Exchange currents in nuclear physics

E. A. Ivanov

Joint Institute for Nuclear Research, Dubna

E. Truhlík

Institute of Nuclear Physics, Czechoslovak Academy of Sciences, Řež, Prague

Fiz. Elem. Chastits At. Yadra 12, 492–547 (March–April 1981)

The low-energy theorems for "soft" currents and pions are used to construct single-nucleon pion production amplitudes and, on their basis, two-particle operators of exchange currents. The connection between the exchange currents and the structure of the nuclear potential is analyzed. A critical comparison of the theory with the experimental data is made.

PACS numbers: 25.40.Rb, 25.10. + s, 11.20.Dj

INTRODUCTION

During the last decade, the physics of intermediate energies has been intensively developed. It studies the interaction of elementary particles with nuclei in the energy range up to 1 GeV, i.e., the region between particle physics and nuclear physics. The rapid development of this branch of physics is due to the following circumstances.

a) The accumulation of a large number of fairly accurate data with error that frequently does not exceed the 10% level. The most important part here has been played by the meson factories commissioned since the beginning of the seventies.

b) The development of the theory of interactions of elementary particles. It is hard to overestimate the influence of ideas from current algebra, the covariant formulation of the theory of interaction of two nucleons (the Bethe–Salpeter equation, dispersion relations), the unified theory of electromagnetic and weak interactions, and, more recently, quark models on the development of the physics of intermediate energies.

c) The development of the theory of systems of several nucleons on the basis of the Faddeev equations, the method of hyperspherical harmonics, and the variational method.

d) The appearance of powerful computational tools. For the exact analysis of experimental data and theoretical calculations this is very important.

In traditional nuclear physics, a system of N nucleons is described by the nonrelativistic Schrödinger equation. As interaction potential, one takes the simplest two-nucleon interaction of exchange type, and potentials of more complicated nature (three-particle, etc.) are not considered. The presence of other particles in the nucleus is also not taken into account explicitly. Solving the Schrödinger equation with appropriate boundary conditions, one can obtain the wave function of the problem and then the values of experimentally observable quantities. To calculate the observables associated with the effect of some perturbation on the system, one takes the interaction operator $\hat{J}$ in the form of a sum of single-particle operators $\hat{J}_i$ over all the nucleons in the system (impulse approximation):

$$\hat{J}=\sum_{i=1}^{N}\hat{J}_i. \tag{1}$$

In such an approach, one immediately faces the extremely nontrivial problem of the accuracy of the solution of the Schrödinger equation; this must always be borne in mind when calculations are compared with experimental data. It is well known that at present the Schrödinger equation can be reliably solved only for two nucleons. The situation in the case of three-nucleon systems is also more or less satisfactory.

More serious difficulties arise in this scheme when the interaction of elementary particles with a system of nucleons is considered. For example, very early in the development of nuclear physics (Refs. 1 and 2; see also Refs. 3–5) it was noted that it is not a simple matter to satisfy the charge conservation law in the description of the interaction of radiation with a nucleus. For clarity, we consider a two-nucleon system. The current perturbing the system can be written in the form

$$\mathbf{J}(\mathbf{r})=\sum_{i=1}^{2}\frac{1}{2}(1+\tau_i^3)\frac{1}{2M}[\mathbf{p}_i,\,\delta(\mathbf{r}-\mathbf{r}_i)]_+ + i\sum_{i=1}^{2}\frac{1}{2}(\mu_i^S+\mu_i^V\tau_i^3)\frac{1}{2M}[\sigma_i\times\mathbf{p}_i,\,\delta(\mathbf{r}-\mathbf{r}_i)]_+, \tag{2}$$

where μ_i^S and μ_i^V are, respectively, the isoscalar and the isovector magnetic moment of nucleon i; σ_i and τ_i are, respectively, the spin and the isospin operator for this nucleon; and $\mathbf{p}_i$ is the momentum operator.

In accordance with the current conservation law, we must have

$$\nabla\cdot\mathbf{J}(\mathbf{r})=-i\,[H,\,\rho(\mathbf{r})]_-. \tag{3}$$

Here, $H=T+V$, $T=\sum_{i=1}^{2}\mathbf{p}_i^2/2M$, and V is the two-particle potential. For nonrelativistic nucleons, the charge density is given by

$$\rho(\mathbf{r})=\sum_{i=1}^{2}\frac{1}{2}(1+\tau_i^3)(\mathbf{r}-\mathbf{r}_i). \tag{4}$$

Calculating the commutator $[T,\rho]_-$, we see that the current $\mathbf{J}$, taken in the impulse approximation (2), satisfies the condition

$$\nabla\mathbf{J}=-i\,[T,\,\rho]_-. \tag{5}$$

But, for example, for a potential of the form

$$V(\mathbf{r}) = (\tau_1\tau_2)\, f_1(\sigma_1, \sigma_2, \mathbf{r}) \exp(-r/r_B)/r \tag{6}$$

with range $r_B = m_B c/\hbar$ the commutator $[V, \rho]_-$ does not vanish:

$$[V, \rho]_- = (\tau_1 \times \tau_2)^3 f_2(\sigma_1, \sigma_2, \mathbf{r}) \exp(-r/r_B)/r. \tag{7}$$

This "nonconservation" of the charge merely indicates that the single-particle current (2) must be augmented by an essentially two-particle part, namely, an exchange current of the same range as the potential employed in the calculations. Thus, in the given case allowance for the exchange currents is a necessary consequence of the requirement of charge conservation.

In practice, it may happen that the exchange terms are small and the experimental data can be described satisfactorily without them. However, there are now definite proofs for the existence of two-particle electromagnetic isovector exchange currents of single-pion range, since allowance for them proved decisive in the explanation of experimental data in numerous cases for which other interpretations are not available.

Less satisfactory is the situation with regard to the experimental confirmation of the presence of exchange currents in weak processes, which is explained by the inadequate number and accuracy of the data even for few-nucleon systems.

There are also a number of uncertainties in the treatment of isoscalar exchange currents. These currents are actually the relativistic correction to the main approximation, so that the currect allowance for their contribution entails the consistent allowance for all the other small corrections of the same type, and at the present stage in the development of theory this cannot be done in practice.

In the present review, we discuss the present theoretical and experimental status of two-particle exchange currents and their significance for the physics of intermediate energies.

In Sec. 1, we briefly review the development of the general formalism within which investigations on this problem have been and are being made. We then discuss in detail the low-energy theorems which are the theoretical basis for the construction of the exchange-current operator.

Section 2 is devoted to methods of constructing effective current operators and a potential. We discuss in detail the method of renormalizing the wave function, which has given rise to numerous disputes in the literature.

In Sec. 3, we consider the problem of constructing a consistent scheme for calculating exchange currents in the framework of the Chew–Low model and we compare the predictions of this model with the predictions of the low-energy theorems for the electromagnetic exchange currents. The complexity of the situation in the case of isoscalar exchange currents is illustrated by calculations of the static moments of the deuteron.

Unfortunately, due to lack of space, we cannot consider the experimental situation with regard to axial exchange currents nor give the results of the corresponding numerical calculations. For the same reason, we have omitted a discussion of some other theoretical approaches to exchange currents, in particular, the scheme based on the method of hard pions.[6-9]

The literature contains several papers of review nature[10-13] on exchange currents. A deeper acquaintance with the subject will be facilitated by the comprehensive book edited by Rho and Wilkinson,[14] which collects together the majority of the material up to the start of 1978. More recently, the problem of exchange currents has been discussed extensively at the Workshop on Few-Nucleon Systems and Electromagnetic Interactions at Frascati,[15] at the Conference on Meson–Nucleus Interactions at Houston,[16] at the Conference on Nuclear Physics and Electromagnetic Interactions at Mainz, and at the Eighth International Conference on High Energy Physics and Nuclear Structure at Vancouver.[17]

1. THE *S*-MATRIX FORMALISM

1.1. General definition of the exchange current

The meson theory of exchange currents first took shape in the classical paper of Chemtob and Rho.[18] They gave a general classification of the structure of the two-particle exchange current and analyzed in detail the case of single-pion exchange, using the results of Adler[19] on the electro-, photo-, and weak production of pions on the nucleon. Among the contributions of the different single-boson exchanges, the contribution of the single-pion operator is as a rule decisive, and therefore the basic concepts of the theory of exchange currents can be introduced conveniently by considering this example. Other exchanges are treated similarly.

The operator of the single-pion exchange current is shown graphically by the Feynman diagram in Fig. 1.

Here, $\hat{J}(k)$ may be either a vector current [isovector $\hat{J}^j_\rho(k)$, isoscalar $\hat{J}^S_\rho(k)$] or an axial-vector current $\hat{J}^j_{5\lambda}(k)$. It is assumed that the standard Feynman rules can be used in the calculations. The fact that the nucleons in the initial or final state are nuclear particles, i.e., they are not free, is taken into account by using nuclear wave functions as the bra and ket vectors.

The general diagram in Fig. 1 is in reality a set of many Feynman graphs corresponding to different elementary contributions to the amplitude for pion production on the nucleon by the current $\hat{J}(k)$. A detailed analysis of this important amplitude using the methods of current algebra will be made later. Here, we describe the basic graphs that are taken into account in some way or other in any model calculation of the operator of the single-pion exchange current (Fig. 2).

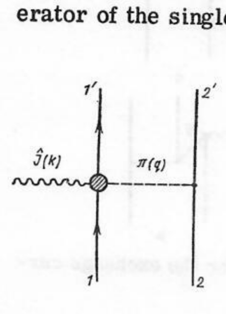

FIG. 1. Exchange current of single-pion range.

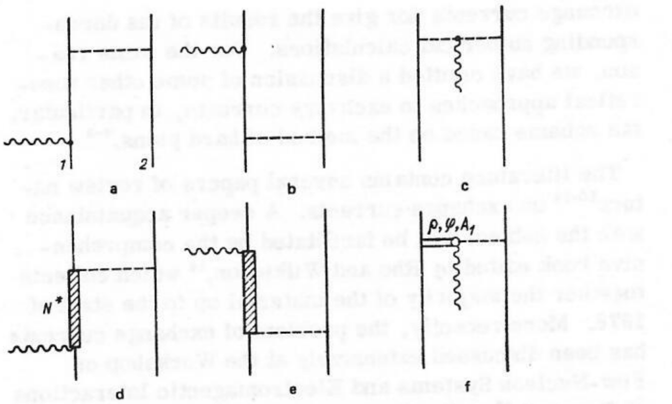

FIG. 2. Individual contributions to the single-pion exchange current.

The first three diagrams are generated by the Born part of the amplitude of pion production by the current $\hat{J}(k)$. The remaining graphs reflect the contribution to this amplitude of low-lying many-particle states—the isobar $\Delta(1236)$, vector mesons $(\rho, \omega, \varphi, A_1, \ldots)$.

We define the exchange-current operator $\tilde{J}^{(2)}_{\lambda(5\lambda)}$ from the S-matrix element of the Feynman diagram as follows:

$$S = -(2\pi i)\,\delta^4(P_i - P_f)\,\tilde{J}^{(2)}_{\lambda(5\lambda)}\varepsilon_\lambda. \tag{8}$$

For the electromagnetic interaction (with a real photon), ε_λ denotes the polarization vector of the photon; for the weak interaction, it denotes the lepton current L_λ.

Since the nuclear wave functions are nonrelativistic, it is necessary to go to the nonrelativistic limit in the exchange-current operator as well. Then instead of diagrams 2a and 2b we obtain a set of time-ordered diagrams (Fig. 3). Diagrams 3a–3d should not be included in the exchange-current operator, since they are already taken into account in the correctly constructed nuclear wave function (together with the part of diagrams 2a and 2b that contains the positive-frequency part of the nucleon propagator).

The part of the exchange current associated with diagrams 3e and 3f is called the recoil current. The problem of the correct allowance for this current has not yet been solved satisfactorily. We shall return to a fuller discussion of it later. The exchange current represented by diagrams 3g and 3h is called the pair term.

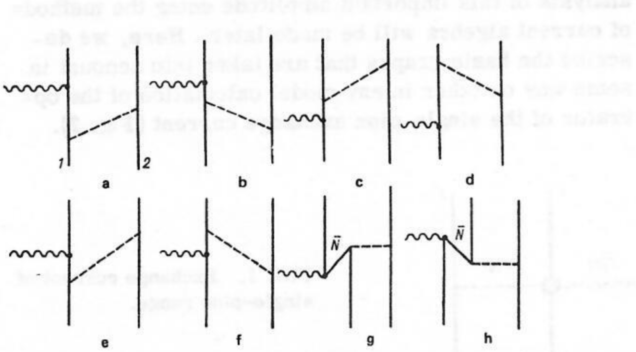

FIG. 3. Nonrelativistic approximation for the exchange current 2a and 2b.

After the transition in (8) to the nonrelativistic limit, the exchange-current operator $\tilde{J}^{(2)}_{\lambda(5\lambda)}$ can be transformed to the coordinate representation $J^{(2)}_{\lambda(5\lambda)}$ in accordance with

$$\begin{aligned} &J^{(2)}_{\lambda(5\lambda)}(k, \mathbf{r}_1, \mathbf{r}_2)\,\delta(\mathbf{r}_1 - \mathbf{r}'_1)\,\delta(\mathbf{r}_2 - \mathbf{r}'_2) = \langle \mathbf{r}'_1, \mathbf{r}'_2 | J^{(2)}_{\lambda(5\lambda)} | \mathbf{r}_1, \mathbf{r}_2 \rangle \\ &= \frac{1}{(2\pi)^{12}} \int d\mathbf{p}'_1\, d\mathbf{p}'_2\, d\mathbf{p}_1\, d\mathbf{p}_2 \exp[-i(-\mathbf{r}'_1\mathbf{p}'_1 - \mathbf{r}'_2\mathbf{p}'_2 + \mathbf{r}_1\mathbf{p}_1 + \mathbf{r}_2\mathbf{p}_2)] \\ &\times (2\pi)^3\,\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k} - \mathbf{p}'_1 - \mathbf{p}'_2)\,\tilde{J}^{(2)}_{\lambda(5\lambda)} + (1 \leftrightarrow 2). \end{aligned} \tag{9}$$

If the current $\tilde{J}^{(2)}_{\lambda(5\lambda)}$ depends only on the difference $\mathbf{q} = \mathbf{p}'_i - \mathbf{p}_i$, then for $J^{(2)}_{\lambda(5\lambda)}$ one can obtain the expression

$$J^{(2)}_{\lambda(5\lambda)} = \frac{\exp(-i\mathbf{k}\mathbf{r}_1)}{(2\pi)^3} \int d^3q \exp(i\mathbf{q}\mathbf{r})\,\tilde{J}^{(2)}_{\lambda(5\lambda)}(\mathbf{q}) + (1 \leftrightarrow 2). \tag{10}$$

The explicit dependence of $\tilde{J}^{(2)}_{\lambda(5\lambda)}$ on the momenta $\mathbf{p}_i$ and $\mathbf{p}'_i$ leads to the appearance in $J^{(2)}_{\lambda(5\lambda)}$ of velocity terms.

This approach to the single-pion exchange currents (it is called the Chemtob–Rho method for the S-matrix method) was widely used in calculations of reactions and static characteristics of light nuclei at the beginning of the seventies. At that time, there were effectively no other approaches to exchange currents; they were developed later. The Chemtob–Rho method proved very fruitful. Its use made it possible in many cases to eliminate discrepancies between experimental data and calculations in traditional nuclear physics. Particularly satisfactory agreement was achieved for processes with the participation of vector isovector exchange currents (isovector magnetic moment of the three-nucleon system, the radiative capture reaction $n + p \to d + \gamma$, $n + d \to t + \gamma$, the reactions $e + d \to n + p + e'$, $\gamma + d \to n + p$). The results of the actual calculations are discussed in the following sections.

1.2. Low-energy theorems for the amplitudes of pion production on the nucleon and the structure of the single-pion exchange current

Having outlined the Chemtob–Rho method, we turn to a detailed study of the amplitude of pion production on the nucleon by the current $\hat{J}(k)$. As we have already noted, this amplitude plays a fundamental part in the construction of the operator of the single-pion exchange current.

Formally, it correspond to the process

$$J(k) + N(p_1) \to \pi^n(q) + N(p_2) \tag{11}$$

and admits a standard representation in the form of a current matrix element[1)]:

$$T^{nj}_{\lambda(5\lambda)}(q, k) = \langle N(p_2)\,\pi^n(q) | J^j_{\lambda(5\lambda)}(0) | N(p_1) \rangle; \tag{12}$$

$$T^{nS}_\lambda(q, k) = \langle N(p_2)\,\pi^n(q) | J^S_\lambda(0) | N(p_1) \rangle. \tag{13}$$

Here, the indices n and j denote the isospin states of the pion and the current. The index S denotes the isoscalar current.

The matrix elements (12) and (13) have the isospin structure

$$T^{nj}_{\lambda(5\lambda)} = \bar{u}(p_2)\{a^{(+)}_{nj} T^{(+)}_{\lambda(5\lambda)} + a^{(-)}_{nj} T^{(-)}_{\lambda(5\lambda)}\}\, u(p_1); \tag{14}$$

$$\left.\begin{aligned} T^{nS}_\lambda &= (1/2)\,\bar{u}(p_2)\,\tau_n T^0_\lambda u(p_1); \\ a^{(\pm)}_{nj} &= (1/4)\,[\tau_n, \tau_j]_\pm. \end{aligned}\right\} \tag{15}$$

[1)]An invariant normalization of the states is used: $\langle N(\mathbf{P}) | N(\mathbf{P}') \rangle = (2\pi)^3 \delta(\mathbf{P} - \mathbf{P}') P_0/M$; $\langle \pi(\mathbf{q}) | \pi(\mathbf{q}') \rangle = (2\pi)^3 2q_0 \delta(\mathbf{q} - \mathbf{q}')$.

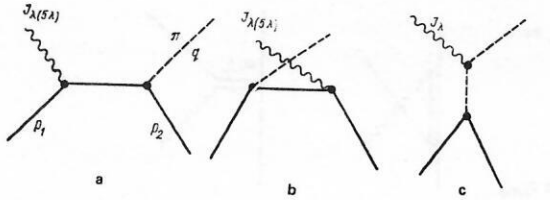

FIG. 4. Amplitude of pion production by the current $\hat{J}_{\lambda(5\lambda)}$ on the nucleon in the Born approximation.

