^3p=0.$$

We calculate first the density by means of the expression (6). The corresponding result will be required to calculate the angular momentum, and it is also of independent interest. To calculate the integrals over the momentum in (6) with the distribution function (13), it is convenient to introduce the new variables $\mathbf{p}'$ and $\mathbf{r}'$ in accordance with

$$\mathbf{p}=\mathbf{p}'+\mu[\boldsymbol{\omega}\times\mathbf{r}'],\quad \mathbf{r}'=\mathbf{r}. \tag{16}$$

Then the arguments in (13) are transformed as follows:

$$\tilde{p}_0^2-p^2=2\mu(\varepsilon_F-V+\mu\omega^2(x^2+y^2)/2)-p'^2\equiv p_0'^2-p'^2. \tag{17}$$

In the p' space, we obtain a Fermi sphere with radius

$$p_0'=\{2\mu(\varepsilon_F-V+[\lambda\omega^2(x^2+y^2)/2]\}^{1/2}. \tag{18}$$

Obviously, Eq. (16) can be understood as the transition to an intrinsic frame, the centrifugal potential $-\mu\omega^2(x^2+y^2)/2$ being added to the potential energy because of the rotation.

Carrying out the transformation (16) in Eq. (6), we obtain for the density $\rho_\omega(r)$ the result

$$\rho_\omega(r)=\frac{1}{3\pi^2}(2\mu)^{3/2}(\varepsilon_F-V_{eff})^{3/2}\times\left[1-\frac{1}{16\mu}\left(\frac{\nabla^2V}{(\varepsilon_F-V_{eff})^2}+\frac{(\nabla V)^2}{4(\varepsilon_F-V_{eff})^3}\right)\right], \quad (19)$$

where we have introduced the effective potential $V_{eff}=V(r)-\mu\omega^2(x^2+y^2)/2$. This expression for the density ρ_ω contains a volume effect of the redistribution of the density which is due to the appearance of the potential V_{eff} and the dependence of the Fermi energy on the rotation frequency, and also smooth quantum effects, which arise because of the gradient terms in (13). A similar relation was obtained in Ref. 1 by Kirkwood's method.

Qualitatively, the influence of rotation on the density distribution can be readily understood in the Thomas–Fermi approximation, when the gradient terms are omitted (Fig. 1). In this approximation, there is a centrifugal barrier, and the Fermi energy is lowered,

$$\Delta\varepsilon_F=\varepsilon_F(\omega)-\varepsilon_F(0)=-\frac{1}{2}\mu\omega^2\int\rho_0^{1/3}(x^2+y^2)\,d^3r\Big/\int\rho_0^{1/3}\,d^3r,$$

which leads to a decrease in the density at the center and to an increase at the edge of the nucleus. However, in this model the behavior of the density is not correctly described in the neighborhood of the turning point, and it is necessary to take into account quantum corrections (see Sec. 3). The results of a numerical investigation of the redistribution of the density are given in Sec. 4.

We now turn to the calculation of the total angular momentum M given by (15) in the order linear in ω. The terms containing the operator $\hat{l}_z$ can then be omitted immediately, since

$$\hat{l}_z\int d^3p' f_\omega(\mathbf{rp}')=O(\omega^2)$$

in accordance with (19).

Applying the transformation (16), we obtain

$$l_z=l_z'+\mu\omega(x^2+y^2);\quad l_z'=x'p_y'-y'p_x'. \quad (20)$$

We represent the single-particle angular momentum as a sum of an intrinsic part and a collective part. From the latter we obtain

$$M^{(0)}=\mu\omega\int(x^2+y^2)\rho_\omega(r)\,d^3r=\omega J, \quad (21)$$

i.e., a generalization of the well-known result that the principal part of the total angular momentum is determined by the classical formula with moment of inertia J determined by the density ρ_ω (which thus contains corrections for the inhomogeneity). Ignoring the dependence of the density on the frequency ω, we obtain the result of the Thomas–Fermi approximation.

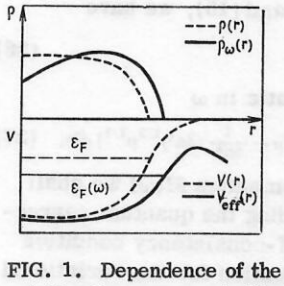

FIG. 1. Dependence of the density and the potential on the rotation frequency ω in the Thomas–Fermi approximation.

It is now interesting to consider the quantum corrections associated with the term l_z' in (15):

$$\delta M=2\int d^3r\int\frac{d^3p'}{(2\pi)^3}l_z'f_\omega(\mathbf{r},\mathbf{p}'). \quad (22)$$

It is obvious that the terms in $f_\omega(\mathbf{r},\mathbf{p}')$ that do not contain the momentum p do not contribute to the correction δM in the order linear in ω. In the remaining terms, the momentum occurs everywhere in the combination $\mathbf{p}\cdot\nabla$, which after the transformation (16) gives

$$\mathbf{p}\nabla=\mathbf{p}'\nabla+i\mu\omega\hat{l}_z.$$

Having $\hat{l}_zp_0^2=0$ for an axisymmetric potential, we immediately conclude that only the third and last terms of the expression (13) for the distribution function make a contribution to δM. From the third term, we obtain

$$\delta M^{(3)}=2\int d^3r\int\frac{d^3p'}{(2\pi)^3}l_z'\left(-\frac{2}{3}\right)\times(\mathbf{p}'\nabla+i\mu\omega\hat{l}_z)\,\mathbf{p}'\nabla p_0^2\delta''(p_0'^2-p'^2).$$

