Limiting fragmentation of nuclei—the cumulative effect (experiment)

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A review is given of Baldin's hypothesis of scaling invariance of the interaction of relativistic nuclei (the cumulative effect) and the currently available experimental data on the probability of production of cumulative particles (the dependences on the energy and the angle of emission of the secondary particles, on the atomic weight of the fragmenting nucleus, and on the energy of the interacting nuclei). The experimental data are considered in the following order: the first data, the most accurate data, and then data as yet unique.

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INTRODUCTION

The study of deep inelastic interactions between electrons and nucleons led to the discovery of the new dynamical principle of scaling invariance, which is well known in the literature as Bjorken scaling. At the beginning of the seventies, scaline ideas were extended to the interactions of hadrons. It was confirmed experimentally that the inclusive spectra of hadrons produced in interactions do indeed scale at different initial energies. In other words, the strong interactions were found to be scale invariant (to be self-similar, or to scale).

In 1971, Baldin conjectured[1] that the spectra of high-energy secondary particles produced by collisions of relativistic nuclei are determined by the local properties of hadronic matter and not by the geometry (form factors) of the colliding objects. This conjecture extended the idea of scaling to collisions of relativistic nuclei.

1. SCALING

Scaling is a law that determines the interaction dynamics.

The problem of studying inclusive spectra was posed for the first time by Logunov and his collatorators[2]; they obtained bounds and consequences of the general principles of quantum field theory. Later, Yang and Feynman formulated hypotheses of limiting fragmentation, which are valid at infinitely high energies. Ther There is no fundamental difference between the hypotheses of Yang and Feynman, and they imply that when the energy (s) of the interacting particles tends to infinity the invariant cross section of the inclusive process tends to a limit:[1)]

$$E\,(d\sigma/d\bar{p})\,(s,\ p_{\parallel},\ p_{\perp}) \rightarrow E\,(d\sigma/d\bar{p})\,(\mathrm{p}). \quad (1)$$

According to Yang,[3] the produced particles for which $p_{\parallel}$ is limited in the laboratory system are to be regarded as fragments of the target. Particles with limited momenta in the rest frame of the incident particle are fragments of it. The production dynamics for the inclusive particle (c) in the interaction of hadrons A_1 and A_2 can be expressed in this scheme in the form

$$\begin{array}{l} A_1 + A_2 \longrightarrow A_1^* + A_2^* \\ \qquad\qquad\qquad\ \ \lfloor \rightarrow c^{(2)} + X_2 \\ \qquad\qquad\ \lfloor \rightarrow c^{(1)} + X_1. \end{array} \quad (2)$$

In the space of rapidities, one can expect a separation of the target ($c^{(2)}$) and projectile ($c^{(1)}$) fragments. It should be noted that such a mechanism does not in general imply periodicity of the interaction in the impact parameter.

It follows from symmetry arguments and Lorentz invariance that for composite hadrons A_1 and A_2 the dependence of the invariant production cross section of particle $c^{(1)}$ on the number of constituents in the hadrons A_1 and A_2 is the same as the dependence of the production cross section of particle $c^{(2)}$ on the number of constituents in hadrons A_2 and A_1:

$$\left.\begin{array}{l} d\sigma\Big/\dfrac{d\bar{p}}{E}\,(c^{(1)}) = F\,(A_1,\ A_2); \\ d\sigma\Big/\dfrac{d\bar{p}}{E}\,(c^{(2)}) = F\,(A_2,\ A_1). \end{array}\right\} \quad (3)$$

It will become clear in what follows that the A dependence of the production cross sections of cumulative particles on nuclei is the most important characteristic of the process. On the basis of the A dependence it is possible to distinguish the cumulative and noncumulative processes and choose a definite mechanism of the cumulative effect.

It also follows from Lorentz invariance that a convenient variable[4] for describing the energy spectra of the inclusive particles in the reaction (2) is the scalar product of the four-momenta of the fragmenting hadron and its fragment:

$$b_1 = (p_{A_1} \cdot p_{c^{(1)}}) \quad \text{or} \quad b_2 = (p_{A_2},\ p_{c^{(2)}}). \quad (4)$$

In the rest frame of the fragmenting particle this variable is equivalent to the fragment energy, and in the moving frame to the ratio $\sim p_c/p_A$.

Formally, limiting fragmentation [the relation (1)] begins when

$$\sum m_i^2/s \rightarrow 0, \quad (5)$$

where m_i are the masses of the interacting particles.

If scaling is the law determining the dynamics of hadron interactions, it is particularly important to

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[1)] $(E/p^2)d\sigma/dpd\Omega$ (mb · GeV^{-2} · c^3 · sr^{-1}) $\equiv E(d\sigma/d\bar{p})$.

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establish the energies at which this law begins to hold. To establish this directly from experiments, we need a good method for choosing a scaled variable; in other words, the finite masses of the interacting particles must be taken into account properly. The variable that became most widely used among the experimentalists studying projectile fragmentation is

$$X = p_c/p_c^{\max} \quad \text{or} \quad X = b/b^{\max}. \tag{5'}$$

In the frame in which the fragmenting particle is at rest and the inclusive particle has limited momentum, the importance of the mass corrections in the choice of the scaled variable can be seen especially clearly. Strictly speaking, this question remains open and there is every reason to pose the problem of finding a scaled variable and understanding its significance from the equality of inclusive cross sections at different energies.

Progress in the description of scaling has advanced furthest in the approaches based on new entities with vanishingly small dimensions—partons. At the present time, parton-quark theories are in a state of stormy development. The quark model, developed on the basis of composite models, has to its credit not only confirmed predictions but also the capacity to describe a remarkably large body of experimental material.

A basic hypothesis of the composite models is that of the existence in nature of binding energies comparable with the masses of the constituent particles. This necessitates an essentially relativistic approach to the bound-state problem—a fundamental problem in its own right. In other words, one of the central problems is that of describing "interhadronic matter" and finding a relativistic description of extended composite objects.

The aim of the nearest review is to draw attention to a new approach to this group of problems initiated at the High Energy Laboratory at Dubna—relativistic nuclear physics.[5] Among the many questions, we consider one, possibly the most striking; it relates to the case when the energy of an inclusive particle produced in the collision of relativistic nuclei greatly exceeds the energy per nucleon of the accelerated nucleus—this is the cumulative effect.

The cumulative effect can be identified most reliably in the case of limiting fragmentation of nuclei, for which there is an additional identification criterion, namely, scaling. The phenomenon is important because the energy of the inclusive particle is appreciably greater (by a factor 10^2) than the binding energy of the nuclear constituents permitted by the kinematics corresponding to collision of free nucleons of the nucleus. Much experimental material has now been accumulated on this question in different laboratories, and the time is ripe for systematization of the data and identification of the main effects.

