Coupling of collective and single-particle motions in transition nuclei

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The present state of the phenomenological model which describes the coupling of an odd nucleon to vibrations of the core is discussed. In nuclei with relatively small mean deformation, the observed phenomena can be described as a consequence of coupling of the motion of a quasiparticle in the spherical average potential to the vibrations of the average field. For large fluctuations of the shape, it is necessary to use a more general model that describes correctly the particle–hole structure of the states in transition nuclei. In the framework of nuclear field theory, which is a theory of many-particle systems that takes into account accurately the coupling of the single-particle and collective degrees of freedom, the concept of coupling of a particle to core vibrations is given a microscopic justification.

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INTRODUCTION

In recent years, nuclear spectroscopy has yielded interesting results on nuclei in transition regions. These results opened up new and interesting possibilities for testing theoretical models. Study of high-spin states has shown that to describe the observed phenomena it is necessary to take into account collective as well as single-particle degrees of freedom. The mechanism of coupling of these elementary modes can also be investigated in even nuclei by studying, for example, the phenomenon of back-bending. However, both modes are unavoidably present in odd nuclei, since in them the odd particle is separated in a natural manner from the main body of the nucleons participating in the collective motion. Thus, odd transition nuclei are the most suitable for studying the single-particle and collective modes and their coupling. The treatment can, at least in the part relating to the coupling of the odd particle to the core vibrations, be restricted to allowance for only the collective motion associated with the quadrupole deformation of the nucleus, which is the most important for transition nuclei.

It is convenient to divide the odd nuclei into those with "small" deformation ($\sqrt{\beta^2} \leq 0.2$) and those with large. This is due to the fact that weakly deformed nuclei can be treated in the framework of the model that takes into account the coupling of the quasiparticles in the spherical average potential to the quadrupole vibrations, as is done in Sec. 3. In nuclei with stable deformation of the average field, such an approach is not applicable. For axially deformed nuclei, it is necessary to take into account the coupling between the single-particle motion and the collective rotation. In this case, it is more convenient to use a deformed single-particle basis, in which allowance is already made for the static quadrupole deformation of the average field. Using such a basis of internal states, one can obtain promising results in the description of the coupling of rovibronic and single-particle degrees of freedom.[1] As a rule, our representation is based on a spherical single-particle basis even for nuclei having stable deformations. However, the general model presented in Sec. 5 provides us with an internal quasiparticle basis corresponding to the given collective motion. Thus, it is possible to take into account correctly the particle–hole structure of the excitations, which is particularly important for large shape fluctuations, for which the traditional methods cannot be used.

In Sec. 6, on the basis of nuclear field theory, the concept of particle–core coupling is given a microscopic justification. We begin with a brief introduction to this theory, discussing its main aspects. We show how in nuclear field theory it is possible to overcome the difficulties usually encountered in the simultaneous treatment of single-particle and collective degrees of freedom; namely, one can eliminate the overcompleteness of the basis, take into account correctly the Pauli principle, etc. We then investigate the influence of the Pauli principle on the particle–core coupling mechanism. We show that the so-called $j-1$ anomaly (see Sec. 6) is directly related to the Pauli principle acting between the odd nucleon and the core nucleons. We analyze the particle–core coupling model presented in Secs. 3 and 4 in terms of Feynman diagrams (Sec. 6), which shows clearly what kind of interaction processes (many-particle correlations) are included in the phenomenologically introduced models of particle–core coupling.

1. COUPLING OF THE MOTION OF AN ODD PARTICLE TO VIBRATIONS (PVC MODEL)

Questions concerning the coupling of collective modes to the single-particle motion arise naturally when one takes into account the circumstance that the collective excitations are constructed from the degrees of freedom of individual nucleons. From the quasiclassical point of view, the collective motion can be interpreted as a variation in time of the density of the nucleon distribution, which determines the average nuclear single-particle potential.[2] Therefore, the collective motion changes the average single-particle potential $V(r)$, which for surface vibrations of multipolarity λ in the lowest order is given by the expression[2]

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$$H_{PVC} = -k(r)\sum_{\mu}\alpha_{\lambda\mu}Y^{*}_{\lambda\mu}(\theta,\varphi), \tag{1}$$

in which the form factor $k(r) = r\partial V r)/\partial r$ has a sharp peak on the surface of the nucleus.

The total Hamiltonian of the generally used model describing particle–core coupling has the form

$$H = H_{\rm sp} + H_{\rm vib} + H_{PVC}. \tag{2}$$

The Hamiltonian (2) was used intensively in earlier years to describe odd nuclei.[3,4,5] A model Hamiltonian similar to (2) was also obtained in the framework of a microscopic theory with pairing and multipole forces (see below). Systematic investigations were made by Kisslinger and Sarensen,[3] Reehal,[4] and Borse[5] (see also Refs. 6–9). However, initially the PCV model did not give satisfactory results, for which there were two main reasons. First, in many cases, the spectra of the vibrational states of the even–even neighbors of the odd nuclei are strongly anharmonic. However, in the theortical treatment the even nuclei are handled in the harmonic approximation, which in microscopic calculations corresponds to the RPA. However, later investigations[10,11] showed that the results obtained in the framework of the PVC model for odd nuclei were very sensitive to the choice of the wave functions of the vibrational states of the even–even nuclei. Indeed, in the framework of a model describing the coupling of an odd particle to the core vibrations, good agreement was obtained[12] with the experimental data in the cases when the data on the core states were taken directly from experiments. In recent years, good results have also been obtained for high-spin states in transition nuclei[10] when the core states are correctly described.

The second reason for the originally unsatisfactory agreement between the results of the PVC model and the experimental data was the complete neglect of the Pauli principle acting between the valence nucleon and the core nucleons. In the usually employed diagonalization of the model Hamiltonian (2) in the basis $|j,(n)R;\,\mathrm{IM}\rangle$, which is constructed from n-phonon states of the core with total spin R and the states $|jm\rangle$ of the odd nucleon, it is difficult to take into account the Pauli principle. However, it can be taken into account accurately in a somewhat modified treatment of the coupling (1) in the framework of nuclear field theory, which is presented in Sec. 6. As will be shown there, when the Pauli principle is taken into account, the PVC mechanism also explains the occurrence of the $j-1$ anomaly (Secs. 4 and 6). In Sec. 2, we review the main aspects of the model of particle–core coupling. The main attention is devoted to the details of the model developed in Ref. 10. The specific feature of this model, which was responsible for its success, was the allowance for the Pauli principle acting between the odd valence particle and the core nucleons.

2. HAMILTONIAN OF THE COUPLING BETWEEN THE ODD PARTICLE AND THE CORE

We begin our treatment with a microscopic model that takes into account pairing and multipole residual forces, since this model has proved itself well in the description of spherical and deformed nuclei and one can have good hope of describing the main features of transition nuclei:

$$H = \sum_{jm} e_j a^{+}_{jm} a_{jm} - \frac{\varkappa}{4}\sum_{\mu} Q_\mu \overline{Q}_\mu - \frac{G}{4}P^{+}P; \tag{3a}$$

$$Q_\mu = \frac{1}{\sqrt{5}}\sum_{jj'}\langle j\,\|\,r^2 Y_2\,\|\,j'\rangle\,(a^{+}_{j}\overline{a}_{j'})_{2\mu}; \tag{3b}$$

$$P^{+} = \sum_{jm} a^{+}_{jm}\overline{a}^{+}_{jm}. \tag{3c}$$

Here, jm is a multiple index representing the set of quantum numbers $(nljm)$ characterizing the spherical single-particle state. In addition, the bar denotes conjugation with respect to the time, i.e., $\overline{a}_{jm} = (-)^{j+m}a_{j-m}$, etc.

The model of particle–core coupling is based on the following assumptions:

1) there exist collective modes associated with the coherent motion of the main body of the nucleons in the system, which is generally called the even–even core;

2) to a good accuracy, it can be assumed that the collective modes act on valence nucleons as a dynamical field and can rearrange the motion of particles which, as was originally assumed, move in orbits in the spherical average potential;

3) the back reaction of the valence nucleons on the collective modes can be ignored.

In accordance with these approximations, the quadrupole operator (36) for the particle–core system splits up into two terms:

$$Q_\mu = Q^{c}_\mu + q_\mu, \tag{4}$$

where Q_μ is the collective quadrupole operator, and q_μ is the quadrupole operator of the valence particle. Therefore, the Hamiltonian (1) can be split into three parts—the collective and single-particle parts and the particle–core interaction:

$$H = H_{\rm c} + h_{\rm sp} - \frac{\varkappa}{2}\sum_{\mu} q_\mu \overline{Q}^{c}_\mu - \frac{\Delta}{2}(P^{+}+P), \tag{5}$$

where the last term is obtained from the pairing Hamiltonian under the same assumptions as were made above. The gap parameter Δ characterizes the coupling between the pairing field of the core and the single-particle modes. A more detailed derivation of the Hamiltonian (5) is given in Refs. 13 and 14. We shall not be concerned here with questions of the consistency of the collective field and the single-particle motion; these play a central part in the fully microscopic theory of particle–core coupling. These questions are considered in Sec. 6. It must be emphasized that the collective quadrupole operator Q^{c}_μ which appears in (5) also contains anharmonicity effects, in contrast to the usual PVC model.[3,4] In the particle–core coupling model formulated in Ref. 10, the results of which are given below, the collective quadrupole operator is given by the expression

$$Q^{c}_\mu = F\,\{b^{+}_\mu\sqrt{N-\hat{n}} + \sqrt{N-\hat{n}}\,\overline{b}_\mu + \sqrt{7/4}\,(b^{+}\overline{b})_{2\mu}\}, \tag{5a}$$

where b^{+}_μ and $\overline{b}_\mu$ are the phonon operators, $\hat{n}$ is the phonon number operator, and F and N are the constants defined in Ref. 10.

The aim of the treatment that follows is to investigate the physical phenomena that can be described by means of the Hamiltonian (5). The presence of pairing forces complicates the problem, since the pairing operators P^+ and $P(3c)$ can generate particle states from hole states, and vice versa, which gives rise to a coupling between particle and hole states in addition to the coupling between the particles and the collective field of the core. In the two limiting cases of weakly deformed and strongly deformed systems, a Bogolyubov transformation gives a good approximation for diagonalizing the contribution of the pairing forces. In these cases, one can always introduce a static average field and determine quasiparticles corresponding to it. An approach to the solution of the problem in the general case is presented in Sec. 5. However, for many applications the collective field can be regarded as a small perturbation of the average spherical potential. In these cases, the pairing forces can be taken into account by transforming the spherical particle operators to quasiparticle operators. The treatment below will be restricted to this important case.

3. CORE-PARTICLE COUPLING IN TRANSITION NUCLEI WITH SMALL AVERAGE DEFORMATION

Eigenvalue Equation. For weakly deformed systems, we can ignore the quadrupole interaction in the lowest approximation, i.e., in the determination of the single-particle basis, and, therefore, diagonalize the pairing Hamiltonian by means of a transformation to quasiparticle operators that preserves the spherical symmetry. This procedure introduces slight changes in the Hamiltonian (5). Instead of the single-particle term h_{sp}, we have the quasiparticle term

$$h_{qp}=\sum_{jm}\tilde{e}_j\alpha^+_{jm}\alpha_{jm}, \tag{6}$$

which includes the quasiparticle energies

$$\tilde{e}_j=\sqrt{(e_j-\lambda)^2+\Delta^2}, \tag{7}$$

where λ is the chemical potential.