In the calculation, it is important to bear in mind that the commutation relations that follow from Eq. (16), $[\Delta_y,p_x']=\mu\omega$ and $[\Delta_x,p_y']=-\mu\omega$, must hold, and then

$$\delta M^{(3)}=\frac{2}{3}\frac{\mu\omega}{\pi^2}\int p_0(r)\,d^3r,$$

or, introducing the density of levels at the Fermi surface,

$$g(\varepsilon_F)=\frac{\mu}{\pi^2}\int p_0(r)\,d^3r,$$

we have

$$\delta M^{(3)}=\frac{2}{3}g(\varepsilon_F)\,\omega. \quad (23)$$

The contribution of the last term in (13) can be calculated similarly:

$$\delta M^{(5)}=2\int d^3r\int\frac{d^3p'}{(2\pi)^3}l_z'i\delta''(p_0'^2-p'^2)\,2\mu\omega\hat{l}_z\mathbf{p}\nabla p_0^2=-g(\varepsilon_F)\,\omega.$$

As a result, we obtain

$$\delta M=\delta M^3+\delta M^{(5)}=-g(\varepsilon_F)\,\omega/3. \quad (24)$$

It is interesting to note that the final expression for δM does not contain a gradient dependence. It can be seen directly that

$$\delta M\propto\int\frac{r^3\,dp_0}{dr}\,dr.$$

It follows from this that the correction δM is due to the gradient terms of the density and is thus a surface effect. It is only after integration by parts that a dependence of the level density arises.

The expression (24) recalls the well-known expression of Landau diamagnetism. The analogy is in fact deeper, as was shown by Dabrowski.[7]

Besides the quantum correction from the orbital motion, it is also necessary to take into account the spin polarization. To do this, we include the term $-\omega\hat{S}_z$ in the Hamiltonian and calculate the expectation value of the total spin,

$$\langle \hat{S}_z \rangle = \mathrm{tr}\,(\delta\hat{\rho}\hat{S}_z) = \sum_{\nu} (\delta\rho)_{\nu\nu} (S_z)_{\nu\nu},$$

and since[12]

$$(\delta\rho)_{\nu\nu} = \frac{d\Theta(\varepsilon_F - \varepsilon_\nu)}{d\varepsilon_F}\omega (S_z)_{\nu\nu},$$

we have

$$\langle \hat{S}_z \rangle = \delta M_S = \frac{\omega}{4}\frac{d}{d\varepsilon_F}\int \rho(\varepsilon_F, r)\, d^3r.$$

Using the quasiclassical expression (19) for the density, we obtain

$$\delta M_S = g(\varepsilon_F)\,\omega/4 \tag{25}$$

in complete analogy with Pauli paramagnetism. Using (21), (24), and (25), we arrive at the complete expression for the terms of first order in ω:

$$M = \omega (J - g(\varepsilon_F))12. \tag{26}$$

To estimate the role of the quantum correction to the total angular momentum, we use the relations

$$J \sim (2/5)\,\mu R^2 A;\quad g(\varepsilon_F) \sim 2\mu (8/3\pi^2)^{1/3} R^2 A^{1/3}.$$

Thus, we find $\delta M/M \sim A^{-2/3}$. As follows from the calculations in Sec. 4, the quantum corrections are a few percent of the total angular momentum. However, as is discussed in the following section, they are important for the correct description of the density in the surface region.

We note finally that the procedure we have described for calculating the density can also be used to determine the current

$$\mathbf{j} = \frac{1}{2\mu i}(\nabla - \nabla')\,\rho(\mathbf{r}, \mathbf{r}')|_{\mathbf{r}=\mathbf{r}'}.$$

It follows from the expression (6) for the mixed density that

$$\mathbf{j} = 2\int \frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}}{\mu} + \frac{\hat{\mathbf{p}}}{\mu}\right) f_\omega(\mathbf{r}, \mathbf{p}). \tag{27}$$

After the transformation (16), we obtain for the collective part of the current $\mathbf{j}_{\mathrm{coll}} = [\omega \times \mathbf{r}]\rho_\omega(\mathbf{r})$. The expression for the quantum correction is rather cumbersome. It is given in Ref. 8, in which a quasiclassical approach similar to Kirzhnits's is used.

3. SELF-CONSISTENCY

In considering the rotation hitherto, we have ignored self-consistency, using the approximation of an external potential. Formally, the relations for the distribution function $f_\omega(\mathbf{r}, \mathbf{p})$ remain valid, but the appearance of the self-consistent potential V_{HF} leads to an additional dependence on the frequency ω. For simplicity, we consider only the direct part of the potential V_H, for which we can write

$$V_H = \frac{\delta}{\delta\rho_\omega^{(1)}}(\mathrm{tr}\,\hat{\rho}^{(1)}\hat{\rho}^{(2)}\hat{V}(12)), \tag{28}$$

and we use the schematic momentum-dependent potential

$$\hat{V}(12) = \alpha g(|\mathbf{r}_1 - \mathbf{r}_2|)(\hat{\mathbf{p}}_1 - \hat{\mathbf{p}}_2)^2. \tag{29}$$