The most realistic description of cumulative processes is provided by the mechanism of excitation of a group of N nucleons with subsequent "decay" into the inclusive particle and anything else:

$$(Nm)^* \to c + Nm. \tag{6}$$

In this representation, the inclusive particles (the fragments) can be divided into two classes: specific fragments (π, k, $\tilde{p}$, etc), for whose production the excitation energy of the group of nucleons $(Nm)^*$ must at least exceed the mass defect, and ordinary fragments (p, d, t, α, etc.), as composite parts of the "remnant" [Eq. (5)]. The binding energy of the specific fragments is equal to their mass; the binding energy of the ordinary fragments is much less than their mass.

Thus, it appears reasonable to follow this classification in the description of the experimental data.

2. BALDIN'S HYPOTHESIS OF SCALING OF THE INTERACTION OF RELATIVISTIC NUCLEI AND ITS EXPERIMENTAL VERIFICATION

According to the hypothesis of the cumulative nuclear effect, inelastic interactions with nucleons of complex composite systems such as nuclei consisting of A nucleons are described by the sum of scale-invariant processes of the type

$$\left.\begin{array}{l} f_1(m+m\to\pi+M_1)=f_1(x_1);\\ f_2(2m+m\to\pi+M_2)=f_2(x_2);\\ \vdots\\ f_N(Nm+m\to\pi+M_N)=f_N(x_N);\\ \vdots\\ f_A(Am+m\to\pi+M_A)=f_A(x_A). \end{array}\right\} \tag{7}$$

The scaled argument x is the ratio of the energy variable of the produced particle to the maximal value of this variable permitted by the kinematics of a collision of N nucleons with one nucleon.

The number N of nucleons of the composite system A that participate in the interaction event depends only on the properties of the nucleus A itself, and the probability of a definite value of N is described, for example, by the binomial distribution. Thus, in general form

$$E\frac{d\sigma}{d\mathbf{p}} = \sum_{N_{\min}}^{A} P_N(N)\, f_N(x_N). \tag{8}$$

The threshold value of $N_{\min}$ is determined by the energy and emission angle of the inclusive particle c. At high energies of the interacting particles in the rest frame of the fragmenting nucleus[6]

$$N_{\min} \approx (E_c - P_c\cos\theta_c)/m, \tag{9}$$

where θ_c is the angle between the momenta of the fragment and the projectile.

Note that the relation (8) does not yet contain specific information. For example, if we assume that

$$f_N(x_N) \equiv f_1(x_1),$$

then the expression (8) can be interpreted in terms of multiple interaction of the projectile with the individual nucleons of the nucleus A, in which case $N_{\min}$ is physically approximately equal to the number of rescatterings; it can also be interpreted as the mechanism of Fermi motion, and then the probability $P(N)$ can be identified numerically with the probability of the kinematically necessary Fermi momentum of a nucleon in the nucleus.

The first prediction of the value of the cross section of the cumulative effect was made by Baldin on the basis of the simplest assumption

$$f_N(x_N) \equiv f_1(x_N). \tag{10}$$

If the invariant cross section $Ed\sigma/d\overline{p}$ scales, then the quantities $P_N(N)$ can only be constant coefficients. This conclusion is obvious for the case of the Dp interaction. In the framework of the hypotheses (10), there can exist different models of the interaction mechanism.

In Ref. 5 there are considered two models for binomially distributed coefficients:

$$P_N(N) = \frac{A!}{N!(A-N)!} p^N (1-p)^{A-N}. \tag{11}$$

V Model. The value of p is determined by the probability of finding N nucleons in the characteristic volume $4\pi r^3/3$:

$$p = (r/r_0)^3 (1/A), \tag{12}$$

where $R = r_0 A^{1/3}$ is the radius of the fragmenting nucleus.

S Model. The value of p in the expression (11) is determined by the probability of finding a nucleon in a "tube" with radius r and length $4r_0 A^{1/3}/3$:

$$p = (r/r_0)^2 (1/A^{2/3}). \tag{13}$$

It is assumed that $r < r_0 \approx 1.2$ F. Therefore, $p \ll 1$ in both the V and the S model, and $P_N(N)$ can be well approximated by the Poisson distribution with $\langle N\rangle = pA$:

$$P_N(N) = (\langle N\rangle^N/N!)\exp(-\langle N\rangle), \tag{14}$$

where for the V model

$$\langle N\rangle = (r/r_0)^3 \tag{15}$$

and for the S model

$$\langle N\rangle = (r/r_0)^2 A^{1/3}. \tag{16}$$

Since the mean number $\langle N\rangle$ of the nucleons participating in the interaction depends differently on the atomic weight of the nucleus, the difference between the models is manifested in the A dependences of the inclusive cross sections. When $\langle N\rangle \ll 1$ and we can restrict ourselves to the lowest N number in the sum (8), we have in the case of the V model

$$E\frac{d\sigma}{d\overline{p}} \approx \frac{4}{3} r_0^3 A \frac{\langle N\rangle^{N_{\min}}}{N_{\min}!} \exp(-\langle N\rangle) f(x_{N\min}) = G_V(N_{\min}) A; \tag{17}$$

and in the S model

$$E\frac{d\sigma}{d\overline{p}} \approx G_S(N_{\min}) A^{2/3} A^{N_{\min}/3}. \tag{18}$$

The relations (17) and (18) show that both models give nontrivial and experimentally verifiable dependences of the inclusive cross sections on the atomic number of a fragmenting nucleus.

Knowing the experimental form of the function $f(x)$ for the nucleon-nucleon interaction and bearing in mind the definite connection [the relation (9)], we obtain a definite prediction for the energy and angular dependences of the production cross sections of cumulative particles [the relation (8)].

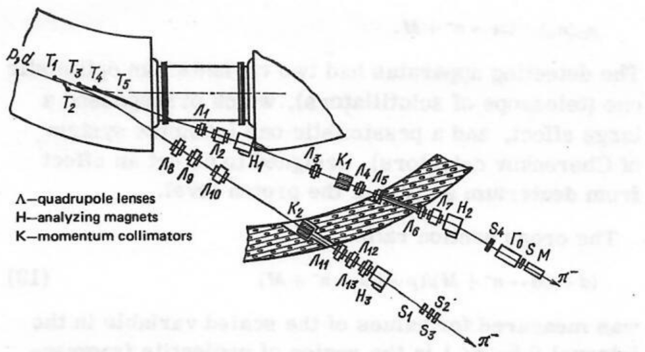

FIG. 1. Scheme of experiment in which the cumulative effect was discovered. (Λ are quadrupole lenses, H analyzing magnets, and K momentum collimators.)

