Magnetic analysis of charged particles in nuclear physics

N. I. Tarantin

Joint Institute for Nuclear Research, Dubna
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The article discusses the development of charged-particle magnetic analyzers used in nuclear physics–alpha and beta spectrometers, mass spectrometers and mass separators, spectrometers of elastically and inelastically scattered particles, and monochromators of accelerated particles. The discussion is carried out in terms of the linear theory of magnetic analysis of charged particles and on the basis of the general laws of particle optics. The review describes a new approach to discussion of linear transformations of particle beams in dipole and quadrupole magnets which permits a unique and simplified representation of all linear-transformation coefficients. Graphical procedures are given for determination of the particle-optics action of dipole and quadrupole magnets. The approach and the procedures are extended to analyzers with crossed magnetic and electric fields.

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INTRODUCTION

In experimental research in nuclear physics charged-particle magnetic analyzers of very diverse types have been used extensively from the very beginning: α and β spectrometers and spectrographs, mass spectrometers and mass separators of stable nuclides and radioactive products of nuclear reactions, spectrometers for scattered particles and particles emitted in nuclear reactions, monochromators of accelerated ions, and similar installations. The development of the theory of magnetic analysis and the attempt to satisfy to an ever increasing degree the growing requirements of experiments have led to the result that at the present time there is used in nuclear physics a rather extensive arsenal of essentially diverse magnetic analyzers which differ from one another in the types of magnets used, the particle-optics schemes, and other parameters.

The development and application of methods of magnetic analysis in various fields of nuclear physics has already been discussed repeatedly. In particular, the review articles by Bainbridge,[1] Hanna,[2] Deutsch and Kofoed-Hansen,[3] and Siegbahn[4] have discussed rather in detail the questions of use of magnetic analyzers in nuclear mass spectrometry and in spectrometry of α and β particles. A number of books[5–10] have been devoted to discussion of the problems of the shaping and analysis of charged-particle beams by means of dipole and quadrupole magnets with application to the problems of nuclear physics, high-energy and elementary-particle physics, and accelerator technology. However, all of these review articles and books were published more than ten years ago and naturally the achievements of recent years in the field of magnetic analysis of charged particles are not reflected in them. Yet only recently we have seen the appearance of the most refined magnetic analyzers such as spectrometers with separated poles and with combined fields, new approaches to discussion of the action of magnetic analyzers have been developed, and new laws of particle optics have been established. The need has arisen to generalize all of these recent achievements of the theory and technique of the magnetic analysis of charged particles.

On the one hand, a generalization of only the results of recent years, which are historically related to past results and which follow from the previous development of the theory and practice of magnetic analysis of charged particles, appears unjustified. On the other hand, discussion in sufficient detail of all known charged-particle analyzers in the present review, which is limited in size is simply impossible and hardly appropriate if we consider the thorough reviews which have already been published on individual aspects of the magnetic analysis of charged particles. Therefore in the review presented here we give only a general idea of the development of magnetic analyzers used in nuclear physics, and this development is illustrated by examples of individual magnetic spectrometers. The discussion is carried out with use of only the linear theory of magnetic analyzers and the general laws of particle optics, which are sufficient for systematization of the magnetic spectrometers used in nuclear physics and for demonstration of their features, advantages, and deficiencies. Accordingly we present in the review the main results of the theory of the linear transformation of charged particles in dipole and quadrupole magnets, described in a new framework. Questions of the further improvement of the performance of magnetic analyzers are as a rule left untouched, since a critical and systematic discussion of them is possible, in the author's opinion, only after exposition of the theory of transformation of particle beams in the second and higher approximations, which is impossible as the result of the limited size of the review.

In the review we consider only methods of magnetic analysis which use a transverse magnetic field; these methods are essentially the only ones used in nuclear physics at the present time.

The generalization of a large assortment of magnetic analyzers of charged particles in terms of a relatively simple linear theory and the rules which result from it, which brings out the main features and possibilities of magnetic analyzers of various types, should be of assistance in work with such analyzers.

1. GENERAL POSTULATES OF PARTICLE OPTICS FOR MAGNETIC ANALYSIS OF CHARGED PARTICLES

The separation and analysis of charged particles in mass, velocity, and charge can be accomplished by

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means of a magnetic or electric field. The fundamental difference in the possibilities of these two methods of analysis is that in the magnetic method the mass and velocity of the particle appear in combination as the momentum, while in the electric method they appear as the kinetic energy. Therefore in mass spectrometers and mass separators where ions accelerated to a rigorously determined energy are used, only magnetic analysis can be used for separation of particles in mass. An electric field, if it is applied in addition to the magnetic field, fulfills in this case auxiliary, although sometimes very important, functions. If the circumstance mentioned is not decisive, then the alternative of using either a magnetic or an electric field in a specific physical problem is solved from considerations of the practical possibilities of realization and the convenience of work with one field or the other. In particular, the following considerations can be used as a general orientational criterion for the choice.

The magnetic field strength B obtained without special difficulty by means of ordinary electromagnets (without use of superconductors) is ~1 T; the maximum electric field strength in vacuum is $E \sim 100$ kV/cm. If for the field strengths mentioned we compare the components of the Lorentz force evB/c and eE which determine the focusing and dispersing power of the magnetic and electric fields, we find that magnetic analysis is more effective for particles moving with velocity $v \geqslant 0.03c$ where e is the charge of the particle and c is the velocity of light in vacuum. For heavy particles—protons, deuterons, α particles, fission fragments, and so forth—this lower limit of velocity corresponds to a minimum particle energy ~0.4 MeV per nucleon, and for electrons and positrons ~0.2 keV.

The main purpose of magnetic analyzers in nuclear physics is the separation of charged particles and the determination of their momentum or energy, mass number, and charge. Here as a rule one requires from a magnetic analyzer the maximum resolution, which determines the degree of separation of the analyzed particles and the accuracy of the measurement, and also the maximum acceptance, on which the sensitivity of the analyzer depends.

The requirement of the maximum possible resolution in turn reduces to the requirement of the greatest dispersion of the analyzer and to the smallest cross section of the beam of analyzed particles in the detector plane of the analyzer. Thus, in analysis of particles attention is concentrated on the crossover or waist of the beam, in contrast to problems of acceleration and guiding of particles; in the latter use the main requirement on the magnetic optics is to bring the beam to a specified place without substantial loss, and therefore the broadest regions of the beam, which are dangerous from the point of view of possible loss of particles, are considered. This difference determines the special approach to discussion of the optical action of magnetic elements in analysis of charged particles.

In the general case the action of a magnetic analyzer on a beam of particles reduces to a directed change in the beam shape, i.e., a definite change in the coordinates of the particle trajectory in the detector plane of the analyzer as a function of the particle coordinates at the source location and other parameters. In general form this transformation can be represented as follows:

$$q_{2i} = \sum_j Q_{ij} q_{1j} + \sum_j \sum_k Q_{ijk} q_{1j} q_{1k} + \dots \quad (1)$$

Here q_{1j} and q_{1k} are the generalized transverse coordinates of the point of emission of the particle from the source, q_{2i} are the generalized transverse coordinates of the point of arrival of the particle at the detector, and $i, j, k = 1, 2, 3, \dots$. By source we mean here the region of volume from which charged-particle emission occurs. This region can be an ion source, radioactive material, or target bombarded by a beam of accelerated ions. The detector is a surface or ion collector, or a scintillation, ionization, or semiconductor detector of charged particles, or a nuclear emulsion, or a dielectric detector.

Some generalized coordinates have the meaning of small parameters characterizing either the particle independently of its location or the magnetic analyzer. For example, such a parameter is the difference δ of the considered particle from the principal (central) particle in mass, velocity, and charge. Such generalized coordinates naturally are taken without the subscript 1 or 2: $q_{1i} = q_{2i} = \delta$.

In order that the transformation (1) have the meaning of successive approximations in the transition from the first term to the second term, etc., the generalized coordinates must satisfy the condition $q_{1i} \ll 1$, $q_{ij} \ll 1$, $q_{1k} \ll 1$, $q_{2i} \ll 1$.

The terms Q_{ij} are called linear transformation coefficients; Q_{ijj} are quadratic coefficients, and Q_{ijk} $(j \neq k)$ are mixed transformation coefficients of second order.

The goal of a correct choice of the parameters of a magnetic analyzer of charged particles consists in providing the largest possible value of the coefficient Q_δ which determines the transverse coordinate of the particle at the detector as a function of δ and in reducing all coefficients on which the transverse size of the beam crossover at the detector plane depends. It is necessary first of all to reduce the linear transformation coefficients of the initial coordinates of the particle trajectory, since for $q_1 \ll 1$ the width of the beam crossover depends on them to the greatest degree. In view of the general relations of magnetic optics it is impossible to make all of the linear coefficients Q_{ij} equal to zero. Therefore the problem is solved of choice of the coefficient which is set equal to zero in such a way as to avoid the greatest contribution to the size of the crossover for a given particle-beam emittance. This is what determines the type of transformation of the particle beam in the magnetic analyzer. Thus, already in the linear approximation we have determined the minimum possible size of the beam crossover and the maximum resolution which can be provided by a given magnetic analyzer for a given beam of particles being analyzed.

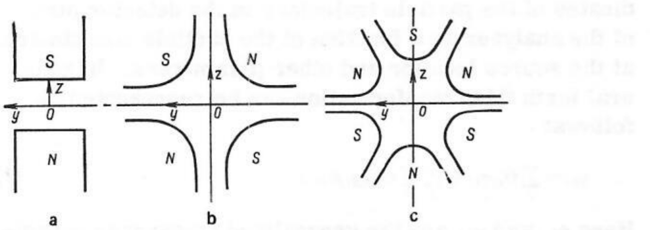

FIG. 1. Types of magnets used in charged-particle analyzers with a transverse magnetic field: a—dipole, b—quadrupole, c—sextupole.

The question of the degree to which this resolution can be achieved depends already on how successfully one can reduce or equate to zero the coefficients of second and higher orders. However, in no case will the resolution achieved be higher than the resolution determined from the initial linear discussion. All this indicates the importance of correct choice of the first-order parameters of the magnetic analyzer, which in addition govern the size and cost of the apparatus and the possibility and suitability of use of the analyzer under specific conditions.

In the general case a charged-particle magnetic analyzer is a combination of dipole, quadrupole, and higher-multipole magnets. In Fig. 1 we have shown a schematic representation of the location of the magnet poles in the cross section of the beam in dipole, quadrupole, and sextupole magnets. A dipole magnet provides transverse dispersion of the charged particles as well as linear and higher-order focusing of the beam. A quadrupole magnet accomplishes only focusing of the particle beam, beginning with the linear term. A sextupole magnet provides focusing of second and higher orders. In order to discuss the action of a charged-particle magnetic analyzer in the linear approximation it is necessary and sufficient to know in the linear approximation the charged-particle beam transformations accomplished by dipole and quadrupole magnets.

2. LINEAR OPTICS OF DIPOLE MAGNETS

In the first studies of the particle optics of dipole magnets, which were carried out at the beginning of the 1930's, a definite tendency was observed to reduce the action of a dipole magnet on a beam of charged particles to the action of known optical elements—prisms and lenses—on a beam of light rays. One of these studies was that by Herzog,[11] in which, for example, a dipole magnet with a uniform field is described as a thick optical lens with definite locations of the principal planes h_i and a definite value of the front and back focal lengths f. The parameters of a dipole magnet necessary for accomplishing a given transformation of a particle beam are given by the equation of a thick lens, which is known from geometrical optics:

$$1/(L_1-h_1)+1/(L_2-h_2)=1/f,$$

where

$$\left.\begin{array}{l} h_i=R_0\dfrac{\cos\varepsilon_i\cos(\Phi-\varepsilon_1-\varepsilon_2+\varepsilon_i)-\cos\varepsilon_1\cos\varepsilon_2}{\sin(\Phi-\varepsilon_1-\varepsilon_2)} \\ (i=1,\,2)\text{ and }\; f=R_0\dfrac{\cos\varepsilon_1\cos\varepsilon_2}{\sin(\Phi-\varepsilon_1-\varepsilon_2)}. \end{array}\right\}$$

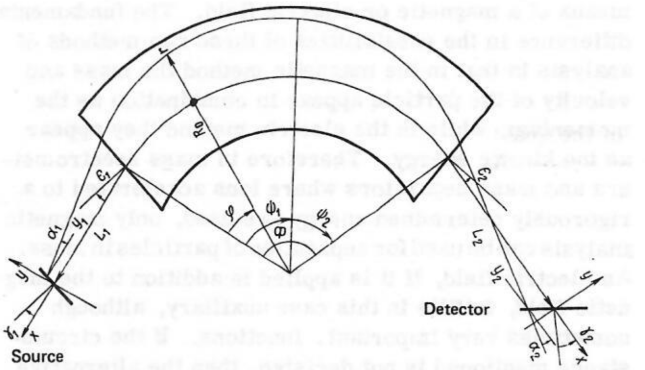

Fig. 2. Trajectory of radial motion of a charged particle in a magnetic dipole element. The arrows show the directions of positive measurement of the quantities.

the symbols used in the formulas have the usual meanings (Fig. 2).

There have been attempts to extend this approach also to the action of a dipole magnet with a nonuniform field.[12] However, this discussion, although it has a simple optical analog, leads to very awkward mathematical expressions.

Another direction of discussion of the action of a dipole magnet attempted to describe the transformations of the particle trajectory by means of matrices of linear algebra. The matrix method of representation of the linear transformation of a particle beam by a dipole magnet was given for the first time by Cotte.[13] Penner[14] and independently Kekk[15] extended this technique by inclusion of the dispersive action of the magnetic field in the transformation matrix.

The possibility of use of two-dimensional planar matrices for discussion of the transformations of a particle beam with inclusion of second-order transformation coefficients, which generally require three-dimensional spaces, was demonstrated by Brown[16] and somewhat later by Takeshita.[17]

The transformation of a particle beam carried out by a dipole magnet is described by the matrix method in the form of the product of several matrices which successively represent the actions of a gap without magnetic field, the edge (fringing) field and the so-called wedge magnet, a sector magnet, and then the elements preceding the sector magnet, but taken in reverse order. Below we give an example of the matrix description of the linear transformation of the coordinates of a particle trajectory in the radial plane of a dipole magnet:

$$\begin{bmatrix}\eta_2\\ \alpha_2\\ \delta\end{bmatrix}=\begin{bmatrix}1 & l_2 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ \operatorname{tg}\varepsilon_2 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\omega\Phi & \dfrac{\sin\omega\Phi}{\omega} & \dfrac{1-\cos\omega\Phi}{\omega^2}\\ -\omega\sin\omega\Phi & \cos\omega\Phi & \dfrac{\sin\omega\Phi}{\omega}\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}1 & 0 & 0\\ \operatorname{tg}\varepsilon_1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & l_1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\eta_1\\ \alpha_1\\ \delta\end{bmatrix}.$$

This representation is not convenient for the initial choice of the linear parameters of a magnetic analyzer. Although each of the matrices entering into the product has a relatively simple form, nevertheless it is impossible to obtain the analytic dependence of the final coordinates on the initial coordinates. Therefore the method of searching for the parameters of a magnetic analyzer reduces to calculations with various values of the quantities entering into the matrices, chosen at random. As a number of studies have noted, there is no guarantee here that the best version of the analyzer will not be missed in the calculations.