Each of the amplitudes $T^{(\pm)}_{\lambda(5\lambda)}$, T^{0}_{λ} can also be decomposed with respect to the invariant operators O_λ in the spin space:

$$T_\lambda=\sum_{i=1}^{6}V_iO_\lambda(V_i);\quad T_{5\lambda}=\sum_{i=1}^{8}A_iO_\lambda(A_i), \tag{16}$$

where the coefficients V_i and A_i depend only on the kinematic invariants of the reaction. The explicit form of the operators O_λ is given in Refs. 18 and 19. Note that when all the particles in the radiative amplitude T_λ are on the mass shell it contains only four terms.[19]

The amplitudes (12) and (13) are made up of two parts—the Born and the non-Born:

$$T^{nj}_{\lambda(5\lambda)}(q,k)=[T^{nj}_{\lambda(5\lambda)}(q,k)]_{\rm B}+\overline{T}^{nj}_{\lambda(5\lambda)}(q,k); \tag{17}$$

$$T^{nS}_{\lambda}(q,k)=[T^{nS}_{\lambda}(q,k)]_{\rm B}+\overline{T}^{nS}_{\lambda}(q,k). \tag{18}$$

By definition, the Born term is a sum of pole graphs containing single-particle (nucleon and pion) intermediate states of renormalized perturbation theory (Fig. 4). In this way, the non-Born part of the amplitude T is also defined.[2)]

Born approximation. The Born amplitudes can be parametrized in a model-independent manner. For pseudoscalar πNN coupling

$$\begin{aligned}[T^{nj}_{\lambda}(q,k)]_{\rm B}=&-g\bar{u}(p_2)\Big\{\tau_n\gamma_5\frac{1}{{\rm i}(\not{p}_1+\not{k})+M}\hat{J}^{j}_{\lambda}(k)\\&+\hat{J}^{j}_{\lambda}(k)\frac{1}{{\rm i}(\not{p}_2-\not{k})+M}\gamma_5\tau_n\\&-2{\rm i}a^{(-)}_{nj}\frac{(2q-k)_\lambda}{(q-k)^2+m^2}F^{V}_{\pi}(k^2)\gamma_5\Big\}u(p_1);\end{aligned} \tag{19a}$$

$$\begin{aligned}[T^{nj}_{5\lambda}(q,k)]_{\rm B}=&-g\bar{u}(p_2)\Big\{\tau_n\gamma_5\frac{1}{{\rm i}(\not{p}_1+\not{k})+M}\hat{J}^{j}_{5\lambda}(k)\\&+\hat{J}^{j}_{5\lambda}(k)\frac{1}{{\rm i}(\not{p}_2-\not{k})+M}\gamma_5\tau_n\Big\}u(p_1);\end{aligned} \tag{19b}$$

$$\begin{aligned}[T^{nS}_{\lambda}(q,k)]_{\rm B}=&-g\bar{u}(p_2)\Big\{\tau_n\gamma_5\frac{1}{{\rm i}(\not{p}_1+\not{k})+M}\hat{J}^{S}_{\lambda}(k)\\&+\hat{J}^{S}_{\lambda}(k)\frac{1}{{\rm i}(\not{p}_2-\not{k})+M}\gamma_5\tau_n\Big\}u(p_1),\end{aligned} \tag{19c}$$

where M and m are the nucleon and pion masses. The operators of the isovector currents $\hat{J}^{j}_{\lambda(5\lambda)}(k)$ and the isoscalar current $\hat{J}^{S}_{\lambda}(k)$ in the spin–isospin space [the Fourier transforms of the single-nucleon parts of the currents $J^{j}_{\lambda(5\lambda)}(x)$, $J^{S}_{\lambda}(x)$] can be written in the form

$$\hat{J}^{j}_{\lambda}(k)=(1/2)\tau_j[F^{V}_{1}(k^2)\gamma_\lambda-(1/2M)F^{V}_{2}(k^2)\sigma_{\lambda\mu}k_\mu]; \tag{20}$$

$$\hat{J}^{j}_{5\lambda}(k)=(1/2)\tau_j[F_A(k^2)\gamma_\lambda-{\rm i}F_p(k^2)k_\lambda]\gamma_5; \tag{21}$$

$$\hat{J}^{S}_{\lambda}(k)=\frac{1}{2}\Big[F^{S}_{1}(k^2)\gamma_\lambda-\frac{1}{2M}F^{S}_{2}(k^2)\sigma_{\lambda\mu}k_\mu\Big]. \tag{22}$$

[2)] In the case of production by the axial current, there exists a special type of pole diagram corresponding to the contribution of the single-pion pole to the axial current. We associate the diagrams of this kind in which the virtual pion line is joined to a nonpole graph with the non-Born amplitude.

Here, the form factors are normalized as follows:

$$\begin{aligned}&F^{S}_{1}(0)=F^{V}_{1}(0)=F^{V}_{\pi}(0)=1;\quad F^{V}_{2}(0)=\varkappa_V=3.70;\\&F^{S}_{2}(0)=\varkappa_S=-0.12;\quad F_A(0)=g_A=1.25,\end{aligned} \tag{23}$$

where $\varkappa_V$ and $\varkappa_S$ are the isovector and isoscalar anomalous magnetic moments of the nucleon, and g_A is the axial constant of β decay of the nucleon. The form factor of the induced pseudoscalar $F_p(k^2)$ can be represented as the sum of a pion pole term and a nonpole part $\tilde{F}_p(k^2)$:

$$F_p(k^2)=2f_\pi g/(k^2+m^2)+\tilde{F}_p(k^2). \tag{24}$$

Here, $f_\pi\approx 94$ MeV is the weak decay constant of the pion, and it is related to g_A by the Goldberger–Treiman relation $f_\pi=(g_A/g)M$.

It is usually assumed that the form factors $F^{V}_{1}(k^2)$ and $F^{V}_{2}(k^2)$ have a dipole dependence on the momentum transfer, i.e., the dependence is described by $(1+k^2/m^2_V)^{-2}$, where $m^2_V=0.71$–0.84 GeV/c.[20] For the axial-vector form factor $F_A(k^2)/F_A(0)=(1+k^2/m^2_A)^{-2}$, but the mass m_A is not determined so well as m_V: $m^2_A=1$–1.5 $({\rm GeV}/c)^2$.[20] If it is assumed that in the limit of a pion with zero mass the axial current is conserved (the hypothesis of partial conservation of the axial current; see below), then the form factor $\tilde{F}_p(k^2)$ has the representation

$$\tilde{F}_p(k^2)=2M[F_A(k^2)-g_A]/k^2 \tag{25}$$

[the relation (25) is satisfied up to small correction terms $\sim m^2$, which arise when the pion pole in the axial current is shifted from the point $k^2=0$ to the point $k^2=-m^2$].

Since the vector currents $J^{j}_{\lambda}(x)$ and $J^{S}_{\lambda}(x)$ are strictly conserved, the corresponding total amplitudes must satisfy the transversality conditions (on the pion mass shell, i.e., for $q^2=-m^2$)

$$k_\lambda T^{nj}_{\lambda}(q,k)=0; \tag{26}$$

$$k_\lambda T^{nS}_{\lambda}(q,k)=0. \tag{27}$$

With allowance for the explicit structure (19a) of the Born amplitude $[T^{nj}_{\lambda}]_{\rm B}$, the condition (26) yields a low-energy theorem of the type of Low's theorem,[21] which relates the non-Born part of the isovector amplitude to the parameters of its Born part:

$$\begin{aligned}k_\lambda\overline{T}^{nj}_{\lambda}(q,k)&=-k_\lambda[T^{nj}_{\lambda}]_{\rm B}\\&=-2{\rm i}g\bar{u}(p_2)a^{(-)}_{nj}\gamma_5[F^{V}_{1}(k^2)-F^{V}_{\pi}(k^2)]u(p_1).\end{aligned} \tag{28}$$

It is readily seen that the Born part of the isoscalar amplitude satisfies the transversality condition by itself. Therefore, the non-Born part is also transverse by virtue of (27):

$$k_\lambda\overline{T}^{nS}_{\lambda}(q,k)=0. \tag{29}$$

Since $F^{V}_{1}(0)=F^{V}_{\pi}(0)=1$, it follows from (28) and (29) that the expansion of the amplitudes $\overline{T}^{nj}_{\lambda}$ and $\overline{T}^{nS}_{\lambda}$ with respect to the 4-momentum k_μ begins with the linear terms. This assertion is essentially equivalent to the Kroll–Ruderman theorem,[22] according to which the amplitude of pion photoproduction at threshold, i.e., at $k_\mu\sim m$, is, up to small terms $\sim m/m_{\rm char}$, given by the Born graphs alone. We emphasize that it is only in the case $F^{V}_{1}(k^2)=F^{V}_{\pi}(k^2)$ that the Born part of the isovector amplitude T_{λ}^{nj} is by itself transverse. This is the situation in the vector-dominance model, in which F_1^V and F_π^V are described by the single monopole formula

$$F_1^V(k^2) = F_\pi^V(k^2) = (1 + k^2/m_V^2)^{-1}.$$

As is shown in Gourdin's review,[20] the behavior of the form factors F_1^V and F_π^V predicted by vector dominance agrees well with the data on pion electroproduction up to $q^2 \approx 2\ (\mathrm{GeV}/c)^2$.

Concluding our discussion of the Born approximation, we make a remark which will be important in connection with the uniqueness of the decomposition of the isovector amplitude T_λ^{nj} into the Born and non-Born parts. It is obvious that on the pion mass shell $q^2 = -m^2$ the decomposition is unique, since all the pole contributions are exhausted by the graphs in Fig. 4, and the vertices in these graphs can be expressed in terms of the residues of the total amplitude at the corresponding poles. In particular, the form factor $F_\pi^V(k^2)$ is the residue at the pion pole (see Fig. 4c). The total contribution of the pole graphs is equal to the amplitude (19a). At the same time, if the 4-momentum q_μ is off the mass shell (it is necessary to leave the mass shell in, for example, the method of current algebra), then not all the pole contributions are taken into account by the expression (19a). The point is that in the general case the graph 4c contains the off-shell amplitude $\pi^m(q-k) + \hat{J}_\lambda^j(k) \to \pi^n(q)$:

$$\varepsilon^{njm}\mathfrak{M}_\lambda(q, k) = \mathrm{i}(q^2+m^2)\int d^4x \exp(-iqx) \times \langle 0 | T\{\pi^n(x) J_\lambda^j(0)\} | \pi^m(q-k)\rangle, \quad (30)$$

which only for $q^2 = -m^2$ reduces to the form factor $F_\pi^V(k^2)$:

$$\mathfrak{M}_\lambda(q^2 = -m^2) = (2q-k)_\lambda F_\pi^V(k)^2. \quad (31)$$

Thus, if the definition (19a) is retained for $[T_\lambda^{nj}]_B$ off the mass shell, then the off-shell part of the non-Born amplitude of pion production will necessarily contain pole terms $\sim(q^2+m^2)/[(q-k)^2+m^2]$ generated by the terms in the expression (30) that remain after separation from it of the vertex (31). Evidently, it is more consistent to define the Born amplitude for $q^2 \neq -m^2$ in such a way that in this case too it contains all the pole terms and on the mass shell is equal to the amplitude (19a). This can be done by replacing the vertex (31) in (19a) by the value of the amplitude (30) at the pion pole $(q-k)^2 = -m^2$. In what follows, we shall adopt such a point of view.

Non-Born amplitudes. We now consider the non-Born parts of the amplitudes (12) and (13). They are determined by the contribution of the many-particle intermediate states, and therefore the analysis of them is a much more complicated and model-dependent problem than the analysis of the Born parts. Here, there exist several approaches. One of them consists of considering diagrams of the type shown in Fig. 5, which take into account the contributions of excited nucleon states (isobars) and heavy mesons $(\omega, \rho, \varphi, A_1)$. Such diagrams are calculated in the most systematic and consistent manner in the hard-pion model.[23,24] This model consists of a list of simple rules that enable one to write down for any process the complete

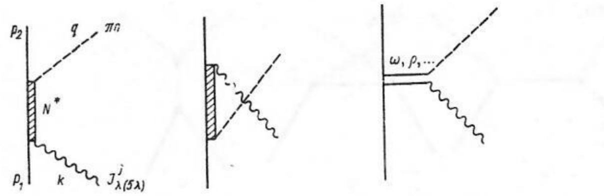

FIG. 5. Non-Born part of the amplitude of pion production by the current $J_{\lambda(5\lambda)}^j$.

set of diagrams required by the dynamical principles of vector dominance and chiral symmetry that underlie this model.[3)] Another approach, which is to a considerable degree model-independent, is based on the low-energy (soft pions) theorems of $SU(2)\times SU(2)$ current algebra (see, for example, Refs. 25 and 26). Although these theorems fix only the leading terms in the momenta k and q in the momentum expansion of the amplitudes $\bar{T}_{\lambda(5\lambda)}^{nj}(q,k)$, $\bar{T}_\lambda^{nS}(q,k)$, this is sufficient in many cases. There are two ways of deriving low-energy theorems. One, the most perspicuous, proceeds from the concept of phenomenological chiral Lagrangians (see, for example, Refs. 24, 27, and 28 and, in application to problems of exchange currents, Refs. 6 and 8). We shall use a different and more traditional method based on the standard reduction technique.[25,26]

There are two types of low-energy theorem: for matrix elements with emission and absorption of "soft" pions and for the matrix elements of "soft" currents with given divergences. The relations of the second type are called generalized Low theorems (their derivation and numerous applications are considered in detail in the classical paper of Adler and Dothan[29]). Both forms of the low-energy theorems[19] apply in this situation. The theorems of the first type can be used to estimate the amplitudes (12) and (13) in the limit $q_\mu \to 0$, while the theorems for the currents are constructive in the limit $k_\mu \to 0$.

The limit $q_\mu = 0$. We consider first the case of isovector currents J_λ^j and $J_{5\lambda}^j$. We reduce the pion in the matrix element (12) off the mass shell, use the condition of partial conservation of the axial current (PCAC),

$$\partial_\mu J_{5\mu}^n(x) = m^2(g_A M/g)\pi^n(x), \quad (32)$$

and go to the limit $q_\mu \to 0$. As a result, we obtain for the amplitude (12) in this limit the representation

$$T_{\lambda(5\lambda)}^{nj}(q,k) \xrightarrow[q\to 0]{} -\frac{g}{Mg_A}\left\{\langle N(p_2)|[F_5^n(0), J_{\lambda(5\lambda)}^j(0)]_-|N(p_1)\rangle + q_\mu\int d^4x \exp(-iqx)\langle N(p_2)|T\{J_{5\mu}^n(x) J_{\lambda(5\lambda)}^j(0)\}|N(p_1)\rangle\right\}, \quad (33)$$

where $F_5^n(0) = \int d^3x J_{54}^n(\mathbf{x}, 0)$ is the axial charge taken for $x_4 = 0$. The expression (33) is the sum of singular pole terms corresponding to the Born graph in Fig. 4 and a nonpole term. Since we define the Born amplitude here in terms of the pseudoscalar πNN coupling, it is necessary to go over in the pole terms from the pseudovector vertices $\sim\bar{q}\gamma_5\tau_n$, which correspond to insertions of the axial current $\hat{J}_{5\mu}^n$ in the external lines of the matrix element $\langle N(p_2)|J_{\lambda(5\lambda)}^j(0)|N(p_1)\rangle$, to the pseudoscalar

[3)]The hard-pion method was applied to the calculation of exchange currents in Refs. 6–8.

vertices $\sim\gamma_5\tau_n$. Finally, we have

$$T^{nj}_{\lambda(5\lambda)}(q,k)\xrightarrow[q\to 0]{}\bar{u}(p_2)\Big\{\frac{g}{Mg_A}\hat{J}^{nj}_{\lambda(5\lambda)}(k-q)$$
$$+\frac{g}{2M}[\gamma_5\tau_n,\hat{J}^{j}_{\lambda(5\lambda)}(k)]_+-2ig\gamma_5 a^{(-)}_{nj}\frac{q_\mu q\mathfrak{M}_{5\mu,\lambda}(q,k)}{(q-k)^2+m^2}C_{\lambda(5\lambda)}$$
$$+g\gamma_5\tau_n\frac{i\bar{q}}{2p_2q}\hat{J}^{j}_{\lambda(5\lambda)}(k)+\hat{J}^{j}_{\lambda(5\lambda)}(k)\frac{i\bar{q}}{2p_1q}g\gamma_5\tau_n\Big\}u(p_1)\psi^*_n+O(q), \quad (34)$$

where $C_\lambda=1$, $C_{5\lambda}=0$, and ψ^*_n is the isospin wave function of the pion.

We now consider the origin of the different terms in (34) and discuss their meaning. The first term is the commutator term:

$$\langle N(p_2)|[F^n_5(0),J^j_{\lambda(5\lambda)}(0)]_-|N(p_1)\rangle$$
$$=-\bar{u}(p_2)\hat{J}^{nj}_{\lambda(5\lambda)}(k-q)u(p_1). \quad (35)$$

By virtue of the current-algebra commuation relations

$$[F^n_5(0),J^j_{\lambda(5\lambda)}(0)]_-=-\varepsilon^{njm}J^m_{5\lambda(\lambda)} \quad (36)$$

this term reduces to the expressions (21) or (20). In particular,

$$\hat{J}^{nj}_\lambda(k-q)=a^{(-)}_{nj}\Big\{F_A[(k-q)^2]\gamma_\lambda-i\Big[\frac{2Mg_A}{(q-k)^2+m^2}$$
$$+\tilde{F}_P[(q-k)^2]\Big](k-q)_\lambda\Big\}\gamma_5, \quad (37)$$

where for the form factor of the induced pseudoscalar the representation (24) is used. The last two terms in (34) are the singular contributions of the Born nucleon graphs (see Figs. 4a and 4b). They arise from the second term on the right-hand side of (33). The anticommutator $[\gamma_5\tau_n,\hat{J}^j_{\lambda(5\lambda)}(k)]_+$ arises on the transition to the pseudoscalar πNN coupling (we shall call it the "PCAC term").[4)] The term with the pion propagator present in the vector amplitude $T^{nj}_\lambda(q,k)$ corresponds to the pion pole contribution to the matrix element of the time-ordered product of the currents in the relation (33) when that relation contains the vector current $J^j_\lambda(0)$ (Fig. 6).

The amplitude $\mathfrak{M}_{5\mu,\lambda}(q,k)$ in the numerator of this term is determined by

$$\varepsilon^{nj t}\mathfrak{M}_{5\mu,\lambda}(q,k)=\frac{1}{f_\pi}\int d^4x\exp(-iqx)$$
$$\times\langle 0|T\{J^n_{5\mu}(x)J^j_\lambda(0)\}|\pi^t(q-k)\rangle, \quad (38)$$

where, as before, $f_\pi=(g_A/g)M$. The structure of this amplitude in the framework of PCAC and current algebra is discussed in detail in the review of Ref. 30. Using the results given there, one can show that

$$\mathfrak{M}_{5\mu,\lambda}(q,k)=-\Big[\delta_{\mu\lambda}+2(k_\mu k_\lambda-k^2\delta_{\mu\lambda})\frac{1-F^V_\pi(k^2)}{k^2}\Big]$$
$$-k_\lambda q_\mu+O(q^2), \quad (39)$$

whence

$$q_\mu\mathfrak{M}_{5\mu,\lambda}=(q-k)_\lambda-(2q-k)_\lambda F^V_\pi(k^2)$$
$$+[(q-k)^2-q^2]k_\lambda\frac{1-F^V_\pi(k^2)}{k^2}-k_\lambda q^2+O(q^3) \quad (40)$$

(we have ignored in (39) the terms $\sim[k_\mu q_\lambda-(qk)\delta_{\mu\lambda}]$,[30] which do not contribute to the contraction $q_\mu\mathfrak{M}_{5\mu,\lambda}$). It is readily seen that the part of the total pion pole contribution to the amplitude (34) that arises from the commutator term (37) and the first two terms in the expression (40) corresponds exactly to the contribution

---

[4)]The sum of the PCAC term and the Born terms is equal to the Born amplitude for pseudovector πNN coupling.

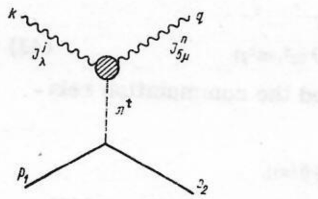

FIG. 6. Pion pole contribution to the matrix element $\int d^4\exp(-iqx)\langle N(p_2)|T\{J^n_{5\mu}(x)J^j_\lambda(0)\}|N(p_1)\rangle$.

of the graph in Fig. 4c to the amplitude (19a) at $q^2=-m^2$. The remaining part is associated with off-shell effects. Note that the fourth term $(-k_\lambda q^2)$ in Eq. (40) can be omitted, since its contribution is exactly compensated by the analogous contribution from the commutator term if the reduction factor $(q^2+m^2)/m^2$ in this last term is retained before the passage to the limit $q_\mu\to 0$. Note that in the pion terms of the vector amplitude the leading dependence on q_μ is retained as in the singular nucleon terms. The point is that to current-algebra accuracy the pion mass is effectively indistinguishable from zero, and for this reason the contribution of the terms with the pion pole $1/[(q-k)^2+m^2]$ in the region $q_\mu\sim 0, k^2\sim 0$ is, strictly speaking, singular. Note also that the limiting value of the vector amplitude is transverse with respect to the 4-momentum k_μ only up to the neglected regular terms linear in q (off the mass shell, current conservation leads not to the transversality condition but to the Ward identities). If, however, we set $q_\mu=0$ and $m^2=0$ in all the terms in (34) that are regular for $k^2\neq 0$ and assume that $k_\mu=(p_2-p_1)_\mu$, then the condition (26) is rigorously satisfied (in a world with massless pions, such an amplitude would be physical and correspond to production of a pion with zero energy by the current J^j_λ).

