A field theory of gravitation and new notions of space and time

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A field theory of gravitation is constructed on the basis of two principles: 1. The existence of conservation laws for the matter and the gravitational field, which is achieved by choosing a pseudo-Euclidean geometry as a natural geometry for the gravitational field. 2. A geometrization principle (identity principle), which asserts that the equations of motion for matter under the influence of the gravitational field in the pseudo-Euclidean space-time can be represented identically as the equations of motion of matter in some effective Riemannian space-time with metric tensor that depends on the gravitational field and on the metric tensor of the pseudo-Euclidean space-time.

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INTRODUCTION

In all physical theories that describe the different forms of matter, one of the most important characteristics of the field is the energy–momentum tensor density, which is usually obtained by varying the Lagrangian L with respect to the components g_{in} of the metric tensor of space-time:

$$T^{in} = -2\delta L/\delta g_{in}$$

This characteristic reflects the existence of a field: If the energy–momentum tensor density vanishes in an infinitesimally small region of space, then the field also vanishes in this region. Further, the energy and momentum of any physical field contributes to the total energy–momentum tensor of the system and does not vanish identically outside the source of the field. This last circumstance makes it possible to study the transport of energy by waves in the spirit of Faraday and Maxwell, namely, one can study the distribution of the field intensity in space, determine the energy fluxes through a surface, calculate the change in the energy and the momentum in emission and absorption processes, and so forth.

In the general theory of relativity, the gravitational field does not have the properties inherent in other physical fields, since it does not possess such a characteristic. By virtue of Einstein's equations, the density of the total symmetric energy–momentum tensor of a system consisting of the gravitational field and matter (we regard as matter all the matter fields except the gravitational) is strictly zero:

$$T^{in} + t^{in} = 0, \tag{A}$$

where T^{in} is the symmetric energy-momentum tensor density of the matter, and

$$t^{in} = -(\sqrt{-g}/8\pi)\,[R^{in} - (1/2)\,g^{in}R].$$

It also follows from (A) that all components of the symmetric energy–momentum tensor density t^{in} of the gravitational field vanish everywhere outside the matter. Therefore, in the expression (A) the tensor t^{in} characterizes the geometry within the matter, whereas the tensor T^{in} characterizes the matter.

Thus, it follows already from the above results that in Einstein's general relativity the gravitational field does not have properties inherent in other physical fields, since outside the source it does not possess an energy–momentum tensor, which is a fundamental physical characteristic.

In Einstein's theory, the gravitational field is characterized by the curvature tensor R^{i}_{nlm}. We owe the clear recognition of this fact to Synge (see p. VIII in Ref. 1): "If we accept the idea that space-time is a Riemannian four-space (and if we are relativists we must), then surely our first task is to get the feel of it just as early navigators had to get the feel of a spherical ocean. And the first thing we have to get the feel of is the Riemann tensor, for it *is* the gravitational field. Yet, strangely enough, this most important element has been pushed into the background.... " Further, Synge notes: "... In Einstein's theory, either there *is* a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with an observer's world-line... "

However, this characteristic of the gravitational field (the curvature tensor) reflects rather the ability of the gravitational field to change the energy and momentum of the matter, i.e., it reflects the forces exerted by the gravitational field on the matter in accordance with the equation[2]

$$\delta^2 n^i/\delta s^2 + R^{i}_{mkl} u^m u^l n^k = 0,$$

where $u^i = dx^i/ds$ is the 4-vector of the velocity, and n^i is the infinitesimally small vector of the geodesic deviation. But the description by means of curvature waves gives no information about the energy flux transported by the waves.

Thus, Einstein's general relativity links together matter and the gravitational field, the former being characterized, as in all physical theories, by the energy–momentum tensor, i.e., a tensor of second rank, while the latter is characterized by the curvature tensor, which is of fourth rank. It follows directly from this that in principle general relativity does not have conservation laws connecting the matter and the gravitational field. Thus, general relativity was created at the price of dispensing with conservation laws for the matter and the gravitational field considered together.

Lorentz and Levi-Civita suggested that the quantities

$$t^{in} = -2\delta L_g/\delta g_{in} = -(\sqrt{-g}/8\pi)\,[R^{in} - (1/2)\,g^{in}R] \tag{1}$$

should be regarded as the components of the energy–momentum tensor density of the gravitational field and the expression (A) as a somewhat unusual conservation law for the total energy–momentum tensor density.

The conservation law (A) is unusual in that it is a local conservation law—from the change in the energy–momentum tensor of the matter at a particular point one can determine the change in the energy–momentum tensor of the gravitational field at the same point:

$$\partial T^{0i}/\partial t = -\partial t^{0i}/\partial t. \tag{2}$$

It should be emphasized that in Einstein's theory changes in the energy and momentum of matter are related directly to only the change in the scalar curvature R and the second-rank tensor R^{in} in the region occupied by the matter. The result of this direct connection is the unusual conservation law (2).

Curvature waves, which are described by the fourth-rank tensor R^{i}_{nlm}, are not directly related in general relativity to the changes in the energy and momentum of the matter but only indirectly, through the metric tensor g_{in}. Therefore, the curvature waves in general relativity are not associated with any conservation laws that link the change in the energy–momentum tensor of the matter (a tensor of second rank) to the change in the curvature tensor (a tensor of fourth rank).

The introduction of a conservation law on the basis of the expression (A) did not satisfy Einstein. He wrote (see p. 645 in Ref. 3): "...of course, one cannot advance a *logical* objection against *such a designation.* However, I find that it is impossible to deduce from Eqs. (A) the consequences that we are accustomed to draw from conservation laws. This is due to the circumstance that in accordance with (A) the components of the tensor of the *total energy* vanish everywhere" [our italics].

Einstein emphasizes further that in accordance with (A) a material system could dissolve completely and leave not a trace behind, since its energy (A) vanishes.

Einstein correctly notes that one cannot deduce from Eq. (A) the consequences that one is accustomed to draw from conservation laws, but we are concerned here not with a *designation* but with the essence of the general theory of relativity.

As is shown in Ref. 5, Einstein's attempts[3,4] to obtain in general relativity conservation laws by the introduction of noncovariant pseudotensors do not solve the problem, since the energy–momentum pseudotensors are not energy characteristics of the gravitational field and are in no way related to the existence of curvature waves. Therefore, the calculation of energy loss by a source and the determination of energy fluxes of gravitational waves using any of the energy–momentum pseudotensors are devoid of physical meaning.

Nevertheless, the energy–momentum pseudotensors have been used for more than 60 years in various energy calculations in the general theory of relativity, although they have as much bearing of the gravitational field (characterized by a fourth-rank tensor, the curvature tensor R^{i}_{nlm}) in Einstein's theory as last year's snow has on the mystery of the Tungusska meteorite.

Why did this happen? Is there some dogmatism here? Perhaps there is; for as Cicero said: "The authority of teachers can often hinder those who wish to learn." Indeed, the recognition of the fundamental fact that in the general theory of relativity there is no unified conservation law for the matter and the gravitational field has taken a long time.

However, as Helvétius wrote: "The truth may be hidden by confusion for a while, but its light will break through the clouds sooner or later."

It is also shown in Ref. 5 that the well-known formula for calculating the total energy radiated by weak gravitational waves,

$$-dE/dt = (G/45c^5)\,\dddot{D}^2_{\alpha\beta}, \tag{3}$$

which was first obtained by Einstein, does not hold in the general theory of relativity, so that its use in Einstein's theory is invalid. Some authors[6] have noted that Eq. (3) is not derived consistently in the general theory of relativity. Others,[7-8] meeting such objections, assert that the rigor in the derivation of Eq. (3) in general relativity exceeds the rigor with which many other questions in mathematical physics are analyzed to the general satisfaction of physicists.

However, as is shown in Ref. 5, such an assertion is incorrect, since we are concerned here, not with mathematical subtleties, but with the essence of the general theory: Eq. (3) does not in principle follow from Einstein's theory.

Because matter is characterized by the energy–momentum tensor (a tensor of second rank), and the gravitational field by the curvature tensor (a tensor of fourth rank), there is no common conservation law linking matter and the gravitational field in Einstein's general theory of relativity.

Thus, the gravitational field in general relativity is entirely different from all other physical fields and is not a field in the spirit of Faraday and Maxwell.

Since the theories of other physical fields do have a common conservation law for the energy and momentum of the different forms of matter and there are at present no experimental data to indicate its violation (and, moreover, the development of physics has always demonstrated its unshakeable validity), we have no grounds for abandoning it.

Therefore, we shall assume that a conservation law connecting the energy and momentum of the different forms of matter must be the basis of any physical theory. Only experimental data could force us to abandon this position. The law must be valid for all matter fields, including the gravitational field.

Mathematically, the energy–momentum and angular-momentum conservation laws are a reflection of definite properties of space-time, namely, homogeneity and isotropy. There exist three types of spaces[9] which possess the properties of homogeneity and isotropy to a degree that permits the introduction of all the energy-momentum and angular-momentum conservation laws leading to all 10 integrals of the motion for a closed system: a space of constant negative curvature (Lobachevskiĭ space), a space of zero curvature (pseudo-Euclidean space), and a space of constant positive curvature (Riemannian space). The first two spaces are infinite and have infinite volume, while the third space is closed and has a finite volume but no boundaries.

Since the experimental data obtained from the study of strong, electromagnetic, and weak interactions indicate that *the geometry of space-time is pseudo-Euclidean*, it is natural to assume, at least at the present stage of our knowledge, that *this geometry is common to all physical processes, including gravitational ones. This assertion provides one of the fundamental propositions of the field approach to the theory of the gravitational interaction that we are developing*. It is obvious that it leads to fulfillment of all the energy–momentum and angular-momentum conservation laws and all the associated 10 integrals of the motion for a system consisting of the gravitational field and the remaining matter fields. In the field approach, the gravitational field, like all other physical fields, is characterized by an energy–momentum tensor, and this makes its contribution to the total energy–momentum tensor of the system. This is a fundamental difference between our approach and Einstein's general theory of relativity.

Another important principle on which the field approach to the theory of the gravitational interaction is based is the geometrization principle (identity principle), which asserts that the equations of motion of matter under the influence of the gravitational field in the pseudo-Euclidean space-time with metric tensor γ_{in} can be represented identically as equations of motion of matter in an effective Riemannian space-time with metric tensor g_{in} that depends on the gravitational field and on the metric tensor γ_{in}.

This principle was introduced and formulated by us in Ref. 10, although it was essentially already given in Ref. 11. In formulating and using this principle, we again differ fundamentally in our field approach from Einstein's theory. This principle reflects the universality of the interaction of the gravitational field with the remaining matter fields; it follows from the results of gravitational experiments and corresponds to a definite choice of the Lagrangian density of the interaction between the gravitational field and the other matter fields.

Naturally, the concept of the gravitational field as a physical field that transmits energy leads us, in combination with the identity principle, to equations to the gravitational field that differ from Einstein's equations and changes our notions of space, time, and gravitation.

It should be emphasized that the field approach to the theory of the gravitational interaction does not particularize in advance the nature of the gravitational field. We do not know what is the nature of the real gravitational field. It is possible, for example, that its adequate description requires the use of spin tensors or, say, a vector field. Only time and experimental facts will make possible a definite choice between variants of the theory.

One possible realization of such an approach, which consists of using a symmetric second-rank tensor field to describe the gravitational field, was made in Refs. 11 and 12. However, the simplest variants, for example, the quasilinear theory, were not logically consistent and required the formulation of additional conditions to ensure positive definiteness of the energy of gravitational waves. In the subsequent papers of Refs. 10 and 13 a field theory of gravitation was proposed on the basis of a symmetric tensor field of second rank whose spin structure was restricted by means of the formalism of projection operators so as to ensure that free gravitational waves have spin equal to two.

The field theory of gravitation makes it possible to describe all the currently existing experimental facts and is one way to realizing the field approach to the theory of the gravitational interaction.

The aim of the present review is to give a clearer exposition of the present variant of the theory and its presuppositions, and also to establish rather more fully the consequences of this theory. In the paper, we use a system of units in which $G=c=1$. In Secs. 1 and 2 we formulate the main propositions of the field approach for the case of a symmetric second-rank tensor field, and we also establish a number of relationships of a general nature not related to the particular choice of the Lagrangian density. In Secs. 3–6, we propose a definite realization of the above ideas. The post-Newtonian approximation of the field theory of gravitation constructed in Sec. 7, and the analysis of the experimental results made in Sec. 9, show that the field theory of gravitation enables one to describe all the currently available experimental facts. Comparison of the post-Newtonian parameters of the field theory of gravitation and Einstein's general theory of relativity shows that these two theories are indistinguishable from the point of view of all experiments made with post-Newtonian accuracy. In Secs. 9 and 10, we investigate the gravitational field of an island source with spherically symmetric motion and distribution of the matter. The conservation laws in the post-Newtonian approximation of the field theory of gravitation are considered in Sec. 11. In Sec. 12, we construct a nonstationary model of a homogeneous universe, which makes it possible to describe the cosmological red shift. Finally, we list the main conclusions of the field theory of gravitation, and also the fundamental differences between it and Einstein's general theory of relativity.

1. A PHYSICAL FIELD AND NATURAL GEOMETRY FOR IT

In any physical theory in which the field variable is a tensor quantity the form of the differential equations for the field must not depend on the choice of the coordinates in which a given process is described. This can be achieved in two ways: either by allowing the field equations to contain only covariant derivatives in the space-time metric that is natural for the process being described, or by constructing a tensor quantity from the field functions and their partial derivatives. In the latter case, the equations will be essentially nonlinear.

In the equations of general relativity, Einstein connected the metric tensor g_{in} of Riemannian space-time to matter. This led to the idea of matter influencing the metric of space-time. However, such an approach does not enable one to regard the gravitational field in general relativity as a physical field possessing energy and momentum densities. In addition, the natural geometry of the gravitational field became in general relativity the geometry of Riemannian space-time, which, in general, did not follow from any experimental facts but was, rather, a hypothesis of a definite type of self-interaction of the gravitational field. However, the self-interaction of the gravitational field need not necessarily reduce to a change in the geometry, although it may be nonlinear. In this connection, we are led to consider what is the natural geometry for the gravitational field.

Any physical field is associated with a natural geometry, so that in the absence of interaction with other fields the front of a free wave of the considered physical field moves along geodesics of the natural space-time.

The propagation of the wave front of a massless field (the equation of the characteristics),[14]

$$g^{in}\,(\partial\psi/\partial x^i)\,(\partial\psi/\partial x^n)=0, \tag{4}$$

and also the motion of free material particles (the Hamilton–Jacobi equation)

$$g^{in}\,(\partial\psi/\partial x^i)\,(\partial\psi/\partial x^n)=1 \tag{5}$$

are determined by the metric tensor of the geometry natural for these processes.

The problem of the choice of the natural geometry is the problem of the effective metric tensor used to contract the highest derivatives in the Lagrangian density. It is entirely possible, as was noted already by Lobachevskii, that different physical phenomena are described in terms of different natural geometries.

It follows from Eqs. (4) and (5) that the natural geometry of a physical theory can be determined experimentally on the basis of data on the motion of test particles and fields. Study of the motion of test particles with mass and of massless fields makes it possible to determine the metric tensor of natural space-time up to a constant factor.[15,16]

Thus, study of the motion of different forms of matter makes it possible to verify experimentally the nature of the space-time geometry of the world. As our knowledge of nature expanded, so did our notions about space and time. For example, Newton's mechanics (mechanical phenomena) in conjunction with the principle of relativity (as we now know) established that space is Euclidean and time is absolute, i.e., the same in all coordinate systems.

Subsequently, Faraday–Maxwell electrodynamics (electromagnetic phenomena) in conjunction with the principle of relativity led to the discovery of the pseudo-Euclidean geometry of the space-time of the world. For this we are in the greatest degree indebted to Minkowski. In his "Space and Time"[17] he wrote: "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." Further, he noted that: "... only the four-dimensional world in space and time is given by phenomena, but the projection in space and in time may still be undertaken with a certain degree of freedom...."

It was Minkowski who first discovered that the essence of the theory of relativity (or, as it is sometimes called, the special theory of relativity) is that the geometry of space-time is a pseudo-Euclidean geometry. The later study of strong, electromagnetic, and weak interactions has shown that, in the absence of gravitation, the natural geometry for the fields associated with these interactions is pseudo-Euclidean geometry.

Thus, Minkowski's geometry has a universal nature and in the absence of gravitation is the natural geometry for all known fields. *It follows from this that pseudo-Euclidean space-time is not a priori and given from the very beginning with an independent existence.* Its existence is inseparable from the existence of matter.

In the case of pseudo-Euclidean geometry, there is a group of coordinate transformations that leave the metric tensor of space-time invariant. This is the Lorentz group. Since the Lorentz transformations correspond to transitions between different inertial frames of reference, all field equations are invariant in such a case in inertial coordinate systems. This means that for all phenomena, fields, and interactions for which the natural geometry is pseudo-Euclidean geometry the principle of relativity holds.

In accordance with the principle of relativity,[18] "... the laws of physical phenomena are the same for an observer at rest and for an observer in a state of uniform translational motion, so that we have no means of establishing whether we are in such motion or not."

The fact that these theories are formulated in inertial frames of reference is not a restriction, since physical phenomena can be described in any other arbitrary system, including an accelerated coordinate system. In this last case, there will exist an infinite set of corresponding accelerated frames of reference in which all the physical processes will take place in the same manner.

Thus, the formulation of the theory in inertial frames of reference ensures the simplest form of description, but it in no way precludes a description of physical processes in any other admissible coordinate system. [By admissable coordinate systems we mean coordinate systems for which the quadratic form $\gamma_{\alpha\beta}dx^{\alpha}dx^{\beta}$ is negative definite and the component γ_{00} satisfies the condition $\gamma_{00} > 0$.]

One of the key questions that arises in the construction of the theory of the gravitational field concerns the nature of the interaction of the gravitational field with matter (by matter we mean all matter fields except the gravitational). When the gravitational field affects the matter, it can change its geometry if it occurs in the terms with higher derivatives in the equations of motion of the matter. Then the motion of material bodies and other physical fields in pseudo-Euclidean space-time under the influence of the gravitational field will be indistinguishable from their motion in some effective Riemannian space-time.

It follows from experiments that the gravitational field has a universal influence on matter, and therefore the effective Riemannian space-time will be common to all matter, irrespective of its form.

This leads us to an assertion that we shall call the identity principle (geometrization principle), defining it as follows: *The equations of motion of matter under the influence of the gravitational field φ_{in} in the pseudo-Euclidean space-time with metric tensor γ_{in} can be represented identically as the equations of motion of matter in an effective Riemannian space-time with metric tensor g_{in} that depends on the gravitational field φ_{in} and on the metric tensor γ_{in}.*

In such an approach, it is a consequence of the identity principle that the gravitational field φ_{in} (as a physical field) seems to be eliminated in the description of the motion of the matter and, figuratively speaking, disappears in forming the effective Riemannian space-time with metric tensor g_{in}. This means that the description of the motion of matter under the influence of the gravitational field in pseudo-Euclidean space-time is physically identical to the description of the motion of matter in the corresponding effective Riemannian space-time. It should be emphasized that the identity principle does not follow from any other physical principles. It is an independent principle, determining, on the one hand, the equivalence of the descriptions of the motion of matter and, on the other, the character of the interaction of the gravitational field with matter.

The identity principle, in particular, establishes a strict connection between the field-theoretical notion of a physical field as a carrier of energy and momentum (developed above all by Faraday and Maxwell) and Einstein's notions about Riemannian space-time.

The identity principle is the basis of the notions developed here on space, time, and gravitation. Indeed, it is in the formulation and use of this principle that our theory has its main difference from Einstein's general theory of relativity. By virtue of the identity principle, the energy and momentum of the gravitational field, which is regarded as a physical field possessing energy and momentum densities, are used to create the Riemannian space-time in which the motion of the matter is described. This means that the geometrical description of the motion of the matter is in complete agreement with the conservation laws of all the physical fields, including the gravitational field. Riemannian space-time can be regarded as a carrier of energy and momentum, and in accordance with the identity principle it contains as much energy as there is in the gravitational field.

The identity principle links together field-theoretical notions about physical fields with geometrical notions. Riemannian space-time for the motion of matter has field origin—in the literal sense, it is a manifestation of the existence of the gravitational field as a physical field possessing energy and momentum densities. The identity principle determines the nature of the interaction of the gravitational field φ_{in} with matter. In particular, it also determines the physical fact that the inertial mass of a point body is equal to its gravitationl mass.

Another key feature in the construction of the theory of gravitation is the choice of the Lagrangian density of the gravitational field.

For linear theories, the natural geometry is the geometry of flat space-time, and the theory of gravitation with linear equations of the free gravitational field is formulated in terms of flat space-time with metric tensor γ_{in}. We shall call theories of gravitation formulated in terms of flat space-time theories of class A. Theories of class A can also be nonlinear, but it is important that this nonlinearity does not occur in the terms with the highest derivatives in the field equations and, thus, does not change the geometry of the natural space-time. Thus, in theories of class A we have a single flat space-time; the Riemannian space-time, in terms of which the motion of the matter is described, is an effective space-time, and arises as a result of the influence of the gravitational field φ_{in} on the matter.

Among the theories of class A, we must mention the subclass of bimetric theories, in which the gravitational field φ_{in} in combination with the metric tensor γ_{in} forms a new field variable—the metric tensor g_{in} of the effective Riemannian space-time—in the Lagrangian density L_g of the gravitational field. The equations of motion of the matter are formulated in terms of this tensor, while the natural geometry for this field variable is the pseudo-Euclidean geometry:

$$L = L_g(\gamma_{in}, g_{in}(\gamma_{lm}, \varphi_{lm})) + L_M(g_{in}, \varphi_A).$$

An example of a nonlinear theory of this subclass is Rosen's theory[19] with Lagrangian density

$$L_g = (\sqrt{-\gamma}/64\pi)\,\gamma^{ik}g^{mn}g^{pl}\,[D_i g_{nl} D_k g_{mp} - (1/2)\,D_i g_{nm} D_k g_{pl}],$$

where γ is the determinant of the metric tensor of the flat space-time, and D_i is the covariant derivative in the flat space-time.

In bimetric theories, the gravitational field φ_{in} is actually absent, since the field variable is the metric tensor g_{in}, so that here there is no sufficiently deep physical justification of the connection between the effective Riemannian space-time and the common flat space-time.

In theories of class A, we actually have two physical space-times: the flat space-time with metric tensor γ_{in}, in terms of which the equations of the gravitational field are formulated, and the non-Euclidean space-time with metric tensor g_{in}, in terms of which the motion of the matter is formulated. Both these space-times are real observable space-times.

If in a nonlinear theory of the tensor field φ_{in} the nonlinear terms occur in the contraction of the derivatives in the Lagrangian density (in the terms with the highest derivatives in the field equations), then for such a theory a non-Euclidean space-time with an effective metric tensor $g_{in}=g_{in}(\gamma_{lm}, \varphi_{lm})$ provides the natural geometry. We shall call theories of gravitation formulated in terms of an effective Riemannian space-time theories of class B. The Lagrangian density of theories of this class has the form

$$L=L_g(g_{in}, \varphi_{in})+L_M(g_{in}, \varphi_A).$$

Theories of this class warrant special consideration, since they include theories possessing a conservation law for the total energy–momentum tensor. It should also be noted that the equations of a gravitational field in theories of class B are necessarily nonlinear.

A subclass of geometrized theories of class B is the set of theories with complete geometrization; in them, the Lagrangian density of the gravitational field depends only on the metric tensor g_{in}:

$$L=L_g(g_{in})+L_M(g_{in}, \varphi_A).$$

Einstein's theory belongs to this subclass of theories and corresponds to the special choice of the Lagrangian density in the form $L_g=\sqrt{-g}R$. In theories with complete geometrization, the flat space-time is completely eliminated from the description of the motion of both the matter and the gravitational field. Neither the gravitational field φ_{in} nor the metric tensor γ_{in} is manifested anywhere in the theory. The quantities g_{in} have in this case a double significance: They are the variables of a physical field and the metric tensor of space-time. This last circumstance has the consequence that in theories of this subclass the gravitational field is not a Faraday–Maxwell field possessing energy and momentum densities.

It should be emphasized that theories of the classes A and B are essentially different theories of gravitation. By no transformation of the field variables or the coordinates is it possible to reduce a theory of one class to a theory of the other class.

A theory leading to linear equations of the free gravitational field is the simplest variant among all theories of class A. In this case, the gravitational field is described in the same physical space-time as all the other physical fields in the absence of gravitation, and as a result the flat space-time is common to all physical fields. And although we shall consider a theory in flat space-time, nevertheless the motion of the matter can be regarded identically as motion of the matter in the pseudo-Euclidean space-time under the influence of the gravitational field, or, alternatively, the gravitational field can be eliminated and the motion of the matter considered in an effective Riemannian space-time. In what follows, we shall call this theory of gravitation a field theory of gravitation.

Thus, in a field theory of gravitation there exist two actually observable space-times. The front of a gravitational wave moves along the geodesics of the flat space-time, and therefore gravitational waves can be used to determine the geometry of the pseudo-Euclidean space-time. The front of an electromagnetic wave moves along the geodesics of the effective Riemannian space-time, and therefore electromagnetic waves and massive particles can be used to determine the geometry of this Riemannian space-time. It should be emphasized that *although a field theory of gravitation has two metric tensors g_{in} and γ_{in}, such a theory is neither conceptually nor on the basis of the field equations a member of the class of so-called bimetric theories, since in such a scheme the metric tensor g_{in} reflects only the effective influence of the gravitational field on the matter.* Note also that the equations of a field theory of gravitation can be formulated not only for inertial but also for noninertial coordinate systems, and on the transition from one noninertial coordinate system to another the field equations are form-invariant for each infinite set of noninertial coordinate systems. In the case of inertial coordinate systems, the field equations are Lorentz-invariant on the transition from one inertial system to another. This leads us to the need to extend the relativity principle, which we formulate as follows: *No physical phenomena, including gravitational phenomena, enable us to determine whether we are at rest or in a state of uniform rectilinear motion.*

We emphasize that the relativity principle does not require constancy of the velocity of propagation of the front of an electromagnetic wave, i.e., the velocity of light. It is natural that in the presence of an interaction with external gravitational fields the velocity of light, like the velocity of all bodies, will not be constant.

Such an approach makes it possible to retain in the theory all the conservation laws in their usual sense, as a result of which the propagation of curvature waves in Riemannian space-time reflects ordinary transfer of energy by gravitational waves in the pseudo-Euclidean space-time. Therefore, in our approach curvature waves in the Riemannian space-time are a direct consequence of the existence of gravitational waves in the spirit of Faraday and Maxwell, and they possess energy and momentum densities.

2. CONSERVATION LAWS FOR THE GRAVITATIONAL FIELD AND MATTER

We shall assume that the gravitational field is universal, i.e., that it acts on all forms of matter in the same way. This universality of the gravitational field is embodied in the identity principle, which gives us the possibility of representing the action of this field on matter in the form of motion of the matter in Riemannian space-time. The gravitational field itself, which possesses energy and momentum, is distinguished from the other forms of matter, and its action on itself, if there is such, is quite different from its action on matter.

This difference is manifested in the circumstance that the geometry of space-time for the gravitational field remains pseudo-Euclidean. If the free gravitational field acts on itself, its equations will be nonlinear, but this nonlinearity does not change the nature of the geometry. In the given representation, the free gravitational field can also be described by linear field equations, and then the free gravitational field has no influence on itself. Of course, the field equations in matter are always nonlinear equations.

The existence of a universal gravitational field as a physical field possessing energy and momentum densities is the fundamental feature that leads us to notions of space, time, and gravitation that differ from those in Einstein's general theory of relativity.

In the present section, we obtain a number of general assertions which hold for all local theories of class A irrespective of the particular choice of the Lagrangian density.

On the basis of the identity principle, we write the Lagrangian density of the system consisting of matter and the gravitational field in the following form for theories of class A:

$$L=L_g(\gamma_{in},\ \varphi_{in})+L_M(g_{in},\ \varphi_A), \tag{6}$$

where γ_{in} is the metric tensor of the pseudo-Euclidean space-time with signature (+, −, −, −), φ_{in} is the gravitational field, and φ_A are the remaining matter fields.

Without loss of generality, we shall assume that the metric tensor g_{in} of the Riemannian space-time is a local function that depends on the metric tensor γ_{in} of the flat space-time, the gravitational field φ_{in}, and their partial derivatives up to second order:

$$g_{lm}=g_{lm}(\gamma_{in};\ \partial_p\gamma_{in},\ \partial_{pl}\gamma_{in},\ \gamma^{in},\ \partial_p\gamma^{in},\ \partial_{pl}\gamma^{in},\ \varphi_{in},\ \partial_p\varphi_{in},\ \partial_{pl}\varphi_{in}), \tag{7}$$

where

$$\partial_{np}\varphi=\partial^2\varphi/\partial x^n\,\partial x^p.$$

We shall assume that the Lagrangian density of the matter L_M depends only on the fields φ_A, their partial derivatives of first order, and the metric tensor g_{in}. It is easy to see that in this case the Lagrangian density of the matter will contain the partial derivatives of the gravitational field up to the second order.