In addition, in the single-particle quadrupole operator we encounter the well-known BCS population factor:

$$q_\mu\to\tilde{q}_\mu=\frac{1}{\sqrt{5}}\sum_{jj'}(u_ju_{j'}-v_jv_{j'})\langle j\,\|\,r^2Y_2\,\|\,j'\rangle(\alpha^+_j\bar{\alpha}_{j'})_{2\mu}, \tag{8}$$

where the brackets denote the vector coupling of the quasiparticle operators with angular momenta j and j' to angular momentum 2. The amplitudes u_j and v_j, which determine the quasiparticle operators α^+_{jm} and $\bar{\alpha}_{jm}$, are given by the well-known expressions for spherical single-particle states.[15]

The Hamiltonian of the particle–core system now takes the form

$$H=H_c+h_{qp}-\frac{\varkappa}{2}\sum_\mu\tilde{q}_m\bar{Q}^c_\mu. \tag{9}$$

The natural basis for diagonalizing this Hamiltonian forms the states

$$|j,\,R;\,IM\rangle=\sum_{m,\,M_R}\langle jm,\,RM_R\,|\,IM\rangle\,\alpha^+_{jm}\,|0\rangle\,|\,RM_R\rangle, \tag{10}$$

where $|0\rangle$ is the quasiparticle vacuum and $|RM_R\rangle$ are the core states characterized by the angular momentum, to which other quantum numbers must be added in the general case. The vector coupling of the angular momenta j and R ensures rotational invariance of the nuclear Hamiltonian. The eigenstates of the Hamiltonian:

$$|j,IM\rangle=\sum_{jR}C^{(i)}_I(jR)\,|\,j,\,R;\,IM\rangle \tag{11}$$

are obtained by solving the equation

$$\sum_{jR}\Big\{E_R+\tilde{e}_j)\,\delta_{jj'}\delta_{RR'}-\frac{\varkappa}{2}(-)^{j'+R+I}\times(u_ju_{j'}-v_jv_{j'})\langle j\,\|\,r^2Y_2\,\|\,j'\rangle\times\langle R\,\|\,Q^c\,\|\,R'\rangle\begin{Bmatrix}j&j'&2\\R'&R&I\end{Bmatrix}\Big\}C^{(i)}_I(j'R')=E^{(i)}_IC^{(i)}_I(jR), \tag{12}$$

where the index j labels the solutions of this system. The collective properties of the core are included in the energies E_R of the core states and the matrix elements $\langle R\,\|\,Q^c\,\|\,R'\rangle$ of the collective quadrupole operator. In the particle–core model, these quantities are regarded as input parameters, which can be calculated in the framework of different collective models. In this sense, the core is an external field that influences the motion of the odd particle. To obtain an appropriate set of quantities characterizing the core, the phenomenological description is very effective. For example, an anharmonic boson Hamiltonian[16] obtained with allowance for the algebraic structure of the fermion operators was used successfully. The parametrization of the collective Hamiltonian used by Leander[17] is close to the original Bohr–Mottelson concept of collective quadrupole motion. These models make it possible to take into account all five quadrupole degrees of freedom describing the interconnection of the rotation and the σ and γ vibrations. The core characteristics obtained in this way are more realistic input parameters than those obtained in the harmonic approximation in the early publications[6-9] on the model of particle–core coupling. It was found[10,11] that the diagonal matrix elements $\langle R\,\|\,Q^c\,\|\,R\rangle$ play a particularly important part. They have a considerable influence on the scheme of the coupling of the particle to the collective field. In the harmonic approximation, they are equal to zero and, therefore, a discussion of this question cannot be based on such a simplified model. It is, however, necessary to note that the treatment in the framework of nuclear field theory presented in Sec. 6 is also based on the harmonic approximation for the collective modes. But the expansion in powers of the perturbation employed in nuclear field theory is very different from the direct diagonalization (12) of the Hamiltonian of the particle–core system. Thus, the assertion made above does not apply to the treatment in the framework of nuclear field theory.

Coupling Schemes. In odd transition nuclei such as Pd, Cd, Sb, and I, quasirotational bands were found from distinguished $E2$ transitions coupling the states of these bands. There were found to be quasirotational sequences of states characterized by the spin selection rule $\Delta I = 1$ like the well-known "decoupled" bands with $\Delta I = 2$. The appearance of the $\Delta I = 2$ quasirotational bands based on states of "opposite" parity like $h_{11/2}$ can be interpreted as a realization of the aligned, or decoupled, coupling scheme introduced by Stephens *et al.*[18] In the adiabatic limit (strong coupling) the particle follows the rotating deformed core, and the angular momentum of the particle is quantized with respect to the symmetry axis. In the rotating system, the adiabatic motion of the particle is perturbed by the Coriolis interaction, which tends to align the angular momentum of the particle along the angular momentum of the core. The decoupling of the particle from the deformation field is then observed as the $\Delta I = 2$ bands. However, this argument is based on the assumption of a strongly deformed field and cannot be applied to transition nuclei, in which the deformation is not stable.

Analysis of the structure of the bands in the framework of the particle–core model showed that the coupling scheme (strong coupling or alignment) can also be used in odd transition nuclei if one considers the properties of yrast states (lowest states in the E–I diagrams). This is because the yrast states reflect more fully than the remaining states the collective nature of the core states which make a coherent contribution to their structure. As a typical example, let us consider the level scheme obtained from the coupling of a quasiparticle in the state $h_{11/2}$ to the ^{106}Cd core. The characteristics of the core states were found by means of an anharmonic boson Hamiltonian.[16]

In Fig. 1, we show the energies of the states of the $h_{11/2}$ multiplets plus the core excitations as a function of the effective coupling parameter

$$A_j = \frac{\varkappa}{4}(u_j^2 - v_j^2)\langle j \| r^2 Y_2 \| j\rangle \langle 2_1^+ \| Q^c \| 2_1^+\rangle \qquad (13)$$

for the level $j = 11/2$.

This coupling constant A_j contains a diagonal matrix element of the collective quadrupole operator for the first 2^+ state and the population factor of the j state on which the band is based. These parameters determine the structure of the resulting bands. Going over in Fig. 1 from positive to negative values of $A_{11/2}$, we obtain a gradual transition in the order of the levels from the strong coupling scheme ($\Delta I = 1$) of the band to aligned ($\Delta I = 2$) bands, passing through an intermediate coupling scheme with doublet structure at $A_{11/2} \approx 0$. This figure is completely analogous to the one obtained by Stephens *et al.*[18] in the approach that takes into account the Coriolis mixing, in which the deformation β plays the same part as the coupling constant A_j.

The expected type of band structure can be determined by means of the following rules:

$$\left.\begin{array}{lll} 1) & A_j > 0, \ \Delta I = 1 & \text{sequence (strong coupling);} \\ 2) & A_j < 0, \ \Delta I = 2 & \text{sequence (aligned bands);} \\ 3) & |A_j| \ll 1, & \text{doublet structure (intermediate coupling).} \end{array}\right\} \qquad (14)$$

These rules relate to the structure of levels coupled

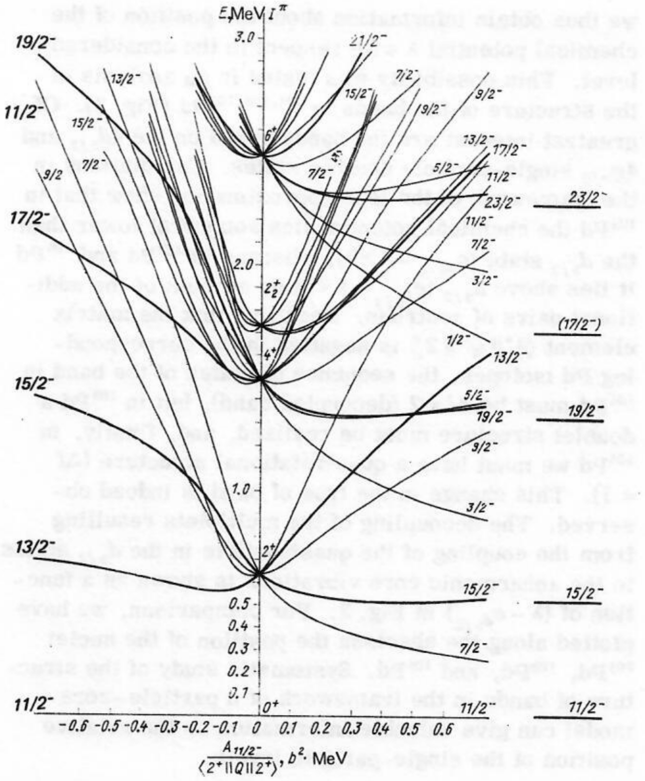


FIG. 1. Dependence of splitting of the multiplet formed by a quasiparticle in the state $h_{11/2}$ and core excitations on $A_{11/2}/\langle 2_1^+ \| Q^c \| 2_1^+\rangle$. On the right, we show the band based on the state $11/2^-$ and observed experimentally in ^{107}Cd. The oscillator length parameter is $b^2 = 1.01 \times A^{1/3}\ F^{-2}$.

by strong $E2(M1)$ transitions, i.e., they belong to yrast cascades. It can be seen from Fig. 1 that the decoupled band (11/2, 15/2...) is not very sensitive to variations of the coupling constant. This is a general result characteristic of any core model, wehreas the levels that do not belong to the yrast band (for example, the first 17/2 state in Fig. 1) depend on the values of A_j as well as on the details of the matrix elements of the core. The rules (14) can be obtained already in the first order of perturbation theory in the particle–core interaction [see (12)], as was emphasized in Ref. 11. It should be noted that the type of band structure that results does not depend on whether the collective motion in the core is rotational or vibrational. The following remark concerns the case of vanishing of the diagonal element $\langle 2_1^+ \| Q^v \| 2_1^+\rangle$, when the rules (14) cannot be applied. In this limiting case, the average deformation is zero and, therefore, the strong coupling scheme ($\Delta I = 1$ sequence) cannot be realized. Thus, the core–particle system always tends to a situation with decoupled bands. Analysis of the experimental structure of the bands by means of the rules (14) gives information about the population factor of the $(u_j^2 - v_j^2)$ state on which the band is based. This is due to the fact that the population factor essentially determines the sign of the coupling constant A_j (13) and, therefore, the type of coupling. In accordance with the relation

$$u_j^2 - v_j^2 = (e_j - \lambda) \Big/ \sqrt{(e_j - \lambda)^2 + \Delta^2} \qquad (15)$$

we thus obtain information about the position of the chemical potential λ with respect to the considered j level. This possibility was tested in an analysis of the structure of the bands in ^{101,103,105}Pd (Fig. 2). Of greatest interest are the bands based on the $4d_{5/2}$ and $4g_{7/2}$ single-particle neutron states. Calculations in the framework of the BCS approximation show that in ^{101}Pd the chemical potential lies somewhat lower than the $d_{5/2}$ state ($e_{d_{5/2}} - \lambda > 0$), whereas in ^{103}Pd and ^{105}Pd it lies above $d_{5/2}$ ($e_{d_{5/2}} - \lambda < 0$) on account of the additional pairs of neutrons. Recalling that the matrix element $\langle 2_1^+ \| Q^c \| 2_1^+ \rangle$ is negative in the corresponding Pd isotopes, the sequence of states of the band in ^{101}Pd must be $\Delta I = 2$ (decoupled band), but in ^{103}Pd a doublet structure must be realized, and, finally, in ^{105}Pd we must have a quasirotational structure ($\Delta I = 1$). This change in the type of band is indeed observed. The decoupling of the multiplets resulting from the coupling of the quasiparticle in the $d_{5/2}$ states to the anharmonic core vibrations is shown as a function of ($\lambda - e_{d_{5/2}}$) in Fig. 3. For comparison, we have plotted along the abscissa the position of the nuclei ^{101}Pd, ^{103}Pd, and ^{105}Pd. Systematic study of the structure of bands in the framework of a particle–core model can give valuable information on the relative position of the single-particle levels.

From the successful description of the properties of a given odd nucleus, one cannot always draw unambiguous conclusions about the shape of the even–even neighbors of the odd nucleus, data on which are used in the model of ^{197}Hg. The experimental level scheme, including states based on $i_{13/2}$, and also the predictions of two models, the particle–triaxial rotor (column a) and particle–anharmonic vibrator (column b),[20] are given in Fig. 4. It can be seen that the theoretical level schemes for core models as different as the rigid triaxial model and the model that is unstable with respect to γ vibrations agree qualitatively equally well with the experimental data. Similarly, the results of calculations of the $B(M1)$ and $B(E2)$ transition probabilities do not permit a choice to be made between these models. It is obvious that the matrix elements of the core are averaged in the particle–core model,

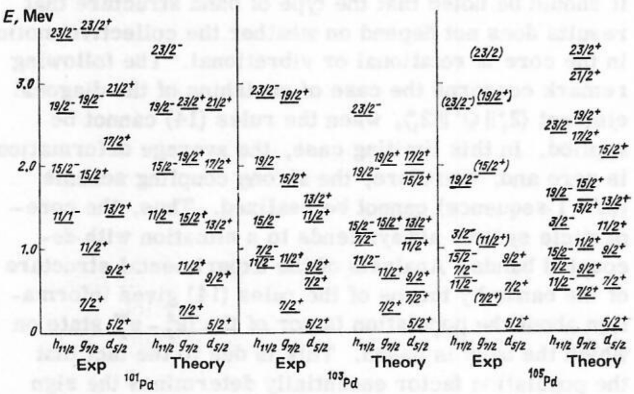


FIG. 2. Theoretical and experimental values of the energies of a number of yrast states of ^{101,103,105}Pd that belong to multiplets formed by a quasiparticle in $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$ states and core excitations.