Thus, Baldin's hypothesis has the following consequences.

1. First, the complicated interaction of composite systems such as nuclei follows a general fundamental law of nature-scaling.

2. The most characteristic features of the inclusive spectra of the secondary particles (dependences on the energy and emission angle and on the atomic weight of the fragmenting nucleus) are basically determined, not by the form factors of the interacting particles, but by the probability function $P_N(N)$ and, in particular, the threshold value $N_{\min}$, which entails a new approach to hadron interactions quite generally. Formally, this approach can be extended to the interaction of "elementary" hadrons πN, NN, etc.

The hypothesis of the cumulative nuclear effect was tested experimentally for the first time by means of relativistic deuterons accelerated in the synchrophaseotron in the High Energy Laboratory to energies of about 5 GeV per nucleon.[7] The arrangement of the experiment is shown in Fig. 1.

The secondary (negative) pions were formed by the usual magnetic channels (quadrupole lenses Λ, analyzing magnets H, and momentum collimators K) used in accelerators.

Since the target (Cu) is situated in the magnetic field of the accelerator, the scaled variable x could be varied only by changing the position of the target in the accelerator. When protons were accelerated to 10 GeV/c, secondary pions were detected from the reaction

$$p + \mathrm{Cu} \to \pi^- + M.$$

When deuterons were accelerated to 10 GeV/c, pions were detected from the reaction

$$d + \mathrm{Cu} \to \pi^- + M,$$

and the pion energy significantly exceeded the kinematic limit permitted by the interaction of individual nucleons of the accelerated deuterium nucleus and the target:

$p_D(n_D)+\mathrm{Cu}\to\pi^-+M.$

The detecting apparatus had two variants: an optimistic one (telescope of scintillators), which presupposed a large effect, and a pessimistic one (complex system of Cherenkov detectors), designed to detect an effect from deuterium at 10^{-8} of the proton level.

The cross section ratio

$$(d+\mathrm{Cu}\to\pi^-+M)/(p+\mathrm{Cu}\to\pi^-+M) \tag{19}$$

was measured for values of the scaled variable in the interval $0.5<x<1$ in the region of projectile fragmentation for $p_\perp=0$ at different energies of the accelerated deuterons. Figure 2 shows the obtained data. The dependence on the variable

$$x=p_{\pi^-}/p_{\pi^-}^{\max}$$

is shown by the continuous curve (Fig. 2a), which is an analytic approximation[7] of the experimental data on nucleon-nucleon interactions, and the points on the curve are the yields of pions from the accelerated deuterons, multiplied by 20.

Figure 2a indicates that the pion yield in dCu interactions follows the same dependence as the pion yield in pCu interactions [the relation (10)]. This confirms the scaling conjecture in the region of kinematic variables in which pion production is possible only when both nucleons of the deuterium nuclei participate in the interaction together. Figure 2b illustrates the fact that the probability of simultaneous participation in the interaction of the two nucleons of the deterium nuclei does not depend on the excess of the energy of the produced meson above the kinematic limit ($x\geqslant 0.5$) corresponding to the interaction of completely free deuterium nucleons. It should be noted that this probability [the coefficient P_2 in (8)] was predicted before the experiment as the probability of finding the two nucleons of the deuterium nucleus (in accordance with the wave function) in a certain volume characteristic of meson production ($r\approx 0.7$ F):

$$P_2\approx(r/r_D)^3\approx 5\cdot 10^{-2}.$$

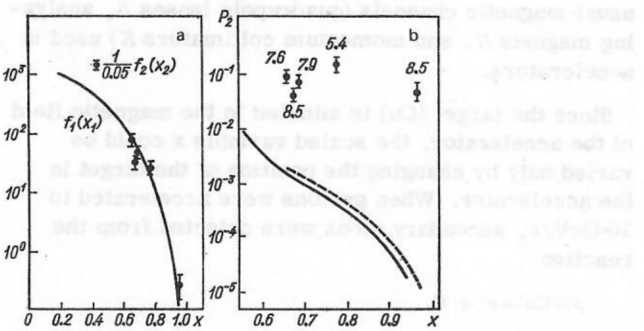

FIG. 2. High-energy yield of pions in dCu interactions. a) Comparison of the x dependence for pCu (continuous curve) and dCu (black circles) interactions; b) ratio of the probability of pion production by a deuteron as a whole to the probability of production by individual nucleons. The possible contribution of the Fermi-motion model is shown (with core, the solid curve; without core, the dashed curve). The kinetic energy of the accelerated deuterons is given next to the experimental points.

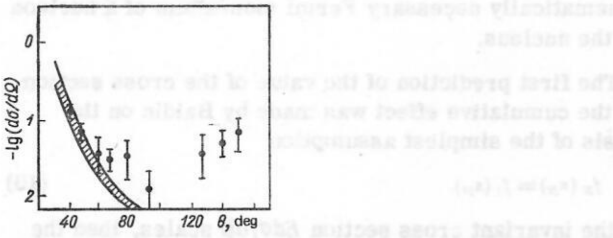

FIG. 3. Differential cross section for scattering of 660-MeV protons by deuterons as a function of the proton scattering angle in the center-of-mass system. The hatched region is a calculation in the impulse approximation.

In the same figure, we have plotted curves corresponding to a calculation in the impulse approximation of the value of p_2 determined by the Fermi motion of the nucleons of the interacting nuclei. The calculation was made by Gerasimov and Giordenescu.[8] It can be seen from the figure that, first, the contribution of the Fermi motion depends on the variable x and, second, in absolute magnitude it is much smaller than the experimentally observed pion yields.

Thus, within the errors of the experiment (±30%) it was confirmed that the interaction of relativistic nucleu scales. The experimentally established phenomenon was called the cumulative effect.

The observed effect was so large that it was natural to consider the experiments in which it could be manifested.

The first indications of a breakdown of the impulse approximation and of the concept of a deuteron wave function at short distances for hadron interactions were apparently obtained in 1956 by Leksin,[9] who investigated the elastic scattering of 660-MeV protons on deuterons with large momentum transfers. Figure 3 shows the experimental data and a calculation in the impulse approximation (hatched region).

