Theoretical models for describing the angular distributions of the products of pre-equilibrium nuclear reactions

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As a rule, nuclear reactions at low and medium energies are accompanied by the emission of particles in a wide energy region. The usual statistical models are incapable of describing the high-energy part of these spectra. An attempt is made in the review to systematize and compare the various theoretical approaches and models created to describe the experimental angular distributions in the pre-equilibrium part of the spectrum.

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INTRODUCTION

It has now been shown experimentally that a significant contribution to the high-energy part of the spectra is made by particles emitted in the initial phase of the nuclear reaction from highly excited states of a compound system.

Ideas about the formation of a long-lived state of statistical equilibrium (compound nucleus) lead, as is well known, to certain discrepancies with these experimental facts. To describe the mechanism of these processes, it is necessary to take into account faster reaction processes, the so-called pre-equilibrium processes. (By this concept, we understand here all processes which take place before the establishment of statistical equilibrium, these including the well-known direct reactions.) Allowance for pre-equilibrium processes in addition to the decay mechanisms of the compound nucleus leads to good agreement with the experiments.

For the theoretical descriptions of these processes, very different formalisms have been developed recently. Modifications of the exciton model[1-4] have become the most widely known. They have made it possible to describe successfully the cross sections of nuclear reactions with nucleons and α particles, the excitation functions, and also the emission spectra of the particles integrated over the angles, this being the case for initial energies of order 10 MeV and higher. On the basis of these simple models, some interesting physical conclusions have been drawn about the reaction mechanisms, for example, the structure of the initial configurations, the lifetimes of the intermediate states, the probabilities for collision of excited particles in nuclear matter, etc. The main advantage of such an approach has been that it has been possible, by means of the exciton model and by solving the master equations for the probabilities of population of the n-exciton states, to describe pre-equilibrium and equilibrium processes in the framework of a single closed physical model, which has not been achieved in any of the other theoretical approaches.

One of the important shortcomings of the exciton model is that its initial formulation took into account only energy conservation but not conservation of the angular momentum. Therefore, it cannot be used to describe the angular distributions of the emitted particles. There have recently been published a number of papers[5-8] in which the attempt is made to include the angular distributions in the formalism of the exciton model and thereby extend its applicability.

At the same time, models based on the well-developed methods of the theory of direct nuclear reactions have been developed. In Refs. 9–13, an averaging over the single-particle excitations in definite energy ranges with allowance for multiple collisions made it possible to describe the angular distributions and thus to solve the posed problem. Finally, the angular distributions have also been described by the cascade model in conjunction with the equations of nonequilibrium statistics and classical thermodynamics.[14-17]

The obvious interest in pre-equilibrium processes and, as a consequence, the variety of approaches are due to the fundamental nature of the problem of elucidating the mechanism of nuclear reactions in this energy region and its significance in applications to the calculations of various nuclear processes. The time is therefore ripe for a systematic analysis and practical evaluation of the individual theoretical models. The present paper should be seen as a first step in this direction.

1. APPROACHES BASED ON THE EXCITON MODEL

Generalized Master Equations. Mantzouranis, Weidenmüller, and Agassi[5] have developed a method for calculating the energy spectra and angular distributions of the pre-equilibrium emission of nucleons on the basis of generalized master equations of the exciton model. One of the main assumptions of this method is that the angular distribution of pre-equilibrium emission is determined only by the direction of motion of the "leading," i.e., incident, particle. They consider the process of two-particle residual interaction between this particle and the nucleons of the nucleus, in which the particle gradually loses not only its energy but also information about its initial direction of motion. In contrast to the original exciton model, the intermediate states of the system are determined in this case not only by the number n of excitons and the excitation energy E but also by the direction of motion Ω of the "leading" particle. Thus, the probabilities $P(n, t, \Omega)$ for population of the n-exciton states depend also on the angle. This follows from the circumstance that in a collision of the "leading" particle with the nucleons of the nucleus the change in its direction of motion is uniquely related to the

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transferred energy needed to produce a particle–hole pair. It is also assumed that the transition probabilities $\lambda_\pm(n, E, \Omega \to \Omega')$ can be represented as products of two factors:

$$\lambda_\pm(n, E, \Omega \to \Omega') = \lambda_\pm(n, E)\, g(\Omega \to \Omega'). \qquad (1)$$

The first factor does not depend on the angle and has the well-known form obtained in the exciton model.[4,18] An angular dependence is contained only in the second factor, which is proportional to the differential cross section for free scattering of nucleons, which has maximal value at 0° in the laboratory system. Thus, the anisotropy of the pre-equilibrium emission reduces to the asymmetry of the scattering of the leading particle with the nuclear nucleons in the first interactions. On the basis of this, the probabilities $P(n, t, \Omega)$ for the population of the n-exciton states are calculated in Ref. 5 on the basis of the generalized master equations

$$\frac{\partial P(n, t, \Omega)}{\partial t} = \int d\Omega' \{\lambda_+(n-2, E, \Omega \to \Omega')\, P(n-2, t, \Omega) + \lambda_-(n+2, E, \Omega \to \Omega')\, P(n+2, t, \Omega')\} - \int d\Omega' P(n, t, \Omega) \{\lambda_+(n, E, \Omega \to \Omega') + \lambda_-(n, E, \Omega \to \Omega')\}. \qquad (2)$$

From this one can obtain an expression for the double differential cross section for pre-equilibrium emission of particles of species x:

$$\frac{d^2\sigma_{0x}^{PE}(\varepsilon_0 \cdot \varepsilon, \theta)}{d\varepsilon\, d\Omega} = \alpha \sum_{\substack{n=n_0 \\ \Delta n=+2}}^{\bar{n}} W_x(n, \varepsilon) \int_0^{t_{eq}} P(n, t, \Omega)\, dt. \qquad (3)$$

Here, $W_x(n, \varepsilon)$ is the probability of emission of particles of species x with energy ε from states with n excitons per unit time, t_{eq} is the relaxation time needed for establishment of the equilibrium state, α is a normalization constant, and ε_0 and ε are, respectively, the kinetic energies of the incident particle and the emitted x particle.

