The problem of *CP* invariance in quantum chromodynamics

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The problem of *CP* and *P* invariance in quantum chromodynamics is reviewed in connection with the complex structure of the vacuum state in non-Abelian gauge field theories. Various possible solutions to the problem of *CP* and *P* conservation are discussed: the axion hypothesis (the existence of a light pseudoscalar particle with semiweak interaction), the massless *u* quark, models with left-right symmetry, and some others.

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INTRODUCTION

Non-Abelian gauge fields and the principle of spontaneous symmetry breaking are the basis for the construction of modern models of the weak, electromagnetic, and also strong interactions. The principle of local gauge invariance was introduced by Yang and Mills as a generalization of the requirement of isotopic symmetry imposed on the behavior of quantum fields at each separate space-time point.

The concept of spontaneous symmetry breaking arose in Bogolyubov's studies in the theory of quantum-statistical systems with degenerate ground state. The method of quasiaverages which he created is today well known and is a universal means for studying quantum systems whose ground state is unstable against small perturbations that break a particular symmetry of the problem. Quasiclassical analysis of non-Abelian theories indicates degeneracy of the ground state associated with the so-called topological charge.[1-3] The presence of degeneracy indicates a complicated structure of the vacuum state in gauge theories, and this has serious physical consequences. In particular, the so-called θ structure of the vacuum in quantum chromodynamics discussed in recent years can lead to the violation of P and CP invariance in strong interactions.

The aim of this review is to consider the problem of CP invariance in quantum chromodynamics and various ways of resolving it. In the review, we give the basic propositions of quantum chromodynamics, which is at present the most popular theory for the strong interactions. We introduce the concept of the θ vacuum and consider its properties. We make estimates of the value of the parameter θ (the parameter of CP violation) in quantum chromodynamics on the basis of experimental bounds for the electric dipole moment of the neutron. We discuss the resolutions of the CP problem associated with the hypothesis that the u quark has no mass. We consider models with an axion, in which the CP problem is solved by the existence of a light pseudoscalar particle that has semiweak coupling to the leptons and the quarks. We discuss models with a discrete symmetry group (in particular, models with right–left symmetry, in which the parameter θ of the CP violation is small). In the review, we consider other "nonorthodox" possibilities discussed in the literature for explaining the observed CP invariance in the strong interactions.

1. QUANTUM CHROMODYNAMICS AS THE THEORY OF COLORED QUARKS AND GLUONS

Hope has been growing stronger in recent years that quantum chromodynamics could serve as the basis for describing strong interactions.[4] In this theory, the strong interactions between hadrons are regarded as the result of interaction between colored quarks, the interactions being transmitted by massless vector bosons (gluons) corresponding to the octet representation of the color gauge group $SU^c(3)$.

The original argument for introducing the new quantum number of color into hadron physics was the solution to the problem of quark statistics in the framework of the composite quark model. In this model, the unexcited baryon states are described by a wave function that is symmetric with respect to the quark and spin indices. For example, the Δ^{++} resonance in the state with $J_3=3/2$ is described by the quark wave function

$$|\Delta^{++},\ J_3=3/2\rangle=|u\uparrow u\uparrow u\uparrow\rangle,$$

and all three quarks are in the energetically most advantageous s state. Then the total wave function of the quarks is symmetric, which is in contradiction with the Fermi statistics of the quarks. To eliminate this contradiction with Fermi statistics, it was suggested in Refs. 5 and 6 that the quark of each type may be in three different (color) states, and the wave function of the quarks in the baryon could be antisymmetric with respect to the color indices. Thus, the Δ^{++} resonance would be described by the complete antisymmetric wave function

$$|\Delta^{++},\ J_3=3/2\rangle=(1/\sqrt{6})\,\varepsilon^{ijk}\,|u_i\uparrow u_j\uparrow u_k\rangle,$$

where i, j, k are the color indices.

Thus, a new symmetry group appeared in hadron physics: the color group $SU^c(3)$. The hadrons so far observed are singlets with respect to the color group $SU^c(3)$. If all strongly interacting particles are color singlets, it is natural to expect the absence in the free state of quarks, diquarks, etc. (the confinement hypothesis). At the present time, the investigation of the confinement mechanism is undoubtedly one of the central problems in the theory of strong interactions. We mention that the hypothesis of "tripling the number of quarks" is brilliantly confirmed in the description of e^+e^- annihilation into hadrons, the decay $\pi^0\to 2\gamma$, and the production of $\mu^+\mu^-$ pairs.[4]

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The Lagrangian of quantum chromodynamics has the form[4]

$$\mathcal{L} = -F^a_{\mu\nu}F^{a\mu\nu}/4 + \bar{q}\,[i\gamma^\mu(\partial_\mu - igA^a_\mu\lambda_a/2) - m]\,q. \quad (1)$$

Here, the symbol q denotes the colored quarks $u, d, s, c, t, \ldots$;

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\nu;$$

and m is the mass matrix of the quarks.

Quantum chromodynamics has the important property of asymptotic freedom (the effective coupling constant tends to zero in the ultraviolet region). This makes it possible to use perturbation theory to describe processes taking place at short distances (e^+e^- annihilation into hadrons, deep inelastic lepton–hadron scattering etc.). In the infrared region, the effective coupling constant $\alpha_s = g^2/4\pi$ increases. The hopes for explaining quark confinement within hadrons are based on this circumstance. However, since the problem of strong coupling has not yet been solved in quantum field theory, quantum chromodynamics remains as yet an incomplete theory.

2. TOPOLOGICAL CHARGE AND THE STRUCTURE OF THE θ VACUUM

One can arrive at a complicated structure of the vacuum in gauge theories either by using topologically nontrivial gauge transformations or by using the path-integral method. To be specific, let us consider a gauge field corresponding to the group $SU(2)$. The Lagrangian of the gauge field

$$\mathcal{L} = -F^a_{\mu\nu}F^{a\mu\nu}/4;$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon_{abc}A^b_\mu A^c_\nu$$

has in the gauge $A^a_0 = 0$ the form

$$\mathcal{L} = (\dot{A}^a_i)^2/2 - (F^a_{ij})^2/4 \quad (2)$$

and recalls an ordinary Lagrangian of classical mechanics if $(\dot{A}^a_i)^2/2$ is identified with the kinetic and $(F^a_{ij})^2/4$ with the potential energy of the system. Vacuum fields are c-number values of the gauge field A^a_i for which the field tensor vanishes, $F^a_{ij} = 0$. A vacuum field is a "pure gauge" field and can be represented in the form

$$\left.\begin{array}{l} A_i \equiv A^a_i T^a = U^{-1}\partial_i U; \\ {[T^a, T^b]} = i\varepsilon_{abc}\sqrt{2}T^c, \end{array}\right\} \quad (3)$$

where $U(\mathbf{x})$ is a unitary matrix.

We impose on the matrices $U(\mathbf{x})$ describing admissible vacuum fields the condition

$$U(\mathbf{x}) \to 1 \quad (4)$$

as $|\mathbf{x}| \to \infty$, irrespective of the direction, i.e., the entire spatial infinity is identified with a single point.[1,3,7] We emphasize that the condition (4), which is very important in obtaining the structure of the θ vacuum, does not follow unambiguously from known physical principles.

