Clusters in multiparticle production

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Multiparticle production processes at high energies are treated as two-stage processes with the formation of bunches of nuclear matter, or clusters (first stage), which decay into the final hadrons (second stage). The main methods of analyzing such processes are presented. It is shown that in addition to resonances clusters with large mass must be produced. A theoretical scheme is considered in which clusters with a wide spectrum of masses are generated. The physical nature of the clusters is discussed.

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INTRODUCTION

The problem of clusters in hadron-hadron collisions[1)] arose in theory in the thermodynamic models of Heisenberg,[1] Fermi,[2] Pomeranchuk,[3] and Landau[4] (1936-1953); experimentally, it arose in the assertion that at energy[2)] $\sqrt{s}\sim 20-100$ GeV in *typical* collisions of nucleons (at a multiplicity of the order of or greater than the mean multiplicity, $n\gtrsim\langle n\rangle$) the production of new hadrons proceeds through an intermediate stage of formation of one or two bunches of nuclear matter, which then decay into hadrons (cosmic rays—Miesowcz *et al.*,[5] Niu,[6] Cocconi,[7] 1958; Dobrotin, Slavatinskiĭ and collaborators,[8] 1960).

The cited theoretical models suffered from a common shortcoming, in that they considered "central" collisions of nucleons, in which the initial particles belong on an equal footing with the final particles to a common thermodynamic system (using modern terminology, this would entail abandonment of asymptotic freedom, as was recognized, for example, by Landau[9]). But it was even then clear (from cosmic-ray experiments[10]) that "peripheral" collisions were at the least predominant (and nonperipheral collisions were even possibly entirely absent). Attempts were made by Fermi[11] and Heisenberg[12] to take into account this peripheral nature of the collisions in the framework of the same quasiclassical models, but these attempts either violated the relativistic condition of a finite velocity of propagation of the interaction[11] or the uncertainty relation for time and energy.[12] This was noted in Ref. 13, in which it was also suggested that multiparticle production should be regarded as decay of thermodynamic subsystems formed in a peripheral collision as a result of a quantum process (pion exchange, etc.; "peripherally thermodynamic" or "statistically peripheral" process). In what follows, we shall adhere to such a modification of the thermodynamic models. Essentially, it is realized in multiperipheral schemes with the participation of clusters (if the decay of these clusters is treated thermodynamically). In the current conception of clusters, it is necessary to distinguish a number of stages or elements.

1. The assertion that multiparticle production of hadrons occurs in two stages. First, on the collision of the initial particles intermediate unstable formations arise (and in what follows we shall refer to these as clusters generically), these then decaying into the final hadrons; the generation and decay of the clusters are basically independent and can be separated, perhaps even in time. (This separation is called factorization; in particular, the clusters formed in NN, πN, or KN collisions are basically the same when the effect of leading particles is ignored.)

2. The intermediate formations may be either hadronic resonances (known or as yet unknown) or nonresonance hadronic entities of a special kind.

It is the choice between the two possibilities (or a combination of them) that is the essence of the problem. Corresponding to this alternative, two approaches are possible.

3. If the clusters are resonances, it must be assumed—trivially—that these are simply few-particle resonances of the type of ρ and ω particles (original multiperipheral model[14] or its subsequent modification taking into account other light mesonic resonances on an equal footing with the ρ mesons[15]).

4. If the clusters are not resonance objects, two important questions arise:

a) What are these objects? Do they, for example, have a place in quantum field theory?

b) How do they decay into the final hadrons—thermodynamically, through a cascade (tree-like diagrams, as in the statistical bootstrap[16]), or in some other way?

Anticipating the exposition that follows, we should like to give our opinion as to what answers to these questions currently appear most justified (if we are considering collisions with multiplicity that is not particularly small, $n\gg 1$, $n\gtrsim\langle n\rangle$).

1. The production process does indeed take place in two stages. The clusters really are formed. In this form, the thesis is generally accepted.

2. If the collision energy is sufficiently high (and the corresponding phase space is not very small), then in the initial stage there are formed few-particle resonances ($\rho,\omega,\ldots$) and heavy clusters (mass $M_{fb}\sim 2-3$

[1)] We shall not consider the problem of clusters in hadron—nucleus collisions.

[2)] $\sqrt{s}$ is the total c.m.s. energy of the colliding particles; for a collision of two nucleons of mass m_N each, $s\approx 2m_N E_{\rm lab}$, where $E_{\rm lab}$ is the energy of the incident nucleon in the laboratory system.

GeV), which (to make a distinction from the word "cluster," which can be applied to any intermediate formation) we shall call fireballs. It is very probable that these are not resonance objects, but the possibility cannot be ruled out that we have here a superposition of many, still unknown heavy resonances with a wide spectrum. Already at $\sqrt{s}\sim 10$ GeV they play a very important part, and at higher energies this increases even more. In processes with $\langle n\rangle \sim 3-4$, there is hardly any prospect of finding fireballs.

3. The suggestion that fireballs should be identified with a superposition of just few-particle resonances appears satisfactory only if it is confronted with a limited group of experiments. If all available data are taken into consideration, it meets serious difficulties and is implausible.

4a. A fireball could be any hadron (i.e., a hadronic object with the quantum numbers of some stable hadron) in a state very far from its mass shell (but it could also be an object that does not have a definite spin, etc.). If one discounts the possible existence of such a nonresonance object, one actually assumes that the spectral function of the propagator in the Lehmann expansion consists solely of resonance peaks of Breit–Wigner type and that there is no smooth background. For this special assumption there are no *a priori* grounds. A fireball could be a bare hadron or a "partly dressed" hadron, in particular a Feynman parton.

4b. The assumption of a thermodynamic nature of the decay of the fireball into the final hadrons gives a remarkably good description of the final mass composition of the particles and their distributions with respect to the transverse momenta. The possibility that a dynamical system goes over into a thermodynamic system is supported theoretically by weighty arguments. However, this still does not permit one to assume that such a nature of the decay is proved.

The picture we have outlined in 2)–4) is by no means generally accepted, although it is steadily gaining ground. As we shall see below, many papers have been published whose authors attempt to show that all processes can be encompassed by the production of the ordinary light resonances. Therefore, at the present time the problem now revolves around the question of the existence and the nature of fireballs, i.e., clusters significantly heavier than "ordinary" light resonances (with mass 0.5–1.3 GeV for pion resonances and 1–1.2 GeV for baryon resonances).

1. SOME BASIC METHODS FOR ESTABLISHING THE EXISTENCE OF CLUSTERS EXPERIMENTALLY

There have by now been developed in detail, and are still being developed and applied, special statistical methods of evaluation of experimental data that employ inclusive and semi-inclusive distributions. The semi-inclusive approach is characterized by, for example, its consideration of events of only a given multiplicity n_{ch} ("topological" correlation coefficients, etc.). In the first place one studies the binary (but also the ternary) correlation coefficients (or definite combinations of them) as functions of the rapidities of the particles and the azimuthal angles of their momenta for different combinations of the charges; one studies the distribution and correlation of the rapidity intervals between "neighboring" and "non-neighboring" particles, the correlation of the total electric charges on either side of certain points on the rapidity axis ("the flux of electric charge along the rapidity axis" and its correlation functions), etc. The aim is to establish the size of the clusters, i.e., the number of (relativistic) particles K or the number of charged (relativistic) particles K_{ch} produced by the decay of one cluster and, through this, the mean number of clusters $\langle n_c\rangle$ in one event. Attempts are also made to determine the mass M, the electric charge Q, and even the spin of the cluster. We hasten to mention other designations for these quantities, which were proposed in Ref. 17 and which are being used more and more widely. Quantities referring to a cluster as a whole are designated by means of a single bar above the symbol, while those designating the products of one cluster are designated by means of two bars:

$$\left.\begin{array}{l} n_c\equiv\bar{n} - \text{is the number of clusters in the event;}\\ K\equiv\bar{\bar{n}},\ K_{ch}\equiv\bar{\bar{n}}_{ch} - \text{is the number of particles in the cluster, etc.}\end{array}\right\} \quad (1)$$

More than a hundred papers have been devoted to detailed investigations of this kind. Many of the authors of these papers do not wish to prejudge the question of the existence of a real decaying object and prefer to speak of a cluster interpretation of the experiments, calling it a convenient and exceptionally "economic" parametrization of the experimental data. Since the main experimental justification for the idea of clustering is the existence of a strong binary correlation at small rapidity intervals between the particles of a pair, one frequently speaks of the directly observed phenomenon of short-range ordering, preferring this phenomenological terminology to a concrete model. But for all that, the cluster model is in mind, the results are compared with model calculations, and it is established that the results agree with some particular physical model of clusters.

The process of looking for such short-range ordering effects or (in the model representation) clustering has recently also been undertaken in terms of the "local compensation" of different physical quantities, for example, the electric charge or the transverse momentum. Here, one studies the statistical characteristics of so-called zones.[18] By zones, the following is meant.

Moving along the rapidity axis, for example, from right to left, we plot along the ordinate the total electric charge remaining to the right of the given abscissa y. This leads to a histogram, as in Fig. 1. The length of each small region in which the deviation has a definite sign is called the length of the zone, and the middle of the region is called its center. It is obvious that the presence of many such zones demonstrates the mutual compensation of the investigated physical quantity (in the given case, the charge) within an interval of rapidities of the order of the length of the zone; it is merely another expression of the phenomenon of short-range ordering and corresponds to a model of clusters with a

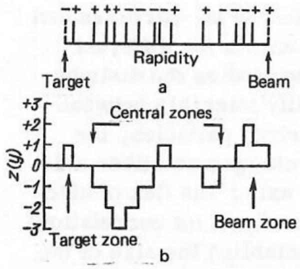

FIG. 1. Typical inelastic event at high energies (a) and representation of it in a zone diagram (b). The particles are indicated by the bars on the rapidity scale; their charges are given above them. The charge is counted from right to left.

small, frequently zero, value of the charge. However, although the detailed study does give a certain indication of an effect of local compensation, the departure from a completely random distribution of the charges of the particles with respect to y is relatively small at the investigated energies ($E_{\rm lab}=100-400$ GeV), and the qualitative conclusions drawn about the cluster properties depend strongly on the chosen model.

A considerable number of papers has been devoted to the local compensation of the transverse momentum. It should be noted that these characteristics of the multiparticle production process are very general and can be reproduced in a number of dynamical models of the process and therefore, taken by themselves, can hardly serve as a critical test of a particular model. In particular, they do not contradict the model of independent emission of clusters and, moreover, give information on, for example, the possible exchange of charge between clusters (see below).

In what follows, we shall consider more definite criteria of particle clustering. To simplify the exposition, we shall first interpret the obtained results in the framework of the simplified model of independent emission of clusters, despite the objection relating to the possible correlation of the clusters and the asymptotic nature of the model (conservation laws!).

In the considered case this is admissible, since the obtained qualitative results can be tested and confirmed in the framework of a quantitative dynamical model that takes into account fully both the conservation laws and the multiperipheral nature of the cluster emission (see Sec. 5).

The methods for identifying clusters have advanced far from the original procedures that led to the assertion that fireballs exist in cosmic rays.[5-8] Earlier, one simply analyzed the groupings of charged particles on the scale of (pseudo) rapidities in individual events, high-multiplicity events (number of charged relativistic particles $n_{\rm ch}\geq 5$) usually being specially selected, so that one could speak in that case of the selection of only relatively heavy clusters. As a result, everything was done at superhigh energies ($\sqrt{s}\gtrsim 100$ GeV), at which one could expect that such clusters are produced with appreciable probability and in a significant fraction of events are completely separated on the rapidity scale.

