

Interaction of heavy ions at energies around 10 MeV/nucleon

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The review is devoted to a topical problem of nuclear physics, namely, the interaction of heavy ions at energies ~ 10 MeV/nucleon. The experimental data on fusion reactions, deep inelastic collisions, and excitation of giant resonances in heavy-ion collisions are analyzed. Theoretical approaches to the description of these processes are discussed.

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INTRODUCTION

During the last five or six years, much research has been done at numerous scientific centers on reactions involving heavy ions. These studies were initiated by successes in the development of accelerator technology with the possibility for accelerating very heavy nuclei. Currently, there are heavy-ion beams with either energies of order 10 MeV/nucleon or relativistic energies around 1 GeV/nucleon. The existing plans, after realization, will open up a new and interesting range for investigation, namely, reactions with heavy ions at energies ~ 100 MeV/nucleon. This will make it possible to construct a "bridge" between the investigations at ~ 10 MeV/nucleon and ~ 1 GeV/nucleon.

By means of beams of heavy ions accelerated to relativistic energies, investigations are currently being made of the properties of nuclear matter under extremal conditions (pion condensation, critical charge, superdense nuclei) and a search made for collective effects in the interaction of nuclei (cumulative effect, threshold effects, shock waves).

Outside the relativistic region at energies up to 10 MeV/nucleon, where collective effects, which determine the course of reactions with heavy ions, play an important part, the investigations are concentrated on elucidating the microscopic picture of the collective effects in the hope of understanding the mechanism of the various reactions. This branch of heavy-ion physics has developed most rapidly in recent years, but it is still far from the construction of a unified picture of such reactions. Many questions are still unanswered, while many new ones are being posed by the experimental investigations.

This review is an attempt to systematize the problems investigated in this branch of physics.¹ The main attention will be concentrated on the problems still unsolved. It is convenient to classify the various types of reaction involving heavy ions on the basis of classical notions about the motion of an ion in a trajectory. Then, depending on the distance of closest approach of the nuclei, different types of reaction will be realized (Fig. 1). In grazing collisions of the nuclei, only direct reactions with characteristic reaction times 10^{-23} – 10^{-22} sec take place.

In the case of closer encounters of the nuclei, deep inelastic collisions of the heavy ions are observed. This

type of nuclear reaction was investigated for the first time at the Laboratory of Nuclear Reactions at Dubna.² We encounter here a fundamentally new phenomenon in nuclear physics—the existence of a double nuclear system. Such a system lives for $\sim 10^{-21}$ sec, without reaching a state of complete equilibrium. This makes it possible to investigate various nonequilibrium processes in nuclear matter. The different partial waves that contribute to the deep inelastic collisions correspond to different interaction times and, accordingly, different angles of deflection of the incident nucleus. Therefore, investigation of the correlations between the reaction characteristics and the angular distributions gives information about the development in time of the nonequilibrium processes. Finally, the nuclei may get so close to each other that fusion reactions become possible. The system formed in such a case evolves toward the establishment of statistical equilibrium. The process ends either with the evaporation of light particles and the formation of a residual nucleus or with fission into two fragments ($\tau \approx 10^{-20}$ – 10^{-18} sec). The fusion reactions are of great interest in connection with the attempts to synthesize superheavy elements. In this direction, fundamentally new results have been obtained at the Laboratory of Nuclear Reactions.³

Because of the low compressibility of nuclear matter at energies of the incident ion up to 10 MeV/nucleon, the mutual penetration of the nuclei is slight, except for light systems, and the interaction occurs basically

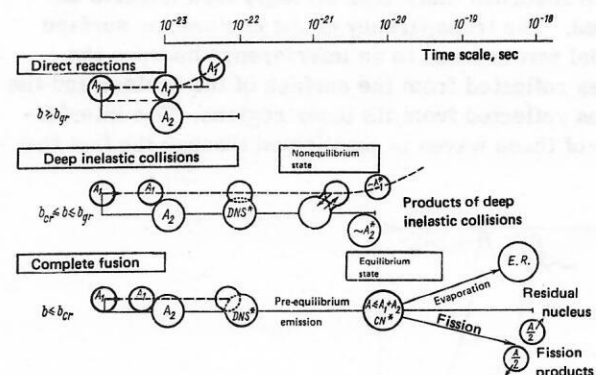


FIG. 1. Classification of heavy-ion reactions according to the value of the impact parameter b and the reaction time; b_{gr} is the value of the impact parameter corresponding to a grazing collision.

on the surface of the nucleus. Therefore, except for the fusion reactions, the reactions considered above can be classified as peripheral.

1. DIRECT REACTIONS

1.1 Elastic scattering

At low energies, there is purely Coulomb scattering of the heavy ions: either elastic, described by the Rutherford formula, or inelastic, accompanied by Coulomb excitation of the nuclei. With increasing energy, the nuclei make closer encounters. Besides the Coulomb interaction, the "tail" of the nuclear potential begins to have an effect. Therefore, the elastic scattering cross section is no longer described by the Rutherford formula. Elastic scattering is conveniently characterized by the ratio of the experimental cross section of elastic scattering to the Rutherford cross section. As can be seen in Fig. 2,⁴ this ratio decreases exponentially above a certain angle. The angle at which $\sigma_{el}/\sigma_{Ruth} = 0.25$ is called the angle θ_{gr} of grazing collision. Knowing it, one can determine the values of the orbital angular momentum l_{gr} and the impact parameter b_{gr} corresponding to a grazing collision. Knowing θ_{gr} , one can in principle also determine the interaction range R_{int} . With increasing energy of the incident ion, θ_{gr} decreases, and the rate of decrease of $\sigma_{el}/\sigma_{Ruth}$ increases (see Fig. 2).

This behavior of the cross section corresponds to a simple classical picture in which all the partial waves with l greater than some critical l_0 are scattered with virtually no absorption. The partial waves with $l < l_0$ are absorbed without contributing to the scattering. The scattering is determined by the "tail" of the nuclear potential and does not depend on the behavior of the potential within the nucleus. This is the reason why optical potentials with the same tails but very different behavior within the nucleus describe the experimental data on elastic scattering equally well.

Such a picture existed for several years prior to the experimental discovery of anomalously strong scattering through large angles and a number of other facts.⁵ To describe this phenomenon, it was necessary to assume that the partial waves with angular momenta near l_0 are absorbed much less strongly than hitherto assumed. The transparency of the nucleus for surface partial waves leads to an interference between the waves reflected from the surface of the nucleus and the waves reflected from its inner regions. The interference of these waves is manifested through the fact that

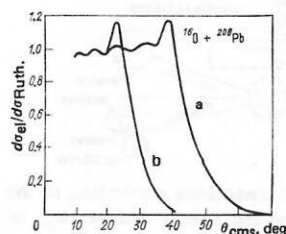


FIG. 2. Cross section for elastic scattering of ^{16}O ions by ^{208}Pb at energies 129.5 (a) and 192 MeV (b).

the elements s_l of the S matrix cease to depend smoothly on l . Their behavior exhibits nonmonotonicity. The weak absorption of the partial waves with large values of l in the surface layer and the occurrence of a standing wave in it sharply increase the backward scattering compared with the case of strong absorption.

The anomalous backward scattering may be due to the transfer of a group of particles (cluster) from one nucleus to the other or to scattering on substructures (for example, α clusters). Many problems associated with anomalous backward scattering are now understood, but a full explanation of this phenomenon has still not been provided.

1.2 Excitation of giant resonances

Among the experiments on the inelastic scattering of heavy ions, investigations into the excitation of giant resonances are of independent interest and also of interest from the point of view of studying the energy-dissipation mechanism in collisions. The probability that a fragment is discovered in a distinguished resonance state depends on the considered system. For heavy nuclei, the probability of multiple excitation of giant resonances is large because of the large losses of kinetic energy. In these reactions, it is difficult to observe a fragment in a distinguished and, therefore, identifiable state. It is therefore necessary to increase the energy per nucleon of the incident nucleus in order to obtain information about the early stage of the energy relaxation process and observe fragments in the distinguished resonance state.

In light systems, the high energies of the giant resonances make the separation of fragments in a distinguished resonance state probable. The question at issue is whether there is selective population of giant resonances above the continuum.

So far, only a few investigations have been made in

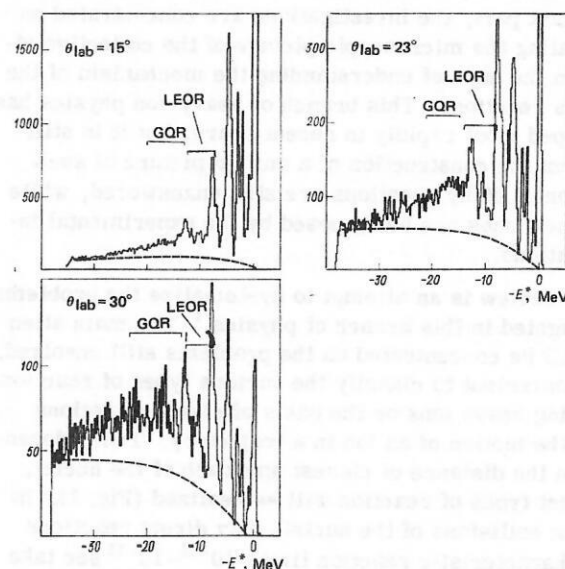


FIG. 3. Energy spectrum of the $^{27}\text{Al} (^{12}\text{C}, ^{12}\text{C}^*)^{27}\text{Al}^*$ reaction at $E_{lab} = 82$ MeV. The excitation regions corresponding to the low-energy octupole resonance (LEOR) and the giant quadrupole resonance (GQR) are shown.

which giant resonances were excited by means of heavy ions.⁶⁻¹⁰ In Ref. 6, the $^{27}\text{Al}(^{12}\text{C}, ^{12}\text{C}^*)^{27}\text{Al}^*$ reaction was investigated at $E_{\text{lab}} = 82$ MeV. The energy spectra, measured at different angles, reveal transitions to groups of states at 10.4, 12.8, and 14–26 MeV (Fig. 3). The groups at the excitation energies 10.4 and 12.8 MeV lie in the region of the low-energy octupole resonance of ^{27}Al , although there are no direct indications that these are octupole transitions. The broad structure covering the range of excitation energies from 14 to 26 MeV coincides with the known position and width of the giant quadrupole resonance in ^{27}Al . The integrated cross section corresponding to excitation of the individual resonances was 10 mb, whereas the total inelastic cross section is 200 mb, which indicates a relatively low probability that the target remains in the resonance state at the end of the reaction. Data do exist¹⁰ on the excitation of isoscalar quadrupole and octupole resonances in ^{208}Pb and ^{58}Ni by 315-MeV ^{16}O ions.

It is evident that under certain conditions heavy ions can be used to excite giant resonances of high multipolarity.