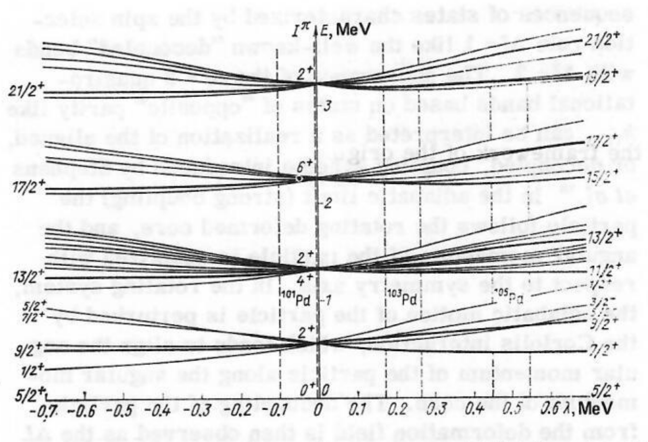


FIG. 3. Dependence of the splitting of a multiplet (quasiparticle in $d_{5/2}$ state and vibrational states of Pd) on the chemical potential λ. The position of the states of the 5/2 band in different odd Pd isotopes is indicated.

so that their specific features are not manifested. It is necessary to find transitions or details in the level scheme that permit a choice to be made between the two competing models. However, it is usually the case that neither the experimental data nor the theoretical calculations (convergence of the calculations with respect to truncation of the basis of states) are sufficiently accurate for such a program to be carried through.

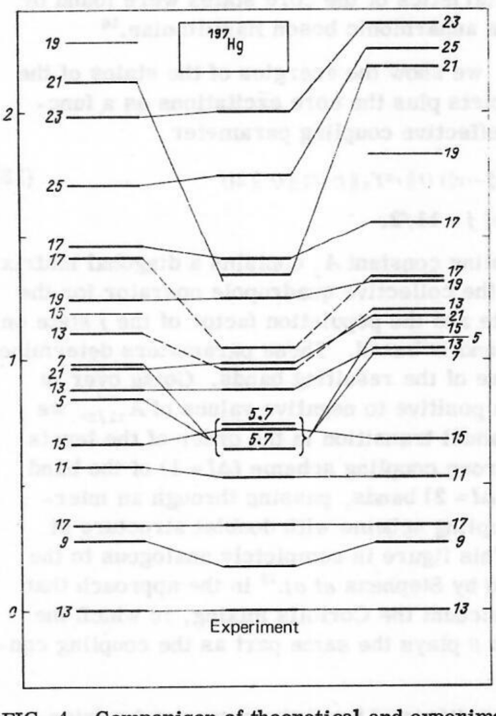


FIG. 4. Comparison of theoretical and experimental excitation energies of positive-parity states in ^{197}Hg. We give twice the spin of the levels and the excitation energies relative to the isomer state $I = 13/2$. In the calculations based on the model of an anharmonic vibrator, the coupling constant $\varkappa = 160A^{-5/3}$ MeV $\cdot\, b^{-2}$ and the experimental ^{198}Hg excitation energies were used. In the calculations based on the rotator model, the parameters $\beta = 0.14$, $\gamma = 38°$, $\Delta = 0.7$ MeV, and $\tilde{\lambda}_F = 0.5$ MeV were used.[25]

4. ANTISYMMETRIZATION EFFECTS

We here discuss the problem of taking into account the antisymmetrization between the outer nucleon and the core nucleons. Of course, this will take us outside the framework of the originally formulated phenomenological model of particle–core coupling, in which the core is an external dynamical field. An exact treatment of the antisymmetrization problem in the framework of nuclear field theory is given in Sec. 6. In the particle–core model, the so-called *kinematic* effects can be taken into account approximately if the collective wave function is expressed in terms of phonons whose microscopic structure is known (RPA or TD phonons). In the following exposition, it is assumed that the collective wave function $|RM_R\rangle$ is a superposition of many-phonon states $|(n)RM_R\rangle$, where n is the number of phonons.

Large exchange contributions are to be expected from the quadrupole interaction, which contains large collective matrix elements. The most important matrix elements, expressed in the basis $\alpha^+_{jm}|(n)RM_R\rangle$, have the form

$$V_{qQ}=-\frac{\varkappa}{2}\sum_{\mu}\langle jm,\ (n)\,RM_R\,|\,q_\mu\overline{Q}^c_\mu\,|\,j'm',\ (n')\,R'M'_{R'}\rangle. \tag{16}$$

Using the completeness of the basis and the fact that the collective quadrupole operator Q^c_μ does not depend on the single-particle states, we can rewrite Eq. (16) as

$$V_{qQ}=-\frac{\varkappa}{2}\sum_{\substack{\mu n''\\ R''M''_R}}\langle jm,\ (n)\,RM_R\,|\,q_\mu\,|\,j'm',\ (n'')\,R''M''_R\rangle \times\langle (n'')\,R''M''_R\,|\,\overline{Q}^c_\mu\,|\,(n')\,R'M'_R\rangle. \tag{17}$$

Let us concentrate on the matrix elements of the operator q_μ. In the quasiparticle representation, this operator has the form

$$q_\mu=\frac{1}{\sqrt{5}}\sum_{KK'}\{f^{(-)}_{KK'}\,(\alpha^+_K\overline{\alpha}_{K'})_{2\mu}+f^{(+)}_{KK'}\,[(\alpha^+_K\alpha^+_{K'})_{2\mu}+(\overline{\alpha}_{K'}\overline{\alpha}_K)_{2\mu}]\}, \tag{18}$$

where

$$f^{(-)}_{KK'}=(u_K u_{K'}-v_K v_{K'})\,\langle K\,\|\,r^2Y_2\,\|\,K'\rangle; \tag{19}$$

$$f^{+}_{KK'}=(u_K v_{K'}+u_{K'}v_K)\,\langle K\,\|\,r^2Y_2\,\|\,K'\rangle. \tag{20}$$

If the fermion structure of the collective wave function is not taken into account, then only the so-called *scatter term*, which is proportional to $f^{(-)}_{KK'}$, contributes to the matrix element (17). In this case, we obtain the general factorized form used in the derivation of Eq. (12):

$$V^{\phi\,\mathrm{fact}}_{qQ}=-\frac{\varkappa}{2\sqrt{5}}\sum_{\mu}f^{(-)}_{jj'}\langle jm,\ j'\overline{m}'\,|\,2\mu\rangle\,\langle (n)\,RM_R\,|\,\overline{Q}^c_\mu\,|\,(n')\,R'M'_R\rangle. \tag{21}$$

This is connected with the fact that the pairing term, which is proportional to $f^{(+)}_{KK'}$, does not conserve the number of quasiparticles. It makes a nonvanishing contribution only in the case when the collective wave function includes quasiparticle operators. We assume that the many-phonon state $|(n)RM_R\rangle$ is constructed from RPA phonons, i.e., it is a polynominal in the creation operators of the collective phonons;

$$b^+_{2\rho}=\sum_{ii'}[r_{ii'}\,(\alpha^+_i\alpha^+_{i'})_{2\rho}+s_{ii'}\,(\overline{\alpha}_i\overline{\alpha}_{i'})_{2\rho}],\quad \rho=0,\ \pm1,\ \pm2. \tag{22}$$

Since the pairing term in q_μ can annihilate not more than one phonon, to find the exchange contribution it is necessary to take into account the quasiparticle structure of only one of the phonon operators included in the intermediate state $|(n'')R''M''_R\rangle$. Therefore, we must separate one phonon from the state $|(n'')R''M''_R\rangle$, using the well-known technique of coefficients of fractional parentage[21]:

$$|(n'')\,R''M'_R\rangle=\sum_{\rho\tilde{R}\tilde{M}_R}\langle (n''-1)\ \tilde{R}\,|\}\,(n'')\,R''\rangle \times\langle\tilde{R}\tilde{M}_R,\ 2\rho\,|\,R''M''_R\rangle\,b^+_{2\rho}\,|\,(n''-1)\ \tilde{R}\tilde{M}_R\rangle. \tag{23}$$

The remaining $n''-1$ phonons are treated as a pure collective mode, whereas the separated phonon participates in the exchange of the outer nucleon with the nucleons of the phonon (22). Substituting (22) and (23) in (17), we obtain the following expression for the exchange term:

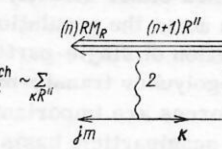
$$V^{\mathrm{exch}}_{qQ}=-\frac{\varkappa}{2\sqrt{5}}\sum_{ii'KK'R}f^{(+)}_{KK'}\,\langle (n)\,|\rangle\,(n+1)\,R''\rangle \times\langle RM_R,\ 2\rho\,|\,R''M''_R\rangle\,\langle 0\,|\,\alpha_{jm}\,[(\alpha^+_K\alpha^+_{K'})_{2\mu} +(\overline{\alpha}_{K'}\overline{\alpha}_K)_{2\mu}]\,\alpha^+_{j'm'}\,[r_{ii'}\,(\alpha^+_i\alpha^+_{i'})_{2\rho}+s_{ii'}\,(\overline{\alpha}_{i'}\overline{\alpha}_i)_{2\rho}]\,|\,0\rangle \times\langle (n+1)\,R''M''_R\,|\,\overline{Q}^c_\mu\,|\,(n')\,R'M'_R\rangle. \tag{24}$$

For simplicity, we assume that the RPA vacuum is simultaneously the quasiparticle vacuum, i.e., we shall ignore correlations in the ground state. This assumption is used implicitly in the derivation of the RPA equations in the usually adopted form (see, for example, Ref. 22).

Thus, after all the calculations, we obtain for the exchange matrix element

$$V^{\mathrm{fact}}_{qQ}=-\frac{\varkappa}{\sqrt{5}}\sum_{KR''}\langle (n)\,R\,|\}\,(n+1)\,R''\rangle \times\langle (n+1)\,R''M''_R\,|\,Q^c_\mu\,|\,(n)\,RM_R\rangle \times\langle RM_R,\ 2\rho\,|\,R''M''_R\rangle\,\langle km_k,\ j'm'\,|\,2\mu\rangle\langle km_k,\ jm\,|\,2\rho\rangle\,f^{(+)}_{kj}r_{kj}. \tag{25}$$

This matrix element can be represented schematically by means of the diagram

$V^{exch}_{qQ}\sim\sum_{\kappa R''}$ 

$f^{(+)}_{\kappa j}r_{\kappa j}\langle (n+1)R''\|Q^c\|(n')R'\rangle.$ (26)

The elementary process included in this exchange term was already described by Mottelson[23]: the phonon line, separated from the intermediate state R'', decomposes into two quasiparticles, one of which, the quasiparticle k, forms a new phonon together with the outer quasiparticle j', whereas the other becomes an outer quasiparticle with quantum number j. Of course, the diagram can be interpreted similarly if the parts played by j and j' are interchanged.

Thus, the fermion structure of the basis phonons is the origin of the additional contribution of the quadrupole interaction to the effective core–particle interaction. We shall not here discuss the problem of how the fermion structure influences the normalization and orthogonality of the collective states. These effects have been studied in Ref. 24. It is shown in Ref. 25 that the exchange interaction mentioned above can be obtained by means of a Dyson transformation from the fermion space to a boson space. We shall not use this representation, since the direct approach chosen above must also be used for the case when the collective wave function cannot be expressed in the form of an expansion in powers of the phonon operators.

In contrast to the ordinary nucleon–phonon interaction, the exchange interaction (23) has nonvanishing diagonal matrix elements, which have great importance in the description of the so-called states with anomalous coupling. In odd spherical nuclei, in which the states of "opposite" parity are partly populated, as, for example, in Te, Rb, Ag, Se, and Ba, a competition between $I=j$ and $I=j-1$ states is observed. This phenomenon, which has been called the $j-1$ anomaly, can be explained qualitatively by the influence of the exchange interaction on the energies of the states of the multiplet constructed from a phonon and quasiparticle in the state j.

The $j-1$ anomaly is explained as a consequence of the Pauli principle in Sec. 6 in the framework of nuclear field theory.

5. GENERALIZED MODEL OF QUASIPARTICLE–CORE COUPLING

Particle–Hole Structure of the Wave Function. In the particle–core model, a necessary assumption, which is frequenctly not stated explicitly, relates to the information on the population of the single-particle states in the core. It is clear that an additional particle can occupy only free single-particle states. An alternative basis for an odd nucleus can be constructed by the excitation of a hole in populated states. Both particle-like and hole states are observed is physical excitations. Because of pairing forces, these modes are coupled. As a rule, in the studies that employ the particle–core model the point of departure is the average nuclear potential, which is used either directly to obtain the necessary information about the population numbers, the sequence of occupation of single-particle states being known, or else a Bogolyubov transformation is first made when pairing forces are important. In the latter case, one obtains a quasiparticle basis, which is a special case of a particle–hole superposition referred by means of the chemical potential to a given number of particles. In Sec. 2, for example, we began with a spherical average potential and used accordingly spherical quasiparticles in order to obtain a basis of states for diagonalizing the Hamiltonian of the particle–core model. In strongly deformed nuclei, we construct quasiparticles in the deformed field, thereby taking into account the influence of quadrupole forces already in the average potential. But with regard to the particle–hole structure, we actually use the same assumptions as for a spherical average potential.

