Space-time description of multiparticle production in nuclear matter and the structure of hadrons

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The results of a series of investigations which have developed new ideas about multiparticle production of hadrons in hadron–nucleus collisions at high energies are presented systematically. This dominant channel for strong interactions is described in space-time terms. The proposed phenomenological model is in fairly good qualitative and quantitative agreement with the greater part of the experimental data.

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INTRODUCTION

The main result of the intensive investigation of hadron–nucleus collisions during the last decade has been the recognition of the important part played by space-time factors in the dynamics of strong-interaction processes. This is a qualitative result. A necessary condition for obtaining quantitative information on the development of a process in space and time is the construction of a model that reflects as fully as possible the real situation and reduces the entire body of data on multiparticle production processes to a minimal number of assumptions and parameters.

The numerous attempts to solve this problem have led to the existence at the present time of an abundance of models of hadron–nucleus interactions. It is, however, readily seen that the multitude of models by no means corresponds to a multitude of fundamentally different physical approaches. In reality, the number of such approaches is very limited. As a rule, a new model is an attempt to eliminate some of the difficulties within one or two of the most popular schemes, which are of cascade or of "tube" type.

In its simplest variant, the cascade model regards the hadron–nucleus interaction as a collection of independent hadron–nucleon collisions. It is assumed that in each such collision particles are produced instantaneously with characteristics identical to those of the hadrons. Such an assumption proved unsuitable for describing the process in nuclear matter; for during the time in which a newly created particle becomes isolated, the intermediate system can traverse a distance of the order of or even greater than the nuclear diameter. This was understood comparatively early,[1] and effectively it entirely rules out the cascade mechanism of production at high energies. Despite this obvious logical contradiction, there exist numerous attempts to preserve elements of the cascade scheme as a basis of the hadron–nucleus interaction mechanism. They are found, in particular, in models with a leading-particle cascade in the framework of approaches which describe multiple scattering (see, for example, Ref. 2). The necessary degree of suppression of the cascade process is achieved in these models by the completely unjustified neglect of the interactions of all the secondary particles in the nucleus except for the most energetic, leading particle. However, even at this price these models cannot be reconciled with the experiments in a number of characteristic features. (This question will be briefly discussed in Sec. 1.)

Some attempts at a partial correction of this situation have been made by transferring the cascade scheme to the parton level.[3] However, at this level too the fundamental difficulties inherent in the cascade mechanism remain, being merely modified, and comparisons already made indicate a disagreement between the results of calculations in the scheme proposed in Ref. 3 and the experiments (this is discussed in more detail in Sec. 2).

The need for a correct treatment of the space-time factors in the production process also follows from the inadequacy of the idea which has been opposed to the cascade mechanism, according to which the process is stretched over time to such an extent that the incident hadron interacts with a tube of nuclear matter as a whole. The best known examples of these approaches are the hydrodynamic model[4] and the coherent-tube model.[5] (The possibility of coherent interaction of the incident particle with the nucleons of a nucleus was apparently pointed out for the first time in the first citation in Ref. 5. In the remaining papers in Ref. 5, this idea is realized in models which are too crude.)

The internal inconsistency of the hydrodynamic scheme in its most widely used and developed form[4] was convincingly demonstrated in Ref. 6 (see also Ref. 7 and the literature quoted there), namely, it has not yet proved possible to overcome the conflict between the formulation of the initial phase of the process and the quantum uncertainty principle. As is correctly noted in the review of Ref. 8, the cascade and hydrodynamic models are opposite extremes, neither of which is realized in nature.

The coherent-tube model also faces serious difficulties in interpreting the experiments that are most sensitive to the degree of collectivization of the hadron-nucleus interactions. It was shown in Ref. 9 that the predictions of this model are in strong disagreement with the experiments on cumulative pion production, the production of $\mu^{\pm}$ pairs, and the production of particles with large transverse momenta on nuclei. The disagreement between the predictions of this model and the data on multiparticle production in hadron–nucleus interactions was also pointed out in Ref. 10.

Thus, the general situation in this field of hadron physics indicates that other approaches must be sought. One such approach is presented in this paper, which is devoted to a systematic exposition of a definite description of multiparticle production in hadron–nucleus interactions. The approach was developed in a series of investigations made during the last six years[11–18] in the framework of a space-time scheme which takes into account the finite duration of the interval within which the process develops. Alternative approaches are discussed only to contrast them with the present scheme.

In contrast with the previous models, the cascade and hydrodynamic models, which have been discussed in numerous reviews, the present approach has not yet been fully presented.

Among the schemes at present in existence, it so far explains the greatest number of characteristics associated with the hadron–nucleus interaction at high energies. Although originally developed phenomenologically, the model has been found to bear an intimate relation to modern ideas about the structure of hadrons and the nature of strong interactions, i.e., the quark-gluon model of the hadron and quantum chromodynamics. Moreover, it can not only confirm a number of important predictions of these ideas but can also become a means for extracting additional useful information about hadron structure and the dynamics of hadronic processes. In our view, these must be the main aims in a study of hadron–nucleus interactions.

We shall consider multiparticle production, the principal channel of hadron–nucleus interactions. We shall not consider numerous interesting questions associated with processes of cumulative type. A space-time approach to their description which is a limiting case of the model presented here was developed in Ref. 19. This limiting case and its relation to the main channel are briefly discussed in the Appendix.

1. PHENOMENOLOGICAL FORMULATION OF THE MODEL

It is helpful to begin the exposition of the model with the phenomenological variant.[11–16] Above all, this formulation is of independent interest.

Moreover, the phenomenological variant is not based on a special picture of hadron structure and can, in the framework of its axioms, exist independently of it. In practice, this means that a possible future radical revision of currently existing notions about hadron structure need not necessarily result in a strong modification of this model of hadron–nucleus interactions.

Furthermore, in the case of success this model can be regarded as embodying certain additional phenomenologically formulated conditions that follow from a large body of experimental data on multiparticle production in hadron–nucleus interactions, which must also not be contradicted by predictions which follow from models of hadron structure.

Basic Propositions. The model is based on a number of assumptions, among which there are two fundamental propositions.

The first is that the hadronic system formed in the collision of the initial hadron with one of the nucleons in the nucleus and responsible for initiating the overwhelmning number of produced particles ("pionization" region of the spectrum) behaves during the first stage of its development in its interaction with the remaining nucleons in the nucleus as a single object, a cluster, in which the separation of particles is not completed.

This assumption is fairly well justified, because the first stage is associated with the localization of strongly excited hadronic matter in a small volume (the characteristic dimension is that of the range of the strong interaction or the diameter of a nucleon). The intensive exchange processes due to the high energy density do not permit the separation in such a system of a definite number of real particles. This argument was in fact used long ago in various models of multiparticle production of hadrons in the elementary event (see, for example, Ref. 20).

Of course, after a certain interval of time, having traversed a definite distance, during which it has expanded and decayed into real particles, the cluster ceases to be a single entity. In the energy range we are considering, the cluster decays, as a rule, outside the nucleus.

Clear evidence that the produced particles are not individualized within the nucleus is the absence of a dependence of the mean transverse momentum of the produced particles on the size of the target nucleus (see, for example, Ref. 21), since multiple interactions would lead to a growth in $\langle p_\perp \rangle$.

The second proposition of the model is reflected in the assertion that the component which appears at the end of the process in the form of a small number of the most energetic particles ("leading" group) interacts much more weakly with the matter than the cluster or a real hadron. In principle, the existence of such a possibility could be inferred purely logically without recourse to definite models of hadron structure as long as the composite nature of the hadron is recognized as a fact. Indeed, the first inelastic interaction of the incident nucleon (or pion) with a nuclear nucleon already plays a selection role in dividing the strongly interacting components (which leads to formation of the cluster) from the comparatively weakly interacting components of the colliding hadrons. The leading system must on the average be the final product of the weakly interacting component (which therefore loses a small fraction of its initial momentum). But then such a system must continue to interact weakly with the nucleons in subsequent collisions within a space-time interval of nuclear order after its production. After a sufficient time has elapsed, the leading system is realized, as a result of internal rearrangement, in the form of a "normal" hadron (or hadrons), which is now capable of "normal" interaction.

The restoration of the hadronic properties requires a time interval not shorter than

$$\tau_0 \approx \langle r_h \rangle / c \approx (2-3) \cdot 10^{-24}\ \text{sec}, \tag{1}$$

where $\langle r_h \rangle$ is the rms radius of the hadron. The mean path traversed by the leading particle before restoration of the normal properties is

$$l \approx \gamma_L \tau_0 c, \tag{2}$$

where γ_L is its Lorentz factor (L stands for leading). Since the mean inelasticity in a pp interaction is ≈0.5, and the leading system can be an excited system with mass $M \approx 2$ GeV,

$$\gamma_L \approx 0.5 E^{\text{in}}/M \approx E^{\text{in}}/4, \tag{3}$$

i.e.,

$$l \gtrsim 0.2 E^{\text{in}}\ \text{F}. \tag{4}$$

Thus, if $E^{\text{in}} \approx 10$ GeV, l exceeds the internucleon distance in nuclei, and if $E^{\text{in}} \approx 100$ GeV it exceeds the diameter of heavy nuclei. Consequently, the idea that the leading particle manifests in its repeated interactions in the nucleus the same properties as a hadron in hadron–nucleon collisions contradicts the causality principle, on which the estimate (1)–(4) is based.

There exist many experimental facts which indicate that the properties of leading particles in repeated interactions within the nucleus are very different from those of normal hadrons.

Let us consider some of them.

a) We have already mentioned that the mean inelasticity $\langle k \rangle$ is a very weakly increasing function of the atomic number A of the target nucleus. If the dependence of $\langle k \rangle$ on A is approximated by $\langle k \rangle \sim A^{\alpha}$, then experimentally $\alpha = 0.06 \pm 0.02$ (Ref. 22) for proton–nucleus interactions and $\alpha = 0.05 \pm 0.01$ (Ref. 23) for pion–nucleus interactions. The value of α obtained in the model of multiple scattering with a constant interaction cross section of the leading system with nucleons in the nucleus ($\sigma \approx 32$ mb) is appreciably larger: $\alpha = 0.12$–0.17 for $\langle k_{NN} \rangle = 0.5$–$0.4$ (the calculation is made under the assumption of a Fermi distribution of the nucleon density in the nucleus). If one assumes that as a result of the first interaction only the cross section of a leading hadron is reduced by a factor $1/n$, while the remaining properties are unchanged, the multiple-scattering model yields the dependence of α on n shown in Fig. 1. It follows from this figure that if agreement with the experiments is to be achieved in the framework of this assumption the cross section of repeated interactions of the leading particle must be reduced by several times. The same conclusion is reached in Ref. 24 in an investigation of pion–nucleus interactions.

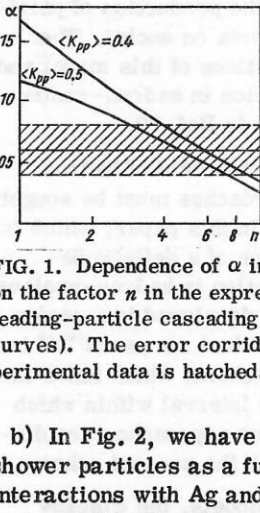

FIG. 1. Dependence of α in the parametrization $\langle k_{pA} \rangle \sim A^{\alpha}$ on the factor n in the expression $\sigma_L = \sigma_{NN}^{\text{in}}/n$ in the description of leading-particle cascading at different $\langle k_{pN} \rangle$ (continuous curves). The error corridor for the values satisfying the experimental data is hatched.

b) In Fig. 2, we have plotted the mean multiplicity of shower particles as a function of the energy for proton interactions with Ag and Br and the results of a calculation of this dependence in the multiple-scattering model for different values of n. Interactions of secondary particles were not taken into account. It can be seen in Fig. 2 that, except for low values of the energy (≈20 GeV), the multiple-scattering model can be reconciled with the experimental data[25] only if the interaction cross section of the leading system is reduced by several times.

c) A similar situation obtains for the energy dependence of the ratio $\langle n_s \rangle / D$, where D is the dispersion of the distribution with respect to the number n_s of relativistic particles. This can be seen in Fig. 3, in which we have plotted the results of a calculation of $\langle n_s \rangle / D$ in the multiple-scattering model and in pEm interactions (the experimental data are taken from Ref. 26).

d) It has been noted frequently that the assumption of normal properties of the leading hadron in succeeding interactions in the nucleus contradicts the data on the spectra of the leading hadrons.

For example, it was shown in Ref. 15 that the number of γ rays from the decay of π^0 mesons in the fragmentation region of the incident particle in π^-C interactions at $E_{\pi^-}^{\text{in}} = 40$ GeV does not depend on the number n_g of accompanying nonrelativistic protons, which characterizes the range of the leading hadron in the nucleus. This behavior of the γ-ray spectra can be explained only by reducing the cross section of a succeeding interaction of the leading particle by several (~8; Ref. 15) times.

The momentum spectra of charged particles were recently analyzed[27] in the same πC interactions at $E_{\pi}^{\text{in}} = 40$ GeV. In particular, they were compared with the results of calculations in the multiple-scattering

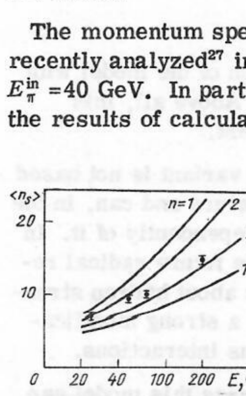

FIG. 2. The dependence of $\langle n_s \rangle$ in the model of a leading-particle cascade for different $\sigma_{LN} = \sigma_{LN}^{\text{in}}/n$.

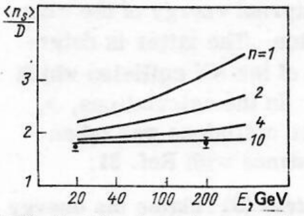

FIG. 3. The dependence of $\langle n_s\rangle / D$ under the same assumptions as in Figs. 1 and 2.

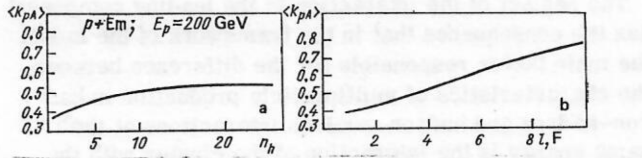

FIG. 4. Dependences of k_{pA} on n_h (Ref. 30) (a) and the thickness of the nuclear matter in the framework of the model of additive quarks (b).

model. Here too it was concluded that the assumption of normal properties of the leading particle in repeated interactions is not realized, since it leads to an underestimation of the production of particles with large momenta. A similar conclusion is drawn in Ref. 28 by Murzin and Sarycheva, who analyzed the spectra of energetic particles produced in the interaction of protons ($E_p^{\text{in}} \approx 20$ GeV) with various clean targets, and by Anzon *et al.*,[24] who investigated the $p_\perp$ distribution of the leading particles in πEm interactions at $E_\pi^{\text{in}} = 50$ GeV.

These facts (and they are by no means the only ones) convincingly confirm the hypothesis put forward earlier in Ref. 29 to the effect that after the first collision in the nucleus the hadron remains for a certain time in a "bare" or juvenile state with properties radically different from the normal ones. It should, however, be noted that the existing attempts at a quantitative description of these properties are still to a large degree conditional. Failure to recognize this has often led to confusion and incorrect conclusions in questions related to the problem of "bare" hadrons.