Because of the momentum dependence, the potential V_H acquires a term proportional to ω^2 in the case of rotation. For the trace in (28), we have in accordance with the definition of the distribution function

$$\mathrm{tr}\,(\hat{\rho}^{(1)}\hat{\rho}^{(2)}\hat{V}(12)) = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} d^3r_1\, d^3r_2\, \alpha g(|\mathbf{r}_1 - \mathbf{r}_2|)\,[(\mathbf{p}_1 - \mathbf{p}_2)^2 + (\hat{\mathbf{p}}_1 - \hat{\mathbf{p}}_2)^2]\, f_\omega(\mathbf{r}_1, \mathbf{p}_1)\, f_\omega(\mathbf{r}_2, \mathbf{p}_2). \tag{30}$$

For the integration over the momenta, we again use the transformation (16), and then

$$\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_1' - \mathbf{p}_2' + \mu(\omega \times (\mathbf{r}_1 - \mathbf{r}_2)) \tag{31}$$

and in (30) we obviously acquire the term

$$\alpha\mu^2 \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} d^3r_1\, d^3r_2\, g(|\mathbf{r}_2 - \mathbf{r}_2|) \times (\boldsymbol{\omega} \times (\mathbf{r}_1 - \mathbf{r}_2))^2 f_\omega(\mathbf{r}_1, \mathbf{p}_1)\, f_\omega(\mathbf{r}_2, \mathbf{p}_2). \tag{32}$$

which contributes to V_H in accordance with (28). However, for the δ-functional interaction $g(|\mathbf{r}_1 - \mathbf{r}_2|) = \delta(\mathbf{r}_1 - \mathbf{r}_2)$ the contribution of the term that depends directly on ω^2 vanishes. The situation here is analogous to the influence of a magnetic field on spin–orbit coupling.[9] Assuming for simplicity $f_\omega(\mathbf{r}, \mathbf{p}) = \Theta(\tilde{p}_0^2 - p^2)$, we obtain for the δ force from Eqs. (28) and (30)

$$V_H = (8\alpha/15\pi^2)(3\pi^2)^{5/3}\rho_\omega^{5/3}(r) - 2\alpha\nabla^2\rho_\omega(r). \tag{33}$$

Thus, the self-consistent potential contains a dependence on the frequency ω through the density $\rho_\omega(r)$. The second term in (36) has the consequence that Eq. (8) for the determination of the Fermi energy can be represented in the form

$$\frac{1}{2\mu}(3\pi^2)^{2/3}\rho_\omega^{2/3} + \frac{8\alpha}{15\pi^2}(3\pi^2)^{5/3}\rho_\omega^{5/3} - 2\alpha\nabla^2\rho_\omega - \frac{1}{2}\mu\omega^2(x^2 + y^2) = \varepsilon_F(\omega), \tag{34}$$

which is the law of conservation of energy. To be consistent, we must include all the gradient terms in the distribution function $f_\omega(\mathbf{r}, \mathbf{p})$, using the expression (13). There then appear additional gradient terms in the kinetic energy, which we have not written out. The self-consistency equation (34) thus becomes a nonlinear second-order differential equation for the determination of the density $\rho_\omega(r)$. It is important to note that the local Fermi momentum ρ_0', which vanishes at the classical turning point, disappears from the treatment. The usual difficulty of the Thomas–Fermi method associated with divergence of the quantum corrections at the turning point is thus eliminated. However, Eq. (34) still contains the centrifugal term $-\mu\omega^2(x^2 + y^2)/2$, which, as we discussed in Sec. 2, leads to the appearance of an unphysical barrier. The quantum correction δM [(24) and (25)] compensates this term in the surface region. To see this, we introduce the energy functional

$$E' = \mathrm{tr}\,(\hat{\rho}\hat{h}'). \tag{35}$$

Then, using the relations (11) and (15), we have

$$M = -\partial E'/\partial\omega. \tag{36}$$

Therefore, in the order quadratic in ω

$$E'(\omega) = E'(0) - \frac{1}{2}\mu\omega^2\int ((x^2 + y^2)\rho - \frac{1}{6\pi^2}(3\pi^2)^{1/3}\rho^{1/3})\, d^3r, \tag{37}$$

where for the total angular momentum $M(\omega)$ we shall use the expression (26), including the quantum correction. We now note that the self-consistency condition (34) is the Euler–Lagrange equation for the variational principle $\delta E' = 0$ with the additional condition

$A = \int \rho\omega(r) d^3r.$

It is easy to see that because of the quantum correction in (37) a rotational term appears in the self-consistency equation (34):

$$-\mu\omega^2[(x^2+y^2) - (3\pi^2)^{1/3}\rho_\omega^{-2/3}/18\pi^2]/2. \quad (38)$$

The quantum correction, which is proportional to $\rho_\omega^{-2/3}$, compensates the classical centrifugal energy in the surface region, where $\rho_\omega \to 0$, and because of this the barrier disappears.

In conclusion, we note that because of the complexity of the direct solution of the variational equation (34), in which it is necessary to take into account systematically the quantum corrections and use a realistic interaction, we shall solve the variational problem approximately. The corresponding results will be presented below.

