Mass parameters in fission

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A review is given of the methods employed to calculate the mass parameters B, which are used in practical calculations of the half-lives T_{sf} of spontaneously fissioning heavy elements. The phenomenological description of B and the cranking model are considered in detail. The hydrodynamic model on which the phenomenological description is based is also discussed. The results are given of calculations of the parameters B and also results for the half-lives T_{sf} based on them.

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INTRODUCTION

In the collective adiabatic description of fission (see, for example, Refs. 1–3) an important part is played by information about both the potential and the kinetic (mass parameters) energy as well as dissipation. We know the potential energy relatively well, but the mass parameters much less so, and in the investigation of dissipation we are still at the initial stage.[4-7]

The potential energy can be described relatively simply by representing it as the sum of a smooth part (usually described by the liquid-drop model) and a relatively small shell correction.[8-13]

It is much harder to describe the mass parameters. They depend more strongly on the details of the internal structure of the nucleus than on the potential energy (see, for example, Refs. 14 and 15). In addition, they are represented by a more complicated entity (a tensor) than the energy (a scalar), and even in a simple model a complete description of this tensor is much more complicated. There exist various methods of fairly general self-consistent description of the inertial parameters. Among them, we have the generating coordinate method[16-18] and the time-dependent Hartree–Fock formalism (see, for example, Ref. 19, in which this formalism is specially adapted for describing collective processes). However, because of their complexity, these methods have not yet been used in practical calculations.

In the present paper, we review only those methods of calculating the mass parameters B that have so far been used in practical calculations of the spontaneous-fission half-lives. This means that we consider the phenomenological description and the description in the framework of the cranking model. The hydrodynamic model has here played some part as well. In the paper, we use simultaneously the expressions "mass or inertial parameters" and "mass or inertial functions."

In Sec. 1, we present a method for calculating the spontaneous-fission half-lives and indicate the part played by the inertial parameters in the method. Section 2 is devoted to the hydrodynamic and phenomenological description of these parameters, and also to the description by means of the cranking model. In Sec. 3, we present and discuss the main results obtained by means of these methods. At the end, we draw some conclusions from these results.

1. CALCULATIONS OF THE HALF-LIVES

The spontaneous-fission half-life is

$$T_{sf} = (\ln 2/n)\, P^{-1}, \tag{1}$$

where n is the number of times the nucleus encounters the barrier per unit time. This number is usually identified with the frequency of vibrations of a type that can lead to fission. If the vibration energy is taken equal to 1 MeV, we obtain $n = 10^{20.38}$ sec^{-1}. The probability P of penetration of the nucleus through the barrier when the barrier is encountered once is usually calculated in the quasiclassical (WKB) approximation:

$$P = [1 + \exp S(L_{\min})]^{-1}, \tag{2}$$

where

$$S(L) = 2\int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar^2}[V(s) - E]\, B(s)}\, ds \tag{3}$$

is the action integral calculated along a trajectory L defined in the space of deformations, $V(s)$ is the potential energy, $B(s)$ is the effective mass parameter along L, and E is the energy of the fissioning nucleus.

The effective mass parameter is

$$B(s) \equiv B_s(s) \equiv B_{ss}(s) = \sum_{ij} B_{\alpha_i\alpha_j} \frac{d\alpha_i}{ds}\frac{d\alpha_j}{ds}, \tag{4}$$

where $B_{\alpha_i\alpha_j}$ are the components of the mass tensor corresponding to the deformation parameters α_i and α_j of the nucleus. To calculate the probability P of penetration through the barrier, we must find the trajectory $L_{\min}$ along which the action integral $S(L)$ takes its minimal value.

An important part is here played by the space of deformations in which the minimum is sought. It must be sufficiently large to describe at least two characteristics of the nuclear shape such as elongation and the formation of a neck, which are important in fission. Thus, the description must contain at least two parameters. For the fission of the majority of actinides, a third property is also important—mirror (or left–right) asymmetry, i.e., asymmetry about a plane perpendicular to the symmetry axis of the nucleus and passing through its center of mass. The departure from axial shape (described, as a rule, by the gamma parameter) is not so important.

It can be seen from Eq. (3) that the part played by the mass parameters in fission is as important as that played by the potential energy. However, as we have already said in the Introduction, these parameters have not been studied so well as the potential energy. In addition, their calculation in the general case is more complicated than that of the potential energy, since we need, not one, but $n(n+1)/2$ components of the symmetric tensor $B_{\alpha_i \alpha_j}(\alpha_1, \alpha_2, \ldots, \alpha_n)$, where $i, j = 1, 2, \ldots, n$, and n is the dimension of the space of deformations. Moreover, the calculation of each component is, as a rule, more difficult than the calculation of the potential energy $V(\alpha_1, \alpha_2, \ldots, \alpha_n)$. This is why the mass parameters are treated very approximately in calculations of T_{sf}; one either calculates fewer components than exist in the employed deformation space or very crude simplifying assumptions are adopted.

Of course, the one-dimensional description of the barrier penetrability is also a simplification. However, it appears to be a comparatively good approximation. Some investigations have been made into the penetrability of a two-dimensional barrier.[20-22] It was shown[21,22] that in the case of a "diagonal" transition through the barrier (i.e., without excitation of the orthogonal degree of freedom), in which we are here interested, the influence of the orthogonal degree of freedom reduces basically to a change in the effective height of the barrier, this being associated with a change in the energy of the zero-point vibrations corresponding to the orthogonal degree of freedom (see also Ref. 23). However, to some extent this fact is already taken into account in the one-dimensional description presented here, since the liquid-drop parameters in the potential energy are chosen to reproduce the experimental barrier heights.