Using the condition (4), one can characterize the vacuum fields by the value of the topological charge[1]

$$n = \frac{1}{12\sqrt{2}\pi^2}\int d^3x\,\mathrm{Tr}\,\varepsilon_{ijk}A_iA_jA_k = 0, \pm 1, \pm 2, \ldots, \quad (5)$$

which labels equivalence classes of vacuum gauge fields with respect to continuous local transformations. We emphasize that in perturbation theory all the vacuum states $|n\rangle$ are equivalent and orthogonal.

In Refs. 2 and 3, on the basis of the quasiclassical approximation, it was shown that there exist transitions between different states $|n\rangle$ and $|m\rangle$, i.e.,

$$\langle n_{\mathrm{out}}|m_{\mathrm{in}}\rangle \neq 0.$$

The presence of the transitions $n \rightleftarrows m$ for $n \neq m$ indicates that the true vacuum in gauge theories must be sought in the form of a linear combination of the vectors $|n\rangle$.

We consider a gauge transformation of the form

$$g(\mathbf{x}) = (\mathbf{x}^2 - 1)/(\mathbf{x}^2 + 1) - 2i\sigma\mathbf{x}/(\mathbf{x}^2 + 1), \quad (6)$$

where σ represents the Pauli matrices.

To this transformation $g(\mathbf{x})$ there corresponds a transformation $T(g)$ on the state space with the properties[7]

$$T^+T = 1, \quad T\hat{n}T^+ = \hat{n} + 1, \quad (7)$$

where $\hat{n}$ is the operator of the topological charge.

From the requirement of invariance of the vacuum state under $\hat{T}$, i.e.,

$$T\,|\,\theta\rangle = \exp(i\theta)\,|\,\theta\rangle \quad (8)$$

we conclude that the vacuum is a superposition of the states $|n\rangle$ of the form[1)]

$$|\theta\rangle = \sum_{n=-\infty}^{n=+\infty} \exp(in\theta)|n\rangle \quad (9)$$

and it is characterized by the parameter $-\pi \leq \theta \leq \pi$.

We now consider how we can obtain a structure of the vacuum of the type (9) by using the path-integral method. The expression for the vacuum matrix element of the operator $F(A)$ has the form[10]

$$\langle F(A)\rangle = \int F(A)\exp[iS(A)]\,(DA)\Big/\int \exp[iS(A)]\,(DA). \quad (10)$$

Here $S(A) = \int d^4x\mathcal{L}(x)$, and (DA) is the measure of integration with respect to the gauge field A^a_μ with allowance for the subsidiary gauge condition and the Faddeev–Popov determinant.

However, the expression (10) is formal, since the fields A^a_μ over which one must integrate are not indicated. The gauge fields A^a_μ are characterized by the topological number[1]

$$\left.\begin{array}{l} q = \int d^4x\,\partial_\mu k_\mu = \frac{g^2}{32\pi^2}\int d^4x\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu}; \\ \tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\lambda\rho}F_{\lambda\rho}/2. \end{array}\right\} \quad (11)$$

As was shown in Ref. 1, the topological number q is integral for fields A_μ that at infinity are pure gauges, the matrix of the gauge transformation having the

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[1)]Note that in the Schwinger model, which has many features in common with non-Abelian gauge fields (topologically nontrivial configurations, anomalous axial current), the structure of the vacuum is more complicated than the expression (9). The vacuum in this model is characterized by two parameters θ_1 and θ_2.[8]

asymptotic behavior (4). Thus, if one integrates over fields with the asymptotic behavior (4) at infinity, it must be borne in mind that all such fields are decomposed into classes characterized by integral topological number q.

We now consider how the parameter θ appears in the language of path integrals. Without violating the gauge invariance, we can add to the action $\mathscr{L}(x) = -F^a_{\mu\nu}F^a_{\mu\nu}/4$ for the gauge field a term of the form $\theta(g^2/64\pi^2)\varepsilon F^a_{\alpha\beta}F^a_{\gamma\delta}$, which is a total derivative. Usually, such terms do not contribute to the path integral when one integrates over fields that decrease at infinity. However, in the case of integration over topologically nontrivial configurations the contribution of this term is nonvanishing. For the Lagrangian

$$\mathscr{L}_\theta = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \theta\frac{g^2}{64\pi^2}\varepsilon^{\alpha\beta\gamma\delta}F^a_{\alpha\beta}F^a_{\gamma\delta} \tag{12}$$

in the case of integration over fields with the asymptotic behavior (4) at infinity, the vacuum expectation value of the gauge-invariant operator $F(A)$ has the form[2,3]

$$\langle\theta|F(A)|\theta\rangle = \frac{\int F(A)\exp[iS_\theta(A)](DA)}{\int\exp[iS_\theta(A)](DA)} = Z_\theta^{-1}\sum_{q=-\infty}^{q=+\infty}\exp(i\theta q)\int F(A)\exp[iS(A)](DA)_q, \tag{13}$$

where

$$S(A) = \int d^4x\left(-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right);\quad S_\theta(A) = \int d^4x\mathscr{L}_\theta(x);\quad Z_\theta = \sum_{q=-\infty}^{q=+\infty}\exp(i\theta q)\int\exp[iS(A)](DA)_q;$$

and $(DA)_q$ is the measure of integration over the fields with topological number q.

To calculate the path integral (13) over a topologically nontrivial sector, one uses a quasiclassical method, which is as follows.[10] The fields A^a_μ are represented in the form $A^a_\mu = A^{a\,\mathrm{cl}}_\mu + A^{a\,\mathrm{qu}}_\mu$, where $A^{a\,\mathrm{cl}}_\mu$ is a solution of the Yang–Mills equations in Euclidean space-time with topological number q. Note that in Ref. 1 Belavin found a solution for $q = \pm 1$, the solution for arbitrary q being found in Refs. 11 and 12. Then the action $S(A_{\mathrm{cl}} + A^{\mathrm{qu}})$ is expanded with respect to the fields A^{qu} up to the terms quadratic in A^{qu}:

$$S(A_{\mathrm{cl}} + A^{qu}) \approx S(A_{\mathrm{cl}}) + \frac{1}{2}\int\frac{\delta^2 S}{\delta A^2}\Big|_{A=A_{\mathrm{cl}}} A_{qu}A_{qu}d^4x.$$

In this approximation, the path integral can be calculated. A detailed discussion of the problems that arise in such a method of calculating the path integral is given in Ref. 10.

We now consider how the problem of CP invariance arises in quantum chromodynamics when allowance is made for topologically nontrivial configurations in the path integral or, equivalently, the complex structure of the vacuum is taken into account.

One of the attractive features of quantum chromodynamics (without allowance for instanton effects) was the exact P and CP invariance of the theory. Indeed, the matrix m in the Lagrangian (1), which may contain CP-noninvariant γ_5 terms because of the corrections for the weak interactions, can be reduced to diagonal form by unitary transformations of the left- and right-handed quark fields.