The new statistical methods are also much more accurate compared with the still used simple construction of the distributions of pairs of particles with respect to the total mass of the pair, which, for example, reveal a weak maximum in the region of the mass m_ρ of the ρ meson and are used to attempt to estimate the fraction of the particles due to the decay of ρ mesons, etc. (as will be shown below, this method by itself does not give unambiguous results).

The need for a more accurate approach is related to the fact that the experimental material is known in sufficient detail only for the current accelerators, i.e., for completely inadequate energies $\sqrt{s}\lesssim 60$ GeV, at which the number $\langle n_{\rm ch}\rangle$ of produced charged particles (in the majority of cases, it is just these that are detected) (and, *a fortiori*, the number of particles with a given sign of the charge, for example, $\langle n^-\rangle$) is very small ($\langle n_{\rm ch}\rangle\lesssim 10$), so that heavy clusters cannot be numerous.

In the interpretation of the experimental results it is frequently assumed (explicitly or implicitly) that there is some unified clustering mechanism, so that the characteristics of the clusters are distributed fairly closely around certain mean or most probable values, i.e., the entire production occurs through clusters of a particular kind. Sometimes, to simplify the calculations, it is assumed that all clusters are simply identical. It is obvious that these are crude assumptions. In reality, there could be production of both heavy nonresonance clusters and (directly) few-nucleon resonances (a corresponding model, worked out in detail and giving a good description of the inclusive and semi-inclusive data, exists[19]; see Sec. 5). Of course, at the present stage of the investigations such a simplification is permitted, but its danger must not be forgotten. It often leads to a lack of understanding of the essence of the process; Sec. 5 is therefore devoted to this question.

In analyzing the experimental data, we shall take into account above all three important circumstances.

1. If an object decays isotropically in its rest frame into $K\,(\equiv\bar{\bar{n}})$ particles, then irrespective of the number of products their distribution dN/dy with respect to y will be excellently approximated by a Gaussian curve:

$$G(y-y_c)\equiv\frac{dN(y)}{dy}=\frac{K}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(y-y_c)^2}{2\sigma^2}\right],\quad \sigma\approx 0.7-0.8, \quad (2)$$

where y_c is the cluster rapidity before decay. Thus, on the y axis the products of one cluster cover the region

$$\delta y\sim 2\sigma\sim 1-2\,(2\langle|y|\rangle=2\sqrt{2/\pi}\,\sigma\approx 1.3)\,.^{3)} \quad (3)$$

2. The total width of the region actually occupied by all produced particles for investigations with current accelerators is so far ($\sqrt{s}\lesssim 60$ GeV) only (Fig. 2)

$$Dy\leqslant 3-4. \quad (4)$$

Indeed, the quasiplateau has width $D^{\rm quasipl}\lesssim 3$, and the curve of $dn/d\eta$ falls to half the height of the quasipla-

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[3)] This is true at least if the additional factor $\exp(-m^2\sinh^2 y/\langle p_T^2\rangle)$ is close to unity, which is well confirmed for pions even for $y(\equiv|y-y_c|)\approx 1$, but for the choice $\sigma\approx 0.7$ it is valid up to $y=2$ (see Fig. 9 in Ref. 20).

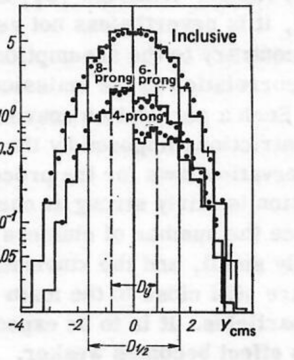

FIG. 2. Inclusive and semi-inclusive spectra of π^- mesons in pp interactions at 69 GeV. The vertical lines indicate the region of the quasiplateau and the half-width of the distribution.

teau at points separated by a distance $D_{1/2} \lesssim 4$ (note that the quasirapidity satisfies $\eta = -\ln\tan\bar{\bar{\theta}} \approx y$ for $\bar{\bar{p}}_\perp \sim \langle \bar{p}_\perp \rangle$ and $y \gtrsim 0.5-1$). All this is true, although the kinematically allowed region is much wider, $2Y \approx 8$.

Thus, the products of only one cluster cover half the entire observed interval of rapidities.[4] Studying the inclusive distributions and assuming that the clusters are small ($K \ll \langle n \rangle$) and that the number of them is accordingly large, we inescapably find ourselves in a situation in which the products of different clusters overlap on the y axis (this means that some products from different clusters approach each other with very small relative velocity); nevertheless, the interaction of the products of different clusters with one another is ignored, which, of course, is very bad[21]; but this is done in many widely adopted theoretical schemes (Fig. 3).

3. There exist at least two different production mechanisms: diffraction dissociation and the main mechanism of multiparticle production—pionization, whose cluster structure is the primary aim of the investigation. Their products (at least, at accessible accelerator energies) cannot be separated in the inclusive distributions over y. However, in, for example, the total coefficient of the two-particle rapidity correlation $C_2(y_1, y_2)$, the inherently independent correlation coefficients C_2^{dd} and C_2^{p} are mixed very oddly. Thus, the integrals f_2, f_2^{p}, and f_2^{dd} of these functions over all y_1 and y_2 (see below) are related by the equation[22]

$$f_2 = c f_2^{dd} + (1-c) f_2^{p} + c(1-c)(\langle n^{dd}\rangle - \langle n^{p}\rangle)^2. \tag{5}$$

where c is the relative probability of the dd process, and n^{dd} and n^{p} are the corresponding multiplicities. It is obvious that even for small c, since $\langle n^{p}\rangle \gg \langle n^{dd}\rangle$, the

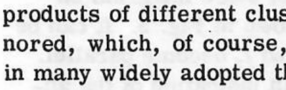

FIG. 3. Schematic representation of the overlap of the decay products of clusters on the rapidity scale.

4) Note that if the decay is not isotropic, δy will be even larger. Thus, for the decay of a ρ meson polarized strictly along the longitudinal axis, the rapidity distance between the two final pions is

$$\delta y \approx 2\ln(2\mathcal{E}_\pi/m_\pi) \approx 2\ln[2(m_\rho/2)/m_\pi] = 3.35.$$

poor resolution of the two components can lead to a strong difference between the directly measured C_2 and the C_2^{p} in which we are interested. But in the pionization itself two mechanisms may be present, for example, that of heavy resonance clusters and few-particle (ρ, ω) resonances. This also leads to a mixing of the correlation coefficients of the type (5). In particular, this explains why in the model of Ref. 19, which allows both types of cluster, they both significantly influence the correlation coefficient even at energies which are not high ($E_{\rm lab} \sim 40-70$ GeV) despite the small cross section for the production of heavy clusters [the coefficient analogous to c in (5) is small].

Thus, it is very difficult to identify the clusters. In analyses, one usually makes the justified assumption that the emission of different clusters is weakly correlated and, *a fortiori*, that the correlation between the decay products of different clusters is weak. However, this assumption is valid only to a certain extent (see below).

We consider some of the main methods.

1. The simplest characteristic of the correlation properties is the binary rapidity correlation coefficient

$$C_2(y_1, y_2) = d^2\sigma(y_1, y_2)/\sigma_{\rm in}\,dy_1\,dy_2 - (d\sigma(y_1)/\sigma_{\rm in}\,dy_1)(d\sigma(y_2)/\sigma_{\rm in}\,dy_2) \tag{6}$$

($d\sigma(y)/dy$ and $d^2\sigma/dy_1dy_2$ are the corresponding inclusive cross sections).

The expected general behavior of C_2 is clear. As long as $|y_1 - y_2| \ll \delta y$, there is a probability that both particles belong to one cluster. Here a correlation can occur, since the momenta of particles from one cluster must compensate each other (in the rest frame of the cluster). In the case of sufficiently massive clusters, one can assume this even if particles with definite signs of the charge are selected, for example, if one studies C_2^{--}, etc. For if the cluster decays isotropically with the rapidity distribution (2), then, as can be seen, for these products

$$\bar{\bar{C}}_2(y_1, y_2) \sim \exp[-(y_1 - y_2)^2/4\sigma^2] \tag{7}$$

and the mean for the cluster is

$$\langle |y_1 - y_2| \rangle = (2\sqrt{\pi})\,\sigma \approx 0.8 - 0.9. \tag{7a}$$

When $|y_1 - y_2| \gg \delta y$, C_2 must be small if the mutual momenta of the widely separated clusters are weakly correlated, which appears justified for a number of reasons. However, if all the final particles were emitted independently, we should have everywhere $C_2 \equiv 0$. Moreover, even if they arise from the decay of *independently emitted* ρ mesons and we detect particles with only definite signs of the charge, we have $C_2^{++} = C_2^{--} \equiv 0$.

To reduce the influence of leading particles in pp collisions, it is frequently suggested that one should study the correlation C_2^{--} of negatively charged particles. Instead of C_2, one actually frequently uses

$$R_2(y_1, y_2) = C_2(y_1, y_2) \Big/ \left(\frac{d\sigma}{\sigma_{\rm in}\,dy_1}\frac{d\sigma}{\sigma_{\rm in}\,dy_2}\right) = \frac{\sigma_{\rm in} d^2\sigma\ dy_1\,dy_2}{(d\sigma/dy_1)\,d\sigma/dy_2} - 1.$$

The experiments show that C_2 not only does not vanish but is unexpectedly large: $R_2^{\rm ch\,ch}(0) \approx 0.6$ and $R_2^{--}(0) \approx 0.3-0.4$ (Fig. 4). This fact alone shows that the clusters cannot be only ρ mesons. Of course, even in the multiperipheral model,[14] which assumes the existence of only ρ mesons, these have a correlation,[23] but of the opposite sign: The particles of the "Feynman gas"[24] repel each other and, moreover, for $\Delta y = 0$ the function R has a dip (and not a peak).[23]

Taking into account the emission of many ρ ladders,[23] one can obtain only a Δy-independent constant positive component in the function R. To explain the peak at small Δy, one must have recourse to a many-component description in the framework of such a model, introducing exchanges of different isospins with different weights,[23] and taking into account the effects due to the indistinguishability of particles for identical charges or the possibility of the production in one resonance for different charges. In the framework of the cluster hypothesis, the peak at small Δy receives a unified description. Moreover, even when $|y_1 - y_2| > \delta y$ [Eq. (3)],

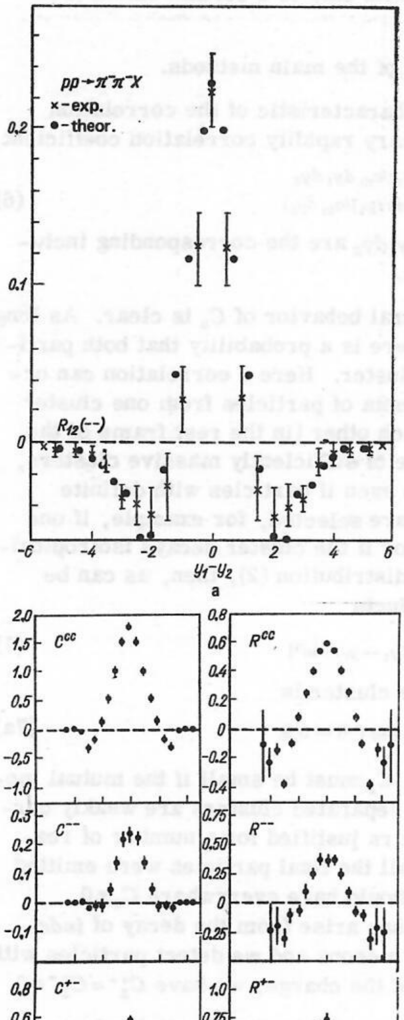

FIG. 4. Two-particle rapidity correlations of pions ($y_2 = 0$) at 69 GeV (a) and 200 GeV (b).

