Interactions of high-energy hadrons, photons, and leptons with nuclei

N. N. Nikolaev

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow

Fiz. Elem. Chastits At. Yadra 12, 162–219 (January–February 1981)

The present state of high-energy nuclear physics is reviewed. Particular attention is devoted to effects associated with the growth in the scales of the longitudinal distances and interaction times at high energies and the manifestation of composite quark structure of hadrons in multiparticle production processes.

PACS numbers: 25.40.Rb, 94.40.Rc

INTRODUCTION

The review is devoted to high-energy nuclear physics, which is a rapidly developing branch of elementary-particle physics. I have set myself two aims. The first is to describe the present state of the physics of the interaction of high-energy hadrons, photons, and leptons with nuclei. This problem has a traditional solution. We must explain why this region is interesting, especially at the present time; consider what new information about the interaction mechanism of elementary particles at high energies and about particle structure can be and already has been given by the study of collisions with nuclei; establish the extent to which our theoretical ideas about the interactions of particles with nuclei agree with the experiments, and consider the prospects for this branch of high-energy physics. In solving this problem, we shall follow closely the modern ideas about the quark–parton structure of hadrons and the multiperipheral nature of inelastic interactions of hadrons at high energies.

In the theory of hadron–nucleus interactions in recent years, there have been frequent rediscoveries of many old, and therefore largely forgotten results, which were frequently obtained more than a quarter of a century ago. I have felt it appropriate to consider the history of this question and remind readers of these classical studies, and therefore my second aim in this review is to indicate, as fully as possible, the sources of the modern theory of hadron–nucleus and lepton–nucleus interactions.

Although nuclear targets have been and are being widely used in high-energy physics, there has until comparatively recently been rather little interest in the processes that actually take place within the nucleus. Nuclei are extended targets for secondary particles with a thickness up to a few interaction lengths. It was assumed that the interactions of the secondary particles within such a thick target would merely, as is the case in real thick targets, distort the picture of the fundamental interaction with one nucleon of the nucleus (or with one nucleus in the case of real thick targets). Such a point of view is not without justification, and processes in nuclei are indeed similar to processes in thick targets. However, it is not this similarity that is interesting but the circumstance that at high energies the interactions of the secondary particles within a nucleus become fundamentally different from the interactions of the secondary particles in a thick target. We shall state the difference in more detail.

The physics of the interaction of high-energy hadrons in thick targets reduces to two equations: the equation

$$w_{abs}=1-\exp(\nu_{abs}) \tag{1}$$

for the probability of absorption of the incident particle in the target, and the cascade transport equation

$$\frac{d}{dt}\left(\frac{dN_s(\varepsilon,t)}{d\varepsilon}\right)=-\frac{dN_s(\varepsilon,t)}{dt}+\int\limits_{\varepsilon}^{E}d\omega\,\frac{dN_s(\omega,t)}{d\omega}\,\frac{dN_s(\omega\to\varepsilon)}{d\varepsilon} \tag{2}$$

for the energy spectrum $dN_s(\varepsilon,t)/d\varepsilon$ of the secondary particles at depth t from the point of absorption of the incident particle. Here, the target thickness ν_{abs} and t are measured in units of the absorption length

$$l_{abs}=1/\sigma_{in}\rho, \tag{3}$$

and the boundary condition at $t=0$ has the form

$$dN_s(\varepsilon,\ 0)/d\varepsilon=dN_s(E\to\varepsilon)/d\varepsilon, \tag{4}$$

where $dN_s(E\to\varepsilon)/d\varepsilon$ is the energy spectrum of the secondary particles for a primary particle with energy E. The dependence of $dN_s(\varepsilon,t)/d\varepsilon$ on t is obvious: The fast particles are absorbed, and the multiplicity of the slow particles increases with increasing t because of the processes of cascade multiplication.

Let us fix ν_{abs} and compress the target, increasing its density ρ right up to the nuclear density ρ_A. Equation (1) is also valid for nuclear densities with small but, in principle, important corrections. The experimentally observed dependence of the nuclear spectra on the thickness ν_{abs} of the nucleus is qualitatively the same as for thick targets. Quantitatively, however, the simple cascade model described by Eq. (2) strongly contradicts the experiments at energies above 10–20 GeV.

The explanation for this is simple. In (2) we have implicitly assumed that the formation lengths l_f of the secondary particles are much less than the absorption length l_{abs}. At high energies, the Compton wavelengths $\lambda=1/k$ are small, and it would seem that the wavelength gives the natural scale for l_f. However, the situation is the reverse of the naive classical situation: In the production of high-energy particles $l\sim1/\lambda$,[1] and in the case of nuclei l_f for fast secondary particles is much greater than both l_{abs} and the radius R_A of the nu-

 0090-4759/81/010063-26$02.80

cleus. The shortcoming of thick targets is transformed in the case of nuclei into an important advantage, namely, the nature of the intranuclear interactions gives unique information about the space-time development of the process of multiparticle production. It is this circumstance that stimulated the increased interest in inelastic collisions with nuclei at high energies.

Such a situation, with $l_f > l_{abs}$ and, generally, with the growth of the characteristic scales of longitudinal distances at high energies, is not new. To the best of my knowledge, a large formation length appeared for the firs time in the paper of I. M. Frank in 1942,[2] in which it was shown that an accelerated electron does not radiate from a point but from an interval $\lambda/(1-v) \approx (E_e/m_e)^2\lambda$ (here, λ is the photon wavelength and v is the velocity of the electron). The influence of formation lengths on bremsstrahlung and pair production in matter was discovered by Ter-Mikaélyan[3] in 1952 and Landau and Pomeranchuk[4] in 1953 for coherent and incoherent processes, respectively. The Landau–Pomeranchuk effect provides a very close analogy to the production of particles in nuclei.

In hadron physics, the importance of large longitudinal scales was first noted by Pomeranchuk and Feĭnberg in 1954.[5] They noted that in the so-called diffraction generation of particles on nuclei, $hA \to h^*A$, longitudinal distances $L \approx E/(m^{*2} - m^2)$ are important, these exceeding the nuclear dimension, $L > R_A$, in the process of coherent generation on nuclei. Note that the last condition was already contained implicitly in Feĭnberg's paper[6] of 1941 (the Russian-language version of this paper was never published, since the corresponding issue of Zhurnal Eksperimental'noĭ i Teoreticheskoĭ Fiziki was lost in the blockaded Leningrad). The more recent history of the question will be discussed in detail below.

The large longitudinal dimensions of nuclei make it possible to use them as space-time analyzers of the production process; they are a kind of miniature, superdense bubble chamber. The large transverse dimensions of nuclei make collisions with nuclei sensitive to hadron structure as well. Let us explain this for the example of dN and dA interactions. In the first case, the impulse approximation is valid to a high accuracy, i.e., only one of the nucleons in the fast deuteron interacts with the target nucleus. Therefore, the particle production processes in dN and NN collisions will be the same, although the nucleons and deuterons have different structures. However, in dA collisions at small impact parameters both nucleons will be absorbed in virtually all cases, i.e., the process of production on nuclei will depend on the number of composite parts of the incident particle. In accordance with modern ideas, fast hadrons are quark–parton systems, and interactions with nuclei give a unique possibility for testing our ideas about the quark–parton structure of hadrons. This possibility is particularly topical in connection with the successes of quantum chromodynamics—the theory of colored quarks and gluons, whose existence is now regarded as beyond doubt—as applied to deep inelastic processes[7] and the physics of heavy particles.[8] All information about how hadrons are composed of quarks and gluons and how this composite structure of the hadrons influences their strong interactions is valuable. It is found that the additive quark model formulated in 1965 by Levin and Frankfurt[9] and applied to interactions with nuclei for the first time by Anisovich[10] is the most successful model.

In Sec. 1, we give the main experimental data on interactions with nuclei. In Sec. 2, we discuss the various manifestations of the growth in the scale of the longitudinal distances at high energies. In Sec. 3, we consider the total cross sections and the effects of inelastic shadowing in the total cross sections and in diffraction dissociation, and also the problem of measuring the cross sections for the interaction of diffraction-produced states. Section 4 is devoted to a discussion of photoproduction and deep inelastic scattering on nuclei with a critical analysis of the applicability of the vector dominance model for describing deep inelastic scattering.

The discussion of multiparticle production processes begins in Sec. 5, in which we analyze the data on the single-particle inclusive spectra and present briefly the main models of interaction with nuclei, which were proposed mainly to describe the inclusive spectra. We also briefly discuss the connection between the total cross sections and production processes. Section 6 is devoted to a discussion of correlation phenomena in multiparticle production on nuclei. We consider the correlations between fast particles and fragments of the nucleus and the manner in which the correlations between the fast particles and the multiplicity distributions of the fast particles depend on fragmentation of the nucleus. A number of models that successfully describe the single-particle spectra are refuted by the correlation data. In Sec. 7, we discuss hard processes: production of massive lepton paris, ψ particles, and particles with large transverse momenta.

A detailed discussion of all the models of inelastic interaction with nuclei hitherto proposed exceeds the scope of a single review. The exposition is subjective in the sense that preference is given to models based on the multiperipheral picture of strong interactions. It is such models, and essentially only these, that make it possible to describe all the main features of hadron–nucleus collisions from a unified point of view. In addition, many other models which go far beyond the framework of quantum field theory in their basic propositions simply do not stand up to comparison with the experiments.

1. BASIC EXPERIMENTAL DATA

The data summarized in this section are given mainly for orientation in the reading of the review. The detailed experimental data are discussed when the theories are considered in the corresponding sections.

1.1. Total cross sections and absorption cross sections. For charged particles, the usual method in which particles are knocked out of a beam enables one to measure only the absorption cross sections σ_{abs}.

The observed cross sections σ_{abs} can be described reasonably well by a generalization[11] of Eq. (1):

$$\sigma^A_{abs}=\int_0^\infty db 2\pi b\{1-\exp[-\nu_{abs}(b)]\}, \qquad (5)$$

where b is the impact parameter, and

$$\nu_{abs}(b)=\sigma^N_{abs}\int_{-\infty}^{+\infty} dz\rho_A(z,b)=\sigma^N_{abs}T(b). \qquad (6)$$

For large σ^N_{abs}, when $\nu_{abs}(b)\gg 1$, Eq. (5) gives $\sigma^A_{abs}=\pi R^2_A \sim A^{2/3}$ (black nucleus), and for $\nu_{abs}(b)\ll 1$ we have $\sigma^A_{abs}=A\sigma^N_{abs}$. Experimentally,[12] the exponent α in the parametrization $\sigma^A_{abs}=\sigma_0 A^\alpha$ is indeed close to $\alpha=2/3$ for large σ^N_{abs} (Fig. 1).

Note that Eq. (1) can be rewritten in the form

$$w=\sum_n w_n=\sum_n \nu^n \exp(-\nu)/n! \qquad (7)$$

One can interpret $w_n=\nu^n\exp(-\nu)/n!$ as the probability of n-fold inelastic interaction of the incident particle, it being assumed that the inelastic interactions take place without absorption of the incident particle (which, of course, is incorrect). However, the parameter $\bar\nu=\langle n\rangle=(\sum_n nw_n)/w$ is a very convenient characteristic of the thickness of the target for the incident particle. In the case of nuclei, it follows from (5) that

$$\bar\nu=A\sigma^N_{abs}/\sigma^A_{abs}. \qquad (8)$$

For photons $\sigma^{\gamma N}_{abs}T(b)\ll 1$, but experimentally $\alpha_{\gamma A}\sim 0.9$.[13] The interpretation is that at high energies the photons are first transformed into a hadronic system of the type of vector mesons, which then interact with the nucleus.[14] In deep inelastic scattering, i.e., for virtual photons, $\alpha_{\gamma * A}\sim 1$ experimentally, but important systematic deviations from unity are observed[15] (see the discussion of this below).

1.2. Mean multiplicities of secondary particles. On nuclei, the mean multiplicities $\langle N_s\rangle$ of the secondary particles are higher than on free nucleons. For hadrons at energies above 50 GeV, an approximate universal formula holds[16] (Fig. 2):

$$R=\langle N_s\rangle_A/\langle N_s\rangle_N\approx a+b\bar\nu, \qquad (9)$$

where $a\approx 0.4$ and $b\approx 0.6$–0.7. For neutrinos and virtual photons, the relation (8) gives $\bar\nu\equiv 1$. Nevertheless, experimentally $R_{\gamma * A}>1$ (Ref. 17) and $R_{\nu A}>1$.[18] Data on photoproduction on nuclei are not available. In accordance with (9), $R\approx 2.5$ for pPb interactions. At energies

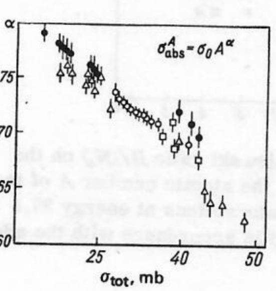

FIG. 1. Compilation of data (Bobchenko *et al.*[12]) on the dependence of the exponent α in the parametrization $\sigma^A_{abs}=\sigma_0A^\alpha$ on σ^N_{tot}.

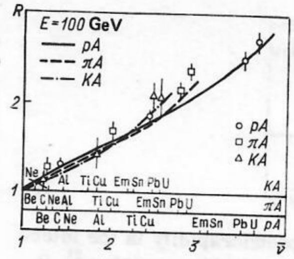

FIG. 2. Dependence of $R=\langle N_S\rangle_A/\langle N_S\rangle_N$ on the thickness ν of the nucleus in KN, πN, and pN interactions.[16]

$\approx$100 GeV, the simple cascade model would give values of R several times larger.[19]

The low nuclear multiplicity was noted long ago after the first cosmic-ray experiments at the beginning of the fifties, although the conclusion drawn then in Ref. 20 of a contradiction between the cascade model and the experiments was based on statistically inadequate data.

1.3. Nuclear inclusive spectra. The relative nuclear inclusive spectra $R_\eta=(dN_s/d\eta)_A/(dN_s/d\eta)_N$ show which regions of the pseudorapidity $\eta=-\ln\tan(\theta_{lab}/2)$ (or rapidities $y=(1/2)\ln[(\varepsilon+k_z)/(\varepsilon-k_z)]$) make a contribution to the multiplicity on nuclei that is in excess compared with the multiplicity on nucleons (θ_{lab} is the laboratory emission angle, and ε is the energy and k_z the longitudinal momentum of the secondary particles). For primary hadrons, it has been established that:

a) in the beam fragmentation region $R_\eta<1$, and R_η decreases with increasing $\bar\nu$ (see, for example, Fig. 3);

b) in the region of fragmentation of the nucleus, R_η increases with increasing $\bar\nu$ in accordance with the approximate law[21]

$$R_\eta=1+S(\eta)(\bar\nu-1),\quad S(\eta)\approx 2.5-3. \qquad (10)$$

In this respect, particle production on nuclei is similar to production in thick targets. Qualitative differences are manifested in the following:

c) in the region of fragmentation of the nucleus, R_η does not depend on the energy[22] (Fig. 4);

d) at high energies, there is an indication in R_η of a

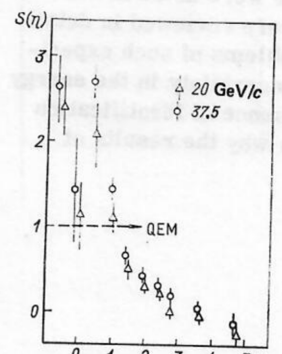

FIG. 3. Dependence on the pseudorapidity η of the slope $S(\eta)$.[21] The eikonal model (QEM) corresponds to $S(\eta)=1$ (see Sec. 5.7).

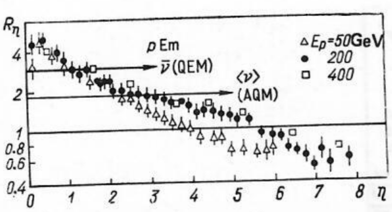

FIG. 4. Dependence of R_η on the pseudorapidity in the interaction of protons with nuclei in photographic emulsion.[23] $R_\eta = \bar{\nu}$ corresponds to the eikonal model, and $R_\eta = \langle\nu\rangle$ to the height of the plateau expected in the additive quark model.[97]

plateau separating the beam fragmentation region and the cascade region[22,23] (see Fig. 4).

Quantitatively, the simple cascade model gives for $\eta \approx 0$ values of R_η which are tens of times higher than the value observed experimentally.[19] In contrast, many models in which cascades are ignored (see below) predict for small η the value $S(\eta)=1$ (see Secs. 5.2 and 5.7 below).

Data on leptoproduction on nuclei are extremely sparse, but they unambiguously indicate that $R_\eta \approx 1$ in the region of large pseudorapidities and that the entire excess multiplicity is due to the cascade region of small η (Fig. 5).[17,18]

1.4. Multiplicity distributions. The ratio $D/\langle N_s\rangle$ ($D^2 = \langle N_s^2\rangle - \langle N_s\rangle^2$) is observed to be virtually independent of the size of the nucleus and the energy (Figs. 6 and 7).[16,21,24]

1.5. Correlations between the secondary particles. The secondary particles with velocities $0.3 < v/c < 0.7$ (or the gray prongs in the photographic emulsions) are predominantly recoil protons. The higher is their number N_g, the greater is the number of intranuclear interactions. Experimentally, the dependences of $\langle N_s\rangle_A$ and $dN_s/d\eta$ on $\bar{\nu}$ and N_g are similar (Fig. 8). There is one significant difference: $D/\langle N_s\rangle$ does not depend on $\bar{\nu}$ but decreases with increasing N_g (see Fig. 7).[21,24] With increasing N_g, there is also a decrease in the value of the two-particle correlation function $R_2(\eta_1,\eta_2) = (d^2N_s/d\eta_1 d\eta_2)/[(dN_s/d\eta_1)(dN_s/d\eta_2)] - 1$ in the central region (Fig. 9).[24,25]

The majority of the general properties of particle production on nuclei listed above were noted in cosmic-ray experiments. These data were reviewed in detail by Feĭnberg.[26] The specific problems of such experiments—the low statistics, the uncertainty in the energy of the primary particle, and absence of identification of the primary particle—explain why the results of

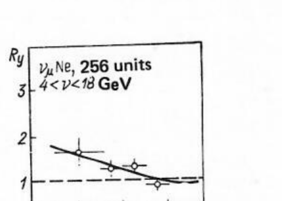

FIG. 5. Dependence of the ratio of the spectra in νNe and νN interactions on the rapidity.[18]

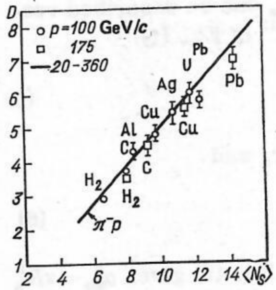

FIG. 6. Comparison of the dependence of the dispersion D on $\langle N_s\rangle$ in $\pi^- p$ and $\pi^- A$ interactions.[16]

these experiments were not treated with confidence. However, it must be emphasized that the analysis of these data led long ago to conclusions that are still basically valid today. The reader is recommended to examine the review papers of Refs. 20, 26, and 27 and compare their conclusions with those of the present review.

2. TIME AND LONGITUDINAL-DISTANCE SCALES AND FORMATION LENGTHS AT HIGH ENERGIES

2.1. The Landau–Pomeranchuk effect.[4] The emission of soft photons is described by the classical equation[1)]

$$\frac{dn}{d^3k} \approx \left| \int [d\mathbf{r}\mathbf{n}] \exp\{i\omega[t - \mathbf{n}\mathbf{r}(t)]\}\right|^2, \tag{11}$$

where ω is the photon frequency, $\mathbf{n} = \mathbf{k}/\omega$, and the integration is along the classical trajectory of the electron. For double scattering $\mathbf{v}_1 \to \mathbf{v}_2 \to \mathbf{v}_3$, we have

$$\frac{dn}{d^3k} \sim \frac{1}{\omega^2}\left|\left[\frac{[\mathbf{v}_1\mathbf{n}]}{1-\mathbf{v}_1\mathbf{n}} - \frac{[\mathbf{v}_2\mathbf{n}]}{1-\mathbf{v}_2\mathbf{n}}\right] + \left[\frac{[\mathbf{v}_2\mathbf{n}]}{1-\mathbf{v}_2\mathbf{n}} - \frac{[\mathbf{v}_3\mathbf{n}]}{1-\mathbf{v}_3\mathbf{n}}\right]\exp(il_{12}/l_f)\right|^2, \tag{12}$$

where v_i is the electron velocity, and we have introduced the formation length

$$l_f = 1/\omega(1 - \mathbf{n}\mathbf{v}_2). \tag{13}$$

If the distance l_{12} between the scattering points is large,

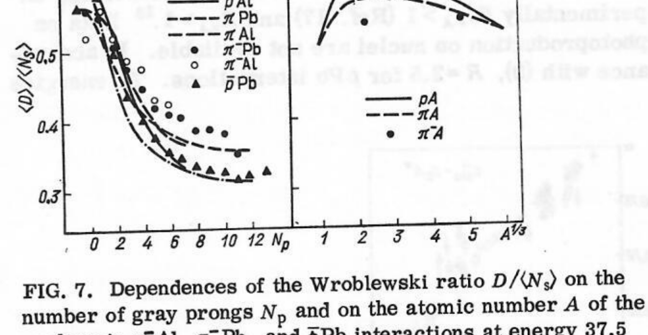

FIG. 7. Dependences of the Wroblewski ratio $D/\langle N_s\rangle$ on the number of gray prongs N_p and on the atomic number A of the nucleus in π^-Al, π^-Pb, and $\bar{p}$Pb interactions at energy 37.5 GeV.[21] The curves are calculated in accordance with the additive quark model.[97]

---

1)The Russian notation for scalar and vector products is retained to save composition costs.

