Weak interaction in quark models with unconfined color

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A study is made of the weak interaction between quarks for the three- and four-triplet models for integral quark charges in the most general form. It is shown that under certain conditions the requirement that the algebra SU(2) be satisfied for the weak charges automatically leads to diagonality of the neutral current. The Lagrangian of nonleptonic interaction for ordinary hadrons is obtained. Some predictions are made for nonleptonic decay of charmed hadrons. These currents are capable of describing the latest experiments on neutrino reactions. The methods proposed for dealing with weak currents may be very helpful in the consideration of all quark schemes.

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INTRODUCTION

Following the discovery of the isotopic symmetry group of the strong interactions, it was found that the group $SU(3)$, specifically its eight-fold way,[1,2] is also a symmetry group. The lowest representation of $SU(3)$ used in this approach has dimension 8 (which gave the name to the entire direction). All the necessary representations can be obtained by decomposition into irreducible representations of the direct product of the eight-dimensional representations, for example, $8\otimes 8 \sim 1\oplus 8\oplus 8\oplus 10\oplus \overline{10}\oplus 27$. But the group also has a series of representations (3, 6, 15, ...) such that the particle multiplets corresponding to them in the framework of the eight-fold way will have fractional values of the quantum numbers. A triplet of such particles, the quarks q_i, have the following charges: $q_1\equiv p$ (2/3), $q_2 \equiv n$ (−1/3), and $q_3\equiv \lambda$ (−1/3). Since the direct products of triplets and antitriplets form multiplets of physical particles, for example, $3\otimes \bar{3} = 1\oplus 8$, Gell-Mann and Zweig[3,4] had the very fruitful idea of regarding the observed particles as composed of quarks. Mesons are formed by quark–antiquark pairs, $M\sim \tilde{q}q$, and baryons by three quarks, $B\sim qqq$; for example, $K^+\sim \tilde{\lambda}p$, and $\Lambda^0 \sim pn\lambda$ or $\Omega^-\sim \lambda\lambda\lambda$. It appears obvious to ascribe a quark the spin $\frac{1}{2}$, and it then follows from the theorem on the connection between spin and statistics that the quarks must be fermions. At this point, the entire scheme becomes contradictory, since it is impossible to put, for example, three identical particles in the S state to form $\Delta^{++}\sim ppp$ or $\Omega^-\sim\lambda\lambda\lambda$. Similar arguments hold for baryons with spins $\frac{1}{2}$. One way out of this dilemma is to introduce the concept of color.[5,6] It was also suggested that the quark fields should be regarded as parafermion fields,[7] which also eliminated the contradiction; however, in this review we shall discuss only color quarks.

The idea common to all color models is that the quarks acquire a new degree of freedom (a new quantum number), which is called color. The quark field acquires a further index. We shall denote the indices of the group $SU(3)$ by $i, j, k, \ldots$, and the color indices by $\alpha, \beta, \gamma\ldots$. The quark triplet (p, n, λ) was written as q_i $(q_1\equiv p, q_2\equiv n, q_3\equiv\lambda)$, and now $q_{i\alpha}$ is represented by

$$\begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix}. \qquad (1)$$

The quarks with different colors are different particles and can form S states. The observed hadrons do not have this new degree of freedom, so that to construct them from color quarks we must contract over the color indices. The generally accepted color group is $SU(3)$, although other possibilities have been discussed.[8] In the group $SU(3)$, the well-known symbols $\delta_{\alpha\beta}$ and $\varepsilon_{\alpha\beta\gamma}$ are invariants, and these serve to construct ordinary hadrons from color quarks. The mesons are constructed by means of the Kronecker delta:

$$M\sim \tilde{q}^{i\alpha} q_{j\alpha}. \qquad (2)$$

For example, K^+ now has the form

$$K^+ = (1/\sqrt{3})\,(\tilde{\lambda}_1 p_1 + \tilde{\lambda}_2 p_2 + \tilde{\lambda}_3 p_3). \qquad (2')$$

The baryons are formed by means of the Levi-Civita symbol:

$$B\sim \varepsilon_{\alpha\beta\gamma} q_{i\alpha} q_{j\beta} q_{k\gamma}; \qquad (3)$$

for example,

$$\Lambda^0 = (1/\sqrt{6})\,\varepsilon_{\alpha\beta\gamma} p_\alpha n_\beta \lambda_\gamma \equiv (1/\sqrt{6})\,[p_1 n_2 \lambda_3 + p_2 n_3 \lambda_1 + p_3 n_1 \lambda_2 - p_1 n_3 \lambda_2 - p_3 n_2 \lambda_1 - p_2 n_1 \lambda_3]. \qquad (3')$$

Similar procedures for averaging over the color are also applied for other physical quantities such as the currents, products of currents, etc. Rather a lot of different color models have been proposed. An exhaustive list of them and a description of the differences are given in the Reviews of Refs. 9–11. Basically, these reviews classify the color hadrons and their strong and electromagnetic interactions. In contrast, the studies concerned with weak interactions are not represented nearly so fully in the quoted reviews. In addition, since the writing of Refs. 9–11 several investigations have been made into the weak interactions, and the time is now ripe for a systematic survey. These were the reasons that prompted the writing of the present review.

The experimental data can be satisfactorily described in the framework of a model with fractionally charged quarks. This circumstance, and also the attractions of such a theory from a purely theoretical point of view have led to a widespread conviction of its validity. However, this does not rule out the possibility of considering alternative schemes with, in particular, integral quark charges. As we shall see below, these schemes have additional possibilities associated with a change of the color in weak interactions, which in a

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number of cases makes it possible to described phenomena whose treatment in the standard model leads to difficulties such as, for example, the problem of P-odd nuclear transitions. In the present review, we shall discuss different aspects of the theory of weak interactions in the framework of a model with integral quark charges, avoiding as far as possible a repetition of the questions discussed in Refs. 9–11.

As the basic model, we choose the original model of Bogolyubov, Struminskiĭ, and Tavkhelidze[5] and Han and Nambu[6] with integral quark charges. Many results can be transferred with virtually no modification to other models with integral charges.[12,13]

1. MODEL AND GENERAL FORM OF THE WEAK HADRON CURRENT

The first of the color models[5,6] was constructed as follows. The group structure is $SU(3)'\otimes SU(3)''$. With respect to the group $SU(3)'$, the quarks transform in accordance with the representation 3, and the quantum numbers of this group are distinguished by an index as follows:

$$\left.\begin{array}{llll} & Y' & I_3' & Q' \\ q_1^\alpha: & 1/3 & 1/2 & 2/3; \\ q_2^\alpha: & 1/3 & -1/2 & -1/3; \\ q_3^\alpha: & -2/3 & 0 & -1/3. \end{array}\right\} \quad (4)$$

In the considered variant, the quarks behave with respect to the group $SU(3)''$ as the antitriplet 3*, the index α fixing their quantum numbers with respect to this group as follows:

$$\left.\begin{array}{llll} & Y'' & I_3'' & Q'' \\ q_i^1: & -1/3 & -1/2 & -2/3; \\ q_i^2: & -1/3 & 1/2 & 1/3; \\ q_i^3: & 2/3 & 0 & 1/3. \end{array}\right\} \quad (5)$$

It is assumed that the ordinary group $SU(3)$ is a diagonal subgroup of the group $SU(3)'\otimes SU(3)''$. This means that the generators of $SU(3)$ (denoted by symbols without primes) are sums of the generators of $SU(3)'$ and $SU(3)''$. Thus,

$$Y=Y'\otimes\mathbb{1}+\mathbb{1}\otimes Y''; \quad I_3=I_3'\otimes\mathbb{1}+\mathbb{1}\otimes I_3''; \quad Q=Q'\otimes\mathbb{1}+\mathbb{1}\otimes Q''. \quad (6)$$

Using the rule (6), we can readily obtain the quantum numbers of the nine quarks in the triplet model. For example, the quark $q_1^1\equiv p^1$ has the quantum numbers $Y=0, I_3=0, Q=0$; the quark $q_2^3\equiv n^3$ has $Y=1, I_3=-\frac{1}{2}, Q=0$; and so forth. As a result, we obtain the table

	Y	I_3	$Q=I_3+Y/2$
p^1	0	0	0
n^1	0	-1	-1
λ^1	-1	$-1/2$	-1
p^2	0	1	1
n^2	0	0	0
λ^2	-1	1/2	0
p^3	1	1/2	1
n^3	1	$-1/2$	0
λ^3	0	0	0

(7)

Frequently, this quark nonet is written in the form of the matrix

$$\begin{pmatrix} p_0^1 & p_+^2 & p_+^3 \\ n_-^1 & n_0^2 & n_0^3 \\ \lambda_-^1 & \lambda_0^2 & \lambda_0^3 \end{pmatrix}, \quad (8)$$

in which the subscript indicates the charge of the quark. We consider also the group $SU(4)'\otimes SU(3)''$ and require that the 12 quarks in it also possess integral charges. The generally accepted charges of the quarks in the group $SU(4)'$ are

$$\left.\begin{array}{ll} & Q' \\ q_1^\alpha: & 2/3; \\ q_2^\alpha: & -1/3; \\ q_3^\alpha: & -2/3; \\ q_4^\alpha: & 2/3, \end{array}\right\} \quad (9)$$

while with respect to $SU(3)''$ they are as in (5). We again form the sum $Q=Q'\otimes\mathbb{1}+\mathbb{1}\otimes Q''$, and then exactly as in the group $SU(3)'\otimes SU(3)''$ we obtain a matrix analogous to (8) ($q_4^\alpha\equiv c^\alpha$):

$$\begin{pmatrix} p_0^1 & p_+^2 & p_+^3 \\ n_-^1 & n_0^2 & n_0^3 \\ \lambda_-^1 & \lambda_0^2 & \lambda_0^3 \\ c_0^1 & c_+^2 & c_+^3 \end{pmatrix}. \quad (10)$$

Such a rule can be extended in an obvious manner to any number of flavors. It is important to note that the electromagnetic current in schemes with integral quark charges is an octet with respect to both the group $SU(3)'$ and $SU(3)''$: $J_{em}=J(8,1)+J(1,8)$, whereas in the scheme with fractional charges $Q=Q'\otimes 1$, and here $J_{em}=J(8,1)$. This important property makes it possible to distinguish experimentally these currents below the color excitation threshold.

We now turn to the construction of the matrices of the weak transitions between the obtained color quarks. In principle, we can stay within the framework of the diagonal subgroup of $SU(3)'\otimes SU(3)''$ and attempt to construct responsible for weak transitions between the obtained nonet of quarks. In the group $SU(3)$, the weak current has the form

$$\left.\begin{array}{l} J_\mu^+=\cos\theta_c J_\mu^{1+i2}+\sin\theta_c J_\mu^{4+i5}; \\ J_\mu^{1+i2}=\tilde{q}\gamma_\mu(1-\gamma_5)\left(\frac{\lambda_1+i\lambda_2}{2}\right)q; \\ J_\mu^{4+i5}=\tilde{q}\gamma_\mu(1-\gamma_5)\left(\frac{\lambda_4+i\lambda_5}{2}\right)q. \end{array}\right\} \quad (11)$$

Here, $\tilde{q}=(\tilde{q}^1,\tilde{q}^2,\tilde{q}^3)$ and λ_i are the Gell-Mann matrices. We rewrite the current (11) in explicit form, omitting the Lorentz structure:

$$J^+=\tilde{q}M^+q=\tilde{q}\begin{pmatrix} 0 & \cos\theta_c & \sin\theta_c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}q. \quad (12)$$

If we stay within the framework of the diagonal subgroup, it is obvious that the currents u in the groups $SU(3)'$ and $SU(3)''$ have the same form (with angles θ' and θ''). In the Han–Nambu model, the generators of the diagonal subgroup are

$$\Lambda_\mu=(\lambda_\mu\otimes\mathbb{1}-\mathbb{1}\otimes\lambda_\mu^T)/2, \quad (6')$$

where λ_μ are the Gell-Mann matrices, and the symbol T denotes the transpose, since we use the antitriplet 3* with respect to the group $SU(3)''$. Using the matrix notation, we then obtain the weak current in the form

$$J^+=\tilde{q}_\alpha^i M_{i\beta}^{+j\alpha} q_j^\beta, \quad (13)$$

where

$$M^+=\begin{pmatrix} 0 & \cos\theta' & \sin\theta' & & & & & & \\ 0 & 0 & 0 & & 0 & & & 0 & \\ 0 & 0 & 0 & & & & & & \\ \cos\theta'' & 0 & 0 & 0 & \cos\theta' & \sin\theta' & & & \\ 0 & \cos\theta'' & 0 & 0 & 0 & 0 & & 0 & \\ 0 & 0 & \cos\theta'' & 0 & 0 & 0 & & & \\ \sin\theta'' & 0 & 0 & & & & 0 & \cos\theta' & \sin\theta' \\ 0 & \sin\theta'' & 0 & & 0 & & 0 & 0 & 0 \\ 0 & 0 & \sin\theta'' & & & & 0 & 0 & 0 \end{pmatrix};$$

$$q=\begin{pmatrix} p^1 \\ n^1 \\ \lambda^1 \\ p^2 \\ n^2 \\ \lambda^2 \\ p^3 \\ n^3 \\ \lambda^3 \end{pmatrix}. \qquad (13')$$

When M^+ is averaged over the color indices, we obtain the usual Cabibbo matrix:

$$M^+=\frac{1}{3}\begin{pmatrix} 0 & 3\cos\theta' & 3\sin\theta' \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (13'')$$

i.e., $\theta'=\theta_c$, and the angle θ'' remains arbitrary. Such a form of the current (or with small modifications) has also been used most frequently to treat weak interactions in the three-triplet model. In such an approach, we effectively remain in the framework of the group $SU(3)$. Here, as in Cabibbo theory, we cannot determine the neutral current by means of the algebra $SU(2)$ of weak currents[14] in such a way as to suppress transitions with $\Delta S\neq 0$.

In what follows, we shall consider all possible weak transitions[15,16,23] between quarks with $\Delta Q=+1$ (for the current J^+_μ). As can be seen from (7), there are 20 such transitions with corresponding coupling constants. The notation for these constants is chosen as follows. The strangeness-conserving transitions with $\Delta Y=0$ have the constants $a_i, i=1,\ldots,8$; the transitions with $\Delta Y=+1$ $-b_i, i=1,\ldots,8$. There are also transitions with $\Delta Y=-1$ and $\Delta Y=2$, whose constants we shall denote by c_i, $i=1,2$, and $d_i, i=1,2$, respectively. The matrix of transitions then takes the form

$$M^+=\begin{pmatrix} 0 & a_1 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & 0 & a_2 & b_2 & 0 & c_1 & a_7 \\ 0 & a_5 & b_8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & a_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_4 & 0 & 0 & 0 & b_7 & d_1 & 0 & a_3 & b_3 \\ 0 & b_5 & d_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_8 & b_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (14)$$

We have not considered here CP, and we therefore assume that the constants in M^+ are real; then $M^-=(M^+)^T$. We also define the weak charge, which is important for what follows:

$$Q^\pm=\frac{1}{2}\int d^3x\tilde{q}\gamma_0(1-\gamma_5)M^\pm q. \qquad (15)$$

An important part in the determination of the constants in (14) will be played by the condition that the algebra $SU(2)$ is closed for the charges (15); we postulate this in the form

$$[[Q^+,Q^-],Q^\pm]=\pm 2Q^\pm. \qquad (16)$$

From this we obtain the following relations for $M^\pm$:

$$[[M^+,M^-],M^\pm]=\pm 2M^\pm. \qquad (16')$$

And from this we can obtain the matrix of neutral transitions, which is defined as

$$M_0=[M^+,M^-]/2. \qquad (17)$$

Thus, the neutral current is

$$J^0_\mu=(1/2)\tilde{q}\gamma_\mu(1-\gamma_5)[M^+,M^-]q. \qquad (17')$$

It should be emphasized that we are here speaking of the part J^0_μ of the total Salam–Weinberg neutral current:

$$J^N_\mu=J^0_\mu-2\sin^2\theta_W J^{\mathrm{em}}. \qquad (18)$$

2. CLOSURE CONDITION ON THE ALGEBRA OF WEAK CHARGES AND ITS CONSEQUENCES FOR M+

In the modern theory of weak interactions, it is, on the one hand, regarded as necessary that this condition be satisfied; on the other, it is extremely useful for choosing a particular form of the current in different schemes. In principle, both the Cabibbo current in $SU(3)$ and the GIM current[17] in $SU(4)$ can be obtained solely on the basis of the condition that the algebra of the weak charges is closed for the corresponding currents. It should, however, be said that in the case of $SU(3)$ the neutral current determined in accordance with (17) cannot be made diagonal, which was one of the reasons for the introduction of the fourth quark c and the construction of the GIM scheme, in which this shortcoming is absent. In addition, in the case of $SU(4)$ the equations obtained from the weak currents of the $SU(2)$ algebra have an additional solution which differs from the one used in the GIM scheme and in Refs. 18 and 19 to construct a current with right-handed helicity. This current has exactly the same form as the ones proposed in other papers[20,21] for quite different reasons. The features of the various solutions also make it possible in other schemes to use some of these solutions for the construction of GIM-like left-helicity currents and others for right-helicity currents. For example, the principle is used in Ref. 22 to construct weak currents in six-quark schemes. We shall discuss this question in more detail in the section on right-helicity currents, and we give here one of the solutions obtained in Ref. 23 (see also Ref. 24) for the considered case of the three-triplet model.