We shall assume that the Lagrangian density of the gravitational field depends on the metric tensor γ_{in}, the gravitational field φ_{in}, and their partial derivatives up to third order.

To obtain the conservation laws, we use the covariant method of infinitesimally small displacements. As is well known, to obtain strong conservation laws it is sufficient to have invariance of the action function under only infinitesimal transformations of the coordinates. Therefore, the strong conservation laws (which do not depend on fulfillment of the field equations) must hold for both the matter Lagrangian density L_M and the Lagrangian density L_g of the gravitational field.

Since the action J is a scalar, an arbitrary infinitesimally small transformation

$$x'^i=x^i+\xi^i(x) \tag{8}$$

of the coordinates will lead to variations of the action of the matter, δJ_M, and the gravitational field, δJ_g, that vanish.

Since the matter Lagrangian density contains both covariant and contravariant components of the metric tensor of the Riemannian space-time, we shall vary the Lagrangian density with respect to them as if they were independent, and then take into account the connection between their variations:

$$\delta g^{np}=-g^{ni}g^{pl}\delta g_{il}.$$

Then the symmetric energy–momentum tensor density of the matter in the Riemannian space-time, T^{in}, will have the form

$$T^{in}=-2\frac{\Delta L_M}{\Delta g_{in}}=-2\left(\frac{\delta L_M}{\delta g_{in}}-g^{im}g^{np}\frac{\delta L_M}{\delta g^{pm}}\right); \tag{9}$$

$$\frac{\delta L}{\delta\varphi}=\frac{\partial L}{\partial\varphi}-\partial_n\left(\frac{\partial L}{\partial(\partial_n\varphi)}\right)+\partial_{np}\left(\frac{\partial L}{\partial(\partial_{np}\varphi)}\right)-\partial_{ipn}\left(\frac{\partial L}{\partial(\partial_{ipn}\varphi)}\right)+\partial_{ilpn}\left(\frac{\partial L}{\partial(\partial_{ilpn}\varphi)}\right)-\ldots, \tag{10}$$

where $\delta L/\delta\varphi$ is the Euler–Lagrange variation.

We write the variation of the matter action integral under the transformation (8) in the form

$$\delta J_M=\int d^4x\left\{\frac{\Delta L_M}{\Delta g_{in}}\delta g_{in}+\frac{\delta L_M}{\delta\varphi_A}\delta\varphi_A+\mathrm{Div}\right\}=0, \tag{11}$$

where $\Delta L_M/\Delta g_{in}$ is determined by the expression (9), and Div denotes divergence terms whose addition leads to relations that are unimportant for the purposes of the present treatment.

The variation of the matter action integral under the transformation (8) can be written in a different form equivalent to (11). We introduce the notation

$$\Delta L/\Delta\gamma_{mn}=\delta L/\delta\gamma_{mn}-\gamma^{ns}\gamma^{mp}\delta L/\delta\gamma^{ps}, \tag{12}$$

where $\delta L/\delta\gamma_{mn}$ and $\delta L/\delta\gamma^{ps}$ are to be understood in the sense of the corresponding partial derivatives defined by the expression (10). Here, the covariant metric tensor γ_{mn} and the contravariant metric tensor γ^{mn} are to be regarded as independent tensors, since their dependence is already taken into account in (12).

Then the variation of the matter action integral under the transformation (8) takes the form

$$\delta J_M=\int d^4x\left\{\frac{\delta L_M}{\delta\varphi_{nm}}\delta\varphi_{nm}+\frac{\Delta L_M}{\Delta\gamma_{mn}}\delta\gamma_{mn}+\frac{\delta L_M}{\delta\varphi_A}\delta\varphi_A+\mathrm{Div}\right\}=0. \tag{13}$$

The variations $\delta\gamma_{mn}$, $\delta\varphi_{mn}$, $\delta\varphi_A$ and δg_{mn} resulting from the coordinate transformations (8) have the form

$$\left.\begin{aligned}&\delta\gamma_{lm}=-\gamma_{ln}D_m\xi^n-\gamma_{mn}D_l\xi^n;\\&\delta\varphi_{lm}=-\varphi_{ln}D_m\xi^n-\varphi_{mn}D_l\xi^n-\xi^nD_n\varphi_{lm};\\&\delta\varphi_A=-\xi^nD_n\varphi_A+F_{A;\,n}^{B;\,l}\varphi_BD_l\xi^n;\\&\delta g_{lm}=-g_{ln}D_m\xi^n-g_{mn}D_l\xi^n-\xi^nD_ng_{lm}.\end{aligned}\right\} \tag{14}$$

Using Eqs. (14), we can write the variation of the matter action integral (13) in the form

$$\delta J_M = \int d^4x \left\{ \xi^n \left[2D_l \left(\frac{\delta L_M}{\delta \varphi_{lm}} \varphi_{mn} \right) - D_l t^l_{Mn} - \frac{\delta L_M}{\delta \varphi_{lm}} D_n \varphi_{lm} - D_l \left(\frac{\delta L_M}{\delta \varphi_A} F^{B;\,l}_{A;\,n} \varphi_B \right) - \frac{\delta L_M}{\delta \varphi_A} D_n \varphi_A \right] + \mathrm{Div} \right\} = 0, \qquad (15)$$

where the symmetric energy–momentum tensor density of the matter in flat space-time is

$$t^{in}_M = -2\Delta L_M / \Delta \gamma_{in}; \; t^i_{Ml} = \gamma_{ln} t^{in}_M.$$

Since the displacement vector ξ^n in (15) is arbitrary, we have the identity

$$D_i t^i_{Mn} - 2D_i \left(\frac{\delta L_M}{\delta \varphi_{im}} \varphi_{mn} \right) + \frac{\delta L_M}{\delta \varphi_{lm}} D_n \varphi_{lm} + D_i \left(\frac{\delta L_M}{\delta \varphi_A} F^{B;\,i}_{A;\,n} \varphi_B \right) + \frac{\delta L_M}{\delta \varphi_A} D_n \varphi_A = 0. \qquad (16)$$

We obtain another important identity by substituting (14) in (11):

$$D_i (g_{nl} T^{il}) - \frac{1}{2} T^{lm} D_n g_{lm} = -D_i \left(\frac{\delta L_M}{\delta \varphi_A} F^{B;\,i}_{A;\,n} \varphi_B \right) - \frac{\delta L_M}{\delta \varphi_A} D_n \varphi_A. \qquad (17)$$

We now express the covariant derivatives on the left-hand side of the identity (17) in terms of the partial derivatives and the connection γ^i_{nl} of the flat space-time:

$$\gamma^i_{nl} = (1/2)\, \gamma^{is} (\partial_n \gamma_{ls} + \partial_l \gamma_{ns} - \partial_s \gamma_{ln}).$$

Noting that T^{in} is a tensor density of weight 1, we obtain

$$\partial_i (g_{nl} T^{il}) - \frac{1}{2} T^{lm} \partial_n g_{lm} = -D_i \left(\frac{\delta L_M}{\delta \varphi_A} F^{B;\,i}_{A;\,n} \varphi_B \right) - \frac{\delta L_M}{\delta \varphi_A} D_n \varphi_A.$$

But the left-hand side of this expression is the covariant divergence in the Riemannian space-time of the energy–momentum tensor density T^i_n of the matter:

$$\partial_i (g_{nl} T^{il}) - \frac{1}{2} T^{lm} \partial_n g_{lm} = \partial_i (g_{nl} T^{il}) - \Gamma^l_{in} T^i_l = \nabla_i T^i_n = g_{np} \nabla_i T^{ip},$$

where, as usual, Γ^l_{in} denotes the connection of the Riemannian space-time:

$$\Gamma^l_{in} = (1/2)\, g^{ls} (\partial_i g_{ns} + \partial_n g_{is} - \partial_s g_{in}).$$

Therefore, Eq. (17) takes the form

$$g_{in} \nabla_l T^{il} = -D_i \left(\frac{\delta L_M}{\delta \varphi_A} F^{B;\,i}_{A;\,n} \varphi_B \right) - \frac{\delta L_M}{\delta \varphi_A} D_n \varphi_A. \qquad (18)$$

Subtracting Eq. (18) from (16), we obtain

$$D_i t^i_{Mn} - 2D_i \left(\frac{\delta L_M}{\delta \varphi_{il}} \varphi_{ln} \right) + \frac{\delta L_M}{\delta \varphi_{lm}} D_n \varphi_{lm} = g_{in} \nabla_l T^{il}. \qquad (19)$$

It must be emphasized that the identity (19) is valid irrespective of the fulfillment of the equations of motion of the matter and the gravitational field.

Similarly, from the invariance of the action of the gravitational field under the transformation (8) we obtain

$$D_i t^i_{gn} - 2D_i \left(\frac{\delta L_g}{\delta \varphi_{il}} \varphi_{ln} \right) + \frac{\delta L_g}{\delta \varphi_{lm}} D_n \varphi_{lm} = 0. \qquad (20)$$

For the symmetric energy–momentum tensor density of the gravitational field, t^i_{gn}, we have, as usual,

$$t^i_{gn} = -2\gamma_{mn} \Delta L_g / \Delta \gamma_{im}.$$

It follows from (10) and (20) that

$$D_i (t^i_{Mn} + t^i_{gn}) - 2D_i \left(\frac{\delta L}{\delta \varphi_{il}} \varphi_{ln} \right) + \frac{\delta L}{\delta \varphi_{lm}} D_n \varphi_{lm} = \nabla_i T^i_n. \qquad (21)$$

If the equations of the gravitational field are satisfied,

$$\delta L / \delta \varphi_{nm} = \delta L_g / \delta \varphi_{nm} + \delta L_M / \delta \varphi_{nm} = 0, \qquad (22)$$

the expression (21) simplifies:

$$D_i (t^i_{Mn} + t^i_{gn}) = g_{in} \nabla_m T^{im}. \qquad (23)$$

Equation (23) is a manifestation of the identity principle. It follows from this equation that the covariant divergence in the pseudo-Euclidean space-time of the sum of the energy–momentum tensor densities of the matter and the gravitational field is transformed into the covariant divergence in the Riemannian space-time with metric tensor g_{in} of the energy–momentum tensor density of the matter alone. Thus, these are different forms of expression of the same relation. If the matter equations of motion are satisfied,

$$\delta L_M / \delta \varphi_A = 0, \qquad (24)$$

then the expression (16) simplifies:

$$D_i t^i_{Mn} - 2D_i \left(\frac{\delta L_M}{\delta \varphi_{il}} \varphi_{ln} \right) + \frac{\delta L_M}{\delta \varphi_{lm}} D_n \varphi_{lm} = 0, \qquad (25)$$

and a covariant conservation law in the Riemannian space-time follows automatically from (17):

$$\nabla_n T^{nm} = 0.$$

This assertion is common to all theories with geometrized Lagrangian density of the matter and is not tied to any particular variant of the theory of gravitation.

Further, we see that if the equations (22) of the gravitational field are satisfied, then (25) and (20) yield a covariant conservation equation for the total symmetric energy–momentum tensor density in the pseudo-Euclidean space-time:

$$D_i (t^i_{Mn} + t^i_{gn}) = 0. \qquad (26)$$

Thus, the gravitational field, considered in the pseudo-Euclidean space-time, behaves like all other physical fields. It has energy and momentum and contributes to the total energy–momentum tensor density of the system.

On the basis of Eq. (26) and the identity (23), we obtain

$$D_i (t^i_{Mn} + t^i_{gn}) = g_{ln} \nabla_i T^{il} = 0.$$

Thus, the covariant energy–momentum conservation law of the matter and of the gravitational field in the pseudo-Euclidean space-time is represented in the form of a conservation law for the energy–momentum tensor density of the matter alone in the Riemannian space-time with metric tensor g_{in}.

The conservation law (26) for the total energy–momentum tensor density and the conservation law in the form

$$\nabla_n T^{ni} = \partial_n T^{ni} + \Gamma^i_{lm} T^{lm} = 0 \qquad (27)$$

are, when the equations (22) of the gravitational field and the equations of motion (24) of the matter are satisfied, simply different forms of expression of the same relation. The conservation law (26) expresses the fact that in the pseudo-Euclidean space-time the total energy–momentum tensor density of the system consisting of the matter and the gravitational field is conserved. This law has the usual form of a conservation law. The conservation law (27) in the Riemannian space-time is not a conservation law in the usual understanding, since the energy–momentum tensor density T^{in} of the matter need not be conserved: $\partial_n T^{ni} \neq 0$.

As Einstein himself wrote (see p. 492 in Ref. 3): "...the presence of the second term on the left-hand side means from the physical point of view that for the matter alone the momentum and energy conservation laws are not satisfied in their original sense; more precisely, they are satisfied only when the g_{ni} are constant, i.e., when the components of the gravitational field intensity vanish. This second term represents the expression for the momentum and, accordingly, the energy that in unit time and in unit volume are transferred from the matter to the gravitational field...."

In the considered case, the second term in (27) expresses the energetic effect of the gravitational field on the matter and shows that matter receives energy that is, as it were, stored in the Riemannian geometry. One could say that the energy of the gravitational field has been used to create the Riemannian geometry. But it cannot be seen from (27) what quantity is conserved.

The absence of conservation laws in their original meaning in Einstein's theory of gravitation forced him to introduce into a covariant theory a noncovariant entity—the energy–momentum pseudotensor of the gravitational field—in order to formulate a conservation law for the sum of the energy–momentum tensor of the matter and the energy–momentum pseudotensor of the gravitational field. However, as is shown in Ref. 5, such an approach does not have any physical meaning.

The absence of conservation laws in their original meaning is common to the complete subclass of gravitational theories with complete geometrization, and not only Einstein's. The Lagrangian density L_g of the gravitational field of theories of this subclass depends on the field φ_{in} and the metric tensor γ_{in} only through the metric tensor g_{in} of the Riemannian space-time. Therefore, in theories of this class we have for the symmetric energy–momentum tensor density of the matter and the gravitational field in the pseudo-Euclidean space-time

$$-\frac{1}{2}t^{in}=\frac{\Delta L}{\Delta\gamma_{in}}=\frac{\Delta L_g}{\Delta\gamma_{in}}+\frac{\Delta L_M}{\Delta\gamma_{in}}=\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial\gamma_{in}}$$
$$-\partial_p\left(\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial(\partial_p\gamma_{in})}\right)+\partial_{pq}\left(\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial(\partial_{pq}\gamma_{in})}\right)$$
$$-\gamma^{is}\gamma^{np}\left\{\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial\gamma^{sp}}-\partial_q\left[\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial(\partial_q\gamma^{sp})}\right.\right.$$
$$\left.\left.-\partial_k\left(\frac{\Delta L}{\Delta g_{lm}}\frac{\partial g_{lm}}{\partial(\partial_{qk}\gamma^{sp})}\right)\right]\right\}.$$

Since in a geometrized theory the equations of the gravitational field have the form

$$\Delta L/\Delta g_{lm}=\delta L/\delta g_{lm}-g^{ls}g^{mp}\delta L/\delta g^{sp}=0,$$

the symmetric energy–momentum tensor density of the matter and of the gravitational field in the pseudo-Euclidean space-time vanishes by virtue of the equations of the gravitational field:

$$\Delta L/\Delta\gamma_{in}=-(1/2)\,t^{in}=0.$$

A similar conclusion about the vanishing of the symmetric tensor density is obtained for the free gravitational field. In Einstein's theory ($L_g=\sqrt{-g}R$), the equations of the free gravitational field have the form $R_{in}=0$.

These equations contain solutions for the variables g_{in} for which the curvature tensor R^i_{nlm} is nonzero. It follows from this that the vanishing of the symmetric energy–momentum tensor density $t^{in}=-2\Delta L/\Delta g_{in}$ of the free gravitational field does not lead to the vanishing of the field φ_{in} and, therefore, there exists a certain fictitious field that does not possess energy or momentum densities but leads to a curving of space-time (formation of a Riemannian geometry).

Thus, our approach shows that in Einstein's general theory of relativity the Riemannian geometry is created on the basis of a fictitious field φ_{in} that does not possess energy and momentum densities; the creation of this geometry does not, therefore, require energy, although the Riemannian geometry itself is a source of energy. Transition to general relativity in the formalism we have developed necessarily leads to the vanishing of all components of the energy–momentum tensor of the free gravitational field. The field φ_{in} is devoid of the most important physical characteristics, and ceases to be an observable physical field carrying energy and momentum. In principle, Einstein's approach rules out the introduction of the concept of a gravitational field possessing energy and momentum.

Thus, we arrive at the following conclusions.

1. In local theories of class A, the gravitational field, described in the pseudo-Euclidean space-time, is a physical field possessing energy and momentum. On the basis of the identity principle, the motion of the matter is described in an effective Riemannian space-time, the creation of which requires the energy and momentum of the gravitational field.

In this approach, the geometrical description arises on the basis of field-theoretical notions about the gravitational field and conservation laws form its basis.

2. In the subclass of theories with complete geometrization, the gravitational field and the matter have a common geometry. Then the gravitational field loses the properties of a physical field and does not possess energy and momentum densities. In such an approach, we do not have the field-theoretical notions of a gravitational field as a field in the spirit of Faraday and Maxwell.

The general theory of relativity realizes this possibility of constructing a theory. It introduced a field of a new type which is described by the curvature tensor and is not a Faraday–Maxwell field. Therefore, in this case there are no conservation laws for the matter and the gravitational field taken together.

3. GAUGE-INVARIANT TENSOR FIELD

For simplicity, we shall in this section consider the formulation of the theory in Cartesian coordinates, although, of course, all the equations and expressions of the theory can be expressed covariantly in an arbitrary curvilinear coordinate system.

We consider theories of class A with Lagrangian density in the form (6). The equations of the gravitational field and the equations of motion of the matter have the form

$$\delta L_g/\delta\varphi_{in}+\delta L_M/\delta\varphi_{in}=0; \quad (28)$$

$$\delta L/\delta\varphi_A=0. \quad (29)$$

Among the set of theories with the Lagrangian density (6), there are theories in which the action integral is invariant under the gauge transformation

$$\varphi_{in}\to\varphi_{in}+\partial_i\alpha_n+\partial_n\alpha_i, \quad (30)$$

where α_i is an arbitrary gauge 4-vector.

It is well known that in electrodynamics[20] the action integral is also invariant under an analogous gauge transformation of the vector potential: $A_i\to A_i+\partial_i f$. The gravitational field, like the electromagnetic field, is a long-range field, i.e., the potential decreases with increasing r as $1/r$. The gauge invariance of electrodynamics has the consequence that the photon mass remains equal to zero even when allowance is made for radiative corrections.

From the invariance of the action $J_g=\int L_g d^4x$ of the free gravitational field under the gauge transformation (30) it follows that

$$\delta J_g=\int\left[-2\alpha_n\partial_i\frac{\delta L_g}{\delta\varphi_{in}}+\mathrm{Div}\right]d^4x=0,$$

where Div denotes divergence terms.

Since the gauge vector α_n is arbitrary,

$$\partial_i\delta L_g/\delta\varphi_{in}=0. \quad (31)$$

From the field equations (28) and the conservation law (31) there follows a conservation equation for the source of the gravitational field:

$$\partial_i\delta L_M/\delta\varphi_{in}=0.$$

It is well known that the gauge invariant of electrodynamics with the Lagrangian density $L=L_M+L_A$ leads to analogous conservation equations:

$$\partial_i\delta L_M/\delta A_i=0;\ \partial_i\delta L_A/\delta A_i=0.$$

Since the source in the field equations of a gauge theory is conserved, it is usually assumed that the source in the equations of a gauge theory of gravitation is the total energy–momentum tensor of the system consisting of the matter plus the gravitational field. This has the consequence that the field equations become nonlinear, and it is usually suggested that the consistent inclusion of such nonlinearities will lead to Einstein's nonlinear geometrized theory of gravitation.[21-23]

However, in reality, such a suggestion leads rather to the circumstance that the gravitational field is not a carrier of energy and momentum. If one assumes the possibility of identifying the source $\delta L_M/\delta\varphi_{in}=(1/2)J^{in}$ with the total energy–momentum tensor $\Delta L_g/\Delta\gamma_{in}+\Delta L_M/\Delta\gamma_{in}$, it follows directly that the energy–momentum tensor of the free gravitational field vanishes when $L_M=0$. Such a theory does not possess the properties characteristic of other physical systems, and we therefore regard it as unacceptable.

In accordance with Noether's theorem, invariance of the action integral under a transformation group entails the existence of definite conserved quantities. Invariance under coordinates transformations leads, as is well known, to conservation of the energy–momentum tensor density t^{in}. The invariance of the action integral under the gauge transformations (30) leads to conservation of the current J^{in}. Since coordinate transformations and gauge transformations are entirely different transformations, t^{in} and J^{in} are, naturally, entirely different physical quantities.

The problem of constructing a gauge-invariant theory of a tensor field is above all the problem of constructing a conserved tensor current J^{in}, or, in other words, the problem of constructing a Lagrangian density L_M of the matter that leads to a conserved variation $\delta L_M/\delta\varphi_{in}$.

To solve this problem it is necessary to consider the question of the spin states of a field described by a symmetric tensor of second rank.

In all relativistic theories of free fields, the principle of invariance under a group of coordinate transformations, the Poincaré group, holds. In accordance with this principle, the wave function realizes a basis of a representation of the Poincaré group. The simplest type of field is determined by an irreducible representation.

The Poincaré group has two invariants:

$$q^2=q_nq^n;$$

$$S^2=(1/2)L_{mn}L^{mn}-(q_nq^p/q^2)L^{nm}L_{pm},$$

where $-iL_{mn}$ is the generator of 4-dimensional rotations.

For irreducible representations, these equations are transformed into identities, and the parameters $m^2=q^2$ and $S^2=s(s+1)$ (the mass and spin) characterize an irreducible representation of the Poincaré group.

The simplest irreducible representation of the Poincaré group is the scalar representation, for which $s=0$. The wave function in this case is a function of q alone and satisfies the wave equation $(q^2-m^2)\varphi(q)=0$.

A vector function $\varphi_i(q)$ forms the basis of a reducible representation D, which decomposes into the sum of two irreducible representations with spin 0 and 1:

$$D=D(0)+D(1),$$

where $D(s)$ denotes the irreducible representation corresponding to the spin s.

An entity that transforms as the product $D^S=D\times\ldots\times D$ is a tensor of rank S. It realizes a reducible representation, which can be decomposed into irreducible representations in accordance with the standard Clebsch–Gordan technique:

$$D\times D=D(2)+3D(1)+2D(0).$$

The irreducible representation $D(s)$ has $2S+1$ dimensions.

Each component of the wave function satisfies the wave equation, whereas the condition $S^2 = s(s+1)$ ensures the vanishing of the projections of the field φ onto the spaces of representations with lower spins. For the case $S=2$, these conditions can be written in the well-known form

$$q_i\varphi^i_n = 0;\ \varphi_{ni} = \varphi_{in};\ \gamma^{in}\varphi_{in} = 0$$

or, briefly, $\eta_i\varphi = 0$.

The subsidiary conditions establish a connection between the different components of the tensor field φ_{in}, restricting thereby the number of independent components. In other words, a tensor of rank S is excessive for the description of a concrete physical field with definite spin.

To eliminate the redundancy in the formalism of tensor fields, it is convenient to work directly with irreducible representations of the tensor field. This is achieved by means of projection operators P_S, which project a tensor of rank S onto an irreducible representation, so that the field $\psi = P_S\varphi$ transforms in accordance with $D(s)$, i.e., $\eta_i\psi = 0$. If a specific tensor φ transforms in accordance with $D(s)$, then $\varphi = P_S\psi$.

The formalism of projection operators is considered, for example, in Refs. 24 and 25.

A symmetric tensor φ_{in} of second rank can be represented as a sum of irreducible representations: one representation with spin 2, one with spin 1, and two representations with spin 0:

$$\varphi_{in} = (P_2 + P_1 + P_0 + P_{0'})^{lm}_{in}\,\varphi_{lm}.$$

The projection operators P_S satisfy the standard relations

$$P_S P^T = \delta^T_S P_S;\ P^{in}_{S\,in} = 2S+1;$$
$$\sum_S P^{lm}_{S\,ik} = 1 = \frac{1}{2}(\delta^l_i\delta^m_k + \delta^m_i\delta^l_k).$$

The operators P_S are conveniently expressed in the momentum representation, when $\partial_n \to iq_n$ and $\Box \to -q^2$.

We introduce the auxiliary operators

$$X_{in} = (1/\sqrt{3})\,(\gamma_{in} - q_i q_n/q^2);\ Y_{in} = q_i q_n/q^2,$$

by means of which the operators P_S can be represented in the form

$$P_0 = X_{in}X^{lm};\ P_{0'} = Y_{in}Y^{lm};$$
$$P_1 = (\sqrt{3}/2)\,(X^l_i Y^m_n + X^m_n Y^l_i + X^m_i Y^l_n + X^l_n Y^m_i);$$
$$P_2 = (3/2)\,(X^l_i X^m_n + X^m_i X^l_n) - X_{in}X^{lm}.$$

In the x representation, the projection operators P_S are nonlocal integrodifferential operators:

$$P^{lm}_{in}\varphi_{lm} = \int d^4y\, P^{lm}_{in}(x, y)\,\varphi_{lm}(y)$$

and, for example, $Y\varphi = Y_{in}\varphi^{in}$ has the form

$$Y_{in}\varphi^{in}(x) = -\frac{\partial}{\partial x^i}\frac{\partial}{\partial x^n}\int G(x-y)\,\varphi^{in}(y)\,d^4y,$$

where $G(x-y)$ is the Green's function of the scalar wave equation:

$$\Box\, G(x-y) = -\delta(x-y);\ \Box = \partial_i\partial^i.$$

It must be emphasized that the operators P_2 and P_0 are conserved:

$$q_l P^{lm}_{2in} = q_m P^{lm}_{2in} = 0;\ q_l P^{lm}_{0in} = q_m P^{lm}_{0in} = 0.$$

Therefore, if the field φ_{in} occurs in the Lagrangian density of the gravitational field only in the form of the combination

$$f_{in} = [(P_2 + \alpha P_0)\,\varphi]_{in}, \tag{32}$$

then the Lagrangian density, and, therefore, the equations of the free gravitational field are invariant under the gauge transformation (30). It should be emphasized that although in the theory all ten components of the gravitational field φ_{in} are independent, only six of the ten components f_{in} are independent, since the projection operators distinguish only six spin states: five states of the representation with spin 2 and one state with spin 0. However, the use of the expression (32) is not entirely convenient, since it is an integro-differential expression and therefore leads to nonlocal field equations.

If the field equations are to be local, it is necessary to have a differential connection between the fields f_{in} and φ_{in}.

This can be achieved, for example, by taking the combination

$$f_{lm} = \Box^2[(P_2 + \alpha P_0)\,\varphi]_{lm}.$$

In this case, the tensor f_{lm} will be expressed in terms of the fourth derivatives of the field functions φ_{lm}. But among all values of α, the value $\alpha = -2$ is distinguished in the sense that it enables one to express f_{lm} in the form of a combination of, not the fourth derivatives of φ_{lm}, but only the second derivatives:

$$f_{in} = \Box\,[(P_2 - 2P_0)\,\varphi]_{in}. \tag{33}$$

It is easy to see that the operator $\Box(P_2 - 2P_0)$ is a gauge-invariant and local operator of the lowest order, namely, in a theory using a symmetric tensor field of second rank there is no other local operator which uses lower derivatives and leads to gauge invariance. Thus, we have

$$\left.\begin{array}{l} f_{in} = \Box\,\theta_{in} - \partial_i\partial^m\theta_{mn} - \partial_n\partial^m\theta_{mi} \\ +\,\gamma_{in}\partial^l\partial^m\theta_{lm};\ \partial^i f_{in} = 0, \end{array}\right\} \tag{34}$$

where we have introduced the notation

$$\theta_{lm} = \varphi_{lm} - (1/2)\,\gamma_{lm}\varphi^n_n. \tag{35}$$

In this case, the vector field and the field of spin 0', which are not invariant under the gauge transformation (30), will be eliminated from the theory.

Since the entire theory must be gauge-invariant, we assume that in the connection equations $g_{in} = g_{in}(\varphi_{lm})$ the fields φ_{lm} occur only through the field f_{lm}. Moreover, we shall assume that the metric tensor g_{in} of the Riemannian space-time is a local function of only the fields f_{lm} and the metric tensor of the flat space-time. Concerning the form of this function, we shall at this stage make no assumptions except the requirment that the quadratic form with the coefficients $g_{\alpha\beta}$ be negative definite and the component g_{00} be a positive quantity. Then the parameter x^0 will have the nature of time, and the parameters x^α the nature of spatial coordinates in the Riemannian space-time as well as in the flat space.