it is found that C_2 does not vanish. Although $|C_2|$ is here smaller than $C_2(0, 0)$, it is nevertheless not very small. This, of course, contrary to the assumption, may be a consequence of correlation in the emission of the clusters themselves. Such a correlation unavoidably exists through the restrictions imposed by the energy and momentum conservation laws for the process as a whole. This correlation is fairly strong at current accelerator energies, since the number of clusters per event is still comparatively small, and the kinematic bounds with respect to y are still close to the main rapidity region filled by the particles. It is to be expected that with increasing s this effect becomes weaker. Diffraction dissociation and its crossed effect with pionization [see (5)] have a similar but even stronger influence. For an appropriate choice of events with weakening of the contribution of diffraction dissociation this correlation becomes less, and the importance of short-range correlation is indeed brought out more clearly.[20]

Besides C_2, characteristic quantities are the already mentioned integrals over y of the correlation coefficients:

$$\left.\begin{aligned} f_1(s) &= \int C_1(y)\,dy \equiv \int \rho_1(y)\,dy = \langle n\rangle;\\ f_2(s) &= \int C_2(y_1, y_2)\,dy_1\,dy_2 \equiv \langle n(n-1)\rangle - f_1^2;\\ f_3(s) &= \langle n(n-1)(n-2)\rangle - (3f_1f_2^2 + f_1^3)\end{aligned}\right\} \qquad (8)$$

(ρ_1 is the density of particles per unit rapidity interval). They are sometimes called "Mueller moments."[20] The experimental dependence of these integrals on $\langle n^-\rangle$, and therefore on s, is shown in Fig. 5. It can be seen that, asymptotically, f_1 and f_2 become parallel. This can be regarded as both an indication of the absence of correlation between the clusters and of factorization of the production and decay of the clusters. Indeed, if these properties hold, then any distribution corresponding to one cluster [for example, f_n^c, which in the given case must be written in the form f_n; see (1)] can be simply averaged over all clusters,

$$f_n = \bar{f}_1 \bar{\bar{f}}_n,$$

where s occurs only in $\bar{f}_1 \equiv \langle n_c\rangle$.[20] However, this last assertion is true only asymptotically, when the energy is sufficiently large for all the participating clusters to have grown to their "normal" mean size (i.e., so that $\bar{\bar{f}}_n$ does not depend on s). Otherwise, the "threshold effect" operates. This is particularly clear if there exist heavy clusters with a broad mass spectrum; for then heavier and heavier clusters are produced with increasing s.[5)]

Therefore, the turning point (the minimum) of, for

[5)]One of the clearest manifestations of the threshold effect can be seen in the growth of

$$\rho_1(0) = \frac{1}{\sigma}\frac{d\sigma}{dy}\Big|_{y=0}$$

with increasing energy (see Fig. 5b). Multiperipheral models with the emission of resonances cannot explain this growth at energies higher than 20 GeV; indeed, they predict a decrease of $\rho(0)$ with increasing energy.[23] But in models with the emission of fireballs with mass up to 3 GeV this growth can continue up to $\sqrt{s} \sim 100$ GeV, being then replaced by a very weak decrease at even higher energies.

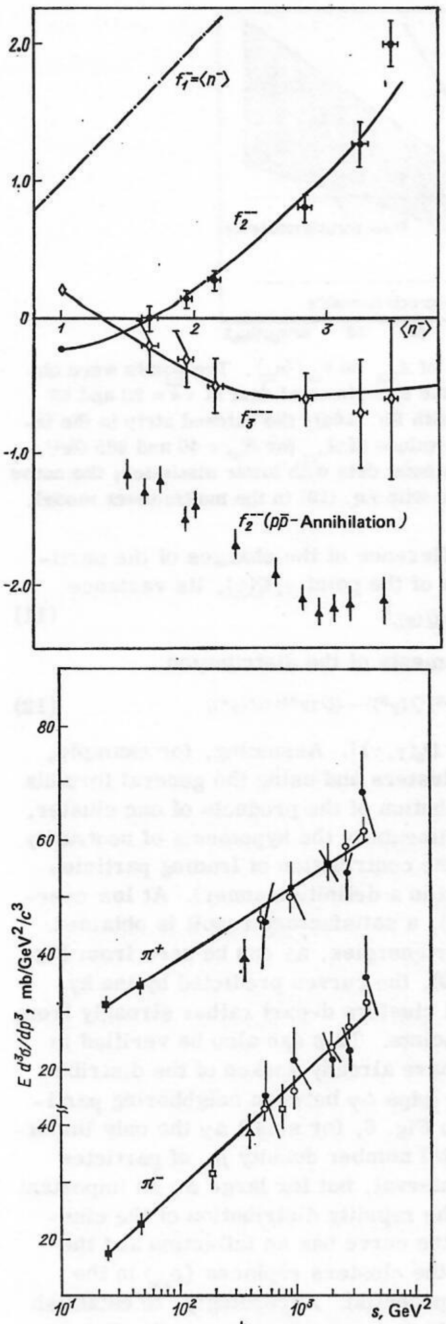

FIG. 5. Dependence of the Mueller moments on the mean number of π^- mesons (a) and the energy dependence (b) of the height of the "plateau" [i.e., $\rho_1(0)$]. The curves are drawn through the experimental data.

example, the curve f_2^{--} is a measure of the number of particles in the effectively participating clusters. The fact that asymptotically f_3^{--} is not parallel to f_1 and f_2 even at $n^- \approx 3$ can be interpreted as an indication that very heavy clusters make an important contribution to the ternary correlations (for more details, see Sec. 6). The data for f_2^{--} in $\bar{p}p$ annihilation,[22] shown in the same figure, also indicate that in annihilation heavier clusters are formed than in pp collisions.

2. Another method is to study the distribution $P(\Delta y)$ of the intervals between particles with neighboring rapidities, $\Delta y = |y_{i+1} - y_i|$, in the resulting y distribution (see Figs. 1 and 3). It can be shown[26] that in the case of independent emission of clusters (the independent cluster emission model) $P(\Delta y)$ has the form shown in Fig. 6, i.e., in the limiting cases $\Delta y \ll 1$ and $\Delta y \gg 1$ it is described by exponential functions in which (when, for example, only charged particles are considered) the argument is the corresponding mean density of the total number of charged particles along the y axis (the number of particles per unit rapidity), $\langle \rho_{ch} \rangle$, and the mean number density of the clusters, $\langle \rho_c \rangle$.

The experimental data do indeed reveal such an inflection, and $\langle \rho_{ch} \rangle \approx 2$ and $\langle \rho_c \rangle \approx 1$. From this one can find both $\langle K \rangle$ and $\langle n_c \rangle$.

However, since in reality δy and Dy (see Fig. 2) differ experimentally by only about a factor 2, the exponentials cover only comparatively small intervals Δy. For $\Delta y \sim 3$, the conservation laws (which we have ignored) begin to have an influence. When these laws are taken into account, the curve sinks, so that here the part played by clusters is somewhat underestimated. As a result, this method is insufficiently accurate[32] under the actual conditions of an accelerator experiment (for $\langle K_{ch} \rangle$, the value 2 was obtained). It may be that clustering is revealed more convincingly by a generalization of this method[27,28]—the rapidity interval method, in which one considers the distribution of rapidity intervals

$$ {}^n r_k = |y_{i+k+1} - y_i|, \quad k \leqslant n-2, \tag{9} $$

between particles in the rapidity intervals between which there are k other particles, events with given n_{ch} being chosen (Fig. 7). The distributions of these quantities $P({}^n r_k)$ obtained from data on collisions at 200 GeV are shown in Fig. 8 and compared with the expected curves for independent pion emission and for independent cluster emission with $K_{ch} = 2$; events with $n_{ch} = 8$ were collected (along the abscissa we have plotted, not the ${}^n r_k$ themselves, but their normalized values—they are divided by their maximal possible value $Y = 2\ln(\sqrt{s}/m_N)$.[28] The clustering is obvious. Similar distributions are constructed for other E_{lab} and $n_{ch}/\langle n_{ch} \rangle$ (see, for example, Ref. 29); at large $n_{ch}/\langle n_{ch} \rangle$ the maximum is shifted to smaller ${}^n r_k$, and this means that better agreement with the curves is obtained for larger K_{ch} (Ref. 30).

Developing such methods of evaluation for semi-in-

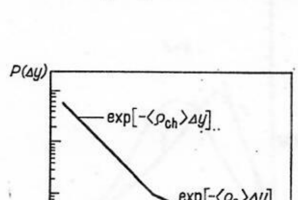

FIG. 6. Schematic representation of the probability of finding a given distance (gap) on the rapidity axis between neighboring particles in a given event and calculated in the independent cluster emission model.

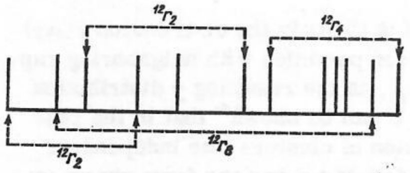

FIG. 7. Examples of the definition of rapidity intervals.

clusive distributions, namely, determining $C_2^{(n_{ch})}(y_1, y_2)$ for given n_{ch} (under the same assumption of independent cluster emission), one can also extract from experiments the value of

$$A_{n_{ch}} = \langle K_{ch}(K_{ch}-1)\rangle / \langle K_{ch}\rangle \tag{10}$$

in events with different n_{ch}.[31] In the evaluation of the experiments, use is made of the formula

$$C_2^{n_{ch}}(y_1, y_2) = A_{n_{ch}}\left(\frac{1}{\sigma_n}\frac{d\sigma_n}{dy}\right) G(y_1-y_2) - \frac{1}{n_{ch}}(1+A_{n_{ch}})\left(\frac{1}{\sigma_n}\frac{d\sigma_n}{dy_1}\right)\left(\frac{1}{\sigma_n}\frac{d\sigma_n}{dy_2}\right), \tag{10a}$$

where G is given by Eq. (2).

As can be seen from Fig. 9, the experimental data admit only at not very large multiplicities the still unrealistic interpretation of the clusters as solely three-pion resonances. For $n_{ch} > (3/2)\langle n_{ch}\rangle$, the contribution of significantly heavier clusters becomes appreciable.

Thus, here too we find evidence that clusters exist and that they cannot be reduced to only ρ and f mesons ($K_{ch} = 1$); it is only at not very large n_{ch} that this experiment does not contradict, for example, a purely ω, η, A_1, and f' mesonic composition ($K_{ch} = 2$–3). There remains, however, in principle the possibility of a purely $B(1235)$ mesonic ($B \to \omega\pi$) and (or) $\rho(1700)$ mesonic ($\rho \to 4\pi$) nature of the clusters. Of course, it is still hard to see why these heavy mesons are produced exclusively, and not the lighter mesons, from $\rho(765)$ to $f(1260)$. Allowance for the multiperipheral nature of the process and the conservation laws leads (see Sec. 5) to the conclusion that the deductions drawn above can in fact be strengthened, and the estimates we have made of the sizes of the clusters are only lower bounds.

3. A new stage in the investigation of short-range ordering was the already mentioned study of the local compensation of the electric charge and, generally, the zone structure of the rapidity distributions.[17,18] We note first the statistical properties of the total charges of all particles situated on either side of a given y.