Proof (*Ref.* 4). The mass term in the Lagrangian (1) has the form

$$\bar{q}mq = \bar{q}_L M q_R + \bar{q}_R M^+ q_L, \tag{14}$$

where M^+ is the Hermitian-conjugate matrix. We represent the matrix M in the form

$$M = M^h U, \tag{15}$$

where M^h is a Hermitian matrix, and U is a unitary matrix. By the redefinition of the right-handed quark fields

$$Uq_R = q'_R \tag{16}$$

(under this redefinition, the term describing the interaction of the quark fields with the gluon fields in not changed), the mass term $\bar{q}mq$ can be described in the form

$$\bar{q}mq = \bar{q}_L M^h q'_R + \bar{q}'_R M^h q_L. \tag{17}$$

The Hermitian matrix M^h can be reduced to diagonal form by a unitary transformation V applied to q_L and q'_R:

$$V^{-1}M^h V = \begin{pmatrix} m_u & 0 & 0 & \\ 0 & m_d & 0 & . \\ 0 & 0 & m_s & \\ & & & . \end{pmatrix}. \tag{18}$$

We finally find that as a result of the redefinition of the quark fields the mass term $\bar{q}mq$ takes the form

$$\bar{q}mq = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \ldots \tag{19}$$

However, when allowance is made for instanton effects, the above proof becomes incorrect. This occurs because a transformation of the form

$$q_{Ri} \to \exp(i\alpha)q_{Ri};\quad q_{Li} \to q_{Li} \tag{20}$$

has an anomaly, i.e., the gauge-invariant current corresponding to this transformation is not conserved in the limit $m_i = 0$.

The divergence of the current is

$$\partial_\mu J_{\mu R} = \frac{Ng^2}{32\pi^2}\operatorname{Tr}(F\tilde{F}), \tag{21}$$

where

$$J_{\mu R} = \sum_{i=1}^{M}\bar{q}_{Ri}j_\mu q_{Ri};$$

N is the number of different quark species.

Therefore, under the transformation (16), an additional term arises in the Lagrangian (1):

$$\operatorname{Arg}(\operatorname{Det}U)\,\partial_\mu J_{\mu R} = N\frac{g^2}{32\pi^2}\operatorname{Tr}(F\tilde{F})\operatorname{Arg}(\operatorname{Det}U), \tag{22}$$

which is important when allowance is made for the topologically nontrivial configurations.

It is readily seen that a term of the form $(\theta g^2/32\pi^2)\operatorname{Tr}(F\tilde{F})$ is P and CP noninvariant. Therefore, inclusion of such a term in the Lagrangian of quantum chromodynamics leads to P and CP violation in strong interactions. As will be shown in the following part of the review, the experimental bound on the neutron dipole moment indicates that the parameter θ must be less than $|\theta| \leqslant 10^{-9}$. It is therefore natural to ask why there is no strong CP violation in strong interactions, i.e., why the parameter θ is so small. The problem of explaining the absence of CP violation in quantum chromodynamics is intimately related to the problem of how the quarks acquire mass. Indeed, as will be shown below, if the theory contains at least one massless quark (the u quark), then the strong interactions will be automatically (for any choice of the parameter θ) CP invariant. In modern unified models of the weak and electromagnetic interactions, the quarks and leptons acquire masses as a result of interaction with Higgs fields. Thus, it is clear that the weak and electromagnetic interactions must have a direct bearing on the problem of CP conservation in strong interactions.

Thus, allowance for the complex structure of the vacuum in quantum chromodynamics, together with the hopes of obtaining confinement and solving the problem of $U(1)$ invariance, has led to a new serious problem—the problem of CP invariance.

3. BOUNDS ON THE PARAMETER θ IN QUANTUM CHROMODYNAMICS

As we have noted above, the Lagrangian of quantum chromodynamics

$$\mathscr{L} = -(F^a_{\mu\nu})^2/4 + \sum_k \bar{q}_k\,[i\hat{D} - m_k]\,q_k, \tag{23}$$

where $\hat{D} = \hat{\partial} - ig(\hat{A}^a/2)\lambda_a$, augmented by the CP-violating interaction

$$\mathscr{L} \to \mathscr{L} + (\theta g^2/32\pi^2)\,\mathrm{Tr}\,F\tilde{F}, \tag{24}$$

leads to CP violation in strong interactions. We shall obtain an estimate of the CP-violation parameter θ on the basis of the known experimental bound on processes with CP violation (bound on the neutron dipole moment).

The parameter which characterizes CP violation in quantum chromodynamics is

$$\langle \mathrm{vac},\ \theta|\ \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F})|\,\mathrm{vac},\ \theta\rangle \equiv k(\theta).$$

From the explicit expression for $k(\theta)$ in terms of a path integral it follows that $k(\theta)=0$ at $\theta=0$ or $\theta=\pi$ (no CP violation).

The equations for the divergence of the axial currents have the form[13]

$$\partial_\mu J^{\mu 5}_k = (g^2/32\pi^2)\,\mathrm{Tr}\,F\tilde{F} + im_k\bar{q}_k\gamma_5 q_k, \tag{25}$$

where $2J^{\mu 5}_k = \bar{q}_k\gamma_\mu\gamma_5 q_k$ is the gauge-invariant axial current. It follows from the translational invariance of the vacuum that

$$\langle \mathrm{vac},\ \theta|J^{\mu 5}_k(x)|\,\mathrm{vac},\ \theta\rangle = 0. \tag{26}$$

From Eqs. (25) and (26) we obtain

$$\langle \mathrm{vac},\ \theta|\ \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F})|\,\mathrm{vac},\ \theta\rangle + im_k\langle \mathrm{vac},\ \theta|\bar{q}_k\gamma_5 q_k|\,\mathrm{vac},\ \theta\rangle = 0. \tag{27}$$

We denote

$$\langle \mathrm{vac},\ \theta|\bar{q}_{L,k}q_{R,k}|\,\mathrm{vac},\ \theta\rangle = 0.5\psi_k\exp(i\theta_k), \tag{28}$$

where $\psi_k = \psi^*_k$ and $\theta_k = \theta^*_k$. In terms of ψ_k and θ_k, Eq. (27) becomes

$$\langle \mathrm{vac},\ \theta|\ \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F})|\,\mathrm{vac},\ \theta\rangle = \psi_k m_k\sin\theta_k,\quad k=1,\ 2,\ \ldots. \tag{29}$$

Note that (13) yields

$$\frac{\partial E(\theta)}{\partial\theta} = \langle \mathrm{vac},\ \theta|\ \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F})\,|\,\mathrm{vac},\ \theta\rangle.$$

Here, $E(\theta)$ is the shift in the energy of the vacuum associated with the introduction into the Lagrangian (23) of the θ term:

$$\exp(i\Omega E_\theta) = \frac{\int DA\exp[i\int\mathscr{L}_\theta d^4x]}{\int DA\exp[i\int\mathscr{L}d^4x]}, \tag{30}$$

where Ω is the 4-dimensional volume in which the system is contained.

In the approximation $m_k = 0$ $(k=1, 2, \ldots, L)$, the Lagrangian (24) has the chiral symmetry group $SU(L)\otimes SU(L)$.