4. EQUATIONS OF THE GRAVITATIONAL FIELD

The Lagrangian density of the gravitational field of a gauge theory using derivatives of the fields f_{lm} of order not higher than the first can be written in the most general form

$$L_g=\frac{1}{64\pi}\{\partial_i f_{lm}\partial^i f^{lm}-b\partial_i f_l^l\partial^i f_n^n-m_g^2[\alpha f_{in}f^{in}+\beta f_i^i f_n^n]\}.$$

If $\alpha \neq 0$ or $\beta \neq 0$, the obtained equations describe a gravitational field whose quantum (the graviton) has a nonvanishing rest mass. Since we expect the front of a gravitational wave to propagate with the fundamental velocity $v=c$, the rest mass of the graviton must be zero. For this, we must take

$$\alpha=\beta=0.$$

In addition, in the present formulation of the theory the symmetric fields play the part of potentials of the gravitational field, and the connection of the Riemannian space-time plays the part of the field "intensities." These field intensities are proportional to the first derivatives of the fields:

$$\Gamma_{i,\,nl}=\frac{1}{2}\left(\frac{\partial g_{in}}{\partial f_{pq}}\frac{\partial f_{pq}}{\partial x^l}+\frac{\partial g_{il}}{\partial f_{qp}}\frac{\partial f_{pq}}{\partial x^n}-\frac{\partial g_{ln}}{\partial f_{pq}}\frac{\partial f_{pq}}{\partial x^i}\right).$$

Therefore, the "observables" are the first derivatives of the fields f_{lm} (for a weak field in the linear approximation $\Gamma_{in,l}$ and Γ^i_{nl} can be expressed solely in terms of the first derivatives of the fields f_{lm}, while in the higher approximations the first derivatives of the fields acquire factors that depend nonlinearly on the potentials f_{lm}).

If the energy–momentum tensor of the gravitational field is to depend only on these observables, the case of a weak field included, it is necessary to require that the Lagrangian density of the free gravitational field contain only the first derivatives of the fields f_{lm}.

Choosing different values of b, one can realize different physical situations. One can show that the energy of the free gravitational field has positive sign if $b \leqslant \frac{1}{2}$. In addition, if $b<\frac{1}{2}$, then a scalar component of gravitational waves is emitted, the value of this component and its energy depending essentially on b, since for $b<\frac{1}{2}$ the scalar component transmits positive energy. However, we shall not require such a degree of generality in what follows, since we assume that gravitational waves (gravitons) are characterized by the spin value $s=2$ and a positive-definite energy. Therefore, in what follows we assume $b=\frac{1}{2}$ to eliminate the emission of the scalar component.

Thus, we arrive at a Lagrangian density of the free gravitational field in the form

$$L_g=\frac{1}{64\pi}\{\partial_i f_{lm}\partial^i f^{lm}-\frac{1}{2}\,\partial_i f_l^l\partial^i f_m^m\}. \qquad (36)$$

This Lagrangian density of the gravitational field is the simplest Lagrangian density invariant under the gauge transformation (30) of the fields φ_{in}. The fields f_{lm} can also be subjected to the gauge transformation

$$f_{lm}\to f_{lm}+\partial_l a_m+\partial_m a_l-\gamma_{lm}\partial_n a^n, \qquad (37)$$

which does not violate the conditions $\partial^l f_{lm}=0$, if the gauge vectors a^n satisfy the homogeneous equations $\Box a^n=0$. Under the gauge transformations (37), the Lagrangian density of the free gravitational field is changed by a divergence:

$$L_g\to L_g+\partial_l C^l,$$

where the vector density C^l has the form

$$C^l=\frac{1}{32\pi}\{2\partial_i f^{lm}\partial^i a_m+\partial_i a_m\partial^i\partial^m a^l+\partial_i a_m\partial^l\partial^i a^m-\partial_i a^i\partial^l\partial_n a^n\}.$$

Therefore, we obtain the field equations under the gauge transformations (37) in the same form as before the transformation (37). The canonical energy–momentum tensor density of the free gravitational field is changed under the gauge transformations (37) by the divergence of an antisymmetric tensor of third rank:

$$\tilde{t}^n_{gi}\to\tilde{t}^n_{gi}+\partial_l\,[\delta^l_i C^n-\delta^n_i C^l].$$

As will be shown in what follows, the presence of this divergence in the canonical energy–momentum tensor density does not affect the observable physical quantities.

It should be emphasized that the symmetric fields f_{lm} are not independent by virtue of the four conditions $\partial^l f_{lm}=0$ which they satisfy, and therefore it is necessary in the derivation of the field equations to take the Euler–Lagrange variation with respect to the field φ_{in}, since it is only for this field that all ten components are independent. However, if the Euler–Lagrange variation is taken with respect to the field f_{lm} in the derivation of the field equations, one must also take into account the four subsidiary conditions $\partial^l f_{lm}=0$ which this field satisfies, i.e., one must consider the problem of a conditional extremum. In both cases, we arrive at equivalent field equations.

From the relation (34) we have

$$\frac{\partial f_{in}}{\partial(\partial_{lm}\varphi_{sq})}=\frac{1}{4}[\delta^q_p\delta^s_k+\delta^s_p\delta^q_k-\gamma_{pk}\gamma^{qs}][\gamma^{lm}(\delta^p_i\delta^k_n+\delta^k_i\delta^p_n)$$
$$-\gamma^{lp}(\delta^m_i\delta^k_n+\delta^k_i\delta^m_n)-\gamma^{mp}(\delta^l_i\delta^k_n+\delta^k_i\delta^l_n)+\gamma_{in}(\gamma^{lp}\gamma^{mk}+\gamma^{lk}\gamma^{mp})]. \qquad (38)$$

Then, using the expression for the Lagrangian density (36) of the gravitational field and taking into account (38), we obtain

$$\frac{\delta L_g}{\delta\varphi_{lm}}=-\frac{1}{32\pi}\{\Box^3\theta^{lm}-\partial^l\partial_n\Box^2\theta^{nm}-\partial^m\partial n\Box^2\theta^{ln}+\gamma^{lm}\partial_n\partial_p\Box^2\theta^{np}\}.$$

The variation of the matter Lagrangian density can be obtained in two ways, either directly, by writing down the Euler–Lagrange variation (10) with respect to the field φ_{lm}, or by using the circumstance that the gravitational field enters the matter Lagrangian density only through the field f_{lm} (34), and the field f_{lm}, in its turn, enters the matter Lagrangian density through the metric tensor of the Riemannian space-time. In both cases, we obtain the same result.

Bearing in mind the definition (9), we can write the variation of the matter Lagrangian density in the form

$$\frac{\delta L_M}{\delta\varphi_{ik}}=-\frac{1}{2}\Big[T^{lm}\frac{\partial g_{lm}}{\partial f_{np}}\frac{\partial f_{np}}{\partial\varphi_{ik}}-\partial_s\Big(T^{lm}\frac{\partial g_{lm}}{\partial f_{np}}\frac{\partial f_{np}}{\partial(\partial_s\varphi_{ik})}\Big)$$
$$+\partial_s\partial_q\Big(T^{lm}\frac{\partial g_{lm}}{\partial f_{np}}\frac{\partial f_{np}}{\partial(\partial_{sq}\varphi_{ik})}\Big)\Big]. \qquad (39)$$

Introducing the notation

$$h^{lm}=\frac{1}{2}T^{np}\frac{\partial g_{np}}{\partial f_{ik}}(\delta^l_i\delta^m_k+\delta^m_i\delta^l_k-\gamma_{ik}\gamma^{lm}) \qquad (40)$$

and using the relation (38), we obtain from the expression (39)

$$\frac{\delta L_M}{\delta \varphi_{lm}} = -\frac{1}{2}[\Box h^{lm} - \partial^l \partial_n h^{mn} - \partial^m \partial_n h^{ln} + \gamma^{lm} \partial_n \partial_p h^{np}].$$

Then the equations (28) of the gravitational field take the form

$$\Box^3 \theta^{lm} - \partial^l \partial_n \Box^2 \theta^{nm} - \partial^m \partial_n \Box^2 \theta^{ln} + \gamma^{lm} \partial_n \partial_p \Box^2 \theta^{np} = -16\pi J^{lm}, \quad (41)$$

where

$$J^{lm} = \Box h^{lm} - \partial^l \partial_n h^{nm} - \partial^m \partial_n h^{nl} + \gamma^{lm} \partial_n \partial_p h^{np}.$$

The field equations (41) can, using the definition (34), be rewritten in the form

$$\Box^2 f^{lm} = 16\pi J^{lm}. \quad (42)$$

It is readily seen that the equations of the gravitational field in either the form (41) or the form (42) are invariant under the gauge transformations (30) with arbitrary gauge vector α^n. If we take the total divergence with respect to one of the indices in the field equations (41) and (42), we obtain the identity $0=0$. Therefore, although the field φ_{lm} has ten independent components, the structure of the equations is such that the four components corresponding to the irreducible representations with spins 1 and 0′ are automatically eliminated from the equations, as a result of which the equations will contain only six independent components, corresponding to spins 2 and 0. For them, we have six independent field equations, since the four conditions (31) hold by virtue of the gauge invariance.

The gravitational field equations (41) can be simplified by using a gauge transformation (30) and imposing subsidiary conditions on the field equations. The arbitrariness in the choice of the gauge means that in the solution of concrete problems we must explicitly determine the gauge conditions in some manner, for example, by imposing additional conditions. The fact that the Euler–Lagrange variation in a gauge theory satisfies the four identities (31) also means that when the field equations are solved in a definite problem it is necessary to impose at least four additional conditions on the field. Under the gauge transformations (30), the fields θ_{lm} undergo the following gauge transformation in accordance with the relation (35):

$$\theta_{lm} \to \theta_{lm} + \partial_l \alpha_m + \partial_m \alpha_l - \gamma_{lm} \partial_n \alpha^n. \quad (43)$$

The most general additional conditions that are linear in the field $\Box^2 \theta^{ln}$ are the conditions

$$\partial_n \Box^2 \theta^{nm} = A \partial^m \Box^2 \theta^n_n. \quad (44)$$

If the conditions (44) are satisfied, the equations of the gravitational field can be written in the form

$$\Box^3 \theta^{lm} - 2A \partial^l \partial^m \Box^2 \theta^n_n + A \gamma^{lm} \Box^3 \theta^n_n = -16\pi J^{lm}.$$

It is readily seen that the left-hand side of these equations is also conserved when allowance is made for the additional conditions (44). For $A=0$, we obtain the equations of the gravitational field in the simplest form

$$\Box^3 \theta^{lm} = -16\pi J^{lm} \quad (45)$$

with the subsidiary conditions

$$\partial_n \Box^2 \theta^{nm} = 0. \quad (46)$$

Thus, the equations of the gravitational field in our case are equations with higher derivatives. At the same time, Eqs. (45) are also invariant under the gauge transformations (43), which do not violate the subsidiary conditions (46).

We introduce the field H^{lm} in accordance with the equation

$$\Box H^{lm} = h^{lm}. \quad (47)$$

Then Eqs. (45) take the form

$$\Box^3 \theta^{lm} = -16\pi \{\Box h^{lm} - \partial^l \partial_n \Box H^{nm} - \partial^m \partial_n \Box H^{nl} + \gamma^{lm} \Box \partial_n \partial_p H^{np}\}.$$

Since we shall in what follows be interested in only causally dependent solutions, we can in accordance with Ref. 26 "divide" these equations by the d'Alembertian. Introducing the notation

$$\psi_{lm} = \Box \theta_{lm} \quad (48)$$

for the causally dependent solutions, we obtain the equations of the gravitational field in the form

$$\Box \psi^{lm} = -16\pi \{h^{lm} - \partial^l \partial_n H^{nm} - \partial^m \partial_n H^{nl} + \gamma^{lm} \partial_n \partial_p H^{np}\}.$$

The tensor current on the right-hand side of this equation satisfies outside the source the condition

$$\Box \{h^{lm} - \partial^l \partial_n H^{mn} - \partial^m \partial_n H^{ln} + \gamma^{lm} \partial_n \partial_p H^{np}\} = 0.$$

Therefore, outside the matter this tensor current can be eliminated by a gauge transformation. Indeed, since the conditions (46) admit a transformation (43) with gauge 4-vector satisfying the equation

$$\Box^3 \alpha^n = 0, \quad (49)$$

we can make the gauge transformation

$$\psi^{mn} \to \psi^{mn} + \partial^m \Box \alpha^n + \partial^n \Box \alpha^m - \gamma^{mn} \partial_l \Box \alpha^l. \quad (50)$$

Outside the source we choose as gauge 4-vector a vector satisfying the condition

$$\Box^2 \alpha^n = 16\pi \partial_m H^{mn}.$$

Since $\Box H^{mn} = 0$ outside the source, the gauge 4-vector also satisfies Eq. (49) in such a region, as a result of which the subsidiary conditions (46) are satisfied automatically. The form of the gauge vector within the source is unimportant for our purposes. After the gauge transformation (50), we obtain the gravitational field equations outside the source in the form

$$\Box \psi^{mn} = 0.$$

This means that the tensor current

$$I^{mn} = h^{mn} - \partial^n \partial_l H^{lm} - \partial^m \partial_l H^{ln} + \gamma^{mn} \partial_l \partial_p H^{lp} \quad (51)$$

is nonvanishing only within the matter. Therefore, in this gauge the gravitational field equations take the form

$$\Box \psi^{mn} = -16\pi I^{mn}. \quad (52)$$

Equations (52) admit the gauge transformations (43) on the class of vectors satisfying the condition

$$\Box^2 \alpha^n = 0.$$

Therefore, we shall solve the equations with the subsidiary conditions $\partial_l \psi^{lm} = 0$, which leave the possibility of making gauge transformations of only the above class. The choice of the subsidiary conditions accords with

Fock's theorem,[27] in accordance with which a solution of the homogeneous wave equation $\Box\partial_l\psi^{lm}=0$ that is bounded in the whole of space and satisfies Sommerfeld's radiation condition vanishes identically:

$$\partial_l\psi^{lm}=0.$$

Thus, we obtain the gravitational field equations

$$\Box\psi^{lm}=-16\pi I^{lm} \tag{53}$$

with the subsidiary conditions

$$\partial_l\psi^{lm}=0. \tag{54}$$

We note further that the expression $\Box f_{lm}$ can be written using the notation (48) in the form

$$\Box f_{lm}=\Box\psi_{lm}-\partial_l\partial^n\psi_{nm}-\partial_m\partial^n\psi_{ln}+\gamma_{lm}\partial^n\partial^p\psi_{np}.$$

The last expression is also invariant under the transformations (50) with any gauge vector a^n, but the operator $\Box f_{lm}$ in the given case will have the original form. We can simplify this operator by noting that in the gauge we have adopted the subsidiary conditions (54) hold. In such a case, we obtain

$$\Box f_{lm}=\Box\psi_{lm}. \tag{55}$$

It should be noted that the obtained expression $\Box f_{lm}$ (55) is also invariant under the gauge transformations (50), which do not violate the subsidiary conditions (54). The relations (55) make it possible to rewrite the gravitational field equations in the form

$$\Box f^{nm}=-16\pi I^{nm} \tag{56}$$

with the subsidiary conditions

$$\partial_n f^{nm}=0. \tag{57}$$

It should be emphasized especially that the tensor current I^{nm} on the right-hand side of Eqs. (56) is concentrated in the matter alone.

The solution of these equations for the case in which gravitational waves are emitted will be given in Sec. 6.

5. CONSERVATION LAWS IN THE FIELD THEORY OF GRAVITATION

In Sec. 2, we obtained conservation laws that hold for all gravitational theories of class A. The existence in theories of this class of the differential conservation law (26) for the total symmetric energy–momentum tensor density of the system in the flat space-time makes it possible to obtain a corresponding integral conservation law.

In Cartesian coordinates, we have

$$\partial_n(t_g^{in}+t_M^{in})=0.$$

Integrating this expression over a volume V for $i=0$, we obtain

$$-\frac{\partial}{\partial t}\int dV\,(t_g^{00}+t_M^{00})=\int dS_\alpha\,(t_g^{0\alpha}+t_M^{0\alpha}). \tag{58}$$

Equation (58) means that the change in the energy of the matter and the gravitational field in a volume V is equal to the flux of the energy through the surface bounding this volume. If one takes as volume of integration a volume occupied by matter and assumes that there is no matter flow through the surface bounding this volume, then the change in the total energy of the matter and the gravitational field in the source will be equal to the energy flux of the gravitational field through the surface of the source.

Thus, when gravitational waves are emitted the energy of the source must change, and if the gravitational waves transmit positive energy, the energy of the source must decrease:

$$-\frac{\partial}{\partial t}\int dV\,(t_g^{00}+t_M^{00})=\int dS_\alpha t_g^{0\alpha}. \tag{59}$$

All these conclusions and relations are also valid for the field theory of gravitation, which is a particular representative of theories of class A.

Since the symmetric and canonical energy–momentum tensors differ by the divergence of an antisymmetric tensor of third rank, the conservation laws (26) and (58) hold for the canonical energy–momentum tensor.

The canonical energy–momentum tensor of the free gravitational field can be obtained as follows. We write down the equation

$$\partial_p L_g=\partial_n\left(\frac{\partial L_g}{\partial(\partial_n f_{lm})}\partial_p f_{lm}\right)-\partial_p f_{lm}\partial_n\left(\frac{\partial L_g}{\partial(\partial_n f_{lm})}\right). \tag{60}$$

In accordance with (56), the free gravitational field satisfies the equation

$$\partial_n(\partial L_g/\partial(\partial_n f_{lm}))=\Box f^{lm}=0,$$

and therefore the expression (60) means that the divergence of the canonical energy–momentum tensor of the free gravitational field vanishes.

Hence,

$$\tilde{t}^{\,n}_{gp}=-L_g\delta_p^n+(\partial L_g/\partial(\partial_n f_{lm}))\,\partial_p f_{lm}. \tag{61}$$

Using the expression (36) for the Lagrangian density of the free gravitational field, we find

$$\tilde{t}^{\,n}_{gp}=\frac{1}{64\pi}\left\{-\delta_p^n\left[\partial_i f_{lm}\partial^i f^{lm}-\frac{1}{2}\partial_i f_l^l\partial^i f_m^m\right]+2\partial_p f_{lm}\partial^n f^{lm}-\partial_p f_l^l\partial^n f_m^m\right\}. \tag{62}$$

To obtain the symmetric energy–momentum tensor t^{in} of the gravitational field, we must write the Lagrangian density L_g of the gravitational field and the expression for f_{in} in explicitly covariant form.

Going over in (36) from the Cartesian coordinate system to an arbitrary curvilinear system, we obtain

$$L_g=(\sqrt{-\gamma}/64\pi)\,\gamma^{ik}(\gamma^{ln}\gamma^{mp}-(1/2)\,\gamma^{lm}\gamma^{np})\,D_i f_{lm}D_k f_{np}. \tag{63}$$

Similarly, from (33),

$$f_{ik}=\gamma^{lm}(D_lD_m\varphi_{ik}-D_iD_l\varphi_{mk}-D_kD_l\varphi_{mi}+D_iD_k\varphi_{lm}+\gamma_{ik}\gamma^{pn}[D_lD_n\varphi_{mp}-D_nD_p\varphi_{lm}]). \tag{64}$$

To shorten the expressions that follow, we also introduce the notation

$$\begin{aligned}\Lambda^{ik}=&-A^{lm}[\partial_l\partial_m\varphi^{ik}-\partial^i\partial_l\varphi_m^k-\partial^k\partial_l\varphi_m^i+\partial^i\partial^k\varphi_{lm}]\\&+(1/2)\,f_n^nA^{ik}+A_n^n[f^{ik}-(1/2)\,\gamma^{ik}f_m^m]\\&+(1/2)\,\partial_s\{\varphi_n^i[-\partial^sA^{kn}+2\partial^nA^{sk}+2\gamma^{sk}\partial_lA^{ln}\\&-\gamma^{kn}\partial_lA^{ls}-\partial^kA^{sn}+\gamma^{kn}\partial^sA_l^l-2\gamma^{sk}\partial^nA_l^l]\\&+\varphi_n^s[\partial^iA^{kn}-\partial^nA^{ik}-\gamma^{ik}\partial_lA^{ln}+\gamma^{ik}\partial^nA_l^l]\\&+2\gamma^{ks}A^{np}\partial^i\varphi_{np}-A^{sn}\partial^i\varphi_n^k-3A^{kn}\partial^i\varphi_n^s\\&+2A^{ks}\partial^i\varphi_n^n-\gamma^{ik}A^{np}\partial^s\varphi_{np}+3A^{kn}\partial^s\varphi_n^{i}\end{aligned}$$

$$-A^{ik}\partial^s\varphi^n_n-2\gamma^{sk}A^{ln}\partial_l\varphi^i_n-2A^{sk}\partial_l\varphi^{li}$$
$$+A^{ns}\partial_n\varphi^{ik}+\gamma^{ik}A^{ln}\partial_l\varphi^s_n+A^{ik}\partial_n\varphi^{ns}$$
$$+A^l_l[2\partial^i\varphi^{ks}-2\gamma^{ks}\partial^i\varphi^n_n-2\partial^s\varphi^{ik}$$
$$+\gamma^{ik}(\partial^s\varphi^n_n-\partial_n\varphi^{ns})+2\gamma^{ks}\partial_n\varphi^{ni}]\}. \quad (65)$$

The symmetric energy–momentum tensor of the gravitational field can be obtained by substituting (63) and (64) in (12). In the Cartesian coordinate system,

$$t^{ik}_g=(1/64\pi)\{-\gamma^{ik}[\partial_l f_{np}\partial^l f^{np}-(1/2)\,\partial_l f^n_n\partial^l f^p_p]+2\partial^i f_{nm}\partial^k f^{nm}$$
$$-\partial^i f^n_n\partial^k f^m_m\}+(1/16\pi)\{\partial_l f^{in}\partial^l f^k_n-(1/2)\,\partial_l f^{ik}\partial^l f^n_n\}$$
$$-(1/32\pi)\,\partial_l\{f^i_p[\partial^l f^{kp}+\partial^k f^{lp}]-f^{ik}\partial^l f^n_n$$
$$+f^k_n[\partial^l f^{in}+\partial^i f^{ln}]-f^l_n[\partial^i f^{kn}+\partial^k f^{in}]\}-2\Lambda^{(ik)}, \quad (66)$$

where, as usual, the parentheses surrounding indices signify symmetrization:

$$\Lambda^{(ik)}=(\Lambda^{ik}+\Lambda^{ki})/2.$$

The tensor A^{mn} which occurs in (65) has in this case the form

$$A^{mn}=-(1/32\pi)\,\Box\,(f^{mn}-(1/2)\,\gamma^{mn}f^l_l).$$

Outside the matter $\Box f_{nm}=0$, and therefore the expression for t^{ik}_g simplifies considerably:

$$t^{ik}_g=\tilde{t}^{ik}_g+(1/32\pi)\,\partial_l\,[f^l\ \ (\partial^i f^{kn}+\partial^k f^{in})-f^i_n\partial^k f^{nl}-f^k_n\partial^i f^{nl}], \quad (67)$$

where $\tilde{t}^{ik}_g$ is the canonical energy–momentum tensor (62) of the free gravitational field.

We show that in the wave zone the symmetric energy–momentum tensor t^{ik}_g of the gravitational field differs from the canonical energy–momentum tensor $\tilde{t}^{ik}_g$ only by nonwave terms which decrease faster than $1/r^2$.

In the wave zone, we have the expansion

$$f_{lm}=a_{lm}\,(t-r,\ \theta,\ \varphi)/r+O\,(1/r^2),$$

so that for an arbitrary function $F(f_{lm})$ we have

$$\partial_\alpha F=n_\alpha\partial F\,(flm)/\partial t+O\,((1/r)\,F\,(f_{lm})),$$

where $n_\alpha=x_\alpha/r$. Therefore, the expression (67) can be written in the form

$$t^{ik}_g=\tilde{t}^{ik}_g+\frac{1}{32\pi}\frac{\partial}{\partial t}\{(f^{0l}+n_\alpha f^{\alpha l})\,(\partial^i f^k_l+\partial^k f^i_l)$$
$$-f^{il}_n\partial^k\,(f^{0n}+n_\alpha f^{\alpha n})-f^k_l\partial^i\,(f^{0l}+n_\alpha f^{\alpha l})\}+O\,(1/r^3).$$

Denoting differentiation with respect to the time by a dot, we find from the subsidiary conditions (57)

$$\dot{f}^{0l}+n_\alpha\dot{f}^{\alpha l}=O\,(1/r^2). \quad (68)$$

Integrating this expression with respect to the time and setting the constants of integration equal to zero, since the waves must not have a time-independent part, we obtain

$$f^{0l}+n_\alpha f^{\alpha l}=O\,(1/r^2). \quad (69)$$

It follows that in the wave zone the symmetric energy–momentum tensor of the gravitational field differs from the canonical energy–momentum tensor by a nonwave quantity that decreases faster than $1/r^2$ with increasing r:

$$t^{ik}_g=\tilde{t}^{ik}_g+O\,(1/r^3). \quad (70)$$

Therefore, in the wave zone calculations made using the symmetric or the canonical energy–momentum tensor of the gravitational field give the same result. These tensors are also equivalent for calculating the integrated characteristics of gravitational radiation.

Indeed, from expression (67),

$$t^{00}_g=\tilde{t}^{00}_g+\frac{1}{16\pi}\,\partial_\alpha\,(f^{\alpha l}\dot{f}^0_l-\dot{f}^{\alpha l}f^0_l).$$

Therefore

$$\int t^{00}_g\,dV=\int\tilde{t}^{00}_g\,dV+\frac{1}{16\pi}\int dS_\alpha\,(f^{\alpha l}\dot{f}^0_l-f^0_l\dot{f}^{\alpha l}).$$

If the boundary of the region of integration is in the wave zone, then by virtue of the relations (68) and (69)

$$f^{\alpha l}\dot{f}^0_l-f^0_l\dot{f}^{\alpha l}=n_\beta\,(f^{\alpha l}\dot{f}^\beta_l-\dot{f}^{\alpha l}f^\beta_l)+O\,(1/r^3).$$

Taking as surface of integration a sphere of radius r ($dS_\alpha=-r^2n_\alpha d\Omega$), we obtain

$$\int t^{00}_g\,dV=\int\tilde{t}^{00}_g\,dV+O\,(1/r). \quad (71)$$

In addition, it follows from (70) that

$$\int t^{0\alpha}_g\,dS_\alpha=\int\tilde{t}^{0\alpha}_g\,dS_\alpha+O\,(1/r). \quad (72)$$

Thus, the equivalence of the canonical and symmetric energy–momentum tensors for calculating integrated characteristics of gravitational radiation follows from (71) and (72).

We consider a source of gravitational waves of island type. We show that outside the source of the waves the components $\tilde{t}^0_{g0}$ and $\tilde{t}^\alpha_{g0}$ have a definite sign and are positive. For this, we take an arbitrary point outside the source and consider around the chosen point a region of space whose linear dimensions are significantly smaller than the radius of curvature of the wave front at that point. Then in such a region the gravitational wave can be regarded as a plane wave. To be specific, we direct the x axis of a Cartesian coordinates system to pass through the chosen point. We place the origin of the Cartesian system at any point of the source. Then in our region the functions f_{lm} will be functions of the difference $u=x-t$ alone. For the components $\tilde{t}^0_{g0}$ and $\tilde{t}^1_{g0}$ in this case we obtain from (62)

$$\tilde{t}^0_{g0}=\tilde{t}^1_{g0}=\frac{1}{32\pi}\,(\dot{f}^2_{00}+\dot{f}^2_{11}+\dot{f}^2_{22}+\dot{f}^2_{33}$$
$$+2\dot{f}^2_{12}+2\dot{f}^2_{13}+2\dot{f}^2_{23}-2\dot{f}^2_{01}-2\dot{f}^2_{02}-2\dot{f}^2_{03}). \quad (73)$$

From the conditions (57) we find

$$\dot{f}_{00}=\dot{f}_{11};\quad \dot{f}_{01}=-\dot{f}_{11};$$
$$\dot{f}_{02}=-\dot{f}_{12};\quad \dot{f}_{03}=-\dot{f}_{13}.$$

Substituting these relations in (73), we obtain

$$\tilde{t}^0_{g0}=\tilde{t}^1_{g0}=(1/16)\,\{\dot{f}^2_{23}+(1/4)\,(\dot{f}_{22}-\dot{f}_{33})^2\}\geqslant 0.$$

Because the chosen point is arbitrary, this last relation holds in the whole of space outside the source. Thus, the energy density of the gravitational wave is positive definite, and only the transverse components of the gravitational wave contribute to the energy. Therefore, it follows from (59) that the energy of the source decreases when waves are emitted.