Using the representation (34), we can write down explicit expressions for the non-Born parts of the amplitude (12) in the limit $q_\mu=0$ in terms of the current form factors J^j_λ and $J^j_{5\lambda}$. If we adopt the standard hypothesis of the possibility of smooth extrapolation of the nonpole amplitudes from the unphysical region $q\sim 0$ to the region $q\sim m$, it is to be expected that the obtained expressions will be good approximations for the physical values of the non-Born terms (i.e., taken at $q^2=-m^2$). For the non-Born amplitudes, we shall use the parametrization

$$\bar{T}^{nj}_\lambda(q=0,k)=\bar{u}(p_2)\{a^{(+)}_{nj}\varphi(k^2)\gamma_5\sigma_{\lambda\mu}k_\mu$$
$$+a^{(-)}_{nj}[\beta(k^2)\gamma_5\gamma_\lambda-i\varkappa(k^2)\gamma_5k_\lambda]\}u(p_1); \quad (41)$$
$$\bar{T}^{nj}_{5\lambda}(q=0,k)=i\bar{u}(p_2)\{a^{(+)}_{nj}\rho(k^2)k_\lambda$$
$$+a^{(-)}_{nj}[i\lambda(k^2)\sigma_{\lambda\nu}k_\nu-2i\alpha(k^2)M\gamma_\lambda]\}u(p_1). \quad (42)$$

To obtain the required relations, it is necessary to compare (41) and (42) with (34), having first separated from (34) the Born contributions. As we have already noted, accuracy is needed in the treatment of the contribution of the Born pion graph (see Fig. 4c) when the external pion line does not lie on the mass shell. In this case, as the vertex for absorption of the vector current by the pion it is necessary to take the value of the amplitude $\mathfrak{M}_\lambda(k,q)$ (30) at the point $(q-k)^2=-m^2$. The condition of conservation of the vector current fixes the amplitude $\mathfrak{M}_\lambda(q,k)$ at this point to terms $\sim q_\mu(q^2+m^2)$:

$$\mathfrak{M}_\lambda(q,k) = (2q-k)_\lambda F_\pi^V(k^2) + k_\lambda (q^2+m^2)[1-F_\pi^V(k^2)]/k^2 + O(q^3, m^2 q) \qquad (43)$$

(in deriving (43), we have used the commutation relation

$$[J_4^j(\mathbf{x},0), \pi^n(0)]_- = -\varepsilon^{jnm}\pi^m(0)\,\delta(\mathbf{x})).$$

It can be seen from comparison of Eqs. (40) and (43) that the pion terms in the limiting amplitude T_λ^{nj} (34) contain not only the purely pole contribution $\sim\mathfrak{M}_\lambda(k,q)/(q-k)^2+m^2$ but also the nonpole contribution $\sim[1-F_\pi^V(k^2)]/k^2$. Comparing the amplitudes (41) and (42) with the expression (34) in the light of this remark, we obtain

$$\left.\begin{aligned}
\varphi(k^2) &= -(g/2M^2)F_2^V(k^2);\\
\beta(k^2) &= -(g/Mg_A)[F_A(k^2)-g_A F_1^V(k^2)];\\
\varkappa(k^2) &= \frac{g}{Mg_A}\tilde{F}_p(k_2) + 2g\frac{1-F_\pi^V(k^2)}{k^2}\\
&= 2\frac{g}{g_A}\frac{F_A(k^2)-g_A F_\pi^V(k^2)}{k^2};
\end{aligned}\right\} \qquad (44)$$

$$\left.\begin{aligned}
\rho(k^2) &= -(g/M)F_p(k^2);\\
\lambda(k^2) &= (g/2M^2 g_A)F_2^V(k^2);\\
\alpha(k^2) &= (g/2M^2 g_A)[F_1^V(k^2)-g_A F_A(k^2)].
\end{aligned}\right\} \qquad (45)$$

In deriving the last of the relations (44), we have used the representation (25). Note that the amplitude (41) with the form factors (44) satisfies the condition (28) for $k_\mu=(p_2-p_1)_\mu$. The Kroll–Ruderman theorem is satisfied explicitly, since it follows from (44) that the amplitude (41) begins with terms linear in k_μ. At the same time, the form factor $\beta(k^2)$ begins with quadratic terms:

$$\beta(k^2) = -(g/Mg_A)[F_A'(0)-g_A F_1^{V\prime}(0)]k^2 + O(k^4).$$

In the limit $k_\mu\to 0$, the form factor $\alpha(k^2)$ in the amplitude (42) reduces to the expression

$$\alpha(0) = (g/2M^2 g_A)(1-g_A^2). \qquad (46)$$

One can show that the low-energy theorem (46) is none other than the well-known Adler–Weisberger relation[31] (see below).

We now consider the isoscalar amplitude $T_\lambda^{nS}(q,k)$ (13). Its Born part is determined by Eq. (19c). The non-Born part in the limit $q_\mu\to 0$ can be calculated on the basis of PCAC and the commutation relation

$$[F_5^n(0), J_\rho^S(0)]_- = 0. \qquad (47)$$

Proceeding as in the derivation of the representation (34), we find

$$T_\lambda^{nS}(q,k) \xrightarrow[q\to 0]{} \bar{u}(p_2)\Big\{\frac{g}{2M}[\gamma_5\tau_n, \hat{J}_\lambda^S(k)]_+ + g\gamma_5\tau_n\frac{\mathrm{i}\bar{q}}{2p_2q}\hat{J}_\lambda^S(k) + \hat{J}_\lambda^S(k)\frac{\mathrm{i}\bar{q}}{2p_1q}g\gamma_5\tau_n\Big\}u(p_1)\psi_n^* + O(q). \qquad (48)$$

Thus, the non-Born part of the isoscalar amplitude in the limit $q_\mu\to 0$ contains only the PCAC term and reduces to the expression

$$\bar{T}_\lambda^{nS}(k, q=0) = -(g/4M^2)\bar{u}(p_2)\tau_n\gamma_5\sigma_{\lambda\mu}k_\mu F_2^S(k^2)u(p_1), \qquad (49)$$

where we have used the definition (22).

The limit $k_\mu=0$. We now turn to the discussion of the other limiting case $q_\mu\neq 0$, $k_\mu\to 0$. This limit is of interest for the following reasons. Since the pion in the graph in Fig. 1 is virtual, there are, in general, no grounds for ignoring the dependence of the corresponding vertices on q_μ (the nuclear wave functions may be sensitive to the momentum dependence of the operators of the exchange currents). At the same time, the 4-momentum k_μ corresponds to real particles (photons, leptons pairs, etc.) and in a number of cases (for example, in β decay) is small.

As we have already noted, the vector amplitudes T_λ^{nj} and T_λ^{nS} reduce in the limit $k_\mu\to 0$ to a sum of Born graphs, i.e., their non-Born parts in this limit vanish. In the case of the axial amplitude, this does not occur. Applying the PCAC condition to the current $J_{5\lambda}^j$, we can relate the non-Born part of the amplitude $T_{5\lambda}^{nj}$ at $k_\mu=0$ to the parameters of the off-shell amplitudes of πN scattering.[18,19]

For the amplitude $\bar{T}_{5\lambda}^{nj}(q,k=0)$ we choose a parametrization analogous to that used in (42):

$$\bar{T}_{5\lambda}^{nj}(q,k=0) = \mathrm{i}\bar{u}(p_2)\{a_{nj}^{(+)}\gamma(q^2)q_\lambda + a_{nj}^{(-)}[\mathrm{i}\beta(q^2)\sigma_{\lambda\nu}q_\nu - 2\mathrm{i}\alpha'(q^2)M\gamma_\lambda]\}u(p_1). \qquad (50)$$

From the PCAC condition (32) we find

$$\begin{aligned}
&\langle N(p_2)\pi^n(q)\,|\,\partial_\mu J_{5\mu}^j(0)|\,N(p_1)\rangle\\
&= -\mathrm{i}k_\mu\langle N(p_2)\pi^n(q)\,|\,J_{5\mu}^j(0)|\,N(p_1)\rangle\\
&= m^2M(g_A/g)\langle N(p_2)\pi^n(q)|\,\pi^j(0)|\,N(p_1)\rangle.
\end{aligned} \qquad (51)$$

The matrix element on the right-hand side of (51) is directly related to the πN scattering amplitude, in which the pion with isotopic index j and momentum k_μ is off the mass shell:

$$\langle N(p_2)\pi^n(q)\,|\,\pi^j(0)|\,N(p_1)\rangle = -[1/(k^2+m^2)]T_\pi^{nj}(q,k), \qquad (52)$$

where

$$T_\pi^{nj}(q,k) = 2\bar{u}(p_2)\{a_{nj}^{(+)}T^{\pi N(+)} + a_{nj}^{(-)}T^{\pi N(-)}\}u(p_1); \qquad (53)$$

$$\bar{u}(p_2)T^{\pi N(\pm)}u(p_1) = -\bar{u}(p_2)[A^{\pi N(\pm)}(\nu,\nu_B,q^2,k^2) - \mathrm{i}\not{k}B^{\pi N(\pm)}(\nu,\nu_B,q^2,k^2)]u(p_1). \qquad (54)$$

Here, ν and ν_B are the standard kinematic invariants

$$\nu \equiv -(p_1+p_2)k/2M;\quad \nu_B \equiv qk/2M. \qquad (55)$$

It is clear that the relation (51) must be satisfied separately for the pole and nonpole terms of the matrix elements on its left- and right-hand sides. Here, however, it is necessary to bear in mind a subtlety. The contraction of the Born amplitude $[T_{5\lambda}^{nj}]_B$ in (19b) with the 4-momentum k_μ contains both pole and nonpole parts. Indeed, using the obvious identities

$$\left.\begin{aligned}
\frac{1}{\mathrm{i}(\not{p}_1+\not{k})+M}\not{k}\gamma_5 &= -\mathrm{i}\gamma_5 + \frac{2\mathrm{i}M}{\mathrm{i}(\not{p}_1+\not{k})+M}\gamma_5;\\
\not{k}\gamma_5\frac{1}{\mathrm{i}(\not{p}_2-\not{k})+M} &= -\mathrm{i}\gamma_5 + \gamma_5\frac{2\mathrm{i}M}{\mathrm{i}(\not{p}_2-\not{k})+M},
\end{aligned}\right\} \qquad (56)$$

we can readily obtain

$$k_\lambda(T_{5\lambda}^{nj})_B = -\mathrm{i}\frac{Mg_A}{g}(T_\pi^{nj})_B\frac{m^2}{k^2+m^2} + \frac{2\mathrm{i}g_A}{M}\bar{u}(p_2)a_{nj}^{(+)}u(p_1) + O(k), \qquad (57)$$

where $(T_\pi^{nj})_B$ is the Born part of the amplitude (53):

$$(T_\pi^{nj})_B = -\bar{u}(p_2)\Big[\mathrm{i}g\tau_n\gamma_5\frac{1}{\mathrm{i}(\not{p}_1+\not{k})+M}\mathrm{i}g\tau_j\gamma_5 + \mathrm{i}g\tau_j\gamma_5\frac{1}{\mathrm{i}(\not{p}_2-\not{k})+M}\mathrm{i}g\tau_n\gamma_5\Big]u(p_1). \qquad (58)$$

On substitution in the relation (51), the pole term in (57) is exactly compensated by the Born part of the off-shell πN amplitude on the right. The nonpole part of the contraction (57) cancels with the zeroth term in k_μ in the non-Born part of the amplitude by virtue of the well-known self-consistency condition of Adler.[32] As a result, we obtain

$$k_\lambda \overline{T}^{nj}_{5\lambda} = -\mathrm{i}\,\frac{M g_A}{g}\,k_\lambda \frac{\partial}{\partial k_\lambda}\overline{T}^{nj}_{\pi}|_{k=0} + O(k^2), \tag{59}$$

where $\overline{T}^{nj}_{\pi}$ is the non-Born part of the amplitude (53). The relation (59) makes it possible to express the form factors γ, β, and α' in (50) in terms of the invariant amplitudes $\overline{A}^{\pi N(\pm)}$ and $B^{\pi N(\pm)}$ at the point $k_\mu = 0°$

$$\left.\begin{aligned} &\gamma(q^2) = (g_A/g)\,[\partial\overline{A}^{\pi N(+)}/\partial\nu_B]_{\nu=\nu_B=k^2=0};\\ &\beta(q^2) = (g_A/g)\,[\partial\overline{A}^{\pi N(-)}/\partial\nu]_{\nu=\nu_B=k^2=0};\\ &\alpha'(q^2) = (g_A/g)\,[\overline{B}^{\pi N(-)} + \partial\overline{A}^{\pi N(-)}/\partial\nu]_{\nu=\nu_B=k^2=0}. \end{aligned}\right\} \tag{60}$$

Comparison of the last of these relations with Eq. (46) shows that (46) really is the Adler–Weisberger relation[31]:

$$(1/g_A^2) - 1 = (2M^2/g^2)\,[\overline{B}^{\pi N(-)} + \partial\overline{A}^{\pi N(-)}/\partial\nu]_{\nu=\nu_B=k^2=0}.$$

It is in fact the condition of consistency of the low-energy expansions of the axial amplitude at the points $q_\mu = 0$ and $k_\mu = 0$.

Exchange currents. We now consider what are the consequences of the structure of the amplitudes $T^{nj}_{\lambda(5\lambda)}$ and T^{nS}_{λ} predicted by current algebra for the calculation of the exchange currents. We consider first the case of the vector isovector current. In the limit of small q_μ, the amplitude T^{nj}_λ, which must be substituted in the general diagram in Fig. 1, is given by the expression (34). As we have already said, the positive-frequency parts of the nucleon propagators in (34) must be included in the nuclear wave function. The remainder of the amplitude (34) generates the exchange current, whose parts corresponding to the different terms in (34) are shown graphically in Fig. 7. The first two graphs (7a and 7b) represent the pair term and arise from the last two terms in (34). Diagram 7d corresponds to the pion pole contribution to the amplitude $(T^{nj}_\lambda)_B$ (see the graph in Fig. 4c). In accordance with what we have said earlier, as vertex $\pi\hat{J}_\lambda\pi$ in this diagram it is necessary to substitute the off-shell amplitude (43). However, at small q_μ and k^2, the off-shell effects can be ignored and this vertex specified to good accuracy by the standard expression (31) [as can be seen from (43), there will be a difference in the small terms $\sim(1/6)m^2\langle r_\pi\rangle^2 \sim m^2/m_V^2$, where $\langle r_\pi\rangle^2$ is the mean-square radius of the pion]. The contact diagram of Fig. 7c can be obtained by substituting in the graph of Fig. 1 the non-Born part of the vector amplitude, i.e., the expression (41) with the form factors (44). At the

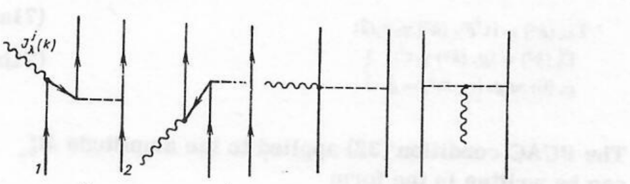

FIG. 7. Operator of the vector isovector exchange current constructed using the amplitude (34). The current is given either by the sum of diagrams a, b, and d or the sum c and d.

threshold, the PCAC term and the commutator term cancel each other almost completely [the Kroll–Ruderman theorem; see the discussion after Eqs. (44) and (45)], so that the contribution of this diagram is negligibly small, i.e., the exchange current is given by the diagram sum 7a+7b+7c. It is interesting to note that there is a different and equivalent graphical representation of the vector isovector exchange current of single-pion range. It can be shown that the PCAC term [the second term in the amplitude T^{nj}_λ (34)] in the nonrelativistic limit at the threshold exactly compensates the total contribution of the negative-frequency parts of the Born nucleon graphs. In other words, the contribution from the pair term (7a+7b) can be compensated by part of the contribution of the contact graph 7c, namely, the part associated with the PCAC term. As a result, the operator of the single-pion exchange current constructed on the basis of the soft-pion isovector amplitude T^{nj}_λ (34) can be represented equivalently either by the diagram sum 7a+7d or by the sum 7c+7d. In the latter case, the graph 7c contains the amplitude $\overline{T}^{nj}_\lambda$ with PCAC term subtracted. The "truncated" amplitude is equal to the non-Born amplitude corresponding to determination of the Born amplitude in terms of the pseudovector πNN coupling, and is given at the threshold by the expression (before the transition to the nonrelativistic limit)

$$\overline{T}^{nj'}_\lambda(q\sim 0, k\sim 0) = -\overline{u}(p_2)\,a^{(-)}_{nj}\,(g/M)\,\gamma_5\gamma_\lambda u(p_1). \tag{61}$$

As we have already noted, the use of such an exchange–current operator explained the majority of discrepancies between the calculations and the experimental data. The remaining discrepancies could be explained (with some reservations) by an exchange current with excitation of an isobar (see Figs. 2d and 2e). In the considered case, the amplitude (34) at the threshold is gauge invariant, and the exchange current in Fig. 7 has the advantage that together with the single-particle current it satisfies the charge conservation law (3) if the potential (6) is determined by single-pion exchange, i.e., has the form

$$V = \frac{1}{4\pi}\,\frac{g^2}{4M^2}\,(\tau_1\tau_2)\,(\sigma_1\nabla)\,(\sigma_2\nabla)\,\frac{\exp(-mr)}{r}. \tag{62}$$

Note that the operator of the vector exchange current with excitation of the isobar $\Delta(1236)$ is transverse by itself.

Summarizing, we can say that the case of the vector isovector currents can be well described by the low-energy theorem (34).

In the case of the axial-vector current, the situation is more complicated. In this case, the single-nucleon matrix element of the sum of the commutator term and the PCAC term has at the threshold the form

$$\overline{u}(p_2)\left\{\mathrm{i}\varepsilon_{njk}\frac{g}{Mg_A}\frac{1}{2}\tau_k\left[(1-g_A^2)\gamma_\lambda - \frac{\varkappa_V}{2M}\sigma_{\lambda\mu}(k-q)_\mu\right] - \mathrm{i}\,\frac{g}{2M}F_p(k^2)\,k_\lambda\delta_{nj}\right\}u(p_1). \tag{63}$$

At the first glance, it might appear that here, as in the vector amplitude, canceling occurs. However, since $g_A^2 \approx 1.5$, it is only partial. Moreover, having obtained the nonrelativistic approximation for the matrix element (63),

$$-\frac{i}{2M^2}\frac{g}{g_A}(1-g_A^2)(\mathbf{p}_1+\mathbf{p}_2)+\frac{1}{2M^2}\frac{g}{g_A}(1-g_A^2+\varkappa_V)\,\sigma\times\mathbf{q}, \quad (64)$$

we see that there is a large contribution from the isovector anomalous magnetic moment of the nucleon ($\varkappa_V = 3.7$).

Taking into account the result (64), one can show that the amplitude $T^{mj}_{5\lambda}(q,k)$ [see (34)] leads to an exchange current whose spatial and time components in the classification[4] in powers of $1/M \sim v/c$ have, respectively, the order $O(1/M^2)$ and $O(1/M)$. In addition, the spatial and time components of the single-particle axial current (21) have the order $\sim O(1)$ and $O(1/M)$. The situation here is the opposite to that which obtains in the case of the vector isovector current, for which the spatial and time components of such an exchange current are $\sim O(1/M)$ and $O(1/M)^2$, whereas the spatial and time components of the single-particle current (20) are $\sim O(1/M)$ and $O(1)$.

With regard to the isoscalar exchange currents, it can be seen from Eqs. (48) and (49) that an appreciable contribution arises from only the pair term. For both the spatial and the time component, this contribution is only the relativistic correction [$\sim O(1/M^2)$] to the impulse approximation (22). As we shall see later, calculation of corrections of this type requires particular care.

1.3. General low-energy theorem for the operator of the two-particle axial exchange current

Above, we have analyzed in detail in the framework of $SU(2)\times SU(2)$ current algebra the structure of the single-nucleon pion production amplitudes that occur as blocks in the operators of the single-pion exchange currents (see Fig. 1). However, the direct use of the results of current algebra for these amplitudes in the two-nucleon operators is not completely consistent. In particular, the kinematics of the process in Fig. 1 is such that part of the total dependence of the corresponding exchange-current operator on the 4-momentum k_μ is associated with the 4-momentum of a virtual pion, whereas this additional source of k dependence was not taken into account in the derivation of the low-energy theorems (34) and (59). Let us explain this. The observable variables in the process in Fig. 1 are the 4-momentum K_μ (corresponding to a photon or a lepton pair) and the nucleon 4-momenta; as a basis set of these momenta, it is convenient to take

$$\left.\begin{aligned}&\hat{p}_1=(p_1+p_1')/2;\quad \hat{q}=(p_1-p_1'+p_2'-p_2)/2;\\&\hat{p}_2=(p_2+p_2')/2.\end{aligned}\right\} \quad (65)$$

In the kinematics (65), the intermediate pion in the diagram in Fig. 1 carries the 4-momentum $(\hat{q}+(1/2)k)_\mu$. Thus, to determine the real dependence of the operator of the single-pion exchange current on k_μ it is necessary to know the dependence of the amplitudes (12) and (13) simultaneously on the 4-momenta k and q (for $q^2 \neq -m^2$). It will become clear in what follows that at small k_μ direct substitution of the amplitude (50) with the form factors (60) in the diagram in Fig. 1 is a very good approximation. However, in advance it is not obvious that in the case of other boson exchanges the procedure of direct substitution in the two-nucleon diagram of the limiting values of the corresponding single-nucleon amplitudes is as justified. It would therefore be desirable to find the general form of the restrictions imposed by current algebra on the operator of the two-particle exchange current. Such a program was realized in the papers of Futami, Fujita, and Ohtsubo[33,34] and also Ohta.[35] They applied current algebra and the PCAC condition directly to the two-particle matrix element of the current

$$M^j_{\lambda(5\lambda)}=\langle N(p_1')\,N(p_2')|\,J_{\lambda(5\lambda)}(0)\,|\,N(p_1)\,N(p_2)\rangle \quad (66)$$

without a preliminary particularization of its structure. In what follows, we shall consider the case of the axial current, which is the most complicated.