4. INVESTIGATION INTO THE EFFECTS OF THE REDISTRIBUTION OF THE DENSITY

To investigate the effects of the redistribution of the density, we use the expression for the energy, defined as a functional of the density.[3] Such an approach in the case without rotation[10] showed that a simple form of the functional makes it possible to recover the averaged results of calculation by the Hartree–Fock method with good accuracy. Generalizing this approach to the case of rotation, we assume

$$E[\rho] = |W_0| \int \left(-\frac{2\rho}{\bar\rho} + \frac{\rho^2}{\bar\rho^2}\right)\rho\, d^3r + \eta \int (\nabla\rho)^2\, d^3r + \frac{1}{2F[\rho]} I^2, \quad (39)$$

where

$$W_0 = -16.44 \text{ MeV}, \ \bar\rho = 0.159 \text{ F}^{-3}, \ \eta = 78\times174 \text{ MeV}\cdot\text{F}^{-5}$$

The functional (39) can be understood as an approximation to the energy in the laboratory system, $E = E' + \omega M$, where E' is given by (35), and $M = \sqrt{I(I+1)} \approx I$. To solve the variational problem $\delta E = 0$ with the additional condition $A = \int \rho d^3r$, we use a parametrized form of the density, determining the set of parameters by requiring $E[\rho]$ to be minimal. Such a procedure for the case without rotation was discussed in Refs. 10 and 11. We use the following analytical forms of the density:

$$\rho_I = \rho_0/\{1 + \exp[(r-R)/a]\}; \quad (40)$$

$$\rho_{II} = \rho_0 \sinh(R/a)/[\cosh(r/a) + \cosh(R/a)];$$

$$\rho_{III} = \rho_I(1 + (9/2)(\mu\omega^2/K)(r^2 - r_\perp^2)); \quad (41)$$

$$r_\perp^2 = (1/A)\int \rho(x^2+y^2)\, d^3r. \quad (42)$$

We introduce the form ρ_{III} to take into account explicitly the volume effect of the redistribution of the density; K is the coefficient of compressibility.

It is clear that the main influence of rotation on the density consists of a deformation of the shape of the nucleus. Accordingly, we take

$$R = R_0(1 - \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta)); \ \beta_2 > 0.$$

Besides the parameter ρ_0 of the central density and the deformation parameters β_2, β_4, we also take into account the possibility of anisotropic variation of the diffuseness parameter a, taking $a = a_0 + a_2\sin^2\theta$. Thus, the complete variational calculation contains the four parameters $\rho_0, \beta_2, \beta_4, a_2$.

For qualitative estimates, it is convenient to expand the energy $E[\rho]$ (39) in powers of a_0/R_0, which corresponds to the drop model (for simplicity, we do not take into account the angular dependence). Using the parametrization (40), we have

$$E/A = -|W_0| + |W_0|(1{,}5 + \eta\rho_0/2a_0^2)(a_0/R_0) + 3|W_0|(a_0^2/R_0^2) + 5I^2/\{4\mu A^2 R_0^2[1 + (7/3)(\pi a_0/R_0)^2]\}. \quad (43)$$

From the relation (43), we obtain an estimate for the decrease in the central density due to rotation with spin I:

$$\delta\rho/\rho = (\rho_0 - \bar\rho)/\bar\rho = -(1/3)E_{\text{rot}}/|E_{\text{vol}}| \sim I^2A^{-8/3}. \quad (44)$$

Accordingly, we have an increase in the radius: $\delta R/R \sim (1/9)E_{\text{rot}}/|E_{\text{vol}}|$. Similarly, fixing $\rho_0 = \bar\rho$, we obtain a maximal estimate for the increase in the thickness of the surface layer:

$$\frac{\delta a}{a_0} \sim \frac{14\pi^2}{3}\frac{E_{\text{rot}}}{E_{\text{surf}}}\left(\frac{a_0^2}{R_0^2}\right) \sim I^2A^{-3}. \quad (45)$$

Including also the deformation, we can see[3] that

$$(5/4\pi)^{1/2}\beta_2 \sim E_{\text{rot}}/E_{\text{surf}}. \quad (46)$$

From the estimates we obtain the expected order of magnitude of the effects of the redistribution of the density and the deformation due to rotation. It is obvious that it is only in light nuclei and at maximal spins $I \sim A$ that the rotation has an appreciable influence on the density distribution.

The result of numerical calculations with variation of the parameters a_2, β_2, β_4 is shown in Table I. It can be seen that without the inclusion of deformation one obtains an appreciable decrease in the central density, which corresponds to (44). When allowance is made for the variation of the parameter a_2, there is an increase in the thickness of the surface layer, which for $A = 40, I$

TABLE I. Dependence of the density distribution and the shape on the spin I for nuclei in the region $40 \leqslant A \leqslant 100$.[3]