The conditions (16′) form a system of 20 equations in the case of the matrix (14). The extremely cumbersome notation of these equations and the procedure for solving them can be avoided by means of the following device. We introduce four five-dimensional "vectors," which are composed of the elements of the matrix M^+:

$$\left.\begin{array}{l} \mathbf{X}=\frac{1}{\sqrt{2}}\begin{pmatrix} a_1 \\ a_5 \\ c_2 \\ b_5 \\ a_8 \end{pmatrix}; \quad \mathbf{Y}=\frac{1}{\sqrt{2}}\begin{pmatrix} b_1 \\ b_8 \\ a_6 \\ d_2 \\ b_6 \end{pmatrix}; \\ \mathbf{Z}=\frac{1}{\sqrt{2}}\begin{pmatrix} a_4 \\ a_2 \\ b_2 \\ c_1 \\ a_7 \end{pmatrix}; \quad \mathbf{V}=\frac{1}{\sqrt{2}}\begin{pmatrix} b_4 \\ b_7 \\ d_1 \\ a_3 \\ b_3 \end{pmatrix}. \end{array}\right\} \qquad (19)$$

The relations (16) are then written as follows:

$$\left.\begin{array}{l} 2\mathbf{X}^2\mathbf{X}+2(\mathbf{XY})\mathbf{Y}-(\mathbf{XZ})\mathbf{Z}-(\mathbf{XV})\mathbf{V}=\mathbf{X}; \\ 2\mathbf{Y}^2\mathbf{Y}+2(\mathbf{XY})\mathbf{X}-(\mathbf{YZ})\mathbf{Z}-(\mathbf{YV})\mathbf{V}=\mathbf{Y}; \\ 2\mathbf{Z}^2\mathbf{Z}+2(\mathbf{ZV})\mathbf{V}-(\mathbf{XZ})\mathbf{X}-(\mathbf{YZ})\mathbf{Y}=\mathbf{Z}; \\ 2\mathbf{V}^2\mathbf{V}+2(\mathbf{ZV})\mathbf{Z}-(\mathbf{XV})\mathbf{X}-(\mathbf{YV})\mathbf{Y}=\mathbf{V}, \end{array}\right\} \qquad (20)$$

where (XY), etc., denote the scalar products of the vectors X and Y, etc. To solve the system (20), we multiply each of the equations in turn by the vectors X, Y, Z, and V. The system (20) is then transformed into a system of 14 equations for the following ten invariants:

$$\mathbf{X}^2,\ \mathbf{Y}^2,\ \mathbf{Z}^2,\ \mathbf{V}^2,\ (\mathbf{XY}),\ (\mathbf{XZ}),\ (\mathbf{XV}),\ (\mathbf{YZ}),\ (\mathbf{YV}),\ (\mathbf{ZV}). \tag{21}$$

In Ref. 23 and here we consider only the case when there is a nonvanishing determinant

$$(\mathbf{XZ})(\mathbf{YV})-(\mathbf{XV})(\mathbf{YZ})\neq 0. \tag{22}$$

We then have a unique solution of the system we have obtained:

$$\mathbf{X}^2=\mathbf{Y}^2=\mathbf{Z}^2=\mathbf{V}^2=1; \tag{23}$$

$$(\mathbf{XY})=(\mathbf{ZV})=0; \tag{24}$$

$$(\mathbf{XZ})=(\mathbf{YV})=\pm\sin\alpha;\quad (\mathbf{YZ})=-(\mathbf{XV})=\pm\cos\alpha, \tag{25}$$

where α is an arbitrary parameter.

We substitute the obtained values of the parameters (23)–(25) in the original system (20) and find its general solution in the very simple form

$$\mathbf{Z}=\pm(\sin\alpha\,\mathbf{X}+\cos\alpha\,\mathbf{Y});\quad \mathbf{V}=\pm(\cos\alpha\,\mathbf{X}-\sin\alpha\,\mathbf{Y}). \tag{26}$$

The ten components of the vector X and Y together with α are 11 parameters, on which the general solution of Eqs. (20) depends (or, which is the same thing, they are the conditions of closure of the algebra of weak charges). In addition, three further conditions are imposed on these parameters [see (23) and (24)]:

$$\mathbf{X}^2=\mathbf{Y}^2=1; \tag{23$'$}$$

$$(\mathbf{XY})=0. \tag{24$'$}$$

It is usual and very convenient to express the constants of weak transitions in terms of angular variables (Cabibbo current, GIM scheme, and the various multiquark schemes). In the present case, it is also possible to parametrize the conditions (23′) and (24′) by angular variables. Some forms of parametrization and methods of their construction are considered in Ref. 23; here, we give one of them:

$$\left.\begin{array}{l}\mathbf{X}=\begin{pmatrix}\cos\theta\cos\chi\\ \cos\theta\sin\chi\\ \sin\theta\cos\varphi\cos\psi\\ \sin\theta\cos\varphi\sin\psi\\ \sin\theta\sin\varphi\end{pmatrix};\\ \mathbf{Y}=w_1\mathbf{Y}_1+w_2\mathbf{Y}_2+w_3\mathbf{Y}_3+w_4\mathbf{Y}_4,\end{array}\right\} \tag{27}$$

where

$$\mathbf{Y}_1=\begin{pmatrix}\sin\theta\cos\chi\\ \sin\theta\sin\chi\\ -\cos\theta\cos\varphi\cos\psi\\ -\cos\theta\cos\varphi\sin\psi\\ -\cos\theta\sin\varphi\end{pmatrix};\quad \mathbf{Y}_2=\begin{pmatrix}-\cos\tilde\theta\cos\chi\\ \sin\tilde\theta\cos\chi\\ \sin\tilde\theta\cos\psi\sin\varphi\\ \sin\tilde\theta\sin\psi\sin\varphi\\ -\sin\tilde\theta\cos\varphi\end{pmatrix};$$

$$\mathbf{Y}_3=\begin{pmatrix}-\sin\tilde\theta\sin\chi\\ \sin\tilde\theta\cos\chi\\ -\cos\tilde\theta\cos\psi\sin\varphi\\ -\cos\tilde\theta\sin\psi\sin\varphi\\ \cos\tilde\theta\cos\varphi\end{pmatrix};\quad \mathbf{Y}_4=\begin{pmatrix}0\\ 0\\ \sin\psi\\ -\cos\psi\\ 0\end{pmatrix}.$$

Thus, the expression for the weak current in the three-triplet model that satisfies the condition for the weak charges to be closed and also the condition (22) depends on nine angular variables. The appearance in (27) of the additional parameter [the relations (26) and (23′)–(24′) leave eight parameters free] is due to the method of construction of the parametrization given above.[23]

3. EXAMPLES OF WEAK COLOR CURRENTS

A. The form of the current obtained in (23′) and (24′) satisfies only the weak-current algebra $SU(2)$, eight parameters remaining free. They can be used to satisfy other physical requirements. The most important condition is that the while part of the color current should be identical to the Cabibbo current. To give explicit expression to this condition, we decompose the current defined by the matrix (14) into the singlet part and octet part in the color space:

$$J^+=\tilde q^i_\alpha M^{+j\alpha}_{i\beta}q^\beta_j=(1/3)\,\mathrm{Sp}\,(M^{+j}_i)\,(\tilde q^i_\gamma q^\gamma_j)+M^{+j\alpha}_{i\beta}\,[\tilde q^i_\alpha q^\beta_j-(1/3)\,\delta^\beta_\alpha\,(\tilde q^i_\gamma q^\gamma_j)]. \tag{28}$$

In matrix notation, the singlet part of (28) has the form

$$(J^+)_{\text{singl}}=\frac{1}{3}\tilde q\begin{pmatrix}0 & a_1+a_2+a_3 & b_1+b_2+b_3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}q, \tag{29}$$

from which we obtain the relations

$$(a_1+a_2+a_3)/3=\cos\theta_c;\quad (b_1+b_2+b_3)/3=\sin\theta_c. \tag{30}$$

We obtain one further condition on the coefficients from the white part of the nonleptonic Lagrangian. We shall consider its construction in detail in Sec. 7, and we merely mention here that it contains $\Delta S=2$ transitions. We set the coefficient that arises in the term with such transitions equal to zero:

$$a_8d_2+b_8c_2=0. \tag{31}$$

To satisfy Eqs. (30) and (31), there is no need to use all the eight free parameters. It is also clear that there exist many ways of taking into account (30) and (31).

Among all the possibilities, we have chosen a set which is interesting in that the parameters are approximately equal to the Cabibbo angle:

$$\mathbf{X}=\mathbf{V}=\frac{1}{\sqrt2}\begin{pmatrix}\cos\varphi\\ -\sin\theta\\ -\sin\varphi\sin\theta\\ \cos\theta\\ \sin\varphi\cos\theta\end{pmatrix};\quad \mathbf{Y}=\mathbf{Z}=\frac{1}{\sqrt2}\begin{pmatrix}\sin\varphi\\ \cos\theta\\ \cos\varphi\sin\theta\\ \sin\theta\\ -\cos\varphi\cos\theta\end{pmatrix}; \tag{32}$$

for $\sin\theta_c=0.235$,

$$\sin\theta=0.230;\quad \sin\varphi=0.244. \tag{33}$$

B. We consider one further fairly interesting example. As defined in the matrix (14), the weak interactions of color quarks permit transitions with $\Delta Y=-1$ and $\Delta Y=2$, which have not been observed in the interactions of the known hadrons. If it is assumed that such transitions are also not allowed for color states, then we must set

$$c_i=d_i=0. \tag{34}$$

Then the weak current can be parametrized as follows:

$$\mathbf{X}=\mathbf{V}=\frac{1}{\sqrt2}\begin{pmatrix}\cos\psi\\ \sin\psi\\ 0\\ \cos\varphi\\ \sin\varphi\end{pmatrix};\quad \mathbf{Y}=\mathbf{Z}=\frac{1}{\sqrt2}\begin{pmatrix}\sin\psi\\ \cos\psi\\ \cos\varphi\\ 0\\ \sin\varphi\end{pmatrix}. \tag{35}$$

Equations (23′), (31), and (34) are satisfied by this set automatically, and Eq. (24′) imposes the following condition on the parameters (35):

$$\sin 2\psi = \sin^2 \varphi. \qquad (36)$$

The solution of the three equations (30) and (36) gives a value of the Cabibbo angle close to the experimental value:

$$\sin \theta_c = 0.211. \qquad (37)$$

4. NEUTRAL CURRENT

As we have already noted, the condition of the algebra $SU(2)$ for the weak charges gives an expression for the $V-A$ neutral current J^0 (part of the total neutral Salam–Weinberg current J^N_μ) in the form

$$J^0_\mu = \tilde{q}\gamma_\mu (1-\gamma_5) M^0 q, \quad M^0 = [M^+, M^-]/2. \qquad (38)$$

We shall first find the general form of the commutator (38) for the matrices (14), this being valid for all types of models of the weak interaction in the three-triplet model. We recall that the relations (26), (23′), and (24′) that we have obtained between the coefficients of the current represent only one of the possible models determined by the condition (22). We take the system of notation for the coefficients of the neutral current to be the same as for the charged currents, except that we replace the lower-case letters by upper-case letters: A_i correspond to transitions with $\Delta Y = 0$; B_i, to transitions with $|\Delta Y| = 1$; and D, to transitions with $\Delta Y = 2$;

$$2M^0 = \begin{pmatrix} A_1 & 0 & 0 & 0 & A_{10} & B_7 & 0 & B_8 & A_{11} \\ 0 & A_2 & B_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B_1 & A_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_4 & 0 & 0 & B_4 & 0 & 0 \\ A_{10} & 0 & 0 & 0 & A_5 & B_2 & 0 & B_5 & A_{12} \\ B_7 & 0 & 0 & 0 & B_2 & A_6 & 0 & D & B_6 \\ 0 & 0 & 0 & B_4 & 0 & 0 & A_7 & 0 & 0 \\ B_8 & 0 & 0 & 0 & B_5 & D & 0 & A_8 & B_3 \\ A_{11} & 0 & 0 & 0 & A_{12} & B_6 & 0 & B_3 & A_9 \end{pmatrix}. \qquad (39)$$

In contrast to the case of the charged current (14), there is here no need to distinguish the transitions with $\Delta Y = +1$ and $\Delta Y = -1$, since M^0 is symmetric, and therefore to each transition $\Delta Y = +1$ there corresponds a conjugate transition ($\Delta Y = -1$) with the same coefficient. The same is true of the $|\Delta Y| = 2$ transitions. The coefficients of the matrix M^0 can be expressed in terms of the coefficients of the charged current as follows:

$$\left.\begin{aligned}
&A_1 = a_1^2 + b_1^2 - a_4^2 - b_4^2; \quad A_2 = -(a_1^2 + a_5^2 + a_8^2 + b_5^2 + c_2^2);\\
&A_3 = -(a_6^2 + b_1^2 + b_6^2 + b_8^2 + d_2^2); \quad A_4 = a_2^2 + a_4^2 + a_7^2 + b_2^2 + c_1^2;\\
&A_5 = a_5^2 + b_8^2 - a_2^2 - b_7^2; \quad A_6 = a_6^2 + c_2^2 - b_2^2 - d_1^2;\\
&A_7 = a_3^2 + b_3^2 + b_4^2 + b_7^2 + d_1^2; \quad A_8 = b_5^2 + d_2^2 - a_3^2 - c_1^2;\\
&A_9 = a_8^2 + b_6^2 - a_7^2 - b_3^2; \quad A_{10} = a_1 a_5 + b_1 b_8 - a_2 a_4 - b_4 b_7;\\
&A_{11} = a_1 a_8 + b_1 b_6 - a_4 a_7 - b_3 b_4; \quad A_{12} = a_5 a_8 + b_6 b_8 - a_2 a_7 - b_7 b_3;\\
&B_1 = -(a_1 b_1 + a_5 b_8 + a_6 c_2 + b_5 d_2 + a_8 b_6);\\
&B_2 = a_5 c_2 + a_6 b_8 - a_2 b_2 - b_7 d_1;\\
&B_3 = a_8 b_5 + b_6 d_2 - a_7 c_1 - a_3 b_3;\\
&B_4 = a_2 b_7 + a_4 b_4 + a_3 c_1 + b_2 d_1 + a_7 b_3;\\
&B_5 = a_5 b_5 + b_8 d_2 - a_2 c_1 - a_3 b_7; \quad B_6 = a_6 b_6 + a_8 c_2 - a_7 b_2 - b_3 d_1;\\
&B_7 = a_1 c_2 + a_6 b_1 - a_4 b_2 - b_4 d_1; \quad B_8 = a_1 b_5 + b_1 d_2 - a_4 c_1 - a_3 b_4;\\
&D = a_6 d_2 + b_5 c_2 - a_3 d_1 - b_2 c_1.
\end{aligned}\right\} \qquad (39')$$

As in the case of the charged currents, it is interesting to separate the white part from the obtained neutral current and to consider what requirements experiment imposes on the coefficients of the matrix (39). In the space of color indices, we decompose the neutral current into the singlet and octet parts:

$$J^0 = \tilde{q}^i_\alpha M^{0j\alpha}_{i\beta} q^\beta_j = (1/3)\, \mathrm{Sp}\, M^{0j}_i (\tilde{q}^i_\gamma q^\gamma_j) + M^{0j\alpha}_{i\beta} [\tilde{q}^i_\alpha q^\beta_j - (1/3)\, \delta^\beta_\alpha (\tilde{q}^i_\gamma q^\gamma_j)]. \qquad (40)$$

Explicitly, we obtain the matrix of neutral transitions of the white current:

$$(M^0)_{\text{singl}} = \begin{pmatrix} A_1 + A_4 + A_7 & 0 & 0 \\ 0 & A_2 + A_5 + A_8 & B_1 + B_2 + B_3 \\ 0 & B_1 + B_2 + B_3 & A_3 + A_6 + A_9 \end{pmatrix}. \qquad (41)$$

In the general case, since there are no experimentally observed transitions $n \leftrightarrow \lambda$, we must require that

$$B_1 + B_2 + B_3 = 0. \qquad (42)$$

As we shall see below, the conditions obtained in Ref. 23 [see the relations (26), (23′), and (24′)] automatically ensure fulfillment of (42). Here, this ratio is given for completeness, since by no means all models (and according to our information there is just a unique one, proposed in Ref. 23) have the property that fulfillment of some other conditions [in our case, this is the condition of the weakcurrent algebra $SU(2)$ and the inequality (22)] automatically entails diagonality of the neutral current. In the considered case, not only the white current with the matrix (41) is diagonal but also the total current, which includes the octet representation in the color space, i.e., the entire matrix (39). Indeed, substituting (26) in (39′) and using the conditions (23′) and (24′), we can readily show that the only nonvanishing elements of the matrix (39) are

$$-A_2 = -A_3 = A_4 = A_7 = 2. \qquad (43)$$

Thus, the weak neutral current of the model is completely diagonal and has the form

$$J^0_\mu = -n_1 \gamma_\mu (1-\gamma_5) n_1 - \tilde{\lambda}_1 \gamma_\mu (1-\gamma_5) \lambda_1 + \tilde{p}_2 \gamma_\mu (1-\gamma_5) p_2 + \tilde{p}_3 \gamma_\mu (1-\gamma_5) p_3. \qquad (44)$$

Note also that the quark structure of the current (44) is identical to that of the electromagnetic current. We also write down the white part of the current (which also, naturally, is identical to the electromagnetic part) corresponding to the ordinary $SU(3)$:

$$(J^0_\mu)_{\text{singl}} = (2/3)\, \tilde{p} \gamma_\mu (1-\gamma_5) p - (1/3)\, \tilde{n} \gamma_\mu (1-\gamma_5) n - (1/3)\, \tilde{\lambda} \gamma_\mu (1-\gamma_5) \lambda. \qquad (45)$$

It is interesting to note the following. It is widely known that in the "pure" $SU(3)$ theory the neutral current cannot be determined in the framework of the weak-current algebra $SU(2)$ without its containing the current $(n\tilde{\lambda})$ with constant $\sin\theta_c \cos\theta_c$. In the case under discussion, the current (45) is completely diagonal and corresponds to the usual $SU(3)$ obtained from the symmetry group $SU(3)' \otimes SU(3)''$. This is not the only case when the conclusions of the Cabibbo $SU(3)$ theory do not agree with the results of the $SU(3)$ theory obtained by averaging over the color indices of the group $SU(3)' \otimes SU(3)''$. The latter has a richer content, and in what follows we shall illustrate this in some examples.