The generalized Low theorem obtained in Refs. 33–35 for the amplitude $M^j_{5\lambda}$ makes it possible to relate the non-Born part of this amplitude in the limit $k_\mu \to 0$ to the pion absorption amplitude, and also to the invariant amplitudes of nucleon–nucleon scattering (and, thus, to relate the exchange-current operator to the potential of the two-nucleon system). We give, with slight modifications, the derivation of this theorem, following basically Ref. 35. It is analogous to the derivation of the low-energy theorem for the amplitude $T^{mj}_{5\lambda}$ in the limit $k_\mu \to 0$ (see the previous section).

We represent the matrix element $M^j_{5\lambda}$ (66) in the form of the sum

$$M^j_{5\lambda}=(M^j_{5\lambda})^{\text{ext}}+\overline{M}^j_{5\lambda}, \quad (67)$$

where $(M^j_{5\lambda})^{\text{ext}}$ includes all the terms with nucleon and pion poles:

$$\begin{aligned}(M^j_{5\lambda})^{\text{ext}}=\bar{u}(p_1')\,\bar{u}(p_2')\Big\{&T(p_1'p_2',Q_1p_2)\frac{1}{i\hat{Q}_1+M}\Gamma^j_{5\lambda}(k^2)\\&+\Gamma^j_{5\lambda}(k^2)\frac{1}{i\hat{Q}_1'+M}T(Q_1'p_2',p_1p_2)+(1\leftrightarrow 2)\Big\}\,u(p_1)\,u(p_2)\\&-\frac{Mg_A}{g}\frac{ik_\lambda}{k^2+m^2}M^j_\pi.\end{aligned} \quad (68)$$

Here, $Q_1=p_1+k$, $Q_1'=p_1'-k$, T is the nucleon–nucleon scattering operator, and M^j_π is the matrix element for the absorption of a pion with 4-momentum k_μ:

$$\begin{aligned}M^j_\pi=\bar{u}(p_1')\,\bar{u}(p_2')\Big\{&T(p_1'p_2',Q_1p_2)\frac{1}{i\hat{Q}_1+M}\Gamma^j_5(k^2)\\&+\Gamma^j_5(k^2)\frac{1}{i\hat{Q}_1'+M}T(Q_1'p_2',p_1p_2)+(1\leftrightarrow 2)\Big\}\,u(p_1)\,u(p_2)+\overline{M}^j_\pi.\end{aligned} \quad (69)$$

As usual, the bar denotes the non-Born part of the amplitude:

$$\overline{M}^j_\pi=i\bar{u}(p_1')\,\bar{u}(p_2')\,\overline{T}^j_\pi(k)\,u(p_1)\,u(p_2), \quad (70)$$

where $\overline{T}^j_\pi(k)$ is the corresponding part of the operator of the source of the pion field in the momentum representation. In the amplitudes (68) and (69),

$$\Gamma^j_{5\lambda}(k^2)=i\tau^jF_A(k^2)\,\gamma_\lambda\gamma_5/2; \quad (71a)$$

$$\left.\begin{aligned}&\Gamma^j_5(k^2)=ig_r(k^2)\,\gamma_5\tau^j;\\&g_r(0)\approx g_r(-m^2)=g.\end{aligned}\right\} \quad (71b)$$

The PCAC condition (32) applied to the amplitude $M^j_{5\lambda}$ can be written in the form

$$k_\lambda M^j_{5\lambda}=i(Mg_A/g)\,[m^2/(k^2+m^2)]\,M^j_\pi. \quad (72)$$

Using identities of the type (56), we readily obtain

$$k_\lambda (M^j_{5\lambda})^{\mathrm{ext}} = \overline{u}(p_1')\overline{u}(p_2')\Big\{T(p_1'p_2', Q_1p_2)\frac{1}{2}\tau^j F_A(k^2)\gamma_5 + \frac{1}{2}\tau^j F_A(k^2)\gamma_5 T(Q_1'p_2', p_1p_2) + (1\leftrightarrow 2)\Big\}u(p_1)u(p_2) + \mathrm{i}M\frac{g_A}{g}[M^j_\pi - \overline{M}^j_\pi] - \mathrm{i}M\frac{g_A}{g}\frac{k^2}{k^2+m^2}M^j_\pi + O(k^2). \tag{73}$$

Substituting (73) in (72), we find

$$k_\lambda \overline{M}^j_{5\lambda} = -\overline{u}(p_1')\overline{u}(p_2')\{T(p_1'p_2', Q_1p_2)(1/2)\tau^j F_A(k^2)\gamma_5 + (1/2)\tau^j F_A(k^2)\gamma_5 T(Q_1'p_2', p_1p_2) + (1\leftrightarrow 2)\}u(p_1)u(p_2) + \mathrm{i}M(g_A/g)\overline{M}^j_\pi. \tag{74}$$

The term of zeroth order in k_μ on the right-hand side of (74) vanishes by virtue of Adler's self-consistency condition, which in the given case has the form[35]

$$\overline{u}(p_1')\overline{u}(p_2')\,\mathrm{i}\overline{T}^j_\pi(0)\,u(p_1)u(p_2) = -\overline{u}(p_1')\overline{u}(p_2')\Big\{T(p_1'p_2', p_1p_2)\frac{1}{2M}\Gamma^j_5(0) + \Gamma^j_5(0)\frac{1}{2M}T(p_1'p_2', p_1p_2) + (1\leftrightarrow 2)\Big\}u(p_1)u(p_2). \tag{75}$$

Differentiating the relation (74) with respect to k, we find that the part of the amplitude $\overline{M}^j_{5\lambda}$ of zeroth order in k_μ can be expressed in terms of the coefficient of the linear term in the k expansion of the right-hand side of (74):

$$\overline{M}^j_{5\lambda}(k=0) = \bar{\overline{M}}^j_{5\lambda}(k=0) + \overline{M}^{\pi j}_{5\lambda}(k=0) = -\overline{u}(p_1')\overline{u}(p_2')\Big\{\frac{\partial}{\partial k_\lambda}T(p_1'p_2', Q_1p_2)|_{k=0}\frac{1}{2}\tau^j g_A\gamma_5 + \frac{1}{2}\tau^j g_A\gamma_5\frac{\partial}{\partial k_\lambda}T(Q_1'p_2', p_1p_2)|_{k=0} + (1\leftrightarrow 2)\Big\}u(p_1)u(p_2) - M\frac{g_A}{g}\overline{u}(p_1')\overline{u}(p_2')\frac{\partial}{\partial k_\lambda}\overline{T}^j_\pi(k)|_{k=0}u(p_1)u(p_2), \tag{76}$$

where $\bar{\overline{M}}^j_{5\lambda}$ and $\overline{M}^{\pi j}_{5\lambda}$ are the first and second terms on the right-hand side of (76).

The relation (76) is one of the possible forms of expression of the low-energy theorem that we wish to obtain. The first term in (76) is determined exclusively by the parameters of the *NN* scattering amplitude and does not depend on the details of meson theory. The second term is determined by the parameters of the non-Born part of the amplitude of pion absorption by the two-nucleon system and is to a large degree model-dependent. It is important to emphasize that theorem (76) is the most general restriction imposed by the PCAC condition on the operator of the two-nucleon axial exchange current and is satisfied in any particular model of current algebra.

A different form of the low-energy theorem (76). In the original papers,[33-35] theorem (76) was formulated directly for the exchange-current operator, which is the most convenient from the point of view of the transition to the static limit. In accordance with what we have said earlier, it is necessary to identify with the exchange-current operator the part of the matrix element $M^j_{5\lambda}$ that remains after subtraction from the Born term of the contributions of the positive-frequency parts of the nucleon propagators (these contributions are already taken into account in the nuclear wave function). In other words,

$$M^{j\,\mathrm{exch}}_{5\lambda} = M^{j\,\mathrm{pair}}_{5\lambda} + \overline{M}^j_{5\lambda}, \tag{77}$$

where $M^{j\,\mathrm{pair}}_{5\lambda}$ is the term with excitation of a nucleon–antinucleon pair obtained from the nucleon part of the matrix element (68) by replacing the total nucleon propagator by its negative–frequency component:

$$\frac{1}{\mathrm{i}\hat{Q}+M} \to -\frac{1}{2\sqrt{M^2+\mathbf{Q}^2}}\frac{\gamma_4\sqrt{M^2+\mathbf{Q}^2}+\mathrm{i}\boldsymbol{\gamma}\mathbf{Q}-M}{Q_0+\sqrt{M^2+\mathbf{Q}^2}}. \tag{78}$$

We denote by $M^{j\,\mathrm{exch}}_\pi$ the part of the total amplitude of pion absorption that remains after separation of the positive-frequency components of the nucleon propagators:

$$M^{j\,\mathrm{exch}}_\pi = M^{j\,\mathrm{pair}}_\pi + \overline{M}^j_\pi = \mathrm{i}\overline{u}(p_1')\overline{u}(p_2')\{T^{j\,\mathrm{pair}}_\pi + \overline{T}^j_\pi(k)\}u(p_1)u(p_2). \tag{79}$$

Further, using identities of the type

$$\frac{1}{2E_Q}\frac{\gamma_4\sqrt{M^2+\mathbf{Q}^2}+\mathrm{i}\boldsymbol{\gamma}\mathbf{Q}-M}{Q_0+\sqrt{M^2+\mathbf{Q}^2}}(\hat{k}-2\mathrm{i}M)\gamma_5 = \mathrm{i}-u(Q)(\overline{u}(Q)\gamma_4), \tag{80}$$

we can readily show that

$$\overline{u}(p_1')\overline{u}(p_2')\frac{\partial}{\partial k_\lambda}T^{j\,\mathrm{pair}}_\pi(k)|_{k=0}u(p_1)u(p_2) = -\frac{g}{Mg_A}\Big\{M^{j\,\mathrm{pair}}_{5\lambda} + \tilde{\overline{M}}^j_{5\lambda}(k=0) - \Delta M^{(2)\,j}_\lambda\Big\} + O(k). \tag{81}$$

Here, $\bar{\overline{M}}^j_{5\lambda}(k=0)$ is defined by (76), and $\Delta M^{(2)j}_\lambda$ by

$$\Delta M^{(2)\,j}_\lambda = \frac{\partial}{\partial k_\lambda}\{\nu(p_1'p_2', Q_1p_2)\rho^{5j}(Q_1, p_1) - \rho^{5j}(p_1', Q_1')\nu(Q_1'p_2', p_1p_2) + (1\leftrightarrow 2)\}|_{k=0}, \tag{82}$$

where

$$\nu(p_1'p_2', p_1p_2) = -\overline{u}(p_1')\overline{u}(p_2')T(p_1'p_2', p_1p_2)u(p_1)u(p_2); \tag{83}$$

$$\rho^{5j}(Q, p) = -\mathrm{i}\overline{u}(Q)\Gamma^{5j}_4 u(p). \tag{84}$$

After the substitution

$$\overline{T}^j_\pi(k) = T^j_\pi(k) - T^{j\,\mathrm{pair}}_\pi(k)$$

and the use of the relation (81), the low-energy theorem (76) can be rewritten directly in terms of $M^{j\,\mathrm{exch}}_{5\lambda}$:

$$M^{j\,\mathrm{exch}}_{5\lambda}(k=0) = \Delta M^{(2)\,j}_\lambda - M\frac{g_A}{g}\overline{u}(p_1')\overline{u}(p_2')\frac{\partial}{\partial k_\lambda}T^j_\pi(k)|_{k=0}u(p_1)u(p_2). \tag{85}$$

The representation (85) is convenient in that it associates the operator of the two-nucleon axial exchange current with experimentally measured quantities: the two-body nuclear potential $V(\mathbf{r}_1, \mathbf{r}_2)$, the single-nucleon density of axial charge

$$\rho^{Aj} = -\frac{g_A}{2M}\tau^j\Big\{-\frac{1}{2}\mathrm{i}\sigma_n(\vec{\nabla}_n - \overleftarrow{\nabla}_n)\Big\}, \tag{86}$$

and the two-nucleon part of the source $J^{(2)}_{\pi j}$ of the pion field. The quantities V, ρ^{Aj}, and $J^{(2)}_{\pi j}$ are determined, respectively, by the Fourier transforms of the matrix elements (83), (84) (for $k_\mu = 0$), and the matrix element of pion absorption:

$$\mathrm{i}\overline{u}(p_1')\overline{u}(p_2')T^j_\pi(k)u(p_1)u(p_2).$$

Going over in the static limit to the configuration space in accordance with the rules[33,34]

$$\left.\begin{aligned}
&\frac{\partial}{\partial \mathbf{p}_s}T(p_1'p_2',\ p_1p_2) \to V(\mathbf{r}_1,\ \mathbf{r}_2)\,\mathrm{i}\,\mathbf{r}_s;\\
&\frac{\partial}{\partial \mathbf{p}_s'}T(p_1'p_2',\ p_1p_2) \to -\mathrm{i}\,\mathbf{r}_s V(\mathbf{r}_1,\ \mathbf{r}_2),\ s=1,2;\\
&\frac{\partial}{\partial p_{s0}}T(p_1'p_2',\ p_1p_2) \approx \frac{\partial}{\partial p_{s0}'}T(p_1'p_2',\ p_1p_2) \approx 0,
\end{aligned}\right\} \tag{87}$$

we can write the low-energy theorem (85) for the total two-particle exchange current in the x representation, this being defined as the integral of the density of the exchange current:

$$\mathbf{J}^{(2)}_j \equiv \tilde{\mathbf{J}}^{(2)}_j(\mathbf{k})|_{k=0} = \int d^3r\,\mathbf{J}^{(2)}_j(\mathbf{r}). \tag{88}$$

Then theorem (85) becomes

$$\mathbf{J}^{(2)}_j = \mathbf{J}^{(2)\,\mathrm{int}}_j + \mathbf{J}^{(2)\,\pi}_j, \tag{89}$$

where the individual terms are determined by the Fourier transforms of the first and second terms in (85):

$$J_j^{(2)\,\mathrm{int}} = i\left[V(\mathbf{r}_1, \mathbf{r}_2), \sum_{s=1}^{2} \frac{1}{2}(\mathbf{r}_s\rho_s^{Aj} + \rho_s^{Aj}\mathbf{r}_s)\right]_-; \quad (90)$$

$$J_j^{(2)\,\pi} = i\,(Mg_A/g)\,\nabla_k J_{\pi j}^{(2)}(\mathbf{k})|_{k=0}; \quad (91)$$

$$J_{\pi j}^{(2)}(\mathbf{k}) = (\mathbf{k}^2 + m^2)\,\varphi_{\pi j}^{(2)}(\mathbf{k}). \quad (91a)$$

Here, $\varphi_{\pi j}^{(2)}(\mathbf{k})$ is the projection of the operator of the pion field (in the momentum representation) onto the subspace of two-nucleon states.

Note that the relation (89) is the analog of Siegert's well-known theorem[2] or the electromagnetic exchange current [in this case, the exchange current contains only the single term $J_j^{(2)\mathrm{int}}$, which is determined by an expression of the type (90) but with replacement of the axial density ρ_s^{Aj} by the charge density $e((1+\tau^3)/2)_s \times \delta(\mathbf{r}-\mathbf{r}_s)$]. This is natural, since Seigert's theorem is simply the nonrelativistic form of the standard Low theorem for the two-nucleon matrix element of the electromagnetic current.

The single-boson exchange model. To conclude this section, we apply theorem (76) to the case of single-boson (in particular, single-pion exchange, and compare the obtained predictions with the results of direct substitution in the diagram in Fig. 1 of the limiting values of the amplitudes of pion production on individual nucleons.

In the single-boson exchange model

$$T(p_1'p_2',\ Q_1p_2) = T(Q_1'p_2',\ p_1p_2)$$
$$= \Gamma_{1\alpha}^n(-\hat{q} - (1/2)k)[(\hat{q} + (1/2)k)^2 + m_B^2]^{-1}\Gamma_{2\alpha}^n(\hat{q} + (1/2)k), \quad (92)$$

where $\Gamma_{i\alpha}^n(q)$ is the invariant vertex function for absorption of a meson with 4-momentum q_μ, isospin index n, and mass m_B (for simplicity, it is assumed that the vertex functions do not contain a dependence on $\hat{p}_1$ and $\hat{p}_2$).

The non-Born part of the operator $\overline{T}_\pi^j(k)$ in the same model has the structure

$$\overline{T}_\pi^j(k) = \overline{A}_{5\alpha}^{nj}(k,\ \hat{q} + (1/2)k)\,1/[(\hat{q} + (1/2)k)^2 + m_B^2]^{-1}$$
$$\times\,\Gamma_{2\alpha}^n(\hat{q} + (1/2)k). \quad (93)$$

The operator $\overline{A}_{5\alpha}^{nj}(k,q)$ is defined such that $-i\overline{u}(p_1')\overline{A}_{5\alpha}^{nj}u(p_1)$ is equal to the non-Born amplitude for production of the meson B with 4-momentum q_μ on the nucleon by a pion with 4-momentum k_μ.

Substitution of the special representations (92) and (93) in the general relation (76) with allowance for Adler's self-consistency condition (75), written for the amplitude $\overline{A}_{5\alpha}^{nj}(k,q)$,

$$\overline{u}(p_1')\,\overline{A}_{5\alpha}^{nj}(0,\ \hat{q})\,u(p_1) = \frac{i}{2M}\,\overline{u}(p_1')\,[\Gamma_\alpha^n(-\hat{q}),\ \Gamma_5^j]_+ u(p_1), \quad (94)$$

leads to the relation

$$\overline{M}_{5\lambda}^j(k=0) = (M_{\lambda\alpha}^{jn\,(\mathrm{cont})} + M_{\lambda\alpha}^{jn\,(\pi)})$$
$$\times\,(q^2 + m_B^2)^{-1}\overline{u}(p_2')\,\Gamma_\alpha^n(\hat{q})\,u(p_2) + (1 \leftrightarrow 2), \quad (95)$$

where

$$M_{\lambda\alpha}^{jn\,(\mathrm{cont})} = i\,\frac{g_A}{2g}\,\overline{u}(p_1')\left[\frac{1}{2}\,\frac{\partial}{\partial\hat{q}_\lambda}\,\Gamma_\alpha^n(-\hat{q}),\ \Gamma_5^j\right]_+ u(p_1); \quad (96)$$

$$M_{\lambda\alpha}^{jn\,(\pi)} = -\frac{Mg_A}{g}\,\overline{u}(p_1')\,\frac{\partial}{\partial k_\lambda}\,\overline{A}_{5\alpha}^{nj}\left(k,\ -\hat{q} - \frac{1}{2}k\right)\Big|_{k=0} u(p_1). \quad (97)$$

The contact term (96) describes the coupling of the axial current to the meson source at the same point. It appears because in the kinematics (65) the single-nucleon vertex for emission of a virtual boson contains a dependence on the current 4-momentum [Eq. (92)].