The symmetry group associated with γ_5 transformations,

$$q \to \exp(i\omega_k\lambda_k\gamma_5)\,q, \tag{31}$$

is spontaneously broken[14] [the vacuum is not invariant under the transformations (31)]. A measure of the spontaneous breaking of the symmetry (31) is the presence of the nonvanishing vacuum expectation values

$$\langle\bar{q}_k q_k\rangle = \langle\bar{q}_i q_i\rangle,\quad i,\ k=1,\ 2,\ \ldots,\ L.$$

If the quarks have nonvanishing mass, the symmetry $SU(L)\otimes SU(L)$ is approximate. In what follows, we shall basically consider the chiral symmetry $SU(2)\otimes SU(2)$, the consequences of which agree best with the experiments, this being due to the fact that the masses of the u and d quarks are small compared with the characteristic scale of the strong interactions. In Ref. 15, on the basis of current algebra, it was shown that

$$\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -f^2_\pi m^2_\pi/[4(m_u+m_d)]. \tag{32}$$

In the lowest approximation in the masses of the u and d quarks,

$$\psi_1 = \psi_2 = -f^2_\pi m^2_\pi/[4(m_u+m_d)]. \tag{33}$$

We consider the case $\theta \ll 1$ (weak CP violation). Then

$$\theta_1 m_1 = \theta_2 m_2. \tag{34}$$

Using the equation $\theta = \theta_1 + \theta_2$, which is valid in the limit $m_u, m_d \to \theta$, we find that

$$\langle \mathrm{vac},\ \theta|\ \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F})\,|\,\mathrm{vac},\ \theta\rangle = \frac{\theta\psi_1}{1/m_u + 1/m_d} = -\frac{f^2_\pi m^2_\pi m_u m_d}{4(m_u+m_d)^2}\,\theta. \tag{35}$$

Note that in the derivation of Eq. (35) current algebra was used only in the calculation of $\langle \mathrm{vac}, \theta|uu|\mathrm{vac}, \theta\rangle$. The same result was obtained earlier[16] by a different method.

To study the spontaneous breaking of chiral symmetry, it is convenient to use the effective-action formalism.[17] In the quantum-chromodynamic Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\sum_{k=1}^{N}\bar{q}_k\hat{D}q_k,$$

which describes the interaction of massless quarks with gluons, we introduce sources bilinear in the quark fields:

$$\mathcal{L} \to \mathcal{L}_J = \mathcal{L} - J^i_j \bar{q}_{Li} q_{Rj} + \text{h.c.}$$

We introduce the generating functional

$$W(J) = \frac{1}{i} \ln \frac{\int (DA)\, d\bar{q}\, dq \exp(iS_J)}{\int (DA)\, d\bar{q}\, dq \exp(iS_{J=0})}. \qquad (36)$$

It is readily seen that

$$\varphi^i_j = \langle \bar{q}_{Li} q^j_R \rangle = \delta W / \delta J^i_j.$$

We make a Legendre transformation (going over from the variables J^i_j to the variables φ^i_j):

$$\Gamma(\varphi^i_j) = W(J^i_j) - \int J^i_j \varphi^j_i d^4x - \int (J^i_j \varphi^j_i)^* d^4x. \qquad (37)$$

It follows from the definition of the functional $\Gamma(\varphi^i_j, \varphi^{*j}_i)$ that

$$\delta\Gamma / \delta\varphi^i_j = -J^j_i. \qquad (38)$$

Spontaneous breaking of chiral invariance means that the following equation has a nontrivial solution:

$$\delta\Gamma / \delta\varphi^i_j = 0. \qquad (39)$$

Since we are interested in translationally invariant solutions of Eq. (39) (φ^i_j does not depend on x), it is convenient to represent the functional $\Gamma(\varphi^i_j, \varphi^{*j}_i)$ as follows:

$$\Gamma(\varphi^i_j, \varphi^{*j}_i) = \int [-V(\varphi^i_j, \varphi^{*j}_i) d^4x + Z(\varphi^i_j, \varphi^{*j}_i)(\partial_\mu \varphi^i_j)(\partial_\mu \varphi^{*i}_j) + \text{terms with higher derivatives}].$$

Equation (39) takes the form

$$\partial V(\varphi^i_j, \varphi^{*j}_i)/\partial\varphi^i_j = 0. \qquad (40)$$

If the quark masses are nonzero, the effective action and the effective potential are

$$\left.\begin{aligned} \Gamma_{m_i}(\varphi^i_j) &= \Gamma(\varphi^i_j, \varphi^{*j}_i) + \int \sum_{i=1}^N m_i \varphi^i_i d^4x + \int \sum_{i=1}^N m_i (\varphi^i_i)^* d^4x; \\ V_{m_i}(\varphi_i) &= V(\varphi_i) - \sum_{i=1}^N m_i \varphi^i_i - \sum_{i=1}^N m^*_i (\varphi^i_i)^*. \end{aligned}\right\}$$

In what follows, we shall seek solutions of the form $\varphi^i_j = \delta^i_j \varphi_i$ [by virtue of the $SU(N) \otimes SU(N)$ symmetry of the functional $\Gamma(\varphi^i_j, \varphi^{*i}_j)$, any solution can be reduced by a $SU(N) \otimes SU(N)$ transformation to such a form].

The effective potential depends on $|\varphi_i|$ and $\theta_i \equiv \arg\varphi_i$ as follows[2)]: $V(|\varphi_i|, \sum_{i=1}^N \theta_i - 0)$ is a periodic function of $(\sum_{i=1}^N \theta_i - 0)$ with period $T = 2\pi$, and it can be represented in the form

$$V(|\varphi_i|, \theta) = V_0(|\varphi_i|) + \cos\theta V_1(|\varphi_i|) + \cos 2\theta V_2 \times (|\varphi_i|) + \ldots,$$

a contribution to $V_0(|\varphi_i|)$ being made by configurations with topological number $|q| = 0$ and to $V_1(|\varphi_i|)$ by configurations with $|q| = 1$, etc.

The equation for determining the minimum of V is

$$\partial V / \partial |\varphi_i| = 0, \quad \partial V / \partial\theta_i = 0. \qquad (41)$$

It follows from the explicit expression for V that the second equation on (41) is satisfied for $\Sigma\theta_i = 0$ or $\sum_{i=1}^N \theta_i = \theta + \pi$.

The condition of stability of the system is

$$\left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\theta = \{{0 \atop \pi}} \geqslant 0.$$

For nonvanishing quark masses, the equations for determining the minimum take the form

$$\partial V / \partial |\varphi_i| - 2m_i \cos\theta_i = 0; \qquad (42)$$

$$\partial V / \partial\theta_i + 2m_i \varphi_i \sin\theta_i = 0. \qquad (43)$$

It follows from Eq. (43) that

$$m_i \varphi_i \sin\theta_i = m_j \varphi_j \sin\theta_j. \qquad (44)$$

This equation was obtained earlier [Eq. (29)] on the basis of the equations for the axial curents. We find a solution of these equations for $\theta, \theta_i \ll 1$.

For small θ_i, Eq. (43) can be written in the form

$$k\Big(\sum_{i=1}^N \theta_i - \theta\Big) = -k_i \theta_i, \qquad (45)$$

where $k \equiv (1/2)(\partial^2 V / \partial\theta^2)|_{\theta=0}$ and $k_i \equiv m_i \varphi_i$.

The solution of Eq. (45) is

$$\theta_i = \frac{\theta}{k_i} \frac{1}{1/k + \sum_{j=1}^N 1/k_j}. \qquad (46)$$

If the quark masses are small, then $k \gg k_i$ and (46) takes the form

$$\theta_i = \frac{\theta}{k_i} \frac{1}{\sum_{j=1}^N 1/k_j}; \quad \theta_i k_i = \theta_j k_j = \theta \Big/ \sum_{i=1}^n 1/k_i, \qquad (47)$$

i.e., we obtain Eq. (35).

Note that using the effective action the σ model is obtained naturally in the lowest approximation.