Comparing the experiment ($\theta_p^*>100°$) and theory, Leksin arrived at the obvious conclusion that "the simultaneous interaction of three particles concentrated in a region whose linear dimensions are of the order of or less than the wavelength of the incident particle renders the impulse approximation invalid." Leksin also noted[9] that "the emission of fast fragments from nuclei can apparently be regarded as due to quasi-inelastic interactions of the incident nucleon with a group of nucleons that is strongly bound in the nucleus at the time of the collision."

Later, Meshcheryakov and his collaborators[10] found a curious phenomenon—almost elastic knock-out by 675-MeV protons of deuterons in the forward direction from light nuclei. Figure 4 shows these experimental data together with the data of Ref. 11.

Thus, the angular dependence (see Fig. 4a) is identical to that for elastic pd scattering and reproduces well the peak in the backward scattering (see Fig. 3). The observed A dependence indicates[12] a surface nature of the reaction mechanism: The cross section is proportional to the perimeter of the geometrical cross section of the target nucleus. It should be noted that the

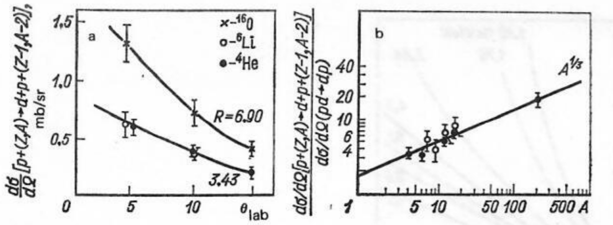

FIG. 4. Quasielastic knock-out of deuterons. a) Angular distributions of the deuterons (the continuous curves show elastic pd scattering multiplied by the constant K); b) ratio of the cross sections of quasielastic knock-out to the cross section of free pd scattering as a function of the mass number A of the target nucleus.

observed phenomenon cannot be interpreted as pickup of a neutron by the incident proton, since the experimentalists also had at their disposal the proton that has "defected" from the quasideuteron.

Later, Leksin *et al.*[13] attempted to observe this phenomenon on carbon and copper nuclei solely on the basis of recoil protons (angle of observation 137°) at much higher energies of the initial protons (up to 6 GeV). Figure 5 shows the measured momentum spectra of the protons for initial 1.15- and 5.7-GeV protons.

This experiment did not observe quasiscattering on clusters, but it did show that the proton yield is anomalously high and virtually independent of the initial proton energy in the range 1–6 GeV. The momentum of the secondary protons reached 1 GeV/c. It was concluded in Ref. 13 that the observed protons could not be evaporated protons or cascade protons. It is now obvious that they were cumulative protons.

The heuristic value of investigations into elastic scattering and quasielastic knock-out of clusters can hardly be overestimated. And it is not the fact that initial protons with energies of several hundred MeV interact with free deuterons and nucleon pairs (clusters) on the surface of the nucleus. The point is that the effect itself is large.

A new and unusual explanation of these phenomena was given by Blokhintsev in his paper "Fluctuations of nuclear matter"[14]:

"...Note that neither pick-up theory nor the impulse approximation is applicable in our case; for in both

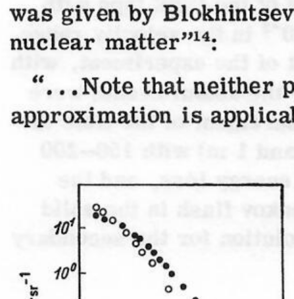

FIG. 5. Spectra of protons at the angle 137° in the reaction $p + {}^{12}C \rightarrow p + M_x$. The arrows show the positions of the expected quasielastic peaks for d, t, and α clusters.

these methods it is assumed that the incident nucleon interacts either only with a single nucleon or with two, but independently. In the considered case, the momentum transfers are so large that the entire process proceeds through very high harmonics of the deuteron wave function, i.e., through states in which the two nucleons are close to each other and their collision with the third nucleon cannot be regarded as two independent collisions." However, we can now understand[12] that in the problem of quasielastic knock-out of clusters ideas about fluctuations of nuclear matter remained in the shadow of the impulse approximation and pole diagrams.

Another effect where collective interactions of nucleons could be manifested is the mechanism of subthreshold production of antiprotons.[15] The threshold energy of antiproton production in a nucleon-nucleon collision is

$$E_{\text{thr}} \approx 6.6 \text{ GeV}.$$

However, the experiments reveal antiproton production events at energies of order 3.9 GeV. At such energies of the initial protons, antiprotons could be produced in two cases:

a) an opposite Fermi momentum $\gtrsim 0.6$ GeV/c;

b) a target mass $\geqslant 3m_N$.

In this experiment L. M. Lederman and his collaborators considered the problem of finding the distribution function of the Fermi momenta in a heavy nucleus from the probability of antiproton production. Then, using a 30-GeV proton beam and taking into account the lowering of the threshold by the Fermi momentum, they looked for new particles (assuming pair production) with masses in the range 3 to 7 GeV.

The result of the experiment was negative.[15] It is possible that the sought new particles do not occur in nature, but, on the other hand, it is obvious that the desired gain in the energy for pair production is not available if in the first experiment with antiprotons the gain was due to the target mass.

A quite unusual manifestation of the cumulative effect is possible in the process of annihilation of an antiproton stopped in a deuteron.[16] Figure 6 shows the distribution of antiproton annihilation in deuterium as a function of the momentum of the observed proton. It can be seen that the calculation completely disagrees with the observed spectrum at high momenta. According to the kinematics, "annihilation" on the "compressed" deuteron can give the observed tail at high energies.

We shall also consider experimental data corresponding to interactions of a relativistic nucleus A with a particle μ in which the detected inclusive particle c with coordinates $\{E_c, \mathbf{p}_c\}$ receives, in accordance with kinematics that "discounts" the Fermi motion, the momentum of a group ($N > 1$) of nucleons of nucleus A.

The obvious separation in rapidity of the fragments of the nucleus A and of the particle μ at high energies is clear in the coordinate system in which the

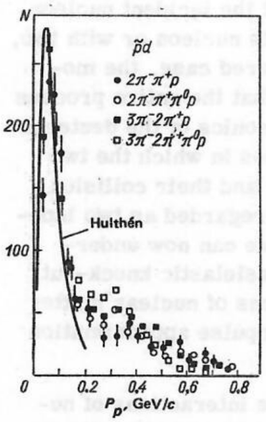

FIG. 6. Distribution of antiproton annihilation events in deuterium as a function of the momentum of the observed proton. The curve is a calculation in accordance with the deuteron wave function.

nucleus A is at rest and the fragments are detected in the backward direction relative to the projectile μ:

$$\mu + A \to c\,(\theta_c > 90^\circ) + X. \tag{20}$$

This formulation of the problem makes it possible to obtain relevant information before there become available beams of relativistic nuclei with $A > 4$ and energy permitting reliable separation of the effects of interaction with a group of nucleons from effects associated with Fermi motion, rescattering, and uncertain separation of target and projectile fragments.