As a first example, we consider single-pion exchange. In this case, $\Gamma_\alpha^n(-\hat{q}) = \Gamma_5^n(\hat{q}^2)$, where the last vertex is determined by (71b). The quantity

$$-i\overline{u}(p_1')\,\overline{A}_{5\alpha}^{nj}(k,q)\,u(p_1)$$

is equal to the non-Born part of the πN scattering amplitude $\overline{T}_\pi^{nj}(k,q)$ introduced earlier [see Eqs. (51)–(54)]. In the framework of the standard current-algebra hypothesis of smooth extrapolation,

$$g_r(q^2) \approx g_r(0) \approx g_r(-m^2) = g,$$

and the derivative $\partial g_r(q^2)/\partial q^2$ must be very small.[5)] Therefore, in the considered model the term (96) is negligibly small. Further, it can be seen by comparison with the low-energy theorem (59) that in the given case the term (97) coincides with the limiting value of the non-Born amplitude for pion production by the axial current $\overline{T}_{5\lambda}^{nj}(\hat{q}, k=0)$ (50), with the only difference that in the form factor of this amplitude $\gamma(\hat{q}^2)$ (60) there is now an additional contribution:

$$2\,\frac{g_A M}{g}\,\frac{\partial\overline{A}^{\pi N\,(+)}}{\partial\hat{q}^2}\Bigg|_{\nu=\nu_B=k^2=0} \quad (98)$$

(because k_μ also occurs in the operator $\overline{A}_5^{nj}$ through the 4-momentum of the virtual pion). To current-algebra accuracy, this contribution can be ignored, since it is of higher order than the expressions (60). Thus, in the model of single-pion exchange the amplitude (95) is actually identical to the results of direct substitution in the diagram in Fig. 1 of the limiting value (50) of the single-nucleon amplitude $T_{5\lambda}^{nj}(k,\hat{q})$ [with form factors (60)].

In the case of the exchange of a ρ meson,

$$\Gamma_\alpha^n(\hat{q}) = (1/2)\,i\,g_{\rho NN}\,[\gamma_\alpha - (\varkappa_V/2M)\sigma_{\alpha\beta}q_\beta]\tau^n, \quad (99)$$

where $g_{\rho NN}$ is the ρ-meson–nucleon coupling constant.

From Eq. (96), we obtain

$$M_{\lambda\alpha}^{jn\,(\mathrm{cont})} = \overline{u}(p_1')\,(i/2)\,(g_{\rho NN}/4M)\,\varkappa_V\delta^{jn}\sigma_{\lambda\alpha}\gamma_5 g_A u(p_1). \quad (100)$$

It can be shown that in this case too the contact term can be ignored. For example, the contribution of the exchange current which it introduces to the β-decay amplitude of tritium (^{3}H and ^{3}He in the S state) is approximately 0.2% of the contribution of the impulse approximation.[35]

Hitherto, we have studied the exchange-current operator under the assumption that the nucleons are free. However, the presence of the additional degrees of freedom in the nucleus not only leads to the appearance of exchange currents but simultaneously changes its wave function. Moreover, some of the exchange currents (the potential exchange currents) are related in the most direct manner to the determination of the po-

---

5) Phenomenologically, the form factor $g_r(q^2)$ is described by the formula $g_r(q^2) = g(\Lambda_\pi^2 - m^2)/(\Lambda_\pi^2 + q^2)$, where $\Lambda_\pi^2 \approx 72m^2$. Hence

$$\partial g_r(q^2)/\partial q^2 \sim g/(\Lambda_\pi^2 - m^2) \sim g/71m^2.$$

tential and the wave function of the problem. We shall now discuss this question.

2. EFFECTIVE OPERATORS

The question of the renormalization of the wave function and its relation to the recoil current has been widely discussed in the literature.[36-42] Ultimately, it was shown to be intimately related to the problem of determining the potential currents. By these currents, one understands the exchange currents which can be obtained by joining the line of the external perturbation to the diagram which occurs in the definition of the potential (elastic interaction of the current with the hadron). For example, the currents in Figs. 2a–2c are potential currents, whereas the currents in Figs. 2d–2f are nonpotential (at the vertex $A\hat{J}B$, the hadrons A and B are different).

At the present time, we cannot solve exactly the dynamical system of equations for the system of nucleons and mesons. One can eliminate the meson degrees of freedom from the wave function of the system (the nucleus), but an effective interaction V_{eff} between the nucleons then arises. It recalls the presence of the mesons in the nucleus. In the case of perturbation of the nuclear system by the single-particle current $\hat{J}(1) \equiv J_\mu(1)$, one can proceed similarly. In addition, besides the effective potential V_{eff} there also arises an effective current operator $\hat{J}_{eff}$, which contains not only the renormalized initial single-particle current $\hat{J}_R(1)$ but also many-particle components $\hat{J}_{ex}$, i.e., exchange currents.

The elimination of the meson degrees of freedom from the wave function of the nuclear system is not a unique procedure. It can be done either by the method of unitary transformations[36-41] or by the projection method.[40-42] In both methods, the meson–nucleon system is described by a Hamiltonian H consisting of the free part H^0 and the meson–nucleon interaction H^I. The wave function $|\Psi\rangle$ of the system satisfies the equation

$$H|\Psi\rangle = E|\Psi\rangle \tag{101}$$

and contains both purely nucleon components $|\tilde{\Phi}_N\rangle = \eta|\Psi\rangle$ (η is the operator of projection onto the nucleon subspace) and the meson part $|\tilde{\Phi}_M\rangle = \Lambda|\Psi\rangle (\Lambda = 1 - \eta)$. These functions satisfy the system of equations

$$\begin{aligned} &H_{\eta\eta}|\tilde{\Phi}_N\rangle + H_{\eta\Lambda}|\tilde{\Phi}_M\rangle = E|\tilde{\Phi}_N\rangle; \\ &H_{\Lambda\eta}|\tilde{\Phi}_N\rangle + H_{\Lambda\Lambda}|\tilde{\Phi}_M\rangle = E|\tilde{\Phi}_M\rangle. \end{aligned} \tag{102b}$$

Here, $H_{\eta\eta} = \eta H \eta$, and the remaining operators are related to H similarly. Using Eq. (102b), we can express $|\tilde{\Phi}_M\rangle$ in terms of $|\tilde{\Phi}_N\rangle$:

$$|\tilde{\Phi}_M\rangle = [1/(E - H_{\Lambda\Lambda})]\, H_{\Lambda\eta}|\tilde{\Phi}_N\rangle. \tag{103}$$

Then from (102a) we obtain an equation for the purely nucleon wave function $|\tilde{\Phi}_N\rangle$:

$$\tilde{H}_{eff}|\tilde{\Phi}_N\rangle \equiv \left(H_{\eta\eta} + H_{\eta\Lambda}\frac{1}{E - H_{\Lambda\Lambda}} H_{\Lambda\eta}\right)|\tilde{\Phi}_N\rangle = E|\tilde{\Phi}_N\rangle. \tag{104}$$

Equation (104) is not a standard Schrödinger equation, since the effective potential

$$\tilde{V}_{eff} \equiv H_{\eta\Lambda}[1/(E - H_{\Lambda\Lambda})]\, H_{\Lambda\eta} \tag{105}$$

depends on the energy. Moreover, the wave function $|\tilde{\Phi}_N\rangle$ satisfies the nonstandard normalization condition

$$\langle\tilde{\Phi}_N|1 + H_{\eta\Lambda}[1/(E - H_{\Lambda\Lambda})]^2 H_{\Lambda\eta}|\tilde{\Phi}_N\rangle = 1, \tag{106}$$

which makes a probability interpretation of it difficult.

For succinct exposition of the formalism, it is convenient to introduce the operator

$$J = 1 + F_{\Lambda\eta}, \tag{107}$$

where

$$\begin{aligned} F_{\Lambda\eta} &= \frac{1}{E - H_{\Lambda\Lambda}} H_{\Lambda\eta} = \frac{1}{E - H^0_{\Lambda\Lambda}} R_{\Lambda\eta} \\ &= \frac{1}{E - H^0_{\Lambda\Lambda}} H^I_{\Lambda\eta} + \frac{1}{E - H^0_{\Lambda\Lambda}} H^I_{\Lambda\Lambda} \frac{1}{E - H^0_{\Lambda\Lambda}} H^I_{\Lambda\eta} + \dots \\ &= F^{(1)} + F^{(2)} + \dots, \end{aligned} \tag{108}$$

$$R = H^I + H^I \frac{1}{E - H^0_{\Lambda\Lambda}} R = H^I + H^I \frac{1}{E - H^0_{\Lambda\Lambda}} H^I + \dots. \tag{109}$$

Equations (103)–(106) can now be written in the more compressed form

$$|\tilde{\Phi}_M\rangle = F_{\Lambda\eta}|\tilde{\Phi}_N\rangle; \tag{103a}$$

$$\tilde{H}_{eff}|\tilde{\Phi}_N\rangle \equiv (H_{\eta\eta} + H_{\eta\Lambda}F_{\Lambda\eta})|\tilde{\Phi}_N\rangle = E|\tilde{\Phi}_N\rangle; \tag{104a}$$

$$V_{eff} = H_{\eta\Lambda}F_{\Lambda\eta}; \tag{105a}$$

$$\langle\tilde{\Phi}_N|1 + (F^\dagger F)_{\eta\eta}|\tilde{\Phi}_N\rangle = \langle\tilde{\Phi}_N|(J^\dagger J)_{\eta\eta}|\tilde{\Phi}_N\rangle = 1; \tag{106a}$$

$$|\Psi\rangle = |\tilde{\Phi}_N\rangle + |\tilde{\Phi}_M\rangle = J|\tilde{\Phi}_N\rangle. \tag{110}$$

Using the definition (108) and Eq. (104a), one can show that F satisfies the nonlinear equation

$$([F, H]_- + FHF)_{\Lambda\eta} = H_{\Lambda\eta}. \tag{111}$$

Equations (103a)–(106a) and (110) are basic for both methods.

The projection method. To reduce the normalization condition (106a) to standard form, we introduce the function $|\Phi'_N\rangle$:

$$|\tilde{\Phi}_N\rangle = Z^{1/2}(E)|\Phi'_N\rangle. \tag{112}$$

It also satisfies Eq. (104a) but it is correctly normalized, i.e.,

$$\langle\Phi'_N|\Phi'_N\rangle = 1, \tag{113}$$

provided

$$\frac{1}{Z(E)} = 1 + \langle\Phi'_N|(F^\dagger F)_{\eta\eta}|\Phi'_N\rangle \equiv 1 + N(E). \tag{114}$$

To eliminate from Eq. (105a) the energy dependence, one assumes that the energy E is near the unperturbed energy, i.e., $E = E_0$. Then in the second order in H^I

$$\tilde{V}^p_{eff} = (H^I F^{(1)})_{\eta\eta}. \tag{115}$$

The effective current operator is determined by the equation

$$\begin{aligned} \langle\Psi|\hat{J}(1)|\Psi\rangle &= Z(E)\langle\Phi'_N|J^\dagger\hat{J}J|\Phi'_N\rangle \\ &\equiv Z(E)\langle\Phi'_N|\hat{J}^p_{eff}|\Phi'_N\rangle, \end{aligned} \tag{116}$$

i.e.,

$$\hat{J}^p_{eff} = J^\dagger\hat{J}(1)J = \hat{J}_R(1) + \hat{J}^p_{EX}. \tag{117}$$

Here, $\hat{J}_R(1)$ is the renormalized current of the impulse approximation, and $\hat{J}^p_{EX}$ is the operator of the many-particle exchange current obtained by the projection method.

If the current $\hat{J}(1)$ can be represented in the form

$$\hat{J}(1) = \hat{J}^N + \hat{J}^M + \hat{J}^{MN}, \tag{118}$$

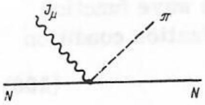

FIG. 8. The contact interaction $\hat{J}^{MN}$.

where $\hat{J}^N$ and $\hat{J}^M$ are the interaction of the current with the nucleon and the meson, and $\hat{J}^{MN}$ is the possible contact interaction (Fig. 8), then in the second order in H^I the two-nucleon part $\hat{J}^p_{EX}$ has the form

$$\hat{J}^p_{EX}(2) = [\hat{J}^{MN}F^{(1)} + F^{(1)\dagger}\hat{J}^{MN} + \hat{J}^M F^{(2)} + F^{(2)\dagger}\hat{J}^M + F^{(1)\dagger}\hat{J}^M F^{(1)} + F^{(1)\dagger}\hat{J}^N F^{(1)}]_{\eta\eta} \tag{119}$$

[$\hat{J}^p_{EX}(2)$ is shown graphically in Fig. 9]. The first two terms in (119) correspond to the contact terms in Figs. 9a and 9b; the third, fourth, and fifth terms are the pion exchange current (Figs. 9c, 9d, and 9e), and the last term is the recoil current (Fig. 9f).

In the same approximation in the interaction,

$$\frac{1}{Z(E_0)} = 1 + \langle\Phi'_N|(F^{(1)\dagger}F^{(1)})_{\eta\eta}|\Phi'_N\rangle \equiv 1 + N^{(2)}(E_0). \tag{120}$$

Equation (120) is the result obtained for $Z(E_0)$ by Chemtob and Rho.[18]

In this manner, the contribution of the exchange currents to the electromagnetic form factors were calculated in the elastic $e-d$ reaction[43] and in $e-{}^4$He scattering.[44] For example, for the first reaction in the case of unpolarized particles one can write down the effective differential cross section in terms of the electromagnetic form factors as follows[45]:

$$d\sigma/d\Omega = (d\sigma/d\Omega)_{\text{Mott}}\{[F_c^2(k^2) + (1/18)(k^2F_Q(k^2))^2] + \left(1 + 2\,\mathrm{tg}^2\frac{1}{2}\theta\right)(k^2/6M^2)(F_M(k^2))^2\}. \tag{121}$$

Here, $(d\sigma/d\Omega)_{\text{Mott}}$ is the effective differential cross section for a point nucleus, θ is the scattering angle, and k^2 is the square of the momentum transfer. The charge, quadrupole, and magnetic form factors are defined as follows:

$$\left.\begin{aligned} &F_c(k^2) = \sqrt{4\pi}\langle 11|M_{00}^{\text{Coul}}(k)|11\rangle;\\ &F_Q(k^2) = 12\frac{\sqrt{5\pi}}{k^2}\langle 11|M_{20}^{\text{Coul}}(k)|11\rangle, \quad F_Q(0) = Q_D;\\ &F_M(k^2) = \frac{2\sqrt{6\pi}\,M}{\mathrm{i}\,k}\langle 11|T_{10}^{\text{mag}}(k)|11\rangle, \; F_M(0) = \mu_D;\\ &M_{JM}^{\text{Coul}}(k) = \frac{1}{e}\int j_J(kr)\,Y_{JM}(\Omega_{\hat{r}})\,\rho_T(\mathbf{r})\,d^3r;\\ &T_{JM}^{\text{mag}}(k) = \frac{1}{e}\int j_J(kr)\mathbf{Y}_{JJ1}^M(\Omega_{\hat{r}})\,\mathbf{J}_T(\mathbf{r})\,d^3r. \end{aligned}\right\} \tag{122}$$

The nuclear state $|11\rangle$ is described by the deuteron wave function with projection $J_z = 1$; ρ_T and $\mathbf{J}_T$ are the total charge density and the total spatial component of the current. In the impulse approximation, they are given by the sum of the corresponding single-particle expressions (20). Finally, Q_D and μ_D are the quadrupole and magnetic moment of the deuteron.

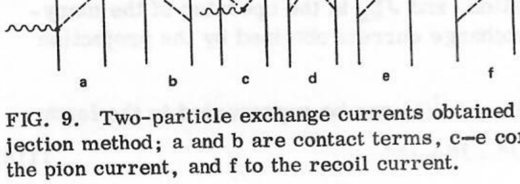

FIG. 9. Two-particle exchange currents obtained by the projection method; a and b are contact terms, c–e correspond to the pion current, and f to the recoil current.

When the corrections from the exchange currents to ρ_T are calculated, a contribution to the form factor $F_c(k^2)$ arises from the recoil current (see Fig. 9f)[43]:

$$\left.\begin{aligned} &F_c^{\text{rec}}(k^2) = (m/2M)^2(g^2/4\pi)\,G_E^S(k^2)\,I_1^R(k^2);\\ &I_1^R = \frac{2}{\pi}\int_0^\infty dr j_0\left(\frac{kr}{2}\right)[u^2\overline{K}_0(x) + 4\sqrt{2}uwK_2(x)\\ &+ w^2(\overline{K}_0(x) - 2K_2(x))], \overline{K}_0(x) = (K_0(x)\\ &- K_1(x))/x, \; x = mr. \end{aligned}\right\} \tag{123}$$

Here, $K_0(x)$ are modified Bessel functions, and u and w are the S and D components of the deuteron wave function.

Since $I_1^R(0) \neq 0$, it might seem that the deuteron has acquired a spurious charge. The situation is saved by the factor $Z(E_0)$, which in accordance with (116) renormalizes the contribution of the impulse approximation $F_c^{IA}(k^2)$. Indeed, since the correction $N^{(2)}$ to (120) is small and $F_c^{IA}(0) = G_E^S(0)$,

$$\begin{aligned}(Z(E_0) - 1)F_c^{IA}(0) &= -N^{(2)}(E_0)\,G_E^S(0)\\ &= -(m/2M)^2(g^2/4\pi)\,I_1^R(0)\,G_E^S(0).\end{aligned} \tag{124}$$

Comparing (123) and (124), we see that for $k = 0$ the contribution to the form factor F_c from the recoil current is completely compensated by the contribution from the renormalization of the wave function. However, as can be seen in Fig. 10, which is taken from Ref. 43, there is no such cancellation far from the threshold, and the effect may be appreciable. It was this result that stimulated criticism in Refs. 36–39, whose authors assert that it is preferable to take into account the renormalization of the wave function by the transformation method. We now consider this method.

Transformation method. Instead of introducing the correctly normalized wave function by Eq. (112), one can make an additional transformation[41] of the wave function $|\tilde{\Phi}_N\rangle$ by an operator U_{11} such that the new wave function $|\Phi_N\rangle$,

$$|\tilde{\Phi}_N\rangle = U_{11}|\Phi_N\rangle; \; U_{11} = [(J^\dagger J)_{\eta\eta}]^{-1/2} \tag{125}$$

satisfies the standard normalization condition

$$\langle\Phi_N|\Phi_N\rangle = 1. \tag{126}$$

The operator J is defined in (107).

In accordance with (110) and (125), $|\Phi_N\rangle$ is related to

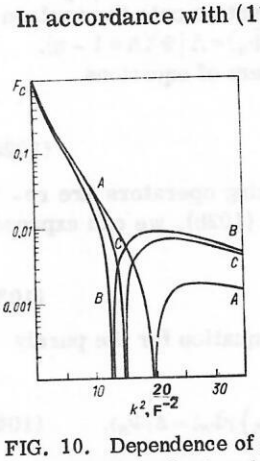

FIG. 10. Dependence of the form factor F_c on k^2. Curve A corresponds to the impulse approximation, curve B to allowance for the pair term, and curve C to allowance for the contributions of the pair term, the recoil current, and the wave-function renormalization.

$|\Psi\rangle$ as follows:

$$|\Psi\rangle = J\,[(J^{+}J)_{\eta\eta}]^{-1/2}\,|\Phi_N\rangle. \tag{127}$$

Equation (104a) in terms of the function $|\Phi_N\rangle$ can be written in the form

$$H^{T}_{\rm eff}|\Phi_N\rangle \equiv U^{-1}_{11}\tilde{H}_{\rm eff}U_{11}|\Phi_N\rangle = E|\Phi_N\rangle. \tag{128}$$

Using (101), (111), and (126)–(128), one can find the connection between $H^{\rm T}_{\rm eff}$ and $\tilde{H}_{\rm eff}$ and the original Hamiltonian H:

$$\begin{aligned} E &= \langle\Phi_N\,|\,H^{T}_{\rm eff}|\,\Phi_N\rangle \equiv \langle\Psi\,|\,H\,|\,\Psi\rangle \\ &= \langle\Phi_N\,|\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2}\,\tilde{H}_{\rm eff}\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2}\,|\,\Phi_N\rangle \\ &= \langle\Phi_N\,|\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2}\,J^{\dagger}HJ\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2}\,|\,\Phi_N\rangle. \end{aligned} \tag{129}$$

The effective potential is determined by the equation

$$H^{T}_{\rm eff} = [(J^{+}J)_{\eta\eta}]^{+1/2}\,J^{\dagger}HJ\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2} \equiv H^{0}_{\eta\eta} + V^{T}_{\rm eff}. \tag{130}$$

The effective current operator $\hat{J}^{\rm T}_{\rm eff}$ is defined by analogy with $H^{\rm T}_{\rm eff}$:

$$\hat{J}^{T}_{\rm eff} = [(J^{\dagger}J)_{\eta\eta}]^{-1/2}\,J^{\dagger}\hat{J}(1)\,J\,[(J^{\dagger}J)_{\eta\eta}]^{-1/2} = \hat{J}_R(1) + \hat{J}^{T}_{EX}. \tag{131}$$

Here, the operator $\hat{J}^{\rm T}_{\rm EX}$ has a meaning analogous to that of $\hat{J}^{\rm p}_{\rm EX}$ in (117).

