# Analog states and resonances

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A semimicroscopic approach to the theory of resonance nuclear reactions based on the shell model is presented. It is applied to the analysis of the basic parameters of analog states and analog resonances in the energy-averaged cross sections for the interaction of nucleons with spherical nuclei. The conclusions of the theory are compared with experimental data and the results of other comparable theoretical approaches.

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## INTRODUCTION

In a period of less than 20 years, analog states and resonances have come to constitute a rapidly developing branch of low-energy nuclear physics. Experimentally, analog states are manifested as narrow resonances in the cross sections for the interaction of nucleons and $\gamma$ rays with nuclei. The specific feature of analog resonances which distinguishes them from other giant resonances is the smallness (on the nuclear scale) of their total widths. This is the basis for asserting that isospin is conserved approximately in medium and heavy nuclei. Thus, the interest in the study of analog resonances arises from the possibility of obtaining information about: 1) the mechanism of nuclear reactions (in the first place, with nucleons), 2) the mechanism of the breaking of the isospin symmetry of the nuclear states, and 3) the structure of the ground state and low-lying states of nuclei whose analogs are excited in reactions or to which analog resonances decay.

In the present review, we propose a description of the main parameters of analog resonances on the basis of the shell approach to the theory of nuclear reactions. The results are compared with some experimental data, and also with the conclusions of other comparable theoretical approaches. In Sec. 1, we present aspects of the theory of resonance nuclear reactions. On the basis of the methods of the theory of finite Fermi systems, we formulate a variant of the shell approach to the theory of nuclear reactions (semimicroscopic approach). We parametrize the energy-averaged amplitudes and cross sections of resonance reactions in the region of an isolated doorway state. In Sec. 2, analog states and resonances are investigated in the framework of the shell model without allowance for their coupling to complex configurations. We consider the isobar symmetry of the Hamiltonian of the shell model and study the Coulomb structure of the symmetry energy, the Coulomb mixing of states with different values of the isospin, collective analog states and isotopic splitting of multipole giant resonances, and the natural proton widths of analog resonances; we also formulate a method for taking into account strong coupling of proton channels of analog-resonance decay. In Sec. 3, we investigate the formation of analog resonances, i.e., the coupling of the analog states to the continuum and configurations of complicated nature. In terms of the shell model and the optical model parametrized in a definite manner, expressions are obtained for the basic parameters of analog resonances (partial proton and neutron widths, widths for "decay" into complicated configurations, etc.), and we investigate the asymmetry and fine structure of analog resonances and also quasielastic $(p,n)$ scattering. In the Conclusions, we discuss some open problems. The bibliography in no way pretends to completeness and is given only to the extent needed for the exposition.

## 1. ASPECTS OF THE THEORY OF RESONANCE NUCLEAR REACTIONS

*Shell Approach to the Theory of Resonance Nuclear Reactions.* Resonance nuclear reactions with nucleons and low-energy $\gamma$ rays constitute the most important source of information about the reaction mechanism and structure of highly excited nuclear states. The diversity of the phenomena (the various types of resonances and the different decay channels of resonances of a given kind) and the large body of experimental data (partial cross sections, strength functions, etc.) make it desirable to have a unified approach to the description of not only the structure of nuclear states but also the reaction mechanisms. For not too high excitation energies, such an approach can in principle be based on the shell model. The shell model and its most systematic formulation in the framework of finite Fermi systems[1] have been fairly well investigated in their application to the ground state and low-lying states of nuclei. Therefore, theoretical and practical interest attaches to an investigation of the possibility of applying the shell model to the description of the parameters of the cross sections of resonance nuclear reactions and, consequently, to the structure of highly excited nuclear states.

A literal application of the shell model to the description of highly excited states of medium and heavy nuclei and the corresponding resonance nuclear reactions with nucleons and $\gamma$ rays encounters certain difficulties, which are associated with the need: 1) to include in the treatment a large number of shell configurations (not only particle–hole configurations but also many-particle configurations: two-particle–two-hole, three-particle–three-hole, etc.); 2) to take into account the continuum states for the nucleons; and 3) to introduce new phenomenological constants to describe correlations of the nucleons at short distances. The existing approaches attack these difficulties in different ways. For example, in the monograph of Ref. 2 the simple configurations are coupled to the many-particle configurations at the level of a parametrization of the energy-

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averaged reaction amplitudes. In Ref. 3, the main attention is devoted to the problems 1) and 3) mentioned above. In the framework of the shell model with continuum, the basis of many-particle configurations is restricted, and therefore such a model is used for sufficiently light nuclei.[4]

One of the ways to apply the shell model to the description of highly excited states of medium and heavy nuclei and the corresponding resonance nuclear reactions is based on the existence at sufficiently high excitation energies of a "hierarchy" of states, which is formed by simple and complicated configurations. Giant resonances in the reaction cross sections correspond to isolated simple configurations. Among the systematically encountered giant resonances there are single-particle resonances, multipole giant resonances, and isobar analog resonances. The coupling of the simple configurations to the more complicated ones leads to a fine structure of the giant resonances and, when the cross sections are averaged, to a modification (from the "natural" values) of the parameters of the giant resonances.

If one eschews a description of the properties of individual resonances of the compound nucleus and goes over to a description of the energy-averaged cross sections for the interaction of nucleons and $\gamma$ rays with nuclei, one can formulate on the basis of the shell model a semiempirical approach to the theory of nuclear reactions. In this approach, the parameters of the average cross sections can be expressed in terms of the shell model and a (phenomenological) optical model based on it. It is important that the optical model is not introduced into the theory *a priori*, since there do not exist *a priori* prescriptions for such an introduction, but it arises as a result of averaging of the "microscopic" equations for the reaction amplitudes. In this approach we have the following features: 1) the shell model is used to describe the simple configurations (with allowance for their coupling to the continuum) and also the correlations of nucleons at large distances; the parameters of this problem are determined by an analysis of the properties of the ground state and low-lying states of the nuclei; 2) the coupling of the simple configurations to the many-particle configurations is taken into account on the average by means of an optical model for the nucleons which is parametrized in a definite manner; the parameters of this problem can be found from an analysis of the cross section for elastic scattering of nucleons by nuclei and the reaction cross sections. Thus, the semimicroscopic approach has the following advantages: 1) it enables one to overcome to a a certain extent the difficulties listed above when the shell model is applied to the descritpion of resonance nuclear reactions with nucleons and $\gamma$ rays; 2) it is fairly "economic" in its choice of tools; 3) it is fairly productive, since it enables one to obtain a quantitative interpretation of various parameters of various giant resonances in the framework of a unified description. An example of the application of the semimicroscopic approach to the analysis of single-particle resonances is contained in the review of Ref. 5.

*Transition to the Optical Model by the Methods of the Theory of Finite Systems.* The transition from the shell model to the optical model to describe the energy-averaged reaction amplitudes is made most consistently by the methods of the theory of finite Fermi systems (Green's-function method). This is not fortuitous, since the single-particle Green's function for a Fermi system contains in principle all information about the relaxation of the single-particle degree of freedom. We give the basic properties of the single-particle Green's function $G(x, x')$ (Ref. 1) needed for what follows. By definition,

$$G(x, x') = -\mathrm{i}\langle 0 \mid \tilde{T}\Psi(x)\Psi^{+}(x') \mid 0\rangle; \quad x = (\mathbf{r}, t), \tag{1}$$

where $\Psi(x)$ is the operator of nucleon annihilation in the Heisenberg representation, $\tilde{T}$ is the time ordering operator, and the expectation value is taken with respect to the ground state of the system of $N$ particles (in the considered case, with respect to the ground state of the shell-model Hamiltonian). The transition to the Fourier representation with respect to $\tau = t - t'$ and expansion with respect to spherical harmonics is made in accordance with the relations ($\hbar = 1$)

$$\left.\begin{aligned} G(\mathbf{r}, \mathbf{r}'; \tau) &= \int G(\mathbf{r}, \mathbf{r}'; \varepsilon)\exp(-\mathrm{i}\varepsilon\tau)\, d\varepsilon/2\pi; \\ G(\mathbf{r}, \mathbf{r}'; \varepsilon) &= \frac{1}{rr'}\sum_{jlm} G_{(jl)}(r, r'; \varepsilon)\Phi_{jlm}(\mathbf{n})\Phi^{\dagger}_{jlm}(\mathbf{n}'), \end{aligned}\right\} \tag{2}$$

where $\Phi_{jlm}(\mathbf{n})$ are spherical spinors. Let $r^{-}\chi_{0Ejl}(r)$ be a regular solution of the radial Schrödinger equation with shell-model potential, so that $(h_{0jl}(r) - E)\chi_{0Ejl}(r) = 0$. (For $E > 0$, the functions $\chi_{0Ejl}$ are assumed to be normalized to a $\delta$ function of the energy.) Then the Green's function $G_{(jl)}(r, r'; \varepsilon)$ can be represented in the form

$$G_{(jl)}(r, r'; \varepsilon) = \sum_{E} \chi_{0Ejl}(r)\chi_{0Ejl}(r')\, G_{Ejl}(\varepsilon). \tag{3}$$

[The assumption that $G_E(\varepsilon)$ is diagonal is discussed below.] For the Green's function $G_E(\varepsilon)$, we have the Lehmann expansion

$$G_E(\varepsilon) = \sum_{c}\frac{b_c(E)}{\varepsilon - \mu - E_c + \mathrm{i}\delta} + \sum_{c'}\frac{\tilde{b}_{c'}(E)}{\varepsilon - \mu + E_{c'} - \mathrm{i}\delta} \equiv G_E^{(+)}(\varepsilon) + G_E^{(-)}(\varepsilon). \tag{4}$$

Here, $\mu$ is the chemical potential, and $E_c$ and $E_{c'}$ are the energies of the excited states of the systems of $N+1$ and $N-1$ particles, respectively. In accordance with (1)–(4), the Green's function corresponding to the motion of the nucleons in the average field of the shell model has the form (here and in what follows, we ignore pairing)

$$\begin{aligned} G_E(\varepsilon) \to G_{0E}(\varepsilon) &= [1 - n(E)]/(\varepsilon - E + \mathrm{i}\delta) + n(E)/(\varepsilon - E - i\delta) \\ &\equiv G_{0E}^{(+)}(\varepsilon) + G_{0E}^{(-)}(\varepsilon), \end{aligned} \tag{5}$$

where $n(E)$ are the population numbers. The Green's function $G(r, r'; \varepsilon)$, which corresponds to motion of a nucleon in the nuclear medium, satisfies the Dyson equation

$$\begin{aligned} G(r, r'; \varepsilon) &= G_0(r, r'; \varepsilon) \\ &+ \int G_0(r, r_1; \varepsilon)\, T_0(r_1, r_2; \varepsilon)\, G(r_2, r'; \varepsilon)\, dr_1\, dr_2 \\ &= G_0(r, r'; \varepsilon) + \int G_0(r, r_1; \varepsilon)\, T(r_1, r_2; \varepsilon)\, G_0(r_2, r'; \varepsilon)\, dr_1\, dr_2. \end{aligned} \tag{6}$$

Here, $T_0$ and $T$ are, respectively, the irreducible and

reducible self-energy parts, and for sufficiently large values of $|\varepsilon-\mu|$ they are rapidly varying functions of their argument because of the virtual excitation of many-particle configurations.

In the analysis of the energy-averaged reaction amplitudes, it is necessary to calculate the Green's function for complex values of $\varepsilon$: $\varepsilon-\mu\to\varepsilon-\mu+iI\,\mathrm{Sgn}(\varepsilon-\mu)$ ($I$ is the averaging interval). In the absence of isolated doorway states, when it is justified to assume that correlations at short distances (of the order of the reciprocal Fermi momentum $K_0^{-1}$) play a dominant part in the formation of $T_0(r,r';\varepsilon-\mu+iI\,\mathrm{Sgn}(\varepsilon-\mu))$, this last can be parametrized in the form

$$T_0(r,r';\varepsilon-\mu+iI\,\mathrm{Sgn}(\varepsilon-\mu))=\Delta h(r;\varepsilon-\mu)\,\delta(r-r'), \tag{7}$$

where $\mathrm{Im}\,\Delta h(r;\ \varepsilon-\mu)=-w(r;\ |\varepsilon-\mu|)\,\mathrm{Sgn}(\varepsilon-\mu)$ (Ref. 1). After this substitution, the function $G(r,r';\ \varepsilon-\mu+iI\,\mathrm{Sgn}(\varepsilon-\mu))\equiv g(r,r';\ \varepsilon-\mu)$ is in accordance with (6) and (7) the Green's function of the radial Schrödinger equation with the optical-model Hamiltonian

$$\left.\begin{aligned}&(h_{jl}-\varepsilon)\,g_{(jl)}(r,r';\varepsilon-\mu)=-\delta(r-r');\\&h_{jl}(r;\varepsilon-\mu)=h_{0jl}(r)+\Delta h(r;\varepsilon-\mu).\end{aligned}\right\} \tag{8}$$

In connection with (7) and (8), we note that: 1) the optical model arises for both positive and negative energies; 2) the optical correction to the shell-model potential is not calculated in the framework of the semimicroscopic approach but rather is parametrized by means of a small number of phenomenological parameters.

The procedure for averaging the amplitude for scattering of a nucleon (or $\gamma$ ray) by a nucleus, $f(E)$, is based on the analytic properties of the amplitude. The assumption that simple poles of the function $f(E)$ correspond to resonances of the compound nucleus leads to the well-known relation (see, for example, Ref. 2):

$$\bar{f}(E)=f(E+iI), \tag{9}$$

where the averaging interval satisfies $I\gg d=\rho^{-1}$ ($d$ is the average energy interval between resonances with definite values of the angular momentum and parity).

We now analyze the average amplitude for scattering of a nucleon by a nucleus in the framework of the shell approach. The reducible self-energy part $T$ determines the diagonal element of the $S$ matrix of a nucleon with energy $E>0$ corresponding to the given values $jl$:

$$S(E)=\exp(2i\delta_0)-2\pi i\int\chi_{0E}^{(+)}(r)\,T(r,r';\varepsilon=E)\,\chi_{0E}^{(+)}(r')\,dr\,dr'. \tag{10}$$

Here, $\delta_0(E)$ is the phase shift for scattering of the nucleon by the shell potential, and $\chi_{0E}^{(\pm)}=\exp(\pm i\delta_0)\chi_{0E}$. Equation (10) is derived by means of the relations (5) and (6) in the same way as for scattering by a potential.[6] In accordance with (9) and (10), we have the expression

$$\begin{aligned}\bar{S}(E)&=S(E+iI)\\&=\exp(2i\delta_0)-2\pi i\int\chi_{0E}^{(+)}(r)\,T_0(r,r';\,E+iI)\,\chi_E^{(+)}(r')\,dr\,dr',\end{aligned} \tag{11}$$

where the function $\chi_E^{(\pm)}(r)$ is determined by the relation $\{T_0\chi_E^{(+)}\}=\{T\chi_{0E}^{(+)}\}$ (the brackets $\{\ldots\}$ denote integration over the configuration space). In accordance with (6)–(8), after the substitution $E\to E+iI$, this function satisfies the equation

$$\chi_E^{(+)}(r)=\chi_{0E}^{(+)}(r)+\int g_0(r,r';\,E)\,\Delta h(r';\,E)\,\chi_E^{(+)}(r')\,dr', \tag{12}$$

i.e., it is an eigenfunction of the optical-model Hamiltonian: $(h-E)\chi_E^{(\pm)}=0$. On the basis of (11) and (12), we conclude that

$$\bar{S}(E)=\exp[2i\xi-2\eta], \tag{13}$$

where $\xi$ and $\eta$ are the real and imaginary parts of the phase shift for nucleon scattering by the optical potential. The assumption that the nucleon decay of the resonances of the compound nucleus occurs through the different channels in a statistically independent way leads to the assertion that the average $T$ (and, therefore, $S$) matrix is diagonal with respect to the indices $a$ that label the nucleon channels:

$$\bar{S}_{aa'}=\delta_{aa'}\exp[2i\xi_a-2\eta_a]. \tag{14}$$

The single-particle Green's function determines a quantity that is very important in applications, namely, the polarizability $\mathscr{P}(\omega)$, of the nucleus corresponding to the external single-particle field $V$. If $V=\sum_i V(i)$ is a multipole operator, then $\mathrm{Im}\,\mathscr{P}(\omega)$ determines the cross section for absorption of $\gamma$ rays of the corresponding multipolarity and is proportional to the imaginary part of the amplitude for scattering of $\gamma$ rays through zero angle. Therefore, in accordance with (9)

$$\mathrm{Im}\,\bar{\mathscr{P}}(\omega)=\mathrm{Im}\,\mathscr{P}(\omega+iI). \tag{15}$$

By definition,

$$\mathscr{P}(\omega)=\sum_s|V_{s0}|^2\left(\frac{1}{\omega-E_s+i\delta}-\frac{1}{\omega+E_s-i\delta}\right), \tag{16}$$

where the subscript $s$ labels the excited states of the system. The polarizability $\mathscr{P}(\omega)$ is determined as follows by the single-particle Green's function[1]:

$$\mathscr{P}=\{V\delta\rho[\tilde{V}]\};\quad \tilde{V}=V+\{\hat{F}\delta\rho[\tilde{V}]\}. \tag{17}$$

Here, $\delta\rho[V]$ is the change in the single-particle density matrix induced by the weak field, and $\tilde{V}$ is an effective field, which differs from the external field $V$ because of the polarization effects due to the effective interaction $\hat{F}$ of the nucleons in the particle–hole channel. The change in the density matrix to first order in the field $V$ is $\delta\rho[V]=\{AV\}$, where $A(\omega)$ is the so-called response function, which is the change in the density matrix induced by a unit field. An explicit expression for this function can be obtained on the basis of the relations (1)–(3):

$$\left.\begin{aligned}&A(r,r';\omega)=\sum_{ab}t_{ab}\chi_a(r)\,\chi_a(r')\,\chi_b(r)\,\chi_b(r')\,A_{ab}(\omega);\\&A_{ab}(\omega)=\int\limits_{\tau\to0-}G_a(\varepsilon)\,G_b(\varepsilon-\omega)\exp(-i\varepsilon\tau)\,d\varepsilon/2\pi,\end{aligned}\right\} \tag{18}$$

where $t_{ab}$ is a geometrical factor that depends on the multipolarity of the external field and the quantum numbers $j_a\,l_a$ and $j_b\,l_b$. For fields of monopole symmetry, which are the ones essentially considered in the present paper, $t_{ab}=t_a\delta_{ab}=(2j_b+1)\delta_{ab}$. In (17), we express $\mathscr{P}(\omega)$ and $\tilde{V}(\omega)$ in terms of the response function, assuming that the effective interaction is taken in the form $\hat{F}(\mathbf{r}_1,\mathbf{r}_2)\to F\delta(\mathbf{r}_1-\mathbf{r}_2)$:

$$\mathscr{P}(\omega)=\{VA\tilde{V}\}\equiv\int V(r)\,A(r,r';\omega)\,\tilde{V}(r;\omega)\,dr\,dr'; \tag{19}$$

$$\tilde{V}(r,\omega)=V(r)+(F/4\pi r^2)\int A(r,r';\omega)\,\tilde{V}(r';\omega)\,dr'. \qquad (20)$$

These relations make it possible to obtain an expression for $\mathrm{Im}\,\mathscr{P}(\omega)$ in a form convenient for applications to problems in which the continuum is taken into account. Multiplying Eq. (20) term by term once by $\tilde{V}^*$, and a second time by $V$, and integrating over the configuration space, we find in accordance with (19) and (20)

$$\mathrm{Im}\,\mathscr{P}(\omega)=\mathrm{Im}\{\tilde{V}^*A\tilde{V}\}. \qquad (21)$$

We now analyze the average response function $\bar{A}(\omega)=A(\omega+iI)$. An explicit expression for this quantity in terms of the shell and optical models with exact allowance for the continuum can be found only in some limiting cases. We take into account initially the damping of only the single-particle (but not the single-hole) states. This approximation corresponds to the following substitution in the expression (18) for $A_{ab}(\omega)$: $G_b(\varepsilon)\to G_{0b}(\varepsilon)$. Calculation of this quantity in this case in accordance with (18), (4), and (5) leads to the formula

$$\begin{aligned}A_{ab}^{ph_0}(\omega)&=n(E_b)\,G_a(E_b+\omega)-G_a^{(-)}(E_b+\omega)\\&=-(1-n(E_b))\,G_a(E_b+\omega)+G_a^{(+)}(E_b+\omega).\end{aligned} \qquad (22)$$

Using these expressions for $A_{ab}^{ph_0}(\omega)$, and also the expansion (3), we represent the response function (18) in the form

$$\left.\begin{aligned}&A^{ph_0}=A_1^{ph_0}+A_2^{ph_0};\\&A_1^{ph_0}(r,r';\omega)=\sum_{(a)b}t_{ab}G_{(a)}(r,r';E_b+\omega)\,(n(E_b)\\&-n(E_b+\omega))\,\chi_b(r)\,\chi_b(r');\\&A_2^{ph_0}(r,r';\omega)=\sum_{ab}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_b+\omega)\,G_a^{(+)}(E_b+\omega)\\&-(1-n(E_b+\omega))\,G_a^{(-)}(E_b+\omega))\,\chi_b(r)\,\chi_b(r').\end{aligned}\right\} \qquad (23)$$

The decomposition of $A^{ph_0}$ into two terms is done in such a way that $\mathrm{Im}\,A_2^{ph_0}=0$, as follows from the Lehman expansion (4). Therefore, after the substitution $\omega\to\omega+iI$, we can calculate $A_2^{ph_0}$ to order $(w/D)^2$ ($D$ is the energy interval between the single-particle levels with equal values of the angular momentum and parity) without allowance for the coupling of the single-particle configurations to the many-particle ones, i.e., by the substitution $G_a^{(\pm)}\to G_{0a}^{(\pm)}$. Thus, on the basis of (23), (8), and (5) we obtain the following expression for the average response function $\bar{A}^{ph_0}=\bar{A}_1^{ph_0}+\bar{A}_2^{ph_0}$ in terms of the shell and optical models:

$$\left.\begin{aligned}&\bar{A}_1^{ph_0}(r,r';\omega)=\sum_{(a)b}t_{ab}g_{(a)}(r,r';E_b+\omega)\,(n(E_b)\\&-n(E_b+\omega))\,\chi_b(r)\,\chi_b(r');\\&\bar{A}_2^{ph_0}(r,r';\omega)=\sum_{ab}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_b+\omega)\\&-n(E_a))\,(E_b-E_a+\omega)^{-1}\,\chi_b(r)\,\chi_b(r').\end{aligned}\right\} \qquad (24)$$

Formula (24) for $\mathrm{Im}\,\bar{A}_1^{ph_0}$ is used for quantitative interpretation of the valence mechanism of the $(\gamma n)$ reaction near the threshold in Ref. 5.