To obtain the symmetric energy–momentum tensor density of the matter in the flat space-time, t^{in}_M, we note that the metric tensor γ_{in} enters the matter Lagrangian density only through the metric tensor $g_{in}=g_{in}(\gamma_{lm},\ \varphi_{lm})$ of the Riemannian space-time. Therefore the tensor density t_M^{in} can be written in the form

$$t_M^{in} = T^{lm} A_{lm}^{in} - 2\Lambda^{(in)}, \tag{74}$$

where

$$A_{lm}^{in} = \partial\tilde{g}_{lm}/\partial\gamma_{in} - \gamma^{is}\gamma^{nq}\,\partial\tilde{g}_{lm}/\partial\gamma^{sq}, \tag{75}$$

and the tilde means that the metric tensor of the Riemannian space-time must be represented before the operation of differentiation in "canonical" form, i.e., as a function of the field f_{in} and the metric γ_{in}:

$$\tilde{g}_{lm} = g_{lm}(\gamma_{in}, f_{in}).$$

We obtain the expression for Λ^{in} from (65) if we set

$$A^{lm} = -(1/2)\,T^{np}\,\partial g_{np}/\partial f_{lm}.$$

Since we do not consider it necessary in the present paper to make a choice of a definite connection equation $g_{in} = g_{in}(\gamma_{lp}, \varphi_{lp})$, we cannot obtain the expression for t_M^{in} explicitly. As will be shown later, using the results of experiments made in the solar system one can obtain the connection equation in the approximation of a weak field up to quadratic terms. This will enable us to obtain an explicit expression for t_M^{in} in the same approximation.

6. EMISSION OF GRAVITATIONAL WAVES

In the gauge we have chosen, the field equations are

$$\Box f^{lm} = -16\pi I^{lm}, \tag{76}$$

the tensor current I^{lm} (51) being defined only in the matter.

Although these equations are nonlinear equations because f^{in} occurs nonlinearly on the right-hand side, their wave solutions can still be analyzed.

Since the metric tensor g_{in}, and also the energy-momentum tensor of the free gravitational field, i.e., the field outside the matter, depend only on the fields f_{in}, we shall also solve Eqs. (76) for f_{in}.

We write the tensors f^{ln} and I^{ln} in the form of Fourier integrals with respect to the time:

$$\left.\begin{aligned} f^{nm}(\mathbf{r}, t) &= \int \exp(-\mathrm{i}\omega t)\,\tilde{f}^{nm}(\omega, \mathbf{r})\,d\omega;\\ I^{nm}(\mathbf{r}, t) &= \int \exp(-\mathrm{i}\omega t)\,\tilde{I}^{nm}(\omega, \mathbf{r})\,d\omega. \end{aligned}\right\} \tag{77}$$

In the spectrum $\tilde{I}^{nm}(\omega, \mathbf{r})$, we separate the static part $\tilde{I}_0^{nm}(\mathbf{r})$. Obviously, the static part of the tensor current $\tilde{I}_0^{nm}(\mathbf{r})$ will give only static solutions, and therefore we omit it. Using the representations (77), we cast the field equations (76) into the form

$$\Delta\tilde{f}^{nm} + \omega^2\tilde{f}^{nm} = 16\pi\tilde{I}^{nm}.$$

We place the origin of a Cartesian coordinate system at any point of the source. Then in this system the solution of the field equations can be written in the form

$$\tilde{f}^{nm} = -4\int \frac{\exp(\mathrm{i}\omega R)}{R}\,\tilde{I}^{nm}(\omega, \mathbf{r}')\,dV, \quad R = |\mathbf{r} - \mathbf{r}'|. \tag{78}$$

Using the Lorentz conditions (57), $\mathrm{i}\omega\tilde{f}^{0l} = \partial_\alpha\tilde{f}^{\alpha l}$, we express the components $\tilde{f}^{0l}$ in terms of the spatial components:

$$\tilde{f}^{00} = -(1/\omega^2)\,\partial_\alpha\partial_\beta\tilde{f}^{\alpha\beta}; \quad \tilde{f}^{0\alpha} = -(\mathrm{i}/\omega)\,\partial_\beta\tilde{f}^{\alpha\beta}.$$

Outside the source of the gravitational waves, we can, by the choice of the gauge

$$f'^{lm} = f'^{lm} + \partial^l\alpha^m + \partial^m\alpha^l - \gamma^{lm}\partial_n\alpha^n, \tag{79}$$

which is compatible with the Lorentz condition (57) for $\Box\alpha^n = 0$, impose on the wave components $\tilde{f}^{lm}$ four further conditions corresponding to the number of independent gauge vectors. As such conditions, we can choose $\tilde{f}_n'^n = 0$ and $\tilde{f}'^{0\alpha} = 0$ (TT gauge).

The gauge vectors leading to these conditions have the form

$$\tilde{\alpha}^0 = (\mathrm{i}/2\omega)\,[\tilde{f}^{00} - (1/2)\tilde{f}_n^n];$$
$$\tilde{\alpha}^\alpha = (\mathrm{i}/\omega)\,\tilde{f}^{0\alpha} + (1/2\omega^2)\,\partial^\alpha\,[\tilde{f}^{00} - (1/2)\tilde{f}_n^n].$$

As a result of this gauge, we obtain

$$\left.\begin{aligned} &\tilde{f}_n'^n = 0; \quad \tilde{f}'^{0n} = 0;\\ &\tilde{f}'^{\alpha\beta} = \tilde{f}^{\alpha\beta} - \frac{1}{2}\gamma^{\alpha\beta}\tilde{f}_n^n - \frac{\mathrm{i}}{\omega}\left(\partial^\beta\tilde{f}^{0\alpha} + \partial^\alpha\tilde{f}^{0\beta}\right)\\ &\quad - \frac{1}{\omega^2}\,\partial^\alpha\partial^\beta\left(\tilde{f}^{00} - \frac{1}{2}\tilde{f}_n^n\right). \end{aligned}\right\} \tag{80}$$

Using the Lorentz conditions (57), we write the expressions (80) in the form

$$\begin{aligned} \tilde{f}'^{\alpha\beta} &= \tilde{P}^{\alpha\beta} - (1/\omega^2)\left(\partial^\beta\partial_\varepsilon\tilde{P}^{\alpha\varepsilon} + \partial^\alpha\partial_\varepsilon\tilde{P}^{\beta\varepsilon}\right)\\ &+ (1/2\omega^2)\,\gamma^{\alpha\beta}\partial_\varepsilon\partial_\tau\tilde{P}^{\varepsilon\tau} + \frac{1}{2\omega^4}\,\partial^\alpha\partial^\beta\partial_\varepsilon\partial_\tau\tilde{P}^{\varepsilon\tau}, \end{aligned} \tag{81}$$

where

$$\tilde{P}^{\alpha\beta} = \tilde{f}^{\alpha\beta} - (1/3)\,\gamma^{\alpha\beta}\tilde{f}_\tau^\tau. \tag{82}$$

Thus, the wave solution of the field equations contains in the general case six nonvanishing spatial components $\tilde{f}'^{\alpha\beta}$, but only two of these components are independent by virtue of the three Lorentz conditions (57) (the fourth Lorentz condition is trivial because of the TT gauge) and the vanishing of the trace, $\tilde{f}_n'^n = 0$. These subsidiary conditions are the well-known subsidiary conditions for an irreducible representation with spin 2 in the TT gauge, and therefore the free gravitational wave has spin 2, and the scalar component corresponding to the irreducible representation with spin 0 is not emitted in the form of gravitational waves. This last fact can also be seen from the expression for the energy-momentum tensor of the gravitational waves, since in the case of the wave solution the scalar component does not contribute to the energy-momentum tensor of the field.

Usually, the wave solutions of the equations of the gravitational field are written in a somewhat different form which demonstrates clearly the quadrupole nature of the emitted gravitational waves.

In our case, it is also possible to express the obtained solution in terms of generalized quadrupole moments of the tensor current I^{lm}. For this, we note that the spatial components $\tilde{f}^{\alpha\beta}$ (78) can, by virtue of the conservation of the tensor current $\partial_n I^{nm}$, be written in the form

$$\begin{aligned} \tilde{f}^{\alpha\beta} = -4\int \frac{\exp(\mathrm{i}\omega R)}{R}\,\tilde{I}^{\alpha\beta}\,dV = 2\omega^2\Big\{&\int \frac{\exp(\mathrm{i}\omega R)}{R}\,\tilde{I}^{00}x^\alpha x^\beta\,dV\\ &+ \frac{2\mathrm{i}}{\omega}\,\partial_\tau\int \frac{\exp(\mathrm{i}\omega R)}{R}\,\tilde{I}^{0\tau}x^\alpha x^\beta\,dV\\ &- \frac{1}{\omega^2}\,\partial_\varepsilon\partial_\tau\int \frac{\exp(\mathrm{i}\omega R)}{R}\,\tilde{I}^{\tau\varepsilon}x^\alpha x^\beta\,dV\Big\}. \end{aligned} \tag{83}$$

This is an exact relation. It simplifies considerably if the linear dimensions of the source are appreciably less than the distance from its center to the point of observation. Omitting the nonwave terms, which decrease faster than $1/r$, we obtain

$$\tilde{f}^{\alpha\beta}=\frac{2\omega^2}{r}\int dV x^{\alpha}x^{\beta}\exp(i\omega R)\,(\tilde{I}^{00}+2n_{\tau}\tilde{I}^{0\tau}+n_{\varepsilon}n_{\tau}\tilde{I}^{\varepsilon\tau}),$$

where $n^{\varepsilon}=x^{\varepsilon}/r$ and $n_{\tau}n^{\tau}=-1$. Then the expression (82) can be written in the form

$$\tilde{P}^{\alpha\beta}=\frac{2\omega^2}{r}\int dV\left(x^{\alpha}x^{\beta}-\frac{1}{3}\gamma^{\alpha\beta}x_{\varepsilon}x^{\varepsilon}\right)\times(\tilde{I}^{00}+2n_{\tau}\tilde{I}^{0\tau}+n_{\varepsilon}n_{\tau}\tilde{I}^{\varepsilon\tau})\exp(i\omega R). \qquad (84)$$

Introducing the projection operators

$$Z^{\alpha\beta}=\gamma^{\alpha\beta}+n^{\alpha}n^{\beta}, \qquad (85)$$

which satisfy the conditions

$$Z^{\alpha\beta}\gamma_{\alpha\beta}=2;\quad Z^{\alpha\beta}Z_{\beta\tau}=Z^{\alpha}_{\tau},$$

we rewrite (81) in the form

$$\tilde{f}'^{\alpha\beta}=(Z^{\alpha}_{\varepsilon}Z^{\beta}_{\tau}-(1/2)Z^{\alpha\beta}Z_{\varepsilon\tau})\tilde{P}^{\varepsilon\tau}. \qquad (86)$$

Substituting (84) in the Fourier integral, we obtain

$$P^{\alpha\beta}=-\frac{2}{r}\frac{d^2}{dt^2}\int dV\left(x^{\alpha}x^{\beta}-\frac{1}{3}\gamma^{\alpha\beta}x_{\tau}x^{\tau}\right)\times[I^{00}+2n_{\tau}I^{0\tau}+n_{\varepsilon}n_{\tau}I^{\varepsilon\tau}]_{\rm ret}. \qquad (87)$$

Here, $[\ldots]_{\rm ret}$ means that the expression in the square brackets is taken at the retarded time $t'=t-R$. If we introduce the traceless tensor of the generalized quadrupole moment

$$\mathcal{D}^{\alpha\beta}=D^{\alpha\beta}+2n_{\tau}D^{\alpha\beta\tau}+n_{\varepsilon}n_{\tau}D^{\alpha\beta\tau\varepsilon}, \qquad (88)$$

where

$$\left.\begin{aligned}D^{\alpha\beta}&=\int dV(3x^{\alpha}x^{\beta}-\gamma^{\alpha\beta}x_{\tau}x^{\tau})[I^{00}]_{\rm ret};\\ D^{\alpha\beta\tau}&=\int dV(3x^{\alpha}x^{\beta}-\gamma^{\alpha\beta}x_{\varepsilon}x^{\varepsilon})[I^{0\tau}]_{\rm ret};\\ D^{\alpha\beta\tau\varepsilon}&=\int dV(3x^{\alpha}x^{\beta}-\gamma^{\alpha\beta}x_{\delta}x^{\delta})[I^{\tau\varepsilon}]_{\rm ret};\end{aligned}\right\} \qquad (89)$$

then the components of the gravitational wave (86) can be written in the form

$$f'^{\alpha\beta}=-(2/3r)[Z^{\alpha}_{\varepsilon}Z^{\beta}_{\tau}-(1/2)Z^{\alpha\beta}Z_{\varepsilon\tau}]\ddot{\mathcal{D}}^{\varepsilon\tau}. \qquad (90)$$

Here and in what follows, the dot denotes the derivative with respect to the time.

Noting that $\partial_{\varepsilon}f_{\alpha\beta}=n_{\varepsilon}\dot{f}_{\alpha\beta}$, for the components of the energy–momentum tensor $\tilde{t}^{\alpha}_{g0}$ of the gravitational wave we obtain the expression

$$\tilde{t}^{\alpha}_{g0}=(1/32\pi)n^{\alpha}\dot{f}_{\tau\varepsilon}\dot{f}^{\tau\varepsilon}.$$

Then for the intensity of emission of gravitational-wave energy in the element of solid angle $d\Omega$ we have

$$dI/d\Omega=(1/32\pi)r^2\dot{f}_{\varepsilon\tau}\dot{f}^{\varepsilon\tau}. \qquad (91)$$

It can be seen from (91) that this is a positive quantity for all values of the components of the tensor $\dot{f}^{\alpha\beta}$ if they do not all vanish. If they all vanish, $\dot{f}^{\alpha\beta}=0$, then $dI/d\Omega=0$ also.

Using the relations (85) and (90), we can write the expression (91) in the form

$$\frac{dI}{d\Omega}=\frac{1}{32\pi}\left\{\frac{1}{4}(\dddot{\mathcal{D}}^{\alpha\beta}n_{\alpha}n_{\beta})^2+\frac{1}{2}\dddot{\mathcal{D}}^{\alpha\beta}\dddot{\mathcal{D}}_{\alpha\beta}+n_{\beta}n^{\tau}\dddot{\mathcal{D}}^{\alpha\beta}\dddot{\mathcal{D}}_{\alpha\tau}\right\}. \qquad (92)$$

The expressions (90) and (91) hold for points of observation far from the source at distances appreciably exceeding the linear dimensions of the source and the wavelength. No restrictions are imposed on the ratio of the source dimensions to the wavelength of the emitted wave.

To calculate the energy loss in all directions per unit time, it remains to integrate (92) over the angles of the point of observation.

We consider in what follows the case of the emission of weak gravitational waves, which is the case most frequently considered in practice. In the usual linear approximation, the tensor current I^{lm} (51) must be taken in the absence of the gravitational field. In this approximation, the only physical symmetric tensor of second rank satisfying a conservation law is the energy-momentum tensor of the matter, and we therefore require that the following correspondence hold: In the zeroth approximation in the gravitational field, the tensor current I^{lm} must go over automatically into the energy–momentum tensor of the matter:

$$I^{lm}(f_{ik}=0)=T^{lm} \qquad (93)$$

This correspondence requirement makes it possible to recover uniquely in the linear approximation the structure of the connection equations $g_{ik}=g_{ik}(\gamma_{lm}, f_{lm})$. To see this, using the expressions (40), (47), and (51), we find that the correspondence requirement (93) leads to the following connection equations in the linear approximation:

$$g_{mn}=\gamma_{mn}+f_{mn}-(1/2)\gamma_{mn}f^{l}_{l}.$$

Note that in the present case $h^{lm}=T^{lm}$ also. In the case of the emission of gravitational waves with wavelengths much greater than the dimensions of the source, the retardation in the system can be ignored, and in Eqs. (89) the expressions in the square brackets can be taken at the time $t'=t-r$.

If the components of the energy–momentum tensor of the matter satisfy the inequalities

$$|\dddot{T}^{00}|\gg|\dddot{T}^{0\alpha}|;\quad|\dddot{T}^{00}|\gg|\dddot{T}^{\alpha\beta}|,$$

then for the energy loss over all directions per unit time we obtain

$$-dE/dt=(G/45c^5)\dddot{D}^{\alpha\beta}\dddot{D}_{\alpha\beta}, \qquad (94)$$

where

$$D^{\alpha\beta}=\int dV(3x^{\alpha}x^{\beta}-\gamma^{\alpha\beta}x_{\tau}x^{\tau})T^{00}(t-r)$$

and we have introduce the gravitational constant G and the velocity of light c.

This formula agrees with the results of indirect measurements of the energy loss of the binary pulsar PSR 1913+16 through the assumed emission of gravitational waves.

The first observations of the pulsar PSR1913+16 showed[28] that it is one component of a binary system and has parameters which permit the observation of a number of relativistic effects. This pulsar has a very short period of orbital revolution, $T=8$ h, a relatively high orbital velocity $v\sim10^{-3}c$, and its orbit has a very high eccentricity $e=0.6$ and small linear dimensions of the order of the radius of the Sun. The other component of the system is not a pulsar. However, from the absence of eclipse of the pulsar signal, and also from the small linear dimensions of the pulsar orbit, it follows that this component must be compact and may be a neutron star, white dwarf, or another compact object.

Subsequent measurements showed[29] that the period of revolution of the pulsar in its orbit decreases [$\dot{T} = (-3.2 \pm 0.6) \times 10^{-12}$ second per second], and the periastron of the system advances ($\delta\varphi = 4.226 \pm 0.002°$ per year), After allowance for various effects, Taylor, Fowler, and McCulloch concluded that the decrease in the orbital period and the advance of the periastron can be explained by assuming that the binary system emits gravitational waves, these carrying away positive energy in accordance with Eq. (94).

Since the calculation of the "energy loss" usually made in general relativity by means of the energy–momentum pseudotensors in the weak-field approximation leads to the expression (94), it was concluded in Ref. 29 that the results of the observations agree with the prediction of Einstein's theory.

However, as is shown in Ref. 5, Eq. (94) is not a consequence of Einstein's general theory of relativity. In Einstein's theory, one can only speak of curvature waves, with which energy transfer to matter is associated, but conservation laws in their usual sense are absent, as a result of which the calculation of the energy loss by a source and the determination of the energy fluxes of gravitational waves in general relativity are impossible.

Thus, if one believes the experimental results of Ref. 29, Einstein's theory is not in a position to explain the results of observation of the binary pulsar PSR 1913+16.

In the field theory of gravitation, the gravitational field, like all other physical fields, possesses energy and momentum, and when a slowly moving source emits weak gravitational waves its energy decreases in accordance with (94). Therefore, the experimental proof of the existence of gravitational waves as a physical field transmitting energy and thereby decreasing the source energy would be a confirmation of the ideas developed here.

To conclude the present section, we discuss briefly the calculation of the Riemann tensor in the field theory of gravitation. In Einstein's theory, one can have a situation[30] when the energy–momentum pseudotensor of the gravitational waves is zero but the components of the Riemann tensor do not vanish. This beautifully illustrates the invalidity of interpreting the energy–momentum pseudotensors as energy characteristics of the gravitational field.

In the field theory of gravitation, the vanishing of the components of the energy–momentum tensor of the gravitational waves entails the identical vanishing of the Riemann tensor as well, i.e., the formation of the Riemannian space-time always requires energy and momentum of the gravitational field. It should be noted that the metric of the Riemannian space-time has meaning only within the matter. One can calculate the components of the metric tensor g_{in}, and also the curvature tensor R^i_{nlm}, at any point, which may be outside the matter, but one must then always take into account the need for an appropriate gauge of the fields f_m outside the matter, since the physical quantities do not depend on the components of the field f_{mn} that change under gauge transformations. These components do not occur in the expression for the energy–momentum tensor of the gravitational field. By an appropriate gauge transformation they can always be made to vanish. Therefore, in a calculation outside the matter of the geometrical characteristics of the space-time such as, for example, the metric tensor g_{in} or the Riemann tensor R^i_{nlm} we must substitute in the connection equation $g_{in} = g_{in}(\gamma_{lm}, f_{lm})$ only those components f_{in} that occur in the energy–momentum tensor of the gravitational field; we shall assume that all the other field components are zero, since they can be made to vanish by an appropriate gauge transformation. Thus, our theory will always be internally consistent.

Suppose all components of the canonical energy–momentum tensor of free gravitational waves vanish:

$$\tilde{t}^n_{gp} = (1/64\pi)\{2\partial_p f_{lm}\partial^n f^{lm} - \partial_p f^l_l \partial^n f^m_m - \delta^n_p [\partial_i f_{lm}\partial^i f^{lm} - (1/2)\,\partial_i f^l_l \partial^i f^m_m]\} = 0. \quad (95)$$

From the vanishing of the trace of this tensor we have

$$\partial_p f_{lm}\partial^n f^{lm} - (1/2)\,\partial_p f^l_l \partial^n f^m_m = 0.$$

For $n = p = 0$, we obtain

$$\dot{f}_{lm}\dot{f}^{lm} - (1/2)\,\dot{f}^m_m \dot{f}^l_l = 0. \quad (96)$$

We show that in the TT gauge all the components of a free gravitational wave vanish identically by virtue of the condition (96). Then in this gauge all the components of the metric tensor of the Riemannian space-time are equal to the components of the metric tensor of the flat space-time: $g_{in} = \gamma_{in}$. Therefore, the curvature tensor also vanishes if the energy–momentum tensor of the gravitational field does.

We consider a point. We orient the x axis of a Cartesian coordinate system to make it pass through the point of observation. Around this point we take a sufficiently small region for the gravitational wave to be regarded as plane in it. Then all its components will depend only on the different $t - x$. The conditions $\partial_n f^{nm} = 0$ take the form

$$\dot{f}^{00} = \dot{f}^{01} = \dot{f}^{11}; \quad \dot{f}^{02} = \dot{f}^{12}; \quad \dot{f}^{03} = \dot{f}^{13}.$$

Integrating these equations and setting the constants of integration equal to zero, since the gravitational waves do not have time-independent parts, we obtain

$$f^{00} = f^{01} = f^{11}; \quad f^{02} = f^{12}; \quad f^{03} = f^{13}.$$

By virtue of the TT gauge, all these components vanish. In addition, from the vanishing of the trace, $f^n_n = 0$, we have $f^{22} = -f^{33}$. From the condition of vanishing of the energy–momentum tensor (95) we obtain

$$2(\dot{f}_{23})^2 + (1/2)(\dot{f}_{22} - \dot{f}_{33})^2 = 0,$$

from which we have the equations

$$\dot{f}_{23} = 0; \quad \dot{f}_{22} = \dot{f}_{33}.$$

Then the transverse components of the gravitational wave also vanish:

$$f_{23} = 0;\ f_{22} = f_{33} = 0.$$

Thus, in the TT gauge the condition of vanishing of the energy–momentum tensor of a free gravitational wave entails the vanishing of all components of this wave. Therefore, all the components of the metric tensor of the Riemannian space-time are equal to the corresponding components of the metric tensor of the pseudo-Euclidean space-time, $g_{in}=\gamma_{in}$, which leads to vanishing of all components of the Riemann tensor:

$$R^i_{nlm}=0.$$

7. POST-NEWTONIAN APPROXIMATION OF THE FIELD THEORY OF GRAVITATION

Until recently, the requirements imposed on possible theories of gravitation reduced to the need to obtain Newton's law of gravitation in the weak-field limit and also to describe the three effects open to observation: the gravitational red shift in the field of the Sun, the bending of a light ray passing near the Sun, and the advance of Mercury's perihelion.

These experiments are intimately related to gravimetric measurements of the equality of the gravitational, M_g, and inertial, M_i, masses made in the last century by Bessel and Eötvös. These measurements established that for bodies of laboratory size the ratio of the gravitational mass to the inertial mass can differ from unity by not more than 10^{-9}, irrespective of the matter of which the body consists. This result made a deep impression on Einstein, and prompted him to formulate the equivalence principle.

However, although this result is regarded as establishing the equality of gravitational and inertial mass to a very high accuracy, it does not mean that large bodies have equal gravitational and inertial masses to the same accuracy. For bodies of laboratory dimensions, the gravitational self-energy of the body, the energy of elastic deformations of the body, and so forth, are very small compared with the total energy of the body. According to the estimates of Nordtvedt,[31] for a body of mass M having characteristic dimension a the ratio of the gravitational self-energy of the body to its total energy is

$$(GM^2/a)/Mc^2=GM/ac^2\sim G\rho a^2/c^2,$$

where ρ is the density of the body.

This ratio is of order 10^{-25} for bodies of laboratory sizes. Therefore, if the measurements have accuracy 10^{-9}, nothing can be said about the distribution of the gravitational energy between the inertial and gravitational masses of the body. And even the gravimetric experiments made with increased accuracy (10^{-11} in the experiments made by Dicke's group,[32] and 10^{-12} in the experiments of Braginskii's group[33]) do not answer the posed problem.

Therefore, one can assert that the gravimetric measurements establish the equality of the gravitational and inertial masses of a point body, i.e., a body having negligibly small size and, therefore, negligibly small gravitational self-energy, elastic-deformation energy, etc.; the measurements do not establish more than that. To establish the equality of the gravitational and inertial masses of an extended body, one must either increase significantly the accuracy of the gravimetric experiments with bodies of laboratory size, or make measurements with larger bodies, for example, planets, for which the ratio of the gravitational self-energy to the total energy is much greater than for bodies of laboratory size. In the case of planets, inequality of the inertial and gravitational masses would be revealed in small perturbations of the orbits. However, the traditional methods of optical astronomy did not permit measurements of the orbital parameters of planets with the accuracy needed for this purpose.

In addition, an appropriate theoretical formalism for such experiments had not been developed. Thus, the existing requirements on possible theories of gravitation were clearly inadequate, since a very large number of theories satisfied these requirements.

For further selection among theories of gravitation, it was necessary to make qualitatively new experiments.

At the present time, due to the development of experimental techniques, especially astronautics, and the greater accuracy of measurements, it has become possible to make more accurate measurement of the orbital parameters of the planets (and above all the Moon), to measure the time delay of radio signals in the gravitational field of the Sun, and to make new experiments within the solar system. These experiments make it possible to restrict further the group of viable gravitational theories. To facilitate the comparison of the results of experiments made within the solar system with the predictions of the various theories of gravitation for which Riemannian geometry is the natural geometry for the motion of matter, Nordtvedt and Will[34] developed a formalism that has become known as the parametrized post-Newtonian (PPN) formalism.

In this formalism, the metric of the Riemannian space-time produced by a body consisting of a perfect fluid is expressed in the form of the sum of all possible generalized gravitational potentials with arbitrary coefficients, which are called post-Newtonian parameters. Using the revised Will–Nordtvedt parameters, the metric of the Riemannian space-time can be written in the form

$$\left.\begin{aligned}
g_{00}&=1-2U+2\beta U^2-(2\gamma+2+\alpha_3+\xi_1)\Phi_1\\
&+\xi_1 A+2\xi_\omega\Phi_\omega-2\,[(3\gamma+1-2\beta+\xi_2)\Phi_2\\
&+(1+\xi_2)\Phi_3+3(\gamma+\xi_4)\Phi_4]-(\alpha_1-\alpha_2-\alpha_3)W^\alpha W_\alpha U\\
&+\alpha_2 W^\alpha W^\beta U_{\alpha\beta}-(2\alpha_3-\alpha_1)W^\alpha U_\alpha;\\
g_{0\alpha}&=(1/2)(4\gamma+3+\alpha_1-\alpha_2+\xi_1)V_\alpha+(1/2)(1+\alpha_2-\xi_1)N_\alpha\\
&+(1/2)(\alpha_1-2\alpha_2)W_\alpha U+\alpha_2 W^\beta U_{\alpha\beta};\\
g_{\alpha\beta}&=(1+2\gamma U)\gamma_{\alpha\beta},
\end{aligned}\right\}\quad(97)$$

where W^α are the spatial components of the velocity of the frame of reference relative to a universal rest frame. For some theories of gravitation, this is the velocity of the center of mass of the solar system with respect to the rest frame of the Universe.

The generalized gravitational potentials have the form

$$\left.\begin{aligned}
&U(\mathbf{r}, t) = \int \frac{\rho_0(\mathbf{r}', t)}{R}\, dV;\; R = |\mathbf{r} - \mathbf{r}'|;\; R^\alpha = x^\alpha - x'^\alpha;\\
&\Phi_1 = -\int \frac{\rho_0 v_\alpha v^\alpha}{R}\, dV;\; \Phi_2 = \int \frac{\rho_0 U}{R}\, dV;\\
&\Phi_3 = \int \frac{\rho_0 \Pi(\mathbf{r}', t)}{R}\, dV;\; \Phi_4 = \int \frac{P}{R}\, dV;\\
&A = \int \frac{\rho_0 v_\alpha v_\beta R^\alpha R^\beta}{R^3}\, dV;\; V_\alpha = -\int \frac{\rho_0 v_\alpha}{R}\, dV;\\
&N_\alpha = \int \frac{\rho_0 v_\beta R^\beta R_\alpha}{R^3}\, dV;\; U_{\alpha\beta} = \int \frac{\rho_0 R_\alpha R_\beta}{R^3}\, dV;\\
&\Phi_\omega = \int \frac{\rho_0(\mathbf{r}', t)\,\rho_0(\mathbf{r}'', t)}{|\mathbf{r}' - \mathbf{r}|^3} \left\{ \frac{\mathbf{r}' - \mathbf{r}''}{|\mathbf{r} - \mathbf{r}''|} - \frac{\mathbf{r} - \mathbf{r}''}{|\mathbf{r}' - \mathbf{r}''|} \right\}\\
&\times (\mathbf{r} - \mathbf{r}')\, d\mathbf{r}'\, d\mathbf{r}'',
\end{aligned}\right\} \tag{98}$$

where ρ_0 is the invariant mass density of the body, v^α are the velocity components of the elements of the ideal fluid, P is the isotropic pressure, and $\rho_0\Pi$ is the density of the internal energy of the ideal fluid.