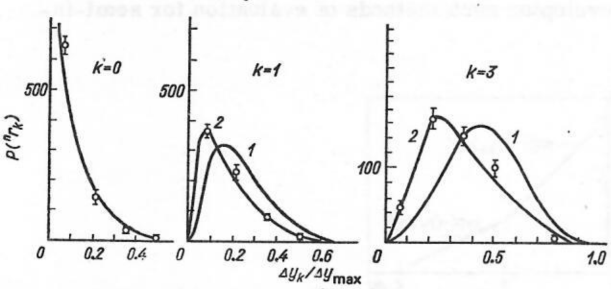

FIG. 8. Distribution of the probability for finding given rapidity intervals at $E_{lab} = 200$ GeV, $n_{ch} = 8$. The open circles are the experiments; 1) independent pion emission; 2) independnet cluster emission ($K_{ch} = 2$).

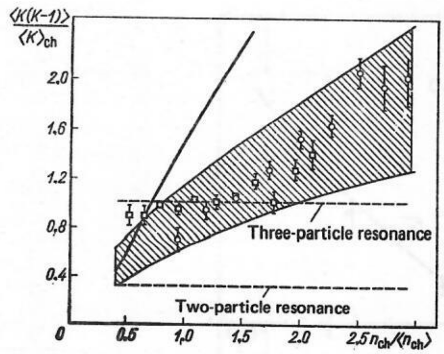

FIG. 9. Dependence of $A_{n_{ch}}$ on $n_{ch}/\langle n_{ch}\rangle$. The points were obtained by analyzing the experimental data at $\sqrt{s} = 23$ and 63 GeV in accordance with Eq. (10a); the hatched strip is the interval of admissible values of $A_{n_{ch}}$ for $E_{lab} = 40$ and 205 GeV, according to experimental data with lower statistics; the curve is $A_{n_{ch}}$ in accordance with Eq. (10) in the multicluster model.

One studies the difference of the charges of the particles on either side of the point y, $Q(y)$, its variance

$$D_2(y) = \langle Q^2(y)\rangle - \langle Q(y)\rangle^2 \tag{11}$$

and the second moments of the distribution

$$D_2(y^A, y^B) = \langle Q(y^A)\, Q(y^B)\rangle - \langle Q(y^A)\rangle \langle Q(y^B)\rangle \tag{12}$$

[obviously, $D_2(y) \equiv D_2(y, y)$]. Assuming, for example, neutrality of the clusters and using the general formula (2) for the y distribution of the products of one cluster, one can test experimentally the hypothesis of neutrality (a correction for the contribution of leading particles must be introduced in a definite manner). At low energies ($E_{lab} = 24$ GeV), a satisfactory result is obtained. However, at higher energies, as can be seen from Fig. 10 (Refs. 31 and 20), the curves predicted by the hypothesis of neutral clusters depart rather strongly from the experimental points. This can also be verified in another way. We have already spoken of the distribution of the rapidity gaps Δy between neighboring particles. According to Fig. 6, for small Δy the only important thing is the total number density ρ_{ch} of particles per unit rapidity interval, but for large Δy an important part is played by the rapidity distribution of the clusters themselves: the curve has an inflection and the number density of the clusters replaces $\langle\rho_{ch}\rangle$ in the argument of the exponential. Accordingly, to establish the charge of the cluster, one investigates the distribution of the gaps Δy in the semi-inclusive case, name-

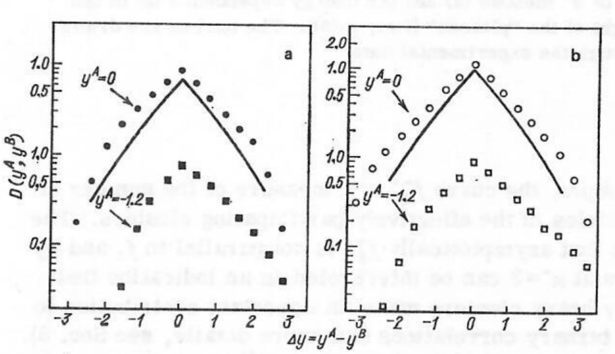

FIG. 10. Second moment of the charge distribution at 102 GeV (a) and 400 GeV (b). The curves are the predictions of the independent cluster emission model.

ly, when the value of the transferred charge $Q(y)$ is fixed. As can be seen from Fig. 11 (Refs. 31 and 20), such an inflection is present when events with $Q=0$ and $|Q|=1$ are chosen, and it corresponds to the inflection in the inclusive distribution (see Fig. 6); however, for $|Q|\geq 2$ there is no inflection. This (in conjunction with Fig. 10) is interpreted as an indication that the clusters may have charge 0 or ±1 but not 2 or more. The clusters can be neither exclusively neutral nor exclusively charged. The results so far obtained agree best with the assumption of a definite combination of charges 0 and ±1, possibly even consistent with an isovector origin. In a comparison with the experiments, detailed model calculations[32] lead, for example, to the conclusion that 50–60% of the clusters are charged and that the charge transferred between neighboring clusters exceeds ±1. It is, however, clear that in Fig. 11 the effect appears as yet as a qualitative indication.

4. The existence of binary rapidity correlations is of course also predicted by the Kancheli-Mueller theorem in the Regge approach to multiparticle production[33] (see also Refs. 34 and 35). Sufficiently far from the kinematic limits (i.e., for the region of the central plateau)

$$C_2(y_1, y_2) \sim \exp[-(\alpha_P(0)-\alpha_{P'}(0))\,|y_1-y_2|], \tag{13}$$

where $\alpha_P(0)$ and $\alpha_{P'}(0)$ are the intercepts of the P and P' trajectories. Thus, $C_2 \sim \exp(-|y_1-y_2|/2)$ and the correlation range is $\langle\Delta y\rangle = \langle|y_1-y_2|\rangle = 2$. This differs appreciably from the value $\langle\Delta y\rangle = 0.8$–$0.9$ [see (2)] expected for isotropic decay of clusters, but corresponds reasonably to the experimentally observed decrease of C_2 (see Fig. 4) at not too small $|y_1-y_2|$. However, one cannot yet reckon with the applicability of Eq. (13) to the region $|y_1-y_2|\lesssim 1$–2. For in its derivation it was assumed that in the inclusive diagram the interaction between particles with given y_1 and y_2 is transmitted by a single Regge pole, which is true only asymptotically at large $|y_1-y_2|$.

Similarly, the distribution of the gaps Δy between neighboring (in rapidity) particles in the Regge approach,

$$P(\Delta y) \sim \exp[(2\alpha_{P'}-\alpha_P-1)\,dy], \tag{14}$$

gives $\exp(-\Delta y)$ for the same values $\alpha_{P'}(0)=1/2$ and $\alpha_P(0)=1$. This agrees well with the observed dependence at $\Delta y \gtrsim 1.5$ (see Fig. 6), the argument of the exponential containing the density of clusters per unit rapidity interval, $\langle\rho_c\rangle \approx 1$. But the Regge formula (14) too is valid only for $\Delta y > 1$–2.

Thus, in both cases one can say that the Regge approach, which is valid only at large rapidity separations, gives results that in this region do not contradict the cluster interpretation. This does not occasion surprise if for no other reason that that in the ladder approximation of the reggeon (including the pomeron) the rungs of the ladder in the elastic scattering amplitude can be arbitrary. In particular, they can be any clusters. Then the inelastic amplitude, obtained from the scattering of the rungs of the ladder, will correspond to the multiperipheral cluster model and give long-range ordering, which, as we have already said, reflects the correlation of clusters and not of individual particles in a cluster.

2. CAN CLUSTERS BE ORDINARY FEW-PARTICLE RESONANCES?

There has been no lack of attempts to reduce the clustering effect observed in the accelerator data to the manifestation of the ordinary resonances: $\rho, \omega, A_2, f, \ldots$. It is sometimes asserted rather categorically (Refs. 23, 36, 37, and 38) that the experimental data can be explained by just these particles and their decay. These assertions are sometimes based on analysis of experiments at the low energies $E_{\mathrm{lab}} = 16$ or 24 GeV. However, we already know how risky are the attempts to extrapolate the low-energy results, when $\langle n\rangle \lesssim 4$–$5$, to an energy region exceeding the investigated region by one or two orders of magnitude. For example, ten years ago, when the accelerator data were limited to the region $E_{\mathrm{lab}} \leq 30$ GeV, many authoritative experimentalists and theoreticians assumed that the process of multiparticle production can be reduced to the excitation and subsequent decay of two colliding hadrons. It was regarded as admissible to discount the cosmic-ray results, according to which at genuinely large multiplicities the overwhelming fraction of pro-

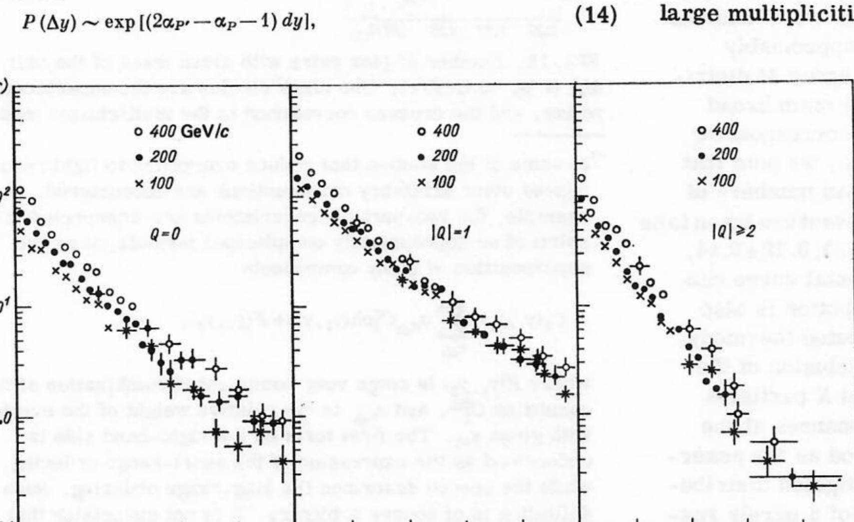

FIG. 11. Distributions of the rapidity gaps for different values of the charge transfer Q.

ducts corresponds to the pionization region, and the excitation and decay of the colliding particles—fragmentation—makes a small contribution. Time was required to establish the full validity of these results. It is just as dangerous to extrapolate to the general case conclusions about the production of few-particle resonances obtained in processes of low multiplicity at low energies. One must reckon with the possibility that in processes of sufficiently high energy there are generated directly not only few-particle resonances but also heavier clusters, whose nature must still be specially elucidated. At low energies, their production probability is of course small. But they make a large contribution to the inclusive distributions, as is shown by detailed model calculations,[19] already at $E_{lab} \sim 40-70$ GeV.

The investigations that reduce clusters to light resonances usually employ a special device to separate a smooth background, on which one identifies the contribution of, say, the ρ mesons; the background is then interpreted by means of certain model conceptions. The difference between the methods of interpretation sometimes leads to directly contradictory conclusions. Let us consider some of the approaches.

1. In studies of this kind, a very common type of analysis considers the integral of the correlation function C_2 at fixed mass of the particle pair:

$$C_2(M)=\int d^3p_1 d^3p_2 \delta(\sqrt{(p_1+p_2)^2}-M)\, C_2(p_1,p_2) \equiv \rho_2(M)=\rho_1^2(M), \tag{15}$$

where p_1 and p_2 are the four-momenta of the final particles with rapidities y_1 and y_2 and transverse momenta $p_{1\perp}$ and $p_{2\perp}$. Different charge combinations are studied, i.e., $C_2^{+-}(M)$, $C_2^{++}(M)$ and $C_2^{--}(M)$.