The transformation U_{11} is a component of the matrix of the unitary transformation $U(F)$ of the function Ψ used in Ref. 36. The operator $U(F)$ transforms the subspaces of functions $|\tilde{\Phi}_N\rangle$ and $|\tilde{\Phi}_M\rangle$ in such a way that the new functions are normalized in accordance with (126), and the new Hamiltonian $H' = UHU^{+}$ is diagonal ($H'_{\Lambda\eta} = H'_{\eta\Lambda} = 0$), Eq. (110) being the condition of diagonality of H'. In deriving Eqs. (130) and (131), we proceeded somewhat differently from Ref. 36 with the aim of following later the parallel between the projection method and the transformation method.

We consider $V^{\rm T}_{\rm eff}$ and $\hat{J}^{\rm T}_{\rm EX}$ in the second approximation in the interaction, i.e., we take F in the approximation

$$F(E_0) = F^{(1)}(E_0) + F^{(2)}(E_0); \tag{132}$$

$$[(J^{\dagger}J)_{\eta\eta}]^{-1/2} \approx 1 - (1/2)\,(F^{\dagger}F)_{\eta\eta}. \tag{133}$$

Then, using (130), we have

$$\begin{aligned} V^{T}_{\rm eff} &= H^{T}_{\rm eff} - H^{0}_{\eta\eta} = (H^{I}F^{(1)} + F^{(1)\dagger}H^{I} \\ &+ F^{(1)\dagger}H^{0}F^{(1)} - (1/2)\,[H^{0},\; F^{(1)\dagger}F^{(1)}]_{-})_{\eta\eta}. \end{aligned} \tag{134}$$

In this case, we have obtained a complicated nucleon interaction containing terms quadratic in p. In the same approximation, proceeding from (131)–(133), we can obtain the two-particle effective current operator $\hat{J}^{\rm T}_{\rm EX}(2)$:

$$\begin{aligned} \hat{J}^{T}_{EX}(2) &= [\hat{J}^{MN}F^{(1)} + F^{(1)\dagger}\,\hat{J}^{MN} + \hat{J}^{M}F^{(2)} \\ &+ F^{(2)\dagger}\,\hat{J}^{M} + F^{(1)\dagger}\hat{J}^{M}F^{(1)} + F^{(1)\dagger}\,\hat{J}^{N}F^{(1)} \\ &- (1/2)\,[\hat{J}^{N},\; F^{(1)\dagger}F^{(1)}]_{+}]_{\eta\eta} = \hat{J}^{p}_{EX}(2) - (1/2)\,([\hat{J}^{N},\; F^{(1)\dagger}F^{(1)}]_{+})_{\eta\eta}, \end{aligned} \tag{135}$$

where $\hat{J}^{\rm p}_{\rm EX}(2)$ is the operator of the two-particle exchange current (119) obtained by the projection method. The anticommutator in (135) is represented graphically in Fig. 11.

In the nonrelativistic limit (the operators of the exchange currents are calculated to terms of order $1/M^2$ inclusive) the recoil current $F^{(1)\dagger}\hat{J}^{N}F^{(1)}$ and the anticommutator in (135) cancel completely irrespective of the value of $\mathbf{k}^2$.[36] Therefore, if we calculate the contribution of the exchange currents to the electromagnetic form factors of the deuteron by the transformation method, curves B and C in Fig. 10 coincide.

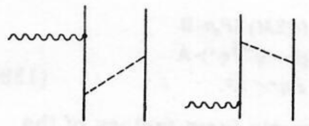

FIG. 11. Contribution to the exchange current from the wave-function renormalization in the transformation method.

In the same limit, the contact term and the pion current of the transformation method are equal to the same currents calculated by the S-matrix method.[36] It is clear from (119) and (135) that this is also true for the analogous currents of the projection method.

It is evident from what has been said that from the theoretical point of view the two methods of defining the wave function are equivalent. They are merely two technically different approaches to the calculation of the matrix elements $\langle\Psi|\hat{O}|\Psi\rangle$. We emphasize that the main difficulty is as follows: It is not clear which effective potential ($V^{\rm T}_{\rm eff}$ or $\tilde{V}^{\rm p}_{\rm eff}$) corresponds to the phenomenological wave functions used in practical calculations of the matrix elements. This defect of the present scheme of calculations must disappear in a more perfect theory, and all the basic elements (potential, wave functions, ...) will be constructed on the basis of a unified field-theoretical approach.

3. HAMILTONIAN FORMALISM

To make practical calculations, it is necessary to know the Hamiltonian H of the system. In the case of interaction with an external electromagnetic field, the Hamiltonian density can be represented in the form

$$\mathscr{H} = \mathscr{H}^{0} + \mathscr{H}^{I}_{s} + \mathscr{H}^{I}_{\rm em}. \tag{136}$$

Here, $\mathscr{H}^{0}$ is the density of the free Hamiltonian, $\mathscr{H}^{\rm I}_{s}$ is the Hamiltonian density of the strong πN interaction, and $\mathscr{H}^{\rm I}_{\rm em}$ is the Hamiltonian density of the electromagnetic interaction.

We consider in more detail the individual terms in (136). The density of the free Hamiltonian of the system of nucleons, mesons, and photons can be represented as a sum of three terms:

$$\mathscr{H}^{0} = \mathscr{H}^{0}_{N} + \mathscr{H}^{0}_{M} + \mathscr{H}^{0}_{\gamma}; \tag{137}$$

$$\mathscr{H}^{0}_{N} = \psi^{+}(\alpha\cdot\mathbf{p} + \beta M)\,\psi; \tag{137a}$$

$$\begin{aligned} \mathscr{H}^{0}_{M} &= \pi^{+}\pi^{-} + \nabla\varphi^{+}\cdot\nabla\varphi^{-} + m^{2}\varphi^{+}\varphi^{-} \\ &+ (1/2)\,[(\pi^{0})^{2} + \nabla\varphi_0\cdot\nabla\varphi_0 + m^{2}\varphi_0^{2}]; \end{aligned} \tag{137b}$$

$$\mathscr{H}^{0}_{\gamma} = (1/2)\,(\mathbf{H}^{2} + \mathbf{E}^{2}); \tag{137c}$$

$$\left.\begin{aligned} &\varphi^{\pm} = (1/\sqrt{2})\,(\varphi_1 \pm i\varphi_2);\quad \pi^{\pm} = (1/\sqrt{2})\,(\pi_1 \mp i\pi_2) = \delta\mathscr{L}/\delta\dot{\varphi}^{\pm}; \\ &[\varphi^{+}(x),\; \pi^{+}(x')]_{-}|_{t=t'} = [\varphi^{-}(x),\; \pi^{-}(x')]_{-}|_{t=t'} = i\delta(\mathbf{x}-\mathbf{x}'); \\ &[\psi_\alpha(x),\; \psi^{+}_\beta(x')]_{+}|_{t=t'} = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}'); \\ &[\psi_\alpha(x),\; \psi_\beta(x')]_{+}|_{t=t'} = [\psi^{+}_\alpha(x),\; \psi^{+}_\beta(x')]_{+}|_{t=t'} = 0. \end{aligned}\right\} \tag{138}$$

In Eqs. (138), $\mathscr{L}$ is the Lagrangian density of the system.

The Hamiltonian density of the electromagnetic interaction, which arises after the substitution $\partial_\mu \to \partial_\mu - ieF_1A_\mu$ for the derivatives of the charged fields in the Lagrangian density and the inclusion of the nonminimal Pauli term, has the form

$$\mathscr{H}^I_{em}=\psi^+[-eF_1\boldsymbol{\alpha}\cdot\mathbf{A}+eF_1\Phi-(e/2M)\beta F_2\boldsymbol{\sigma}\cdot\mathbf{B}+i(e/2M)\beta F_2\boldsymbol{\alpha}\cdot\mathbf{E}]\psi+ie(\varphi^+\nabla\varphi^--\varphi^-\nabla\varphi^+)\cdot\mathbf{A}+ie(\pi^+\varphi^+-\pi^-\varphi^-)\Phi+e^2\varphi^+\varphi^-\mathbf{A}^2. \quad (139)$$

Here, F_i are the electromagnetic form factors of the nucleon:

$$F_i=(1/2)(F_i^S+F_i^V\tau_3),\quad i=1,\ 2; \quad (140)$$

A_μ is the potential of the electromagnetic field:

$$A_\mu\equiv(\mathbf{A},\ i\Phi). \quad (141)$$

The Hamiltonian density of the strong πNN interaction is uniquely fixed at low energies by the principle of chiral invariance,[24] in the framework of which the πNN vertex is a pseudovector quantity:

$$\left.\begin{array}{l}\mathscr{H}^I_s=i\frac{f}{m}\Psi^+\beta\gamma_\mu\gamma_5\partial_\mu\phi\psi;\ \phi\equiv\tau\cdot\varphi=\tau^+\varphi^-+\tau^-\varphi^++\tau_3\varphi_0;\\ f/m=g/2M;\ f^2/4\pi=0.08.\end{array}\right\} \quad (142)$$

However, we choose the pseudoscalar πNN coupling

$$\mathscr{H}^I_s=-ig\psi^+\beta\gamma_5\phi\psi, \quad (143)$$

to which one can always make a transition from the coupling (142) by an equivalence transformation. It was this interaction that was the point of departure for Friar[38,46,47] and other authors[43–45] in their calculation of the exchange corrections to the electromagnetic form factors of the lightest nuclei.

Dyson transformation. It is well known that, using a Dyson equivalence transformation, one can go over from the pseudoscalar πNN coupling not only to the pseudovector πNN coupling (and back again) but also to a certain mixture of a pseudoscalar and a pseudovector πNN interaction. This transformation has the form

$$H'=\exp(iS)(H-i\partial/\partial t)\exp(-iS);\quad \Psi'=\exp(iS)\Psi, \quad (144)$$

where

$$S=\lambda\int d^3x\psi^+\gamma_5\phi\psi(\mathbf{x}),\quad \lambda=-\mu(g/2M), \quad (145)$$

and μ is the mixing parameter of the two types of πNN coupling.

Applying the transformation (144)–(145) to the Hamiltonian H (143) containing the pseudoscalar πNN coupling, we obtain in the first order in g

$$\begin{aligned}H'&\equiv\int d^3x\mathscr{H}'=H^0+H^I_{em}-ig\int d^3x\psi^+\beta\gamma_5\phi\psi(\mathbf{x})\\&+i\mu g\int d^3x\psi^+\left(\beta\gamma_5\phi+\frac{1}{2M}\beta\gamma_\mu\gamma_5\partial_\mu\phi\right)\psi(\mathbf{x})\\&+i\mu e\frac{g}{2M}\int d^3x\psi^+\Big\{(\boldsymbol{\sigma}\cdot\mathbf{A}+\gamma_5\Phi)[F_1,\ \phi]_-\\&+\frac{1}{2M}\beta(\boldsymbol{\alpha}\cdot\mathbf{B}-i\boldsymbol{\sigma}\cdot\mathbf{E})[F_2,\ \phi]_+\Big\}\psi(\mathbf{x}).\end{aligned} \quad (146)$$

For $\mu=1$, only the pseudovector coupling remains in H'.

Although the transformed Hamiltonian H' depends explicitly on μ, the elements of the S matrix do not contain a dependence on μ. This is the physical meaning of the equivalence theorem.

Foldy–Wouthuysen transformation. It is natural to require that the wave function of the nuclear system contain only mesons and nucleons (and not antinucleons). The meson–nucleon component of the wave function can be separated by means of a new unitary transformation, which decomposes the original state space into the subspace of meson–nucleon states and its orthogonal complement. The structure of this transformation can be found from the requirement that the transformed Hamiltonian density $\mathscr{H}''$ should not, to terms $\sim 1/M^n$ and higher, contain odd Dirac matrices coupling the negative- and positive-frequency components of the nucleon wave function ψ. Such a transformation was proposed in Ref. 48 by Foldy and Wouthuysen. It has the form

$$H'\to\exp(iT_n)\ldots\exp(iT_1)(H'-i\partial/\partial t)\exp(-iT_1)\ldots\exp(-iT_n),\quad \Psi'\to\exp(iT_n)\ldots\exp(iT_1)\Psi, \quad (147)$$

where

$$T_i=\int d^3x\,\psi^+t_i\psi(x);\quad t_i=-\frac{i}{2M}\beta O_{i-1}, \quad (148)$$

and O_{i-1} is the odd part of the Hamiltonian obtained at step $i-1$, and

$$\begin{aligned}O_0&=\boldsymbol{\alpha}\cdot\mathbf{p}-ig(1-\mu)\beta\gamma_5\phi+\mu(g/2M)\gamma_5\dot{\phi}\\&-eF_1\boldsymbol{\alpha}\cdot\mathbf{A}+i(e/2M)\beta\boldsymbol{\alpha}\cdot\mathbf{E}F_2+i\mu e(g/2M)\gamma_5[F_1,\ \phi]_-\Phi\\&+i\mu e(g/4M^2)\beta[F_2,\ \phi]_+\boldsymbol{\alpha}\cdot\mathbf{B}.\end{aligned} \quad (149)$$

Applying this reduction to the Hamiltonian H' (146), we find that to terms $\sim M^{-2}$

$$H''=\int(\mathscr{H}_{CL}+\mathscr{H}_{NS})d^3x; \quad (150)$$

$$\begin{aligned}\mathscr{H}_{CL}&=\psi^+(M-(f/m)\boldsymbol{\sigma}\cdot\nabla\phi+ie(f/m)[F_1,\ \phi]_-\boldsymbol{\sigma}\\&\times\mathbf{A}+eF_1\Phi-\mu_N\boldsymbol{\sigma}\cdot\mathbf{B})\psi(\mathbf{x})+\mathscr{H}^0_M\\&+ie(\varphi^+\nabla\varphi^--\varphi^-\nabla\varphi^+)\mathbf{A}+ie(\pi^+\varphi^+-\pi^-\varphi^-)\Phi;\end{aligned} \quad (151)$$

$$\begin{aligned}\mathscr{H}_{NS}&=\frac{1}{2M}\psi^+(\mathbf{p}^2-eF_1[\mathbf{p},\ \mathbf{A}]_+-(f/2m)(1+\mu)[(\boldsymbol{\sigma}\cdot\mathbf{p}),\ \dot{\phi}]_+\\&+e(f/m)[F_2,\ \phi]_+(\boldsymbol{\sigma}\cdot\mathbf{E})-ie(f/2m)(1+\mu)[(\boldsymbol{\sigma}\cdot\mathbf{p}),\ [F_1,\ \phi]_-\Phi]_+\\&+e(f/2m)(1-\mu)[F_1,\ \dot{\phi}]_+(\boldsymbol{\sigma}\cdot\mathbf{E})\\&+e(f/2m)(1+\mu)[F_1,\ \dot{\phi}]_+(\boldsymbol{\sigma}\cdot\mathbf{A}))\psi(\mathbf{x}).\end{aligned} \quad (152)$$

Here, $g/2M=f/m$ and $\mu_N=(e/2M)(F_1+F_2)$. The Hamiltonian $\mathscr{H}_{CL}$ corresponds to Chew–Low theory[49] (see also Refs. 50–52) as applied to the description of pion photoproduction on a nucleon at rest. Note that $\mathscr{H}_{CL}$ does not depend on μ. It is convenient to write it in the form

$$\mathscr{H}_{CL}=(M-(f/m)\boldsymbol{\sigma}\cdot\nabla\phi)\delta(x)+\mathbf{j}_I\cdot\mathbf{A}+\rho_N\Phi+\mathbf{j}_N\cdot\mathbf{A}+\mathscr{H}^0_M+e\rho_\pi\Phi+\mathbf{j}_\pi\cdot\mathbf{A}, \quad (153)$$

where

$$\left.\begin{array}{l}\mathbf{j}_I=e(f/m)\boldsymbol{\sigma}(\tau_1\varphi_2-\tau_2\varphi_1)\delta(x);\\ \rho_N=eF_1\delta(x);\ \mathbf{j}_N=\mu_N\nabla\times\boldsymbol{\sigma}\delta(x);\\ \mathbf{j}_\pi=e(\varphi_2\nabla\varphi_1-\varphi_1\nabla\varphi_2);\\ \rho_\pi=e(\pi_2\varphi_1-\pi_1\varphi_2).\end{array}\right\} \quad (154)$$

In the nucleon part of the Hamiltonian $\mathscr{H}_{CL}$, we have gone to the limit of a point source.

Such a theory is gauge invariant, and the total current $\mathbf{j}=\mathbf{j}_N+\mathbf{j}_\pi+\mathbf{j}_I$ and the total charge density $\rho=\rho_N+\rho_\pi$ satisfy the continuity equation

$$\dot{\rho}(x,\ t)+\nabla\cdot\mathbf{j}(x,\ t)=0. \quad (155)$$

The third term in (153) is an example of the contact interaction shown in Fig. 8, and $\mathbf{j}_\pi\cdot\mathbf{A}$ is the interaction of the field with the meson current.

The Chew–Low theory predicts the following amplitude of photoproduction of a pion with momentum $\mathbf{q}$ and isospin projection β on a nucleon at rest[52,53]:

$$\varepsilon \cdot \Gamma(\mathbf{q}, \mathbf{k})_\beta = \mathrm{i}e\,(f/m)\left[(\sigma\varepsilon) - 2(\mathbf{q}\varepsilon)\frac{\sigma\cdot(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2+m^2}\right]\varepsilon_{\beta\delta 3}\tau_\delta - \mathrm{i}\frac{e}{2M}\frac{\varkappa_V}{2}\frac{m}{f}4\pi\sum_\alpha P_\alpha(\mathbf{q}, \mathbf{k})\,h_\alpha(\omega). \quad (156)$$

Here, k is the photo momentum, and P_α is the operator of projection onto the state with quantum numbers $\alpha \equiv (2J, 2T)$, where J and T are the total angular momentum and isospin of the πN system. Also,

$$h_\alpha(\omega) = (1/q^3)\exp(\mathrm{i}\delta_\alpha)\sin\delta_\alpha;\quad \omega^2 = \mathbf{q}^2 + m^2, \quad (157)$$

where δ_α is the phase shift of πN scattering in the channel with quantum numbers α. The first term in the square brackets in (156) is due to the contact interaction $\mathbf{j}_\mathrm{I}\cdot\mathbf{A}$ in (153). It is equal to the threshold value of the negative-frequency part of the nucleon Born amplitude in Figs. 4a and 4b. The second term corresponds to the contribution of the vertex $\mathbf{j}_\pi\cdot\mathbf{A}$. In this approximation, the amplitude (156) is equal to the Born approximation (19a) for the pion photoproduction amplitude (without allowance for the contribution from the magnetic moment of the nucleon).

The third term in (156) takes into account the strong interaction of the p-wave pion with the nucleon in the final state; this interaction is satisfactorily described by the Chew–Low theory in the effective-range approximation.

Isovector exchange currents. Using the amplitude (156), one can construct the exchange-current operator. Graphically, it is represented in Figs. 2c–2e and Figs. 3g and 3h. Such an operator of exchange type was used by Gari and Huffman[54] and then by Thakur and Foldy[53] in calculations of the reaction

$$n + p \rightarrow d + \gamma \quad (158)$$

with thermal neutrons ($v_n = 2200$ m/sec).

In this reaction, it was noted[55] that the experimental cross section[56]

$$\sigma_{\mathrm{exp}} = (334.2 \pm 0.5)\ \mathrm{mb} \quad (159)$$

differs by ≈10% from the effective theoretical cross section calculated in the impulse approximation:

$$\sigma_{IA} = (302.5 \pm 4.0)\ \mathrm{mb}. \quad (160)$$

This difference was explained for the first time by Riska and Brown.[57] They took into account an exchange current similar to the current used in Refs. 53 and 54 but took the Born part of the photoproduction amplitude from the low-energy theorem and calculated the current with isobar excitation in the quark model.

The calculations made in Refs. 53 and 54 (see also Ref. 58) confirmed the result of Riska and Brown. It was found that ~65% of the effect is determined by the first two terms in (156) and the remaining ~35% by the current with isobar excitation. The decisive point here, however, is allowance for the admixture of the D wave in the deuteron wave function, since the exchange-current corrections arise basically because of this wave. As an example of a concrete calculation for the reaction (158), we give the results of Ref. 59 obtained with allowance for violation of the charge independence of the nuclear forces in the $^1S_0$ channel. The potential of the np interaction was chosen to make it describe the experimental value of the np scattering lengths: $a_{np} = -23.72$ F.