An intermediate version, between expression of the action of a dipole magnet by a single formula similar to the lens formula, and the description by the matrix method which separately represents the action of the individual component parts of the magnetic analyzer, was the discussion of a dipole magnet with use of special parameters having a specific interpretation. The first such attempts were made in Refs. 18–21. In this treatment the mathematical expressions are greatly simplified and it becomes possible to use simple graphical constructions for selection of the necessary parameters and calculation of the characteristics of a magnetic analyzer. Among the latter are the well-known rules of Barber,[22] Cartan,[23] and Judd,[24] which determine the location of the crossover of the particle beam as a function of the source position. The most complete development of this direction is found in Ref. 25, the results of which are discussed briefly below.

The dipole magnet in an analyzer, as a rule, is considered together with the source and particle detector, on the locations of which with respect to the magnet many characteristics of the analyzer depend. Therefore the dipole magnet itself and the space without magnetic field—from the entrance edge of the magnet to the source, and from the exit edge to the detector—comprise a single whole—a *magnetic dipole element*. A magnetic dipole element is shown in Fig. 2. There we have shown the particle trajectory in the usual notation. The z axis is directed from the figure to the reader. As the generalized coordinates entering to Eq. (1) we use the following: $q_1=\eta=y/R_0$; $q_2=\alpha$; $q_3=\zeta=z/R_0$; $q_4=\beta$; $q_5=\delta=(R-R_0)/R_0=\Delta m/m_0+\Delta v/v_0-\Delta e/e_0$, where β is the angle of inclination of the particle trajectory to the median plane of the dipole magnet; R_0 is the radius of curvature of the optical axis of the magnet, which is equal to the radius of curvature of the trajectory of the principal particle in a magnetic field B_0 ($R_0=m_0v_0c/B_0e_0$); R is the radius of curvature of the trajectory of the particle discussed ($R=mvc/B_0e$). The remaining symbols are clear from Fig. 2.

The axial component of the magnetic field in the median plane is specified in the form $B_z(\rho,\varphi,0)=B_0(1+a_1\rho)$, where $-1\leqslant a_1\leqslant 0$ corresponds to a magnetic field which does not defocus in either the axial or radial directions, $\rho=(r-R_0)/R_0$.

The radial equation of the particle trajectory with accuracy to first order in the small quantities ρ and δ has the following form:

$$\rho''+\omega^2\rho-\delta=0, \tag{2}$$

where $\omega=\sqrt{1+a_1}$ and the prime indicates differentiation with respect to φ.

In solution of Eq. (2) use is made of a special procedure which in essence is described below.

A solution of Eq. (2) is found which corresponds to the following initial conditions:

$$\rho_1(\varphi_{\lim})=\eta_1+\alpha_1 l_1;\quad \rho_1'(\varphi_{\lim})=\eta_1 t_1+\alpha_1(1+l_1t_1), \tag{3}$$

where $\eta_1=y_1/R_0$, $l_1=L_1/R_0$, $t_1=tg\varepsilon_1$. The particular solution of Eq. (2) with the boundary conditions (3) has the form

$$\rho_1(\varphi)=(\eta_1+\alpha_1 l_1-\delta/\omega^2)\cos\omega\varphi + (1/\omega)[\eta_1 t_1+\alpha_1(1+l_1t_1)]\sin\omega\varphi+\delta/\omega^2. \tag{4}$$

We find another particular solution which gives the trajectory of the particle arriving at the detector at a point with coordinate y_2 and at an angle α_2. The boundary conditions at the exit edge in this case can be written as follows:

$$\rho_2(\omega_{\lim})=\eta_2-\alpha_2 l_2;\quad \rho_2'(\omega_{\lim})=-\eta_2 t_2-\alpha_2(1+l_2t_2), \tag{5}$$

where $\eta_2=y_2/R_0$, $l_2=L_2/R_0$, $t_2=tg\varepsilon_2$. The solution of Eq. (2) corresponding to the boundary conditions (5) is

$$\rho_2(\varphi)=(\eta_2-\alpha_2 l_2-\delta/\omega^2)\cos\omega(\Phi-\varphi) + (1/\omega)[\eta_2 t_2-\alpha_2(1+l_2t_2)]\sin\omega(\Phi-\varphi)+\delta/\omega^2, \tag{6}$$

where Φ is the angle of deflection of the principal particle ($\delta=0$), which moves along the optical axis.

In order to relate the exit coordinates of the particle η_2 and α_2 with the entrance coordinates η_1, α_1, and δ, we extend the particle trajectory (4) expressed in terms of the initial coordinates of the trajectory (6) determined by the final coordinates, setting

$$\rho_1(\psi_1)=\rho_2(\psi_1)\ \text{and}\ \rho_1'(\psi_1)=\rho_2'(\psi_1), \tag{7}$$

where ψ_1 is an arbitrary value of the angle φ.

Limiting ourselves to the linear approximation in the expansion (1),

$$\eta_2=H_\eta\eta_1+H_\alpha\alpha_1+H_\delta\delta;\quad \alpha_2=A_\eta\eta_1+A_\alpha\alpha_1+A_\delta\delta,$$

where the generalized linear coefficients Q_{ij} have been designated by the capital letters A and H of the Greek alphabet of the same type as the symbols of the exit generalized coordinates η_2 and α_2, and equating to each other the coefficients of η_1, α_1, and δ in the left and right sides of the equalities (7), we obtain three systems of equations for determination of the linear transformation coefficients. We give the solution of one of these systems:

$$H_\alpha=[l_1c_1+(1+l_1t_1)s_1/\omega][-\omega l_2s_2+(1+l_2t_2)c_2] + [-\omega l_1s_1+(1+l_1t_1)c_1][l_2c_2+(1+l_2t_2)s_2/\omega], \tag{8}$$

where $s_i=\sin\omega\psi_i$, $c_i=\cos\omega\psi_i$, $i=1,2$, $\psi_2=\Phi-\psi_1$.

Experience shows that a magnetic dipole element, depending on the emittance of the source and the final shape of the particle beam, is used to accomplish one of the four transformations shown in Fig. 3, which by analogy with well-known optical devices are called a *burning glass* ($H_\eta=0$), a *projector* ($H_\alpha=0$), a *telescope* ($A_\eta=0$), and a *condenser* ($A_\alpha=0$).

Each of the types of particle-beam transformations mentioned above can be accomplished by a dipole element by two different means differing in the different behavior of the particle trajectories in the field of the

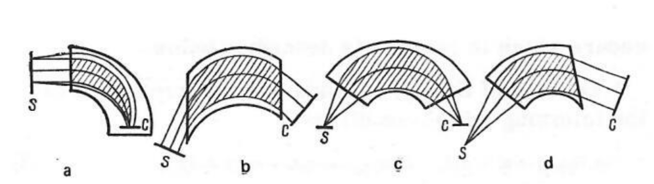

FIG. 3. Types of transformation of a charged-particle beam by a magnetic dipole element: a—burning glass, b—telescope, c—projector, d—condenser. S is the location of the particle source and C is the location of the detector.

dipole magnet and in the different values of the dispersion and isochronism of the dipole element. This is determined by the fact that each of the linear transformation coefficients H_η, H_α, A_η, and A_α can be made to vanish under two conditions. For example, $H_\alpha = 0$ under the conditions

$$-\omega l_i s_i + (1 + l_i t_i)\, c_i = 0 \tag{9}$$

or the conditions

$$l_i c_i + (1/\omega)\,(1 + l_i t_i)\, s_i = 0, \quad i = 1, 2. \tag{10}$$

If the conditions (9) are satisfied, there is a quasi-parallel behavior of the principal trajectories for $\varphi = \psi_1$, i.e., $\rho_1'(\psi_1) = \rho_2'(\psi_2) = 0$, and this determines the value of the angle ψ_1. In this case the magnetic dipole element has a nonzero dispersion which determines the possibility of its use as a magnetic analyzer ($H_\delta \neq 0$ for a burning glass and a projector; $A_\delta \neq 0$ for a telescope and a condenser).

The condition (10) signifies crossing of the principal trajectories of the particles in the region of the magnetic field for $\varphi = \psi_1 (\rho_1(\psi_1) = \rho_2(\psi_1) = 0$ and simultaneously determines the second value of ψ_1. In this case the dispersion of the magnetic dipole element is substantially less than in the first case. These two cases of transformation are logically called the chromatic and quasichromatic variants of the transformation.

The coefficients and conditions for various types of transformations of a charged-particle beam in a chromatic dipole element are given in Table I (the following notation is used: $s = \sin\omega\Phi$, $c = \cos\omega\Phi$). The values of the parameters entering into the expressions for the transformation coefficients are restricted by the following conditions: $0 \leq l_i < \infty$; $-\pi/2 < \varepsilon_i < \pi/2$; $-\pi/2 < \omega\psi_i \leq \pi/2$, $i = 1, 2$, $\psi_1 + \psi_2 > 0$.

TABLE I. Conditions and coefficients of linear transformation of a charged-particle beam by a chromatic magnetic dipole element in the radial plane.

Type of transformation	Burning glass	Projector	Telescope	Condenser
Conditions of transformation	$\frac{s_1}{c_1} = \frac{t_1}{\omega}$; $\frac{s_2}{c_2} = \frac{1}{\omega}\left(t_2 + \frac{1}{l_2}\right)$	$\frac{s_i}{c_i} = \frac{1}{\omega}\left(t_i + \frac{1}{l_i}\right)$, $i = 1, 2$	$\frac{s_i}{c_i} = \frac{t_i}{\omega}$, $i = 1, 2$	$\frac{s_1}{c_1} = \frac{1}{\omega}\left(t_1 + \frac{1}{l_1}\right)$; $\frac{s_2}{c_2} = \frac{t_2}{\omega}$
H_η	0	$-\frac{l_2 c_1}{l_1 c_2}$	$\frac{c_2}{c_1}$	$\frac{c_2}{c_1} - \frac{l_2 c_1}{l_1 c_2} - \frac{s}{\omega l_1}$
H_α	$\frac{l_2 c_1}{c_2}$	0	$\frac{l_1 c_2}{c_1} + \frac{l_2 c_1}{c_2} + \frac{s}{\omega}$	$\frac{l_1 c_2}{c_1}$
H_δ	$\frac{l_2 (s_1 + s_2)}{\omega c_2}$		$\frac{l_2 (s_1 + s_2)}{\omega c_2} + \frac{1 - c}{\omega^2}$	
A_η	$-\frac{c_2}{l_2 c_1}$	$-\frac{c_1}{l_1 c_2} - \frac{c_2}{l_2 c_1} + \frac{s}{\omega l_1 l_2}$	0	$-\frac{c_1}{l_1 c_2}$
A_α	$\frac{c_1}{c_2} - \frac{l_1 c_2}{l_2 c_1} - \frac{s}{\omega l_2}$	$-\frac{l_1 c_2}{l_2 c_1}$	$-\frac{c_1}{c_2}$	0
A_δ	$\frac{s_1 + s_2}{\omega c_2} - \frac{1 - c}{\omega^2 l_2}$		$\frac{s_1 + s_2}{\omega c_2}$	

TABLE II. Coefficients of linear transformation of a charged-particle beam by a chromatic magnetic dipole element for $l_1 = c_i = 0$.

Type of transformation	Burning glass	Projector			Condenser
Values of i	2	1	2	1.2	1
H_η	0	$-\frac{\omega l_2}{c_2}$	$-\frac{c_1}{\omega l_1}$	-1	$-\frac{\omega l_2}{c_2} - s_2$
H_α	$\frac{c_1}{\omega}$	0	0	0	$\frac{c_2}{\omega}$
H_δ	$\frac{1 + s_1}{\omega^2}$	$\frac{l_2 (1 + s_2)}{\omega c_2}$	$\frac{1 + s_1}{\omega^2}$	$\frac{2}{\omega^2}$	$\frac{l_2 (1 + s_2)}{\omega c_2} + \frac{1 + s_2}{\omega^2}$
A_η	$-\frac{\omega}{c_1}$	$\frac{s_2}{l_2} - \frac{\omega}{c_2}$	$\frac{s_1}{l_1} - \frac{\omega}{c_1}$	0	$-\frac{\omega}{c_2}$
A_α	$-\frac{\omega l_1}{c_1} - s_1$	$-\frac{c_2}{\omega l_2}$	$-\frac{\omega l_1}{c_1}$	-1	0
A_δ	$\frac{c_1}{\omega}$	$\frac{1 + s_2}{\omega c_2} - \frac{1 + s_2}{\omega^2 l_2}$	$\frac{c_1}{\omega}$	0	$\frac{1 + s_2}{\omega c_2}$

For values $l_i = 0$ and $\omega\psi_i = \pi/2$ in the expressions for certain coefficients, indeterminacies of the type 0/0 or 1/0 − 1/0 appear. An expansion of the indeterminacies for these cases is given in Table II.

The linear transformation coefficients correspond to the requirement of conservation of the phase space of the particle beam in its transformation, following from the general theorem of Liouville. Indeed, let us find the phase space of the beam after the transformation s_{ABCD}, considering the area of the rectangle $ABCD$ (Fig. 4) obtained as the result of transformation of the rectangular element $\Delta\eta_1 \Delta\alpha_1$ corresponding to the initial phase space of the beam. The area of the rectangle $ABCD$ is the geometric sum of the areas of the triangles AOB, BOC, COD, and DOA. For example, the area of the triangle AOB is $s_{AOB} = (\mathrm{OB} \cdot \mathrm{OA})/2 = (OB_\eta OA_\alpha - OB_\alpha OA_\eta)/2 = [H_\eta \eta_{1\,\max} + H_\alpha\, \alpha_{1\,\min})(A_\eta \eta_{1\,\min} + A_\alpha\, \alpha_{1\,\min}) - (H_\eta \eta_{1\,\min} + H_\alpha\, \alpha_{1\,\min})(A_\eta \eta_{1\,\max} + A_\alpha\, \alpha_{1\,\min})]/2 = (H_\eta A_\alpha - H_\alpha A_\eta)\,\Delta\eta_1\, \alpha_{1\,\min}/2$. We have similar expressions for the other triangles. The area of the rectangle is

$$s_{ABCD} = (\mathrm{H}_\eta \mathrm{A}_\alpha - \mathrm{H}_\alpha \mathrm{A}_\eta)\, \Delta\eta_1 \Delta\alpha_1, \tag{11}$$

where $\Delta\eta_1 = \eta_{1\,\max} - \eta_{1\,\min}$ and $\Delta A_1 = \alpha_{1\,\max} - \alpha_{1\min}$.