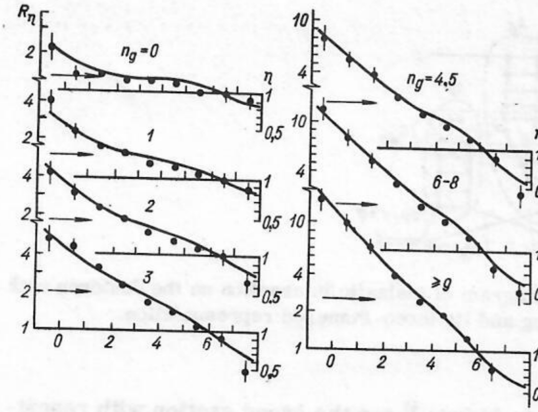

FIG. 8. Dependence of R_η on the number of gray prongs in the interactions of protons with nuclei in photographic emulsion at energy 400 GeV.[23] The curves are calculated in accordance with the additive quark model.[97] The arrows indicate the height of the plateau expected in the quark model.

$l_{12} \gg l_f$, then averaging over l_{12} in an amorphous medium leads to the classical picture of intensity addition (the terms in the square brackets in (12) are the amplitudes for emission on individual centers). For for $l_{12} \ll l_f$ there remains in (12) the emission by only the initial and final electrons, and there is not sufficient time for emission by the intermediate electron over distances much shorter than the formation length l_f. Following Feĭnberg, one can say that after the first scattering the electron "shakes off" its field and until this field is re-established the scattering in the external field takes place without emission.[28,29] This interpretation has a deep analogy in hadron processes. An elegant exact solution to the Landau–Pomeranchuk problem was given by Migdal,[30] and a physically very perspicuous discussion of the effect can be found in Ref. 31.

Equation (13) has radical consequences for, for example, transition radiation. In cases of practical interest in the optical range, l_f has the order of laboratory dimensions, and it is impossible to amplify a weak radiation intensity by increasing the number of radiators.[32]

2.2. Scales of longitudinal distances in strong interactions. The Gribov–Ioffe–Pomeranchuk method. Feĭnberg's analysis[28,29] shows that there is an analog of the Landau–Pomeranchuk effect in strong interactions as well. However, the appeal to electrodynamics is not rigorous. A more rigorous approach was proposed by Gribov, Ioffe, and Pomeranchuk[33] in 1965.

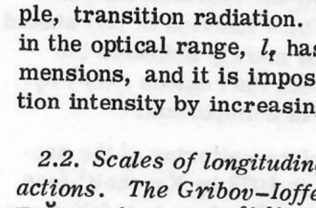

FIG. 9. Dependence of the rapidity correlation $R_2^A(0, 0)$ in the pionization region on the number N_g of gray prongs in interactions of pions and protons with nuclei in photographic emulsion.[24,25] The curves are calculated in accordance with the additive quark model.

The absorption part of the hadron–hadron $k + p \to k + p$ scattering amplitude can be expressed in terms of the current commutator:

$$\mathrm{Abs}\, F(s, 0) = \frac{1}{2} \int d^4x \exp(ikx) \langle p \mid [J(x), J(0)] \mid p\rangle. \tag{14}$$

Consider (14) as a function of k^2. At high energies,

$$kx = \varepsilon (t - z) + (k^2/2\varepsilon) z. \tag{15}$$

In (14), the 4-vector x can be interpreted as the difference between the points of absorption of the incident particle and production of the scattered particle. A dependence of $F(s, 0)$ on k^2 would mean that in elastic scattering longitudinal distances

$$\Delta z \sim \varepsilon/m^2, \tag{16}$$

which increase with increasing energy, are important. In hadron scattering, the external masses are fixed. To test the k^2 dependence of the amplitude $F(s, 0)$, one cannot use the departure from the mass shell in bremsstrahlung, as was first proposed in Ref. 33. The first rigorous proof that the longitudinal distances are proportional to the energy was given by Ioffe[34] for deep inelastic scattering, for which the square of the photon mass, Q^2, can be varied. In this case, it follows from the scaling behavior of the total cross section that

$$\Delta z \sim \varepsilon/Q^2. \tag{17}$$

2.3. Elastic multiscattering in the multiperipheral approach. The multiperipheral model is based on the hypothesis, motivated by the experiments, that the hadron amplitudes decrease as one leaves the mass shell. It is natural (see Sec. 2.2) that large longitudinal distances arise in the multiperipheral approach. In the simplest variant of the approach, elastic scattering is described by the diagram in Fig. 10a. The time of interaction with the target is short, $\Delta t \sim 1/\mu$. Therefore, this diagram can be interpreted as follows. The incident hadron, decaying successively, forms a parton fluctuation[2)] and a wee parton of the fluctuation interacts with the target. The time during which decay is possible is determined by the uncertainty principle: $\tau \sim 1/\Delta E$. For relativistic particles in the transition $1 \to 2 + 3$,

$$\Delta E \sim (m^2 + \mathbf{k}_\perp^2)/E, \tag{18}$$

and this gives $\tau \sim E$.

Besides single scattering, one can have double scattering. It is obvious that in the limit $E \to \infty$ the planar diagram in Fig. 10b makes a vanishing contribution, since the formation during a time $\Delta t \sim 1/\mu$ of two successive fluctuations with lifetimes $\tau \sim E/\mu^2$ is impossible.[3)] However, one can have two parallel fluctuations, which lead to the nonplanar diagram of Fig. 10c. In the high-energy limit, the hadron can be represented generally as a superposition of parton ladders, the notion of a parton wave function being introduced.[36,38] It

---

[2)]It is appropriate to recall the well-known von Weizsäcker–Williams method in electrodynamics; the equivalent photons are partons.

[3)]This result is due to Mandelstam,[35] and it is discussed in space-time language by Gribov[36] and Ansel'm.[37]

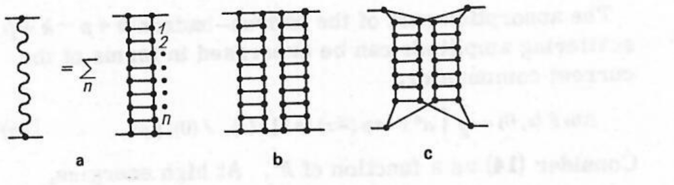

FIG. 10. Multiperipheral diagrams for the pole amplitude of elastic scattering (a), the planar amplitude of double scattering (b), and the Mandelstam nonplanar amplitude of double scattering (c).

is important that the hadron is transformed into a parton fluctuation not only at the threshold of the interaction with the target (the probability for such guessing of the necessary instant of time would become zero with increasing energy), and slow partons are present in the hadron, replacing each other over times $\sim 1/\mu$ with probability near unity.[36,37] A detailed discussion of this question, which has been inadequately considered in the literature except in the excellent lectures by Gribov[36] and Ansel'm[37] and the recent papers of Levin and Ryskin,[39] Grassberger,[40,41] and Kopeliovich *et al.*,[42] would take us too far from our path. We shall merely show that one can demonstrate that fluctuations are indeed formed before the target.

In the Feynman diagram of Fig. 10c, which is considered in the laboratory system, we introduce a time ordering:

$$\theta(t_1-t_2)=\frac{i}{2\pi}\int\frac{d\omega}{\omega+i0}\exp[-i\omega(t_1-t_2)], \tag{19}$$

adding to the diagram a spurion,[43] with respect to whose energy ω it is necessary to integrate. A straightforward but fairly lengthy analysis of the structure of the singularities shows that for time ordering $t_B > t_A$ the diagram of Fig. 10c makes a vanishing contribution.[44] In this, essential use is made of the fact that the amplitudes decrease on departure from the mass shell. In Refs. 45 and 46, the last condition was neglected and the exactly opposite (and erroneous) conclusion that there is no time ordering in the nonplanar Mandelstam diagram was drawn.

2.4. Time scales in production processes. The interaction of the secondary particles produced on one nucleon of a deuteron with the second nucleon is described by the diagram in Fig. 11. The total cross section of the process with rescattering is given by a diagram which formally is similar to the three-Pomeron diagram with an effective constant G_{eff}. If G_{eff} is equal to the usual three-Pomeron constant G_{PPP}, which is small, then the contribution of the diagram in Fig. 11 is negligibly small. However, in G_{PPP} the particles are far from the mass shell. In the problem of scattering on separated particles, particle b in Fig. 12 is almost real; more precisely, either b or b' is simply on the mass shell, and the Green's function of the second particle is $1/2(\mathbf{k}\cdot\mathbf{q})$, where $\mathbf{q}$ is the relative momentum of the nucleons in the deuteron. For $|\mathbf{k}\cdot\mathbf{q}| < \mu^2$, the amplitude A_6 is transformed into the amplitude that occurs in the Kancheli–Mueller optical theorem for the inclusive spectrum of particle b. Using

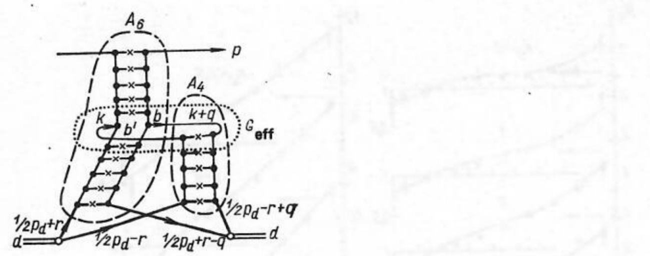

FIG. 11. Diagram of inelastic interaction on the deuteron with rescattering and its three-Pomeron representation.

Gribov's technique,[47] for the cross section with rescattering one can readily obtain the result[48]

$$\sigma_{RE}=\int d^3\mathbf{k}\,\frac{d\sigma_b}{d^3\mathbf{k}}\,\sigma_{abs}^{bN}\int\frac{d^3\mathbf{q}\,\rho_D(\mathbf{q}^2)}{(2\pi)^3(q_z+i\varepsilon)}, \tag{20}$$

where the domain of integration is restricted by the requirement that the intermediate particles be near the mass shell, $|k_z q_z| < \mu^2$; $\rho_D(\mathbf{q}^2)$ is the deuteron form factor. In the coordinate representation, (20) takes the form

$$\sigma_{RE}\approx\sigma_{abs}^{hd}\int d\varepsilon\,(dn_b/d\varepsilon)\,\sigma_{abs}^{bN}\int dr\,|\psi_d(r)|^2\,\theta(r-\varepsilon/\mu_0^2). \tag{21}$$

In (20) and (21), we do not need any information about the Regge structure of the cross section for the production of particle b on nucleons and the cross section of the bN interaction. The relation (21) contains directly measured quantities, and (21) itself has a simple probability meaning, namely, for secondary particles with formation length

$$l_f\approx\varepsilon/\mu_0^2 < R_d, \tag{22}$$

inelastic interaction with the secondary particle has the simple geometrical probability

$$w_{RE}\approx\sigma_{abs}^{bN}(1/4\pi)\langle 1/r_d^2\rangle, \tag{23}$$

while for particles with $l_f > R_d$ the rescattering probability is negligibly small. The θ function in (21) should not be understood literally; it merely indicates the nature of the transition between the regions $l_f(\varepsilon) \lessgtr R_d$. The exact form of the cutoff is unimportant.

Thus, the simple picture of rescattering of the secondary particles formulated in 1973 by Kancheli[1] (see also the earlier paper of Ref. 49) arises in a felicitous manner: The secondary hadrons with energy ε have formation length $l_f(\varepsilon)\approx\varepsilon/\mu_0^2$; at distances greater than $l_f(\varepsilon)$, the motion of the particles can be regarded as classical (a proof of this assertion in the language of wave packets was also given recently by Gottfried and Low[50]), i.e., the probability treatment is valid; the

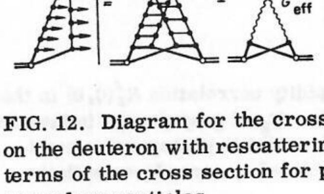

FIG. 12. Diagram for the cross section of inelastic interaction on the deuteron with rescattering and its representation in terms of the cross section for production and scattering of secondary particles.

contribution of the regions within the formation zone, in which the detailed structure of the amplitudes and interference phenomena are important, can be ignored; one of the contributions of this region, corresponding to the three-Reggeon diagram, can be calculated, but it gives a small contribution to the cross section that can be ignored at all real energies, though this contribution will be dominant at superasymptotic energies of the order of the mass of the Universe. In accordance with the above derivation, μ_0^2 is determined by the rate of decrease of the amplitude with departure from the mass shell or, which is the same thing, by the mean square of the transverse mass of the so-called direct secondary particles. The natural scale is $\mu_0^2 \approx \mu_\rho^2$.

2.5. Formation lengths in the language of the parton model. Equation (13) can be rewritten as

$$l_t \approx (E_e/m_e)(1/\omega_0), \qquad (24)$$

where ω_0 is the frequency of the photon in the rest frame of the emitting electron, i.e., a large l_t corresponds simply to the Lorentz time dilatation $T \approx 1/\omega_0$ of the emission of the photon by the slow electron. Similarly, (22) corresponds to the Lorentz-transformed Yukawa relation for hadrons: $T \sim 1/\mu$.

Let us consider the evolution of a parton fluctuation after inelastic interaction with the target. Colliding with the target, the wee parton, which by definition is equivalent to a slow hadron, is knocked out by the fluctuation, causing a transition to a hadron (hadronization) and the knocking out of the fluctuation in a time $\sim\varepsilon_2/\mu_0^2$ of the parton with the next following rapidity. This initiates hadronization of the next parton, which occurs in a time $\sim\varepsilon_3/\mu_0^2$, etc. The excitation is passed through the rapidity scale up to hadronization of the fastest parton of the fluctuation. With this, the process of formation of the final state is completed. In any frame of reference, the slow secondary particles are formed first and then the faster particles in increasing order of energy. Such a picture of inelastic interaction has also been frequently discussed by Bjorken.[51]

2.6. Collective interactions of secondary particles. If the secondary particles were produced at a point, the particles with momentum k would be separated from the beam at a distance $\Delta b \sim 1/\mu_\pi$ in the impact-parameter plane, and it would be meaningful to regard them as separate particles only at a distance

$$L \approx (k/\langle k_\perp\rangle)(1/\mu_\pi), \qquad (24')$$

the beam of faster particles resembling rather a single collective particle. Then the number of secondary interactions would be reduced. Such a suggestion was made by Barashenkov *et al.*[52] to improve the agreement between the experiments and the simple cascade model, which can be extended in this manner to energies of several tens of giga-electron-volts.

From the point of view of the multiperipheral model, the notion of particle production from a point is incorrect, since the partons in the parton fluctuation and the produced hadrons are distributed in the impact-parameter plane in accordance with the random-walk law with constant mean distance between neighboring particles, so that all the secondary particles are produced with separation in the impact parameters.[36,37]

2.7. Influence of the nuclear medium on the formation lengths. Absorption would lead in (11) to a term of the form $-\gamma t$ in the exponential in the integrand. For $\omega(1-\mathbf{n}\cdot\mathbf{v}) < \gamma$, growth of the formation lengths would cease. In the case of hadrons, as is seen from the discussion in Sec. 2.4, this does not occur, since the newly produced hadron does not interact until it has been formed. In a series of papers, Bialkowski, Chin, and Tow[53] postulate a decrease in the formation lengths proportional to the amount of nuclear matter traversed by the secondary particle:

$$1/l_f(\varepsilon) = 1/\gamma l_0 + \lambda\rho_A l_f(\varepsilon). \qquad (25)$$

This assumption is arbitrary and does not have a theoretical justification.

3. TOTAL CROSS SECTIONS AND DIFFRACTION PROCESSES

3.1. Absorption, elastic scattering, and diffraction dissociation. For an elementary particle incident on a black nucleus, there are two processes: total absorption for impact parameters $b \leqslant R_A$ and elastic diffraction scattering due to this absorption. For a composite particle (deuteron), the absorption cross section begins to depend on the internuclear distance b_d in the impact-parameter plane,

$$\sigma_{abs} = \pi(R_A + b_d)^2, \qquad (26)$$

and in the transmitted wave $|f\rangle$ the fraction of states with small b_d is higher than in the deuteron. After expansion

$$|f\rangle = C_d\,|d\rangle + \sum_i C_i\,|i\rangle \qquad (27)$$

with respect to a system of eigenfunctions, we obtain continuum states $|i\rangle$, i.e., besides elastic scattering there is an essentially new process—diffraction dissociation[5,54]:

$$dA \to (np)A. \qquad (28)$$

The nucleus remains in the ground state and the process is coherent if $q_L = (m_{np}^2 - m_d^2)/2E$ is small: $q_L R_A < 1$. This means that the characteristic longitudinal distances are large[5,54]:

$$L \approx 1/q_L > R_A. \qquad (29)$$

3.2. The eigenstate method and the Glauber formalism. The deuteron is a diagonal state in the mass spectrum but not of the scattering operator. Diagonal scattering states, or eigenstates, are states that are absorbed and scattered only elastically. In the case of the deuteron, there is no need for the special introduction of eigenfunctions of the scattering operator, although this can be readily done.[54] In the case of hadrons, the system of eigenfunctions is unknown, since there is no theory of strong interactions, but the formalism of eigenstates itself is very helpful.

Thus, suppose that for the state vector $|A\rangle$ of the incident particle there is an expansion with respect to a system of eigenfunctions of the scattering operator:

$$|A\rangle = \sum_i C_i |i\rangle. \quad (30)$$

Such an expansion will be meaningful to the extent that the lifetimes of the parton fluctuations, which will be identified below with the eigenstates, are long compared with the time of interaction with the target (see Sec. 2.3).

For fixed impact parameter, we have for the matrix element of the T matrix

$$\langle A | \mathrm{Im}\, T | A\rangle = \sum_i |C_i|^2 t_i = \langle t\rangle. \quad (31)$$

To diffraction dissociation there corresponds the component

$$|D\rangle = \mathrm{Im}\, T |A\rangle - \langle t\rangle |A\rangle \quad (32)$$

of the final state. This gives

$$\frac{d\sigma_D}{d^2\mathbf{b}} = \langle D | D\rangle = \sum_i |C_i|^2 t_i^2 - \langle t\rangle^2 = \langle t^2\rangle - \langle t\rangle^2. \quad (33)$$

Here, we have followed the recent paper of Ref. 55, although this formalism is contained essentially already in Refs. 5, 54, and 56.

The generalization to the case of nuclei is elementary. For short wavelengths, the quasiclassical treatment is valid, and for the probability that an eigenstate passes through a nucleus without interaction we have

$$P = \exp[-\sigma_{\mathrm{tot}}\ T(b)] = \exp[-\nu(b)]. \quad (34)$$

For the transition to quantum scattering theory, we identify P with $|S(b)|^2$ and, assuming that the scattering amplitudes are purely imaginary, we obtain

$$S(b) = \sqrt{P} = \exp[-\nu(b)/2], \quad (35)$$

i.e.,[57]

$$\sigma^A_{\mathrm{tot}} = 2\int_0^\infty db 2\pi b \{1 - \langle \exp[-\nu(b)/2]\rangle\}. \quad (36)$$

We note that (35) agrees with the $S(b)$ given by the well-known Glauber formalism.[58] What is the relationship between the two approaches?