2. DESCRIPTION OF THE MASS PARAMETERS

In this section, we discuss in some detail both descriptions of the mass parameters used in practical calculations of the half-lives T_{sf}, i.e., the phenomenological description and the description by means of the cranking model. We begin by presenting the hydrodynamic approximation on which the phenomenological description is based.

Hydrodynamic Approximation. In this approximation, nuclear matter is regarded as an incompressible and irrotational fluid. To find the kinetic energy of the fluid, we must find its velocity field within the nucleus for a given normal component of the velocity on its boundary. Mathematically, this is a Neumann problem for the velocity potential.

For a simple shape of the nucleus, for example, an ellipsoid, this problem can be solved analytically.[24] For an ellipsoid of revolution, the mass function corresponding to the Nilsson deformation parameter ε is[25]

$$B^{\rm ir}_{\varepsilon\varepsilon}(\varepsilon) = 2/15 mAR_0^2 (1+2\varepsilon^2/9)(1-2\varepsilon/3)^{-2}(1-\varepsilon^2/3-2\varepsilon^3/27)^{-4/3}, \qquad (5)$$

where m is the mass of a nucleon, A is the mass number, and R_0 is the radius of a sphere of the same volume as the considered nucleus.

For zero deformation (sphere)

$$B^{\rm ir}_{\varepsilon\varepsilon}(0) = (2/15)\, mr_0^2 A^{5/3} = 0.00463 A^{5/3} \hbar^2 \ \mathrm{MeV}^{-1} \qquad (6)$$

for $R_0 = r_0 A^{1/3} = 1.2 A^{1/3}$ F.

The dependence of all three functions $B_{\beta\beta}$, $B_{\beta\gamma}$, and $B_{\gamma\gamma}$ (and also all three moments of inertia I_x, I_y, I_z) on the deformation parameters β and γ describing an arbitrary ellipsoid is discussed, for example, in Ref. 14.

For a more complicated shape of the nucleus, the mass parameters are calculated numerically in exactly the same way as they are approximately (see, for example, Refs. 26 and 27).

With regard to the numerical values of the hydrodynamic inertial functions, they are too small to describe vibration and rotation[15,28] and also fission.[29] They are of interest only as a point of departure, particularly in the construction of phenomenological mass parameters.

Phenomenological Description. In the phenomenological description, the starting point is a definite (as simple as possible) expression for the effective mass function [Eq. (4)] with one or two free parameters. These parameters are chosen so as to reproduce best the experimental half-lives T_{sf}.

In the early calculations[29] of T_{sf}, the effective function B_s was chosen in the simple form

$$B^{\rm phen}_{\varepsilon}(\varepsilon) = \mathrm{const}\, A^{5/3}, \qquad (7)$$

i.e., independent of the deformation and with the same mass-number dependence as is obtained in the hydrodynamic approximation (6). In these calculations, the parameter s describing the position on the fission trajectory was the Nilsson parameter ε: $s \equiv \varepsilon$. The condition of reproducing the actinide half-lives yielded

$$B^{\rm phen}_{\varepsilon} = 0.054 A^{5/3} \hbar^2 \ \mathrm{MeV}^{-1}, \qquad (7a)$$

i.e., a value approximately 12 times greater than the hydrodynamic value (6).

The simple form (7) does not satisfy the asymptotic condition that the effective parameter B_{rr} (where r is the distance between the centers of mass of the fission fragments) should tend to the reduced mass μ when r tends to the value corresponding to the scission point, i.e.,

$$B_{rr}(r) \xrightarrow[r \to r_{\rm scission}]{} \mu = M_1 M_2/(M_1 + M_2). \qquad (8)$$

One can, of course, assume that this condition is not that important for analyzing the T_{sf}, since the principal part is here played by the properties of the nucleus near the saddle point, and not near the scission point, which is very far from the saddle point, especially in nuclei that are not too heavy. However, the condition (8) can be used as a heuristic argument in determining B as a function of the deformation.

In Ref. 30, precisely this condition was used to construct the phenomenological function B. The hydrodynamic approximation played an auxiliary part. The essence of the matter is as follows. Suppose the distance r between the centers of mass of the incipient fission fragments is determined for any shape of the fissioning nucleus up to the fission point. For each shape investigated in practice, this is not difficult to do. If the motion of the nucleus is regarded as the motion of an incompressible fluid, it follows from Kelvin's theorem[31] that the mass parameter B_r corresponding to this motion cannot be less than the irrotational value:

$$B_r \geqslant B_r^{\mathrm{ir}}. \tag{9}$$

This follows from the fact that among all the motions corresponding to the given boundary-value problem the kinetic energy is smallest for irrotational motion. But B_r^{ir} is greater than the reduced mass, since the motion of the fluid includes not only the relative motion of the centers of mass of the fragments but also the internal motion in the fragments. Thus,

$$B_r^{\mathrm{ir}} \geqslant \mu. \tag{10}$$

Equality can here hold only at the scission point and at large r, when the variable r no longer describes the internal motion of the fragments.