For example, in Ref. 36 $\rho_2^{+-}(M)$ was studied at $\sqrt{s} \sim 50$ GeV. The experiment gives a broad distribution with a maximum at $M\sim 0.5$ GeV, and a weak shoulder is found at the mass of the ρ meson, $M\sim m_\rho$. With a definite method for subtracting the contribution of this shoulder, one analyzes the remainder as the contribution of pairs of charged π mesons from three-particle decays of the ω, $K^*(890)$, and other such resonances. In fact, the ω meson, which decays into $\pi^+\pi^-\pi^0$ and has mass close to m_ρ, gives $\pi^+\pi^-$ pairs of appreciably smaller mass $M_{\pi^+\pi^-}$, with moreover a broad M distribution, and can in principle explain the main broad maximum. Indeed, by a choice of the corresponding production probabilities (as an example, we note that five coefficients are specified—the mean numbers of the resonances $\rho^0, \omega, K^{*0}+\bar{K}^{*0}, \eta, f$ per event are taken to be equal to $1.19\pm0.25, 1.43\pm0.26, 0.76\pm0.23, 0.22\pm0.14, 0.24\pm0.13$, respectively) the experimental curve can be well described (and a good $p_\perp$ distribution is also obtained, if the resonances are distributed thermodynamically with respect to $p_\perp$). The conclusion of the authors that more than 60% of the π and K particles arise from the decay of the listed resonances at the 95% confidence level must be understood as the assertion that the data of two or three investigated distributions do not contradict the assumption of a purely resonance (light resonances!) origin of the clusters.

We must draw attention to the following feature of such an approach. Frequently, the experiments give the curve of Fig. 12, in which one can discern the statistically confirmed shoulder at $M_{\pi^+\pi^-}=m_\rho$. How can the contribution of the ρ mesons be extracted from this? Two approaches are possible. If one simply straightens the curve smoothly (the dashed curve in Fig. 12) and ascribes the remaining bulge to the ρ mesons, their contribution is small. The remaining "background" can be ascribed, for example, to heavy clusters decaying thermodynamically. Their products will of course be grouped in the region of masses ~0.5 GeV. In the other approach one assumes *a priori* an appreciable admixture of, for example, ω mesons, which, as we have already said, give the necessary maximum in a broad region around the same mass ~0.5 GeV. A specific point here is that if we ascribe a large contribution to ω mesons, then we also obtain a large contribution of ρ mesons.

The authors of Ref. 36 and papers of that ilk adopt the second approach. As we have already emphasized, their conclusion is that the M distribution *does not contradict* the selected "cocktail" of the known light resonances. But this ignores the results of the analysis of other correlation characteristics of the process (for example, curves such as those of Figs. 5, 6, 8, and 9); there is no analysis with selection of different n_{ch}, etc. (we note also that, because of the experimental conditions, pions with small $|y^*|$ in the center-of-mass system were eliminated from the analysis of Ref. 36; but the contribution of heavy, and therefore slowly moving (in the center-of-mass system), clusters must here be especially large).[6)]

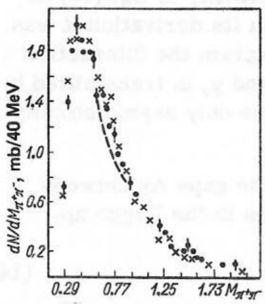

FIG. 12. Number of pion pairs with given mass of the pair $M_{\pi\pi}$ (π^-p, 40 GeV/c). The black circles are the experimental points, and the crosses correspond to the multicluster model.

[6)]In some of the studies that reduce everything to light resonances other arbitrary constructions are encountered. For example, the two-particle correlations are interpreted in the spirit of an appropriately complicated formula (5) as the superposition of many components:

$$C_2(y_1,y_2)=\sum_{n_{ch}} a_{n_{ch}} C_2^{n}\mathrm{ch}(y_1,y_2)+F(y_1,y_2),$$

where $F(y_1,y_2)$ is some very complicated combination of the quantities $C_2^{n_{ch}}$, and $a_{n_{ch}}$ is the relative weight of the events with given n_{ch}. The first term on the right-hand side is understood as the expression of the short-range ordering, while the second describes the long-range ordering. Such a definition is of course arbitrary. It is not surprising that after this the comparison with the experiments leads to an absurd conclusion—the maximum at $y_1=y_2$ in Fig. 4 is a consequence of the "long-range correlation" $F(y_1, y_2)$.

All this renders the final conclusion conditional, and it cannot be regarded as unambiguous or binding. This is especially emphasized by analysis in the framework of the multiperipheral cluster model[19] (see Sec. 5), in which this "ρ shoulder" can be readily explained by a small relative contribution of resonances, while the main peak in the region of masses ~0.5 GeV/c^2 is due to pions from the decay of fireballs.

The ρ peak is more pronounced in the $C_2^{+-}(M)$ curve (Fig. 13), and this offered hope for success of the conception we are discussing.[39] However, when it was implemented with allowance for the effect of the Pauli principle,[40] Thomas[41] did not regard it as possible to ascribe high probabilities to the production of light resonances, these explaining not more than 30% of the area under the curve. At the same time, as was demonstrated in Ref. 42, the histogram of Fig. 13 for C_2^{+-} can be explained well solely by clusters with $\langle M\rangle \approx 1.3$ GeV (it was assumed that their decay has a tree-like nature, as in the statistical bootstrap) with $\langle K_{ch}\rangle = 2.1$.

Nor can we fail to point out the following fact. In the histogram in Fig. 13, the correlation of particles with *opposite* signs, C_2^{+-}, has a peak at the ρ-meson mass. It is interpreted as indicating appreciable direct production of ρ mesons. But there is a similar peak for C_2^{--}, in which such an interpretation is impossible. In both cases, the deviation is within <3 standard errors, and it demonstrates how carefully one must approach a suggestive interpretation.

2. Strong evidence against the reduction of clusters to few-particle resonances and in favor of heavy clusters is provided by measurements of the azimuthal correlation. Besides the older papers,[43] we must draw particular attention to the investigation of Ref. 44 made with colliding pp beams and with very high statistics (>10^5 events). If at each point of the axis of the rapidities $y=(1/2)\ln[(E+p_{\parallel})/(E-p_{\parallel})]$, which are above all a characteristic of the longitudinal motion of the final particles (of energy E and momentum p), we imagine a plane perpendicular to that point, the azimuthal angle φ of emission of a particle with transverse momentum $\mathbf{p}_{\perp}$ is measured from a fixed direction (Fig. 14). The subject of study is the dependence on $\varphi=\varphi_1-\varphi_2$ of the number of pairs with $y=y_1(\varphi=\varphi_1)$ and $y_2(\varphi=\varphi_2)$,

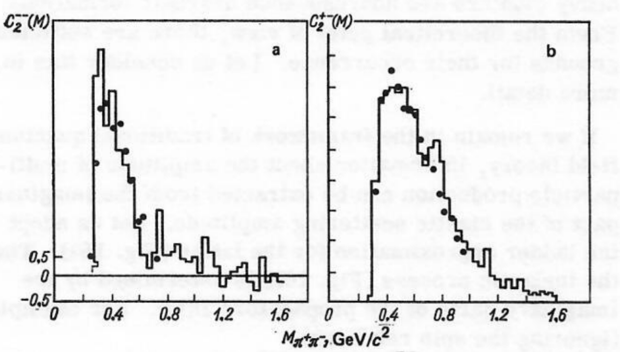

FIG. 13. Mass correlations of $\pi^-\pi^-$ (a) and $\pi^+\pi^-$ (b) pion pairs at E_{lab} = 200 GeV in pp interactions. The histograms are the experimental data and the points were obtained by calculation.

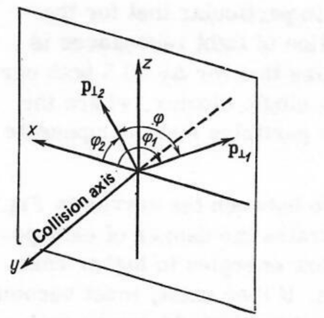

FIG. 14. Projection of the momenta of the decay products of clusters onto the azimuthal plane.

$|y_1-y_2|=\Delta y$. The simplest measure is the difference between the numbers of particles with $0\leqslant|\varphi|\leqslant\pi/2$ and with $\pi/2\leqslant\varphi\leqslant\pi$:

$$B(\Delta y)=\frac{N(|\varphi|<\pi/2)-N(|\varphi|>\pi/2)}{N(|\varphi|<\pi/2)+N(|\varphi|>\pi/2)}. \tag{16}$$

Figure 15a shows some results of measurement of the azimuthal asymmetry from Ref. 44, and Fig. 15b shows the previously obtained results[45] for π^-p collisions. The few-particle resonances must give some almost constant (or slightly decreasing with increasing Δy) background, and at small Δy the Pauli principle must cause the curve to sink. This is observed for π^{++} and π^{--} pairs at low energies (see Fig. 15b); however, for different particles, $\pi^+\pi^-$, there is here a slight rise in the region of small Δy. It is a remarkable fact that at high energies (see Fig. 15a) in the region $\Delta y\leqslant 0.5$ there is a clear excess of B above the background for all pairs, $\pi^+\pi^+$, $\pi^+\pi^-$, and $\pi^-\pi^-$ (under the sampling conditions described in Ref. 44).

A feature of Ref. 44 was that to eliminate the region subject to the influence of the Pauli principle, pairs with $\Delta y<0.8$ were taken into account only if for them $\Delta p_{\perp}>0.2$ GeV (it was confirmed that there is then a strong decrease of the difference between the curves for $\pi^+\pi^+$ and $\pi^-\pi^-$ on the one hand and $\pi^+\pi^-$ on the other; the remaining small difference lay entirely in the region of the light resonances and could therefore be as-

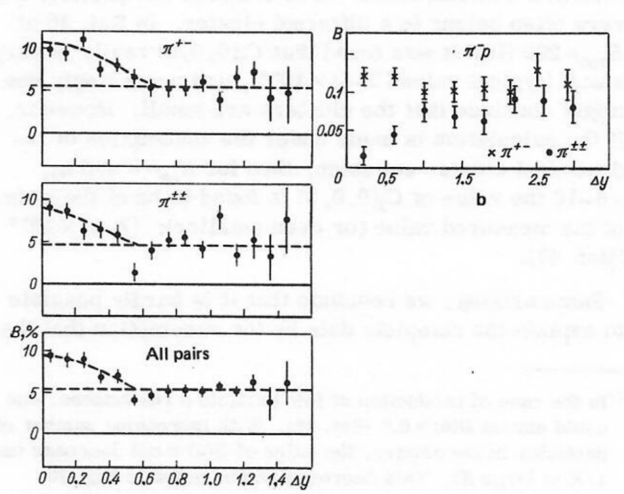

FIG. 15. Data on azimuthal correlation in pp processes at $\sqrt{s}$ = 52.5 GeV $n_{ch}\geqslant 6$ (a) and in π^-p processes at E_{lab} = 40 GeV (b).

cribed to them; it follows in particular that for the chosen $n_{ch} \geqslant 6$ the contribution of light resonances is small). Such a rise indicates that for $\Delta y < 0.5$ both particles lie in the region of a single cluster, where the transverse momenta of the particles must compensate each other.

The qualitative difference between the curves in Fig. 15a and 15b again demonstrates the danger of extrapolating results obtained at low energies to higher energies (where heavy clusters, if they exist, must become more important). The results of Ref. 44 can be regarded as evidence in favor of heavy clusters. In Ref. 44, they are called superclusters, evidently to distinguish them from few-particle resonances.