5. THE KATUYA–KOIDE MODEL

In Secs. 2–4, we have considered one of the variants of the weak hadronic current in the three-triplet Han–Nambu model. In the literature, one can find numerous variants of the currents in the color scheme (a fairly detailed list of papers on this subject can be found in Ref. 10), but, as a rule, these do not use the rich possibilities of the current (14) from the point of view of the number of parameters. To the best of our knowledge, the papers of the Japanese authors Katuya and Koide are an exception. Comparatively recently, they proposed a fairly general and relatively parameter-rich variant of the weak interaction for color hadrons in Ref. 25. In their earlier papers (see the references in Ref. 25), they considered a very large number of physical problems, using for this the matrix M^+ [see (14)] directly. However, in Ref. 25 they proposed a very elegant form of quark mixing. The mixing parameters serve as the coefficients of the weak current, which describes rather economically the structure of the weak interactions of ordinary and charmed hadrons. We have found it necessary to present this scheme in some detail in the present review for the following reasons. Namely, it is one of the few schemes that combine generality and elegance. It has many properties of the model described in Ref. 23 but differs radically from it; a comparison of these two models reflects fairly well the general situation in the study of the weak interaction of color hadrons, i.e., the many choices of model. The Katuya–Koide model (KK model) applies equally to the scheme $SU(3)'\otimes SU(3)''$ and the scheme $SU(4)'\otimes SU(3)''$, and this is an advantage that the model described in Ref. 23 (ART) does not possess. We shall show below that the weak-interaction scheme presented in Sec. 2 cannot be extended to the group $SU(4)'\otimes SU(3)''$, and its method of construction can be applied only in the case of the groups $SU(n)'\otimes SU(n)''$. In the Appendix, this method is illustrated for the group $SU(4)'\otimes SU(4)''$.[26]

We now turn directly to the KK model, omitting where possible the details, which are given very fully in the original paper. We here describe its main features needed for the following discussion.

One considers an $SU(4)'\otimes SU(3)''$ scheme with 12 quarks whose charges are determined in the matrix (8). All the quarks are distributed over left-handed doublets of the gauge group $SU(2)_L$ as follows:

$$\begin{pmatrix} p_1^0 \\ n_1^{\prime -} \end{pmatrix};\quad \begin{pmatrix} c_1^0 \\ \lambda_1^{\prime -} \end{pmatrix};\quad \begin{pmatrix} p_2^+ \\ n_2^{\prime 0} \end{pmatrix};\quad \begin{pmatrix} c_2^+ \\ \lambda_2^{\prime 0} \end{pmatrix};\quad \begin{pmatrix} p_3^+ \\ n_3^{\prime 0} \end{pmatrix};\quad \begin{pmatrix} c_3^+ \\ \lambda_3^{\prime 0} \end{pmatrix}. \tag{46}$$

The primed fields are combinations of unprimed fields after the manner of the GIM scheme. The transformations are specified as follows:

$$\left.\begin{aligned} &\begin{pmatrix} n_1' \\ \lambda_1' \end{pmatrix} = \begin{pmatrix} \cos\varphi_1 & \sin\varphi_1 \\ -\sin\varphi_1 & \cos\varphi_1 \end{pmatrix}\begin{pmatrix} n_1 \\ \lambda_1 \end{pmatrix};\\ &\begin{pmatrix} n_2' \\ \lambda_2' \\ n_3' \\ \lambda_3' \end{pmatrix} = A_1A_2A_3\begin{pmatrix} n_2 \\ \lambda_2 \\ n_3 \\ \lambda_3 \end{pmatrix} \equiv A\begin{pmatrix} n_2 \\ \lambda_2 \\ n_3 \\ \lambda_3 \end{pmatrix}, \end{aligned}\right\} \tag{47}$$

where

$$\left.\begin{aligned} A_1 &= \begin{pmatrix} \cos\psi_2 & \sin\psi_2 & 0 & 0 \\ -\sin\psi_2 & \cos\psi_2 & 0 & 0 \\ 0 & 0 & \cos\psi_3 & \sin\psi_3 \\ 0 & 0 & -\sin\psi_3 & \cos\psi_3 \end{pmatrix};\\ A_2 &= \begin{pmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & \cos\beta & 0 & \sin\beta \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & -\sin\beta & 0 & \cos\beta \end{pmatrix};\\ A_3 &= \begin{pmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & \cos\varphi_3 & \sin\varphi_3 \\ 0 & 0 & -\sin\varphi_3 & \cos\varphi_3 \end{pmatrix}. \end{aligned}\right\} \tag{48}$$

Then the weak charged current J^+ associated with the W boson has the form (we omit the Lorentz structure of the current)

$$\begin{aligned} J^+ = \tilde{p}_1(\cos\varphi_1 n_1 + \sin\varphi_1\lambda_1) &+ \tilde{c}_2(-\sin\varphi_1 n_1 + \cos\varphi_1\lambda_1)\\ &+ \tilde{p}_2(A_{11}n_2 + A_{12}\lambda_2 + A_{13}n_3 + A_{14}\lambda_3)\\ &+ \tilde{c}_2(A_{21}n_2 + A_{22}\lambda_2 + A_{23}n_3 + A_{24}\lambda_3)\\ &+ \tilde{p}_3(A_{31}n_2 + A_{32}\lambda_2 + A_{33}n_3 + A_{34}\lambda_3)\\ &+ \tilde{c}_3(A_{41}n_2 + A_{42}\lambda_2 + A_{43}n_3 + A_{44}\lambda_3), \end{aligned} \tag{49}$$

where A_{ij} are the elements of the matrix A defined by the relation (48). [For the decomposition of the current (49) into $SU(3)''$ octet and singlet parts, see Ref. 25.] To establish a correspondence between the notation for the constants of the KK weak current and the constants (14), we write out M^+ in this case. We denote the constants of currents containing fields of charmed quarks by primed letters, and the remaining system of notation is the same as in (14). We have

$$M^+ = \begin{pmatrix} 0 & a_1 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1' & b_1' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & a_4' & 0 & a_2 & b_2 & 0 & 0 & c_1 & a_7 & 0 \\ 0 & a_5 & b_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & a_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_5' & 0 & 0 & a_6' & 0 & a_2' & b_2' & 0 & 0 & c_1' & a_7' & 0 \\ b_4 & 0 & 0 & b_4' & 0 & b_7 & d_1 & 0 & 0 & a_3 & b_3 & 0 \\ 0 & b_5 & d_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_8 & b_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_5' & 0 & 0 & b_6' & 0 & b_7' & 0 & 0 & 0 & a_3' & b_3' & 0 \end{pmatrix};\quad q = \begin{pmatrix} p_1 \\ n_1 \\ \lambda_1 \\ c_1 \\ p_2 \\ n_2 \\ \lambda_2 \\ c_2 \\ p_3 \\ n_3 \\ \lambda_3 \\ c_3 \end{pmatrix}. \tag{50}$$

The matrix (50) contains 36 transitions, whereas in the current (49) there are 20. Moreover, the complete current (49) is determined by just seven parameters: $\alpha, \beta, \varphi_1, \varphi_2, \varphi_3, \psi_2, \psi_3$. Note that even in the more restricted case $SU(3)'\otimes SU(3)''$ the ART model is determined by nine parameters after the requirement of the $SU(2)$ algebra for the weak currents has been satisfied. Such a structure of the current (49) is determined by the specific feature of the doublet representations for the quark fields in the gauge group $SU(2)\otimes U(1)$. This variant must be regarded as one of the possible variants; for example, the 12 quarks (46) can be combined in triplets, doublets, and singlets with respect to $SU(2)_L$, and to each choice there will correspond different weak currents. In what follows, we shall require the coefficients of the matrix A explicitly. We write down expressions for them, giving them simultaneously in their correspondence with the elements of the matrix (50):

$$\left.\begin{aligned} &a_1 = \cos\varphi_1;\quad b_1 = \sin\varphi_1;\quad a_1' = -\sin\varphi_1;\quad b_1' = \cos\varphi_1;\\ &a_2 = A_{11} = \cos\psi_2\cos\alpha\cos\varphi_2 - \sin\psi_2\cos\beta\sin\varphi_2;\\ &b_2 = A_{12} = \cos\psi_2\cos\alpha\sin\varphi_2 + \sin\psi_2\cos\beta\cos\varphi_2;\\ &a_3 = A_{33} = \cos\psi_3\cos\alpha\cos\varphi_3 - \sin\psi_3\cos\beta\sin\varphi_3;\\ &b_3 = A_{34} = \cos\psi_3\cos\alpha\sin\varphi_3 + \sin\psi_3\cos\beta\cos\varphi_3;\\ &a_7 = A_{14} = \cos\psi_2\sin\alpha\sin\varphi_3 + \sin\psi_2\sin\beta\cos\varphi_3;\\ &b_7 = A_{31} = -\cos\psi_3\sin\alpha\cos\varphi_2 + \sin\psi_3\sin\beta\sin\varphi_2;\\ &c_1 = A_{13} = \cos\psi_2\sin\alpha\cos\varphi_3 - \sin\psi_2\sin\beta\sin\varphi_3;\\ &d_1 = A_{32} = -\cos\psi_3\sin\alpha\sin\varphi_2 - \sin\psi_3\sin\beta\cos\varphi_2;\\ &a_2' = A_{21} = -\sin\psi_2\cos\alpha\cos\varphi_2 - \cos\psi_2\cos\beta\sin\varphi_2;\\ &b_2' = A_{22} = -\sin\psi_2\cos\alpha\sin\varphi_2 + \cos\psi_2\cos\beta\cos\varphi_2;\\ &a_3' = A_{43} = -\sin\psi_3\cos\alpha\cos\varphi_3 - \cos\psi_3\cos\beta\sin\varphi_3;\\ &b_3' = A_{44} = -\sin\psi_3\cos\alpha\sin\varphi_3 + \cos\psi_3\cos\beta\cos\varphi_3;\\ &a_7' = A_{24} = -\sin\psi_2\sin\alpha\sin\varphi_3 + \cos\psi_2\sin\beta\cos\varphi_3;\\ &b_7' = A_{41} = \sin\psi_3\sin\alpha\cos\varphi_2 + \cos\psi_3\sin\beta\sin\varphi_2;\\ &c_1' = A_{23} = -\sin\psi_2\sin\alpha\cos\varphi_3 - \cos\psi_2\sin\beta\sin\varphi_3;\\ &d_1' = A_{42} = \sin\psi_3\sin\alpha\sin\varphi_2 - \cos\psi_3\sin\beta\cos\varphi_2. \end{aligned}\right\} \tag{51}$$

The physical requirements imposed on the parameters of the current are the same as in the case of the scheme $SU(3)'\otimes SU(3)''$. Averaging over the color indices of the matrix (50), we obtain the current of weak transitions of the $SU(4)$ theory:

$$J^{+}=\frac{1}{3}\tilde{q}\begin{pmatrix}0 & a_1+a_2+a_3 & b_1+b_2+b_3 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & a_1'+a_2'+a_3' & b_1'+b_2'+b_3' & 0\end{pmatrix}q, \tag{52}$$

where

$$q=\begin{pmatrix}p\\ n\\ \lambda\\ c\end{pmatrix}. \tag{52'}$$

In this case, the agreement with the Cabibbo theory is expressed exactly as in (30):

$$\cos\theta_c=(a_1+a_2+a_3)/3;\quad \sin\theta_c=(b_1+b_2+b_3)/3. \tag{53}$$

Substituting in (53) the expressions (51), we readily show (remark of the authors in Ref. 25) that Eqs. (53) can be satisfied only under the conditions

$$\left.\begin{array}{l}\sin\alpha=\sin\beta=0;\\ a_2=a_3=\cos\varphi_1=\cos(\varphi_2+\psi_2)=\cos(\varphi_3+\psi_3)=\cos\theta_c,\\ b_2=b_3=\sin\varphi_1=\sin(\varphi_2+\psi_2)=\sin(\varphi_3+\psi_3)=\sin\theta_c.\end{array}\right\} \tag{54}$$

But in this case there is absolutely no octet part of the color current, i.e., nondiagonal 4×4 blocks of the matrix (50), which can be readily seen by direct substitution. To avoid contradiction with the Cabibbo theory and to preserve the $SU(3)''$ octet part of the current (49), we can proceed as follows. We replace Eqs. (30) by

$$(a_1+a_2+a_3)=3\varkappa\cos\theta_c;\quad (b_1+b_2+b_3)=3\varkappa\sin\theta_c, \tag{55}$$

where $\varkappa<1$. It is also necessary to redefine the weak hadronic constant G, replacing it by $\varkappa G$ and regarding it as an experimentally determined quantity. However, it is well known that purely leptonic processes, for example, μ decay, take place with the same weak coupling constant G, and the total weak current is the sum of the hadronic and leptonic currents with a common factor G (universality of the weak interaction). Thus, having redefined the weak hadronic constant, we must also redefine the leptonic part of the current in exactly the same way. It is also necessary to treat the weak-current algebra $SU(2)$ for the leptons (the hadronic part is chosen in such a way that it already satisfies this condition). All the listed requirements can be reconciled (see Refs. 23, 25, 27, and 28) by a mixing of ordinary leptons with heavy leptons, charged or neutral, these being placed in weak $SU(2)$ doublets; for example,

$$\begin{pmatrix}\nu_e\\ e^-\cos\varphi+E^-\sin\varphi\end{pmatrix};\quad \begin{pmatrix}\nu_\mu\\ \mu^-\cos\varphi+M^-\sin\varphi\end{pmatrix}, \tag{56a}$$

or

$$\begin{pmatrix}\nu_e\cos\varphi+N_e\sin\varphi\\ e^-\end{pmatrix};\quad \begin{pmatrix}\nu_\mu\cos\varphi+N_\mu\sin\varphi\\ \mu^-\end{pmatrix}, \tag{56b}$$

where $\cos\varphi=\varkappa$. (For more details about the consequences of the introduction of the factor $\varkappa$, see the papers quoted above and Ref. 29.)

We must make one further remark concerning the terminology in the KK model. Katuya and Koide themselves call it a GIM-like model. If this were so, one would expect the matrix (52) to correspond to the GIM scheme. Then for the remaining elements of this matrix we would have the relations

$$(a_1'+a_2'+a_3')/3=-\sin\theta_c;\quad (b_1'+b_2'+b_3')/3=\cos\theta_c. \tag{57}$$

Equations (57) are satisfied for the same values of the parameters as in (54) (with replacement of the unprimed a_i and b_i by the primed a_i' and b_i') with corresponding consequences, i.e., vanishing of the octet part of the current. In practical calculations, we shall not replace (57) by other equations in the way that (53) was replaced by Eqs. (55), and we shall not fix the relations (57) by the value of the Cabibbo angle. We shall also not include them in the system of equations for determination of the parameters of the model.

We write down further the electromagnetic and neutral currents of the KK model and their white parts. In the gauge group $SU(2)\otimes U(1)$, the electromagnetic current associated with the photon is

$$J_\mu^{\text{em}}=-\tilde{n}_1\gamma_\mu n_1-\tilde{\lambda}_1\gamma_\mu\lambda_1+\tilde{p}_2\gamma_\mu p_2+\tilde{p}_3\gamma_\mu p_3+\tilde{c}_2\gamma_\mu c_2+\tilde{c}_3\gamma_\mu c_3. \tag{58}$$

Its white part is identical to the current of the $SU(4)$ scheme:

$$(J_\mu^{\text{em}})_{\text{singl}}=2(\tilde{p}\gamma_\mu p)/3-(\tilde{n}\gamma_\mu n)/3-(\tilde{\lambda}\gamma_\mu\lambda)/3+2(\tilde{c}\gamma_\mu c)/3. \tag{58'}$$

In the general case, the neutral current is obtained by the commutation $2M^0=[M^+,M^-]$, but in the case of the KK model one can proceed in the following simpler manner. The matrix A which mixes the fields n_i and λ_i is orthogonal, and the Salam–Weinberg Lagrangian corresponding to the neutral transitions contains these fields only in the combination $\sum_{i=1}^{4}\tilde{q}_i'q_i'$. Then

$$\sum_{l,j,i=1}^{4}\tilde{q}_l A_{li}^{\mathrm{T}}A_{ij}q_j=\sum_{l,j=1}^{4}\tilde{q}_l(A^{\mathrm{T}}A)_{lj}q_j=\sum_{i=1}^{4}\tilde{q}_iq_i. \tag{59}$$

Restoring the color index of the quarks, we therefore obtain a completely diagonal neutral current

$$J_\mu^0=\frac{1}{2}\sum_{\alpha=1}^{3}[(\tilde{p}_\alpha\gamma_\mu(1-\gamma_5)p^\alpha)-(\tilde{n}_\alpha\gamma_\mu(1-\gamma_5)n^\alpha) -(\tilde{\lambda}_\alpha\gamma_\mu(1-\gamma_5)\lambda^\alpha)+(\tilde{c}_\alpha\gamma_\mu(1-\gamma_5)c^\alpha)]. \tag{60}$$

Its colorless part is identical to the current of the usual four-quark model:

$$(J_\mu^0)_{\text{singl}}=[(\tilde{p}\gamma_\mu(1-\gamma_5)p)-(\tilde{n}\gamma_\mu(1-\gamma_5)n) -(\tilde{\lambda}\gamma_\mu(1-\gamma_5)\lambda)+(\tilde{c}\gamma_\mu(1-\gamma_5)c)]/2, \tag{60'}$$

but it differs from the corresponding current of the ART model for the $SU(3)$ part [see (45)], which in it coincides with the electromagnetic part. This difference can be used in the construction of color models with right-helicity currents. We shall discuss this question later.