They are identical. In the limit $A \gg 1$, the Glauber approach is equivalent to the optical model with potential[59]

$$V(\mathbf{r}) = -4\pi f(0)\rho_A(\mathbf{r}). \quad (37)$$

In the presence of diffraction dissociation, $f(0)$ must be replaced by the matrix $f_{ik}(0)$ of the amplitudes of the diffraction processes and one must solve the equation[60]

$$(\nabla^2 + k_i^2)|i, \mathbf{r}\rangle = -\sum_k 4\pi f_{ik}(0)\rho_A(\mathbf{r})|k, \mathbf{r}\rangle. \quad (38)$$

If the condition (29) is satisfied, i.e., in the limit $|\mathbf{k}_i - \mathbf{k}_j| R_A \ll 1$, the scattering eigenstates are states that diagonalize the matrix $f_{ik}(0)$.

One further method of deriving the Glauber formalism is, following Gribov,[47] to sum all the multiple scattering diagrams of Fig. 13. Equation (36) corresponds to exact summation of all such diagrams with allowance for all intermediate states h^*.

3.3. Inelastic shadowing in the total cross sections. The simple optical model gives

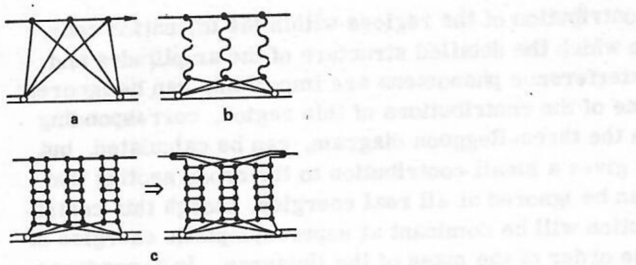

FIG. 13. Gribov diagrams of multiple scattering for the amplitude of elastic scattering on a nucleus (a), the planar amplitudes of m-Pomeron exchange corresponding to them (b), and the assumed equivalence between the planar eikonal diagrams and the nonplanar diagrams of m-fold scattering on the nucleus.

$$\sigma^A_{\mathrm{tot}} = 2\int_0^\infty db 2\pi b \left\{1 - \exp\left[-\tfrac{1}{2}\sigma^N_{\mathrm{tot}} T(b)\right]\right\}. \quad (39)$$

In the eigenstate method, $\sigma^N_{\mathrm{tot}} T(b) = \langle \nu(b)\rangle$. Since

$$\langle \exp[-\nu(b)/2]\rangle \geqslant \exp[-\langle \nu(b)\rangle/2], \quad (40)$$

diffraction dissociation reduces the total nuclear cross sections compared with those expected in the simple optical model. This decrease in σ^A_{tot} is unambiguously confirmed by the experimental data on the total cross sections of nA (Ref. 61) and $K_L A$ (Ref. 62) scattering (Figs. 14 and 15).

To estimate the contribution of diffraction dissociation (inelastic shadowing) to σ^A_{tot}, one usually employs the Karmanov–Kondratyuk formula[63]:

$$\Delta\sigma = -4\pi \int db 2\pi b\, (d\sigma_D/dt)_{t=0} T^2(b) \exp\left[-\tfrac{1}{2}\sigma^N_{\mathrm{tot}} T(b)\right]. \quad (41)$$

Here, $(d\sigma_D/dt)_{t=0}$ is the total differential cross section of all the diffraction processes at $t=0$. In the derivation of (41), only the transitions $h \to h^* \to h$ were taken into account. This is a perfectly sensible approximation, since $|f_{hh^*}| \ll |f_{hh}|$. However, the second approximation

$$f_{h^*h^*} = f_{hh} \quad (42)$$

is inconsistent, since the diffraction dissociation is due

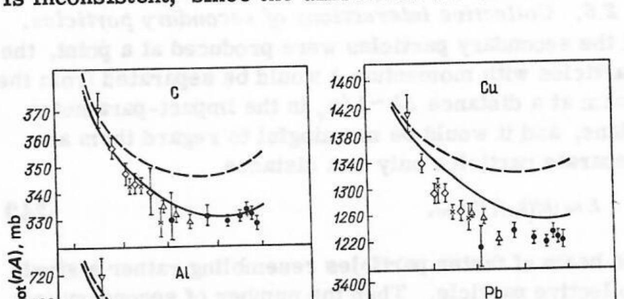

FIG. 14. Dependence on the energy and the atomic number of the total cross section of nA interaction.[61] The broken curve is calculated in accordance with the Glauber model, and the continuous curve takes into account the Karmanov–Kondratyuk corrections.

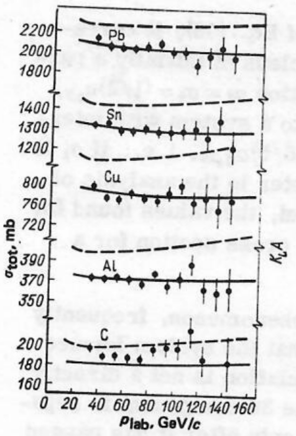

FIG. 15. Dependence on the energy and the atomic number of the total cross section of K_LA interaction.[62] The broken curves are the calculation in accordance with the simple Glauber model, and the continuous curves are the calculation with allowance for the inelastic correction in the model with passive quark states.[78]

to fluctuations of the cross section. Despite this, the Karmanov–Kondratyuk formula is reasonably well satisfied experimentally (see Figs. 14 and 15).

This can be explained as follows.[64] In accordance with (36) and (40), $\Delta\sigma$ can be expressed in the form

$$\Delta\sigma = -4\pi \int_0^\infty db\, b\, \{\langle \exp[-\nu(b)/2]\rangle - \exp[-\langle\nu(b)\rangle/2]\}. \tag{43}$$

Expansion in a series gives

$$\langle \exp[-\nu(b)/2]\rangle - \exp[-\langle\nu(b)\rangle/2] \approx (1/8)\exp[-\langle\nu(b)\rangle/2]\,(\langle\nu(b)^2\rangle - \langle\nu(b)\rangle^2). \tag{44}$$

Comparing (44) with (43) and noting that $\langle\nu(b)\rangle = \sigma_{tot}^N T(b)$, we arrive at (43).

The fluctuations of the cross sections σ_i are of the order of the cross sections themselves. At small impact parameters, $\nu(b)$ is greater than unity, and the expansion (44) is invalid. But the contribution of this region to the inelastic shadowing is suppressed by the exponential factor $\exp[-\nu(b)/2]$. Therefore, the bad approximation (42) leads to the satisfactory quantitative estimate (41).

In the two-channel approximation with $f_{11} = f_{22}$, it follows from (38) that

$$f_{a,b} = f_{11} \pm f_{12}, \tag{45}$$

i.e., the cross section is small for one of the eigenstates $|a\rangle$ and $|b\rangle$. In the multichannel problem, it may even be zero. If one bears in mind the analogy between the parton model and the von Weizsäcker–Williams method, a passive state containing no wee partons is perfectly natural. Grassberger has noted[41] that in theories with increasing total cross sections the weight $P = |C_0|^2$ of the passive state increases with increasing energy, approaching an asymptotic value from below.

Experimentally, $\sigma_{tot}(nN)$ and $\sigma_{tot}(K_LN)$ rise with increasing energy, which leads to an increase in σ_{tot}^A as well for light nuclei. For heavy nuclei the situation is different. For a black nucleus, it follows from (36) that[57]

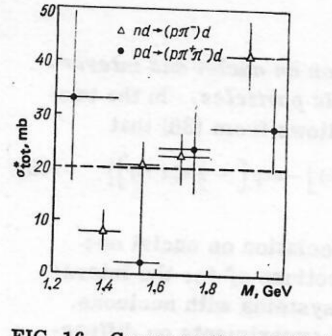

FIG. 16. Cross sections of $(p\pi^-)N$ and $(p\pi^+\pi^-)N$ interactions obtained by Glauber analysis of diffraction dissociation on deuterons.[67]

$$\sigma_{tot}^A = (1-P)\,2\pi R_A^2, \tag{46}$$

i.e., a *decrease* of σ_{tot}^A with increasing energy is expected. Note that (46) corresponds to a relative inelastic correction that does not decrease with increasing R_A:

$$\Delta\sigma/\sigma_{tot}^A \approx P, \tag{47}$$

whereas the Karmanov–Kondratyuk correction (41) for heavy nuclei decreases on account of the factor $\exp[-\sigma_{tot}^N T(b)/2]$ in the integrand. This difference between (41) and (47) is natural, since the expansion (47) is invalid for heavy nuclei in the presence of a passive state.

The data on $\sigma_{tot}(nA)$ and $\sigma_{tot}(K_LA)$ in Figs. 16 and 17 confirm such a change in the energy dependence of the nuclear cross sections on the transition from light to heavy nuclei. The data on the neutron cross sections also indicate that (41) underestimates the inelastic correction and that experimentally $\Delta\sigma$ increases with the energy more rapidly than follows from (41). It is possible that the passive state is manifested in this. It is very important to continue precise measurement of the neutron cross sections into the region of Tevatron energies.

In the derivation of (36) and (41), all the diffraction amplitudes were assumed to be purely imaginary, which is valid in the limit $S\to\infty$ for the excitation of any finite mass M, since the contributions of the nonvacuum exchanges tend to zero. This is not the case in the production of large ($M^2/s \approx$ const) masses. Masses $M^2/s \lesssim 1/R_A m_N$ contribute to the inelastic shadowing. As was noted for the first time by Anisovich *et al.*,[65] the production of large masses as a result of a number of nonvacuum exchanges makes an antishadowing contribution to $\Delta\sigma$, whose relative magnitude does not decrease as $S\to\infty$. Quantitatively, the antishadowing contribution to the total cross section of scattering on deuterons is ~10% of the inelastic correction.[65] The question of their contribution to the total nuclear cross sec-

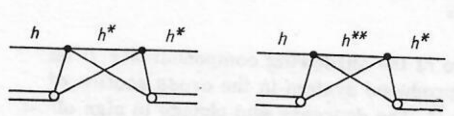

FIG. 17. Diagonal and nondiagonal shadowing in diffraction dissociation on deuterons.

tion remains open.

3.4. Diffraction dissociation on nuclei and interaction cross sections of unstable particles. In the two-channel approximation, it follows from (38) that

$$t_{12}^{A}(\mathbf{b})=\frac{2f_{12}^{N}(0)}{\sigma_2^*-\sigma_1}\left\{\exp\left[-\frac{1}{2}\sigma_1 T(b)\right]-\exp\left[-\frac{1}{2}\sigma_2^* T(b)\right]\right\}, \qquad (48)$$

and from the diffraction dissociation on nuclei one could determine the cross sections σ_2^* for the interaction of diffraction-produced systems with nucleons. This stimulated a number of experiments on diffraction dissociation on nuclei. The results were discouraging, since experimentally σ_2^* for many-particle systems was found to be smaller than or of the order of σ_{NN} or $\sigma_{\pi N}$ instead of the naively expected sum of the cross sections over all particles of the system.[66] An example of a recent determination of σ_2^* for the $N\pi$ and $N\pi\pi$ systems from the diffraction dissociation of nucleons on deuterons[67] is shown in Fig. 18.

The various interpretations of this phenomenon all reduce ultimately to the fact that the two-channel approximation (48) is invalid. We shall demonstrate this by the following simple example.[68] Suppose the scattering eigenstates $|1\rangle$, $|2\rangle$, and $|3\rangle$ have interaction cross sections $\sigma_1=\sigma_0$, $\sigma_2=2\sigma_0$, and $\sigma_3=3\sigma_0$, and suppose the physical states, called nominally $|\pi\rangle$, $|3\pi\rangle$, and $|5\pi\rangle$, are related to the eigenstates by the decomposition

$$\left.\begin{aligned} &|\pi\rangle=(1/2)\,|1\rangle+(1/\sqrt{2})\,|2\rangle+(1/2)\,|3\rangle;\\ &|3\pi\rangle=-(1/2)\,|1\rangle+(1/\sqrt{2})\,|2\rangle-(1/2)\,|3\rangle;\\ &|5\pi\rangle=-(1/\sqrt{2})\,|1\rangle+(1/\sqrt{2})\,|3\rangle.\end{aligned}\right\} \qquad (49)$$

In accordance with (49),

$$\sigma_{\pi N}=\sigma_{(3\pi)N}=2\sigma_0;\quad \sigma_{(5\pi)N}=(5/2)\,\sigma_0 \qquad (50)$$

and for the amplitudes of diffraction dissociation we obtain the expressions

$$\left.\begin{aligned} &t_{\pi\to 3\pi}^{A}\sim\exp\left[-\sigma_0 T(b)\right]-\frac{1}{2}\exp\left[-\frac{1}{2}\sigma_0 T(b)\right]\\ &-\frac{1}{2}\exp\left[-\frac{3}{2}\sigma_0 T(b)\right];\\ &t_{\pi\to 5\pi}^{A}\sim\exp\left[-\frac{1}{2}\sigma_0 T(b)\right]-\exp\left[-\frac{3}{2}\sigma_0 T(b)\right],\end{aligned}\right\} \qquad (51)$$

which have nothing in common with the cross sections (50) given by Eq. (48). Especially noteworthy is Eq.

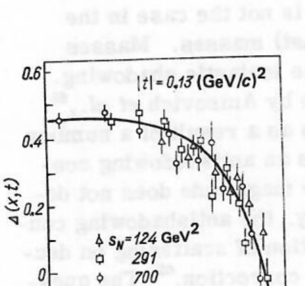

FIG. 18. Dependence of the shadowing component $\Delta(x,t)$ on the mass m_X of the produced system in the cross section of $pd\to Xd$ dissociation.[70] The decrease and change in sign of $\Delta(x,t)$ with increasing x correspond to the transition from shadowing to antishadowing. The curve is calculated in accordance with the parton model of Ref. 70.

(51) for $t_{\pi\to 5\pi}^{A}$. In the language of Eq. (48), it corresponds to propagation in the nucleus of initially a particle with interaction cross section $\sigma_1=\sigma_0=(1/2)\sigma_{\pi N}$, this particle then going over into a system with interaction cross section $\sigma_2^*=3\sigma_0=(6/5)\sigma_{(5\pi)N}$, i.e., if σ_1 is also regarded as a free parameter in the analysis of diffraction dissociation on nuclei, the values found for σ_1 will differ from the physical cross section for a beam particle.[69]

The physical reason for the phenomenon, frequently emphasized by Feĭnberg,[29] is that the system formed as a result of diffraction dissociation is not a directly observable, say, 3π state. The 3π component is separated from the excited system only after it has passed through the nucleus, in the process of which nondiagonal transitions of the type $\pi\to 5\pi\to 3\pi$ are extremely important. Moreover, nondiagonal transitions can even lead to $\sigma_2^*<0$. We shall demonstrate this for the example of diffraction dissociation on the deuteron.

The inelastic shadowing of the total cross section is quadratic in f_{hh*} and has fixed sign [see Eq. (41)]. The inelastic correction to the amplitude of the diffraction dissociation $h\to h^*$ contains the terms $f_{hh}f_{hh*}$ and $f_{hh*}f_{h*h*}$ (see Fig. 17a), which give shadowing, as well as terms of the form $ff_{hh*}f_{h**h*}$ (see Fig. 17b), whose sign is model-dependent. Thus, in the simple parton model[70] $\mathrm{Im}f_{hh*}<0$ for all nondiagonal transitions ($\mathrm{Im}f_{h*h*}>0$ by virtue of unitarity), whereas the nondiagonal inelastic corrections have *antishadowing* sign. The relative contribution of the diagram in Fig. 17b increases with increasing mass of the produced system, and shadowing is replaced by antishadowing (see Fig. 18).[70] The possibility that the shadowing in the diffraction dissociation on the deuteron could differ from the shadowing given by the simple Glauber model was pointed out for the first time by Levin *et al.*[71]

To determine σ_2^*, one could also use incoherent diffraction dissociation with disintegration of the nucleus. In this case, at low energies[66]

$$\left.\frac{d\sigma_{12}^{A}}{dt}\right|_{t=0}=\left.\frac{d\sigma_{12}^{N}}{dt}\right|_{t=0}\frac{1}{\sigma_2^*-\sigma_1}\int db\,2\pi b\,\{\exp\left[-\sigma_1 T(b)\right]-\exp\left[-\sigma_2^* T(b)\right]\}. \qquad (52)$$

A general multichannel analysis of incoherent dissociation has not yet been made. It can, however, be stated that the values of σ_2^* for the same system determined in coherent and incoherent dissociations may differ strongly.[69] In an experiment on the coherent dissociation $p\to p\pi^+\pi^-$ the cross sections σ_2^* for the mass intervals $M_{p\pi^+\pi^-}$=1.4–1.6, 1.6–1.8, and 1.8–2.2 GeV/c^2 were found to be 24, 25, and 18 mb, respectively,[72] which should be compared with $\sigma_2^*=0$, 8, and 7 mb ($\sigma_2^*<24$, 32, and 10 mb at the 95% confidence level) found from incoherent dissociation.[73]

Thus, one cannot use diffraction dissociation to determine σ_2^* on the basis of the two-channel formulas (48) and (52). It would not be correct to interpret this as invalidity of the Glauber approach altogether[55]; on the contrary, the eigenstate method and the Glauber approach are the same.[58] However, both formulations are incomplete—in one case, the explicit system of complete functions is unknown, in the other the amplitudes f_{h*h*} and f_{h*h**}. Recently, however, there has been progress in the construction of quark–parton models for scattering eigenfunctions.[42]

We note finally that there are no objections against applying the analog of Eq. (52) to nondiffraction processes of the type $\pi\to\rho$ and $N\to\Delta$ on nuclei at high energies. Therefore, the cross sections of ρN, ΔN, etc., interactions determined in this manner must be equal to the physical cross sections (for references to experimental studies, see, for example, the reviews of Refs. 66 and 74).

3.5. $K_L\to K_S$ *regeneration on nuclei*. The regeneration of K_S mesons on nuclei is a unique example of a coherent nondiffraction process. In the experiments of Telegdi's group,[75] it was found that neither the regeneration amplitude f^A_{LS}, nor its phase φ^A_{LS}, nor its energy dependence agreed with the Glauber approximation (Fig. 19). In particular, it was found that for the effective ω trajectory in f^A_{LS}

$$\alpha_\omega(0)=0.39\pm0.01, \tag{53}$$

whereas it follows from the data on KN and $\overline{K}N$ interactions that

$$\alpha_\omega(0)=0.44\pm0.01. \tag{54}$$

Bertocchi and Treleani were the first to note[76] that in calculations of f^A_{LS} it is necessary to take into account nondiagonal transitions $h\to h^*$ in exchange of either a Pomeron or ω trajectory. The introduction of such transitions makes it possible to reproduce the measured amplitudes f^A_{LS}.[77]

The part played by the inelastic corrections is most readily analyzed in the framework of the eigenstate method. For the difference between the cross sections of the K^0A and $\overline{K}^0A$ interactions, which determines $\mathrm{Im} f^A_{LS}$, we have[78]

$$\Delta\sigma_A=\int db\,2\pi bT(b)\sum_n|C_n|^2\,\Delta\sigma_n\exp\left[-\tfrac{1}{2}\sigma_nT(b)\right]. \tag{55}$$

In quantum chromodynamics, a purely gluon parton

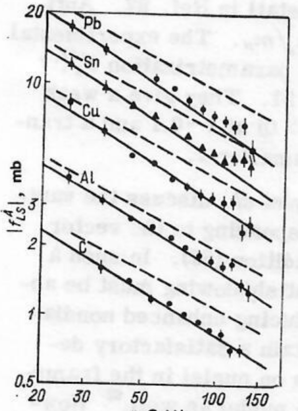

FIG. 19. Dependence of the amplitude of $K_LA\to K_SA$ regeneration on the energy and the atomic number of the nucleus.[75] Calculation in accordance with the Glauber model without (broken curves)[75] and with allowance (continuous curves)[78] for inelastic shadowing.

fluctuation corresponds to the Pomeron.[79] Nonvacuum exchanges are possible only through fluctuations in which one of the wee partons is a valence quark. Therefore, regeneration gives a unique possibility for studying experimentally the structure of such fluctuations.