First, the interaction cross section of the leading system in the nucleus is estimated under the assumption that the remaining properties (inelasticity, mean multiplicity, and so forth) are the same as for normal hadrons. As a result, the estimates of the cross section of succeeding interaction of the leading system with the nuclear nucleons made on the basis of different observable characteristics may differ strongly. It could be that the leading system interacts with a very large cross section, even approaching σ_{NN}^{in}, but basically elastically, with small momentum transfers. Such interactions do not contribute to the mean multiplicity or the inelasticity, but they have a strong influence on the distributions of the leading particles with respect to the transverse momentum and some other characteristics.

Second, not all interactions of hadrons lead to a strong disturbance of their structure. For example, in diffraction processes, for which the partial cross section is approximately $(0.2-0.3)\sigma_{NN}^{\text{in}}$, the leading hadron is in the ground state or a weakly excited state. It is improbable that its properties should differ strongly from those of ordinary hadrons. Neglect of this circumstance leads to an overestimation of the cross section of repeated interactions of leading particles produced in the pionization process.

It is possible that the main contribution to the A dependence of the inelasticity and other characteristics associated with repeated interaction of the leading particles is made by the events in which a diffraction process takes place in the first interaction of the incident hadron with a nucleon of the nucleus. This hypothesis is confirmed by the following:

a) Estimates of the interaction cross section of the leading particle based on an analysis of all events lead to values $\sigma_L \approx (1/6-1/3)\sigma_{NN}^{\text{in}}$ (Refs. 24 and 27), which are close to the cross section $(0.2-0.3)\sigma_{NN}^{\text{in}}$ of diffraction processes.

b) With increasing n_h for $n_h > 10$, the inelasticity ceases to increase[30] (Fig. 4) ($n_h = n_g + n_b$; here, n_g and n_b are the numbers of gray and black tracks in accordance with the terminology used in the photoemulsion technique). This can be attributed to the suppression of the diffraction channels at ranges in the nucleus corresponding to $n_h \lesssim 5$ and the negligibly small interaction of the leading particles produced in the pionization process.

It follows that attempts to estimate the interaction cross section of the leading particle in events with small energy loss ($x_L \approx 0.8-1$) can lead to the result $\sigma_L \approx \sigma^{\text{in}}$, since the dominant contribution to events with small energy release is made by peripheral collisions, in which the structure of the incident hadron is not destroyed.

Thus, the basic propositions of the model require explicit allowance for space-time factors in the description of the process of multiparticle production of hadrons in nuclear matter. Further, from the practical point of view the second proposition enables one in a first approximation to ignore the effects due to the secondary interactions of the leading system with the nuclear nucleons.

These two propositions are interpreted from the point of view of modern ideas about hadron structure in Sec. 2.

We make one further comment. It is assumed in the model that the process of multiparticle production in hadron–nucleus interactions commences with a collision of the incident hadron with one of the nucleons of the nucleus. Data on the production of $\mu^\pm$ pairs on nuclei indicate that such a situation is realized to a high accuracy. It was shown in Ref. 10 that even a small addition (≈6%) of interactions with dinucleons leads to a pronounced discrepancy with the experiments.

Equations of the Model. The construction of the model requires the development of a mathematical formalism for making concrete calculations. The problem is not simple and it is hardly possible to use a ready-made scheme in its solution.

The neglect of the interaction of the leading component has the consequence that in the framework of the model the main factor responsible for the difference between the characteristics of multiparticle production in hadron–hadron and hadron–nucleus interactions at the same energy is the interaction of the cluster with the nuclear matter.

The special role of the cluster, and also the difference between its properties and those of ordinary particles resulting from its being a strongly excited and absolutely unstable system, made it necessary to formulate special equations describing its motion in matter.

It follows from the basic properties of the cluster[11] that the equations must describe a picture which adequately reflects the motion in a medium of an object which has increasing mass and transverse dimensions and loses some of its energy on excitation of the medium. Such equations were formulated in Ref. 11. Ignoring elastic processes, we can represent these equations for a homogeneous nuclear medium in the form[1)]

$$dE/dz = -\bar{\varepsilon}\rho\sigma_{KN}(z); \tag{5a}$$

$$dE_0/dz = T_{KN}(z)\,\rho\sigma_{KN}(z), \tag{5b}$$

where

$$\sigma_{KN}(z) = \pi\Big[r_0 + \int_{z_0}^{z} dz'/\sqrt{\gamma^2(z')-1}\,\Big]^2; \tag{6}$$

$$\gamma(z) = E(z)/E_0(z); \tag{7}$$

$$T_{KN}(z) = \langle k_{KN}\rangle\{\sqrt{2m\gamma(E)\,E_0 + E_0^2 + m^2} - (E_0 + m)\}. \tag{8}$$

In (5) and (7), E and E_0 are the total and the internal energy of the cluster, γ is its Lorentz factor in the laboratory system, z_0 is its point of production in the nucleus, z is the coordinate along the trajectory of the motion, ρ is the density of the nucleons in the nucleus, and $r_0 \approx \hbar/\mu_\pi c$ is the radius of the region occupied by the cluster at its time of production.

In Eq. (5a), $\bar{\varepsilon}$ is the mean kinetic energy of the recoil nucleon in the cluster–nucleon interaction. In Refs. 11–15, $\bar{\varepsilon}$ is regarded as a parameter of the model to be fixed by comparison with experiments. However, systematic use of the conservation laws permits determination of $\bar{\varepsilon}$ from the kinematics of the cluster–nucleon collision, i.e., the mass of the cluster, the inelasticity of this collision, and the transverse momentum transferred to the recoil nucleon[16]:

$$\bar{\varepsilon} = \sqrt{\bar{p}_\parallel^2 + \bar{p}_\perp^2 + m^2} - m, \tag{9}$$

where m is the mass of the nucleon, and $p_\parallel$ and $p_\perp$ are the longitudinal and transverse momenta of the recoil nucleon, the former being given by

$$p_\parallel = \frac{p\,[2m(E+m) - (E_0^{*2} - E_0^2)]}{2\,(2mE + E_0^2 + m^2)}$$
$$\times\Big\{1 - \sqrt{1 - \frac{4(E+m)^2(\bar{p}_\perp^2 + m^2) - [2m(E+m) - (E_0^{*2} - E_0^2)]^2}{p^2\,[2m(E+m) - (E_0^{*2} - E_0^2)]^2}}$$
$$\times\sqrt{(2mE + E_0^2 + m^2)}\Big\}. \tag{10}$$

[1)] *Translator's Note.* Here and throughout the paper, K (from klaster, the Russian form of "cluster") denotes cluster.

In (10), E_0 and E_0^* are the internal energy of the cluster before and after the collision. The latter is determined by E_0 and the dynamics of the KN collision which provides the basis of Eq. (5b). In the calculations, $p_\perp$ for the recoil nucleon in hadron collisions was taken equal to ~0.45 GeV/c in accordance with Ref. 31.

The first equation of the system (5) relates the energy lost by the cluster in the matter per unit path length to the cross section σ_{KN} of its interaction with a nucleon. The second term in the brackets in the expression for σ_{KN} takes into account the change in the transverse dimension of the cluster, which expands in its rest frame at a nearly luminal velocity. In the second equation, $T_{KN}(z)$ is the amount by which the mass of the cluster increases as a result of the cluster–nucleon interaction (8); $\langle K_{KN}\rangle$ is the mean value of the inelasticity of this interaction. Thus, the second equation describes the change in the internal energy of the cluster in the matter. The possibility of a classical approximation is due to the smallness of the de Broglie wavelength of the cluster compared with the internucleon distance, and the rectilinearity of the trajectory is due to the small mean transverse momentum of the cluster. The motion of the cluster is shown schematically in Fig. 5.

We now consider the number of parameters of the model which determine the solution of Eqs. (5).

At the first glance, one could include $E_0(0)$, $E(0)$, and r_0 among them. Since in what follows we shall be interested in the most probable channel of multiparticle production, which is determined basically by average quantities,

$$E_0(0) \approx \langle k_{NN}\rangle\sqrt{s},\ \langle k_{NN}\rangle \approx 0.4 - 0.5, \tag{11}$$

where $\langle k_{NN}\rangle$ is the mean inelasticity in NN collisions. It is usually assumed that the experimental data do not indicate a significant dependence of $\langle k_{NN}\rangle$ on E_{lab}. Thus, $E_0(0)$ and its dependence on E_{lab} are completely determined by the relation (11).

Further, it follows from $\gamma(0) \approx \gamma_{cms}$ that $E(0) = E_0(0)\gamma_{cms}$. Therefore, $E(0)$ and $E_0(0)$ are not parameters of the model but are determined by E_{lab} and the inelasticity of the initial interaction.

Equation (5) also contains r_0, the initial transverse dimension of the produced cluster, i.e., the dimension of the region filled with hadronic matter, which is distributed at the time of the collision over the volumes of

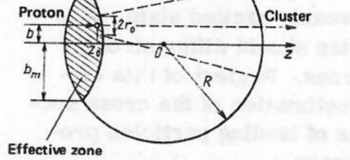

FIG. 5. General scheme of cluster motion in the nucleus. The hatched region is the effective region of the points of production of a cluster capable of causing "complete" disintegration of the nucleus.

the colliding nucleons. In accordance with the idea of an extended hadron, the radius of this region is bounded below by $r_0 \approx \langle r_N \rangle$ (in the case of complete overlap) and above by $r_0 \approx 2\langle r_N \rangle$ (in peripheral collisions). Using the well-known value $\langle r_N \rangle \approx 0.8$ F, we readily find the mean value $r_0 \approx 1.24$ F. This value (or rather $r_0 = 1.2$ F) is used in the concrete calculations.

It is also helpful to make a remark concerning the possible dependence of r_0 on E_{lab}. It follows from a number of microscopic models (for example, Ref. 32) that the range of the strong interaction must increase with energy, albeit very weakly, logarithmically. The possible rate of increase of r_0 in the energy range 20–400 GeV in which we are interested (the energies accessible to accelerators and at which we expect our model to "work") can be estimated from the increase in the total pp interaction cross section. This increase (the Serpukhov effect) amounts to only a few percent. Therefore, the growth of r_0 itself is even slower. Clearly, this effect is at the limits of the accuracy of the model. It is therefore sensible to regard r_0 as a constant.

The quantity $\langle k_{KN} \rangle$, the mean inelasticity of the cluster–nucleon interaction, is not known in advance. It must be regarded as a parameter. In some cases, the system of equations (5) can be conveniently represented in the dimensionless form[11]

$$d\eta/d\xi = -\left[s_0 + \int_0^{\xi} d\xi'/\sqrt{\gamma^2(\xi') - 1}\right]^2; \tag{12a}$$

$$d\varkappa/d\xi = L\left\{\sqrt{\varkappa^2 + 2\frac{m}{E_0(0)} + \frac{m^2}{(E_0(0))^2}} - \varkappa - \frac{m}{E_0(0)}\right\} \times\left[s_0 + \int_0^{\xi} d\xi'/\sqrt{\gamma^2(\xi') - 1}\right]^2, \tag{12b}$$

where

$$\eta(\xi) = E(\xi)/E_0(0);\ \varkappa(\xi) = E_0(\xi)/E_0(0); \tag{13}$$

$$\left.\begin{array}{l}\xi = (z - z_0)\sqrt[3]{\pi\rho\bar{\varepsilon}/E_0(0)},\ s_0 = r_0\sqrt[3]{\pi\rho\bar{\varepsilon}/E_0(0)};\\ L = \langle k_{KN}\rangle E_0(0)/\bar{\varepsilon}.\end{array}\right\} \tag{14}$$

In a comparison of the results of the model with experimental data, it is necessary to bear in mind a circumstance which will permit the elimination of confusion later. We are referring to quantities that are directly determined in the model.

As we have seen, the mathematical formalism of the model is grounded on Eqs. (5). Solving (5), we must as a result obtain the functions $E(z)$ and $E_0(z)$, i.e., the energy and mass of the cluster which leaves the nucleus. The relation (13) also permits an estimate of the Lorentz factor of the cluster. On the basis of these solutions, one can also obtain the mean values of $\bar{E}$ and $\bar{E}_0$ per interaction. Thus, to compare the predictions of the model with experiments, it is necessary to determine experimentally precisely these quantities, and for this we need information on the total energy of the products of the disintegration of the nucleus and on the spectral and angular characteristics of all the produced particles in each interaction event.

However, the experimental methods currently used to investigate hadron–nucleus interactions do not, as a rule, give such a complete picture of the process. The transition from the characteristics of the process calculated in the model to quantities that are directly observed in an experiment requires the use of some additional assumptions. The actual transition may be governed by the particular experiment and is not an organic part of the model.

Transition from Solutions of the System of Equations (5) to Observable Quantities. The connection between the characteristics of relativistic particles and the solutions of the system (5) is based on the assumption that the normalized spectra of the particles produced in the decay of the clusters in hA and hp collisions (E^{in} = const) are approximately the same in their respective rest frames:

$$\frac{1}{\langle n_{hA}\rangle}\left(\frac{d^3 f(\mathbf{p})}{dp^3}\right)_{h,A} \approx \frac{1}{\langle n_{hp}\rangle}\left(\frac{d^3 f(\mathbf{p})}{dp^3}\right)_{h,p}. \tag{15}$$

From (15), it follows directly that the mean energies $\bar{\varepsilon}_c$ of the produced particles are equal. Since the mean number of particles resulting from the decay of a cluster with mass E_0 is

$$\langle n\rangle \approx E_0/\bar{\varepsilon}_c, \tag{16}$$

we obtain from (16)

$$\langle n_s\rangle_{hA}/\langle n\rangle_{hp} \approx E_{0,hA}/E_0(0). \tag{17}$$

In the case of a single-cluster production mechanism, the assumption (15) is equivalent to assuming the existence of a universal phase transition of the quark–gluon matter of the clusters into hadrons, which can be called hadronization. Therefore, in a first approximation (15) does not depend on the cluster masses.

As an example of a special model realization of multiparticle production in hp interactions which illustrates the relations (15) and (17), we can mention the scheme considered in Ref. 33. It is based on two assumptions: a) thermodynamic decay of the cluster with momentum distribution characteristic of black-body radiation, and b) constancy of the mean hadronization time τ_h, which for $E_{lab} \leq 200$–300 GeV leads to constancy of the mean decay volume V_h of the cluster (for more details, see Ref. 33).

If assumption a) is applied to clusters with masses that do not differ too strongly, it corresponds to (15). Using the model of Ref. 33, we can also estimate the extent to which the relation (17) is satisfied. It follows from Ref. 33 that

$$\langle n\rangle \sim V_h^{0.18} E_0^{0.82}. \tag{18}$$

The dependence (18) departs very little from linearity. For example, for photoemulsion nuclei $\bar{\varkappa} = E_0/E_0(0) \approx 1.6$–1.7. This leads to a difference between (17) and (18) not greater than 8–9%. The estimate is an upper bound, since a "nuclear" cluster decays at a larger volume due to the delay of hadronization as a result of the cluster's succeeding inelastic interactions. Then the dependences (17) and (18) become even closer.

When we bear in mind the accuracy of the model and the experiments, we can regard the dependences as identical.

Further, to calculate the distribution with respect to the number of relativistic particles in the model, another very general principle is used; namely, it is assumed that we have approximate Koba–Nielsen–Olesen scaling:

$$\text{KNO scaling.} \tag{19}$$

We assume in a first approximation that the profile of the n_s distribution does not depend on the cluster mass. Of course, in actual calculations of the characteristics of the relativistic particles it is necessary to use the actual form of the functions that describe the spectral and angular distributions (15), the distributions which follow from KNO scaling, and also to introduce some necessary corrections.