Indeed, the fact that $B(0)$ differs a little (but reliably) from the "background" $B(\Delta y > 0.5)$ indicates that there are many particles in the cluster[7)] and that fixing the momentum of one particle imposes a fairly weak constraint (since it derives from the necessity of mutual compensation of the momenta of all particles of the cluster) on the chosen second particle.

It is not fortuitous that these clusters could be identified in precisely the high-multiplicity events, $n_{ch} \geqslant 6$. Moreover, the comparatively small rapidity interval of the correlation, $\Delta y < 0.5$ [whereas the decay products of the cluster cover the much larger interval (2)], means that the products of different clusters overlap on the rapidity scale. Only a few particles (two or three charged particles according to a rough estimate) do not "overlap."

3. The accumulation of experimental data makes it possible to consider the analysis of many-particle correlations. The rapidity interval method, presented in Figs. 7 and 8, essentially already uses a certain combination of higher correlation moments. But one can also measure directly $C_3(y_1, y_2, y_3)$ or its integrated measure $f_3(s)$, shown in Fig. 5. It can be seen that f_3 is on the whole smaller in modulus than f_2. At the first glance, it would seem that this smallness of C_3 implies small clusters; for if the clusters are small, then in a combination of three clusters one particle will very often belong to a different cluster. In Ref. 46 at $E_{lab} = 200$ GeV it was found that $C_3(0,0,0)$ really is very small (typical value $(5-6) \times 10^{-4}$), and accordingly one might conclude that the clusters are small. However, if the calculation is made under the assumption of independent cluster emission, then for $K_{ch} = 3$ and $n_{ch} = 6-10$ the value of $C_3(0,0,0)$ is found to be of the order of the measured value (or even smaller): $(2-1) \times 10^{-4}$ (Ref. 47).

Summarizing, we conclude that it is hardly possible to explain the complete data by the assumption that the clusters are the known few-particle resonances decaying into two or three pions.

3. NATURE OF THE HEAVY CLUSTERS

The need to include heavy ($M \gtrsim 2$ GeV) clusters besides the known few-particle resonances to interpret the multiparticle production data appears, on the whole, justified. However, the question of their nature is still very debatable. Two cases are possible.

1. The heavy clusters are as yet undiscovered resonances of essentially the same type as the known few-particle resonances ρ, $\omega, \ldots$. They may be situated on the same Regge trajectories and, in view of the considerable widths of these resonances, may overlap on the mass scale, so that effectively their spectrum will appear as continuous and fairly smooth. This idea is supported by recent results of an analysis of data on a different but similar process—coherent $\pi \to 3\pi$ production on nuclei. Until recently, only a general, comparatively broad distribution with respect to the masses $M_{3\pi}$ of the final state, peaking around $M_{3\pi} \approx 1.1$ GeV, was observed. However, the accumulation of higher statistics makes it possible to undertake a decomposition of the final state into partial waves, determine from $M_{3\pi}$ the energy distribution of the amplitudes of these partial waves, and measure the relative phases. This analysis is very complicated (one must take into account up to ten partial waves) and contains significant uncertain elements. For all that, the result of Ref. 49 appears important; according to it, the distribution can be decomposed into the contribution of the two resonances $A_1(1070), J^P = 1^+$ ($\Gamma \approx 0.3$ GeV) and $A_3(1650), J^P = 2^-$ ($\Gamma \approx 0.4$ GeV) and a small nonresonance background. However, the 0^- state also makes appreciable contributions to both peaks. In addition, the relative phase changes in the necessary manner on passing through the middle of the resonance.

This example shows that heavy clusters in incoherent production could also be a superposition of broad resonances (heavier than the known ones). However, no additional arguments in favor of this possibility (apart from the dual resonance concept with an infinite series of narrow resonances on each trajectory) are as yet known.

2. There is, however, a different possibility: The heavy clusters are nonresonance hadronic formations. From the theoretical point of view, there are sufficient grounds for their occurrence. Let us consider this in more detail.

If we remain in the framework of traditional quantum field theory, information about the amplitude of multiparticle production can be extracted from the imaginary part of the elastic scattering amplitude. Let us adopt the ladder approximation for the latter (Fig. 16a). Then the inelastic process (Fig. 16b) is determined by the imaginary parts of the propagators $D(k^2)$. For example (ignoring the spin relations),

$$D(k^2) = \frac{Z_3}{k^2 + m^2} + \int_{\varkappa_0^2}^{\infty} \frac{\rho(\varkappa^2)\, d\varkappa^2}{\varkappa^2 + k^2}; \tag{17}$$

$$\operatorname{Im} D(k^2) = -\pi\delta(k^2 + m^2) - \pi\rho(k^2), \tag{18}$$

[7)] In the case of production of few-particle ρ resonances, one could expect $B(0) \approx 0.2$ (Ref. 48). With increasing number of particles in the cluster, the value of $B(0)$ must decrease (as $1/K$ at large K). This decrease can be noted in Fig. 20, which shows data at 40 and 200 GeV, analyzed as in Ref. 44 and described by the multicluster model.[47] For comparison, Fig. 20b gives the curve corresponding to the data of Ref. 44 at $\sqrt{s} = 52.5$ GeV.

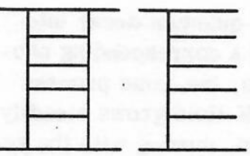

FIG. 16. Ladder diagram of elastic scattering (a) and the inelastic process determining it (b).

where the spectral function $\rho(\varkappa^2)$ is positive and must decrease sufficiently rapidly as $\varkappa^2 \to \infty$, ensuring convergence of the integral. From it, one term, corresponding to the lightest of the possible particles with the given quantum numbers, is separated; Z_3 is a renormalization constant,

$$Z_3 = 1 - \int_{\varkappa_0^2}^{\infty} \rho(\varkappa^2)\, d\varkappa^2. \tag{19}$$

For the particle with the internal quantum numbers of the pion $m = m_\pi$ and $\varkappa_0 = 3m_\pi$. Further, k^2 is the square of the mass of the produced cluster. In Fig. 16b, $\rho(k^2)$ determines in accordance with (18) the mass spectrum of these clusters.

The assumption that only (known or still unknown) resonances are directly produced implies in accordance with (17) that $\rho(k^2)$ is a collection of individual peaks (Fig. 17a). However, it must be emphasized that there are no *a priori* grounds for this; it is a special hypothesis. In general, it is no less probable that $\rho(\varkappa^2)$ has a smooth structure or, at least, a smooth background besides the peaks (see Figs. 17b and 17c). This would mean that the steps in the amplitude of the inelastic process (see Fig. 16b) have a continuous mass spectrum $\sqrt{k^2}$ and that they decompose into a corresponding number of final hadrons.

The physical interpretation of the spectral function is suggested by the fact that the integral of $\rho(k^2)k^2$ determines the difference between the mass m_0 of the bare particle and the mass m of the physical particle:

$$m_0^2 = m^2 + \int_{m^2}^{\infty} \rho(\varkappa^2)(\varkappa^2 - m^2)\, d\varkappa^2, \quad \frac{d^2(m_0^2)}{d(m^2)^2} = \rho(m^2). \tag{20}$$

Since $\rho(\varkappa^2) > 0$, the mass of the bare particle is greater than that of the physical one. Indeed, m is the mass of the lowest stable state of the particle with the given quantum numbers. The contribution to the integral from larger and larger masses $\varkappa$ must be interpreted as the contribution of ever deeper parts of the particle-field system.

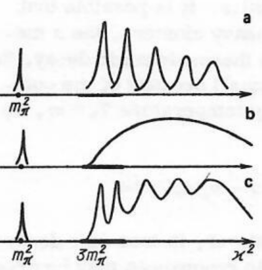

FIG. 17. Form of the spectral density: a) only resonances; b) smooth structure (background); c) combination of resonances and background.

An example of the use of the concept of a bare particle is provided by the parton scheme.[8)] It is customary to regard a parton as some point-like object of a definite kind.

However, it follows from (17) and (18) that in reality it may be necessary to take into account unstable hadrons of different masses as particles that to various degrees are dressed and are manifested in processes of lepton-hadron scattering to different degrees at different momentum transfers. But it can already be seen from (17) that if k^2 is very large, much larger than the $\varkappa^2$ for which $\rho(\varkappa^2)$ is nonzero (see Fig. 17), then (17) with allowance for (19) reduces to $D(k^2) = 1/k^2$, which corresponds to asymptotic freedom and, hence, to a completely bare particle—a true "point-like" parton. In this case, the mass spectrum of the clusters, $\sigma(M) \sim \rho(M^2)$, will end at the corresponding mass of the bare particle, i.e., the parton.

Thus, the nonresonance cluster—the fireball—could be simply a particle very far from the mass shell, a half-naked, or, in the limiting case, bare parton in the sense of Bjorken and Feynman. Of course, this is just one, albeit very plausible possibility and is attractive because it finds a simple expression in field theory. In contrast to this, a fireball could be a formation with indefinite spin (in particular, it could be decomposable into states with definite spins, as is apparently possible for the diffraction dissociation process $\pi \to \pi^* \to 3\pi$ mentioned above[48]). We shall not consider here what its relationship to quarks could be.

4. DECAY OF CLUSTERS

Reflecting the two ideas about the nature of the clusters there are two approaches to the mechanism of their decay into final hadrons.

When the known comparatively light resonances decay, this occurs through a cascade, for example, $B(1235) \to \omega\pi, \omega \to 3\pi$; $A_3(1640) \to f\pi, f(1260) \to \pi\pi$; etc. A similar scheme is well justified for the decay of coherently produced clusters in, for example, the process $\pi \to \pi^* \to 3\pi$; evidently, the process proceeds through the stages $\pi^* \to \rho\pi, \rho \to 2\pi$, etc.[50] Such a "tree-like" decay (Fig. 18) is assumed in the dual resonance model, in which each heavy resonance goes over into the neighboring, lighter resonance with the emission of one particle, and also in the statistical bootstrap model,[16] in which at each vertex there is emitted, not one, but, on the average, 1.4 light final particles.

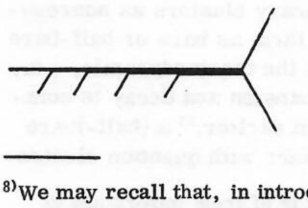

FIG. 18. "Tree" diagram of fireball decay.

8)We may recall that, in introducing patrons, Feynman argued that since the only good relativistically covariant theory is quantum electrodynamics, it can be taken as a convenient paradigm and a parton can be understood as a bare particle corresponding in electrodynamics to a bare electron.[24]

In the traditional thermodynamic model, which takes into account the strong interaction of the particles in the final state, the process is regarded as the expansion in space and subsequent cooling to the final critical decay temperature $T_k \approx m_\pi$ of the original nonresonance hadronic bunch. If the mass of this bunch is very large, the process can take a long time and may need a hydrodynamic description; the decay of the different elements of the bunch into the final hadrons then takes place at different times, when the temperature of a given element in its associated rest frame has sunk to T_k. Thus, in thermodynamic decay too the final particles are actually emitted, not simultaneously, but successively. It is hardly surprising that the momentum spectra in the statistical bootstrap, which is based on the concept of a limiting temperature, and in the ordinary thermodynamic model, which allows an arbitrarily high temperature at the start of the process, come out to be virtually identical.[9)]

In the dual resonance model with heavy resonances of zero width, the spectrum is exponential in the energy[51]:

$$dN \sim \exp[-E(\mathbf{p})/\theta]\,d^3p, \quad \theta = \frac{\sqrt{6}}{2\pi\sqrt{\alpha'(0)D}}, \tag{21}$$

where $\alpha'(0)$ is the slope of the trajectory, assumed constant, $\alpha'(0) \sim 1\ \mathrm{GeV}^{-2}$, and D is the number of dimensions of the oscillators in the theory. If $D=4$–7, we obtain $\theta = 0.18$–$0.15\ \mathrm{GeV} \approx m_\pi$.