Similarly, the response function can be calculated with allowance for the damping of only the single-hole (but not single-particle) states. This approximation corresponds to the following substitution in the expression (18) for $A_{ab}(\omega)$: $G_a(\varepsilon)\to G_{0a}(\varepsilon)$. In this case, calculation of the corresponding quantity in accordance with (18), (4), and (5) leads to the formula

$$\begin{aligned}A_{ab}^{p_0h}(\omega)&=-(1-n(E_a))\,G_b(E_a-\omega)+G_b^{(+)}(E_a-\omega)\\&=n(E_a)\,G_b(E_a-\omega)-G_b^{(-)}(E_a-\omega).\end{aligned} \qquad (25)$$

Using this expression for $A_{ab}^{p_0h}$, and also the expansion (3), we represent the response function (18) in the form

$$\left.\begin{aligned}&A^{p_0h}=A_1^{p_0h}+A_2^{p_0h};\\&A_1^{p_0h}(r,r';\omega)=-\sum_{a(b)}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_a-\omega)\\&-n(E_a))\,G_{(b)}(r,r';E_a-\omega);\\&A_2^{p_0h}(r,r';\omega)=\sum_{ab}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_a-\omega)\,G_b^{(+)}(E_a-\omega)\\&-(1-n(E_a-\omega))\,G_b^{(-)}(E_a-\omega))\,\chi_b(r)\,\chi_b(r'),\end{aligned}\right\} \qquad (26)$$

where $\mathrm{Im}\,A_2^{p_0h}=0$. After the substitution $\omega\to\omega+iI$ and on the basis of (26), (8), and (5), we obtain the following expression for the average response function $\bar{A}^{p_0h}=\bar{A}_1^{p_0h}+\bar{A}_2^{p_0h}$ in terms of the shell and optical models:

$$\left.\begin{aligned}&\bar{A}_1^{p_0h}(r,r';\omega)=-\sum_{a(b)}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_a-\omega)\\&-n(E_a))\,g_{(b)}(r,r';E_a-\omega);\\&\bar{A}_2^{p_0h}(r,r';\omega)=-\sum_{ab}t_{ab}\chi_a(r)\,\chi_a(r')\,(n(E_b)\\&-n(E_a-\omega))\,(E_a-E_b-\omega)^{-1}\,\chi_b(r)\,\chi_b(r').\end{aligned}\right\} \qquad (27)$$

Naturally, if the effects of damping of quasiparticles are included, Eqs. (24) and (27) lead to the well-known expression that takes into account exactly the contribution of the continuum to the response function:

$$\begin{aligned}A^{p_0h_0}(r,r';\omega)&=\sum_{(a)b}t_{ab}g_{0(a)}(r,r';E_b+\omega)\,n(E_b)\,\chi_b(r)\,\chi_b(r')\\&+\sum_{a(b)}t_{ab}g_{0(b)}(r,r';E_a-\omega)\,n(E_a)\,\chi_a(r)\,\chi_a(r'),\end{aligned} \qquad (28)$$

where $g_{0(jl)}$ is the Green's function that satisfies Eq. (8) with $\Delta h=0$. In this case, the substitution $\omega\to\omega+iI$ defines the rule for avoiding the poles in the expansions of the Green's function with respect to the eigenfunctions of the Hamiltonian $h_0$. The given expression for the response function is the basis for analyzing collective excitations in the random phase approximation with allowance for the continuum.[7,8]

One can take into account simultaneously the continuum and the damping of the particle and hole configurations in the approximation corresponding to an additive contribution of the damping of the particles and holes to the imaginary part of the response function. This approximation is valid in an energy interval outside a giant resonance (with the given multipolarity) with energy $\omega_g$:

$$(\omega-\omega_g)^2\gg w^2;\quad |\,\mathrm{Re}\,\bar{A}\,|\gg|\,\mathrm{Im}\,\bar{A}\,|. \qquad (29)$$

This approximation corresponds to the replacement in the formula for $A_{ab}(\omega)$ of the product of the two exact Green's functions by a sum[9]:

$$G_aG_b\to G_aG_{0b}+G_{0a}G_b-G_{0a}G_{0b}. \qquad (30)$$

This relation is a variant of perturbation theory in $T_0$ in the Dyson equation (6). In this approximation, we find the average response function in accordance with (30), combining Eqs. (24) and (27) for $\bar{A}^{ph_0}$ and $\bar{A}^{p_0h}$, respectively, and the two equivalent representations for $\bar{A}^{p_0h_0}$ (28) corresponding to these expressions:

$$\begin{aligned}\bar{A}&=\bar{A}^{ph_0}+\bar{A}^{p_0h}-\bar{A}^{p_0h_0}\\&=\bar{A}_1^{ph_0}+\bar{A}_1^{p_0h}+\bar{A}_2^{ph_0}+\bar{A}_2^{p_0h}-\left\{\begin{matrix}\bar{A}_1^{ph_0}+\bar{A}_2^{ph_0}\\ \bar{A}_1^{p_0h}+\bar{A}_2^{p_0h}\end{matrix}\right\}_{\Delta h\to 0}=\bar{A}_1+\bar{A}_2,\end{aligned} \qquad (31)$$

where $\mathrm{Im}\,\bar{A}_2 = 0$. We give the explicit expression for one of the two equivalent representations of $\mathrm{Im}\,\bar{A}$:

$$\mathrm{Im}\,\bar{A}(r, r'; \omega) = \mathrm{Im}\{\sum_{(a)b} t_{ab} g_{(a)}(r, r'; E_b+\omega)(n(E_b) - n(E_b+\omega))\chi_b(r)\chi_b(r') - \sum_{a(b)} t_{ab}\chi_a(r)\chi_a(r')(n(E_a-\omega) - n(E_a))(g_{(b)}(r, r'; E_a-\omega) - g_{0(b)}(r, r'; E_a-\omega))\}. \quad (32)$$

For comparison with the results of other papers, it is of interest to go to the limit in this expression in the case when the existence of the continuum is ignored. Using a first iteration of the integral equation for the Green's function $g(r, r'; \omega)$ and the approximation $w_{ab} = w_{aa}\delta_{ab}$, we obtain on the basis of (32)

$$\mathrm{Im}\,\bar{A}_{ab}(\omega) = [(n(E_b) - n(E_b+\omega))w_{aa} + (n(E_a-\omega) - n(E_a))w_{bb}(E_a - E_b - \omega)]^{-2}. \quad (33)$$

In such a form, a formula for $\mathrm{Im}\,\bar{A}_{ab}(\omega)$ was obtained in Ref. 10. Formula (33) differs from the value obtained in Ref. 11 (see also Ref. 5):

$$\mathrm{Im}\,\bar{A}'_{ab}(\omega) = (1-n(E_a))n(E_b)\,\mathrm{Im}\,\bar{A}_{ab}(\omega). \quad (34)$$

This difference, which arises from the neglect of the correlations in the ground state, leads to a quantity $\mathrm{Im}\,\bar{A}'(r, r'; \omega)$ that is approximately half of $\mathrm{Im}\,\bar{A}(r, r'; \omega)$

Another approximation for $\bar{A}(r, r'; \omega)$ in the case of arbitrary $\omega$ is the relation (24), in which the Green's function $g_{(a)}(r, r', E_b+\omega)$ is calculated with modified imaginary part of the optical potential: $w(E_b+\omega-\mu) - w(E_b+\omega-\mu) + w(\mu - E_b)$. Such a modification, which amounts to an approximate allowance for the damping of both the particle and the hole states, can be obtained by the methods used in Refs. 5 and 11. When the continuum is ignored, for small $\omega$ we obtain in this approximation the expression

$$\mathrm{Im}\,\bar{A}_{ab}(\omega) = [(n(E_b) - n(E_b+\omega))(w_{aa}(E_b+\omega-\mu) + w_{aa}(\mu - E_b))](E_a - E_b - \omega)^{-2},$$

which agrees better with (33) than Eq. (34). Therefore, instead of $\bar{A}'(r, r'; \omega)$ it is preferable to use the expression (24) modified in the indicated manner for the semiquantitative description of multipole giant resonances.

We note finally that the assumption of diagonality of the Green's function $G_E(\varepsilon)$, like the approximation $w_{ab} = w_{aa}\delta_{ab}$, corresponds to the inequality $w < D$. This inequality has a simple physical meaning: The relaxation time of a single-particle state is long compared with the time taken by a nucleon to traverse a distance of the order of the nuclear diameter. The opposite inequality corresponds to a "black" nucleus, when it is not justified to take the shell basis as the point of departure.

*Alternative Method for Transition to the Optical Model and Parametrization of the Average S Matrix for Nucleons in the Region of an Isolated Doorway State.* We present here an alternative method for going over to the optical model based on explicit allowance for many-particle configurations. The perspicuity of the results, the possibility of establishing a connection with comparable approaches, and also of obtaining a parametrization of the average $S$ matrix for scattering of nucleons by nuclei justify the interest in such an approach.[29,5] At the same time, some results of the method can be "controlled" on the basis of the more systematic approach presented above.

At sufficiently high excitation energies, there exist not only single-particle configurations but also many-particle configurations: two-particle–one-hole, three-particle–two-hole, etc. (to be specific, we consider odd compound nuclei). As a rule, the matrix elements of the interaction of the many-particle configurations exceed the energy gaps between them. It is therefore convenient to introduce the so-called "unrenormalized" levels of the compound nucleus $|\lambda\rangle$, which are determined by diagonalizing the shell-model Hamiltonian on the basis of the many-particle configurations. Resonance scattering of nucleons is realized through the coupling of the single-particle continuum states $|E\rangle$ to the states $|\lambda\rangle$, this being due to a certain effective interaction $H'$. Expressions for a diagonal element of the $T$ matrix in Eq. (10) can be obtained by summing a perturbation series, i.e., by the methods of the theory of quantum transitions[12]:

$$\langle E|T|E\rangle = \sum_{\lambda}\langle E|H'|\lambda\rangle(E-E_\lambda)^{-1}\langle\lambda|U|E\rangle; \quad (35)$$

$$\langle\lambda|U|E\rangle = \langle\lambda|H'|E\rangle + \sum_{\lambda'}\Pi_{\lambda\lambda'}(E)(E-E_{\lambda'})^{-1}\langle\lambda'|U|E\rangle, \quad (36)$$

$$\Pi_{\lambda\lambda'}(E) = \sum_{E'}\langle\lambda|H'|E'\rangle(E-E'+i\delta)^{-1}\langle E|H'|\lambda'\rangle. \quad (37)$$

Note that, literally, the summation over the virtual single-particle states $E'$ in (37) is extended only to unoccupied states. However, allowance for the correlations in the ground state (in the considered case, allowance for virtual excitation of single-hole configurations) makes it necessary to extend the summation over $E'$ to all states. We need not dwell on the proof of this assertion, since it is confirmed by the result which follows and is already well known. Indeed, if we define the wave function $\chi^{+}_{E}(r)$ by the relation $\langle\lambda|U|\chi^{(+)}_{0E}\rangle = \langle\lambda|H'|\chi^{(+)}_{E}\rangle$, then in accordance with (36) and (37) this function satisfies the integral equation (12):

$$\chi^{(+)}_{E}(r) = \chi^{(+)}_{0E}(r) + \int g_0(r, r'; E)T_0(r', r''; E)\chi^{(+)}_{E}(r'')\,dr'\,dr'', \quad (38)$$

where $T_0(r, r'; E) = \sum_\lambda (H'|\lambda\rangle\langle\lambda|H')(E-E_\lambda)^{-1}$ is a model representation of the irreducible self-energy part for $E > \mu$ [strictly speaking, from the expression given for $T_0$ we should subtract an energy-independent quantity to achieve $T_0(E-\mu-0)$]. After the substitution $E \to E + iI$, and on the basis of (35) and (38) and also the relations (10)–(13), we conclude that the $S$ matrix for scattering of a nucleon by the nucleus, averaged over the resonances of the compound nucleus, is identical to the $S$ matrix for nucleon scattering by the optical potential. The assumption of the statistical independence of the different decay channels of the resonances of the compound nucleus has the consequence that the average $S$ matrix is diagonal with respect to the indices that label the decay channels.

We now turn to the description of resonance scattering of nucleons with excitation of an isolated doorway state, ignoring initially the coupling to the many-particle configurations. Let $|d\rangle$ be the wave function of the doorway state (a collective particle–hole state "constructed" on the single-particle state $|b\rangle$), and $\mathcal{V}(\mathbf{r})$ be the field which realizes the coupling of the

doorway state to the single-particle continuum states. The amplitude of the nucleon width of the doorway state is determined by the matrix element of this field: $\langle d|H'|E\rangle \sim \int \chi_b(r)\mathscr{V}(r)\chi_{0E}^{(+)}(r)\,dr$. The actual form of the field $\mathscr{V}$ depends on the nature of the doorway state. For collective particle–hole excitations, this field is determined by the so-called transition density and the effective interaction in the particle–hole channel; for analog states, $\mathscr{V}$ is determined by the average Coulomb field (see below). Equations of the form (35)–(37), in which the substitution $\langle\lambda|H'|E\rangle \to \langle d|H'|E\rangle$, in which the lead in the single-channel case to the single-level Breit–Wigner formula

$$S_{aa}(E) = \exp(2i\delta_{0a})\,[1 - i\Gamma_{0a}^{\uparrow}(E - E_d - (\Pi_0)_{dd})^{-1}], \tag{39}$$

where

$$\Gamma_{0a}^{\uparrow} = -2\,\mathrm{Im}\,(\Pi_0)_{dd} = 2\pi t_{ab} n(E_b)\left|\int \chi_b \mathscr{V}\chi_{0E_a}^{(+)}\,dr\right|^2 \tag{40}$$

and it may be called the natural nucleon width of the doorway resonance.

We now take into account the coupling to configurations of complicated nature of both the doorway state and the continuum states. In this case, a diagonal element of the $T$ matrix is determined by a sum of two terms in accordance with the two possible mechanisms of attachment of the nucleon to the nucleus:

$$\begin{aligned}\langle E|T|E\rangle &= \langle E|H'|d\rangle(E - E_d)^{-1}\langle d|U|E\rangle \\ &+ \sum_{\lambda}\langle E|H'|\lambda\rangle(E - E_\lambda)^{-1}\langle\lambda|U|E\rangle.\end{aligned} \tag{41}$$

The transition amplitudes $\langle d|U|E\rangle$ and $\langle\lambda|U|E\rangle$ are characterized by a system of coupled equations that is obtained by summing perturbation series:

$$\left.\begin{aligned}&\langle d|U|E\rangle = \langle d|H'|E\rangle + (\Pi_0)_{dd}(E - E_d)^{-1}\langle d|U|E\rangle \\ &+ \sum_{\lambda}\langle d|\mathscr{H}'|\lambda\rangle(E - E_\lambda)^{-1}\langle\lambda|U|E\rangle; \\ &\langle\lambda|U|E\rangle = \langle\lambda|H'|E\rangle + \sum_{\lambda'}\Pi_{\lambda\lambda'}(E - E_{\lambda'})^{-1}\langle\lambda'|U|E\rangle \\ &+ \langle\lambda|\mathscr{H}'|d\rangle(E - E_d)^{-1}\langle d|U|E\rangle.\end{aligned}\right\} \tag{42}$$

The effective matrix element of the interaction $\langle d|\mathscr{H}|\lambda\rangle$ consists of two terms, one of which corresponds to "direct" interaction of the states $|d\rangle$ and $|\lambda\rangle$, and the second to interaction through the continuum:

$$\left.\begin{aligned}&\langle d|\mathscr{H}'|\lambda\rangle = \langle d|H'|\lambda\rangle + \Pi_{d\lambda}; \\ &\Pi_{d\lambda} = \sum_{E'}\langle d|H'|E'\rangle(E - E' + i\varepsilon)^{-1}\langle E'|H'|\lambda\rangle.\end{aligned}\right\} \tag{43}$$

Since the scattering outside an isolated doorway resonance is accompanied by the excitation of only many-particle configurations, it is natural to seek a solution of the system (42) in the form

$$\langle\lambda|U|E\rangle = \langle\lambda|U^{bg}|E\rangle + \langle\lambda|U^{res}|E\rangle$$

and, therefore,

$$\langle E|T|E\rangle = \langle E|U^{bg}|E\rangle + \langle E|T^{res}|E\rangle,$$

where the "background" amplitudes are determined by the relations (35)–(37). Solving the system (42), we obtain the following expression for a diagonal element of the $S$ matrix:

$$S_{aa}(E) = S_{aa}^{bg} - 2\pi i\langle d|H'|E\rangle_{eff}^2(E - E_d - \Pi_{dd})^{-1}, \tag{44}$$

where, in particular,

$$\langle d|H'|E\rangle_{eff} = \langle d|H'|E\rangle + \sum_{\lambda}\langle d|\mathscr{H}'|\lambda\rangle(E - E_\lambda)^{-1}\langle\lambda|U^{bg}|E\rangle. \tag{45}$$

After the substitution $E \to E + iI$ in (44) and with allowance for (14), the expression for the average $S$ matrix can be represented in the form

$$\overline{S}_{aa}(E) = \exp(2i\xi_a)\{\exp(-2\eta_a) - i\exp(2i\phi_a)\,\Gamma_a^{\uparrow}(E - E_r + (i/2)\,\Gamma^{-1}\}, \tag{46}$$

where $\Gamma_a^{\uparrow}$ is the effective (elastic) nucleon width of the resonance, $\phi_a$ is the resonance mixing phase, and $E_r$ and $\Gamma$ are the energy and total width of the resonance, respectively. If the statistical hypothesis

$$\overline{\langle d|H'|\lambda\rangle\langle\lambda|H'|E\rangle} = 0 \tag{47}$$

is valid or this equation is a consequence of specific selection rules, then it follows from (38), (40), (43), and (45) that $\phi_a$ and $\Gamma_a^{\uparrow}$ can be expressed in terms of the shell and optical models:

$$\exp[2i(\phi_a + \xi_a)]\,\Gamma_a^{\uparrow} = 2\pi t_{ab} n(E_b)\left(\int \chi_b \mathscr{V}\chi_{E_a}^{(+)}\,dr\right)^2. \tag{48}$$

The generalization of (39) and (46) for the case when several nucleon decay channels are open is straightforward:

$$\begin{aligned}\overline{S}_{aa'}(E) &= \exp[i(\xi_a + \xi_{a'})]\{\exp[-(\eta_a + \eta_{a'})]\,\delta_{aa'} \\ &- i\exp[i(\phi_a + \phi_{a'})]\,(\Gamma_a^{\uparrow})^{1/2}(\Gamma_{a'}^{\uparrow})^{1/2}(E - E_r + (i/2)\,\Gamma)^{-1}\}.\end{aligned} \tag{49}$$

In the approximation of independent nucleon channels, the expressions for the inelastic nucleon widths are given by relations of the kind (48). Allowance for coupling of the channels has the consequence that the field $\mathscr{V}$ becomes complex. In this case, Eq. (48) is modified:

$$\exp[2i(\phi_a + \xi_a)]\,\Gamma_a^{\uparrow} = 2\pi t_{ab} n(E_b)\int \chi_{E_a}^{(+)}\mathscr{T}^{*}\chi_b\,dr\int \chi_b \mathscr{V}\chi_{E_a}^{(+)}\,dr. \tag{50}$$

In the absence of many-particle configurations, such a modification ensures unitarity of the $S$ matrix.

Thus, the proposed method permits one to parametrize the average $S$ matrix for nucleons with excitation of an isolated doorway state on the "background" of the excitation of the many-particle configurations and also to indicate a method for the transition to the optical model in the formulas for the effective nucleon widths of the corresponding giant resonance.