Parameters	β_2	β_4	a_2, F	R_0, F	$\sqrt{\langle r^2\rangle}$, F	ρ_0, F^{-3}	E, MeV	I [$\hbar$]
$A=40$	—	—	—	3.91	3.41	0.144	—395.01	0
$a_0=0.42$ F	—	—	—	4.07	3.52	0.128	—283.31	40
	—	—	0.11	3.93	3.55	0.136	—290.07	40
	0.52	—	0	4.00	3.64	0.128	—303.28	40
	0.53	—	0.06	3.92	3.66	0.132	—304.20	40
	0.54	0.06	—	3.99	3.65	0.127	—303.85	40
	0.56	0.08	0.07	3.90	3.69	0.132	—304.97	40
	0.57	0.08	0.07	3.90	3.70	0.132	—303.27	40
$A=60$	—	—	—	4.46	3.80	0.148	—640.65	0
$a_0=0.43$ F	—	—	—	4.59	3.89	0.137	—512.35	60
	—	—	0.06	4.52	3.91	0.141	—513.36	60
	0.48	—	—	4.52	4.01	0.136	—529.28	60
	0.49	—	0.05	4.46	4.03	0.139	—530.02	60
	0.49	0.05	—	4.51	4.02	0.136	—529.84	60
	0.50	0.06	0.05	4.45	4.05	0.139	—530.77	60
	0.52	0.07	0.05	4.45	4.06	0.139	—529.22	60
$A=80$	—	—	—	4.90	4.13	0.151	—895.66	0
$a_0=0.43$ F	—	—	—	5.01	4.20	0.141	—750.26	80
	—	—	0.04	4.97	4.21	0.144	—750.93	80
	0.45	—	—	4.94	4.32	0.140	—768.63	80
	0.46	—	0.04	4.90	4.33	0.143	—769.26	80
	0.46	0.05	—	4.94	4.33	0.141	—769.19	80
	0.47	0.06	0.04	4.89	4.35	0.143	—769.99	80
	0.48	0.06	0.04	4.89	4.36	0.143	—768.56	80
$A=100$	—	—	—	5.28	4.40	0.152	—1156.93	0
$a_0=0.44$ F	—	—	—	5.38	4.47	0.144	—997.15	100
	—	—	0.03	5.34	4.48	0.146	—997.63	100
	0.43	—	—	5.31	4.59	0.144	—1016.66	100
	0.44	—	0.03	5.27	4.60	0.146	—1017.21	100
	0.44	0.04	—	5.31	4.60	0.144	—1017.22	100
	0.45	0.05	0.04	5.26	4.61	0.146	—1017.92	100
	0.46	0.05	0.04	5.26	4.62	0.146	—1016.58	100

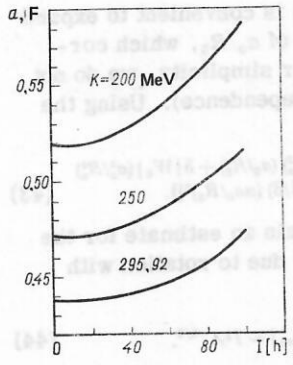

FIG. 2. Dependence of the diffuseness parameter at the equator $a=a_0+a_2$ on the spin I for different values of the coefficient of compressibility K.

$=40$ is 25% on the equator. At the same time, the central density is increased. The inclusion of deformation decreases the anisotropy of the diffuseness, and the central density again decreases.

All the results in Table I were obtained for the density parametrization (41). For the density ρ_{I}, similar data are obtained in accordance with (40). However, the density ρ_{III} (42) gives in a number of cases at $I=A$ a decrease in the binding energy by 10–15 MeV, the variations in the parameters being approximately the same as for the density ρ_{II}.

Naturally, the results we have given here are sensitive to the choice of the coefficient of compressibility. The parametrization of the energy functional in the form (39) corresponds to $K=295.92$ MeV. To demonstrate the influence of compressibility, Fig. 2 shows the spin dependence of the diffuseness thickness and at the equator $a=a_0+a_2$ for different values of the coefficient of compressibility. Finally, let us briefly discuss the influence of the Coulomb energy. Assuming $\rho_p=\rho_n$, we include[3]

$$E_{\mathrm{coul}}=\frac{e^2}{2}\int\frac{\rho_p(r)\,\rho_p(r')}{|r-r'|}\,d^3r\,d^3r'$$

in the energy functional (39). Not only the deformation but also the effects of the density redistribution are then increased, as can be seen from Table II. Thus, for nuclei with $A<100$ the decrease in the central density and the increase in the density on the equator can lead to interesting consequences. For heavier nuclei, the assumption of additivity of the drop and deformation energies assumed in Ref. 4 is completely justified.

5. ALLOWANCE FOR SHELL EFFECTS

Hitherto, we have considered only the smooth part $M(\omega)$, ignoring the shell effects. However, for the case of the rotation of a spherical nucleus the appearance of an angular momentum $M(\omega)$ is a purely quantum effect due to the crossing of levels as the frequency ω increases. When the term $\omega\hat{l}_x$ is included in the single-particle Hamiltonian, the spherical shells (nlm) are split with respect to the quantum number m (Fig. 3).

As long as there is no crossing $(\omega<\omega_1)$, the total angular momentum is zero. When the lowest unfilled level crosses the Fermi limit at $\omega=\omega_1$, an angular momentum appears abruptly, corresponding to excitation of a particle–hole state. With further increase in the frequency there are new jumps, which directly reflect the shell structure. It is only after averaging on the scale of the shell splitting that a smooth dependence of the angular momentum on the frequency appears. A shortcoming of our description is the introduction of averaging from the very start. It can be shown that one can carry through a quasiclassical treatment with the inclusion of shell effects[3] by generalizing the method developed to calculate the shell effects in the density.[6] It is here necessary to take into account quantization in the framework of the Bohr–Sommerfeld method.