6. COMPARISON OF THE TWO MODELS OF THE WEAK INTERACTION

The two models of the weak interaction in the color scheme considered in the previous sections demonstrate well the richness of the color scheme with regard to the choice of the definite form of the weak currents. In both models, one can readily satisfy the physical requirements imposed by the experiments. In the first place, this concerns the condition that the color-singlet part of the current be identical to the Cabibbo weak current. However, already in this respect there are differences between the models. As we have seen (see Sec. 3), this requirement can be readily satisfied in the ART model without recourse to a change in the considered conditions. In the KK case it is in principle impossible to satisfy these conditions unless the color-octet part of the current disappears completely [see (54) and (55)]. In the case of the ART model, exactly the same necessity for changing the equations arises only when we consider the additional conditions on the coefficients that arise from the requirement that the selection rule $\Delta T=\frac{1}{2}$ be satisfied in nonleptonic decays (see below). In both cases, the introduction of the factor $\varkappa$ in (30) and (54) forces us to introduce the same factor in the leptonic current in order to preserve universality, and this, in its turn, leads to the need to introduce a heavy (charged or neutral) lepton into the scheme. However, some consequences of the introduction of the factor $\varkappa$ are such that too small values of it are undesirable. For example, in the usual gauge scheme the mass of the W boson is related to the factor by a direct proportionality:

$$M_W=(\pi\alpha/\sqrt{2}G_F)^{1/2}\varkappa/\sin\theta_W, \tag{61}$$

and for very small $\varkappa$ we obtain a very small value of M_W, which justifies our assertion above. We make one further remark in connection with the expressions for the neutral current in both models. Earlier [see (59)] we wrote down in general form the matrix of the neutral current in the $SU(4)'\otimes SU(3)''$ scheme not only for the purpose of exposition but also because in this case one can readily compare the different models of the currents. In particular, in a comparison of the models under discussion here we can note the following. If we attempt to go over from the KK model to the ART model by simply ignoring the coefficients which arise when $SU(3)'\otimes SU(3)''$ is generalized to $SU(4)'\otimes SU(3)''$, i.e., a'_i, b'_i, c'_i, d'_i, but for the unprimed a_i, b_i, c_i, d_i take their expressions in the KK model [see (51)], we cannot use them to render the current (39) diagonal. On the other hand, the current (39) is completely diagonal for the values of the coefficients given by the relations (26), (23′), and (24′). This indicates a deep difference between both the models and the methods of their construction. An important part is here played by the determinant (22), whose analog in the KK model vanishes. For more details on this, see Appendix 2.

7. THE ΔT = ½ RULE IN COLOR MODELS

One of the problems that arose in the discovery of strange particles concerned the selection rules for the isotopic spin in their decays into nonstrange particles without a leptonic accompaniment. It was found, for example, that the decay $K_s^0\to\pi^+\pi^-$ occurs approximately 500 times more often than the decay $K^+\to\pi^+\pi^0$. In the first of the reactions, the isotopic spin changes by $\frac{1}{2}$, and in the second by 3/2. The same phenomenon was observed in all measured nonleptonic decays of strange particles for both mesons and baryons, i.e., the amplitudes of transitions that change the isospin by 3/2 are suppressed by 20–25 times compared with the amplitudes in which the change is $\frac{1}{2}$. This phenomenon became known as the $\Delta T=\frac{1}{2}$ rule and later, with the discovery of $SU(3)$ symmetry, was given the equivalent name of "octet dominance." This latter name is explained as follows. The weak current j_μ transforms in $SU(3)$ as an octet. The Lagrangian of the nonleptonic decay is constructed as a product of currents, $L\sim j^+_\mu j_\mu$. If j_μ is the Cabibbo current, the Lagrangian L contains only the representations 8 and 27. In turn, from the isotopic point of view the representation 8 does not contain the spin $\frac{3}{2}$, which appears only in the representation 27. This was the reason for the name.

Various suggestions were made for explaining the $\Delta T=\frac{1}{2}$ rule in terms of the structure of the weak hadronic currents and on the basis of dynamical enhancement of the octet part by the strong interactions. In particular, in the framework of quantum chromodynamics it has been noted that there is indeed an enhancement of the octet part compared with the representation 27 by 4–5 times.[35,36] In the subsequent papers of Ref. 37 it was shown that allowance for the exchange of gluons increases this difference to almost the experimentally observed value under certain additional assumptions. These arguments cannot be transferred to the model with integral quark charges. We shall show here how the problem of the $\Delta T=\frac{1}{2}$ rule can be solved in the framework of the model under discussion.

Immediately after the introduction of the color degree of freedom it was noted[30,32] that at least for baryon decays the $\Delta T=\frac{1}{2}$ rule is satisfied automatically. This can be shown as follows. For ease of demonstration, we consider first the variant proposed in Refs. 33 and 34, in which it is postulated that the quark fields satisfy Bose statistics. The Lagrangian of the nonleptonic decays of the strange particles is constructed as a product of currents. In Cabibbo theory,

$$L=(G/\sqrt{2})\,[(j_\mu^{\Delta S=0})^+ j_\mu^{\Delta S=1}+\text{h.c.}], \tag{62}$$

where

$$\begin{aligned} j_\mu^{\Delta S=0}&=(\tilde{n}\gamma_\mu(1-\gamma_5)\,p)\equiv(\tilde{q}^2\gamma_\mu(1-\gamma_5)\,q_1);\\ j_\mu^{\Delta S=1}&=(\tilde{p}\gamma_\mu(1-\gamma_5)\,\lambda)\equiv(\tilde{q}^1\gamma_\mu(1-\gamma_5)\,q_3);\end{aligned} \tag{62′}$$

here, 1, 2, 3 are $SU(3)$ indices. When q_i are Bose fields, the Lagrangian (62) can be replaced by the following equivalent Lagrangian by means of a Fierz transformation:

$$\begin{aligned} L=\frac{G}{2\sqrt{2}}\,&[(\tilde{q}^1\gamma_\mu(1-\gamma_5)\,q_2)\,(\tilde{q}^1\gamma_\mu(1-\gamma_5)\,q_3)\\ &-(\tilde{q}^1\gamma_\mu(1-\gamma_5)\,q_3)\,(\tilde{q}^1\gamma_\mu(1-\gamma_5)\,q_2)].\end{aligned} \tag{63}$$

Note that in the case of Fermi fields q_i we would obtain in this expression +, i.e., a symmetric combination. In our case, the combination (63) is antisymmetric with respect to the two $SU(3)$ indices, i.e., 2 and 3. At the same time, the representation 27 is completely symmetric with respect to the $SU(3)$ indices; this means that it is not present in the expression (63) and, hence, the Lagrangian does not contain an amplitude of transitions with $\Delta T=3/2$.

A similar procedure can be carried through with color indices as well.[30-32] We rewrite the Langrangian (62), ascribing color indices to the fields q_i. Assuming that the quarks are fermions, we obtain

$$L=(G/2\sqrt{2})\,[(\widetilde{q}^{1}_{\alpha}\gamma_\mu(1-\gamma_5)\,q^{\beta}_{2})\,(\widetilde{q}^{1}_{\gamma}\gamma_\mu(1-\gamma_5)\,q^{\delta}_{3}) + (\widetilde{q}^{1}_{\alpha}\gamma_\mu(1-\gamma_5)\,q^{\delta}_{3})\,(\widetilde{q}^{1}_{\gamma}\gamma_\mu(1-\gamma_5)\,q^{\beta}_{2})]. \tag{64}$$

The Lagrangian (64) is now symmetric with respect to the $SU(3)$ indices 2 and 3 and therefore contains the representation 27. However, we recall that colorless baryons contain quarks in completely antisymmetric states with respect to the color indices: $B_{[\alpha\beta\gamma]} \sim \varepsilon_{\alpha\beta\gamma}q^{\alpha}q^{\beta}q^{\gamma}$. Therefore, denoting the Lagrangian (64) by $L_{\{\beta\gamma\}}$ (retaining in it only the two indices that are important in the present context), we obtain for the matrix elements of the baryon decays

$$\langle X\mid L^{\{\beta\gamma\}}\mid B^{[\alpha\beta\gamma]}\rangle=0 \tag{65}$$

as a result of contraction of the fields in the Lagrangian and any bra. The effect is found to be exactly the same as in the case of Bose quarks, namely, the symmetric part of the Lagrangian of the nonleptonic decays does not contribute to the matrix elements of the transitions between the baryons if the latter are composed of colored quarks. We see that the introduction of the color degree of freedom by itself can help toward understanding of the $\Delta T=\frac{1}{2}$ rule. The arguments we have just given do not apply in the case of meson decays.

In Refs. 23 and 25 use was made of one further possibility of a color degree of freedom for describing the $\Delta T=\frac{1}{2}$ rule. As we have already said, if one constructs a color model of the weak interaction and then goes over to the colorless theory, the results of such a procedure do not agree with Cabibbo theory. This applies in the first place to the structure of the neutral current, which is completely diagonal. A similar argument applies to the Lagrangian of the nonleptonic decays, namely, the white part of the Lagrangian constructed from color currents differs strikingly from the Lagrangian constructed from the Cabibbo current. In the latter case, one must have recourse to additional measures and principles in order to suppress the representation 27, but in the case of the color scheme this suppression can be achieved without going beyond the framework of the scheme itself. The large number of parameters of the current (14) give us such a possibility. After separation of the white part from the color Lagrangian, which, as in the case of Cabibbo theory, is postulated in the form of the current product $L \sim j^{+}_{\mu}j_{\mu}$, one can proceed in two basically equivalent ways. One can decompose the obtained white Lagrangian into the representations 8 and 27 in accordance with the ordinary $SU(3)$, and set equal to zero the coefficients of the representation 27 expressed in terms of the parameters of the current (14) (Ref. 16). In this case, one obtains a fairly cumbersome system of nonlinear algebraic equations. As is shown in Appendix 1, this system does not have solutions even in the case of the ART model richest in parameters. Simpler relations are obtained[38] if the resulting Lagrangian is decomposed into the isotopic amplitudes with $T=\frac{1}{2}$ and $T=\frac{3}{2}$ and the coefficients of these amplitudes are compared. In Refs. 39 and 40 in the scheme $SU(4)'\otimes SU(3)''$, the white Lagrangian is decomposed into the representations 20 and 84 in accordance with the flavor group $SU(4)$.

Following Ref. 38, we obtain a Lagrangian of the nonleptonic decays that takes into account explicitly the $\Delta T=\frac{1}{2}$ rule. Just as we separated the singlet part of the current in the relation (28), we separate it from the current product. Contracting the color (Greek) indices, we obtain from the product of currents (13) the following expression for L:

$$\begin{aligned} L_{\text{singl}} &= [(\widetilde{q}^{i}_{\alpha}M^{+j\alpha}_{i\beta}q^{\beta}_{j})\,(\widetilde{q}^{k}_{\gamma}M^{-l\gamma}_{k\delta}q^{\delta}_{l})]_{\text{singl}} \\ &= (1/8)\,[(\delta^{\beta}_{\gamma}\delta^{\delta}_{\alpha}-\delta^{\beta}_{\alpha}\delta^{\delta}_{\gamma}/3)\,\mathrm{Sp}\,(M^{+j}_{i}M^{-l}_{k}) \\ &+ (\delta^{\beta}_{\alpha}\delta^{\delta}_{\gamma}-\delta^{\beta}_{\gamma}\delta^{\delta}_{\alpha}/3)\,\mathrm{Sp}\,M^{+j}_{i}\,\mathrm{Sp}\,M^{-l}_{k}]\,(\widetilde{q}^{i}_{\alpha}q^{\beta}_{j})\,(\widetilde{q}^{k}_{\gamma}q^{\delta}_{l}), \end{aligned} \tag{66}$$

where a summation is understood over all repeated indices and the traces are taken over the color indices. In the expression (66), to simplify the notation, we have ignored the Lorentz structure. In this connection, we must make the following remark. In the considered case, we have postulated a V–A structure of the currents, so that, for example, the product

$$\delta^{\beta}_{\gamma}\delta^{\delta}_{\alpha}\,(\widetilde{q}^{i}_{\beta}q^{\alpha}_{j})\,(\widetilde{q}^{k}_{\delta}q^{\gamma}_{l}) \tag{67}$$

can be written by means of a Fierz transformation in the form $(q^{i}_{\alpha}q^{\alpha}_{l})(q^{k}_{\delta}q^{\delta}_{j})$, since the structure $\gamma_\mu(1-\gamma_5)$ is not changed in this case. In the other variants of the weak interaction, it is necessary to consider rigorously terms of the type (67). The white part of the nonleptonic Lagrangian has a very cumbersome form. However, we shall write it out in full, since in what follows we shall need not only the strangeness-changing part but also the $\Delta S=0$ part; this is so because we shall be considering problems related to parity-violating nuclear transitions. To trace the origin of each term of the Lagrangian, we decompose it into three parts:

$$L=L_1+L_2+L_3, \tag{68}$$

where L_1 corresponds to the first term in the expansion (66), L_2 to the second, and L_3 to the sum of the third and fourth terms in (66). After simple but lengthy transformations, we obtain

$$\begin{aligned} 8L_1 &= (a_1^2+a_2^2+a_3^2+b_7^2+c_1^2)\,(\widetilde{p}p)\,(\widetilde{n}n) \\ &+ (b_1^2+b_2^2+b_3^2+a_7^2+d_1^2)\,(\widetilde{p}p)\,(\widetilde{\lambda}\lambda) \\ &+ (a_1b_1+a_2b_2+a_3b_3+a_7c_1+b_7d_1)\,(\widetilde{p}p)\,(\widetilde{n}\lambda) \\ &+ (a_4^2+b_4^2)\,(\widetilde{p}p)\,(\widetilde{p}p)+(a_5^2+b_5^2)\,(\widetilde{n}n)\,(\widetilde{n}n) \\ &+ (b_8^2+d_2^2)\,(\widetilde{n}n)\,(\widetilde{\lambda}\lambda)+(a_8^2+c_2^2)\,(\widetilde{n}n)\,(\widetilde{\lambda}\lambda) \\ &+ (a_6^2+b_6^2)\,(\widetilde{\lambda}\lambda)\,(\widetilde{\lambda}\lambda)+(a_4a_5+b_4b_5)\,(\widetilde{p}n)\,(\widetilde{n}p) \\ &+ (a_4c_2+a_8b_4)\,(\widetilde{p}\lambda)\,(\widetilde{n}p)+(a_4b_8+b_4d_2)\,(\widetilde{p}n)\,(\widetilde{\lambda}p) \\ &+ (a_4a_6+b_4b_6)\,(\widetilde{p}\lambda)\,(\widetilde{\lambda}p)+(a_5c_2+a_8b_5)\,(\widetilde{n}\lambda)\,(\widetilde{n}n) \\ &+ (a_5b_8+b_5d_2)\,(\widetilde{n}n)\,(\widetilde{\lambda}n)+(a_5a_6+b_5b_6)\,(\widetilde{n}\lambda)\,(\widetilde{\lambda}n) \\ &+ (a_8d_2+b_8c_2)\,(\widetilde{n}\lambda)\,(\widetilde{n}\lambda)+(a_6b_8+b_6d_2)\,(\widetilde{n}\lambda)\,(\widetilde{\lambda}\lambda) \\ &+ (a_6c_2+a_8b_6)\,(\widetilde{\lambda}\lambda)\,(\widetilde{n}\lambda)+\text{h.c.} \end{aligned} \tag{68a}$$

It is readily seen that L_2 can be obtained from L_1 by a Fierz transformation; precisely,

$$L_2=-L_1^{\text{F.T.}}/3, \tag{68b}$$

and therefore we shall not write out the second term, though we shall always take it into account in the calculations. We write out the third, fairly short term fully, although in it too one part can be obtained from the other by a Fierz transformation:

$$\begin{aligned} 8L_3 &= (a_1+a_2+a_3)^2\,(\widetilde{p}n)\,(\widetilde{n}p)+(b_1+b_2+b_3)^2\,(\widetilde{p}\lambda)\,(\widetilde{\lambda}p) \\ &+ (a_1+a_2+a_3)\,(b_1+b_2+b_3)\,[(\widetilde{p}n)\,(\widetilde{\lambda}p)+(\widetilde{p}\lambda)\,(\widetilde{n}p)] \\ &- (1/3)\,\{(a_1+a_2+a_3)^2\,(\widetilde{p}p)\,(\widetilde{n}n)+(b_1+b_2+b_3)^2\,(\widetilde{p}p)\,(\widetilde{\lambda}\lambda) \\ &+ (a_1+a_2+a_3)\,(b_1+b_2+b_3)\,[(\widetilde{p}p)\,(\widetilde{\lambda}n)+(\widetilde{p}p)\,(\widetilde{n}\lambda)]\}. \end{aligned} \tag{68c}$$

We mention once more that the obtained Lagrangian of the nonleptonic interaction differs significantly from the Cabibbo Lagrangian in the richness of its structure. We see that it contains the neutral current $(\tilde{n}\lambda)$, which in the case of the Cabibbo current must be introduced artificially in order to construct amplitudes that transform as the isospin $T=\frac{1}{2}$. The second remark refers to the term with $\Delta S=2$:

$$8L^{\Delta S=2}=(a_8 d_2+b_8 c_2)(\tilde{n}\lambda)(\tilde{n}\lambda)+\text{h.c.} \tag{69}$$

Since such processes are strongly suppressed, we set the coefficient of this current equal to zero, obtaining an additional relation for the coefficients of the matrix M^+. This requirement was also used in Sec. 3 in the construction of the approximately weak currents of the ART model.