Another advantage of this way of studying the interaction between nuclei and nucleons is that the energy per nucleon is raised by a factor 2; for in a proton accelerator with momentum p_0 it is possible to accelerate nuclei to momenta $p_0/2$ per nucleon.

On the other hand, the study of interactions between nuclei and hadrons such as antinucleons, strange particles, and even pions is only possible when the problem is formulated in this way.

Experimentally, it is necessary to measure the invariant cross sections of the inclusive process (20) for different energies of particle c and emission angles. According to the hypothesis [the relation (8)], the inclusive cross section is basically determined by the threshold value $N_{\min}$, i.e., by the minimal, kinematically allowed number of nucleons of nucleus A participating in the reaction. We find the minimal value of the target mass $N_{\min}\, m$ (m is the nucleon mass) from the condition for the production of particle c with coordinates E_c and P_c at the production threshold.[6] In general form, the solution of the problem for the reaction

$$\mu + N_{\min} m \to m_c + [(N_{\min} m) + m_b] \tag{21}$$

gives the relation

$$N_{\min} = Q = \left(\frac{E_c - \beta_\mu p_c \cos\theta_c}{m}\right)\left(1 - \frac{E_c + m_b}{E_\mu}\right)^{-1} + \frac{m_b^2 - m_c^2 - \mu^2}{2m\,(E_\mu - E_c - m_b)}. \tag{22}$$

In Table I we give the values of the masses for definite reactions. When the projectile energy E_μ is high and its velocity β_μ tends to unity, we obtain the relation (9).

TABLE I. Values of the masses for definite reactions.

Reaction	μ	m_c	m_b
$Ap \to \pi^0$	m	m_π	m
$Ap \to p$	m	m	0
$A\pi \to p$	m_π	m	$-m$
$A\pi \to \Lambda$	m_π	m_Λ	$m_K - m$

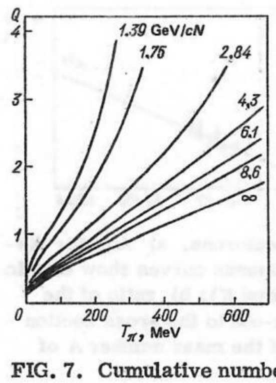

FIG. 7. Cumulative number as a function of the pion kinetic energy.

In what follows, we shall denote $N_{\min}$ by Q and call it the cumulative number. Figure 7 shows the cumulative number as a function of the kinetic energy of pions emitted at 180° for different momenta per nucleon of the interacting nuclei.

3. SPECIFIC FRAGMENTS. CUMULATIVE PRODUCTION OF PIONS AND HYPERONS

The arrangement of the first systematic experimental investigation of the cumulative production of mesons emitted in the backward direction [the process (20)] is shown in Fig. 8a.[17] Figure 8b shows the scheme of an up-dated experiment, which uses the same magnetic elements and makes it possible to investigate the particle spectra at different angles.[18]

In the first experiment, the pions were detected by a differential Cherenkov counter of the DISK type with velocity resolution $\Delta\beta = \pm 3 \times 10^{-2}$ in the velocity range 0.7–1.0. In the second variant of the experiment, with an on-line BÉSM-4 computer, the useful events were identified by independent measurement of the time of flight over two flight paths (4 and 1 m) with 150–200 psec accuracy, the ionization energy loss, and the intensity of the Vavilov–Cherenkov flash in the solid radiator. The momentum resolution for the secondary

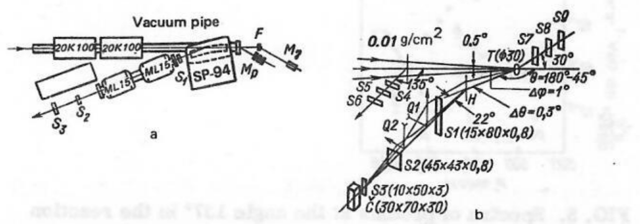

FIG. 8. Arrangement of experiment to measure the production cross sections of cumulative pions.

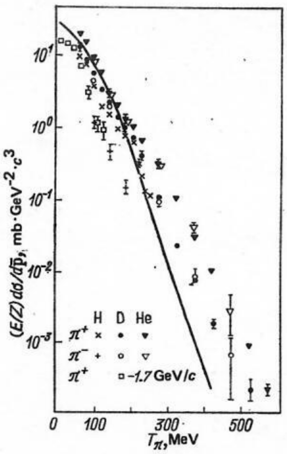

FIG. 9. Invariant cross sections for the production of pions in interactions of protons with H, D, and He nuclei. The open boxes are data from Ref. 26 for the *pd* interaction at $T_p = 1.7$ GeV/*cN*.

particles was ± 6%, and the admittance solid angle was 2×10^{-4} sr. These experiments provided the experimental material[19-25] on which this section is based.

Pion Energy Spectra. Figure 9 shows the experimental data on the invariant inclusive cross sections normalized to the charge Z of the fragmenting nucleus as a function of the kinetic energy of pions (positive and negative) emitted at angle 180° relative to the initial 8.9-GeV/c proton beam.[2)]

It can be seen from the figure that:

a) the yield of pions of opposite signs is the same (+5%) for deuterium and helium nuclei in the complete range of kinetic energies:

$$\left.\begin{aligned} &E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{D}\to\pi^+)=E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{D}\to\pi^-);\\ &E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{He}\to\pi^+)=E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{He}\to\pi^-);\end{aligned}\right\}\tag{23}$$

b) in the noncumulative range of pion energies ($T_\pi < 270$ MeV), the yield of pions from the helium nuclei is three times greater than the yield from deuterium nuclei:

$$E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{He}\to\pi)\approx 3E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{D}\to\pi^-)\tag{24}$$

while in the cumulative region of pion energies ($T_\pi > 270$ MeV):

$$E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{He}\to\pi)\approx 10E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{D}\to\pi);\tag{25}$$

c) the yield of positive pions from hydrogen nuclei exceeds the yield of negative pions by about five times:

$$E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{H}\to\pi^+)\approx 5E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{H}\to\pi^-);\tag{26}$$

d) for $T_\pi < 250$ MeV

$$E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{D}\to\pi)\approx E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{H}\to\pi^+)+E\frac{d\sigma}{d\bar{\mathbf{p}}}(\mathrm{H}\to\pi^-).\tag{27}$$

It can be seen from the figure that all the dependences on the pion energy are exponential. Moreover, such a dependence is observed for the deuterium nuclei up to the kinematic limit corresponding to interaction with the nucleus as a whole ($T_\pi \approx 600$ MeV). Since the connection between the cumulative number Q and the pion kinetic energy is almost linear (see Fig. 7), the dependence of the inclusive cross section of pion production on the cumulative number is also exponential.