After substitution into Eq. (11) of the coefficients determined from Tables I and II, we obtain

$$\mathrm{H}_\eta \mathrm{A}_\alpha - \mathrm{H}_\alpha \mathrm{A}_\eta = 1, \text{ i.e., } s_{ABCD} = \Delta\eta_1 \Delta\alpha_1. \tag{12}$$

It follows from Eq. (12) that all of the indicated four coefficients of a linear transformation cannot simul-

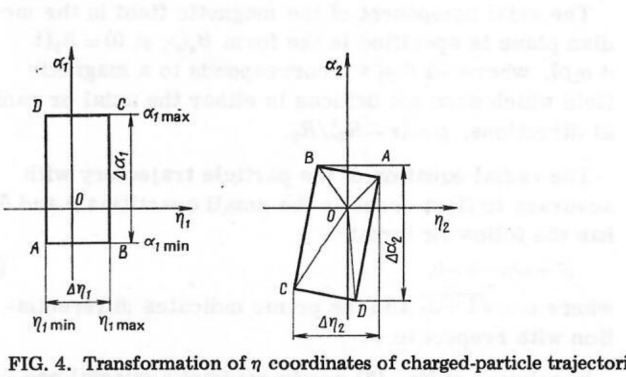

FIG. 4. Transformation of η coordinates of charged-particle trajectories by a magnetic dipole element in the $\nu\alpha$ phase plane.

taneously be equal to zero, and in addition any two coefficients in different terms are not simultaneously zero.

The question of the choice of the coefficient which is equal to zero, which reduces in essence to the choice of one of the four indicated types of particle-beam transformation, is solved by proceeding from the requirements of providing minimum size in the linear approximation of the beam crossover at the analyzer detector. This choice is determined by the initial emittance of the particle beam and by the variant of use of the dipole magnet in the analyzer. It is easy to see that for $l_1|\Delta\alpha_1| > |\Delta\eta_1|$ the minimal size of the beam crossover is obtained for a transformation of the projector type, and in the case when $|\Delta\eta_1| > l_1|\Delta\alpha_1|$ the minimal beam crossover occurs for a transformation of the burning-glass type. Here $\Delta\alpha_1$ is the angular opening of the trajectories of the particles captured by the magnetic analyzer; $\Delta\eta_1$ is the source width used, which is determined by the collimating diaphragm.

Dipole magnets with a transformation of the telescope or condenser type are used together with following focusing elements, for example, lenses which reduce the parallel beam to a spot. In this case to obtain the smallest beam size at the detector from a dipole magnet it is necessary to provide a minimal angular spread of the particles, which specifies the use of a telescope for $|\Delta\eta_1| > l_1|\Delta\alpha_1|$ and a condenser for the reverse relation.

The question of the optimal choice of the type of transformation of a particle beam is discussed also in Refs. 21 and 26.

We note that the coefficient H_α determines the quantity $\Delta\eta_2$ for a transformation of the burning-glass type. However, it cannot be called the magnification or demagnification in the sense in which this term is used with respect to H_η in a transformation of the projector type, since in a burning-glass transformation there is no unique image of the region $\Delta\eta_1$ onto the region $\Delta\eta_2$. The coefficient H_η in the case of a projector transformation also must be considered as an aberration, since in particle optics the goal pursued is to obtain the smallest width of the beam crossover, and not to produce an image of the object, and therefore any difference of $\Delta\eta_2$ from zero is a distortion of what is required, i. e., an aberration.

The coefficient Ξ_ξ is sometimes included in the linear transformation coefficient; here $\xi = x/R_0$ is determined by the coordinate x of the point of emission of a particle from the source volume. This coefficient expresses the change in the location of the particle-beam crossover for a change in the position of the source on the x axis. However, with respect to the coordinate η_2 it enters always together with one of the other linear transformation coefficients, for example, as $\Xi_\xi A_\alpha$, which in the case of η_2 leads eventually to second-order effects which are not considered here.

The equation of axial motion of a charged particle in the magnetic field considered has the form

$$\zeta'' + \nu^2\zeta = 0, \tag{13}$$

where $\nu = \sqrt{-a_1}$. One of the special solutions of Eq. (13) is found by proceeding from the conditions at the entrance edge of the magnetic field. These conditions are determined by the coordinates of exit of the particle from the source ζ_1 and the angle β_1 between the projection of the trajectory on the xz plane and the median plane, and also by the bending of the trajectory[13,27] in traversing the fringing field.

The special solution of Eq. (13) is

$$\zeta_1(\varphi) = (\zeta_1 + \beta_1 l_1)\cos\nu\varphi + (1/\nu)[-\zeta_1 t_1 + \beta_1(1 - l_1 t_1)]\sin\nu\varphi.$$

Another special solution of the same equation, determined by the conditions at the exit edge, has the form

$$\zeta_2(\varphi) = (\zeta_2 - \beta_2 l_2)\cos\nu(\Phi - \varphi) - (1/\nu)[\zeta_2 t_2 + \beta_2(1 - l_2 t_2)]\sin\nu(\Phi - \varphi).$$

The linear coefficients of the axial transformation of a particle trajectory, determines by means of the method indicated above, are:

$$\left.\begin{aligned}
Z_\zeta &= (c_1 - t_1 s_1/\nu)[-\nu l_2 s_2 + (1 - l_2 t_2)c_2] \\
&\quad - (\nu s_1 + t_1 c_1)[l_2 c_2 + (1/\nu)(1 - l_2 t_2)s_2] \\
&= (l_2 t_1 t_2/\nu - t_1/\nu - \nu l_2)s + [1 - l_2(t_1 + t_2)]c; \\
B_\zeta &= (\nu s_1 + t_1 c_1)(-c_2 + t_2 s_2/\nu) - (c_1 - t_1 s_1/\nu)(\nu s_2 + t_2 c_2) \\
&= (t_1 t_2/\nu - \nu)s - (t_1 + t_2)c; \\
Z_\beta &= [l_1 c_1 + (1/\nu)(1 - l_1 t_1)s_1][-\nu l_2 s_2 + (1 - l_2 t_2)c_2] \\
&\quad + [-\nu l_1 s_1 + (1 - l_1 t_1)c_1][l_2 c_2 + (1/\nu)(1 - l_2 t_2)s_2] \\
&= [(1/\nu)(1 - l_1 t_1)(1 - l_2 t_2) - \nu l_1 l_2]s + [l_1 + l_2 - l_1 l_2(t_1 + t_2)]c; \\
B_\beta &= [-\nu l_1 s_1 + (1 - l_1 t_1)c_1](c_2 - t_2 s_2/\nu) \\
&\quad - [l_1 c_1 + (1/\nu)(1 - l_1 t_1)s_1](\nu s_2 + t_2 c_2) \\
&= -[\nu l_1 + (1/\nu)(1 - l_1 t_1)t_2]s + [1 - l_1(t_1 + t_2)]c,
\end{aligned}\right\}$$

where the capital Greek letters Z and B designate the coefficients of a linear expansion of the exit axial coordinates ζ_2 and β_2 according to the principle (7) chosen for the radial coordinates, and $s_i = \sin\nu\chi_i$, $c_i = \cos\nu\chi_i$, $i = 1, 2$, $s = \sin\nu\Phi$, $c = \cos\nu\Phi$. $\chi_1 + \chi_2 = \Phi$. The axial transformation coefficients satisfy a relation similar to Eq. (12):

$$Z_\zeta B_\beta - Z_\beta B_\zeta = 1.$$

These linear transformation coefficients determine the axial size and angular divergence of a particle beam in the detector plane of a magnetic dipole element whose parameters are chosen on the basis of requirements on the transformation of a particle beam in its radial section, which as a rule are dominant. However, very frequently in addition to the required transformation of a beam in a radial section, a definite transformation is necessary also in the axial section, for example, to obtain an anastigmatic, i. e., stigmatic, transverse section of the beam crossover. A definite type of beam transformation is then necessary for the axial transformation of the beam.

We recall that in the radial section we considered only one means of beam transformation in a magnetic analyzer, namely, with quasiparallel behavior of the trajectories for $\varphi = \psi_1$ as providing the greatest dispersion. Since an axial transformation does not involve a similar dispersion condition, here the other means indicated above for equating the transformation coefficients to zero is also possible. In the general case it is necessary to distinguish two forms of particle-beam transformation in the axial section: with *even* (including zero) and *odd* numbers of crossings of the median plane of the dipole magnet by the particle trajectory, not counting the crossings at the source and detector locations, within the limits of the auxiliary angles χ_1 and χ_2.

The conditions and the linear transformation coefficients of a particle beam in the axial section for an even number of crossings are expressed also by the formulas of Table I, but with use of the axial parameters ν and χ_i instead of the radial characteristics ω and ψ_i, and also with a change of sign in front of t_i. For example, the conditions of an axial transformation of the projector type are $\sin\nu\chi_i/\cos\nu\chi_i = (1/\nu)(1/l_i - t_i)$, $i=1, 2$, and the coefficient is $Z_\xi = -l_2\cos\nu\chi_1/l_1\cos\nu\chi_2$. By a similar substitution we find the limiting expressions for the linear transformation coefficients for $l_i=0$ and $\cos\nu\chi_i$ from the formulas of Table II.

For axial transformations with an odd number of crossings, the conditions and coefficients are determined also from Tables I and II, with with the additional replacement of s_i by $-(-1)^i c_i$ and of c_i by $(-1)^i s_i$. In the particular case of a uniform field $\nu=0$ the condition of accomplishment of a transformation of the projector type goes over to $1/\chi_i + 1/l_i = t_i$, which is the well-known formula for a thin lens with a focal length $f=R_0/t_i$ first obtained by Cotte[13] and Khurgin.[27]

The forms of the conditions and transformation coefficients given have a convenient graphical representation. In Fig. 5 we have given this representation for a radial transformation of the projector type. We have drawn an arbitrary diagram of a magnetic dipole element in which we have used the quantities ωL_i, $\omega\psi_i$, and $\mathrm{arctg}(\mathrm{tg}\varepsilon_i/\omega)$ instead of the corresponding parameters L_i, ψ_i, and ε_i of the magnetic dipole element. In the case of a uniform field ($\omega=1$) the former coincide with the latter. The source and detector locations satisfy the condition that the three normals—namely, the normal to the effective edge at the point of entry (or exit) of the principal trajectory, the normal to the radius-vector for $\varphi=\psi_1$ at the point O, and the normal to the principal trajectory at the location of the source (or detector)—intersect at a single point A (or B). Here $H_\eta = -OB/OA$; $A_\alpha = -OA/OB$; $H_\delta = (OC+OD)/R_0\omega^2$,

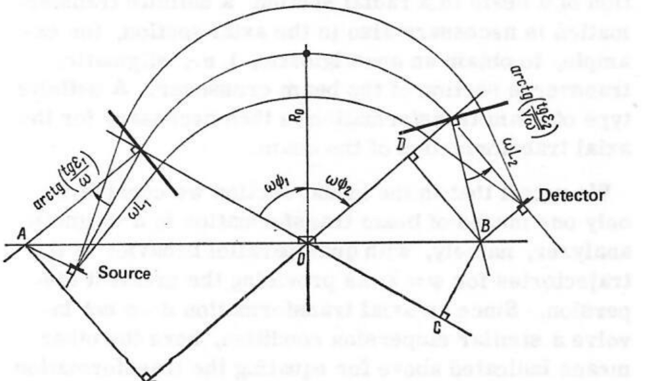

FIG. 5. Method of graphical determination of the source and detector locations and also of the principle coefficients of linear transformation of a charged-particle beam in the radial section of a chromatic magnetic dipole element. The squares denote right angles.

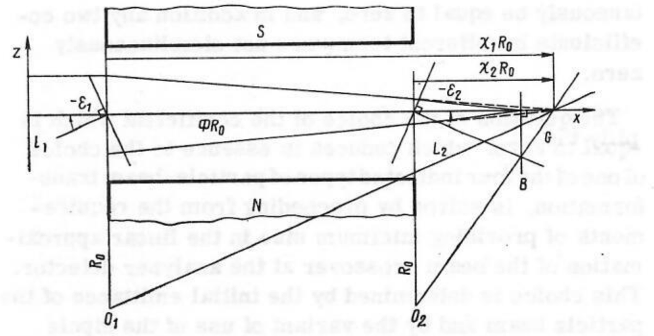

FIG. 6. Method of graphical determination of the source and detector locations and also of the principle coefficients of linear transformation of a charged-particle beam in the axial section of a magnetic dipole element with a uniform field.

and the values of l_1c_2/c_1 and l_2c_1/c_2 necessary for determination of the coefficient A_η are expressed by the segments AE and BC.

A similar construction also exists for determination of the parameters and coefficients of the magnetic dipole element for an axial transformation with an even number of crossings. The difference is that in the arbitrary diagram we plot $-\mathrm{arctg}(\mathrm{tg}\varepsilon_1/\nu)$. For an odd number of intersections of the particle trajectories with the median plane, we use in the construction, instead of the normal to the radius vector, the radius vector itself for its extension.

For a uniform field the construction is simplified. In Fig. 6 we have shown a method of graphical determination of the location of the axial crossover of a particle beam for a transformation of the burning-glass type for an odd number of crossings (one) of the median plane within the limits of the angles χ_1 and χ_2. In the constructions we use the quantities L_2, χ_i, and $-\varepsilon_i$ instead of the quantities νL_2, $\nu\chi_i$, and $-\mathrm{arctg}(\mathrm{tg}\varepsilon_i/\nu)$ required by the general rule. The quantity $\chi_1 R_0$ (χ_1, χ_2, and Φ are in radians) is determined by the point of intersection G of the principal trajectory, represented by the straight line, the x axis, with the straight line drawn parallel to the entrance normal through the point O_1 located at a distance R_0 along the normal to the principal trajectory from the point of entry of the optical axis. The location of the point of axial crossover of the particle beam is determined by the condition that the normal to the axis edge and the normal to the optical axis at the crossover point intersect at a point B lying on the straight line O_2G, where O_2 is a point located at a distance R_0 from the point of exit of the optical axis.

The coefficients of the axial transformation are readily determined by means of the segments obtained in the construction:

$$Z_\beta = -l_2\chi_1/\chi_2;\quad B_\beta = -\chi_1/\chi_2 + l_1\chi_2/l_2\chi_1 - \Phi/l_2;\quad B_\zeta = \chi_2/l_2\chi_1.$$

3. LINEAR OPTICS OF QUADRUPOLE MAGNETS

The quadrupole magnet as a strong focusing system has become very well known following the work of Courant, Livingston, and Snyder.[28] The action of a quadrupole magnet is usually discussed by means of the matrix representation. The search for parameters of a quadrupole magnet and the calculation of its optical action in this representation are a rather cumbersome problem. Therefore it is no accident that the literature discusses also other procedures intended to simplify the calculation of quadrupole magnets. In particular, a number of useful procedures which facilitate calculation of the optical action of a quadrupole magnet or of a system of quadrupole magnets can be found in Refs. 29–36. This list of studies shows the urgency of seeking simpler schemes for discussion of the action of a quadrupole.