In conclusion, we note that a similar approach to the description of the coupling of simple and complex configurations is widely used in Ref. 2. Evidently, an unfortunate choice of the basis (in particular, the states $|\lambda\rangle$ could include single-particle states of the discrete spectrum, contradicting the definition of the irreducible self-energy part) prevented the authors of the monograph of Ref. 2 from establishing the connection between the shell approach and the predictions of the optical model, and this reduced the practical value of this variant of the shell approach.

*Average Cross Sections of Resonance Reactions.* The energy-averaged cross sections for the interaction of nucleons with nuclei are determined by the average $S$ matrix when the resonances of the compound do not overlap. In an energy interval near such a resonance (which is a component of the fine structure of a giant resonance), the $S$ matrix can be represented in the form

$$S_{aa'}(E) = \exp[i(\psi_a + \psi_{a'})]\{|S_{aa'}^{bg}| - i\exp[i(\varphi_a + \varphi_{a'})]\gamma_{ca}^{1/2}\gamma_{ca'}^{1/2}(E - E_c + i\gamma_c/2)^{-1}\}. \quad (51)$$

In the absence of direct processes, $|S_{ad}^{bg}| \neq \delta_{aa'}$ and $\varphi_a$ reflect the existence of an isolated doorway state in the considered energy interval. The cross sections of elastic and inelastic scattering are determined by the elements of the $S$ matrix:

$$\sigma_{aa} = |1 - S_{aa}|^2; \ \sigma_{aa'} = |S_{aa'}|^2. \quad (52)$$

(The kinematic factors are assumed to be included in $\sigma$.) The unitarity condition $\sum_{a'} |S_{aa'}(E)|^2 = 1$ leads to the following connections between the parameters in Eq. (51):

$$1 = \sum_{a'} |S_{aa'}^{bg}|^2; \ \sum_{a'} |S_{aa'}^{bg}| \gamma_{ca}^{1/2}\gamma_{ca'}^{1/2} \sin(\varphi_a + \varphi_{a'}) = 0,$$
$$\gamma_c = \sum_a \gamma_{ca}; \ \sum_{a'} |S_{aa'}^{bg}| \gamma_{ca}^{1/2}\gamma_{ca'}^{1/2} \cos(\varphi_a + \varphi_{a'}) = \gamma_{ca}. \quad (53)$$

Averaging $S_{aa'}$ and $|S_{aa'}|^2$ over an energy interval $I \gg \rho^{-1}$, we find in accordance with (51)

$$|\overline{S}_{aa'}|^2 = |\,|S_{aa'}^{bg}| - \exp[i(\varphi_a + \varphi_{a'})]\pi\rho\overline{\gamma_{ca}^{1/2}\gamma_{ca'}^{1/2}}|^2,$$
$$\overline{|S_{aa'}|^2} = |S_{aa'}^{bg}|^2 + 2\pi\rho\gamma_a\gamma_{a'}/\gamma - 2\pi\rho|S_{aa'}^{bg}|\overline{\gamma_{ca}^{1/2}\gamma_{ca'}^{1/2}}\cos(\varphi_a + \varphi_{a'}), \quad (54)$$

where $\gamma_a = \overline{\gamma}_{ca}, \gamma = \overline{\gamma}_c$. The relations (53) and (54) make it possible to relate the so-called penetrability coefficient $\tilde{T}_a$ to the corresponding partial strength function $\rho\gamma_a$:

$$\tilde{T}_a \equiv 1 - \sum_{a'} |\overline{S}_{aa'}|^2 \cong 2\pi\rho\gamma_a. \quad (55)$$

In accordance with (54) and (55), the fluctuation cross section of the partial reaction is determined by the strength functions and, therefore, by the coefficients $\tilde{T}_a$:

$$\sigma_{aa'}^{fl} = \overline{|S_{aa'}|^2} - |\overline{S}_{aa'}|^2 \approx 2\pi\rho\gamma_a\gamma_{a'}/\gamma. \quad (56)$$

Thus, in accordance with (52), (55), and (56), the average cross sections of elastic and inelastic scattering are determined as follows by the elements of the average $S$ matrix (see, for example, Ref. 2):

$$\bar{\sigma}_{aa} = \sigma_{aa}^{opt} + \sigma_{aa}^{fl} = |1 - \overline{S}_{aa}|^2 + \tilde{T}_a^2/\sum_{a'}\tilde{T}_{a'},$$
$$\bar{\sigma}_{aa'} = \sigma_{aa'}^{opt} + \sigma_{aa'}^{fl} = |\overline{S}_{aa'}|^2 + \tilde{T}_a\tilde{T}_{a'}/\sum_{a''}\tilde{T}_{a''}. \quad (57)$$

In connection with the relations given here for the average cross sections, let us make two comments: 1) In the absence of a justified method of calculation, formulas (57) can also be used to analyze average cross sections in the case of overlapping resonances of the compound nucleus [only the single-channel case $|S|^2 = 1$ admits treatment of arbitrarily overlapping resonances, when $T = 1 - \exp[-2\pi\rho\gamma]$ (see Refs. 16 and 5)]; 2) away from an isolated doorway resonance, when the assumption of statistical independence of the decay of the resonances of the compound nucleus through the different channels is justified, i.e., when $\overline{S}_{aa} \sim \delta_{aa'}, |S_{aa'}^{bg}| = \delta_{aa'}, \tilde{T} \to T_a$, the relations (57) go over into the well-known Hauser–Feshbach formulas for the case when the number of open channels is large and the relative importance of each channel is small ($\gamma_a \ll \gamma = \sum_{a''}\gamma_{a''}$). Here and in what follows, these conditions are assumed to be satisfied.

## 2. ANALOG STATES IN THE SHELL MODEL

*Isobar Symmetry of the Shell Hamiltonian. Symmetry Energy.* The basis of the quantitative treatment of analog states which follows is the shell-model Hamiltonian

$$H = H_0 + H^{int}, \quad (58)$$

where $H_0$ is the single-particle Hamiltonian and $H^{int}$ is the Hamiltonian of the pairing interaction. Analysis of the single-particle motion (bound states, elastic scattering) shows that for nuclei with equal numbers of neutrons and protons ($N = Z$) the Hamiltonian $H_0$ can be taken in the form[13]

$$H_0 = \sum_i (K_i + U_0(r_i)) + \frac{1}{2}\sum_i (1 - \tau_i^{(3)})V_C(r_i). \quad (59)$$

Here, $K_i$ is the kinetic energy operator (we ignore the mass difference of the neutron and proton), $U_0$ is the isoscalar part of the shell potential (with allowance for the spin–orbit interaction), and $V_C(r)$ is the Coulomb energy of the interaction of the proton with the nucleus found in the Hartree approximation. The interaction Hamiltonian $H^{int} = \hat{F} + H_C^{int}$ contains the isotopically scalar nuclear part $\hat{F}$ and the "residual" Coulomb interaction of the protons $H_C^{int} = (e^2/8)\sum_{i\neq k}(1 - \tau_i^{(3)})(1 - \tau_k^{(3)}) \times r_{ik}^{-1} - 1/2\sum_i (1 - \tau_i^{(3)})V_C(r_i)$. Since the neutron excess in medium to heavy nuclei is small compared with the total number of particles, the Hamiltonian $H$ can be used to describe the states of these nuclei (with excitation energy lower than the Fermi energy). In the determination of the spectrum of single-particle excitations in this case it is necessary to separate from the interaction Hamiltonian the terms that contribute to the average field, this contribution being proportional to the neutron excess (see below).

Let us consider the parametrization of the Hamiltonian $H$. In accordance with the short range of the nuclear forces, the isoscalar part of the average field is parametrized in the same way as the nuclear density: $U_0(r) \sim f(r, R, a) = [1 + \exp((r - R)a^{-1}]^{-1}$. The Coulomb energy $V_C(r)$ is determined by the density distribution of the protons, for which one usually takes a "step" with radius $R_C$. An example of the shell potential (a set that makes it possible to reproduce the binding energy of the last nucleon for nuclei in a wide range of atomic masses) is given in Ref. 14. Below, the results of numerical calculations illustrating the conclusions of the theory are given on the basis of this set of parameters. The parametrization of the nuclear interaction $\hat{F}$ depends on the nature of the considered excitations. For long-wavelength excitations of particle–hole type, when large (of order $R$) relative distances between the colliding particles are important, the interaction can be parametrized in the form[1]

$$F = \frac{1}{4}\sum_{i\neq k}[F + G\sigma_i\sigma_k + (F' + G'\sigma_i\sigma_k)\tau_i\tau_k]\delta(r_i - r_k), \quad (60)$$

where $F$, $G$, $F'$, and $G'$ are coordinate-dependent phenomenological parameters characterizing the intensity of the interaction: $F = [(4/3)E_0\Omega/A]f = 348f\,\text{MeV}\cdot\text{F}^3$, etc. ($A$ is the number of nucleons, $\Omega$ is the volume of the nucleus, and $E_0$ is the Fermi energy); $f$, $g$, etc.,

are the dimensionless values of these parameters. The sets of corresponding quantities found by analyzing a large number of data on the properties of the ground and low-lying states of medium to heavy nuclei can be found in Ref. 15. It follows from these studies in particular that the parameter $f'$, which determines the intensity of the isovector part of the interaction, can be assumed to be independent of the coordinates.

If the Coulomb energy $V_C(r)$ is replaced by an average value

$$V_C(r) = \Delta E_C + \mathscr{V}_C(r), \tag{61}$$

then the Hamiltonian $H$ can be represented in the form

$$H = \overline{H} + \Delta H;\ \Delta H = \mathscr{V}_C + H_C^{\text{int}};\ \mathscr{V}_C = \frac{1}{2}\sum_i (1-\tau_i^{(3)})\mathscr{V}_C(r_i). \tag{62}$$

For the Hamiltonian $\overline{H}$ defined in this way, the following commutation rules hold:

$$[\overline{H},\ \mathbf{T}^2] = 0;\quad [\overline{H},\ T^{(-)}] = \Delta E_C T^{(-)}, \tag{63}$$

where $\mathbf{T} = 1/2\sum_i \tau_i = 1/2\mathscr{T}$ is the isospin operator of the nucleus, and $T^{(-)} = (T^{(1)} - iT^{(2)})/2$. If the isospin-nonconserving part of the Coulomb interaction $\Delta H$ is ignored, then on the basis of (62) and (63) we can conclude the following: 1) The states of the nucleus $(N, Z)$ can be classified by means of the isospin $T$ [and its third projection, which is equal to $(N-Z)/2 = T_0$]; 2) the isospin of the ground state $|0\rangle$ of the nucleus is equal to the value of the third projection, i.e., $T = T_0$, since $T^{(+)}|0\rangle = 0$, so that $|0\rangle = |T_0, T_0\rangle$; 3) among the excited states of the nucleus $(N-1, Z+1)$ there exists an analog state with isospin $T_> = T_0$ which exceeds by unity the isospin of the ground state of this nucleus, which is equal to $T_< = T_0 - 1$, with the wave function

$$|A\rangle = (2T_0)^{-1/2}\mathscr{T}^{(-)}|T_0,\ T_0\rangle = |T_0,\ T_0 - 1\rangle. \tag{64}$$

Thus, the analog state is the neighboring component of the isobar multiplet to the ground state of the parent nucleus; the states that follow in energy with isospin $T_>$ in the nucleus $(N-1, Z+1)$ are analogs of the first, second, etc., states of the parent nucleus; 4) the analog of the ground state is the lowest state in the nucleus $(N-1, Z+1)$ with isospin $T_>$, and therefore states of complicated nature with energy near an analog state have "normal" isospin $T_<$; 5) the mixing of states with different values of the isospin occurs through the interaction $\Delta H$ (62), i.e., both through the variable part of the average Coulomb field $\mathscr{V}_C$ and through the residual Coulomb interaction (Coulomb scattering amplitude). Some of the states listed above with different values of the isospin are shown schematically in Fig. 1. In accordance with (61)–(64), the energy $E_A$ of the analog state, measured from the ground state of the parent nucleus, is, up to terms of first order in $\Delta H$ inclusively,

$$E_A = \Delta E_C = (2T_0)^{-1}\langle 0|[\mathscr{T}^{(+)}[H,\ \mathscr{T}^{(-)}]]|0\rangle = \Delta E_C^{\text{dir}} + \Delta E_C^{\text{exc}}. \tag{65}$$

The first term $\Delta E_C^{\text{dir}}$, which is due to the direct (nonexchange) part of the Coulomb interaction $V_C$, is in accordance with (62) and (65) equal to

$$\Delta E_C^{\text{dir}} = (2T_0)^{-1}\langle 0|\sum_i V_C(r_i)\tau_i^{(3)}|0\rangle = (2T_0)^{-1}\int V_C(r)\,n(r)\,d^3r, \tag{66}$$

where

$$n(r) = \langle 0|\sum_i \tau_i^{(3)}\delta(\mathbf{r}-\mathbf{r}_i)|0\rangle \tag{67}$$

can be called the density of the excess neutrons, since $\int n(r)d^3r = N - Z$. Thus, $\Delta E_C^{\text{dir}}$ is the average (with respect to the neutron excess) Coulomb energy of the interaction of a proton with the nucleus. A rough estimate of this quantity, $\Delta E_C^{\text{dir}} = (6/5)Ze^2/R = 1.4\,ZA^{-1/3}$ MeV, follows from (66) and (67) under the assumption that the densities of the protons and the excess neutrons take the form of "steps" with radius $R$. The exchange part of the energy of the analog state is due to the interaction $H_C^{\text{int}}$. The value of $\Delta E_C^{\text{exc}}$ is nonzero through the correlation of the protons due to the Pauli principle. Since the Coulomb interaction has a long range, and the correlation range, which is equal in order of magnitude to $K_0^{-1}$, is short, it follows that $|\Delta E_C^{\text{exc}}/\Delta E_C^{\text{dir}}| \sim (K_0R)^{-2} \sim A^{-2/3}$. Numerical calculations made in accordance with the Fermi gas model for nuclei with $(N-Z) \gg 1$ lead to the following result[17]: $\Delta E_C^{\text{exc}} \cong -0.9Z/A$ MeV. The relative smallness of the part played by exchange effects in the determination of $E_A$ also permits one to conclude that the mixing of states with different values of the isospin in nuclei with $(N-Z) \gg 1$ occurs

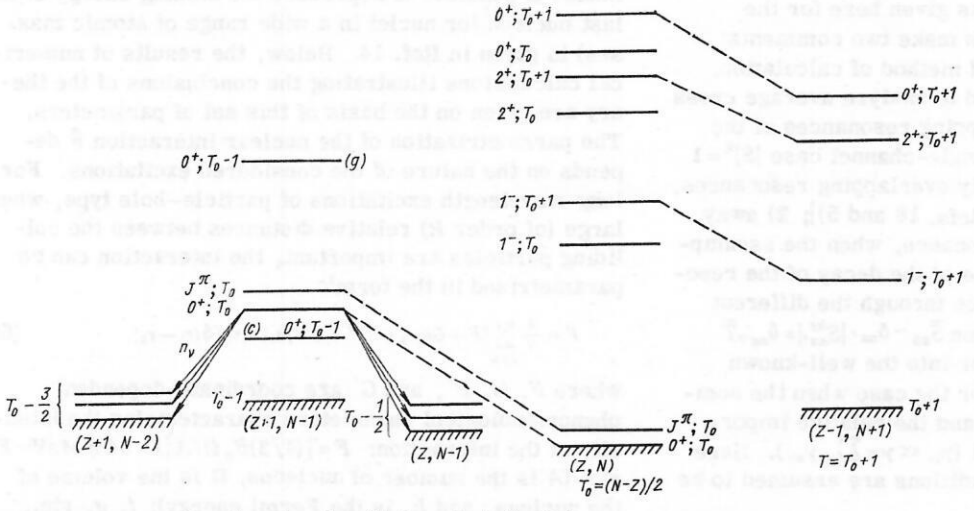


FIG. 1. Scheme of nuclear levels with isospin indicated.

mainly through the field $\mathscr{V}_C$, and is not due to the Coulomb scattering amplitude $H_C^{\text{int}}$, this being because of the predominance of the coherent effect over the incoherent effect.

In nuclei with $N>Z$, the average field contains not only $U_0$ but also terms proportional to the neutron excess. For nuclei with $A \gg (N-Z) \gg 1$, these terms can be separated from the charge-exchange part of the interaction (60) in the Hartree approximation[18]:

$$T^{sc}(r)=\tau^{(3)}v(r)/2; \quad v(r)=F'n(r), \tag{68}$$

where $n(r)$ is the density of the excess neutrons (67) and determines the so-called symmetry energy $v(r)$. Then in accordance with (58)–(60), for nuclei with $N>Z$ the single-particle Hamiltonian for the neutrons and protons has the form

$$h_{0n}=K+U_0(\mathbf{r})+v(r)/2; \quad h_{0p}=K+U_0(\mathbf{r})-v(r)/2+V_C(r). \tag{69}$$

Phenomenologically, the symmetry energy is taken in the form

$$v(r)=U_1 f(r, R, a)(N-Z)/A, \tag{70}$$

where $U_1 \approx 66$ MeV (Refs. 13 and 14). The single-particle wave functions of the discrete spectrum for neutron and proton states with equal quantum numbers calculated in accordance with (69) and (70) are very close to each other (near the Fermi limit, the overlap integrals for these states differ from unity by less than 1%). In accordance with (67)–(69), the definition of the single-particle wave functions for neutrons and protons, $\varphi^{n,p}(\mathbf{r})$, presupposes that a self-consistency procedure has been carried out. This procedure is also particularly simple when correlations are ignored in the ground state, when $n(r)-n_0(r)=\sum_b (n^n(E_b)-n^p(E_b)) \times |\varphi_b|^2$. In this case, the functions $\varphi_b(\mathbf{r})$ are determined by means of the symmetry energy $v_0(r)=F'n(r)$, which in its turn is determined by the functions $\varphi_b$. Since $|v_0| \ll |U_0|$, the corresponding iteration procedure, which can be begun conveniently with the expression (70) for $v_0(r)$, converges rapidly.[19] One of the ways of determining the force constant $F'$ (or $f'$) is as follows. In nuclei with large neutron excess $[(N-Z) \gg A^{1/3})]$, the density of the excess neutrons $n_0(r)$ is virtually constant over the volume of the nucleus, so that

$$v_0(r)=F'n_0(r) \approx F'\bar{n}(r); \quad \bar{n}(r)=(N-Z)\Omega^{-1}\bar{f}(r, R, a); \quad \int \bar{f}d^3r/\Omega=1. \tag{71}$$

Comparing the expressions (70) and (71), we find for nuclei in the neighborhood of Pb that $f'=1.53(1+2.55A^{-2/3})$. This value agrees satisfactorily with the values of $f'$ found in Ref. 15.

In nuclei with relatively small neutron excess $[(N-Z) \sim A^{1/3}]$, where the density $n_0(r)$ changes significantly over the volume of the nucleus, it is necessary to take into account in the calculation of the symmetry energy $v(r)$ [or the density $n(r)$] the correlations in the ground state or the polarization effect, which in this case is not small. In the random phase approximation, the core polarization effect is described by an integral equation of the form (20) for a static effective field of monopole symmetry:

$$v(r)=v_0(r)+\frac{F'}{4\pi r^2}\int A^{p_0h_0}(r, r'; \omega=0)v(r')\,dr', \tag{72}$$

where to accuracy $\sim |v_0|/D$ the response function $A(\omega=0)$ can be determined for one (for example, the neutron) subsystem. Since the radial dependence of $v_0(r)$ differs appreciably from $r^2$, in solving Eq. (72) we must take into account transitions in which the radial quantum number changes by more than one. This makes the numerical solution of Eq. (72) very difficult. A more constructive approach is to solve this equation in the quasiclassical approximation (see Appendix, §1). In the case of $n(r)$, the result of such a solution is[20]

$$n(r)=(1+f')^{-1}[n_0(r)+f'\bar{n}(r)]. \tag{73}$$

This solution has a reasonable interpolation nature: For $f' \ll 1$ (weak polarization) we have $n \approx n_0$; for $f' \gg 1$ (strong polarization) $n \approx \bar{n}$, while for $n_0 \approx \bar{n}$ there is no polarization. In Fig. 2, as an example, we give the functions $n_0$, $\bar{n}$, and $n$ and the binding energies corresponding to them for the last neutron, $|E_b^0|$, $|\bar{E}_b|$, and $E_b|$, for the nucleus $^{90}$Zr, in which the excess neutrons fill the subshell $1g_{9/2}$, so that $n_0(r)=(4\pi r^2)^{-1}(2j+1) \times \chi^2_{0E_b jl}(r)$. The functions $\chi_0(r)$ and $v(r)$ are determined self-consistently. As follows from Fig. 2, the core polarization effect in this case is indeed not small.