In the thermodynamic model, neglecting the hydrodynamic motion, a distribution of exactly the same type is obtained from the Bose and Fermi formulas, since in them one can usually ignore the term ± 1 in the denominator. Indeed, because T_k is small, $\sim m_\pi$, this is always possible for particles heavier than the pion. In addition, it is permissible for relativistic pions with energy $E \gtrsim (2$–$3)m_\pi$ (unfortunately, there do not yet exist sufficiently accurate measurements for the spectra of nonrelativistic pions, when the difference between the Bose and Boltzmann distributions should be manifested; there are only some unpromising indications). Thus, the coincidence of (21) with the thermodynamic distribution

$$dN \sim \exp(-E(\mathbf{p})/T_h)\,d^3p, \quad T_h \approx m_\pi \tag{22}$$

is a consequence of the fortuitous numerical coincidence of the dynamical quantity θ and the thermodynamical quantity T_k. [It is well known that (22) describes excellently the $p_\perp$ and mass distributions of all particles, from the π mesons to the ψ and ψ' particles, produced in high-energy collisions.[52]]

Note that if we regard the heavy clusters as nonresonance particles and describe them as bare or half-bare particles of large mass, then the thermodynamic, i.e., quasiclassical, picture of expansion and decay is completely natural. As was shown earlier,[53] a (half-)bare electron restores, in accordance with quantum electrodynamics, its normal clothing by quantum decay into a dressed (physical) electron and a corresponding photon. In classical electrodynamics, the same process takes place differently[54]: The self-field grows steadily as the front of the increasing field, moving with the velocity of light, moves away from the electron and forms a light wave. A similar picture must apply for the decay of a hadron far from the mass shell, bare or half-bare, in particular for a parton. Indeed, if it decays into *many* hadrons, then this decay can be described quasiclassically as steady expansion from an initial small volume (which may have characteristic dimension $\sim M^{-1}$, where M is the mass).

5. DEVELOPMENT BEYOND THE SIMPLIFIED SCHEME OF PRODUCTION OF IDENTICAL ("AVERAGE") CLUSTERS

The experimental data, which have only partly been discussed above, give a clear indication that clusters heavier than the known meson resonances participate in multiparticle production. It is also highly probable that ordinary resonances are also produced directly in the collision process. The participation of ρ mesons is already indicated by the corresponding peaks in the experimentally obtained pair mass distributions of the produced pions (Refs. 36–38).[10)] There are no grounds for thinking that direct production of other ordinary resonances is absent either. On the other hand, it does not seem possible to explain the complete data by them alone (see Sec. 2).

Numerous evaluations of different correlation data obtained at the accessible accelerator energies, $\sqrt{s} \lesssim 60$ GeV, are made *on the basis of the model of independent emission of clusters of some common "average" mass*. An invariable deduction from these evaluations (see, for example, Refs. 20, 32, 42, 43, 55, and 56) is that this "average" cluster has the mass

$$\langle M_c \rangle \approx 1.3-2.0\ \mathrm{GeV} \tag{23}$$

and decays into

$$\langle K \rangle \approx 4\ \text{pions}. \tag{24}$$

However, this concept of an average cluster can, perhaps, be allowed as a first step in order to get one's bearing, but it is inadequate for understanding the clustering mechanism and in many respects is confusing. Let us consider this question in more detail.

If we assume that not only heavy clusters but also light ρ and ω mesons are directly emitted, it follows

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9)Note that in such an approach it is of great importance to justify the application of the concept of thermodynamic equilibrium, the possibility of a classical description of the hydrodynamic stage in the expansion of the bunch, and so forth; we shall not consider these questions here.

10)However, this is not quite conclusive. It is possible that initially there are produced only heavy clusters, the ρ mesons arising from their decay. In thermodynamic decay, the admixture of ρ mesons would be small because of the comparatively high mass; at the decay temperature $T_k \approx m_\pi$ we obtain

$$\frac{\langle n_\rho \rangle}{\langle n_\pi \rangle} = (2I+1)\left(\frac{1}{3}\sqrt{\frac{2}{\pi}}\right)\left(\frac{m_\rho}{m_\pi}\right)^{3/2}\exp(-m_\rho/m_\pi) \approx 0.05$$

(I is the spin of the ρ meson). However, in tree-like decay (see Fig. 18) the contribution of the resonances may be greater. In any case, the above-mentioned observation that in coherent $\pi \to \pi^* \to 3\pi$ production the decay takes place preferentially as $\pi^* \to \rho\pi \to 3\pi$ (Ref. 50) indicates such a possibility.

from (23) and (24) that much heavier clusters, fireballs, must also be emitted. Symbolically, we can write

$$\langle \text{cluster} \rangle = \langle \text{light resonances} \rangle + \langle \text{fireballs} \rangle \qquad (25)$$

(and we need not prejudge the question of the nonresonance nature of the fireballs).

An averaging of the type (25) has in fact a quite different nature for different investigated effects. For example, two components can occur together very oddly in the binary correlation, as we have already said in connection with Eq. (5). With regard to the mean multiplicity, fireballs with $\langle K^{fb} \rangle = 8$, "added" to ρ mesons ($\langle K^{\rho} \rangle \equiv K^{\rho} = 2$), give, for example, the relation (24): $\langle K \rangle = 4$ if the relative contribution of the ρ particles is $\alpha = 2/3$:

$$\langle K \rangle = 2\alpha + 8(1-\alpha) = 4. \qquad (26)$$

In other words, it is sufficient if one fireball of eight particles is produced for two ρ mesons.

In the light of this circumstance, we must above all recognize the importance of the results of early cosmic-ray investigations in which a special choice of events of not too small multiplicity was made. For example, it was found that in events with $n_{ch} \geqslant 5$ (i.e., $n \geqslant 7-8$) at energies above the currently available accelerator energies (we recall that at $\sqrt{s} \approx 50$ GeV, $\langle n_{ch} \rangle \approx 12$) there are produced two fireballs with $K \approx 7-10$ (Refs. 5–7). At $\sqrt{s} \approx 15$ GeV with a different experimental technique the production on the average of one such fireball was established.[8]

These results were sharply criticized, but it can now be seen that in many respects at least the criticism was unjustified. For example, doubt was expressed concerning the employed methods of identifying nucleon-nucleon collisions in experiments with a nuclear target [emulsion,[5-7] or LiH (Ref. 8)], but they were sufficiently reliable. The observed fact of weak absorption of fireballs in nuclei was correct; this was already long ago interpreted as a sign that the fireball interaction cross section directly after production is small.[5] Correct too was the distribution, found admittedly with large errors, of the inelasticity coefficients,[57] etc.

The accelerator data on the inclusive distributions [see Fig. 2 and Eqs. (2) and (3)] enable one to understand why it is not possible that there are produced only light resonances or light partons decaying into two or three pions.[21] If only light clusters were produced, then because of the rapidity overlap of pions from different clusters they would have to enter into multiple interaction with one another, so that their decay products would be emitted from a small volume, of the order of a single hadron: $4\pi/3m_{\pi}^3$. With high probability, such a system can be regarded as a heavy fireball. In other words, the purely ρ-mesonic (and ω-mesonic) multiperipheral model (and also the parton model, if the partons are light, i.e., if they decay into only two or three pions) is not self-consistent.

To get an idea of the consequences that could be obtained from a more realistic scheme of multiparticle production (involving light resonances and fireballs), we turn to the concrete multiperipheral model already mentioned earlier.[19] Here, in πN and NN collisions one assumes at the vertices of the chain production of not only ρ and baryon (at πN vertices) resonances, but also fireballs with some continuous mass spectrum, $M \gtrsim 2-3$ GeV (Fig. 19). Of course, it is necessary here to fix a number of parameters, and for the fireballs it is also necessary to select a definite type of decay (Pomeranchuk's statistical model is adopted). However, the simple requirement that this model reproduce the correct behavior of the total cross sections $\sigma_{\pi N}(s)$ and $\sigma_{NN}(s)$ for $\sqrt{s} \lesssim 15$ GeV already strongly restricts the arbitrariness. In particular, it is impossible to achieve this without a significant admixture of clusters with mass $M \gtrsim 2-3$ GeV. This concrete, analytically and numerically developed model describes very well with a single set of parameters all the available and, in recent years, newly published inclusive and semi-inclusive data, and also the binary correlation coefficients for πN and NN collisions at $E_{lab} = 40$, 70, and 200 GeV. The complete listing of all confrontations with the experiments would occupy too much space. We restrict ourselves to pointing out that for the chosen set of parameters a computer was used to generate (at a given E_{lab}) 50 000 exclusive events. They constitute a fund of events from which even now one can extract all inclusive and semi-inclusive distributions. Moreover, one can now see which particles arose from clusters of a given mass, which from ρ mesons, and so forth. Therefore, one can see what contribution to any particular characteristic is made by resonances (ρ, Δ^{++}, etc.) and fireballs of any mass. One can sample events corresponding to the conditions of a definite experiment, etc. Many of the confrontations with the experiments have been carried out (and are being carried out) on the publication of the corresponding experimental data on the basis of the same fund of computer-simulated events (and therefore, without any change of the parameters). Note, in particular, that the points in Fig. 2 give the inclusive and topological rapidity distributions calculated in this manner for pp collisions with energy 70 GeV (the histograms correspond to an experiment at 69 GeV); the points in Fig. 4 give the binary correlation coefficients (the "wings" of the curve, on which $C_2 < 0$, were obtained in the model of Ref. 19 before they were discovered experimentally and at first, when the experiment had only low statistics, were taken to contradict observation); and the points in Fig. 12 are the distributions $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ recently extracted from the same fund for $E_{lab} = 200$ GeV.[47] In this last case, a peak was not found at $M = m_\rho$, but it is also insufficiently confirmed experimentally.

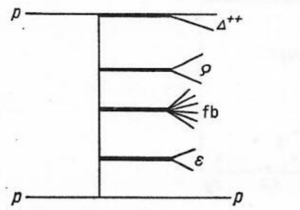

FIG. 19. Example of a multicluster diagram.

All this enables us to assume that a physical world in which the model of Ref. 19 is exactly valid (see Fig. 19) would not differ too strongly from our actual world, and this model can (on the basis of the 50 000 simulated events corresponding to it) be used to test various hypotheses about the process of multiparticle production of hadrons.

Recently, after the publication of the data given above on the azimuthal correlations[44] (see Fig. 14) there was extracted from the same fund, for the same data selection as in Ref. 44 ($n_{ch} \geqslant 6$, $\Delta p_\perp > 0.2$ GeV/c if $\Delta y < 0.8$) but at lower energy $\sqrt{s} = 20$ GeV a curve $B(\Delta y)$ analogous to the one shown in Fig. 14. It is shown in Fig. 20 (Ref. 47). The curve is of the same kind as the experimental curve in Ref. 44. It was noted above (in connection with Fig. 15) that the small deviation at $\sqrt{s} = 52.5$ GeV of $B(0)$ from the background can be explained if the clusters are heavy. This can, even if only very approximately, be verified as follows using the material of the simulated events and Fig. 19 (Ref. 47).