From the total Lagrangian (68), we now write down the part corresponding to $\Delta S=1$ (taking into account L_2!):

$$8L^{\Delta S=1}=A(\tilde{n}p)(\tilde{p}\lambda)+B(\tilde{p}p)(\tilde{n}\lambda)+C(\tilde{n}n)(\tilde{n}\lambda)+D(\tilde{\lambda}\lambda)(\tilde{n}\lambda), \tag{70}$$

where

$$A=a_4(b_8+c_2)+b_4(a_8+d_2)+(a_1+a_2+a_3)(b_1+b_2+b_3) - (a_1b_1+a_2b_2+a_3b_3+a_7c_1+b_7d_1)/3; \tag{70a}$$

$$B=a_1b_1+a_2b_2+a_3b_3+a_7c_1+b_7d_1 - [a_4(b_8+c_2)+b_4(a_8+d_2)+(a_1+a_2+a_3)(b_1+b_2+b_3)]/3; \tag{70b}$$

$$C=2[a_5(b_8+c_2)+b_5(a_8+d_2)]/3; \tag{70c}$$

$$D=2[a_6(b_8+c_2)+b_6(a_8+d_2)]/3. \tag{70d}$$

Further, we split the Lagrangian into pure isotopic amplitudes with $T=1/2$ and $3/2$, for which we first transform it identically to the form

$$\begin{aligned}8L^{\Delta S=1}=\frac{1}{2}\Big\{&[A-(B-C)]\Big[(\tilde{n}p)(\tilde{p}\lambda)-\frac{1}{\sqrt{2}}\frac{(\tilde{p}p)-(\tilde{n}n)}{\sqrt{2}}(\tilde{n}\lambda)\Big]\\&+[A+(B-C)]\Big[(\tilde{n}p)(\tilde{p}\lambda)+\frac{1}{\sqrt{2}}\frac{(\tilde{p}p)-(\tilde{n}n)}{\sqrt{2}}(\tilde{n}\lambda)\Big]\Big\}\\&+\frac{1}{3}\Big\{\Big[\frac{B+C}{2}-D\Big][(\tilde{p}p)+(\tilde{n}n)-2(\tilde{\lambda}\lambda)](\tilde{n}\lambda)\\&+(B+C+D)[(\tilde{p}p)+(\tilde{n}n)+(\tilde{\lambda}\lambda)](\tilde{n}\lambda)\Big\}.\end{aligned} \tag{71}$$

Here, the first term is the pure isospin state with $T=1/2$, the second is a mixture of states with spin $T=1/2$ and $3/2$, and the last two have spin $1/2$. So as not to overburden the paper with calculations, we do not include the last two terms in the system of equations determining the parameters of the current.

Decomposing the second term in a Clebsch–Gordan series and summing all the coefficients of the amplitudes with $\Delta T=1/2$ and $\Delta T=3/2$ with allowance for (70a) and (70b), we obtain the following decomposition of the strangeness-changing nonleptonic Lagrangian with respect to its isotopic structure:

$$L^{\Delta S=1}=\frac{1}{24}\Big[\Big(\frac{1}{\sqrt{6}}A_2+\sqrt{6}A_2\Big)\Big|\frac{1}{2},\ \frac{1}{2}\Big\rangle+\frac{2}{\sqrt{3}}A_2\Big|\frac{3}{2},\ \frac{1}{2}\Big\rangle\Big], \tag{72}$$

where

$$A_1=(a_1+a_2+a_3)(b_1+b_2+b_3) - (a_1b_1+a_2b_2+a_3b_3+a_7c_1+b_7d_1) + (a_4+a_5/2)(b_8+c_2)+(b_4+b_5/2)(a_8+d_2); \tag{72a}$$

$$A_2=(a_1+a_2+a_3)(b_1+b_2+b_3)+a_1b_1+a_2b_2+a_3b_3+a_7c_1+b_7d_1 + (a_4-a_5)(b_8+c_2)+(b_4-b_5)(a_8+d_2). \tag{72b}$$

In Sec. 3, we regarded the relations (23′), (24′), (26), and (31) as a system of equations for finding the coefficients of the current. To these equations, we now add the equation that follows from the requirement that the ratio of the coefficient of the $\Delta T=1/2$ amplitude to the coefficient of the $\Delta T=3/2$ amplitude be equal to the experimentally measured ratio ~25. This gives

$$A_1/A_2=11.6, \tag{73}$$

where A_1 and A_2 are determined by Eqs. (72a) and (72b). The requirements (23′), (24′), (26), (31), and (71) constitute the system for finding the values of the parameters of the current.

Because of the complexity of this system, its solution was found on a computer. It was found that Eqs. (9) cannot be satisfied. However, the system has a solution if

$$(a_1+a_2+a_3)=3\varkappa\cos\theta_c; \quad (b_1+b_2+b_3)=3\varkappa\sin\theta_c. \tag{74}$$

The presence of the common factor in the hadronic current destroys the universality of the leptonic current. Therefore, to avoid contradiction with the experimental data on semileptonic interactions, it is necessary to carry out a mixing of the ordinary leptons and heavy leptons (charged or neutral) with angle $\varphi=\cos^{-1}\varkappa$. One can say that the $\Delta T=1/2$ rule is due to the existence of heavy leptons in the framework of the three-triplet model. None of these requirements are satisfied for definite values of the parameters. From the possible sets, we have chosen one for which A_1 (the main coefficient of $L\Delta T=1/2$) has the largest value. Thus, with a high degree of accuracy all the physical requirements are satisfied for the parameter values

$$\left.\begin{array}{ll} a_1=0.575; & b_1=-0.290;\\ a_5=0.795; & b_8=0.501;\\ c_2=0.715; & a_6=-0.999;\\ b_5=-0.577; & d_2=-0.815;\\ a_8=0.439; & b_6=0.0285;\\ \varkappa=0.740; & \alpha=\theta_c+\pi/2=1.802 \end{array}\right\} \tag{75}$$

(α is given in radians). At the same time,

$$A_1=2.77; \quad A_2=0.238. \tag{76}$$

The contributions of the other two $\Delta T=1/2$ amplitudes (not included in the system of equations) are also not small compared with the contributions of the $\Delta T=3/2$ amplitudes. With the same normalization of the Lagrangian as in (72), the coefficients of the last two amplitudes in the relation (71) are, respectively, equal to 0.724 and –1 for the parameter values (75). Thus, we have succeeded in constructing a Lagrangian of the weak nonleptonic interaction which contains explicitly the necessary relationship between the magnitudes of the $\Delta T=1/2$ and $\Delta T=3/2$ transitions. In addition, we have thereby fixed the values of the coefficients of the hadronic current, which is now completely determined in the three-triplet model with integral quark charges.

8. DISCUSSION OF THE LAGRANGIAN WITH ΔS = 0

In recent years, there has been much discussion about P-odd effects associated with weak interactions in nuclear transitions. In principle, in such experiments one can determine the Lorentz structure of the neutral current and, in particular, verify the Salam–Weinberg model. However, the need to take into account the strong interactions carefully must raise doubts about the possibility of choosing between different forms of Hamiltonians for the weak interactions,[39] although work in this direction is carried on intensively.[40] However, even before the attempts to determine the structure of the weak neutral current in such effects, the first experiment[41] presented its own difficulty—the theoretical estimates of the degree of circular polarization of the photons emitted in these reactions are unquestionably lower than the experimental observations (see, for example, Ref. 42 and the papers quoted above). As yet, it is difficult to determine which of the interactions in these calculations is "underestimated"—the weak or the strong. However, with regard to the contribution of the weak interaction it is necessary to say the following. In the case of nonleptonic decays of hadrons the difficulty in their theoretical interpretation resides not only in the pronounced disparity of magnitude between the amplitudes with different isotopic structures but also in the phenomenon of the so-called octet enhancement, namely, the decays take place as if in the product $L \sim \sin\theta_c \cos\theta_c\, j_\mu^{(\Delta S=0)} j_\mu^{(\Delta S=1)}$ the factor $\sin\theta_c \cos\theta_c$ "does not work." It is very probable that a phenomenon of this kind occurs also in the $\Delta S=0$ interactions, the terms with coefficients of the type $\sin\theta_c$ suppressing the contributions of the corresponding currents to the theoretical estimates. In the first place, this affects the amplitudes of the $\Delta T=1$ transitions, which in the weak-interaction Lagrangian arise only from the product of strangeness-changing currents with $T=1/2$. The amplitudes of $\Delta T=2$ transitions also cannot be enhanced, since they do not occur in the octet part of the Lagrangian.

We write down the strangeness-conserving nonleptonic Lagrangian in our case, using the expressions (68):

$$L^{\Delta S=0} = 5.0\pi^-\pi^+ + 1.08\pi^+\pi^- + 4.93\pi^0\pi^0 + 1.32\eta\eta + 1.48\pi^0\eta + 0.82\eta\pi^0. \qquad (77)$$

This Lagrangian is written down by means of the symbols of the particles, for example, $\pi^-\pi^+$ means $V_\mu^{\pi^-}A_\mu^{\pi^+} + A_\mu^{\pi^-}V_\mu^{\pi^-}$, where $\pi^- \equiv (\tilde{p}n)$, $\pi^+ \equiv (\tilde{n}p)$, etc. The values of the coefficients in (77) are obtained from the parameters (75). Since the expression (77) is given only to demonstrate the Lagrangian obtained in the color scheme, no allowance is made for the contribution of λ quarks, and also in the case of the calculation of the fraction of the neutral current in (77) the electromagnetic part of this current is not taken into account.

All the terms of the Lagrangian are roughly of the same importance, and, as in the $L^{\Delta S=1}$ case, there is no Cabibbo suppression of the individual contributions. This circumstance could play a certain part in increasing the theoretical estimates of the effects of the weak interactions in nuclear transitions.

9. SOME PROPERTIES OF NONLEPTONIC INTERACTIONS OF CHARMED PARTICLES

The Lagrangian of the nonleptonic interactions including the c quark is constructed in the same way as the Lagrangian (68) with replacement of the matrix M^+ of the $SU(3)'\otimes SU(3)''$ scheme by the matrix M^+ of the $SU(4)'\otimes SU(3)''$ scheme from the expression (50). As in the case of the white $SU(3)$ group, the parameters of the obtained Lagrangian must be determined from a number of well-established physical requirements. As can be seen from the foregoing, the strongest restriction on the parameters is imposed by the $\Delta T=1/2$ rule: whereas without it an acceptable weak current of the ART model can be constructed on the basis of only two parameters (see Sec. 3), we have seen in Sec. 7 that even a large number of parameters is inadequate to satisfy all the necessary requirements without a change of the conditions (30). Thus, in a certain sense we can say that the properties of the color current are basically determined by the properties of the nonleptonic decays of the ordinary hadrons. The same remarks also apply to the KK scheme. Of the two models we have considered, only this last is suitable for describing decays of charmed particles, since, as we have already noted, the ART model is based on principles that do not permit its extension to the group $SU(4)'\otimes SU(3)''$. Therefore, we shall construct the Lagrangian of the nonleptonic interaction of charmed hadrons on the basis of the KK model. In Refs. 25 and 49, the nonleptonic Lagrangian of the $SU(4)'\otimes SU(3)''$ scheme is used to fix the parameters of the current, which is then used to discuss neutrino reactions, including two-muon production. In Ref. 44, an attempt is made to use the KK model to describe the decay (weak) properties of charmed hadrons.

As we have already said, in considering the $\Delta T=1/2$ rule one may require either dominance of the isotopic $T=1/2$ amplitude, or octet dominance in $SU(3)$, or dominance of the representation 20 in $SU(4)$ over the representation 84. In accordance with the adopted point of view, the nonleptonic Lagrangian is also decomposed into the necessary representations. We shall here fix the parameters of the current on the basis of the ratio of the amplitudes of the $\Delta T=1/2$ and 3/2 transitions for the $SU(3)$ part of the KK model, i.e., we use the relation (73). The white part of the KK Lagrangian has the form

$$\begin{aligned} L^c = (G/8)\{&(C_2 - C_1/3)(\tilde{p}n)(\tilde{n}c) + (C_1 - C_2/3)(\tilde{p}c)(\tilde{n}n) \\ &+ (C_4 - C_3/3)(\tilde{p}\lambda)(\tilde{n}c) + (C_3 - C_4/3)(\tilde{p}c)(\tilde{n}\lambda) \\ &+ (C_6 - C_5/3)(\tilde{p}n)(\tilde{\lambda}c) + (C_5 - C_6/3)(\tilde{p}c)(\tilde{\lambda}n) \\ &+ (C_8 - C_7/3)(\tilde{p}\lambda)(\tilde{\lambda}c) + (C_7 - C_8/3)(\tilde{p}c)(\tilde{\lambda}\lambda) \\ &+ (C_{10} - C_9/3)(\tilde{c}n)(\tilde{\lambda}c) + (C_9 - C_{10}/3)(\tilde{c}c)(\tilde{\lambda}n) \\ &+ \text{h.c.} + \text{self-adjoint terms of the form } (\tilde{c}n)(\tilde{n}c). \text{ etc.}\}, \end{aligned}$$

where the coefficients C_i can be expressed as follows in terms of the parameters of the current in (50) and (51):

$$\left.\begin{aligned} C_1 &= a_1a_1' + a_2a_2' + a_3a_3' + c_1c_1' + b_7b_7'; \\ C_2 &= (a_1 + a_2 + a_3)(a_1' + a_2' + a_3'); \\ C_3 &= b_1a_1' + b_2a_2' + b_3a_3' + a_7c_1' + d_1b_7'; \\ C_4 &= (b_1 + b_2 + b_3)(a_1' + a_2' + a_3'); \\ C_5 &= a_1b_1' + a_2b_2' + a_3b_3' + c_1a_7' + b_7d_1'; \\ C_6 &= (a_1 + a_2 + a_3)(b_1' + b_2' + b_3'); \\ C_7 &= b_1b_1' + b_2b_2' + b_3b_3' + a_7a_7' + d_1d_1'; \\ C_8 &= (b_1 + b_2 + b_3)(b_1' + b_2' + b_3'); \\ C_9 &= a_1'b_1' + a_2'b_2' + a_3'b_3' + a_7'c_1' + b_7'd_1'; \\ C_{10} &= (a_1' + a_2' + a_3')(b_1' + b_2' + b_3'). \end{aligned}\right\} \qquad (78)$$

In the left-hand column of (78) we have written down a Lagrangian like the one obtained in the GIM model, and the right-hand column is obtained from the left by a Fierz transformation. To obtain the actual values of the coefficients of the Lagrangian, we use only the three equations (55) and (73) and do not include Eqs. (57). In the KK model, these three equations are

$$\cos\varphi_1 + A_{11} + A_{33} = 3\varkappa\cos\theta_c; \tag{79a}$$

$$\sin\varphi_1 + A_{12} + A_{34} = 3\varkappa\sin\theta_c; \tag{79b}$$

$$A_1/A_2 = 11.6, \tag{79c}$$

where

$$A_{1,2} = (\cos\varphi_1 + A_{11} + A_{33})(\sin\varphi_1 + A_{12} + A_{34}) - (+)\,[(1/2)\sin 2\varphi_1 + A_{11}A_{12} + A_{33}A_{34} + A_{14}A_{13} + A_{31}A_{32}]. \tag{79d}$$

Computer calculations give a set of parameters satisfying (79):

$$\left.\begin{array}{l}\alpha\approx 1.64;\quad \beta\approx 0.0025;\quad \varphi_1\approx\varphi_2\approx\varphi_3=2\pi;\\ \psi_2\approx 0.223,\quad \psi_3\approx 0.018;\quad \varkappa=0.301.\end{array}\right\} \tag{80}$$

Note the rather small value of $\varkappa$. We have already mentioned that this is undesirable, since the mass of the W boson is proportional to $\varkappa$.

With the obtained values of the parameters (80), we find the following form of the weak nonleptonic Lagrangian of charmed particles:

$$\begin{aligned}8L^c = {}&0.203\,(\tilde{p}n)(\tilde{n}c) - 0.245\,(\tilde{p}c)(\tilde{n}n) + 0.023\,(\tilde{p}\lambda)(\tilde{n}c)\\ &+0.015\,(\tilde{p}c)(\tilde{n}\lambda) + 2.31\,(\tilde{p}n)(\tilde{\lambda}c) + 0.025\,(\tilde{p}c)(\tilde{\lambda}n)\\ &+0.546\,(\tilde{p}\lambda)(\tilde{\lambda}c) - 0.005\,(\tilde{p}c)(\tilde{\lambda}\lambda)\\ &+0.409\,(\tilde{c}n)(\tilde{\lambda}c) - 0.001\,(\tilde{c}c)(\tilde{\lambda}n).\end{aligned} \tag{81}$$

In the calculation, we have fixed one parameter, the coefficient of the term $(\tilde{p}n)(\tilde{n}c)$, which has a value ≈ 0.2; all the remaining numbers are determined by the system of equations (79a)–(79c). The left-hand column of the expression (81) reproduces the main features of the Lagrangian obtained from the GIM current, although it differs quantitively somewhat from it. This circumstance is interesting in that we have not used any conditions to fix the coefficients of the currents $(\tilde{n}c)$ and $(\tilde{\lambda}c)$ of the $SU(4)$ scheme. Although the quantitative differences are significant, they are not yet distinguishable at the contemporary accuracy of the experiments. A general feature of both Lagrangians is the dominance of the K mesons compared with the pions in D decays.