Strictly speaking, in transition nuclei there is no well-defined static average potential. This is due to the strong fluctuations of the shape of such nuclei. If the considered single-particle orbits are close to the Fermi surface, the particle–hole structure of the state is determined by the dynamics and is not based on the static potential. We shall illustrate this situation by considering deformed single-particle orbits for fixed deformation β but for variable parameter γ of the departure from axial symmetry: from $\gamma=0^\circ$ (prolate shape) to $\gamma=60^\circ$ (oblate shape). Figure 5 shows the family of states of the $h_{11/2}$ subshell in a nonaxial deformed potential of this kind. At the extreme points, at which the nuclei have an axial shape, the levels $\nu=1, 2, \ldots, 6$ are also labeled by the corresponding values of K, which in this case are good quantum numbers. Suppose the Fermi surface lies somewhat lower than $\varepsilon=0$. When the shape of the core fluctuates between $\gamma=0$ and 60°, which can actually happen in transition nuclei, the particle states associated with prolate shapes go over into hole states associated with oblate shapes because of the complete inversion of the sequence of K levels. Clearly, for such fluctuations of the core, the ordinary definition of the static particle–hole superposition corresponding to the BCS method cannot be used. The idea of the generalized quasiparticle–core coupling model consists of defining the particle–hole structure with respect to given properties of the core, and not with respect to a static average potential.

Eigenvalue Equation. The generalized model of particle–core coupling is based on the following expression for the wave function:

$$\Psi_{IM}^{(A)}=\sum_{jR}(jm,\ RM_R|IM)\times[u_I(jR)\,a_{jm}^{+}\Phi_{RM_R}^{(A-1)}+v_I(jR)\,\overline{a}_{jm}\Phi_{RM_R}^{(A+1)}]. \quad (27)$$

Here, a_{jm}^{+} and a_{jm} are the operators of creation and annihilation of particles in spherical single-particle states; the indices $A\pm1$ indicate that the states RM belong to families of collective states of the even–even nuclei that are the neighbors of the investigated odd-A nucleus; $u_I(jR)$ and $v_I(jR)$ are the amplitudes of the probabilities that the state $\Psi_{IM}^{(A)}$ is constructed from a particle coupled to the collective state $\Phi_{RM_R}^{(A-1)}$ of the core of $A-1$ nucleons or a hole coupled to the collective state of the heavier core $A+1$ nucleons. As in the

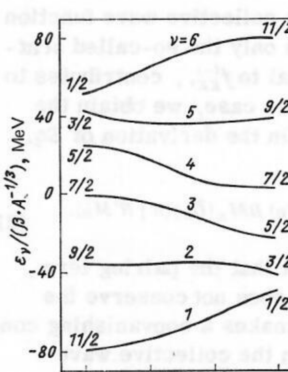


FIG. 5. Dependence of the energy ε_ν of single-particle states of the $j=11/2$ subshell in a triaxial deformed potential (β = const) on the parameter γ. In the limiting cases $\gamma=0$ and 60° we indicate the corresponding values of the projections of the angular momentum $K=1/2, \ldots, 11/2$, which in this case are good quantum numbers.

general BCS theory, these amplitudes determine the particle–hole structure of the quasiparticle excitations. However, the trial function (27) is more general as the point of departure for the generalized particle–core model, since the particle–hole structure is determined by the diagonalization of the total Hamiltonian, and it is not given by the static potential.

A number of authors[26,27] have obtained a closed system of equations for the amplitudes u and v. They used the circumstance that the state R has a collective nature, and they ignored all terms that do not make a coherent contribution. For the Hamiltonian (5), the following equations are obtained:

$$(H)\begin{bmatrix} u \\ v \end{bmatrix}_I = E_I\begin{bmatrix} u \\ v \end{bmatrix}_I, \tag{28}$$

where the amplitudes $u_I(jR)$ and $v_I(jR)$ form a vector. The matrix (H) is given by the expression

$$(H)=(H_{sp})+(H_c)+(H_{int}), \tag{28a}$$

where

$$(H_{sp})=\begin{pmatrix} e-\lambda & 0 \\ 0 & -e+\lambda \end{pmatrix};\quad (H_c)=\begin{pmatrix} E_c^{(A-1)} & 0 \\ 0 & E_c^{(A+1)} \end{pmatrix};\quad (H_{int}) = \begin{pmatrix} \Gamma^{(A-1)} & \Delta \\ \Delta & \Gamma^{(A+1)} \end{pmatrix}. \tag{28b}$$

The diagonal matrices contain $(e-\lambda)=\delta_{jj'}\delta_{RR'}(e_j-\lambda)$, $(\Delta)=\delta_{jj'}\delta_{RR'}\Delta$ and $(E_i^{(A\pm1)})=\delta_{jj'}\delta_{RR'}E_R^{(A\pm1)}$, the single-particle energies of spherical states, the parameters λ and Δ of the pairing correlations, and the energies of core states. The value of λ is determined by the mean number of particles by means of the condition

$$N(\lambda)=\langle (A+1)_{g.s}|\hat{N}|(A+1)_{g.s}\rangle=\sum_{jI}(2I+1)v_I^2(j,\ R=g.s)+1, \tag{29}$$

which is analogous to what is obtained in the framework of the general BCS approach. The quadrupole field occurs in the nondiagonal matrix elements $\Gamma^{(A\pm1)}$, which are given by the expressions

$$\Gamma^{(A+1)}_{jR,\,j'R'}=-\frac{\varkappa}{2}(-1)^{j'+R+I}\begin{Bmatrix} j & j' & 2 \\ R' & R & I \end{Bmatrix} \times\langle R(A\pm1)\,\|\,Q^c\,\|\,R'(A\pm1)\rangle\, g_{jj'}. \tag{30}$$

The diagonal blocks of the matrices (28) are the same as those obtained by considering the coupling of a pure particle or hole state to the collective state R. The pairing field generates a coupling of the particle and hole blocks, making the particle–hole structure different in the general case for different states $\Psi_{IM}^{(A)}$.

The core characteristics, $(E_R^{(A\pm1)}, \langle R(A\pm1)\,\|\,H'(A\pm1)\rangle$ and Δ), which play the part of a core field, are, as before, assumed known (see Sec. 2).

The use of Eq. (27) entails difficulties relating to the correct treatment in such an approach of the particle–hole structure of the states. To elucidate the nature of the difficulty, let us consider the simple case when the space of collective states includes only the ground states $R=0$ of the nuclei $A\pm1$. Then the matrices $\Gamma^{(A\pm1)}$ vanish, and the solutions of Eq. (27) take the form

$$E^{\pm}_{I=j}=\pm\sqrt{(e_j-\lambda)^2+\Delta^2}. \tag{31}$$

The positive solution can be interpreted as the ordinary quasiparticle energy. Considering the corresponding amplitudes, we find that

$$\begin{bmatrix} u^- \\ v^- \end{bmatrix}_j=\begin{bmatrix} -v^+ \\ u^+ \end{bmatrix}_j, \tag{32}$$

i.e., the solutions are related by particle–hole conjugation. Thus, the negative solution determines a quasihole state with energy $-E^+_{I=j}$. The quasiparticle excitations in which we are interested belong to positive eigenvalues, whereas we do not need the quasihole excitations for our purposes.

Unfortunately, in the general case the situation is much more complicated, since there do not exist simple symmetry relations analogous to those used in the trivial case (31) making it possible to divide the eigenstates of Eq. (27) into two classes, namely, states describing real excitations in the odd system and redundant states. However, one can construct an adequate quasiparticle basis for given core field which can also be used to formulate selection rules that make it possible to separate single-quasiparticle excitations.

Construction of Quasiparticle Basis. For simplicity, we assume that the core of $A+1$ and the core of $A-1$ nucleons have the same characteristics. The general case is considered below. First, we introduce the adiabatic Hamiltonian (the notation is as before and we have omitted the indices $A\pm1$)

$$(H_{af})=\begin{pmatrix} e-\lambda-\Gamma & \Delta \\ \Delta & -(e-\lambda+\Gamma) \end{pmatrix}. \tag{33}$$

It is obtained from the Hamiltonian (H) [see (28)] in the limit of infinitely large mass parameter, i.e., when all the energies E_R of the core states tend to zero. This corresponds to the adiabatic approximation, when the collective motion (the frequencies are equal to $E_R/\hbar$) is assumed to be slow compared with the single-particle motion.

The corresponding quasiparticle basis for given core field is determined as the set of positive-energy solutions of the Hamiltonian H_{af}, which is part of the total Hamiltonian. In contrast to the total matrix (H), the matrix (H_{af}) has an important symmetry—it is antisymmetric under the transformation

$$\sigma=\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix};\quad 1-\delta_{jj'}\delta_{RR'}, \tag{34}$$

i.e., $(\sigma H_{af}\sigma^+)=-(H_{af})$. It follows from the form of the trial wave function (27) that the matrix σ corresponds to the particle–hole transformation $a^+_{jm}\leftrightarrow a_{jm}$. Because of this symmetry, the eigenvalues of (H_{af}) form pairs, as in the simple case (31). The adiabatic Hamiltonian (H_{af}) can be diagonalized in two stages. First, for given spin I it is necessary to diagonalize the block $(e+\Gamma)$ of the matrix (33). We denote the corresponding eigenvalues by $\varepsilon_{\nu I}$ and the amplitudes by $C_{\nu I}(jR)$. In the next stage, the 2×2 matrix (in the transformed basis)

$$(H_{af})_{\nu\nu'}=\begin{pmatrix} \varepsilon_{\nu I}-\lambda & \Delta \\ \Delta & -\varepsilon_{\nu I}+\lambda \end{pmatrix}\delta_{\nu\nu'} \tag{35}$$

is diagonalized, giving the eigenvalue pairs

$$E_{\nu I}^{(\pm)}=\pm\sqrt{(\varepsilon_{\nu I}-\lambda)^2+\Delta^2} \tag{36a}$$

and the amplitude pairs

$$\left.\begin{aligned}\begin{bmatrix}u^{+}_{\nu I}\\ v^{+}_{\nu I}\end{bmatrix}&=C_{\nu I}(jR)\left[{}^{1}/_{2}\left(1\pm\frac{\varepsilon_{\nu I}-\lambda}{E^{+}_{\nu I}}\right)\right]^{1/2};\\ \begin{bmatrix}u^{-}_{\nu I}\\ v^{-}_{\nu I}\end{bmatrix}&=\begin{bmatrix}-v^{+}_{\nu I}\\ u^{+}_{\nu I}\end{bmatrix}.\end{aligned}\right\}\qquad(36b)$$

Thus, the positive and negative solutions can again be interpreted as quasiparticle and quasihole solutions, respectively, in the field of the core, which is characterized by the quantities Γ and Δ, as in the simple case considered above.

When the core field is, for example, the field of an axial rotator, the well-known quasiparticle solutions for an axial deformed potential are obtained, and the field Γ is equal to

$$(\Gamma)_{jR,\,j'R}=-\hbar\omega_0\beta(-1)^{j'+R+I}\begin{Bmatrix}j & j' & 2\\ R' & R & I\end{Bmatrix}\times\sqrt{(2R+1)(2R'+1)}\begin{pmatrix}R & R' & 2\\ 0 & 0 & 0\end{pmatrix}\bar{q}_{jj'},\qquad(37)$$

where $\bar{q}_{jj'}$ is the reduced matrix element of the quadrupole operator in units of the oscillator length. Note that the quasiparticle states determined by the amplitudes (36) are not internal states of the deformed nucleus in the usual sense. In our approach, the collective and single-particle degrees of freedom are treated simultaneously, and therefore the quasiparticle solutions (36) correspond to the total adiabatic wave function, i.e., in the special case of the function

$$(D^{I}_{MK}\alpha^{+}_{K}+(-1)^{I+K-1/2}D^{I}_{M-K}\alpha^{+}_{-K})|\,BCS\rangle,\qquad(38)$$

where the operator α^{+}_{K} generates a deformed (internal) quasiparticle state. Nevertheless, we shall call the eigenstates (H_{af}) quasiparticle states, bearing in mind that they are equivalent to the total adiabatic wave function, in which the collective and single-particle degrees of freedom are intertwined.