*Schematic Theory of Isovector Giant Resonances. Isotopic Splitting. Internal Coulomb Mixing.* The solution of problems in the theory relating to, for example, the purity of the isospin of nuclear states and the isotopic splitting of multipole giant resonances requires quantitative interpretation of isovector giant resonances. If we ignore the continuum and the coupling to many-particle configurations, multipole giant resonances can be regarded as collective excitations of particle–hole type, whose wave functions (transition densities) and energies are determined by the solutions of homogeneous equations of the type (20) and (A.12) (Ref. 1). Let us consider first the classification of these excitations, beginning for the sake of definiteness with the ground state of an even–even parent nucleus $|0\rangle - |0^+; T_0, T_0\rangle$ (see Fig. 1). For nuclei with large neutron excess, analog states can be regarded as collective excitations of monopole symmetry of the proton–neutron-hole type $(p\bar{n})$ with wave function $|A\rangle - |\text{CS}0^+; T_0, T_0-1\rangle$ (64) with energy $E_A=\Delta E_C$. In contrast to an analog state, and also to a "double analog" $|T_0+1,$

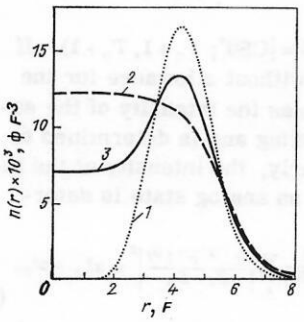


FIG. 2. Radial dependences of the single-particle (1), averaged (2), and effective (3) densities of the excess neutrons for the nucleus $^{90}$Zr The corresponding binding energies of the last neutron are 10.7, 11.3, and 11.0 MeV.

$T_0-1\rangle \sim (\mathscr{T}^{(-)})^2|T_0+1, T_0+1\rangle$, the $(p\bar{n})$ excitations are not characterized by a definite isospin. To construct states with definite isospin, it is necessary to extend the particle–hole basis. However, for nuclei with large neutron excess, one can to accuracy $\sim(N-Z)^{-1}$ speak of a definite isospin of a $(p\bar{n})$ state, this being equal to $T_0-1$ in accordance with the largest coefficient of the vector addition. We shall denote the wave functions and energies of monopole collective $(p\bar{n})$ states by $|\mathrm{CS0}^+; T_0-1, T_0-1\rangle$ and $E^{p\bar{n}}(0^+)$, respectively. States with energy less than $E_A$ are called configuration (or anti-analog) states; states with energy greater than $E_A$ correspond to $\Delta T_3=-1$, a monopole giant resonance. For $(n\bar{p})$ excitations, we "deem" isospin projection $\Delta T_3=1$. Therefore, such excitations can be characterized by the definite isospin $T_0+1$. We shall denote the wave functions and energies of the corresponding collective states by $|\mathrm{CS}J^\pi; T_0+1, T_0+1\rangle$ and $E^{n\bar{p}}(J^\pi)$. These giant resonances can be excited, for example, by the capture of negative muons by nuclei. The isovector excitations in the parent nucleus can have isospin $T_0$ or $T_0+1$. Collective $(n\bar{n}, p\bar{p})$ excitations, with which one usually associates isovector multiple giant resonances and which are excited with large cross sections in processes of absorption of $\gamma$ rays and inelastic scattering of electrons, can be characterized to accuracy $\sim(N-Z)^{-1}$ by the isospin $T_0$. We shall denote the wave functions and energies of the corresponding collective states by $|\mathrm{CS}J^\pi; T_0, T_0\rangle$ and $E^{N\bar{N}}(J^\pi)$. The components of multipole giant resonances with isospin $T_0+1$ can be regarded as analogs of the highly excited states considered above of the nucleus $(Z-1, N+1)$: $|\mathrm{CS}J^\pi; T_0+1, T_0+1\rangle$, i.e., as analog collective states (ACS). In accordance with (64) and (65), the wave functions and energies of these states are

$$|\mathrm{ACS}J^\pi; T_0+1, T_0\rangle=(2T_0+2)^{-1/2}\mathscr{T}^{(-)}|\mathrm{CS}J^\pi; T_0+1, T_0+1\rangle; \quad E^{\mathrm{ACS}}(J^\pi)=E^{n\bar{p}}(J^\pi)+\Delta E_C. \tag{74}$$

Then the isotopic splitting of a multipole giant resonance is

$$\Delta E(J^\pi)=E^{\mathrm{ACS}}(J^\pi)-E^{N\bar{N}}(J^\pi)=E^{n\bar{p}}(J^\pi)-E^{N\bar{N}}(J^\pi)+\Delta E_C. \tag{75}$$

In accordance with (74) and (62), the isospin purity of the ground state of the parent nucleus can be characterized by

$$\alpha^2_{\mathrm{gr.st}}=\sum_{T_>}\left|\frac{\langle T_>|\mathscr{V}_C|0\rangle}{E_>-E_0}\right|^2=\frac{1}{2T_0+2}\sum_{s'}\left|\frac{\langle s'|\mathscr{V}_C^{(+)}|0\rangle}{E_{s'}+\Delta E_C-E_0}\right|^2, \tag{76}$$

where $\mathscr{V}_C^{(+)}=\sum_i\tau_i^{(+)}\mathscr{V}_C(i)$, $|s'\rangle=|\mathrm{CS0}^+; T_0+1, T_0+1\rangle$. If the $(n\bar{p})$ states are defined without allowance for the continuum, (67) characterizes the intensity of the so-called internal Coulomb mixing and is determined to lowest order in $\mathscr{V}_C$. Similarly, the intensity of the internal Coulomb mixing for an analog state is determined by the quantity

$$\alpha_A^2=\sum_{T_<}\left|\frac{\langle T_<|\mathscr{V}_C|A\rangle}{E_<-E_A}\right|^2=\frac{1}{2T_0}\sum_s\left|\frac{\langle s|\mathscr{V}_C^{(-)}|0\rangle}{E_s-E_A}\right|^2=\alpha^2_{(c)}+\alpha^2_{(g)}, \tag{77}$$

where $\mathscr{V}_C^{(-)}=\sum_i\tau_i^{(-)}\mathscr{V}_C(i)$; $|s\rangle=|\mathrm{CS0}^+; T_0-1, T_0-1\rangle$. The decomposition of $\alpha_A^2$ into two terms corresponds to the contribution to the intensity of the internal mixing of the configuration states and the $\Delta T_3=-1$ monopole giant resonance. In the approximation $E_s\approx E_{s'}\gg\Delta E_C$, we have in accordance with (76) and (77)

$$\alpha^2_{(g)}=\alpha^2_{\mathrm{gr.st.}}(2T_0+2)/2T_0\approx\alpha^2_{\mathrm{gr.st.}}. \tag{78}$$

The schematic theory of isovector multipole giant resonances based on a quasiclassical analysis of the equations of the random phase approximation and treating collective excitations as certain single-phonon states enables one, in particular, to obtain a semi-quantitative interpretation of (75)–(77). The method of quasiclassical summation in equations of such type was proposed by Migdal[21] and was applied to the analysis of various isovector giant resonances in Refs. 22 and 23. The shortcomings of the schematic theory—the insufficient smallness of the parameters $A^{-1/3}$ and $(N-Z)^{-1}$ for a quasiclassical treatment, the use of a schematic shell potential, and the neglect of the spin-orbit interactionand pairing—are offsetby an important advantage, namely, the possibility of determining analytically the characteristics of collective states as functions of the parameters of the shell model (the radius of the nucleus, the constants of the effective interaction, and the number of nucleons and the neutron excess). It is natural to expect the relative and difference effects to depend less strongly on the shortcomings than do the absolute quantities. The method of quasiclassical summation in the RPA equations and its application to the derivation of the relations given below can be found in the Appendix.

1. *Analog States.* According to Eq. (A.4) in the Appendix, the energy of the analog state and the symmetry energy are

$$E=\Delta E_C=(1+f')\Delta_0; \quad v=f'\Delta_0 \quad \Delta_0=\frac{4}{3}\frac{N-Z}{A}E_0, \tag{79}$$

where $E_0\approx 33$ MeV is the Fermi energy defined for $A/2$ nucleons and radius $R=1.245A^{1/3}$ F of the nucleus. The condition of $\beta$ stability, $\Delta E_C=\beta(N-Z)/A$ ($\beta\approx 100$ MeV is a constant characterizing the symmetry energy in the semiempirical formula[13] for the nuclear binding energy), together with (79), leads to the following estimate for the contant $f'$: $f'=(3\beta/4E_0)-1\approx 1.25$ (Ref. 1), which agrees satisfactorily with the value given above.

2. *Monopole Collective $(\bar{p}n)$ and $(\bar{n}p)$ Excitations. Internal Coulomb Mixing.* In accordance with Eqs. (A.4)–(A.6), the energy and transition density for the configuration state with energy nearest $E_A$ are

$$\rho^c(r)=\frac{c_0}{1+f'(1-c_0)}\frac{(2T_0)^{1/2}V_{L=0}(r)/\Omega}{(\overline{V}^2_{L=0})^{1/2}}; \quad c_0=4/9; \qquad E_c=E_A[1+f'(1-c_0)]^{-1}; \quad V_{L=0}(r)\sim\mathscr{V}_C(r). \tag{80}$$

In the approximation $E^{n\bar{p}}(0^+)\approx E^{p\bar{n}}(0^+)\gg\Delta E_C$, the energy and transition density for an isovector monopole giant resonance are

$$\rho^g(r)=(1-c_0)^{1/2}\frac{V_{L=0}}{\Omega}\left[\frac{3}{8}\frac{A}{\overline{V}^2_{L=0}}\frac{E^3_{L=0}}{E_0E_g}\right]^{1/2}; \qquad E_g=E_{L=0}[1+f'(1-c_0)]^{1/2}, \tag{81}$$

where $E_{L=0}$ is the energy of the monopole giant resonance in a system of noninteracting nucleons (A.11). Since the matrix element for excitation of the collective state is determined by the transition density, $\langle s|\mathscr{V}_C|0\rangle$

$= \int \rho^s(r) \mathcal{V}_C(r) d^3r$, the relations (79)–(81) permit one to calculate the intensity of the "internal" Coulomb mixing for the ground state in accordance with (76):

$$\alpha^2_{\text{gr. st}} = \frac{1}{224} \frac{1-c_0}{1+f'(1-c_0)} \frac{A}{N-Z+2} \frac{(\Delta E_C)^2}{E_0 E_g} \approx 0.35 \cdot 10^{-6} Z^2 A^{2/3}/(N-Z+2). \quad (82)$$

In order of magnitude, the quasiclassical estimate $\alpha^2_{\text{gr.st}}$ (82) agrees with the value obtained in the hydrodynamic model using the empirical value of the rigidity for vibrations of the neutrons relative to the protons.[13] Note also that use of the $\beta$-stability condition in (82) leads to the value $\alpha^2_{\text{gr.st}} \approx 0.2 \times 10^{-4} Z$, which both in magnitude and in the $Z$ dependence differs from the results of calculations made in Ref. 25 in a simplified single-particle model. The intensity of internal mixing for analog states can be calculated similarly:

$$\alpha^2_{(c)} = c_0/84 f'^2 (1-c_0)^2 \approx 0.6 \cdot 10^{-2};\ \alpha^2_{(g)} \approx \alpha^2_{\text{gr. st}};\ \alpha^2_A = \alpha^2_{(c)} + \alpha^2_{(g)}. \quad (83)$$

For heavy nuclei $[(N-Z) \sim A^{2/3}]$, the quantities $\alpha^2$ are not literal smallness parameters. The numerical smallness of the intensity of the internal Coulomb mixing is explained by the smoothness and alternating sign within the nucleus of the field $v_C(r)$. If the correlations between the nucleons are ignored, an explicit expression for $\alpha^2_{(c)}$ follows from Eq. (77):

$$\alpha^2_{(c)} = (2T_0)^{-1} \sum_b [1-n^p(E_b)]\, n^n(E_b)\, (2j_b+1)\, (\mathcal{V}_C)^2_{bb}\, v^2_{b\bar{b}}. \quad (84)$$

A numerical calculation in accordance with this formula for the nucleus $^{208}$Pb leads to the value $\alpha^2_{(c)} \approx 2.5 \times 10^{-3}$, which agrees qualitatively with the quasiclassical estimate (83).

3. *Correction to the Energy of an Analog State.* To lowest order in $\mathcal{V}_C$, the correction to the energy

$$\Delta_A = (E_A - E_c)\, \alpha^2_{(c)} - (E_g - E_A)\, \alpha^2_{(g)}$$

of an analog state is in accordance with (80)–(83)

$$\Delta_A = \frac{1}{168} \frac{E_A}{1+f'(1-c_0)} \left[ \frac{2c_0}{f'(1-c_0)} - (1-c_0)(1+f') \right] \approx -1.5 \cdot 10^{-3} E_A. \quad (85)$$

Note the approximate compensation of the two terms in this relation.

4. *Isotopic Splitting of Multipole Giant Resonances.* In accordance with Eq. (A.2), the energies of isovector giant resonances of $(n\bar{n})$ and $(p\bar{p})$ type in the model of independent particles, $E^{NN}(J^\pi) - E_L$, can be determined by means of corresponding sum rules. It is obvious that in the model of independent particles the energies of some isovector giant resonances of $(n\bar{p})$ and $(p\bar{n})$ type are in accordance with (79) and (A.11) equal to $E^{n\bar{p}}(J^\pi) - E_L - \Delta_0$, $E^{p\bar{n}} - E_L + \Delta_0$, and if $L = 0.2$, then $\Delta_0 \ll E_L$. For dipole excitations, if $E_{L=1} \approx \Delta_0$, the $(n\bar{p})$ states cannot be described quasiclassically. Allowance for the interaction of the nucleons leads to a shift in the energies of isovector collective states [see the Appendix, Eqs. (A.3) and (A.5)]:

$$\left.\begin{aligned} &E^{N\bar{N}}(J^\pi) = E_L[1+f'(1-c_L)]^{1/2}; \\ &E^{n\bar{p}}(1^-) = -\Delta_0 - v/2 + [v^2/4 + (E^{N\bar{N}}(1^-))^2]^{1/2}, \\ &(E_{L=1} - \Delta_0 \gg \Delta_0/(N-Z)), \\ &E^{n\bar{p}}(J^\pi) = -\Delta_0 - v(1-\alpha_L)/2 + E^{N\bar{N}}(J^\pi);\ J^\pi = 0^+,\ 2^+, \\ &(E_L \gg \Delta_0), \end{aligned}\right\} \quad (86)$$

where $\alpha_L = c_L[1+f'(1-c_L)]^{-1}$; in the brackets, we give the accuracy with which these relations are obtained. Equations (75), (79), (85), and (86) enable one in the quasiclassical approximation to calculate the magnitude of the isotopic splitting of isovector giant resonances[24]:

$$\Delta E(1^-) = v/2 + [v^2/4 + (E^{N\bar{N}}(1^-))^2]^{1/2} - E^{N\bar{N}}(1^-) \approx v/2; \quad (87)$$

$$\Delta E(J^\pi) = v(1+\alpha_L)/2;\ J^\pi = 0^+,\ 2^+. \quad (88)$$

The relation (87) together with (70) is in good agreement with the empirical formula $\Delta E(1^-) = 30(2T_0+2)A^{-1}$ MeV (Ref. 26). The relation (88) makes it possible to estimate the expected values of the $T_>$ component of a quadrupole giant resonance if one uses the empirical value of the energy of the $T_<$ component: $E^{N\bar{N}}(2^+) = 130A^{-1/3}$ MeV. The method of quasiclassical analysis of the equations for the energy of a collective state also enables one to understand why the Tamm–Dankcoff approximation, which does not have quantitative accuracy, gives for $\Delta E(1^-)$ a result close to that obtained in Ref. 27. In this approximation,

$$\left.\begin{aligned} &E^{N\bar{N}}(1^-) = E_{L=1}(1+f'/2); \\ &E^{n\bar{p}}(1^-) = -\Delta_0 - v/2 + E^{N\bar{N}}(1^-), \end{aligned}\right\} \quad \Delta E(1^-) = v/2, \quad (89)$$

as follows from Eq. (A.5).

*Analog States in the Continuum. Coupling of Proton Decay Channels. External Coulomb Mixing.* In medium to heavy nuclei, the analog states are in the continuum, since their excitation energy exceeds the nucleon binding energy. In other words, the analog states can be manifested as resonances in the nucleon–nucleus interaction cross sections. Since their discovery in 1964 (Ref. 28), analog resonances have been intensively investigated in the elastic and inelastic scattering of protons. The total widths of the analog states do not exceed a few hundred keV in the heaviest nuclei (see, for example, Ref. 17). This means that one can regard as least the first few analog resonances as being isolated. The use of tandem generators with high beam intensity and high energy resolution makes it possible to perform precision experimental investigations of analog resonances in proton reactions. The theoretical interpretation of the proton decay of an analog resonance as a single-stage process is the simplest. For these reasons, we shall devote our main attention below to reactions with protons.

In accordance with the conclusions drawn above, if we ignore the field $\mathcal{V}_C$ (62) the analog state is an exact state of the system. Therefore, processes of nucleon decay of analog resonances, like the coupling of the analog states to the many-particle configurations, are due to the field $v_C$. The coupling of an analog state and the $T_<$ states $|s\rangle$ is determined by the matrix elements of this field, which can in accordance with (62) and (64) be represented in the form [see also (77)]:

$$\langle s | \mathcal{V}_C | A \rangle = (2T_0)^{-1/2} \langle s | v_C^{(-)} | 0 \rangle, \quad (90)$$

so that the analog states are mixed directly with $0^+$ excitations of $(p\bar{n})$ type. It should be emphasized that the $T_<$ states of $(p\bar{n})$ type can be found with allowance for the field $\mathcal{V}_C$ in all perturbation orders, i.e., it can be determined with allowance for the continuum [to the so-called internal mixing considered above there cor-

responds the zeroth order in $\mathscr{V}_C$ in the determination of the $(p\bar{n})$ states]. At the same time, the coupling of the $T_>$ and $T_<$ states can be taken into account by (90) to lowest order in $\mathscr{V}_C$. With allowance for these comments, we represent the (complex) correction to the energy of an analog state due to the coupling between the analog and $T_<$ states in the form

$$\delta E_A = \sum_s \frac{|\langle s|\mathscr{V}_C^{(-)}|A\rangle|^2}{E_A - E_s + i\delta} = \frac{1}{2T_0}\sum_s \frac{|\langle s|\mathscr{V}_C^{(-)}|0\rangle|^2}{E_A - E_s + i\delta}. \quad (91)$$

If we introduce the Coulomb polarizability of the nucleus in accordance with the relation (16), in which we make the substitution $V \to \mathscr{V}_C$, then

$$\operatorname{Im}\delta E_A = (2T_0)^{-1}\operatorname{Im}\mathscr{P}_C(\omega = E_A). \quad (92)$$

The effective field $\tau^{(-)}\tilde{\mathscr{V}}_C(r;\omega)$, which determines in accordance with (17) and (19) the polarizability $\mathscr{P}_C(\omega)$, satisfies the integral equation (20), in which $F \to F'$ in accordance with (60).

As throughout in Sec. 2, we take into account the coupling of an analog state to only $T_<$ states of $(p\bar{n})$ type, ignoring the coupling of the latter to many-particle configurations. In this approximation, $\delta E_A - \delta E_{0A}$ determines the "natural" proton width $\Gamma^{\uparrow}_{0p}$ of the analog resonance and its "natural" energy shift $\Delta_{0A}$ in accordance with the relations

$$\left.\begin{aligned}\Gamma^{\uparrow}_{0p} &= -2\operatorname{Im}\delta E_{0A} = -2(2T_0)^{-1}\operatorname{Im}\mathscr{P}_{0C}(\omega = \Delta E_C);\\ \Delta_{0A} &= \operatorname{Re}\delta E_{0A}.\end{aligned}\right\} \quad (93)$$

On the basis of (93) and Eqs. (21) and (28), which relate the imaginary part of the polarizability to the effective field and the response function, we obtain the following expression for the natural proton width of the analog resonance[9]:

$$\Gamma^{\uparrow}_{0p} = -2(2T_0)^{-1} \times \operatorname{Im}\int \tilde{\mathscr{V}}^{*}_C(r;\omega)\, A^{p_0h_0}(r, r';\omega)\tilde{\mathscr{V}}_C(r';\omega)\, dr\, dr'|_{\omega=\Delta E_C}; \quad (94)$$

$$\left.\begin{aligned}\Gamma^{\uparrow}_{0p} &= \sum_b \Gamma^{\uparrow}_{0(b)};\\ \Gamma^{\uparrow}_{0(b)} &= 2\pi(2T_0)^{-1} n(E_b)\, t_b \times \left|\int \chi_b(r)\tilde{\mathscr{V}}_C(r;\omega = \Delta E_C)\,\chi_{0E(b)}(r)\, dr\right|^2.\end{aligned}\right\} \quad (95)$$

Here, the index $b$ labels the bound states of the neutrons, and, therefore, the proton decay channels of the analog resonance corresponding to these states; $E_{(b)} = E_b + \Delta E_C$ is the kinetic energy of the protons. The integral equation for the effective Coulomb field $\tilde{\mathscr{V}}_C(r;\omega)$, which takes into account the polarization effects due to the isovector part of the effective interaction, has in accordance with (20) the form

$$\tilde{\mathscr{V}}_C(r;\omega) = \mathscr{V}_C(r) + (F'/4\pi r^2)\int A^{p_0h_0}(r, r';\omega)\tilde{\mathscr{V}}_C(r';\omega)\, dr'. \quad (96)$$

[In (94) and (96), the response function is determined by Eq. (28), in which the indices $b$ and $a$ label the neutron and proton states, respectively.] In other words, the integral equation (96) takes into account the mutually interfering effects of the coupling of the real and virtual proton decay channels of the analog resonance.