The differential cross sections of the reactions ^{115}In(p,n) and ^{181}Ta(p,n) at initial energy 18 MeV calculated in the framework of this formalism are shown in Fig. 1. It can be seen that the experiment is described satisfactorily. A similar agreement is obtained in some other cases.[5]

Hybrid Model with Allowance for the Direction of the Particles. The leading-particle formalism described above is very similar to the hybrid model, in which the entire probability of pre-equilibrium emission is attributed to just a single particle in the continuum.[3] On the other hand, for the exciton model it was shown[4] that under certain conditions, for example, when $\lambda_\pm \gg \lambda_0 \gg \lambda_-$, which are well satisfied in the initial phase of the nuclear reaction that is important for pre-equilibrium decay, the solution to the master equations for the probability of pre-equilibrium emission can be obtained in a simple analytic form. On the basis of the generalized master equations (2) and the analytic form of the exciton model, Mantzouranis[6] developed a formalism for describing the angular distributions—the hybrid model. As in Ref. 5, the states of the system are classified in accordance with n, ε, and Ω. Ignoring transitions of the type λ_0 and λ_-, one can write expressions for the population probabilities $P(n, \Omega)$ in the analytic form

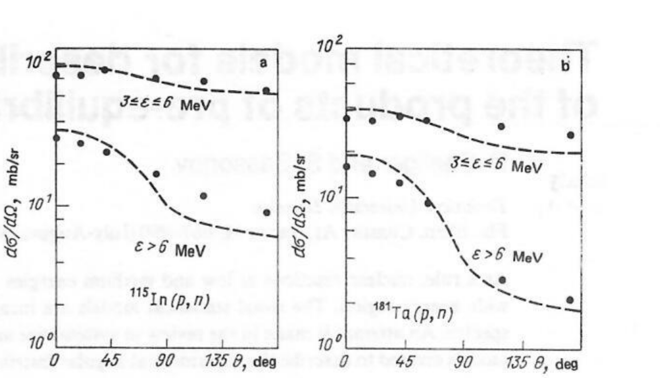

FIG. 1. Comparison of the differential cross section calculated[5] by means of the generalized master equations at initial energy $\varepsilon_0 = 18$ MeV with experiments for two energy ranges of the emitted neutrons: a) the reaction ^{115}In(p,n); b) the reaction ^{181}Ta(p,n).

$$P(n, \Omega) = \int d\Omega_\mu W(\Omega_\mu \to \Omega) \int d\Omega_1 W(\Omega_1 \to \Omega_2)\, P(n_0, \Omega_1), \qquad (4)$$

where $\mu = (n - n_0)/2$ is the number of two-particle transitions, $\Delta n = +2$, leading to population of the n-exciton states; $W(\Omega \to \Omega')$ is the probability of transitions of the type $n \to n' = n+2$ with a change in direction $\Omega \to \Omega'$ of the leading particle; and $P(n_0, \Omega_1)$ is the probability of population of the initial state. The functions $P(n, \Omega)$ in (4) can be compared with the integrals $\int_0^{t_{eq}} P(n, t^*\Omega)dt^*$ for the time-dependent population probabilities in the master equations (2). It follows from (4) that with increasing number of excitons the angular dependence weakens rapidly, i.e., already after a few two-particle interactions the system "forgets" the initial direction of motion of the incident particle. Only the particle emission after the first or second collision makes an appreciable contribution to the asymmetry of the angular distribution.

Using the population probabilities of the n-exciton states obtained from Eq. (4), Mantzouranis obtained on the basis of the hybrid model a simple analytic formula for the double differential cross section of pre-equilibrium processes with emission of nucleons[6]:

$$\frac{\partial^2\sigma_{0x}^{PE}(\varepsilon_0, \varepsilon, \vartheta)}{\partial\varepsilon\, \partial\Omega} = \sigma_0^{abs}(\varepsilon_0)\, g \sum_{\substack{n=n_0 \\ \Delta n=+2}}^{\bar{n}} D_n \frac{W(p-1, h, U)}{W(p, h, E)}\, nf_x P(n, \Omega) \frac{\lambda_x^c(\varepsilon)}{\lambda_x^c(\varepsilon) + \lambda_+(\varepsilon)}. \qquad (5)$$

Here, σ_0^{abs} is the cross section for absorption of the initial particle 0, D_n is an attenuation factor, which takes into account the decrease in the flux of the probability for transition to more complicated states through pre-equilibrium emission from the preceding states, nf_x is the relative contribution of nucleons of species x to the n-exciton state, $\lambda_x^c(\varepsilon)$ is the probability of emission of particle x into the continuum per unit time, and $\lambda_+(\varepsilon)$ is the probability of a collision of the leading particle x resulting in a transition to a more complicated state. The normalized integral $\int P(n, \Omega)d\Omega$ must be equal to unity. It follows from this that the cross section (5) when integrated over the angles is identical to the cross section $d\sigma_{0x}^{PE}(\varepsilon_0; \varepsilon)/d\varepsilon$ obtained in the hybrid model. It is this that makes it possible to apply this approach to calculations of the absolute values of the cross sections.

As an example of the use of this method, Fig. 2 shows the angular distribution calculated in Ref. 6 for protons in the reaction $^{59}Co(\alpha, p)$ at initial energy 54.8 MeV and a comparison with experiment. Unfortunately, the analytic variant of the hybrid model has not yet been widely used in concrete calculations. Only very recently has there been published Ref. 19, which contains a successful analysis of the angular distribution of the neutron spectra for neutrons emitted when 14-MeV neutrons are scattered by various nuclei.[20] It is well known that the hybrid model has already been used in many programs for the calculation of nuclear data,[4] and therefore its comparatively simple analytic variant is particularly suitable for use and generalization of the existing computational programs when one wants to take into account the angular distributions of the reaction products. For this, we require a further detailed investigation of the applicability of this approach.

The Momentum as an Additional Quantum Number. A further possibility for calculating the angular distribution was proposed by Mädler and Reif.[7] The states of the compound system are here classified, as usual, by the quantum numbers n and E, and, in addition, by the total momentum $\mathbf{p}$ of the system. The most important step in this method is the determination of the densities of n-exciton states with definite momentum. For this, a statistical approach is used, in which the initial partition function contains an additional term which depends on the mean velocity $\mathbf{v}$ of the excitons and on their momenta $\mathbf{p}_k$:

$$Z_{ph}(\beta, \mathbf{v}) = \sum_k \exp(-\beta\varepsilon_k + \mathbf{v}\mathbf{p}_k). \tag{6}$$

From the partition function, by the usual methods of statistics, one determines the characteristic mean values of the physical quantities, and also $\omega(p, h, \varepsilon, \mathbf{p})$, the density of states of the n-exciton system with definite momentum. It is expedient to represent this density as a function of the parallel component $\mathbf{p}_\parallel$ of the momentum and the component $\mathbf{p}_\perp$ perpendicular to the direction of the incident particle. (This is so because $\mathbf{p}=0$ prior to the emission of a particle from the compound system.)