Introducing one free parameter k, we can give the following expression[30] as the function B^{phen}:

$$B_r^{\mathrm{phen}} - \mu = k(B_r^{\mathrm{ir}} - \mu). \tag{11}$$

This expression enables us to give a simple interpretation of the free parameter k. Namely, its departure from unity characterizes the departure of the motion of the nuclear matter from irrotational motion. Analysis of the half-life of the nucleus ^{240}Pu in Ref. 30 led to the value $k = 8.9$. In the calculations of the kinetic energy of the irrotational motion (B_r^{ir}) allowance was made for only the symmetric shape of the nucleus, described[13] by the parameter y introduced by Hill and Wheeler.[16] (However, the potential barrier was calculated with allowance for the mirror asymmetry of the deformation.) The diameter B_r^{phen} is shown as a function of the distance r for $k = 8.9$ in Fig. 1.

The representation (11) for the parameter B_r^{phen} was used in Ref. 32 to analyze T_r^{sf} in the complete range of actinides. However, instead of a detailed calculation of the function $B_r^{\mathrm{ir}} - \mu$, the following exponential form was used:

$$B_r^{\mathrm{ir}}(r) - \mu = (17/15)\,\mu \exp[-(r - 3R/4)/d]. \tag{12}$$

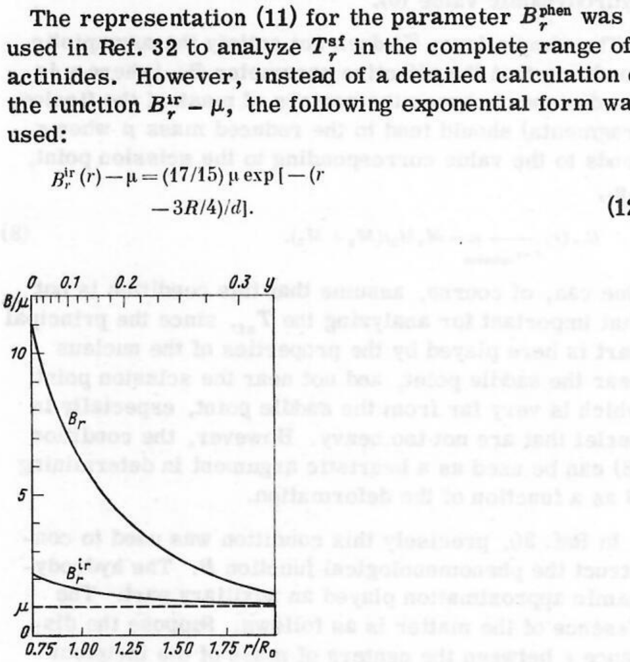

FIG. 1. Dependence of the phenomenological inertial function B_r on the distance between the fragment centers of mass for shape of the nucleus described by the parameter y. For comparison, the function for an irrotational fluid and the reduced mass μ of the fragments are also given.[30]

The value $(17/15)\mu$ of the coefficient was obtained under the assumption that the nucleus has an ellipsoidal shape at small deformations. In this case, for a spherical nucleus ($r = 3R/4$)

$$B_r^{\mathrm{ir}}(r = 3R/4) = (32/15)\,\mu.$$

If the parameter d of the rate of decrease is fitted so as to reproduce the values of $B_r^{\mathrm{ir}}(r) - \mu$ obtained for the shape described by the variable y (the lower part of Fig. 1), we obtain

$$d = d_0 = R/2.452. \tag{13}$$

Analysis of the experimental half-lives T_{sf} of even-even actinides based on B^{phen} [see (11)–(13)] yielded $k = 6.5$ (Ref. 32). However, after some improvements[33] of the potential barrier, this value was increased to $k = 11.5$.

In Ref. 33, the possibility of regarding d as a free parameter was investigated. It was found that besides the parameter set

$$k = 11.5, \quad d = d_0, \tag{12a}$$

we obtain also the set

$$k = 6.5, \quad d = 2d_0, \tag{12b}$$

which actually permits a somewhat better reproduction of the experiment. Namely, it makes it possible to reproduce the half-lives T_{sf} of nuclei with $Z = 92-104$ with accuracy up to a factor of ~25 on the average.

Microscopic Calculations. In practice, to calculate the spontaneous-fission half-lives T_{sf} from microscopic mass parameters only parameters obtained in the adiabatic approximation in the cranking model have been used. In this approximation,[34] it is assumed that the time-dependent deformation of the nucleus is specified independently, and the response (inertia) of the nucleus to this perturbation is investigated. The additional assumption of adiabaticity means that the deformation of the nucleus (the collective motion) is slow compared with the single-particle motion.

In the adiabatic approximation in the cranking model, the expression for the inertial function corresponding to the components α_i and α_j of the deformation of the nucleus has the form[1,35]

$$B_{\alpha_i\alpha_j} = 2\hbar^2 \sum_{k \neq 0} \frac{\langle 0 | \partial/\partial\alpha_i | k\rangle \langle k | \partial/\partial\alpha_j | 0\rangle}{\varepsilon_k - \varepsilon_0}, \tag{14}$$

where $|0\rangle$ and $|k\rangle$ are the ground state and excited state of the nucleus, and ε_0 and ε_k are their energies. If these states describe the nucleus as a system of nucleons placed in a field with deformed potential with pairing forces acting between them, the expression (14) becomes[12,36,37]

$$B_{\alpha_i\alpha_j} = 2\hbar^2 \sum_{\nu, \nu'} \frac{\langle \nu | \partial H/\partial\alpha_i | \nu'\rangle \langle \nu' | \partial H/\partial\alpha_j | \nu\rangle}{(E_\nu + E_{\nu'})^3} \times (u_\nu v_{\nu'} + u_{\nu'} v_\nu)^2 + P^{ij}. \tag{15}$$

The pairing interaction is here taken into account in the BCS approximation; H is the single-particle Hamiltonian, which describes the motion of a nucleon in the deformed potential; u_ν and v_ν are the variational parameters of the BCS function corresponding to the single-particle state $|\nu\rangle$ with energy e_ν, and $E_\nu=\sqrt{(e_\nu-\lambda)^2+\Delta^2}$ is the energy of a quasiparticle in this state. The quantities λ and 2Δ are the Fermi energy and the energy gap.