Since the weight P of the passive state increases with increasing energy, the mean interaction cross section of the active state

$$\langle\sigma\rangle_{\text{act}}=\sigma_{hN}/(1-P) \tag{56}$$

increases with increasing energy faster than σ_{hN}. This growth leads to an addition decrease in $\Delta\sigma_A$ with increasing energy:

$$\exp\left[-(1/2)\,\sigma_n\,T(b)\right]\approx\exp\left[-(1/2)\,\sigma_n(Y_0)\,T(b)\right]s^{-\delta_n}, \tag{57}$$

where

$$\delta_n=(1/2)\,T(b)\,d\sigma_n/d\ln s, \tag{58}$$

which gives the correct sign of the observed renormalization of $\alpha_\omega(0)$ and makes it possible[78] to describe quantitatively the data on f^A_{LS} (see Fig. 19).

4. PHOTOPRODUCTION, ELECTROPRODUCTION, AND NEUTRINO REACTIONS ON NUCLEI

4.1. Hadronic properties of photons. By virtue of the the uncertainty principle, a photon of high energy ν can go over into a hadronic system with mass M at distance $\Delta z=\nu/M^2$ from a nucleus, this hadronic system then interacting with the nucleus. Such a picture of the interaction of photons with nuclei was formulated for the first time in 1954 by Pomeranchuk[80] for the example of the photoproduction of $\pi^+\pi^-$ pairs on nuclei. A simple generalization of Ref. 80 leads to the conclusion that there is shadowing in the total cross sections of photoproduction as well.

Pomeranchuk's work was forgotten, and the new history of the problem began a decade later with the papers of Bell[81] and Stodolsky.[14] Adler's relation[82]

$$d^2\sigma_{\nu A}/dq_0\,dq^2=K(q_0,q^2)\,\sigma_{\pi A}(E_\pi=q_0) \tag{59}$$

relates the neutrino scattering cross section in collinear kinematics with momentum transfer q proportional to the neutrino momentum and with $|q^2|\lesssim m_\pi^2$ to the pion interaction cross section [here, $K(q_0,q^2)$ is a kinematic factor]. Bell noted that although $\sigma_{\nu N}$ is negligibly small, it follows from (59) that in this kinematic region $\sigma_{\nu A}\sim\sigma_{\pi A}\sim A^{2/3}$. Stodolsky[14] interpreted (59) as π dominance of neutrino interactions for $|q^2|\lesssim m_\pi^2$, and transferred this interpretation to photons, formulating the vector dominance model for the interaction of high-energy photons.

In the simplest case, the vector dominance model gives (Fig. 20)

$$\sigma_{\gamma^*N}=\Gamma_{\gamma\rho}\frac{m_\rho^2}{m_\rho^2+Q^2}\sigma_{\rho N}\frac{m_\rho^2}{m_\rho^2+Q^2}\Gamma_{\gamma\rho}, \tag{60}$$

where $Q^2=-q^2$. If allowance is made for all the hadronic states V into which the photon can pass, and also nondiagonal transitions $V_1\to V_2$, the generalization of (60) is Gribov's dispersion representation[83]

$$f_{\gamma^* A} = \int \frac{dM^2 M^2}{M^2+Q^2} \frac{dM'^2 M'^2}{M'^2+Q^2} \Gamma_{\gamma^* V} f_{VV'} \Gamma_{\gamma^* V'}. \quad (61)$$

The vertices $\Gamma_{\gamma^* V}$ in (61) can in principle be measured in the annihilation $e^+e^- \to \gamma^* \to V \to$ hadrons.

4.2. Bjorken's paradox and criticism of vector dominance. For hadrons, inelastic shadowing was quantitatively small, and by analogy we restrict ourselves in (61) to the diagonal approximation:

$$\sigma_{\gamma^* A} \sim \int \frac{dM^2 M^4}{(M^2+Q^2)^2} \sigma_{VA} \sigma_{e^+e^-}(M^2). \quad (62)$$

Using

$$\sigma_{e^+e^-}(M^2) \sim \alpha/M^2 \quad (63)$$

and

$$\sigma_{VA} \sim \pi R_A^2, \quad (64)$$

we obtain[84]

$$\sigma_{\gamma^* A} \sim \alpha\pi R_A^2 \ln(\nu/Q^2) \quad (65)$$

in strong contradiction to the scaling behavior

$$\sigma_{\gamma^* A} \sim 1/Q^2. \quad (66)$$

In the framework of the diagonal vector dominance model, the paradox is resolved if[85]

$$\sigma_{VA} \sim 1/M_V^2. \quad (67)$$

This corresponds to the absence of nuclear shadowing in the electroproduction cross section at large Q^2.

Field-theoretical treatment gives a different solution.[85,87] In both the scaling and more modern quantum chromodynamic parton models, the main contribution to the amplitude of photon–hadron scattering is made by diagrams of multiperipheral type (see Fig. 20b).[88] The amplitude f_{qN} in the diagram of Fig. 20b is not small if the quark is near the mass shell: $|p_2|^2 \lesssim m^2$. This condition leads to a pronounced asymmetry of the $q\bar{q}$ pair: $p_1 \approx \nu, p_2 \approx \nu m^2/Q^2$. Moreover, only the wee quark of the pair interacts with the target.[86] The virtual photon interacts as a single quark, and this is different from the vector dominance model, in which the vector states V consist of symmetric $q\bar{q}$ pairs. Allowance for this asymmetry introduces into (62) an additional cutoff $|p_\perp^2| \lesssim m^2$ with respect to the transverse momenta of the quarks of the pair, and this eliminates the contradiction with scaling.[87]

Of course, the amplitude corresponding to the diagram in Fig. 20b can be expressed in the form of a Gribov dispersion integral, which has the form[89]

$$f_{\gamma^* N} \sim \int \frac{dM^2}{Q^2+M^2} \frac{dM'^2}{Q^2+M^2}$$
$$\times \Big\{ \delta(M^2 - M'^2) \Big[M^2 + \theta(M^2-\Lambda^2)\Big(M^2 - \Lambda^2 \ln \frac{eM^2}{\Lambda^2}\Big)$$
$$- \frac{\Lambda^2}{(M^2-M'^2)^2} [M^2\theta(M'^2 - M^2 - \Lambda^2) + M'^2\theta(M^2 - M'^2 - \Lambda^2)\Big]\Big\}, \quad (68)$$

where Λ is the parameter which determines the rate of decrease of the amplitude f_{qN} on departure from the mass shell. The diagonal term in (68) corresponds exactly to the nonscaling cross section (65). The nondiagonal term leads to exactly the same nonscaling contribution, but with opposite sign. After their compensation, (68) naturally corresponds to the scaling cross section $\sim 1/Q^2$.

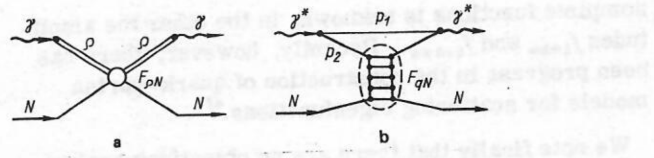

FIG. 20. Amplitude of photon–hadron interaction in the vector dominance model (a) and the multiperipheral diagram of deep inelastic scattering (b).

Thus, the generalized vector dominance model can be saved only at the price of such a compensation, which is extremely artificial from the point of view of hadron physics. The reason is simple: The intermediate states of mass M in electroproduction are not the same as the states of the same mass produced in annihilation, since only the wee parton of the pair succeeds in hadronization during the time ν/Q^2. Thus, the vector dominance model is inapplicable for the description of electroproduction cross sections.

4.3. Shadowing and antishadowing in deep inelastic scattering. In accordance with Sec. 4.2, nuclear shadowing of the virtual photon is similar to shadowing in the interaction cross section of a single quark. In terms of structure functions, shadowing means that for $x = Q^2/2m\nu \lesssim (m_\pi/m_N)A^{-1/3}$

$$F_2^A(x) < AF_2^N(x). \quad (69)$$

At the same time, we have the sum rule[87]

$$\int_0^1 F_2^A(x)\,dx = A\int_0^1 F_2^N(x)\,dx, \quad (70)$$

which means that the total momentum of the quarks in the nucleus is equal to the sum of the momenta of the quarks in the nucleons that constitute the nucleus. For $x \gtrsim m_\pi/m_N$, we have $F_2^A(x) = AF_2^N(x)$ (incoherent scattering), and (69) and (70) are compatible if there is an antishadowing region in which[87]

$$F_2^A(x) > AF_2^N(x). \quad (71)$$

The actual mechanism through which antishadowing occurs—the fusion of the parton clouds of the nucleons in the nucleus—is discussed in detail in Ref. 87. Antishadowing is expected at $x \sim m_\pi/m_N$. The experimental data[15] on the exponent ε in the parametrization $F_2^A(x) = A^{1+\varepsilon}F_2^N(x)$ are shown in Fig. 21. They give a weak indication of antishadowing ($\varepsilon > 0$) at $x \sim 0.1$ and a transition to shadowing ($\varepsilon < 0$) at smaller x.

In Refs. 90 and 91, Brodsky *et al.* discuss the variant of the parton model corresponding to the vector dominance model with the condition (67). In such a model, both shadowing and antishadowing must be absent when $Q^2 \gg m^2$. By introducing enhanced nondiagonal transitions, one can obtain a satisfactory description of electroproduction on nuclei in the framework of the vector dominance model as well.[92] However, the value of such a description is doubtful.

4.4. Connection between photoproduction and electroproduction. In photoproduction with $Q^2 = 0$, appreciable nuclear shadowing of the total cross sections is observed: $\sigma_{\gamma A} \sim A^{0.9}$. The rate at which the shadowing

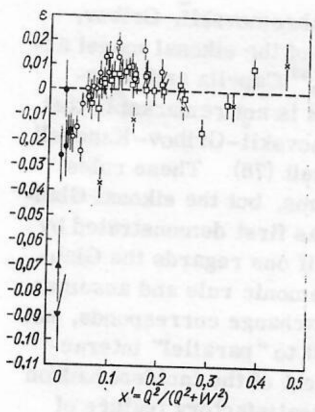

FIG. 21. Compilation[85] of the data of Ref. 15 on the exponent $\varepsilon(x)$ in the parametrization $\sigma_{\gamma * A}(x) = A^{1+\varepsilon(x)}\sigma_{\gamma * N}(x)$.

disappears with increasing Q^2 is remarkable and hitherto unexplained theoretically. An example of the dependence of $A_{eff}/A = \sigma_{\gamma * A}/A\sigma_{\gamma * N}$ on Q^2, measured in the recent experiment of Ref. 93, is shown in Fig. 22. The continuous curve shows the Q^2 dependence given by the vector dominance model.[92] Experimentally, the shadowing disappears over the interval $|Q^2| \leq 0.2$ (GeV/c)^2. A similar or even more rapid dependence of the shadowing on Q^2 is possible in neutrino reactions, where by virtue of Adler's theorem one expects $A_{eff} \approx A^{0.75}$ for $Q^2 \leq m_\pi^2$ (see Sec. 4.1). But in electroproduction it is hard to find a scale that differs strongly from the masses of the vector mesons.

The simple vector dominance model with ρ, ω, and φ dominance overestimates, in strong contradiction to

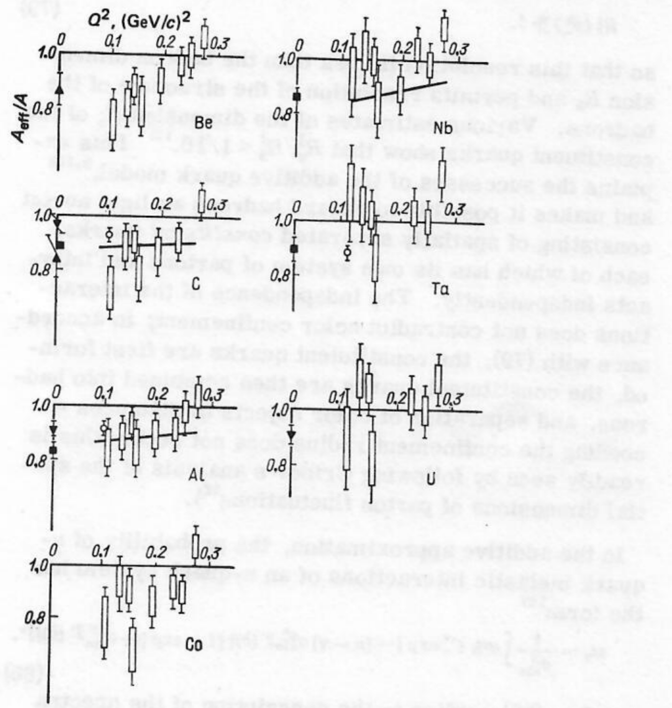

FIG. 22. The Q^2 dependence of the shadowing in the cross section of electroproduction on nuclei.[93] The curves are calculated in accordance with the vector dominance model.[92]

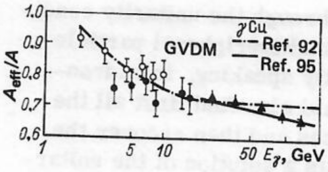

FIG. 23. Comparison of shadowing in the cross section of photoabsorption on the copper nucleus[94] with calculations in accordance with the vector dominance model.

the experiment, the degree of shadowing. True, it reproduces only 80% of the total cross section of photoabsorption on nucleons (see the detailed discussion in the recent reviews of Ref. 85). The contradiction with experiment is not eliminated, though it is reduced, even if one assumes arbitrarily that the remaining 20% of $\sigma_{\gamma N}$ corresponds to nonshadowed "point" component of the photon.[85] There is better agreement with the new data[94] on $\sigma_{\gamma A}$ at energies up to 150 GeV, which indicate an increase in the degree of shadowing with increasing energy (Fig. 23). In Fig. 23, we also show two variants of the description of A_{eff}/A for the Cu nucleus in the framework of vector dominance. Overall, the question of the applicability of the vector dominance model to the quantitative description of fine details in photoproduction remains open.

5. INCLUSIVE PRODUCTION OF PARTICLES ON NUCLEI

5.1. Nuclear cascade equation. The generalization of Eq. (2) with allowance for formation lengths is in integral form (see Sec. 2.4)[96]

$$\frac{dN_s(\varepsilon, t)}{d\varepsilon} = \frac{dN_s(E\to\varepsilon)}{d\varepsilon}\theta(t-l_f(\varepsilon))\exp[-(t-l_f(\varepsilon))] + \int_0^{t-l_f(\varepsilon)} d\tau \exp[-(t-\tau-l_f(\varepsilon))]\int_\varepsilon^E d\omega \frac{dN_s(\tau,\omega)}{d\omega}\frac{dN_s(\omega\to\varepsilon)}{d\varepsilon}. \quad (72)$$

In the limit $l_f(\varepsilon) \ll 1$, Eq. (72) goes over into (2). If the incident particle is composite, then (72) must be applied to the description of the interactions of the constituent particles (quarks in the quark model).[97]

Equation (72) corresponds to a picture of the interaction analogous to the picture of interaction with a thick target. The fast particle incident on the nucleus is absorbed, interacting inelastically with one of the nucleons of the nucleus, after which the secondary particles move through the nucleus, their interactions being describable in accordance with Sec. 2.4 by probability laws.

5.2. How many times does the incident particle interact inelastically with the nucleus? In retaining the analog of Eq. (1) for the total nuclear cross sections [see Eqs. (34)–(36) in Sec. 3.2] and replacing Eq. (2) by Eq. (72), we implicitly assumed that the multiple scattering of the secondary particles in inelastic interactions does not influence the total cross section. The justification for this is the probabilistic nature of the interactions of the secondary particles in processes of production on extended targets (Sec. 2.4). In hadron-hadron collisions, the Regge form of the elastic scattering amplitude is dictated through the unitarity condition by the dominance of the multiperipheral particle production processes. Strictly speaking, in hadron-nucleus interactions one should also find first all the important production processes and then recover the elastic scattering amplitude as a solution of the unitarity condition. This program has not been fully implemented.

There has, however, been wide discussion in the literature of the solution of the problem "the other way around." It reduces to interpreting

$$w_n = (v^n_{abs}/n!)\exp(-v_{abs}) \tag{73}$$

in the expansion

$$w_{abs} = \exp(-v_{abs})[\exp(v_{abs}) - 1] = \sum_n \frac{v^n_{abs}}{n!}\exp(-v_{abs}) \tag{74}$$

of Eq. (1) as the probability of n-fold inelastic interaction of the incident particle. This is incorrect, since by the very definition the first and only inelastic interaction signifies the disappearance of the incident particle. References to the leading-particle effect are unfounded, since, for example, in the case of incident deuterons the final states in inelastic collisions do not contain deuterons at all in any significant number.

We now discuss a more "rigorous" derivation of (73). In the normalization $\mathrm{Im} f_{hN} = \sigma^N_{tot}$, we write (39) in the form of the series

$$\frac{i}{2} f_{hA} = \int d^2b \sum_{m=1}^{\infty} \frac{1}{m!}\left[\frac{i}{2} f_{hN} T(b)\right]^m, \tag{75}$$

identifying the terms of the series with the amplitudes of m-Pomeron exchange (see Fig. 13). The production processes correspond to the discontinuities of f_{hA} across the energy cut, which we write in the form of a series in the discontinuities of F_{hN} (with respect to "cut" Pomerons). The contribution of m-Pomeron exchange with k cut Pomerons has the form

$$\sigma_{m,k} = \frac{1}{m!}\int d^2b\, C^k_m [\sigma^N_{tot} T(b)]^k \left\{\left[\left(\frac{i}{2} f_{hN}\right) + \left(\frac{i}{2} f_{hN}\right)^*\right] T(b)\right\}^{m-k}. \tag{76}$$

Here, σ^N_{tot} corresponds to the cut Pomerons, and $[(i/2)f_{hN}]$ and $[(i/2)f_{hN}]^* = [(i/2)f_{hN}]$ to the Pomerons to the right and left of the cut Pomerons, respectively (scattering in the initial and final states). Substituting $(\sigma^N_{tot})^k = \sum_n C^n_k (\sigma^N_{abs})^n (\sigma^N_{el})^{k-n}$ in (76) and summing over k, we obtain the cross section of n-fold inelastic interaction,

$$\sigma^{in}_{m,n} = \frac{1}{m!}\int d^2b\, C^n_m [\sigma^N_{abs} T(b)]^n [-\sigma^N_{abs} T(b)]^{m-n}, \tag{77}$$

and after summation over m this gives

$$\sigma^{in}_n = \int d^2b \frac{1}{n!}[\sigma^N_{abs} T(b)]^n \exp[-b_{abs} T(b)]. \tag{78}$$

The interaction picture which corresponds to (77) is cascading of the leading particle in the complete absence of intranuclear interactions of all the remaining secondary particles, and (78) is the generalization to the case of nuclei of the expansion (74), which, as we have seen, is erroneous.

Formally, the described derivation of (78) corresponds to the cutting rules of Abromovskiĭ, Gribov, and Kancheli[98] and is the basis of the eikonal model actively developed by Shabel'skiĭ,[99] Capella and Krzywicki,[100,101] and others.[102,103] It is not remarkable that the undoubtedly correct Abramovskii–Gribov–Kancheli rules lead to the incorrect result (78). These rules apply only to nonplanar diagrams, but the eikonal Glauber amplitude is planar, as was first demonstrated by Gribov.[47] One can retain (78) if one regards the Glauber expansion as a purely mnemonic rule and assumes that the amplitude of m-fold exchange corresponds, not to successive interactions, but to "parallel" interaction of the m-particle component of the incident hadron (see Fig. 13c).[100,102,104] An unsatisfactory feature of this is that the structure of the hadron then depends on the target and the interaction amplitudes of the constituent particles must be taken equal to the interaction amplitude of the hadron itself. There is also no direct proof that the planar Gribov–Glauber diagrams are compensated by the nonplanar diagrams as is assumed in Fig. 13c.

5.3. Multiple rescattering and the quark model. If multiple rescattering is to be attributed to the composite structure of the hadron, it is natural to consider the quark model. In deep inelastic scattering at large Q^2, the valence quarks and gluons and also the "sea" $q\bar{q}$ pairs are relevant within the hadron. It is important that the parton wave function of hadrons is additive, i.e., its different parts can be ascribed to valence quarks. The mixing of the partons belonging to different valence quarks is suppressed at large Q^2.