When current algebra is used, it is more convenient not to consider the Lagrangian (24) but, redefining the phases of the quark fields, to reduce the CP-noninvariant term to the form[18]

$$-i\theta m_u m_d m_s (m_u m_d + m_u m_s + m_d m_s)^{-1} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s) \equiv \delta\mathcal{L}_{CP}. \qquad (48)$$

The CP-violating term (58) leads to triple vertices for pseudoscalar mesons with coupling constants[19]

$$G_{abc} = \langle 0 | \delta\mathcal{L}_{CP} | M^a M^b M^c \rangle, \qquad (49)$$

where M^a are the fields that describe the octet of pseudoscalar mesons. Using current algebra and the inequalities $m_s \gg m_u$ and $m_s \gg m_d$, we can write the expression (37) in the form

$$G_{abc} = -\theta \frac{m_u m_d}{(m_u + m_d)} F_\pi^{-3} \langle 0 | [Q^a_5, [Q^b_5, [Q^b_5, \bar{q}\gamma_5 q]]] | 0 \rangle, \qquad (50)$$

where F_π is the decay constant of the π mesons, and Q^a_5 are the corresponding $SU^A(3)$ generators.

Calculating the commutators in (50), one can obtain[19] the corresponding effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{6} G_{abc} M^a M^b M^c = -\frac{1}{6} \theta m_\pi^2 F_\pi^{-1} \frac{m_u m_d}{(m_u + m_d)^2} \mathrm{Tr} M^3. \qquad (51)$$

Here, $M = \lambda^a M^a$.

It follows from (51) in particular that the effective Lagrangian for $\eta \to \pi\pi$ decay is

$$\mathcal{L}_{\text{eff}} = -\frac{\theta m_\pi^2}{\sqrt{3} F_\pi} \frac{m_u m_d}{(m_u + m_d)^2} \pi^2 \eta. \qquad (52)$$

[2)] Such a dependence follows from the definition (37) of the effective action.

The strongest bound on the parameter θ can be obtained on the basis of the known experimental bound on the dipole moment of the neutron[20]: $d_n \leqslant 10^{-24}$ cm.

In Ref. 19, current algebra was used to calculate the neutron dipole moment[3)]: $d_n \approx 5\times10^{-16}\theta$ cm. It follows from (42) and (41) that $\theta \leqslant 2\times10^{-9}$.

A simpler estimate of θ can be obtained using the following arguments.[21] The parameter θ occurs in the CP-violating Lagrangian (36) in conjunction with the factor

$$m_u m_d m_s (m_u m_d + m_u m_s + m_d m_s)^{-1}. \quad (53)$$

Therefore, the neutron dipole moment can be estimated as

$$d_n = \frac{1}{\Lambda^2}\frac{m_u m_d m_s}{(m_u m_d + m_u m_s + m_d m_s)}\,\theta, \quad (54)$$

where Λ is the parameter characterizing the strong interaction. For $\Lambda = 200$ MeV and quark masses $m_s = 150$ MeV, $m_u = 4.2$ MeV, and $m_d = 7.5$ MeV (Ref. 22) we find that $\theta \leqslant 10^{-9}$.

We now consider what restriction on the parameter θ can be obtained on the basis of the stability of the system against a small external perturbation.[21] We add to the parameter θ in the expression (24) the source $J(x)$ and make a Legendre transformation with respect to the source $J(x)$:

$$\Gamma(\varphi) = E(\theta + J) - \int J\varphi\, d^4x; \quad \varphi = \delta E(\theta + J)/\delta J. \quad (55)$$

The equation for determining the equilibrium φ_0 is

$$\delta\Gamma/\delta\varphi|_{\varphi=\varphi_0} = 0. \quad (56)$$

The stability condition of the system at the equilibrium point has the form

$$\delta^2\Gamma(\varphi)/\delta\varphi^2|_{\varphi=\varphi_0} \geqslant 0 \quad (57)$$

or

$$\frac{\partial^2 E}{\partial\theta^2} \geqslant 0. \quad (58)$$

In the approximation of a rarefied instanton gas,[3]

$$E(\theta) = A(1 - \cos\theta), \quad A \geqslant 0. \quad (59)$$

The inequality (50) then takes the form

$$\cos\theta \geqslant 0, \quad (60)$$

i.e., $|\theta| \leqslant \pi/2$.

4. IS THE MASS OF THE u QUARK ZERO?

If the mass of at least one of the quarks (the u quark) is zero, then the theory is CP invariant for any parameter θ in the Lagrangian (9). This is readily seen by noting that by making a transformation of the quark field under which the classical Lagrangian (1) of quantum chromodynamics does not change, $u \to \exp(i\theta j_5)u$, i.e., by redefining the phases of the u and the u_L and u_R quark fields, one can eliminate the CP-noninvariant term in the Lagrangian, i.e., in this case the theory depends trivially on the parameter θ.

In the path-integral language, the absence of CP violation is due to the circumstance that the Dirac operator $\hat{D} = \hat{\partial} - ig\hat{A}$ in an external, topologically nontrivial field has normalizable zero modes (tunneling is suppressed), and[23]

$$q = n_R - n_L,$$

where n_R is the number of right-handed normalizable modes of the operator $\hat{D}$, n_L is the number of left-handed normalizable modes, and q is the topological number. Therefore, the topological nontrivial configurations contribute to only the chirality-changing Green's functions.

The fact that the theory for $m_u = 0$ in CP invariant can also be understood by the following considerations. The parameter which determines the CP violation in quantum chromodynamics is the vacuum expectation value of the operator $(g^2/32\pi^2)\,\mathrm{Tr}(F\tilde{F})$.

The divergence of the gauge-invariant current $\sqrt{2}J^5_\mu = \bar{u}j_\mu j_5 u$ is

$$\partial_\mu J^5_\mu = (g^2/32\pi^2)\,\mathrm{Tr}(F\tilde{F}). \quad (61)$$

From the condition of translational invariance of the vacuum in the gauge-invariant sector we have

$$\langle \mathrm{vac}|J_{\mu 5}|\mathrm{vac}\rangle = 0. \quad (62)$$

Using Eqs. (62) and (61), we obtain

$$\left\langle \mathrm{vac}\left|\frac{g^2}{32\pi^2}\,\mathrm{Tr}(F\tilde{F})\right|\mathrm{vac}\right\rangle = 0, \quad (63)$$

i.e., the absence of CP violation in the case of a massless u quark.

The quark masses can be calculated by using the equations of motion for the axial current. The equations have the form

$$\partial_\mu A^\alpha_\mu = -i[Q^\alpha_A, H_m(0)]. \quad (64)$$

where $H_m = \bar{m}_u uu + \bar{m}_d dd + \bar{m}_s ss$; $A^\alpha_\mu = \bar{q}\lambda_a j_\mu j_5 q$, $q = (u, d, s)$; λ_a are the Gell-Mann matrices; and Q^α_A are the generators of $SU(3)$ axial transformations of the form $q \to \exp(i\omega_i\lambda_i j_5)q$.