For example, in (17) it is necessary to take into account a correction for the contribution of the particles of the leading component, the number of which is approximately constant and equal to 1.6. They do not participate in the increase of the cluster mass. Making this correction, we obtain instead of (17)

$$\langle n_s\rangle_{hA} = [\langle n_s\rangle_{hp} - 1.6]\, E_{0,hA}/E_0(0) + 1.6. \tag{20}$$

In accordance with the assumptions (15) and (19), the following procedure is used in the model for making the transition to observable quantities.

1. For the multiplicity distribution of the s particles, the expression obtained in Ref. 34 for pp collisions, which does not contradict KNO scaling, is used:

$$\left.\begin{array}{l} F(n_s) = n_s(\psi + \mu w)\ \exp(-\nu w), \\ \nu = 1.31;\ \psi = 0.93;\ \mu = 0.49;\ w = (\pi/4)(n_s/\langle n_s\rangle)^2, \end{array}\right\} \tag{21}$$

where $\langle n_s\rangle$ is connected by the relation (17) to E_0, the solution of the system of equations (5).

2. The distribution with respect to $\lambda = \log\tan\theta$ is given by

$$F(\lambda) \sim \exp\{-[(\lambda - \log \varkappa)/\bar{\lambda}]^2/2\sigma^2\}, \tag{22}$$

where $\bar{\lambda}$ is the mean value of λ in a pp collision, $\sigma = 0.65$, and $\varkappa = E_0/E_0(0)$ is the solution of Eqs. (5). Such an approximation of $F(\lambda)$ for pp collisions was made in Ref. 31.

3. The number of g particles, which characterizes the reaction of the nucleus to the multiparticle production process taking place in it, is determined as follows.

Under the assumption that a nucleon emitted on the disintegration of the nucleus is a proton with probability Z/A, the probability of emission of a given number n_g of g particles as a result of the cluster's transfer of the energy ΔE to the nucleus can be represented in the form

$$C^n_{n_g}(Z/A)^{n_g}(1 - Z/A)^{n-n_g}, \tag{23}$$

where

$$n = \Delta E/\bar{\bar{\varepsilon}},\quad \bar{\bar{\varepsilon}} \approx \bar{\varepsilon}_g + \bar{\varepsilon}_{\mathrm{bd}} + (1.5 - 2.0)(\bar{\varepsilon}_b + \bar{\varepsilon}_{\mathrm{bd}}); \tag{24}$$

here $\bar{\varepsilon}_b$ and $\bar{\varepsilon}_{bd}$ are the mean energy of the b particles and the mean binding energy of a nucleon in the nucleus. The coefficient 1.5–2 in (24) reflects the experimentally observed relation $n_b \approx (1.5–2)n_g$, which holds on the average for nuclei of different masses. It does not hold in the rare events which result in almost "complete disintegration" of the nucleus.[35] However, because $\bar{\varepsilon}_b$ is small, the corresponding correction in $\bar{\bar{\varepsilon}}$ is not important.

It is held that the mean energy $\bar{\varepsilon}_g$ of the g particles is virtually independent of the energy of the initial hadron. As yet, this is confirmed by the experiments. The mean energy is approximately $\bar{\varepsilon}_g = 0.13$ GeV. Setting $\bar{\varepsilon}_b \approx \bar{\varepsilon}_{bd} \approx 0.01$ GeV, we obtain

$$\bar{\bar{\varepsilon}} \approx 0.18\,\mathrm{GeV}. \tag{25}$$

The relations (24) presuppose that $\bar{\varepsilon}_g$ does not depend on the number of g particles, which does not contradict the experimental data.[36]

We mention that in the first approximate version of the model,[11] the mean energy $\bar{\varepsilon}$ of the recoil nucleon, the mean energy $\bar{\varepsilon}_g$, and $\bar{\bar{\varepsilon}}$ were identified. This was due to the neglect of dissipation of part of the energy on the production and emission of b particles by the nucleus.

We emphasize that $\bar{\bar{\varepsilon}}$ is not a parameter of the model. It can be calculated, as we have seen above, and is used only to represent a result of the model, the energy ΔE, in terms of the number of g particles.

4. The correlations between the numbers of relativistic, s, and nonrelativistic, g, particles are obtained from the product of the distributions (21) and (23) by averaging with respect to one of the variables: with respect to n_s for $\langle n_s\rangle = f_1(n_g)$, or with respect to n_g for $\langle n_g\rangle = f_2(n_s)$. For example, to calculate the correlation function $\langle n_g\rangle = f_2(n_s)$, this product must be multiplied by n_g and summed with respect to this variable. It is readily seen that, like the distributions (21)–(23), the function

$$\langle n_g(l)\rangle = \sum_{n_g} F[n_s(l)]\, F(n_g, l)\, n_g \tag{26}$$

depends on the distance l traversed by the cluster in the nucleus and thus on the point (b, z_0) at which it is produced; b is the impact parameter of the initial incident hadron.

Other correlation functions can be constructed similarly for a given value of l.

For the calculation of the distributions and correlation functions for the nucleus as a whole, the expressions (21)–(23) and (26) and their ilk are, as usual, averaged with a weight over the production points:

$$\frac{2\pi}{\sigma^{\mathrm{in}}_{pA}}\,\sigma^{\mathrm{in}}_{pN}\rho\left\{\exp\left[-\sigma^{\mathrm{in}}_{pN}\int_{-\infty}^{z_0}\rho(b, z')\,dz'\right]\right\} b\,db\,dz_0. \tag{27}$$

Main Consequence of the Model. The qualitatively new facts about the hadron–nucleus interaction accumulated in recent years have posed theoreticians some interesting questions.

One can confront the model of Ref. 11 with the greater part of these facts without detailed solution of the system of equations (5). For this, it is sufficient to go to some limits and use the axioms of the model.

Let us consider these consequences.

1. If we ignore the interactions of the leading component, the inelasticity of the hadron–nucleus and hadron–hadron interactions must be nearly equal; there should be no dependence of this coefficient on A or on $n_h = n_g + n_b$. For the fraction of the energy of the initial particle expended on the production of the cluster is distributed over a greater number of s particles the greater is its range in nuclear matter (i.e., n_h or n_g). This is confirmed by the experiments with the required accuracy[30] (see Fig. 4a).

2. In the model, the cluster decays, as a rule, into real particles outside the nucleus, so that a direct consequence of the original premises is an equality of the mean transverse momentum of the particles produced in pp and pA interactions.

If the cascade scheme were true, one would expect a significant increase in $\langle p_\perp \rangle$. The observed behavior of $\langle p_\perp \rangle$ does not support the scheme. This is widely recognized. For the sake of objectivity, we should here mention Ref. 37, in which this is disputed. The argument goes that this behavior of $\langle p_\perp \rangle$ is not an "anti-cascade thesis," since it is observed for all particles, whereas for π mesons with $p_\pi > 1$ GeV/c the value of $\langle p_\perp \rangle$ increases with increasing A.

However, this effect cannot be taken as an indication of cascading of fast particles—it is a trivial consequence of the kinematics of the process. It is well known that the transverse and longitudinal components of the momentum, referred to the center-of-mass system of the colliding particles, are correlated, the correlation being positive in the central region. This is shown schematically in Fig. 6. As the cluster is decelerated in the nucleus, its Lorentz factor is decreased by a factor $\varkappa$, $\varkappa$ being a solution of the system (12). Therefore, if in the elementary interaction pions with momentum $p > p_{\text{lab}}$ are in the range

$$p_\parallel^* > p_a \approx p_{\text{lab}}(\gamma_{\text{cms}} - \sqrt{\gamma_{\text{cms}}^2 - 1}),$$

then in hadron–nucleus collisions such pions must have longitudinal momentum

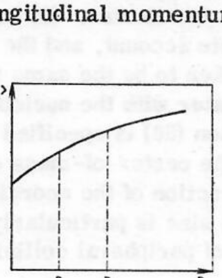

FIG. 6. Scheme illustrating the correlation $\langle p_\perp \rangle \leftrightarrow p_\parallel$ in the central region.

$$p_\parallel^* > p_b \approx p_{\text{lab}}(\varkappa\gamma_{\text{cms}} - \sqrt{\varkappa^2\gamma_{\text{cms}}^2 - 1}) \approx p_a + (p_{\text{lab}}/2\gamma_{\text{cms}})(\varkappa - 1).$$

It is natural that $\langle p_\perp \rangle$ for them is larger (see Fig. 6).

It is worth noting here that the observed behavior of $\langle p_\perp \rangle$ on the transition from hadron–hadron to hadron–nucleus interactions is hard to understand in the framework of parton cascade models.[3]

3. A result which is fully in accord with the model was obtained recently in Ref. 38, in which Veĭsenberg *et al*. investigated the production of antiprotons on Be, Al, Cu, and Au nuclei by protons with $E_p^{\text{in}} = 10$ GeV. They showed that to explain the observed $\bar{p}$ production cross sections it is necessary to eliminate the possibility of absorption of the antiproton through annihilation over a distance $\Delta z \approx 5$–9 F from the point of production. This is natural, since the products of the initial interaction pass through virtually the entire nucleus in the cluster state, in which the antiproton does not yet exist as such. The attempt in Ref. 38 to explain this result in the parton model of Ref. 3 lacks foundation, since the path interval over which the partons are "dressed" is estimated[39] from an analysis of multiparticle production processes to be $\Delta z \approx 0.07\gamma_0$ (γ_0 is the Lorentz factor). For the $\bar{p}$ observed in Ref. 38 with momentum 0.71 GeV/c, this leads to $\Delta z \approx 0.1$ F.

4. It was noted in Ref. 40 that the behavior of the ratio of the square of the mean multiplicity to the dispersion, $\langle n \rangle^2/D^2$, depends on the assumptions made about the multiparticle production process in the nucleus. If the process consists of a number of independent events, this ratio must exhibit appreciable growth with increasing n_g (or n_h). If the production is due basically to the decay of a single system (for example, a cluster), then, apart from the correction for the interaction of the leading particle, the ratio $\langle n \rangle^2/D^2$ should be virtually constant. It was shown in Ref. 14 that allowance for the correction can increase the ratio at large n_g by 20–30%.

At high energies ($E_{\text{lab}} \approx 200$ GeV), a new factor may become relevant—the transition to a new production regime. This effect could also cause a slight rise of the ratio as a function of n_g.

Overall, the experiments agree with the cluster interpretation of the multiparticle production.

5. It has been established[41] that at high energies the ratio of the mean multiplicities on nuclei and on a hydrogen target does not depend on E_{lab}. This is a simple consequence of the system of equations (12). If E_{lab} is large, so is γ_{cms}. Then in (12) we can ignore the terms which take into account the expansion of the cluster, and the system of equations "decouples." From (12b), ignoring the term $m/E_0(0)$ and going over to the variable z, we obtain

$$d\varkappa/dz = \rho\pi r_0^2 \langle k_{KN} \rangle (\sqrt{\varkappa^2 + 1/\langle k_{NN} \rangle} - \varkappa). \tag{28}$$

The quantities on the right-hand side of (28) are virtually independent of the energy E_{lab}. To this accuracy, we have

$$\langle n_s\rangle_{pA} / \langle n_s\rangle_{pp} \approx \langle n_s\rangle_{pp} \langle \varkappa\rangle / \langle n_s\rangle_{pp} = \langle \varkappa\rangle = \mathrm{const}(E_{\mathrm{lab}}). \quad (29)$$

6. Further, at sufficiently high energies, we obtain from the system (12) the equation

$$(1/\langle n_s\rangle)\, dn_s/d\Delta E \approx (\langle k_{KN}\rangle/\bar{\varepsilon})\left(\sqrt{\varkappa^2+1/\langle k_{NN}\rangle}-\varkappa\right). \quad (30)$$

Its right-hand side is, apart from a small change in the value of $\bar{\varepsilon}$, the same at different energies. It follows that

$$dn_s(E_{\mathrm{lab},1})/dn_s(E_{\mathrm{lab},2}) \approx \langle n_s\rangle_{pp}(E_{\mathrm{lab},1})/\langle n_s\rangle_{pp}(E_{\mathrm{lab},2}) \quad (31)$$

and further

$$n_s(E_{\mathrm{lab},1})/n_s(E_{\mathrm{lab},2}) \approx \langle n_s\rangle_{pp}(E_{\mathrm{lab},1})/\langle n_s\rangle_{pp}(E_{\mathrm{lab},2}). \quad (32)$$

Thus, the ratio of the multiplicities is approximately independent of the energy transferred to the nucleus (i.e., is independent of n_h and n_g) and is equal to their ratio for the multiple production process in pp collisions at the same energies.

The relation (32) corresponds to the experimental data of Ref. 30.

7. Intimately related to the foregoing is another fact established experimentally.[30] We write Eq. (30) in the form

$$dn_s/d\Delta E = \langle n_s\rangle_{pp}(E_{\mathrm{lab}})\langle k_{KN}\rangle\left(\sqrt{\varkappa^2+1/\langle k_{NN}\rangle}-\varkappa\right). \quad (33)$$

Then for the slopes of the correlation functions at different energies, we obtain

$$\frac{dn_s}{d\Delta E}(E_{\mathrm{lab},1}) \Big/ \frac{dn_s}{d\Delta E}(E_{\mathrm{lab},2}) \approx \frac{\langle n_s\rangle_{pp}(E_{\mathrm{lab},1})}{\langle n_s\rangle_{pp}(E_{\mathrm{lab},2})}. \quad (34)$$

It follows from (34) that the slope of the correlation function $n_s(n_h)$ increases monotonically with the energy. This is observed experimentally.[30]

8. In a number of papers, considerable attention has been devoted to a fact established in an investigation into the angular distribution of particles produced at $E_{\mathrm{lab}} = 200$ and 300 GeV on a nuclear target.[30] Their number in the forward cone corresponding to $\theta_{1/2}^{(s)}$, the half-angle in the elementary interaction, does not depend on the energy ($\sim n_h$) transferred to the nucleus.

The model also permits a simple interpretation of this fact. First, fast particles produced by the decay of the leading component, which interacts weakly with the nucleus, contribute in the forward cone. Second, the integrated contribution of the particles—the decay products of the cluster in this cone—also depends very weakly on n_h. Assuming that the angular distribution of the decay products of the cluster in its center-of-mass system is described by the expression

$$(dn_s/d\theta)_{\mathrm{cms}} \sim 1 + d\cos^2\theta,$$

and one can show that the ratio of Δn_s, the number of particles in this interval of angles, to $(\Delta n_s)_{pp}$, the number of particles in it for pp collisions, has the form

$$\Delta n_s/(\Delta n_s)_{pp} \approx [2\varkappa/(\varkappa^2+1)][1+d(\varkappa^2-1)/6]. \quad (35)$$

The function $\varkappa$, the solution of Eqs. (12), varies from 1 to 2.5–3.0, depending on the range of the cluster in the photoemulsion nucleus. Therefore, (35) for $d=0$ varies from 1 to 0.7–0.6 accordingly. For $d \neq 0$, this change is even smaller. This weak change is explained by the approximate compensation of two effects. On the one hand, the "growth" of the cluster decreases its γ, i.e., increases $\theta_{1/2}^{(s)}$, but on the other hand it increases the number of s particles in the changed angular interval.

The combined influence of these two factors leads to the absence of a dependence $\Delta n_s(n_h)$.

It can therefore be seen that the listed results demonstrate a good agreement between the calculations of observable quantities with those obtained from the model, very simple means being used.

Features of the Solution of Eq. (5) for the Motion of Clusters in Real Nuclei. However, the model also permits deduction of numerous other results for the means ($\langle n_s\rangle$ and $\langle n_g\rangle$ as functions of A, etc.) and differential quantities (for example, the distributions $F(n_s)$ and $F(\log\tan\theta)$, the correlation functions $\langle n_s\rangle(n_g)$, $\langle n_g\rangle(n_s)$, etc.). In Refs. 11–15, these quantities are obtained on the basis of a semianalytic and very rough solution to Eqs. (5).