2. DEEP INELASTIC COLLISIONS

The elucidation of the mechanism of deep inelastic collisions of heavy ions is one of the most important problems in heavy-ion physics. In such reactions, a huge number of internal degrees of freedom of the nucleus are excited, and it is impossible to follow the evolution of each of them. Instead, one considers the most important, collective degrees of freedom associated with the change in the state of the motion of a large number of nucleons, and one considers their interaction with the large number of internal degrees of freedom. Such collective macroscopic degrees of freedom are: the kinetic energy of the relative motion; the total excitation energy and its distribution between the fragments; the mass or charge distribution between the fragments; the relative angular momentum; the internal angular momenta of the fragments and their orientation; the relationship between the numbers of neutrons and protons in the fragments; and the deformation of the system.

The coupling between the collective and single-particle degrees of freedom is a source of irreversible processes in the nucleus such as friction and mass and charge transfer. Although different irreversible processes have already been investigated for a long time in many branches of macroscopic physics, deep inelastic collisions have for the first time opened up the possibility of studying them in a small quantum-statistical system such as an atomic nucleus. The study of deep inelastic transfer reactions is also interesting in that we obtain information on macroscopic properties of nuclear matter such as the viscosity, thermal conductivity, and mass and charge diffusion.

The various macroscopic quantities have different relaxation times. One of the most important advantages of deep inelastic collisions of heavy ions is the possibility of studying a nuclear system at different stages in the

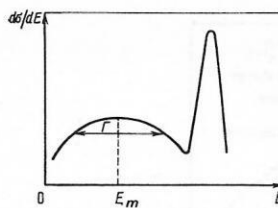


FIG. 4. Energy spectrum of products of deep inelastic heavy-ion collisions. E_m is the position of the center of the deep inelastic maximum, and Γ is its width.

relaxation processes on account of the connection between the angular distributions and the interaction times. We shall consider below in more detail the various relaxation processes which take place in nuclei in deep inelastic collisions.

2.1 Dissipation of the kinetic energy

One of the most characteristic features of deep inelastic collisions of heavy ions is the large loss of kinetic energy. Study of the process of kinetic-energy dissipation must provide the answer to questions such as the following: which nuclear states are excited preferentially; what is the excitation mechanism; what is the characteristic time of "discharge" of the main part of the kinetic energy; how is the excitation energy distributed between the fragments; how rapidly, if at all, is thermal equilibrium established?

Experimental data. The energy spectrum $d\sigma/dE$ of the reaction products is shown schematically in Fig. 4. Its most characteristic features are the narrow quasi-elastic peak and the broad peak corresponding to deep inelastic collisions with large energy loss up to several hundred MeV. An increase in the energy of the incident ion leads to a growth in the width Γ of the deep inelastic peak.^{11,12} At the same time, the most probable

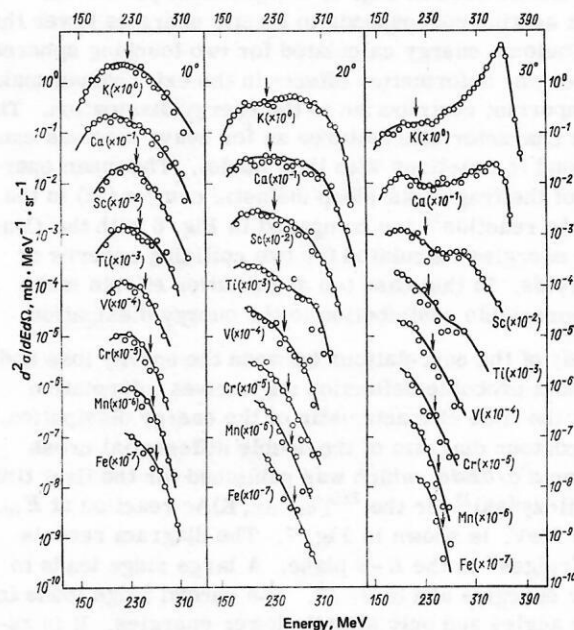


FIG. 5. Energy spectra of K, Ca, Sc, Ti, V, Cr, Mn, and Fe.¹ The arrows indicate the exit Coulomb barriers for the final products.

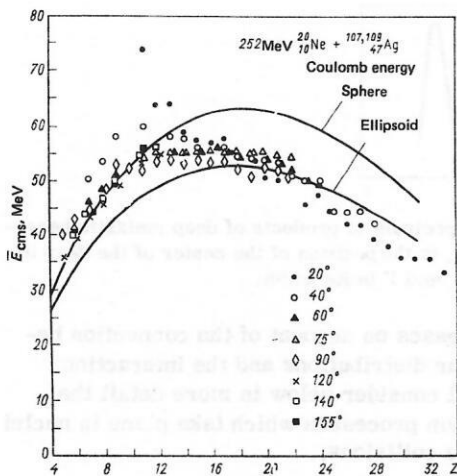


FIG. 6. Dependence of the mean kinetic energies in the center-of mass system of the products of the $^{20}\text{Ne} + ^{107,109}\text{Ag}$ reaction ($E_{\text{lab}} = 252$ MeV) on the Z of one of the fragments for different angles. The curves correspond to the calculated kinetic energies of the fragments obtained under the assumption of Coulomb repulsion of two spheres in contact (upper curve) or two spheroids (lower curve).

kinetic energies E_m of the fragments are in many cases independent of the energy of the incident ion. This last fact indicates that the energy dissipation is a fast process.

Important information can be obtained by investigating the correlations between the energy losses and the mass transfer. The differential cross section $d^2\sigma/dEdZ$ is shown as a function of the kinetic energy for different elements in Fig. 5. For Z near Z_p of the incident nucleus, the spectrum contains an appreciable high-energy component. With increasing $|Z - Z_p|$, the spectrum becomes softer.

As can be seen in Fig. 5, a significant part of the cross section corresponds to kinetic energies lower than the Coulomb energy calculated for two touching spheres. Therefore, deformation effects in the exit channel make an important contribution to the energy dissipation. The same characteristic features as for heavy systems can be found in reactions with light nuclei. The mean energies of the fragments (deep inelastic component) in the $\text{Ne} + \text{Ag}$ reaction¹² are compared in Fig. 6 with the Coulomb energies calculated for two colliding spheres or spheroids. In this case too deformation effects make an appreciable contribution to the energy dissipation.

Study of the correlations between the energy loss and the most probable deflection angle gives information about the time characteristic of the energy dissipation. The contour diagram of the double differential cross section $d^2\sigma/dEd\theta$, which was published for the first time by Wilczyński¹³ for the $^{232}\text{Th}(^{40}\text{Ar}, \text{K})\text{Ac}$ reaction at $E_{\text{lab}} = 388$ MeV, is shown in Fig. 7. The diagram reveals two "ridges" in the $E-\theta$ plane. A large ridge leads to lower energies and to $\theta = 0^\circ$. The second ridge leads to large angles and only slightly lower energies. It is assumed that it continues the first ridge to the region of negative angles.

To determine the interaction time, it is necessary to

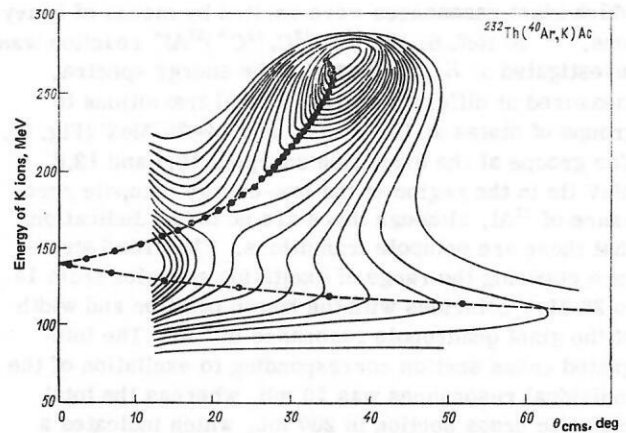


FIG. 7. Contour diagram of the double differential cross section $d^2\sigma/dEd\theta$ for the $^{232}\text{Th}(^{40}\text{Ar}, \text{K})\text{Ac}$ reaction at laboratory energy 388 MeV. The black circles are the angular momentum for 1, in the range from 250 to 180.

construct the experimental deflection function $\theta(l)$. Such a construction is based on the assumption that the E axis in the Wilczyński diagram can be converted to the l axis. The difference $\Delta\theta$, between the experimental and Coulomb deflection functions characterizes the angle through which the intermediate system is turned during the interaction time τ_{int} , which can be calculated approximately: $\tau_{\text{int}}(l) = \Delta\theta J(l)/\hbar l$, where $J(l)$ is the moment of inertia of the intermediate system. As follows from the evaluation²² of the experimental data, $\tau_{\text{int}}(l)$ decreases exponentially with increasing l (Fig. 8).

Information on the time characteristic of energy dissipation can also be obtained by investigating the correlations between the energy loss and the width of the charge distribution σ_Z^2 of the reaction products, since the transfer of nucleons requires a definite time. The dependence of ΔE on σ_Z^2 for the $\text{Xe} + \text{Sn}$ reaction ($E_{\text{lab}} = 779$ MeV) is shown in Fig. 9.¹⁴ The connection between the scale of σ_Z^2 with l and τ_{int} is indicated. However, such a graph does not give a full picture of how rapidly the energy loss proceeds. For example, for $\tau_{\text{int}} = 2 \times 10^{-21}$ sec, $\Delta E = 135$ MeV. However, on the basis of this graph one cannot assert that 70% of the ΔE is lost in $\approx 1.1 \times 10^{-21}$ sec. For example, calculations in a model using a phenomenological frictional force¹⁵ show that 70% of the kinetic energy is lost in a

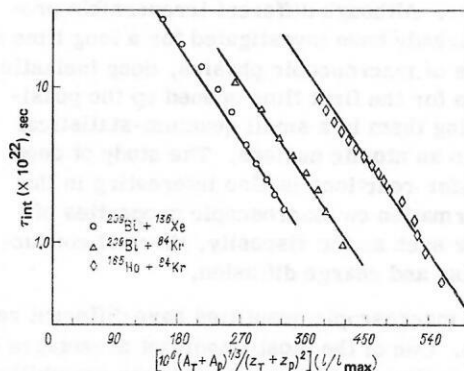


FIG. 8. Dependence of the interaction time on the angular momentum l .