It appears interesting to consider the properties of the obtained Lagrangian with respect to the group $SU(4)$. For this, we decompose the Lagrangian (81) in accordance with the representations 20 and 84 of this group. The decomposition is

$$L^c = L^{(20)}/12 + L^{(84)}/24, \tag{82}$$

where

$$\begin{aligned}L^{c(20),(84)} = {}&(C_2 \mp C_1)\,[(\tilde{p}n)(\tilde{n}c) \mp (\tilde{p}c)(\tilde{n}n)]\\ &+(C_4 \mp C_3)\,[(\tilde{p}\lambda)(\tilde{n}c) \mp (\tilde{p}c)(\tilde{n}\lambda)]\\ &+(C_6 \mp C_5)\,[(\tilde{p}n)(\tilde{\lambda}c) \mp (\tilde{p}c)(\tilde{\lambda}n)]\\ &+(C_8 \mp C_7)\,[(\tilde{p}\lambda)(\tilde{\lambda}c) \mp (\tilde{p}c)(\tilde{\lambda}\lambda)]\\ &+(C_{10} \mp C_9)\,[(\tilde{c}n)(\tilde{c}\lambda) \mp (\tilde{c}c)(\tilde{\lambda}n)].\end{aligned} \tag{82'}$$

Let us consider the ratios of the coefficients of the representation 20 to the corresponding coefficients of the representation 84, taking into account the values of the parameters (80):

$$\left.\begin{array}{l}2\dfrac{C_2-C_1}{C_2+C_1}\approx 10.6;\quad 2\dfrac{C_4-C_3}{C_4+C_3}\approx 0.20;\\ 2\dfrac{C_6-C_5}{C_6+C_5}\approx 0.98;\quad 2\dfrac{C_8-C_7}{C_8+C_7}\approx 1.02;\quad 2\dfrac{C_{10}-C_9}{C_{10}+C_9}\approx 1.0.\end{array}\right\} \tag{83}$$

These ratios do not reveal dominance of any representation of $SU(4)$ in the nonleptonic decays of charmed hadrons.

The attempt undertaken in this section to obtain the Lagrangian for describing decays of charmed particles must be recognized to be rather unsuccessful. The main reason for this is the small value of $\varkappa$. We have also not determined the weak current for semileptonic interactions of the c quark as the GIM current. The attempt to obtain, for example, the GIM scheme for this interaction reduces $\varkappa$ even more. From (80), we obtain the following current of the c quark: $0.16(\tilde{c}n) + 2.97(\tilde{c}\lambda)$. Hence we have the ratio $r = \Gamma(D^0 \to e^+\nu\pi^-)/\Gamma(D^0 \to e^+\nu K^-) \approx 0.003$; for the GIM scheme, $r = \tan^2\theta_c \approx 0.05$. For nonleptonic decays, we have $r \equiv \Gamma(D^0 \to \pi^+\pi^-)/\Gamma(D^0 \to K^-\pi^+) \approx 0.088$, and in the GIM scheme we have as before $r \sim \tan^2\theta_c$, while experimentally $r < 8\%$. Nevertheless, the KK scheme can be used, for example, to construct right-helicity currents, for which $\varkappa = 0$ by definition. However, it must be said that the existence of such currents is at present in doubt, although contemporary experiments do not rule out their existence. In Sec. 11, we shall consider this question in more detail.

10. EXCITATION OF COLOR IN NEUTRINO REACTIONS

The question of the manifestation of the color degree of freedom has been considered in many papers (see Refs. 45 and 46; references to earlier papers can be found in Refs. 9 and 10). As a rule, the weak color currents in the quoted papers have a very simplified form of the type (13). However, the nature of these currents reflects in its general features the essence of the effects of color manifestation, and we shall therefore consider below such simple currents. Two main points of view about the manifestation of color have developed. One of them, proposed in a number of papers by Pati and Salam,[47] is that although color is a physical property it is as yet not manifested in experiments for various kinematic reasons, the essence of which will be explained below. The other viewpoint, represented strongly in the papers of Tsai *et al.*,[46] consists of the assertion that color has already been observed in contemporary experiments and in conjunction with, for example, chromodynamics successfully explains the departure from scaling in high-energy lepton–hadron inclusive processes. Let us consider first as an example neutrino processes with physical color, following Refs. 45 and 46.

The model considered by Tsai is the $SU(4)'\otimes SU(3)''$ scheme [in earlier papers, $SU(3)'\otimes SU(3)''$] with quark charges determined by Eqs. (10). The weak current of this scheme has the form

$$J_\mu = J_\mu^0 + J_\mu^1 + J_\mu^2, \tag{84}$$

where

$$J^0_\mu=\sum_{\alpha=1}^{3}[\cos\theta_c(\widetilde{p}_\alpha\gamma_\mu(1-\gamma_5)n^\alpha)+\sin\theta_c(\widetilde{p}_\alpha\gamma_\mu(1-\gamma_5)\lambda^\alpha)], \quad (84a)$$

$$J^1_\mu=\sum_{\alpha=1}^{3}[-\sin\theta_c(\widetilde{c}_\alpha\gamma_\mu(1-\gamma_5)n^\alpha)+\cos\theta_c(\widetilde{c}_\alpha\gamma_\mu(1-\gamma_5)\lambda^\alpha)] \quad (84b)$$

are the color-singlet currents constructed in accordance with the GIM scheme, and

$$J^2_\mu=\sum_{q=p,\,n,\,\lambda,\,c}\{c_-[\cos\theta'(\widetilde{q}^2\gamma_\mu(1-\gamma_5)q_1)+\sin\theta'(\widetilde{q}^3\gamma_\mu(1-\gamma_5)q_1)]+c_+[\cos\theta''(\widetilde{q}^2\gamma_\mu(1+\gamma_5)q_1)+\sin\theta''(\widetilde{q}^3\gamma_\mu(1-\gamma_5)q_1)]\} \quad (84c)$$

is the $SU(4)'$ singlet part (summation over the flavors), this recalling in a certain sense the $SU(3)''$ Cabibbo current. In the currents (84a) and (84b), the angle θ is taken equal to the Cabibbo angle, and in the color current (84c) the four parameters θ', θ'' and c_-, c_+ must be determined experimentally. However, for c_- and c_+ theoretical restrictions follow from the universality principle, namely, there are three possibilities: $(c_-, c_+)=(1,0)$, $(1,1)$, and $(0,1)$.

Attention should be drawn to the presence in the expression (84c) of right-helicity currents. It can be seen from the considered example that such currents could play a part in neutrino reactions; the general situation with regard to right-helicity currents is considered in Sec. 11.

With regard to the model (84) itself, it is necessary to make the following comment. The attempt to introduce this scheme into the $SU(2)\otimes U(1)$ gauge theory can lead to the following conclusion (this question is discussed more fully in Ref. 45): If the combinations $(p^\alpha, n^\alpha\cos\theta+\lambda^\alpha\sin\theta)$ and $(c^\alpha, -n^\alpha\sin\theta+\lambda^\alpha\cos\theta)$ are taken as left-helicity doublets, it is then necessary to place the combination $(q_2\cos\theta''+q_3\sin\theta'', q_1)$ in a right-helicity doublet in order to obtain the current (84b), the relations $c_-=0$ and $c_+=1$ then holding. Thus, the left-helicity currents for the color-singlet part of the current and the octet part cannot coexist. However, the considered example is interesting in that, as we have already said, it reflects the general situation in color models, and therefore in the paper we shall also discuss the case $c_-\neq 0$.

The inclusive cross sections for ν and $\bar{\nu}$ can be considered in the framework of the usual quark–parton model. Here it is only necessary to bear in mind that because the target nucleons are color singlets the momentum distribution functions satisfy the equations

$$\left.\begin{array}{l} q_1(x)=q_2(x)=q_3(x)=q(x)/3;\\ \bar{q}_1(x)=\bar{q}_2(x)=\bar{q}_3(x)=\bar{q}(x)/3.\end{array}\right\} \quad (85)$$

In the analysis of the data one uses not only the usual variables $x=q^2/2M\nu$ and $y=\nu/E$ but also the variables z_1 and z_2 which take into account the mass m_1 of the charmed quark:

$$z_1=x+m_1^2/2MEy, \quad (86)$$

and the effective mass m_2 of the quark bound in excited color states:

$$z_2=x+m_2/2MEy.$$

The corresponding thresholds for the production of the charmed quark and the colored quark are denoted by W_1 and W_2. The threshold effects are manifested in the cross sections by appropriate θ functions with values $W_1\approx 2.25$ GeV and $W_2\approx 10$ GeV. The masses have the values $m_1\approx 1.25$ GeV and $m_2\approx 5$ GeV. Note also that in the target nucleons the effects of the charmed quark are not taken into account. This last remark applies to the usual division of quarks into valence quarks and sea quarks for targets with different numbers of protons and neutrons, for which we have

$$\left.\begin{array}{l} p(z)+n(z)-\bar{p}(z)-\bar{n}(z)\equiv F_V(z);\\ \bar{p}(z)+\bar{n}(z)=2\bar{\lambda}(z)=2\lambda(z)\equiv F_S(z).\end{array}\right\} \quad (87)$$

Corresponding to the three currents in (84), the differential inclusive cross section decomposes into three parts:

$$\frac{d^2\sigma}{dx\,dy}=\sum_{i=0,\,1,\,2}\left(\frac{d^2\sigma}{dx\,dy}\right)_i, \quad (88)$$

where

$$\left(\frac{d^2\sigma}{dx\,dy}\right)_0=\frac{G^2ME}{\pi}\left(1+\frac{q^2}{m_W^2}\right)^{-2}\{F_{1,0}(x)+F_{2,0}(x)(1-y)^2\}; \quad (88a)$$

$$\left(\frac{d^2\sigma}{dx\,dy}\right)_1=\frac{G^2ME}{\pi}\left(1+\frac{q^2}{m_W^2}\right)^{-2}\left\{F_{1,1}(z_1)\left(1-\frac{m_1^2}{2MEz_1}\right)+F_{2,1}(z_1)\left[(1-y)^2+\frac{m_1^2}{2MEz_1}(1-y)\right]\right\}\theta(W-W_1); \quad (88b)$$

$$\left(\frac{d^2\sigma}{dx\,dy}\right)_2=\frac{G^2ME}{\pi}\left(1+\frac{q^2}{m_W^2}\right)^{-2}\left\{F_{1,2}(z_2)\left(1-\frac{m_2^2}{2MEz_2}\right)+F_{2,2}(z_2)\left[(1-y)^2+\frac{m_2^2}{2MEz_2}(1-y)\right]\right\}\theta(W-W_2). \quad (88c)$$

The factor $(1+q^2/m_W^2)^{-2}$ is introduced into the cross sections to take into account the decrease in the energy slope of the total cross sections with increasing energy: σ^ν/E at high energies[48] is approximately 20% smaller than the corresponding quantity obtained from the Gargamelle data.[49] Although this quantity does indicate an effect of the W-boson propagator, m_W must for the time being be regarded as a phenomenological parameter. The structure functions in the formulas for the cross sections can be expressed as follows in terms of the quark distribution functions.

For νp scattering

$$F_{1,0}(x)=2x[n(x)\cos^2\theta_c+\lambda(x)\sin^2\theta_c]; \quad (89)$$

$$F_{2,0}(x)=2x\bar{p}(x); \quad (90)$$

$$F_{1,1}(z)=2z[n(z)\sin^2\theta_c+\lambda(z)\cos^2\theta_c]; \quad (91)$$

$$F_{1,2}(z)=(2/3)z\{|c_-|^2[p(z)+n(z)+\lambda(z)]+|c_+|^2[\bar{p}(z)+\bar{n}(z)+\bar{\lambda}(z)]\}; \quad (92)$$

$$F_{2,2}(z)=(2/3)z\{|c_-|^2[\bar{p}(z)+\bar{n}(z)+\bar{\lambda}(z)]+|c_+|^2[p(z)+n(z)+\lambda(z)]\}. \quad (93)$$

The expressions for νn scattering are obtained from (89)–(93) by the substitutions $p(z)\to n(z)$ and $\bar{p}(z)\to\bar{n}(z)$. The expressions for $\bar{\nu}p$ and $\bar{\nu}n$ scattering are obtained from those given above by the substitution $q(z)\to\bar{q}(z)$, with $q=p, n, \lambda$.

The results of the analysis of the neutrino data by means of the introduced color current can be summarized as follows. The recent data on inclusive scattering[48] of neutrinos indicate a definite departure from scaling; for example, σ^ν/E and $\sigma^{\bar{\nu}}/E$ at high energies[48] are approximately 20% smaller than the quantities obtained from the Gargamelle data.[49] This departure from scaling, which Tsai calls "negative," can be

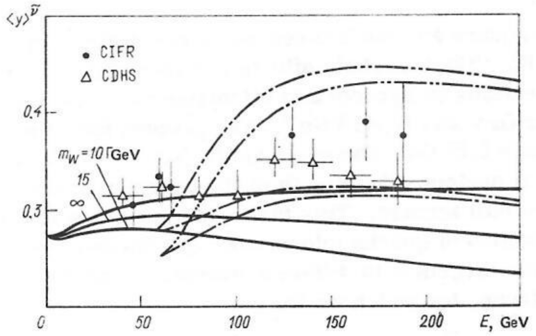

FIG. 1. Energy dependence of $\langle y\rangle^{\bar\nu}$. The solid curves are the predictions of the standard model; the chain curves show the effect of excitation of color with $W_2=10$ GeV and $m_2=5$ GeV for the case $m_W=10$ GeV. The lower curves are obtained with a cutoff ($E'\geqslant 5$ GeV) and the upper curves without one.

readily explained by the introduction of the suppression factor $(1+q^2/m_W^2)^{-2}$ in the expression for the cross section with $m_W\approx 10$–15 GeV. It should be borne in mind that the most widely accepted method for describing this departure from scaling is based on asymptotically free theories.[50] From the point of view of possible excitation of color, greater interest attaches to "positive" departure from scaling, for which there are some indications in the BEBC and CIFR results.[48] This departure from scaling can be described by a color current with $c_-=1$ and $c_+=0$ and threshold $W\approx 10$ GeV. Of course, if such a departure from scaling does occur, it can also be explained by the production of new quarks, i.e., $b, t, \ldots$, but this can correspond fully to excitation of the color degree of freedom. With regard to the nonvanishing values of c_+, the analysis made by Tsai shows that the data do not support the existence of right-helicity currents. Some data also cannot be described by a current with $c_-\neq 0$, as, for example, can be seen in Fig. 1, although in Fig. 2 there is a definite agreement between Tsai's calculations and experiment. Tsai's conclusions can be augmented as follows. A certain disagreement between the data and the theoretical curves could be explained by the choice of the form of the color current (84). If one considers the most general structure of this current, i.e., (14) and (50), then, using the abundance of parameters, one could undoubtedly obtain satisfactory theoretical curves. However, the observed decrease in the energy slope of the cross section can be explained by the appearance of the factor

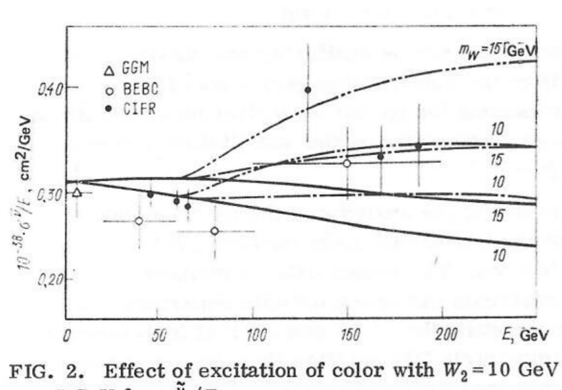

FIG. 2. Effect of excitation of color with $W_2=10$ GeV and $m_2=5$ GeV for $\sigma^{\bar\nu}/E$.

$\varkappa$ in front of the color current. Finally, one may assume that in contemporary experiments the color structure has not yet been manifested. Also, following Pati and Salam,[47] one can assume that color is not manifested for certain "kinematic" reasons, the essence of which is as follows. In the unified scheme of weak, electromagnetic, and strong interactions proposed by Pati and Salam, the mixing of the fields of the intermediate bosons occurs in such a way that the interaction of the leptonic current with the color current has the form

$$J^{\rm lept}\,[1/q^2-1/(q^2-m_U^2)]\,J^{\rm color}, \tag{94}$$

where the first term in the square brackets is the photon propagator and the second is the propagator of a massive gluon, this being due to the departure from color symmetry. The matrix element acquires an additional kinematic factor

$$\Delta(q^2)=q^2\,[1/q^2-1/(q^2-m_U^2)]=-m_U^2/(q^2-m_U^2), \tag{95}$$

which tends asymptotically to zero for $q^2\gg m_U^2$. Thus, there is a cancellation of the contributions of the photon and the gluon. Since the gluon mass U is 2–5 according to the estimate of the authors, the factor (95) can reconcile the low threshold for production of color states with the fact that they are not observed.

11. ON THE POSSIBLE EXISTENCE OF RIGHT-HANDED CURRENTS

In the previous section, right-handed color currents were used to analyze neutrino interactions. The possible existence of such currents warrants study.