In inelastic hadron–hadron interactions the resolution scale is the $\langle p^2_\perp\rangle \approx m^2_\rho$ of the direct secondary particles. It is found that

$$R^2_h \langle p^2_\perp\rangle \gg 1, \tag{79}$$

so that this resolution is less than the hadron dimension R_h and permits resolution of the structure of the hadrons. Various estimates of the dimension R_q of the constituent quarks show that $R^2_q/R^2_h < 1/10$.[105] This explains the successes of the additive quark model,[9,106] and makes it possible to regard hadrons as light nuclei consisting of spatially separated *constituent* quarks, each of which has its own system of partons and interacts independently. The independence of the interactions does not contradict color confinement; in accordance with (79), the constitutent quarks are first formed, the constitutent quarks are then combined into hadrons, and separation of color objects to distances exceeding the confinement radius does not occur (this is readily seen by following Gribov's analysis of the spatial dimensions of parton fluctuations[36]).

In the additive approximation, the probability of ν-quark inelastic interactions of an n-quark system has the form[107]

$$w_\nu = \frac{1}{\sigma^A_{abs}}\int d^2b\, C^\nu_n \exp[-(n-\nu)\sigma^{qN}_{abs} T(b)]\{1 - \exp[-\sigma^{qN}_{abs} T(b)]\}^\nu, \tag{80}$$

and Eq. (72) applies to the description of the spectra of the particles produced by one quark.[97] Such a picture of the interaction, based on the analogy with a thick target, gives a more adequate description of the physics of the phenomenon than the eikonal model. As we shall see below, it is in good quantitative agreement with the experiments.

Several successive absorptions of the same quark are impossible, but this does not eliminate the problem of cuts. In hadron–nucleon interactions, cuts are necessary, being needed in particular to describe the multiplicity distributions[108] and the two-particle correlations with respect to the rapidities.[109,110] In the additive quark model, these cuts can be attributed partly to a small admixture of many-quark interactions, and partly, apparently, to cuts in the quark–quark amplitudes themselves. The question of the change in the contribution of the latter on the transition to nuclear targets remains open.

Equation (72) corresponds to allowance for the most important nonplanar diagrams (see Sec. 2.4). Because of the large longitudinal dimensions of nuclei, planar diagrams can also make a nonvanishing contribution,[111] and the question of the part they play and their influence on the connection between production processes and the total cross section remains open.[112] In their recent paper of Ref. 113, Levin and Ryskin assert that the growth of the formation lengths changes not only Eq. (2) but also Eqs. (1) and (34)–(36).

5.4. Quark counting rules for beam fragmentation on nuclei. If Eq. (72) is applied directly to hadrons, it leads to the conclusion that the multiplicity of fast particles is independent of the target[1,96]:

$$R_y=\left(\frac{dN_s(\varepsilon,\,t)}{d\varepsilon}\right)_A\Big/\left(\frac{dN_s(E\to\varepsilon)}{d\varepsilon}\right)_N=1,\ \ l_t(\varepsilon)>R_A. \qquad (81)$$

In 1976, it seemed that the experiments agreed with (81).[96] The more detailed data published later showed that in the beam fragmentation region $R_y<1$ at all energies (see Figs. 3 and 4). Such effective absorption of fast particles is natural in the quark model (Fig. 24).[107,114] Let us consider, for example, NA interactions. The fast secondary nucleons are produced by the recombination of two spectator quarks with one

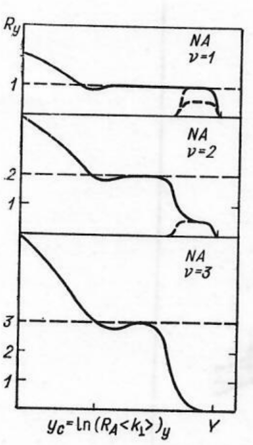

FIG. 24. Qualitative picture of the ratio R_y of the spectra on nuclei and nucleons in one-, two-, and three-quark nucleon–nucleus collisions. The decrease in the contribution of the spectators to the fragmentation spectrum with increasing ν is clearly shown.

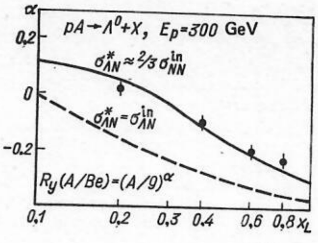

FIG. 25. The exponent $\alpha(x)$ in the parametrization $R_y=A^{\alpha(x)}$ for $p\to\Lambda$ fragmentation.[117] The broken curve is calculated in accordance with the eikonal model, and the continuous curve in accordance with the additive quark model.[97]

of the newly produced quarks, and for the production of a fast pion it is sufficient to have one spectator quark in the final state. Therefore, for fast fragments

$$R_y^{N\to N}\approx w_1;\ R_y^{N\to\pi}\approx w_1+aw_2. \qquad (82)$$

In all recombination models,[107,115,116] one obtains $a\approx 1$. In a rough approximation, (82) says that the effective cross section σ^*_{hN} for absorption in the nucleus of a fast beam fragment is given by the cross section for the absorption of the spectator quarks that recombine into the given fragment[97,114]:

$$\left.\begin{aligned}&\sigma^*_{NN}\approx\sigma^*_{\Lambda N}\approx(2/3)\,\sigma_{NN};\\&\sigma^*_{\pi N}\approx(1/3)\,\sigma_{NN}\approx(1/2)\,\sigma_{\pi N}.\end{aligned}\right\} \qquad (83)$$

In pion beams and for secondary pions and baryons, $\sigma^*_{\pi N}\approx\sigma^*_{NN}\approx(1/2)\sigma_{\pi N}$. The experimentally observed[117] absorption of Λ^0 hyperons in $p\to\Lambda^0$ fragmentation is indeed weaker than what is calculated with the physical value of $\sigma^{\Lambda^0N}_{\rm abs}$, and agrees well with the result given by (83) (Fig. 25).[97] The values of $\sigma^*_{\pi N}$ found in Ref. 118 by analyzing $p\to\pi$ fragmentation also agree with (83) (Fig. 26). The experimental data on neutron fragmentation[119] also agree with (82) and (83), but their accuracy is not high (Fig. 27).

In comparison with the experiments, it must be borne in mind that (82) and (83) apply to direct particles. Decay products of resonances will have σ^*_{hN} equal to the absorption cross section of the parent resonance.[114] This may explain why the A dependences of $p\to K^0$ and $p\to\Lambda^0$ fragmentation are found experimentally to be nearly the same.[120] In the experiment of Ref. 120, it was also found that the absorption of $\overline{\Lambda}^0$ hyperons is close to the absorption of K^0 and Λ^0. This fact is very interesting and requires explanation. Its significance for the quark model is obscure, since this model does not apply to rare processes such as the production of fast $\overline{\Lambda}^0$ hyperons, which do not contain valence quarks. The rules (82) and (83) correspond to additive interaction of the quarks. It is noted

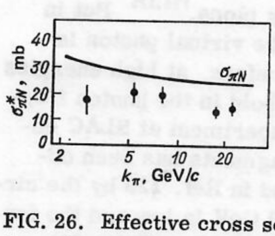

FIG. 26. Effective cross section $\sigma^*_{\pi N}$ for interaction of secondary pions in the nucleus with $x\approx 1$.[118]

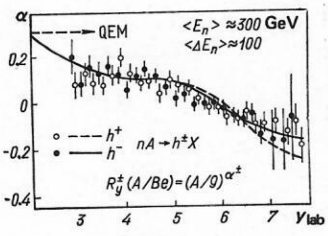

FIG. 27. Data on fragmentation of neutrons into charged particles on nuclei.[119] The curves are calculated in accordance with the additive quark model.[97] The eikonal model corresponds to the central region $\alpha_\pm = 0.3$.

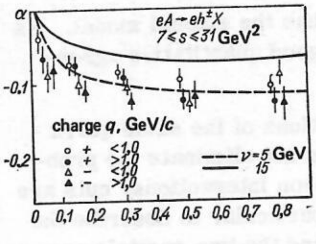

FIG. 28. The exponent $\alpha(z)$ in the parametrization $R_y = A^{\alpha(z)}$ of the spectrum in the region of fragmentation of the virtual photon (z is the Feynman variable in the center-of-mass system).[126] The curves are calculations of $\alpha(z)$ in the quark model with allowance for the formation lengths.[125]

in Ref. 114 that if the quarks have a collective behavior (for example, if the hadrons interact through a gluon component common to all the valence quarks[121]), the A dependences of the fragmentation spectra will be universal for all secondary particles. For better understanding of the fragmentation mechanisms, an experimental study of a large number of fragmentation processes in both baryon and meson beams is needed.

Anisovich *et al.*[105,107] assume that the constitutent quarks have a narrow distribution with respect to the fraction x of the hadron momentum with a sharp peak at $x \approx 1/3$, and therefore (82) applies only to $x_\pi \approx 1/3$ and $x_{p,\Lambda^0} \approx 2/3$. But experimentally the points $x_\pi = 1/3$ and $x_{p,\Lambda^0} = 2/3$ are in no way distinguished in the spectra. The approach used in Ref. 97 appears more realistic; it takes into account the observed profile of the spectra in the smooth shift of the spectra to smaller values of x with decreasing number of spectator quarks in the fragment hadron. Thus, in accordance with Ref. 97 one obtains for $R_y(p \to p)$ a dependence of the form $[y = \ln(\varepsilon/\mu_\pi), Y = \ln(E/\mu_\pi)]$

$$R_y(p \to p) = \{w_1[1 + (Y - y)] + w_2[(Y - y) + (Y - y)^2/2] + w_3(Y - y)^2\}/2[1 + (Y - y)], \quad (84)$$

which agrees well with the experiment (see Fig. 30 later). The dependence (84) means that $R_y(p \to p)$ for all nuclei at nearly equal values of $\Delta = (Y - y) \cong 1.0\text{–}1.5$, which agrees well with the experiment.[122]

In Ref. 97, Eq. (84) is obtained with allowance for conservation of the baryon charge. One cannot, as is done, for example, in Ref. 123, assume that the spectator quarks and the quarks which have interacted fragment independently into the final hadrons, since this leads to a contradiction with the conservation law of, for example, baryon charge.

To the extent that the vector dominance model is valid, the photon behaves as a two-quark system (see Sec. 4.4), and photoproduction of hadrons on nuclei must be similar to production by pions.[124,125] But in the case of electroproduction, the virtual photon is similar to a single quark. Therefore, at high energies the Kancheli relation (81) must hold in the photon fragmentation region.[124,125] In an experiment at SLAC energies, absorption of photon fragments has been observed,[126] and this was explained in Ref. 125 by the circumstance that the energy $\nu \approx 10$ GeV is low and the formation lengths are correspondingly short (Fig. 28). With increasing energy, the absorption must disappear, and this would be the most unambiguous confirmation of growth of the formation lengths. We note here that growth of the formation lengths makes the measurement of the cross section of quark–nucleon elastic scattering discussed by Bialas[127] impossible.

5.5. *Central pionization region and mean multiplicities*. In the central region (Fig. 29) $R_y = \langle\nu\rangle = \sum_\nu \nu w_\nu$, which can also be written in the form[128]

$$R_y = \bar{\nu}_{hA}/\bar{\nu}_{qA}. \quad (85)$$

In the limit of a black nucleus, $\sigma^A_{abs} = \pi R_A^2$ for all hadrons and (85) goes over into Anisovich's relation[10]

$$R_y = \nu_{max} = \begin{cases} 3, & pA \\ 2, & \pi A. \end{cases} \quad (86)$$

The experimental data[22,23,119] give an indication of a plateau at energies above 200–400 GeV (see Figs. 4 and 27). These data can be well described by the quark

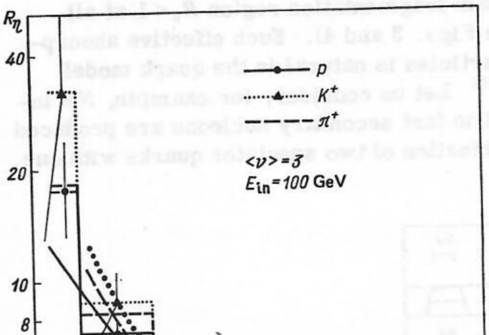

FIG. 29. Dependence of the ratio R_η on the rapidity for nuclei with $\bar{\nu} = 3$.[16] The eikonal model corresponds to $R = \bar{\nu} = 3$. The heights of the plateau expected in the additive quark model are indicated by the horizontal arrows.

model, but do not make it possible to verify (85) reliably. As we have seen above, the beam fragmentation region occupies a rapidity interval of order $2\Delta \approx 3$. At $\mu_0^2 = 0.7$ GeV^2, the cascade region corresponds to rapidities $y \lesssim y_{cas} = \ln(R_A \mu_0^2/\langle k_\perp \rangle) \approx 3$–$4$.[97] Therefore, at energies 200–400 GeV no room remains for the plateau. The conclusion of Bialas *et al.*[128,129] to the effect that the data of Ref. 16 make it possible to verify (85) and even to determine in accordance with (85) the number of constituent quarks in the nucleon is unfounded, since at $E = 100$ GeV there are no indications at all of a plateau in R_η (see Fig. 29).

The cascade and fragmentation regions give contributions to the mean multiplicity that are independent of the energy, so that at high energies

$$\langle N_s(E)\rangle_A = \langle \nu \rangle \langle N_s(E)\rangle_N + \text{const.} \tag{87}$$

The analysis made in Refs. 130 and 131 of the dependence of $\langle N_S(E)\rangle_A$ on $\langle N_S(E)\rangle_N$ shows that the slope in (87) is closer to $\bar{\nu}$, which is obtained from (8), than to $\langle \nu \rangle$ (Fig. 30). Shabelsky and Shekhter[131] interpret this as proof that each quark interacts inelastically $\bar{\nu}_{qA}$ times.

This conclusion cannot be regarded as reliable, since the data on $\langle N_s\rangle_A$ used in Refs. 130 and 131 refer to energies at which there is no plateau in R_y and (87) is invalid. If the slope in (87) is equal to $\bar{\nu}$, then the ratio $R = \langle N_s\rangle_A / \langle N_s\rangle_N$, which at energy 100 GeV has experimentally[16] the dependence $R = 0.4 + 0.67\bar{\nu}$, must increase with increasing energy and tend to $R = \bar{\nu}$. But if the slope in (87) is $\langle \nu \rangle$, then R must decrease slowly with increasing energy,[97] since $R > \langle \nu \rangle$ from (9). The most complete data at energy above accelerator energies were obtained in cosmic-ray experiments at the Tskhra-Tskharo facility[132] (Fig. 31). They indicate a slow decrease or constancy of R with increasing energy rather than a growth.

5.6. Scaling with respect to $\bar{\nu}$ of the inclusive spectra and the multiplicities. Experimentally, R and R_η have approximately the same dependence on $\bar{\nu}$ for all hadrons (see Figs. 2 and 29).[16] This is widely interpreted (see Refs. 16, 100, 101, and 103) as proof that only the primary particle interacts within the nucleus. Approximate $\bar{\nu}$ scaling also holds in the quark model, since the mean number of interacting quarks $\langle \nu \rangle$ is a parameter with significance similar to $\bar{\nu}$ (see Figs. 2

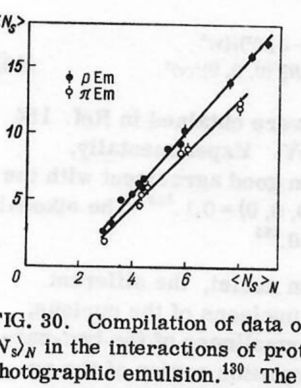

FIG. 30. Compilation of data on the dependence of $\langle N_s\rangle$ on $\langle N_s\rangle_N$ in the interactions of protons and pions with nuclei in photographic emulsion.[130] The straight lines correspond to the asymptotic ratio $R = 0.91\bar{\nu}$.

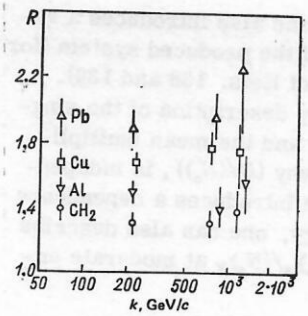

FIG. 31. High-energy behavior of $R = \langle N_s\rangle_A / \langle N_s\rangle_N$ in accordance with the data from the Tskhra–Tskharo facility.[132]

and 29).[97] In electroproduction, for which $\bar{\nu} \equiv 1$, $\bar{\nu}$ scaling would mean the complete absence of nuclear effects, whereas in the quark model cascade effects give both $R > 1$ and $R_\eta \geqslant 1$, in good agreement with experiment (see Figs. 5 and 32).

.7. Experiments and the eikonal model. In the eikonal model, $R_\eta = \bar{\nu}$ in the central region and in the fragmentation region of the nucleus, i. e., in (10) the slope is $S(\eta) \equiv 1$ (Refs. 99, 101, 102, 103, and 133). The same behavior of R_η is predicted in the quark model with eikonal scattering of the quarks.[131] Calculations in accordance with the eikonal model give a good description of the mean multiplicities $\langle N_s\rangle_A$ and even of multiplicity distributions,[99,101] but $R_\eta = \bar{\nu}$ has not been observed in any of the experiments (see Figs. 3, 4, 27, and 29). The agreement in $\langle N_s\rangle_A$ between the experiment and a model that does not describe the inclusive spectra cannot be given any significance.

5.8. Nonmultiperipheral models. A typical representative of this class of models is the coherent tube model, which has been frequently rediscovered in slightly different variants during the last 25 years (see Refs. 124, 135, 136, and 137). In the model, it is assumed that the incident particle interacts simultaneously with all N_T nucleons in a tube of cross-sectional area $S \approx \sigma_{hN}$. The interaction with the tube is assumed to be equivalent to interaction with a nucleon at energy

$$E \to EN_T. \tag{88}$$

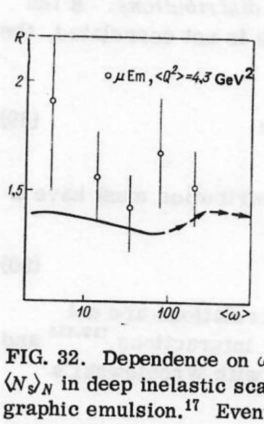

FIG. 32. Dependence on $\omega = 2m\nu/Q^2$ of the ratio $R = \langle N_s\rangle_{Em} / \langle N_s\rangle_N$ in deep inelastic scattering of muons on nuclei in photographic emulsion.[17] Events with three or more ($N_h \geqslant 3$) particles with $v/c < 0.7$ are selected. The curve is calculated in accordance with Ref. 125. The arrows indicate the expected growth of R on the transition from incoherent to diffraction scattering.

In the hydrodynamic variant, one also introduces a renormalization of the volume of the produced system (for more detail, see the reviews of Refs. 138 and 139). The model gives a satisfactory description of the single-particle inclusive spectra and the mean multiplicities and explains naturally why $(D/\langle N_s\rangle)_A$ is independent of the target.[138,139] If one introduces a dependence of the tube length on the energy, one can also describe the growth in the ratio $R=\langle N_s\rangle_A/\langle N_s\rangle_N$ at moderate energies.[140]

However, the model does not stand up to a more detailed comparison with the experiments. For example, (88) requires a broadening of the rapidity distributions with increasing N_T. But experimentally they shrink.[141]

Since $(D/\langle N_s\rangle)_N$ does not depend on the energy, (88) means[142] that $(D/\langle N_s\rangle)_A$ is independent of N_T. This also applies to the rapidity correlations $R_2^A(0,0)$ in the central region. The data in Figs. 7 and 9 on $(D/\langle N_s\rangle)_A$ and $R_2^A(0,0)$ $(N_T\sim N_{p,s})$ unambiguously refute any variant of the tube model.

There has also been wide discussion in the literature of the cluster model of Kalinkin and Shmonin.[143] That it contradicts the experiments was convicingly demonstrated in Ref. 144. One may add that the equations in Ref. 143 for excitation of a cluster contradict the energy conservation law. This error was not noted in Ref. 144.

In discussions of the experimental data, reference is frequently made to the model of Gottfried.[145] Its predictions do not agree with the experiments for either the inclusive spectra or the mean multiplicities. However, Gottfried's paper in Ref. 145 and, in particular, his review in Ref. 146 played an important part in attracting attention to interaction with nuclei at high energies. An example of comparison of Gottfried's model with the experiments and one further non-field-theoretical model, proposed by Fishbane and Trefil,[147] can be found in Ref. 148.