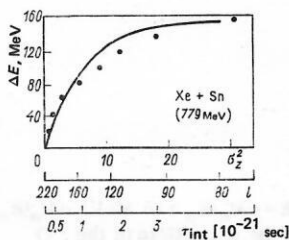


FIG. 9. Dependence of the kinetic-energy loss ΔE on the width σ_z^2 of the charge distribution of the light fragment in the Xe + Sn reaction ($E_{\text{lab}} = 779$ MeV).

time $\sim 1.2 \times 10^{-22}$ sec. Therefore, the energy can be lost in a very short time (amounting to a small fraction of τ_{int}) despite the correlations shown in Fig. 9, which are also reproduced by calculations in the framework of phenomenological models. It cannot be concluded from these correlations that the "discharge" of the kinetic energy and the diffusion of the nucleons take place in a comparable time. It can be seen from Fig. 9 that for short interaction times a high rate of energy dissipation is characteristic.

Depending on the interaction time, and therefore on the number of transferred nucleons, the nature of the angular distributions also changes (Fig. 10).¹⁰⁵ Angular distributions with a sharp peak at θ_{tr} are encountered only for elements with Z near Z_p . For $|Z - Z_p| \geq 6$, the angular distributions become smoother and acquire features characteristic of systems with relatively long lifetimes.

Great interest attaches to the investigation of the distribution of the excitation energy between the fragments in deep inelastic reactions. The experiments so far made^{25, 26} indicate a distribution of the excitation energy

proportional to the masses of the fragments. However, the question of structural effects in the distribution of the excitation energy associated, for example, with the manifestation of large shells remains open.

Theoretical models. To describe the process of dissipation of kinetic energy, it was suggested¹⁶ that classical equations of motion with phenomenological frictional forces should be used. Choosing the value and radial dependence of the friction tensor, one can achieve a satisfactory description of the experimental data¹⁷⁻¹⁹ if allowance is made for deformation effects in the exit channel. However, such an approach does not give information about the energy-dissipation mechanism.

This mechanism was analyzed in the framework of classical notions in Refs. 20 and 21. Since nucleons are characterized by a long mean free path if the excitation energies of the nuclei are not high, they can be treated in a first approximation as an ideal gas in a closed shell. Collisions between the nucleons and the surface lead to an exchange of energy between the motion of the nucleons and the collective motion of the surface, i.e., to energy dissipation. Another possibility is associated with transition of nucleons from one nucleus to another. The relative momentum of the nucleon will be dissipated, going over into momentum of internal excitation. In this last case, the energy loss per nucleon must decrease linearly with increasing dissipation of the energy. The experimental data confirm such a dependence.²² However, the observed energy losses are much greater than the classical model predicts. Exchange of nucleons explains 15-30% of the energy loss. The remainder must be due to excitation of nucleons in the same nucleus. Here one could have incoherent particle-hole excitations²³ or collective excitations, i.e., giant resonances.²⁴ Of course, the giant resonances have a fairly large width and are coupled to other excitations of the nuclei of more complicated nature (two-particle-two-hole, etc.), but they could be "doorway" states in the reaction.

Emission of light particles. We now consider the rate of thermalization of excitation energy, i.e., the rate at which thermal equilibrium is established throughout the volume of the nucleus. This question is intimately related to the emission of light particles in deep inelastic reactions. It is to be expected that the particles emitted in the interaction process will carry information about the degree of equilibration achieved in the collision process if they are emitted prior to the disintegration of the compound system. If they are emitted after disintegration of the compound system by one of the fragments, they may carry information about the excitation energy transferred to the fragment and the rate of establishment of thermal equilibrium. A deep inelastic reaction takes place in a time that is only a certain fraction of the characteristic rotation time. Therefore, if emission takes place in a time interval comparable with the interaction time of the nuclei, one cannot expect manifestation of characteristic features of evaporation in the emission spectra of the light particles. Such processes must be direct or preequilibrium processes.

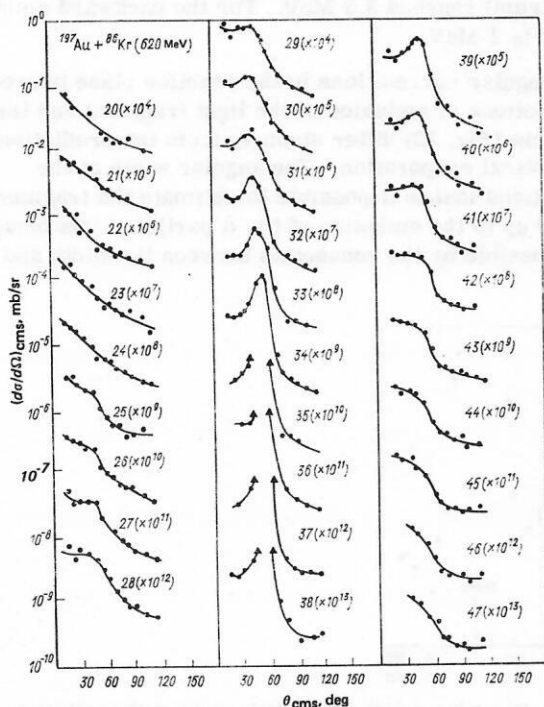


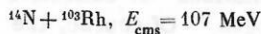
FIG. 10. Dependence of the angular distribution on the Z of the light fragment in the $^{197}\text{Au} + ^{86}\text{Kr}$ reaction ($E_{\text{lab}} = 620$ MeV).

In the initial stage of the collision process, a large fraction of the excitation energy can be concentrated in the overlap region of the interacting nuclei. It is very probable that one or several particles will be emitted from this region of higher "temperature." As in pre-equilibrium emission, such particles will predominantly have high energies. Then, if the lifetime of the system is sufficiently long, the remainder of the excitation energy will be uniformly distributed over the complete compound system until a common temperature is established.

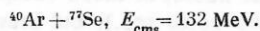
Are there experimental data which confirm the above considerations? Measurements of various types have so far been made.

Observation of light charged particles (p , α) in reactions with heavy ions without coincidence with one of the heavy fragments. It was already shown in the early paper of Ref. 27 that the spectrum of charged particles emitted in heavy-ion reactions is not always symmetric about 90° , as should be the case for emission from compound nuclei. The observed forward peak indicates the existence of an additional reaction mechanism. The spectrum of nonequilibrium α particles is enhanced more than the spectrum of nonequilibrium protons and deuterons in reactions induced by ^{12}C , ^{14}N , and ^{16}O ions. The kinetic energies of the α particles are also consistent with disintegration of the incident ion in the field of the target nucleus. The greater yield of α particles in the reactions with ^{12}C and ^{16}O compared with those induced by ^{14}N ions is support for the existence of the disintegration mechanism.

Unexpected results were obtained in Ref. 28. Two reactions leading to the same intermediate systems were investigated:



and



The emission of pre-equilibrium α particles in the reaction with Ar was found to be small compared with that in the reaction with ^{14}N . The reasons for this difference are still not clear. In addition, the high mean energies of the protons and the high-energy tail in the α -particle spectrum precluded an explanation of the experiment on the basis of the disintegration mechanism. However, without measurement of the emitted light particles in coincidence with one of the heavy fragments it is difficult to say at which stage of the collision process the emission occurred.

Experiments with coincidence between deep inelastic fragments and light particles. If a light particle is detected in coincidence with a massive fragment, this does not yet give a direct indication of the stage in the reaction in which the particle was emitted nor by which fragment. To determine which nucleus emitted the particle, it is necessary to know the recoil energy of the parent nucleus. Particularly interesting in this connection is the case of strongly asymmetric breakup when the velocity of the light fragment is much greater than that of the heavy fragment.

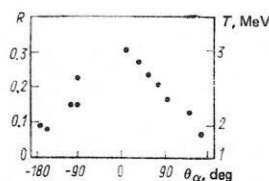


FIG. 11. Dependence of the ratio $R = dM_\alpha(E_\alpha > 15 \text{ MeV}) / dM_\alpha(E_\alpha < 15 \text{ MeV})$ on the emission angle of the α particle in the $^{16}\text{O} + ^{58}\text{Ni}$ reaction ($E_{\text{lab}} = 96 \text{ MeV}$). The temperature corresponding to the given value of R is indicated on the right.

The specific features of high-energy light particles emitted in deep inelastic reactions depend on the incident-ion-target combination and E_{lab} . For example, in reactions with light systems, α particles are emitted predominantly from the heavy fragment. Later results obtained for heavy systems show that α particles are also emitted from the light fragment.²⁹

In Ref. 30, a study was made of the emission of α particles accompanying deep inelastic $^{16}\text{O} + ^{58}\text{Ni}$ collisions. The particles were detected in coincidence with the light fragment. It was found that the α particles are emitted predominantly by the heavy fragment.

The coincidence spectrum of the α particles in the rest frame of the heavy fragment has a Maxwellian profile with peak energy independent of the emission angle of the α particle. However, at forward angles the observed α spectrum has an extended high-energy tail (Fig. 11). It can be seen from Fig. 11 that the ratio $R = dM_\alpha(E_\alpha > 15 \text{ MeV}) / dM_\alpha(E_\alpha < 15 \text{ MeV})$ has a maximum at forward angles. The high-energy part of the α -particle spectrum carries information about the nuclear "temperature." The temperature of the region from which the α particles are emitted predominantly forward (determined from the asymptotic behavior of the α spectrum) reaches 3.5 MeV. For the backward emission, it is 2 MeV.

The angular correlations in the reaction plane between the directions of emission of the light fragment and the α particle (Fig. 12) differ strongly from the predictions of statistical evaporation. The angular width of the correlations makes it possible to estimate the fragment lifetime up to the emission of the α particle, this being made possible by the connection between the width and

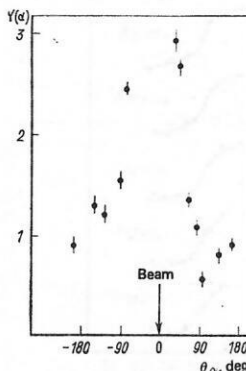


FIG. 12. Dependence of the α -particle yield on the emission angle with respect to the beam direction in the reaction plane ($^{16}\text{O} + ^{58}\text{Ni}$).

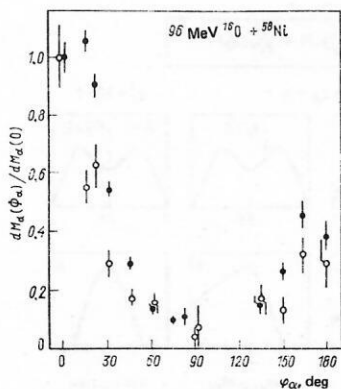


FIG. 13. Dependence of the α -particle yield on the emission angle φ_α with respect to the plane of the reaction ($^{16}\text{O} + ^{58}\text{Ni}$).

the rotation time: $\tau \approx 2\sigma_\theta J_{\text{rigid}} / \langle I \rangle \approx 2 \times 10^{-21}$ sec.

It can be seen from the angular correlations outside the reaction plane (Fig. 13) that coincidences are concentrated in the reaction plane. The observed anisotropy can in principle indicate alignment of the fragment spins perpendicular to the reaction plane. But the strong anisotropy in the forward direction ($\Phi_\alpha < 90^\circ$) cannot be described in the model of a rotating nucleus with homogeneous temperature and indicates a pre-equilibrium emission of the α particles.