The density-of-states functions of the compound system $\omega(p, h, \varepsilon, p_\parallel, 0)$ and of the residual nucleus $\omega(p-1, h, U, p_\parallel, p_\perp)$ calculated in this manner uniquely determine the momentum transfer to the nucleus and the direction of motion of the emitted particle. Using the obtained density-of-states functions, which depend on the momenta, one can in the framework of the simple analytic exciton model calculate the double differential cross section for pre-equilibrium emission of nucleons. The possibilities of this method are illustrated in Figs. 3 and 4. For the reaction $^{93}Nb(n, n')$ at initial energy 14 MeV the description is not satisfactory for large angles. The reason for this is the insufficient number of particle–hole states in the residual nucleus in this energy and momentum range. With increasing excitation energy, the number of states increases and one obtains much better agreement with experiment, which is illustrated by Fig. 4 for the reaction $^{27}Al(p, p')$ at $\varepsilon_0 = 62$ MeV. For comparison, we also show the results of Ref. 11, which in both cases give satisfactory agreement. It should also be noted that use of a momentum dependence of the density of states need not necessarily be restricted to the analytic form of the exciton model. It would also appear to be promising to use it in the framework of the master-equation formalism. So far, this has not been done and further work is needed in this direction.

Allowance for Spin Dependence in the Pre-Equilibrium Processes. In Ref. 8, Plyuiko classifies the exciton states by the number n of excitons, the excitation energy E, and the spin I. It is assumed that states with equal number of excitons and equal spin but different orbital angular momentum and total spin do not interfere. In addition, the correlation between states with different numbers of excitons and total momentum is also ignored. Thus, it is implicitly assumed that the n-exciton states are relatively long-lived quasistationary quantum states like the states of the compound nucleus in the statistical theory of nuclear reactions. The result corresponds to the expectation. The angular distribution calculated in this way is symmetric about 90°,

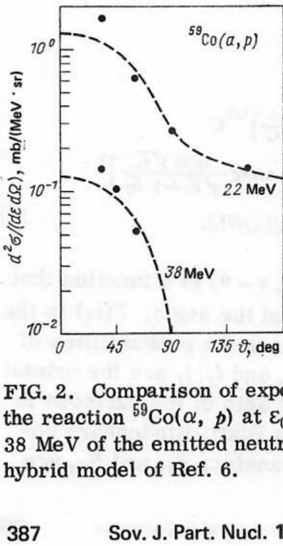

FIG. 2. Comparison of experimental angular distributions for the reaction $^{59}Co(\alpha, p)$ at $\varepsilon_0 = 54.8$ MeV for energies $\varepsilon = 22$ and 38 MeV of the emitted neutrons with calculations based on the hybrid model of Ref. 6.

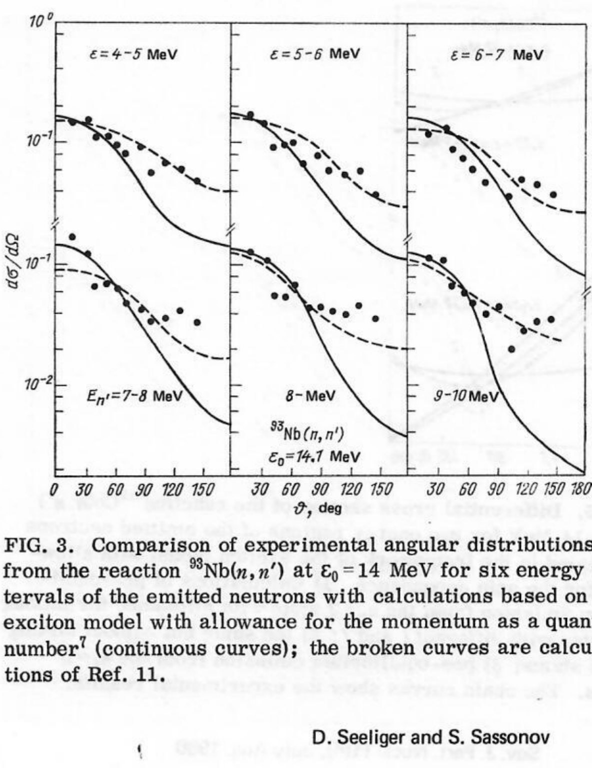

FIG. 3. Comparison of experimental angular distributions from the reaction $^{93}Nb(n, n')$ at $\varepsilon_0 = 14$ MeV for six energy intervals of the emitted neutrons with calculations based on the exciton model with allowance for the momentum as a quantum number[7] (continuous curves); the broken curves are calculations of Ref. 11.

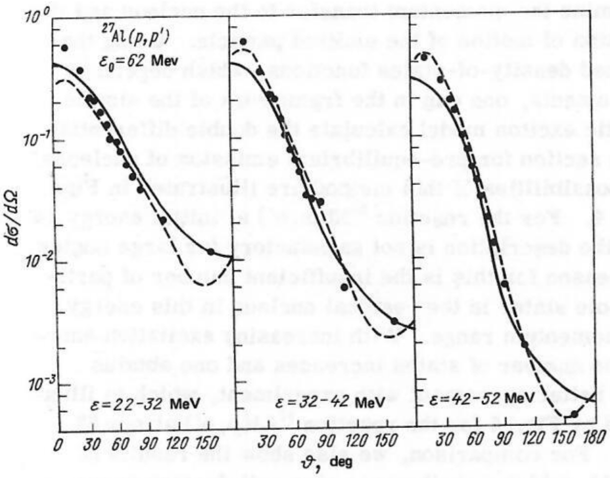

FIG. 4. The same as in Fig. 3 for the reaction $^{27}Al(p,p')$ at $\mathcal{E}_0=62$ MeV.

which follows from the introduction of a coupling between the spins of the individual states. However, these results contradict the experiments. It is obvious that the description of intermediate exciton states as quasistationary states is not justified because of their very short lifetimes of order 10^{-21}–10^{-22} sec (see, for example, Ref. 4).

Asymmetry of the angular distribution is obtained when allowance is made for the interference between the partial waves with different orbital angular momenta in the initial stage of the reaction with $n=n_0$ excitons. The results of calculations for the reaction $^{59}Co(n,n')$ at initial energy 14 MeV are shown in Fig. 5. The asymmetry increases with increasing energy of the emitted particles. The profile of the angular distribution also depends strongly on the initial number of excitons n_0.