Sometimes the post-Newtonian expansion (97) of the metric is expressed in terms of Will's parameters $\gamma, \beta, \beta_1, \beta_2, \beta_3, \beta_4, \xi, \Delta_1, \Delta_2, \xi_\omega$. These parameters are related linearly to the revised Will–Nordtvedt parameters:

$$\alpha_1 = 7\Delta_1 + \Delta_2 - 4\gamma - 4;\quad \alpha_2 = \Delta_2 + \xi - 1;$$
$$\alpha_3 = 4\beta_1 - 2\gamma - 2 - \xi;\quad \xi_1 = \xi;$$
$$\xi_2 = 2\beta + 2\beta_2 - 3\gamma - 1;\quad \xi_3 = \beta_3 - 1;$$
$$\xi_4 = \beta_4 - \gamma.$$

In this paper, we shall use the revised Will–Nordtvedt parameters.

Every theory of gravitation in which the natural geometry for describing the motion of matter is Riemannian geometry will have its corresponding set of values of the ten post-Newtonian parameters $\beta, \gamma, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4, \xi_\omega$; from the point of view of experiments made in the solar system, one theory of gravitation will differ from another only in the values of these parameters. Naturally, it is assumed that the metric tensor g_{in} of each theory of gravitation is expressed in the same coordinate system as the metric tensor (97), since otherwise the comparison of the post-Newtonian parameters becomes meaningless, different coordinate systems corresponding to different sets of parameters. Therefore, after the determination of the metric tensor g_{in} produced by the gravitational field of the solar system, it is necessary to go over to a "canonical" coordinate system, in which the metric tensor g_{in} has the form (97).

To find theories of gravitation that in the post-Newtonian limit make it possible to describe all experiments made in the solar system, it is sufficient to determine from all these experiments the values of the ten post-Newtonian parameters and select only those theories of gravitation whose post-Newtonian approximation leads to values of the parameters equal to those obtained in the experiments. Then all such theories of gravitation will be indistinguishable from the point of view of all experiments made with post-Newtonian accuracy.

A further selection between theories of gravitation adequate for the description of reality entails either an increase in the accuracy of the measurements to the post-post-Newtonian level or the search for possibilities of studying the properties of gravitational waves and also phenomena in strong gravitational fields.

We now determine what set of values of the post-Newtonian parameters corresponds to the field theory of gravitation.

We write the gravitational field equations of this theory for the calculation of the post-Newtonian approximation in the form

$$\Box^2 f^{nm} = -16\pi J^{nm};\; \Box = \partial_i \partial^i. \tag{99}$$

If we use the notation (40), then for the tensor current J^{mn} we obtain the expression

$$J^{nm} = \Box h^{nm} - \partial^n \partial_p h^{pm} - \partial^m \partial_p h^{pn} - \gamma^{nm} \partial_l \partial_p h^{lp}. \tag{100}$$

The metric tensor g_{in} of the Riemannian space-time is a local function that depends only on the fields f_{in} and on the metric tensor of the flat space-time. Using the correspondence requirement, in Sec. 6 we obtained an expression for the metric tensor of the Riemannian space-time in the linear approximation in the field f_{nm}:

$$g_{nm} = \gamma_{nm} + f_{nm} - (1/2)\, \gamma_{nm} f_l^l. \tag{101}$$

One might assume that the relation (101) is an exact connection equation and is satisfied always and not only in the linear approximation in the weak field f_{nm}. But then a theory with such a connection equation will belong to the class of so-called quasilinear theories of gravitation (in the terminology of Will). However, as is shown in Ref. 35, any quasilinear asymptotically Lorentz-invariant theory of gravitation contradicts the results of experiments. Therefore, the relation (101) must be only an expansion of the connection equation up to terms linear in the weak field f_{nm}. Thus, the connection equation $g_{nm} = g_{nm}(\gamma_{lm}, f_{lm})$ must be a nonlinear equation in the field f_{lm}.

In the present paper, we shall not consider any definite connection equation, since the construction of the post-Newtonian approximation of the field theory of gravitation and the description of all effects does not require such a choice to be made. It is only necessary to write down the expansion of this connection equation up to quadratic terms. As we shall see later, even these very general assumptions about the connection equation are sufficient to describe the complete set of existing gravitational experiments.

In the case of a weak field, the expansion of the connection equation up to quadratic terms can be written in the most general form as

$$g_{nm} = \gamma_{nm} + f_{nm} - (1/2)\, \gamma_{nm} f_l^l + (1/4)\, [b_1 f_{nl} f_m^l + b_2 f_{nm} f_l^l + b_3 \gamma_{nm} f_{il} f^{il} + b_4 \gamma_{nm} f_l^l f_i^i] \tag{102}$$

with as yet undetermined coefficients b_1, b_2, b_3, and b_4.

In the language of the theory of interacting fields, the connection equation (102) is equivalent to writing down the Lagrangian density for the interaction of the gravitational field with the remaining matter fields only in the approximation of a weak gravitational field up to quadratic terms.

Following Fock,[27] to construct a post-Newtonian approximation valid in the solar system we shall consider a problem of astronomical type. We shall assume that the components of the energy–momentum tensor of the matter vanish in the whole of space except for some regions. Within each such region, the energy–momentum tensor must correspond to our model of an ideal fluid and must satisfy the covariant conservation equation in the Riemannian space-time. Besides the physical properties of the model of the celestial bodies, the energy–momentum tensor of the matter will also depend on the metric of the Riemannian space-time. Therefore, to write down the expressions for the components of the energy–momentum tensor of the matter it is necessary to know the metric. But to determine the metric tensor, it is necessary to solve Eq. (99), which is possible if the energy–momentum tensor of the matter is known. Thus, the construction of the energy–momentum tensor of the matter and the determination of the metric tensor of the Riemannian space-time must be done simultaneously.

We use the circumstance that within the solar system the maximal values of the gravitational potential, the square of the characteristic velocity v^2 (the velocity of the celestial bodies with respect to the center of mass of the solar system), the specific pressure P/ρ_0, and the specific internal energy Π have approximately the same order $\sim\varepsilon^2$, where $\varepsilon \sim 10^{-3}$ is a small dimensionless parameter. Therefore, in the solar system we have the estimates

$$\left.\begin{array}{l} U = O(\varepsilon^2);\ v^\alpha = O(\varepsilon); \\ \Pi = O(\varepsilon^2);\ P/\rho_0 = O(\varepsilon^2). \end{array}\right\} \qquad (103)$$

In addition, we shall consider the field in the near zone, i.e., at distances from the Sun significantly less than the wavelength of gravitational waves emitted by objects in the solar system, which move with characteristic velocity $v^\alpha = \varepsilon$: $R/\lambda \sim R\partial/\partial t \sim \varepsilon$. In such a case, the variations of all quantities with the time are due in the first place to the motion of the matter. Therefore, the partial derivatives with respect to the time are small compared with the partial derivatives with respect to the coordinates:

$$\partial/\partial t = 0(\varepsilon)\, \partial/\partial x^\alpha. \qquad (104)$$

We shall solve the problem of simultaneous determination of the energy–momentum tensor of the matter and the metric tensor of the Riemannian space-time in successive stages, each of which corresponds to expansion of the exact equations of the problem in powers of the dimensionless parameter ε.

We have the following exact relations: the energy–momentum tensor of the ideal fluid is

$$T^{nm} = \sqrt{-g}\,[(P+\mathscr{E})\, u^n u^m - P g^{nm}]; \qquad (105)$$

the covariant continuity equation is

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^l} (\sqrt{-g}\, \rho_0 u^i) = 0, \qquad (106)$$

and the conservation equation for the energy–momentum tensor density of the matter in the Riemannian space-time is

$$\nabla_n T^{nm} = \partial_n T^{nm} + \Gamma^m_{ni} T^{in} = 0, \qquad (107)$$

where $\mathscr{E}$ is the total energy density of the ideal fluid, u^i is the 4-velocity, and Γ^m_{ni} is the connection of the Riemannian space-time.

For our purposes, the gravitational field equations (99) and the connection equation (102) are more conveniently written in the form

$$\Box^2 \chi^{nm} = -16\pi A^{nm}; \qquad (108)$$

$$g_{nm} = \gamma_{nm} + \chi_{nm} + (1/4)\,[b_1 \chi_{nl}\chi^l_m + b_3 \gamma_{nm}\chi^{li}\chi_{li} - (b_1+b_2)\,\chi_{nm}\chi^l_l + (b_4 + b_2/2 + b_1/4)\,\gamma_{nm}\chi^l_l\chi^i_i], \qquad (109)$$

where

$$\chi^{mn} = f^{mn} - (1/2)\,\gamma^{mn} f^l_l; \qquad (110)$$

$$A^{nm} = \Box\left(h^{nm} - \frac{1}{2}\gamma^{nm} h^l_l\right) - \partial^n \partial_l h^{lm} - \partial^m \partial_l h^{ln}.$$

We expand all the quantities in Eqs. (105)–(108) in series in the small parameter ε. If we ignore the energy loss due to emission of gravitational waves, these expansions must also be valid when the sign of the time is reversed. When this occurs, i.e., when we make the coordinate transformation $x'^0 = -x^0$, the components v^α, $\chi^{0\alpha}$, $T^{0\alpha}$, $g_{0\alpha}$, $A^{0\alpha}$, $\partial/\partial x^0$ reverse their sign. Since $v^\alpha \sim \varepsilon$ and $\partial/\partial x^0 \sim \varepsilon\, \partial/\partial x^\alpha$, the dimensionless parameter ε also changes sign under time reversal. It follows that if energy loss due to emission of gravitational waves is ignored the expansions of the components v^α, $\chi^{0\alpha}$, $T^{0\alpha}$, $g_{0\alpha}$, $A^{0\alpha}$ contain only odd powers of the parameter ε, and the expansions of the remaining components contain only even powers of the parameter ε.

We write the expansions of the tensor current A^{nm} and the field χ^{nm} in the form

$$\chi^{mn} = \overset{(1)}{\chi}{}^{mn} + \overset{(2)}{\chi}{}^{mn} + \ldots; \qquad (111)$$

$$A^{mn} = \overset{(0)}{A}{}^{mn} + \overset{(1)}{A}{}^{mn} + \overset{(2)}{A}{}^{mn}, \qquad (112)$$

where the components of the zeroth $A^{(0)mn}$, first $A^{(1)mn}$, and second $A^{(2)mn}$ approximations have the orders

$$\overset{(0)}{A}{}^{0\alpha} = O(\varepsilon);\quad \overset{(0)}{A}{}^{00} = O(1);\quad \overset{(0)}{A}{}^{\alpha\beta} = O(1);$$

$$\overset{(1)}{A}{}^{0\alpha} = O(\varepsilon^3);\quad \overset{(1)}{A}{}^{00} = O(\varepsilon^2);\quad \overset{(1)}{A}{}^{\alpha\beta} = O(\varepsilon^2);$$

$$\overset{(2)}{A}{}^{0\alpha} = O(\varepsilon^5);\quad \overset{(2)}{A}{}^{00} = O(\varepsilon^4);\quad \overset{(2)}{A}{}^{\alpha\beta} = O(\varepsilon^4).$$

We rewrite the gravitational field equations (108) using the expansions (111) and (112) and the estimates (104) in the form of successive approximations:

$$\left.\begin{array}{l} \Delta^2 \overset{(1)}{\chi}{}^{mn} = -16\pi \overset{(0)}{A}{}^{mn}; \\ \Delta^2 \overset{(2)}{\chi}{}^{mn} = -16\pi \overset{(1)}{A}{}^{mn} + 2\partial^2 \Delta \overset{(1)}{\chi}{}^{mn}/\partial t^2. \end{array}\right\} \qquad (113)$$

From (40) and (102), we obtain

$$h^{mn} = T^{mn} + (b_1/4)\,[T^{nl}\chi^m_l + T^{ml}\chi^n_l] + (b_3/2)\,\gamma_{lp} T^{lp}\chi^{mn} - [(b_1+b_2)/4]\, T^{mn}\chi^p_p - [(b_1+b_2)/4]\,\gamma^{mn} T^{li}\chi_{li} + (b_2/4 + b_1/8 + b_4/2)\,\gamma^{mn}\gamma_{lp} T^{lp}\chi^i_i. \qquad (114)$$

Then for the tensor current A^{nm}, we have

$$\left.\begin{array}{l} \overset{(0)}{A}{}^{nm} = -\Delta\,[\overset{(0)}{T}{}^{nm} - (1/2)\,\gamma^{nm}\gamma_{il}\overset{(0)}{T}{}^{il}]; \\ \overset{(1)}{A}{}^{mn} = \dfrac{\partial^2}{\partial t^2}\left[\overset{(0)}{T}{}^{mn} - \dfrac{1}{2}\gamma^{mn}\gamma_{lp}\overset{(0)}{T}{}^{lp}\right] + \partial^n (\overset{(1)}{\Gamma}{}^m_{lp}\overset{(0)}{T}{}^{lp}) \\ \quad + \partial^m (\overset{(1)}{\Gamma}{}^n_{lp}\overset{(0)}{T}{}^{lp}) - \Delta\left[\overset{(1)}{T}{}^{mn} - \dfrac{1}{2}\gamma^{mn}\gamma_{lp}\overset{(1)}{T}{}^{lp} + \dfrac{b_1}{4}(\overset{(0)}{T}{}^{nl}\overset{(1)}{\chi}{}^m_l \right. \\ \quad + \overset{(0)}{T}{}^{ml}\overset{(1)}{\chi}{}^n_l) - \dfrac{b_1+b_2}{4}\overset{(0)}{T}{}^{mn}\overset{(1)}{\chi}{}^l_l \\ \quad + \dfrac{b_2}{4}\gamma^{mn}\overset{(0)}{T}{}^{il}\overset{(1)}{\chi}_{il} + \dfrac{b_3}{2}\gamma_{il}\overset{(0)}{T}{}^{il}\overset{(1)}{\chi}{}^{mn} \\ \quad \left. - \left(\dfrac{b_2}{8} + \dfrac{b_3}{4} + \dfrac{b_4}{2}\right)\gamma^{mn}\overset{(0)}{T}{}^{il}\gamma_{il}\overset{(1)}{\chi}{}^p_p\right], \end{array}\right\} \qquad (116)$$

where $\Delta = -\partial_\alpha\partial^\alpha$.

To determine the post-Newtonian parameters, it is sufficient to find the components $g_{\alpha\beta}$ up to ε^2, the components $g_{0\alpha}$ up to ε^3, and the component g_{00} up to ε^4.

It follows from the connection equation (109) that for this it is necessary to determine the field components $\chi^{\alpha\beta}$ to accuracy ε^2, $\chi^{0\alpha}$ to accuracy ε^3, and χ^{00} to accuracy ε^4.

In the initial approximation, we assume that the metric tensor of the Riemannian space-time is equal to the metric tensor of the pseudo-Euclidean space-time, i.e., we ignore gravitational forces. Then Eqs. (106) and (107) take the form

$$\left.\begin{aligned}&\partial(\rho u^i)/\partial x^i = O(\varepsilon^2);\\&\partial_n T^{n0} = O(\varepsilon^3);\\&\partial_n T^{n\alpha} = O(\varepsilon^2).\end{aligned}\right\} \quad (117)$$

Using the estimates (104), we obtain from Eqs. (117)

$$u^0 = 1 + O(\varepsilon^2);\quad u^\alpha = v^\alpha(1 + O(\varepsilon^2));$$
$$T^{00} = \rho_0(1 + O(\varepsilon^2));\quad T^{\alpha\beta} = \rho_0 O(\varepsilon^2);$$
$$T^{0\alpha} = \rho_0 v^\alpha(1 + O(\varepsilon^2)).$$

Therefore, the components of the tensor current A^{mn} in the zeroth approximation can be written in the form

$$\overset{(0)}{A}{}^{00} = -(1/2)\Delta\rho_0;\quad \overset{(0)}{A}{}^{0\alpha} = \Delta(\rho_0 v^\alpha);\quad \overset{(0)}{A}{}^{\alpha\beta} = (1/2)\gamma^{\alpha\beta}\Delta\rho_0. \quad (118)$$

Then from Eq. (113) we obtain

$$\overset{(1)}{\chi}{}^{00} = -2U;\quad \overset{(1)}{\chi}{}^{\alpha\beta} = 2\gamma^{\alpha\beta}U;\quad \overset{(1)}{\chi}{}^{0\alpha} = 4V^\alpha. \quad (119)$$

As a result, the components of the metric tensor (109) of the Riemannian space-time can be written in the first approximation in the form

$$\left.\begin{aligned}&g_{00} = 1 - 2U + O(\varepsilon^4);\\&g_{0\alpha} = 4V_\alpha(1 + O(\varepsilon^2));\\&g_{\alpha\beta} = \gamma_{\alpha\beta}(1 + 2U) + O(\varepsilon^4).\end{aligned}\right\} \quad (120)$$

The knowledge of the metric in this approximation makes it possible to determine the components of the energy–momentum tensor of the matter in the following approximation. Using the expressions (120), we find

$$\begin{aligned}&\sqrt{-g} = 1 + 2U + O(\varepsilon^4);\\&u^0 = 1 + U - (1/2)v_\alpha v^\alpha + O(\varepsilon^4);\\&\Gamma^0_{00} = -\partial U/\partial t + O(\varepsilon^5);\\&\Gamma^\alpha_{00} = \gamma^{\alpha\beta}\partial U/\partial x^\beta + O(\varepsilon^4);\\&\Gamma^0_{0\alpha} = -\partial U/\partial x^\alpha + O(\varepsilon^4);\\&\Gamma^\alpha_{\beta\tau} = O(\varepsilon^2);\quad \Gamma^\alpha_{0\beta} = O(\varepsilon^3);\quad \Gamma^0_{\alpha\beta} = O(\varepsilon^3).\end{aligned} \quad (121)$$

We also introduce the conserved mass density ρ in accordance with the equation $\rho = \sqrt{-g}\rho_0 u^0$. To obtain the metric in the following approximation, we must construct the energy–momentum tensor density of the matter satisfying the conservation equations (107),

$$\partial_0 T^{00} + \Gamma^0_{00}T^{00} + \partial_\alpha T^{0\alpha} + 2\Gamma^0_{0\alpha}T^{0\alpha} = O(\varepsilon^5),$$
$$\partial_0 T^{0\alpha} + \partial_\beta T^{\beta\alpha} + \Gamma^\alpha_{00}T^{00} = O(\varepsilon^4)$$

by virtue of the covariant continuity equation

$$(1/\sqrt{-g})[\partial\rho/\partial t + \partial(\rho v^\alpha)/\partial x^\alpha] = 0$$

and the equation of motion of the ideal fluid in the Newtonian approximation[27]:

$$\rho\left(\frac{\partial v^\alpha}{\partial t} + v^\beta\frac{\partial v^\alpha}{\partial x^\beta}\right) = \gamma^{\alpha\beta}\left(-\rho\frac{\partial U}{\partial x^\beta} + \frac{\partial P}{\partial x^\beta}\right);$$
$$\rho\left(\frac{\partial\Pi}{\partial t} + v^\beta\frac{\partial\Pi}{\partial x^\beta}\right) = -P\partial_\alpha v^\alpha.$$

It is easy to show that these conditions are satisfied by the following components of the energy–momentum tensor density of the matter:

$$\left.\begin{aligned}&T^{00} = \rho(1 - v_\alpha v^\alpha/2 + \Pi + U) + \rho O(\varepsilon^4);\\&T^{0\alpha} = \rho v^\alpha(1 - v_\beta v^\beta/2 + \Pi + U) + Pv^\alpha + \rho O(\varepsilon^5);\\&T^{\alpha\beta} = \rho v^\alpha v^\beta - \gamma^{\alpha\beta}P + \rho O(\varepsilon^4).\end{aligned}\right\} \quad (122)$$

At the same time, we have the following relation between the conserved mass density ρ and the invariant mass density ρ_0:

$$\rho = \rho_0[1 + 3U - (1/2)v_\alpha v^\alpha] + \rho O(\varepsilon^4). \quad (123)$$

From (122) and (123), we obtain

$$\left.\begin{aligned}&\overset{(0)}{T}{}^{00} = \rho_0;\quad \overset{(0)}{T}{}^{\alpha\beta} = 0;\quad \overset{(0)}{T}{}^{0\alpha} = \rho_0 v^\alpha;\\&\overset{(1)}{T}{}^{00} = \rho_0(4U + \Pi - v_\alpha v^\alpha);\\&\overset{(1)}{T}{}^{\alpha\beta} = \rho_0 v^\alpha v^\beta - \gamma^{\alpha\beta}P.\end{aligned}\right\} \quad (124)$$

To obtain the post-Newtonian approximation, it remains to determine $\chi^{(2)}{}_{00}$. Equation (114) for the component $\chi^{(2)}{}_{00}$, with allowance for (116), (119), (121), and (124), takes the form

$$\Delta^2\overset{(2)}{\chi}{}^{00} = 8\pi\partial^2\rho_0/\partial t^2 + 16\pi\Delta\{(3/2)P$$
$$+ \rho_0[\Pi/2 - v_\alpha v^\alpha - 2(b_1 + b_2 + b_3 + b_4)U]\}.$$

Solving this last equation, we obtain

$$\overset{(2)}{\chi}{}^{00} = -\frac{\partial^2}{\partial t^2}\int\rho_0 R\,dV - 4\Phi_1 - 2\Phi_3$$
$$-6\Phi_4 + 8(b_1 + b_2 + b_3 + b_5)\Phi_2. \quad (125)$$

Using (109), (119), and (125), we write the metric of the Riemannian space-time in the post-Newtonian approximation in the form

$$\left.\begin{aligned}&g_{00} = 1 - 2U + 2BU^2 - 4\Phi_1 + 4(B-2)\Phi_2\\&\quad - 2\Phi_3 - 6\Phi_4 - \frac{\partial^2}{\partial t^2}\int\rho_0 R\,dV + O(\varepsilon^6);\\&g_{0\alpha} = 4V_\alpha + O(\varepsilon^5);\\&g_{\alpha\beta} = \gamma_{\alpha\beta}(1 + 2U) + O(\varepsilon^4),\end{aligned}\right\} \quad (126)$$

where we have introduced the notation

$$B = 2(b_1 + b_2 + b_3 + b_4).$$

To determine the values of the post-Newtonian parameters of our theory, it is necessary to go over to the coordinate system in which the post-Newtonian expansion of the metric (97) is expressed. If we make the coordinate transformation

$$x'^n = x^n + \xi^n(x) \quad (127)$$

and assume that

$$\xi^\alpha(x) = O(\xi^2);\quad \xi^0(x) = O(\varepsilon^3),$$

then the metric (126) in the new coordinate system will have the form

$$\left.\begin{aligned}&g'_{00} = g_{00} - 2\partial_0\xi_0 + O(\varepsilon^6);\\&g'_{0\alpha} = g_{0\alpha} - \partial_0\xi_\alpha - \partial_\alpha\xi_0 + O(\varepsilon^5);\\&g'_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\xi_\beta - \partial_\beta\xi_\alpha + O(\varepsilon^4).\end{aligned}\right\} \quad (128)$$

As "canonical" coordinate system, it is customary to choose a coordinate system in which the nondiagonal components of the spatial part of the metric tensor g_{in} vanish:

$$g'_{12}=g'_{23}=g'_{13}=0;$$

and, in addition, the component g_{00} does not contain terms of the form

$$\frac{\partial^2}{\partial t^2}\int \rho_0 R\, dV.$$

These requirements make it possible to determine the 4-vector with the required accuracy. In our case, for the transition to the "canonical" coordinate system it is necessary to choose the following 4-vector ξ^n:

$$\xi^\alpha(x)=0;\quad \xi^0(x)=-\frac{1}{2}\frac{\partial}{\partial t}\int \rho_0 R\, dV.$$

Using the continuity equations (106), we obtain

$$\partial_\alpha \xi_0=(V_\alpha-N_\alpha)/2.$$

Thus, we have finally the following expression for the metric tensor of the effective Riemannian space-time:

$$\left.\begin{aligned} &g'_{00}=1-2U+2BU^2-4\Phi_1+4(B-2)\Phi_2\\ &\quad -2\Phi_3-6\Phi_4+O(\varepsilon^6);\\ &g_{0\alpha}=(7/2)V_\alpha+(1/2)N_\alpha+O(\varepsilon^5);\\ &g_{\alpha\beta}=\gamma_{\alpha\beta}(1+2U)+O(\varepsilon^4). \end{aligned}\right\} \quad (129)$$

Thus, the post-Newtonian approximation of the field theory of gravitation leads to the metric (129) of the Riemannian space-time, and this contains only the one arbitrary constant B.

For the case when the source of the gravitational field is a static, spherically symmetric body of radius a, this metric takes the form

$$\left.\begin{aligned} &g_{0\alpha}=0;\\ &g_{00}=1-2M/r+2BM^2/r^2+O(M^3/r^3);\\ &g_{\alpha\beta}=\gamma_{\alpha\beta}(1+2M/r)+O(M^2/r^2), \end{aligned}\right\} \quad (130)$$

where the total mass of the field source is

$$M=4\pi\int_0^a \rho_0\left[1+\Pi+3\frac{P}{\rho_0}+2(2-B)U\right]r^2\,dr. \quad (131)$$

It follows from (129) and (97) that in the field theory of gravitation the post-Newtonian parameters have the values

$$\left.\begin{aligned} &\gamma=1;\ \beta=B=2(b_1+b_2+b_3+b_4);\\ &\alpha_1=\alpha_2=\alpha_3=0;\\ &\xi_1=\xi_2=\xi_3=\xi_4=\xi_\omega=0. \end{aligned}\right\} \quad (132)$$

For comparison, we note that in the general theory of relativity the post-Newtonian parameters have the values[34]

$$\left.\begin{aligned} &\gamma=1;\ \beta=B=1;\\ &\alpha_1=\alpha_2=\alpha_3=0;\\ &\xi_1=\xi_2=\xi_3=\xi_4=\xi_\omega=0. \end{aligned}\right\} \quad (133)$$

Thus, for $B=1$ the post-Newtonian parameters of the field theory of gravitation and Einstein's general theory of relativity agree completely, and, therefore, these two theories will be indistinguishable from the point of view of *all* experiments made in the gravitational field of the solar system (with post-Newtonian accuracy of the measurements).

As is shown in Ref. 36, the vanishing of the three parameters α has a definite physical meaning; any theory of gravitation in which $\alpha_1=\alpha_2=\alpha_3=0$ has no preferred universal rest frame in the post-Newtonian limit. In this case, the metric of the Riemannian space-time in the post-Newtonian limit is form-invariant on the transition from the universal rest frame to a moving system, and the velocity W^α of the new coordinate system with respect to the universal rest frame does not occur explicitly in the metric.

It follows from (132) that in the field theory of gravitation there is no universal preferred rest frame.

A linear dependence of the parameters ξ and α also has a definite physical meaning. As is shown in Ref. 37, when

$$\left.\begin{aligned} &\alpha_1=0;\ \xi_3=0;\\ &\alpha_2-\xi_1-2\xi_\omega=0;\ \xi_2=\xi_\omega;\\ &\alpha_3+\xi_1+2\xi_\omega=0;\\ &3\xi_4+2\xi_\omega=0;\ \xi_1+2\xi_\omega=0 \end{aligned}\right\} \quad (134)$$

one can determine from the post-Newtonian equations of motion quantities that do not depend on the time in the post-Newtonian approximation.

However, these quantities can be interpreted as the energy, momentum, and angular momentum of the system (i.e., as integrals of the motion) only in the theories of gravitation that have conservation laws for the energy–momentum tensor of the matter and the gravitational field.

For example, in Einstein's theory the relations (134) are satisfied, but the quantities that do not depend on the time in the post-Newtonian approximation are not, as a detailed analysis shows, integrals of the motion of the system consisting of the matter and the gravitational field.

In the field theory of gravitation, an isolated system has all ten conservation laws in their usual sense in the pseudo-Euclidean space-time, and in the post-Newtonian approximation these lead to ten integrals of the motion of the system; therefore, in the post-Newtonian approximation the field theory of gravitation has time-independent quantities. The fulfillment of the relations (134) in the field theory of gravitation confirms this conclusion.

8. GRAVITATIONAL EXPERIMENTS IN THE SOLAR SYSTEM

Let us consider what restrictions are imposed by the experiments on the values of the post-Newtonian parameters, and determine the value of the parameter B for which the results of the experiments agree with the predictions of the field theory of gravitation.