In the recent review of Ref. 51, Harari has discussed in considerable detail right-handed couplings of the c quark. On the basis of experimental facts, he concludes: a) the right-handed current $(\bar c n)_R$ does not exist; b) contemporary data are inadequate to confirm or disprove the existence of the current $(\bar c\lambda)_R$.

However, other points of view have been expressed. The papers of Refs. 52 and 53 give a number of arguments to the effect that the current $(\bar c\lambda)_R$ is needed to explain the $\Delta T=1/2$ rule[52] and the observed decay widths of the D mesons.[53]

Right-handed currents are also discussed in Refs. 18–21: In Ref. 20, they are used for the explicit introduction of CP violation in the weak-interaction Hamiltonian; in Ref. 21, mainly in connection with the $\Delta T=1/2$ rule; and, finally, in Refs. 18 and 19, in a study of the general conditions for the fulfillment of the algebra $SU(2)$ for weak charges. In our view, in this last case there is a definite prescription for distinguishing left- and right-handed currents in accordance with the types of solutions to the equations that arise from the requirement that the weak-current algebra $SU(2)$ be satisfied. Let us consider the simplest case of the group $SU(4)$, in which the matrix of weak $\Delta Q=+1$ transitions has the form

$$M^+=\begin{pmatrix}0&a&b&0\\0&0&0&0\\0&0&0&0\\0&c&d&0\end{pmatrix};\qquad q=\begin{pmatrix}p\\n\\\lambda\\c\end{pmatrix}. \tag{96}$$

If, as in the case (14), we introduce the "vectors" $\mathbf{x}=\binom{a}{b}$ and $\mathbf{y}=\binom{c}{d}$, then the condition $[M^+,[M^+,M^-]]=2M^+$ yields the system of equations

$$\left.\begin{array}{ll} a\mathbf{x}^2+c\,(\mathbf{xy})=a; & b\mathbf{x}^2+d\,(\mathbf{xy})=b; \\ c\mathbf{y}^2+a\,(\mathbf{xy})=c; & d\mathbf{y}^2+b\,(\mathbf{xy})=d, \end{array}\right\} \quad (97)$$

which is analogous to the system (21) for the case of the group $SU(3)'\otimes SU(3)''$. If the (same) determinant of the first and second pair of equations (97) does not vanish,

$$ad-bc\neq 0, \quad (98)$$

then the solution of the system is

$$\mathbf{x}^2=\mathbf{y}^2=1; \quad (\mathbf{xy})=0, \quad (99)$$

which corresponds completely to the solution (23) and (24) of the system (21) for the color model. The conditions (99) are none other than the conditions on the coefficients of the weak current of the GIM scheme, and can be readily parametrized

$$M^+=\begin{pmatrix} 0 & \cos\theta_c & \sin\theta_c & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sin\theta_c & \cos\theta_c & 0 \end{pmatrix}. \quad (100)$$

If $ad-bc=0$, the system (97) has a solution parametrized as follows:

$$M^+=\begin{pmatrix} 0 & \cos\alpha\cos\beta & \sin\alpha\cos\beta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \cos\alpha\sin\beta & \sin\alpha\sin\beta & 0 \end{pmatrix}. \quad (101)$$

From this representation, we can readily obtain different forms of right-handed currents,[18,19] provided two physical requirements are satisfied: 1) $\sin\theta_c=0$, $\cos\theta_c=0$; 2) the neutral current is diagonal. The requirement 1) is satisfied for $\beta=\pi/2$. We now write down the matrix of the neutral current $M^0=[M^+,M^-]/2$ for $\beta=\pi/2$:

$$M^0=\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\cos^2\alpha & -(1/2)\sin 2\alpha & 0 \\ 0 & -(1/2)\sin 2\alpha & -\sin^2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (102)$$

The element $-(1/2)\sin 2\alpha$ corresponds to the unobserved transition $(\tilde{n}\lambda)$ and vanishes for two values of the parameter α, which correspond to the two known forms of right-handed currents in $SU(4)$. For suppose $\alpha=0$; then as can be seen from (101) and (102),

$$J_R^+=(\tilde{c}n)_R; \quad (103)$$

$$J_R^0=-(\tilde{n}n)_R/2+(\tilde{c}c)_R/2, \quad (104)$$

which corresponds to the DGG scheme.[21] For $\alpha=\pi/2$,

$$J_R^+=(\tilde{c}\lambda)_R; \quad (105)$$

$$J_R^0=-(\tilde{\lambda}\lambda)_R/2+(\tilde{c}c)_R/2, \quad (106)$$

i.e., we obtain the right-handed current $AF-FM$ introduced and studied in detail in Refs. 52 and 53.

We have made this detailed comparison of the two solutions of the system to show that it is possible to obtain the right-handed currents considered in the literature from the "second" solution (with determinant $ad-bc=0$). In its turn, the analogous procedure for constructing a left-handed current in the color model, i.e., the solution (23)–(25) with determinant $(\mathbf{XZ})(\mathbf{YV})-(\mathbf{XV})(\mathbf{YZ})\neq 0$, corresponds to the solution (99) of the four-quark model, i.e., the GIM scheme. In the KK scheme, the determinant vanishes, $(\mathbf{XZ})(\mathbf{YV})-(\mathbf{XV})(\mathbf{YZ})=0$, and the expression corresponding to it for the weak current (49) recalls the structure of the right-handed current in the group $SU(4)$. An additional argument in favor of this assertion is the extremely small value of $\varkappa$ [see (80)] that is obtained from consideration of the decays of the c quark. For $\varkappa=0$, we have $\sin\theta_c=0$ and $\cos\theta_c=0$ in the white part of the KK current. The possible separation of the currents considered in the present review into left-handed currents (ART scheme) and right-handed currents (KK scheme) may be very helpful for the further development of weak-interaction schemes in color models. For example, if the conditions of the $\Delta T=1/2$ rule are satisfied, we necessarily obtain $\varkappa<0$, although without these conditions it is perfectly all right to use $\varkappa=1$ in the ART scheme. Therefore, if we use the KK scheme (in which we always have $\varkappa<1$) for the right-handed current, i.e., with $\varkappa=0$, to satisfy the conditions imposed by the $\Delta T=1/2$ rule, then in the left-handed current (for which the ART scheme can be used) there is no need for the factor $\varkappa$. In this case, the entire scheme becomes much more flexible because of the freedom in the choice of certain parameters.

CONCLUSIONS

The main content of the present review has been the exposition of the most complete schemes of the weak interaction in models with the group structures $SU(3)'\otimes SU(3)''$, $SU(4)'\otimes SU(3)''$, and $SU(4)'\otimes SU(4)''$ and unconfined color. In the case of the groups $SU(n)'\otimes SU(n)''$ we have shown that under the single assumption [Eq. (22) for the group $SU(3)'\otimes SU(3)''$ and the analogous equation for the group $SU(4)'\otimes SU(4)''$] that the $SU(2)$ algebra is satisfied for the weak charges diagonality of the neutral current follows. We have considered the most general form of the hadronic current (14), i.e., we have allowed all possible transitions between the nine color quarks. The method of constructing this weak-interaction scheme of the three-tiplet model is such that it cannot in principle be used for the case of the four-triplet model. Therefore, physical processes with the participation of the charmed color quark are described by means of the KK scheme,[25] in which the octet part of the current in the color space is generated by quark mixing [see Eqs. (46)–(48)].

To describe the ordinary hadrons, we have used physical quantities (currents and Lagrangians) averaged over the color indices. The neutral currents and Lagrangians of the weak nonleptonic interactions obtained as a result of such averaging differ radically from the corresponding expressions constructed directly from the Cabibbo currents. In particular, the neutral current is completely diagonal and in its quark structure is identical to the electromagnetic current for the three-triplet model,[23] but differs from the KK neutral current. One can in a natural manner construct a Lagrangian of the weak nonleptonic interaction describing the decay processes of ordinary hadrons in which the experimentally observed ratio between the amplitudes of transitions with $\Delta T=1/2$ and $\Delta T=3/2$ is specified. Fixing in this manner the parameters of the currents, we then obtain the nonleptonic Lagrangian for strangeness-conserving interactions. In principle, such a Lagrangian describes nuclear P-odd transitions. Its main feature is that the coefficients of the amplitudes of transitions with different ΔT (0, 1, and 2) are of the same order. It should be noted that in Cabibbo theory the $\Delta T = 1$ amplitude is suppressed by the factor $\sin^2\theta_c$.

An attempt has been made to describe the weak nonleptonic decays of charmed particles by the Lagrangian obtained from the requirement of fulfillment of the $\Delta T = 1/2$ rule, by means of which the parameters of the KK current can be fixed. In their general features, the results do not differ strongly from the consequences of the GIM theory. However, some features of the KK scheme revealed by this analysis suggest that this scheme could give a good description of right-handed currents if they exist. Additional theoretical arguments in favor of this assumption follow from the analogy with the group $SU(4)$.

To describe high-energy neutrino reactions, we have used a model that differs from the two main models considered in the review. Although it does not fully accord with the principles of the gauge group $SU(2) \otimes U(1)$, this model has the basic features inherent in all color models of the weak interaction. We have shown that in addition to other models, for example, quantum chromodynamics, weak currents with physical color are also able to describe the contemporary neutrino experiments.

All the questions considered in the paper have been discussed under the assumption that color is observable. It is possible that color quarks have fractional charges, for which there are already indirect experimental indications.[45] In addition, a model with confined color is at present also assumed to be preferable from the theoretical point of view, this being supported, for example, by the considerable successes of quantum chromodynamics. On the other hand, no serious arguments can be advanced at the present time for either the theoretical or experimental point of view against a model with integral charges. Moreover, as was shown in the construction of the Lagrangian responsible for the P-odd nucleus transitions, the considered model has definite advantages over the model with fractional charges, which contains within it the basic features of Cabibbo theory with regard to the weak interactions. Attention should be drawn to the fact that in the case of three triplets the neutral current, averaged over the color indices, is freed from the most disagreeable feature of the Cabibbo current, i.e., it becomes completely diagonal. In almost all work with color currents we encounter such results, namely, virtually all the physical quantities obtained by averaging over the color indices, for example, the nonleptonic Lagrangians, differ in the most radical manner from the corresponding quantities constructed without allowance for the color degree of freedom.

An important simplification in work with a large number of quarks is achieved by means of the method developed in the paper for solving the equations obtained from the algebra $SU(2)$ for the weak charges. These methods make it possible to decompose the obtained currents into "GIM-like currents" (in which diagonality of the neutral current follows from the solution itself) and "non-GIM-like currents." The latter could be responsible for right-handed couplings of heavy quarks if such couplings exist. The methods we have discussed can be readily transferred to other schemes. In the case of the various six-quark schemes, such a classification of the currents has been made, for example, in Ref. 22. In Appendix 2 we illustrate the possibility of a relatively compact description of schemes with an extremely large number of parameters. Thus, the developed methods of construction and subsequent study of weak hadronic currents can be successfully used in all other multiquark schemes.

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APPENDIX 1[1)]

We show here that the condition of octet dominance in the form proposed by Katuya[15] is not satisfied by the current of the ART model. The properties of the latter are expressed by Eqs. (26), (23′), and (24′). To these equations, it is necessary to add the requirements (30). The requirement of octet dominance is expressed by the equations

$$(a_1+a_2)^2+(a_2+a_3)^2+(a_1+a_3)^2-(a_4-a_5)^2-(b_4-b_5)^2+b_7^2+c_1^2=0; \quad \text{(A.1a)}$$
$$(b_1+b_2)^2+(b_2+b_3)^2+(b_1+b_3)^2-(a_4-a_6)^2-(b_4-b_6)^2+a_7^2+d_1^2=0; \quad \text{(A.1b)}$$
$$(a_1+a_2+a_3)(b_1+b_2+b_3)+a_1b_1+a_2b_2+a_3b_3+a_7c_1$$
$$+b_7d_1+(a_4-a_6)(b_8+c_2)+(b_4-b_6)(a_8+d_2)=0; \quad \text{(A.1c)}$$
$$(a_5-a_6)^2+(b_5-b_6)^2-a_8^2-b_8^2-c_2^2-d_2^2=0; \quad \text{(A.1d)}$$
$$(a_5-a_6)(b_8+c_2)+(b_5-b_6)(a_8+d_2)=0; \quad \text{(A.1e)}$$
$$a_8d_2+b_8c_2=0. \quad \text{(A.1f)}$$

The quantities on the left-hand sides of Eqs. (A.1) are the coefficients of the parts of the Lagrangian with the property of transforming in accordance with the representation 27 in $SU(3)$.

To solve the system (26), (23′), (24′), (30), and (A.1), we represent it in a different form. We note first that (A.1d)–(A.1f) yield

$$a_8+d_2=\pm(a_5-a_6); \quad \text{(A.2a)}$$
$$b_8+c_2=\mp(b_5-b_6); \quad \text{(A.2b)}$$
$$(a_8-d_2)^2+(b_8-c_2)^2=(a_5-a_6)^2+(b_5-b_6)^2. \quad \text{(A.2c)}$$

We introduce further the variables

$$\left.\begin{array}{ll} \tilde{x}=(a_5-a_6)/2; & \tilde{\xi}=(a_5+a_6)/2; \\ \tilde{y}=(b_5-b_6)/2; & \tilde{\eta}=(b_5+b_6)/2; \\ \tilde{z}=(a_8-d_2)/2; & \tilde{v}=(b_8-c_2)/2, \end{array}\right\} \quad \text{(A.3)}$$

in which the system takes a simpler form. Making once more the substitution

$$x=\sin\alpha\tilde{x}+\cos\alpha\tilde{y}; \quad y=\cos\alpha\tilde{x}-\sin\alpha\tilde{y}, \quad \text{(A.4a)}$$

and the same substitution for the pairs (z, v) and (ξ, η), we obtain finally the system of equations

$$a_1+z+\xi=3C; \quad \text{(A.5a)}$$
$$b_1+v+\eta=3S; \quad \text{(A.5b)}$$
$$a_1b_1+2x(v+\eta)-2y(z+\xi)=0; \quad \text{(A.5c)}$$
$$(a_1^2-b_1^2)/4+x(z+\xi)+y(v+\eta)=0; \quad \text{(A.5d)}$$
$$9CS+a_1b_1-2xv+2yz-2a_1y+2b_1x=0; \quad \text{(A.5e)}$$

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1)Appendix 1 is the outcome of work done jointly by the authors and A. V. Kulikov.

$$9(C^2-S^2)/4-xz-yv+(a_1^2-b_1^2)/4+a_1x+b_1y=0; \tag{A.5f}$$
$$(a_1^2+b_1^2)/2+2x^2+2y^2+z^2+v^2+\xi^2+\eta^2-2=0; \tag{A.5g}$$
$$9+x^2+y^2+z^2+v^2-a_1^2-b_1^2+4a_1\xi+4b_1\eta=0; \tag{A.5h}$$
$$z^2+v^2=x^2+y^2, \tag{A.5i}$$

where $C\equiv\cos\theta_c$ and $S=\sin\theta_c$. Note first that in the variables x, y, z, v, ξ, η Eqs. (A.5) do not contain the parameter α. Thus, the number of equations is larger than the number of variables, which suggests that the system is not soluble. Let us show this. From (A.5c) and (A.5d), using (A.5a) and (A.5b), we find

$$x=-(1/2P)\,[(3S-b_1)\,a_1b_1+(a_1^2-b_1^2)\,(3C-a_1)/2]; \tag{A.6a}$$
$$y=-(1/2P)\,[-(3C-a_1)\,a_1b_1+(a_1^2-b_1^2)\,(3S-b_1)/2], \tag{A.6b}$$

where $P=9-6(Ca_1+Sb_1)+a_1^2+b_1^2$. We also write down an equation which will be useful in what follows:

$$x^2+y^2=(a_1^2+b_1^2)/16P. \tag{A.7}$$

To find z and v, we use Eqs. (A.5e) and (A.5f):

$$\begin{aligned}(x^2+y^2)\,z&=[9(C^2-S^2)/4+(a_1^2-b_1^2)/4+a_1x+b_1y]\,x\\&-[(9CS+a_1b_1)/2+b_1x-a_1y]\,y;\end{aligned} \tag{A.8a}$$
$$\begin{aligned}(x^2+y^2)\,v&=[9(C^2-S^2)/4+(a_1^2-b_1^2)/4\\&+a_1x+b_1y]+[(9CS+a_1b_1)/2+b_1x-a_1y]\,x.\end{aligned} \tag{A.8b}$$

As a result, there remain at our disposal only the two variables a_1 and b_1, which occur only in the combinations

$$a=Ca_1+Sb_1;\quad b=Cb_1-Sb_1. \tag{A.9}$$

Indeed,

$$P=9-6a+(a^2+b^2); \tag{A.10}$$
$$a_1z+b_1v=(a^2+b^2)^{-1}\,[2(a^2+b^2)^2+9(a^2-b^2)-3a(a^2+b^2)-27a]; \tag{A.11}$$
$$\begin{aligned}z^2+v^2=(a^2+b^2)^{-2}\,\{&4(a^2+b^2)^3+3[-4a^5-4ab^4-8a^3b^2+15a^4\\&-9b^4+6a^2b^2-54a^3+18ab^2+81a^2-27b^2-162a+243]\};\end{aligned} \tag{A.12}$$
$$Cz+Sv=(a^2+b^2)^{-2}\,[2a(a^2+b^2)^2-3(a^2+b^2)-27(a^2-b^2)+9a^3-27ab^2]. \tag{A.13}$$

It is now easy to see that the obtained system of the three equations (A.5g), (A.5h), and (A.5i) is incompatible. For this it is sufficient to consider the one equation $z^2+v^2=x^2+y^2$, using (A.6) and (A.13) for the values $-\sqrt{2}\leqslant a, b\leqslant+\sqrt{2}$.