We first determine the total angular momentum in

TABLE II. Dependence of the density distribution and the deformation on the spin I when allowance is made for the Coulomb energy.[3]

Parameters	β_2	β_4	a_2, F	R_t, F	$\sqrt{\langle r^2\rangle}$, F	ρ_0, F^{-3}	E, MeV	E_{coul}, MeV	E_{rot}, MeV	I, [$\hbar$]
$A=40$	—	—	—	3.96	3.45	0.138	—315.94	78,52	—	0
$a_0=0.42$ F	0.61	0.08	—	4.04	3.74	0.121	—229.11	74.10	72.29	40
	0.63	0.10	0.08	3.93	3.79	0.127	—230.17	73.66	70.06	40
$A=60$	—	—	—	4,54	3.86	0.140	—482.19	157.19	—	0
$a_0=0.43$ F	0.58	0.08	—	4.58	4.14	0.128	—477.90	150.10	87.89	60
	0.61	0.09	0.07	4.49	4.18	0.132	—379.45	149.25	85.43	60
$A=80$	—	—	—	5.01	4.21	0.141	—636.48	256.33	—	—
$a_0=0,44$ F	0.56	0.06	—	5.02	4.47	0.132	—519.93	246.25	101.84	80
	0.61	0.09	0.07	4.93	4.53	0.135	—521.31	244.53	96.60	80
$A=100$	—	—	—	5,36	4,48	0.145	—777,63	375,90	—	0
$a_0=0,45$ F	0.60	0.07	—	5.37	4.81	0.133	—652.43	358.56	108.68	100
	0,65	0.1	0.07	5,28	4,87	0.137	—653,56	355.65	103.28	100

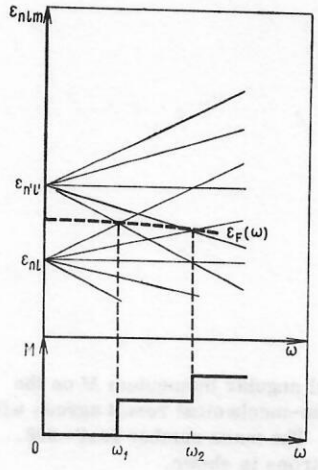

FIG. 3. Crossing of different single-particle levels as the rotation increases.

accordance with the quantum-mechanical formula

$$M(\omega) = 2 \sum_{\nu, m} \Theta(\varepsilon_F - \varepsilon_\nu + m\omega)\, m \int |\psi_{\nu m}|^2 d^3r. \quad (47)$$

Here, $\nu \equiv (n, l)$, and the limits of summation can be seen in Fig. 3. To calculate the wave function, we must use the WKB approximation,

$$\psi_{\nu m} = (C_\nu / r\sqrt{p_\nu}) \sin(S_\nu + \pi/4)\, Y_{lm}, \quad (48)$$

where S_ν is the truncated action, $S_\nu = \int_{r_1}^{r} p_\nu(r')dr'$, p_ν is the classical radial momentum $p_\nu = [2\mu(\varepsilon_\nu - V) - (l+1/2)^2/r^2]^{1/2}$, and r_1 is determined by the left-hand turning point for motion in the potential well with orbital angular momentum l. Finally, the normalization coefficient c_ν in (48) is found by means of the relation $C_\nu^2 = 2\mu/\tau_{0\nu}$, where $\tau_{0\nu}$ is the time required for the particle to pass from the left-hand turning point (r_1) to the right-hand turning point (r_2), i.e.,

$$\tau_{0\nu} = \int_{r_1}^{r_2} \mu\, dr/p_\nu; \quad (49)$$

we now replace the sum over the principal quantum number n in (47) by an integral, using Poisson's formula:

$$\sum_{n=a}^{b} f_n = \sum_{k=-\infty}^{\infty} \int_a^b f(n) \cos(2k\pi n)\, dn. \quad (50)$$

Using the Bohr–Sommerfeld quantization condition

$$S_{0\nu} = \int_{r_1}^{r_2} p_\nu(r)\, dr = \omega(n + 1/2), \quad (51)$$

we then obtain for the total angular momentum M from (47)–(49)

$$M(\omega) = 2\mu \sum_{lm} m \sum_{k=-\infty}^{\infty} \int dr\, d\Omega \int \frac{dn}{\tau_{0\nu} p_\nu} \cos(2k\pi n) \times (1 + \sin 2S_\nu) |Y_{lm}|^2. \quad (52)$$

In what follows, we shall ignore the contribution of the term containing $\sin 2S_\nu$, since it describes shell fluctuations of the density, which are unimportant after integration over the radius.

In (50), we integrate, replacing the variable n by the energy ε, and we determine the upper limit by means of the relation

$$\tilde{\varepsilon} = \varepsilon_F(\omega) + m\omega, \quad (53)$$

and we find the lower limit ε_{min} from the requirement $p_l(\varepsilon_{min}) = 0$. Bearing in mind that $\partial S_{0\nu}/\partial \varepsilon_\nu = \tau_{0\nu} = \pi dn/d\varepsilon_\nu$, we obtain

$$M(\omega) = \frac{2\mu}{\pi} \sum_{lm} m \sum_{k=-\infty}^{\infty} (-)^k \int dr \int_{\varepsilon_{min}}^{\tilde{\varepsilon}} \frac{d\varepsilon}{p_l(\varepsilon)} \cos(2kS_{0l}(\varepsilon)). \quad (54)$$

We divide the sum over k into two parts. In the first, we set $k = 0$. Then denoting the corresponding fraction of the total angular momentum by $M^{(0)}$, we obtain after integration

$$M^{(0)}(\omega) = \frac{2}{\pi} \sum_{lm} m S_{0lm}, \quad (55)$$

where S_{0lm}, which depends on the projection m, is the action

$$S_{0lm} = \int_{r_1}^{r_2} (2\mu(\tilde{\varepsilon} - V) - (l+1/2)^2/r^2)^{1/2}\, dr. \quad (56)$$