Taking the matrix elements of Eq. (64) between the vacuum and the single-meson states, we obtain

$$m^2_\pi f_\pi = Z^{1/2}_\pi \frac{m_u + m_d}{2}; \quad m^2_{K^+} f_K = Z^{1/2}_K \frac{m_u + m_s}{2},$$
$$m^2_{K^0} f_K = Z^{1/2}_K \frac{m_d + m_s}{2} \quad (65)$$

where $Z^{1/2}_\pi = \langle 0|\nu^i|\pi\rangle$, $i = 1, 2, 3$; $Z^{1/2}_K = \langle 0|\nu_i|K\rangle$, $K = 4, 5, 6, 7$; $\nu^i = \bar{q}\lambda^i j_5$; f_π and f_K are the constants of the weak decays of the π meson and K meson, respectively: $(\langle 0|A^l_\mu|\pi_l\rangle = if_\pi P_{\mu\pi}$, $l = 1, 2, 3$; $\langle 0|A^l_\mu|K_l\rangle = if_K P_{\mu K}$, $l = 4, 5, 6, 7)$. From the relations (65), we obtain

$$\left.\begin{aligned}
\frac{m_d}{m_u} &= \frac{m^2_{\pi^0} - (\Delta m^2_K)_{u_3}(f_K Z^{1/2}_\pi / f_\pi Z^{1/2}_K)}{m^2_{\pi^0} + (\Delta m^2_K)_{u_3}(f_K Z^{1/2}_\pi / f_\pi Z^{1/2}_K)};\\
\frac{m_s}{m_d} &= -\frac{(f_K Z^{1/2}_\pi / f_\pi Z^{1/2}_K)\,[2m^2_{K^0} - (\Delta m^2_K)_{u_3} - m^2_{\pi^0}]}{(f_K Z^{1/2}_\pi / f_\pi Z^{1/2}_K)(\Delta m^2_K)_{u_3} - m^2_{\pi^0}},
\end{aligned}\right\} \quad (66)$$

where $(\Delta m^2_K)_{u_3}$ is the square of the mass difference of

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[3)]A similar result was obtained in Ref. 16, in which a bag model was used.

the K^+ and K^0 mesons without electromagnetic corrections.

It was shown in Ref. 24 that in the limit of $SU(3) \otimes SU(3)$ symmetry

$$(m_{K^+}^2 - m_{K^0}^2)_\gamma = (m_{\pi^+}^2 - m_{\pi^0}^2)_\gamma. \quad (67)$$

Here, the subscript γ denotes the square of the mass difference due to the electromagnetic interaction. Since the mass difference of the π^+ and π^0 mesons is due solely to the electromagnetic interaction, $(\Delta m_K^2)_{u_3} = m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2 = -0.0053$ GeV^2. If we assume that $SU(3)$ symmetry holds, then

$$f_\pi / Z_\pi^{1/2} = f_K / Z_K^{1/2}. \quad (68)$$

In this case, we arrive at the following expressions for the ratios of the quark masses[22]:

$$\left.\begin{aligned} \frac{m_d}{m_u} &= \frac{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2}{m_{K^+}^2 - m_{K^0}^2 + 2m_{\pi^0}^2 - m_{\pi^+}^2} = 1.8; \\ \frac{m_s}{m_d} &= \frac{m_{K^0}^2 + m_{K^+}^2 - m_{\pi^+}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^0}^2} = 20.1. \end{aligned}\right\} \quad (69)$$

Immediately after the problem of CP conservation in quantum chromodynamics had been noted, many authors[25-29] investigated the possibility $m_u = 0$.

As was shown in Ref. 25, assuming $m_u = 0$, we obtain strong breaking of $SU(3)$ symmetry: $Z_\pi \neq Z_K$. Indeed, if we set $m_u = 0$ in Eqs. (66) and assume that (67) holds, then

$$\left.\begin{aligned} & m_s/m_d = -1 - m_{K^0}^2/(\Delta m_K^2)_{u_3} = 46.8; \\ & Z_K^{1/2} f_\pi / Z_\pi^{1/2} f_K = -(\Delta m_K^2)_{u_3}/m_{\pi^0}^2 = 0.36. \end{aligned}\right\} \quad (70)$$

Note that for such a ratio of the masses of the s and d quarks the predictions for the $SU(2)$ mass splitting of the baryons are virtually identical to the predictions obtained for $Z_K^{1/2} f_\pi = Z_\pi^{1/2} f_K$, and they agree reasonably with the experiments. However, as was shown in Ref. 26, such strong breaking of $SU(3)$ symmetry ($Z_K^{1/2} f_\pi / (Z_\pi^{1/2} f_K) = 0.36$) is in contradiction with the nonrenormalization theorems.[30] In Refs. 27 and 28, another possibility in which one can still take $m_u = 0$ was investigated, but this leads to a mass difference for the $\Delta I = 1$ baryon splittings which is too large.

5. AXION MODELS

In the modern models of the weak and electromagnetic interactions, the masses of the quarks and the leptons arise as a result of spontaneous symmetry breaking through the interaction of Higgs fields with the quark and lepton fields. It was shown in Ref. 31 that in models in which there is an additional chiral $U(1)$ symmetry group the problem of CP conservation is solved automatically even for nonvanishing quark masses. The price to be paid for the resolution of the CP problem in this case is the appearance in the theory of a light pseudoscalar meson (an axion).[32,33]

We consider the simplest model in which the mass of the quark q is produced by interaction with a complex scalar field φ:

$$\mathcal{L}_{q\varphi} = h\bar{q}_L q_R \varphi + h\bar{q}_R q_L \varphi^+ + V(\varphi, \varphi^+), \quad (71)$$

where $V(\varphi, \varphi^+) = \lambda(\varphi^+\varphi - \varphi_0^2)^2$, the quark q interacting with the gluon field A_μ^a in the usual manner:

$$\mathcal{L}_{qA} = -(F_{\mu\nu}^a)^2/4 + i\bar{q}\hat{D}q + (\theta g^2/32\pi^2)\,\mathrm{Tr}\,(F\tilde{F}).$$

It is readily seen that the classical Lagrangian of the model is invariant under transformations of the form

$$q_L \to q_L \exp(-i\alpha);\ q_R \to q_R \exp(i\alpha);\ \varphi \to \varphi \exp(-2i\alpha). \quad (72)$$

However, the effective potential $V_\theta(\varphi, \varphi^+)$ of the model (71) is not invariant under the transformation $\varphi \to \varphi \exp(i\theta)$ when instanton effects are taken into account. On the basis of the definition of the effective potential, one can show that

$$V_\theta(\varphi, \varphi^+) = V_{\theta=0}(\varphi \exp(2i\theta),\ \varphi^+ \exp(-2i\theta)).$$

At the point of the minimum of the effective potential

$$\partial V_\theta / \partial\varphi = \partial V_\theta / \partial\varphi^+ = 0;\ \varphi_{\min} = |\varphi_{\min}| \exp(-2i\theta),$$

and therefore there is no CP violation at the point of the minimum.

This result can also be understood on the basis of the following arguments. The gauge-invariant current

$$J_\mu(x) = \bar{q}\gamma_\mu\gamma_5 q/2 - i(\varphi^+\partial_\mu\varphi - \varphi\partial_\mu\varphi^+),$$

which corresponds to the transformation (72), has the anomaly

$$\partial_\mu J_\mu(x) = (g^2/32\pi^2)\,\mathrm{Tr}\,(F\tilde{F}). \quad (73)$$

It follows from Eq. (73), and also from the translational invariance of the vacuum, that

$$\left\langle \mathrm{vac},\ \theta \left| \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F}) \right| \mathrm{vac},\ \theta \right\rangle = 0, \quad (74)$$

i.e., there is no CP violation in the model.