Thus, we have shown that the consequences that follow from the requirements on the hadronic current in the color model of Ref. 23 contradict the conditions for the vanishing of the amplitudes of $\Delta T=3/2$ transitions.[15]

APPENDIX 2

We here consider the weak interaction between quarks and leptons forming a fundamental multiplet in the $SU(4)'\otimes SU(4)''$ scheme, following the papers of Pati and Salam.[47] The general form of this multiplet is

$$\Psi=\begin{pmatrix}p_1^0 & p_2^+ & p_3^+ & p_4^0\\ n_1^- & n_2^0 & n_3^0 & n_4^-\\ \lambda_1^- & \lambda_2^0 & \lambda_3^0 & \lambda_4^-\\ \chi_1^0 & \chi_2^+ & \chi_3^+ & \chi_4^0\end{pmatrix}. \tag{A.14}$$

Here, the fourth row and fourth column can stand for charmed quarks and leptons (known ones and heavy ones).

We now write down the matrix of weak transitions between the constituents of the multiplet (A.14). The notation in this matrix is the same as in the matrices (14) and (50), and the newly introduced elements [i.e., the coefficients of transitions in which the quarks and leptons of the fourth column of (A.14) participate] carry two primes. Thus,

$$M^+=\begin{pmatrix}
0 & a_1 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_{17} & a''_{18} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & a'_1 & b'_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_{15} & a''_{16} & 0\\
a_4 & 0 & 0 & a'_4 & 0 & a_2 & b_2 & 0 & 0 & c_1 & a_7 & 0 & a''_{13} & 0 & 0 & a''_{14}\\
0 & a_5 & b_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_{11} & a''_{12} & 0\\
0 & c_2 & a_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c''_1 & c''_2 & 0\\
a'_5 & 0 & 0 & a'_6 & 0 & a'_2 & b'_2 & 0 & 0 & c'_1 & a'_7 & 0 & a_9 & 0 & 0 & a''_{10}\\
b_4 & 0 & 0 & b'_4 & 0 & b_7 & d_1 & 0 & 0 & a_3 & b_3 & 0 & b''_3 & 0 & 0 & b''_4\\
0 & b_5 & d_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b''_1 & b''_2 & 0\\
0 & a_8 & b_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_5 & a''_6 & 0\\
b'_5 & 0 & 0 & b'_6 & 0 & b'_7 & d'_1 & 0 & 0 & a'_3 & b'_3 & 0 & a''_7 & 0 & 0 & a''_8\\
0 & a''_{19} & b''_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_1 & a''_2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & a''_{20} & b''_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a''_3 & a''_4 & 0
\end{pmatrix}. \tag{A.15}$$

In the matrix (A.15), there are 64 nonvanishing elements, and therefore the system obtained from the requirement of fulfillment of the weak-current algebra $SU(2)$, which is $[M^+,[M^+,M^-]]=2M^+$, contains 64 equations. As in the case of the matrix (14), it is here also convenient to group the elements of the matrix M^+ into corresponding "vectors." We recall that the vectors **X** and **Y** in the set (19) are the second and third columns of the matrix (14) without the zeroth elements and divided by $\sqrt{2}$, while the vectors **Z** and **V** are the fourth and seventh rows. In accordance with exactly the same principle we construct the vectors in the present case; namely, we denote by the vector **X** the second column (without the xeroth elements) of the matrix (A.15) divided by $\sqrt{2}$. The vectors **Y**, $\mathbf{X}_1$, and $\mathbf{Y}_1$ are the third, 14th, and 15th columns, respectively. By the vectors **Z**, $\mathbf{Z}_1$, **V**, and $\mathbf{V}_1$ we denote, respectively, the fifth, eighth, ninth, and 12th rows with order of increasing numbers of the coordinates from the left to the right without allowance for the vanishing elements. The necessary system of 64 equations for finding the coupling between the elements of the matrix M^+ can then be written in the following compact form of just eight equations:

$$\left.\begin{aligned}
&2\mathbf{X}^2\mathbf{X}+2(\mathbf{XY})\mathbf{Y}+2(\mathbf{XX}_1)\mathbf{X}_1+2(\mathbf{XY}_1)\mathbf{Y}_1-(\mathbf{XZ})\mathbf{Z}\\
&\quad-(\mathbf{XV})\mathbf{V}-(\mathbf{XZ}_1)\mathbf{Z}_1-(\mathbf{XV}_1)\mathbf{V}_1=\mathbf{X};\\
&2\mathbf{V}^2\mathbf{Y}+2(\mathbf{XY})\mathbf{X}+2(\mathbf{YX}_1)\mathbf{X}_1+2(\mathbf{YY}_1)\mathbf{Y}_1\\
&\quad-(\mathbf{YZ})\mathbf{Z}-(\mathbf{YV})\mathbf{V}-(\mathbf{YZ}_1)\mathbf{Z}_1-(\mathbf{YV}_1)\mathbf{V}_1=\mathbf{Y};\\
&2\mathbf{Z}^2\mathbf{Z}+2(\mathbf{ZV})\mathbf{V}+2(\mathbf{ZZ}_1)\mathbf{Z}_1+2(\mathbf{ZV}_1)\mathbf{V}_1\\
&\quad-(\mathbf{ZX})\mathbf{X}-(\mathbf{ZY})\mathbf{Y}-(\mathbf{ZX}_1)\mathbf{X}_1-(\mathbf{ZY}_1)\mathbf{Y}_1=\mathbf{Z};\\
&2\mathbf{V}^2\mathbf{V}+2(\mathbf{ZV})\mathbf{Z}+2(\mathbf{Z}_1\mathbf{V})\mathbf{Z}_1+2(\mathbf{VV}_1)\mathbf{V}_1\\
&\quad-(\mathbf{VX})\mathbf{X}-(\mathbf{VY}_1)\mathbf{Y}_1-(\mathbf{VY})\mathbf{Y}-(\mathbf{VX}_1)\mathbf{X}_1=\mathbf{V};\\
&2\mathbf{X}_1^2\mathbf{X}_1+2(\mathbf{X}_1\mathbf{X})\mathbf{X}+2(\mathbf{X}_1\mathbf{Y})\mathbf{Y}+2(\mathbf{X}_1\mathbf{Y}_1)\mathbf{Y}_1\\
&\quad-(\mathbf{X}_1\mathbf{Z})\mathbf{Z}-(\mathbf{X}_1\mathbf{V})\mathbf{V}-(\mathbf{X}_1\mathbf{Z}_1)\mathbf{Z}_1-(\mathbf{X}_1\mathbf{V}_1)\mathbf{V}_1=\mathbf{X}_1;\\
&2\mathbf{Y}_1^2\mathbf{Y}_1+2(\mathbf{Y}_1\mathbf{X})\mathbf{X}+2(\mathbf{Y}_1\mathbf{Y})\mathbf{Y}+2(\mathbf{Y}_1\mathbf{X}_1)\mathbf{X}_1\\
&\quad-(\mathbf{Y}_1\mathbf{Z})\mathbf{Z}-(\mathbf{Y}_1\mathbf{V})\mathbf{V}-(\mathbf{Y}_1\mathbf{Z}_1)\mathbf{Z}_1-(\mathbf{Y}_1\mathbf{V}_1)\mathbf{V}_1=\mathbf{Y}_1;\\
&2\mathbf{Z}_1^2\mathbf{Z}_1+2(\mathbf{Z}_1\mathbf{Z})\mathbf{Z}+2(\mathbf{Z}_1\mathbf{V})\mathbf{V}+2(\mathbf{Z}_1\mathbf{V}_1)\mathbf{V}_1\\
&\quad-(\mathbf{Z}_1\mathbf{X})\mathbf{X}-(\mathbf{Z}_1\mathbf{Y})\mathbf{Y}-(\mathbf{Z}_1\mathbf{X}_1)\mathbf{X}_1-(\mathbf{Z}_1\mathbf{Y}_1)\mathbf{Y}_1=\mathbf{Z}_1;\\
&2\mathbf{V}_1^2\mathbf{V}_1+2(\mathbf{V}_1\mathbf{Z})\mathbf{Z}+2(\mathbf{V}_1\mathbf{V})\mathbf{V}+2(\mathbf{Z}_1\mathbf{V}_1)\mathbf{Z}_1\\
&\quad-(\mathbf{V}_1\mathbf{X})\mathbf{X}-(\mathbf{V}_1\mathbf{Y})\mathbf{Y}-(\mathbf{X}_1\mathbf{V}_1)\mathbf{X}_1-(\mathbf{V}_1\mathbf{Y}_1)\mathbf{Y}_1=\mathbf{V}_1.
\end{aligned}\right\} \tag{A.16}$$

The method of solving the system (A.16) is exactly the same as that for the system (20), i.e., we multiply each of Eqs. (A.16) in turn by each of the vectors **X**, **Y**, **Z**, **V**, $\mathbf{X}_1$, $\mathbf{Y}_1$, $\mathbf{Z}_1$, and $\mathbf{V}_1$ and we again obtain a system of 64 equations, but now for the invariants composed of these vectors, i.e., a system analogous to the system (21). Because the resulting system is so cumbersome, we shall not write it out here, but instead we draw attention to an important condition under which we solve it. This condition is the complete analog of the condition (22) and contains it as a special case. Namely, we require

$$\mathrm{Det}\begin{pmatrix} (\mathbf{XZ}) & (\mathbf{XV}) & (\mathbf{XZ}_1) & (\mathbf{XV}_1) \\ (\mathbf{YZ}) & (\mathbf{YV}) & (\mathbf{YZ}_1) & (\mathbf{YV}_1) \\ (\mathbf{X}_1\mathbf{Z}) & (\mathbf{X}_1\mathbf{V}) & (\mathbf{X}_1\mathbf{Z}_1) & (\mathbf{X}_1\mathbf{V}_1) \\ (\mathbf{Y}_1\mathbf{Z}) & (\mathbf{Y}_1\mathbf{V}) & (\mathbf{Y}_1\mathbf{Z}_1) & (\mathbf{Y}_1\mathbf{V}_1) \end{pmatrix} \neq 0. \tag{A.17}$$

As in the case of $SU(3)'\otimes SU(3)''$, the condition (A.17) ensures diagonality of the neutral current obtained by commuting the charged currents: $M^0=[M^+,M^-]/2$. If the condition (A.14) is satisfied, the required invariants have the values

$$\left.\begin{aligned}
&\mathbf{X}^2=\mathbf{Y}^2=\mathbf{Z}^2=\mathbf{V}^2=\mathbf{X}_1^2=\mathbf{Y}_1^2=\mathbf{Z}_1^2=\mathbf{V}_1^2=1;\\
&(\mathbf{XY})=(\mathbf{XX}_1)=(\mathbf{XY}_1)=(\mathbf{Y}_1\mathbf{X}_1)=(\mathbf{YY}_1)=(\mathbf{YX}_1)=(\mathbf{ZZ}_1)\\
&=(\mathbf{ZV})=(\mathbf{ZV}_1)=(\mathbf{VV}_1)=(\mathbf{VZ}_1)=(\mathbf{Z}_1\mathbf{V}_1)=0;\\
&(\mathbf{XZ})=2\cos\alpha\cos\beta;\quad (\mathbf{YZ})=2\cos\alpha\sin\beta;\quad (\mathbf{X}_1\mathbf{Z})=2\sin\alpha\cos\gamma;\\
&(\mathbf{Y}_1\mathbf{Z})=2\sin\alpha\sin\gamma;\quad (\mathbf{XZ}_1)=2\sin\alpha\cos\beta;\quad (\mathbf{YZ}_1)=2\sin\alpha\sin\beta;\\
&(\mathbf{X}_1\mathbf{Z}_1)=-2\cos\alpha\cos\gamma;\quad (\mathbf{Y}_1\mathbf{Z}_1)=-2\cos\alpha\sin\gamma;\quad (\mathbf{XV})=2\cos\delta\sin\beta;\\
&(\mathbf{YV})=-2\cos\beta\cos\delta;\quad (\mathbf{X}_1\mathbf{V})=2\sin\gamma\sin\delta;\quad (\mathbf{Y}_1\mathbf{V})=-2\cos\gamma\sin\delta;\\
&(\mathbf{XV}_1)=2\sin\beta\sin\delta;\quad (\mathbf{YV}_1)=-2\cos\beta\sin\delta;\quad (\mathbf{X}_1\mathbf{V}_1)=-2\sin\gamma\cos\delta;\\
&(\mathbf{Y}_1\mathbf{V}_1)=2\cos\gamma\cos\delta,
\end{aligned}\right\} \tag{A.18}$$

where α, β, γ, and δ are arbitrary parameters.

Substituting the obtained values of the parameters (A.18) into the original system of equations (A.16), we obtain its general solution:

$$\left.\begin{aligned}
&\mathbf{Z}=\cos\alpha\cos\beta\mathbf{X}+\sin\alpha\cos\beta\mathbf{Y}+\sin\beta\cos\gamma\mathbf{X}_1+\sin\beta\sin\gamma\mathbf{Y}_1;\\
&\mathbf{Z}_1=\cos\alpha\sin\beta\mathbf{X}+\sin\alpha\sin\beta\mathbf{Y}+\cos\beta\cos\gamma\mathbf{X}_1-\cos\beta\sin\gamma\mathbf{Y}_1;\\
&\mathbf{V}=\sin\alpha\cos\delta\mathbf{X}-\cos\alpha\cos\delta\mathbf{Y}+\sin\gamma\sin\delta\mathbf{X}_1-\cos\gamma\sin\delta\mathbf{Y}_1;\\
&\mathbf{V}_1=\sin\alpha\sin\delta\mathbf{X}-\cos\alpha\sin\delta\mathbf{Y}-\sin\gamma\cos\delta\mathbf{X}_1+\cos\gamma\cos\delta\mathbf{Y}_1.
\end{aligned}\right\} \tag{A.19}$$

In addition, ten conditions [see (A.18)] are imposed on the 32 parameters that make up the vectors $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{X}_1$, and $\mathbf{Y}_1$:

$$\mathbf{X}^2=\mathbf{Y}^2=\mathbf{X}_1^2=\mathbf{Y}_1^2=1; \tag{A.20}$$

$$(\mathbf{XY})=(\mathbf{XX}_1)=(\mathbf{XY}_1)=(\mathbf{YX}_1)=(\mathbf{YY}_1)=(\mathbf{X}_1\mathbf{Y}_1)=0. \tag{A.21}$$

Thus, together with the parameters α, β, γ, and δ, we obtain a current that depends on 26 parameters after the condition of the algebra $SU(2)$ for the weak charges is satisfied.

The obtained solution (A.19)–(A.21) for the matrix M^+ must now be substituted in the expression for the neutral current $M^0=[M^+,M^-]/2$. By direct calculations we can readily show that the obtained current is completely diagonal and that its quark structure is identical to that of the electromagnetic current, i.e., it has the form

$$\begin{aligned}
J_\mu^0=&(\tilde{p}^2\gamma_\mu(1-\gamma_5)p_2)+(\tilde{p}_3\gamma_\mu(1-\gamma_5)p_3)-(\tilde{n}_1\gamma_\mu(1-\gamma_5)n_1)\\
&-(\tilde{\lambda}_1\gamma_\mu(1-\gamma_5)\lambda_1)-(\tilde{n}_4\gamma_\mu(1-\gamma_5)n_4)-(\tilde{\lambda}_4\gamma_\mu(1-\gamma_5)\lambda_4)\\
&+(\tilde{\chi}_2\gamma_\mu(1-\gamma_5)\chi_2)+(\tilde{\chi}_3\gamma_\mu(1-\gamma_5)\chi_3).
\end{aligned} \tag{A.22}$$

Thus, the solution (A.19)–(A.21) is completely analogous to the solution (26), (23′), and (24′) of the group $SU(3)'\otimes SU(3)''$, and the requirement (A.17) corresponds to the requirement (22).

The transition from the group $SU(4)'\otimes SU(4)''$ to $SU(3)'\otimes SU(3)''$ is made in the most direct manner: We set $\mathbf{X}_1$, $\mathbf{Y}_1$, $\mathbf{Z}_1$, and $\mathbf{V}_1$ equal to zero and "truncate" $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$, and $\mathbf{V}$, i.e., we take $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$, and $\mathbf{V}$ from the definition (19). The determinant (A.19) goes over into the determinant (22).

It is now easy to see that such a transition to the group $SU(4)'\otimes SU(3)''$ is not possible without violating the condition analogous to the conditions (A.17) and (22) that arises in this case. For if we attempt to obtain a solution by the method just described, it is necessary in the case of the matrix M^+ of the group $SU(4)'\otimes SU(3)''$ to introduce corresponding "vectors," i.e., to denote by $\mathbf{X}$ (with appropriate reservations) the second column of the matrix (50), by $\mathbf{Y}$ the third column, and take the vectors $\mathbf{Z}$, $\mathbf{Z}_1$, $\mathbf{V}$, and $\mathbf{V}_1$ to be the fifth, eighth, ninth, and 12th rows of the same matrix (50). The vectors $\mathbf{X}_1$ and $\mathbf{Y}_1$ are then zero. If we substitute them in the determinant (A.17), we obtain a contradiction to (A.17), since this determinant then necessarily vanishes, and we do not have a possibility of obtaining a solution analogous to (A.19)–(A.21). Thus, the method described in the present Appendix and in the text is not applicable in the case of the model of three quartets. One of the examples of the weak current that can be obtained in this case is the current of the KK model considered in the review.

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