6. CORRELATION PHENOMENA IN MULTIPLE PRODUCTION ON NUCLEI

6.1. Connection between the two-particle rapidity correlations and the multiplicity distributions. If the production of secondary particles is not correlated, the rapidity correlation function

$$R_2(y_1,y_2)=\left(\frac{d^2N_s}{dy_1dy_2}\right)\Big/\left(\frac{dN_s}{dy_1}\frac{dN_s}{dy_2}\right)-1 \tag{89}$$

vanishes, and the multiplicity distribution must have a Poisson form with

$$D=(\langle N_s^2\rangle-\langle N_s\rangle^2)^{1/2}=\langle N_s\rangle^{1/2}. \tag{90}$$

The experimentally observed correlations are not small, $R_{ch,ch}(0,0)\approx 0.6$ for all hN interactions,[149,150] and D depends on $\langle N_s\rangle$ in accordance with Wroblewski's law[151]

$$D=a(\langle N_s\rangle-b) \tag{91}$$

for all interactions, $a\approx 0.5-0.6$, and b is small: $b\approx 0.5-1$.[149,150]

Analysis of the dependence of $R_2(y_1,y_2)$ on the rapidities and the initial energy shows that $R_2(y_1,y_2)$ can be divided into short-range correlations $R_S(y_1,y_2)$ with range $\Delta=|y_1-y_2|\approx 2$ and long-range correlations $R_L(y_1,y_2)$, which do not (in the central region) depend on the difference between the rapidities. Experimentally, $R_S(0,0)\approx R_L(0,0)\approx 0.3$ (for more detail, see the reviews in Refs. 149 and 150). Near the boundary of the kinematic region, $R_2(y_1,y_2)<0$, which is a consequence of energy conservation.

It is easy to derive relations[152] connecting a and b in (91) to $R_L(0,0)$ and $R_S(0,0)$:

$$a=(R_L(0,0))^{1/2}; \tag{92}$$

$$b=(2\delta R_L(0,0)=1-C\Delta R_S(0,0))/2R_L(0,0). \tag{93}$$

In (93), $\delta\approx 2\ln 2$, and allowance has been made for the vanishing of the two-particle spectrum near the boundary of the kinematic region; $C=(dN_s/dy)_{y=0}$. The correct order of magnitude of both the slope a and the intercept b follows from (92) and (93).

6.2. Multiparticle correlations for production on nuclei. In the central region of rapidities, the following relations hold[152,153]:

$$R_2^A(0,0)=(\langle\nu^2\rangle-\langle\nu\rangle^2)/\langle\nu\rangle^2+R_2^N(0,0)/\langle\nu\rangle; \tag{94}$$

$$(D/\langle N_s\rangle)_A^2=(\langle\nu^2\rangle-\langle\nu\rangle^2)/\langle\nu\rangle^2+(D/\langle N_s\rangle)_N^2/\langle\nu\rangle, \tag{95}$$

and these relate the correlations and the Wroblewski ratios for interactions with nuclei to the distributions with respect to the number ν of interacting quarks in the quark model or the number of interactions of the incident particle in the eikonal model. The dependences of $R_2^A(0,0)$ and $(D/\langle N_s\rangle)_A$ on the atomic number of the nucleus that follow from (94) and (95) are fairly weak (see Fig. 7). This is due to the mixing of peripheral and central collisions [the term $(\langle\nu^2\rangle-\langle\nu\rangle^2)/\langle\nu\rangle^2$ in (94) and (95)]. Calculations in accordance with the additive quark model agree well with the existing data. But there may also be corrections to the simple formulas (94) and (95) due to energy conservation. For example, in (95) one should take into account the decrease in the multiplicity in the beam fragmentation region. In the additive quark model, this decrease is compensated by the production of particles in the cascade region. In the eikonal model, in which cascades are ignored, the values of $(D/\langle N_s\rangle)_A$ can be lower than those given by Eq. (95).[101]

The three-particle correlation function for nuclei is[153]

$$R_3^A(0,0,0)=(\langle\nu^3\rangle-3\langle\nu^2\rangle\langle\nu\rangle+2\langle\nu\rangle^3)/\langle\nu\rangle^3 + 3((\langle\nu^2\rangle-\langle\nu\rangle^2)/\langle\nu\rangle^3)R_2^N(0,0)+R_3^N(0,0,0)/\langle\nu\rangle^2. \tag{96}$$

The first data on $R_3^A(0,0,0)$ were obtained in Ref. 154 for π^-C interactions at 40 GeV. Experimentally, $R^{---}(0,0,0)=-0.02\pm 0.04$ is in good agreement with the quark model, in which $R^{---}(0,0,0)\approx 0.1$.[153] The eikonal model gives $R^{---}(0,0,0)\approx 0.40$.[153]

In the case of production on nuclei, the different quarks interact on different nucleons of the nucleus, and therefore the kinematic smallness of the two-particle spectrum in the fragmentation region of the nucleus is absent. In particular, in the fragmentation region of the nucleus $R_2^A(y_1, y_2)$ is positive.[25,155] Further, in (93) it is necessary to replace δ by $\delta/2$, which decreases b. When all the corrections are taken into account, Eq. (93) gives $b_{Em} = b_N - 0.4$, in good agreement with the data in the review of Ref. 24.

6.3. Connection between production of fast particles and fragmentation of the nucleus. Hitherto, we have discussed the A dependences of the spectra and the correlations. They are inconvenient in that even for heavy nuclei the fraction of peripheral interactions is large, and the check on the number of intranuclear interactions by the size of the nuclei is not effective. A more direct measure of the number N_{int} of intranuclear interactions is the number of fast protons knocked out of the nucleus or, following the tradition of emulsion experiments, the number N_g of gray tracks, which correspond to charged particles with velocities $0.3 \leq v/c \leq 0.7$. Semiempirical relations connecting N_{int} to N_g were obtained in the framework of the quark model (here, N_{int} includes both the number ν of interacting quarks of the incident particle as well as the number of cascade interactions) in Ref. 152 and in the framework of the eikonal model (here, N_{int} is equal to the number ν of inelastic interactions of the incident particle) in Ref. 156.

In the additive quark model, events with $N_g \gg \langle N_g \rangle$ correspond to the interaction of all constituent quarks: $\nu = \nu_{max}$. In the relative spectra of R_η a plateau is predicted with height

$$R_\eta = \nu_{max}, \tag{97}$$

which does not depend on the atomic number of the nucleus. As was already noted in Sec. 5.5, very high energies are needed to observe the plateau. The calculations in accordance with the quark model agree well with the experiments, but for the direct verification of (97) energies of 400 GeV are insufficient (see Fig. 8). In the eikonal model[99-103] or in the model with eikonal rescattering of the quarks,[131] the values of ν are not bounded, and R_η must increase with increasing N_g unboundedly.

It is clear from Fig. 8 that the limiting form of the spectra has not yet been established. The attempts of Ref. 157 to guess the limiting properties of the N_g dependences of the spectra by extrapolating the low-energy data are not reliable. Data on the spectra at higher energies are very necessary.

The additive quark model, which gives a good description of the N_g dependences of the spectra, also gives the correct N_g dependence of the mean multiplicities.[152] A satisfactory description of the N_g dependences of $\langle N_s \rangle_A$ is also obtained for the connection between N_g and ν given by the eikonal model if one takes $\langle N_s \rangle_A = 0.5(1+\nu)\langle N_s \rangle_N$.[156] But if one adopts, which would be more correct, the empirical dependence (9), then for $N_g \gg \langle N_g \rangle$ the eikonal model gives a high value of the multiplicity $\langle N_s \rangle_A$ in contradiction to the experiments. In photoproduction, the N_g dependences of the spectra must be close to the dependences in πA interactions, while in electroproduction it is predicted that the spectra are independent of N_g in the beam fragmentation region.[125] Such behavior of the inclusive spectrum was observed in an experiment on deep inelastic scattering of muons on photographic-emulsion nuclei,[17] but the statistics (86 events) was too low for unambiguous conclusions. In electroproduction, the excess of $\langle N_s \rangle_A$ over $\langle N_s \rangle_N$ is due solely to cascades, so that the dependences of R on N_g and on the atomic number of the nucleus must be much weaker than for hadrons.[125]

6.4. Dependences of the correlation between the fast particles on the fragmentation of the nucleus. For $N_g \gg \langle N_g \rangle$, it follows from (94) and (95) that

$$R_2^A(0, 0) = R_2^N(0, 0)/\nu_{max}; \tag{98}$$

$$(D/\langle N_s \rangle)_A = (D/\langle N_s \rangle)_N / \sqrt{\nu_{max}}. \tag{99}$$

As in (97), the limiting values in (98) and (99) do not depend on the atomic number of the target nucleus. The dependences of $R_2^A(0, 0)$ and $(D/\langle N_s \rangle_A$ on N_g obtained from (94) and (95) [in Fig. 7, N_p is the uncorrected (for the efficiency) number of gray prongs observed in the experiment of Ref. 21, and in Table I, Q is the observed charge of the secondary particles in π^-C interactions, $Q \approx N_g - 1$] agree well with those observed experimentally (see Figs. 7 and 9). The data in Fig. 7 indicate that in the limit $N_p \to \infty$ the ratio $(D/\langle N_s \rangle)_A$ tends to a limiting value of $(D/\langle N_s \rangle)_A$ for protons or antiprotons is lower than for pions $[(D/\langle N_s \rangle)_{pN} \approx (D/\langle N_s \rangle)_{NN}]$, in excellent agreement with the experimental data in Fig. 7. At large Q, the correlations in the π^-C interactions also decrease by a factor of about two (see Table I). The azimuthal correlations are particularly interesting in that the dispersion of the distributions with respect to ν does not contribute to the azimuthal asymmetry

$$A_\varphi = (N(\varphi < 90°) - N(\varphi > 90°))/(N(\varphi < 90°) + N(\varphi > 90°)) \tag{100}$$

(Refs. 153 and 158). Therefore

$$A_\varphi^A = A_\varphi^N / \langle \nu \rangle. \tag{101}$$

At large Q, the value of A_φ^A in π^-C interactions decreases by a factor of approximately two, which agrees well with the quark model[152,154] (it is difficult to use the data on $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs because of the uncertainties associated with the strong influence of the identity of the particles).

With increasing N_g, the dispersion of the ν distribution must obviously decrease in the eikonal model as

TABLE I. Dependences of the correlations of the rapidities and the azimuthal asymmetry for different charge combinations on the charge Q of the final particles in π^{-12}C interactions.

Correlation	Q: −1	Q: 0	Q: 1
R_2^{++} (0, 0)	0.22±0.07	0.03±0.05	0.03±0.05
R_2^{--} (0, 0)	0.34±0.06	0.31±0.06	0.25±0.08
R_2^{+-} (0, 0)	0.89±0.09	0.54±0.07	0.43±0.08
R_2^{cc} (0, 0)	0.63±0.05	0.35±0.04	0.28±0.04

	Q: 0	Q: 1	Q: 2
$A_\varphi^{+-} \times 10$	0.89±0.03	0.64±0.04	0.49±0.05

well. Since the values of ν are not bounded above, the eikonal model should not have limiting values for either $R_2^A(0, 0)$ or $(D/\langle N_s \rangle)_A$ when $N_g \gg \langle N_g \rangle$. The relationship between ν and N_g proposed in Ref. 156 corresponds to values $\nu \gtrsim 6$ for $N_g \gtrsim 8$ or $N_p \gtrsim 6$. At these values of ν, (98) and (99) strongly contradict the experimental data given in Figs. 7 and 9.

While the eikonal model gives a decrease of the correlations with increasing N_g that is too rapid, in the coherent-tube model the correlations should not depend on N_g at all. As was noted in Sec. 5.8, the experimentally observed decrease of $(D/\langle N_s \rangle)_A$ and $R_2^A(0, 0)$ with increasing N_g unambiguously refutes all variants of the tube model and the hydrodynamic model.

In deep inelastic scattering of leptons on nuclei $\nu \equiv 1$, so that in the central region the correlations should not depend on N_g.[125] In this, deep inelastic scattering differs from photoproduction which must be similar to πA interactions.

At the start of the cascade region, an effective boundary of the kinematic region arises. In particular, the minimum in R_y (see Fig. 24) is associated with this. The effect of the boundary of the kinematic region leads to a negative contribution to the correlation function in the region of the minimum.[124] In accordance with the estimates made in Ref. 48, $\delta R_2(y_1, y_2) \approx 0.5$–$0.15$ in events with rescattering on deuterons. For heavier nuclei, estimates of the effecthave not been made.

6.5. *Associated multiplicities*. In accordance with (82), a contribution to $pA \to p + X$ fragmentation into a fast proton with $x \sim 1$ is made by only the single-quark interactions,

$$\langle \nu\,(p \to p\,(x \sim 1))\rangle \approx 1. \tag{102}$$

In $pA \to \pi + X$ fragmentation, we have from (82)

$$\langle \nu\,(p \to \pi\,(x \sim 1))\rangle \approx (w_1 + 2aw_2)/(w_1 + aw_2) > 1. \tag{103}$$

This means that the multiplicity $\langle N_X \rangle$ of the particles X associated with the fragmentation $p \to \pi(x \sim 1)$ is higher than $\langle N_X \rangle$ in the fragmentation $p \to p(x \sim 1)$.[159] For lead nuclei, $w_1 \approx w_2$ and in accordance with (103) we have $\langle N_X \rangle_{p\to\pi} \approx 1.5 \langle N_X \rangle_{p\to p}$.

At realistic energies, the production of antiprotons is unimportant. Therefore, slow protons are produced mainly in three-quark interactions:

$$\langle \nu\,(p \to p\,(x \lesssim (0.1 - 0.05)))\rangle > \approx 3. \tag{104}$$

For pions on lead nuclei,

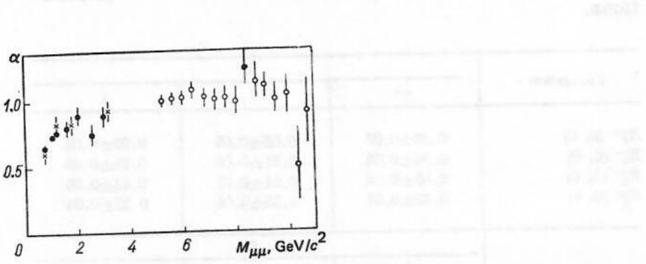

FIG. 33. Compilation made in Ref. 166 of the data of Ref. 167 on the exponent $\alpha(M_{\mu\mu})$ for the cross section for the production of muon pairs by protons.

$$\langle \gamma\,(p \to \pi\,(x \lesssim (0.1 - 0.05)))\rangle \approx 2, \tag{105}$$

i.e., the inequality between $\langle N_X \rangle_{p\to p}$ and $\langle N_X \rangle_{p\to\pi}$ changes sign as x decreases.[159]

In $\pi \to p$ and $\pi \to \pi$ fragmentation, the numbers of spectator quarks and interacting quarks are equal, and the associated multiplicities $\langle N_X \rangle_{\pi\to\pi}$ and $\langle N_X \rangle_{p\to\pi}$ are expected to be close to each other.[159]

7. HARD PROCESSES IN HADRON-NUCLEUS INTERACTIONS

7.1. *Production of lepton pairs and ψ particles*. The experimentally observed additivity $\sigma_{\gamma^* A} = A\sigma_{\gamma^* N}$ of the deep inelastic scattering cross sections (Secs. 4.3 and 4.4) indicates additivity, at least for $x \gtrsim 0.05$, of the densities of the partons in the nucleons of the nucleus. The hard processes which take place at short distances are described by the incoherent interaction of partons in the colliding hadrons, namely, by annihilation $q\bar{q} \to l^+ l^-$ of quarks–partons in the production of massive lepton pairs[160] (for a review of the present situation with regard to this question, see Ref. 161), quark fusion $q\bar{q} \to \psi$ or gluon fusion $gg \to \varkappa_1$ with subsequent decay $\varkappa_1 \to \psi\gamma$ in the production of ψ particles,[162] and scattering of quarks–partons or gluons through a large angle in the production of particles with large transverse momenta.[163] It is important for us that the cross sections of the hard processes are proportional to the product of the densities of the appropriate partons in the colliding hadrons. The additivity of the densities of the partons in the nuclear nucleons means that the cross sections of the hard processes must be proportional to A^1.[164,165]

In the laboratory system, the A^1 law means that the fast partons of the incident particle pass through the nucleus without strong absorption. This is satisfied at momenta of the pair above $(5-10) \times A^{1/3}$ GeV. A compilation[166] of the data of Ref. 167 on the A dependence in pA interactions is shown in Fig. 33. It can be seen that the A^1 law is established at pair masses $M_{\mu\mu} \gtrsim 4$ GeV/c^2. The A^1 law is even regarded as a criterion for applicability of the Drell–Yan approach.[161] Actually, $M_{\mu\mu} \gtrsim 4$ GeV/c^2 is the currently generally accepted limit of the Drell–Yan continuum, since Drell–Yan scaling $d\sigma/dM_{ll} = M_{ll}^{-3} f\,(M_{ll}^2/s)$ in the production cross sections itself begins to be satisfied at the same masses. A dependence of α on $M_{\mu\mu}$ close to that given in Fig. 33 is also observed in πA collisions,[168] for which $\alpha = 1.03 \pm 0.03$ according to the latest and most accurate measurements at CERN for heavy pairs.[169]

Hard rescattering of quarks before their annihilation can lead to a certain broadening of the distribution with respect to the transverse momenta of the lepton pair. Estimates in the framework of quantum chromodynamics show that the possible effect is negligibly small.[170] The nature of the transition from $\alpha \approx 2/3$ at $M_{\mu\mu} \approx m_\rho$ to $\alpha = 1$ at $M_{\mu\mu} \gtrsim 4$ GeV/c^2, like the production mechanism of such pairs, is not yet understood.

In the production of ψ particles, α is near k but still less than unity: $\alpha_\psi = 0.927 \pm 0.030$.[171,172] It is possible that the reason for this is that the parton mechanisms of production of the ψ particles do not apply well, since $\alpha(M_{\mu\mu}=M_\psi)<1$ for lepton pairs as well. There is another reason. For particles containing heavy quarks, the characteristic scale of the virtual masses is the mass of the heavy quarks (see, for example, Ref. 8). Therefore, for the ψ particles the formation lengths are short, $l_t \sim k/M_\psi^2$, and their absorption in nuclear matter may make a certain contribution to the decrease in α_ψ. Various estimates indicate that the cross section of ψN interaction is small, of the order of one or two millibarns.[173]

7.2. *Production of particles with large transverse momenta.* The situation with regard to the production of particles with large $p_\perp$ is very confused. There is no consistent theoretical interpretation of the observed cross sections in the region of $p_\perp$ not exceeding 4–5 GeV/c. Calculations in the framework of quantum chromodynamics can be reconciled with the experiments beginning at $p_\perp \approx 3$–4 GeV/c only by the introduction of an appreciable initial transverse momentum of the partons in the hadrons, which increases the cross section by more than an order of magnitude.[163] In this situation, the interpretation of the experimental data on the production of particles with large $p_\perp$ is very difficult. The data themselves are intriguing: In the production of massive muon pairs $\alpha(M_{\mu\mu} \gtrsim 4\ \mathrm{GeV}/c^2)=1$, whereas in the production of particles with large transverse momenta the exponent α in the parametrization $d\sigma/d^3p \sim A^{\alpha}(p_\perp)$ increases with increasing $p_\perp$ for all particles (Fig. 34).[174] This was first observed by Cronin's group as early as 1974, has been confirmed subsequently in a number of experiments,[175] and has not yet found a satisfactory theoretical explanation. Indications of an even more rapid growth of $\alpha(p_\perp)$ were obtained in experiments on the production of, not individual particles, but jets of particles with large $p_\perp$.[176]

The fact that $\alpha(M_{\mu\mu})=1$ in the production of massive lepton pairs immediately rules out all explanations of the Cronin effect based on the hypothesis[177] that, for one reason or another, the density of hard partons in the nucleus increases faster than A. There remain the models that use the mechanism of multiple hard scattering of the partons. One of the possibilities is double hard scattering of a parton in the nucleus with subsequent fragmentation into the observed hadron. In this case, the differential cross section would have the form

$$d\sigma = A d\sigma^{(1)} + A^{4/3} d\sigma^{(2)} + \ldots, \tag{106}$$

and the Cronin effect could be explained if $d\sigma^{(2)} \approx d\sigma^{(1)}$.