Thus, the spectrum of α particles accompanying deep inelastic collisions contains the two components—equilibrium and pre-equilibrium—with comparable yields.

The equilibrium component is completely responsible for the α particles emitted backward in coincidence. Its statistical nature is clear from the most probable α -particle energy, which is near the Coulomb barrier, from the profile of the α spectrum, which corresponds to $T=2$ MeV, and from the anisotropy outside the reaction plane at backward angles, which agrees with the predictions of the equilibrium model for a rotating nucleus at $T=2$ MeV. The pre-equilibrium component is characterized by forward-backward asymmetry, by an emission time $\tau \approx 2 \times 10^{-21}$ sec, which is an order of magnitude shorter than the lifetime of a fragment with the excitation energy realized in the reaction, and by the anisotropy outside the reaction plane, which cannot be described in the model of a rotating nucleus in thermal equilibrium.

One of the possible explanations of these experiments is the existence of a spatially localized region with temperature higher than the remainder of the nucleus.^{31,32} The large energy transfer in a short time could lead to the existence of such regions. However, it is clear that this cannot be a simple particle-hole excitation, since it is not localized in any narrow interval along the radius of the nucleus and, having a definite angular momentum, is not localized with respect to the angle. It must be a collective excitation localized in the surface region of the nucleus. One requires a spectrum of such excitations with different l to achieve angular localization. The experimental data correspond to an area of the strongly heated region corresponding to about one fifth of the area of the sphere.

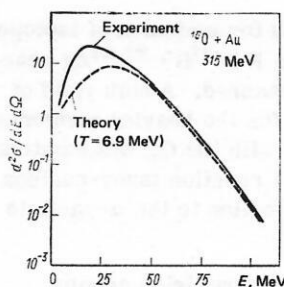


FIG. 14. Energy spectrum of protons emitted in the $^{16}\text{O} + ^{208}\text{Pb}$ reaction ($E_{\text{lab}} = 315$ MeV).

Investigation of the phenomenon of local "heating" is extremely interesting, since not only the space-time evolution of the reaction but also a property of nuclear matter such as thermal conductivity becomes observable. The phenomenon itself has some similarity to a fireball, and its investigation, especially at high excitation energies, makes it possible to bridge the gap to the physics of heavy-ion interactions at high energies. As an example, we give the results of measurement of the proton spectrum in the $^{16}\text{O} + ^{197}\text{Au}$ reaction at ~ 20 MeV/nucleon.³³ This spectrum (Fig. 14) has a profile characteristic of a statistical evaporation process that corresponds to a temperature of 6.9 MeV. The $^{16}\text{O} + ^{197}\text{Au}$ compound nucleus should have temperature 3.3 MeV. As an alternative to the above model, one can propose direct knockout of an α particle. This presents no difficulties in the explanation of the angular asymmetry and anisotropy. But to explain the statistical features of the spectrum, the knockout particle must have undergone rescattering. One consequence of the existence of a region of enhanced temperature is the possibility of achieving very high temperatures in a small region. The problem of the existence of a region with enhanced temperature does not exhaust all the problems associated with the emission of fast α particles in heavy-ion collisions.

Measurements of α particles in coincidence with the light fragment in the $^{32}\text{S} + ^{197}\text{Au}$ reaction at 12 MeV/nucleon did not explain the forward emission of a large fraction of the α particles.²⁹ True, the heavy-ion detector was at a fixed angle near θ_{gr} . It is possible that the ratio of α particles measured in coincidence with the light fragment to the total number of α particles will increase with decreasing detection angle of the light fragment.

In reactions with energy per nucleon reaching 20 MeV, disintegration of the incident ion may make a large contribution to the cross section of α -particle coincidence. For example, investigation of the $^{12}\text{C} + ^{160}\text{Gd}$ reaction at 7.5 and 16.7 MeV/nucleon²⁷ revealed not only complete fusion and quasielastic and deep inelastic scattering but also incomplete ($^{12}\text{C}, \alpha$) and ($^{12}\text{C}, 2\alpha$) fusion processes and disintegration of the incident nucleus. The α particles detected in coincidence have on the average the velocity of the incident beam. The angular distributions of these α particles have a forward peak. The absence of γ - α coincidences accompanying complete disintegration indicates the absence of excitation of the heavy fragment.

In Ref. 42, the cross sections for emission of isotopes of various elements from He to F in $^{11}\text{B} + {}^{107, 109}\text{Ag}$ reactions at $E_{\text{lab}} = 86$ MeV were measured. A high yield of α particles was observed. As for the heavier elements, the results for He agreed fully with the Q_{gg} systematization. This indicates that in this reaction many-nucleon transfers make the main contribution to the α -particle yield.

Interest in the emission of light particles accompanying heavy-ion collisions has increased considerably in recent years. However, the accumulated experimental material is clearly still inadequate. Little is known on the emission of protons, deuterons, and tritons. The emission mechanism of the pre-equilibrium particles in the heavy-ion reactions is completely obscure, and one could imagine that it is precisely in this region that the most interesting results will be obtained in the immediate future.

2.2 Many-nucleon transfers

One of the best studied regions of deep inelastic heavy-ion collisions is that of the processes involving mass (and charge) transfer between the incident nucleus and the target nucleus. The broad mass (and charge) distributions of the reaction products indicate a statistical nature of these reactions. However, when the mass (and charge) distributions are integrated over the energy and angle, they reveal maxima near the masses of the incident nucleus and the target, which indicates that a "memory" of the entrance channel is not entirely lost and, therefore, the deep inelastic collisions have a pre-equilibrium nature. It was for this reason that a diffusion model was used to describe the evolution of such a microscopic system as a double nuclear system. Such diffusion processes are well known in other branches of physics and are described by a Fokker-Planck equation or a master equation. Nörenberg³⁵ was the first to use a Fokker-Planck equation to describe the diffusion of nucleons between two nuclei in deep inelastic reactions. Later,^{36, 37} a master equation was used for this purpose.

In using transport equations to describe the transfer of nucleons, we take as a basis the fact that the deep inelastic collision process can be divided into three stages.

1. Very rapid dissipation of the kinetic energy and formation of the double nuclear system.

2. Evolution of the strongly excited double nuclear system and intense exchange of mass (and charge) between its two parts.

3. Decay of the double nuclear system. The diffusion models of Refs. 35-37 were used to describe the second stage with no allowance for the relative motion of the two ions. In Refs. 38 and 39, attempts were made to treat simultaneously the relative motion and the mass diffusion by solving a corresponding equation of motion and a Fokker-Planck equation for the mass transfer.

Let us discuss the process of charge diffusion using a Fokker-Planck equation for the probability $P(Z_1, t)$

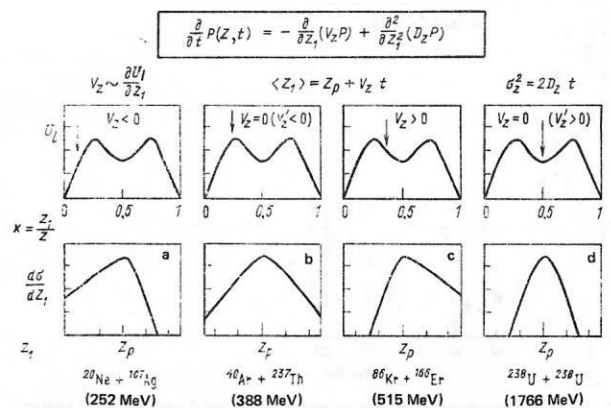


FIG. 15. Description of the process of transfer of nucleons on the basis of a Fokker-Planck equation.

of finding the system at time t with charge asymmetry Z_1 (Fig. 15). To characterize the charge asymmetry $x = Z_1 / (Z_1 + Z_2)$ of the system, we shall use the charge Z_1 of the fragment nearer the incident ion. We also assume that the drift coefficient is proportional to the derivative with respect to the charge-asymmetry parameter of the ground-state energy of a system of two touching spherical nuclei at the point of initial asymmetry. For simplicity, we shall assume that the transport coefficients v_z and D_z are constants. The solution of the Fokker-Planck equation has the form of a Gaussian with mean value $\langle Z_1 \rangle = Z_p + v_z t$ (Z_p is the charge of the incident nucleus) and variance $\sigma_z^2 = 2D_z t$ (D_z is the diffusion coefficient). The charge distribution of the products of the deep inelastic reaction is obtained from this solution by integrating over all contributions of the partial waves corresponding to different interaction times.

For a strongly asymmetric initial distribution (see Fig. 15a), the drift velocity v_z is negative and, therefore, the mean value $\langle Z_1 \rangle$ decreases with increasing interaction time. For longer interaction times, the diffusion process leads to fusion ($x=0$). The charge distributions $d\sigma/dZ_1$ are not symmetric about Z_p and the cross section is greater in the direction of greater asymmetry.

In the case of Fig. 15b, the drift velocity is zero, $v_z = 0$, which leads to a symmetric distribution of $d\sigma/dZ_1$ about $\langle Z_1 \rangle = Z_p$. The mass diffusion is due to statistical fluctuations.

In the case $v_z > 0$ (see Fig. 15c), there is drift in the direction of a symmetric configuration ($x=0.5$), and, therefore, the charge distribution again becomes asymmetric with cross section greater in the direction of lower asymmetry, i.e., $Z_1 > Z_p$.

If the initial configuration is symmetric (see Fig. 15d), the drift velocity is again zero, which leads to a symmetric charge distribution $d\sigma/dZ_1$. In this case, the statistical fluctuations will decrease the drift coefficient in the case of small deviations from a symmetric configuration. Therefore, the charge distribution will be narrower than that shown in Fig. 15b.

As is indicated in Fig. 15, such charge distributions

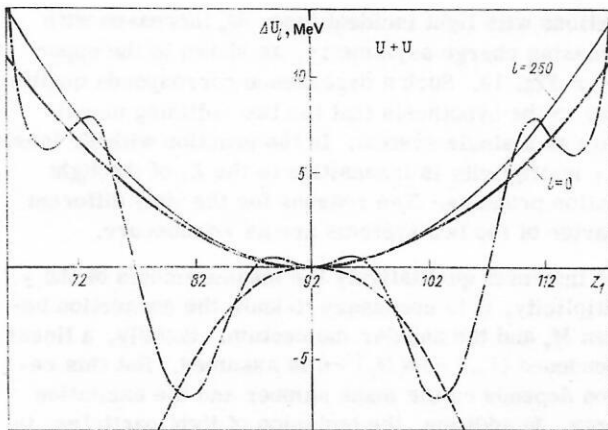


FIG. 16. Potential energy ΔU_l of a system of two touching nuclei calculated in the liquid-drop model for $l = 0$ and 250 (continuous curves) and with allowance for shell corrections (broken curve).

have been observed experimentally, although there are deviations from the idealized picture.