Here, as in Ref. 16, we use a more correct solution. Of course, there is no point in striving for an accuracy greater than that of the initial data. However, bearing in mind the success of the model already in the rough approximation, it is desirable to take into account physical factors which could influence the result at the contemporary level of the errors (20%). What are these physical factors?

1. The diffuseness of the boundary of the nucleus was not taken into account in Refs. 11–15. The nucleus was assumed to be a sphere of radius R with a sharp edge. In Ref. 16, a more realistic Fermi distribution of the nuclear matter density was adopted:

$$\rho(r) = B/\{1+\exp[(r-R)/a]\}, \quad (36)$$

where $R = 1.08A^{1/3}$ F, $a = 0.55$ F, and B is a normalization constant. Allowance for the diffuseness significantly changes the solutions of the system of equations (5).

2. Since the transverse dimensions of the cluster may reach appreciable values, it is also necessary to take into account the change in the matter density within the nucleus that is overlapped by the cluster. (In Refs. 11–15, this fact was not taken into account, and the density of nuclear matter was taken to be the same in the zone of interaction of the cluster with the nucleons.) This can be done if the distribution (36) is specified and we know the coordinates of the center-of-mass of the cluster and its radius as a function of the coordinates. Allowance for the cluster size is particularly important when the contribution of peripheral collisions is being estimated.

3. To use the model in a wider range of energies and mass numbers A, it is also necessary to take into ac-

count the boundedness of the cluster–nucleon interaction cross section

$$\sigma_{KN}(z)=\pi\left[r_0+\int_{z_0}^{z}\frac{dz'}{\sqrt{\gamma^2(z')-1}}\right]^2, \quad (37)$$

which occurs in the system of equations (5). The importance of this has been noted several times.[11,12] For sufficiently long ranges and low initial energy (E_{lab} =20–40 GeV), the cluster, expanding at a near-luminal velocity, can decelerate strongly, i.e., its Lorentz factor tends to $\gamma=1$. Then, as can be seen from (37), the cross section tends formally to infinity, which is physically meaningless. The cross section must be bounded. This can be readily understood by taking into account the following two arguments.

First, the formation and expansion of the cluster can be regarded as a point explosion, which takes place at near-luminal velocity. However, during the subsequent expansion the pressure within the cluster drops rapidly and becomes comparable with the reaction of the medium to the explosion, which is manifested as a counter pressure (we use the terminology of the monograph of Ref. 42 for the description of explosions in media). As a result, the expansion virtually ceases. Such a situation arises when the cluster radius reaches values near the decay radius[33]:

$$r_{K}\approx r_0+c\tau_h\approx 2.5\text{ F}. \quad (38)$$

(In Ref. 11, a similar estimate was obtained on the basis of Pomeranchuk's model.[20])

Second, under the same conditions the boundedness of σ_{KN} follows from the fact that the cluster must, in the next stage, be realized in s particles, basically pions. It is then obvious that

$$(\sigma_{KN})^{\max}\leqslant\langle n_s\rangle\,\sigma_{\pi N}, \quad (39)$$

where $\sigma_{\pi N}$ is the πN interaction cross section. The inequality in (39) corresponds to the presence of a partial mutual screening of the newly produced pions in their collisions with the nuclear nucleons.

In accordance with this factor, we must keep a check on the size of the cluster at each stage in the solution of Eqs. (5) and require that its radius satisfy

$$r\leqslant r_{K}(\langle n_s\rangle), \quad (40)$$

where $\langle n_s\rangle$ can be estimated in accordance with the model of Ref. 33. It is clear that these three factors can be effectively taken into account only in a numerical solution of the system of equations (5). The illustrations presented in Ref. 16 indicate the desirability of simultaneous allowance for these factors, since it has an appreciable influence on the behavior of ΔE and ΔE_0. Underestimation of some of them leads to a distortion of the consequences of the model. An example of such underestimation is Ref. 43.

From Eqs. (5) and the commentaries on them, it can be seen that the interaction of the cluster with the nucleons of the target nucleus can be regarded as consisting of two-particle interactions. This assumption is certainly reasonable when the cluster radius is still small, of order r_0. However, when its size and mass have increased strongly, the assumption is not so indisputable.

It is helpful to elucidate the sensitivity of the obtained results to the assumption made about the nature of the interaction between the cluster and the nuclear matter. Suppose the cluster interacts simultaneously with a group of nucleons within the range of its forces. The volume V_K within this range is bounded by the transverse section of the cluster and an interval r_l in the longitudinal direction. In this case Eq. (12b) has the form

$$d\varkappa/dz=(\langle k_{KN}\rangle\,|\,r_l)\left[\sqrt{\varkappa^2+2\eta M/E_0(0)-M^2/(E_0(0))^2}-\varkappa-M/E_0(0)\right], \quad (41)$$

where

$$M=m\int\rho\,dV_K$$

is the mass of the group of nucleons with which the interaction takes place.

A numerical calculation made for (41) with r_l up to 2 F, which in order of magnitude approaches the decay value of the cluster radius at high energies, leads to results that differ from the earlier results by 5–7%. This difference for ΔE and ΔE_0 is observed only in the region of comparatively low energies (20–30 GeV), where the relative importance of cluster configurations with large radii is greater.

We now say a few words about the limits of applicability of the model.

It is clear from the foregoing that the model can be sensibly applied in situations in which there is a well-developed pionization process. This determines the limits of its region of applicability for the range of initial-particle energies E_{lab} and the set of channels of the production process.

For example, it is not suitable for describing channels with low multiplicity of the relativistic particles. It follows in particular that in an investigation of the distribution with respect to n_s, and also of the correlation function $\langle n_g\rangle=f(n_s)$, the region of small n_s must, strictly speaking, be separately treated.[13] For the same reason, the model should be used at E_{lab} =20–30 GeV and above to describe the mean characteristics of the hadron–nucleus interaction.

However, the model can also be used at lower energies $E_{lab}\approx 10$ GeV, but only for the channel with inelasticity $k_{NN}\to 1$, which corresponds to the production of cluster-type objects.

With regard to the upper limit, the model, in the form presented here, is barely yet in contradiction with experiments at $E_{lab}\approx 200$ GeV. Generally speaking, the upper limit to the applicability of such a model relates to the possibility of its use without significant modifications, i.e., when the pionization part of the spectrum can be described at least approximately (or effectively) in the framework of the production of a single cluster. However, as will be shown in Sec. 3, a simple modification permits use of the model at even higher energies.

Results Obtained by Numerical Solution of the System of Equations (5). The solution of Eqs. (5) and the use of the method described above for making the transition to the distribution and correlation functions makes it possible to obtain a number of results for the most important characteristics of the process. In the calculations, it was assumed that $\langle k_{KN}\rangle = 0.4$.

In Fig. 7, we have plotted the distributions with respect to the multiplicity n_s for photoemulsions at different energies E_p^{in} (Refs. 25 and 44).

It is interesting to note that if we use (21), but for $F_{\text{Em}}(n_s)$ with $\langle n_s\rangle_{\text{Em}}$ calculated in accordance with the model, we obtain the broken curves in Fig. 7 ($E_p^{\text{in}} = 300$ and 50 GeV). Thus, in the framework of the model there is approximate nuclear scaling of the multiplicity distributions.

The results for the distribution $F(\lambda)$ are shown in Fig. 8 (Ref. 25). In a number of papers (see, for example, Ref. 45) in which the distributions with respect to $y = -\ln\tan(\theta/2)$ were analyzed, it has been noted that the difference $y_{\max} - y_0$, where $y_{\max} = \ln(2E^{\text{in}}/m)$ and y_0 is determined by the condition

$$\frac{1}{\sigma_{pA}}\frac{d\sigma_{pA}}{dy}\Big|_{y=y_0} = \frac{1}{\sigma_{pN}}\frac{d\sigma_{pN}}{dy}\Big|_{y=y_0},$$

does not depend on the energy. The experimental data from Ref. 45 for $y_{\max} - y_0$ and $r_y = \sigma_{pp}(d\sigma_{pA}/dy)/\sigma_{pA}(d\sigma_{pp}/dy)$ are compared with the model presented here (Figs. 9a and 9b, continuous curves) and the parton-cascade model of Ref. 3 (Figs. 9a and 9b, broken curves). The parton-cascade model is in strong contradiction with the experiments (results taken from Ref. 45).

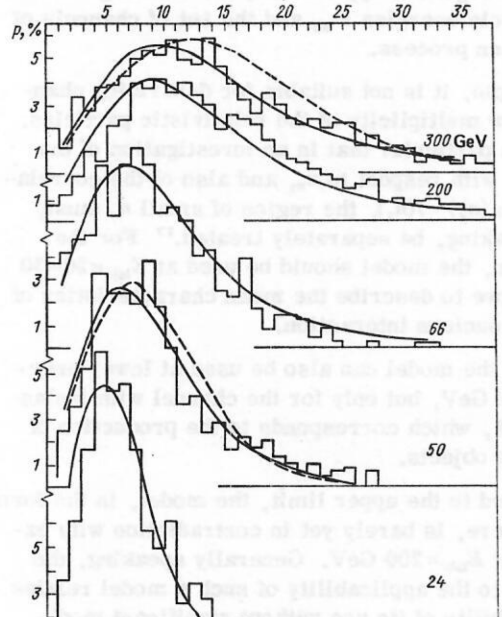

FIG. 7. The distributions $F_{\text{Em}}(n_s)$ for different values of E_p^{in}.

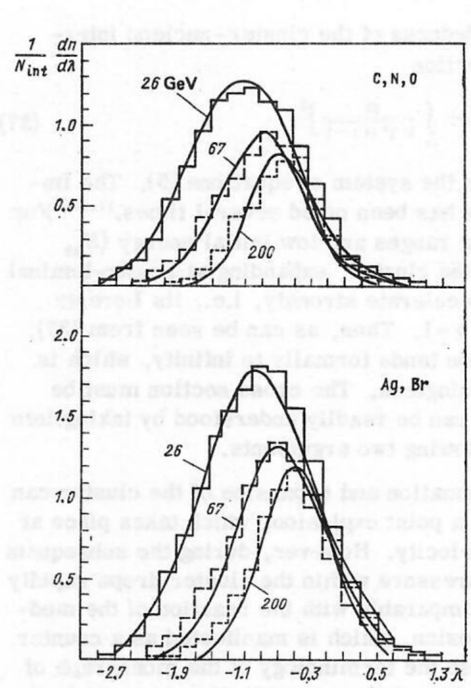

FIG. 8. The distributions $F_A(\lambda)$ for A_1 = C, N, O and A_2 = Ag, Br for different E_p^{in}.

For the calculation of r_y in the fragmentation region of the incident particle, it is necessary to take into account the contributions from the decay of the leading system and from the nonpionization channels. These are estimated in Ref. 14.

It is also easy to establish a connection between $\langle y\rangle$ and the number n_g:

$$\langle y(n_g)\rangle = \langle y_{pp}\rangle - \ln\langle \varkappa(n_g)\rangle, \qquad (42)$$

where $\langle \varkappa(n_g)\rangle$ is the solution of Eqs. (12) averaged over the points (b, z_0) for given n_g.

In Fig. 9c, this dependence is compared with the experimental data of Ref. 45 obtained in pEm interactions at $E_p^{\text{in}} = 400$ GeV.

The correlations $\langle n_s\rangle(n_g)$ and $\langle n_g\rangle(n_s)$ are shown in Fig. 10 (experimental data from Refs. 46 and 47). In

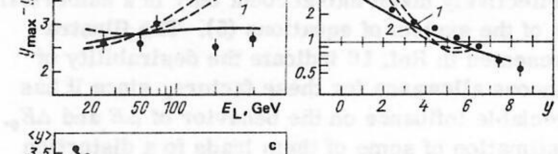

FIG. 9. Dependences of y_0 on E_L (a), $r_y = \sigma_{pp}(d\sigma_{pA}/dy)/\sigma_{pA}(d\sigma_{pp}/dy)$ on y (b), and $\langle y\rangle$ on n_g (c). The functions r_y and $\langle y\rangle$ are obtained for pEm interactions at $E_p^{\text{in}} = 400$ GeV.

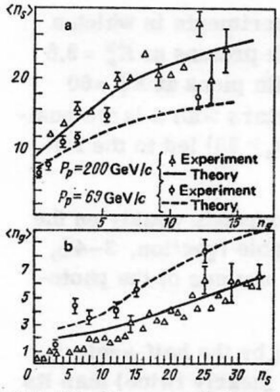

FIG. 10. The correlations $\langle n_s \rangle \leftrightarrow n_g$ (a) and $\langle n_g \rangle \leftrightarrow n_s$ (b).

Fig. 11, we have plotted the distributions $F(n_g)$ for light (C, N, O) and heavy (Ag, Br) photoemulsion nuclei at $E_p^{in} = 200$ GeV (Ref. 14). The form of these distributions reflects the expansion of the cluster during its motion in the nucleus adopted in the model. In this connection, we mention Ref. 48, in which a general analysis of such distributions is made without recourse to a definite model of the hadron–nucleus interactions. This paper showed that the observed $F(n_g)$ does indeed indicate a growth of the total cross section of hadronic systems traversing the nucleus during the development of multiple production.

This result indicates the inadequacy of some models which ignore the growth of the transverse dimensions of the cluster and, *a fortiori*, the models which presuppose a decrease of them.[49]

In Fig. 12, we show the A dependences of $\langle n_s \rangle$ and $\langle n_g \rangle$ and also $\langle n_g \rangle$ and $\langle n_s \rangle$ as functions of E_{lab} (experimental data from Refs. 25 and 50). It can be seen that $\langle n_g(E_{lab}) \rangle_{theor}$ in the interval $20 \leqslant E_{lab} \leqslant 200$ GeV is approximately constant and corresponds to the experiment, though for $E_{lab} > 200$ GeV a discrepancy is observed, the theoretical results lying lower than the experimental points. The decrease in $\langle n_g \rangle$ is due to the increase in γ for the cluster, which leads to an insufficiently fast growth of its transverse dimensions at such high energies. This can be taken as an indication that the description in the framework of a model with one effective cluster is too crude in this region.

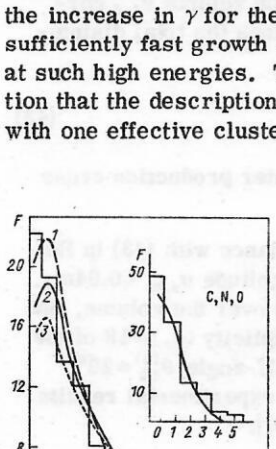

FIG. 11. The distribution $F(n_g)$. Curves 1 and 3 are obtained for $\delta = 0.28 \pm 0.08$.

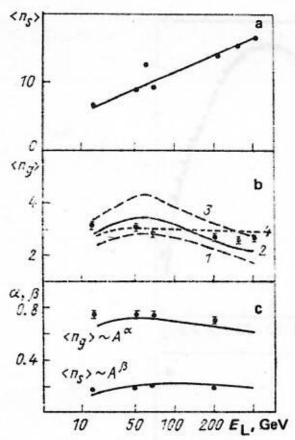

FIG. 12. The dependences $\langle n_s \rangle$ (E_p^{in}) (a) and $\langle n_g \rangle$ (E_p^{in}) (b), and also the A dependences of these quantities (c). Curves 1 and 3 correspond to the quark—gluon interpretation with $\delta = 0.28 \pm 0.08$, and curve 2 to the phenomenological variant of the model; curve 4 corresponds to allowance for the collective motion in the cluster at large E.