Important information is given by the experimental data obtained in Ref. 26 for the *pd* interaction at 1.7 GeV/*cN*. It can be seen from the figure that at low pion energies the inclusive cross section of pion production is virtually independent of the pion energy (there is a plateau).

In Fig. 9, the curve shows a calculation in the impulse approximation for the process of pion production in a deuteron-proton collision at 8.4 GeV/*cN* made by different authors.[27,28] These calculations were made before the discussed experimental data had been obtained (the measurements of Ref. 19 up to pion energies ~330 MeV were available). The new experimental data of Ref. 25, shown in Fig. 9, strongly contradict the calculations at high pion energies.

In Ref. 29, a new approach to the formulation of the problem in the impulse approximation was proposed. Frankfurt and Strikman obtained an "enhancement" of the contribution of the high-frequency component of the deuteron wave function. Calculations of the cumulative production of pions by deuterium nuclei required experimental data on the *NN* interaction at limiting values of the scaled variable. Such data are now available,[25] and it is possible to test (true, as always, at finite energies of the interacting particles) the extent to which the relativistic space-time description of the interaction of the deuteron proposed in Ref. 29 agrees with the experiment.

New features in the production of cumulative pions deduced from the hypothesis (8) were observed in the investigation of various nuclei (^{6,7}Li, C, Al, Cu, ^{144,154}Sm, Pb) (Refs. 19 and 22).

Figures 10 and 11 show the experimental data on the invariant cross section of fragmentation (normalized to the atomic weight of the fragmenting nucleus A) as a function of the kinetic energy of negative pions emitted in the backward direction (180°) relative to initial 8.9- and 6.3-GeV/c protons, respectively. Figure 12 gives the analogous quantities at momentum 4.4 GeV/c per nucleon of the relativistic nuclei for the reaction $dA\to\pi^-$ (180°) with relativistic 8.9-GeV/c deuterons.

Figure 13 shows, for comparison, the experimental data of Ref. 30 for 1.39-GeV/c initial protons (pion emission angle 150°).

The experimental data shown in these figures are

2) The difference between this value of the momentum and the one given in the earlier publications results from new measurements of the value of the magnetic field of the accelerator and direct measurements made by the groups of É. O. Okonov and L. N. Strunov.

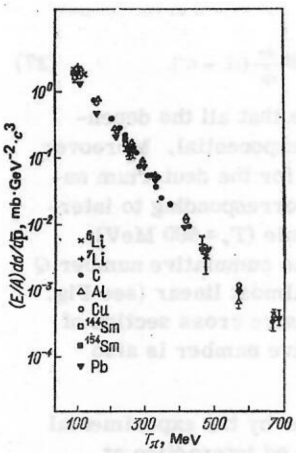

FIG. 10. Dependence on the pion kinetic energy of the fragmentation cross section normalized to the atomic weight of the nucleus at momentum 8.9 GeV/cN in the reaction $pA \to \pi^-$ (180°).

above all interesting in that, in accordance with the laws of conservation of energy and momentum [see Eq. (22) and Fig. 7], the observed pions correspond to interaction of the initial particle with a group of nucleons of the nucleus. Of course, the conservation laws cannot distinguish between two possibilities: 1) the initial particle interacts with a group of nucleons separated by such short distances that it is meaningless to regard them as independent, and 2) the initial particle interacts with and excites independently (successively) nucleons of the nucleus that then combine in a group, and this group emits the observed pion, which carries away the excitation energy of the complete group of nucleons. However, either of these possibilities permitted by the conservation laws corresponds to a new formulation of the problem in the physics of hadron interactions, and we call this cumulative interaction.

It can be seen from Figs. 10–13 that the energy spectra of the pions in a first approximation is exponential:

$$E\,d\sigma/d\bar{p} = a\exp(-T/T_0). \tag{28}$$

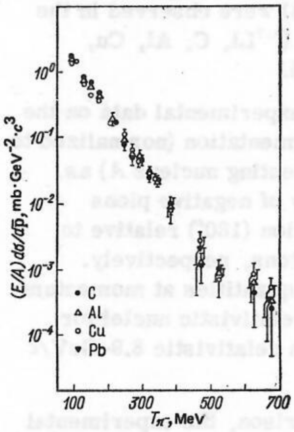

FIG. 11. Dependence on the pion kinetic energy of the fragmentation cross section normalized to the atomic weight of the nucleus at momentum 6.3 GeV/cN in the reaction $pA \to \pi^-$ (180°).

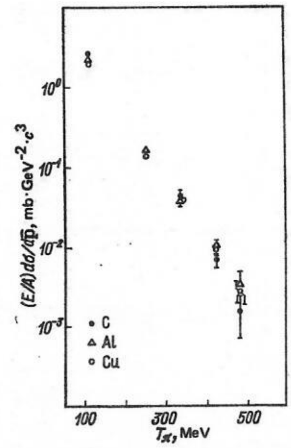

FIG. 12. Dependence on the pion kinetic energy of the fragmentation cross section normalized to the atomic weight of the nucleus at momentum 4.4 GeV/cN in the reaction $dA \to \pi^-$ (180°).

Figure 14 shows the dependence on the momentum per nucleon of the interacting nuclei of the parameters T_0 obtained by fitting to the experimental data the dependence (28) for different nuclei.

It can be seen from Fig. 14 that the parameter T_0 initially increases strongly with increasing momentum per nucleon of the interacting nuclei, with a tendency for a plateau to be formed at 4 GeV/cN. At high energies, an increase is observed in the fragmenting nucleus.

At momentum 8.9 GeV/c of the initial protons, measurements were made of the inclusive cross sections for the production of positive and negative pions emitted in the backward direction from C, Al, Cu, and Pb nuclei.[19] Figure 15 shows the experimental values of the ratio $[\sigma(\pi^-) - \sigma(\pi^+)]/[\sigma(\pi^-) + \sigma(\pi^+)]$ as a function of the pion kinetic energy.

The experimental data indicate a small excess of the yield of negative pions over that of positive pions.