For example, for the very heavy nuclei in the $^{238}\text{U} + ^{238}\text{U}$ reaction ($E = 7.42$ MeV/nucleon), an unexpectedly large diffusion of protons was observed⁴⁰ compared with a different symmetric system such as $^{208}\text{Pb} + ^{208}\text{Pb}$.⁴¹ The reason could be the influence of shell effects on diffusion of the nucleon. In Fig. 16, we compare the potential energies for a system of two touching nuclei obtained in the liquid-drop model and with allowance for shell effects calculated on the basis of Strutinskii's method.⁴¹ It can be seen from the figure that one can expect strong drift in the direction of $Z_1 = 82$, although the shell effects may also disappear with increasing excitation energy. To estimate an upper limit of the influence of the shell effects, we can ignore their dependence on the excitation energy.³⁹ The results are given in Fig. 17, in which the theoretical correlations of the kinetic-energy loss with the width of the charge distribution are compared with the experimental data of Ref. 40. The broken curve lies above the experimental data, although it does correspond to the upper limit for the shell effects. Therefore, the shell effects cannot be the only source of the observed broad distribution of elements in the $^{238}\text{U} + ^{238}\text{U}$ reaction, though they are partly responsible for the differences between the reactions $^{238}\text{U} + ^{238}\text{U}$ and $^{208}\text{Pb} + ^{208}\text{Pb}$ manifested in the $\Delta E - \sigma_z^2$ correlations. Thus, an explanation of the large diffusion of protons in the $^{238}\text{U} + ^{238}\text{U}$ reaction has not been provided. The static deformation of

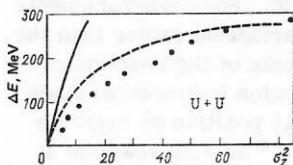


FIG. 17. Correlations between the kinetic-energy loss ΔE and the width of the charge distribution σ_z^2 of the reaction products. The points are the experimental values, and the continuous curve and the broken curve are calculated without and with allowance for shell corrections.

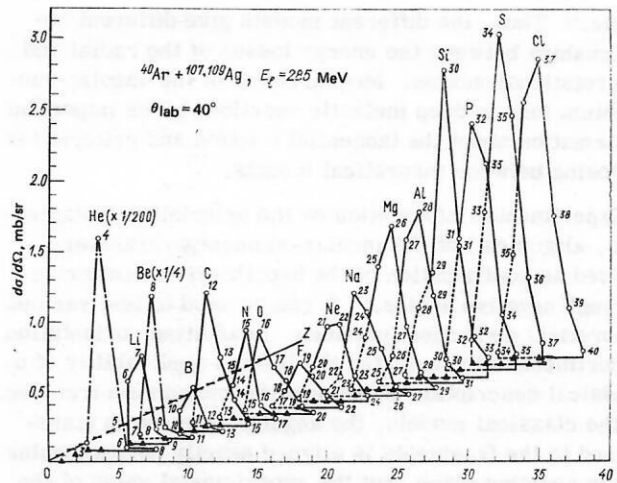


FIG. 18. Differential cross section $(d\sigma/d\Omega)_{40^\circ}$ for production of isotopes with $2 \leq Z \leq 17$ in the $^{40}\text{Ar} + \text{Ag}$ reaction (285 MeV).

the nuclei leads to an effective increase in the overlap region of the colliding nuclei. This could be an additional source of the broad charge distribution. However, the influence of deformation effects on the diffusion process has not yet been investigated.

There is a further experiment⁴² that indicates a possible manifestation of shell effects in reactions with light nuclei, namely, $^{40}\text{Ar} + \text{Ag}$ ($E_{\text{lab}} = 285$ MeV). As can be seen in Fig. 18, the yield of the nuclei decreases rapidly as Z_1 decreases from 18 to 9 (fluorine). But then the yield begins to increase with decreasing Z_1 and reaches its largest value for α particles. In addition, there is an enhanced yield of nuclei with closed shells such as ^{16}O , ^{12}C , and ^{15}N . These experimental results were analyzed in Ref. 43, and it was shown that allowance for the shell effects makes it possible to reproduce qualitatively the tendency in the behavior of the cross section $d\sigma/dZ_1$.

2.3 Dissipation of the relative orbital angular momentum. Orientation

The transition of the relative angular momentum into internal angular momentum of the fragments is one of the most interesting relaxation phenomena observed in heavy-ion reactions. Experimental information on the angular-momentum transfer is important for understanding the mechanism of energy dissipation in deep inelastic collisions. As we have already noted, the energy dissipation can be described in the framework of classical notions by means of frictional forces proportional to the velocity. These forces are decomposed into two components—radial and tangential. The radial friction is responsible for the deceleration of the relative motion of the nuclei. The tangential friction leads to additional energy losses associated with dissipation of the relative angular momentum. Various models are used to describe the loss of angular momentum, the models differing in the significance and radial dependence of the coefficient of tangential friction. The parameters of these models are chosen so as to describe either the fusion cross section^{44,45} or the correlations between the kinetic-energy losses and the deflection

angle.⁴⁶ Thus, the different models give different relationships between the energy losses of the radial and the rotational motion. Measurement of the angular-momentum loss in deep inelastic reactions gives important information about the tangential friction and criteria for choosing between theoretical models.

Experimental information on the orientation (polarization, alignment) of the angular-momentum transfer served as confirmation of the hypothesis of scattering through negative angles.⁴⁷ It can be used to test various theoretical deflection functions. In addition, orientation experiments demonstrated the limited applicability of a classical description of the angular-momentum transfer. In the classical models, the angular momentum transferred to the fragments is aligned exactly perpendicular to the reaction plane, but the experimental value of the alignment is less than unity. Thus, the orientation experiments become an important means for studying the statistical features of the mechanism of angular-momentum dissipation in deep inelastic collisions.

The experiments hitherto made to determine the value and orientation of the angular momentum can be divided into four groups.

Measurement of the γ multiplicity. This is the most general approach to obtaining any information about the angular momentum transferred to the fragments. The method consists of determining the number of γ rays emitted by both fragments of the deep inelastic reaction. The number of γ rays is related to the internal angular momenta of the fragments. Attempts were made⁴⁸ to distinguish the γ rays emitted by the light and the heavy fragment by measuring the mean energies of the γ rays. The basic result of measurement of the mean γ multiplicity can be formulated as follows⁴⁸⁻⁵⁴: In deep inelastic collisions, the angular-momentum transfer is large and close to the limit obtained under the assumption of "rigid interlocking" of the two colliding nuclei that form the double nuclear system.

The mean γ -ray multiplicity is low for the products of a quasielastic reaction, i.e., for small kinetic-energy loss. For large energy loss, the γ multiplicity is high and depends weakly on the total energy loss.

The γ multiplicity integrated over the energy, M_γ , increases with increasing number of nucleons transferred in the reaction. An unexpected difference in the dependence of M_γ on the charge asymmetry of the reaction products is found for light and heavy systems. In

reactions with light incident ions, M_γ increases with increasing charge asymmetry, as shown in the upper part of Fig. 19. Such a dependence corresponds qualitatively to the hypothesis that the two colliding nuclei rotate as a single system. In the reaction with Kr ions, the γ multiplicity is insensitive to the Z_1 of the light reaction products. The reasons for the very different behavior of the two systems are as yet obscure.

To interpret qualitatively the measurements of the γ multiplicity, it is necessary to know the connection between M_γ and the angular momentum. Usually, a linear dependence $\langle I_{\text{tot}} \rangle = m \langle M_\gamma \rangle + n$ is assumed. But this relation depends on the mass number and the excitation energy. In addition, the emission of light particles, in particular α particles, can dissipate an appreciable fraction of the angular momentum, which must also be taken into account.

Measurement of angular correlations in the case of discrete γ transitions. Alignment. The angular correlations outside the reaction plane of γ rays of known multipolarity were studied in deep inelastic collisions of ^{16}O with ^{27}Al in coincidence with the light reaction fragment.⁵⁵ If the angular momentum is initially perpendicular to the reaction plane, the dipole γ rays will have an intensity maximum in the direction perpendicular to the reaction plane, and the quadrupole γ rays will have an intensity maximum in the reaction plane. These predictions are confirmed experimentally, and the observed anisotropy is close to the value expected for complete alignment of the angular momentum perpendicular to the reaction plane.

Measurement of angular correlations in the case of a γ -ray continuum. The investigation of Ref. 49 found a 9–22% decrease in the yield of γ rays outside the reaction plane compared with the yield in the reaction plane in deep inelastic Cu + Au collisions. In the $^{40}\text{Ar} + ^{89}\text{Y}$ reaction, a 30% decrease was found.⁴⁸ The quantitative interpretation of these data requires a detailed knowledge of the γ cascade, including, in particular, the numbers of dipole and quadrupole γ rays. Since the dipole and quadrupole γ rays have anisotropy of opposite signs, there can be appreciable mutual cancellation, resulting in a small total anisotropy of the γ rays.⁵⁶ Thus, the existence of even slight anisotropy outside the reaction plane in deep inelastic collisions is a consequence of constancy of the alignment until the final stage in the de-excitation of the reaction products.

Measurement of circular polarization in the case of a γ -ray continuum. The circular polarization of the γ rays emitted during quasielastic and deep inelastic reactions was measured in Ref. 57. Such measurements give information about the polarization rather than the alignment of the angular momenta of the reaction products. The sign of the polarization indicates whether scattering has occurred through positive or negative angles. It was found that in the $^{40}\text{Ar} + \text{Ag}$ reaction at two energies of the incident ion scattering through positive angles corresponds to the quasielastic component of the spectrum, whereas deep inelastic collisions are accompanied by scattering through negative angles. This fact can be regarded as an experimental confirma-

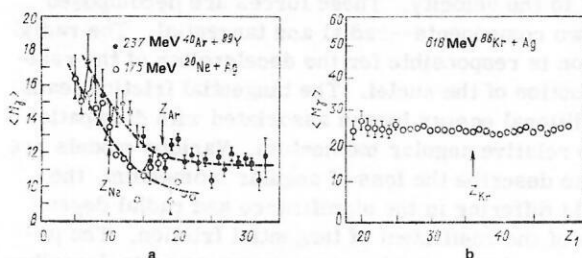


FIG. 19. Dependence of the mean γ -ray multiplicity (M_γ) on the charge asymmetry of the products in the reactions $^{40}\text{Ar} + ^{89}\text{Y}$, $^{20}\text{Ne} + \text{Ag}$ (a) and $^{86}\text{Kr} + \text{Ag}$ (b).

tion of the hypothesis of scattering through negative angles in deep inelastic collisions.