Suppose that in a cluster of $\langle K \rangle$ particles, half of them are emitted into one hemisphere, and the other half into the other. Further, suppose that in the limit $\Delta y \to 0$ particles of only one given cluster play a part. Fixing one particle with given y_1 and φ_1, for the particles with $y_2 = y_1 + \Delta y$ and $\varphi_2 = \varphi_1 + \varphi$ we obtain $\langle K \rangle/2$ particles with $\varphi > \pi/2$ and $\langle K \rangle/2 - 1$ particles with $\varphi < \pi/2$, i.e.,

$$B(0) = \frac{\langle K \rangle/2 - (\langle K \rangle/2 - 1)}{\langle K \rangle/2 + (\langle K \rangle/2 - 1)} = \frac{1}{\langle K \rangle - 1}. \qquad (27)$$

Therefore, $B(0) = 0.22$ (see Fig. 20) implies $\langle K \rangle = 5.7$. On the other hand, having a set of exclusive events for which we know from which cluster a given particle arose, we can determine $\langle K \rangle$ for the events included in a given sample $n_{ch} \geqslant 6$, etc. We find $\langle K \rangle = 5.5$, which agrees well with the number (5.7) extracted from the curve in Fig. 20. Of course, such a close coincidence is fortuitous; in fact, particles of not just one cluster play a part in $B(0)$; at such small B, we cannot restrict ourselves to only the mean $\langle K \rangle$, and fluctuations are important, etc. Therefore, applying formula (27) to Fig. 14, where $B(0) \approx 0.10$, and obtaining $\langle K \rangle = 11$, we must not ascribe too much significance to this number. It is, however, clear that $\langle K \rangle$ is here large, and one can understand why the word "supercluster" was used in Ref. 14. The same theoretical fund of events was also used to calculate[47] the dependence of $\langle K(K-1) \rangle / \langle K \rangle|_n$ on $n/\langle n \rangle$ in semi-inclusive pp collisions at 205 GeV. It was found to be fairly close to the value obtained by analyzing the experimental data in the framework of the independent cluster emission model (see Fig. 9), although it is evident that the estimates obtained by means of this model slightly underestimate the number of particles in a cluster.

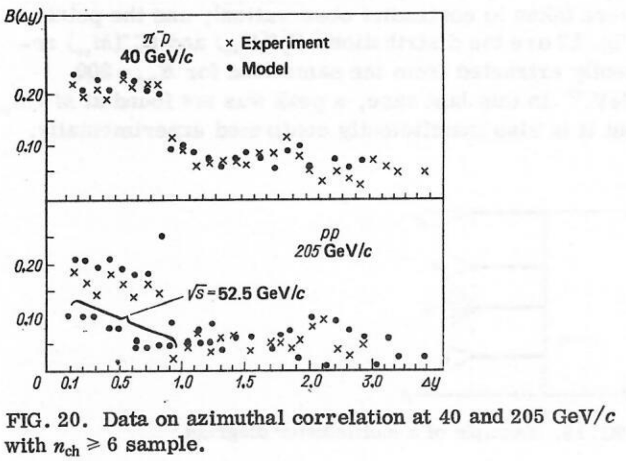

FIG. 20. Data on azimuthal correlation at 40 and 205 GeV/c with $n_{ch} \geqslant 6$ sample.

Recalling again the old cosmic-ray data,[5-7] in which under the condition $n_{ch} \geqslant 5$ (and not $n_{ch} \geqslant 6$ as in Ref. 14) but at much higher energy the existence of fireballs with $\langle K \rangle \approx 8$–10 was indicated, we can regard the results of the experiment of Ref. 44 and the investigations of Ref. 47 as a confirmation of them.

In turn, all this demonstrates once again *how important it is to take into account a nonuniform mass composition of the clusters*, and *how important it is to go beyond a model with some average cluster* (23),(24), $\langle K \rangle \approx 4$.

What is the contribution of the fireballs at different energies? From cosmic-ray data 20 years old,[5-8] in each event there is produced on the average at $E_{lab} \approx 200$ GeV one fireball, and at $E_{lab} \approx (2\text{–}10) \times 10^3$ GeV, two, these having approximately the same characteristics.

We can now base ourselves on the data of the model of Ref. 19 and the fund of events simulated by means of it, according to which the cross section for the production of a pion fireball with $M \gtrsim 2$–3 GeV at $E_{lab} \approx 30$ GeV (in NN collisions) is only 1.5 mb.[58] However, massive fragmentation clusters play an important part at these energies, and their contribution cannot be ignored in the study of many inclusive and semi-inclusive characteristics.[11)] Already at $E_{lab} = 200$ GeV a pion fireball is present on the average in the amount ~1.3 per event, which agrees well with Ref. 8.

From the analysis of the simulated events it follows that in the E_{lab} range from ~40 to 2000 GeV the contribution of the diagrams in Fig. 19 with a small and weakly E_{lab}-dependent number of vertices 3–4 make the dominant contribution. But at low energies we have almost exclusively resonances, while with increasing energy the mass of the clusters increases.

CONCLUSIONS

Numerous analyses of the extensive data on multiparticle production of hadrons in hadron-hadron collisions obtained in the last six or seven years with accelerators at $E_{lab} = 40$–2000 GeV are compatible with the conclusion that the production process can be divided into two stages ("factorized"). The two stages are the production of clusters and their subsequent decay into the final hadrons.

[11)] In this energy region, fragmentation fireballs,[58] i.e., highly excited states associated with colliding nucleons, play an important part. The velocity of the fragmenting nucleons is low ($\gamma_c \lesssim 1.5$). The pions from their decay can simulate a central pion cluster, since they are not separated in rapidity. Thus, one can explain[58] the assertion that at 28.5 GeV pion fireballs are already manifested.[59]

The usual methods for revealing this process use the concept of a certain average cluster, for which one obtains the mass value $\langle M_c \rangle \approx 1.3-2.0$ GeV, multiplicity $\langle K \rangle \approx 3-4$ of the hadrons (almost exclusively pions) into which it decays,[56] and electric charge $Q=0$ and ± 1 (the data do not contradict the assumption of almost equal contributions of these charges, but they must be regarded as preliminary). The nature of the decay of the cluster is not particularized, but the total (inclusive) mass and transverse momentum distributions of the products (in the region of the principal peaks) agree with the thermodynamic formulas (these two distributions are insensitive to the longitudinal motion, which could only be taken into account by particularizing the model and introducing new and not definitely known parameters). This, of course, rules out neither the possibility of a different type of decay nor a resonance nature of even the heaviest clusters.

However, the concept of an average cluster is not satisfactory. One must include among the clusters not only the known resonances, but also heavier formations which could be either as yet unknown resonances or nonresonance objects with a broad mass spectrum, $M_{\text{fb}} \gtrsim 2$ GeV. The heavy clusters are called fireballs. One can hope to explain the observed characteristics of the processes (the correlations, etc.) by light resonances $(\rho, \omega, \ldots)$ alone only at low multiplicities, $n \lesssim \langle n \rangle$, and at relatively low energy (tens of GeV). At larger $\langle n \rangle$ and $n/\langle n \rangle$ the important part played by fireballs is clearly revealed. Because of the considerable width of the fireball mass distribution, the characteristic multiplicity associated with fireball decay depends on the event selection criteria. For events with $n_{\text{ch}} \geqslant 6$ and energy $E_{\text{lab}} = 200-2000$ GeV, K may go up to ~10. Obviously, it is meaningless to look for such fireballs in events with low multiplicity.

Thus, let us now summarize the main evidence supporting the existence of clusters considerably heavier than three-pion resonances.

Indirect Evidence.

1. If the mean multiplicity increases logarithmically with increasing s and we restrict ourselves to the simple multiperipheral model (ladder approximation), then in the formula $\langle n \rangle = a \ln(s/s_0) + b$ the coefficient a is directly related to the mass of an individual rung (and hence to the multiplicity on its decay).[60] The experiments give $a \approx 2.5$, whereas the resonance ladders lead to $a \approx 0.9-1.2$ (Ref. 49).

2. If the total collision cross section $\sigma_{\text{tot}} \sim s^{\lambda}$ is described by the same (as in 1)) simple model, the smallness of λ requires large masses of the rungs[61] (it is well known that if this simple model is given up, in Regge field theory the smallness of λ can also be ensured otherwise, for example, by summing a series of branching combs[62]).

3. The rapid growth of the pion spectra with energy in the central region cannot be explained as a threshold effect in the multiperipheral models with emission of light resonances,[49] and requires for its explanation the introduction of fireballs.[60]

4. The model of Ref. 19, which takes into account the emission of heavy clusters, gives a good description of all the inclusive and semi-inclusive data.

"Direct" Evidence. By this we mean the extraction of information about clusters from the experiments described in the present review, and others like them.

1. The strong maximum of the two-particle rapidity correlation $R_2(0,0)$ (see Fig. 2).

2. The presence of an inflection in the distribution of rapidity gaps (see Fig. 6).

3. The results of the analysis of the rapidity intervals (9) at 200 and 69 GeV (see Fig. 8), which show that $K_{\text{ch}} \gtrsim 2$, this value increasing with increasing $n_{\text{ch}}/\langle n_{\text{ch}} \rangle$.

4. The results for the semi-inclusive two-particle rapidity correlation, which show that $K_{\text{ch}} \geqslant 2$ for $n_{\text{ch}}/\langle n_{\text{ch}} \rangle \gtrsim 3/2$, the value then increasing with a further increase of this ratio (see Fig. 9).

5. The azimuthal correlation (see Fig. 15a) measured with high statistics for $\sqrt{s} = 5.25$ GeV with sampling of many-particle events ($n_{\text{ch}} \geqslant 6$), which indicates the existence of heavy clusters, in agreement with the old cosmic-ray data[5-7] obtained with a $n_{\text{ch}} \geqslant 5$ sample.

The Absence of Contradictions with Data Successfully Interpreted without Heavy Clusters. The absence of such contradictions can sometimes be established only by taking into account the possibility of a spread of the masses (and $K_{\text{ch}} \equiv n_{\text{ch}}$) of the clusters—from light resonances to fireballs.

1. The $C_2(M)$ curves (see Fig. 12)[39] can be interpreted[42] for $K_{\text{ch}} = \langle K_{\text{ch}} \rangle = 2.1$.

2. The $\rho_2(M)$ curves[36] can be interpreted equally successfully by a set of five light resonances (with specially chosen weights)[36] and a model with the participation of heavy clusters.[19,63]

3. The smallness of the semi-inclusive two-particle correlations does not contradict a significant role of fireballs. In contrast, analysis in the framework of the independent cluster emission model indicates the presence of such objects [see 4) in the "direct" evidence].

4. The smallness of the triple correlation $C_3^{---}(0,0,0)$,[46] interpreted as evidence for the absence of many-particle clusters, can in fact be obtained in the independent cluster emission model even for $K_{\text{ch}} = 3$ (Ref. 47).

We emphasize once more that the relative importance of the directly produced few-particle resonances, on the one hand, and of the fireballs, on the other, depends extremely strongly on the nature of the studied effect, on the sample of events (especially on the choice of n_{ch}), etc.

At 200 GeV, the number of pion fireballs per event is $n_c \equiv \bar{n}_c \approx 1.3$ (Ref. 47) (in agreement with the old cosmic-ray results, for which $n_c \approx 1$ at the same energy[8]).

As we have already noted, for some effects the contribution of the fireballs must be taken into account even when their number is small, for example, at

40–70 GeV.

The existing experimental estimates of the cluster charge give $|Q|=0$ and 1 (perhaps, in approximately equal fractions), though the accuracy is as yet low and there are as yet no data on the separation into charges for light and heavy clusters.

Thus, summarizing, we can say that the experiments on multiparticle production at high energies reveal the existence of massive correlated groups of pions (fireballs), whose main qualitative characteristics are already fairly clear. Questions relating to the nature and the decay of the fireballs require further investigation.

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