Nonadiabatic Effects. The nonzero energies of the core states E_R, which are not included in the adiabatic Hamiltonian (33), lead to a mixing of the quasiparticle states (of the positive solutions), and for sufficiently high energies of the collective states to a mixing of the quasiparticle and quasihole states (of the positive and negative solutions). For the axial rotator considered already, the influence of the energies of the core states is well known. In accordance with

$$E_R=\frac{\hbar^2}{2\Im}\hat{R}^2=\frac{\hbar^2}{2\Im}(\hat{I}-\hat{j})^2=\frac{\hbar^2}{2\Im}(\hat{I}^2+\hat{j}^{22})-\frac{\hbar^2}{\Im}\hat{j}\hat{I}\qquad(39)$$

they lead to the Coriolis interaction contained in the term $\hat{j}\hat{I}$.

The general procedure for treating these forces is as follows. Under the assumption that there exists a deformed quasiparticle vacuum, which is conserved during the collective rotation of the system, the Coriolis interaction will lead to mixing of the quasiparticle states (the positive solutions), whereas the quasihole states must be omitted. Therefore, the approach based on allowance for the Coriolis interaction is actually, if the centrifugal term $\hat{j}^2$ is not taken into account, equivalent to diagonalization of the Hamiltonian (28) in the subspace of quasiparticle states (positive solutions) or to projection of the Hamiltonian of the particle–core model onto this subspace and the subsequent solution of the remaining eigenvalue equations. Such an approximation can be used directly to study the coupling of an odd particle to an arbitrary core field, for example, to a triaxial rotator, a γ-unstable core,[28] or to a core executing anharmonic vibrations, the basis introduced above being used. In practice, it is necessary to represent the part of the Hamiltonian including the core energy (H_c) [Eq. (28)] in the quasiparticle representation, using the amplitudes (36), and diagonalize it after the addition of the diagonal quasiparticle energies $E^{+}_{\nu I}$ (35). Before we illustrate the method in some model examples, let us consider the general problem of the choice of the solutions. The projected solutions constructed by analogy with the treatment of the Coriolis interaction have a well-defined quasiparticle structure. From this point of view, all the eigenstates $\Psi^{(A)}_{IM}$ found from the exact diagonalization of the Hamiltonian (28) have an admixture of quasihole components. Therefore, the assumption made in the Coriolis approach, namely, the existence of a vacuum state, is not exact in the general case. However, the projected solutions can be used to choose exact eigenfunctions in which we are interested: We regard each solution $\Psi^{(A)}_{TM}$ which overlaps strongly with one of the projected solutions as a single-quasiparticle + collective state, which corresponds directly to a real excitation of this type in the odd nucleus.

It is clear that this scheme for selecting the solutions can be used only in the cases when the projected solutions approximate sufficiently well the exact solutions $\Psi^{(A)}_{IM}$, i.e., when the main nonadiabatic effects are already contained in the projected Hamiltonian.

Applications. First of all, we compare the results of the Coriolis treatment of an axial rotator with the results of complete diagonalization and the choice of physical solutions by means of the overlapping criterion.

The axial deformed core is characterized by the parameters $\hbar\omega_0=230$ MeV, $\beta=0.28$, $\hbar^2/2\Im=0.0133$ MeV, and $\Delta=0.9$ MeV. We consider the influence of rotation of the core on the motion of an odd particle in the state $i_{13/2}$. The chemical potential λ is situated 1.73 MeV below the spherical state $i_{13/2}$. The energies of the different excited states (measured from the energy of the state $I=11/2$) are given as functions of the angular momentum in Fig. 6.

The eigenvalues for the projected states (dashed curve) agree with the results of the calculation Coriolis mixing when the centrifugal term is included. The physical solutions (continuous curve) were selected by means of the criterion of overlapping with the projected solutions (the overlap integral must be greater than 0.7). The adiabatic limit corresponds to the rotational energies $\hbar^2/2\Im I(I+1)$. The energy for the the complete solution systematically exceeds the results of the calculations with Coriolis mixing.

The largest effect is observed for the band based on the state $K=11/2$, which is the one closest to the Fermi surface, and decreases with increasing distance

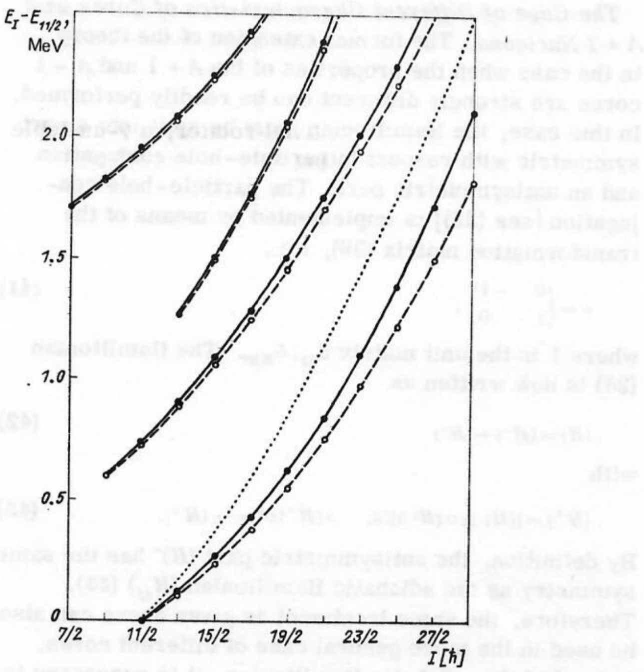


FIG. 6. Coupling of $i_{13/2}$ quasiparticle to a rigid axial rotator. The excitation energy (measured from the $I = 11/2$ ground-state energy) is shown as a function of the angular momentum. The open circles are the excitation energies obtained using the single-quasiparticle solutions. The black circles are obtained by diagonalizing the Hamiltonian H. The states that are usually grouped into bands are connected by continuous and dashed curves, respectively.

from it. This weakening of the influence is due to the change in the quasiparticle structure with increasing angular momentum: To increase the collective angular momentum, it is necessary to excite the core, and in this case the quasiparticles will be determined relative to a modified vacuum. This effect is manifested in the fact that the admixtures of quasihole states (negative solutions) become more and more appreciable with increasing angular momentum. A similar reducing mechanism was already discussed in Ref. 29. In our calculations, this dynamical effect gives for yrast states a quantity of the order of 10% of the experimentally observed value, which is too small to explain the weakening of the Coriolis mixing in strongly deformed nuclei.[30]

We now consider two models for the core field that are relevant to transition nuclei. We compare the coupling of quasiparticle in the state $h_{11/2}$ to a $\gamma = 30°$ triaxial rotator with the coupling of this quasiparticle to a γ-unstable core. The potential energy surface for the second case is shown in Fig. 7. We analyzed only the energies of the projected solutions, since the general particle–triaxial rotator model is already based

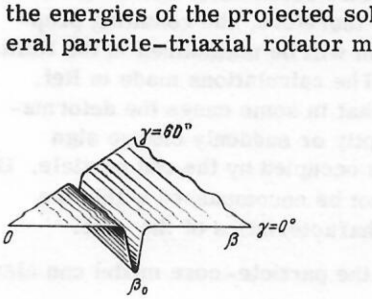


FIG. 7. Potential energy surface for γ-unstable core.

on the Coriolis approach.[31] The resulting excitation energies $E_I - E_{11/2}$ for yrast states are shown as functions of the position λ of the Fermi surfaces in Fig. 8. The parameters of the calculations are given in the caption to the figure. Since the spectrum does not depend on the sign of λ, only the halves of the symmetric diagrams are shown.

Quasiparticle–core coupling leads to similar results, especially near $\lambda = 0$. The differences between the excitation energies are mainly due to the energies of the core states, which for a $\gamma = 30°$ triaxial core are given by the expression (in units of $\hbar^2/2\mathfrak{I}$)

$$E_{nR} = (3/4)\,[R\,(R+4) + 3n\,(I-n)], \tag{40}$$

where the quantum number n of the band is restricted by the condition that $R - n$ must be a positive integer, and for a γ-unstable core $E_{\nu R} = (9/4)\nu(\nu + 3)$, where $\nu = 0, 1, 2, \ldots$ are the seniorities of the phonon multiplets. The unusual inversion of the 11/2 and 9/2 levels is encountered in both models and must therefore be regarded as an indication of appreciable departures from axial shape, though not necessarily as a consequence of static γ deformation.

In the region near ^{190}Pt, large deviations from axial shape are expected. It would be interesting to use an odd particle to obtain information about these shapes. We investigated the coupling of an $i_{13/2}$ quasineutron to the two types of core considered above. In Fig. 9, the calculated energies are compared with the experimental scheme of the positive-parity levels in ^{191}Pt (Ref. 32). The experimental excitation energies of the states with $R'' = 0^+, 2^+, 2^+, 2^+, 4^+, 4^+, 6^+, 6^+, 3^+, 8^+$, and 10^+ in ^{192}Pt are taken instead of the theoretical values of the energies of the core states, where they are ascribed to the lowest excitations of a $\gamma = 30°$ rotator and a γ-unstable core, respectively. The energies of the remaining states with $R \leq 14$, which are

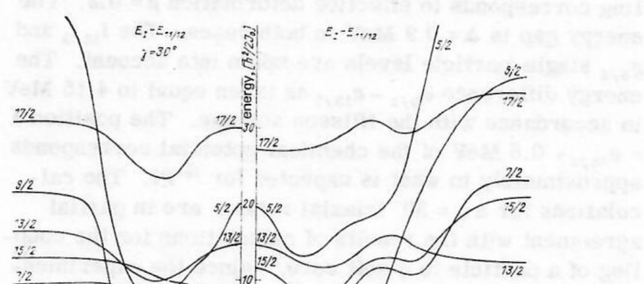


FIG. 8. Coupling of $h_{11/12}$ quasiparticle to $\gamma = 30°$ triaxial core (on the left) and γ-unstable core (on the right). The excitation energies (relative to the $I^\pi = 11/2^-$ state) are shown as functions of the chemical potential ($e_{11/2} = 0$). The energies E_I depend only on the absolute values $|\lambda|$ of the chemical potential. The unit of energy is $h/2 = 204\beta^{-2}A^{-7/3}$ MeV; the gap parameter is $\Delta = 135/A$ MeV, and the deformation is $\beta = 7.5A^{-2/3}$. For comparison, we have plotted on the abscissa the positions of the Nilsson states ε_k ($K = 1/2, \ldots, 11/2$) for the same deformation β but with $\gamma = 0^0$.

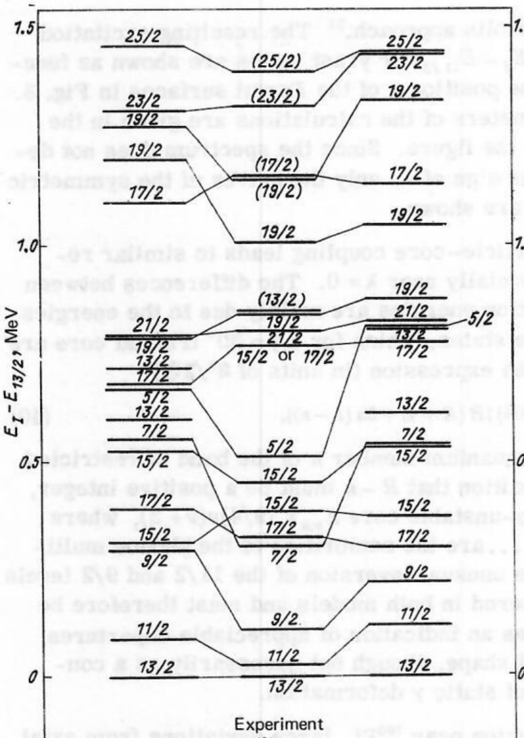


FIG. 9. Excitation energies E_I of even-parity states in ^{191}Pt calculated relative to $(13/2^+)_1$. The calculated energies (a) and (b) are obtained with projected Hamiltonian with allowance for the single-particle states $i_{13/2}$ (0 MeV) and $g_{9/2}$ (4.15 MeV) and core states up to the value $R = 14$ of the collective angular momentum. The gap parameter is the same as in Fig. 8. The scheme (a) corresponds to a triaxial core with $\gamma = 30°$, whereas scheme (b) corresponds to a γ-unstable core. In both cases, $\beta = 0.2$.

also taken into account, are taken from the relations (40) and (41), respectively. The strength of the coupling corresponds to effective deformation $\beta = 0.2$. The energy gap is $\Delta = 0.9$ MeV in both cases. The $i_{13/2}$ and $g_{9/2}$ single-particle levels are taken into account. The energy difference $e_{9/2} - e_{13/2}$ is taken equal to 4.15 MeV in accordance with the Nilsson scheme. The position $\lambda = e_{13/2} + 0.6$ MeV of the chemical potential corresponds approximately to what is expected for ^{191}Pt. The calculations for a $\gamma = 30°$ triaxial rotator are in partial agreement with the results of calculations for the coupling of a particle to a soft core,[33] since the experimental values of the energy were substituted. Good agreement with the experimental data is obtained for both the triaxial rotator and the γ-unstable core. The form of the spectrum definitely rules out the possibility of axial shape for the core, but it is hard to say which shape—triaxial rotator or γ-unstable core—is more realistic. Thus, we have arrived at the conclusion already drawn above, namely, that only a very detailed analysis of the wave functions by means of spectroscopic factors and the probabilities of electromagnetic transitions, which are the most sensitive to the differences between the quadrupole matrix elements of the two models, can answer this question.