The results of calculations for the reaction $n+p\rightarrow d+\gamma$ with inclusion of the exchange currents are given below[59] (the contribution from these currents is given as a percentage, and the contribution δ_Δ^{SD} from the exchange current with isobar excitation is represented separately; the deuteron wave function is calculated using the Reid hard-core potential,[60] and σ_{IA} is the effective cross section in the impulse approximation):

σ_{IA}, mb	δ^{SS}	δ^{SD}	σ_I, mb	δ_Δ^{SD}	σ_T, mb
305.6	1.71	1.30	324.3	1.84	335.9

Note that this value of σ_{IA} agrees well with the value (160), which Noyes[55] obtained for σ_{IA}.

To a greater extent than in the reaction (158), the exchange currents are manifested in the analogous reaction of thermal neutron capture by deuterium:

$$n + d \rightarrow t + \gamma. \quad (161)$$

However, the theoretical and experimental uncertainties are here greater than in the reaction (158). The results of measurements of the effective cross section are shown in Table I, which is taken from Ref. 61. It can be seen that the data obtained by the activation method from the tritium yield are in satisfactory agreement with one another (they are equal to within three standard deviations). The accuracy of the data obtained from the yield of γ rays is not nearly so good.

It should be noted that the data of Ref. 61 are preliminary.

The calculations of Hadjimichael[66] (see also Ref. 67) showed that the cross section in the impulse approximation is

$$\sigma_{IA} = 0.29\ \mathrm{mb}, \quad (162)$$

whereas allowance for the exchange currents calculated in the projection method led to the result

$$\sigma_I = 0.52 \pm 0.05\ \mathrm{mb}. \quad (162\mathrm{a})$$

We see that the exchange currents here change the result by approximately 100%. As in the case of the reaction (158), it proved to be important to take into account the tensor forces in calculations of the nuclear wave function, and the D-wave admixture explains ~40% of the effect. The sum of the contributions from the recoil current and the wave-function renormalization leads to a 9% correction to σ_T.

The isovector electromagnetic currents also satisfactorily explain the experimental value of the isovector magnetic moment $\mu_V^{\mathrm{exp}} = 2.55354$ nuclear magnetons of the system of three bound nucleons:

$$\mu_V^{\mathrm{exp}} = \frac{1}{2}[\mu(^3\mathrm{H}) - \mu(^3\mathrm{He})] = \frac{1}{2}(1+\varkappa_V)\left(p_S - \frac{1}{3}p_{S'} + \frac{1}{3}p_D\right) - \frac{1}{6}p_D + \delta_{\mu_V}. \quad (163)$$

TABLE I. Results of measurements of the effective σ_{exp} for the reaction $n+d\rightarrow t+\gamma$.

Year	1952	1963	1963	1968	1979
σ_{exp}, mb	0.570 (10)	0.353 (35)	0.600 (50)	0.520 (9)	0.487 (24)
Method	Activation	n, γ	n, γ	Activation	n, γ
Reference	Ref. 62	Ref. 63	Ref. 64	Ref. 65	Ref. 61

Here, the admixtures of the S, S', and D waves in the wave function of the three bound nucleons are normalized by the condition $p_S+p_{S'}+p_D=1$. The first two terms in (163) correspond to calculations with the current in the impulse approximation, and $\delta_{\mu V}$ is a possible correction. In Ref. 68, the admixtures of the S, S', and D waves were obtained by solving the Faddeev equations with the Reid soft-core (RSC) potential[60].

$$p_S=0.897;\quad p_{S'}=0.017;\quad p_D=0.086. \tag{164}$$

Then from (163) for the correction $\delta^{\exp}_{\mu V}$ we have the value

$$\delta^{\exp}_{\mu_V}=0.401 \text{ nuc. magn.} \tag{165}$$

The calculation in Ref. 68 (see also Ref. 69) of the correction using the operator of the two-particle current obtained in the S-matrix method is in satisfactory agreement with (165):

$$\delta^{T}_{\mu_V}=0.419 \text{ nuc. magn.} \tag{166}$$

In the above calculations of the electromagnetic isovector currents, the exchange effects are 10–15% of the contribution of the impulse approximation, and for the reaction (161) are even comparable with this contribution.

In the deuteron electrodisintegration reaction

$$e+d\to e'+n+p \tag{167}$$

the effect of the exchange currents may exceed the impulse approximation by even an order of magnitude.[70-72] This reaction is convenient in that the disintegration of the deuteron occurs because of a virtual photon, whose energy and momentum are not related to each other. This extra degree of freedom compared with the reaction $\gamma+d\to n+p$ makes it possible to choose a geometry of the experiment in which the contribution of the exchange currents to the cross section $d^2\sigma/d\Omega dE_f$ (E_f is the energy of the scattered electron) is manifested maximally; for the electrons are scattered backward ($\theta=180°$), and the relative energy E_{np} of the np system does not exceed a few MeV.

For the reaction (167), the exchange-current operator has been calculated in the framework of S-matrix theory[70] or in the static Chew–Low model,[71] i.e., the pion and contact currents and the current with excitation of the $\Delta(1236)$ isobar have been taken into account. In Ref. 72, the isobar-excitation current was not considered, but instead admixtures of $N\Delta(1236)$, $NN'(1470)$, and $\Delta(1236)\Delta(1236)$ configurations in the deuteron wave function were taken into account. Since the momentum transfer can be appreciable in this reaction, it is necessary to retain a dependence on it in the vertices at which the photon interacts with a nucleon or a meson.

In Fig. 12, which we have taken from Ref. 72, we show the results of calculations for the reaction (167). The figure shows well that the prediction of the impulse approximation for momentum transfer $k^2=10$ F^{-2} is approximately ten times smaller than the experimental value of $k_1^2 d^2\sigma/d\Omega dE_f$, which agrees well with calculations that take into account the exchange currents (k_1 is the momentum of the incident electron).

The exchange currents in the reaction (167) are

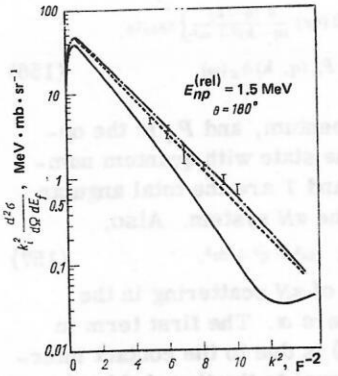

FIG. 12. Double differential cross section for the reaction $e+d\to n+p$. The continuous curve is the impulse approximation and the broken curve is obtained with allowance for the exchange currents.[72]

clearly manifested in the $d^2\sigma/d\Omega dE_f$ spectrum and at other values of the electron scattering angle ($\theta\geqslant 90°$) for energies E_{np} near the threshold energy.[73,74]

It obviously follows from what we have said that the existence of isovector exchange currents has been established rather reliably.

Isoscalar exchange currents. The picture is quite different in the case of isoscalar exchange currents. The part they play is well illustrated by the example of the deuteron magnetic moment. In the impulse approximation,

$$\mu_D^{IA}=(1+\varkappa_S)\,(1-(3/2)\,p_D)+(3/4)\,p_D \text{ nuc. magn.,} \tag{168}$$

where p_D is the D-wave admixture in the deuteron:

$$p_D=\int_0^\infty w^2(r)\,dr. \tag{169}$$

The experimental value of the moment is

$$\mu_D^{\exp}=\mu_D^{IA}+\delta\mu_D=0.8574 \text{ nuc. magn.} \tag{170}$$

The value of p_D is not known too well (4–8%), since an appreciable part of p_D is due to the contribution of short distances, where the wave function has been inadequately studied. If it is assumed that $\delta\mu_D=0$, then from (170) we obtain $p_D=3.9\%$. However, realistic potentials[60,75-77] lead to the range of p_D values

$$5.5\%\leqslant p_D\leqslant 7\%. \tag{171}$$

At these values of p_D, the correction $\delta\mu_D$ varies in the interval

$$0.009\leqslant\delta\mu_D\leqslant 0.0176 \text{ nuc. magn.,} \tag{172}$$

i.e., the correction for the nonadditivity of the nucleon magnetic moments in the deuteron is of order 1–2%, which is much less than the radiative corrections in the case of isovector currents. It is difficult to calculate such a small quantity, since appreciable contributions are made to it by not only the single-pion exchange currents but also the single-boson (ρ, ω) exchange currents, the relativistic corrections to the impulse approximation, and, possibly, two-boson exchange currents. Although many studies (Refs. 7–9, 47, 78, and 79) have been devoted to this problem, it cannot be regarded as solved, since it is directly related to the interaction dynamics of two nucleons at short distances, for which a satisfactory theory is at present not available. The complexity of the problem can be gauged from the calculations made in the recent papers of Refs. 47, 78, and 79.

In Ref. 47, Friar attempted to solve self-consistently the problem of the deuteron magnetic moment using the Hamiltonian (150)–(151); in H_{NS}, he also included terms $\sim f/M^2$. They also depend on the mixing parameter μ (the question of unitary freedom was also considered in Ref. 37).

In contrast to the isovector exchange currents, the isoscalar exchange currents are a relativistic correction to the impulse approximation (and therefore the contribution from them is small) and depend essentially on the parameter μ. Since the matrix elements must not depend on μ, it is important that the wave function be constructed in a self-consistent manner (the potential must be constructed using the same Hamiltonian as the exchange currents). For $\delta\mu_D$, Friar obtained the expression[47]

$$\delta\mu_D^F=\delta\mu_D^0+\frac{3}{2}\left(\varkappa_S+\frac{1}{2}\right)\frac{\sqrt{2}}{4\pi}\frac{f^2}{mM}\mu\int_0^\infty w\,[u\,(h_0''-3h_0'/r)+2u'h_0']\,dr. \tag{173}$$

For a point nucleon, $h_0=\exp(-mr)/mr$. The term $\delta\mu^0$ does not depend on μ. The problem is to find the wave function for at least one value of μ. The simplest choice is $\mu=-1$, since the nuclear potential constructed with the Hamiltonian (150)–(152) has in this case a form close to a static potential of Reid type[60] (if $\mu=-1$, then $\mathscr{H}_{NS}$ (152) does not have terms containing $\dot{\phi}$ and $\sigma\cdot\mathbf{p}$). Then[47]

$$\Psi(\mu)\approx(1-iU_E)\,\Psi(-1)\equiv\Psi_0+\Delta\Psi, \tag{174}$$

where

$$U_E=-\frac{3}{32\pi}\frac{f^2}{mM}(1+\mu)\{[(\sigma_1\cdot\mathbf{p}),\ (\sigma_2\cdot\nabla h_0)]_+ +[(\sigma_2\cdot\mathbf{p}),\ (\sigma_1\cdot\nabla h_0)]_+\}, \tag{175}$$

$$\Delta p_D=2\int w_0\Delta w\approx\frac{\sqrt{2}}{4\pi}\frac{f^2}{mM}(1+\mu)\int_0^\infty w_0\,[u_0(h_0''-3h_0'/r)+2u_0'h_0']\,dr. \tag{176}$$

It can be seen from (168), (170), and (176) that the terms which depend on μ in $\delta\mu_{\rm D}^{\rm F}$ and $\Delta p_{\rm D}$ compensate each other. The structure of the transformation $U_{\rm E}$ indicates that a change in the wave function occurs only at short distances. If $\mu=1$, then for the RSC wave function[60] we obtain the change $\Delta p_{\rm D}=+1.7\%$. The value of $\delta\mu_{\rm D}^{\rm F}$ decreases by the same amount.

In this case, only the total matrix element, which determines the deuteron magnetic moment, does not depend on μ, and, therefore, it is the only observable quantity. Both the D-wave admixture in the deuteron and the correction $\delta\mu_{\rm D}^{\rm F}$ are separately unobservable and are only computational quantities.

In the same way, one can find the correction $F_{\rm c}^{\pi}(k^2)$ from the exchange currents to the charge form factor:

$$\left.\begin{aligned}&F_c(k^2)=F_c^{IA}(k^2)+F_c^{\pi}(k^2);\\&F_c^{IA}(k^2)=G_E^S\int_0^\infty C(r)\,j_0(kr/2)\,dr;\\&F_c^{\pi}(k^2)=\frac{f^2}{16\pi mM}[4G_M^S-(\mu+1)\,G_E^S]\\&\times\int_0^\infty h_0'(r)\,kj_1(kr/2)\,[C(r)+4\sqrt{2}\,Q(r)]\,dr;\end{aligned}\right\} \tag{177}$$

$$C(r)=u^2(r)+w^2(r);\quad Q(r)=w(r)\,[u(r)-(1/\sqrt{8})\,w(r)],$$

where $G_{\rm E}^{\rm S}$ and $G_{\rm M}^{\rm S}$ are the isoscalar electric and magnetic form factors, respectively. Again, it is only the sum of $F_{\rm c}^{\rm IA}(k^2)$ and $F_{\rm c}^{\pi}(k^2)$ that does not depend on μ (a μ dependence enters $F_{\rm c}^{\rm IA}$ through the wave function).

The sensitivity of the calculations to the value of the parameter μ requires self-consistent calculations of the wave functions (potentials) and all the remaining quantities. However, at the present stage in the development of theory this is unrealistic. Therefore, case is needed in the interpretation of the numerical results in the cases when the intermediate calculations depend strongly on the choice of μ. Apart from the already considered case of the deuteron magnetic moment, such a situation can also be expected to obtain in the case of the isoscalar magnetic moment of the bound state of three nucleons.

Hadjimichael[78] calculated the corrections to the impulse approximation for the magnetic and quadrupole moments of the deuteron. He constructed the operators of the exchange currents of single-boson (π, ρ, ω) exchange in the projection method. In addition, he took into account the corrections from the terms of higher powers in $1/M$ in the operator of the single-particle current, the correction from the minimal substitution $\mathbf{p}\rightarrow\mathbf{p}-ie\mathbf{A}$ in the part of the nuclear potential that depends on the momenta of the nucleons, and also the corrections from the isobar configurations in the deuteron. In his calculations, Hadjimichael used numerous realistic potentials—semiphenomenological and of the single-boson exchange type, these giving a $p_{\rm D}$ admixture in the range

$$4.57\%<p_D<7.52\%. \tag{178}$$

However, he did not succeed in achieving agreement between the theoretical predictions and the experimental data simultaneously for the magnetic and quadrupole moments by any of the potentials. For example, using the RSC deuteron wave function[60] ($p_{\rm D}=6.47\%$), he obtained

$$\delta\mu_D^T=0.0152;\quad \delta Q_D^T=0.0131. \tag{179}$$

These values must be compared with

$$\delta\mu_D=\mu_D^{\rm exp}-\mu_D^{IA}=0.0143;\quad \delta Q_D=Q_D^{\rm exp}-Q_D^{IA}=0.0060. \tag{180}$$

The difference between the calculated value $\delta Q_{\rm D}^{\rm T}$ and $\delta Q_{\rm D}$ greatly exceeds the experimental error in $Q_{\rm D}^{\rm exp}=0.2860\pm0.0015$ F^2. On the basis of this, Hadjimichael commented that the situation could be rectified by taking into account the contributions from the two-boson exchange currents. We note that his calculations correspond to the value $\mu=-1$.

The correction from the two-boson exchange currents to the deuteron magnetic moment was calculated by Jaus[79] in the formalism of quasipotential equations.[12,80–88] In principle, one can in this scheme also construct in a self-consistent manner the potential (wave function) and the matrix element of the current operator between two bound states.[89,90]

To be specific, we consider the interaction of a bound state of two nucleons (deuteron) with a current.

The deuteron wave function satisfies the equation

$$\left(2M - M_D + \frac{\mathbf{p}^2}{M}\right)\varphi(\mathbf{p}) = -\frac{1}{(2\pi)^3}\int d^3k V(\mathbf{p}, \mathbf{k})\varphi(\mathbf{k}),$$

$$V(\mathbf{p}, \mathbf{k}) = (M^2/E_pE_k)^{1/2}\bar{u}_1(p)\bar{u}_2(-p)\tilde{V}(\mathbf{p}, \mathbf{k})u_1(k)u_2(-k), \quad (181)$$

where M_D is the mass of the deuteron, and $E_q^2 = \mathbf{q}^2 + M^2$. Equation (181) was obtained from the Bethe–Salpeter equation[91] by the Blankenbecler–Sugar reduction.[81]

The matrix element of the current operator $\hat{J}_\mu$ between the two bound states has the form

$$\langle P|\hat{J}_\mu(0)|Q\rangle = \frac{1}{2M_D}\frac{1}{(2\pi)^6}\int d^3p'\, d^3p''\varphi^*\left(\mathbf{p}'' - \frac{1}{4}\mathbf{q}\right) \times \Lambda_\mu^{++}\left(\mathbf{p}'' - \frac{1}{4}\mathbf{q},\ \mathbf{p} + \frac{1}{4}\mathbf{q}\right)\varphi\left(\mathbf{p}' + \frac{1}{4}\mathbf{q}\right);$$

$$\Lambda_\mu^{++}(\mathbf{p}, \mathbf{q}) = \left(\frac{M^2}{E_pE_q}\right)^{1/2}\bar{u}_1(p)\bar{u}_2(-p)\hat{\Lambda}_\mu(\hat{p}, \hat{q})u_1(q)u_2(-q); \quad (182)$$

where

$P \equiv ((1/2)\mathbf{q},\ iM_D)$; $Q \equiv (-(1/2)\mathbf{q},\ iM_D)$; $\hat{p} \equiv (\mathbf{p}, 0)$; $\hat{q} \equiv (\mathbf{q}, 0)$.

The potential $\tilde{V}$ and the current $\hat{\Lambda}_\mu$ can be represented as a series in the numbers of boson exchanges:

$$\tilde{V} = \tilde{V}^{(1)} + \tilde{V}^{(2)} + \ldots; \quad (183a)$$

$$\hat{\Lambda}_\mu = \hat{\Lambda}_\mu^{(0)} + \hat{\Lambda}_\mu^{(1)} + \hat{\Lambda}_\mu^{(2)} + \ldots. \quad (183b)$$

Here, $\hat{\Lambda}_\mu^{(0)}$ is the single-particle current. If in (182) we substitute $\hat{\Lambda}_\mu = \hat{\Lambda}_\mu^{(0)}$, we obtain the impulse approximation. By analogy, $\hat{\Lambda}_\mu^{(1)}$ and $\hat{\Lambda}_\mu^{(2)}$ represent the exchange current in the single- and two-boson approximations, respectively. The operators of the exchange currents can be constructed on the basis of the field-theoretical amplitude of the process

$$N + N \to N + N + \gamma.$$

The current operator of two-boson exchange[79] is shown in Fig. 13.

From Fig. 13a, it is necessary to remove the part already taken into account in the wave function. Note that it is necessary to include the dependence on the momentum transfer (to introduce form factors) in the elementary vertices ($\pi NN, \rho NN, \ldots$). This dependence ensures convergence of the diagrams. For example, Jaus described the vertex $N(p_1) + \pi^n(q) \to N(p_2)$ by the pseudovector interaction

$$-(f/m)\tau_n\gamma_\mu q_\mu\gamma_5 F_\pi(-q^2), \quad (184)$$

where

$$F_\pi(-q^2) = (\Lambda_\pi^2 - m^2)/(\Lambda_\pi^2 + q^2); \quad \Lambda_\pi^2 \approx 72m^2. \quad (185)$$

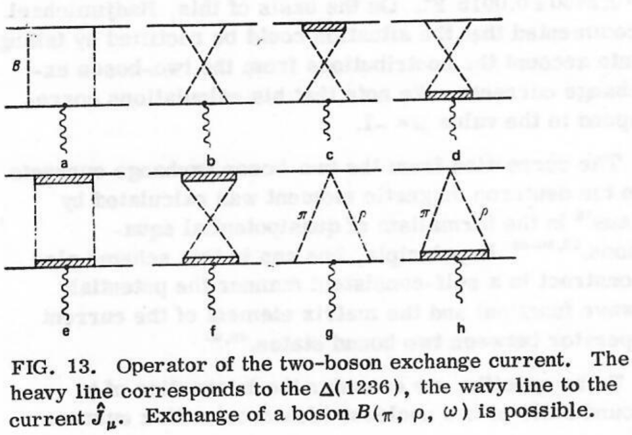

FIG. 13. Operator of the two-boson exchange current. The heavy line corresponds to the $\Delta(1236)$, the wavy line to the current $\hat{J}_\mu$. Exchange of a boson $B(\pi, \rho, \omega)$ is possible.