The additional term P^{ij} is due to the dependence of λ and Δ on the deformation and has the form[12,36,37]

$$P^{ij}=\frac{1}{4}\hbar^2\sum_\nu\frac{1}{E_\nu^5}[\Lambda_\nu^i\Lambda_\nu^j-\Delta(\Lambda_\nu^i r_{\nu\nu}^i+\Lambda_\nu^j r_{\nu\nu}^i)], \tag{15a}$$

where

$$\Lambda_\nu^i=\Delta\partial\lambda/\partial\alpha_i+[(e_\nu-\lambda)/2\Delta]\,\partial\Delta^2/\partial\alpha_i;\quad r_{\nu\nu'}^k=\langle\nu'|\partial H/\partial\alpha_i|\nu\rangle;$$

$$\partial\lambda/\partial\alpha_i=(ac_i+\Delta^2 b\,d_i)/D;\quad \partial\Delta^2/\partial\alpha_i=2\Delta^2(ad_i-bc_i)/D;$$

$$a=\sum_\nu\frac{e_\nu-\lambda}{E_\nu^3};\quad b=\sum_\nu\frac{1}{E_\nu^3};\quad D=a^2+\Delta^2b^2;$$

$$c_i=\sum_\nu\frac{(e_\nu-\lambda)\,v_{\nu\nu}^i}{E_\nu^3};\quad d_i=\sum_\nu\frac{v_{\nu\nu}^i}{E_\nu^3}.$$

3. RESULTS AND DISCUSSION

Dependence on the Deformation. The dependence of the microscopic parameters on the deformation has been most fully investigated. It has been studied for components corresponding to mirror-symmetric (Refs. 12, 10, 25, 38, and 39), mirror-asymmetric,[40,41] and axially asymmetric[14,15,42] shapes of the nucleus.

In Ref. 42, six inertial functions in nuclei in the actinide region were investigated $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$, $B_{\varepsilon_4\varepsilon_4}$, $B_{\beta\beta}$, $B_{\beta\gamma}$, and $B_{\gamma\gamma}$. The first three of them were investigated on the mesh shown in Fig. 2. At the corners of the mesh, the shapes of the nucleus corresponding to these corners are shown; the dashed curve is the shape of the nucleus after allowance for the axisymmetric deformation $\varepsilon_3+\varepsilon_5$.

An example of the results is given in Fig. 3 for ^{258}Fm. The variable ε_4^* is introduced instead of the parameter ε_4 of the hexadecapole deformation:

$$\varepsilon_4=\varepsilon_4^*+0.2\varepsilon-0.06 \tag{16}$$

the aim being that for actinides the mean trajectory leading to fission,

$$\varepsilon_4=0.2\varepsilon-0.06, \tag{17}$$

should be expressed simply as

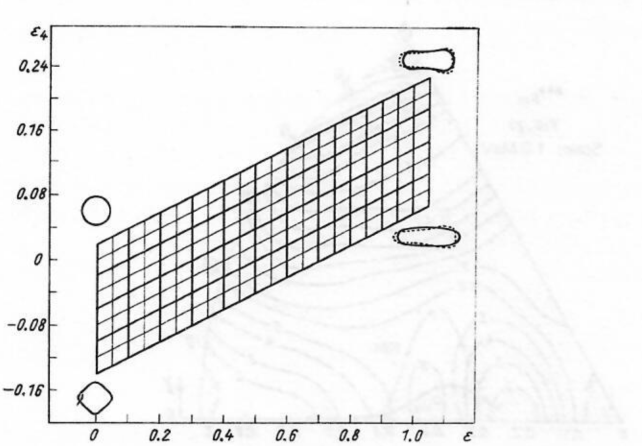

FIG. 2. Mesh of points for which the functions $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$, and $B_{\varepsilon_4\varepsilon_4}$ were calculated.[42]

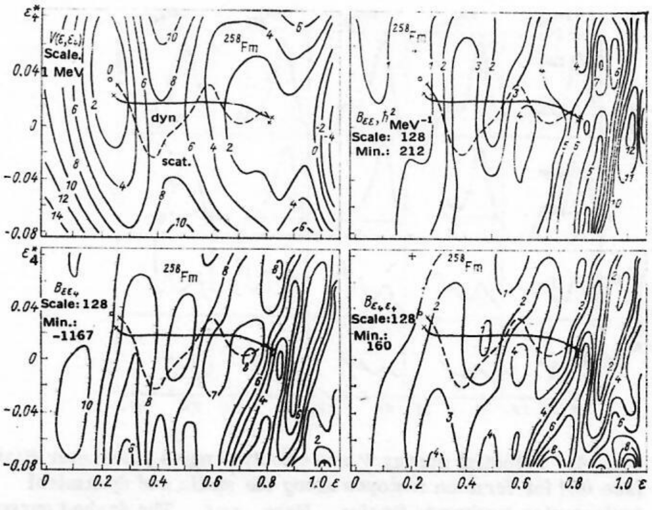

FIG. 3. Dependence of the potential energy V and the inertial functions $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon}$, and $B_{\varepsilon_4\,4}$ on the deformations ε and ε_4. The values of the potential energy are measured from the first minimum (zero), and the values of the inertial functions from the smallest value attained at the point indicated by the cross (min.). The static (dashed curve) and dynamical (continuous curve) trajectories leading to fission are shown.[42]

$$\varepsilon_4^*=0. \tag{17a}$$

One can also say that the transformation (16) transforms the oblique mesh $(\varepsilon,\varepsilon_4)$ in Fig. 2 into the rectangular mesh $(\varepsilon,\varepsilon_4^*)$ in Fig. 3.