In the present review the linear optics of a quadrupole magnet is presented in accordance with our earlier reports[37] in the same approach as that used above for the particle optics of a dipole magnet.[25]

It is well known that the equations of the trajectory of a charged particle in the field of a quadrupole magnet with accuracy to first order in the small quantities α, β, and δ have the form

$$y'' + k^2 y = 0; \tag{14}$$

$$z'' - k^2 z = 0, \tag{15}$$

where y and z are the transverse coordinates of the particle trajectory; α and β are the angles of inclination of the particle trajectory to the optical axis of the quadrupole magnet in the two mutually perpendicular principal planes, $k = \sqrt{eG/cmv}$, and G is the magnetic field gradient. We note that the conditions of obtaining linear equations of the particle trajectory do not require smallness of the quantities y and z. In this respect Eqs. (14) and (15) differ from the linear form of the radial equation (2) of a particle trajectory in a dipole magnet where the condition of linearization of the equation requires that $\rho < 1$.

One of the particular solutions of the equation for a charged-particle trajectory in the collecting plane of a quadrupole magnet (14) is found by proceeding from the boundary conditions at the entrance edge. This solution is

$$\eta_1(x) = (\eta_1 + \alpha_1 l_1)\cos \varkappa x + (\alpha_1/\varkappa)\sin kx, \tag{16}$$

where l_1 is the distance from the entrance edge of the magnet to the source and η_1 is the coordinate of the point of exit of the particle from the source (expressed in units of the length L_0 of the quadrupole magnet); α_1 is the emission angle of the particle from the source (Fig. 7), and $\varkappa = kL_0$.

We shall give another particular solution which represents the trajectory of a particle arriving at the detector at a point with coordinate η_2 and at an angle α_2:

$$\eta_2(x) = (\eta_2 - \alpha_2 l_2)\cos k(L_0 - x) - (\alpha_2/\varkappa)\sin k(L_0 - x), \tag{17}$$

where l_2 is the distance from the exit edge to the particle detector.

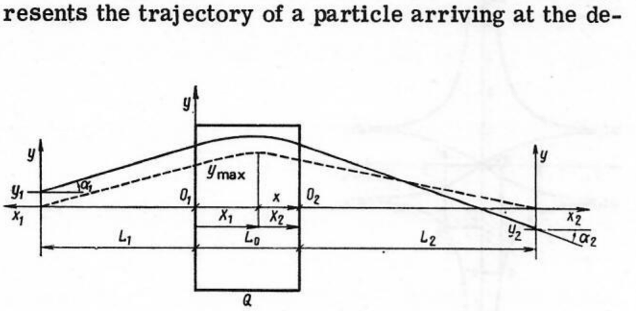

FIG. 7. Trajectories of charged particles in the converging (focusing) plane of a magnetic quadrupole element. The dashed line shows the trajectory characterizing a transformation of the projector type.

We shall express the exit coordinates η_2 and α_2 of the particle in terms of the entrance coordinates η_1 and α_1, continuing the trajectory (16) by the trajectory (17):

$$\eta_1(X_1) = \eta_2(X_1); \quad \eta_1'(X_1) = \eta_2'(X_1), \tag{18}$$

where X_1 is an arbitrary value of x, and using a linear representation of the exit coordinates in terms of the entrance coordinates:

$$\eta_2 = H_\eta \eta_1 + H_\alpha \alpha_1; \quad \alpha_2 = A_\eta \eta_1 + A_\alpha \alpha_1. \tag{19}$$

Then, substituting Eqs. (16), (17), and (19) into (18) and equating the coefficients of the independent variables η_1 and α_1 of the left-hand and right-hand sides of the expanded equality (18), we obtain two systems of equations which determine H_η, H_α, A_η, and A_α. The solutions of these systems of equations are

$$\left.\begin{aligned}
&H_\eta = c_1(-\varkappa l_2 s_2 + c_2) - \varkappa s_1(l_2 c_2 + s_2/\varkappa) = c - \varkappa l_2 s;\\
&H_\alpha = (l_1 c_1 + s_1/\varkappa)(-\varkappa l_2 s_2 + c_2) + (-\varkappa l_1 s_1 + c_1)(l_2 c_2 + s_2/\varkappa)\\
&= (l_1 + l_2)c + (1/\varkappa - \varkappa l_1 l_2)s;\\
&A_\eta = -\varkappa(s_1 c_2 + c_1 s_2) = -\varkappa s;\\
&A_\alpha = (-\varkappa l_1 s_1 + c_1)c_2 - (l_1 c_1 + s_1/\varkappa)\varkappa s_2 = c - \varkappa l_1 s;
\end{aligned}\right\} \tag{20}$$

here $s_i = \sin kX_i$; $c_i = \cos kX_i$, $i = 1, 2$; $X_2 = L_0 - X_1$; $s = \sin\varkappa$; $c = \cos\varkappa$.

The determinant of the transformation matrix (19), the elements of which are expressed by the formulas (20), is equal to unity: $H_\eta A_\alpha - A_\eta H_\alpha = 1$, which agrees with Liouville's theorem.

Vanishing of the first, second, etc. coefficients corresponds respectively to beam transformations of the burning-glass, telescope, projector, and condenser types and imposes definite conditions on the values of the parameters X_1 and X_2, where $x = X_1$ is the coordinate of the coupling of the two particular solutions (16) and (17).

Table III shows the linear transformation coefficients expressed in terms of the parameters X_1 and $X_2 = L_0 - X_1$ for the different types of transformation of a

TABLE III. Conditions and coefficients of linear transformation of a charged-particle beam by a magnetic quadrupole element in the converging plane.

Type of transformation	Burning glass	Projector	Telescope	Condenser
Conditions of transformation	$s_1 = 0$; $\varkappa l_2 = \mathrm{ctg}\,\varkappa$	$\varkappa l_i = \mathrm{ctg}\,kX_i$; $k(X_1 + X_2) = \varkappa$, $i = 1, 2$	$c_1 = c_2 = 0$	$\varkappa l_1 = \mathrm{ctg}\,\varkappa$; $s_2 = 0$
H_η	0	$-\frac{s_1}{s_2} = -\frac{l_2 c_1}{l_1 c_2}$	-1	$c\left(1 - \frac{l_2}{l_1}\right)$
H_α	$\frac{1}{\varkappa s} = \frac{l_2}{c}$	0	$-(l_1 + l_2)$	$\frac{1}{\varkappa s} = \frac{l_1}{c}$
A_η	$-\varkappa s = -\frac{c}{l_2}$	$-\varkappa s$	0	$-\varkappa s = -\frac{c}{l_1}$
A_α	$c\left(1 - \frac{l_1}{l_2}\right)$	$-\frac{s_2}{s_1} = -\frac{l_1 c_2}{l_2 c_1}$	-1	0

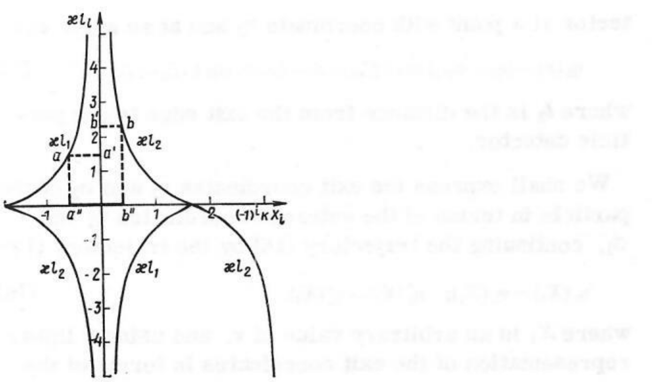

FIG. 8. First method of graphical determination of the parameters l_1, l_2, and $\varkappa$ of a magnetic quadrupole element in the converging plane.

charged-particle beam by a magnetic quadrupole element. We have also shown in Table III the conditions necessary for accomplishment of a given type of transformation. The parameter values lie within the following limits: $-\infty < l_i < \infty$, $-\pi/2 \leq kX_i < \infty$, $i = 1, 2$. A negative value of X_i means that X_i is measured in the direction opposite to the motion of the particles.

The parameters of a magnetic quadrupole element determined by the formulas of the second row of Table III are easily found by calculation or by means of the graphs of Fig. 8. For given values, for example, of the parameters of a magnetic quadrupole element of the projector type l_1 and $\varkappa$, the value of l_2 is found by construction of the broken line $a'a''b''bb'$. Here $a' = \varkappa l_1$ and $a''b'' = \varkappa$, and the point a is plotted on one of the two curves denoted by the symbol $\varkappa l_1$, depending on the sign of l_1. The value of $\varkappa l_2$ is given by the ordinate b' of the point b plotted on the curve labeled by the symbol $\varkappa l_2$. The curves are constructed on the basis of the simple formula $\varkappa l_i = \mathrm{ctg}[(-1)^i kX_i]$.

The form given for the conditions which are necessary for accomplishment of the indicated transformation of a particle beam permits use also of another graphical method which gives values of the parameters l_1 and l_2 directly for given values of k and L_0, and also values of the linear transformation coefficients (Fig. 9). In Fig. 9 the value of l_1 with inclusion of its sign is plotted along the x_1 axis. From the point l_1 a straight line is drawn through the point A on the ordinate axis which corresponds to the specified converging strength of the quadrupole magnet kL_0. At point A we have laid out in a clockwise direction the angle l_1AB which in radians is equal to the value of the converging strength kL_0. The point of intersection of the straight line BA with the x_2 axis gives the value of l_2 for a transformation of the projector type. Values of the linear coefficients for transformation of a particle beam by a magnetic quadrupole element are also determined from the following construction: $H_\eta = -l_2A/l_1A$; $A_\eta = -1/AD$; $A_\alpha = l_1A/l_2A$.

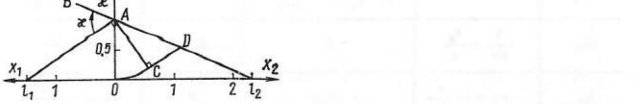

FIG. 9. Second method of graphical determination of the parameters l_1, l_2, and $\varkappa$ and the linear transformation coefficients of a charged-particle beam in the converging plane of a magnetic quadrupole element.

The procedure described above when used for solution of the equation for a particle trajectory in the diverging plane of a quadrupole magnet (15) leads to the following analogous expressions for the linear transformation coefficients $\zeta_2 = Z_\zeta\zeta_1 + Z_\beta\beta_1$ and $\beta_2 = \beta_\zeta\zeta_1 + B_\beta\beta_1$:

$$\left.\begin{aligned} Z_\zeta &= c_1(\varkappa l_2 s_2 + c_2) + \varkappa s_1(l_2c_2 + s_2/\varkappa) = c + \varkappa l_2 s;\\ Z_\beta &= (l_1c_1 + s_1/\varkappa)(\varkappa l_2 s_2 + c_2) + (\varkappa l_1 s_1 + c_1)(l_2c_2 + s_2/\varkappa)\\ &= (l_1 + l_2)c + (1/\varkappa + \varkappa l_1 l_2)s;\\ B_\zeta &= \varkappa(s_1c_2 + c_1s_2) = \varkappa s;\\ B_\beta &= (\varkappa l_1 s_1 + c_1)c_2 + (l_1c_1 + s_1/\varkappa)\varkappa s_2 = c + \varkappa l_1 s, \end{aligned}\right\} \quad (21)$$

where $s_i = \mathrm{sh}kX_i$; $c_i = \mathrm{ch}kX_i$; $i = 1, 2$; $X_1 + X_2 = L_0$; $kL_0 = \varkappa$; $s = \mathrm{sh}\varkappa$; $c = \mathrm{ch}\varkappa$.

The transformation coefficients (21) are identical to the coefficients (20) in the form in which they are written, but they differ from them substnatially in that they are expressed in terms of hyperbolic functions rather than circular functions. Therefore the possibilities of vanishing of the coefficients (21), which are necessary for achievement of a definite transformation, are limited. In particular, a transformation of the telescope type is impossible.

The conditions and coefficients of the transformation of a charged-particle beam in the diverging plane of a quadrupole magnet for transformations of the quasi-burning-glass, quasiprojector, and quasicondenser types are also given in Table III but with replacement of the circular functions by hyperbolic functions and with a change of sign in front of the hyperbolic function in the conditions of accomplishment of the transformations. In the case $\varkappa|l_i| < 1$ for a transformation of the quasiprojector type it is necessary in addition to use the following: $\varkappa l_i = -\mathrm{th}kX_i$; $Z_\zeta = 1/B_\beta = c_1/c_2 = l_2s_1/l_1s_2$.

In searching for the parameters of a magnetic quadrupole element in the diverging plane one can use the graphs of Fig. 10, in which we have given values of $\varkappa l_i$ as a function of kX_i, plotted by means of the relations $\varkappa l_i = \mathrm{th}[-(-1)^i kX_i]$ and $\varkappa l_i = \mathrm{cth}[-(-1)^i kX_i]$. The method of determination of the parameters of a magnetic quadrupole element in the diverging plane follows the similar method for the converging plane.

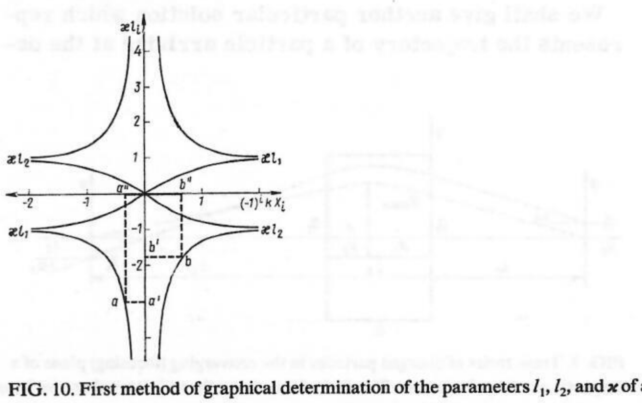

FIG. 10. First method of graphical determination of the parameters l_1, l_2, and $\varkappa$ of a magnetic quadripole element in the diverging (defocusing) plane.

Thus, systematization of the transformations which can be carried out by magnetic dipole and quadrupole elements, and also introduction of new parameters—the components of the total deflection angle of the principal particle in a dipole magnet and the components of the total length of a quadrupole magnet determined by the entrance and exit parameters of the magnetic dipole or quadrupole elements in the new approach to discussion of their action—have permitted simplification of the form of the linear transformation coefficients of particle trajectories in magnetic elements, as well as a systematization and ordering of the entire set of coefficients. Thus, the complete system of coefficients of linear transformation of the transverse coordinates of the particle trajectory and of the slope angles, including 24 coefficients of radial motion and 16 coefficients of axial motion in a magnetic dipole element and also 32 coefficients which characterize the converging and diverging actions of a magnetic quadrupole element, is given essentially by Table I. (We note that Table III is a special case of Table I corresponding to $\varepsilon_1=\varepsilon_2=0°$.)