It can be shown that $M^{(0)}$ is the smooth part of the total angular momentum M and is approximately equal to M_{TF} [see Eq. (66) below]. This could be expected, since for $k = 0$ we have

$$\sum_n f_n = \int f(\varepsilon)\, \rho(\varepsilon)\, d\varepsilon,$$

which leads to the neglect of the shell fluctuations. To calculate the remainder in (52) corresponding to $k \neq 0$, it is only necessary to take into account the first term after integration by parts. Then

$$\int_{\varepsilon_{min}}^{\tilde{\varepsilon}} \frac{d\varepsilon}{p_l(\varepsilon)} \cos(2kS_{0lm}) = \frac{1}{2k\tau_{0l} p_l(\tilde{\varepsilon})} \sin(2kS_{0lm}).$$

Taking into account the relation (49), we obtain

$$M^{(1)} = \frac{2}{\pi} \sum_{lm} m \sum_{k=-\infty}^{\infty}{}' (-)^k \frac{\sin(2kS_{0lm})}{2k}. \quad (57)$$

To calculate the sum over k, we note that the argument $\tilde{\varepsilon}$ can be equal to one of the eigenvalues ε_ν for definite values of the projection m and the frequency ω. Therefore, using the quantization rule (57) for the interval $\varepsilon_{nl} \leq \tilde{\varepsilon} \leq \varepsilon_{n+1,l}$ we can write

$$S_{0lm} = [S_{0lm}] + (n+1)\pi, \quad (58)$$

where $[S_{0lm}]$ varies in the range $-\pi/2 < [S_{0lm}] < \pi/2$. Using the definition of the function $[S_{0lm}]$, we obtain from (57)

$$\sum_{-\infty}^{\infty}{}' \frac{(-)^k \sin(2kS_{0lm})}{2k} = \sum_{-\infty}^{\infty}{}' (-)^k \frac{\sin(2k[S_{0lm}])}{2k} = -[S_{0lm}], \quad (59)$$

from which we obtain the shell correction in the form

$$M^{(1)} = -\frac{2}{\pi} \sum_{lm} m [S_{0lm}]. \quad (60)$$

Together with the smooth part (55), this gives

$$M = \frac{2}{\pi} \sum_{lm} m (S_{0lm} - [S_{0lm}]). \quad (61)$$

To understand the significance of this expression, we consider briefly the determination of the particle number by the same method. For the one-dimensional case without rotation we have by analogy with (61)

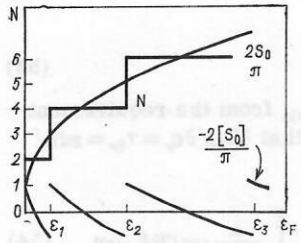

FIG. 4. Dependence of the particle number N and the functions S_0 and $[S_0]$ on the Fermi energy ε_F for one-dimensional motion.

$$N=\frac{2}{\pi}(S_0(\varepsilon_F)-[S_0(\varepsilon_F)])\equiv N^{(0)}+N^{(1)}. \tag{62}$$

This relation makes it possible to determine the number of particles as a function of the position of the Fermi energy. As a function of ε_F, the expression $[S_0(\varepsilon_F)]$ has discontinuities on the passage through the eigenvalue $\varepsilon_F=\varepsilon_n$, while the action S_0 continuously increases. The combined behavior of both parts of the function $N(\varepsilon_F)$ can be seen in Fig. 4. Each time a new level passes through the Fermi energy, the number of particles increases discontinuously.

For the angular momentum given by Eq. (61), the behavior is similar. We consider first the values of the frequency ω for which there is no crossing of the shells (see Fig. 3, $\omega<\omega_1$). For all possible values of m, the argument $\tilde{\varepsilon}=\varepsilon_F+m\omega$ then takes values in the interval $\varepsilon_{nl}<\tilde{\varepsilon}<\varepsilon_{n'l'}$ (ε_{nl} is the last occupied shell, and $\varepsilon_{n'l'}$ is the first unoccupied shell), and in accordance with (58) and (61)

$$M=\frac{2}{\pi}\sum_{lm} m(n+1)\pi=0$$

in agreement with the quantum-mechanical result. We recall that for a closed shell without crossings the smooth part is nonzero, $M^{(0)}\neq 0$, so that we have a case when the shell correction completely compensates the smooth part. In the general case

$$\partial M^{(0)}/\partial\omega=-\partial M^{(1)}/\partial\omega,\ \tilde{\varepsilon}\neq\varepsilon_\nu, \tag{63}$$

from which it follows that $M=\text{const}, \tilde{\varepsilon}\neq\varepsilon_\nu$. We now consider the first crossing of the levels at $\omega=\omega_1(\varepsilon_{n'l'}-\varepsilon_{nl})/(l'+l)$ (see Fig. 3). Then the lower value of the argument ε, $\tilde{\varepsilon}_{\min}=\varepsilon_F-\omega l(m=-l)$, passes through the point ε_{nl}, and simultaneously $\tilde{\varepsilon}_{\max}=\varepsilon_F+\omega l'$ ($m'=l'$) intersects $\varepsilon_{n'l'}$. These intersections are accompanied by discontinuities of the functions $[S_{0lm}]=\pm\pi$ at $m=-l$ and $m'=l'$, whereas S_{0lm} remains a continuous function in the neighborhood of $\varepsilon_{nl},\varepsilon_{n'l'}$. Using Eq. (61), we obtain the total angular momentum $M=2(l+l')$ in agreement with the quantum-mechanical result.