It can be seen that all three inertial functions depend strongly on the deformation. In particular, their appreciable fluctuations indicate strong shell effects. Besides the inertial functions, to complete the picture we show in Fig. 3 the potential energy as well. This was calculated with allowance for the asymmetric deformation $\varepsilon_3+\varepsilon_5$.

Because of the dependence of the inertial functions on the deformation, the dynamic trajectory leading to fission differs from the static trajectory. It can be seen from Fig. 3 that the difference is appreciable. Its influence on the potential barrier is relatively small (since the static trajectory corresponds to the minimum of the potential energy, and deformation from it changes this energy comparatively little), but its influence on the effective mass parameter $B(s)$ is large. This can be seen directly in the example of several fermium isotopes (Fig. 4); namely, the potential barrier along the dynamical trajectory differs little from the barrier along the static trajectory, but the corresponding effective parameter $B^{\text{dyn}}(s)$ is on the average appreciably smaller than $B^{\text{stat}}(s)$. In particular, it is appreciably smaller in the region in which the potential barrier is high, i.e., in the region making a large contribution to the action integral (3). To summarize, the dynamical calculations give values of the spontaneous-fission half-lives T_{sf} of the transuranium elements that are 1.5–2.0 orders of magnitude shorter than the static calculations.[42]

We illustrate the dependence of the mass functions on the nonaxial deformation for the lighter nucleus ^{126}Ba. Figure 5 shows the functions $B_{\beta\beta}$, $B_{\gamma\gamma}$, and $B_{\beta\gamma}$ in their

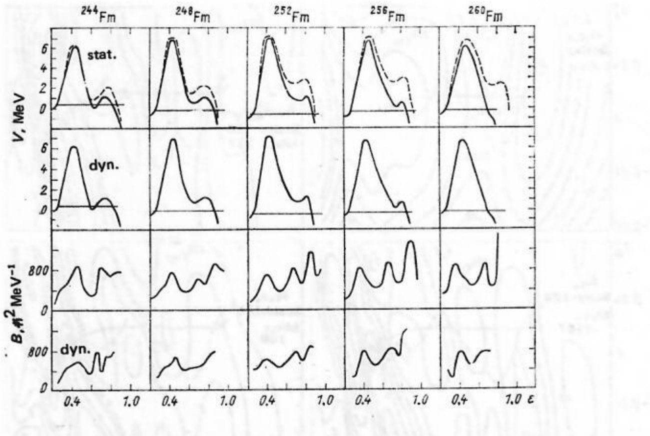

FIG. 4. Potential energy V and effective mass parameter $B(s)$ [see (4)] for fermion isotopes along the static and dynamical trajectories leading to fission. Here, $s = \varepsilon$. The dashed curve is the potential energy with smooth component in the liquid-drop model.[42]

dependence on the deformation parameters β and γ. For completeness, we give the dependence of the potential energy on these parameters. The deformation region is chosen with the aim of analyzing low-lying collective states. Here it can also be seen that shell effects are clearly expressed for all three parameters B; for example, the amplitude of the fluctuation of the parameter $B_{\gamma\gamma}$ is of the order of 40% of the mean value. It is interesting that the dependence of $B_{\gamma\gamma}$ on the deformation γ reveals a certain maximum (barrier), which separates the oblate and prolate shapes. The height of this barrier is about 30% of the value obtained for an axisymmetric shape. This barrier can lead to an axisymmetric shape of the nucleus, although the potential energy by itself gives only a weak preference for such a shape. In Fig. 5, the microscopic inertial functions (and the potential energy) are compared with the functions obtained for an irrotational fluid. The functions corresponding to an irrotational fluid depend very smoothly on the deformation. In addition, their dependence on the deformation is weaker (by two orders of magnitude) than the dependence of the microscopic functions (cf. the scales of the corresponding figures).

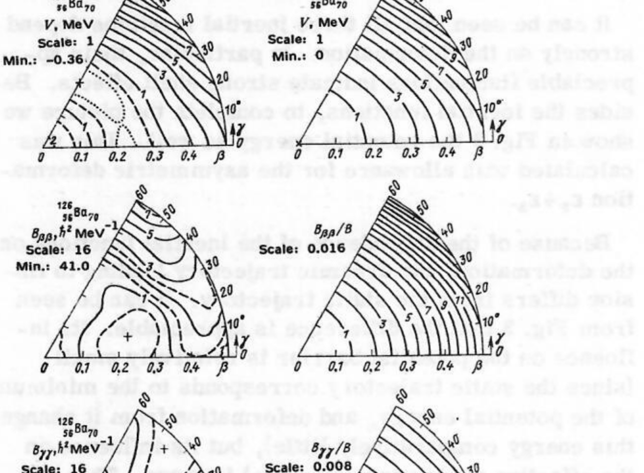

FIG. 5. Dependence of the mass parameters $B_{\beta\beta}$, $B_{\gamma\gamma}$, and $B_{\beta\gamma}$ on the deformations β and γ for ^{126}Ba. On the left, the microscopic values B^{micr}; on the right, the values B^{ir} for an irrotational fluid normalized by means of the values $B = B^{\text{ir}}_{\beta\beta}(\beta=0) = B^{\text{ir}}_{\gamma\gamma}(\beta=0)$. For completeness, diagrams of the potential energy V are also given.[14]