The further discussion of this effect will be continued in Sec. 3. It can also be seen from Fig. 12c that the A dependences of the processes of production of s particles and emission of g particles are very different despite the common origin of their production, i.e., the motion of the cluster in the nucleus. Note that the difference in the A dependences is sometimes erroneously interpreted as an indication of a very weak connection between these processes (see the second citation in Ref. 31, p. 90).

On the whole, the illustrations demonstrate a good agreement between the model and the experimental data.

The major part of the data presented in this section was obtained by the photoemulsion method. A number of characteristics of multiple processes involving charged and neutral pions were investigated by means of a two-meter propane chamber at Dubna. Initially, they were compared with the model on the basis of semianalytic approximate solutions of it.[11] Recently, a comparison has been made under new conditions, viz., between experimental material with much higher statistics and the model in the form presented here.[51] In Ref. 51, good agreement between the theory and the experiments (Fig. 13) is noted.

An interesting application of the model has also been realized in Ref. 52, which investigates the production of antiprotons of heavy nuclei in the pA process. Allowance is made for additional factors relevant in this problem (for example, the possibility of absorption of antiprotons emitted by a cluster strongly decelerated in the nucleus), i.e., the cases of low initial energy of the incident proton and long cluster ranges in the nucleus. Mering and Tayurskiĭ[52] conclude that "the model gives the decrease in the production of fast particles and the increase in the production of slow particles compared with pp collisions that is observed experimentally." Other characteristics are also obtained in Ref. 52. In Fig. 14a, we compare the experimental spectra of antiprotons as functions of the angle of ob-

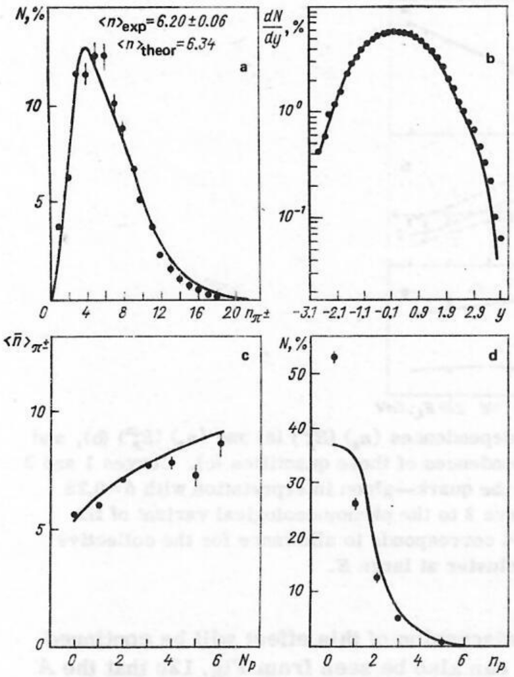

FIG. 13 Some characteristics of multiple production in the process $\pi^{12}C \to \pi + X$ (Ref. 51): a) the $n_{\pi\pm}$ distribution, b) the y distribution, c) the $\langle n\rangle_{\pi\pm}$—$n_p$ correlation (here $n_p \equiv n_g$), d) the distribution with respect to $n_p(\equiv n_g)$; E^{in}_π=40 GeV.

servation, and in Fig. 14b the A dependences $(Ed^3\sigma/dp^3 \sim A^{\alpha(p_\perp)})$ of the production of slow antiprotons are compared for different $p_\perp$ emitted backward in the center-of-mass system of a pN collision. It can be seen that these characteristics too are in reasonable agreement with the model calculations.

Thus, the phenomenological variant of the proposed model is capable of interpreting a large body of qualitatively different experimental data on hadron–nucleus interactions at high energies.

Complete Disintegration of Nuclei by Relativistic Hadrons. To conclude this section, we note that the model permits a satisfactory description of not only the average multiparticle production event in nuclear matter. It also helps one to understand the basic features of even such exotic processes as the complete disintegration of nuclei by relativistic particles.

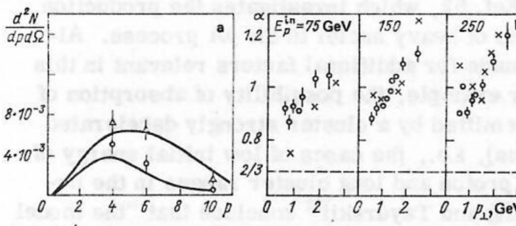

FIG. 14. Comparison of the model with some experimental data on the production of antiprotons on nuclei. a) The spectra at 0.017 rad (open triangles) and 0.087 rad (open circles) in the process $p\text{Pb} \to \bar{p} + \ldots$, E^{in}_p=24 GeV; b) A dependence of the form $A^{\alpha(p_\perp)}$ for different E^{in}_p: the open circles are the experimental points and the crosses are the calculations using the model.

This effect was observed in experiments in which a photoemulsion was irradiated with protons at E^{in}_p = 9.6 and 69 GeV (Ref. 35), and also with pions at E^{in}_π = 60 GeV (Ref. 53). The selection of stars with a large number of nonrelativistic particles ($n_h \geq 28$) led to the following results.

The probability of the process depends weakly on the energy E_p and is a fairly appreciable fraction, 3–4%, of all interactions with the heavy nucleus of the photoemulsion.

The angular cone characterized by the half-angle $\theta_{1/2}$ of the s particles is much larger (nearly twice) than its value for an "average" star.

The half-angle for g and b particles depends weakly on the energy E_p, and these particles are emitted almost isotropically.

The following facts are also of interest: in the spectrum of the b particles, many are "below-barrier," and the mean energy of the b particles decreases on the transition from E_p = 9.6 GeV to E_p = 69 GeV. Their number is also somewhat decreased. The total energy transferred to the nucleons of the nucleus in such an event is 3.5–4.0 GeV according to the estimates of Ref. 35.

This limiting case of hA interaction was considered in detail from the point of view of the cluster model in Ref. 12. For brevity, we shall consider only the experiment at E_p = 69 GeV (Ref. 35), and only the principal characteristics.

The process develops in accordance with the scheme shown in Fig. 5. It is obvious that if the maximal energy is to be transferred to the nucleus the cluster must be produced in a collision with small impact parameter near the front edge of the nucleus. The region corresponding to such events in the (b, z_0) plane is hatched in Fig. 5. Its size is determined by the condition $\Delta E \geq 3.5$ GeV, which is imposed on the solutions of the system of equations (5). The volume $V_{t.d}$ corresponding to this region determines the total disintegration cross section:

$$\sigma_{t.d} \approx \rho V_{t.d}\sigma_{p.cl}, \tag{43}$$

where $\sigma_{p.cl}$ is the pionization cluster production cross section.

The estimate obtained in accordance with (43) in Ref. 12 gives the correct order of magnitude $\sigma_{p.cl} \approx 0.04\sigma_{pA}$. Averaging the solution of Eqs. (5) over the volume, one can readily obtain the mean multiplicity $\langle n_s\rangle$ = 18 of the relativistic particles and their half-angle $\theta^{(s)}_{1/2} \approx 29°$. These values agree well with the experimental results [$(\langle n_s\rangle_{exp} \approx 17$, $(\theta^{(s)}_{1/2})_{exp} \approx 30°$ (Ref. 35)].

2. CONNECTION WITH MODERN STRUCTURE MODELS OF THE HADRON

Important advances in our understanding of multiparticle production have been made in connection with the development of parton and especially quark–gluon models of the hadron.

The connection between the description of hadron–nucleus interactions and the main results of the investigation of hadronic structure is of fundamental importance. On the one hand, the absence of contradictions to these results is an additional criterion in the choice of the hadron–nucleus interaction mechanism. On the other hand, it is only when there is an intimate connection between such a model and hadronic structure that one can hope to extract additional information about the dynamics of multiple processes. A qualitative interpretation of the basic propositions of the phenomenological variant from the point of view of hadronic structure has been discussed on a number of occasions (see, for example, Refs. 11 and 54). The paper of Ref. 17 contains a variant of the model with a quark–gluon description of the hadronic structure and permits a quantitative estimate of the characteristics. We shall here briefly discuss these questions.

We begin with a qualitative comparison of the two basic propositions of the model and the parton description. They appear very natural. We shall use a scheme frequently employed by adherents of such an approach. The interaction of hadrons is to be treated as the sum of interactions of the partons which constitute them. In the center-of-mass system of the colliding hadrons N_1 and N_2, the spectra of their partons is in a first approximation as shown in Fig. 15a.

Proceeding from very general and not very stringent assumptions, one can say that the parton interaction cross section has the form[55]

$$\sigma_{\text{part}} \sim \exp(-\Delta y), \tag{44}$$

where Δy is the difference between the parton rapidities.

It then follows from (44) that it is the "soft" partons (for which Δy is small) that interact with the greatest probability (in Fig. 15a, the corresponding regions in the spectra of the colliding nucleons have been hatched). These partons, which interact and are decelerated, initiate the formation of the cluster system. It is readily seen that for the same reason this system will, in its further collisions with other nuclear nucleons,

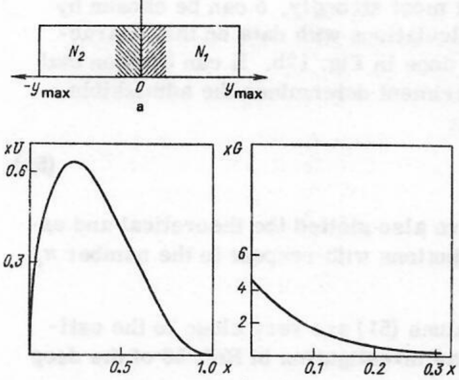

FIG. 15. Illustration of the connection with the parton (a) and quark–gluon (b) models of the hadron.

interact not more weakly than was the case in the initial nucleon–nucleon interaction event. This justifies the first proposition of the phenomenological variant of the hA interaction.

In contrast, the fast partons, for which Δy is large, hardly interact in the first event and are thus separated and must also interact weakly in the following collisions during a certain finite interval of time which is needed for rearrangement of these partons. It is obvious that the fast part of the parton spectrum initiates the leading system. This then justifies the second proposition.

A more detailed comparison of the characteristic features of multiparticle production processes with hadronic structure properties is permitted by the quark–gluon model of hadronic structure.

This model takes into account explicitly the fact that only a certain fraction $1-\delta$ of the proton momentum distributed among the quarks contributes to deep inelastic (ep, μp, and νp) processes. The remaining part δ does not contribute to such processes, since it is associated with the presence of the neutral gluon field. In the variant of the model presented in Ref. 54 (the connection of which to the model discussed in Sec. 1 was established at the qualitative level) it is assumed that the effective interaction between the quarks, and also between the quarks and the gluons, is weaker than the interaction between the gluons.

This assumption is justified by the difference between the structure functions of the quarks and gluons in a hadron. Examples of such functions are shown in Fig. 15b (see the first citation in Ref. 56). As can be seen in Fig. 15b, the quarks are distributed over almost the whole of the interval $0<x<1$. In contrast, the gluons are mainly concentrated in the region of small x, and therefore their interaction as pointlike particles is effectively stronger. For the same reason, the quark–gluon interaction occupies an intermediate position as regards effectiveness between the quark–quark and gluon–gluon interactions.

It is also important that the large-angle scattering cross section in gluon–gluon interactions is more than an order of magnitude greater than the analogous cross section for the interaction of quarks and almost an order of magnitude greater than for the quark–gluon interaction (see the second citation in Ref. 56).

The relative passivity of the quarks is confirmed by the fact that the leading particle conserves preferentially the quantum numbers of the initial hadron, these being transmitted by the valence quarks. The weakness of the quark–gluon interaction is apparently also confirmed by an analysis of charmonium levels.[57]

One of the most important arguments for passivity of the quarks in hadronic collisions is, in our view, to be seen in the connection obtained in Ref. 54 between the spectrum of the leading protons in nondiffraction pp interactions and the spectra of valence quarks of the proton extracted from deep inelastic scattering data. Put briefly, the result is as follows. Let $g(x_1, x_2, x_3)$ be the distribution of the valence quarks in the proton. Then the momentum distribution of one quark is determined by the expression

$$f_i(x)=\int g(x_1, x_2, x_3)\,\delta(x-x_i)\,dx_1\,dx_2\,dx_3. \quad (45)$$

The functions $f_i(x)$ occur in the expressions for the structure functions of hadrons, which are determined in deep inelastic scattering. For example, for *ep* scattering

$$\nu W_2=(8/9)\,f_u(x)+(1/9)\,f_d(x), \quad (46)$$

where $f_u(x)$ and $f_d(x)$ are the x distributions of the u and d quarks in the proton.

At the same time, the spectrum of the leading protons is also determined by the function $g(x_1, x_2, x_3)$ in accordance with

$$F(x)=\int g(x_1, x_2, x_3)\,\delta(x-x_1-x_2-x_3)\,dx_1\,dx_2\,dx_3. \quad (47)$$

In Ref. 54, Van Hove constructed a function $g(x_1, x_2, x_3)$ which describes simultaneously the data on deep inelastic *ep* scattering and the spectrum of the leading protons in *pp* interactions.

Therefore, the valence quarks which form the leading hadron do not significantly change their momentum during the multiparticle production process in hadronic collisions. This means that in hadron–nucleus interactions the component represented by the valence quarks is relatively passive not only in repeated collisions, but also in the first interaction of the incident hadron with a nucleon of the nucleus. Of course, the leading system, after an interval of time τ_h sufficient for its hadronization ("dressing"), recovers its capacity for normal interaction and behaves as an ordinary hadron. In the laboratory system, the "dressing" takes place over a path length (for more details, see Ref. 33)

$$L_h\approx c\tau_h\gamma_L\approx 1.2\gamma_L\ \mathrm{F},$$

where γ_L is the Lorentz factor of the leading system. It can be seen from this expression that L_h exceeds the diameter of heavy nuclei already at $\gamma_L\approx 10$.

Thus, the leading-particle effect in hadron–hadron and hadron–nucleus interactions is a manifestation of the hadronic structure and confirms the data on this structure obtained from deep inelastic processes.

The strong interaction of the cluster with the nucleons of the nucleus is associated with the strong gluon–gluon interaction, and the actual mechanism of the cluster–nucleon interaction adopted in the model described in Ref. 17 consists of the stripping of the gluon fields of the nucleons of the nucleus by the cluster.

In accordance with this picture, the initial radius r_0 of the cluster is determined by the radius of the hadron, as is assumed in the model, since the slow gluons occupy the entire volume of the hadron. The results of a calculation with smaller r_0 strongly contradict the experiments.

However, the quark–gluon model can be compared with the phenomenological variant not only qualitatively.

Regarding the cluster–nucleon interaction as capture by the cluster of the gluon field of the nucleon, it is easy to show that the change T_{KN} in the cluster mass E_0 as a result of such interaction is described by the expression[17]

$$T_{KN}=\sqrt{(E_0+E_N-E_N')^2-(p_N-p_{N\parallel}')^2-p_{N\perp}'^2}-E_0, \quad (48)$$

where

$$p_N'=\sqrt{p_{N\parallel}'^2+p_{N\perp}'^2}=(1-\delta)\,p_N. \quad (49)$$

Here, E_N and p_N are the energy and momentum of the nucleon before the collision in the cluster rest frame, and E_N' and p_N' are the same quantities after the collision. The transverse momentum $p_{N\perp}'$ of the nucleon acquired as a result of the interaction is, as before, taken equal to the mean transverse recoil momentum in nucleon collisions: $\langle p_{N\perp}'\rangle=0.45$ GeV/c.