We shall analyze these experiments in the following order: We first consider the standard effects—the deflection of light and radio waves in the field of the Sun, the advance of Mercury's perihelion, and the measurement of the time delay of radio signals in the gravitational field of the Sun. The planned experiment to measure the precession of a gyroscope in orbit is intimately related to these effects.

After this, we consider the Nordtvedt effect, the measurement of the ratio of the active and passive masses, and also effects associated with nonvanishing of the parameters α_1, α_2, α_3, and ξ_ω.

We shall not consider the red shift in the gravitational field of the Sun, since this effect is completely described in the Newtonian approximation.[38]

In calculations of the standard effects in the gravitational field of the Sun, one usually considers as an idealized model of the Sun a static, spherically symmetric sphere of radius R. The metric for this case can be written in the form ($r > R$)

$$\left.\begin{aligned} &g_{0\alpha}=0; \\ &g_{00}=1-2M/r+2\beta M^2/r^2; \\ &g_{\alpha\beta}=\gamma_{\alpha\beta}(1+2\gamma M/r), \end{aligned}\right\} \tag{135}$$

where the total mass of the Sun is

$$M=4\pi\int_0^R \rho_0\left[1+(3\gamma+1-2\beta)U+\Pi+3\gamma\frac{P}{\rho_0}\right]r^2\,dr. \tag{136}$$

It follows from comparison of (130) and (135) that in the field theory of gravitation the parameters γ and β have the values

$$\gamma=1;\ \beta=B=2(b_1+b_2+b_3+b).$$

1. Deflection of light and radio waves in the gravitational field of the Sun. In accordance with Ref. 39, light rays and radio waves, regarded as massless particles with impact parameter b, are deflected in the gravitational field of the Sun through the angle

$$\delta\varphi=2(1+\gamma)M/b.$$

Analysis of the experimental results obtained from the observations of the bending of light rays of distant stars, and also of radio waves emitted by quasars, in the gravitational field of the Sun suggests[40] that

$$\gamma=1\pm 0.2.$$

2. Time delay of radio signals in the field of the Sun. Another independent way of determining the post-Newtonian parameter γ is to measure the time delay of radio signals in the field of the Sun.[41]

This effect is as follows. The propagation time of radio signals sent from the Earth to a reflector situated in a different part of the solar system and back to the Earth (and measured by a clock on the Earth) differs from the time of this process in the absence of a gravitational field.

If the origin of the coordinate system is placed in the Sun, the Earth has the coordinates (x_1, y), and the reflector the coordinates (x_2, y), then the interval of proper time which elapses on the journey to the reflector and back and measured by a clock on the Earth is

$$T=2(x_1+x_2)\left(1-M/\sqrt{x_1^2+y^2}\right)+2(1+\gamma)M\ln\left[\left(x_1+\sqrt{x_1^2+y^2}\right)\left(x_2+\sqrt{x_2^2+y^2}\right)/y^2\right].$$

In experiments to measure the time delay of radio signals in the gravitational field of the Sun, the surfaces of the planets and also radio apparatus on satellites have been used.

As a result of these experiments,[42] the value $\gamma=1\pm 0.005$ has been obtained. In the field theory of gravitation, as in Einstein's general relativity, $\gamma=1$, which is in good agreement with the results of these experiments.

3. Precession of a gyroscope moving in an orbit. If $\alpha_1=\alpha_2=\alpha_3=0$, measurement of the precession of a gyroscope moving in an orbit around the Earth will provide a third independent method of measuring the parameter γ.

In accordance with Ref. 43, the angular velocity of the precession of a gyroscope moving in a circular orbit in the field of the Earth is

$$\Omega=\frac{2\gamma+1}{2}m\frac{[\mathbf{rv}]}{r^3}+\frac{\gamma+1}{2r^3}\left(-\mathbf{J}+\frac{3\mathbf{r}(\mathbf{Jr})}{r^2}\right),$$

where m is the mass of the Earth, $\mathbf{v}$ is the linear velocity of the gyroscope with respect to the center of the Earth, $\mathbf{J}$ is the angular momentum of the Earth, and $\mathbf{r}$ is the radius vector of the point at which the gyroscope is situated.

The level of the present technology[44] offers hope that this experiment will be realized in the near future.

4. Advance of Mercury's perihelion. The advance of Mercury's perihelion is influenced by various factors in addition to the post-Newtonian corrections in the equation of motion. These include the attraction exerted by the planets in the solar system and a quadrupole moment of the Sun. The only undetermined factor among them is the quadrupole moment of the Sun; the influence of all other factors can be calculated with sufficient accuracy.

The total advance of Mercury's perihelion due to the presence of a quadrupole moment J_2 of the Sun and the post-Newtonian corrections in the equation of motion is[42]

$$\delta\varphi=42.98[(2+2\gamma-\beta)/3]+1.3\cdot 10^5 J_2$$

(in seconds of arc per century).

It follows from the results of the observations[40] that $\delta\varphi=41.4\pm 0.9$ seconds of arc per century. Measurements of the apparent shape of the Sun made by Dicke and Goldenberg[45] gave for J_2 the value $J_2=(2.5\pm 0.2)\times 10^{-5}$, while the later measurements of Hill *et al.*[42] showed that $J_2<0.5\times 10^{-5}$. Comparison of the observed displacements of the perihelia of Mercury and Mars[46] gave a bound for J_2: $J_2<3\times 10^{-5}$.

Thus, because of the absence of direct measurements of the quadrupole moment of the Sun, a large uncertainty remains in the value of β as determined from the advance of Mercury's perihelion:

$$\beta=1^{+0.4}_{-0.2}.$$

Note that in Einstein's general theory of relativity the parameter β has the value $\beta=1$, whereas in the field theory of gravitation $\beta=B$.

5. The Nordtvedt effect and Lunar laser ranging. In any theory of gravitation, as was noted by Bondi,[47] we can distinguish three kinds of mass, depending on the measurements by means of which they are determined: the inertial mass m_{i}, the passive gravitational mass m_{p}, and the active gravitational mass m_{a}.

The inertial mass is the mass that occurs in (and is defined by) Newton's second law:

$$m_{\mathrm{i}}a^\alpha=F^\alpha.$$

The passive gravitational mass is the mass on which a gravitational field acts, i.e., the mass determined by the expression

$$F^\alpha = -m_p \nabla^\alpha V.$$

The active gravitational mass is the mass that is the source of the gravitational field.

In Newtonian mechanics, Newton's third law requires equality of the active and passive masses, $m_a = m_p$, irrespective of the size and composition of the bodies; the equality of the inertial mass to the two remaining masses is regarded as an empirical fact.

In Einstein's theory for point bodies, the inertial and passive gravitational masses are equal. Equality of the active and passive gravitational masses is not postulated.

In some theories of gravitation all three masses may be different for a given body. It is therefore necessary to establish by means of experiments the correspondence between these three masses.

As was noted in Sec. 7, the first attempts to measure the ratio of the passive gravitational mass to the inertial mass for bodies of laboratory size[32,33] gave only a partial answer to the posed question, since the accuracy of the experiments was certainly insufficient to determine the ratio that the gravitational self-energy of the body, the elastic-deformation energy, etc., makes in these masses.

Since the ratio of the gravitational self-energy of a body to its mass increases with increasing size of the body, it is expedient to use extended bodies for such purposes. On the basis of the parametrized post-Newtonian expansion (97) of the metric, Nordtvedt and Will[48,49] showed for the example of the Earth and the Sun that the motion of an extended body (the Earth) in the gravitational field of a massive point body (the Sun) will not take place along a geodesic for a large class of theories, i.e., in an external field, different extended bodies will move with different accelerations. This effect has become known as the Nordtvedt effect. The presence of such an effect in the theory of gravitation means that the passive gravitational mass of a body differs from its inertial mass. The equation of motion of an extended body in an external gravitational field has in this case the form

$$ma^\alpha = m_p^{\alpha\beta}\, \partial U/\partial x^\beta,$$

where m is the inertial mass of the body, a^α are the components of the acceleration of its center of mass, U is the external gravitational potential, and $m_p^{\alpha\beta}$ is the tensor of the passive gravitational mass.

Using the post-Newtonian expansion (97) of the metric, from the conservation equation (8.11) we can obtain the following expression for the tensor of the passive gravitational mass of the Earth:

$$m_p^{\alpha\beta} = -m\{\gamma^{\alpha\beta}[1-(\alpha_1-\alpha_2+\gamma+3+\xi_1-4\beta)\,\Omega/m] + (\xi_2+\alpha_2-\xi_1)\,\Omega^{\alpha\beta}/m\},$$

where

$$\Omega = -\Omega^\alpha_\alpha;$$

$$\Omega^{\alpha\beta} = -\frac{1}{2}\int \frac{\rho(x)\,\rho(x')\,(x^\alpha - x'^\alpha)(x^\beta - x'^\beta)}{|x-x'|^3}\, dx\, dx'.$$

In Einstein's general theory of relativity

$$m_p^{\alpha\beta} = -m\gamma^{\alpha\beta},$$

and in the field theory of gravitation

$$m_p^{\alpha\beta} = -m\gamma^{\alpha\beta}[1-4(1-B)\,\Omega/m].$$

Thus, in the field theory of gravitation the passive gravitational mass of the Earth is equal to its inertial mass when $B=1$, as in Einstein's theory. Departure of the gravitational mass from the inertial mass could lead to a number of observable effects. One such effect is the polarization of the Moon's orbit in the direction of the Sun. This polarization leads to an eccentricity of the orbit, which is elongated in the direction of the Sun with amplitude

$$\delta r = c_0 \eta \cos\theta_0,$$

where c_0 is a constant of order 10 m, θ_0 is the difference between the longitude of the Moon and the Sun, and

$$\eta = 4\beta - \gamma - 3 - \alpha_1 + (2/3)\alpha_2 - (2/3)\xi_1 - (1/3)\xi_2 - (10/3)\xi_\omega.$$

Measurements made using the lunar laser ranging[50,51] showed that

$$c_0\eta = 0 \pm 4 \text{ cm}.$$

From this, one can obtain an estimate for the value of η:

$$\eta = 0 \pm 0.03.$$

In Einstein's theory, $\eta = 0$; in the field theory of gravitation, $\eta = 4(B-1)$.

Since no restrictions are placed on B in the field theory of gravitation, it is natural to determine its value from experiments. Using the results of the lunar laser ranging experiment, we obtain

$$|B-1| \leqslant 0.008.$$

We conclude that to within the errors of the measurements the value $B=1$ makes it possible to describe the lunar laser ranging experiments.

6. *Measurement of the ratio of the active and passive gravitational masses.* For the active gravitational mass in the post-Newtonian approximation, Nordtvedt[52] obtained the expression

$$m_a = m + 2(\alpha_3/2 + \xi_1/2 - \xi_4)\,E_k + \xi_3 E_{\rm int} + (4\beta - 2\xi_2 - 3 - \gamma)\,\Omega - \xi_1 E^{\alpha\beta} e_\alpha e_\beta, \qquad (137)$$

where e_α is a unit vector joining the massive body to the point at which its field is measured,

$$E_k = -\frac{1}{2}\int \rho v_\alpha v^\alpha\, d^3x;\quad E_{\rm int} = \int \rho\Pi\, d^3x;$$

m is the total mass of the body,

$$m = \int \rho\left(1 - \frac{1}{2}v_\alpha v^\alpha + \Pi - \frac{1}{2}U\right) d^3x;$$

$$E^{\alpha\beta} = -\frac{1}{2}\int \rho v^\alpha v^\beta\, d^3x.$$

In the field theory of gravitation, the expression (137) for the active gravitational mass takes the form

$$m_a = m\,[1 - 4(1-B)\,\Omega/m].$$

In the general theory of relativity, we obtain from (137)

$$m_a = m.$$

One of the few experiments in which the active gravitational mass has been measured is Kreuzer's experiment.[53] This experiment was as follows. In a tank containing a specially chosen liquid, a teflon cylinder was immersed, and this could execute motions from one end of the tank to the other. The liquid was chosen to make the cylinder, completely immersed in it, have zero buoyancy. In such a case, the densities of the passive gravitational masses of the liquid and the cylinder will be equal, since the Archimedean forces due to the gravitational attraction of the Earth are proportional to the difference between the densities of the passive gravitational masses. If the densities of the active gravitational masses of the liquid and the cylinder were different, the periodic motion of the cylinder from one end of the tank to the other would change the gravitational field. To measure these changes in the gravitational field, a torsion balance was used in the experiment.

Analysis of the results of this experiment gave the following estimates for the parameter values[54]:

$$|\alpha_3/2+\xi_1/3-\xi_4|<0.4; \quad |\xi_3|<0.5.$$

7. *Effects associated with the presence of a preferred frame of reference.* Theories of gravitation in which at least one of the parameters α_1, α_2, α_3 is nonzero have a preferred frame of reference. The predictions of such theories of gravitation for the standard effects can agree with the results of observations only if the solar system is a preferred frame of reference. It is more sensible to assume that the solar system, which moves with respect to other stellar systems, is not distinguished compared with them, and therefore cannot be a preferred universal rest frame for such theories.

Since a preferred rest frame must be distinguished in some way from other systems, it is more sensible to associate the rest frame with the center of mass of the Galaxy or even the Universe. Then the solar system will be in motion relative to the preferred rest frame with velocity $\sim 10^{-3}c$, which is of the same order as the orbital velocity of the solar system around the center of the Galaxy. In such a case, it will be possible to observe a number of effects associated with the motion relative to the preferred rest frame,[55] which will make it possible to estimate the parameters α_1, α_2, α_3.

In theories of gravitation with a preferred rest frame, the gravitational constant G measured in gravimetric experiments will depend on the motion of the Earth with respect to such a frame.

For the relative value $\Delta G/G$, we have

$$\Delta G/G\approx(\alpha_2/2+\alpha_3-\alpha_1)(\mathbf{W}\mathbf{v})+(\alpha_2/4)[(\mathbf{v}\mathbf{e}_r)^2+2(\mathbf{W}\mathbf{e}_r)(\mathbf{v}\mathbf{e}_r)+(\mathbf{W}\mathbf{e}_r)^2],$$

where $\mathbf{v}$ is the orbital velocity of the Earth around the Sun, $\mathbf{W}$ is the velocity of the Sun with respect to the preferred rest frame, and $\mathbf{e}_r$ is a unit vector pointing from the gravimeter to the center of the Earth.

Because of the rotation of the Earth about its axis, the vector $\mathbf{e}_r$ changes its orientation with respect to the vectors $\mathbf{v}$ and $\mathbf{W}$, which leads to a periodic change of the scalar products $(\mathbf{v}\cdot\mathbf{e}_r)$ and $(\mathbf{W}\cdot\mathbf{e}_r)$ with period approximately equal to 12 h. This leads to corresponding periodic changes in the values of the acceleration of free fall, and for a point of observation at latitude θ we have

$$\Delta g/g\approx 3\alpha_2\cdot 10^{-8}\cos^2\theta.$$

Will,[56] analyzing the results of gravimetric experiments, found that the relative variations of g do not exceed $|10^{-9}|$: $|\Delta g/g|<10^{-9}$. Hence we obtain an estimate for α_2: $|\alpha_2|<3\times10^{-2}$.

The motion of the Earth around the Sun also leads to a periodic change in $(\mathbf{W}\cdot\mathbf{v})$ with a period of the order of a year. This variation gives rise to a contraction and expansion of the Earth, which in turn leads to periodic changes in the angular velocity of the rotation of the Earth due to the change in its moment of inertia:

$$\Delta\omega/\omega\approx 3\cdot10^{-9}[\alpha_3+(2/3)\alpha_2-\alpha_1].$$

It follows from the results of observations that

$$|\alpha_3+(2/3)\alpha_2-\alpha_1|<0.2.$$

The motion of the solar system with respect to the center of the Universe may lead to an anomalous advance $\delta\varphi_0$ of the perihelia of the planets.

For Mercury,[55] the additional contribution to the advance of the perihelion (in seconds of arc per century) has the form

$$\delta\varphi_0=35\alpha_1+8\alpha_2-4\cdot10^4\alpha_3.$$

Comparison with observations and combination of all these estimates of the parameters α gives

$$|\alpha_1|<0.2;\ |\alpha_2|<3\cdot10^{-2};\ |\alpha_3|<2\cdot10^{-5}.$$

In the field theory of gravitation, as in Einstein's general relativity, $\alpha_1=\alpha_2=\alpha_3=0$, and therefore all these effects are absent.

8. *Effect of anisotropy with respect to the center of the Galaxy.* In theories of gravitation for which the parameter ξ_ω does not vanish, one can have anisotropy effects due to the influence of the gravitational field of the Galaxy.[35]

If it is assumed that the mass M of the galaxy is concentrated at the center of the Galaxy at distance R from the solar system, the gravitational field of the Galaxy will lead to periodic changes in the readings of a gravimeter with period 12 h:

$$\Delta G/G=\xi_\omega(1-3K/mr^2)(M/R)(\mathbf{e}_r\mathbf{e}_R),$$

where K is the moment of inertia, m is the mass, and r is the radius of the Earth, $\mathbf{e}_r$ is the unit vector pointing from the gravimeter to the center of the Earth, and $\mathbf{e}_R=\mathbf{R}/R$.

Another effect is the anomalous advance of the perihelia of planets due to the anisotropy produced by the Galaxy:

$$\delta\varphi_0=(\pi\xi_\omega/2)(M/R)\cos^2\beta\cos^2(\omega-\lambda),$$

where λ and β are the angular coordinates of the center of the Galaxy, and ω is the angle of the perihelion of the planet in geocentric coordinates.

Comparison with observations gives an upper limit on ξ_ω: $|\xi_\omega|<10^{-2}$. In the field theory of gravitation, as in Einstein's general theory of relativity, $\xi_\omega=0$, and all the anisotropy effects due to the gravitational field of the Galaxy are absent.

Concluding our review of gravitational experiments, we conclude that the field theory of gravitation for $B=1$ makes it possible to describe the complete set of experimental facts.

It should be noted that in the post-Newtonian limit the quadratic terms in the connection equation (102) are indistinguishable; for no experiment in the gravitational field of the solar system at the post-Newtonian level can enable one to determine the coefficients b_1, b_2, b_3, and b_4 separately.

As will be shown in Sec. 12, measurement of the deceleration parameter of an expanding homogeneous Universe in the contemporary epoch makes it possible to determine the value of a different linear combination of these coefficients, namely, $b_1+3b_2+3b_3+9b_4$. It will be possible to determine the coefficients b_1, b_2, b_3, and b_4 using experiments in the gravitational field of the solar system only when the accuracy of the measurements has been increased to the post-post-Newtonian level.

To conclude this section, we note that in the field theory of gravitation the equivalence principle is valid only for point bodies. For extended bodies moving in a weak gravitational field it is satisfied approximately to the accuracy with which the gravitational field can be assumed to be homogeneous in the region occupied by the body. In this case, we can "eliminate" the gravitational field by going over to a coordinate system in which $g_{in}=\gamma_{in}$ in the region occupied by the matter. As follows from the experiment of Braginskiĭ[33] with bodies of laboratory size in a sufficiently uniform gravitational field, the equivalence principle holds for the strong, electromagnetic, and weak interactions to the accuracy achieved in these experiments. However, for extended bodies when allowance is made for the gravitational field this principle is valid strictly in neither Einstein's general theory of relativity nor the field theory of gravitation, though it is satisfied in the post-Newtonian approximation.

9. STATIC, SPHERICALLY SYMMETRIC GRAVITATIONAL FIELD

In the case of a static source of radius a with spherically symmetric matter distribution, the gravitational field equations (56) and the expression for the tensor current (51) simplify appreciably.

On the basis of the symmetry of the problem, we determine what components of the tensors I_{lm} and h^{lm} will be nonzero in this case.

We place the origin of a spherical coordinate system at the center of the source. When this coordinate system is rotated through an arbitrary angle, the physical situation must not change because of the spherical symmetry of the matter distribution. Therefore, the components of the tensors I_{lm} and h_{lm} must be the same functions of the transformed argument after a rotation transformation as the original functions were of their original arguments, i.e., these tensors must be form-invariant under a rotation of the coordinate system. It follows that in the spherical coordinate system the only components of the tensors I_{lm} and h_{lm} that can be nonvanishing are

$$I_{lm}=\{I_{00};\ I_{0r};\ I_{rr};\ I_{\theta\theta};\ I_{\varphi\varphi}=I_{\theta\theta}\sin^2\theta\};$$
$$h_{lm}=\{h_{00};\ h_{0r};\ h_{rr};\ h_{\theta\theta};\ h_{\varphi\varphi}=h_{\theta\theta}\sin^2\theta\},$$

since it is only in this case that the tensors I_{lm} and h_{lm} are form-invariant under a rotation transformation.

It follows from the expressions (51) and (40) that

$$I_{0r}=0.$$

Therefore, for the case of a static, spherically symmetric distribution of the matter the tensor I_{lm} has the components

$$I_{lm}=\{I_{00};\ I_{rr};\ I_{\theta\theta};\ I_{\varphi\varphi}=I_{\theta\theta}\sin^2\theta\}.$$

The gravitational field equations (56) are written in the form of the system of ordinary second-order differential equations

$$\left.\begin{aligned}&f''_{00}+(2/r)f'_{00}=16\pi I_{00}(r);\\&f''_{rr}+(2/r)f'_{rr}-(4/r^2)f_{rr}+(4/r^2)(f_{\theta\theta}/r^2)=16\pi I_{rr}(r);\\&(f_{\theta\theta}/r^2)''+(2/r)(f_{\theta\theta}/r^2)'-(2/r^2)(f_{\theta\theta}/r^2)\\&\quad+(2/r^2)f_{rr}=16\pi I_{\theta\theta}(r)/r^2.\end{aligned}\right\}\tag{138}$$

Here and in what follows, the prime denotes the derivative with respect to r.

As boundary conditions for these equations, we require the functions f_{00}, f_{rr}, and $(1/r^2)f_{\theta\theta}$ to be bounded at $r=0$ and to vanish as $r\to\infty$.

Then the solution of the gravitational field equations (138) will be unique:

$$\left.\begin{aligned}&f_{00}=-16\pi\left\{\frac{1}{r}\int_0^r r_0^2 I_{00}\,dr_0+\int_r^a r_0 I_{00}\,dr_0\right\};\\&f_{rr}=-(16\pi/3)[A+(2/5)B];\\&f_{\theta\theta}/r^2=-(16\pi/3)[A-(1/5)B],\end{aligned}\right\}\tag{139}$$

where

$$\left.\begin{aligned}&A=\frac{1}{r}\int_0^r r_0^2\,dr_0\left(I_{rr}+\frac{2}{r_0^2}I_{\theta\theta}\right)+\int_r^a r_0\,dr_0\left(I_{rr}+\frac{2}{r_0^2}I_{\theta\theta}\right);\\&B=\frac{1}{r^3}\int_0^r r_0^4\,dr_0\left(I_{rr}-\frac{I_{\theta\theta}}{r_0^2}\right)+r^2\int_r^a\frac{dr_0}{r_0}\left(I_{rr}-\frac{I_{\theta\theta}}{r_0^2}\right).\end{aligned}\right\}\tag{140}$$

However, the components of the tensor current in (140) are not independent because of the conditions $D^nI_{nm}=0$. In our case, these conditions take the form

$$I'_{rr}+\frac{2}{r}\left(I_{rr}-\frac{1}{r^2}I_{\theta\theta}\right)=0.\tag{141}$$

We use Eq. (141) to express the component $I_{\theta\theta}$ and substitute it in (140). Integrating the obtained expressions by parts and noting that outside the source $I_{rr}=0$, we write the components of the gravitational field in the form

$$\left.\begin{aligned}&f_{00}=-16\pi\left\{\frac{1}{r}\int_0^r r_0^2\,dr_0\,I_{00}+\int_r^a r_0\,dr_0\,I_{00}\right\};\\&f_{rr}=-\frac{16\pi}{3}\left\{\frac{1}{r^3}\int_0^r r_0^4\,dr_0\,I_{rr}+\int_r^a r_0\,dr_0\,I_{rr}\right\};\\&\frac{f_{\theta\theta}}{r^2}=-\frac{16\pi}{3}\left\{-\frac{1}{2r^3}\int_0^r r_0^4\,dr_0\,I_{rr}+\int_r^a r_0\,dr_0\,I_{rr}\right\}.\end{aligned}\right\}\tag{142}$$

We consider the exterior $(r>a)$ solution. Introducing

$$M=4\pi\int_0^a r_0^2\,dr_0\,I_{00};\quad \mu=\frac{4\pi}{3}\int_0^a r_0^4\,dr_0\,I_{rr},\tag{143}$$

we obtain for the exterior solution the expressions

$$f_{00}=-4M/r;\ f_{rr}=-4\mu/r^3; \quad (1/r^2)f_{\theta\theta}=2\mu/r^3;\ f_{\varphi\varphi}=f_{\theta\theta}\sin^2\theta. \tag{144}$$

As we have already noted in Sec. 4, the field f_{lm} can be subjected to a gauge transformation:

$$f_{lm}=\bar{f}_{lm}+D_l a_m+D_m a_l-\gamma_{lm}D_n a^n \tag{145}$$

with gauge vector a_n satisfying the equation $D_m D^m a_n=0$. Under this transformation, the Lagrangian density of the gravitational field changes only by a four-dimensional divergence, which is unimportant for the theory, and the change in the metric tensor g_{ik} of the Riemannian space-time produced by the transformation (145) corresponds to a coordinate transformation of the Riemannian space-time and can always be eliminated by a suitable choice of the coordinates.

We use the gauge transformation (145) to simplify the exterior solution (144). By virtue of the symmetry of the problem, we choose the gauge vector a_n, which satisfies the condition $D_m D^m a_n=0$, in the form $a_r=-\mu/r^2$, $a_0=a_\theta=a_\varphi=0$. Then as a result of this gauge transformation, we obtain for the exterior solution

$$f_{00}=-4M/r;\ f_{rr}=f_{\theta\theta}=f_{\varphi\varphi}=0. \tag{146}$$

To obtain the metric tensor in the case of a static, spherically symmetric source, it remains to substitute the components of the gravitational field f_{lm} in the connection equation $g_{ni}=g_{ni}(\gamma_{lm}, f_{lm})$. However, in the field theory of gravitation the connection equation is known only in the weak-field approximation (102), and in the general case we have not yet determined the structure of this equation. For this reason, nothing can be said in the field theory of gravitation about the possible existence of objects like black holes, since this corresponds to the region of strong fields.

Therefore, we consider the metric of a static, spherically symmetric source for a weak field. Substituting (146) in the post-Newtonian connection equation (102), we obtain for the exterior solution

$$g_{00}=1-2M/r+2BM^2/r^2+O(M^3/r^3); \quad g_{\alpha\beta}=\gamma_{\alpha\beta}(1+2M/r)+O(M^2/r^2). \tag{147}$$

From the relations (143) and (51), we have

$$M=8\pi\int_0^a r_0^2\,dr_0\left[I_{00}-\frac{1}{2}I_n^n\right]=8\pi\int_0^a r_0^2\,dr_0\left(h_{00}-\frac{1}{2}h_n^n\right).$$

To obtain the post-Newtonian expansion of M, it is necessary, as usual, to make the calculations in successive stages. We must first obtain an expression for M in the Newtonian approximation, when we completely ignore the influence of gravitation on the energy–momentum tensor of the matter, and then, using the Newtonian approximation, we find the post-Newtonian expression. As a result, we obtain

$$M=4\pi\int_0^a r_0^2\,dr_0\rho_0\left[1+\Pi+2(2-B)U+3\frac{p}{\rho_0}+O(\varepsilon^4)\right]. \tag{148}$$

As was expected, for a static, spherically symmetric body the post-Newtonian expansion of the total mass is equal to the expression (131), and the metric (147) is equal to the metric (130).

10. GRAVITATIONAL FIELD OF A NONSTATIC, SPHERICALLY SYMMETRIC SOURCE

In Einstein's theory, the gravitational field outside a nonstatic, spherically symmetric source is, by virtue of Birkhoff's theorem, a static field with metric corresponding to the Schwarzschild solution.

We shall show that in the field theory of gravitation in the case of a nonstatic, spherically symmetric source the gravitational field outside the matter is also a static field whose components can be expressed by Eqs. (143) and (144). We consider the case when the matter is distributed in a sphere of radius a spherically symmetrically and its motion also occurs spherically symmetrically in the radial directions.

By the symmetry of the problem, the nonvanishing components of the tensors T^{lm}, h^{lm}, I_{lm}, and f_{lm} are the diagonal components, and also the components T^{0r}, h^{0r}, I_{0r}, and f_{0r}.