The choice of the form factor for the ρNN vertex is important. Vector dominance predicts the following momentum dependence of the form factor:

$$F_\rho(-q^2) = \Lambda_\rho^2/(\Lambda_\rho^2 + q^2), \quad \Lambda_\rho \approx 1450 \text{ MeV}. \quad (186)$$

This choice is clearly preferable to the normalization

$$F_\rho(-q^2) = (\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 + q^2), \quad (187)$$

since calculations with the form factor (187) are very sensitive to the value of Λ_ρ. For the form factor (186) and Λ_ρ taken in the interval

$$1200 \text{ MeV} \leqslant \Lambda_\rho \leqslant 1800 \text{ MeV}, \quad (188)$$

the results are almost stable.

Using the RSC deuteron wave function,[60] Jaus obtained from the two-boson current the correction

$$\Delta\mu_D^{(2)} = 0.0093 \text{ nuc. magn.} \quad (189)$$

In addition, he calculated the relativistic correction to the impulse approximation,

$$\Delta\mu_D^{(0)} = -0{,}0093 \text{ nuc. magn.}, \quad (190)$$

which completely compensates the effect from the two-boson exchange current (189). Because of this cancellation, the entire effect reduces to the contribution from the single-boson exchange current,[45,92]

$$\Delta\mu_n^{(1)} = 0.0116 \text{ nuc. magn.}, \quad (191)$$

and the wave-function renormalization due to the higher corrections:

$$\Delta\mu_D = [\mu_D/(1 + \varkappa_S - \Delta\mu'')]\Delta\mu''; \quad \Delta\mu'' = -0.0216. \quad (192)$$

As a result, for the correction to the deuteron magnetic moment we obtain the value

$$\delta\mu_D^J = 0.013 \text{ nuc. magn.}, \quad (193)$$

which lies in the middle of the interval (172).

However, in Jaus's paper,[79] the range of admissible p_D values is narrower:

$$6.17\% \leqslant p_D \leqslant 6.75\%. \quad (171a)$$

The extreme points of this interval correspond to the values of p_D from the potentials of Refs. 76 and 77, which are calculated with partial use of dispersion theory. Accordingly, for $\delta\mu_D$ we have instead of the interval (172)

$$0.012 \leqslant \delta\mu_D \leqslant 0.016 \text{ nuc. magn.} \quad (172a)$$

For values $\Lambda_\pi < 1100$ MeV, the correction $\delta\mu_D^J$ is outside the interval (172a).[79]

Note that calculations of this kind go well beyond the scope of the low-energy theorems. In addition, one must consider the consistency of the procedure of direct introduction of form factors at the vertices of Feyman loop diagrams in the light of the fundamental requirements of gauge invariance and unitarity. This problem is not discussed by Jaus. Even if Jaus's approach is correct from this point of view, it is not clear why a restriction should be made to the two-boson approximation. Indeed, comparison of the contributions of the single- and two-boson approximations (191) and (189) shows that they are of the same order. The choice of the renormalization (192) must also be justified.

Large momentum transfers. Hitherto, we have discussed mainly the effect from the exchange currents in the region of threshold energies. Despite the indicated difficulties, many attempts (see Refs. 43–45, 93, and 94 and also Refs. 10–17) have been made to take into account the contribution of the exchange currents to the electromagnetic form factors of the lightest nuclei at large momentum transfers. Fairly detailed calculations were made[43,45] for the electromagnetic form factors $A(k^2)$ and $B(k^2)$ of the deuteron, which occur in the differential cross section of elastic *ed* scattering (121):

$$d\sigma/d\Omega=(d\sigma/d\Omega)_{\text{Mott}}\,[A(k^2)+B(k^2)\tan^2(\theta/2)]. \quad (194)$$

These calculations showed that at large values of k^2 the contribution from the isoscalar exchange currents can be appreciable and lead to better agreement with the experimental data up to $k^2 \sim 40\ \text{F}^{-2}$. However, equally good agreement can be achieved[93] in the framework of the quasipotential equation of Gross without allowance for exchange currents, which, in the light of our discussions, emphasizes once more the uncertainties associated with allowance for the isoscalar exchange currents.

In connection with the treatment of form factors at large momentum transfers, there has been much discussion of the possibility of describing them in terms of the new, quark nuclear physics.[95-97] In the framework of the quark counting rule,[98-99] the form factor of a system consisting of N quarks has in the asymptotic region the behavior

$$F_{N_q}(k^2)\sim(k^2)^{1-N}. \quad (195)$$

Such behavior for F_{N_q} is also predicted in the model of a relativistic harmonic oscillator[100]:

$$F_{N_q}(k^2)=\left(1+\frac{k^2}{2M^2_{N_q}}\right)^{1-N}\exp\left(-\frac{N-1}{4\alpha_N}\frac{k^2}{1+k^2/2M^2_{N_q}}\right), \quad (196)$$

where M_{N_q} is the mass of the system of N quarks, $\alpha_N = N^{3/2}k$, and k is the elasticity coefficient of the oscillator.

In Ref. 93, the results are given of measurements of the elastic form factors of hadrons (π, n, p) and the lightest nuclei [normalized to the asymptotic behavior (195)]. As can be seen in Fig. 14, the asymptotic regime is reached for the deuteron at values $k^2 > 4\ \text{GeV}^2$; for the nuclei ^{3}He ($N=9$) and ^{4}He ($N=12$) there are data

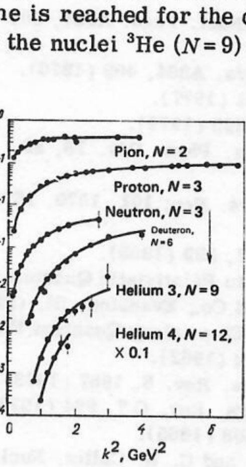

FIG. 14. Elastic form factors of hadrons and the lightest nuclei for large k^2 divided by the asymptotic behavior (195).[93]

in only the pre-asymptotic region. One can therefore say that the quark structure of the nuclei ^{3}He and ^{4}He is not manifested appreciably down to distances ~0.1 F. In addition, comparison of the predictions of the quark model with the available data for the deuteron form factors is premature, since these data are actually still at the edge of the asymptotic region.

CONCLUSIONS

We shall here attempt to list the main points of the material presented in the review and give a brief comparative characterization of the different approaches to the construction of the operators of the exchange currents.

In the review, we have mainly considered the problems of vector exchange currents. Following the historical development, we began by discussing the Chemtob–Rho formalism (the *S*-matrix formalism; see Sec. 1). On the basis of a low-energy theorem of the type of Adler's theorem, we studied the properties of the basic element that occurs in the operator of the two-particle exchange current, namely, the amplitude for pion production on a nucleon by the axial and vector currents. We considered two limits: the soft-pion limit and the soft-current limit. A similar approach was applied directly to the operator of the exchange current. This made it possible to discuss Siegert's theorem on a unified basis for both the vector and the axial current.

In the *S*-matrix method, the final and initial pair of nucleons, whose coordinates occur in the definition of the exchange-current operator, are treated as free. It was shown in Secs. 2 and 3 that the successive elimination of the meson degrees of freedom from the nuclear wave function in the framework of the Hamiltonian formalism leads to the requirement that the nuclear potential and the exchange-current operator be constructed in a self-consistent manner. Two difficulties of the theory were noted.

1. At the present time, it is not possible to eliminate in a unique manner the meson degrees of freedom from the nuclear wave function, which ultimately must be a solution to a Schrödinger equation with a two-particle potential (cf. the transformation method and the projection method in Sec. 2). This lack of uniqueness can also appear in the construction of the exchange-current operator, particularly in the case of large momentum transfers.

2. The nature of the πNN coupling has not been fully studied, and therefore one can, using a Dyson transformation, always introduce a linear combination of pseudoscalar and pseudovector πNN coupling with an undetermined mixing parameter (unitary freedom). Since the πNN vertex occurs in both the potential-energy operator and the exchange-current operator, these operators must, in general, be constructed in a self-consistent manner. At the present stage in the development of the theory, this also cannot yet be done.

Note that there exists a value of the parameter μ for which the exchange-current operator constructed in the framework of the Hamiltonian formalism is equal to the exchange-current operator obtained by the S-matrix method.

If it is sufficient to know the exchange-current operator to terms $\sim(v/c)^2$, the unitary freedom is not manifested (there is no dependence on μ). At the same time, the contact term (see Fig. 7c) and the pion current (Fig. 7d) constructed by the S-matrix method are identical to the same operators constructed by the method of unitary transformations and the projection method (see Figs. 9a–9e). Such a situation obtains for the vector isovector exchange currents.

For the example of the calculations of the deuteron magnetic moment in Sec. 3 we have demonstrated that unitary freedom can also be manifested significantly in the treatment of the isoscalar exchange currents.

In comparing the predictions of the theory with the experimental data (Sec. 3), we first discussed vector isovector exchange currents. Here, the conclusion is unambiguous, and their existence has been proved beyond doubt. They need to be known only to accuracy $(v/c)^2$; the predictions of both the S-matrix method and the method based on the Hamiltonian formalism (static Chew–Low theory for the pion production amplitude) are virtually identical and agree well with the existing data on the reactions $n+p\to d+\gamma$, $n+d\to {}^3\mathrm{H}+\gamma$ (the neutrons are thermal), $e+d\to n+p+e'$ (large scattering angles), and also with the data on the isovector magnetic moment of the bound state of three nucleons. The vector isovector exchange currents can be regarded as firmly established in nuclear physics.

The situation is quite different in the case of isoscalar vector exchange currents. They constitute only a small relativistic correction to the impulse approximation, and their contribution depends strongly on the parameter of the unitary freedom. Therefore, they are difficult to distinguish on the background of the other relativistic effects of the same order. Here, there is in fact no reliable theory as yet. Evidently, the problem of the isoscalar exchange currents is directly related to the problem of constructing a self-consistent relativistic nuclear physics.

[1]E. V. Condon and G. Breit, Phys. Rev. 49, 904 (1936).
[2]A. J. F. Siegert, Phys. Rev. 52, 787 (1937).
[3]M. Chemtob, in: Mesons in Nuclei, North-Holland, Amsterdam (1979), p. 555.
[4]J. L. Friar, Preprint LA-UR-79-2658, Los Alamos (1979); J. L. Friar and S. Fallieros, Phys. Rev. C13, 2571 (1976).
[5]J. H. Dubach *et al.*, Nucl. Phys. A271, 279 (1976).
[6]E. Ivanov and E. Truhlík, Nucl. Phys. A316, 437, 451 (1979).
[7]G. E. Dogotar, R. A. Eramzhyan, and E. Truhlík, Nucl. Phys. A326, 225 (1979).
[8]J. Hošek, and E. Truhlík, Preprint, Institute of Nuclear Physics, Czechoslovak Academy of Sciences, Řež (1979).
[9]E. Truhlík, in: Intern. Symposium on Few Particle Problems in Nuclear Physics, Dubna (1979).
[10]E. Hadjimichael *et al.*, in: Proc. of the June Workshop in Intermediate Energy Electromagnetic Interaction with Nuclei, MIT, Cambridge (1977).
[11]A. M. Green, Rep. Prog. Phys. 39, 1109 (1976).
[12]V. A. Karmanov and I. S. Shapiro, Fiz. Elem. Chastits At. Yadra 9, 327 (1978) [Sov. J. Part. Nucl. 9, 134 (1978)].
[13]H. Baier, Fortschr. Phys. 27, 208 (1979).
[14]Mesons in Nuclei, North-Holland, Amsterdam (1979).
[15]Lecture Notes in Physics, Vol. 86, Springer, Berlin (1978).
[16]AIP Conf. Proc. 54 (ed. E. V. Hungerford), New York (1979).
[17]Lecture Notes in Physics, Vol. 108, Springer, Berlin (1979); High Energy Physics and Nuclear Structure, North-Holland, Amsterdam (1980).
[18]M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971).
[19]S. L. Adler, Ann. Phys. (N.Y.) 50, 189 (1968).
[20]M. Gourdin, Phys. Rep. 11, 29 (1974).
[21]F. E. Low, Phys. Rev. 110, 974 (1958).
[22]N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).
[23]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).
[24]J. Schwinger, Phys. Lett. B24, 473 (1967); J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); B. W. Lee and H. J. Nieh, Phys. Rev. 166, 1507 (1968); V. I. Ogievetsky and B. M. Zupnik, Nucl. Phys. B24, 612 (1970).
[25]S. L. Adler and R. F. Dashen, Current Algebras and Applications to Particle Physics, New York (1968) [Russian translation published by Mir, Moscow (1970)].
[26]A. I. Vaĭnshteĭn and V. I. Zakharov, Usp. Fiz. Nauk 100, 225 (1970) [Sov. Phys. Usp. 13, 73 (1970)].
[27]S. Weinberg, Phys. Rev. 177, 2604 (1969).
[28]S. Gasiorovicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969).
[29]S. L. Adler and Y. Dothan, Phys. Rev. 151, 1267 (1966).
[30]D. Yu. Bardin and E. A. Ivanov, Fiz. Elem. Chastits At. Yadra 7, 726 (1976) [Sov. J. Part. Nucl. 7, 286 (1976)].
[31]S. L. Adler, Phys. Rev. B 140, 736 (1965); W. I. Weisberger, Phys. Rev. 143, 1302 (1966).
[32]S. L. Adler, Phys. Rev. B 137, 1022 (1965); Phys. Rev. B 139, 1638 (1965).
[33]H. Ohtsubo *et al.*, Prog. Theor. Phys. 44, 1595 (1970).
[34]Y. Futami *et al.*, Prog. Theor. Phys. Suppl. 60, 83 (1976).
[35]K. Ohta, Nuc. Phys. A314, 333 (1979).
[36]M. Gari and H. Hyuga, Z. Phys. A277, 291 (1976).
[37]M. Gari, in: Lecture Notes in Physics, Vol. 86, Springer-Verlag, Berlin (1978), p. 41; H. Hyuga and M. Gari, Nucl. Phys. A274, 333 (1976).
[38]J. L. Friar, Ann. Phys. (N.Y.) 104, 380 (1977).
[39]K. Ohta and M. Wakamushu, Prog. Theor. Phys. 55, 131 (1976).
[40]M. Chemtob, in: Mesons in Nuclei, North-Holland, Amsterdam (1979), p. 555.
[41]H. Arenhövel, Nukleonika 24, 273 (1979).
[42]D. O. Riska, in: Mesons in Nuclei, North-Holland, Amsterdam (1979), p. 755.
[43]A. D. Jackson, A. Lande, and D. O. Riska, Phys. Lett. B55, 23 (1975).
[44]J. Borysowicz and D. O. Riska, Nucl. Phys. A254, 301 (1975).
[45]M. Gari and H. Hyuga, Nucl. Phys. A264, 409 (1976).
[46]J. L. Friar, Phys. Lett. B69, 51 (1977).
[47]J. L. Friar, Phys. Rev. C 20, 325 (1979).
[48]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).
[49]G. F. Chew and F. E. Low, Phys. Rev. 101, 1570, 1579 (1956).
[50]G. C. Wick, Rev. Mod. Phys. 27, 339 (1955).
[51]S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Row, Peterson and Co., Evanston, Ill. (1961).
[52]E. M. Henley and W. Thirring, Elementary Quantum Field Theory, McGraw-Hill, New York (1962).
[53]J. Thakur and L. L. Foldy, Phys. Rev. 8, 1957 (1973).
[54]M. Gari and A. H. Huffman, Phys. Rev. C 7, 994 (1973).
[55]H. P. Noyes, Nucl. Phys. 74, 508 (1965).
[56]A. E. Cox, S. A. R. Wynchank, and C. H. Collie, Nucl. Phys. 74, 497 (1965).
[57]D. O. Riska and G. E. Brown, Phys. Lett. B38, 193 (1972).

[58]M. Collocci, B. Mosconi, and P. Ricci, Phys. Lett. B45, 224 (1973).
[59]M. W. Kermod and A. McKerrell, in: Lecture Notes in Physics, Vol. 82, Springer–Verlag, Berlin (1978), p. 63.
[60]R. V. Reid, Jr., Ann. Phys. (N.Y.) 50, 411 (1968).
[61]V. P. Alfimenkov *et al.*, Preprint RZ-12667 [in Russian], JINR, Dubna (1979).
[62]L. Kaplan, G. R. Ringo, and K. E. Wilzbach, Phys. Rev. 87, 785 (1952).
[63]C. C. Trail and S. Raboy, Preprint ANL-6797 (1963), p. 247.
[64]E. T. Jurney and H. T. Motz, Preprint ANL-6797 (1963), p. 236.
[65]J. S. Merritt, J. G. V. Taylor, and A. W. Boyd, Nucl. Sci. Eng. 34, 195 (1968).
[66]E. Hadjimichael, Phys. Rev. Lett. 31, 183 (1973).
[67]A. C. Phillips, Nucl. Phys. A184, 237 (1972).
[68]Y. E. Kym and A. Tubis, Ann. Rev. Nucl. Sci. 24, 69 (1974).
[69]A. Barroso and E. Hadjimichael, Nucl. Phys. A238, 422 (1975).
[70]J. Hockert *et al.*, Nucl. Phys. A217, 14 (1978).
[71]J. A. Lock and L. L. Foldy, Ann. Phys. (N.Y.) 93, 276 (1975).
[72]W. Fabian and H. Arenhövel, Nucl. Phys. A258, 461 (1976).
[73]G. G. Simon *et al.*, Phys. Rev. Lett. 37, 739 (1976).
[74]G. G. Simon *et al.*, Nucl. Phys. A324, 277 (1979).
[75]T. De Tourreil, B. Rouben, and D. W. L. Sprung, Nucl. Phys. A242, 445 (1975); K. Holinde and R. Machleidt, Nucl. Phys. A247, 495 (1975).
[76]M. Lacombe *et al.*, Phys. Rev. D 12, 1495 (1975).
[77]A. D. Jackson, D. O. Riska, and B. Verwest, Nucl. Phys. A249, 397 (1975).
[78]E. Hadjimichael, Nucl. Phys. A312, 341 (1978).
[79]W. Jaus, Nucl. Phys. A314, 287 (1979).
[80]G. E. Brown and A. D. Jackson, Nucleon–Nucleon Interaction, North-Holland, Amsterdam (1979) (Russian translation published by Atomizdat, Moscow (1979)).
[81]E. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).
[82]A. A. Longunov and A. N. Tavkhelidze, Nuovo Cimento 29, 370 (1963).
[83]R. H. Thompson, Phys. Rev. D 1, 110 (1970).
[84]K. Holinde, K. Erkelenz, and R. Alzetta, Nucl. Phys. A194, 161 (1972).
[85]V. G. Kadyshevsky, Nucl. Phys. B6, 125 (1968).
[86]F. Gross, Phys. Rev. 186, 1448 (1969).
[87]M. H. Partovi and E. L. Lomon, Phys. Rev. D 2, 1999 (1970).
[88]M. Chemtob, J. W. Durso, and D. O. Riska, Nucl. Phys. B38, 141 (1972).
[89]S. Mandelstam, Proc. R. Soc. London, Ser. A 233, 248 (1955).
[90]R. N. Faustov, Ann. Phys. (N.Y.) 78, 176 (1973).
[91]E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1932 (1951).
[92]M. Gari, H. Hyuga, and B. Sommer, Phys. Rev. C 14, 2196 (1976).
[93]R. G. Arnold, in: Lecture Notes in Physics, Vol. 108, Springer–Verlag, Berlin (1979), p. 76.
[94]Y. E. Kim, in: Lecture Notes in Physics, Vol. 86, Springer–Verlag, Berlin (1978), p. 231.
[95]C. E. De Tar, in: Lecture Notes in Physics, Vol. 87, Springer–Verlag, Berlin (1978), p. 113.
[96]G. E. Brown and M. Rho, Phys. Lett. B82, 177 (1979).
[97]V. K. Lukyanov and A. I. Titov, in: Intern. Conf. on Extreme States in Nuclear Systems, Dresden (1980).
[98]S. J. Brodsky and G. Farrar, Phys. Rev. Lett. 31, 1153 (1973); Phys. Rev. D 11, 1309 (1975).
[99]V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973).
[100]Y. Kizukuri, M. Namiki, and K. Okano, Prog. Theor. Phys. 61, 559 (1979).

Translated by Julian B. Barbour