The calculations made in the framework of this approach are as yet methodological in nature and have an undoubted heuristic value, since they convincingly demonstrate that the n-exciton states cannot in the framework of the exciton model be quasistationary states of an intermediate system but are, rather, short-lived "scattering" states in nuclear matter.

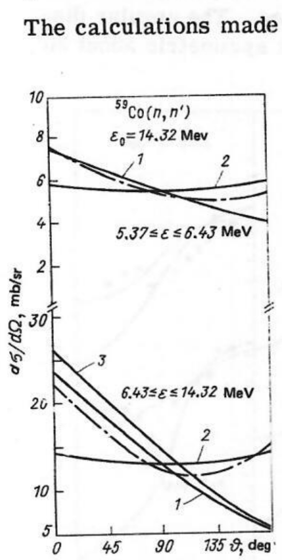

FIG. 5. Differential cross section of the reaction $^{59}Co(n,n')$ at $\mathcal{E}_0=14$ MeV for two energy regions of the emitted neutrons calculated in the framework of the exciton model with allowance for the spin dependence. 1) Calculations of pre-equilibrium emission from the $n_0=3$ state with allowance for mixing of states with different l and I; 2) the same but without mixing of the states; 3) pre-equilibrium emission from the $n_0=5$ states. The chain curves show the experimental results.

2. CALCULATIONS BASED ON THE THEORY OF DIRECT NUCLEAR REACTIONS

Phenomenological Approaches in the Plane-Wave Born Approximation (PWBA). Luk'yanov *et al.*[9] developed a scheme for parametrizing the double differential cross sections of the inelastic scattering of neutrons on the basis of the experimental spectra at initial energies 14.4 and 9.1 MeV. The characteristic deviations of the experimental spectra from the results of the statistical model were interpreted in Ref. 9 as the contribution of the direct inelastic interaction of the leading particle associated with the excitation of single-particle transitions in the nucleus. The calculated spectra are made up of the isotropic or symmetric spectrum of Maxwell evaporation and the contribution of the direct processes, which is averaged over the single-particle transitions within the experimental energy resolution ($\Delta\varepsilon\approx1$ MeV).

The angular dependence of the contribution of the pre-equilibrium processes to the spectra is calculated by the method of the theory of direct nuclear reactions in the PWBA approximation. The corresponding values of the transferred orbital angular momenta are determined in the framework of the simple single-particle shell model in a corresponding interval ΔU of averaging of the excitation energy of the nucleus. The energy dependence is determined by the empirical function $(\varepsilon/\varepsilon_0)^{1/2}U$, which is obtained from comparison with the experimental spectra. Thus, by the choice of this function one finds the contributions of the "direct" spectra, which to a considerable extent are similar to the spectra obtained in the framework of the exciton model for emission of particles from the initial state $n=n_0$ (in the latter case, the profile of the spectra is determined by the function εU).

The double differential cross sections calculated in this scheme and averaged over the energy interval ΔU have the form

$$\frac{d^2\sigma(\varepsilon_0;\,\varepsilon,\,\theta)}{d\varepsilon\,d\Omega}=\left.\frac{d^2\sigma^{DI}(\varepsilon_0;\,\varepsilon,\,\theta)}{d\varepsilon\,d\Omega}\right|_{\rm PWBA}+\alpha(\varepsilon_0;\,\varepsilon,\,\vartheta)\,\varepsilon\exp(-\varepsilon/T),\qquad(7)$$

where

$$\left.\frac{d^2\sigma^{DI}(\varepsilon_0;\,\varepsilon,\,\theta)}{d\varepsilon\,d\Omega}\right|_{\rm PWBA}\sim\left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2}U$$
$$\times\sum\beta_{f_i}\frac{n_i(2j_f+1-n_f)}{(2j_i+1)(2j_f+1)}\left[\frac{\sqrt{B_i}+\sqrt{B_f}}{(q/k)\sqrt{\varepsilon_0}}\,\mathrm{arctg}\,\frac{(q/k)\sqrt{\varepsilon_0}}{\sqrt{B_i}+\sqrt{B_f}}\right]^2$$
$$\times\sum_i Z^2(l_fj_fl_ij_i/1/2L)\,j_L^2(qRf_i).\qquad(8)$$

In (7) and (8), $\alpha(\varepsilon_0;\varepsilon,\theta)=\alpha(\varepsilon_0;\varepsilon,\pi-\theta)$ is a function that depends weakly on the energy and the angle, $T(\varepsilon_0)$ is the nuclear temperature, n_i and n_f are the probabilities of population of shells i and f; l_i, j_i and l_f, j_f are the orbital angular momentum and the total spin of the nucleons in the corresponding shells, Z is a Blatt–Biedenharn coefficient, q is the momentum transfer, β_{fi} and R_{fi} are free parameters obtained by comparison with the experiments, $j_L(qR)$ is a Bessel function, and B_i and B_f are energies of single-particle levels in the interval

$$(U-\Delta U/2)<(B_i-B_j)<(U+\Delta U/2),$$

where $U=\varepsilon_0-\varepsilon$.

Thus, the double differential cross section of inelastic scattering of neutrons, integrated over the angles, was calculated in Ref. 9 for numerous cases. Allowance for the contribution of definite (comparatively few in number, i.e., 1–3) single-particle transitions leads to the appearance of structure in the angular distributions. Figure 6 is a comparison with the experiments on the angular distributions of neutrons in the reactions $^{56}Fe(n,n')$ and $^{93}Nb(n,n')$ at the initial energies 14.4 and 9.1 MeV (Refs. 21 and 22).

The method described above is essentially phenomenological in nature, since use is made of the experimental energy spectra and angular distributions. An advantage of this method is its simplicity and considered possibility for application in the interpretation of experimental data. It is assumed in Ref. 9 that the Maxwellian expression for the spectrum contains not only statistical equilibrium processes but also contributions from the emission of particles from more complicated pre-equilibrium states. Thus, the pre-equilibrium spectrum is divided into a direct contribution and a more complicated contribution, which is still not an equilibrium one.