Formula (95) for the natural partial proton width $\Gamma^{\uparrow}_{0(b)}$ of the analog resonance agrees with the general formula (40) for the natural partial nucleon width of a doorway resonance if we set

$$\tilde{\mathscr{V}}_C(r) = (2T_0)^{-1/2}\tilde{\mathscr{V}}_C(r); \quad t_{ab} = t_b\delta_{ab}; \quad t_b = (2j_b+1). \quad (97)$$

Using the relations [see (69)]

$$V_C(r) = h_{0p} - h_{0n} + v; \quad \tilde{\mathscr{V}}_C - \mathscr{V}_C \equiv \delta v \quad (98)$$

we can express the width $\Gamma^{\uparrow}_{0(b)}$ (95) in terms of the symmetry energy[30]:

$$\Gamma^{\uparrow}_{0(b)} = 2\pi(2T_0)^{-1} n(E_b)\, t_b \left|\int \chi_b(v+\delta v)\,\chi^{(+)}_{0E(b)}\, dr\right|^2, \quad (99)$$

so that $\delta v$ is a dynamical correction to the symmetry energy. Note that for an unfilled neutron shell in the parent nucleus $n(E_b) = S_bN_b(2j_b+1)^{-1}$, where $N_b$ is the number of neutrons in level $j_b$, and $S_b$ is the spectroscopic factor, so that $n(E_b)t_b = S_bN_b$.

The relations (99) enable us to establish to what extent we can describe the width $\Gamma^{\uparrow}_{0(b)}$ by means of the Hamiltonian for the system consisting of the nucleon and the nucleus:

$$H = K + U_0(\mathbf{r}) + V_L(r)\,\mathbf{tT} + (1/2 - t^{(3)})\,V_C(r), \quad (100)$$

where $V_L$ is Lane's phenomenological potential.[31] It follows from a comparison of this expression with (69) that $T_0V_L(r) = v(r)$. The shell structure of Lane's potential (the symmetry energy) has been elucidated above. With allowance for this remark, the formulas for the partial widths $\Gamma^{\uparrow}_{0(b)}$, obtained on the basis of (100) by the distorted-wave Born approximation coincide with (99) only if the channel coupling is ignored, when $\delta_v \to 0$.

To estimate the influence of polarization effects on the widths $\Gamma^{\uparrow}_{0(b)}$, let us consider the quasiclassical solution of the integral equation (97) using a schematic shell potential [Eq. (A.4)]:

$$|\tilde{\mathscr{V}}_C(r;\omega = \Delta E_C)/\mathscr{V}_C(r)|^2 = [1 + f'C_{L=0}(\omega = \Delta E_C)]^{-2} = [1 - c_0 + f'(1-c_0)]^{-2} \approx 0.5. \quad (101)$$

This gives a rough estimate of the relative importance of the polarization effects. In Ref. 32, an alternative method is proposed for taking into account the general coupling, in which instead of (96) in the Tamm–Dankcoff approximation one solves an integral equation for an effective response function $\tilde{A}$, which is defined by $\{A^{p_0h_0}\tilde{\mathscr{V}}_C\}$. Literally by means of $\tilde{A}$ one calculates the total (but not partial) natural proton widths of the analog resonance. The decrease in the total widths $\Gamma^{\uparrow}_{0p}$ due to the polarization effects found in Ref. 32 agrees with the estimate (101).

If we ignore the Coulomb correction to the ground-state energy, then on the basis of (91) and (16) we obtain the following approximate expression for the shift (relative to $\Delta E_C$) of the energy of the analog resonance with allowance for the continuum and the polarization effects:

$$\Delta_{0A} \approx (2T_0)^{-1}\operatorname{Re}\mathscr{P}_{0C}(\omega = \Delta E_C). \quad (102)$$

Numerical calculations have not yet been made using this formula, so that we cannot yet draw any conclusion about the part played by the continuum (or the role of "external" mixing of the analog and $T_<$ states) in the formation of $\Delta_{0A}$. The estimate (85) is obtained with allowance for only "internal" mixing.

If the matrix element (90) determines the Coulomb mixing of an analog and a $T_<$ state, the matrix element

$$(2T_0+2)^{-1/2}\langle s'|\mathscr{T}^{(+)}\mathscr{V}_C|0\rangle=(2T_0+2)^{-1/2}\langle s'|\mathscr{V}_C^{(+)}|0\rangle \qquad (103)$$

determines the Coulomb mixing of the ground and $T_<$ states in the parent nucleus. The relations (90) and (103) enable us in the single-particle approximation to calculate directly the so-called transition density $\rho^A(r)$ to lowest order in $\mathscr{V}_C$ $(2T_0 \gg 1)$:

$$\left.\begin{aligned}&\rho^A(r)=\sum_{ab}\varphi_a^*(\mathbf{r})\,\varphi_b(\mathbf{r})\,\rho_{ab}^A=\langle A'|\sum_i\tau_i^{(-)}\delta(\mathbf{r}-\mathbf{r}_i)|0'\rangle;\\&\rho_{ab}^A=(2T_0)^{-1/2}A_{ab}^{p_bh_0}(\omega=\Delta E_C)(\mathscr{T}\mathscr{V}_C)_{ba}.\end{aligned}\right\} \qquad (104)$$

Using the expression (28) for the response function, on the basis of (104) we obtain the following representation for the transition density:

$$\rho^A(r)=\frac{1}{4\pi r^2}\Big\{\sum_b t_b n(E_b)\,\delta\chi_{(b)}(r)\,\chi_b(r)+\sum_a t_a n(E_a)\,\chi_a(r)\,\delta\chi_{(a)}(r)\Big\}, \qquad (105)$$

where the quantities

$$\delta\chi_{(b)}(r)=(2T_0)^{-1/2}\int g_{0(b)}(r,\,r';\;E_{(b)}=E_b+E_A)\,\mathscr{T}\mathscr{V}_C(r')\,\chi_b(r')\,dr'; \qquad (106)$$

$$\delta\chi_{(a)}(r)=-(2T_0)^{-1/2}\int g_{0(a)}(r,\,r';\;E_{(a)}=E_a-E_A)\,\mathscr{T}\mathscr{V}_C(r')\,\chi_a(r')\,dr' \qquad (107)$$

can be interpreted as the proton and neutron corrections to the wave functions of the analog state and the ground state as found in the coordinate representation.[29,30] Allowance for polarization effects leads to the substitution $\mathscr{V}_C \to \tilde{\mathscr{V}}_C(\omega=E_A)$ in these formulas. The relations (98) enable us to express the Coulomb corrections (106) and (107) in terms of the symmetry energy:

$$\delta\chi_{(b)}(r)=(2T_0)^{-1/2}\Big\{\int g_{0(b)}(r,\,r';\;E_{(b)})\,(v(r')+\delta v(r'))\,\chi_b(r')\,dr'-\chi_b(r)\Big\}; \qquad (108)$$

$$\delta\chi_{(a)}(r)=(2T_0)^{-1/2}\Big\{-\int g_{0(a)}(r,\,r';\;E_{(a)})\,(v(r')+\delta v(r'))\,\chi_a(r')\,dr'-\chi_a(r)\Big\}. \qquad (109)$$

All the processes that are forbidden by the isospin and are associated with the excitation or decay of analog states are realized in the measure indicated by the corrections (106)–(109). Since the continuum is taken into account exactly in these formulas, these corrections take into account simultaneously the internal and external mixing of the states with different values of the isospin. One cannot divide the mechanisms of mixing into internal and external mechanisms in such a way as to eliminate interference between these two mechanisms. Therefore, in the framework of the shell approach, such a separation is nominal.

Note that the relations (96) and (105) make it possible to obtain an integral equation for $\delta_v$ (Ref. 30):

$$\delta v(r)=\delta v_0(r)+(F'/4\pi r^2)\int A^{p_0h_0}(r,\,r';\;\omega=\Delta E_C)\,\delta v(r')\,dr',$$

where

$$(2T_0)^{-1/2}\delta v_0=F'\rho^A[\mathscr{T}\mathscr{V}_C];\quad (2T_0)\tilde{\mathscr{T}}^{1/2}\,\delta v=F'\rho^A[\tilde{\mathscr{T}}\mathscr{V}_C(\omega=\Delta E_C)].$$

To conclude this section, we point out the possibility of obtaining a simple interpretation of the Coulomb corrections $\delta\chi$ by means of the Hamiltonian (100) (Ref. 30).

Let $|n_b\rangle$ be the ground state of an odd-neutron nearly magic nucleus, when $n(E_b)t_b=1$, $|d_b\rangle=|n_bA\rangle$ is the wave function of a doorway state, this being the main component of the wave function of the analog state:

$$|An_b\rangle=|n_bA\rangle+(2T_0)^{-1/2}|p_b\rangle, \qquad (110)$$

where $|p_b\rangle$ is the wave function of a single-proton state with the same coordinate dependence as the wave function $|n_b\rangle$. Through the charge-exchange part of the Hamiltonian (100), there is an admixture in the state $|d_b\rangle$ of a continuum of single-proton states $|p_E\rangle$. The difference between this admixture and $(2T_0)^{-1/2}|p_b\rangle$ is essentially the correction $\delta_{\chi(b)}$ (108) found in the single-particle approximation. In the same approximation, the matrix element of the charge-exchange part of the Hamiltonian (100) for the transition $|d_b\rangle \to |p_{E(b)}\rangle$ is the amplitude of the partial proton width (99).

## 3. FORMATION OF ANALOG RESONANCES

*Effective Proton Widths of Analog Resonances. Resonance Mixing Phases.* An isolated analog resonance is the purest example of a giant resonance because of the extreme simplicity of the shell structure of an analog state. [In accordance with (64), an analog resonance has no gross structure.] As was noted in Sec. 1, the existence of giant resonances is due to the coupling of a simple shell configuration (or a collection of simple configurations with nearly equal energy when there is a gross structure) to levels of a complicated nature. For nonoverlapping resonances of a compound nucleus, a giant resonance exhibits a fine structure. When these resonances overlap (or when one is investigating the cross sections averaged with respect to the energy), the nucleon widths of the resonance are modified compared with the "natural" widths, and the total width of the resonance increases. In accordance with the conclusions drawn in Sec. 1, the modification to the parameters of an analog resonance due to the coupling of the analog state to many-particle configurations can be expressed in terms of the optical model.

If the contribution of direct processes is ignored, the parametrization of the average $S$ matrix for proton scattering in the region of an isolated analog resonance is given by Eq. (49). In accordance with (50) and (97), the effective partial proton widths $\Gamma_a^\uparrow \equiv \Gamma_{(b)}^\uparrow$ of the analog resonance and the resonance mixing phases $\phi_a \equiv \phi_{(b)}$ in the formula for the average $S$ matrix are determined by the relations

$$\exp[2i(\phi_{(b)}+\xi_{(b)})]\,\Gamma_{(b)}^\uparrow = 2\pi(2T_0)^{-1}N_bS_b\int\chi_{E_{(b)}}^{(+)}\tilde{\mathscr{T}}^*\mathscr{V}_C\chi_b\,dr\int\chi_b\tilde{\mathscr{T}}\mathscr{V}_C\chi_{E_{(b)}}^{(+)}\,dr. \qquad (111)$$

Here, $r^{-1}\chi_b$ is the radial wave function of a neutron determined by the Hamiltonian $h_{0n}$ (69), $r^{-1}\chi_{E(b)}^{(+)}$ is the radial wave function of a proton in the continuum determined by the optical-model Hamiltonian (8): $h_p=h_{0p}+\Delta h_p$; and the effective field $\mathscr{V}_C(r;\,\omega=\Delta E_C)$ is determined by the integral equation (95), in which the response function is found with allowance for the damping of the particle–hole configurations, i.e., $A^{p_0h_0}(\omega) \to A^{ph}(\omega+iI)$. Using the relations (98), we can express $\Gamma_b^\uparrow$ and $\phi_b$ in Eq. (111) explicitly in terms of the sym-

metry energy:

$$\exp[2i(\phi_{(b)}+\xi_{(b)})]\,\Gamma^{\uparrow}_{(b)} = 2\pi(2T_0)^{-1}N_bS_b \times \int \chi^{(+)}_{E_{(b)}}(\check{v}_{(b)}+\delta v^*)\chi_b\,dr \int \chi_b(\check{v}_{(b)}+\delta v)\chi^{(+)}_{E_{(b)}}\,dr, \quad (112)$$

where $\check{v}_{(b)} = v - \Delta h_p(E_{(b)})$.

We now analyze the relations (111) and (112). Numerical calculations of $\Gamma^{\uparrow}_{(b)}$ and $\phi_{(b)}$ with allowance for channel coupling have not hitherto been made. The relative importance of the polarization effects in the determination of these quantities can be estimated in accordance with (101). If channel coupling is ignored, we obtain instead of (111) and (112), respectively,

$$\exp[2i(\phi_{(b)}+\xi_{(b)})]\,\Gamma^{\uparrow}_{(b)} = 2\pi(2T_0)^{-1}N_bS_b\left(\int \chi_b\mathscr{V}_c\chi^{(+)}_{E_{(b)}}\,dr\right)^2 \quad (113)$$

$$\exp[2i(\phi_{(b)}+\xi_{(b)})]\,\Gamma^{\uparrow}_{(b)} = 2\pi(2T_0)^{-1}N_bS_b\left(\int \chi_b\check{v}_{(b)}\chi^{(+)}_{E_{(b)}}\,dr\right)^2. \quad (114)$$

In this approximation, $\Gamma^{\uparrow}_{(b)}$ are determined solely by the potentials of the shell and optical models and depend weakly on the absolute magnitude of the symmetry [or in accordance with (68), on the constant $f'$]. An example of this dependence is given in Fig. 3 (Ref. 33). The *a priori* approach to the optical model in the formula for the natural partial proton width (99) [but not in formula (95)!] by means of the substitution $\chi^{(+)}_{0E_{(b)}} \to \chi^{(+)}_{E_{(b)}}$ leads to the incorrect result

$$\exp[2i(\bar{\phi}_{(b)}+\xi_{(b)})]\,\bar{\Gamma}^{\uparrow}_{(b)} = 2\pi(2T_0)^{-1}N_bS_b\left(\int \chi_bv\chi^{(+)}_{E_{(b)}}\,dr\right)^2, \quad (115)$$

i.e., to Eq. (114), in which $\check{v}_{(b)} \to v$. We explain the origin of this result by taking the example of an odd-neutron nearly magic parent nucleus. In accordance with the conclusions of Sec. 1, the relation (115) corresponds to choosing as the field which realizes the coupling of the doorway state $|d_b\rangle = |n_bA\rangle$ to the single-proton continuum states the quantity $\mathscr{V} = (2T_0)^{-1/2}v$. Since the "nuclear" part of the matrix element $\langle An_b|H'|\lambda\rangle$ for a transition between states with different values of the isospin is zero, in the considered case the statistical hypothesis (47) is incorrect. On the basis of (110), we obtain instead of (47) (Refs. 11 and 33)

$$\overline{\langle d_b|H'|\lambda\rangle\langle\lambda|H'|p_{E_{(b)}}\rangle} = -(2T_0)^{-1/2}\overline{\langle p_b|H'|\lambda\rangle\langle\lambda|H'|p_{E_{(b)}}\rangle} \neq 0. \quad (116)$$

The relations (45) and (116) lead not only to the substitution $\chi^{(+)}_{0E} \to \chi^{(+)}_{E}$, but also to a modification of the field $\mathscr{V}$: $v \to v - \Delta h_p$ and, therefore, to (114) obtained with allowance for approximate isospin conservation.

To estimate the extent and nature of the dependence of the widths $\Gamma^{\uparrow}_{(b)}$, and the phases $\Gamma^{\uparrow}_{(b)}$ on the optical correction to the shell potential $\phi_{(b)}$, $\bar{\phi}_{(b)}$

To estimate the extent and nature of the dependence of the widths $\Gamma^{\uparrow}_{(b)}$, $\bar{\Gamma}^{\uparrow}_{(b)}$ and the phases $\phi_{(b)}$, $\bar{\phi}_{(b)}$ on the

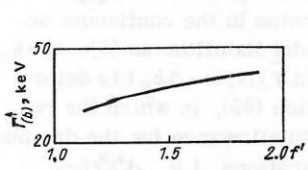


FIG. 3. Dependence of the width $\Gamma^{\uparrow}_{(b)}$ on the symmetry energy (the constant $f'$) calculated for $9/2^+$—analog resonance in the reaction $^{208}$Pb$(p,p_0)$.

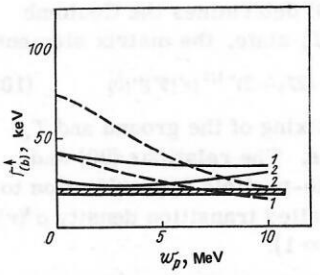


FIG. 4. Dependences of the elastic single-particle proton widths $\Gamma^{\uparrow}_{(b)}$ (continuous curves) and $\bar{\Gamma}^{\uparrow}_{(b)}$ (broken curves) on the intensity of (volume) absorption $w_p$ and the optical shift $\Delta_p$, both calculated for $9/2^+$—analog resonance in the reaction $^{208}$Pb$(p,p_0)$. The hatched strip is the region of the experimental values of the widths $\Gamma^{\uparrow}_{(b)}$.

optical correction to the shell potential $\Delta h_p = -iw_p + \Delta_p$, we take into account this quantity in Eqs. (114) and (115) by perturbation theory:

$$\Gamma^{\uparrow}_{(b)} = \Gamma^{\uparrow}_{0(b)}\left\{1 + 2\,\mathrm{Re}\int (2T_0)^{1/2}\delta\chi_{(b)}(\Delta_p - iw_p) \times \chi_{0E_{(b)}}\,dr \Big/ \int \chi_bv\chi_{0E_{(b)}}\,dr\right\}; \quad (117)$$

$$\phi_{(b)} = -\,\mathrm{Re}\int (2T_0)^{1/2}\delta\chi_{(b)}w_p\chi_{0E_{(b)}}\,dr \Big/ \int \chi_bv\chi_{0E_{(b)}}\,dr. \quad (118)$$

As one would expect, the influence of real and virtual excitation of the many-particle configurations in the process of proton decay of the analog resonance on $\Gamma^{\uparrow}_{(b)}$ and $\phi_{(b)}$ is suppressed by the isospin selection rules and is realized to an extent measured by the Coulomb corrections $\delta\chi_{(b)}$ (101) to the wave functions of the analog and the ground state. In the same approximation, the formulas for $\bar{\Gamma}^{\uparrow}_{(b)}$ and $\bar{\phi}_{(b)}$ are obtained from (117) and (118) by the substitution $(2T_0)^{1/2}\delta\chi_{(b)} - (2T_0)^{1/2}\delta\chi_{(b)} + \chi_b$, so that this influence is overestimated by these formulas. For this reason, the widths $\Gamma^{\uparrow}_{(b)}$ are more stable against variations of the parameters of the optical potential than $\bar{\Gamma}^{\uparrow}_{(b)}$, and the phases $\phi_{(b)}$ are much less than $\bar{\phi}_{(b)}$. In Fig. 4, this conclusion is illustrated by the results of a calculation in accordance with (114) and (115) of the dependence on $\Delta h_p$ of the listed quantities for the elastic proton decay channel of the analog of the ground state of the parent nucleus $^{209}$Pb (Refs. 33 and 29).

The self-consistent calculation of the widths $\Gamma^{\uparrow}_{(b)}$ (and the phases $\phi_{(b)}$) requires the parameters $\Delta h_p$ to be chosen on the basis of an analysis of the elastic scattering of protons and the reaction channels near the analog resonance. In the absence of such data, we must use in the calculations the so-called "realistic" parameters. In Table I, we give the results of calculations in accordance with Eq. (114) of the elastic partial proton widths of an analog resonance for scattering by the nuclei $^{207}$Pb and $^{208}$Pb (it is assumed that $S_b = 1$). We also give the experimental values of the widths $\Gamma^{\uparrow}_{(b)}$ and the results of a calculation in accordance with a formula obtained by means of the projection-operator technique and equivalent to (113) (Ref. 17). The systematic excess of the calculated widths $\Gamma^{\uparrow}_{(b)}$ over the experimental values must be attributed to the neglect of the channel coupling in accordance with the estimate (101).