Similar unique expressions exist for the coefficients of longitudinal transformation of the coordinates of the particle trajectory, the iso-equidistance of the trajectories, and the maximum deflections of the particle trajectories in magnetic dipole and quadrupole elements; however, these are not discussed here.

In discussing the questions of particle optics of magnetic dipole and quadrupole elements we have not considered the interactions of charged particles with each other or with the residual gas atoms, diaphragm edges, etc. In many cases of application of magnetic analyzers in nuclear physics these questions are unimportant as a result of the low intensity and high momentum of the analyzed particles, the adequately high vacuum, and so forth. For example, in mass separators the interaction of the separated ions with each other in intense beams is avoided to a significant degree as a result of neutralization of the space charge of the beam by electrons produced as the result of ionization of the residual gas atoms.

4. GENERAL RELATIONS GOVERNING THE BASIC CHARACTERISTICS OF MAGNETIC ANALYZERS OF VARIOUS TYPES

As we have mentioned above, in charged-particle spectrometry use is made of magnetic analyzers of very diverse types which have substantially different characteristics. These characteristics depend on many parameters of the magnetic analyzer, and this hinders to a substantial degree the identification of the main factors which determine the quality of the analyzer. Among the principal characteristics of a magnetic analyzer, as we have already mentioned, we must include its resolution and acceptance, and it is of interest to determine the dependence of these quantities on the principal parameters of the analyzer.

The first person to turn his attention to the simple relation between the dispersion and the area of the radial section of the beam in a magnetic analyzer was Bruck.[38] Later a similar relation was obtained independently by the author of the present review[39] for magnetic analyzers with an axially symmetric inhomogeneous field and analyzers with separated poles. Still later an article on this question was published by Wollnik,[40] who was familiar with the results of our work[39] as the result of discussions with the author. In the present review this general relation, which determines the principal characteristics of magnetic analyzers of very different types, is presented on the basis of Ref. 41.

In the discussion we make use of the area of the radial section of the particle beam in the field of a dipole magnet bounded by the limiting trajectories characteristic of a given type of beam transformation of the principal particles ($\delta=0$) and by the entrance and exit edges of the dipole magnet, which with accuracy to the first order of small quantities is determined by the relation

$$\Sigma=R_0^2\Big[\int_0^{\psi_1}(\rho_{1\max}-\rho_{1\min})\,d\varphi+\int_{\psi_1}^{\Phi}(\rho_{2\max}-\rho_{2\min})\,d\varphi\Big].$$

In Fig. 3 this area has been crosshatched. For a burning-glass transformation we have also shown the total area of the radial section of the particle beam, which differs insignificantly from the crosshatched area.

For example, in a magnetic dipole element with a transformation of the burning-glass type this area, according to Eqs. (4) and (6) and the conditions $\Delta\eta_1 \gg \Delta\alpha_1 l_1$, $H_\eta=0$, $\delta=0$, is found to be

$$\Sigma=R_0(s_1+s_2)\,\Delta y_1/\omega c_1,$$

where $\Delta y_1=y_{1\,\max}-y_{1\,\min}$ is the radial width utilized in the source and where it is taken into account that $\Delta\alpha_2=A_\eta\Delta\eta_1=-c_2\Delta y_1/l_2c_1R_0$. The limiting resolution of a magnetic analyzer in this case, i.e., the resolution obtained after exclusion of aberrations of second and higher orders, is

$$R_m=\mathrm{H}_\delta/\mathrm{H}_\alpha(\alpha_{1\max}-\alpha_{1\min})=(s_1+s_2)/\omega c_1\Delta\alpha_1.$$

From comparison of the last two relations it follows that

$$R_m\Delta y_1\Delta\alpha_1=R_m A_r=\Sigma/R_0. \tag{22}$$

The quantity $A_r=\Delta y_1\Delta\alpha_1$ is the radial acceptance of the magnetic analyzer, which is equal to the utilized emittance or luminosity of the source.

Equation (22) is obtained by the same method also for a magnetic dipole element with a projector transformation if it is taken into account that an analyzer of this type is used under the condition $\Delta\alpha_1 l_1\gg\Delta\eta_1$.

Magnetic dipole elements of the telescope and condenser type provide angular but not linear separation of particle beams with different values of δ. This angular resolution, which is determined by the ratio $A_\delta/|A_\alpha|\Delta\alpha_1$ for a telescope and by $A_\delta/|A_\eta|\Delta\eta_1$ for a condenser, can be transformed to the equivalent spatial distribution of particles by means of a focusing and even a nondispersive element which focuses the particles and transforms the angular divergence of particles of various types into a linear spread. Thus, for example, for an analyzer of the condenser type R_m

$= R_0(s_1+s_2)l_1/\omega c_1 \Delta y_1$, $\Sigma/R_0 = R_0(s_1+s_2)l_1\Delta\alpha_1/\omega c_1$ and Eq. (22) is satisfied.

Equation (22) is valid also for analyzers consisting of several dipole magnets and other particle-optics elements. As an illustration we shall consider an analyzer consisting of a magnetic quadrupole element of the condenser type ($A_{\alpha 1}=0$), a magnetic dipole element of the telescope type ($A_{\eta 2}=0$), and another quadrupole element but this time of the burning-glass type ($H_{\eta 3}=0$). Here and below the second subscript on the symbols designates the order of the element in the compound analyzer.

The area of the radial section of a beam in a dipole magnet which characterizes a transformation of the telescope type is $\Sigma = R_0(s_1+s_2)\Delta y_{12}/\omega c_1 = R_0(s_1+s)_2 H_{\alpha 1}\Delta\alpha_{11}/\omega c_\lambda$. However,

$$\Delta\eta_{23} = H_{\alpha 3}\Delta\alpha_{22} = H_{\alpha 3}(A_{\alpha 2}\Delta\alpha_{12} + A_{\delta 2}\delta) = H_{\alpha 3}A_{\alpha 2}A_{\eta 1} + H_{\alpha 3}A_{\delta 2}\delta$$

and consequently for the analyzer as a whole we have $H_\eta = H_{\alpha 3}A_{\alpha 2}A_{\eta 1}$ and $H_\delta = H_{\alpha 3}A_{\delta 2}$. Hence $R_m = H_\delta/|H_\eta|\Delta\eta_{11} = R_0 A\delta_2/|A_{\alpha 2}A_{\eta 1}|\Delta y_{11}$. Finally, $R_m\Delta y_{11}\Delta\alpha_{11} = R_m R_r = \Sigma/R_0$, since for a condenser $A_{\eta 1} = -1/H_{\alpha 1}$ (see Table III) and for a telescope $A_{\delta 2} = (s_1+s_2)/\omega c_2$ and $A_{\alpha 2} = c_1/c_2$ (see Table I).

We finally obtain $R_m\Delta y_{11}\Delta\alpha_{11} = R_m A_r = \Sigma/R_0$.

We note that the area of the radial section of a beam in the field of a dipole magnet is taken with inclusion of the sign of solutions (4) and (6), and the sign of the area reverses after a radial crossover of the beam.

The area of the beam cross section in the field of a quadrupole magnet, even if it is treated as a dipole magnet with the uniform component of the field equal to zero and a nonzero linear component, is not taken into account, since in this case $R_0 = \infty$.

From Eq. (22) it follows that the achievable resolution of a magnetic analyzer and its acceptance, regardless of the form of particle-beam transformation and of the specific parameters of the magnetic field and the parameters of the dipole element, are determined by the ratio of the area of the magnet poles to the radius of curvature of its optical axis. Therefore there is no justification for the statement sometimes encountered that some magnetic analyzer or other has substantially higher resolution and higher acceptance as the result of the specific properties of the linear transformation scheme chosen for the particle beam, for dimensions equal to other spectrometers.

One of these statements is the conclusion, which was drawn in Ref. 42, of the possibility of a magnetic analyzer with a substantially smaller width of sector-type guide field than the width ordinarily used but with the same acceptance and allegedly the same resolution. The author[42] does not taken into account the effect of the focusing lenses introduced into the analyzer (although they are nondispersive) on the resolution of the analyzer as a whole. Examples of the specific consideration of the influence of quadrupole magnetic lenses on the dispersion of a dipole magnet can be found in Ref. 43.

Equation (22) permits us to understand and generalize the main trends for improvement of magnetic analyzers. In particular, we can see that one of the possibilities of increasing the resolution of a magnetic analyzer is in a similar increase of its size. In this case the area of the radial section of the beam increases as R_0^2 and the ratio Σ/R_0 increases in proportion to R_0. The other consequences of Eq. (22) will be discussed below.

Rüdenauer[42] suggested that the quantity $R_m A_r A_z$, where A_z is the azimuthal acceptance, be called the quality index (*Güteziffer*) of a magnetic analyzer. As follows from the discussion above, this index, in its principal part $R_m A_r$, is determined by the quantitative parameters of the analyzer—the area of the poles of the dipole magnet and the radius of curvature of its optical axis.

To evaluate the quality of a linear magnetic analyzer arrangement one can use the parameter $H_\delta/H_\eta l$ which was used in Refs. 39 and 44–46. This parameter expresses the dispersion of a magnetic analyzer with reference simultaneously to the two principal factors which spoil the resolution: the coefficient H_η which determines the width of the particle beam at the analyzer detector, and the length l of the flight path from the source to the detector. This parameter, which can be called the doubly relative or specific dispersion, or for simplification of the terminology just the relative dispersion, characterizes the resolution of various magnetic analyzers with respect to the same source of particles and presents a measure of the stability of this resolution with respect to various perturbing factors such as scattering of the analyzed particles by residual gas molecules, electrostatic repulsion of the particles, distortion of the particle trajectories as the result of inaccuracies in the magnetic field topography, and so forth.

5. TYPES OF CHARGED-PARTICLE MAGNETIC ANALYZERS USED IN NUCLEAR PHYSICS

Use of the methods of magnetic analysis in nuclear physics was begun in the earliest years in experiments undertaken in many laboratories at the end of the nineteenth century to establish the nature of the radiations emitted by radioactive materials, which were later designated α, β, and γ rays.

Magnetic Analyzers with 180° Focusing. The analyzing properties of a very simple topography—a uniform magnetic field—were realized by Rutherford and Robinson[47] in the semicircular spectrograph, the principle of action of which was known from the experiments of Classen[48] in 1908 on determination of the charge-to-mass ratio for electrons. As is well known, the semicircular magnetic analyzer is characterized by parameters $\Phi = 180°$, $a_1 = 0$, $l_1 = l_2 = 0$, and its radial linear transformation coefficients $H_\eta = -1$, $H_\alpha = 0$, $H_\delta = 2$ are given by Table II. There is no axial focusing in a semicircular analyzer, which is an important deficiency reducing its acceptance in analysis of isotropically emitted particles. Another deficiency of the semicircular analyzer is the need of placing the source and detector in the gap between the poles. The advantages of analyzers of this type are simplicity of shaping and accurately measuring the uniform magnetic field, and the absence of entrance and exit edges.

Dempster[49] first used a semicircular mass spectrometer to determine the isotopic composition of a number of light elements. Rosenblum[50] described the use of a semicircular spectrometer to measure the first α-particle spectra.

The semicircular magnetic analyzer found extensive use as an isotope separator. In the first part of 1940 in the USA a number of isotope separators were constructed under the guidance of E. O. Lawrence at the University of California Cyclotron Laboratory and therefore named Calutron.[51] First the Calutrons were used to separate kilogram quantities of ^{235}U, and later to carry out an extensive program of production of enriched stable and radioactive isotopes.

The main distinguishing feature of the Calutrons was a rather large mass dispersion determined by the rather large radius of curvature of the optical axis: $R_0 H_{\Delta m/m_0} = 1.22$ cm for a 1% mass change at $R_0 = 122$ cm. We note that for a magnetic analyzer working in the isotope separator mode the mass dispersion coefficient is $H_{\Delta m/m_0} = 0.5 H_\delta = 1$, since the ions being separated have identical energy, which is determined in the same accelerating potential difference, and consequently the lower the velocity, the larger the mass:

$$H_{\Delta m/m_0} = H_\delta (\Delta m/m_0 - \Delta m/2m_0)/(\Delta m/m_0) = H_\delta/2 = 1,$$

since $\Delta m/m_0 + 2\Delta v/v_0 = 0$.

Another feature of the Calutrons was the high luminosity value utilized in the ion source, which permitted collection from the source of a current up to 100 mA and placement of several ion sources in the same gap of the electromagnet in order to utilize the latter more efficiently. Here one semicircular beam intersects two others located next to it.

Similar semicircular separators for enrichment of isotopes have been constructed in the USSR.[52] The largest radius of curvature of the optical axis of the separators is 162.5 cm, and the opening angle of the ion beam is 25–30°.

The stocks of enriched isotopes, both in the USA and in the USSR, contain about 250 isotopes of fifty elements,[51,52] which are widely used in various scientific studies. According to the data given by Love,[51] in 1972 more than 60% of the experimental studies published in the journal Nuclear Physics were carried out with use of isotopes enriched by the electromagnetic method. As an example we shall point out the work of Flerov *et al.*,[53] in which in a study of new possibilities of synthesis of superheavy elements use was made of accelerated ions of the enriched isotope ^{48}Ca (natural content only 0.18%) and targets one of which was enriched to 99.80% in the isotope ^{204}Pb (natural content 1.4%).

To reduce the intrinsic background of the analyzed particles, semicircular analyzers sometimes are used doubly or triply, bending the beam in one direction by 2π or 3π, respectively. In this case the analyzer dispersion H_δ is respectively 2, 0, and 2 after the first, second, and third half revolution. This is easily seen from Eq. (22) if we take into account that the sign of the added area of the beam section changes after each beam crossover.

The first β spectrometer with triple focusing (three crossovers) in a uniform field was described by Dzhelepov *et al.*[54] In the beam crossovers after the first and third half revolutions there were counters connected in coincidence and located on the helical trajectory of the β particles with a step along the magnetic lines of force.

An isotope separator with a similar form of triple focusing has been described by Love *et al.*[55]

A mass spectrometer with double semicircular focusing in a single revolution of the beam is also known.[56]

Sector Magnetic Analyzers with a Uniform Field. The term sector-type analyzer is applied to analyzers with a source and detector taken outside the magnetic field, i. e., with $l_1 \neq 0$, $l_2 \neq 0$, and an angle Φ less than the angle to the first crossover in the magnetic field provided. The convenience of analyzers of this type lies in the possibility of locating the source and detector outside the narrow gap between the poles. However, in this case the need arises of taking into account the effect of the complicated fringing field on the motion of the charged particles in the analyzer.

It is sometimes assumed that sector-type analyzers have dipole magnets of substantially smaller size than semicircular analyzers with the same optical characteristics. It is easy to see that this opinion is not at all accurate, for with identical R_0 a dipole magnet with a shorter length must have a larger width if the sector and semicircular analyzers provide the same acceptance angle $\Delta\alpha_1$. The difference in the areas of the poles is determined only by the difference in the size of the nonworking zones of the dipole magnet, which are somewhat larger in the semicircular version.