With regard to the interpretation of the deformation dependence of the mass parameters, it was noted quite some time ago that the inertia (resistance) of a nucleus with respect to some motion of it (for example, deformation) is connected to the rearrangement that this motion produces. In the single-particle description without residual interaction and under the assumption of adiabatic motion, this rearrangement is localized at the points of intersection of the single-particle levels on the Fermi surface. It is shown in Ref. 43 that the mass parameter calculated in the cranking model has singularities precisely at the points of intersection of the levels and takes the hydrodynamic values far from these points. In the representation of a single-particle Schrödinger fluid[44] intersection of the levels corresponds to compressible (locally, dynamically compressible) flow.

Allowance for residual interaction, for example, pairing, eliminates the singularities, leaving smaller or larger fluctuations depending on the strength of this interaction.

For the nuclei in the actinide region investigated in Ref. 42, the dynamical trajectory leading to fission is closer to the direction $\gamma = 0$ (corresponding to axisymmetric shape) than the static trajectory (Fig. 6).

A comparatively weak dependence on the deformation is put into the phenomenological inertial function B_r^{phen}

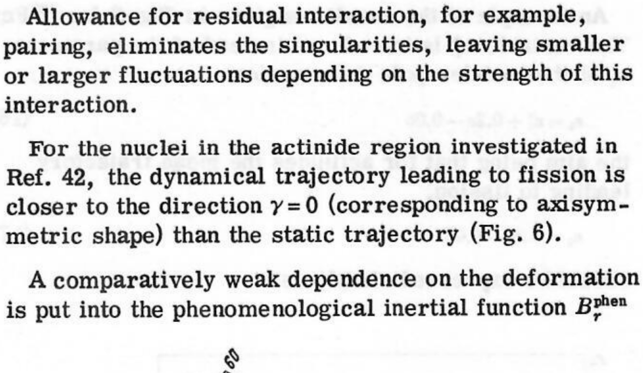

FIG. 6. Static and dynamical trajectories leading to fission on the plot of the potential energy $V(\varepsilon, \gamma)$ for ^{244}Fm (Ref. 42).

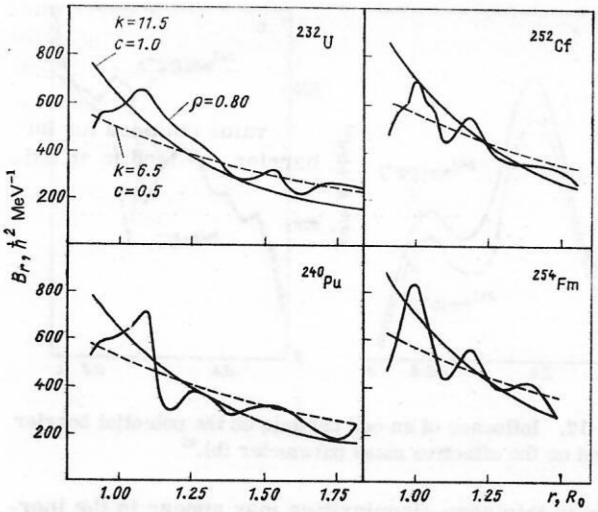

FIG. 7. Comparison of the deformation dependences of the microscopic function B_r reduced by the factor $\rho = 0.80$) and two effective phenomenological inertial functions B_r. The value $c = 1.0$, from (11) and (12), means that $d = d_0$ [see (12a)], and $c = 0.5$ means $d = 2d_0$ [see (12b)] (Ref. 33).

[see (11)]. This is a smooth function of the type of the dependence obtained for an irrotational fluid (see Fig. 1). It takes into account only one deformation parameter—the distance r between the centers of mass of the fission fragments—and it can therefore be compared with the deformation dependence of the microscopic function $B^{\rm micr}$ only with respect to this variable (Fig. 7). The microscopic function $B_r^{\rm micr}$ is calculated on the basis of the single component $B_{\varepsilon\varepsilon}$ by means of the expression

$$B_r^{\rm micr}(r) = \rho B_{\varepsilon\varepsilon}^{\rm micr}(\varepsilon, \varepsilon_4)\,[d\varepsilon/dr\,(\varepsilon, \varepsilon_4)]^2, \qquad (18)$$