The vacuum expectation value $\langle \mathrm{vac} | \varphi | \mathrm{vac} \rangle$ is not invariant under the transformations (72), so that if the symmetry (72) were exact it would follow by Goldstone's theorem that there is a massless particle in the theory. However, when instanton effects are taken into account, the symmetry (72) is no longer exact, and the pseudoscalar $a(x) = -[\varphi(x) - \varphi^+(x)]/2i$ acquires a mass:

$$m_a^2 = \frac{\partial^2 V}{\partial\varphi^2}\bigg|_{\varphi=\varphi_0} \sim \int \left\langle \mathrm{vac} \left| \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F}(x)), \frac{g^2}{32\pi^2}\,\mathrm{Tr}\,(F\tilde{F}) \right| \mathrm{vac} \right\rangle d^4x. \quad (75)$$

The Lagrangian of the interaction of the axion field $a(x)$ with the quark field q is

$$\mathcal{L}_{qa} = ih\bar{q}\gamma_5 q a.$$

A more realistic model[32,33] is based on the use of the gauge group $SU(2) \otimes U(1)$ of the weak and electromagnetic interactions with two isodoublets of scalar fields. The Lagrangian of the interaction of the scalar isodoublets φ_1 and φ_2 with the quark fields u and d is chosen in the form

$$\mathcal{L}_I = g_1 \overline{(u\,d)}_L \varphi_1 u_R + g_2 \overline{(u\,d)}_L \varphi_2 d_R + \text{h.c.} + V(\varphi_1, \varphi_2), \quad (76)$$

where

$$V(\varphi_1, \varphi_2) = V(\varphi_1 \exp(i\alpha),\ \varphi_2 \exp(i\beta));\ \overline{\varphi_2} = i\tau_2\varphi_2.$$

The Lagrangian of the model is invariant under the transformations

$$\left.\begin{aligned} & u_R \to u_R \exp(i\alpha);\ \varphi_1 \to \varphi_1 \exp(-i\alpha); \\ & d_R \to d_R \exp(i\beta);\ \varphi_2 \to \varphi_2 \exp(-i\beta), \end{aligned}\right\} \quad (77)$$

which leads to automatic CP invariance for any θ.

We denote the vacuum expectation values by $\langle\varphi_2\rangle = v_2$ and $\langle\varphi_1\rangle = v_1$. The field $\alpha(x) = -\sin\lambda\,\mathrm{Im}\,\varphi_1^0 + \cos\lambda\,\mathrm{Im}\,\varphi_2^0$; with $\tan\lambda = v_2/v_1$, interacts with the quark and lepton fields:

$$\mathscr{L}_{\mathrm{int}} = i2^{1/4}G_F^{1/2}\alpha(x)\,[\mathrm{tg}\,\lambda \sum_{q=2/3} m_i\bar{q}_i\gamma_5 q_i + \mathrm{ctg}\,\lambda \sum_{q=-1/3} m_i\bar{q}_j\gamma_5 q_j + \mathrm{ctg}\,\lambda \sum_{Q=-1} m_R\bar{e}_R\gamma_5 e_R]. \quad (78)$$

A rough estimate[32] of the axion mass gives $m_a \approx G_F^{1/2}\Lambda^2 = 100\ \mathrm{keV}\cdot 10^{\pm 1}$. Estimates based on current algebra[33] give $m_a = 23\,N/\sin(2\lambda)$ keV (N is the number of quarks with charge $q = 2/3$). If the axion mass is $m_a < 2m_l$, where m_l is the electron mass, then the axion will decay mainly into 2γ, and

$$\Gamma(a \to \gamma\gamma) = \frac{\alpha^2 G_F\sqrt{2}}{9\pi^3}\frac{N^2}{\sin^2 2\lambda}m_a^3.$$

But if $m_a > 2m_l$, then the main decay mode of the axion will be decay into an electron–positron pair, and

$$\Gamma(a \to e^+e^-) = (G_F\sqrt{2}/8\pi)\,m_a m_l^2\,\mathrm{ctg}\,\lambda\,(1 - 4m_l^2/m_a^2)^{1/2}.$$

An extensive literature[32-52] has been devoted to questions related to the axion. However, it was shown in Refs. 38, 40, 43, and 46 that the existence of a light axion with mass less than a few mega-electron-volts contradicts the experiments.

It was shown in Ref. 40 that the existence of an axion with mass $140\ \mathrm{keV} < m_a < 2m_e$ contradicts reactor data. The point is that a light axion produced in a nuclear reactor could initiate deuteron disintegration, $a + D \to n + p$, or the Compton effect $a + e \to \gamma + e$. Using the bounds on the cross sections of these processes, Ellis and Gaillard[40] arrived at the above restriction. On the other hand, in a beam-dump experiment axions can be produced by mixing with π^0 and η, whose production cross sections are well known.

The parameter that determines the mixing of the axion with the π^0 meson is

$$\xi_\pi = \xi\left[\left(\frac{3m_d - m_u}{m_d + m_u}\right)\mathrm{tg}\,\lambda - \left(\frac{3m_u - m_d}{m_u + m_d}\right)\mathrm{ctg}\,\lambda\right].$$

$$\xi = \frac{1}{4}2^{1/4}G_F^{1/2}f_\pi.$$

In Ref. 40, in which the parameter ξ_π, which determines the a–π mixing, was used, it was shown that

$$\sigma(pp \to a + x)\,\sigma(a + p \to x) \geqslant O(10^{-65})\ \mathrm{cm}^4. \quad (79)$$

This product of the cross sections exceeds by more than two orders of magnitude the experimental upper bound $O(10^{-67})$ cm^4.

It follows that the existence of an axion with mass $m_a < 2m_e$ contradicts experiments.[40]

In Ref. 45 it was found that the axion mass satisfies $m_a > 200$ keV on the basis of models of red supergiants (from the very existence of supergiants), In Ref. 37, a model with a heavy quark q that does not participate in the usual $SU(2)\otimes U(1)$ weak interactions was considered. The heavy quark q gets its mass through interacting with a charged scalar field φ, the Lagrangian of the interaction of the quark q with the scalar field φ having the form (71). The model predicts the existence of absolutely stable baryons containing the new quarks.

The axion mass, $m_a \sim \Lambda^2/\varphi_0$, is small in this model ($m_a \approx 1$–10 eV for $m_q = 100$ GeV). The Lagrangian which describes the interaction of the axion with the gluon field has the form

$$\mathscr{L}_{aA} \sim \frac{a(x)}{\varphi_0}\frac{g^2}{32\pi^2}\mathrm{Tr}\,(F\tilde{F}).$$

The smallness of this interaction is achieved by choosing a large vacuum expectation value.

6. MODELS WITH DISCRETE SYMMETRY GROUP

To explain the small CP violation in quantum chromodynamics, models of the weak and electromagnetic interactions were based in Refs. 49 and 53 on the gauge groups $SU_L(2)\otimes SU_R(2)\otimes U(1)\otimes U(1)$ and $SU_L(2)\otimes SU_R(2)\otimes U(1)$, respectively. The subscripts L and R mean that the corresponding $SU(2)$ gauge field interacts only with left- or right-handed fermion isodoublets, respectively. Before the spontaneous breaking, the discrete symmetry $L \rightleftarrows R$ is imposed. This symmetry means that the parameter of CP violation is zero before the spontaneous breaking.