The energy transferred to the nucleus in the interaction of the cluster with a nucleon, i.e., the kinetic energy of the recoil nucleons in the rest frame of the nucleus, is determined by

$$\varepsilon=(E_N'-\beta_K p_{N\parallel}')\,\gamma_K-m_N, \quad (50)$$

where γ_K and β_K are the Lorentz factor and velocity of the cluster in units of the velocity of light in the laboratory system. The equations of motion of the cluster in the nucleus remain as before.

The initial energies $E(z_0)$ and $E_0(z_0)$ are (bearing in mind that in the quark–gluon model δ and the inelasticity of the *NN* interaction are effectively the same thing) determined by the relations

$$E(z_0)\approx\delta E^{\mathrm{in}};\quad E_0(z_0)\approx\delta\sqrt{s}.$$

The assumption that the cluster expansion velocity is near-luminal corresponds in the modern language of quantum chromodynamics to the fact that the gluon field is massless.

Thus, in the framework of the quark–gluon model the system of equations does not contain free parameters peculiar to hadron–nucleus interactions.

Of course, the results depend on δ, which is characteristic of the quark–gluon model of the hadron. Since $\langle n_g\rangle$ depends on it most strongly, δ can be chosen by comparing the calculations with data on this characteristic. This is done in Fig. 12b. It can be seen that fitting to the experiment determines the admissible range of δ values:

$$\delta\approx 0.28\pm 0.08. \quad (51)$$

In Fig. 11, we have also plotted the theoretical and experimental distributions with respect to the number n_g as functions of δ.

Note that the values (51) are very close to the estimate obtained in an investigation in Ref. 58 of the deep inelastic μp process:

$$\delta\approx 0.30\pm 0.08. \quad (52)$$

The results for the other characteristics and distributions considered in Sec. 1 agree with the new calculations to an accuracy of a few percent for $\delta = 0.28$ (for more details, see Ref. 17).

Thus, the features observed in hadron–nucleus interactions agree with the assumed two-component structure of hadrons in the quark–gluon model, and the phenomenological model is intimately related to it.

The discussion of the present section should not give the impression that the data on hadron–nucleus interactions agree with all the existing approaches to hadronic structure and their associated assumptions about the interaction mechanism.

For example, the basic propositions of the phenomenology of hadron–nucleus interactions can be qualitatively explained by the parton model. However, as we have seen above (see also Refs. 27 and 59), the quantitative results for a number of important characteristics of the hA interaction obtained in the parton-cascade model definitely contradict the experiments. In the given case, this discrepancy indicates not only that the structure model itself is too simple, but also that cascading of partons is not a mechanism capable of explaining the interaction. It is evident that the time of the hadron–hadron collision and the characteristic time of branching at a point of the parton "comb" are of the same order and a cascade cannot develop. But this difficulty for the cascade mechanism in strong interactions must not be regarded as unexpected or strange. It already follows from the analysis of the failures of the mechanism of cascading of nucleons and mesons in nuclei at high energies. Therefore, the mechanical transfer of an unsuccessful interaction scheme to the analysis of a situation of considerable similarity is certainly not expedient.

As another example, we can take the model of additive quarks, which has recently been mentioned quite often in the literature in connection with the problems of the hA interaction. We do not share the opinion of various authors (see, for example, Refs. 59–61) who believe that this model is in good agreement with the experiments. We shall mention only some arguments.

1. The history of the investigation of hA interactions indicates that if the validity of a model is to be judged it is absolutely necessary to compare its predictions with experiments for a wide range of qualitatively different characteristics of the multiple processes. Unfortunately, this model has hitherto yielded too few results.

Primarily, these relate to the properties of the function $R(y) = (1/\sigma_{hA})(d\sigma_{hA}/dy)/(1/\sigma_{hp})d\sigma_{hp}/dy$ for the particles in the fragmentation region, namely, its form and A dependence. The data are shown in Fig. 16 (Ref. 61). It can be seen that on the transition to the pionization region the model disagrees strongly with the experiments. Another weak point of the model of additive quarks is the complete absence of predictions for the reaction of the target nucleus to the process taking place in it.

2. If the axioms of this model are used in their

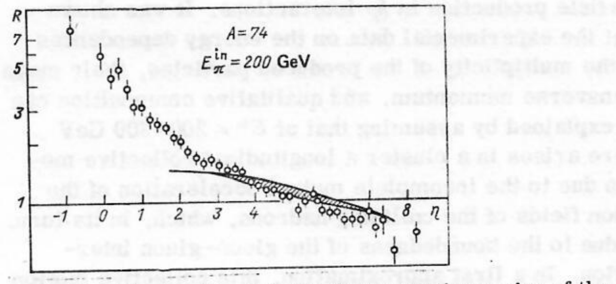

FIG. 16. The function R in the fragmentation region of the incident particle.

modern form, i.e., it is assumed that the cross section of the quark–nucleon inelastic interaction σ_q is approximately equal to $\sigma_{NN}^{in}/3$, and the inelasticity of such a collision is effectively unity, one can estimate the behavior of the mean inelasticity of the hA interaction as a function of A. In the model, one specifies the probabilities of interaction in the nucleus of one, two, and three quarks[60]:

$$\left.\begin{aligned} V_1(A) &= \frac{3}{\sigma_{pA}} \int d^2b \exp[-2\sigma_q T(b)]\{1-\exp[-\sigma_q T(b)]\}; \\ V_2(A) &= \frac{3}{\sigma_{pA}} \int d^2b \exp[-\sigma_q T(b)]\{1-\exp[-\sigma_q T(b)]\}^2; \\ V_3(A) &= \frac{1}{\sigma_{pA}} \int d^2b [1-\exp[-\sigma_q T(b)]]^3, \end{aligned}\right\} \quad (53)$$

where

$$T(b) = A \int \rho(b, z)\, dz.$$

Using (53), for the inelasticity in the hA interaction we obtain

$$\langle k \rangle = (1/3)V_1(A) + (2/3)V_2(A) + V_3(A) \sim A^{\alpha}. \quad (54)$$

The calculations give

$$\alpha \approx 0.20. \quad (55)$$

This value strongly disagrees with the experimental value $\alpha_{exp} = 0.05$–0.06 (see Fig. 1).

3. The same effect is manifested in the dependence of k on n_h. With increasing range of the hadron in the nucleus, the model value of k increases monotonically, tending asymptotically to unity. As can be seen in Fig. 4b, the experimental value of k behaves differently.

It follows that the model of additive quarks in its present form is too crude, and the nucleus " does not accept" this simplification.

It is probable that after a number of improvements it could be used to consider some questions in a limited field (fragmentation of the incident particle) but not the problem of the hA interaction in its entirety.

3. DESCRIPTION OF MULTIPARTICLE PRODUCTION IN HADRON-NUCLEUS INTERACTIONS AT $E^{in} \gtrsim 200$-300 GeV

Using the connection between the phenomenology and the quark–gluon model, we can attempt to interpret the experimental data at higher energies.

In Ref. 33, the standpoint of the quark–gluon model was adopted to analyze some characteristics of multiparticle production in hp interactions. It was shown that the experimental data on the energy dependences of the multiplicity of the produced particles, their mean transverse momentum, and qualitative composition can be explained by assuming that at $E^{\text{in}} \gtrsim 200-300$ GeV there arises in a cluster a longitudinal collective motion due to the incomplete mutual deceleration of the gluon fields of the colliding hadrons, which, in its turn, is due to the boundedness of the gluon–gluon interaction. In a first approximation, this collective motion can be represented as the relative motion of the two parts of the cluster corresponding to the two gluon fields of the initial hadrons. In the framework of such a scheme, one can separate within the total internal energy E_0 of the cluster the energy E_{coll} of the collective motion, and thereby determine the masses and Lorentz factors of the two parts of the cluster.[33]

It was pointed out in Ref. 33 that as such a cluster moves through the nucleus its parts, which move with different velocities, will interact differently with the medium. This is due above all to the different rates of growth of the transverse dimensions of the cluster parts, which are decelerated to corresponding Lorentz factors γ_1 and γ_2.

The circumstance that the slow part of the cluster in the laboratory system interacts more strongly with the medium enhances this mode of the collective motion in the cluster. The increase in the distance between the centers of mass of the parts of the cluster in rapidity space leads to a broadening of the distribution with respect to the quasirapidity and, if the effect is sufficiently large, to the appearance of two peaks in this distribution. This is then the mechanism behind the bimodality in the $\eta = \ln\tan(\theta/2)$ distribution in π^-Em interactions at $E_\pi^{\text{in}} = 200$ GeV, which is not present in $\pi^- p$ interactions at the same energy.[18] The first quantitative estimates of this effect were made in Ref. 18.

The complexity which arises in the description of the motion of a cluster with internal motion through a medium forces one to simplify the problem in some manner. As such a simplification, we shall ignore the exchange of energy between the parts of the cluster and shall regard them as passing independently through the nucleus and take into account only the mutual screening. Then the system of the equations of motion of the cluster in the nucleus can be represented in a form analogous to (5) (there are now twice as many equations as in the previous variant):

$$dE_i/dz = -\bar{\varepsilon}_i \rho \sigma_i(z); \qquad (56a)$$

$$dE_{i,0}/dz = T_{KN}(z)\rho\sigma_i(z), \qquad (56b)$$

where (56a) and (56b) describe, respectively, the changes in the total and the internal energy of part i of the cluster ($i=1$ and $i=2$ for the fast and slow parts, respectively). The remaining notation is as before. The energy transferred to the nucleus by the interaction of the cluster with a nucleon, i.e., the kinetic energy of the recoil nucleons in the rest frame of the nucleus, is determined by

$$\varepsilon_{1,2} = (E_N - \beta_{1,2} p'_{N,\parallel})\gamma_{1,2} - m_N, \qquad (57)$$

where γ and β are the Lorentz factor and velocity of one of the two parts of the cluster in the laboratory system, and $\sigma_i(z)$ is the cross section for interaction of part i of the cluster with a nucleon of the nucleus leading to capture of the nucleon's gluon field:

$$\sigma_1(z) = \eta\pi\left[r_0 + \int_{z_0}^{z}\frac{dz'}{\sqrt{\gamma_1^2(z')-1}}\right]^2; \qquad (58)$$

$$\sigma_2(z) = \pi\left[r_0 + \int_{z_0}^{z}\frac{dz'}{\sqrt{\gamma_2^2(z)-1}}\right]^2 - \sigma_1(z). \qquad (59)$$

In Eqs. (5) and (6), $r_0 \approx 1$ F is the initial radius of the cluster, z_0 is the point of production of the cluster, and z is the running coordinate of the motion of its center of mass. The expressions (58) and (59) take into account the growth of the cluster cross section due to its expansion with near-luminal velocity in its rest frame. In writing down the expressions (59), we have assumed that the interaction cross section of the slow part of the cluster leading to stripping of the nucleon's gluon field is equal to the geometrical cross section. A similar assumption was made earlier for the cluster as a whole. The competition from the fast part of the cluster is taken into account by subtracting in (59) its effective cross section from the geometrical cross section of the slow part.

The additional factor $\eta < 1$ introduced in (58) takes into account the following circumstance. Since both the fast and the slow part of the cluster move with relativistic velocities in the rest frame of the nucleus, i.e., their Lorentz factors are $\gamma_{1,2} \gg 1$, the difference between their velocities is

$$\beta_1 - \beta_2 \ll 1.$$

As a result, when the cluster moves through the nucleus its two parts are not spatially separated and they interact "simultaneously" with nucleons of the nucleus in the region in which they overlap. The factor η is the probability of capture of the gluon field of such a nucleon by the fast part of the cluster. Accordingly, $1-\eta$ is the probability of its capture by the slow part. The value of the parameter η and its possible energy dependence must be established by a comparison with experiments. This will give information about the partial cross section for the interaction of the gluon fields leading to their fusion into a single statistical system. Since such a system is not formed at energies of the order of a few hundred GeV,[33] it is to be expected that $\eta \ll 1$. The initial conditions for the system (56) are determined by the cluster energy E_0 and the fraction of it associated with the collective motion E_{coll}; the values of these for pp and πp interactions are given in Ref. 33. The total energies of the parts of the cluster in the center-of-mass system of the colliding hadrons are given by

$$\varepsilon_i = E_i - \sqrt{(1-\delta_i)^2 p^2 + m_i^2}, \qquad (60)$$

where E_i are the energies and p the momenta of the hadrons before the collision, and m_i are the masses of the leading hadrons after the collision.

The masses μ_i of the parts of the cluster can be found from

$$\mu_1+\mu_2=E_0-E_{\rm coll},\qquad (61)$$

where

$$\mu_i=\sqrt{\varepsilon_i^2-q_i^2};\qquad (62)$$

$$q_i=\delta_i p-Q\qquad (63)$$

(Q is the momentum lost by the mutual deceleration of the gluon fields). Substituting (62) and (63) in (61), we determine Q, and then, using (62) and (63), the masses of the parts of the cluster and their momenta. We find the initial conditions for the system (56) by a Lorentz transformation of ε_i and q_i to the laboratory system.

Let us now compare this description with the experiments.

It was shown in Ref. 33 that collective motion occurs at lower energies of the incident particle in $\pi^- p$ interactions than in pp interactions. This is due to the greater fraction of the momentum belonging to the pion gluon field (according to Ref. 17, $\delta_\pi/\delta_p \approx 1.4$). It is therefore not surprising that at $E_\pi^{\rm in}=200$ GeV collective motion in a cluster is manifested in pion–nucleus interactions in the form of bimodality of the distribution with respect to the quasirapidity, whereas in proton-nucleus interactions of the same energy it is absent. In pA interactions, the first indications of the existence of two peaks in the quasirapidity distribution have been noted at $E_p^{\rm in}=400$ GeV (Ref. 45). However, they are not so pronounced as in πA collisions at $E_\pi^{\rm in}=200$ GeV (Ref. 62). Therefore, we shall make the comparison with the experiments using mainly data on πA collisions at $E_\pi^{\rm in}=200$ GeV.

In the adopted approximate variant of the description of the collective motion in the cluster, the assertions (20), (21), and (22), which determine the transition to observable variables, must be applied to each of the two parts of the cluster separately. Of course, this procedure is very rough, since the transition from the production of one cluster to two virtually independent clusters spans a wide interval in the energy scale.[33] Nevertheless, it appears justified, since it is applied to the part of the energy of the system which is directly expended on the production of new particles. In contrast, when applied to the cluster system as a whole (with allowance for the collective part of the energy), the procedure leads to results which do not agree with the experiments [compare, for example, the form of the functions $F(y)$ in Figs. 17 and 18 for $n_g=3$, 4, and 5].

Assuming that each of the two parts of the cluster decays isotropically in the rest frame for the particles produced by the decay of the cluster, we obtain the quasirapidity distribution $F(y)$ shown in Fig. 17. The masses and Lorentz factors of the parts of the cluster

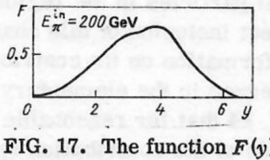

FIG. 17. The function $F(y)$ for the $\pi^- p$ process at $E_\pi^{\rm in}=200$ GeV.