i.e., in accordance with Eq. (4) with $s = r$ and $\alpha_i \equiv \alpha_j \equiv \varepsilon$. The values of $B_{\varepsilon\varepsilon}$ are taken along the static trajectory leading to fission. It can be seen that the dependence $B^{\rm phen}(r)$ is close to the corresponding dependence $B^{\rm micr}(r)$ after averaging of the latter with respect to the shell fluctuations. The values of $B^{\rm micr}(r)$ themselves become close to the values $B^{\rm phen}(r)$ only after they have been reduced by a normalizing factor $\rho = 0.80$. However, if we take into account not only the component $B_{\varepsilon\varepsilon}$ but also the components $B_{\varepsilon\varepsilon_4}$ and $B_{\varepsilon_4\varepsilon_4}$, we obtain $\rho \approx 1$, and, thus, the normalizing factor becomes superfluous (Fig. 8). The microscopic function $B_r^{\rm micr}$ is calculated here in accordance with Eq. (4) for $s = r$ and $(\alpha_i, \alpha_j) \equiv (\varepsilon, \varepsilon_4)$ along the static, $B_r^{\rm stat}$, and dynamic, $B_r^{\rm dyn}$, trajectories. One can see that there are large fluctuations of the function $B^{\rm micr}$ (shell effects). They are smaller along the dynamical trajectory than along the static trajectory (allowance for the dynamics smooths the fluctuations of the effective inertial function B). It can also be seen that the effective B constructed from the "more complete" tensor $B^{\rm micr}$ (i.e., from a larger number of components of this tensor) is close to $B^{\rm phen}$ (after averaging of the fluctuations). The phenomenological function $B^{\rm phen}$ ($B^{\rm semi\text{-}empi}$) is here the same as in Fig. 1 ($k = 8.9$).

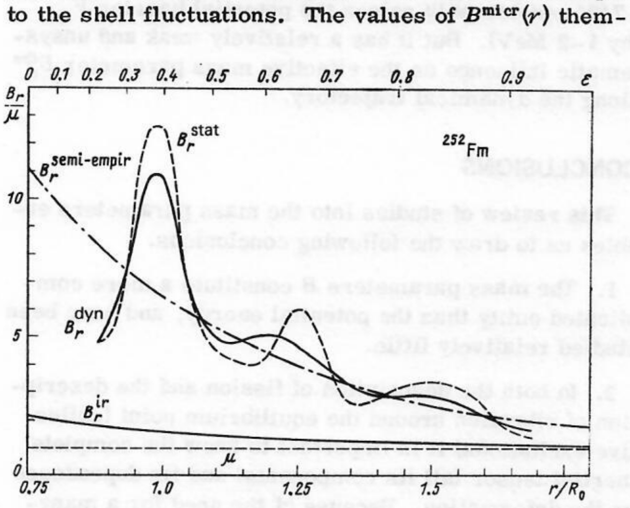

FIG. 8. Deformation dependence of the effective microscopic inertial function B_r constructed from the three components $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$, and $B_{\varepsilon_4\varepsilon_4}$. The phenomenological function $B_r^{\rm semi\text{-}empir}$ here is the same as in Fig. 1. All functions are expressed in units of μ (the reduced mass).[45]

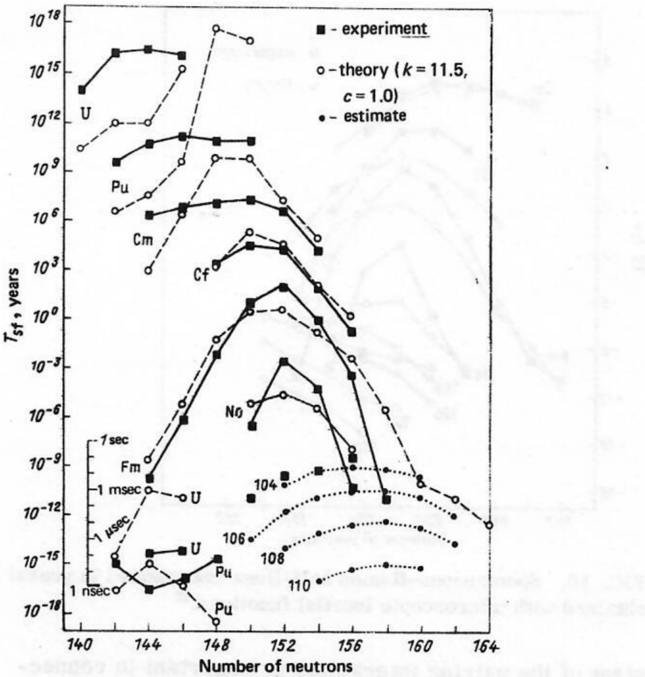

FIG. 9. Spontaneous-fission half-lives obtained with phenomenological inertial functions.[33]

Figure 9 shows the spontaneous-fission half-lives T_{sf} obtained using the phenomenological inertial functions with two free parameters fitted to these half-lives. The values of T_{sf} were calculated along the static trajectories, and their experimental values were taken from Refs. 46 and 47. In contrast to Fig. 9, Fig. 10 shows the values of T_{sf} obtained in dynamical calculations using microscopic inertial functions without free parameters. It can be seen that the dynamical calculations make it possible to obtain T_{sf} values that agree with the experimental half-lives with a relatively good accuracy without introducing (either in B or in the potential energy V) any free parameters. At least, this is true for not too light actinides.

Dependence on the Constant of the Pairing Interaction. Study of the dependence of the parameter B on the con-

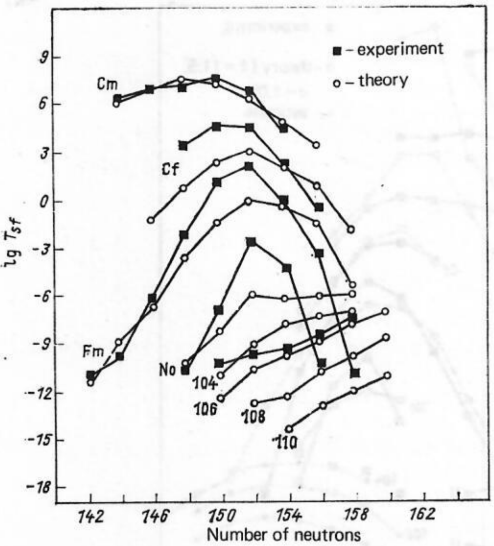

FIG. 10. Spontaneous-fission half-lives (meansured in years) obtained with microscopic inertial functions.[42]

stant of the pairing interaction is important in connection with the fact that this constant is not too well known, especially in the region of large deformations (in the neighborhood of the saddle point), which is the most important for fission problems.