In Ref. 10 there is a similar but more systematic division of the spectra into three components. The double differential cross section of inelastic scattering of neutrons is here represented as the sum of components

$$\frac{\partial^2\sigma(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}=\frac{1}{4\pi}\frac{d\sigma^{CN}}{d\varepsilon}(\varepsilon_0;\varepsilon)+\frac{1}{4\pi}\frac{d\sigma^{PE}(\varepsilon_0;\varepsilon)}{d\varepsilon}\bigg|_{n\geqslant5}+\frac{\partial^2\sigma^{DI}(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}\bigg|_{\mathrm{PWBA}},\qquad(9)$$

where $d\sigma^{CN}/d\varepsilon$ is the contribution of the equilibrium reaction, i.e., of emission from the compound nucleus, $d\sigma^{PE}/d\varepsilon$ is the pre-equilibrium emission from states of greater complexity ($n\geqslant5$) calculated in the framework of the hybrid model, and $\partial^2\sigma^{DI}/\partial\varepsilon\partial\Omega$ is the contribution of direct processes, this corresponding physically to pre-equilibrium emission from an initial state with $n=n_0=3$, and it is calculated in the PWBA approximation.

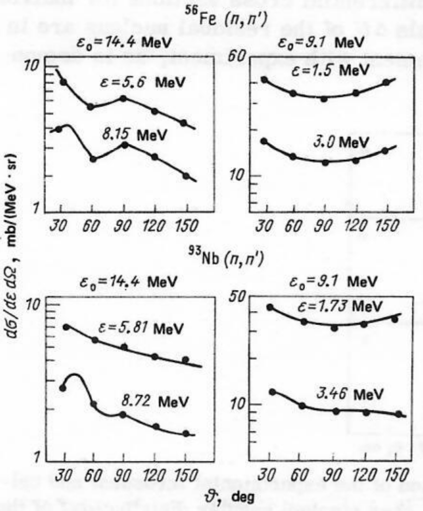

FIG. 6. Angular distributions of the neutrons in the reactions $^{56}Fe(n,n')$ and $^{93}Nb(n,n')$ at the initial energies 9.1 and 14.4 MeV compared with the calculations of Ref. 9.

In Ref. 10, as in Ref. 9, it is assumed that the anisotropy of the angular distribution is completely determined by the first, direct phase of the reaction. The expression for the direct contribution of the reaction in Ref. 10 differs from (8) by the form of the energy dependence:

$$\frac{\partial^2\sigma^{DI}(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}\sim\left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2}(2l_x+1)\sum_L(l_0l_x00\,|\,L0)\,j_L^2(qR),\qquad(10)$$

where $(l_0l_x00|L0)$ are Clebsch–Gordan coefficients.

As an actual example, this approach was applied to the experimental data on the reaction $^{56}Fe(n,n')$ at energy 14 MeV of the incident neutrons.[20] In this special case, only the term with $L=2$ was taken into account in the sum (10), since it is dominant compared with the other values of the angular-momentum transfer. Thus, the obtained angular distribution has the typical profile of a direct reaction with $L=2$. In Fig. 7 we show the results, which demonstrate that in this special case good agreement with the experiment is obtained.

A kind of hybrid exciton model and PWBA theory for describing the energy spectra and angular distributions was presented in Ref. 11. In the framework of the exciton model, one introduces the emission rates $\lambda_{lx}^c(\varepsilon)$, which depend on the angular momentum. Thus, one obtains the pre-equilibrium emission spectra for each orbital angular momentum l_x of the emitted particle. The angular dependence is obtained from calculations of the cross sections in the framework of the theory of direct nuclear reactions in the PWBA approximation for the corresponding values of the transferred orbital angular momentum L. In this formalism, the following expression is obtained in Ref. 11 for the double differential cross section of pre-equilibrium emission:

$$\frac{\partial^2\sigma_{0x}^{PE}(\varepsilon_0,\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}=\sigma_0^{abs}(\varepsilon_0)\sum_{l_x}\sum_{\substack{n=n_0\\ \Delta n=+2}}\sum_{J_c=|I+I_0+s|_{\min}}\sum_{I_R=0\,1/2}H\times\frac{\lambda_{lx}^c(\varepsilon)}{\lambda_{lx}^c(\varepsilon)+\lambda_+(n,\varepsilon)}\,\frac{\omega(p-1,\ h,\ U,\ I_R)}{\omega(p,\ h,\ \varepsilon,\ J_c)}\,\frac{\partial^2\sigma^{DI}(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}\bigg|_{\mathrm{PWBA}},\qquad(11)$$

only the relative form of the angular distribution being taken from PWBA theory:

$$\frac{\partial^2\sigma^{DI}(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega}\bigg|_{\mathrm{PWBA}}\sim(2l_x+1)\sum_L(l_0l_x00\,|\,L0)\,j_L^2(qR).\qquad(12)$$

The absolute magnitude of the cross section is determined by the normalization of the free parameter H in Ref. 11.

In Ref. 11, there is also proposed a somewhat different expression obtained on the basis of a modification of the exciton model. This formalism is used to obtain a good description of the angular distribution of the neutrons in the reaction $^{93}Nb(n,n')$ at initial energy 14 MeV (Fig. 8) and also for the α particles in the (p,α) reaction (Fig. 9).

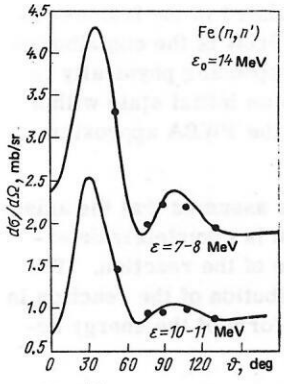

FIG. 7. Angular distribution of neutrons from the reaction $^{56}Fe(n,n')$ at $\mathcal{E}_0 = 14$ MeV for two intervals of the emission energy and comparison of the experiment of Ref. 20 with the calculations of Ref. 10.

Naturally, this formalism must be regarded as a method of parametrizing the angular distributions rather than a closed physical model.

Microscopic Description in the DWBA *Approximation.* On the basis of microscopic wave functions of the ground state and excited states, Reif[12] obtained the differential cross sections for the emission of particles in the continuum in the DWBA approximation. In the framework of this approach, one projects the $1p1h$ components from the total wave functions $|IM\pi U\rangle$, since only these components can be excited by a single-particle–hole residual interaction $\sum_{i=1}^{A} v_{i0}$ of Gaussian type:

$$|IM\pi U\rangle \sum_{p,h} Cp^{I}, h^{\pi}(U)\,|(1p1h)\,IM\pi\rangle + |C\rangle. \quad (13)$$