Angular correlations of the α particles accompanying deep inelastic collisions. The emission of α particles by excited reaction fragments carries information about the magnitude and alignment of the angular-momentum transfer. It is, however, necessary to distinguish these α particles from the α particles produced by a direct process such as disintegration of the incident ion. In Ref. 58, the angular correlations in and out of the reaction plane for α particles in coincidence with the light fragment of the $^{16}\text{O} + ^{58}\text{Ni}$ reaction were studied. The results are shown in Fig. 13. It can be seen that the evaporation component has a large anisotropy outside the reaction plane, from which one can deduce the mean value $\langle I \rangle \approx 15$ for the angular momentum transferred to the fragments. The alignment of the angular momentum was also determined,⁵⁸ and it was found to be $A = 0.77$.

Angular correlations of fission products accompanying deep inelastic collisions. This method consists of measuring the yields of the fission products of the heavy fragment in and out of the reaction plane, which is determined by the simultaneous detection of the light reaction fragment. The angular correlations of the fission products of the heavy fragment of the $^{86}\text{Kr} + ^{238}\text{U}$ reaction ($E_{\text{lab}} = 610$ MeV) with respect to the recoil direction in the reaction plane and outside it were investigated in Ref. 59. The angular-momentum transfer deduced from the experimental data agreed with the prediction of the model based on the assumption of "rigid" rotation of the double nuclear system.

The $^{86}\text{Kr} + ^{238}\text{U}$ reaction ($E_{\text{lab}} = 730$ MeV) was studied in Ref. 60. The alignment and the angular momentum transferred to the heavy fragment were determined at different Q values of the reaction (Fig. 20). It can be seen that the magnitude and alignment of the angular momentum decrease rapidly with increasing energy loss.

Angular anisotropy of β particles. The angular anisotropy of the β particles emitted by the light reaction product contains information about the polarization of the light nucleus. The spin polarization of ^{12}B in the

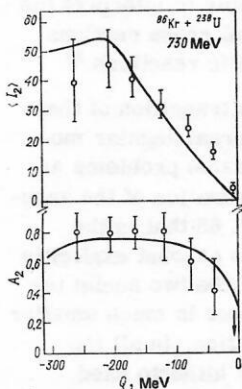


FIG. 20. Dependence of the mean angular momentum of the heavy fragment (I_2) and its alignment A_2 in the $^{86}\text{Kr} + ^{238}\text{U}$ reaction (730 MeV) on the Q value of the reaction.

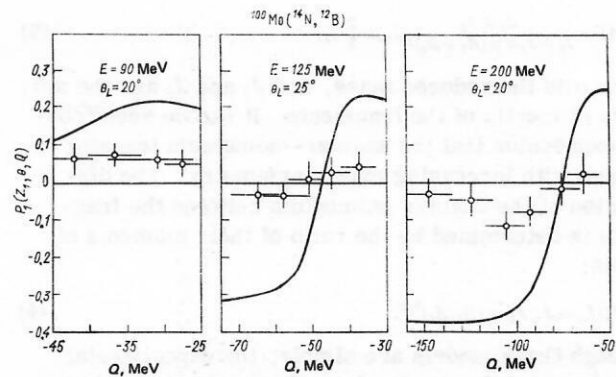


FIG. 21. Dependence of the spin polarization P_1 of the nucleus ^{12}B in the $^{100}\text{Mo}(^{14}\text{N}, ^{12}\text{B})$ reaction on the Q value of the reaction.

$^{100}\text{Mo}(^{14}\text{N}, ^{12}\text{B})$ reaction at incident-ion energies $E = 90$, 125, and 200 MeV and fixed emission angle of the ^{12}B nucleus was determined in Ref. 61. The following results were obtained:

- The polarization is negative and largest in absolute magnitude at the largest values of Q realized in the reaction ($Q \geq -25$ MeV). The polarization decreases with decreasing Q . Such behavior at these energies can be described under the assumption of direct transfer of protons.
- The polarization becomes positive in the region $-25 \text{ MeV} \geq Q \geq -60$ MeV.
- The polarization again becomes negative and is close to -0.1 at $Q \leq -70$ MeV. The polarization decreases with decreasing Q (Fig. 21). The results given in the figure at low values of Q correspond to deep inelastic collisions.

Additional measurements were made in Ref. 62 for the $^{232}\text{Th}(^{14}\text{N}, ^{12}\text{B})$ reaction at $E_{\text{lab}} = 129$ MeV. For Q values corresponding to deep inelastic events the polarization was found to be positive and to decrease with decreasing Q .

Theoretical treatment of transfer of angular momentum. Classical models. To get a qualitative idea of the fraction of the initial orbital angular momentum l_i that can be transferred to the fragments, it is helpful to consider some macroscopic models. When two spherical objects touch each other, they can either roll on their surfaces, when the sliding friction is infinitely large, or they can be rigidly joined and rotate as a rigid body if the rolling friction is infinitely large.

In the limit of infinite sliding friction, the angular momentum $\Delta l = l_i - l_f = I_1 + I_2$ transferred to the fragments is given by the expression

$$\Delta l = 2/7 l_i. \quad (1)$$

It does not depend on the mass asymmetry of the reaction products. The ratio of the angular momenta transferred to the fragments is

$$I_1/I_2 = R_1/R_2 = (A_1/A_2)^{1/3}. \quad (2)$$

In the limit when the double nuclear system rotates as a single system, the transferred angular momentum is

$$\Delta l = \frac{J_1 + J_2}{J_1 + J_2 + \mu(R_1 + R_2)^2} l_i \geq \frac{2}{7} l_i, \quad (3)$$

where μ is the reduced mass, and J_1 and J_2 are the moments of inertia of the fragments. It can be seen from this expression that the angular-momentum transfer increases with increasing mass asymmetry. The distribution of the angular momentum between the fragments is determined by the ratio of their moments of inertia:

$$I_1/I_2 = J_1/J_2 = (A_1/A_2)^{5/3}. \quad (4)$$

Although these models are simple, the experimental data show that the maximal angular momentum Δl transferred in deep inelastic collisions is close to the limit (3).

Besides these two simple models, there are many dynamical models with friction.⁴⁴⁻⁴⁶ In some of them,⁴⁴ inertial rotation of the fragments is ignored. The calculated angular-momentum transfer can exceed the limit (3). In other models, the internal rotation of the fragments is taken into account, and, therefore, the limits (1) and (3) arise automatically. The parameters of these models are chosen to reproduce correctly the fusion cross section, and also the turning angle and the energy loss. We shall not go into details of the problem and will only mention that in the framework of such models it is very difficult to find a set of parameters satisfying the requirements listed above and, in addition, giving the correct value of the internal angular momentum.

Thus, the maximal value of the angular-momentum transfer in deep inelastic collisions is close to the classical limit (3), although it should be noted that the observed differences in the dependence of $\langle M_z \rangle$ on the mass asymmetry for the light and heavy products (see Fig. 19) do not find an explanation in the framework of the simple models. Further, all classical models predict complete alignment of the angular momentum, which is not confirmed experimentally. This indicates the presence of statistical fluctuations, which are important for understanding the mechanism of angular-momentum dissipation in deep inelastic reactions. The statistical fluctuations destroy the alignment of the internal angular momentum in the collision process. These effects are taken into account in the statistical approaches of Refs. 63-66.

Kinetic approach based on a single-particle model. In such an approach, the dissipation of the angular momentum is treated as a transport phenomenon described by a Fokker-Planck equation.⁶³ The following physical picture provides the basis of such an approach. During the initial stage of the collision, the energy of the radial motion and a small fraction of the angular momentum are very rapidly dissipated. Then the excited compound system begins to rotate, and the relative angular momentum is dissipated gradually during the entire interaction time until the breakup. This stage of the reaction is described by the Fokker-Planck equation for the z component of the internal angular momentum with transport coefficients calculated in the single-particle model.⁶⁴ In such a model, the angular momentum is transmitted to particle-

hole excitations. The statistical fluctuations are associated with Fermi motion of the nucleons. Using phenomenological models to calculate the deflection functions and the mean interaction time, one can calculate the mean values, the mean-square fluctuations, the alignment, and the polarization of the internal angular momentum as functions of the initial angular momentum.⁶⁵ The calculated values of the internal angular momentum are in reasonable agreement with the experimental data, though there is a general tendency to underestimation of the angular-momentum transfer in the region of large ΔE . One of the possible reasons for this discrepancy is the breakdown of the one-to-one correspondence between ΔE and l_i at large ΔE . To overcome this difficulty, a fully dynamical treatment is required. In addition, a dynamical model makes it possible to calculate the magnitude, polarization, and alignment of the internal angular momentum as functions of the scattering angle, the energy transfer, and the mass number of the reaction products, which permits direct comparison with the experimental data. Such a model will be described below.

Kinetic approach based on linear-response theory. The basic equation in this approach⁶⁶ is the Fokker-Planck equation in the phase space of the collective degrees of freedom obtained in Ref. 67 on the basis of linear-response theory. This equation describes both the dissipation and the fluctuations of the dynamical variables. It takes into account the connection between classical motion along a trajectory and the internal motions. The latter are characterized by a time-dependent temperature $T(t)$. The solution of the Fokker-Planck equation contains information about the mean values and the fluctuations of the dynamical variables. The mean values satisfy equations of Newtonian type with frictional forces. The transition of the orbital angular momentum into internal angular momentum is brought about by the tangential component of the frictional forces. The statistical fluctuations of the angular-momentum transfer are determined by the diffusion coefficient D , which is obtained by means of Einstein's relation between the diffusion coefficient and the friction coefficient γ :

$$D = \gamma T(t).$$

Hitherto, only phenomenological frictional forces have been used in the realization of such an approach. Nevertheless, these models make it possible to interpret the single, double, and triple differential cross sections for the product yields of deep inelastic reactions.³⁹

However, in the description of the transition of the relative angular momentum into internal angular momentum, one comes up against the same problems as in the classical treatment of the dissipation of the angular momentum. It was shown in Ref. 68 that in the framework of models which take into account explicitly the rotational degrees of freedom of the two nuclei the calculated angular-momentum transfer is much smaller than the experimental value. In addition, in all the formulations of the theory of Ref. 67 hitherto used, complete alignment of the angular-momentum transfer is predicted, as in the classical model.

These difficulties can be overcome by assuming that⁶⁶:

1) the tangential friction is three-dimensional; 2) the forces of the tangential friction depend on the angular momentum. This presupposes the introduction of a form factor which depends on the angular momentum; this has the result that when the classical limit (3) is attained the forces of tangential friction are zero.⁶⁹

The results of such a model calculation are compared in Fig. 20 with the experimental data obtained by measuring the angular correlations of the fission products of the heavy fragment in the $^{86}\text{Kr} + ^{238}\text{U}$ reaction.⁶⁰ The figure gives the mean values of the angular momentum of the heavy fragment and its alignment for different Q values of the reaction. Good agreement with the experimental data is achieved without fitting of free parameters.