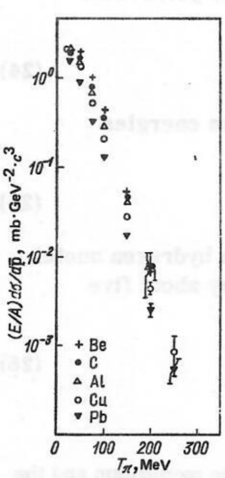

FIG. 13. Dependence on the pion kinetic energy of the fragmentation cross section normalized to the atomic weight of the nucleus at momentum 1.39 GeV/cN in the reaction $pA \to \pi^-$ (180°).

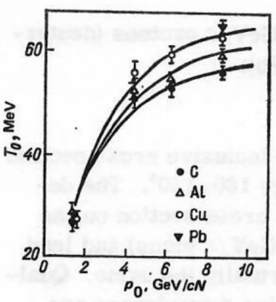

FIG. 14. The parameter T_0 (temperature of the pion spectrum) for C, Al, Cu, and Pb nuclei as a function of the momentum p_0 per nucleon of the interacting nuclei.

In Ref. 20, an investigation was made of isotopically conjugate production reactions of positive and negative mesons in the interaction of relativistic 8.9-GeV/c deuterons with ^{6}Li and ^{12}C nuclei. For pions emitted from the nuclei at angle 180° with kinetic energy 117 MeV

$$[\sigma(\pi^+)-\sigma(\pi^-)]/\sigma(\pi^+)=0\pm 0.1 \quad \text{for the lithium nucleus;}$$

$$[\sigma(\pi^+)-\sigma(\pi^-)]/\sigma(\pi^+)=0.05\pm 0.08 \quad \text{for the carbon nucleus.}$$

Concluding our consideration of the pion energy spectra, we emphasize their principal feature, which follows from the hypothesis of the cumulative nuclear effect [see Eq. (8)]. The probability of interaction with a group of N nucleons, $P_N(N)$, is a function that decreases fairly strongly with increasing N. Extending the hypothesis of the cumulative nuclear effect to an infinite number of constituents, we note that the cross section of inclusive production will be a function of only $N_{\min}$, i.e., Q, and that this dependence will be exponential:

$$E\, d\sigma/d\bar{p}=G\exp\{-Q/\langle Q\rangle\}. \tag{29}$$

The experimental data from 1.4 to 8.9 GeV/cN of the interacting nuclei that we have considered satisfy the exponential dependence (see Fig. 2a). This is a new and important experimentally established fact. The experiment gives a certain "over-compensation": in contrast to the parameter T_0 (the temperature), the number $\langle Q\rangle$ (the mean number of nucleons in the volume associated with the cumulative effect) decreases weakly with increasing energy of the interacting nuclei.

In agreement with the experiment, the number $\langle Q\rangle$ is equal to the product of the mean nucleon density of a nucleus with radius $r_0A^{1/3}$ and the "cumulative" volume:

$$\langle Q\rangle=\frac{A}{(4/3)\pi r_0^3 A}\cdot\frac{4}{3}\pi R_0^3=\left(\frac{R_0}{r_0}\right)^3, \tag{30}$$

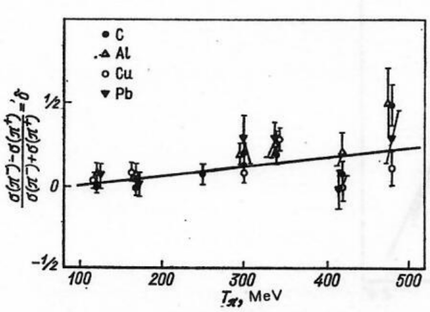

FIG. 15. Ratio of the yields of positive and negative pions as a function of the pion energy.

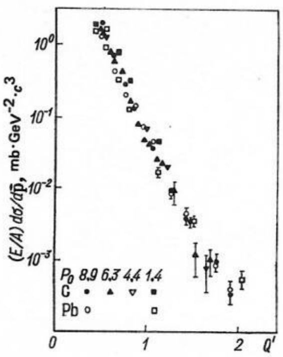

FIG. 16. Inclusive fragmentation cross sections of C and Pb nuclei normalized to the atomic weight as a function of the cumulative number Q'.

where R_0 is the radius of the cumulative volume.

We determine the "fitting" parameter to make $\langle Q\rangle$ decrease with increasing energy of the interacting nuclei with momentum per nucleon p_0. We introduce an increase of the cumulative radius by $\bar{\lambda}_0$:

$$\bar{\lambda}_0=h/p_0.$$

Then

$$\langle Q\rangle|_{p_0}=[(R_0+\bar{\lambda}_0)/r_0]^3 \tag{31}$$

and we must expect equality of the cross sections at the cumulative number

$$Q'=Q\,[R_0/(R_0+\bar{\lambda}_0)]^3. \tag{32}$$

Note that the Q' distribution is the distribution with respect to the cumulative number at infinitely high energy of the interacting nuclei—we are dealing with Yang's limiting fragmentation.

Figure 16 shows experimental data on the inclusive cross sections of the fragmentation of carbon and lead nuclei at different momenta per nucleon of the interacting nuclei as a function of the cumulative number Q' [the relation (32)] for $R_0=0.75$ F. For 4.4 GeV/c, the experimental values of $(1/2)\sigma(dA\to\pi^-)$ were taken at momentum 8.9 GeV/c of the deuterons.[20] It can be seen from Fig. 16 that the cross sections of the inclusive processes, normalized to the atomic weight, can be described with good accuracy (spread by a factor of not more than 2) by the exponential depen-

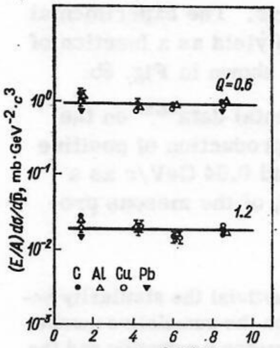

FIG. 17. Fragmentation cross sections of the nuclei C, Al, Cu, and Pb normalized to the atomic weight as a function of the momentum per nucleon of the interacting nuclei.