The Case of Different Characteristics of Cores with A ± 1 Nucleons. The formal extension of the theory to the case when the properties of the $A + 1$ and $A - 1$ cores are strongly different can be readily performed. In this case, the Hamiltonian must be split into a part symmetric with respect to particle–hole conjugation and an antisymmetric part. The particle–hole conjugation [see (32)] is implemented by means of the transformation matrix (36), i.e.,

$$\sigma = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{41}$$

where 1 is the unit matrix $\delta_{jj'}, \delta_{RR'}$. The Hamiltonian (28) is now written as

$$(H) = (H^-) + (H^+) \tag{42}$$

with

$$(H^{\pm}) = [(H) \pm \sigma (H) \sigma]/2; \quad \sigma (H^{\pm}) \sigma^+ = \pm (H^{\pm}). \tag{43}$$

By definition, the antisymmetric part $(H)^-$ has the same symmetry as the adiabatic Hamiltonian (H_{af}) (33). Therefore, the same treatment as given above can also be used in the more general case of different cores. Instead of the adiabatic Hamiltonian, it is necessary to take the antisymmetric part (H^-), and replace the core Hamiltonian (H_c) by the symmetric part (H^+). The matrices $(H^{\pm})$ are given by the expressions

$$(H^-) = \begin{pmatrix} e - \lambda + E_c^- + \Gamma^+ & \Delta \\ \Delta & -(e - \lambda + E_c^- + \Gamma^+) \end{pmatrix}; \tag{44}$$

$$(H^+) = \begin{pmatrix} E_c^+ + \Gamma^- & 0 \\ 0 & E_c^+ + \Gamma^- \end{pmatrix}, \tag{45}$$

where

$$E_c^{\pm} = \frac{1}{2} (E_c^{(A-1)} \pm E_c^{(A+1)}); \quad \Gamma^{\pm} = (\Gamma^{(A-1)} \pm \Gamma^{(A+1)})/2. \tag{46}$$

From the physical point of view, direct use of this formalism is problematic. The particle–core model is based on the assumption that the presence of the additional particle does not significantly change the collective properties of the core. But this assumption is obviously not satisfied in the considered case, since the addition of two particles on the transition from a core with $A - 1$ nucleons to a core with $A + 1$ nucleons significantly changes important quantities. The mechanism of such transitions can be understood by means of Strutinsky's method,[34] in which one takes into account the influence of the odd particle on the core properties. For example, it was found in an investigation of the even and odd isotopes of Hg (A = 185–187) that blocking of an orbit by the particle leads to a radical change in the equilibrium shape of the system. Such effects of dynamic polarization cannot be described by means of the formalism given above. The different characteristics of the $A \pm 1$ cores enter the model in an averaged form and, therefore, the resulting properties of the odd system will be maintained in the limits defined by the cores. The calculations made in Ref. 33 showed, however, that in some cases the deformation can increase abruptly or suddenly change sign when a definite orbit is occupied by the odd particle. Of course, this case cannot be encompassed within the limits defined by the characteristics of the core.

Let us consider how the particle–core model can also be used in these individual cases of transition nuclei. The simplest way of taking into account the effects of dynamic polarization is to modify the matrix Γ by including empirical polarization factors in the quadrupole core–particle interaction. With this aim, we introduce in Eq. (30) an additional factor $(p_j + p_{j'})/2$, i.e.,

$$\Gamma^{(A\pm1)}_{jR,\,j'R'} \to (p_j + p_{j'})\,\Gamma^{(A\pm1)}_{jR,\,j'R'}/2, \quad (47)$$

where the polarizabilities p_j are parameters that can change the value and sign of the quadrupole matrix elements of the core, depending on the quantum number j of the single-particle orbit. In practice, the p_j must be chosen in accordance with the results of calculations by Strutinsky's method for blocked single-particle level j. This method was used to describe ^{123}I. The even–even neighbors of this nucleus are ^{122}Te and ^{124}Xe. The energies of the lowest 2^+ states of these nuclei, 564 keV and 354 keV, respectively, demonstrate the significant difference in the properties of these cores. It follows from calculations by Strutinsky's method[36] for ^{123}I that polarization effects play an important part. Accordingly, the polarization factors for the single-particle levels $4g_{9/2}$, $4d_{5/2}$, $4g_{1/2}$, $4s_{1/2}$, $4d_{3/2}$, and $5h_{11/2}$ included in the calculations were taken equal to $p_j = -2$, 1, 1, −1, −1, and −1, respectively. The calculated level scheme is shown in Fig. 10 together with the experimental data,[36] which reveals excellent agreement between the theoretical results and the experimental data.

It should be noted that the generalized particle–core model contains a number of new possibilities compared with the traditional approaches. In particular, this model makes it possible to consider the case of large shape fluctuations, when the structure of the quasiparticle can change both in accordance with the shape of the nucleus, which varies during the collective motion, and as a consequence of the finite velocity of

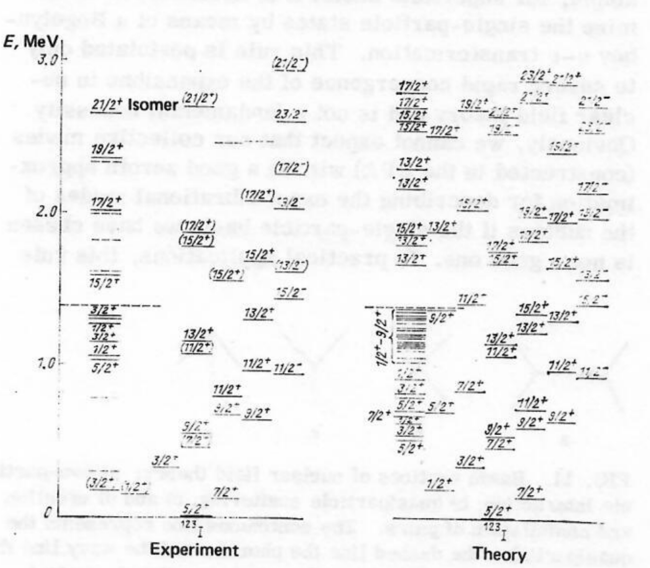


FIG. 10. Comparison of experimental and theoretical level schemes of ^{123}I. The calculations include the collective states of ^{122}Te $(A-1$ core) and ^{124}Xe $(A+1$ core).

the collective motion. A further extension of the formalism is possible in the direction of investigating intersection of bands, which is manifested in the phenomenon of *back-bending*.

6. NUCLEAR FIELD THEORY

The aim of this section is to give a microscopic justification of the particle–core model developed in the previous sections essentially at the phenomenological level. In the microscopic theory of particle–core coupling, one usually encounters two difficulties associated with the microscopic structure of the collective vibrational mode (the phonon). The single-particle excitations and the collective modes are constructed from the same nucleon degrees of freedom. As a consequence, we encounter the following two problems:

1) a basis that includes the single-particle (quasiparticle) and collective (vibrational) states is overcomplete;

2) the nucleons that participate in the quasiparticle and phonon excitations are identical (Pauli principle).

Fortunately, one can show that both problems are resolved in the so-called *nuclear field theory*.[37-39] The name "nuclear field theory" was originally introduced by the Copenhagen group in a series of papers (see Ref. 39 and references there), in which the coupling of a particle to vibrations was considered in the framework of perturbation theory based on a diagram technique. In 1967, Mottelson first pointed out the possibility of taking into account in such a treatment the influence of the Pauli principle acting between the odd nucleon and the nucleons participating in the collective motion.[23] He showed that the effects of the Pauli principle are very important in the description of the spectroscopic properties of weakly excited states of spherical and odd transition nuclei (see below). The correct treatment of the Pauli treatment principle is one of the fundamental achievements of nuclear field theory. Originally, this approach was developed by the Copenhagen group for some simplified and exactly solvable models.[37,39] The justification of the approach from the point of view of many-body theory was given in Refs. 38 and 40. Finally, in Refs. 41 and 42 nuclear field theory was derived rigorously by means of functional integration.

The idea on which nuclear field theory is based in as follows. The spectrum of excitations of a nucleus contains not only single-particle excitations but also a set of collective modes, i.e., surface, pairing, spin, and isospin vibrations. Thus, it is very natural, and in practical calculations convenient, not to describe the nuclear system exclusively in the fermion (Fock) space, but also to include appropriately collective modes (phonons) in the unperturbed basis. Therefore, in addition to the degrees of freedom of the individual nucleons in the single-particle (quasiparticle) states $|\alpha\rangle = a^+_\alpha|0\rangle$, a collective field is introduced in nuclear field theory. The quanta of the collective field are phonons satisfying Bose statistics, and they are characterized by creation, b^+_n, and annihilation, b_n, operators of phonons. In the first stage (in the first approximation) the single-particle (quasiparticle) excitations and the phonons are assumed to be independent excitation modes, i.e., $[a_\alpha^+, b_n] = 0$, etc., and the unperturbed Hamiltonian, which is given by the expression

$$H_0 = H_{sp} + H_{ph}, \tag{48}$$

where

$$H_{sp} = \sum_\beta \tilde{e}_\beta a_\beta^+ a_\beta; \tag{49}$$

$$H_{ph} = \sum_n \omega_n b_n^+ b_n, \tag{50}$$

includes the independent motion of the quasiparticles and the phonons, respectively. However, if these additionally introduced phonons are intended for describing collective excitations of the nucleus, they must be constructed from the degrees of freedom of the nucleons. But then the two types of elementary excitation mode are no longer independent, and the coupling of the modes, the so-called particle–vibration coupling

$$H_{PVC} = -\frac{\varkappa}{2} \sum_\mu Q_\mu^{ph} Q_{-\mu} (-1)^\mu \tag{51}$$

is introduced by means of the residual interaction[1)]

$$H_{QQ} = -\frac{\varkappa}{4} \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}. \tag{52}$$

Here, $Q_\mu = q_\mu$ is the mass quadrupole operator in the quasiparticle representation determined by Eq. (17), and Q_μ^{ph} is the collective part of the quadrupole operator, i.e., the part that can be expressed in terms of the phonon operators:

$$Q_\mu^{ph} = Q_\mu^{ph} (b^+, b). \tag{53}$$

For example, if the collective vibrational modes are determined in the random phase approximation, the collective part of the quadrupole operator is simply a linear combination of the phonon creation and annihilation operators and is written in the form

$$Q_\mu^{ph} = \sqrt{\frac{2}{\varkappa}} \sum_n \Lambda_n (b_{n\mu}^+ + \bar{b}_{n\mu}), \tag{54}$$

where Λ_n is the strength of the particle–vibration coupling and can be expressed as follows in terms of the characteristics of the single-particle states[41]:

$$\Lambda_n = \left[\frac{2}{5} \sum_{ab} \frac{(f_{ab}^+)^2 (\tilde{e}_a + \tilde{e}_b) \omega_n}{[(\tilde{e}_a + \tilde{e}_b)^2 - \omega_n^2]^2}\right]^{-1/2}. \tag{55}$$

Here, ω_n is the frequency of the n-th root of the random phase approximation determined from the RPA dispersion relation

$$f(\omega_{ji}) = 1 \tag{56}$$

with

$$f(\omega) = \frac{\varkappa}{10} \sum_{ab} \frac{f_{ab}^{(+)2} (\tilde{e}_a + \tilde{e}_b)}{(\tilde{e}_a + \tilde{e}_b)^2 - \omega_n^2}. \tag{57}$$

---

1)For reasons of simplicity and correspondence with the first part of this paper (Secs. 1–5), we restrict ourselves here to a separable QQ interaction. But the results are also exact for the general case of an arbitrary two-particle interaction. In addition, we denote the single-particle quadrupole operator by Q_μ to indicate that it acts on all nucleons, including those of the core.