The remaining part of the wave function $|C\rangle$ does not make any contribution to the matrix elements T_{fi}^{DWBA} of the direct interaction. These matrix elements have the form

$$T_{fi}^{DWBA} = \sum_{p,h} C_{ph}^{I,\pi}(U)\left(\chi_f^{(-)} \mid \langle(1p1h)\,IM\pi \mid \sum_{i=0}^{A} v_{i0} \mid 0\rangle\,\chi_i^{(+)}\right), \quad (14)$$

where the functions $\chi_i^{(+)}$ and $\chi_f^{(-)}$ denote, as usual, distorted waves in the entrance and exit channels, respectively, and are obtained by analyzing elastic scattering in the framework of the optical model. The matrix elements (14) are averaged over the energy interval ΔU determined by the experimental resolution. As a result, Reif obtains the differential cross sections in each of the distinguished energy intervals:

$$\left\langle \frac{\partial^2\sigma(I\pi U)}{\partial U\,\partial\Omega}\right\rangle_{DWBA} \Delta U = \sum_{p,h} |C_{ph}^{I\pi}(U)|^2\,\Delta U\,\frac{d\sigma^{ph}(I\pi)}{d\Omega}, \quad (15)$$

where

$$\frac{d\sigma^{ph}(I,\pi)}{d\Omega} = \left(\frac{m}{2\pi h}\right)\left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2}\frac{1}{2s+1} \times \sum_{\text{Projection}} \left|\left(\chi_f^{(-)} \mid \langle(1p1h)\,IM\pi \mid \times \sum_{i=1}^{A} v_{i0} \mid 0\rangle \mid \chi_i^{(+)}\right)\right|^2. \quad (16)$$

The calculated differential cross sections for individual energy intervals ΔU of the residual nucleus are in satisfactory agreement with experiment, as is demon-

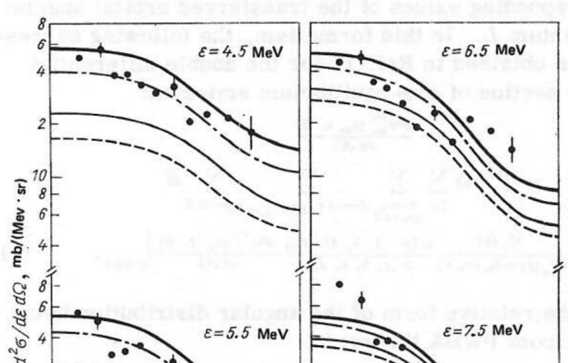

FIG. 8. Comparison of the angular distribution of the reaction $^{98}Nb(n,n')$ at $\mathcal{E}_0 = 14$ MeV for four energies of the emitted neutrons with the results of Ref. 11. In the figure, we compare general results of calculations based on two variants of the formalism (heavy continuous curves and chain curves) and also the corresponding contributions from the $n_0 = 3$ states (thin continuous curves and broken curves).

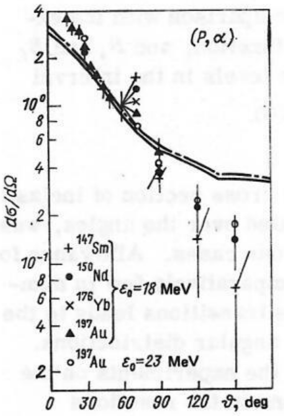

FIG. 9. Comparison of the angular distribution of the (p,α) reaction with the calculations of Ref. 11. The curves are as in Fig. 8.

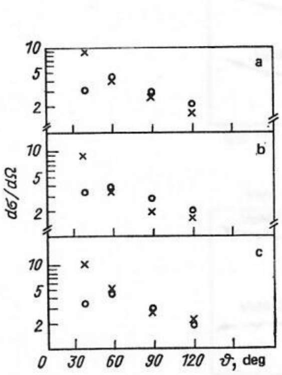

FIG. 10. Comparison of the experimental (crosses) and calculated (in Ref. 12, open circles) angular distributions of the reaction $^{40}Ca(n,n')$ at $\mathcal{E}_0 = 14$ MeV for the following energy intervals of the emitted nucleons: 9–10 (a), 8–9 (b), and 7–8 MeV (c).

strated in Figs. 10 and 11 for the reaction $^{40}Ca(n, n')$ at $\varepsilon_0 = 14$ MeV (Ref. 20) and $^{116,119}Sn(p, p')$ at $\varepsilon_0 = 17$ MeV.

It should be noted that this formalism is not applicable for excitation energies of the residual nucleus above a few MeV, in which one expects a greater influence of multiple collisions and also the emission of particles from quasibound states. Use of the theory of single-stage direct reactions to describe a reaction with excitation energy of the residual nucleus of a few MeV leads to an underestimation of the absolute contribution of these processes and to an overestimation of their angular anisotropy compared with experiment. In addition, this method requires a comparatively extended mathematical formalism in order to calculate the wave functions of the nucleus in the highly excited states.

Use of the Multistep Theory of Direct Reactions. Tamura *et al.*[13] made further use of the theory of direct nuclear reactions. Again in the DWBA approximation, but on the basis of single- and two-step excitations of direct type, they calculated the spectra and angular distributions of pre-equilibrium emission. They investigated excitations of $1p1h$ and $2p2h$ states of the residual nucleus, which makes it possible to encompass the experimentally observed behavior at higher excitation energies of the residual nucleus (compared with approaches in which only single-stage excitation is considered).

The model was formulated and cross sections were calculated for (p, p') reactions, but they are also valid for other processes. The results of calculations for the reactions $^{27}Al(p, p')$ and $^{209}Bi(p, p')$ at initial proton energy 62 MeV are given in Fig. 12 (Ref. 23). Besides the experimental angular distributions in intervals of the excitation energy of width 10 MeV, this figure shows the calculated curves for the contributions of the excitation of $1p1h$ states and the sum of the $1p1h$ and $2p2h$ states. It can be seen that the contribution of the two-stage processes increases with increasing excitation energy of the residual nucleus, i.e., with decreasing energy of the emitted particles. Thus, Ref. 13 is a further stage in the application of the theories of direct nuclear reactions.

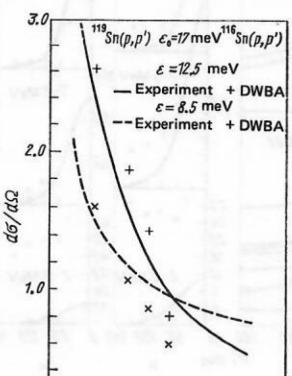

FIG. 11. Comparison of the experimental angular distribution of protons from the reaction $^{119}Sn(p, p')$ at initial energy $\varepsilon_0 = 17$ MeV and $\varepsilon = 12.5$ MeV (solid curve) and $\varepsilon = 8.5$ MeV (broken curve) with microscopic calculations in the DWBA approximation.