TABLE I. Results of calculations made in the single-particle approximation of the effective proton widths and resonance mixing phases for analog resonances excited in the elastic scattering of protons by $^{207,208}$Pb nuclei. The widths calculated using volume absorption are given in brackets.

| $b\{nlj\}$ | $E_r$, MeV | $E_b$, MeV | $(\Gamma^{\uparrow}_{(b)})_{\text{exp}}$, keV (Ref. 17) | $(\Gamma^{\uparrow}_{(b)})_{\text{th}}$, keV (Ref. 17) | $(\Gamma^{\uparrow}_{(b)})_{\text{th}}$, keV | $\sin 2\phi_{(b)}$ |
|---|---|---|---|---|---|---|
| $p+^{208}$Pb, $\Delta_p=5$ MeV, $(w_p)_{SF}=5$ MeV, $(w_p)_{VOL}=5$ MeV | | | | | | |
| $2d_{3/2}$ | 17.48 | −0.68 | 46±15 | 64 | 88 | 0.115 |
| $2g_{7/2}$ | 17.43 | −1.0 | 36±15 | 36 | 56 (51) | 0.180 |
| $4s_{1/2}$ | 16.96 | −1.2 | 48 | 74 | 92 (65) | 0.116 |
| $3d_{5/2}$ | 16.50 | −1.7 | 58±10 | 62 | 90 (70) | 0.152 |
| $1j_{15/2}$ | 16.34 | −1.3 | (1) | 1.2 | 1.6 (1.6) | 0.096 |
| $1i_{11/2}$ | 15.72 | −3.9 | 2±1.6 | 0.8 | 2.0 (2.2) | 0.189 |
| $2g_{9/2}$ | 14.92 | −3.6 | 19.6±0.8 | 20.5 | 32 (31) | 0.206 |
| $p+^{207}$Pb; $\Delta_p=3$ MeV; $(w_p)_{SF}=5$ MeV; $(w_p)_{VOL}=5$ MeV | | | | | | |
| $3p_{1/2}$ | 11.49 | −7.7 | 25±4 | 27 | 43 (40) | 0.293 |
| $2f_{5/2}$ | 10.92 | −8.7 | 28.0±1.4<br>3<br>4.2±0.5 | 4.7 | 6.4 (6.4) | 0.269 |
| $3p_{3/2}$ | 10.59 | −8.5 | 10±3<br>15.8±0.9 | 16.5 | 28 (26) | 0.269 |
| $1i_{13/2}$ | 9.74 | −8.5 | — | 0.001 | 0.016 (0.016) | 0.103 |
| $2f_{7/2}$ | 9.15 | −10.7 | 0.6±0.1 | 0.53 | 1.2 (1.2) | 0.252 |
| $1h_{9/2}$ | 8.06 | −12.0 | — | 0.001 | 0.004 | 0.176 |

The microscopically justified derivation of (114) makes makes it possible to give an interpretation to the following generalization of the phenomenological Hamiltonian (100) for the "nucleon + nucleus" system to the case when optical-model effects are taken into account:

$$H=K+\check{U}_0+\check{V}_L\mathbf{tT}+(1/2-t^{(3)})V_C, \tag{119}$$

where $\check{U}_0$ and $\check{V}_L$ are the effective values of the isoscalar and isovector potentials. The requirements that this Hamiltonian 1) go over for the "neutron + nucleus in the ground state" system into the Hamiltonian $h_{0n}$(69) and 2) go over for the "proton in the continuum + nucleus in the ground state" system into the Hamiltonian $h_p$ (8) for the protons, $h_{0p}$ being determined by (69), lead to a modification of Lane's potential:

$$T_0\check{V}_L=T_0V_L-\Delta h_p=\check{v} \tag{120}$$

and, consequently, to (114) for $\Gamma^{\uparrow}_{(b)}$, $\phi_{(b)}$. This phenomenological approach leaves open not only its justification but also questions such as that relating to the shell structure of the symmetry energy, and the method for taking into account the coupling of the proton decay channels of the analog resonance, an interpretation of which is possible in the framework of the shell approach. Note in this connection that an arbitrary modification of the radial dependence of the symmetry energy can significantly change (by a factor 2) the absolute magnitude of the widths $\Gamma^{\uparrow}_{(b)}$ (Ref. 34).

The relative simplicity of the interpretation, and the possibility of obtaining by a comparison with experimental data information about the spectroscopic factors $S_b$, stimulated interest in the theoretical investigation of the effective proton widths of analog resonances. In the first studies, the widths $\Gamma^{\uparrow}_{(b)}$ were analyzed on the basis of formula (115) obtained by an *a priori* transition to the optical model using either the phenomenlogical Lane potential (100) (Ref. 35) or the symmetry energy in accordance with (71) (Refs. 24 and 36). An attempt at a microscopically justified transition to the optical model on the basis of the statistical hypothesis (47) again led to (115) (Refs. 2, 37, and 38). With neglect of channel coupling, the widths $\Gamma^{\uparrow}_{(b)}$ were analyzed on the basis of (113) in Refs. 39 and 17, on the basis of the equivalent formula (114) using the phenomenological Lane potential (119) and (120) in Ref. 34, and using the symmetry energy in accordance with (71) in Ref. 33. Unrealized aspects of the theory are in practice the analysis of $\Gamma^{\uparrow}_{(b)}$ and $\phi_{(b)}$ in accordance with (111) and (112) with allowance for coupling of the proton decay channels of the analog resonances, and also allowance for the shell structure of the symmetry energy in accordance with (68) and (72) for nuclei with relatively small excess.

*Relaxation Widths of Analog Resonances.* The damping of analog resonances is due to processes of direct nucleon decay and the coupling of the analog states to complicated configurations. The strength of this coupling is characterized by the so-called relaxation width $\Gamma^{\downarrow}$, which exists because of the Coulomb mixing of states with different isospins. In accordance with (90) and (103), the analog state and its corresponding state in the parent nucleus are directly mixed by the Coulomb field $\mathscr{V}_C$ with the configurations of $(p\bar{n})$ and $(n\bar{p})$ type, which, in their turn, "decay" by virtue of the nuclear interaction either into configurations of a complicated nature or directly to the continuum. In accordance with the conclusions of Sec. 1, the relaxation of the particle–hole configurations through the first mechanism can be described on the average in terms of the optical model. Therefore, the width $\Gamma^{\downarrow}$ can be expressed in terms of the shell and optical models.

In accordance with (91) and (92), the total width of an analog resonance in the expression for the average $S$ matrix (49) is determined by the imaginary part of the Coulomb polarizability of the nucleus:

$$\Gamma=-2(2T_0)^{-1}\,\mathrm{Im}\,\mathscr{P}_C(\omega+iI)|_{\omega=\Delta E_C}. \tag{121}$$

In its turn, the average polarizability is determined by the average response function in accordance with (19)–(21). For the analysis that follows, we shall use the approximate expression for the average response function (32) corresponding to additive contribution of the damping of the particle and hole configurations to the imaginary part of this function. In the considered case, the condition (29) becomes

$$|E_A-E_c|^2\gg w^2, \tag{122}$$

where $E_c$ is the energy of the nearest monopole giant resonance of $(p\bar{n})$ type to the analog state. In accordance with the quasiclassical estimate for the energy of the configuration state (80), the condition (122) can be assumed to be satisfied with a certain accuracy. In this approximation, in accordance with the second of the inequalities (29) and (21) it is possible in a calculation of the total width to ignore the damping of the particle–hole configurations in the equation for the effective Coulomb field $\mathscr{V}_C(\omega=\Delta E_C)$ (20) and, therefore, to use the solution to Eq. (96). With allowance for these remarks, and also the formula for the effective partial width (111), we represent the expression for the relaxation width of an analog resonance in the form[9]

$$\Gamma^{\downarrow} = \Gamma - \Gamma^{\uparrow}_{p} = \Gamma^{\downarrow}_{(p)} + \Gamma^{\downarrow}_{(\bar{n})}; \tag{123}$$

$$\Gamma^{\downarrow}_{(p)} = \sum_b \Gamma^{\downarrow}_{(b)} = \sum_b (2T_0)^{-1} t_b [n(E_b) - n(E_{(b)})]$$
$$\times \Big\{ -2\,\mathrm{Im} \int \chi_b \tilde{\mathscr{V}}^* \tilde{c} g_{(b)}(E_{(b)}) \tilde{\mathscr{V}} c \chi_b \, dr \, dr'$$
$$- 2\pi \Big| \int \chi^{(+)}_{E_{(b)}} \tilde{\mathscr{V}}^* \tilde{c} \chi_b \, dr \int \chi_b \mathscr{V} c \chi^{(+)}_{E_{(b)}} dr \Big| \Big\}; \tag{124}$$

$$\Gamma^{\downarrow}_{(\bar{n})} = \sum_a \Gamma^{\downarrow}_{(a)} = \sum_a (2T_0)^{-1} t_a [n(E_{(a)}) - n(E_a)]$$
$$\times \Big\{ 2\,\mathrm{Im} \int \chi_a \tilde{\mathscr{V}}^* \tilde{c} g_{(a)}(E_{(a)}) \mathscr{V} c \chi_a \, dr \, dr' - 2\pi \Big| \int \chi_{(a)} \tilde{\mathscr{V}} c \chi^{(+)}_{0E_a} dr \Big|^2 \Big\}, \tag{125}$$

where $E_{(a)} = E_a - \Delta E_C$; $r^{-1}\chi_a$ and $r^{-1}\chi^{(+)}_{0E_a}$ are the radial wave functions of single-proton states of the discrete spectrum and the continuum, i.e., eigenfunctions of the Hamiltonian $h_{0p}$ (69). The terms (124) and (125) correspond to the contribution to the width $\Gamma^{\downarrow}$ (123) of the damping of the single-proton and neutron-hole states, respectively. The summation over the neutron-hole states $b$ and the single-proton states $a$ in these terms is over the energy interval $\Delta E_C$ near the Fermi limit, the imaginary part of the optical potential varying from zero to $w(|\Delta E_C - \mu|)$. The maximal imaginary part corresponds to the elastic proton channel with energy $E_{(b)} = E_b + \Delta E_C$, when $E_b \approx \mu_n$, the corresponding neutron-hole state being at the Fermi limit, and also to the "neutron-hole channel" with energy $E_{(a)} = E_a - \Delta E_C$, when $E_a \approx \mu_p$, the corresponding single-proton state being at the Fermi limit. To first order in $\Delta h_p$ and $\Delta h_n$, we obtain instead of (124) and (125) expressions for the widths $\Gamma^{\downarrow}_{(p)}$ and $\Gamma^{\downarrow}_{(\bar{n})}$ from which there directly follows our assertion above:

$$\Gamma^{\downarrow}_{(p)} = \sum_b \Gamma^{\downarrow}_{(b)} = \sum_b t_b [n(E_b) - n(E_{(b)})] \int 2w(r;\ E_{(b)} - \mu_p) |\delta\chi_{(b)}|^2 \, dr; \tag{126}$$

$$\Gamma^{\downarrow}_{(\bar{n})} = \sum_a \Gamma^{\downarrow}_{(a)}$$
$$= \sum_a t_a [n(E_{(a)}) - n(E_a)] \int 2w(r;\ \mu_n - E_{(a)}) |\delta\chi_{(a)}(r)|^2 \, dr. \tag{127}$$

Here, $\delta\chi$ are the Coulomb corrections to the wave functions of the analog and ground states (106)–(109). In accordance with the model representation of the irreducible self-energy part $T_0$ [see (38)], the width $\Gamma^{\downarrow}_{(b)}$, like the other partial widths of this type, can be represented in the form $\Gamma^{\downarrow}_{(b)} \sim 2\pi |\langle \lambda | H' | \delta\chi_{(b)} \rangle|^2 \rho$. Therefore, the relaxation width $\Gamma^{\downarrow}$ of an analog resonance can be interpreted as the width for "decay" of the analog resonance into configurations of a complicated nature due to the Coulomb mixing of states with different isospins. In this respect too, the analog resonances differ from giant resonances with "normal isospin," whose relaxation widths are determined by the nuclear interaction with the many-particle configurations and in order of magnitude are equal to $2w$. On the basis of an analysis of the "self-energy" of the doorway resonance $\Pi_{dd}$ and the statistical hypothesis (47) (see Sec. 1), we can, as in the case of the effective proton widths of the analog resonances, show that neglect of the approximate isospin conservation leads to the following substitution in (126) and (127): $(2T_0)^{1/2}\delta\chi - (2T_0)^{1/2}\delta\chi + \chi$, $\Gamma^{\downarrow} - \overline{\Gamma}^{\downarrow}$. Thus, the ratio $\beta^2 = \Gamma^{\downarrow}/\overline{\Gamma}^{\downarrow}$ can serve as a measure of the extent to which the isospin prohibits a "decay" of an analog state to configurations of complicated nature. In the single-particle approximation, calculation of $\beta^2$ for the analog of the ground state of the nucleus $^{208}$Pb leads to values of the order 0.04–0.06 for various choices of the radial and energy dependence of $w(r; |E - \mu|)$. The smallness of $\beta^2$ (compared with unity) predetermines the specific feature of analog resonances noted in the Introduction of having a small (compared with $2w$) relaxation width. The inequality $\beta^2 > \alpha^2$ [see the estimate (83)] indicates an important part of the continuum in destroying the isospin purity of the analog state. Note in this connection that for the considered analog resonance the partial-width ratio $\Gamma^{\downarrow}_{(b)}/\overline{\Gamma}^{\downarrow}_{(p)}$ varies in a wide range, significantly exceeding $\beta^2$ for the elastic and some inelastic proton channels, and that $\Gamma^{\downarrow}_{(p)} \gg \Gamma^{\downarrow}_{(\bar{n})}$.

Using the relations (98), we can explicitly express the widths $\Gamma^{\downarrow}_{(p)}$ and $\Gamma^{\downarrow}_{(\bar{n})}$ in terms of the symmetry energy[30]:

$$\Gamma^{\downarrow}_{(p)} = \sum_b \Gamma^{\downarrow}_{(b)} = \sum_b (2T_0)^{-1} t_b [n(E_b) - n(E_{(b)})]$$
$$\times \Big\{ -2\,\mathrm{Im} \Big[ \int \chi_b (\check{v}_{(b)} + \delta v^*) g_{(b)}(E_{(b)}) (\check{v}_{(b)} + \delta v) \chi_b \, dr \, dr'$$
$$- \int \check{v}^2_{(b)} \chi^2_b \, dr \Big] - 2\pi \Big| \int \chi^{(+)}_{E_{(b)}} (\check{v}_{(b)} + \delta v^*) \chi_b \, dr$$
$$\times \int \chi_b (\check{v}_{(b)} + \delta v) \chi^{(+)}_{E_{(b)}} dr \Big| \Big\}; \tag{128}$$

$$\Gamma^{\downarrow}_{(\bar{n})} = \sum_a \Gamma^{\downarrow}_{(a)} = \sum_a (2T_0)^{-1} t_a [n(E_{(a)}) - n(E_a)]$$
$$\times \Big\{ 2\,\mathrm{Im} \Big[ \int \chi_a (\check{v}_{(a)} + \delta v^*) g_{(a)}(E_{(a)}) (\check{v}_a + \delta v) \chi_a \, dr \, dr'$$
$$+ \int \check{v}_{(a)} \chi^2_a \, dr \Big] - 2\pi \Big| \int \chi_{(a)} (\check{v}_{(a)} + \delta v) \chi^{(+)}_{0E_a} dr \Big|^2 \Big\}, \tag{129}$$

where $\Gamma^{\downarrow}_{(p)}$. The results of calculations of the widths $\Gamma^{\downarrow}_{(p)}$ made in accordance with (128) in the single-particle approximation ($\delta v = 0$) for the analog of the ground state of the $^{208}$Pb nucleus as a function of the parameters of the optical correction $\Delta h_p = -iw_p + \Delta_p$ are given in Fig. 5 (Ref. 41). It follows from the dependences shown in this figure that for "realistic" values of the parameters $w_p$ and $\Delta_p$ the calculated value of the width $\Gamma^{\downarrow}_{(p)}$ overestimates by about a factor 3 the experimental value $\Gamma^{\downarrow}_{exp} \approx 80$ keV (Ref. 17). Other microscopic approaches to the description of analog resonances have achieved a similar accuracy.[17,34,39] This discrepancy, which also occurs for other nuclei,[17] must be attributed in the first place to the neglect of polarization effects. In accordance with the estimate (101), allowance for coupling of the channels offers hope of eliminating this difficulty

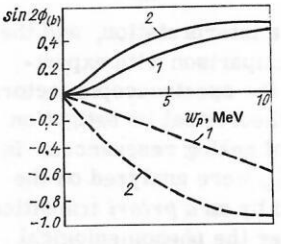


FIG. 5. Dependences of $\sin 2\phi_{(b)}$ (solid curves) and $\sin 2\overline{\phi}_{(b)}$ (broken curves) on the intensity of (volume) absorption $w_p$ and the opitical shift $\Delta_p$ calculated for the indicated analog resonance. The curves with labels 1 and 2 correspond to $\Delta p = 0$ and $\Delta p = 5$ MeV, respectively.

in the quantitative interpretation of the widths $\Gamma^{\downarrow}$.

*Resonance (pn) Reaction. Fine Structure of Analog Resonances.* As resonances in the excitation functions, analog resonances were detected for the first time in the investigation of the $(pn)$ reaction.[28] This reaction is forbidden by the isospin selection rule $\Delta T = 3/2$ (see Fig. 1). Investigation of the resonance $(pn)$ reaction gives information about the mechanism of excitation of analog resonances in the elastic proton channel and about the mechanism of Coulomb mixing of analog states and states with "normal" isospin, as a result of which neutron decay of the analog resonances occurs. There are two possible mechanisms of neutron decay of an analog resonance: direct and statistical. If the proton decay of an analog resonance can be regarded as a direct single-stage process (see Sec. 2), the direct neutron decay is a two-stage process: because of the field $\mathscr{V}_C$, the analog state is mixed with $T_<$ configurations of $(p\bar{n})$ type, which, as a result of the nuclear interaction, decay directly with the emission of a neutron into the continuum. Thus, direct neutron decay can occur without the stage of excitation of many-particle configurations (hence the expression "direct decay"). The statistical mechanism of neutron decay of an analog resonance is due to the Coulomb mixing of the analog state with many-particle configurations. The intensity of this mixing is characterized by the width $\Gamma^{\downarrow}$. Since the neutron decay of complicated configurations is the main decay channel, the width $\Gamma^{\downarrow}$ is the neutron width of the analog resonance corresponding to the statistical mechanism. With allowance for these comments, we represent the total width of the analog resonance in the form

$$\Gamma = \Gamma_p^{\uparrow} + \Gamma_n^{\uparrow} + \Gamma^{\downarrow}, \tag{130}$$

where $\Gamma_n^{\downarrow}$ is the width for the direct neutron decay of the analog resonance (the existence of other direct decay channels besides the single-nucleon channels is ignored). The direct neutron decay of analog resonances is treated concretely below.

Experimentally, analog resonances in the $(pn)$ reaction were observed on nuclei with $A < 150$ (Ref. 40). The reason why the resonance $(pn)$ reaction is not observed in heavier nuclei is that there is a significant increase in the nonresonance part of the cross section, which effectively coincides with the background cross section of the reactions as compared with the resonance part of the cross section at the maximum:

$$\sigma_{pn}^{bg} \approx \pi\lambda_p^2 \sum_{lj} (j+1/2)\, T_{lj} \gg \sigma_{pn}^{res}(E_r), \tag{131}$$

where, in accordance with the conclusions drawn in Sec. 1, $T_{lj} = 1 - |\bar{S}_{lj}|^2$ are the optical-model values of the penetrability coefficients for the protons. A characteristic feature of analog resonances in the $(pn)$ reaction is the small asymmetry of the cross sections $\sigma_{pn}(E)$, the asymmetry being maximal as a rule for resonances with small angular momentum. As is usual in the investigation of resonance reactions, two questions arise, namely, the parametrization of the energy dependence of the total cross section of the $(pn)$ reaction and the quantitative (or qualitative) interpretation of the resonance parameters. In principle, answers to these questions can be obtained in the framework of the semimicroscopic approach.

When a sufficiently large number of neutron channels is open, we can in accordance with (56) ignore the fluctuation cross section of the $(pp)$ reaction compared with the optical cross section. In this case, from the unitarity of the $S$ matrix we obtain an expression for the average total cross section of the resonance $(pn)$ reaction:

$$\sigma_{pn}^{J\pi}(E)/g\pi\lambda_p^2 = 1 - \sum_{b'} |\bar{S}_{(b)(b')}|^2, \tag{132}$$

where the indices $b$ label the proton decay channels. Substitution in (132) of an expression for the average $S$ matrix of the form (49) with allowance for (130) and the conclusions drawn above concerning the partial proton widths and the resonance mixing phases leads to the result

$$\frac{\sigma_{pn}^{J\pi}}{g\pi\lambda_p^2} = T_{(b)} \frac{(E-E_r-\delta_{(b)})^2 + \varepsilon_{pn}^2}{(E-E_r)^2 + (1/4)\,\Gamma^2}, \tag{133}$$

where the optical-model penetrability coefficient $T_{(b)}$, the asymmetry parameter $\delta_{(b)}$, and the symmetrizing factor $\varepsilon_{pn}^2$ are given by the expressions

$$T_{(b)} = 1 - \exp[-4\eta_{(b)}]; \quad \delta_{(b)} = \Gamma_{(b)}^{\uparrow} \sin 2\phi_{(b)}/2 \operatorname{sh} 2\eta_{(b)}; \tag{134}$$

$$\varepsilon_{pn}^2 = T_{(b)}^{-1}\Gamma_{(b)}^{\uparrow}\{\Gamma e^{-2\eta_{(b)}} \cos 2\phi_{(b)} - \Gamma_p^{\uparrow} - \sin^2 2\phi_{(b)}\Gamma_{(b)}^{\uparrow}(T_{(b)}^{-1} - 1)\}. \tag{135}$$

The parametrization (133) of the cross section $\bar{\sigma}_{pn}$ (133) was proposed in Ref. 38. Away from the analog resonance, the cross section (133) goes over into the background value, which effectively coincides with the background value of the reaction cross section for a resonance partial proton wave in accordance with (131). Therefore, the resonance part of the cross section can be represented in the form

$$\frac{\bar{\sigma}_{pn}^{res}}{g\pi\lambda_p^2} = T_{(b)} \left\{ \frac{(E-E_r-\delta_{(b)})^2 + \varepsilon_{pn}^2}{(E-E_r)^2 + (1/4)\,\Gamma^2} - 1 \right\}. \tag{136}$$

Thus, as for the elastic scattering of protons, the asymmetry of an analog resonance in the total cross section of the $(pn)$ reaction

$$2\frac{\sigma_{pn}^{res}(E=E_r+(1/2)\,\Gamma) - \sigma_{pn}^{res}(E=E_r-(1/2)\,\Gamma)}{\sigma_{pn}^{res}(E=E_r+(1/2)\,\Gamma) + \sigma_{pn}^{res}(E=E_r-(1/2)\,\Gamma)} = -\frac{2\Gamma\delta_{(b)}}{\varepsilon_{pn}^2 - (1/4)\,\Gamma^2 + \delta_{(b)}^2} \approx \frac{4\phi_{(b)}\,\Gamma}{\Gamma - \Gamma_p^{\uparrow}} \tag{137}$$

is determined in accordance with (133) and (134) by the "elastic" resonance mixing phase $\phi_{(b)}$. In accordance with the conclusions drawn in Sec. 1 and in this section, such a quantity appears because of the interference between the excitation by the protons of the analog states and the many-particle configurations in the doorway channel of the reaction and exists to an extent measured by the Coulomb correction $\delta\chi_{(b)}$ [(106) and (108)] corresponding to the elastic proton channel. At fixed energy $E_{(b)}$, the value of $|\delta\chi_{(b)}|$ increases with decreasing $l_b$ because of the decrease in the centrifugal barrier. Examples of the calculation of the phases $\phi_{(b)}$ in the single-particle approximation are given in Fig. 4 and Table I. Other examples can be found in Ref. 17.