The total Hamiltonian of nuclear field theory is given by the sum[41,43,44]

$$H_{NFT} = H_{sp} + H_{ph} + H_{QQ} + H_{PVC}. \tag{58}$$

It contains in addition to the total fermion Hamiltonian $H = H_{sp} + H_{QQ}$ the free Hamiltonian H_{ph} of the collective modes [Eq. (50)] and the coupling of the single-particle (quasiparticle) and collective degrees of freedom H_{PCV} [Eq. (51)]. [The Hamiltonian (58) is derived rigorously in Ref. 41.]

The Hamiltonian (58) is considered in nuclear field theory in the framework of perturbation theory using a diagram technique in which the Pauli principle is taken into account appropriately. This treatment using perturbation theory cannot be implemented, however, in the usual manner. This is due to the additionally introduced collective modes; first, because the basis employed, which includes single-particle and collective states, is strongly overcomplete; second, the Pauli principle is violated. In addition, in the total Hamiltonian (58), the correlations already included in the definition of the collective modes are taken into account twice. This is already clear from the construction of the Hamiltonian (58) and is a direct consequence of the overcompleteness of the basis. In nuclear field theory, the overcompleteness of the basis and the double allowance for the correlations, which are already included in the collective modes, are eliminated by means of definite restrictions imposed on the diagrams that are formed to handle H_{NFT} perturbatively (see Refs. 37–41). These restrictions are absent in the usual Feynman diadiagram technique. The main elements of the diagram treatment of H_{NFT} (58) are illustrated in Fig. 11. The rules for constructing the diagrams in nuclear field theory are as follows.

1. The Hartree–Fock–Bogolyubov contribution of the original two-particle interaction must be included in the single-particle Hamiltonian H_{sp} (49), which ensures stability of the unperturbed ground state. For example, for superfluid nuclei it is necessary to determine the single-particle states by means of a Bogolyubov u–v transformation. This rule is postulated only to ensure rapid convergence of the expansions in nuclear field theory and is not a fundamental necessity. Obviously, we cannot expect that our collective modes (constructed in the RPA) will be a good zeroth approximation for describing the exact vibrational modes of the nucleus if the single-particle basis we have chosen is not a good one. In practical applications, this rule

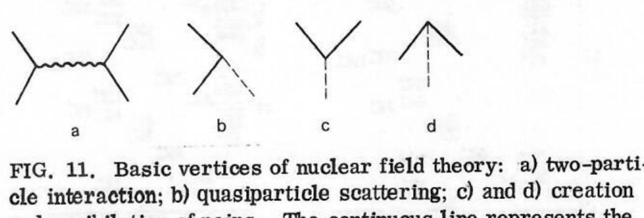


FIG. 11. Basic vertices of nuclear field theory: a) two-particle interaction; b) quasiparticle scattering; c) and d) creation and annihilation of pairs. The continuous line represents the quasiparticle, the dashed line the phonon, and the wavy line the two-particle interaction (a). The corresponding dynamical contributions are as follows: a) $\varkappa/2\, f_{\alpha a}^{(\pm)} f_{ab}^{(\pm)}$; b) $\Lambda f_{ab}^{(-)}$; c) and d) $\Lambda f_{ab}^{(+)}$.

merely means that our single-particle basis must be a good zeroth approximation for excitations of single-particle type.

2. The vibrational modes (phonons) must be constructed microscopically from the fermion degrees of freedom (for example, in the RPA). This is necessary in order to know what correlations are included in the definition of the collective modes [see the rule (4)]. In nuclear field theory, in addition, one can use exact vibrational modes of the nucleus, i.e., the phonon frequencies ω_n and the coupling strengths Λ_n can be taken from experiments.[2)] In this case, all the phonon self-energy diagrams must be eliminated.

3. The two-particle residual interaction H_{QQ} (52) and the particle–vibration coupling Hamiltonian H_{PCV} (51) can act in all orders, giving diagrams of different complexity and order.

4. To avoid the double allowance, the diagrams that are already included in the definition of the collective modes [the rule (2)] must not be taken into account in the nuclear field theory. For example, if the collective modes are determined in the RPA, the loop diagrams must not be taken into account (Fig. 12). (*The loop diagrams* describe processes in which a pair of quasiparticles is created and then annihilated without participation in other interactions of the quasiparticles.)

Thus, the diagrams in Fig. 13b are allowed, whereas the diagrams in Fig. 13a must be eliminated in nuclear field theory. It must be noted that although the RPA is frequently very convenient, it is not the most general way for determining the collective vibrational modes used in nuclear field theory. When a more general determination of the collective modes is employed, it is necessary to eliminate from the treatment in the framework of nuclear field theory a class of diagrams larger than the loop diagrams [see also the comments at the end of the rule (2)].

5. In the initial and final states, the diagrams included in nuclear field theory, which are now called *Mottelson diagrams*, can include fermions and phonons, except for the quasiparticle configurations that can be replaced by a combination of collective modes. (For example, if RPA phonons are used, no two-quasiparticle states having the quantum numbers of the RPA phonons can be included in the initial and final states of the Mottelson diagrams.)

With these restrictions, nuclear field theory is an exact many-particle description of finite Fermi systems and corresponds more accurately to the physical situation existing in nuclei than the usual description by means of fermion diagrams that include only fermion degrees of freedom. Actually, it is these restrictions that make the description in the framework of nuclear field theory more appropriate than the usual

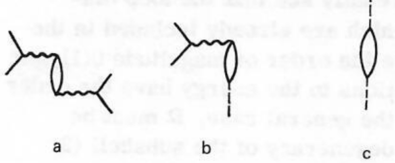


FIG. 12. Loop diagram of the lowest order of magnitude (second order) that must be eliminated from the diagrams considered in nuclear field theory.

treatment of residual forces by means of diagrammatic perturbation theory. For example, an appreciable fraction of the residual interaction is already included in the definition of the collective modes, and the corresponding diagrams that must be eliminated from the treatment in nuclear field theory (the loop diagrams) are, as it happens, the ones that are most strongly divergent. Thus, the expansion in nuclear field theory must converge much more rapidly than the corresponding diagram expansion in powers of the two-particle interaction, which in many cases actually becomes meaningless. Since nuclear field theory uses expansions, we must consider what is the small parameter of the theory. For a schematic two-level model, it was shown in Refs. 39 and 46 that nuclear field theory generates an expansion in powers of $1/\Omega$, where $2\Omega = 2j+1$ is the degeneracy of the subshell. This result must be understood as follows. If $\varkappa$ is the coupling constant, the RPA equations (determining the collective modes) depend only on the constant $\rho = \varkappa\Omega$, which is appreciably smaller than the distance ε between the shells (otherwise, the unperturbed ground state would be unstable). Therefore, with respect to the RPA modes (unperturbed) the residual interaction has the order

$$\varkappa = \rho/\Omega = O(1/\Omega). \tag{59}$$

Further, one can show[39,46] that the particle–phonon coupling constant Λ has the order of magnitude

$$\Lambda = O(1/\Omega^{1/2}). \tag{60}$$

In addition, in the Feynman diagrams each closed fermion loop presupposes free (independent) summation over all fermion states of the loop. This gives a factor Ω in the value of the diagram. Indeed, using Eqs.

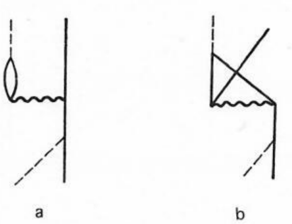


FIG. 13. Forbidden (a) and allowed (b) diagrams describing quasiparticle-phonon interactions. Diagram (a) is eliminated, since it contains the loop diagram in Fig. 12b. This diagram is included in the diagram in Fig. 15a. Diagram b, which is an exchange diagram, is allowed. (This diagram can be eliminated if the exchange part of the QQ forces is included in the RPA).

[2)]The coupling constants Λ_n can be determined from the experimental values of $B(E_\lambda;\ \lambda \to 0_1^+)$ (Ref. 45).

(59) and (60), we can readily see that the loop diagrams (see Fig. 2), which are already included in the unperturbed basis, have the order of magnitude 0(1), and the first-order corrections to the energy have the order of magnitude $1/\Omega$. In the general case, Ω must be regarded as the mean degeneracy of the subshell $(2j+1)$. In many cases, the degeneracies of the subshells are such that the corrections of order $1/\Omega$ already give a qualitative, and frequently even a quantitative description of the spectrum of the low-lying excited states of nuclei.[39,43,47] The approach described above is not restricted to the use of only separable forces. In Refs. 38, 40, and 41, nuclear field theory was developed for Fermi systems with a general two-particle interaction. The Hamiltonian of nuclear field theory (58) and the corresponding rules (1), ..., (5) for constructing the diagrams can be rigorously derived[41,42] by means of the technique of functional integration. We refer the reader who wishes to become acquainted with the details to Refs. 38–41.

Let us summarize: Nuclear field theory is an exact many-particle description adequate for the physical situation existing in nuclei. The theory is not restricted to odd nuclei, and it gives a unified description of even and odd nuclei. Applied to even–even nuclei nuclear field theory shows that anharmonicity is generated to a considerable extent by particle–vibration coupling.[43,48] Thus, particle–vibration coupling plays an important part in all nuclei.

Below, the framework of nuclear field theory, we study the mechanism of particle–core coupling introduced phenomenologically in Sec. 2. In addition, we shall show that in some cases the term of leading order in the expansion in nuclear field theory describes the basic features of the spectrum of the low-lying excited states of nuclei.

The j − 1 Anomaly. We begin by discussing the properties of the quasiparticle–phonon multiplet.

In many vibrational nuclei, one observes a family of collective states with excitation energy approximately equal to the energy of the quadrupole vibrational mode of the neighboring even–even nuclei.[1,2] A typical example is shown in Fig. 14. The diagrams of the leading order $[0(1/\Omega)]$, which describe quasiparticle–phonon coupling in nuclear field theory, are shown in Figs. 15 and 16. We consider first of all these diagrams. In the usual microscopic calculations of particle–vibration coupling, one considers only the diagrams shown in Figs. 15a and 15b, which include only the vertex corresponding to scattering (see Fig. 11c). However, the exchange diagrams (Figs. 15b and 15d) in superfluid nuclei are actually more important. This is because their contribution is proportional to $(f_{ab}^{(+)})^2$, whereas the contribution of the direct diagrams in Figs. 15a and 15c is proportional to $(f_{a,b}^{(-)})^2$. But for states a and b near the Fermi surface

$$|f_{ab}^{(+)}| \gg |f_{ab}^{(-)}|. \qquad (61)$$

The diagrams shown in Figs. 15b and 15d describe the effect of the Pauli principle and to order $1/\Omega$ ensure antisymmetrization between the odd quasiparticle and the nucleons that form the vibrational mode. In the phenomenological models presented in Secs. 3 and 4, these processes are implicitly included in the so-called kinematic interaction (see Sec. 4 and Ref. 25). Mottelson was the first to note that the exchange diagrams in Figs. 15b and 15d have considerable importance for explaining the position of the levels in the particle–phonon multiplets.[2,23]

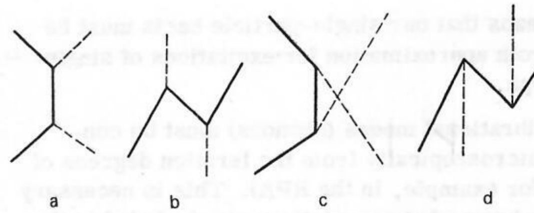


FIG. 15. Mottelson diagrams making a contribution $\sim 1/\Omega$ to the splitting of the quasiparticle–phonon multiplet.

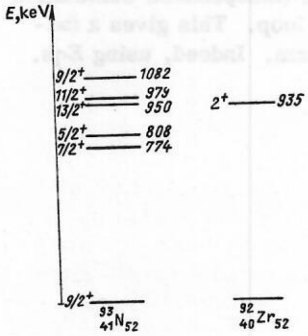


FIG. 14. Typical spectrum of low-lying excited states of odd vibrational nuclei. The multiplet structure can be clearly seen. Note the characteristic sinking of the $I=j-1$ state relative to the other memebers of the quasiparticle–phonon multiplet. For comparison, we give the energy of the 2_1^+ state of the neighboring even–even nucleus ^{92}Zr. The experimental energies are taken from Ref. 50 (^{93}Nb) and Ref. 51 (^{92}Zr).