The hypothesis can be tested in the production of pairs of particles with large $p_\perp$. The observation is that the term $\sim d\sigma^{(1)}$ in (106) corresponds to the production of symmetric pairs, while the term $\sim d\sigma^{(2)}$ corresponds to the production of two particles (jets) with transverse momentum $\sim p_\perp/2$ in the arm opposite to the trigger with large $p_\perp$.[97] Only the term $\sim d\sigma^{(1)}$ makes a contribution to the production of pairs of particles with $p_{\perp_1} \gg p_{\perp_2}$, i.e., for symmetric pairs

$$\alpha(p_{\perp_1} \gg p_{\perp_2}) \approx 1. \tag{107}$$

In the production of asymmetric pairs, the contribution of double scattering is predominant, so that

$$\alpha(p_{\perp_1} \gg p_{\perp_2}) \approx 4/3. \tag{108}$$

Such a dependence of the pair exponent was indeed observed in the experiment of Lederman's group,[178] but it was not confirmed in the experiment described in Ref. 179. True, in the second experiment the investigated $p_\perp$ were lower than those in Ref. 178. The situation remains uncertain not only because of this contradiction between the two experiments. For reliable testing of the rescattering mechanism on the basis of the production of pairs of particles or jets, it is necessary that both scatterings correspond to $p_\perp \gtrsim 3$–4 GeV/c, since at smaller $p_\perp$ the production mechanism itself has not yet been understood.

In Fig. 34, it is noteworthy that α is larger for secondary particles ($\bar{p}, K^-$) that do not contain valence quarks. For protons, α is also large, but this can be attributed to scattering through a large angle (also unexplained) of the incident proton itself. It can be assumed that the production of $\bar{p}$ and K^- is due to large-angle scattering of gluons. In quantum chromodynamics, the cross sections of gg and gq scattering are greater than the cross sections of qq scattering (above the gluon color charge). Therefore, double scatterings enhance the yield of gluons with large $p_\perp$, in agreement with the experiments. The estimates made by Krzywicki *et al.*[180] give the correct order of magnitude for α in the region $p_\perp \gtrsim 5$ GeV/c.

The second explanation of the Cronin effect can be called the pseudojet mechanism. Here, one considers two simultaneous single scatterings of two partons in the incident particle in the same direction. The estimates made by Zmushko[181] and Takagi[182] show that the cross sections for the production of such pseudojets are not small. The specific feature of these pseudojets is that the fraction of fast hadrons in the pseudojets is smaller, and the total multiplicity of the particles is higher than in an ordinary quark jet.[182] Both conclusions are confirmed by the results of Bromberg *et al.*[176] Note that if the pseudojet mechanism is indeed correct, then multijet events must be observed on nuclei with a large cross section, namely, the cross sec-

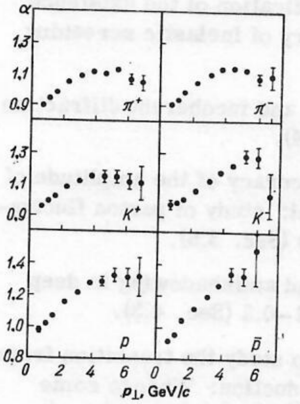

FIG. 34. Dependence on $p_\perp$ of the exponent $\alpha(p_\perp)$ in the production by protons of particles with large $p_\perp$ on nuclei.[174]

tion for the production of four uncorrelated (with respect to the azimuthal angle) jets with momenta $\approx p_\perp$ must be of the order of the cross section for one jet with momentum $\approx 2p_\perp$.

To test all the explanations of the Cronin effect, study of the correlations in the production of particles with large $p_\perp$ on nuclei is extremely important.

CONCLUSIONS

Let us summarize. Retrospectively, it is helpful to compare the present review with the review of Nikitin, Rozental', and Sergeev,[139] which was published in 1977. The two reviews overlap in their titles (strongly), in the questions discussed (to about 50%), and in the qualitative conclusions (very weakly). But in the *quantitative* interpretation of the experimental data there is virtually no overlap. This is an indication of the significant advance in both theory and experiment.

The most realistic model of multiple processes on nuclei—the additive quark model—was in fact developed after 1977. The indications from the interactions with nuclei that the sizes of the constituent quarks in the hadrons are small and that the quarks behave additively are extremely important and must be taken into account in the construction of realistic quantum-chromodynamic models of hadrons. If we do not take into account Gribov's remark to the effect that the purely gluon model of the Pomeron explains the small slope of the vacuum trajectory and, accordingly, the small size of the quarks by the mass scale in gluon physics,[183] a connection between the additive quark model and quantum chromodynamics has not yet been established.

It is no secret that the main attention is now concentrated on the physics of deep inelastic processes. But however good may be the phenology of jets in terms of the fragmentation of quarks and gluons, the further development will still lead us to ask: But how are the hadrons formed from the gluons and quarks? This is a problem of large distances, and to solve this problem any information about the behavior of quarks in strong interactions may be extremely helpful, and here the unique possibilities of experiments with nuclei are not to be considered at the end of the list.

There has been considerable progress in understanding correlation phenomena, for the discussion of which there was neither a quantitative basis nor experimental data two or three years ago. These data evidently rule out all chances that the collective models—the hydrodynamic and the coherent-tube models—will survive.

The first experimental data on electroproduction and neutrino production on nuclei have been published. They confirm the existence of intranuclear cascades, long the subject of discussion.

The development has also been significant in the long classical field of the theory of diffraction processes. In part, it has been negative; for we now understand that diffraction production on nuclei does not give the interaction cross sections of unstable particles. But we have got closer to the solution of a more important problem, one that is equivalent to some extent to the determination of these cross sections, namely, the elucidation by means of diffraction processes of the parton structure of the hadrons. The linking of this phenomenology to quantum chromodynamics is an important task for the future.

The attempt to generalize to interactions with nuclei the now customary concepts of hadron–hadron interactions reveals serious gaps in the theory of hadron interactions. Our understanding and interpretation of multiple scattering in nuclei are far from satisfactory. The growth of the formation lengths in production processes and the explanation by this of the suppression of cascade multiplication of secondary particles have been reliably established. But there is no complete understanding of why the simple optical model of the total cross sections with the not so simple, but still comprehensible, inelastic corrections works equally well at all energies. One would think that the change in the nature of the inelastic processes at energies $E \geq E_{cas} = Rm^2$ would also, by virtue of the unitarity condition, change the dependence of the total cross sections.

It was unavoidable that the review should be fragmentary in part. Thus, we have omitted a discussion of interactions with the deuteron, which could make up an independent review. The deuteron is the simplest nuclear target, and many predictions take their most definite form for rescattering in the deuteron. But the fraction of such events in the deuteron is small, and one needs experiments with very high and as yet unattainable statistics. A discussion of the theory of particle production on deuterons can be found in Ref. 48 and in the extensive survey of Ref. 187.

Because of limited space, we have also completely omitted the discussion of fragmentation of the nucleus itself and the extremely interesting cumulative effect and nuclear scaling. There are, however, the extensive reviews of Frankfurt and Strikman,[184] Stavinskiĭ,[185] and Baldin[186] on this subject.

Finally, we list the experiments that are of greatest interest for the existing theoretical ideas.

1. Accurate measurements of the total cross sections of nA interactions: verification of the existence of passive states and the theory of inelastic screening (Sec. 3.3).

2. Comparison of coherent and incoherent diffraction dissociation on nuclei (Sec. 3.4).

3. Measurement to high accuracy of the amplitude of $K_L \to K_S$ regeneration on nuclei: study of parton fluctuations with wee valence quarks (Sec. 3.5).

4. Search for shadowing and antishadowing in deep inelastic scattering for $x \lesssim 0.1-0.2$ (Sec. 4.3).

5. Accurate experiments to study the transition from photoproduction to electroproduction: Whence come the small scales of Q^2 (Sec. 4.4)?

6. Search for a plateau in the ratio of the nucleus–

nucleon spectra at energies ~1 TeV as a measurement of the number of inelastic absorptions in the nucleus (Secs. 5.2 and 5.5).

7. Testing of the quark counting rules for the fragmentation spectra, especially for different incident particles. Measurement of the associated multiplicities (Secs. 5.4 and 6.5).

8. Accurate comparison of the N_g dependences of the correlations and the Wroblewski ratio in pA and πA interactions at high energies (Sec. 6.4).

9. Comparison to high accuracy of electroproduction, neutrino production, and photoproduction on nuclei with the production of hadrons. Search for the absence of absorption of the beam fragments in electroproduction at high energies (Secs. 5.4, 5.5, and 6.3).

10. Study of the correlations in the production of parparticles and jets with large $p_\perp$ on nuclei (Sec. 7.2).

For all the points listed above, quantitative theoretical predictions have appeared in the last two or three years, and their testing is important for the further development of theory.

I am grateful to A. M. Baldin, whom I have to thank for the coming into being of this review. In writing this review, and also in working on the problems discussed in it, I have had very helpful discussions with V. V. Anisovich, L. Enik, V. G. Grishin, B. Z. Kopeliovich, G. A. Leksin, E. M. Levin, M. G. Ryskin, M. Fessler, Yu. M. Shabel'skiĭ, and V. M. Shekhter. Discussions and correspondence with E. L. Feĭnberg were especially valuable.

[1]O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 469 (1973) [JETP Lett. **18**, 274 (1973)].
[2]I. M. Frank, Izv. Akad. Nauk SSSR, Ser. Fiz. **6**, 1 (1942).
[3]M. L. Ter-Mikaélyan, Dis. na soisk. uchen. stepeni kand. fiz.-mat. nauk (Candidate's Dissertation in the Physical and Mathematical Sciences), P. N. Lebedev Physics Institute, Moscow (1952).
[4]L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 535, 735 (1953).
[5]I. Ya. Pomeranchuk and E. L. Feĭnberg, Dokl. Akad. Nauk SSSR **93**, 439 (1953).
[6]E. L. Feinberg, J. Phys. **5**, 177 (1941).
[7]A. Peterman, Phys. Rep. **C53**, 157 (1979); A. De Rujula *et al.*, Preprint Ref. TH. 2778-CERN (1979); J. Ellis and C. T. Sachrajda, Preprint Ref. TH. 2782-CERN (1979).
[8]A. I. Vaĭnshteĭn *et al.*, Usp Fiz. Nauk **123**, 217 (1977) [Sov. Phys. Usp. **20**, 796 (1977)].
[9]E. M. Levin and L. L. Frankfurt, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 105 (1965) [JETP Lett. **2**, 65 (1965)].
[10]V. V. Anisovich, Phys. Lett. **B55**, 87 (1975).
[11]H. A. Bethe, Phys. Rev. **57**, 1125 (1940).
[12]J. V. Allaby *et al.*, Yad. Fiz. **12**, 538 (1970) [Sov. J. Nucl. Phys. **12**, 295 (1971)]; Yu. P. Gorin *et al.*, Yad. Fiz. **18**, 336 (1973) [Sov. J. Nucl. Phys. **18**, 173 (1973)]; S. P. Denisov *et al.*, Nucl. Phys. **B79**, 62 (1973); A. S. Carrol *et al.*, Phys. Lett. **B80**, 319 (1979); B. W. Allardyce *et al.*, Nucl. Phys. **A209**, 1 (1973); B. M. Bobchenko *et al.*, Yad. Fiz. **30**, 1553 (1979) [Sov. J. Nucl. Phys. **30**, 805 (1979)].
[13]V. Heynen *et al.*, Phys. Lett. **B34**, 651 (1971); G. R. Brooks *et al.*, Phys. Rev. D **8**, 2826 (1973); D. O. Caldwell *et al.*, Phys. Rev. D **7**, 1362 (1973); S. Michalowski, Phys. Rev. Lett. **39**, 737 (1977).
[14]L. Stodolsky, Phys. Rev. Lett. **18**, 135 (1967).
[15]W. R. Ditzler *et al.*, Phys. Lett. **B57**, 201 (1975); S. Stein *et al.*, Phys. Rev. D **12**, 1884 (1975; M. May *et al.*, Phys. Rev. Lett., **34**, 407 (1975); J. Eickmeyer *et al.*, Phys. Lett. **B63**, 104 (1976); Phys. Rev. Lett. **36**, 289 (1976).
[16]W. Busza *et al.*, Phys. Rev. Lett. **34**, 836 (1975); C. Halliwell *et al.*, Phys. Rev. Lett. **39**, 1499 (1977); J. E. Elias *et al.*, Phys. Rev. Lett. **41**, 285 (1978); J. E. Elias *et al.*, Preprint FERMILAB-PUB-79/47-EXP (1979).
[17]L. Hand *et al.*, Acta Phys. Pol. **B9**, 1978 (1978); Z. Phys. **C1**, 139 (1979).
[18]T. H. Burnett *et al.*, Preprints VTL-PUB-44 (1977); VTL-PUB-50 (1978).
[19]H. W. Bertini, Phys. Rev. C **17**, 1382 (1978).
[20]I. L. Rozental' and D. S. Chernavskiĭ, Usp. Fiz. Nauk **52**, 185 (1954).
[21]M. A. Faessler *et al.*, Nucl. Phys. **B157**, 1 (1979).
[22]V. S. Murzin and L. I. Sarycheva, Kosmicheskie luchi i ikh vzaimodeĭstviya (Cosmic Rays and Their Interactions), Atomizdat, Moscow (1968); V. S. Barashenkov and V. D. Toneev, Vzaimodeĭstvie vysokoénergeticheskikh chastits i yader s yadrami (Interaction of High Energy Particles and Nuclei with Nuclei), Atomizdat, Moscow (1972).
[23]Alma-Ata–Gatchina–Moscow–Tashkent Collaboration, Yad. Fiz. **28**, 704 (1978) [Sov. J. Nucl. Phys. **28**, 362 (1978)]; S. A. Azimov *et al.*, Yad. Fiz. **27**, 1011 (1978) [Sov. J. Nucl. Phys. **27**, 535 (1978)].
[24]K. G. Gulamov, U. G. Gulyamov, and G. M. Chernov, Fiz. Elem. Chastits At. Yadra **9**, 554 (1978) [Sov. J. Part. Nucl. **9**, 226 (1978)].
[25]Alma-Ata–Gatchina–Moscow–Tashkent Collaboration, Yad. Fiz. **28**, 989 (1978) [Sov. J. Nucl. Phys. **28**, 507 (1978)].
[26]E. L. Feinberg, Phys. Rep. **C5**, 240 (1972).
[27]E. L. Feĭnberg, Usp. Fiz. Nauk **70**, 333 (1960) [Sov. Phys. Usp. **3**, 147 (1960)].
[28]E. L. Feĭnberg, Zh. Eksp. Teor. Fiz. **50**, 202 (1966) [Sov. Phys. JETP **23**, 132 (1966)].
[29]E. L. Feĭnberg, in: Problemy teoreticheskoĭ fiziki (Problems of Theoretical Physics), Nauka, Moscow (1972), p. 248; E. L. Feĭnberg, in: Trudy shkoly molodykh uchenykh, Sukhumi (Proc. of the School for Young Scientists, Sukhumi), JINR, Dubna (1973), p. 56.
[30]A. B. Migdal, Zh. Eksp. Teor. Fiz. **32**, 633 (1956) [Sov. Phys. JETP **5**, 527 (1957)].
[31]V. M. Galitsky and I. I. Gurevich, Nuovo Cimento **32**, 633 (1964).
[32]V. L. Ginzburg and V. N. Tsytovich, Usp. Fiz. Nauk **126**, 553 (1978) [Physics Reports 49(1), 1–89 (January 1979)]
[33]V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Yad.
[34]B. L. Ioffe, Phys. Lett. **30**, 123 (1969).
[35]S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).
[36]V. N. Gribov, in: Trudy VIII shkoly LIYaF (Proc. Eighth School at the Leningrad Institute of Nuclear Physics), Vol. 2, Leningrad (1973), p. 5.
[37]A. A. Ansel'm in: Trudy VIII shkoly LIYaF (Proc. Eighth School at the Leningrad Institute of Nuclear Physics), Vol. 2, Leningrad (1973), p. 37.
[38]R. P. Feynman, Photon–Hadron Interactions, Addison-Wesley, Reading, Mass. (1972) [Russian translation published by Mir, Moscow (1975)].
[39]V. M. Levin and M. G. Ryskin, Zh. Eksp. Teor. Fiz. **69**, 1537 (1975) [Sov. Phys. JETP **42**, 783 (1975)].
[40]P. Grassberger, Nucl. Phys. **B125**, 84 (1977).
[41]P. Grassberger, Preprint WUB-79-15, Wuppertal (1979) (and references therein).
[42]A. B. Zamolodchikov, B. Z. Kopeliovich, and L. I. Lapidus, Zh. Eksp. Teor. Fiz. **77**, 451 (1979) [Sov. Phys. JETP **50**, 229 (1979)].
[43]V. G. Kadyshevskiĭ, Zh. Eksp. Teor. Fiz. **46**, 654 (1964) [Sov. Phys. JETP **19**, 443 (1964)].