A sector magnetic analyzer with a uniform field in its simplest form of normal entrance and exit of the beam ($\varepsilon_1 = \varepsilon_2 = 0°$) was used by Bainbridge and Jordan as part of a more complicated spectrograph with double focusing in angle and velocity.[57] Another example of an analyzer of this type is Nier's mass spectrometer,[58] whose parameters were $a_1 = 0$, $\Phi = 60°$, $l_1 = l_2 = 1.73$, $\varepsilon_1 = \varepsilon_2 = 0°$, and $R_0 = 12.5$ cm. The symmetric version of such an analyzer provides the same basic optical characteristics $H_\alpha = 0$, $H_\eta = -1$, $H_{\Delta m/m_0} = 1$ as a semicircular analyzer but it has a lower relative dispersion. For example, for the Nier mass spectrometer the relative dispersion is $D_{\text{rel}} = 0.22$, while for the Calutron this parameter is 0.32.

A magnetic analyzer of the sector type with a uniform field and normal entry and exit of the beam does not have axial focusing. This deficiency is avoided in analyzers with inclined entrance and exit edges. The axial transformation of the particle trajectories by the fringing field of the dipole magnet is used both to increase the aperture of the analyzer and to obtain a stigmatic beam section at the detector. The possibility of axial edge focusing of a particle beam was first demonstrated in Refs. 13 and 27. The question of double focusing in analyzers with a uniform field was discussed in more detail by Camac[18] and in more general form by Cross.[19]

The first magnetic analyzer with a uniform field in which double focusing was accomplished was the spectrometer of Lovati and Tyren,[59] which was intended for analysis of charged particles emitted in nuclear reactions. For selection of the parameters and calculation of the coefficients of the analyzer, these authors used the rather complicated formulas of Cartan.[23] For example, the conditions of a double transformation of the projector type had the form

$$H_\alpha = \frac{\cos^2\varepsilon_1\cos^2\varepsilon_2 - [l_1\sin\Omega - \cos(\Phi-\varepsilon_2)\cos\varepsilon_1][l_2\sin\Omega - \cos(\Phi-\varepsilon_1)\cos\varepsilon_2]}{\cos\varepsilon_1\cos\varepsilon_2\sin\Omega} = 0;$$

$$Z_\beta = \Phi + l_1(1-\Phi\,\mathrm{tg}\,\varepsilon_1) + l_2(1-\Phi\,\mathrm{tg}\,\varepsilon_2) - l_1 l_2[\mathrm{tg}\,\varepsilon_1 + (1-\Phi\,\mathrm{tg}\,\varepsilon_1)\,\mathrm{tg}\,\varepsilon_2] = 0,$$

where $\Omega = \Phi - \varepsilon_1 - \varepsilon_2$, which is significantly more cumbersome than the relations obtained from the formulas of Table I,

$$\mathrm{arctg}(\mathrm{tg}\,\varepsilon_1 + 1/l_1) + \mathrm{arctg}(\mathrm{tg}\,\varepsilon_2 + 1/l_2) = 1/(\mathrm{tg}\,\varepsilon_1 - 1/l_1) + 1/(\mathrm{tg}\,\varepsilon_2 - 1/l_2) = \Phi.$$

After considering various versions of the spectrometer, which required laborious calculations, Lovati and Tyren[59] chose the following parameters for the magnetic analyzer: $R_0 = 94$ cm, $\Phi = 90°$, $l_1 = l_2 = 1$, $\varepsilon_1 = 55°5'$, and $\varepsilon_2 = -30°7'$, which provide $H_\eta = -0.41$, $H_\delta = 1.40$, and $Z_\zeta = 3.50$. The relative dispersion of the spectrometer is $D_{\rm rel} = 0.96$, which is larger than the corresponding quantity for the semicircular analyzer (0.64).

The scheme for linear transformation of the beam in the first magnet of the spectrometer turned out to be sufficiently optimal. Indeed, the positive slope of the entrance edge produces axial focusing of the beam, in this way increasing the axial acceptance of the spectrometer. The same slope weakens the radial focusing of the beam, which leads to a greater divergence of the beam in its radial section and to a larger area determining the quantity $R_m A_r$. The negative slope of the exit edge increases the radial convergence of the beam, thereby decreasing l_2 and the entire flight path of the particles from the source to the detector.

We note that a similar scheme for transformation of the beam is used in most contemporary charged-particle magnetic analyzers.[60,61]

A somewhat difference scheme of linear transformation of the particle beam is used in the magnetic analyzers of mass separators. In mass separators an initial beam parallel in the axial section is usually used, and therefore axial focusing at the entrance edge is not required. The necessary stigmatism of the beam at the detector of a mass separator is acheieved by axial focusing at the exit edge. The positive slope of the exit edge in this case increases the dispersion of the mass separator, a quantity which has special and independent significance in mass separators. The mass separators described in Refs. 62–64, for example, have been built according to such a scheme. In particular, the parameters of the magnetic analyzer of the mass separator from Ref. 62 are $R_0 = 70$ cm, $l_1 = 1.86$, $l_2 = 1.16$, $\varepsilon_1 = 0°$, $\varepsilon_2 = 45°$, $H_\alpha = Z_\zeta = 0$, $H_\eta = -1.15$, $H_{\Delta m/m_0} = 1.66$, $Z_\beta = 1.16$, and $D_{\rm rel} = 0.32$. This mass separator provides a resolution $R = 2300$ with an ion source with surface ionization and $R = 500$ with a gas-discharge source[65] with an angular divergence of the beam $\Delta\alpha_1 = 7°$. A mass separator with the ion-optics arrangement described above has been built in two versions: as a separator of the products of nuclear reactions in a heavy-ion cyclotron beam[65] and as a separator of radioactive nuclides, used outside the particle beam.[66]

Examples of the discussion of other types of beam transformation—burning glass and condenser in the radial and axial sections of the beam using a dipole magnet with a uniform field and inclined edges—can be found in Ref. 67, which describes the system for lowering, switching, and monochromatization of the ion beam of the U-400 cyclotron.

An important aspect of sector analyzers is the inclusion of the effect of the edge field of the dipole magnet on the motion of the particles, which appears already in the linear approximation. In particular, the edge field is responsible for the axial bending of the particle trajectories and it affects the actual value of the angle Φ, on which the linear transformation coefficients depend.

The effect of the edge field in the radial action on the particle was already taken into account in the first approximation by Nier[58] by introduction of so-called effective edges of the magnet instead of the real edges. Subsequently the effect of the edge field of a dipole magnet has been discussed many times.[21,68-78]

In discussing the axial motion in the linear approximation, account is sometimes taken of the small change of the focusing strength of the axial lens of the edge field due to the finite width of the region of falloff of the field at the edge. Following the work of Afanas'ev,[70] the changed focal length of the axial lens of the edge field should be equal to $F = R_0/[\mathrm{tg}\varepsilon - c(1+\sin^2\varepsilon)/6R_0\cos^3\varepsilon]$, where c is the width of the edge-field region in the linear approximation, instead of $F = R_0/\mathrm{tg}\varepsilon$ which follows from Refs. 13 and 27, which do not take into account the extent of the edge field. However, a discussion[78] shows that the focal length of the edge-field lens, which determines the location of the beam crossover in the axial section, has a more complicated expression:

$$F = R_0 \Big/ \left\{ \mathrm{tg}\,\varepsilon - \frac{c(1+\sin^2\varepsilon)}{R_0\cos^3\varepsilon}\left[0.20 - 0.49\left(\frac{\zeta}{c}\right)^2 - 0.32\left(\frac{\zeta}{c}\right)^4 - \ldots\right]\right\}. \quad (23)$$

It follows from Eq. (23) that for particles passing through the edge field far from the median plane, for example at $\zeta = 0.6c$, the focal length becomes equal to, and for larger ζ even less than, the focal length $F = R_0/\mathrm{tg}\varepsilon$ obtained without inclusion of the extent of the edge field. This fact must be kept in mind in an accurate calculation of the axial motion in a dipole magnet.

In our report[78] we made use of a new definition of the effective edge of the real field of a dipole magnet which, in contrast to the definition of Matsuda and Wollnik,[76] does not require introduction of any special assumptions regarding the nature of the variation of the field near the magnet edge, which are rather complicated in the case of a nonuniform main field. The position of the effective edge is given by the relation

$$\varphi_{\lim}(\rho)=\varphi_{II}(\rho)-\int_{\varphi_I(\rho)}^{\varphi_{II}(\rho)}\frac{B_z(\rho,\ \varphi,\ 0)}{(1+a_1\rho)B_0}d\varphi, \qquad (24)$$

where $\varphi_{\lim}(\rho)$ is the expression for the effective edge in a cylindrical coordinate system, given in Fig. 2; $B_z(\rho, \varphi, 0)$ is the real field of the dipole magnet at $z=0$; $\varphi_I(\rho)$ and $\varphi_{II}(\rho)$ are the initial and final angular coordinates of the region of the edge fields, determined by the conditions $\varphi_I(\rho)=0$ and $\varphi_{II}(\rho)=(1+a_1\rho)B_0$.

Magnetic Analyzers with a Radially Inhomogeneous Axially Symmetric Field. Another possibility of axial focusing of charged particles in analyzers appears as the result of use of a magnetic field which falls off with increase of the radial coordinate.

A feature of such a field is that the radial focusing of the particles turns out to be weakened in comparison with the radial focusing of a uniform field (the more so, the stronger is the axial focusing), and in view of the action of Eq. (22) this leads to an enhancement of the analyzing ability of the field. The relation between the appearance of an axially focusing component of the field and the weakening of the radially focusing component follows from the Maxwell equations curlB = 0 and divB = 0, which are valid for the gap between the poles provided that the current of the analyzed particles is always negligible in comparison with the current in the electromagnet coils.

Magnetic spectrometer with $\Phi=\pi\sqrt{2}$. The use of a radially inhomogeneous magnetic field to provide double focusing of particles was suggested by Svartholm and Siegbahn[79] on the basis of the well-known results of the theory of betatron oscillations.[80] It was shown that a magnetic field with an exponent $a_1=-0.5$ provides a beam transformation of the projector type simultaneously in the radial and axial sections for $\Phi=\pi\sqrt{2}$ $=254.56°(H_\alpha=Z_\beta=0)$. The principal linear transformation coefficients of a particle beam in such a spectrometer are $H_\eta=Z_\zeta=-1$ and $H_6=2/\omega^2=4$ (see Table II). The dispersion, which is a factor of two greater than that of a semicircular spectrometer, is explained by the factor of two larger areas of the radial section of the beam in the magnetic field of such an analyzer for a given $\Delta\alpha_1$ as a consequence of the $\sqrt{2}$ times larger extent and width of the beam. The relative dispersion of a spectrometer with $\Phi=\pi\sqrt{2}$ is 0.90, i.e., $\sqrt{2}$ times larger than in a semicircular analyzer.

The first β spectrometers constructed according to this principle were those described in Refs. 81–83. The first α spectrometer of this type was described in Ref. 84.

The mass separator, very similar to an analyzer with $\Phi=\pi\sqrt{2}$, was constructed by L. A. Artsimovich and his colleagues.[85] The mass separator utilized a magnetic field with a radial-focusing angle $\Phi=225°$, which is somewhat less than $\pi\sqrt{2}$ as the result of the necessity of placing within a single turn of the particle beam the ion source and collector for separated isotopes. The focusing angle is provided by a field exponent $a_1=(180°/225°)^2-1=-0.36$. This mass separator has a dispersion and a relative dispersion intermediate between the Calutron and a separator with $\Phi=\pi\sqrt{2}: H_{\Delta m/m_0}=1/\omega^2=1.5$ and $D_{\rm rel}=0.40$.

Sector magnetic analyzers with an inhomogeneous field. The conditions for transformation of a particle beam by a sector analyzer with an inhomogeneous magnetic field were first considered for certain special cases in Refs. 86–88 and in more general form by Sternheimer.[89]

Snyder *et al.*[90] described a spectrometer for protons, deuterons, and α particles with $\Phi=180°$, $\varepsilon_1=\varepsilon_2=0°$, and $a_1=-0.5$, for which with $l_1=1.15$ and $l_2=0.35$ a double transformation of the projector type is obtained. The spectrometer has $H_\eta=Z_\zeta=-0.8$, $H_6=3.60$, and $D_{\rm rel}=0.94$. We note that a magnetic field with exponent $a_1=-0.5$ has identical focusing strengths in the radial and axial sections ($\omega=\nu=\sqrt{0.5}$). Therefore in the absence of the action of the fringing field of the magnet edges ($\varepsilon_1=\varepsilon_2=0°$) for particles leaving a point source, the point of axial crossover will always coincide with the point of radial crossover. For the symmetric version $l_1=l_2=\sqrt{2}\,{\rm ctg}(\Phi/2\sqrt{2})$ and $H_6=2/\omega^2=4$.

A mass spectrometer of the sector type with a strongly inhomogeneous field $a_1=-8/9$ was constructed by Alekseevskii *et al.*[91] The parameters of the spectrometer were $\Phi=180°$; $l_1=l_2=5.2$; $\varepsilon_1=\varepsilon_2=0°$. As emphasized by the authors,[91] the dispersion of the spectrometer exceeds by a factor of nine the dispersion of a similar apparatus with a uniform field. We note that the relative dispersion of such a device, $D_{\rm rel}=0.775$, is only 2.4 times greater than the value for a semicircular analyzer.

The limiting variant of analyzers with a highly inhomogeneous field is a spectrometer with a field of the type $B_z\sim 1/r$, for which the field exponent is $a_1=-1$. In such a field there is no radial transformation of the particle trajectories at all, and therefore radial focusing of the beam is accomplished as the result of use of edges of the dipole magnet with negative slope. This gives the possibility of constructing magnetic analyzers with arbitrarily large particle-deflection angle without intermediate crossovers, i.e., with arbitrarily large positive area of the radial section of the beam in the magnetic field.

The particle-optics action of an analyzer with $a_1=-1$ is easily determined by means of the graphical construction described above, which is simplified in this case. The conditions for a particle-beam transformation, for example, of the projector type in the radial section have the simple form $l_i=-{\rm ctg}\varepsilon_i$. The axial construction is easily accomplished in view of the fact that $\nu=1$.

As is reported in Siegbahn's book,[4] Bender and Bainbridge were the first to propose use of such a magnetic field and to construct a β spectrometer with a field of this type. The properties of analyzers with $a_1=-1$ have been analyzed in detail in Refs. 92–94 as well as in other papers.

Malov *et al.*[95] described a mass separator with a field of the above type. Its parameters are $\Phi=114.6°$, $l_1=l_2=2$; the slope angles of the entrance and exit edges of the magnet, determined from the appropriate formula of Table I, are $\varepsilon_1=\varepsilon_2=-26.6°$; the mass dispersion coefficient is $H_{\Delta m/m_0}=l_2\Phi/2=-\Phi\,\mathrm{ctg}\,\varepsilon_2/2=2$; the relative dispersion $D_{\mathrm{rel}}=0.33$ is only slightly greater than in the Calutron.