This dependence is shown in Fig. 11, in which we have plotted the parameter $B_{\varepsilon\varepsilon}$ calculated for several superheavy nuclei[48] at deformation close to the deformation of the saddle point. It is also discussed in Ref. 49. It can be seen that this dependence is very strong. It also depends strongly on the nucleus (i.e., on the position of the single-particle levels near the Fermi level λ). An increase in the constant G of the pairing interaction by 5% reduces the mass parameter B by 10–30%, depending on the nucleus. Since an increase in G also decreases the barrier, although to a lesser extent, the half-lives T_{sf} are rapidly reduced.

The strong dependence of the mass parameters B on the constant of the pairing interaction is confirmed in several papers, for example, Refs. 25, 38, 49, 40, and 48. It is also considered for excited nuclei in Ref. 50.

Influence of an Odd Particle. It is difficult to discuss the influence of an odd particle in the cranking model,

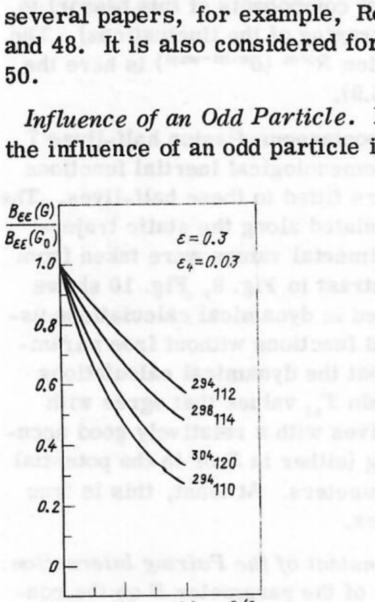

FIG. 11. Dependence of the mass parameter B on the constant G of the pairing interaction.[48]

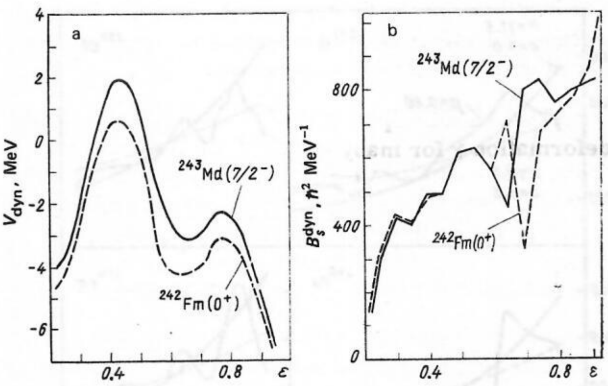

FIG. 12. Influence of an odd particle on the potential barrier (a) and on the effective mass parameter (b).[55]

since in this case singularities may appear in the inertial functions at the points of intersection of the single-particle levels. However, at the level of the dynamical analysis of fission the dramatic nature of this situation is considerably defused, since the dynamical trajectory leading to fission gives a wide berth to the singular points and in general the region of large values of B.

A simple preliminary estimate of the influence of an odd particle on the parameter B, and through it on the half-lives T_{sf}, was already made in the early paper of Ref. 51. If allowance is made for the blocking effect,[52,53] the odd particle significantly increases B already through the weakening of the pairing interaction due to the blocking of the state occupied by the odd particle. Microscopic calculations showed that the corresponding increase in B is on the average about 30% (Ref. 54). If, however, the blocking effect is ignored, the odd particle has relatively little influence on B and its main influence in increasing T_{sf} is through an increase of the potential barrier (specification energy). The results of Ref. 55 obtained in this case are shown in Fig. 12.

It can be seen that an odd proton in a state with $\Omega\pi = 7/2^-$ considerably raises the potential barrier V_{dyn} (by 1–2 MeV). But it has a relatively weak and unsystematic influence on the effective mass parameter B_s^{dyn} along the dynamical trajectory.

CONCLUSIONS

This review of studies into the mass parameters enables us to draw the following conclusions.

1. The mass parameters B constitute a more complicated entity than the potential energy, and have been studied relatively little.

2. In both the description of fission and the description of vibration around the equilibrium point (collective excitations) it is important to know the complete inertial tensor (all its components) and its dependence on the deformation. Because of the need for a many-parameter description of the deformations that occur in these motions, the number of components of the tensor is not small, especially in the case of fission.

3. In the cranking model, the dependence of the

inertial functions on the deformation is strong. In this connection, the effect of the dynamical description, as compared with the static, is appreciable.

4. Because of the dependence of the parameters B on the deformation γ for many nuclei in the region of the actinides, the dynamical trajectory leading to fission is found to be closer to the line corresponding to axial symmetry of the nucleus than in the case of the static trajectory. As a rule, the dynamical trajectory is shorter and smoother than the static trajectory.

5. If blocking is not taken into account, the influence of an odd particle on the dynamical effective mass parameter is small.

6. The influence of residual interaction on B is large. For example, an increase in the constant of the pairing interaction by 5% reduces B by 10–30%.

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