All the components of these tensors except the $(\varphi\varphi)$ components depend on r and t. For the $(\varphi\varphi)$ components, we have

$$I_{\varphi\varphi}=I_{\theta\theta}\sin^2\theta;\ f_{\varphi\varphi}=f_{\theta\theta}\sin^2\theta;\quad h^{\varphi\varphi}=h^{\theta\theta}/\sin^2\theta;\ T^{\varphi\varphi}=T^{\theta\theta}/\sin^2\theta.$$

The 4-velocity of the matter has the form

$$u^i=\{u^0(r,t),\ u^r(r,t),\ 0,\ 0\}.$$

We expand the components of the tensor current I_{lm} and the gravitational field f_{lm} in Fourier integrals with respect to the time:

$$f_{lm}(r,t)=\int\exp(-i\omega t)f_{lm}(\omega,r)\,d\omega;$$
$$I_{lm}(r,t)=\int\exp(-i\omega t)I_{lm}(\omega,r)\,d\omega.$$

In the spectrum $I_{lm}(\omega,r)$, we separate the static part $I_{lm}(r)$. It is obvious that the static part will give the static solutions considered in the previous section. Therefore, in what follows we shall regard $I_{lm}(\omega,r)$ as the nonstatic part.

The field equations (56) for the considered case have the form of ordinary differential equations:

$$\left.\begin{aligned}
&f_{00}''+(2/r)f_{00}'+\omega^2 f_{00}=16\pi I_{00}(\omega,r);\\
&f_{0r}''+(2/r)f_{0r}'+(\omega^2-2/r^2)f_{0r}=16\pi I_{0r}(\omega,r);\\
&f_{rr}''+(2/r)f_{rr}'-(4/r^2)f_{rr}+(4/r^2)(f_{\theta\theta}/r^2)+\omega^2 f_{rr}\\
&\quad=16\pi I_{rr}(\omega,r);\\
&(f_{\theta\theta}/r^2)''+(2/r)(f_{\theta\theta}/r^2)'-(2/r^2)(f_{\theta\theta}/r^2)+(2/r^2)f_{rr}\\
&\quad+\omega^2 f_{\theta\theta}/r^2=(16\pi/r^2)I_{\theta\theta}.
\end{aligned}\right\} \tag{149}$$

As boundary conditions for these equations, it is natural to require the functions f_{00}, f_{0r}, f_{rr}, and $(1/r^2)f_{\theta\theta}$ to be bounded at $r=0$ and fulfillment of the radiation conditions as $r\to\infty$. From the conditions of conservation of the tensor current, $D^l I_{lm}=0$, we have

$$\left.\begin{aligned}
&i\omega I_{00}+I_{0r}'+(2/r)I_{0r}=0;\\
&i\omega I_{0r}+I_r'+(2/r)I_{rr}-(2/r^3)I_{\theta\theta}=0.
\end{aligned}\right\} \tag{150}$$

Solving Eqs. (149) using (150), we obtain

$$f_{rr}=(1/3)(A_2+2B_2);$$
$$f_{\theta\theta}/r^2=(1/3)(A_2-B_3);$$
$$f_{00}=-\frac{8\pi^2}{\sqrt{r}}\left\{H_{1/2}^{(1)}(\omega r)\int_0^r r_0^{3/2}\,dr_0 I_{0r}J_{3/2}(\omega r_0)\right.$$
$$\left.+J_{1/2}(\omega r)\int_r^a r_0^{3/2}\,dr_0 I_{0r}H_{3/2}^{(1)}(\omega r_0)\right\};$$

$$f_{0r}=-\frac{8\pi^2 i}{\sqrt{r}}\left\{H^{(1)}_{3/2}(\omega r)\int_0^r r_0^{3/2}\,dr_0 I_{0r}J_{3/2}(\omega r_0)\right.$$
$$\left.+J_{3/2}(\omega r)\int_r^a r_0^{3/2}\,dr_0 I_{0r}H^{(1)}_{3/2}(\omega r_0)\right\},$$

where

$$A_2=-\frac{8\pi^2 i\omega}{\sqrt{r}}\left\{H^{(1)}_{1/2}(\omega r)\int_0^r r_0^{5/2}\left[iI_{0r}J_{1/2}(\omega r_0)+I_{rr}J_{3/2}(\omega r_0)\right]dr_0\right.$$
$$\left.+J_{1/2}(\omega r)\int_r^a r_0^{5/2}\left[iI_{0r}H^{(1)}_{1/2}(\omega r_0)+I_{rr}H^{(1)}_{3/2}(\omega r_0)\right]dr_0\right\};$$

$$B_2=\frac{4\pi^2 i\omega}{\sqrt{r}}\left\{H^{(1)}_{5/2}(\omega r)\int_0^r r_0^{5/2}\left[iI_{0r}J_{5/2}(\omega r_0)-I_{rr}J_{3/2}(\omega r_0)\right]dr_0\right.$$
$$\left.+J_{5/2}(\omega r)\int_r^a r_0^{5/2}\left[iI_{0r}H^{(1)}_{5/2}(\omega r_0)-I_{rr}H^{(1)}_{3/2}(\omega r_0)\right]dr_0\right\}.$$

Using outside the source the gauge condition

$$f'_{in}=f_{in}-D_i a_n-D_n a_i+\gamma_{in}D_l a^l,$$

we impose on the components of the gravitational field the two conditions

$$f'^{n}_{\alpha}=0;\ f'^{00}=0.$$

If the gauge transformation is not to violate the condition $D^l f_{lm}=0$, the gauge 4-vector must satisfy outside the matter the equation $\Box a_l=0$. Choosing the gauge vectors in the form

$$a_0=\frac{2\pi^2 i}{\omega\sqrt{r}}H^{(1)}_{1/2}(\omega r)\int_0^a r_0^{3/2}\,dr_0\{I_{0r}[J_{3/2}(\omega r_0)$$
$$+\omega r_0 J_{1/2}(\omega r_0)]-i\omega r_0 I_{rr}J_{3/2}(\omega r_0)\};$$
$$a_r=\frac{2\pi^2}{\sqrt{r}}H^{(1)}_{3/2}(\omega r)\int_0^a r_0^{5/2}\,dr_0\left[I_{0r}J_{5/2}(\omega r_0)+iI_{rr}J_{3/2}(\omega r_0)\right];$$
$$a_\theta=a_\varphi=0,$$

we can readily see that all the components of the nonstatic gravitational field outside the matter vanish:

$$f'_{in}=0.$$

Thus, in the case of a nonstatic source with spherically symmetric distribution and motion of the matter the gravitational field outside the matter will be a static field whose components are determined by Eqs. (143) and (144).

11. CONSERVATION LAWS IN THE POST-NEWTONIAN APPROXIMATION OF THE FIELD THEORY OF GRAVITATION

In the field theory of gravitation, the gravitational field, regarded in pseudo-Euclidean space-time, behaves like all other physical fields. It possesses energy and momentum and contributes to the total energy–momentum tensor density of the system. The covariant conservation law for the total energy–momentum tensor density in the pseudo-Euclidean space-time, written down in a Cartesian coordinate system, has the usual form

$$\partial_i(t_g^{ni}+t_M^{ni})=0, \tag{151}$$

where t_g^{ni} is the symmetric energy–momentum tensor density (66) of the gravitational field, and t_M^{ni} is the symmetric energy–momentum tensor density (74) of the matter.

Using the differential conservation law (151), we can obtain the corresponding integral conservation law:

$$-\frac{\partial}{\partial t}\int dV\,(t_g^{0n}+t_M^{0n})=\int dS_\alpha\,(t_g^{\alpha n}+t_M^{\alpha n}).$$

If there is no energy flux of the matter and the gravitational field through the surface bounding a volume,

$$\int dS_\alpha\,(t_g^{\alpha n}+t_M^{\alpha n})=0, \tag{152}$$

then we arrive at a conservation law for the total 4-momentum of an isolated system:

$$dP^n/dt=0,$$

where

$$P^n=\int dV\,(t_g^{0n}+t_M^{0n}). \tag{153}$$

In this case, because of the symmetry of the total energy–momentum tensor density, the angular-momentum tensor of the system is also conserved:

$$dM^{in}/dt=0,$$

where

$$M^{in}=\int dV\,[x^i(t_g^{0n}+t_M^{0n})-x^n(t_g^{0i}+t_M^{0i})]. \tag{154}$$

By virtue of the conservation of the components

$$M^{0\alpha}=x^0\int dV\,(t_g^{0\alpha}+t_M^{0\alpha})-\int dV\,x^\alpha(t_g^{00}+t_M^{00})$$

the center of mass of an isolated system, defined by

$$X^\alpha=\int dV x^\alpha(t_g^{00}+t_M^{00})/\int dV\,(t_g^{00}+t_M^{00})=(P^\alpha t-M^{0\alpha})/P^0, \tag{155}$$

executes uniform rectilinear motion with velocity

$$dX^\alpha/dt=P^\alpha/P^0.$$

Thus, to describe the motion of an isolated system consisting of matter and the gravitational field, it is sufficient to determine the 4-momentum P^i (153). It should be noted that in any real system, because of the motion of its composite parts, thermal motion of the matter, etc., gravitational waves may be emitted; any real system exchanges matter with other systems in the form of electromagnetic radiation and also particles, atoms, etc. Therefore, in the most general case one cannot ignore the energy fluxes of the matter and the gravitational field; there are a large number of astrophysical processes in which these energy fluxes play a leading role, and allowance for them enables one to understand and predict many astrophysical processes. Nevertheless, for systems for which the energy fluxes of the matter and the gravitational field are small, the condition of isolation (152) is satisfied to a certain degree of accuracy. Then to the same degree of accuracy we can assert that the 4-momentum of such a system is conserved. It is such a situation that obtains for a system to which the post-Newtonian formalism applies. In this case, the condition (152) for isolation of the system in the post-Newtonian approximation is satisfied, and we can determine the conserved 4-momentum of the system.

We find the post-Newtonian expression for the 4-momentum of an isolated system in the field theory of gravitation. The total symmetric energy–momentum tensor density in the flat space-time has the form

$$t^{ik}=t_g^{ik}+t_M^{ik}=\frac{1}{64\pi}\Big\{-\gamma^{ik}(\partial_l f_{np}\partial^l f^{np}-\frac{1}{2}\partial_l f\partial^l f)$$
$$-\partial^i f\partial^k f+2\partial^i f_{np}\partial^k f^{np}\Big\}$$
$$-\frac{1}{32\pi}\{f^{in}\Box f_n^k+f_n^k\Box f^{in}-f^{ik}\Box f\}$$
$$-\frac{1}{32\pi}\{\partial_l[f_n^i\partial^k f^{ln}+f_n^k\partial^i f^{ln}-f_n^l(\partial^i f^{kn}+\partial^k f^{in})]\}$$
$$-2\Lambda^{(ik)}+T^{np}A_{np}^{ik}, \qquad (156)$$

where A_{np}^{ik} are defined by the expression (75), Λ^{ik} by the expression (65), and the tensor A^{ln} in this case has the form

$$A^{ln}=-\frac{1}{32\pi}\left\{\Box\left(f^{ln}-\frac{1}{2}\gamma^{ln}f\right)+16\pi\left(h^{ln}-\frac{1}{2}\gamma^{ln}h_m^m\right)\right\}. \qquad (157)$$

Since in the post-Newtonian approximation

$$A_{np}^{ik}=\frac{1}{2}(\delta_n^i\delta_p^k+\delta_n^k\delta_p^i)\left(1-\frac{1}{2}f_m^m\right)+\frac{1}{2}\gamma_{np}f^{ik}+O(\varepsilon^4),$$

it follows from the expressions (126), (156), and (157) that the components t^{00} and $t^{0\alpha}$ of the total symmetric energy–momentum tensor of the system can be determined up to terms $t^{00}\sim\rho O(\varepsilon^2)$ and $t^{0\alpha}\sim\rho O(\varepsilon^3)$. Therefore, we shall omit all quantities of higher order, for example, Λ^{00} and $\Lambda^{0\alpha}$, since $\Lambda^{00}\sim\rho O(\varepsilon^4)$, $\Lambda^{0\alpha}\sim\rho O(\varepsilon^5)$.

Bearing in mind that

$$\partial_\alpha\partial^\alpha U=4\pi\rho;\ \ \partial V^\beta/\partial x^\beta=\partial U/\partial t,$$

we obtain from (156) and (119)

$$\left.\begin{array}{l} t^{00}=\rho(1+v^2/2+\Pi-U/2)-(1/8\pi)\,\partial_\alpha(U\partial^\alpha U)+\rho O(\varepsilon^4);\\ t^{0\alpha}=\rho v^\alpha(1+v^2/2+\Pi+U)+pv^\alpha+2\rho V^\alpha\\ +\frac{1}{4\pi}\{(\partial U/\partial t)\,\partial^\alpha U+2\partial_\beta(U\partial^\alpha V^\beta-V^\beta\partial^\alpha U)\}+\rho O(\varepsilon^5).\end{array}\right\} \qquad (158)$$

To find the 4-momentum of the system in the post-Newtonian approximation, we integrate the expressions (158) over the whole of space. Using the equations

$$\int\frac{\partial U}{\partial t}\partial^\alpha U\,dV=2\pi\int\rho\,(Uv^\alpha+N^\alpha)\,dU;$$
$$\int\rho V^\alpha\,dV=-\int\rho Uv^\alpha\,dV;$$
$$\int\partial_\alpha(U\partial^\alpha U)\,dV=\int dS_\alpha U\partial^\alpha U=0,$$

we obtain finally

$$P^0=\int dV\rho\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U\right);$$
$$P^\alpha=\int dV\left\{\rho v^\alpha\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U\right)+Pv^\alpha+\frac{1}{2}\rho N^\alpha\right\}. \qquad (159)$$

Using (159) and (154), we can readily obtain in the post-Newtonian approximation the conserved angular-momentum tensor of the system:

$$\left.\begin{array}{l} M^{0\alpha}=\int dV\,\rho x^\alpha\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U\right)-P^\alpha t;\\ M^{\alpha\beta}=\int dV\,\rho\Big\{x^\alpha\Big[v^\beta\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U+\frac{P}{\rho}\right)+\frac{1}{2}N^\beta\Big]\\ -x^\beta\Big(v^\alpha\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U+\frac{P}{\rho}\right)+\frac{1}{2}N^\alpha\Big]\Big\}\end{array}\right\} \qquad (160)$$

and the coordinates of the center of mass of the system:

$$X^\alpha=\frac{1}{P^0}\int dV\,x^\alpha\rho\left(1+\Pi+\frac{1}{2}v^2-\frac{1}{2}U\right). \qquad (161)$$

In contrast to the conservation law (151), the covariant conservation equation for the energy–momentum tensor density of the matter in the Riemannian space-time,

$$\nabla_n T^{ni}=\partial_n T^{ni}+\Gamma_{mn}^i T^{mn}=0\,, \qquad (162)$$

does not express in explicit form the conservation of any quantity, but merely reflects the fact that the energy–momentum tensor density of the matter is not conserved:

$$\partial_n T^{ni}\neq 0.$$

However, as is shown in Sec. 2 of the present paper, the conservation laws (151) and the conservation equation (162) are simply different forms of expression of the same conservation law in the field theory of gravitation. This general result obtained in Sec. 2 can also be verified at any stage of the approximate calculations. Therefore, in the field theory of gravitation the integrals of the motion (159)–(161) in the post-Newtonian approximation can also be obtained from the conservation equation (162).

We show, for example, that the post-Newtonian integrals of the motion obtained in the field theory of gravitation from the covariant equation (162) in the Riemannian space-time are identical to the integrals of the motion (159)–(161) obtained in the pseudo-Euclidean space-time from the conservation law (151). It should be emphasized that to find the integrals of the motion the calculations must be made in both cases in the same coordinate system, since different expressions for the integrals of the motion correspond to different coordinate systems.

Therefore, we shall make the calculations in a "noncanonical" coordinate system of the Riemannian space-time, in which the metric tensor has the form (126). This coordinate system of the Riemannian space-time corresponds to the coordinate system of the pseudo-Euclidean space-time in which we found the integrals of the motion (159)–(161).

We note also that the transition to the canonical coordinate system, in which the metric tensor g_{ni} has the form (129), does not, as can be shown, change the post-Newtonian expressions for the integrals of the motion in the field theory of gravitation. But in the general case different expressions for the integrals of the motion will correspond to different coordinate systems.

Using the post-Newtonian expansion of the metric tensor (126) and the definition (105), we find the components of the energy–momentum tensor density of the matter to post-Newtonian accuracy:

$$\left.\begin{array}{l} T^{00}=\rho\,[1+\Pi+v^2/2+U]+\rho O(\varepsilon^4);\\ T^{0\alpha}=\rho v^\alpha[1+\Pi+v^2/2+U]+Pv^\alpha+\rho O(\varepsilon^5);\\ T^{\alpha\beta}=\rho v^\alpha v^\beta[1+\Pi+v^2/2+U]\\ +Pv^\alpha v^\beta-P\gamma^{\alpha\beta}+\rho O(\varepsilon^6).\end{array}\right\} \qquad (163)$$

We write out Eq. (162) in components:

$$\left.\begin{array}{l} \partial_0 T^{00}+\partial_\alpha T^{0\alpha}+\Gamma_{00}^0 T^{00}+2\Gamma_{0\alpha}^0 T^{0\alpha}+\Gamma_{\alpha\beta}^0 T^{\alpha\beta}=0;\\ \partial_0 T^{0\alpha}+\partial_\beta T^{\alpha\beta}+\Gamma_{00}^\alpha T^{00}+2\Gamma_{0\beta}^\alpha T^{0\beta}+\Gamma_{\beta\gamma}^\alpha T^{\beta\gamma}=0.\end{array}\right\} \qquad (164)$$

Since the components of the energy–momentum density of the matter are known to accuracy

$$T^{00}\sim\rho O(\varepsilon^4);\ \ T^{0\alpha}\sim\rho O(\varepsilon^5);\ \ T^{\alpha\beta}\sim\rho O(\varepsilon^6),$$

the connection of the Riemannian space-time must be determined to accuracy

$$\Gamma^0_{00}\sim O(\varepsilon^5);\ \Gamma^0_{0\alpha}\sim O(\varepsilon^4);\ \Gamma^0_{\alpha\beta}\sim O(\varepsilon^3);$$
$$\Gamma^\alpha_{00}\sim O(\varepsilon^6);\ \Gamma^\alpha_{0\beta}\sim O(\varepsilon^5);\ \Gamma^\alpha_{\beta\tau}\sim O(\varepsilon^4).$$

Using the post-Newtonian expansion of the metric (126), we determine the connection of the Riemannian space-time to the required accuracy:

$$\left.\begin{aligned}
&\Gamma^0_{00}=-\partial U/\partial t+O(\varepsilon^5);\\
&\Gamma^0_{0\alpha}=-\partial U/\partial x^\alpha+O(\varepsilon^4);\\
&\Gamma^\alpha_{\beta\tau}=\delta^\alpha_\tau\,\partial U/\partial x^\beta+\delta^\alpha_\beta\,\partial U/\partial x^\tau-\gamma_{\beta\tau}\gamma^{\alpha\eta}\,\partial U/\partial x^\eta+O(\varepsilon^4);\\
&\Gamma^\alpha_{0\beta}=\delta^\alpha_\beta\,\partial U/\partial t+2\gamma^{\alpha\tau}(\partial V_\tau/\partial x^\beta-\partial V_\beta/\partial x^\tau)+O(\varepsilon^5);\\
&\Gamma^0_{\alpha\beta}=O(\varepsilon^3);\\
&\Gamma^\alpha_{00}=4\frac{\partial V^\alpha}{\partial t}+\gamma^{\alpha\beta}(1-2U)\frac{\partial U}{\partial x^\beta}\\
&-\frac{1}{2}\gamma^{\alpha\beta}\frac{\partial}{\partial x^\beta}[2BU^2-4\Phi_1+4(B-2)\Phi_2\\
&-2\Phi_3-6\Phi_4-\omega]+O(\varepsilon^6),
\end{aligned}\right\}\qquad(165)$$

where we have introduced the notation

$$\omega=\partial^2Q/\partial t^2;\ Q=\int\rho R\,dV.$$

Substituting (163) and (165) in the first equation of (164), we obtain

$$\frac{\partial}{\partial t}\left[\rho\left(1+\Pi+U+\frac{1}{2}v^2\right)\right]+\partial_\alpha\left[\rho v^\alpha\left(1+\Pi+U+\frac{1}{2}v^2\right)+Pv^\alpha\right]$$
$$-\rho\frac{\partial U}{\partial t}-2\rho v^\alpha\frac{\partial U}{\partial x^\alpha}+\rho O(\varepsilon^5)=0.\qquad(166)$$

The second equation of (164) takes, when (163) and (165) are used, the form

$$\frac{\partial}{\partial t}\left[\rho v^\alpha\left(1+\Pi+U+\frac{1}{2}v^2\right)+Pv^\alpha\right]+4\rho\frac{\partial V^\alpha}{\partial t}$$
$$+\frac{\partial}{\partial x^\beta}\left[\rho v^\alpha v^\beta\left(1+\Pi+U+\frac{1}{2}v^2\right)+Pv^\alpha v^\beta-P\gamma^{\alpha\beta}\right]$$
$$+\rho\left(1+\Pi+U+\frac{1}{2}v^2\right)\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}-(2+2B)\rho U\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}$$
$$+\gamma^{\alpha\beta}\rho\frac{\partial}{\partial x^\beta}\left[2\Phi_1-2(B-2)\Phi_2+\Phi_3+3\Phi_4+\frac{1}{2}\omega\right]$$
$$+2\rho v^\alpha\frac{\partial U}{\partial t}+4\rho v^\beta\gamma^{\alpha\tau}\left(\frac{\partial V_\tau}{\partial x^\beta}-\frac{\partial V_\beta}{\partial x^\tau}\right)$$
$$+P\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}+2\rho v^\alpha v^\beta\frac{\partial U}{\partial x^\beta}+\rho v^2\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}+\rho O(\varepsilon^6)=0.\qquad(167)$$

To simplify these expressions, we use the continuity equation of an ideal fluid,

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x^\alpha}(\rho v^\alpha)=0,$$
$$\rho=\rho_0[1+3U+v^2/2+O(\varepsilon^4)],$$

the Newtonian equations of motion of an elastic body,

$$\rho\,dv^\alpha/dt=\gamma^{\alpha\beta}(-\rho\,\partial U/\partial x^\beta+\partial P/\partial x^\beta);$$
$$\rho\,d\Pi/\partial t=-P\,\partial v^\alpha/\partial x^\alpha;$$
$$d/dt=\partial/\partial t+v^\beta\partial/\partial x^\beta,$$

and also the relations

$$\partial V_\alpha/\partial x^\beta-\partial V_\beta/\partial x^\alpha=\partial N_\alpha/\partial x^\beta-\partial N_\beta/\partial x^\alpha;$$
$$\partial_\alpha\partial^\alpha U=4\pi\rho_0;$$
$$\rho\frac{\partial U}{\partial t}=-\frac{1}{2}\frac{\partial}{\partial t}(\rho U)+\frac{1}{8\pi}\frac{\partial}{\partial x^\alpha}\left(\gamma^{\alpha\beta}\frac{\partial U}{\partial t}\frac{\partial U}{\partial x^\beta}-\gamma^{\alpha\beta}U\frac{\partial^2U}{\partial t\,\partial x^\beta}\right).$$

As a result of these transformations, we obtain from (166)

$$\frac{\partial}{\partial t}\left[\rho\left(1+\Pi-\frac{1}{2}U+\frac{1}{2}v^2\right)\right]+\frac{\partial}{\partial x^\alpha}\left[\rho v^\alpha\left(1+\Pi+U+\frac{1}{2}v^2\right)\right.$$
$$\left.+Pv^\alpha-\frac{1}{8\pi}\frac{\partial U}{\partial t}\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}+\frac{1}{8\pi}\gamma^{\alpha\beta}U\frac{\partial^2U}{\partial t\,\partial x^\beta}\right]=\rho O(\varepsilon^4).\qquad(168)$$

We reduce the expression (167) to the form

$$\frac{\partial}{\partial t}\left[\rho v^\alpha\left(1+\Pi+U+\frac{1}{2}v^2\right)+Pv^\alpha\right]$$
$$+\frac{\partial}{\partial x^\beta}\left[\rho v^\alpha v^\beta\left(1+\Pi+U+\frac{1}{2}v^2\right)+Pv^\alpha v^\beta-P\gamma^{\alpha\beta}-2\gamma^{\alpha\beta}PU\right]$$
$$+4\rho\frac{dV^\alpha}{dt}+2\rho\frac{d}{dt}(Uv^\alpha)+\rho_0\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}+\rho(4-2B)U\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}$$
$$+\rho(\Pi+2v^2)\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}+3P\gamma^{\alpha\beta}\frac{\partial U}{\partial x^\beta}$$
$$+3\rho\gamma^{\alpha\beta}\frac{\partial\Phi_4}{\partial x^\beta}-4\rho v^\beta\gamma^{\alpha\tau}\frac{\partial V_\beta}{\partial x^\tau}$$
$$+\rho\gamma^{\alpha\beta}\frac{\partial}{\partial x^\beta}\left(2\Phi_1+\Phi_3-2(B-2)\Phi_2+\frac{1}{2}\omega\right)=\rho O(\varepsilon^6).\qquad(169)$$

We integrate these expressions over the whole of space. We note first that

$$\left.\begin{aligned}
&\int dV\,\rho_0\left(\Pi\frac{\partial U}{\partial x^\alpha}+\frac{\partial\Phi_3}{\partial x^\alpha}\right)=0;\\
&\int dV\,\rho_0\left(U\frac{\partial U}{\partial x^\alpha}+\frac{\partial\Phi_2}{\partial x^\alpha}\right)=0;\\
&\int dV\left(P\frac{\partial U}{\partial x^\alpha}+\rho_0\frac{\partial\Phi_4}{\partial x^\alpha}\right)=0;\\
&\int dV\,\rho_0\left(v^2\frac{\partial U}{\partial x^\alpha}+\frac{\partial\Phi_1}{\partial x^\alpha}\right)=0.
\end{aligned}\right\}\qquad(170)$$

Let us consider, for example, the first relation. Using the expression (98), we have

$$\int dV\,dV'\,\rho_0\rho_0'\left[\frac{\Pi(x_\alpha-x'_\alpha)}{|x-x'|^3}+\frac{\Pi'(x_\alpha-x'_\alpha)}{|x-x'|^3}\right].$$

Since the integrand is antisymmetric under the substitutions $\rho_0\leftrightarrow\rho_0'$ and $x_\alpha\leftrightarrow x'_\alpha$, the integral vanishes. The remaining relations (170) are proved similarly.

In addition, we use the obvious equations

$$\int\rho_0v^\alpha\frac{\partial V_\alpha}{\partial x^\beta}dV=\int\rho_0v^\alpha\frac{\partial N_\alpha}{\partial x^\beta}dV=0;$$
$$\int\rho_0\frac{\partial U}{\partial x^\alpha}dV=0;$$
$$\int\rho v^\beta\frac{\partial^3Q}{\partial t\,\partial x^\alpha\,\partial x^\beta}dV=0;$$
$$\frac{d}{dt}\int\rho f(U,V^\alpha,v^\alpha)dV=\int dV\,\rho\left[\frac{d}{dt}f(U,V^\alpha,v^\alpha)+fO(\varepsilon^2)\right].$$

We also use the circumstance that volume integrals of a spatial divergence vanish when they are transformed into surface integrals.

As a result, we obtain from (168) in the post-Newtonian approximation of the field theory of gravitation the energy integral $dP^0/dt=0$, where

$$P^0=\int dV\,\rho(x,t)\left[1+\Pi-\frac{1}{2}U+\frac{1}{2}v^2\right]=\text{const.}\qquad(171)$$

Noting that

$$\rho\gamma^{\alpha\beta}\frac{\partial}{\partial x^\beta}\frac{\partial^2}{\partial t^2}Q=\frac{\partial}{\partial t}\left(\rho\gamma^{\alpha\beta}\frac{\partial^2}{\partial t\,\partial x^\beta}Q\right)$$
$$+\frac{\partial}{\partial x^\beta}\left(\rho v^\beta\gamma^{\alpha\tau}\frac{\partial^2}{\partial t\,\partial x^\tau}Q\right)$$
$$-\rho v^\beta\gamma^{\alpha\tau}\frac{\partial^3Q}{\partial t\,\partial x^\tau\,\partial x^\beta},$$

we obtain from (169) $dP^\alpha/dt=0$, from which it follows that

$$P^\alpha=\int dV\left[\rho v^\alpha\left(1+\Pi+3U+\frac{1}{2}v_2\right)+Pv^\alpha\right.$$
$$\left.+4\rho V^\alpha+\frac{1}{2}\rho\gamma^{\alpha\beta}\frac{\partial^2Q}{\partial x^\beta\,\partial t}\right]=\text{const.}\qquad(172)$$

Since

$$\gamma^{\alpha\beta}\frac{\partial^2Q}{\partial x^\beta\,\partial t}=N^\alpha-V^\alpha;$$
$$\int dV\,\rho V^\alpha=-\int dV\,\rho Uv^\alpha,$$

the expression (172) can be reduced to the form

$$P^{\alpha}=\int dV\,\rho\left[v^{\alpha}\left(1+\Pi-\frac{1}{2}U+\frac{1}{2}v^2+\frac{P}{\rho}\right)+\frac{1}{2}N^{\alpha}\right]=\text{const.} \tag{173}$$

Thus, in the field theory of gravitation the post-Newtonian integrals of the motion obtained from the conservation law (151) in the pseudo-Euclidean space-time and from the covariant equation (162) in the Riemannian space-time are identical. This is a direct consequence of the circumstance that in the field theory of gravitation the conservation law (151) and Eq. (162) are different forms of expression of the same conservation law. In the field theory of gravitation, the gravitational field is a physical field that possesses energy and momentum densities and contributes to the total energy–momentum tensor of the system. It is the presence in the field theory of gravitation of ordinary conservation laws that enables us to make different energy calculations, including the finding of the post-Newtonian expressions for the integrals of the motion.