With allowance for mass diffusion during the reaction, a calculation was also made of the ^{12}B polarization in the $^{100}\text{Mo}(^{14}\text{N}, ^{12}\text{B})$ reaction⁶¹ for given emission angle as a function of Q (see Fig. 21). It can be seen that the calculations correctly reproduce the tendency in the variation of the polarization as a function of Q . The change in the sign of the polarization with increasing Q is correctly reproduced. However, the absolute value of the polarization is overestimated. This could be due to two reasons:

- a) the angular momentum cannot be treated classically in reactions with light fragments;
- b) no allowance is made for the modes of the motion in fission that lead to depolarization.

2.4 Establishment of equilibrium between the number of neutrons and number of protons

The ratio N/Z is usually different for the incident ion (N_p/Z_p) and the target nucleus (N_T/Z_T). But for the products of deep inelastic reactions this ratio is equal to neither N_p/Z_p nor N_T/Z_T but corresponds to the compound system. This alone indicates that the collective mode associated with equilibration with respect to N/Z is a fast mode.

The usual way of studying this fast collective mode is to measure the distribution with respect to the atomic number for fixed mass asymmetry. This distribution has a Gaussian form and is characterized by two moments, namely, the most probable value of N/Z and the width of the distribution. The experimental data show that for light systems ($\text{Ar} + \text{Ca}, \text{Ni}, \text{Zn}$) equilibrium with respect to N/Z is already established in an early stage of the relaxation process.

When heavy ions are scattered by heavy nuclei, the number of nucleons that must be transferred before the N/Z equilibration is greater than for light nuclei. Therefore, for heavy systems, particularly at relatively low energies, it is convenient to study the transition from nonequilibrium to equilibrium ratio N/Z .

In the $^{129, 132, 136}\text{Xe} + ^{197}\text{Au}$ reactions at energies 25 MeV above the Coulomb barrier, the mean value of N/Z and the mean number of emitted neutrons have been determined for each element.⁷⁰ What results could one expect? In the absence of equilibrium with respect to

N/Z , the mean value of A for $Z=79$ should not change on the transition from ^{129}Xe to ^{132}Xe and ^{136}Xe (memory of the entrance channel: ^{197}Au). The mean values of A for fragments with $Z=54$ should be widely distributed for the three reactions: $N/Z=1.39, 1.46$, and 1.52 . If equilibration with respect to N/Z does occur, one should observe a parallel shift in the dependences of $\langle N/Z \rangle$ on Z on the transition from the one reaction to the next. The experimental results show a parallel shift in the case of a transfer $Z > 1$. For $Z=79$, this shift is sharply reduced, and the value of N/Z tends to its value for ^{197}Au . For $Z=54$, there is a broad spread of N/Z values for the three reactions. These values approach N/Z , respectively, for ^{129}Xe , ^{132}Xe , and ^{136}Xe . Thus, $\Delta Z=1$ is the boundary between N/Z equilibration and nonequilibration. For $\Delta Z > 1$, more than four neutrons are emitted. The mean excitation energy per evaporation neutron is 9–11 MeV, i.e., 40–50 MeV is transferred before equilibration. The rate of energy transfer is 4×10^{23} MeV/sec,⁷¹ and therefore $\tau_{\text{eq}} = 1.2 \times 10^{-22}$ sec. An estimate based on the diffusion coefficient gives 1.4×10^{-22} sec.

Another approach to study of equilibration with respect to N/Z is based on investigating how the width of the Z distribution depends on the excitation energy for a fixed mass asymmetry. The potential energy of the compound system as a function of Z for fixed mass asymmetry has the form of a parabola. Therefore, the collective motion in which we are interested can be treated in the harmonic approximation. Strictly speaking, such an oscillator is not isolated but coupled to the other degrees of freedom of the nucleus. We include them in the treatment as a thermal reservoir with temperature T . We shall investigate the influence of the temperature on the fluctuations of the neutron excess, i.e., the influence of the excitation energy on the width of the charge distribution for fixed mass asymmetry. It is

$$\sigma_Z^2 = \langle Z^2 \rangle - \langle Z \rangle^2 = \frac{1}{C} \left[\left(\frac{1}{2} \hbar \omega + \hbar \omega \right) / (\exp(\hbar \omega / T) - 1) \right],$$

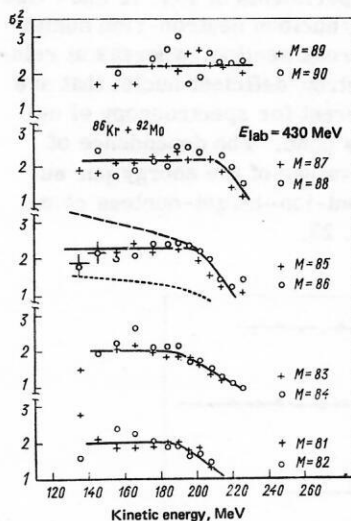


FIG. 22. Dependence of the width of the charge distribution σ_Z^2 of the products of the $^{86}\text{Kr} + ^{92}\text{Mo}$ reaction ($E_{\text{lab}} = 430$ MeV) for fixed mass asymmetry (M) on the value of the kinetic energy in the exit channel.

where C is the rigidity, and $\hbar\omega$ the energy of the vibrational excitation. If $\hbar\omega \ll T$, then $\sigma_z^2 = T/C$. In this case, we are dealing with statistical fluctuations. The width of the charge distribution will increase as $T^{1/2}$ with increasing energy loss. For $\hbar\omega \gg T$, $\sigma_z^2 = \hbar\omega/2C$, i.e., the width of the distribution does not depend on the energy loss and there are quantum fluctuations.

In a study⁷² of the $^{86}\text{Kr} + ^{92}\text{Mo}$ reaction it was found (Fig. 22) that the width of the distribution initially increases with increasing energy loss but then reaches a plateau at $\Delta E = 30$ MeV. The total energy loss in this reaction is 100 MeV. It follows that the relaxation time for the N/Z mode is of order 10^{-22} sec.

What could be the microscopic source of the mode associated with the neutron excess? Very probably it is a giant dipole resonance. In a spherical nucleus, $\hbar\omega = 78/A^{1/3}$ MeV. But it is natural to assume that the N/Z mode is associated, not with one nucleus, but with the compound system. In this case

$$\hbar\omega \sim 78/(A_1^{1/3} + A_2^{1/3}) \text{ MeV}.$$

For the considered system, this gives $\hbar\omega = 8.6-8.7$ MeV.

Figure 22 also shows the expected dependence of σ_z^2 on the energy loss for statistical fluctuations (normalized to the experiment at $\Delta E = 30$ MeV). Also given is the theoretical curve corresponding to the rigidity C calculated in the liquid-drop model for giant dipole vibrations. The experimental width of the distribution is appreciably greater than the theoretical value and exhibits a different dependence on the energy loss.

Under the assumption of a quantum nature of the fluctuations, a value of order 8 MeV is obtained for $\hbar\omega$ for the same value of the rigidity C , this agreeing with the theoretical estimate obtained under the assumption of giant dipole vibrations in the double nuclear system.

It is of interest to continue the investigation of N/Z equilibration at different energies per nucleon of the incident nucleus. The experiments of Ref. 73 show that at energies up to 10 MeV/nucleon neutron-rich nuclei are obtained with large cross section, whereas at relativistic energies it is neutron-deficient nuclei that are obtained, which is of interest for spectroscopy of nuclei far from the stability band. The dependence of $\langle N/Z \rangle$ on Z for different values of the energy per nucleon and different incident-ion-target-nucleus combinations is shown in Fig. 23.

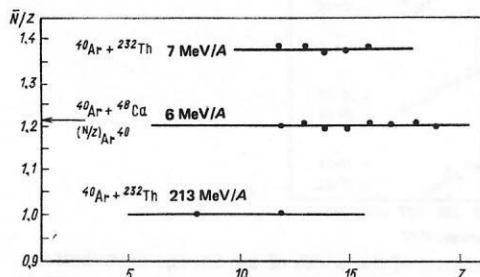


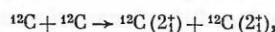
FIG. 23. Dependence of the mean value of the ratio N/Z on Z in the light reaction product for different energies per nucleon of the incident ion.

3. COMPLETE FUSION REACTIONS

In recent years, interesting results have also been obtained in the field of fusion reactions, which is a traditional field of heavy-ion physics. Below, we shall discuss only some of the results, which have not yet found an exhaustive explanation.

Fusion of light nuclei. The energy dependence of the cross section for fusion of light nuclei has been intensively studied in recent years for two main reasons.

First, oscillations were found in the excitation functions for complete fusion in the reactions $^{12}\text{C} + ^{12}\text{C}$ (Ref. 74), $^{12}\text{C} + ^{16}\text{O}$ (Ref. 75), and $^{16}\text{O} + ^{16}\text{O}$ (Ref. 76). No oscillations were found in the reactions $^{12}\text{C} + ^{18}\text{O}$, $^{12}\text{C} + ^{14}\text{N}$, and $^{12}\text{C} + ^{19}\text{F}$.⁷⁷ This resonance structure was also observed in other inelastic channels. In Fig. 24a, we give the energy dependence for the cross section of inelastic $^{12}\text{C} + ^{12}\text{C}$ scattering⁷⁸:



in the energy range from 15 to 45 MeV. The fusion cross section is shown in Fig. 24b. Correlations in the positions of the oscillations are clearly visible. The reasons for the oscillations are not yet clear. One of the possible explanations is the existence of nuclear or α -particle molecules.⁷⁹ But the occurrence of doorway states of a different nature is also possible.^{80,81}

A second unexpected result concerns the absolute

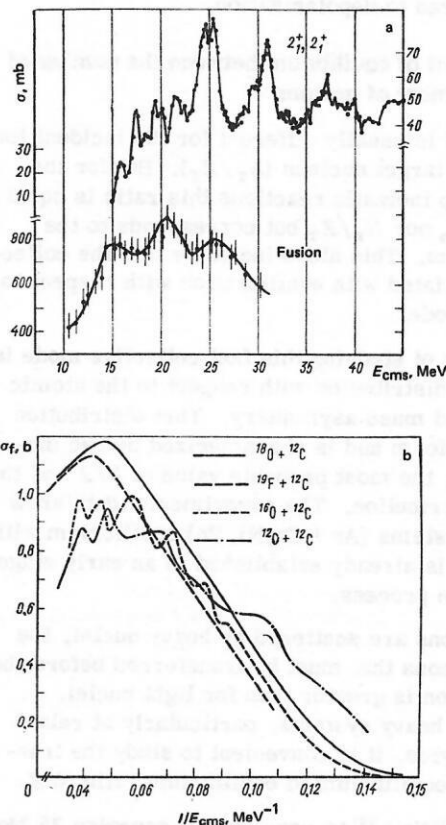


FIG. 24. Energy dependence of the cross section of the inelastic scattering $^{12}\text{C}(\text{g.s.}) \rightarrow ^{12}\text{C}(2_1^+) + ^{12}\text{C}(2_1^+)$ (upper curve) and complete fusion (lower curve) (a); the energy dependence of the fusion cross section in the reactions $^{18}\text{O} + ^{12}\text{C}$, $^{19}\text{F} + ^{12}\text{C}$, $^{16}\text{O} + ^{12}\text{C}$, and $^{12}\text{C} + ^{12}\text{C}$ (b).

magnitude of the fusion cross section when one of the partners goes over from the p shell to the s - d shell. There is a sharp increase by about 20% at the maximum of the excitation function for $^{12}\text{C} + ^{18}\text{O}$, $^{12}\text{C} + ^{19}\text{F}$, $^{12}\text{C} + ^{27}\text{Al}$, and $^{16}\text{O} + ^{24}\text{Mg}$ compared with $^{12}\text{C} + ^{16}\text{O}$ (see Fig. 24b). The result can be interpreted as a shell effect. However, a clear understanding has not yet been achieved.