It is convenient to investigate the general case numerically.[3] The result of such a calculation for $A=208$ is shown in Fig. 5. For the potential $V(r)$, we have used the Woods–Saxon formula with parameters taken from Ref. 12. The Fermi energy was fixed by specifying A. Figure 5 shows only the total angular momentum of the neutrons, and the spin–orbit coupling was not taken into account. In the calculation, the sum over (l,m) takes all values for which levels exist. The maximal value of l has the order Rp_F. Up to frequencies $\omega\leq 0.6$ MeV/$\hbar$, the quasiclassical calculation in

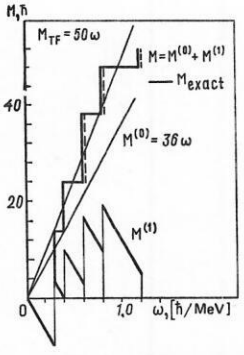

FIG. 5. Dependence of the total angular momentum M on the frequency ω. The exact quantum-mechanical result agrees with $M=M^{(0)}+M^{(1)}$ for $\omega\leq 0.6$ MeV. The mass number is $A=208$. Only the contribution of the neutrons is shown.

accordance with (61) agrees with the exact quantum-mechanical expression. It is interesting to note that although the smooth part $M^{(0)}$ also contains a sum over (l,m) [see (55)] it is numerically found to be exactly straight, though $M^{(0)}<M_{TF}$. This can be understood by seeing how M_{TF} is obtained in the present approach.

Using (61), we can readily derive approximate expressions for $M^{(0)}$ and $M^{(1)}$. To calculate $M^{(0)}$, it is sufficient to expand the action S_{0lm} with respect to ω subject to the condition that this function is everywhere continuous. From the equation

$$S_{0lm}=S_{0l}(\varepsilon_F)+\mu m\omega\int\frac{dr}{p_l(\varepsilon_F)} \tag{64}$$

we obtain, retaining for convenience the factor $\int|Y_{lm}|^2d\Omega$ in (54),

$$M^{(0)}=\frac{2}{\pi}\mu\omega\int dr\,d\Omega\sum_{lm} m^2|Y_{lm}|^2\frac{1}{p_e(\varepsilon_F)}. \tag{65}$$

By means of the quasiclassical expression[13]

$$\sum_{m=-l}^{l} m^2|Y_{lm}|^2=\frac{(l+1/2)^3}{2\pi^2}\frac{\pi}{2}\sin^2\theta \tag{66}$$

and after replacement of the sum over l by an integral from $l=0$ to $l=Rp_F$, we obtain

$$M^{(0)}=\frac{\mu\omega}{3\pi^2}\int r^4\sin^2\theta\,dr\,d\Omega\,(2\mu(\varepsilon_F-V))^{3/2}=M_{TF}, \tag{67}$$

which agrees with the result (21) of the Thomas–Fermi method. Note that the upper limit of integration is chosen in such a way that the contribution of the corresponding level can be ignored. This explains the difference between the numerical value of M_{TF} and $M^{(0)}$ (see Fig. 5).

For the approximate calculation of the correction $M^{(1)}$, we use the expression (59) and again expand S_{0lm} around ε_F [see (64)]. It follows from (60) that

$$M^{(1)}=\frac{2}{\pi}\sum_{lm} m\int|Y_{lm}|^2dr\,d\Omega\sum_{k=1}^{\infty}\cos 2kS_{0l}(-)^k\frac{\sin(2k\omega m\tau_{0l})}{k}. \tag{68}$$

As in the case of the relation (65), we can now calculate the sum over m in the quasiclassical approximation,

$$\sum_{m=-l}^{l} m\sin(2k\omega m\tau_{0l})|Y_{lm}|^2=\frac{(l+1/2)^2}{2\pi}\sin\theta J_1(2k\omega\tau_{0l}\sin\theta(l+1/2)), \tag{69}$$

and finally integrate over θ, which gives

$$M^{(1)}=\frac{2}{\pi}\sum_{l}\sum_{k=1}^{\infty}(-)^{k}\int dr\,(l+1/2)^{2}\cos 2kS_{0l}\,\frac{2j_{1}(2k\omega\tau_{0l}(l+1/2))}{k}. \tag{70}$$

This expression can be used for asymptotic estimates. Its structure is similar to that of the expression obtained in Ref. 2 by summation of the classical trajectories.

For low frequencies $\omega\to 0$, it follows from (69) after summation and integration over l that $M^{(1)}=-M^{(0)}$ in accordance with the general result (63). For estimates, we can restrict ourselves in (70) to the contribution of the last occupied (nl) and first unoccupied shell $(n'l')$ and take into account only the first term of the sum over k. Then, taking $l'=l$, we obtain for the amplitude of the shell correction $M^{(1)}\sim(4/\pi)l^{2}2j_{1}(2\omega\pi_{0l}l)$. The first minimum of this expression is reached at $\omega=\omega_{\min}$, and $\omega_{\min}=0.8\pi/(2\tau_{0l}l)$. Bearing in mind that $\tau_{0l}\sim\pi\Delta r/\Delta\varepsilon$, where $\Delta\varepsilon$ is the mean difference between the levels near the Fermi surface, $\Delta\varepsilon=2\varepsilon_{F}/(3A)$ MeV, we obtain $\omega_{\min}=0.8\Delta\varepsilon/\alpha l\sim\omega_{1}$, where ω_{1} is determined by the first crossing of the levels: $\omega_{1}=(\varepsilon_{n'l'}-\varepsilon_{nl})/(l+l')$. As the frequencies increase, $M^{(1)}$ is damped, reflecting the decrease in the quantum fluctuations at high spins.

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