In the general theory of relativity, the gravitational field is not a field in the spirit of Faraday and Maxwell, with the consequence that in Einstein's theory one cannot make calculations of the energy of the gravitational field. However, in general relativity the post-Newtonian integrals of the motion of an isolated system are usually obtained from the covariant equation (162) and lead to the expressions (159) and (160).

However, as is shown in Ref. 5, no integral of the motion apart from the Einstein equations themselves follow from the covariant equation (162) in general relativity. Therefore, in general relativity the integrals of the motion are identically equal to zero. This general result, obtained in Ref. 5, can also be confirmed at any stage of approximate calculations with any manner of specializing the coordinate system.

A detailed analysis shows that the traditional way of obtaining nonvanishing "energy–momentum integrals" in Einstein's theory leads, not to integrals of the motion, but only to quantities that in the post-Newtonian approximation do not depend on the time and in general relativity have no physical meaning. Therefore, the interpretation of them in Einstein's theory as the energy and momentum of the isolated system is incorrect.

To conclude this section, we note that in the Newtonian approximation the energy of a static field in the field theory of gravitation calculated using the canonical energy–momentum tensor (62) is positive,

$$\int \tilde{t}_g^{00}\,dV=-\frac{1}{8\pi}\int dV\,\partial_{\alpha}U\partial^{\alpha}U>0,$$

while when the symmetric energy–momentum tensor (66) is used it is negative:

$$\int t_g^{00}\,dV=\frac{3}{8\pi}\int dV\,\partial_{\alpha}U\partial^{\alpha}U<0.$$

It is well known that in electrodynamics the opposite situation obtains: The energy of an electrostatic field calculated on the basis of the canonical energy–momentum tensor is negative, but on the basis of the symmetric tensor it is positive. From this analogy, it can be concluded that a static gravitational field is the field of attractive forces, since in electrodynamics charges of the same sign produce a field of repulsive forces.

The calculation of the total energy of matter and a static gravitational field in the post-Newtonian approximation gives the same result when either the canonical or the symmetric energy–momentum tensor is used:

$$P^0=\int dV\,(\tilde{t}_g^{00}+\tilde{t}_M^{00})=\int dV\,(t_g^{00}+t_M^{00})$$
$$=\int dV\,\rho\left(1+\Pi-\frac{1}{2}U+\frac{1}{2}v^2\right).$$

It follows from the last expression that the energy of two particles at rest increases with increasing distance between them, which is also an indication that attractive forces act between the particles.

12. NONSTATIONARY MODEL OF A HOMOGENEOUS UNIVERSE

The field theory of gravitation makes it possible to construct nonstationary models of the Universe capable of describing the cosmological red shift and free of divergences of the Newtonian type. These models correspond to a flat Universe.

Astronomical observations show that the matter in the Universe is distributed very nonuniformly; the main mass of the matter is contained in planets and stars, while the fraction of interstellar gas and radiation is only a small part of the total mass.

However, if one averages over regions of space whose linear dimensions are appreciably greater than the distances between clusters of galaxies, the matter density of the part of the Universe accessible to observation is a constant quantity that does not depend on the position of the center of the region of averaging. Therefore, as a first step, it is natural from the physical point of view to consider a homogeneous isotropic model for the Universe.

In such an approach, the inhomogeneity of the matter distribution found if one averages over smaller regions of space (clusters of galaxies, galaxies, etc.) can be taken into account by the introduction of small inhomogeneous perturbations in the background cosmological field of the homogeneous Universe.

A homogeneous model of the Universe in the field theory of gravitation admits both monotonic and nonmonotonic behavior. The behavior of the model, the age of the Universe, and other characteristics depend strongly on the connection equation $g_{in}=g_{in}(\gamma_{lm}, f_{lm})$.

However, if we are interested in the evolution of the Universe and effects that arise in a small neighborhood ($\Delta\tau\sim 10^8$ years) of the present epoch of the proper time of the Universe, knowledge of the exact connection equation is not required. In such a case, it is quite sufficient to expand the metric tensor of the Riemannian space-time with respect to the weak field up to quadratic terms.

The homogeneous isotropic Universe is described by the interval

$$ds^2=U dt^2-V\,(dx^2+dy^2+dz^2), \tag{174}$$

where the functions U and V depend only on the time variable t:

$U = U(t); \quad V = V(t).$

We shall regard the matter in the Universe as an ideal fluid with energy–momentum tensor density

$$T^{in} = \sqrt{-g}\,[(\varepsilon + P)\, u^i u^n - P g^{in})].$$

By virtue of the homogeneity and isotropy of the Universe,

$$\varepsilon = \varepsilon(t);\ P = P(t);\quad u^\alpha = 0;\ u^0 \neq 0;\quad u^0 u^0 g_{00} = 1.$$

Then the components of the energy–momentum tensor density of the matter take the form

$$\left.\begin{aligned} &T^{00} = \varepsilon \sqrt{V^3/U};\\ &T^{\alpha\beta} = -P\sqrt{UV}\,\gamma^{\alpha\beta}. \end{aligned}\right\} \tag{175}$$

Using (174) for the interval, we determine the connection of the Riemannian space-time:

$$\left.\begin{aligned} &\Gamma^0_{00} = \dot U/2U;\quad \Gamma^0_{0\alpha} = 0;\\ &\Gamma^0_{\alpha\beta} = -(\dot V/2U)\,\gamma_{\alpha\beta};\quad \Gamma^\alpha_{00} = 0;\\ &\Gamma^\alpha_{0\beta} = (\dot V/2V)\,\delta^\alpha_\beta;\quad \Gamma^\alpha_{\beta\varepsilon} = 0; \end{aligned}\right\} \tag{176}$$

where the dot denotes simple differentiation with respect to t.

In our case, the covariant conservation equation (162) for the energy–momentum tensor density of the matter has the form

$$\dot T^{00} + \Gamma^0_{00} T^{00} + \Gamma^0_{\alpha\beta} T^{\alpha\beta} = 0.$$

Substituting (175) and (176) in this equation, we obtain

$$d(\varepsilon\sqrt{V^3})/dt + P\,d\sqrt{V^3}/dt = 0. \tag{177}$$

The solution of Eq. (177) has the form

$$\ln V = -\frac{2}{3}\int_{\varepsilon_0}^{\varepsilon} \frac{d\varepsilon'}{\varepsilon' + P(\varepsilon')}. \tag{178}$$

The connection equation can be expressed in the most general form as follows:

$$g_{in} = \gamma_{in} f_1 + f_{in} f_2 + f_{im} f_{nl} A^{lm}, \tag{179}$$

where f_1 and f_2 are certain scalar functions of the invariants $I_1 = f^m_m$, $I_2 = f_{nm} f^{nm}$, etc., and the tensor A^{lm} is constructed from the tensors γ^{lm}, $f^{ln} f^m_n \ldots$ and invariants.

For a weak field, the exact connection equation (179) must go over into the expression (102). This enables us to determine the expansions of the functions f_1 and f_2 and the tensor A^{lm} in the case of a weak field:

$$\left.\begin{aligned} &f_1 = 1 - (1/2)\,I_1 + (b_3/4)\,I_2 + (b_4/4)\,(I_1)^2 + O(f^3_{lm});\\ &f_2 = 1 + (b_2/4)\,I_1 + O(f^2_{lm});\\ &A^{ln} = (b_1/4)\,\gamma^{ln} + O(f^{ln}). \end{aligned}\right\} \tag{180}$$

We shall give the following treatment in the most general case on the basis of the connection equation (179).

For a homogeneous Universe, the gravitational field equations take the form

$$\ddot H_{in} = h_{in};\quad \ddot f_{00} = \ddot f_{0\alpha} = 0;\quad \ddot f_{\alpha\beta} = -16\pi\,(h_{\alpha\beta} + \gamma_{\alpha\beta} h_{00}). \tag{181}$$

The subsidiary conditions $\partial^l f_{ln} = 0$ give

$$\dot f_{00} = \dot f_{0\alpha} = 0.$$

It follows from the expression (9) that the components f_{00} and $f_{0\alpha}$ of the gravitational field vanish:

$$f_{00} = f_{0\alpha} = 0.$$

By virtue of the isotropy of the Universe, the remaining components of the gravitational field must have the form

$$f_{\alpha\beta} = \gamma_{\alpha\beta} F(t).$$

Then

$$g_{00} = U = f_1;\quad A^{\alpha\beta} = \gamma^{\alpha\beta} f_3;$$
$$g_{\alpha\beta} = \gamma_{\alpha\beta} V;\quad V = f_1 + F f_2 + F^2 f_3;$$
$$I_1 = 3F;\quad I_2 = 3F^2;\ \ldots;\quad I_n = 3F^n.$$

One can show that

$$\partial g_{00}/\partial f_{\alpha\beta} = (1/3)\,\gamma^{\alpha\beta}\,dU/dF;$$
$$\gamma^{\varepsilon\tau}\,\partial g_{\varepsilon\tau}/\partial f_{\alpha\beta} = \gamma^{\alpha\beta}\,dV/dF.$$

Therefore, the field equations (181) take the form

$$\ddot F = \frac{64\pi}{3}\left\{\varepsilon\sqrt{V^3}\frac{d}{dF}\sqrt{U} - P\sqrt{U}\frac{d}{dF}\sqrt{V^3}\right\}. \tag{182}$$

As initial conditions for Eq. (182), we take the conditions at the present time $t = 0$:

$$\varepsilon = \varepsilon_0;\ U = V = 1,\ dV/dt = 2H, \tag{183}$$

where H is the Hubble constant. It follows from the experiments[57] that 20×10^9 years $> 1/H > 7.5\times 10^5$ years. With such a choice of the initial conditions, the cosmological field at the present epoch will be the pseudo-Euclidean background on which we observe all other physical processes.

It follows from the conditions (183) that

$$F(0) = 0;\quad dF/dt|_{t=0} = 2H/(dV/dF).$$

We reduce Eq. (182) to the form

$$\frac{d}{dt}(\dot F^2 + C_1) = \frac{128\pi}{3}\left(\varepsilon\sqrt{V^3}\frac{d}{dt}\sqrt{U} - P\sqrt{U}\frac{d}{dt}\sqrt{V^3}\right).$$

Using the conservation equation (177), we obtain

$$\dot F^2 + C_1 = (128\pi/3)\,\varepsilon\sqrt{UV^3}.$$

It is interesting to note that this equation, obtained from the gravitational field equations (181), is a modified expression of the conservation law for the energy density of the matter and the gravitational field of the Universe in the flat space-time. To see this, we use the definitions (156) and (157), the connection equation (179), the components of the energy–momentum tensor density (175) of the matter in the Riemannian space-time, and also note that $f_{00} = f_{0\alpha} = 0$, $f_{\alpha\beta} = \gamma_{\alpha\beta} F$, obtaining then

$$t^{0\alpha}_M = t^{0\alpha}_g = 0;\quad t^{00}_M = \varepsilon\sqrt{V^3 U};\quad t^{00}_g = -(3/128\pi)\,\dot F^2.$$

Therefore, the conservation law (151) for the energy–momentum tensor density of the matter and the gravitational field in the flat space-time for $n = 0$ will have the form

$$\frac{\partial}{\partial t}(t^{00}_M + t^{00}_g) = 0.$$

It follows from this that

$$t^{00}_M + t^{00}_g = \text{const}.$$

Using the initial conditions (183), we have

$$\varepsilon\sqrt{V^3 U} - (3/128\pi)\,\dot F^2 = (3/128\pi)\,C_1,$$

where $C_1 = 16H^2(1 - \alpha)$.

Thus, the total energy density of the matter and the gravitational field of the Universe in the flat space-time is constant at all stages of its evolution. This means that the energy of the Universe is not changed during its evolution but is merely redistributed between the matter and the gravitational field.

Using the initial conditions, we write the solution of this equation in the form

$$t=-\frac{1}{4H}\int_0^F \frac{dF'}{\sqrt{1-\alpha+(\alpha\varepsilon/\varepsilon_0)\sqrt{V^3(F')\,U(F')}}}, \qquad (184)$$

where we have introduced the notation

$$\left.\begin{array}{l}\alpha=8\pi\varepsilon_0/3H^2;\quad U=f_1(F);\\ V(F)=f_1(F)+Ff_2(F)+F^2f_3(F).\end{array}\right\} \qquad (185)$$

The expressions (178), (184), and (185) determine parametrically the entire evolution of the homogeneous isotropic model of the Universe, including the singular state (or hot Universe) for arbitrary equation of state $P=P(\varepsilon)$ of the matter and connection equation (179) specified in the most general form.

In (184) and (174) we go over to the proper time. In a time interval when $U(t)$ is nonzero, one can go over to a proper time $\tau(t)$ such that

$$\sqrt{U(t)}\,dt=d\tau.$$

Then the interval has the form

$$ds^2=d\tau^2-V(\tau)\,[dx^2+dy^2+dz^2]. \qquad (186)$$

Assuming that $\tau(0)=0$ is the contemporary time, we obtain parametric expressions that determine the evolution of the Universe:

$$\tau=-\frac{1}{4H}\int_0^F \frac{\sqrt{U(F')}\,dF'}{\sqrt{1-\alpha+\frac{\alpha\varepsilon(F')}{\varepsilon_0}\sqrt{U(F')V^3(F')}}}; \qquad (187)$$

$$U=f_1(F);\ V=f_1(F)+Ff_2(F)+F^2f_3(F); \qquad (188)$$

$$\ln V(F)=-\frac{2}{3}\int_{\varepsilon_0}^{\varepsilon(F)}\frac{d\varepsilon'}{\varepsilon'+P(\varepsilon')}. \qquad (189)$$

We investigate these solutions in the neighborhood of the contemporary ($|\tau|\ll 1/4H$) proper time. The case $|\tau|\ll 1/4H$ corresponds to small values of F such that the functions $U(F)$ and $V(F)$ differ little from unity, and therefore the expansion (180) of the metric in the weak-field approximation is valid. In our case ($F\ll 1$), these expansions take the form

$$\left.\begin{array}{l}f_1=1-(3/2)F+(1/4)(9b_4+3b_3)F^2+O(F^3);\\ f_2=1+(3/4)b_2F+O(F^2);\\ A^{\alpha\beta}=(1/4)b_1\gamma^{\alpha\beta}+\gamma^{\alpha\beta}O(F);\\ f_3=(1/4)b_1+O(F).\end{array}\right\} \qquad (190)$$

In addition, we assume that in the neighborhood of the contemporary epoch of the proper time the pressure is negligibly small compared with the energy density: $p\ll\varepsilon$. Therefore, from (189) we obtain

$$\varepsilon(F)=\varepsilon_0/\sqrt{V^3(F)}.$$

From the expansions (190) and (188) we have

$$U=1-(3/2)F+(1/4)(9b_4+3b_3)F^2+O(F^3);$$
$$V=1-(1/2)F+(1/4)(b_1+3b_2+3b_3+9b_4)F^2+O(F^3).$$

Substituting the expressions for U, V, and ε in the integral (187) and integrating, we obtain

$$\tau=-(1/4H)\{F-(3/8)(1-\alpha/2)F^2+O(F^3)\}.$$

Determining F from this relation and substituting it in the expression for $V(F)$, we obtain

$$V(\tau)=1+2H\tau+H^2\tau^2\,[(3/2)\alpha-3+4(b_1+3b_2+3b_3+9b_4)]+O(H^3\tau^3).$$

The metric (186) with the cosmological scale factor $V(\tau)$ leads to experimentally observable effects. One of them is the cosmological red shift discovered in 1929 by Hubble.[53] This consists of a red shift of the spectral lines emitted by distant galaxies, the red shift being directly proportional to the distance from the galaxy to the Earth. In the general theory of relativity, this effect was predicted by the Soviet scientist A. A. Friedmann in 1922.[59]

We now show that the homogeneous model of the Universe in the field theory of gravitation in the neighborhood of the contemporary epoch (for $H\tau\ll 1$ or for $\tau\ll 10^{10}$ years) also describes a linear cosmological red shift. Suppose the observer is at the point with coordinates $(x_1,0,0)$. Suppose that at the point $(x_0<x_1,0,0)$ for $\tau_0<0$ two events occur separated by the time interval $\Delta\tau_0$ (for example, the emission of an electromagnetic wave with period $\Delta\tau_0$). From the expression $ds^2=d\tau^2-V(\tau)[dx^2+dy^2+dz^2]$ for the interval, we find the equation of motion of the wave front along the x axis: $d\tau=dx\sqrt{V(\tau)}$. Therefore, the first signal reaches the observer at the time τ_1, which is determined by the equation

$$\int_{\tau_0}^{\tau_1}\frac{d\tau}{\sqrt{V(\tau)}}=x_1-x_0.$$

The second signal reaches the observer an interval $\Delta\tau_1$ after the first:

$$\int_{\tau_0+\Delta\tau_0}^{\tau_1+\Delta\tau_1}\frac{d\tau}{\sqrt{V(\tau)}}=x_1-x_0.$$

Subtracting the second expression from the first, we obtain

$$\int_{\tau_0}^{\tau_0+\Delta\tau_0}\frac{d\tau}{\sqrt{V(\tau)}}-\int_{\tau_1}^{\tau_1+\Delta\tau_1}\frac{d\tau}{\sqrt{V(\tau)}}=0.$$

Using the approximate equation $1/\sqrt{V(\tau)}\approx 1-H\tau$, which holds for $H\tau\ll 1$, we obtain

$$\Delta\tau_0\,[1-H\tau_0-(1/2)H\Delta\tau_0]=\Delta\tau_1\,[1-H\tau_1-(1/2)H\Delta\tau_1].$$

If $\Delta\tau_0\ll\tau_0$ and $\Delta\tau_1\ll\tau_1$, then

$$\Delta\tau_1=\Delta\tau_0\,(1-H\tau_0)/(1-H\tau_1)\approx\Delta\tau_1\,(1-H(\tau_0-\tau_1)).$$

Since $\tau_0<\tau_1$, we have $\Delta\tau_1>\Delta\tau_0$, i.e., the time interval between the two signals received by the observer at the point x_1 will be greater than the time interval between these signals at their point of emission x_0. Therefore, as electromagnetic waves propagate from some galaxy, their frequency decreases because of the cosmological red shift:

$$\omega=\omega_0\,[1+H(\tau_0-\tau_1)].$$

For galaxies not very far from the observer ($r\ll 1/H\sim 10^{10}$ light years) this red shift will depend linearly on the distance L between the observer and the galaxy: $\omega_1=\omega_0(1-HL)$.

The deceleration parameter of the "expanding" Universe, $q_0=1-2V\ddot{V}/\dot{V}^2$, is equal in the neighborhood of the present time $\tau=0$ to

$$q_0=2-8(b_2+b_3+4b_4)-(3/2)\alpha+2(1-B).$$

For comparison, we point out that in Einstein's theory the deceleration parameter for the homogeneous Universe is $q_0=\alpha/2$. In Einstein's theory of gravitation, the deceleration parameter is one of the most important quantities characterizing the homogeneous Universe as a whole: For deceleration parameter $q_0<\frac{1}{2}(\alpha<1)$, the Universe is open, while for $q_0>\frac{1}{2}(\alpha>1)$ the Universe is closed, having a finite volume but no boundaries. In the field theory of gravitation, there is no such connection—the Universe has infinite volume for all values of α and q_0.

It follows from estimates of the mass of matter in galaxies[60] that $\varepsilon_0=3\times10^{-31}$ g/cm^3. In this case, $\alpha=0.06$. Then the deceleration parameter in Einstein's theory must be equal to $q_0=0.03$, and the Universe will be open, expanding for every. However, measurements of the deceleration parameter gave a different result.

For example, it is concluded in Ref. 61 that the value of q_0 lies in the range from 2 to 32, the most probable value being $q_0=5$. Thus, in Einstein's theory the value of the deceleration parameter obtained from observations conflicts with the observed matter density in the galaxies, which is appreciably lower than is required for agreement. To eliminate this discrepancy between the characteristics of the cosmological solution of Einstein's theory and their values obtained experimentally, attempts are at present being made to increase the value of ε_0 (in the search for hidden matter in the galaxies, the "hidden-mass mystery"), as well as to decrease the value of q_0 obtained experimentally (by assuming a strong evolution of the luminosity function of the galaxies with the red shift). These attempts have not hitherto resolved this question.

In the field theory of gravitation, in contrast to Einstein's general theory of relativity, the decerlation parameter is determined not only by the mean matter density ε_0 (the parameter $\alpha=8\pi\varepsilon_0/3H^2$) but also by the post-Newtonian coefficients b_1, b_2, b_3, and b_4, and therefore measurement of q_0 would enable one, without using post-post-Newtonian experiments in the solar system, to measure the value of $8(b_2+b_3+4b_4)$:

$$8(b_2+b_3+4b_4)=q_0+2-(3/2)\alpha+2(1-B). \qquad (191)$$

The behavior of the homogeneous model of the Universe in the distant past depends strongly on the form of the connection equation in strong gravitational fields. Since we have not yet made any concrete choice of the connection equation, we shall consider the behavior of the model in the distant past purely qualitatively.

If the equation $V(F)=0$ has a solution, then for $F=F_1$ the determinant of the metric tensor and also its spatial components vanish. Therefore, it is natural to assume that for $F=F_1$ a singular state of the Universe is realized. Near the singular state in the Universe, ultrarelativistic particles, whose equation of state has the form $P=\varepsilon/3$, are dominant. Substituting this equation in (189), we obtain

$$\varepsilon=\varepsilon_0/V^2. \qquad (192)$$

It follows that when the function $V(F)$ vanishes the density of the total energy of the Universe becomes infinite, and for $F=F_1$ a singular state of the Universe is actually realized.

A certain time in the past $\tau=\tau_m$ corresponds to the smallest positive root F^* of the equation $V(F)=0$. It is natural to call the time $T=-\tau_m$ the age of the Universe. It is equal to

$$T=\frac{1}{4H}\int_0^{F^*}\frac{\sqrt{U(F)}\,dF}{\sqrt{1-\alpha+\alpha(\varepsilon/\varepsilon_0)\sqrt{UV^3}}}.$$

It is obvious that the age of the Universe and also the features of its evolution depend strongly on the connection equation (179).

We introduce the time $\tau_0=T+\tau$, which is measured from the singular state:

$$\tau_0=\frac{1}{4H}\int_F^{F^*}\frac{\sqrt{U(F')}\,dF'}{\sqrt{1-\alpha+\alpha(\varepsilon/\varepsilon_0)\sqrt{UV^3}}}.$$

In the neighborhood of the singular state (for $F\sim F^*$), the relation (192) holds, and therefore we obtain

$$\tau_0=\frac{1}{4H}\int_F^{F^*}\frac{\sqrt{U(F')}\,dF'}{\sqrt{1-\alpha+\alpha\sqrt{U/V}}}. \qquad (193)$$

The expression (193) determines the dependence of the proper time in the neighborhood of the singular state on the gravitational field F and, thus, makes it possible to determine the behavior of the function $V(\tau)$ in the given neighborhood.

It should be noted that if the equation $V(F)=0$ does not have a solution, then the model of the Universe does not have a singular state.

The behavior of the functions U and V in the neighborhood of the singular state essentially determines the flux densities and spectral characteristics of the primordial electromagnetic, neutrino, and gravitational radiation. Therefore, measurement of the flux densities and spectral characteristics of these primordial forms of radiation makes it possible to determine the behavior of the connection equation in strong gravitational fields. It should be noted that in the cosmological models in which the equation $V(F)=0$ does not have a solution an Olbers paradox arises, namely, the integrated luminosity of all the stars diverges. Indeed, the total energy of starlight at the present time $\tau=0$ is[57]

$$\rho=\int_{-\infty}^{0}Z(\tau)[V(\tau)]^2\,d\tau. \qquad (194)$$

Here, $Z(\tau)$ is the proper density of the luminosity of the stars:

$$Z(\tau)=\int n(\tau,L)\,dL$$

[$n(\tau,L)$ is the density of stars with absolute luminosity L at the time τ]. For the convergence of the integral (194), it is necessary that either there be a singular state of the Universe [$V(F_1)=0$] at finite F_1, so that the integral (194) is effectively truncated at the lower limit at a certain $\tau=\tau(F_1)$, or $V[F(\tau)]$ decrease sufficiently rapidly to zero with increasing $|\tau|$:

$$\tau V(F(\tau))Z(\tau) \to 0$$
$$|\tau| \to 0 \ (F \to \infty).$$

CONCLUSIONS

We have considered the formulation of a theory of gravitation as a theory of a symmetric tensor field of second rank in a flat space-time. In the theory, the usual notions of energy transport by physical fields have a rigorous meaning, and the gravitational field, like all other physical fields, transmits positive-definite energy and momentum. The equations of motion of the matter are formulated in terms of an effective Riemannian space-time with metric tensor g_{ni}, which ensures equality of the inertial and gravitational masses of a point body in the theory. The combination of the idea of the gravitational field as a physical field that transmits energy and is analogous to other physical fields with the identity principle leads to new equations of the gravitational field and changes our notions of space and time. The equations of the gravitational field in matter are nonlinear because of the nonlinear dependence of the source on the components of the gravitational field. The source in the gravitational field equations is the matter, and the gravitational field is itself a source only to the extent that the expression $T^{ni}\partial g_{ni}/\partial f_{lm}$ depends on the components of the gravitational field. Outside the matter, the field equations are linear. Because of the gauge invariance, the gravitational field equations, which are partial differential equations of fourth order, go over into second-order equations outside the matter. The post-Newtonian approximation of the field theory of gravitation shows that all the parameters of the theory, except the parameter β, agree with the parameters of Einstein's theory. No restrictions are imposed on the value of β in the field theory of gravitation, and it was therefore deduced from the correspondence between the predictions of the theory and the results of experiments that $\beta = 1$.

Thus, the field theory of gravitation and Einstein's theory are indistinguishable from the point of view of all gravitational experiments made with post-Newtonian accuracy in the gravitational field of the solar system. In the field theory of gravitation, there is no preferred rest frame, since the geometry of the pseudo-Euclidean space-time is not an *a priori* geometry but the natural geometry for all physical fields, including the gravitational field. The Riemannian space-time for the motion of matter is an effective space-time, reflecting only the influence of the gravitational field on the matter in the pseudo-Euclidean space-time. Therefore, the field theory of gravitation does not belong to the class of the so-called bimetric theories of gravitation in either its meaning or its field equations.

In the field theory of gravitation, the concept of the energy–momentum tensor is common to all physical fields, and therefore the existence of curvature waves in the Riemannian space-time reflects the transport of energy and momentum by gravitational waves in the pseudo-Euclidean space-time. Therefore, in the field theory of gravitation one can make energy calculations. In the field theory of gravitation, the integrals of the motion of an isolated system in the post-Newtonian approximation coincide with the Newtonian integrals of the motion; in the case of the emission of weak gravitational waves by a slowly moving source, the source energy decreases in accordance with the formula

$$-dE/dt = (G/45c^5)\,\dddot{D}^2_{\alpha\beta}. \qquad (195)$$

In contrast to the field theory of gravitation, there are no conservation laws in their usual meaning in general relativity, as a result of which Einstein's theory has only vanishing integrals of the motion. In general relativity, it is impossible to calculate the energy loss by a source or to determine the energy fluxes of gravitational waves, since in Einstein's theory there are no conservation laws relating the change in the energy–momentum tensor of the matter to the existence of curvature waves. Therefore, Eq. (195) does not in principle follow from general relativity.

The field equations of the field theory of gravitation differ from the equations of Einstein's theory, which leads to very different descriptions in these theories of effects in strong gravitational fields, and also in the properties of the gravitational field. Among these differences, there is in the field theory no bending of a gravitational ray passing near a massive body, so that massive bodies do not have a focusing influence on gravitational waves. In addition, in contrast to Einstein's theory, the frequency of free gravitational waves emitted by some source changes in the field theory only on account of relative motion of the source and observer (Doppler effect), since there is no gravitational red shift for free gravitational waves in vacuum.

As in Einstein's theory, in the field theory of gravitation the gravitational field of a nonstatic, spherically symmetric source outside the matter is a static field. Nonstationary homogeneous models of the Universe in the field theory of gravitation describe the cosmological red shift and admit both monotonic and nonmonotonic behavior. In contrast to Einstein's theory, the deceleration parameter is determined not only by the mean matter density in the Universe but also by the post-Newtonian coefficients, and therefore in the field theory of gravitation one does not have the difficulties which occur in Einstein's theory in connection with the insufficient mean matter density to obtain the observed deceleration parameter.

Our review of the various gravitational experiments shows that the field theory of gravitation makes it possible to describe all the existing experimental facts.

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