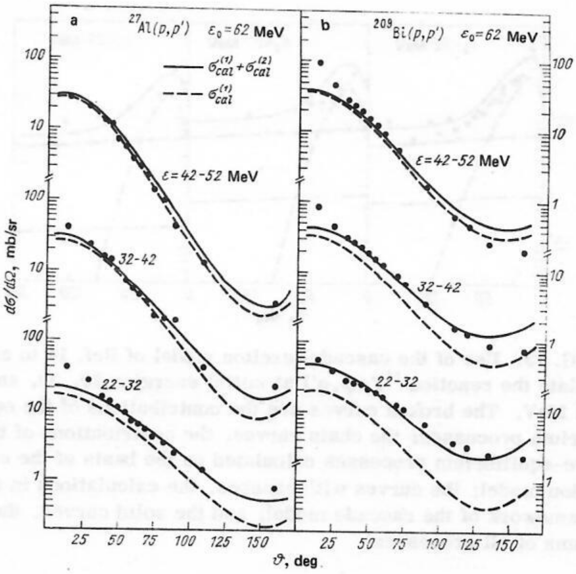

FIG. 12. Comparison of experimental angular distributions of the reactions $^{27}Al(p, p')$ and $^{209}Bi(p, p')$ at $\varepsilon_0 = 62$ MeV with calculations in accordance with the multistep theory of direct reactions.[13] The broken curves give the contribution of $1p1h$ processes; the solid curves are the contributions to the reactions with allowance for processes from $1p1h$ and $2p2h$ states.

Recently, Ignatyuk *et al.* have published their paper of Ref. 24, in which they use microscopic wave functions of phonon type in the DWBA approximation with allowance for one- and two-phonon excitations. They obtain very good agreement with, for example, the experiments of Refs. 20–22 on the neutron spectra.

A general shortcoming of all the proposed methods in the framework of the theory of direct reactions (apart from the mathematical difficulties associated with the microscopic description) is the need to match them to calculations based on a radically different theory (in the simplest case, the statistical model). It is only then that one can obtain a satisfactory description of the spectra and angular distributions at all emission energies.

3. USE OF GEOMETRICAL REPRESENTATIONS OF THE PARTICLE TRAJECTORIES

To interpret nuclear processes at high energies, one uses the popular cascade model of nuclear reactions, which is based on a description of the motion of the incident particle in the nucleus along a semiclassical trajectory, in which it makes successive collisions with nucleons of the nucleus. The model thus gives information about the energy spectra and angular distributions of the emitted particles. The success of the use of this model at high energies led to an investigation of the applicability of this formalism for describing the initial phase of the interaction at lower initial energies. With this aim, Gudima and Toneev[14] developed a cascade–exciton model, which combines semiclassical ideas from the cascade and exciton models with the complete statistical theory of nuclear reactions.

The formalism of the cascade model is used to de-

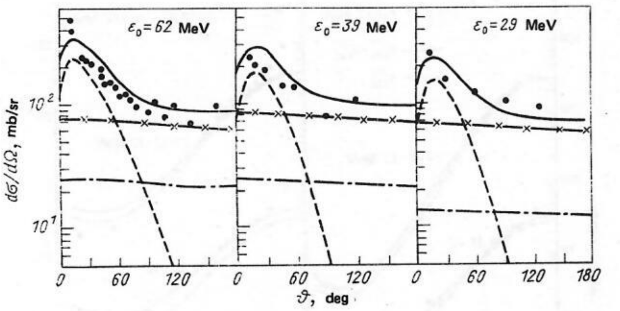

FIG. 13. Use of the cascade–exciton model of Ref. 14 to calculate the reaction $^{54}Fe(p, p')$ at initial energies 29, 39, and 62 MeV. The broken curves are the contributions of the equilibrium processes; the chain curves, the contributions of the pre-equilibrium processes calculated on the basis of the exciton model; the curves with crosses, the calculations in the framework of the cascade model; and the solid curves, the sums of all processes.

scribe the collisions of the incident particle with nucleons of the nucleus leading to the production of excited particles and holes in the initial phase of the nuclear process. The angular distribution of the secondary particles in this time interval, which is of order 10^{-21} sec, determines the anisotropy of the pre-equilibrium emission.

Beginning with an initial number of excitons $n_0 = p + h$, which are present in the nucleus after this first phase has ended, the further development of the compound system is described in the framework of the master equations of the exciton model until equilibrium is established with $n = \bar{n} = (2gE)^{1/2}$. The pre-equilibrium emission in the second phase is assumed to be isotropic. After statistical equilibrium has been reached, the subsequent fate of the nuclear system is described in the framework of the complete statistical theory of nuclear reactions. Thus, the scheme for decomposing the spectrum of emitted particles in this method is reminiscent of the method used by other authors.

To determine the mean free path of the particles in the nucleus in the cascade phase, one uses the imaginary part of the optical potential, which is optimized by comparison with experiment. All the remaining parameters are fixed. The obtained differential cross sections of the reaction $^{54}Fe(p, p')$ at initial energies 29, 39, and 62 MeV and the contributions of the individual processes are shown in Fig. 13. Here, and also in some other cases, good agreement with experiment was obtained.

The advantages of this formalism are offset by two important deficiencies: It applies only at initial energies of the nucleons above 30 MeV, and to describe a single experiment it is here necessary to fit together three completely different theories.

4. METHODS BASED ON NONEQUILIBRIUM STATISTICS AND THERMODYNAMICS

Description in the Framework of Nonequilibrium Statistics. In Ref. 15, Mädler, Reif, and Toneev describe the energy spectra and the angular distributions on the basis of nonequilibrium statistics in conjunction with the formalism of the exciton model. By analogy with the analytic approximation of the exciton model, the differential cross section of pre-equilibrium emission is

$$\frac{\partial^2 \sigma_{0x}^{pE}(\varepsilon_0;\ \varepsilon,\ \vartheta)}{\partial\varepsilon\,\partial\Omega} = \sigma_0^{abs}(\varepsilon_0) \sum_{\substack{n=n_0\\ \Delta n=2}}^{\bar{n}} W_x(n,\ \varepsilon)\, t_n F(n,\ \vartheta), \qquad (17)$$

where $W_x(n, \varepsilon)$ is the probability for emission of particles of species x from an n-exciton state, and t_n is the mean lifetime of this state.