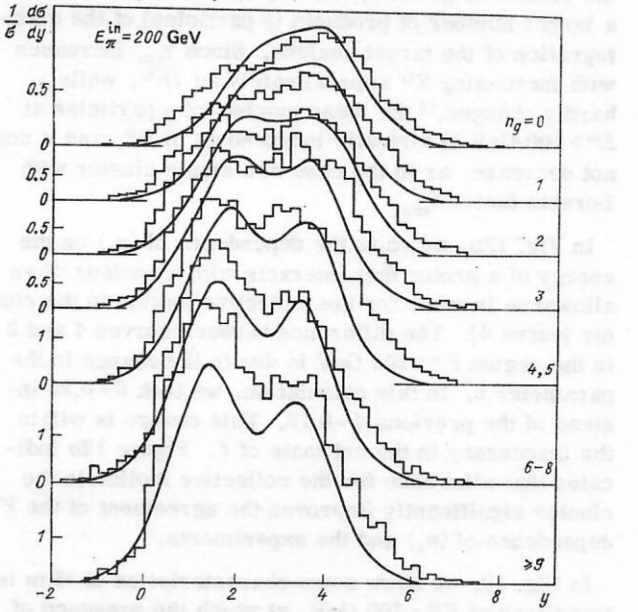

FIG. 18. The y dependence of $d\sigma/dy$ in the process π^-Em at $E_\pi^{\rm in}=200$ GeV.

in the laboratory frame are calculated in accordance with (60)–(63):

$$\mu_1=2.77,\quad \mu_2=2.26,\quad \gamma_1=23.4,\quad \gamma_2=5.67.\qquad (64)$$

By means of the distribution $F(y)$ it is impossible to give a detailed description of the quasirapidity distribution in π-N interactions, since it does not include particles of extracluster origin (leading particles), and it also does not take into account various factors such as possible fluctuations of the parameter δ and fluctuations of the decay volume. Such is not our aim at the present level of investigation. The task is to separate the principal effect, namely, the influence of the medium on the "bare" collective motion in the cluster and its ability to affect experimentally observable tendencies.

In Fig. 18, we show the y distributions for different n_g. In obtaining the model distributions, we assumed that the increase in the number of particles produced by the decay of the considered part of the cluster is proportional to the increase in its mass. It can be seen from Fig. 18 that with increasing n_g, i.e., with increasing range of the cluster in the nucleus, the distance between the centers of the parts of the clusters in rapidity space increases. This then leads to the appearance of two peaks in the $F(y)$ distribution. The best agreement with experiment is attained for $\eta=\frac{1}{4}$.

Another characteristic sensitive to the presence of collective motion in the cluster is the number n_g of gray prongs, which reflects the reaction of the medium to the production process taking place in it. Since the Lorentz factor γ_2 of the slow part of the cluster is smaller than the Lorentz factor $\gamma_{\rm cms}$ of its center-of-mass motion, its rate of expansion is higher than that of a single cluster with the same $\gamma_{\rm cms}$ without collective motion. Thus, in the presence of collective motion the number of nucleons of the nucleus which interact with the cluster is greater, and experimentally this leads to a larger number of products (g particles) of the disintegration of the target nucleus. Since γ_{cms} increases with increasing E^{in} approximately as $\sqrt{E^{in}}$, while γ_2 hardly changes,[18] the mean number of g particles at $E^{in} > 100$ GeV is virtually independent of E^{in}, and it does not decrease, as in the case of a single cluster with Lorentz factor γ_{cms}.

In Fig. 12b, we show the dependence of $\langle n_g \rangle$ on the energy of a proton that interacts with a nucleus when allowance is made for the collective motion in the cluster (curve 4). The difference between curves 4 and 2 in the region $E^{in} < 200$ GeV is due to the change in the parameter $\bar{\varepsilon}$. In this calculation, we took $\bar{\varepsilon} = 0.20$ instead of the previous $\bar{\varepsilon} = 0.18$. This change is within the inaccuracy in the estimate of $\bar{\varepsilon}$. Figure 12b indicates that allowance for the collective motion in the cluster significantly improves the agreement of the E^{in} dependence of $\langle n_g \rangle$ and the experiments.

In Fig. 19, we show some characteristics of πEm interactions at $E_\pi^{in} = 200$ GeV, at which the presence of collective motion in the cluster is not so strongly expressed. Here too there is completely satisfactory agreement between the results of the model and the experimental data.

These results of our investigation into the influence of collective motion in the cluster on the development of the multiparticle production process in matter permits the following conclusions to be drawn.

1. The description formulated in Ref. 18 of the internal collective motion in a cluster formed in hadronic collisions does not contradict the existing experimental data on hadron–nucleus interactions at energies of the order of a few hundred GeV.

2. Some characteristics of hadron–nucleus interactions are sensitive to the presence of collective motion in the cluster and can be used to investigate it.

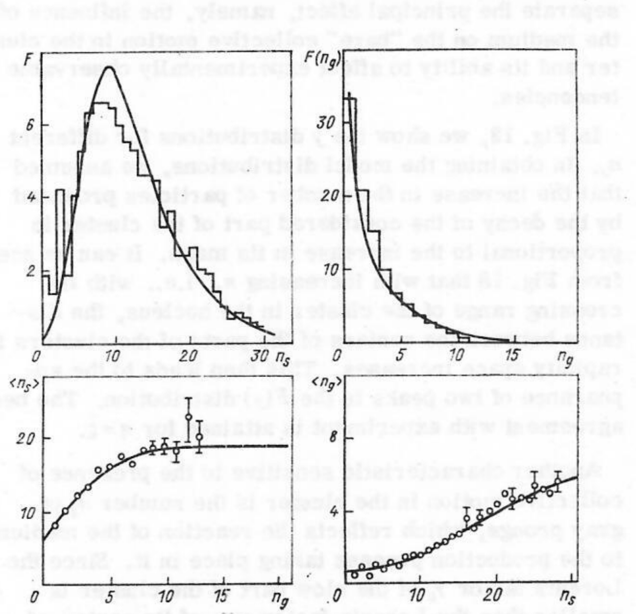

FIG. 19. Some characteristics of multiparticle production in the π^-Em process at $E_\pi^{in} = 200$ GeV with allowance for the collective motion in the cluster. The notation is the same as in the previous figures.

3. The value found for the probability of capture by the fast cluster of the gluon field of a nucleon by comparison with experiments is in agreement with the probability of production of compound systems in hadronic interactions determined from analysis of cumulative processes.[19]

Limitations of the Model. As we have seen, the level of the model is fully adequate to the level of the experimental data.

However, we cannot fail to see the limitations which it faces or could face.

Above all, it is clear that its accuracy is restricted by the accuracy of the data on the elementary interaction, the accuracy of the experiments on nuclear targets, and also the possibilities of the model itself.

Like any other, this model is not intended to explain absolutely all information on multiparticle production on nuclei and associated phenomena. This is due to the adopted system of assumptions and approximations, and also the formulation of the problem, all of which aim at a simplification within limits so as to have a solvable problem. Clearly, the approximations are different. With some it will no doubt be possible to live for a considerable time; others may, when necessary, be eliminated; yet others will probably prove resistant to organic inclusion in the formalism of the model and they must be regarded as additional, correction factors.

Let us discuss briefly some aspects of this question.

1. It is assumed in the model that pionization occurs in the first interaction of the incident hadron with a nucleon of the nucleus. Such an approximation is justified for describing the inclusive characteristics of the produced particles in the central region. But we then do not encompass channels of nonpionization nature, whose contributions to the region of the inclusive spectrum at large x and to events with low multiplicity are evidently dominant. Therefore, in the present formulation the model cannot pretend to a correct description of these characteristics.

However, it does in principle permit the inclusion of processes of this kind in its scheme. Since stripping of the gluon field does not occur in the nonpionization process, the properties which the incident hadron manifests in the next collision should not differ appreciably from its "normal" properties. Therefore, it can initiate pionization, which can then be described by the model. Since the cross section of the pionization channel is large compared with the nonpionization channel, the latter must be "suppressed" on a nucleus approximately as $A^{-1/3}$. Just such an A dependence is observed for the inclusive spectra of fast particles in the region $x \approx 0.7$ (Ref. 60). For the correct inclusion of this channel, it is necessary to have information on its contribution to the characteristics observed in the elementary interaction. It is shown in Ref. 14 that for reasonable assumptions about the magnitude of the contribution one can satisfactorily describe the changed spectra in the proton fragmentation region in pEm collisions at E_p^{in} =200 GeV. However, these estimates have as yet only methodological significance.

2. In the equations of motion (1) for a cluster in nuclear matter, we use $\langle k_{KN}\rangle$, the mean inelasticity of the KN interaction [or δ in Eqs. (49)]. Such a description corresponds to a major part of the information on hA interactions. However, some characteristics, for example, $D/\langle n_s\rangle$, can be influenced by fluctuations of $\langle k_{KN}\rangle$ or δ. The higher accuracy of future experiments will undoubtedly necessitate allowance for these fluctuations. It is readily seen that such allowance is already possible. Unfortunately, we do not yet have at our disposal data or reliable arguments concerning the nature and magnitude of these fluctuations.

3. As we have seen, the interaction of the leading particles is not taken into account in the model. This is justified by the results obtained in this approximation, the theoretical arguments given above, and also the facts adduced in the present review, which constitute a logical chain of indirect proof. However, the relative smallness of the effects produced by the interaction of the leading particles is not a reason for still ignoring them in future investigations. It seems to us that this interaction will not lead within nuclear distances to the formation of cluster-type objects, since valence quarks are the main component of the leading system. Therefore, the description of such interaction and its results goes beyond the scope of the model's formalism and must be taken into account additionally by means of other methods.

4. In the model, there is no treatment of the interaction with the nuclear matter of the recoil nucleons produced in the cluster–nucleon collisions. For this too there is good reason—such an interaction is a later stage in the general hierarchy of times when all the "pieces" have been performed in the relativistic "act." Schematically, the hierarchy of times can be presented in the following form: a) the fast relativistic process, which leads to multiparticle production and the occurrence of recoil nucleons; b) the slower transformation of the energy carried by the recoil nucleons into the energy of the g particles through the interaction of the former with the nuclear matter; c) finally, the completely slow process of de-excitation of the residual nucleus with the emission of low-energy b particles, which in some cases can be represented as "evaporation."

Thus, from the point of view of the fulfillment of the minimal conditions for a sufficiently clear understanding of the actual process of multiparticle production, the particularization of the mechanism of dissipation in which energy is transferred from the relativistic system, the cluster, to the nuclear matter in a subsequent stage in time is superfluous.

Therefore, to make the transition in the model to the number of g particles, which characterize the reaction of the target nucleus, we have used the energy-balance relation (24). As a consequence of this step we lose information about the spectral and angular characteristics of the "gray" particles.

Thus, in our opinion it is sufficient for the multiparticle production model if we know the total energy of the particles that are the fragmentation products of the target nucleus in at least a first and very reasonable approximation.

With regard to the study of the spectral and angular characteristics of these fragments, this problem is of independent interest and belongs really to nuclear physics of intermediate energies. The appropriate initial conditions for the problem are, as we have seen, formulated by means of the results obtained in the framework of the model described here.

In this connection, we make a comment. It seems to us that it is also incorrect to use a cascade scheme to describe the passage of the recoil nucleons through the nuclear matter and the exchange of energy with it. One of the arguments against such an approach is given in Ref. 19.

CONCLUSIONS

In discussing the most important aspects of multiple production, we have drawn a number of conclusions. We shall now recall the most important of these and then, without any pretence at completeness, mention as examples some important consequences.

1. By taking into account the space-time aspects of the process of multiparticle production in hadron–nucleus interactions one can formulate a phenomenological model with a high physical plausibility. It contains just one parameter, the inelasticity of the cluster–nucleon interaction, the value of which is determined by comparison with experiment.

2. The satisfactory agreement between the results of the model and a large body of qualitatively different experimental data means that we can regard its axioms as additional requirements on structural models of the hadron formulated on the basis of an analysis of a particular class of interactions, i.e., hadron–nucleus interactions.

3. The use of the approach in such a way leads to the conclusion that the two-component quark–gluon model satisfies these requirements.

At the same time, the simple parton model and the model of additive quarks are too crude and are not capable of explaining the principal characteristics of production in nuclear matter.

4. The results obtained in this direction of investigation can be used to construct new variants of the multiparticle production mechanism in the elementary interaction.

For example, two consequences of the analysis presented here were taken as the basis of such a model for pp collisions.[33] First, the possibility of using the single-cluster approximation in pA interactions right up to 200–300 GeV. Second, the proposition that the cluster expands in its own frame with near-luminal velocity. If the hadronization time is assumed to be universal, this is equivalent to approximate constancy of the decay volume of the cluster.

We shall now discuss two questions that illustrate (but do not exhaust) the possibilities based on the consequences that flow from the logical scheme of the proposed description.

The first question relates to the transition to the study of multiparticle production processes initiated by the collision of relativistic nuclei.

The confirmation of the hypothesis of the significantly different behavior of the two components of a hadron in production processes at high energies can provide the basis for the development of a collision mechanism of relativistic nuclei. In accordance with the conclusions about the properties of quarks and gluons, the mechanism of collision of overlapping parts of nuclei can be regarded as collective stripping of the gluon fields of the nucleons with the formation of one gluon cluster in the central region and two baryon clusters in the fragmentation regions of the nuclei. The first attempt to construct a description of the process in accordance with such a scheme was undertaken in Ref. 63. The results do not contradict the experimental data for either the pionization[64] or the fragmentation[65] part of the process.

It is natural to expect that when the energy is increased to several hundred GeV per nucleon a collective motion will develop in the pionization cluster analogous to the one in hp and hA collisions (see Sec. 3).

The second question is the problem of describing multiparticle production in hadron–hadron collisions at very high (up to 10^5 GeV) and even ultrahigh ($E^{in} > 10^5$ GeV) energies.

By now, quite a few experimental results indicate a change in the multiparticle production regime at energies $\gtrsim$100 TeV (see, for example, the reviews of Ref. 66). The most important conclusions drawn from analysis of these results are the following:

a) the inelasticity in the production processes becomes close to unity;

b) the leading particles do not carry such a high proportion of the energy, but their number increases;

c) the mean transverse momentum of the secondary particles increases;

d) the multiplicity increases sharply, and the law of variation of the multiplicity approaches $(E^{in})^{1/2}$;

e) an appreciable fraction of the energy of the hadronic cascade ($\gtrsim$20%) is transmitted to particles which are absorbed in matter several times less strongly than pions.

An important feature of the new production regime is that the transition to it appears as a threshold effect, and the qualitative changes in the characteristics of the process take place over a section of growth of the energy by only an order of magnitude.

Using the main results of the present review in their quark–gluon variant, we can arrive at a quite definite interpretation of these phenomena.

On the one hand, it follows from this description that the use of nuclei rather than protons as the target cannot be the reason for the effect. On the other hand, simple extrapolation of the model of Ref. 33 does not lead to the observed changes in the inelasticity and the nature of the leading-particle effect and gives a growth in $\langle n_s \rangle$ and $\langle p_\perp \rangle$ with increasing energy that is too slow (see the broken curves in Figs. 20a and 20b).

To reconcile the description of the process with the above conclusions, only one possibility remains. It is necessary to assume that at energies $E^{in} \gtrsim 100$ TeV the quarks shed the role of passive spectators and begin to interact inelastically with a large cross section. Such behavior of the qq interaction is neither strange nor unexpected. In contrast, it would be extremely surprising if it were found that the quark level is the last in the hierarchy of the structure of matter. The only question is that of the energy at which the next level in the structure of matter will be found. Judging from the experimental data, the corresponding threshold is surmounted at 10^4–10^5 GeV.

The threshold nature of the transition to the new regime is most clearly manifested in experiments which measure the mean absorption length of showers in a calorimeter as a function of the energy,[66] an abrupt change in the shower length from 700 to 1100 g/cm² in lead occurring between 50 and 100 TeV (Fig. 21). Since the mean quark momentum is $p_q \approx (1-\delta)p_h/3$, we have $\sqrt{s_{qq}} = 70$ and 100 GeV for $E^{in} = 50$ and 100 TeV. If the disintegration of a quark near the threshold takes the form of the production of a pair of heavy particles, the mass of such particles is $M \approx \sqrt{s}/2 \approx 35$–50 GeV (this is close to the theoretical estimates of the mass of the hypothetical W bosons).