[44]T. Jaroszewicz *et al.*, Phys. Lett. **B79**, 127 (1978).
[45]L. Caneschi, I. G. Halliday, and A. Shcwimmer, Nucl. Phys. **B144**, 397 (1978).
[46]P. Heney and J. Pumplin, Preprint UCSD 10P10-195 (1978).
[47]V. N. Gribov, Zh. Eksp. Teor. Fiz. **56**, 892 (1969) [Sov. Phys. JETP **29**, 483 (1969)].
[48]N. N. Nikolaev and V. R. Zoller, Nucl. Phys. **B147**, 336 (1979).
[49]O. V. Kancheli and S. G. Matinyan, Yad. Fiz. **11**, 1305 (1970) [Sov. J. Nucl. Phys. **11**, 726 (1971)].
[50]K. Gottfried and F. Low, Phys. Rev. D **17**, 2487 (1978).
[51]J. D. Bjorken, Lectures at DESY Summer School (1975); Preprint SLAC-PUB-1756 (1976).
[52]I. Z. Artykov, V. S. Barashenkov, and S. M. Eliseev, Nucl. Phys. **87**, 823 (1966); Nucl. Phys. **B6**, 11 (1968); V. S. Barashenkov *et al.*, Usp. Fiz. Nauk **109**, 91 (1973) [Sov. Phys. Usp. **16**, 31 (1974)].
[53]G. Bialkowski, C. Chin, and D. Tow, Phys. Rev. D **17**, 862 (1978); M. Hossain and D. M. Tow, CPT-Preprint ORO 3992-357, University of Texas (1979); P. Valanju, E. C. G. Sudarshan, and C. B. Chiu, CPT-Preprint ORO 3992-365, University of Texas (1979).
[54]E. L. Feinberg and I. Ya. Pomeranchuk, Suppl. Nuovo Cimento **111**, 652 (1956); A. I. Akhiezer and I. Ya. Pomeranchuk, Usp. Fiz. Nauk **65**, 593 (1978); A. G. Sitenko, Usp. Fiz. Nauk **67**, 377 (1959) [Sov. Phys. Usp. **2**, 195 (1959)].
[55]H. I. Miettinen and J. O. Pumplin, Phys. Rev. D **18**, 1696 (1978); H. I. Miettinen and J. Pumplin, Phys. Rev. Lett. **42**, 204 (1979).
[56]M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).
[57]B. Z. Kopeliovich and L. I. Lapidus, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 664 (1978) [JETP Lett. **28**, 614 (1978)]; in: Mnozhestvennoe rozhdenie i predel'naya fragmentatsiya yadra (Multiple Production and Limiting Fragmentation of Nuclei), Dubna (1979), p. 469.
[58]R. J. Glauber, Lectures in Theoretical Physics, Vol. 1, Interscience, New York (1959); R. Glauber, Review paper at the Third Intern. Conf. on High Energy Physics and Nuclear Structure, Columbia University (1969); Russian translation: Usp. Fiz. Nauk **103**, 641 (1971); V. M. Kolybasov and M. S. Marinov, Usp. Fiz. Nauk **109**, 137 (1973) [Sov. Phys. Usp. **16**, 53 (1974)].
[59]A. Y. Abul-Magd *et al.*, Phys. Lett. **B30**, 182 (1969).
[60]H. Lesniak and L. Lesniak, Phys. Lett. **B40**, 167 (1972).
[61]A. I. Babaev *et al.*, Yad. Fiz. **20**, 71 (1974) [Sov. J. Nucl. Phys. **20**, 37 (1975)]; P. V. R. Murthy *et al.*, Nucl. Phys. **B92**, 269 (1975); V. A. Lyubimov, Usp. Fiz. Nauk **121**, 193 (1977) [Sov. Phys. Usp. **20**, 97 (1977)].
[62]A. Gsponer *et al.*, Phys. Rev. Lett. **42**, 9 (1979).
[63]V. A. Karmanov and L. A. Kondratyuk, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 451 (1973) [JETP Lett. **18**, 266 (1973)].
[64]N. N. Nikolaev, Yad. Fiz. **32**, 1159 (1980) [erratum].
[65]V. V. Anisovich, L. G. Dakhno, and P. E. Volkovitsky, Phys. Lett. **B42**, 224 (1972); A. B. Kaidalov and L. A. Kondratyuk, Nucl. Phys. **B56**, 90 (1973).
[66]H. H. Bingham, in: Trudy Mezhdunar. seminara po vzaimodeĭstviyu chastits vysokoĭ énergii s yadrami (Proc. Intern. Seminar on the Interaction of High Energy Particles with Nuclei), No. 1, Atomizdat, Moscow (1974), p. 1; A. V. Tarasov, Fiz. Elem. Chastits At. Yadra **7**, 771 (1976) [Sov. J. Part. Nucl. **7**, 306 (1976)]; C. Bemporad *et al.*, Nucl. Phys. **B33**, 397 (1971); **B42**, 627 (1972); V. K. Dolinov *et al.*, Yad. Fiz. **26**, 1230 (1977) [Sov. J. Nucl. Phys. **26**, 649 (1977)].
[67]B. Goggi *et al.*, Preprint CERN-EP/79-43 (1979).
[68]W. Czyz, Phys. Rev. D **8**, 3219 (1973).
[69]N. N. Nikolaev, in: Multiparticle Production on Nuclei at Very High Energies, IAEA-SMR-21, Trieste (1977).
[70]A. B. Zamolodchikov *et al.*, Zh. Eksp. Teor. Fiz. **77**, 451 (1979) [Sov. Phys. JETP **50**, 229 (1979)].
[71]E. M. Levin *et al.*, Nucl. Phys. **B124**, 1152 (1977).
[72]R. M. Edelstein, in: High Energy Collisions Involving Nuclei, Bologna (1975).
[73]P. Bruton *et al.*, Phys. Lett. **B59**, 490 (1975).
[74]Yu. M. Zaĭtsev, in: Trudy II shkoly fiziki ITÉF (Proc. Second School of Physics at the Institute of Theoretical and Experimental Physics), No. 1, Atomizdat, Moscow (1975), p. 45; B. M. Kolybasov, *ibid.*, p. 59.
[75]A. Gsponer *et al.*, Phys. Rev. Lett. **42**, 13 (1979); W. R. Molzon *et al.*, Phys. Rev. Lett. **41**, 1213 (1978); J. Roekrig *et al.*, Phys. Rev. Lett. **38**, 1116 (1977).
[76]L. Bertocchi and D. Treleani, Nuovo Cimento **A50**, 338 (1979).
[77]B. Diu, A. Ferraz, and F. De Camargo, Preprint PAR-LPTHE 79-10 (1979).
[78]B. Z. Kopeliovich and N. N. Nikolaev, Z. Phys. **C5**, 333 (1980).
[79]Ya. Ya. Balitskiĭ, L. N. Lipatov, and V. S. Fadin, in: Materialy XIV zimneĭ shkoly LIYaF (Proc. 14th Winter School at the Leningrad Institute of Nuclear Physics), Leningrad (1979), p. 109; J. Bartels, Preprint DESY 79/68 (1979).
[80]I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **96**, 265, 481 (1954).
[81]J. S. Bell, Phys. Rev. Lett. **13**, 57 (1964).
[82]S. J. Adler, Phys. Rev. B **135**, 963 (1964).
[83]V. N. Gribov, Zh. Eksp. Teor. Fiz. **57**, 1306 (1969) [Sov. Phys. JETP **30**, 709 (1970)].
[84]J. D. Bjorken and J. Kogut, Phys. Rev. D **8**, 1341 (1973).
[85]G. Grammer and J. D. Sullivan, in: Electromagnetic Interactions, Vol. 2, Plenum, New York (1978); T. H. Bauer *et al.*, Rev. Mod. Phys. **50**, 261 (1978).
[86]S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D **6**, 177 (1972).
[87]V. I. Zakharov and N. N. Nikolaev, Yad. Fiz. **21**, 434 (1975) [Sov. J. Nucl. Phys. **21**, 227 (1975)]; N. N. Nikolaev and V. I. Zakharov, Phys. Lett. **B55**, 397 (1975).
[88]Yu. L. Dokshitser, D. I. D'yakonov, and S. I. Troyan, in: Fizika élementarnykh chastits (Trudy XIII zimneĭ shkoly LIYaF) [Physics of Elementary Particles (Proc. 13th Winter School at the Leningrad Institute of Nuclear Physics)], Leningrad (1978), p. 3.
[89]S. J. Brodsky *et al.*, Report, SLAC (1978) (unpublished).
[90]S. J. Brodsky, J. F. Gunion, and J. Kühn, Phys. Rev. Lett. **39**, 1120 (1977).
[91]S. J. Brodsky, Preprint SLAC-PUB-2395 (1979).
[92]L. E. Ibanez and J. L. Sanchez-Gomez, Nucl. Phys. **B156**, 427 (1979); H. Fraas, B. J. Read, and D. Schildknecht, Nucl. Phys. **B88**, 301 (1975); **B86**, 346 (1975); P. Ditsas and G. Shaw, Nucl. Phys. **B113**, 246 (1976).
[93]J. Bailey *et al.*, Nucl. Phys. **B151**, 367 (1979).
[94]D. O. Caldwell *et al.*, Phys. Rev. Lett. **42**, 553 (1979).
[95]L. Bertocchi and D. Treleani, Preprint IC/79/62, ICTP, Trieste (1979).
[96]G. V. Davidenko and N. N. Nikolaev, Yad. Fiz. **24**, 772 (1976) [Sov. J. Nucl. Phys. **24**, 402 (1976)].
[97]N. N. Nikolaev and A. Ya. Ostapchuck, Preprint Ref. TH 2575-CERN (1978); N. N. Nikolaev, A. Ya. Ostapchuck, and V. R. Zoller, Preprint Ref. TH. 2541-CERN (1978).
[98]V. A. Abramovskiĭ, V. N. Gribov, and O. V. Kancheli, Yad. Fiz. **18**, 595 (1973) [Sov. J. Nucl. Phys. **18**, 308 (1973)].
[99]Yu. M. Shabel'skiĭ, Yad. Fiz. **26**, 1084 (1977) [Sov. J. Nucl. Phys. **26**, 573 (1977)].
[100]Yu. M. Shabelsky, Nucl. Phys. **B132**, 491 (1978); A. Capella and A. B. Kaidalov, Nucl. Phys. **B111**, 477 (1977).
[101]L. Bertocchi and D. Treleani, J. Phys. G **3**, 147 (1977); A. Capella and A. Krzywicki, Phys. Rev. D **18**, 3357 (1978).
[102]E. Lehman, Nucl. Phys. **B127**, 331 (1978).
[103]A. Bialas, M. Blezynski, and W. Czyz, Acta Phys. Pol. **B8**, 389 (1977); K. Konoshita, A. Minaka, and Z. Sumiyoshi,

Preprint KYSHU-79-HE-5/KAGOSHIMA-HE-79-5 (1979); K. Kinoshita, A. Minaka, and N. Sumiyoshi, Prog. Theor. Phys. 61, 165 (1979); T. Jaroszewicz *et al.*, Z. Phys. C **1**, 181 (1979).
[104]L. Caneschi and A. Schwimmer, in: Proc. 12th Rencontre de Moriond Flaine, Vol. II (1977), p. 173.
[105]V. N. Anisovich, in: Materialy IX zimneĭ shkoly LIYaF (Proc. Ninth Winter School at the Leningrad Institute of Nuclear Physics), Part 1, Leningrad (1974); V. V. Anisovich, E. M. Levin, and M. G. Ryskin, Yad. Fiz. **29**, 1311 (1979) [Sov. J. Nucl. Phys. **29**, 674 (1979)]; V. V. Anisovich, in: Materialy XIV zimneĭ shkoly LIYaF (Proc. 14th Winter School at the Leningrad Institute of Nuclear Physics), Leningrad (1979), p. 3.
[106]E. M. Levin and L. L. Frankfurt, Usp. Fiz. Nauk **94**, 243 (1968) [Sov. Phys. Usp. **11**, 106 (1968)].
[107]V. V. Anisovich, Yu. M. Shabelsky, and V. M. Shekhter, Nucl. Phys. **B133**, 477 (1978); V. V. Anisovich, F. G. Lepekhin, and Yu. M. Shabel'skiĭ, Yad. Fiz. **27**, 1639 (1978) [Sov. J. Nucl. Phys. **27**, 861 (1978)].
[108]K. A. Ter-Martirosyan, Phys. Lett. **B44**, 377 (1973).
[109]A. B. Kaĭdalov, in: Élementarnye chastitsy. Trudy II shkoly fiziki ITÉF (Elementary Particles. Proc. Second School of Physics at the Institute of Theoretical and Experimental Physics), No. 3, Atomizdat, Moscow (1975).
[110]E. M. Levin and M. G. Ryskin, Yad. Fiz. **21**, 396 (1975) [Sov. J. Nucl. Phys. **21**, 206 (1975)].
[111]J. Koplik and A. E. Mueller, Phys. Rev. D **12**, 3638 (1975).
[112]S. Voloshin and Yu. P. Nikitin, Yad. Fiz. **29**, 1003 (1979) [Sov. J. Nucl. Phys. **29**, 518 (1979)]; 30, 765 (1979) [**30**, 394 (1979)].
[113]E. M. Levin and M. G. Ryskin, Yad. Fiz. **31**, 429 (1980) [Sov. J. Nucl. Phys. **31**, 225 (1980)].
[114]N. N. Nikolaev, Phys. Lett. **B60**, 363 (1976); **B70**, 95 (1977).
[115]V. V. Anisovich and V. M. Shekhter, Nucl. Phys. **B55**, 455 (1973).
[116]N. N. Nikolaev and S. Pokorski, Phys. Lett. **B80**, 290 (1979).
[117]K. Heller *et al.*, Phys. Rev. D **16**, 2737 (1977).
[118]Yu. D. Bayukov *et al.*, Yad. Fiz. **29**, 947 (1979) [Sov. J. Nucl. Phys. **29**, 487 (1979)].
[119]D. Cahney *et al.*, Phys. Rev. Lett. **40**, 71 (1978).
[120]P. Skubic *et al.*, Phys. Rev. D **18**, 3115 (1978).
[121]L. Van Hove and S. Pokorski, Acta Phys. Pol. **B5**, 229 (1974); Nucl. Phys. **B86**, 287 (1975).
[122]S. A. Azimov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 450 (1977) [JETP Lett. **25**, 421 (1977)].
[123]A. Bialas and E. Bialas, Preprint FERMILAB-PUB-79/48-THY (1979).
[124]G. V. Davidenko and N. N. Nikolaev, Nucl. Phys. **B135**, 333 (1978).
[125]N. N. Nikolaev, Preprint Ref. TH 2792-CERN (1979); Z. Phys. C (1980) (in press).
[126]L. S. Osborn *et al.*, Phys. Rev. Lett. **40**, 1624 (1978).
[127]A. Bialas and W. Czyz, Nucl. Phys. **B137**, 359 (1978).
[128]A. Bialas, W. Czyz, and W. Furmanski, Acta Phys. Pol. **B8**, 585 (1977).
[129]A. Bialas, Invited Talk at the First Workshop on Ultra-Relativistic Nuclear Collisions, LBL, Berkeley, May 1979; Preprint FERMILAB-Conf-79/35-THY (1979).
[130]B. Anderson, I. Otterlund, and E. Stenlund, Phys. Lett. **B84**, 87 (1979); I. Otterlund, Preprint TECHNION-PH-79-46 (1979).
[131]Yu. M. Shabelsky and V. M. Shekhter, Preprint LINP-524 (1979).
[132]D. M. Kotlyarevskiĭ, Lecture at the Seventh All-Union School on Inelastic Interactions, Bakuriana (1980).
[133]N. N. Nikolaev and A. Ya. Ostapchuk, Lett. Nuovo Cimento **23**, 273 (1978).
[134]F. C. Roesler and C. B. A. McCusker, Nuovo Cimento **10**, 127 (1953); G. Cocconi, Phys. Rev. **93**, 1107 (1954); E. L. Feĭnberg, Zh. Eksp. Teor. Fiz. **28**, 241 (1955) [Sov. Phys. JETP **1**, 176 (1955)].
[135]A. Z. Patashinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 654 (1974) [JETP Lett. **19**, 296 (1974); É. V. Shuryak, Yad. Fiz. **24**, 630 (1976) [Sov. J. Nucl. Phys. **24**, 330 (1976)].
[136]G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D **13**, 1616 (1976).
[137]S. Fredriksson, Nucl. Phys. **B111**, 167 (1976); S. Fredriksson, Preprint Ref. TH. 2720-CERN (1979).
[138]I. L. Rozental', Usp. Fiz. Nauk **116**, 271 (1975) [Sov. Phys. Usp. **18**, 430 (1975)].
[139]Yu. P. Nikitin, I. L. Rozental', and F. M. Sergeev, Usp. Fiz. Nauk **121**, 3 (1977) [Sov. Phys. Usp. **20**, 1 (1977)].
[140]O. V. Zhirov and É. V. Shuryak, Yad. Fiz. **28**, 485 (1978) [Sov. J. Nucl. Phys. **28**, 247 (1978)].
[141]S. A. Azimov *et al.*, Phys. Lett. **B73**, 339 (1978).
[142]N. N. Nikolaev, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 383 (1976) [JETP Lett. **24**, 348 (1976)].
[143]B. N. Kalinkin and V. L. Shmonin, Yad. Fiz. **21**, 628 (1975) [Sov. J. Nucl. Phys. **21**, 325 (1975)].
[144]K. G. Gulamov and A. A. Ushinskiĭ, Yad. Fiz. **26**, 1095 (1977) [Sov. J. Nucl. Phys. **26**, 579 (1977)].
[145]K. Gottfried, Phys. Rev. Lett. **32**, 957 (1974).
[146]K. Gottfried, in: Proc. of the Fifth Intern. Conf. on High Energy Physics and Nuclear Structure, Uppsala (1974), p. 79.
[147]P. M. Fishbane and J. S. Trefil, Phys. Lett. **B51**, 139 (1974).
[148]F. K. Aliev *et al.*, Lett. Nuovo Cimento **23**, 212 (1978).
[149]J. Whitmore, Phys. Rep. **C27**, 187 (1976).
[150]V. G. Grishin, Usp. Fiz. Nauk **127**, 51 (1979) [Sov. Phys. Usp. **22**, 1 (1979)].
[151]A. Wroblewski, Acta Phys. Pol. **B4**, 857 (1973).
[152]N. N. Nikolaev, Nucl. Phys. B (1980) (to be published).
[153]E. M. Levin, M. G. Ryskin, and N. N. Nikolaev, Preprint Ref. TH.2780-CERN (1979); Z. Phys. (1980) (in press).
[154]S. Batsbovich *et al.*, Preprint RI-12777 [in Russian], JINR, Dubna (1979); L. Enik, Ph.D. thesis, Central Research Institute for Physics, Budapest (1980).
[155]G. Baroni *et al.*, Nucl. Phys. **B103**, 213 (1976); G. Baroni, E. Lamana, and S. Petera, Nucl. Phys. **B135**, 405 (1978).
[156]J. Babecki and G. Nowak, Acta Phys. Pol. **B9**, 401 (1978); B. Andersson, I. Otterlund, and E. Stenlund, Phys. Lett. **B73**, 343 (1978).
[157]B. Andersson, G. Nilsson, and I. Otterlund, Nucl. Phys. **B153**, 493 (1979).
[158]E. M. Levin, M. G. Ryskin, and S. I. Troyan, Yad. Fiz. **23**, 423 (1976) [Sov. J. Nucl. Phys. **23**, 222 (1976)].
[159]N. N. Nikolaev, Phys. Rev. Lett. (1980) (in press).
[160]S. Drell and T. M. Yan, Phys. Rev. Lett. **25**, 316 (1970).
[161]R. Stroynowski, Preprint SLAC-PUB-2402 (1979).
[162]J. F. Gunion, Phys. Rev. D **12**, 1345 (1975); M. B. Green, M. Jacob, and P. V. Landshoff, Nuovo Cimento **A29**, 123 (1976).
[163]D. Sivers, S. Brodsky, and R. Blankenbecler, Phys. Rep. **C23**, 1 (1976); S. Ellis and R. Stroynowski, Rev. Mod. Phys. **49**, 753 (1977); M. Jacob and P. V. Landshoff, Phys. Rep. **C48**, 285 (1978); M. Jacob, Rapporteur's talk at the EPS Intern. Conf., Geneva (1979); Preprint Ref. TH 2700-CERN (1979).
[164]N. N. Nikolaev and V. A. Zakharov, Preprint, Chermogolovka (1975).
[165]G. R. Farrar, Phys. Lett. **B56**, 185 (1975).
[166]F. Vannucci, Preprint CERN-EP/79-151 (1979).
[167]M. Binkley *et al.*, Phys. Rev. Lett. **37**, 571 (1976); J. G. Branson *et al.*, Phys. Rev. Lett. **38**, 457 (1977); D. M. Kaplan *et al.*, Phys. Rev. Lett. **40**, 435 (1978).
[168]K. J. Anderson *et al.*, Phys. Rev. Lett. **42**, 944 (1979).
[169]J. Badier *et al.*, Phys. Lett. **B89**, 145 (1979).
[170]C. Michael and D. M. Webber, Phys. Lett. **B83**, 243 (1979).
[171]Yu. M. Antipov *et al.*, Phys. Lett. **B76**, 235 (1978).
[172]K. J. Anderson *et al.*, Phys. Rev. Lett. **42**, 944 (1979);

J. Badier *et al.*, Preprint CERN/EP 79-61 (1979).
[173] R. L. Anderson *et al.*, Phys. Rev. Lett. 38, 263 (1977); B. L. Ioffe, Preprint ITER-124, Moscow (1975); A. Yu. Khodzhamiryan and I. S. Tsukerman, Preprint ITÉF-96 [in Russian], Moscow (1976).
[174] K. Kluberg *et al.*, Phys. Rev. Lett. 38, 670 (1977); D. Antreasyan *et al.*, Phys. Rev. D 19, 764 (1979); J. W. Cronin *et al.*, Phys. Rev. D 11, 3105 (1975).
[175] U. Becker *et al.*, Phys. Rev. Lett. 37, 1731 (1976); D. A. Garbutt *et al.*, Phys. Lett. B67, 355 (1977); V. V. Abramov *et al.*, Preprint 79-131 [in Russian], Institute of High Energy Physics, Serpukhov (1979).
[176] C. Bromberg *et al.*, Phys. Rev. Lett. 42, 1202 (1979); 43, 1057 (1979).
[177] Y. Afek *et al.*, Phys. Rev. D 15, 2622 (1977); A. Krzywicki, Phys. Rev. D 14, 152 (1976).
[178] R. L. McCarthy *et al.*, Phys. Rev. Lett. 40, 213 (1978).
[179] D. A. Finley *et al.*, Phys. Rev. Lett. 42, 1031 (1979).
[180] A. Krzywicki *et al.*, Phys. Lett. B85, 407 (1979).
[181] V. V. Zmushko, Preprints OÉF-79-145, OÉF-69-157 [in Russian], Institute of High Energy Physics, Serpukhov (1979).
[182] F. Takagi, Phys. Rev. Lett. 43, 1296 (1979).
[183] V. A. Novikov *et al.*, Nucl. Phys. B (1980) (in press).
[184] L. L. Frankfurt and M. I. Strikman, in: Élementarnye chastitsy. VI shkola fizikov ITÉF (Elementary Particles. Sixth School of Physics at the Institute of Theoretical and Experimental Physics), No. 2 (1979), p. 16.
[185] V. S. Stavinskiĭ, Fiz. Elem. Chastits At. Yadra 10, 949 (1979) [Sov. J. Part. Nucl. 10, 373 (1979)].
[186] A. M. Baldin, Fiz. Elem. Chastits At. Yadra 8, 429 (1977) [Sov. J. Part. Nucl. 8, 175 (1977)].
[187] L. Bergström and S. Fredriksson, Preprint, Royal Institute of Technology, Stockholm (1979).

Translated by Julian B. Barbour