Very similar to an axially symmetric field with an exponent $a_1=-1$ in its particle optics is a cylindrically symmetric field varying according to a law

$$B\sim B_0/d, \tag{25}$$

where d is the distance from a straight line passing through the source and detector. Such a field is produced, for example, by a straight current-carrying wire or a toroidal coil.

The possibility of use of such a field in a charged-particle spectrometer was first discussed by Richardson[96] for the case in which the source and detector are placed in the magnetic field. A sector variant of this field more convenient for use was discussed by Kofoed-Hansen, Lindhard, and Nielsen.[97] The formulas which determine the transformation of the particle trajectory in this field, which are expressed in terms of Bessel functions, are not very convenient. Therefore for an approximate description of the optical action of a spectrometer with a field of the type (25) we can use the methods and formulas given above, arbitrarily considering this spectrometer for the case $\Phi<\pi$ as a spectrometer with an axially symmetric field with a_1 close to -1. It follows from this discussion that the toroidal spectrometer has rather strong axial focusing of the beam and very weak radial focusing. Therefore to accomplish radial focusing in such spectrometers it is necessary to use negative slope angles of the entrance and exit edges.

Several β spectrometers with fields of the form (25) have been built. The first of these was a spectrometer with six gaps of the orange-segment type,[98] which provides a high transmission of the particle beam (15–20% of 4π). An iron-free version of such a spectrometer with use of a toroidal coil has been described by Vladimirskiĭ.[99]

An additional example of the need for use of gradient and edge focusing in a dipole magnet is the telescope transformation of a particle beam simultaneously in the two sections. To search for the parameters and to calculate the main coefficients of a dipole magnet which will provide the same telescopic action in the radial and axial sections of the beam, it is convenient to use the construction given above (Fig. 11).

A drawing of the construction is given for a dipole magnet with a field $a_1=-0.5$ which provides equal focusing action in the two beam sections $\omega=\nu=\sqrt{0.5}$. The conventional constructions for radial and axial transformations which are special cases of the general con-

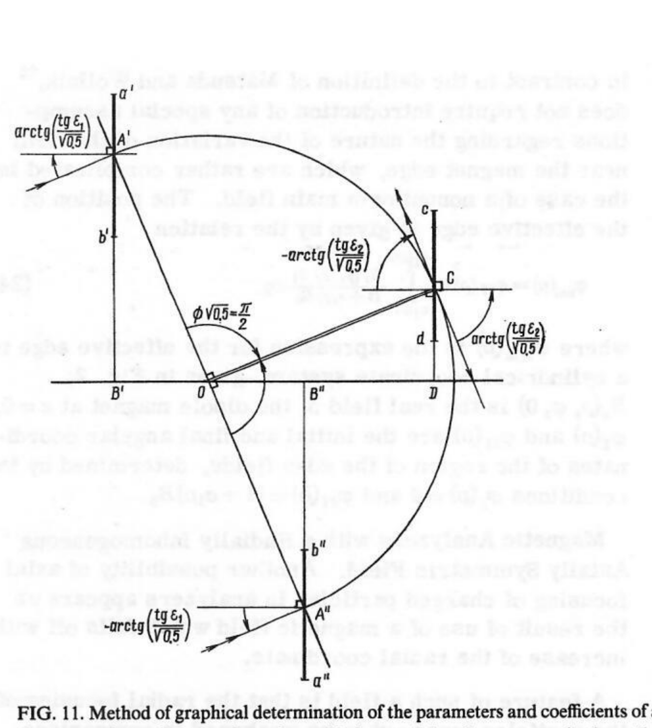

FIG. 11. Method of graphical determination of the parameters and coefficients of a linear transformation of a charged-particle beam of the telescope type in the radial and axial sections of a magnetic dipole element.

structions (see Fig. 5), are shown in the upper and lower halves of the figure. The edges of the dipole magnet in these constructions are given by the lines $a'b'$, $a''b''$, and cd, which in accordance with the conditions of a telescope transformation are parallel, $\Phi\sqrt{0.5}=\pi/2$, i.e., $\Phi=127.5°$. The principal linear transformation coefficients of the particle beam are $H_\eta=1/A_\alpha=c_2/c_1=CD/A'B'$ and $A_\delta=(s_1+s_2)/\omega c_2=(B'O+OD)/\sqrt{0.5}CD$ in the chromatic variant of the transformation, and $H_\eta=1/A_\alpha=-s_2/s_1=-CD/A''B''$ and $A_\delta=(c_1-c_2)/\omega s_2=(B''O-OD)/\sqrt{0.5}A''B''$ in the quasichromatic variant. For example, for $\varepsilon_2=\pm 60°$, which corresponds respectively to $\varepsilon_1=\pm 16.1°$, we have $H_\eta=1/A_\alpha=\pm 0.41$, $Z_\zeta=1/B_\beta=\mp 0.41$, i.e., the dipole magnet compresses the beam by about 2.5 times. The angular dispersion coefficient is $A_\delta=4.87$ in the chromatic variant of the beam transformation ($\varepsilon_1=16.1°$ and $\varepsilon_2=60°$) and $A_\delta=-0.84$ in the quasichromatic variant ($\varepsilon_1=-16.1°$ and $\varepsilon_2=-60°$). For a backward motion of the particles the magnetic dipole element shown in Fig. 11 increases the transverse size of the beam.

Magnetic spectrometers with $\Phi>\pi\sqrt{2}$ and double focusing of the particle beam. The first person to notice that double focusing of a particle beam can be achieved in an inhomogeneous magnetic field with a_1 values not equal to -0.5 was Lee-Whiting.[100] He obtained the conditions of double focusing of a particle beam of the projector type in the radial and axial sections $H_\alpha=Z_\beta=0$ and also the principal transformation coefficients. Daniel[101] discussed the problem of double focusing for the case of projector transformation in the radial section and condenser transformation $B_\beta=0$ in the axial section.

In the general case the conditions of double focusing in an inhomogeneous magnetic field for the chromatic variant have the form $\omega\Phi=\pi$, $\nu\Phi=n\pi$. Here n is a positive integer if $Z_\beta=0$, and $n-1$ is equal to the num-

TABLE IV. Parameters and characteristics of magnetic analyzers with $\phi \geqslant \sqrt{2}$ and double focusing of the particle beam.

n	1	3/2	2	5/2	3	7/2	4
a_1	—1/2	—9/13	—4/5	—25/29	—9/10	—49/53	—16/17
Φ/π	$\sqrt{2}$	$\sqrt{13}/2$	$\sqrt{5}$	$\sqrt{29}/2$	$\sqrt{10}$	$\sqrt{53}/2$	$\sqrt{17}$
H_δ	4	6.5	10	14.5	20	26.5	34
D_{rel}	0.89	1.15	1.42	1.71	2.02	2.32	2.63
Ω	1.00	0.92	0.80	0.69	0.60	0.53	0.47
S_b/S_{an}	0.63	0.63	0.71	0.86	1.01	1.16	1.31

ber of crossings of the median plane by the axial trajectory of the particle. If $B_\beta = 0$, then n is a positive half-integer. It follows from this that $\Phi = \pi\sqrt{1+n^2}$; $a_1 = -n^2/(1+n^2)$; $H_\delta = 2/\omega^2 = 2(1+n^2)$; $H_\eta = -1$, $Z_\zeta = (-1)^n$ for integral n and $Z_\zeta = 0$ for half-integral n.

The values of the principal parameters and characteristics of magnetic spectrometers with $\Phi \geqslant \pi\sqrt{2}$ and double focusing of the beam are given in Table IV. The first five lines of the table require no explanation. In the sixth line we have given relative values of the solid angles Ω of the spectrometers for a fixed width of the magnetic guide field and magnet gap height, obtained from the formula $\Omega \sim \Delta\alpha_1 \Delta\beta_1 \sim \omega\nu \sim n/(1+n^2)$. It can be seen that with increase of the radial falloff of the magnetic field the solid angle of a spectrometer with double focusing decreases as the result of the more rapid weakening of the radial focusing in comparison with the strengthening of the axial focusing. In the last line we have given the ratio of the area of the radial section of the beam S_b for a given $\Delta\alpha_1$ to the area of the working part of the ring-shaped magnetic guide field S_{an}. This ratio is constant as long as $\Phi \leqslant 360°$, and beyond that it increases, reflecting the measure of repeated use of the magnetic guide field of the analyzer.

One of the examples of realization of analyzers of this type is the β spectrometer with $\Phi = (\pi/2)\sqrt{13} = 324.5°$.[102] The necessary profile of magnetic field is created by ten current-carrying coils without use of an iron yoke.

Baranov *et al.*[103] have described a β spectrometer with $\Phi = \pi\sqrt{10} = 507°$. For $R_0 = 40$ cm the spectrometer provides a dispersion of 8 cm for 1% change of momentum. The spectrometer gives an eight-fold gain in usable aperture at the target for a resolution equal to other spectrometers of similar size of the $\pi\sqrt{2}$ type.

In spectrometers with $\Phi > 2\pi$, to avoid particle hitting the detector in the initial stage of the motion, one uses as the axial trajectory of the beam an orbit with an initial deflection either in α_1 (Ref. 103) or in η_1 (Ref. 101), or screening or diaphragming of the beam in ζ_1.[104]

An important deficiency of the spectrometers discussed above is the very high requirements on the accuracy of the magnetic field topography, due to the rapid dependence of the principal linear characteristics of the spectrometer on a_1 for a value of this parameter close to −1. The latter circumstance produces serious technical difficulties in construction of such magnetic analyzers, especially analyzers with iron-core electromagnets, in which hysteresis and nonlinearity in the $B-H$ dependence occur.

These difficulties are reduced to a significant degree in magnetic analyzers with separated poles and a uniform field, in which large deflection angles Φ and large dispersion are also possible.

Magnetic Analyzers with Uniform Field and Separated Poles. A charged-particle magnetic spectrograph with separated poles was described by Spencer and Enge.[105] The spectrometer pole consists of two parts surrounded by a single exciting coil. In each part of the spectrograph magnet the field is uniform, and in the gap between the parts the field is approximately 10% of the field in the main gaps. The parameters of the spectrograph, taken for the principal trajectory, are as follows: $\Phi = 40°$, $\varepsilon_1 = 37°$, $\varepsilon_2 = 14°$ for the first part and $\Phi = 74°$, $\varepsilon_1 = 35°$, $\varepsilon_2 = -19$: for the second part. The maximum radius of curvature of the partical trajectories is 90 cm.

The existence of four edges instead of the usually used two edges in the path of the particles from the source to the detector gives additional possibilities for improvement of first- and second-order characteristics. The slopes and radii of curvature of the edges are chosen in such a way as to provide stigmatic focusing of the particle beam over the entire detecting plane of the spectrograph.

Various designs for magnetic analyzers with uniform field and separated poles have been discussed.[39,106] It has been shown that such analyzers in their dispersion and relative dispersion are essentially not inferior to, and sometimes better than, similar analyzers with highly inhomogeneous fields, differing advantageously from them in the simplicity of shaping of the uniform field. For example, an analyzer with separated poles which has a deflection angle $\Phi = 360°$ produced by six 60-degree sector magnets located along a circle with $\varepsilon_1 = \varepsilon_2 = 22.8°$ has $H_\delta = 7.5$ and $D_{rel} = 1.1$, which can be compared with $H_\delta = 8$ and $D_{rel} = 1.27$—the characteristics of a spectrometer with an axially symmetric inhomogeneous field with $\Phi = 360°$ for $a_1 = -0.75$.

Combined Magnetic Analyzers. Combination of fields of different types in a single analyzer frequently extends its possibilities and improves its particle-optics characteristics.

One of the examples of use of two dipole fields of different types (uniform field and field with $a_1 = -1$) is the gas-filled fission-fragment separator.[107] The possibilities of combination of such fields was first discussed by Egidy.[108]

The total deflection angle of the particle beam in the separator of Ref. 107 is $\Phi = 312°$, of which 66° at the beginning and 66° at the end of the sector are occupied by a uniform field and 180° in the central portion by a nonuniform field with exponent $a_1 = -1$. The separator has $l_1 = l_2 = 0.92$ and edge slope angles $\varepsilon_1 = \varepsilon_2 = 46°$, which can be changed by ±8° by use of movable end portions of the magnet poles. The separator provides symmetric ($H_\eta = Z_\zeta = -1$) double focusing of the projector type ($H_\alpha = Z_\beta = 0$) and has $H_\delta = 11$. The relative dispersion of the separator is rather large, $D_{rel} = 1.51$. As noted by Lawin *et al.*,[107] the choice of this combination of fields for the separator was due to the necessity of deflecting the beam of fission fragments by an angle 60° because of conditions on the location of the apparatus. Of the two possible versions of the magnetic analyzer, with $\Phi = 60°$ and $\Phi = 300°$, the latter was chosen, as having approximately four times better particle-optics characteristics.

A feature of the separator described by Lawin *et al.*[107] is the use of focusing of the fission fragments on the basis of their velocity and ionic charge as a result of their charge exchange in collisions with the gas filling the separator chamber to a low pressure (~500 Pa).

A combination of uniform and nonuniform magnetic fields of this type was used in an electron spectrometer for study of the (n, e) reaction.[109] A combination of two uniform fields of different strengths with one intermediate edge, intended to improve the spectrographic characteristics, was used in the charged-particle spectrograph of Ref. 110.

Another example of the combination of magnetic fields is the location in front of a dipole magnet of a quadrupole lens whose converging plane coincides with the median plane of the magnet. This leads to increase of the area of the radial section of the particle beam and of the quantity $R_m A_r$. For a given source emittance the analyzer resolution is increased, or the same resolution is obtained for a larger radial extent of the source. At the same time the quadrupole lens focuses particles in the axial section of the dipole magnet, increasing the analyzer aperture.

This combination of a quadrupole lens and a dipole magnet was first described by Enge.[111]

The improvement of the parameter $R_m A_r$ for the quadrupole-dipole combination discussed appears in terms of a decrease of the coefficient H_η.

In fact, the total coefficient of the system is $H_{\eta(Q+D)} = H_{\eta Q} H_{\eta D}$, where the symbols Q and D denote the assignment of the coefficients. According to the formulas given above $H_{\eta Q} = c_1/c_2$ for $k|l_1| < 1$ and $H_{\eta Q} = -s_1/s_2$ for $k|l_1| > 1$, where $s_i = \mathrm{sh} kX_i$; $c_i = \mathrm{ch} kX_i$, $i = 1, 2$; $X_2 = L_0 - X_1$. For a particle source located in front of a quadrupole defocusing magnet, $X_1 < 0$, and therefore $|X_1| < X_2$. Hence it follows that $c_1/c_2 < 1$, $s_1/s_2 < 1$, and $H_{\eta(Q+D)} < H_{\eta D}$. For the analyzer described in Ref. 11, $H_{\eta(Q+D)} = -0.56$.