For small asymmetry (137), Eq. (136) goes over into the Breit–Wigner formula:

$$\sigma_{pn}^{res}/(g\pi\lambda_p^2) = \Gamma_{(b)}^{\uparrow}\Gamma_n/[(E-E_r)^2 + (1/4)\,\Gamma^2], \tag{138}$$

so that the width $\Gamma_n \equiv \Gamma - \Gamma_p^\downarrow = \Gamma^\downarrow + \Gamma_n^\downarrow$ is the total neutron width of the analog resonance, or the width for the decay of the analog resonance through the isospin-forbidden channels.

If the magnitude and energy dependence of the cross section $\sigma_{pn}^{bg}(E)$ are described by means of the optical model in accordance with (131) or more accurately by the Hauser–Feshbach method (see Sec. 1), comparison of (136) with the experimental excitation function makes it possible to determine the experimental values of the parameters $E_r$ and $\Gamma$ and the parameter combinations $T_{(b)}\delta_{(b)}$ and $T_{(b)}(\varepsilon_{pn}^2 - (1/4)\Gamma^2 + \delta_{(b)}^2)$. If, in addition, we use the value of the penetrability coefficient $T_{(b)}$ found by means of optical-model "fitting" of the cross section $\sigma_{pn}^{bg}$, we can determine separately the parameters $\delta_{(b)}$ and $\varepsilon_{pn}$. Such a procedure was carried out for the first time in Ref. 42. The hitherto most complete experiment realized for resonance scattering of protons by nuclei with $A = 100 - 130$ consists of the simultaneous analysis of not only the cross section $\sigma_{pn}(E)$ but also the differential cross section for elastic scattering of protons.[43] Such an analysis makes it possible to determine as well the experimental values of $\Gamma_{(b)}^\downarrow$ and $\phi_{(b)}$.

Given the experimental values of the parameter $\delta_{(b)}$ and $\Gamma_{(b)}^\downarrow$, and also the penetrability coefficient $T_{(b)}$, one can draw a qualitative conclusion about the width $\Gamma_{(b)}^\downarrow$ [(124) and (128)], which characterizes the intensity of "elastic" mixing, i.e., the Coulomb mixing of states with different isospins in the doorway channel of the reaction. To see this, we consider the following parameter combination:

$$\check{\Gamma}_{(b)}^\downarrow \equiv \delta_{(b)}^2 T_{(b)}/\Gamma_{(b)}^\uparrow + \Gamma_{(b)}^\uparrow T_{(b)}/4. \tag{139}$$

Restricting ourselves to the realistic case $\phi_{(b)} \ll 1$, $T_{(b)} \ll 1$, we find in accordance with (134), (95), (7), (11), and (118) to first order in $\Delta h_p$

$$\begin{aligned}\check{\Gamma}_{(b)}^\downarrow &\approx 4\phi_{(b)}^2 \Gamma_{0(b)}^\uparrow/T_{(b)} + \Gamma_{(b)}^\uparrow T_{(b)}/4 \\ &= 2N_b S_b \left| \int \delta\chi_{(b)} w_p \chi_{0E_{(b)}}\, dr \right|^2 \Big/ \int w_p \chi_{0E_{(b)}}^2\, dr.\end{aligned} \tag{140}$$

Comparing this expression with (126), we see that $\check{\Gamma}_{(b)}^\downarrow = \Gamma_{(b)}^\downarrow$, if in the region where $w_p(r) \neq 0$, $\mathrm{Re}\,\delta\chi_{(b)} = \mathrm{const}\,\chi_{0E(b)}$. In reality, these widths are equal only in order of magnitude. A calculation of the ratio $\Gamma_{(b)}^\downarrow/\Gamma_{(b)}^\downarrow$, made in accordance with (140) for the first $0^+$ analog resonance in the nucleus $^{208}$Bi gives the value 0.8–1.4, depending on the form of the function $w_p(r)$. In connection with Eqs. (139) and (140), we note that the variant of the shell approach in whose framework there is no microscopically justified transition to the optical model leads to the equality $\Gamma_{(b)}^\downarrow = \Gamma_{(b)}^\downarrow$ (Refs. 2 and 38).

With regard to the parameters $\delta_{(b)}^2$ and $\varepsilon_{pn}^2$ determined directly by the analysis of the cross section $\sigma_{pn}^{res}$ (136), one can draw a conclusion about the relative importance of the elastic Coulomb mixing in the formation of the total neutron widths $\Gamma_n = \Gamma_n^\downarrow + \Gamma^\downarrow$. To see this, we consider the expression (135) for the parameter $\varepsilon_{pn}^2$ in the limit $\phi_{(b)} \ll 1$, $T_{(b)} \ll 1$:

$$\varepsilon_{pn}^2 \approx \Gamma_{(b)}^\downarrow \check{\Gamma}'/T_{(b)} - (\Gamma - \Gamma_{(b)}^\downarrow)^2/4; \quad \check{\Gamma}' = \Gamma_n - \check{\Gamma}_{(b)}^\downarrow. \tag{141}$$

In the approximation $\check{\Gamma}_{(b)}^\downarrow = \Gamma_{(b)}^\downarrow$, the width $\check{\Gamma}' = \Gamma'$ is the width for the decay of the analog state through the channels forbidden by isospin, the width for elastic mixing being subtracted. Therefore, under these approximations, the ratio

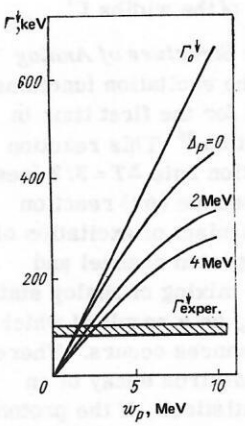


FIG. 6. Dependence of the width $\Gamma^\downarrow$ on the intensity of (surface) absorption $w_p$ and the optical shift $\Delta_p$ for the first $0^+$ analog resonance in the nucleus $^{208}$Bi. The width $\Gamma_0^\downarrow$ is calculated in accordance with (126). The hatched strip is the region of the experimental values of $\Gamma^\downarrow$.

$$\delta_{(b)}^2/\varepsilon_{pn}^2 \approx \check{\Gamma}_{(b)}^\downarrow/\check{\Gamma}' \approx \Gamma_{(b)}^\downarrow/\Gamma' \tag{142}$$

does indeed characterize the relative importance of the elastic mixing in the formation of the width of the analog resonance for decay through the isospin-forbidden channels. An analysis of the experimental cross sections $\sigma_{pn}^{res}$ in accordance with (136) for nuclei in a wide range of atomic masses[42,43] shows that $\delta_{(b)}^2 \ll \varepsilon_{pn}^2$ for all the investigated analog resonances. This assertion is inconsistent with the suggestion made in the first years of the investigation of analog resonances that elastic (external) mixing plays a dominant part in forming the width $\Gamma_n$ (Ref. 44). Such an assumption ($\delta_{(b)}^2 \gg \varepsilon_{pn}^2$) leads also to values of $T_{(b)}$ that are several times smaller than the corresponding optical-model values.[45,42]

In experiments in which proton scattering by comparatively light nuclei is investigated with high resolution, in which the energy $\Delta E_c$ is small, a fine structure of the analog resonances is revealed (see, for example, Ref. 46). The existence of the fine structure means that in the region of the analog resonance the nucleon widths of the compound-nucleus resonances (with the same spin and parity as the analog state) exceed the background values. The enhancement of the widths arises because of Coulomb mixing of states with different isospins. If the number of fine-structure components is large, then from the experimental data on the elastic scattering of protons one can find an "elastic" proton strength function $(SF)_{(b)} = \rho\gamma_{(b)}$. In accordance with (55), this quantity, which contains information about the parameters of the analog resonances, is determined by the penetrability coefficient:

$$\bar{T}_{(b)} = 1 - \sum_q |\bar{S}_{(b)q}|^2 \approx 2\pi\,(SF)_{(b)}, \tag{143}$$

where the summation is over all nucleon decay channels. Proceeding as in the derivation of Eqs. (133)–(135), for the "induced" part of the strength function we obtain the expression

$$2\pi\{(SF)_{(b)}-(SF)_{(b)}^{bg}\}=T_{(b)}\left\{\frac{(E-E_r-\delta_{(b)})^2+\varepsilon_{(b)}^2}{(E-E_r)^2+\Gamma^2/4}-1\right\}. \quad (144)$$

Here, in accordance with Sec. 1, $2\pi(SF)_{(b)}^{bg}=T_{(b)}$; the value of the parameter $\varepsilon_{(b)}^2$ can be found from (135) by the substitution $\Gamma_p^\uparrow \to \Gamma_p^\uparrow+\Gamma_n^\uparrow \equiv \Gamma^\uparrow$ ($\Gamma^\uparrow$ is the total nucleon width of the analog resonance); the remaining parameters are the same as in (133)–(135). Therefore, $\delta_{(b)}^2/\varepsilon_{(b)}^2 \approx \Gamma_{(b)}^\uparrow/(\Gamma^\uparrow-\Gamma_{(b)}^\uparrow)$ characterizes the relative importance of elastic mixing in the formation of the relaxation width of the analog resonance. For small asymmetry, Eq. (144) simplifies:

$$2\pi\{(SF)_{(b)}-(SF)_{(b)}^{bg}\}\approx\Gamma_{(b)}^\uparrow\Gamma^\uparrow[(E-E_r)^2+\Gamma^2/4]^{-1},$$

so that the amount by which the proton strength function exceeds the background value is determined by the width $\Gamma^\uparrow$ (Ref. 2).

Measurement of not only $\bar{\sigma}_{(b)(b)}$ but also the average polarization of the protons in the region of an analog resonance makes it possible to determine the parameters of the average diagonal element of the $S$ matrix and, therefore, in accordance with (57) the fluctuation cross section of elastic scattering:

$$\sigma_{(b)(b)}^{fl}=\bar{\sigma}_{(b)(b)}-\sigma_{(b)(b)}^{opt}=g\pi\lambda_p^2\bar{T}_{(b)}-\bar{\sigma}_r. \quad (145)$$

When only one proton channel is open ($\Gamma_{(b)}^\uparrow \approx \Gamma_p^\uparrow$), the relation (145) makes it possible to determine the penetrability coefficient (143) if the cross section $\bar{\sigma}_{pn}\approx\bar{\sigma}_r$ is known. Such a procedure for determining $\bar{T}_{(b)}$ was used in Ref. 46, and the ratio $\delta_{(b)}^2/\varepsilon_{(b)}^2$ was small.

*Direct Neutron Decay of Analog Resonances.* In recent years, experimental searches have been made for analog resonances in the cross sections of neutron–nucleus interactions[47,48,49] and by analyzing the partial cross sections of the $(pn)$ reaction with the excitation of analog resonances.[50] These processes contain information on the partial neutron widths of the analog resonances. The expressions for the average cross sections of the partial reactions with nucleons are determined by Eqs. (57) and (49) and in the approximation of small asymmetry of the analog resonances and small proton penetrability coefficients $T_{(b)}$ have a simple form:

$$\frac{\bar{\sigma}_{b(\nu)}}{g\pi\lambda_p^2}\approx\frac{\Gamma_{(b)}^\uparrow\Gamma_\nu}{(E-E_r)^2+\Gamma^2/4};\quad \frac{\bar{\sigma}_{\nu_0 p}}{g\pi\lambda_n^2}\approx\frac{\Gamma_{\nu_0}\Gamma_p^\uparrow}{(E-E_r)^2+\Gamma^2/4},$$

where $\Gamma_\nu=\Gamma_\nu^\uparrow+\Gamma^\downarrow$ ($T_\nu/\sum_{\nu'}T_{\nu'}$) is the partial neutron width of the analog resonance. Thus, in the investigation of the average cross sections one cannot separate the direct and statistical mechanisms of neutron decay of the analog resonances.

Although the reduced widths $\theta^2$ (the widths divided by the penetrability of the potential barrier for nucleons) for direct proton and neutron decays of the analog resonances are determined by the field $\mathscr{V}_C$, they differ appreciably because of the difference noted above between the decay mechanisms:

$$\theta_{(\nu)}^2/\theta_{(b)}^2\sim\theta_{int}^2/\theta_{sp}^2\sim A^{-4/3}, \quad (146)$$

where $\theta_{sp}^2\sim E_0A^{-1/3}$ is the single-particle reduced width, and $\theta_{int}^2\sim A^{-4/3}\theta_{sp}^2$ is the reduced width for decay of the simple (three-quasiparticle) configuration through the nucleon channel. The difference between the penetrabilities of the potential barriers for neutrons and protons to some extent compensates the small value of the ratio (146).

The partial widths for direct neutron decay of analog resonances are determined, as for the isospin-forbidden two-stage process, by the Coulomb corrections to the wave functions of the analog and the ground state (106)–(109) and the effective interaction of the nucleons, for which one can take the interaction (60). For resonance scattering of neutrons by an even–even nucleus $(N, Z)$ with isospin $T_0$ there is excited in the nucleus $(N+1, Z)$ the analog of the parent nucleus $(N+2, Z-1)$ with isospin $T_0+3/2$. We shall restrict ourselves to a treatment of the simplest configurations of the target nucleus and the parent nucleus. Such a configuration in the parent nucleus is a two-neutron–proton-hole configuration: $(j_b^2)0+j_a^{-1}$. In this case, the amplitude of the neutron width is determined by the proton and neutron Coulomb corrections $\delta\chi_{(b)}$ and $\delta\chi_{(a)}$. Direct calculation leads to the following expression for the partial neutron widths[51-53,58]:

$$\Gamma_{0\nu}^\uparrow=\frac{2j_b+1}{64\pi}\left|\int[F+F'-3(G+G')]\times[\delta\chi_{(b)}\chi_b\chi_a+\chi_b^2\delta\chi_{(a)}]\,\chi_{0E_n j_a l_a}^{(+)}r^{-2}\,dr\right|^2, \quad (147)$$

where $r^{-1}\chi_{0E_n j_a l_a}(r)$ is the radial wave function of the neutron in the continuum [an eigenfunction of the Hamiltonian $h_{0n}$ (69)]. For resonance scattering of a neutron by an odd-proton nucleus with configuration $j_a^{-1}$ with excitation of the analog of the parent nucleus with configuration $(j_b^2)_{0^+}$, a formula for the partial neutron width $\Gamma_{0\nu}^\uparrow$ can be obtained from (147) by multiplying by the factor $(2j_a+1)^{-1}$. Other cases of direct neutron decay can be described by formulas more cumbersome than (147) (Refs. 52 and 53).

The influence of the excitation of many-particle configurations in the process of direct neutron decay of analog resonances on the width $\Gamma_{0\nu}^\uparrow-\Gamma_\nu^\uparrow$ must be taken into account in two respects: 1) the wave function of a neutron in the continuum is determined by means of the optical-model Hamiltonian for the neutrons $h_n+h_{0n}+\Delta h_n$: $\chi_{0E_n}^{(+)}$; 2) the effective correction to the wave function of the analog state, $\delta\tilde{\chi}_{(b)}$, is determined with allowance for the excitation of many-particle configurations in the process of real and virtual proton decay of the analog resonance through the channel $b$: $\delta\chi_{(b)}-\delta\tilde{\chi}_{(b)}$ (Ref. 29), where $\delta\chi_{(b)}$ is the correction (106) and (108), and

$$\delta\tilde{\chi}_{(b)}(r)=\delta\chi_{(b)}(r)+\int g_{(b)}(r, r';\ E_{(b)})\,\Delta h_p(r';\ E_{(b)})\,\delta\chi_{(b)}(r')\,dr'.$$

A similar relation holds for the correction $\delta\tilde{\chi}_{(a)}$.

The results of calculations of the widths $\Gamma_\nu^\uparrow$ in the single-particle approximation for elastic scattering of neutrons by the nuclei $^{90}$Zr, $^{206,208}$Pb with excitation of the analogs of proton-hole configurations of the parent nuclei $^{91}$Y, $^{207,209}$Tl, respectively, are given in Table II.[51-53] The calculated values of the elastic neutron widths do not contradict the data of Refs. 48 and 50. The results of the calculations also confirm the conclusion that follows from (147) that it is better to study the excitation of analog resonances by neutrons on nuclei with small neutron excess, since $|\delta\chi|^2\sim(2T_0+3)^{-1}$.

TABLE II. Calculations made in the single-particle approximation of the effective neutron widths for direct excitation of analog resonances in the elastic scattering of neutrons by the nuclei $^{90}$Zr, $^{206,208}$Pb.

| Target nucleus | $^{90}$Zr ($j_b^\pi = 5/2^+$) | | | | | | $^{206}$Pb ($j_b^\pi = 1/2^-$) | | | $^{208}$Pb ($j_b^\pi = 9/2^+$) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j_a^\pi$ | $1/2^-$ | $9/2^+$ | $3/2^-$ | $5/2^-$ | $7/2^-$ | $5/2^+$ | $1/2^+$ | $3/2^+$ | $5/2^+$ | $1/2^+$ | $3/2^+$ |
| $E_n$, MeV | 5.21 | 5.63 | 5.72 | 6.15 | 6.39 | 6.48 | 12.8 | 13.1 | 14.4 | 18.1 | 18.4 |
| $\Gamma_\nu^\dagger$, keV | 1.48 | 0.07 | 1.40 | 0.53 | 0.33 | 1.66 | 0.62 | 0.11 | 0.08 | 0.02 | 0.003 |

*Quasielastic (pn) Scattering.* Analog resonances were found for the first time as narrow peaks in the hard part of the neutron spectrum in the direct (pn) reaction.[54] The direct (pn) reaction with excitation of an analog state came to be called quasielastic (pn) scattering, since the total and differential cross sections of this reaction can be described approximately in the framework of the distorted-wave Born approximation by means of the potential $\hat{V}_L = V_L(r)\mathbf{tT}$, which is a direct generalization of the part of the shell (optical) potential that depends on the neutron excess.[31,35] The shell approach to the theory of analog resonances leads to a number of conclusions about the cross section of quasielastic (pn) scattering which require either experimental verification or an additional analysis of the existing experimental data. These conclusions relate to: 1) the choice of the effective charge-exchange field acting on a nucleon in the process of quasielastic (pn) scattering; 2) allowance for absorption of nucleons in this process; 3) the existence of a strong even-odd effect in the total cross section for nuclei with minimal neutron excess.