Fusion of heavy nuclei. The failure to synthesize superheavy elements in reactions with heavy incident ions such as Zn, Ge, and Kr stimulated experiments with ^{48}Ca . For the synthesis of superheavy elements, ^{48}Ca has advantages such as the minimal excitation energy and maximal number of neutrons in the compound nucleus formed in the fusion reaction. So far, all attempts to discover superheavy elements by using the neutron-rich incident nucleus ^{48}Ca and the neutron-rich targets ^{243}Am , ^{242}Pu , and $^{246,248}\text{Cm}$ have yielded only upper limits on the lifetime and cross sections for the formation of superheavy elements.⁸²

Nevertheless, study of the fusion reactions with heavy incident nuclei such as Ca and Ar has led to the discovery of new elements and isotopes. In addition, the investigations into the possibility of synthesizing superheavy elements in fusion reactions involved the systematic study of the reaction mechanism, including, in particular, the influence of the excitation energy and the angular momentum on the "survival" of the compound nuclei. The energy dependence of the fusion cross section was also studied for different nucleus-target combinations.

We shall consider two examples. In Ref. 83, ^{40}Ar ions were used to bombard ^{243}Am nuclei and a study was made of the mass distribution of the products of the fusion reaction with subsequent fission at different energies of the incident ion, i.e., at different excitation energies of the compound system in the range 40–116 MeV. Figure 25 shows contour charts of the reaction-product yields for different values of their total kinetic energies. It can be seen that with decreasing energy of the incident nucleus the mass distribution becomes asymmetric.

The fusion cross sections for different incident-nucleus-target-nucleus combinations are shown in Fig. 26 as functions of V_B/E_{cms} (V_B is the Coulomb barrier and E_{cms} the kinetic energy in the center-of-mass system). For the majority of combinations, the fusion cross sections correspond to a general dependence, but for the doubly magic system $^{48}\text{Ca} + ^{208}\text{Pb}$ the fusion cross section is 20% lower. This reduction may indicate an increase in the barrier for complete fusion in such systems. It is interesting to note that a reduction in the fusion cross section for magic systems was predicted in Ref. 85.

Theoretical models. The fusion phenomenon is still far from being understood from the theoretical point of view. A reflection of this fact is the existence of a huge number of phenomenological models based on different assumptions and describing the experimental data to greater or lesser extent. These models can be classi-

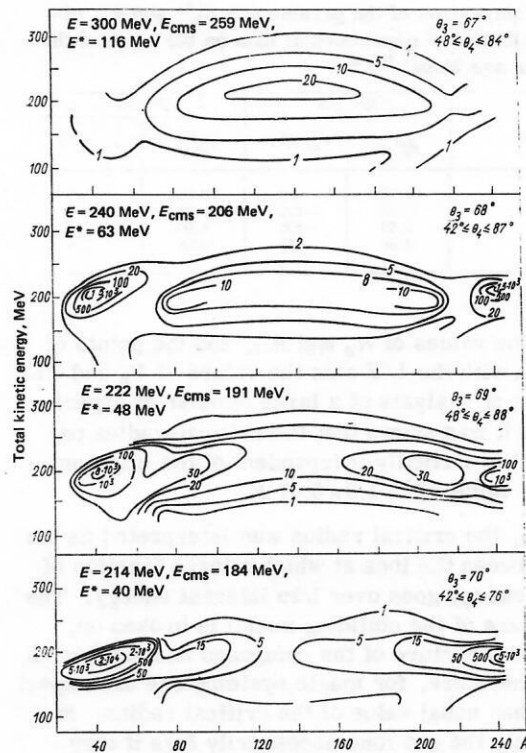


FIG. 25. Contour diagrams of the yields of the products of the $^{40}\text{Ar} + ^{243}\text{Am}$ reaction at different kinetic energies.

fied according to various criteria.⁸⁶ In some models, limits on the fusion cross section are deduced from the instability of the produced system due to the large amount of angular momentum involved in the reaction or, alternatively, the disappearance of the fission barrier.^{87–84}

In other models,^{92–102} the possibility of fusion is associated with the existence of a minimum in the interaction potential $V(l, R)$ of the nuclei.

In a third group,^{85, 103} a limit on the fusion cross section is associated with the existence of a critical radius. This concept arose as an alternative to the description of fusion in terms of a critical angular momentum. It was shown in Ref. 85 that the concept of a critical radius is realized only for high energies of the relative motion of the colliding nuclei. At low energies, the limits are due to the interaction barrier:

$$\begin{aligned}\sigma_{\text{cf}}(E) &= \pi R_B^2 (1 - V_B/E) \text{ at "low" energies,} \\ \sigma_{\text{cf}}(E) &= \pi R_{\text{cr}}^2 (1 - V_{\text{cr}}/E) \text{ at "high" energies,} \\ R_{\text{cr}} &= r_{\text{cr}}^{(0)} (A_1^{1/3} + A_2^{1/3}).\end{aligned}$$

In such a model, two energy regions are distinguished in the behavior of $\sigma_{\text{cf}}(E)$. The graphs of the dependence of σ_{cf} on $1/E$ reveal two straight lines, whose slopes

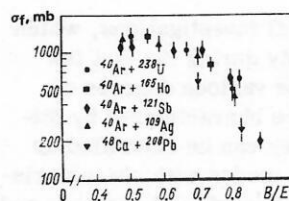


FIG. 26. Energy dependence of the fusion cross section.

TABLE I. Comparison of the parameters $r_{cr}^{(0)}$ and V_{cr} obtained by evaluation of experimental data on the basis of the model of Glas and Mosel.^{104,86}

Reaction	Ref. 104		Ref. 86	
	$r_{cr}^{(0)}$, F	V_{cr} , MeV	$r_{cr}^{(0)}$, F	V_{cr} , MeV
$^{12}\text{C} + ^{27}\text{Al}$	0.75	-70	0.98	-13.7
$^{16}\text{O} + ^{27}\text{Al}$	0.76	-100	1.00	-10.1
$^{20}\text{Ne} + ^{27}\text{Al}$	0.69	-100	1.00	-5.9
$^{32}\text{S} + ^{27}\text{Al}$	0.56	-55	0.98	7.7

determine the values of R_B and R_{cr} , and the points of intersection with the $1/E$ axis the values of V_B and V_{cr} . On the basis of analysis of a large number of experimental data it was shown that the critical-radius parameter $r_{cr}^{(0)}$ is virtually independent of the mass number and has value $r_{cr}^{(0)} = 1.0 \pm 0.07$ F.

In Ref. 85, the critical radius was interpreted as the distance between the ions at which a large fraction of the kinetic energy goes over into internal energy. The shell structure of the colliding nuclei is broken up, and the shell structure of the compound nucleus begins to form. Therefore, for magic systems one can expect a smaller than usual value of the critical radius. In such a model, the two ions necessarily fuse if they reach the distance R_{cr} . There are no other restrictions.

However, in some cases analysis of the experimental data on the basis of such a model has revealed some physical discrepancies. At high energies, r_{cr} begins to depend on the energy, and to achieve agreement with the experiments it is necessary to specify unphysically small values of r_{cr} and a very deep potential V_{cr} . This was a strong argument against the critical-radius concept. However, in Ref. 86 a model was proposed introducing a third energy region in the behavior of σ_{cr} , this being associated with the existence of restrictions on the stability of the double nuclear system formed when the critical radius is reached. It can be seen from Table I that the model makes it possible to interpret the experimental results with reasonable values of r_{cr} and V_{cr} . At the present time, the critical-radius concept is widely used to analyze experimental data on fusion. However, the microscopic foundations of this concept have not yet been elucidated. It is to be hoped that the answer to this question will make it possible to understand the mechanism of energy loss in deep inelastic collisions and fusion reactions.

CONCLUSIONS

The investigation of heavy-ion reactions has made it possible for the first time to study various relaxation processes in nuclear matter.

As a result of the experimental investigations, which have been carried out intensively during the last few years, it has been found that the various degrees of freedom of a nuclear system are characterized by different relaxation times, and they can be classified as fast and slow modes. The slow modes with characteristic relaxation times $\sim 10^{-21}$ sec (transfer of nucleons and angular momentum) have been carefully studied in

heavy-ion reactions at energies ~ 10 MeV/nucleon, and a qualitative understanding of the observed phenomena has been achieved.

The degrees of freedom associated with dissipation of the kinetic energy and equilibration between the number of neutrons and the number of protons are faster processes with characteristic relaxation times $\sim 10^{-22}$ – 10^{-23} sec. In these processes, much remains obscure, and heavy-ion beams with ~ 20 – 100 MeV/nucleon are needed for further investigations. Such beams will make it possible to study processes characterized by short relaxation times ($\tau < 10^{-21}$ sec).

As was noted in the Introduction, intensive investigations have been made in the last few years of reactions with heavy ions in two basic energy regions: 10 and 1000 MeV/nucleon. The investigation of processes with heavy ions having ~ 100 MeV/nucleon will make it possible to understand how the interaction mechanism of the nuclei changes with increasing kinetic energy per nucleon.

In heavy-ion physics there is at present a rapid growth in the experimental information obtained in various correlation experiments. Such information gives more stringent criteria for the selection of theoretical models and is therefore more valuable, but more intense heavy-ion beams are required for correlation experiments. For these reasons, further progress in understanding the interaction mechanism of nuclei will, in our opinion, be associated with an increase in the intensity of the heavy-ion beams and transition to the energy range ~ 100 MeV/nucleon.

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