When the chemical potential lies near a state with large angular momentum and "opposite" parity ($j \gg 1$, $\varepsilon_j \approx \lambda$), the dynamics of the nuclear system at low excitation energies is basically determined by this distinguished j level. In this case, the main contribution ΔE_I to the splitting of the quasiparticle–phonon multiplet is made by the exchange diagrams[3)]:

$$\Delta E_I = \Lambda^2 (f_{jj}^{(+)})^2 \left(\frac{\delta_{Ij}}{(2I+1)^2} \frac{1}{\omega+2\varepsilon_j} - \begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix} \frac{1}{2\varepsilon_j - \omega} \right), \qquad (62)$$

since the scatter term vertex is $f_{jj}^{(-)} \sim (\varepsilon_j - \lambda) \approx 0$ and can be ignored in this case. For the $6-j$ symbol with $j>2$ we have the following sign rule:

$$\begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix} \begin{matrix} >0, & I=j-1; \\ <0, & I \neq j-1. \end{matrix} \qquad (63)$$

Thus, in the considered case the state with $I=j-1$ is unique, which significantly reduces the quasiparticle–vibration interaction. This state is frequently

3)Below, we shall omit the phonon index n, since we shall be concerned with only one collective mode.

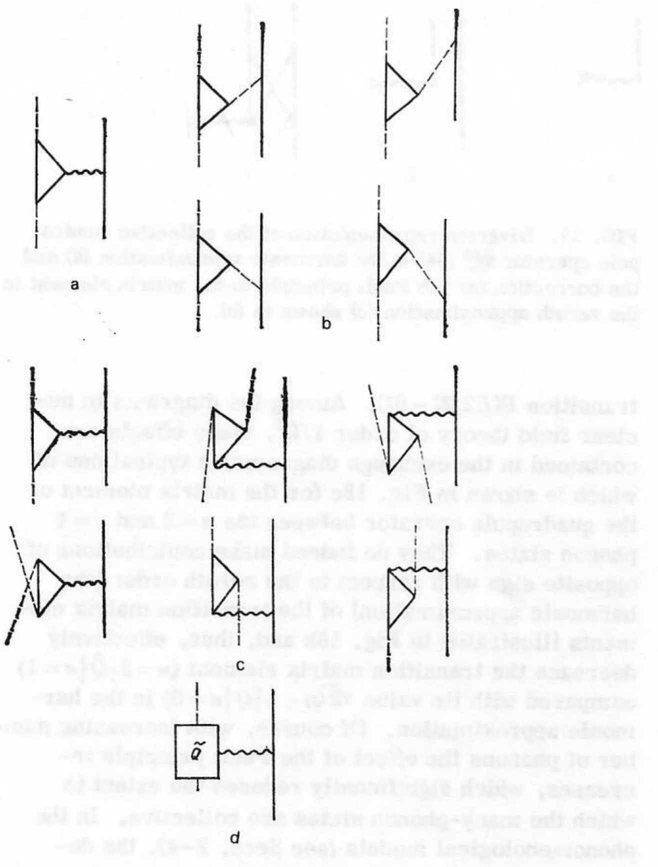


FIG. 16. Diagrams describing the interaction of an odd nucleon with the anharmonic core of the neighboring even–even nuclei: a) the process of direct interaction through QQ forces; b) the corresponding collective process (through phonon exchange); c) six different triangle diagrams describing the direct process; d) schematic representation of the diagrams that describe the interaction of the odd nucleon with the mass quadrupole moment of the even core.

called a state with anomalous coupling. For very strong particle–vibration coupling, as, for example, in transition nuclei that are "soft" with respect to a change of shape, states with anomalous coupling may even sink below the single-quasiparticle states and become ground states. Indeed, in a number of nuclei in which a single-particle state with large angular momentum and "opposite" parity is partly occupied, the ground states are those with spin $I=j-1$. Examples are the isotopes $^{77-81}$Se, ^{103,105}Rh, $^{107-111}$Ag, and ^{125,127}Xe (Ref. 27).

In the general case when several j states contribute to the states formed as a result of the quasiparticle–phonon coupling, the antisymmetrization effect becomes less important. This is because an individual j level makes only a small contribution to the collective state. In addition, in the case of summation over the intermediate states of the diagram, because of the differences between the phases of the geometrical coefficients, the contributions of the different single-particle j states compensate each other. However, this assertion must be carefully verified, since sometimes the sequence of states has a strong influence on the magnitude of the exchange contribution.[11]

Besides the diagrams considered above (see Fig. 15), there is a further set of diagrams of the leading order ($1/\Omega$), which are shown in Fig. 16 and which are completely ignored in the usual spectroscopic calculations of particle–core coupling.[3,4] This is due to the fact that in the usual microscopic approach the vibrational modes of the neighboring even–even nuclei are treated in the harmonic approximation (RPA) (Refs. 6–9, 3, and 4). Whereas the diagrams in Fig. 15 also exist for harmonic vibrations, the processes shown in Fig. 16 are due solely to the anharmonicity in the collective motion. They can be interpreted as the interaction of the odd quasiparticle with the quadrupole moment of the even core. In the phenomenological model considered in Sec. 3, these processes were taken into account by including the anharmonicity in the collective quadrupole operator (5a) and in the phonon space. We shall show below that the diagrams whose principal members are shown in Fig. 16 really do generate the coupling of quasiparticles to an anharmonic core introduced phenomenologically by Eq. (9).

In these diagrams, the odd quasiparticle can interact directly with the quasiparticles that form the phonon through QQ forces (direct processes) (see Fig. 16a) or through the exchange of phonons (collective process) (see Fig. 16b). The direct process is given by the six different diagrams in Fig. 16c. Each of them corresponds to four diagrams with exchange of phonons, these differing only in the time sequence of the vertex parts. Thus, we have altogether $6\times(1+4)=30$ triangle diagrams. Using the factorization theorem,[49] one can show that the four diagrams with phonon exchange lead to renormalization of the QQ interaction in the corresponding direct processes (see Fig. 16), which gives an effective quadrupole interaction constant[43]:

$$\tilde{\varkappa}(E)=\varkappa-2\Lambda^2 D(E), \qquad (64)$$

where

$$D(E)=2\omega/(E^2-\omega^2) \qquad (65)$$

is the phonon propagator, and E is the energy by which the vertex part of the outer (odd) quasiparticle is removed from the mass shell.[43] In the given case, when the phonon has the same energy in the initial and the final state, E is given by the difference between the energies of the odd nucleon in the initial and final states: $E=E_i-E_f$. For given vibrational band, the odd quasiparticle is in approximately the same state ($i\approx f$), and $E=0$ irrespective of the quasiparticle state of the odd nucleon. Note that the renormalization of the quadrupole constant due to phonon exchange (the collective process, as described above) is not restricted to the triangle diagrams but holds for the complete expansion in nuclear field theory.

Therefore, one can determine an effective (anharmonic) quasiparticle–core coupling:

$$H_{\rm coup}=-\frac{\varkappa}{2}\sum Q^c_\mu \tilde{Q}_{-\mu}(-1)^\mu \qquad (66)$$

with $Q^c_\mu=Q^{ph}_\mu+Q^{anh}_\mu$ and $Q^{anh}_\mu=\sqrt{15}Q(b^+b)_{2\mu}$, where $\tilde{Q}$ are the contributions of all the diagrams which participate

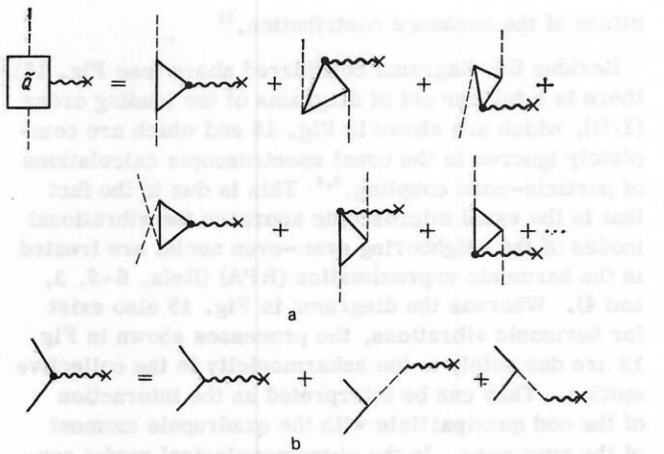


FIG. 17. a) Diagrams of leading order for the effective quadrupole moment $\tilde{Q}$ of the core. The single-particle quadrupole operator is represented by the wavy line with a cross. b) The effective single-particle quadrupole operator obtained after application of the factorization theorem.

in the formation of the mass quadrupole moment of the 2_1^+ state (see Fig. 16). Assuming a uniform mass and charge distribution in the nucleus, one can also take the matrix elements $\tilde{Q}$ from the experimental data on the electric quadrupole moments.

Thus, analyzing the diagrams in nuclear field theory, we obtain a microscopic justification of the particle–anharmonic core model of Sec. 2. More precisely, $\tilde{Q}$ determines the coefficient in front of the anharmonic term $\sim b^+b$ in the collective quadrupole operator defined by Eq. (5a).

In the leading order of the expansion in nuclear field theory ($\sim 1/\Omega$), the anharmonic coupling term $\tilde{Q}$ is given by the contribution of the triangle diagrams calculated on the "mass shell" with respect to the intermediate vertex (Fig. 17):

$$\tilde{Q}=\frac{\tilde{\varkappa}(E=0)}{\varkappa}\Lambda^2\sum_{abc} f^{(+)}_{ca} f^{(-)}_{ab} f^{(+)}_{bc} \begin{Bmatrix}2&2&2\\a&b&c\end{Bmatrix} E_0(\varepsilon_a\varepsilon_b\varepsilon_c), \tag{67}$$

where $E_0(\varepsilon_a, \varepsilon_b, \varepsilon_c)$ is the sum of the energy denominators of the six triangle diagrams shown in Fig. 17a.

We now analyze the "quasiharmonic" term in the selective quadrupole operator (5a):

$$Q^h_\mu = F\left(b^+\sqrt{N-\hat{n}}+\text{h. c.}\right). \tag{68}$$

We use the adjective *quasiharmonic*, since this term corresponds to the harmonic approximation for the collective quadrupole operator (Fig. 18a) if the square root is ignored. Phenomenologically, the square root takes into account the effect of the Pauli principle in the many-phonon states that is associated with the microscopic structure of the nuclear vibrational mode.[4)] Note that the square root does not affect the

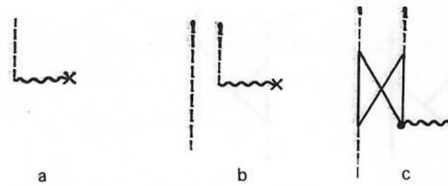


FIG. 18. Diagram representation of the collective quadrupole operator Q^{ph}_μ (54) in the harmonic approximation (a) and the correction for the Pauli principle to the matrix element in the zeroth approximation (c) shown in (b).

transition $B(E2;2_1^+-0_1^+)$. Among the diagrams in nuclear field theory of order $1/\Omega^2$, these effects are contained in the exchange diagrams, a typical one of which is shown in Fig. 18c for the matrix element of the quadrupole operator between the $n=2$ and $n=1$ phonon states. They do indeed make contributions of opposite sign with respect to the zeroth order (the harmonic approximation) of the transition matrix elements illustrated in Fig. 18b and, thus, effectively decrease the transition matrix element $\langle n=2|\hat{Q}|n=1\rangle$ compared with its value $\sqrt{2}\langle n=1|Q|n=0\rangle$ in the harmonic approximation. Of course, with increasing number of phonons the effect of the Pauli principle increases, which significantly reduces the extent to which the many-phonon states are collective. In the phenomenological models (see Secs. 2–4), the decrease of the quadrupole transitions to many-phonon states due to the Pauli principle is taken into account implicitly by the "square-root" dependence on the phonon number operator in Eq. (5a).

Finally, we must emphasize the following. Although the particle–vibration coupling term H_{PVC} (57) contained in the Hamiltonian (59) of nuclear field theory has formally the same structure as the quasiparticle–core coupling term in the Hamiltonian of the phenomenological model (9), there is a large difference between them. In (9), the single-particle quadrupole operator $\tilde{q}_\mu$ is given by the scatter term ($\sim f^{(-)}_{ab}$) and acts only on the odd nucleon. In contrast, in H_{PVC} (58) we encounter the total quadrupole operator Q_μ, which acts on all the nucleons, including the core nucleons, but the collective quadrupole operator Q^{ph}_μ is given by the harmonic approximation. Nevertheless, our above analysis of the diagrams has shown that, starting with the Hamiltonian of nuclear field theory, which is exact when the rules (1)–(5) are applied, one can obtain an effective Hamiltonian of the coupling in which the single-particle operator acts only on the odd nucleon. In this case, the collective quadrupole operator must include anharmonic terms, which is a consequence of the elimination of the core nucleons from the fermion space.

[4)]The square root can also be associated with a generalization of the Holstein–Primakoff boson transformation.[5]

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