1. The total cross section of quasielastic (pn) scattering by an even-even nucleus is determined by the effective field (97), which realizes a coupling of the analog state to the continuum:

$$\sigma_{pn}^{\text{dir}} \sim k_p^{-2} \sum_{jl} \left(j+\frac{1}{2}\right) |J_{E_pE_njl}|^2. \tag{148}$$

In the absence of absorption, the radial integrals $J^0_{E_pE_njl}$ are determined by two equivalent expressions:

$$(2T_0)^{1/2} J^0_{E_pE_njl} = \int \chi^{(+)}_{0E_njl} \tilde{\mathcal{T}}^- c \chi^{(+)}_{0E_pjl}\, dr = \int \chi^{(+)}_{0E_njl} (v+\delta v)\, \chi_{0E_pjl}\, dr. \tag{149}$$

As for the partial proton widths of analog resonances, the use of the phenomenological Hamiltonian (100) to describe quasielastic (pn) scattering (without allowance for the effects of absorption) is justified only in the single-particle approximation. Allowance for real and virtual proton decay of the analog resonances leads in accordance with (149) to the need to use an effective symmetry energy; $T_0 V_L = v \to v + \delta v$ (or an effective Coulomb field: $\mathcal{V}_C \to \tilde{v}_C \equiv v_C + \delta v$). This is possibly one of the reasons for the systematic decrease of the potential $T_0 V_L$ obtained by analyzing the absolute cross sections $\sigma_{pn}^{\text{dir}}$ as compared with (70) found from the systematics of single-particle levels.[13]

The weak dependence of the partial proton widths on the absolute value of the symmetry energy (see above) can be explained by the circumstance that in accordance with (79) the resonance kinetic energy of the protons $E_{(b)} = E_b^n + \Delta E_C = E_b^p + v$ exceeds by the symmetry energy the energy of the corresponding single-particle resonance. For quasielastic (pn) scattering, there is no such connection. Therefore, in accordance with (148) and (149) the reduced cross section $\sigma_0 = \sigma_{pn}^{\text{dir}}/(N-Z)$ is approximately independent of $(N-Z)$. Such isotopic dependence of experimental cross sections is realized with various degrees of accuracy for even-even nuclei.[56,57]

2. As for the effective partial proton widths of analog resonances (see above), allowance for the absorption of nucleons in the process of quasielastic (pn) scattering is made by going over to the wave function of the optical model for the nucleons in the first (but not the second!) of the relations (149):

$$J_{E_pE_njl} = (2T_0)^{-1/2} \int \chi^{(+)}_{E_njl} \tilde{\mathcal{T}}^{-(+)} c \chi_{E_pjl}\, dr. \tag{150}$$

In accordance with (98), the radial integral (150) can be expressed in terms of the symmetry energy:

$$J_{E_pE_njl} = (2T_0)^{-1/2} \int \chi^{(+)}_{E_njl} [v + \delta v - \Delta h_p + \Delta h_n]\, \chi^{(+)}_{E_pjl}\, dr.$$

In the single-particle approximation ($\delta v - 0$), this expression can also be obtained by means of an obvious generalization of the Hamiltonian (119) for the "nucleon + nucleus" system. The distorted-wave Born approximation, with neglect of the approximate conservation of isospin (see above), leads to a different result:

$$\bar{J}_{E_pE_njl}(\delta v \to 0) = (2T_0)^{-1/2} \int \chi^{(+)}_{E_njl} v \chi^{(+)}_{E_pjl}\, dr,$$

so that the ratio $(\sigma_{pn}^{\text{dir}}/\sigma_{pn}^{\text{dir}})_{\delta v \to 0} \approx |v/(v - \Delta h_p + \Delta h_n)|^2$ may differ appreciably from unity.[58]

3. The experimental data on the cross section $\sigma_{pn}^{\text{dir}}$ reveal the existence of a significant even-odd effect in the isotope dependence of this cross section for nuclei with small neutron excess:

$$\sigma_0^{\text{odd}} = \sigma_0^{\text{even}}(1+\beta), \tag{151}$$

where $\beta > (N-Z)^{-2}$ (Ref. 56). The anomalous contribution of the odd neutron to the cross section can be explained by the spin-isospin part of the effective interaction of the nucleons (60), since for scattering by an even-even nucleus ($0^+ \to 0^+$ transition) this interaction does not occur in the cross section. For an odd target nucleus ($J^\pi \to J^\pi$ transition) the effective field acting on a nucleon in the process of quasielastic (pn) scattering is modified (we ignore dynamical polarization effects):

$$\mathcal{T}(r) = (2T_0)^{-1/2} \langle 0^+ | F' \sum_i \tau_i^{(3)} \delta(\mathbf{r} - \mathbf{r}_i) | 0^+ \rangle$$
$$\to (2T_0)^{-1/2} \langle J^\pi M_f | \sum_i \tau_i^{(3)} (F' + G' \boldsymbol{\sigma}\boldsymbol{\sigma}_i)\, \delta(\mathbf{r} - \mathbf{r}_i) | J^\pi M_i \rangle.$$

If the spin and parity of the target nucleus are determined by the odd neutron, a direct calculation of the cross section $\sigma_{pn}^{\text{dir}}$ leads to the following expression for the parameter $\beta$ in Eq. (151) (Refs. 58 and 59):

$$\beta = (N-Z)^{-2} (J+1/2) [\alpha_1 + 3\alpha_2 (G'/F')^2], \tag{152}$$

where the coefficients $\alpha_{1,2} \lesssim 1$ are the ratio of the sums of the squares of the radial integrals weighted with kinematic factors. Since $G' \gtrsim F'$ (Ref. 15), it follows from (8) that the anomalous contribution of the odd neu-

tron to the cross section $\sigma_{pn}^{dir}$ is due basically to the spin–isospin part of the effective interaction of the nucleons and is emphasized by kinematic factors. The analysis made in Refs. 58 and 59 of the experimental data for the Ti isotopes confirms this conclusion. On the basis of (151) and (152), we can also conclude that for nuclei with minimal neutron excess $N-Z=1, 2,$ or 3 the cross section $\sigma_{pn}^{dir}$ can differ appreciably from the values calculated by means of the potential $V_L$ determined from analysis of quasielastic $(pn)$ scattering by even–even nuclei.

## CONCLUSIONS

The analysis of the basic parameters of analog states and analog resonances in the cross sections of nucleon-nucleus interactions shows that the presented variant of the shell approach to the theory of nuclear reactions (semimicroscopic approach) is a constructive method of theoretical investigation of highly excited states of nuclei and the corresponding resonance nuclear reactions. We shall consider briefly the questions that require further investigation, these relating to both the approach itself and the theory of analog resonances. These questions include the following:

1) Systematic derivation in terms of the shell and optical models of an expression for the response function (18) with simultaneous allowance for the continuum and damping of the particle–hole configurations without recourse to a perturbation theory of the form (30) or other approximations. The solution of this problem is needed for the quantitative interpretation of multipole giant resonances (the interpretation of analog resonances is changed insignificantly) and, in particular, for the description of the excitation of these resonances by electrons and nucleons.

2) The search for an exact analytic formula for the shift relative to $\Delta E_C$ of the energy of an analog resonance, this taking into account not only the coupling of the analog resonance to the continuum (and many-particle configurations) but also polarization effects. The solution of this question may remove the problem of the "anomaly in the Coulomb energy."[60] A similar problem also exists in the estimate of the isospin purity in the ground state of the nucleus [in the estimate (76) of $\alpha^2_{gr.st}$].

3) A theoretical analysis of the basic parameters of analog resonances [the widths $\Gamma^{\uparrow}_{(b)}\Gamma^{\uparrow}$, the shift $\Delta_A$, and the phases $\phi_{(b)}$ (or the asymmetry parameters $\delta_{(b)}$)] based on numerical solution of the integral equation for the effective Coulomb field (96). Such an analysis makes possible a detailed comparison of the conclusions of the semimicroscopic approach for the basic parameters of analog resonances and experimental data.

4) Quantitative interpretation of E1 radiative decay of analog resonances on the basis of the solution to the problems noted in 1) and 3) above. A qualitative analysis of the problem is given in the framework of the shell approach in Ref. 61.

5) Improvement of the scheme for calculating the partial widths for direct neutron decay of analog resonances with inclusion in the analysis of more complicated (than single-particle) configurations of the parent nuclei and the product nuclei.

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## APPENDIX

### *Quasiclassical Analysis of the Equations of the Random Phase Approximation*

Here, we give a brief survey of the method and results of the quasiclassical analysis of the equations of the random phase approximation, as applied to isovector giant resonances. The advantages and disadvantages of the method are discussed in Sec. 2, where we give references to papers in which this method is applied.

1. The method of quasiclassical solution of the equation for the effective density of excess neutrons (72)

$$n(r)=n_0(r)+F'\sum_{ab}\varphi_a^*(\mathbf{r})\,\varphi_b(\mathbf{r})\,A_{ab}^{p_0h_0}(\omega=0)\,n_{ba} \tag{A.1}$$

is as follows. The matrix elements of the smooth function of the coordinates $n(r)$ are significantly different from zero for transitions $|E_a-E_b|\sim D<E_0$. Therefore, with accuracy $D/E_0\sim A^{-1/3}$

$$A_{ab}^{p_0h_0}(\omega=0)=(n_a-n_b)/(E_a-E_b)\approx-\delta(E_a-E_0). \tag{A.2}$$

This relation, together with the condition of completeness of the system of functions $\{\varphi_a(\mathbf{r})\}$, makes it possible to transform the integral term of (A.1) to the form

$$\sum_{ab}\varphi_a^*(\mathbf{r})\,\varphi_b(\mathbf{r})\,A_{ab}^{p_0h_0}(\omega=0)\,n_{ba}=-n(r)\,\rho_0(r)+\sum_a|\varphi_a(\mathbf{r})|^2\,n_{aa}\,\delta(E_a-E_0), \tag{A.3}$$

where $\rho_0(r)=\sum_a|\varphi_a(r)|^2\delta(E_a-E_0)$ is the density of particles with the Fermi energy. We replace the positive quantities $n_{aa}$ by some constant determined by the normalization condition $\int n(r)d^3r=N-Z$. If as the shell potential we use a rectangular potential well, when $F'\rho_0=f'$, then in accordance with (A.1)–(A.3) we obtain

$$n(r)=(1+f')^{-1}[n_0(r)+f'\bar{n}], \tag{A.4}$$

where $\bar{n}=(N-Z)/\Omega$. Since $F'$ changes little in crossing the boundary of the nucleus, in the application of the relation (A.4) to real nuclei it is natural to choose $\bar{n}(r)$ in accordance with (71).

2. The quasiclassical analysis of the equations for an effective field of the form (20) can be carried out by finding the eigenfunctions of the kernel of these integral equations. These functions $V_L$, which can be classified in accordance with the transferred angular momentum $L$, are determined by the homogeneous equation

$$\{A^{p_0h_0}(\omega)\,V_L\}\equiv\sum_{ab}A_{ab}^{p_0h_0}(\omega)\,(V_L)_{ba}\,\varphi_a^*(\mathbf{r})\,\varphi_b(\mathbf{r})=C_L(\omega)\,\rho_0(r)\,V_L(\mathbf{r}), \tag{A.5}$$

where for excitations of the same kind $A_{ab}^{p_0h_0}=(n_a-n_b)\times(E_a-E_b-\omega)^{-1}$; $C_L(\omega)$ are eigenvalues. The eigenfunctions have essentially nonzero matrix elements for transitions with energy $|E_a-E_b|$ near the characteristic energies $E_L$. Using the relation (A.2) and the completeness condition for the functions $\{\varphi_a(\mathbf{r})\}$ with quasiclassical accuracy $\sim A^{-1/3}$, we find the eigenvalues

$C_L(\omega)$ on the basis of (A.5):

$$C_L(\omega) = -(1-c_L)\frac{E_L^2}{E_L^2-\omega^2}; \quad c_L = \frac{\sum_a |\varphi_a(r)|^2 (V_L)_{aa}\,\delta(E_a-E_0)}{\rho_0(r)\,V_L(r)}. \tag{A.6}$$

To determine the characteristic energies $E_L$, which are the energies of the giant resonances in the model of independent particles, we calculate the sum

$$\sum_L \equiv \sum_s |(V_L)_{s0}|^2 (E_s-E_0); \quad V_L = \sum_i V_L(r_i) \tag{A.7}$$

in two independent ways: 1) by means of the polarizability (16) corresponding to the field $V_L(\mathbf{r})$:

$$\Sigma_L = -\frac{1}{\pi}\int \omega\,\mathrm{Im}\,\mathcal{P}(\omega)\,d\omega = -\frac{1}{\pi}\int \omega\,\mathrm{Im}\sum_{ab}|(V_L)_{ab}|^2 A_{ab}^{p_0h_0}(\omega)\,d\omega \tag{A.8}$$

[in the absence of velocity forces, $\sum_L$ does not depend on the interaction of the nucleons, and therefore this quantity can be calculated to accuracy $\sim (N-Z)/A$ for a system of $A/2$ noninteracting nucleons]; 2) by means of a sum rule, by expressing $\sum_L$ in terms of the commutator with the Hamiltonian:

$$\sum_L = \tfrac{1}{2}\langle 0|[V_L^+, [H, V_L]]|0\rangle$$
$$= \frac{1}{2M}\left\langle 0\left|\sum_i |\nabla_i V_L(r_i)|^2\right|0\right\rangle = \frac{1}{2M}\int |\nabla V_L(r)|^2 \rho(r)\,d^3r, \tag{A.9}$$

where $\rho(r)$ is the nucleon density. Calculating the sum in (A.8) by means of (A.5), we obtain in accordance with (A.8) and (A.9) the following expression for the characteristic energies:

$$E_L^2 = (A/2M)\int |\nabla V_L(r)|^2 \rho(r)\,d^3r/(1-c_L)\int \rho_0 |V_L(r)|^2 d^3r. \tag{A.10}$$

The actual expressions for the eigenvalues $C_L(\omega)$ and the characteristic energies depend on the form of the shell potential. Below, we give the expressions for the lowest energies $E_L$ and the corresponding eigenfunctions of monopole, dipole, and quadrupole symmetry found by means of Eqs. (A.5), (A.6), and (A.10) for two forms of the potential (oscillator and rectangular well):

$$\left.\begin{array}{ll}
V_{L=0}(r) = 1 - M\overset{\circ}{\omega}{}^2 r^2/E_0,\ c_{L=0}; & V_{L=0}(r) = 1-5r^3/3R^2,\ c_{L=0} = 4/9; \\
E_{L=0} = 2\overset{\circ}{\omega} = 80A^{-1/3}\ \text{MeV}; & E_{L=0} = (42E_0/MR^2)^{1/2} = 195A^{-1/3}\ \text{MeV}; \\
V_{L=1}(\mathbf{r}) = rY_{10}(\cos\vartheta),\ c_{L=1} = 0, & V_{L=1}(\mathbf{r}) = rY_{10}(\cos\vartheta),\ c_{L=1} = 0; \\
E_{L=1} = \overset{\circ}{\omega} = 40A^{-1/3}\ \text{MeV}; & E_{L=1} = (10E_0/3MR^2)^{1/2} = 55A^{-1/3}\ \text{MeV}; \\
V_{L=2}(\mathbf{r}) = r^2Y_{20}(\cos\vartheta),\ c_{L=2} = 1/2; & V_{L=2}(\mathbf{r}) = r^2Y_{20}(\cos\vartheta),\ c_{L=2} = 2/9; \\
E_{L=2} = 2\overset{\circ}{\omega} = 80A^{-1/3}\ \text{MeV}; & E_{L=2} = (12E_0/MR^2)^{1/2} = 105A^{-1/3}\ \text{MeV}.
\end{array}\right\} \tag{A.11}$$

3. The energies of isovector giant resonances of $(n\bar{n}, p\bar{p})$ type with "normal" isospin are determined by solving a homogeneous equation for the effective field (20):

$$V(E^{N\bar{N}}) = F'\{A^{p_0h_0}(E^{N\bar{N}})\}. \tag{A.12}$$

In the quasiclassical approximation for a rectangular potential well, this equation reduces in accordance with (A.5) to the form

$$1 = f' C_L(E^{N\bar{N}}). \tag{A.13}$$

Hence, with allowance for (A.6), the first of formulas (86) for the energies of the isovector giant resonances follows.

4. The energies of isovector giant resonances of $(p\bar{n})$ type are determined by Eq. (A.12), in which the response function is used:

$$A_{ab}^{p_0h_0}(\omega) = (n_a^p - n_b^n)(E_a^p - E_b^n - \omega)^{-1}. \tag{A.14}$$

In this case, to accuracy $(N-Z) \gg A^{1/3}$ we have instead of (A.2)

$$n_a^p - n_b^n = n_a^p - n_a^n + n_a^n - n_b^n = -[\rho_0^{-1}(N-Z) + E_a - E_b]\,\delta(E_a-E_b), \tag{A.15}$$

where $\rho_0 = \int \rho_0(r) d^3r$. For a shell potential of rectangular form, which is considered below, we have the difference

$$E_a^p - E_a^n = E_0^n - E_0^p = (4/3)E_0(N-Z)/A = (N-Z)\rho_0^{-1} \equiv \Delta_0. \tag{A.16}$$

Using (A.14)–(A.16), we obtain an expression for the eigenvalues of Eq. (A.5) in the form

$$-C_L(\omega) = (1-c_L)\frac{E_L^2 + \Delta_0(\omega-\Delta_0)}{E_L^2 - (\omega-\Delta_0)^2} + c_L\frac{\Delta_0}{\Delta_0-\omega}. \tag{A.17}$$

In addition, for monopole excitations of $(p\bar{n})$ type there is a new eigenfunction $V_A$ = const with eigenvalue $C_A(\omega) = -\Delta_0/(\Delta_0-\omega)$. Equation (A.13) determines, in particular, the energy of an analog state (79) and the energy of monopole exictations $E_c$ and $E_g$. In the approximation $\Delta_0 \ll E_{L=0}$, we arrive in accordance with (A.13) and (A.17) at Eqs. (80) and (81) for these energies. Note also that in accordance with (68), (71), and (A.16) the symmetry energy $v$ is equal to $f'\Delta_0$ with quasiclassical accuracy.

5. The energies of isovector giant resonances of $(\bar{n}p)$ type are determined by Eq. (A.13), where in the expression (A.17) for $C_L(\omega)$ the substitution $\Delta_0 \to -\Delta_0$ is made. For $1^-$ excitations, when $c_{L=1} = 0$, we arrive at the second of formulas (86). Approximately solving this equation for monopole and quadrupole excitations with allowance for the terms linear in the small ratio $\Delta_0/E_{L=0;2}$, we arrive at the last of formulas (86). Neglect of the "inverse" transitions $E_a - E_b = -E_L$ in the equation for the effective field (A.12) corresponds to the Tamm–Dankcoff approximation. In this approximation, instead of (A.6) and (A.17) we have, respectively,

$$\left.\begin{array}{l}
-C_L(\omega) = \frac{1}{2}(1-c_L)\frac{E_L}{E_L-\omega}; \\
-C_L(\omega) = \frac{1}{2}(1-c_L)\frac{E_L-\Delta_0}{E_L-\Delta_0-\omega} - c_L\frac{\Delta_0}{\Delta_0+\omega}.
\end{array}\right\} \tag{A.18}$$

Substitution of these expressions in (A.13) leads to (87) for the energies of $1^-$ isovector giant resonances in the Tamm–Dankcoff approximation.

6. The matrix elements for transitions to the excitation of a collective state $|s\rangle$ are determined by the transition density $\rho^s(\mathbf{r})$, the expression for which can be represented in the form

$$\left.\begin{array}{l}
\rho^s(r) = \sum_{ab}\varphi_a^*(r)\varphi_b(r)\rho_{ab}^s; \\
\rho_{ab}^s = \tilde{V}_{ba}^s A_{ab}^{p_0h_0}(E_s); \quad \rho^s(r) = (F')^{-1}\tilde{V}^s(r).
\end{array}\right\} \tag{A.19}$$

The amplitude of the effective self-field $\tilde{V}^s(\mathbf{r})$ is determined by the normalization condition

$$1 = \sum_{ab}|\tilde{V}_{ab}^s|^2 [A_{ab}^{p_0h_0}(E_s)]^2 (n_b - n_a). \tag{A.20}$$

Below, we consider $0^+$ excitations of $(p\bar{n})$ type. The actual calculations are made in the approximation $\Delta_0 \ll E_{L=0}$. Setting $\tilde{V}^s(r) = B_s V_{L=0}(r)$ and using the formulas of §4 above and (A.5) and (A.6), we find

$$B_c^{-2} = \sum_{ab} (V_{L=0})_{ab}^2 [A_{ab}^{p_0 h_0}(E_c)]^2 (n_b^n - n_a^p)$$

$$\approx \sum_a (V_{L=0})_{aa}^2 \Delta_0 (\Delta_0 - E_c)^{-2} \delta(E_a - E_0) = c_{L=0} \overline{V_{L=0}^2} \rho_0 \Delta_0 (\Delta_0 - E_c)^{-2}; \quad \text{(A.21)}$$

$$\left.\begin{aligned} B_g^{-2} &= 2(1 - c_{L=0}) \overline{V}_{L=0}^2 \rho_0 E_g E_{L=0}^2 [E_{L=0}^2 - E_g^2]^{-2}; \\ \overline{V}_{L=0}^2 &= \int \frac{V_{L=0}^2(r)\, d^3r}{\Omega} = \frac{4}{21}. \end{aligned}\right\} \quad \text{(A.22)}$$

The nonzero matrix elements for mixing of the analog and monopole states of $(\bar{p}n)$ type by the Coulomb field $\mathscr{V}_C$:

$$\mathscr{V}_C(r) = V_C(r) - \Delta E_C = \Delta E_C V_{L=0}(r)/4; \quad \Delta E_C = (6/5) Ze^2/R = E_A \quad \text{(A.23)}$$

are determined by the transition densities $\rho^s(r) = B_s = B_s V_{L=0}(r)/F'$ [see (80) and (81)], obtained in accordance with (A.19)–(A.22):

$$\langle s | \hat{\mathscr{V}}_C | 0 \rangle = \int \rho^s(r)\, \hat{\mathscr{V}}_C(r)\, d^3r = \frac{1}{4} B_s \overline{V}_{L=0}^2 \Omega \frac{\Delta E_C}{F'}. \quad \text{(A.24)}$$

Calculation of the amplitudes of the admixture of $T_>$ monopole states to the ground state and $T_<$ states to the analog state, (76) and (77), by means of (A.21)–(A.24) leads to (82) and (83), respectively.


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