The function $F(n, \varphi)$ contains complete information about the angular distribution, and it is calculated in Ref. 16 on the basis of the theory of nonequilibrium statistics. For this purpose, in the description of the time dependence of the relaxation process in the compound system, the latter system is decomposed into energy subsystems that are described by corresponding time-dependent thermodynamic parameters. The function $F(n, \varphi)$ is then obtained in the form

$$F(n,\ \vartheta) = f(n) \exp[f(n) \cos\vartheta]/(4\,\mathrm{sh}\,[f(n)]) \qquad (18)$$

with the normalization

$$\int_{4\pi} F(n,\ \vartheta)\, d\Omega = 1 \qquad (19)$$

and

$$f(n) \approx \frac{2(n-1)}{p}\left(1+\frac{\varepsilon_F}{\varepsilon}\right)^{1/2} \int_0^1 \left(x+\frac{\varepsilon_F}{\varepsilon}\right)^{1/2} (1-x)^{n-2}\, dx. \qquad (20)$$

Equations (17)–(20) are used to calculate the energy spectra and the angular distributions of the neutrons in the reactions $^{93}Nb(n, n')$ and $^{115}In(n, n')$ at initial energy 14 MeV and to compare them with the experiment of Ref. 20 (Fig. 14). The results show that the high-energy part of the spectra is determined essentially by emission from states with $n = 3$, whereas in the low-energy part of the spectra, and especially at large emis-

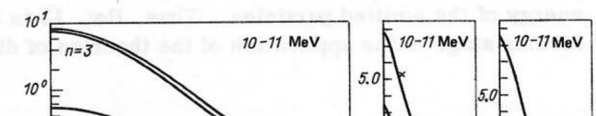

FIG. 14. Comparison of experimental angular distributions with the calculations of Ref. 15. a) The reaction $^{115}In(n, n')$ at $\mathcal{E}_0 = 14.5$ MeV; the individual curves correspond to the contributions of the n-exciton states in the angular distribution of the pre-equilibrium emitted neutrons; the heavy solid curves are the sums of all the contributions. b) The angular distribution of the reaction $^{115}In(n, n')$ at $\mathcal{E}_0 = 14.5$ MeV. c) The same for the reaction $^{93}Nb(n, n')$.

sion angles, decay from states with $n \geq 5$ makes a significant contribution. The anisotropy of the angular distributions is due in the first place to the allowance for momentum conservation in the derivation of Eqs. (18) and (20).

In contrast to the description of the n-exciton states as quasistationary equilibrium intermediate states,[8] this formalism, which is based on nonequilibrium statistics, does not lead to the conclusion that the emission from states with a definite value of n must be symmetric about the angle 90°. Therefore, in the framework of the purely statistical approach to the angular distributions of pre-equilibrium emission one can obtain results similar to those in the approach based on the theory of direct nuclear reactions. It must, however, be pointed out that the latter, because of the more accurate allowance for the coupling of the angular momenta in them, and also the inclusion of the effects of distortion of the waves, give in principle more accurate and detailed structure in the angular distributions of the reaction products.

Formalism of a Locally Heated Nucleus. Weiner and Weström[17] have developed a method for describing the pre-equilibrium decay of a compound system on the basis of classical thermodynamic ideas. In the initial phase of the reaction, the nucleus is assumed to be locally heated. Through diffusion, the localized nuclear excitation extends in a finite time over the complete nucleus until thermodynamic equilibrium is established. The energy spectrum and angular distribution of the particles emitted by the "heated" nucleus are determined by the local, time-dependent nuclear temperature. It is found by solving the diffusion equation with appropriate initial and boundary conditions. The relaxation time is determined by the energy transfer constant in nuclear matter.

Without going into more detail about the mathematical formulation of this approach, we give some of the obtained results (Fig. 15). This method permits only qualitative description of pre-equilibrium processes, and therefore a detailed comparison with experiment is not yet available. It should be noted that such a method, like the cascade–exciton model,[14] is applicable only at

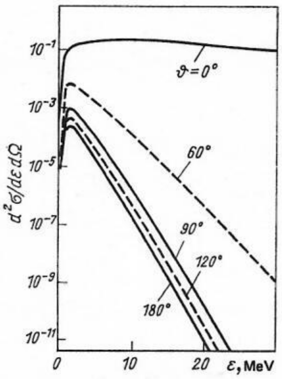

FIG. 15. Spectra of neutrons emitted at different angles from pre-equilibrium states of nuclei. The results of calculations in accordance with the model of local heating[17] for a nucleus with mass number $A \approx 100$ and initial energy 60 MeV.

sufficiently high energies ($\varepsilon_0 > 100$ MeV), at which the semiclassical treatment of the local and bounded heating of the nucleus is justified. A new and important physical aspect here is the linking of pre-equilibrium effects to processes of energy transfer in nuclear matter, which are very important in heavy-ion physics.

CONCLUSIONS

The problems considered in this review have been intensively discussed at the relevant symposia for many years already (see, for example, Ref. 25). This, and also the diversity of the mechanisms and approaches developed in recent years for the description of the angular distributions of the spectra of pre-equilibrium emission of particles, is a measure of the topicality of the problem and also the considerable interest taken in it by many groups. This is due, on the one hand, to the desire to achieve a better understanding and description of the mechanism of nuclear reactions at low and medium energies and, on the other, to the need to calculate nuclear data for applied purposes.

A final evaluation and comparison of all the methods, like a recommendation for a definite one among them as the best universal method, is at present impossible, since the methods have been developed to different degrees and the authors choose different conditions for applying them. Moreover, on purely physical grounds it is evident that in the future too it will be justified to interpret pre-equilibrium emission on the basis of entirely different theoretical methods and models. Thus, the further development of the exciton model[5-8] as well as the theory of direct nuclear reactions[9-13] or the methods of nonequilibrium thermodynamics and statistics[15-17] is entirely justified and necessary. Particularly attractive are the approaches and models based on unified closed physical methods of description as, for example, in Refs. 5, 6, 7, 12, 13, and 17. The "hybrids" obtained by combining different models (Refs. 10, 11, 14, and 15) are in this respect inferior.

One can arrive at a different conclusion by considering the actual applicability of the models for extensive practical calculations of nuclear data. Then other criteria come to the fore, namely the relative simplicity. Particularly promising are therefore the approaches presented in Refs. 9–11, and also in Ref. 14 for high initial energies. Approaches in the framework of an extension of the exciton model[6,7] may acquire a similar significance if in the future they can be improved and included in the existing programs for calculating spectra and excitation functions of various nuclear reactions on the basis of the exciton model that give a good description of extensive experimental material. For the further investigation and comparison of the individual approaches, it is important that they should be used under the same experimental conditions.

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