The high "specific energy" of quark disintegration suggests that at least within one or two orders of magnitude in the energy above the threshold the produced system constitutes a very heavy cluster which is on the average at rest in the center-of-mass system of the colliding hadrons. Virtually all its mass is expended on the production of particles, and the decay is nearly isotropic. We shall call this a Q cluster, to distinguish it from the G clusters formed from the gluon fields of colliding hadrons.

The succession of multiparticle production regimes in the main channel is shown schematically on the energy scale in Fig. 22. At $E^{in} \approx 20$–200 GeV, the pioni-

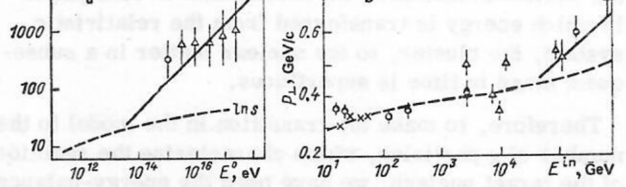

FIG. 20. The dependences of $\langle n_s \rangle$ (a) and $\langle p_\perp \rangle$ (b) on E^{in}.

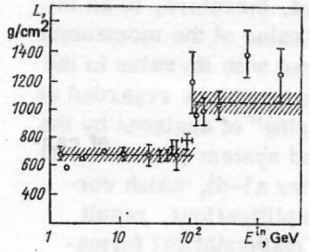

FIG. 21. Dependence of the mean shower length in lead on E^{in}.

zation process with production of one virtually equilibrium G cluster (see Fig. 22a) is predominant. At energies 200–300 GeV, collective motion arises in the G cluster, and this becomes stronger with increasing E^{in} (Fig. 22b). However, right up to 3–5 TeV in a pp collision the G cluster can still be regarded as a single object. Finally, at $E^{in} \gtrsim 5$ TeV, the main mass of the G cluster disintegrates into two parts which decay more or less independently (Fig. 22c). In the framework of the model of Ref. 33, these three stages succeed one another in a natural manner. The qualitative change in the production regime at $E^{in} \approx 100$ TeV is due to the formation of the Q cluster (Fig. 22d). If we adopt such a point of view, it is not difficult to explain the main features of this change.

The closeness to unity of the inelasticity follows trivially from the very definition of the Q cluster.

The change in the nature of the leading-particle effect can be seen from comparison of Figs. 22d and 22c. In the first case, the leading hadrons are produced on the hadronization of the system of valence quarks, which decay into two or three particles; in the second case, they are formed on the hadronization of part of the G cluster, which is emitted in the forward hemisphere. The number of charged particles produced by its decay is 10–12, and the Lorentz factor is appreciably smaller than the Lorentz factor of the leading system formed by the valence quarks. Thus, the number of leading particles increases, but they do not carry such a large proportion of the energy.

The mean transverse momentum and the mean multiplicity of the charged particles produced by the decay of a Q cluster can be estimated in accordance with the

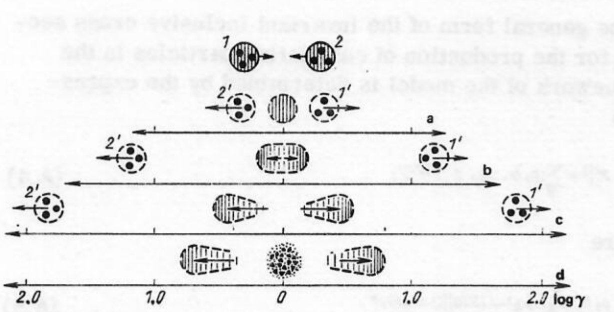

FIG. 22. Schematic representation of the qualitative changes in the multiparticle production mechanism with increasing energy: a) 10 to $(2-3)\times10^2$ GeV, b) $(2-3)\times10^2$ GeV to 3–5 GeV, c) 3–5 GeV to $n\times10$ TeV, d) $\gtrsim 100$ TeV. The G clusters are hatched.

scheme described in Ref. 33 by assuming that the connection between the energy density ε and the temperature T is determined by the relationship between ε and T for a quark–gluon plasma[67]:

$$\varepsilon \approx 2\cdot10^3 T^4.$$

The results of such an estimate for pions are shown in Fig. 20. The estimate of $\langle n_s\rangle$ takes into account the contributions from the decay of ρ mesons and the G cluster. Overall, the model describes satisfactorily the observed behavior.

What is the reason for the formation of long-range showers, i.e., the massive production of hadrons that interact weakly with matter? Among the known hadrons, charmed particles have such properties.[68] The appearance of them in large numbers confirms the assumption that a Q cluster is realized.

Indeed, the formation of the Q cluster is accompanied by a colossal release of energy in a small volume corresponding to the Lorentz-contracted volume of the nucleon. An estimate of the initial temperature of such states leads to $T = 1.7$ GeV at $E^{in} = 100$ TeV and $T = 3$ GeV at $E^{in} = 10\,000$ TeV. At such temperatures, the mass of the charmed quarks plays no part, and they are present in the quark–gluon plasma in the same amount as the u, d, or s quarks. Therefore, approximately a quarter of the entire energy of the Q cluster is allotted to them. As a result of the rapid fall in T due to the expansion of the cluster, the c quarks, which have a large mass, exchange energy weakly with the other components of the cluster and carry away the fraction of the plasma energy which they acquired during the earlier stage in the form of charmed particles.

Thus, the unfreezing of a new level in the structure at $E^{in} \approx 100$ TeV can be naturally included in the description of multiple processes adopted on the basis of an analysis of hadron–nucleus and hadron–hadron collisions at lower energies. All the main propositions are retained: the two-component structure of the hadron, the spatial expansion of the cluster, and the constancy of the time and the statistical nature of hadronization.

It cannot be doubted that the further study of this problem will lead to results which will be of exceptional importance for hadron physics.

APPENDIX

Application of space-time description of multiparticle production to processes of cumulative type

The model in Ref. 19 of cumulative production of particles is the result of using the space-time description of multiparticle production to interpret processes characterized by an extremal situation that is very different from the "average" event realized with the greatest probability. In the approach described in the present review, this case corresponds, at the least, to large values of the following two quantities that characterize the formation of the intermediate system and its decay.

1. The inelasticity k for the production of the cluster system and for its interaction with the nuclear nucleons tends to unity:

$$[illegible]_{A-H} \qquad \text{(A.1)}$$

2. In the final state, we consider configurations in which there is a produced particle whose momentum satisfies

$$p \gg \langle p \rangle; \quad p > p_{hh}^{\max}, \qquad \text{(A.2)}$$

i.e., it is not only much greater than its mean value but also exceeds the maximal value of the momentum permitted by the kinematics of the hadron–hadron collision.

A number of important consequences flow from the restrictions (A.1) and (A.2).

a) It follows from (A.1) that the produced cluster includes nucleons that collided at the beginning and nucleons captured by the cluster during its interaction with the nucleus. In contrast to the pionization cluster produced in the "average" multiparticle production event with baryon number $B=0$, such a cluster has $B \geqslant 2$. If allowance is also made for (A.2), then $B \geqslant 3$.

b) In contrast to the case of the pionization process, the production cross section σ_c of such a cluster system must be small, since this channel is rare, i.e.,

$$\sigma_c \ll \sigma_{NN}^{\text{in}}. \qquad \text{(A.3)}$$

Comparison with experiment gives $\sigma_c \approx (0.20-0.25)\sigma_{NN}^{\text{in}}$.

c) The fulfillment of the condition (A.2) presupposes the choice of a channel in which the number of degrees of freedom realized in the cluster system is small compared with pionization. Energy dissipation cannot proceed far in the system, and the process itself is completed in a time much shorter than in an "average" pionization event. This must be reflected, in particular, in an inability of the cluster state to change in size significantly from the initial size before its decay. Therefore, in a rough approximation it can be assumed that (A.3) also holds in the subsequent two or three collisions of the cluster with nuclear nucleons.

d) For calculations of processes that take place in, at least, the first two or three cumulation orders, it is not necessary to use the complicated equations (5), which presuppose the presence of a rapidly increasing cluster–nucleon interaction cross section. Ignoring the energy losses on the excitation of the residual nucleus, one can construct a fairly simple algorithm for calculating the characteristics of the process (see Ref. 19).

Of course, the differences between the cumulative process, regarded as a limiting case of an "average" event, as indicated under a)–d) above do not signify a radical change in the approach to the process. Here too it is assumed that the cluster which is formed in the initial event and survives for a certain finite interval of time is capable of undergoing further inelastic collisions and capturing nuclear nucleons. As before, this leads to a growth in its mass and, therefore, to an increase in the maximal possible value of the momentum of the produced particle compared with its value in the NN collision. Thus, the cumulation can be regarded as a space-time process of "gathering" of nucleons by the initial particle into the compound system (cluster) with increasing mass. The differences a)–d), which correspond to a limit and some simplifications, result merely in a modification of the mathematical formalism.

The unity of the approaches in the two cases becomes even more obvious if we consider also the method used to describe the decay of the state.

We recall that in describing the decay in the "average" event, we assumed that the normalized spectra of the particles produced by the decay of the clusters in the rest frames in hA and hp interactions are approximately the same [see (15) and the comments on it]. Of course, this assumption applied to the pionization part of the spectrum. Similarly, in the description of the invariant inclusive cross section for the production of cumulative particles it was also assumed in Ref. 19 that the spectra $F(x)$ in hA and hp interactions are the same, but now, as the actual problem requires, in the fragmentation part of the spectrum. Therefore, in this case the most natural variable is the Feynman variable $s = p_{\parallel}/p_{\parallel}^{\max}$, and the principle of identical spectra is realized by assuming scaling of $F(x)$ (at sufficiently high energies) of the "identity" at equal masses of the intermediate systems in hA and hp interactions (at low energies of a few GeV).

Thus, the cumulative model of Ref. 19 is a special limiting case of the space-time description of the development of multiparticle production in matter based on allowance for the most general physical principles as presented in this review. This is one of its most important differences from the other frequently discussed cumulation models, which are formulated specially and exclusively to explain this process; its very possibility is made to depend on definite properties of the target nucleus (for example, the presence of Fermi motion of its nucleons or the occurrence of fluctuations in the density of the hadronic matter in a small volume).

We describe briefly the structure of the model in Ref. 19 and the first results obtained by means of it.

The general form of the invariant inclusive cross section for the production of cumulative particles in the framework of the model is determined by the expression

$$R_A^{(i)} = \sum_n \rho_i(s, x_n, p_\perp) W_A^{(n)}, \qquad \text{(A.4)}$$

where

$$\rho_i(s, x_n, p_\perp) = (E/\sigma_{hh}^{\text{in}})\, d^3\sigma/dp^3 \qquad \text{(A.5)}$$

are the invariant inclusive densities for the production of particles of species i in hh collisions. Frequently, ρ_i are taken in the approximate factorized form

$$\rho_i(s, x, p_\perp) \approx F_i(s, x) \exp(-a_i p_\perp^2). \tag{A.6}$$

Scaling corresponds to $F(s, x) \approx \mathrm{const}(s)$.

In (A.4), $W_A^{(n)}$ are the partial cross sections for the production of compound systems with n "gathered" nucleons of a nucleus with mass number A. According to the model, they can be expressed in the form

$$W_A^{(n)} \approx \frac{\pi}{2} \frac{(\sigma_c\rho)^n}{\prod\limits_{\varkappa=1,\ldots,n}(\sigma_c\rho + a_\varkappa)} \left\{2R^2 + \sum_k^n C_k^{(n)} \frac{\gamma[2;\, 2(\sigma_c\rho + a_k)R]}{(\sigma_c\rho + a_k)^2}\right\}, \tag{A.7}$$

where $a_1 \equiv 0$,

$$C_{k\neq 1}^{(n)} = \prod_{\substack{\lambda=1,\ldots,n\\ \lambda\neq k}} (\sigma_c\rho + a_\lambda)\Big[\prod_{\substack{r=1,\ldots,n\\ r\neq k}} (a_k - a_r)\Big]^{-1}; \tag{A.8}$$

$$C_{k=1}^{(n)} = -\Big[1 + (-1)^n \sum_{k=2}^{n} C_k^{(n)}\Big]. \tag{A.9}$$

The quantities a_n are defined by

$$a_n = \frac{1}{\tau_0 C} \frac{[(n')^2 + 1]\, m_p^2 + 2n' m_p E_p^{\mathrm{in}}}{(E_p^{\mathrm{in}\,2} - m_p^2)}, \tag{A.10}$$

where E_p^{in} is the energy of the initial proton, m_p is its mass, and τ_0 is a parameter which determines the scale of the time interval during which the cluster remains capable of effectively emitting particles with momentum in the cumulative region.

We also emphasize that the values of x_n, the Feynman variable for the cumulative process, are calculated in the kinematics of the "gathering" mechanism of n-th order, i.e., there is an actual check on the fulfillment of the conservation laws.

Thus, the model has just the two free parameters σ_c and τ_0. Nevertheless, it is capable of explaining the spectra of the particles, their A dependence, the angular distributions, and even the absolute yield. Moreover, the model permits a unified calculation of the spectra of cumulative particles which differ in mass and other quantum numbers. The experiments which have been performed are analyzed in Ref. 19, where some predictions for reactions not yet investigated are made.

Unfortunately, some authors (see, for example, Ref. 69) have erroneously identified this model with the one described in Ref. 70, which is purely thermodynamic and does not have an absolute normalization (i.e., it does not give the absolute yields of the particles). This model is also lacking in a check on the actual fulfillment of the conservation laws, which permits considerable arbitrariness in the interpretation of experiments. Further, whereas in Ref. 19 the kinematics of the decaying cluster is calculated in each stage of the cumulation process, the basic relation (9) in Ref. 70, which leads to determination of the "critical" cluster velocity, is essentially only a condition for initiation of the cumulative process, and this, naturally, does not permit its description in the complete range of velocities in which the initial conditions permit it to act effectively.

The models described in Refs. 19 and 70 can be called identical only on the basis of a very superficial comparison of them, the identity being seen in the presence of the term "cluster" in both. But here too there is an important difference, since Gorenshtein *et al.*[70] use the notion of a pionization cluster, referring to our paper of Ref. 11 on multiple production (but, as is well known, well-developed pionization is absent at the energies $E^{\mathrm{in}} \approx 10$ GeV, at which the majority of experiments on cumulation have been performed), whereas the model described in Ref. 19 uses a cluster with $B \geqslant 3$. This circumstance is very important for the entire kinematics of the process.

Analysis of the available experimental data by means of the cumulation model described here has already, at its present phenomenological level, permitted the extraction of important information.

An estimate has been obtained for the time of formation of the cumulative particles in the compound system formed by the collective interaction of the hadrons.

In a wide range of initial energies it has been possible to establish for the first time the magnitude and basic behavior of the cross section for production of hadronic compound systems.

The conclusion that the results of decay of the intermediate system in hadron–hadron and in collective interactions are the same does not contradict the facts. This may mean, in particular, that an important part in the formation of the spectrum of the produced hadrons is played not only by the initial internal (parton) spectrum of the colliding hadrons (as is frequently assumed), but also by the nature of the hadronization process, i.e., the actual phase transition of quark-gluon matter into hadrons in the final stage of the reaction.

Thus, the study of cumulative processes in this approach can also be an additional source of information about the dynamics of strong interactions.

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