The interaction of the scalar fields with one another and with the fermions is chosen to make the mass matrix for the quarks at the tree and single-loop level satisfy the condition

$$\mathrm{Det}\,M^{\pm} = (\mathrm{Det}\,M^{\pm})^*, \quad (80)$$

where M^+ and M^- are the quark mass matrices for the quarks with charges $Q = 2/3$ and $Q = -1/3$, respectively. If the condition (80) is satisfied, there is no renormalization of the parameter θ at the tree and single-loop level. The renormalizaton of the parameter θ at the two-loop level in these models is $\delta\theta \approx 10^{-12}$, which does not contradict the inequality (58). In these models the CP violation is superweak.[54]

We mention also the model of Ref. 51, which is based on the use of the gauge symmetry group $SU(2)\otimes U(1)\otimes U(1)$ of the weak and electromagnetic interactions. The CP violation in this model arises as a result of spontaneous symmetry breaking (the vacuum expectation values of the scalar fields are complex). The CP-noninvariant interaction is transmitted by a superheavy boson of the $U(1)'$ group. The scalar fields are chosen to ensure that at the tree and single-loop level there is no renormalization of the parameter θ.

7. OTHER POSSIBILITIES FOR EXPLAINING *CP* INVARIANCE IN QUANTUM CHROMODYNAMICS

In Ref. 56, a model was proposed that makes it possible to solve the CP problem by extending the gauge symmetry group of the strong interactions to the group G_s. The gauge symmetry group of the strong, weak, and electromagnetic interactions in this model is $G_s\otimes SU(2)\otimes U(1)$, where $SU(2)\otimes U(1)$ is the usual symmetry group of the electroweak interactions.

The Lagrangian describing the interaction of the quark fields u_α and D^α (α is the group index of the color group G^s) with the scalar isodoublets of the group $SU(2) \otimes U(1)$ is identical to the corresponding Lagrangian for the Weinberg–Wilczek model[32,33] and has the form

$$\mathcal{L}_{\rm int} = g_u\,(\overline{u_L^\alpha} D_L^\alpha)\,\phi_u u_R^\alpha + g_d\,(\overline{u_L^\alpha} D_L^\alpha)\,\tilde{\phi}_d D_R^\alpha + \text{h.c.} + V(\phi_u,\ \phi_d), \qquad (81)$$

where

$$V(\phi_u,\ \phi_d) = V(\phi_u \exp(i\alpha),\ \phi_d \exp(i\beta)).$$

The Lagrangian (81) has chiral $U(1)$ symmetry, and therefore CP invariance is ensured automatically. The introduction of the larger symmetry group G_s of the strong interactions makes it possible to obtain a heavy axion. The group G_s is broken by means of the Higgs mechanism to $SU^c(3) \otimes G_m$ (G_m is a semisimple group). The CP-noninvariant parameter θ occurs in the Lagrangian in the standard manner:

$$\mathcal{L}_\theta = \mathcal{L} + \frac{\theta}{32\pi^2}\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu},$$

where $F_{\mu\nu}$ is the field tensor of the gauge fields of the group G_s. The instantons corresponding to the group G_m lead to the appearance of an additional contribution to the axion mass,

$$m_a^2 \sim g_u g_d \Lambda_m^2,$$

where Λ_m is the characteristic scale of the strong interactions of the group G_m, and in order of magnitude it is equal to the vacuum expectation value of the scalar fields responsible for the spontaneous breaking:

$$G_s \to SU^c(3) \times G_m.$$

For example, if $\Lambda_m \approx 10$ TeV, the axion mass is $m_a \approx 200$ MeV. The existence of such an axion does not contradict experimental data. Such a model predicts the existence of new pseudoscalar mesons, "metapion," with mass $m_\pi \approx 10$–100 GeV. [56]

8. THE PROBLEM OF *CP* CONSERVATION IN TWO- AND THREE-DIMENSIONAL MODELS

It is well known that in two-dimensional models with an Abelian gauge group (two-dimensional quantum electrodynamics, the Higgs model) a complex structure of the vacuum also arises. In such models, Wilson's criterion for the confinement of classical external charges is

$$\left\langle \exp(iq) \oint A_\mu(x)\,dx^\mu \right\rangle = \exp[-E(R)\,T],\quad T \gg R,$$

where $E(R)$ is the potential energy of the interaction of the classical charges. By virtue of the equation

$$\oint A_\mu(x)\,dx^\mu = \frac{1}{2}\int \varepsilon_{\mu\nu} F_{\mu\nu}\,d^2x$$

the potential energy $E(R)$ is intimately related to the energy of the θ energy. Namely, $E(R) = R[E_\theta - E_{\theta+(q/e)2\pi}]$. It can be seen from this that for external fractional charges the model possesses the confinement property, but for integral charges there is no confinement. Note that in the case $E_\theta = 0$ (Schwinger model) the classical external charges can be in a free state. Therefore, one can say that in two dimensions the requirements of confinement for classcal charges and natural CP invariance are mutually exclusive.

We now consider how matters stand with the problem of CP invariance in three-dimensional space-time. It was shown in Ref. 55 that in the Georgi–Glashow model there is confinement. The action in this model is

$$S = \int d^3x \left[\frac{1}{4e^2}\mathbf{F}_{\mu\nu}^2 + (\nabla_\mu \varphi)^2 + \lambda(\varphi^2 - \eta^2)\right];$$

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + \mathbf{A}_\mu \times \mathbf{A}_\nu;\ \nabla_\mu \varphi = \partial_\mu \varphi + \mathbf{A}_\mu \times \varphi.$$

The "instantons" in this model are monopoles, which have a long-range interaction. The action for a monopole–antimonopole-type configuration with allowance for the interaction between the monopoles is

$$S = \frac{m_W}{e^2}\,\varepsilon\left(\frac{\lambda}{e^2}\right)\sum_a q_a^2 + \frac{\pi}{2e^2}\sum_{a \neq b}\frac{q_a q_b}{|\mathbf{x}_a - \mathbf{x}_b|}.$$

In this model, the topological charge is

$$Q = \int \partial_\mu H_\mu\,d^3x;\ H_\mu(x) = \varepsilon_{\mu\nu\lambda}\varphi \mathbf{F}_{\mu\nu}\frac{1}{m_W};\ m_W = e\langle\varphi\rangle.$$

If we define in this case the generalized action

$$S_\theta = S + i\theta \int Q(x)\,d^3x$$

and consider what is the dependence on θ, then[55] when allowance is made for the interaction between the monopoles the generating functional

$$Z_\theta = \int \exp[iS_\theta]\,DA\,d\varphi$$

does not depend on θ, i.e., the problem of CP violation does not arise in this model when allowance is made for the monopole–monopole interaction. The monopole–antimonopole plasma is electrically neutral, which leads to an absence of a dependence on θ for the physical quantities. It is possible that such a mechanism could also work in four dimensions, but this question requires additional study.

CONCLUSIONS

Summarizing, we can say that the problem of CP conservation in quantum chromodynamics has not yet, in our view, found an unambiguous convincing solution. In listing the ways in which this problem could be solved, we should also bear in mind that this status of the θ vacuum itself in quantum chromodynamics is not entirely clear. It is possible that allowance for field configurations responsible for quark confinement will give the key to the solution of the problem of CP conservation.

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