Use of a doublet of defocusing and focusing quadrupole lenses in front of a dipole magnet has also been discussed.[112] This doublet extends the possibilities for optimal guiding of a particle beam through a narrow pole gap, but in this case the defocusing action of the first quadrupole magnet is reduced and consequently the increase of the radial section of the beam and the resulting improvement of the analyzer characteristics are reduced.

Different results can be obtained if in front of the dipole magnet one uses two quadrupole lenses with defocusing planes which coincide with the median plane of the dipole magnet. As is well known, the combined action of two successively placed lenses is determined by the formula $1/f = 1/f_1 + 1/f_2 - d/f_1 f_2$, where f is the focal length of the system of two lenses, f_1 and f_2 are the focal lengths of the first and second lenses, and d is the distance between the lenses. According to this formula the focal length of two lenses in the radial section of the analyzer for $f_{r1} < 0$ and $f_{r2} < 0$ is always negative, i.e., the action of two quadrupole magnets in this plane is always defocusing. The action of two quadrupole magnets in the axial direction may be focusing ($f_z > 0$) if $f_{z1} + f_{z2} > d (f_{z1} > 0,\ f_{z2} > 0)$, neutral ($f_z = \infty$) if $f_{z1} + f_{z2} = d$, and defocusing ($f_z < 0$) if $f_{z1} + f_{z2} < d$, with a wide variation of optical strength. This combination of quadrupole magnets is proposed for use in front of dipole magnets in a system for monochromatization of a heavy-ion cyclotron beam, to increase the resolution of the analyzer.[67]

Analyzers constructed with the schemes QDD[113] and $QDDD$[114] are also well known. Such analyzers are essentially a combination of the two improvements discussed above: a defocusing quadrupole lens in front of a dipole magnet, and separated poles of the dipole magnet. The relative dispersion of such installations is rather high (~2 for the principal particle).

A further variant of the combined use of lenses and a dipole magnet is the prism spectrometer proposed by Kel'man and Kaminskii.[115] The spectrometer consists of the following successively placed elements: a focusing lens (electrostatic or magnetic) with a transformation of the condenser type, a dipole magnet of the telescope type, and a lens of the burning-glass type. The dipole magnet has a uniform field and straight entrance and exit edges parallel to each other. In this case, as we can easily see, a beam which is parallel in its radial section remains parallel after passing through the dipole magnet for any value of Φ, since the conditions for a transformation of the telescope type are satisfied: $\Omega = \psi_1 + \psi_2 - \varepsilon_1 - \varepsilon_2 = 0$, and consequently $\mathrm{tg}\psi_i = \mathrm{tg}\varepsilon_i$, $i = 1, 2$.

A feature of the prism spectrometer is the absence in the dipole magnet of angular aberrations of second and higher orders which depend on the radial width of the beam ($A_{\eta\eta} = At_{\eta\eta} = \ldots = 0$ in view of the complete identity of the corresponding trajectories in a uniform field bounded by plane parallel edges), and the absence of aberrations of second order in the focusing lenses, which have planar symmetry (symmetry of revolution). This circumstance provides the possibility of using a relatively wide particle beam in the dipole magnet field and consequently of having a relatively large value of Σ/R_0, which is realized without additional measures to avoid aberrations.

In fact, according to the general scheme of construction of the prism spectrometer $A_{\alpha 1} = A_{\eta 2} = H_{\eta 3} = 0$, where the second subscript to a symbol indicates the order number of the element in the prism spectrometer, and with accuracy to the principal aberration terms of second order we have $\Delta\eta_{23} = H_{\alpha 3} \Delta\alpha_{13} = H_{\alpha 3} \Delta\alpha_{22}$

$$=H_{\alpha 3}(A_{\alpha 2}\Delta\alpha_{12}+A_{\alpha\alpha 2}\Delta\alpha_{12}^2)=H_{\alpha 3}(A_{\alpha 2}\Delta\alpha_{21}+A_{\alpha\alpha 2}\Delta\alpha_{21}^2)$$
$$=H_{\alpha 3}A_{\alpha 2}A_{\eta 1}\Delta\eta_{11}+H_{\alpha 3}A_{\alpha\alpha 2}A_{\eta 1}^2\Delta\eta_{11}^2.$$

It is evident from the expression obtained that the width of the beam crossover at the detector of a prism spectrometer in which transformations of the projector type are used entirely does not depend on $\Delta\alpha_{11}$ within the accuracy adopted in the discussion. This gives the possibility of using a particle beam with a rather large initial angular spread in the radial section and of having a relatively large value of Σ/R_0.

A discussion of different forms of prism β spectrometers and mass spectrometers can be found in the review article by Peregud.[116]

We note that the first spectrometer constructed by analogy with an optical prism spectrograph, described by Klemperer,[117] was constructed in the quasiachromatic version with a beam crossover in the magnetic field region; this spectrometer had a very low dispersion.

Among the combined magnetic analyzers we must also class spectrometers in which magnetic and electric fields are used simultaneously for focusing and analysis. Such fields provide focusing in velocity, in addition to spatial focusing. This was first shown by Bartky and Dempster.[118] A more detailed discussion of the particle-optics action of combined magnetic and electric fields is given in Refs. 11 and 119–121.

The particle-optics action of combined magnetic and electric fields is similar to the action of a magnetic field. The linear equation of a particle trajectory in a magnetic field $B_z=B_0(1+a_1\rho)$ and an electric field $E_r=E_0(1+b_1\rho)$ crossed at right angles and acting in opposite directions on a charged particle has a form similar to Eq. (2):

$$\rho''+R_0[(1+a_1)/R_B+(3+b_1)/R_E-R_0/R_BR_E]\rho-\Delta=0. \quad (26)$$

Here $R_B=m_0v_0c/e_0B_0$ and $R_E=-m_0v_0^2/e_0E_0$ are the radii of curvature of the principal particle, (m_0, v_0, e_0) in a magnetic field of strength B_0 and in an electric field of strength E_0; $\Delta=\Delta m/m_0+\Delta v/v_0-\Delta e/e_0+(R_0/R_E)\Delta v/v_0$, $1/R_0=1/R_B+1/R_E$. The boundary conditions for the particular solutions of Eq. (26) used in the method described above to determine the linear transformation coefficients of a particle beam are identical to the conditions (3) and (5). It is therefore easy to see that the requirements for accomplishing the four types of beam transformation described above and the linear transformation coefficients themselves in analyzers with crossed magnetic and electric fields should be expressed by the formulas of Table I in which instead of $\omega=\sqrt{1+\alpha_1}$ and δ we use respectively $\Omega=\{R_0[(1+a_1)/R_B+(3+b_1)/R_E-R_0/R_BR_E]\}^{1/2}$ and $\Delta=\delta+(R_0/R_E)\Delta v/v_0$. For such analyzers Eq. (22) remains valid with the condition that the resolution of the analyzer is determined in the general case with respect to the parameter Δ:

$$R_m(\Delta)A_r=\Sigma_{BE}/R_0, \quad (27)$$

where $R_m(\Delta)$ is the maximum achievable resolution in Δ; Σ_{BE} is the area of the radial section of the particle beam in the crossed magnetic and electric fields of the analyzer; R_0 is the radius of curvature of the optical axis of the analyzer, determined by the relation $1/R_0=1/R_B+1/R_E$.

For mass spectrometers and mass separators where ions are used which have a constant kinetic energy $mv^2/2=\text{const}$ and consequently $\Delta v/v_0=-\Delta m/2m_0$, $\Delta=(\Delta m/2m_0)R_0/R_B$, $H_{\Delta m/m_0}=H_\Delta\Delta/(\Delta m/m_0)=(R_0/2R_B)H_\Delta$, Eq. (27) takes the form

$$R_m(\Delta m/m_0)A_r=\Sigma_{BE}/2R_B, \quad (28)$$

where $R_m(\Delta m/m_0)$ is the maximum achievable resolution in mass. As follows from Eq. (28), on addition to a magnetic field of an electric field which in itself does not produce a dispersion in mass of ions with constant energy $[H_{\Delta m/m_0}=H_\Delta(1-R_0/R_E)=0$, since $R_0/R_E=1]$, the parameter which determines the maximum resolution of the analyzer with crossed magnetic and electric fields remains the radius of curvature of the principal trajectory in the magnetic field. However, this does not affect the resolution of the analyzer. On the other hand, this dependence shows that for given dimensions of an analyzer with combined fields its resolution can be substantially greater than the resolution of an analyzer of similar size with a magnetic field if $R_B<R_0$, which requires that $R_E<0$, i. e., an electric field which deflects the particle in the direction opposite to the magnetic field.

A mass separator with crossed magnetic and electric fields has been described, for example, by Wahlin.[122] This separator uses a special case of crossed fields with $R_0=\infty$, i. e., with $E_0=-(v_0/c)B_0$, which is known as a velocity selector of the Wien type.[123] Introduction of an electric field in this case increases the relative area of the radial section of the beam and the parameter $R_m(\Delta m/m_0)A_r$.

Crossed and also successively located magnetic and electric fields provide the possibility of focusing particles in velocity, which avoids the dependence of the width of the crossover at the detector on the spread of the velocities and in this way improves the resolution. A particular case of such focusing of particles in velocity in the vicinity of v_0 occurs in crossed fields for $R_0/R_E=-1$, i. e., for $E_0=-0.5(v_0/c)B_0$, since in this case $H_{\Delta v/v_0}=H_\Delta(1+R_0/R_E)$. The dispersion and resolution of a mass spectrometer with such a field are twice the values in a mass spectrometer with a magnetic field and the same radial section of the beam: $H_{\Delta m/m_0}=H_\Delta R_0/2R_B=H_\Delta$ instead of $H_{\Delta m/m_0}=H_\delta/2$.

For mass spectrometers in which a combination of successively placed dipole magnet and electrostatic deflector is used, for the resolution in velocity and the radial acceptance we have the relation

$$R_m(\Delta v/v_0)A_r=\Sigma_B/R_B+2\Sigma_E/R_E. \quad (29)$$

It follows from this relation that zero dispersion of a combined analyzer in velocity is obtained for the condition that the area of the radial section of the particle beam in the magnetic field, divided by R_B, is equal in absolute value and opposite in sign to twice the area of the similar beam cross section in the electric field, divided by R_E.

Equation (29) forms the basis for the particle-optics schemes of all analyzers with focusing of particles in velocity: mass spectrometers, fission-fragment separators, and nuclear reaction product separators. In particular, for a mass spectrometer with focusing in velocity[124] which is a variant of the Dempster mass spectrometer,[25] we have $\Phi_E = 90°$, $b_1 = -1$ (cylindrical condenser), $\Omega = \sqrt{2}$, $R_E = 20$ cm, $l_{1E} = 0.15$, $l_{2E} = 0.609$, $\Phi_B = 180°$, $a_1 = 0$, $\omega = 1$, $R_B = 23$ cm, $l_{1B} = l_{2B} = 0$, and the deflection of the particle beam in the electrostatic deflector and the dipole magnet are accomplished in the same direction. From this we obtain

$$\Sigma_E/R_E = \Delta\alpha_{1E} R_E \int_0^{\Phi_E} \left(l_{1E} \cos\Omega\varphi + \frac{1}{\Omega}\sin\Omega\varphi \right) d\varphi = 17.5\Delta\alpha_{1E}\ \text{cm};$$

$$\Sigma_B/R_B = \Delta\alpha_{1E} A_{\alpha E} R_B \int_0^{\Phi_B} \sin\omega\varphi\, d\varphi = -35.0\Delta\alpha_{1E}\ \text{cm},$$

where $A_{\alpha E} = -l_{1E}\cos\Omega\psi_{2E}/l_{2E}\cos\Omega\psi_{1E} = -0.76$. Finally we obtain $2\Sigma_E/R_E + \Sigma_B/R_B = 0$.

Another variant of the combination of magnetic and electric fields is known, namely that with a mutually perpendicular arrangement of the median planes of these fields. This combination of fields provides stigmatic focusing of a beam of particles with separated mass, velocity, and charge and dispersion of the particles in momentum and energy in two mutually perpendicular directions. The geometrical focus of the points of arrival of particles with fixed mass and ionic charge at the analyzer detector is a parabola. This also determines the name of the analyzers in which such fields are used. Parabolic analyzers do not provide focusing of particles in velocity in the literal sense of the word, but they provide the possibility of analyzing particles in mass and ionic charge with a rather large spread in their velocity, and in addition of determining these velocities.

A parabolic mass spectrometer intended for separation and study of fragments from fission of heavy nuclei induced by neutrons has been described by Moll.[126] It differs from the classical version of the parabolic spectrometer of Thomson,[127] in that the magnetic and electric fields are separated from each other, and further in that it provides stigmatic focusing of the beam. The size of the mass spectrometer can be judged from its parameters: $R_B = 4$ m, $\Phi_B = 45°$, $R_E = 5.6$ m, $\Phi_E = 35.35°$, total length of optical axis 23.1 m, and dispersion in mass and energy respectively 3.24 and 6.52 cm for 1% variation of the quantities analyzed.

CONCLUSION

Use of the parameters of relative dispersion D_{rel} and relative area of the radial section of the particle beam Σ/R_0 has made possible systematization and comparison of magnetic analyzers with a transverse field with very different parameters and characteristics. It has been shown that as magnetic analyzers have been improved from the simplest semicircular spectrometer with a uniform magnetic field to spectrometers with complicated combined fields, the value of the relative dispersion has increased, reflecting in this way the improvement of the quality of the analyzers. In addition we have illustrated the direct dependence of the principal characteristics of a magnetic analyzer—its resolution and radial acceptance—on the relative area of the radial section of the particle beam in the magnetic field of the analyzer. We have also analyzed in some detail just which spectrometer elements—magnetic field gradient, inclined edges of the dipole magnet, additional quadrupole or electric fields—provide the increase of this area and lead to improvement of the magnetic analyzer.

We have described a new approach to discussion of the linear transformations of charged-particle beams in dipole and quadrupole magnets which permits all linear transformation coefficients of a particle beam to be represented in a unified and simplified form.

We have given graphical procedures for determination of the principal results of transformation of a charged-particle beam and of the linear transformation coefficients. This approach and the graphical procedures have been extended to analyzers containing both magnetic and electric fields.

The simplified formulas and procedures permit one to determine relatively simply without cumbersome and laborious calculations the principal results of the particle-optics action of an analyzer which includes dipole and quadrupole magnets and also electrostatic deflectors, and also to evaluate the limiting resolution. Thus, already in the linear approximation the possibility appears of choosing an analyzer arrangement in which further refinement will permit the necessary particle-optics characteristics to be obtained and, on the other hand, to see in what cases the arrangement chosen will not provide the required results.

In this review on the basis of a unified concept we have considered all principal versions of charged-particle magnetic analyzers used in nuclear physics. The features of certain types of magnetic spectrometers (such as spectrometers with compensation of the kinetic effects of nuclear reactions, charged-particle energy-loss spectrometers, and spectrometers employing particle time of flight) have not been discussed individually in the review, since their basic schemes follow the types of magnetic analyzers which have been discussed.

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