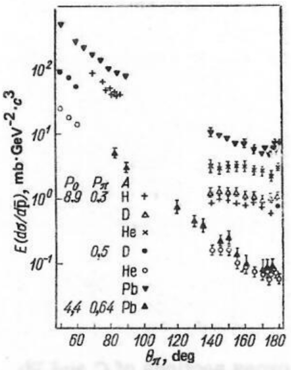

FIG. 18. Inclusive cross section for production of positive pions of different momenta as a function of the emission angle of the meson for different momenta per nucleon of the interacting nuclei; p_0 is the momentum per nucleon of the interacting nuclei (in GeV/c); θ_π is the angle between the incident proton and the secondary pion.

dence

$$(E/A)\, d\sigma/d\bar{\mathbf{p}} = G \exp(-Q'/0.16), \tag{33}$$

where the number Q satisfies the relation

$$\langle Q\rangle = \langle Q\rangle|_{p_0\to\infty} = 0.16. \tag{34}$$

The exponential dependence [see (33) and Fig. 16] is interesting in that it not only gives a possibility of predicting the feasibility of experimental investigations at high cumulative numbers, but also indicates an exponential nature of the function $P_N(N)$ in the model of a large number of constituents.[3)]

Figure 17 shows experimental data on pion fragmentation of the nuclei C, Al, Cu and Pb for the cumulative numbers 0.6 and 1.2 as a function of the momentum per nucleon of the interacting nuclei.

The same figure reveals the deviation of the cross sections from the universal dependence (33). At $p_0 > 4$ GeV/c and $Q' > 1$, the inclusive cross section of pion production really is proportional to the atomic weight of the nucleus, but at small momenta p_0 the cross section is proportional rather to $A^{2/3}$. In this sense, we can assume that limiting fragmentation commences at momenta of order 4 GeV/c per nucleon of the interacting nuclei.

Dependence of the Inclusive Cross Sections of Pion Production on the Emission Angle. The experimental set-up for investigating the pion yield as a function of the emission angle (50–180°) is shown in Fig. 8b.

Figure 18 shows the experimental data[24,25] on the inclusive cross section for the production of positive pions with momenta 0.3, 0.5, and 0.64 GeV/c as a function of the emission angle θ_π of the mesons produced by the interaction of 8.9-GeV/c protons (deuterons) with nuclei A (H, D, He, Pb):

$$p(d) + A \to \pi^+ + X.$$

For the 0.3-GeV/c pions, the inclusive cross section is virtually constant in the range 180–140°. The dependence of the pion production cross section on the angle θ_π for helium nuclei (0.5-GeV/c pions) and lead nuclei (0.64-GeV/c pions) is virtually the same. Qualitatively, all the observed angular dependences are similar, although the absolute values of the cross section for the lead nucleus (0.5 and 0.64 GeV/c) differ by almost a factor 100.

Together, the experimental angular dependences of the inclusive cross sections contradict the hypothesis of multiple interactions (cascades) as an explanation of the cumulative effect.[32] In addition, it is shown in Ref. 33 that there are great difficulties in the attempt to describe by means of the cascade mechanism the experimental angular and energy spectra of g particles (basically, cumulative particles).

The experimental dependences of the invariant cross sections on the emission angle of the pion find a remarkably simple explanation in the model of the cumulative nuclear effect [see Eq. (8)]. As we already know, the energy dependence of the production of cumulative pions is well explained by the minimal cumulative number Q' [see Fig. 16 and Eq. (33)]. It follows from the dependence (22) of the cumulative number on the emission angle of the particle that Q' decreases with decreasing emission angle; therefore, the inclusive cross section must increase in accordance with the relations (8) and (33).

Figure 19 shows the experimental angular dependences of the cross section for fragmentation of lead nuclei (see Fig. 18) as a function of the cumulative number Q' [the relation (32)]. In the same figure, we show the experimental data[30] for fragmentation of lead nuclei with production of negative pions at emission angles 90–150°. The straight line in the figure corresponds to the experimental energy dependence of the fragmentation cross section (see Fig. 16). It can be seen from Fig. 19 that the experimental angular

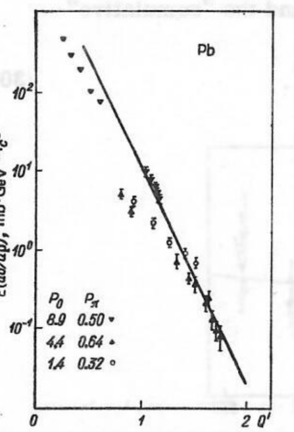

FIG. 19. Inclusive cross section for production of pions on Pb as a function of the cumulative number Q' for different emission angles of the pions (see Fig. 18).

[3)] It is hard to regard as purely superficial the similarity between the exponential dependence on the cumulative number of the inclusive cross section for meson production and the exponential dependence, the Q_{gg} systematics of Volkov, of transfer reactions with heavy ions.[31] In accordance with the physical conception, the cumulative number goes over into the energy Q_{gg} when the fragment energy tends to zero.

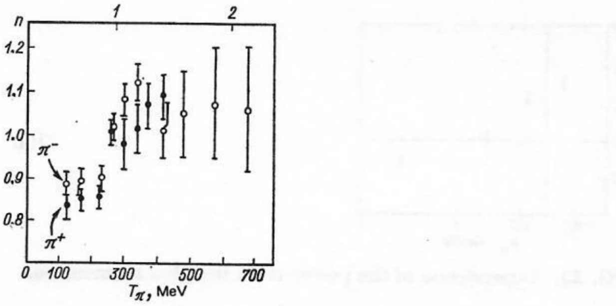

FIG. 20. Value of the exponent n as a function of the kinetic energy of positive (black circles) and negative (open circles) pions. The upper scale is the cumulative number and the lower scale is the pion kinetic energy. The momentum of the initial protons was 8.9 GeV/c.

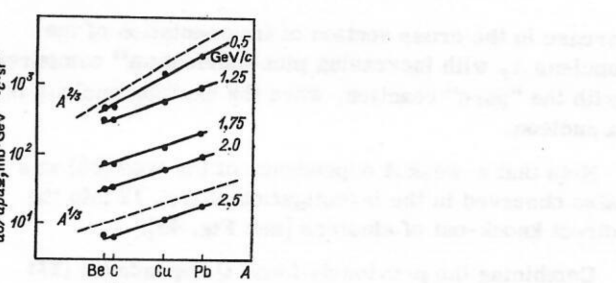

FIG. 21. Cross section for fragmentation of helium nuclei with production of negative pions as a function of the atomic weight of the exciting nucleus for different momenta of the secondary pions.

dependences, at least for large pion emission angles (120–180°), as a function of the cumulative number Q' simply coincide with the data on the energy dependence as a function of the cumulative number. It may be hoped that a concrete model of the cumulative effect, in which the cross section of the inclusive process will be determined, not by the minimal cumulative number, but by an effective value close to it, will describe as a function of this number the complete set of experimental energy and angular dependences of the particle production. Thus, the inclusive cross section for particle production will be a function of not two variables (for example, $P_\parallel$ and $P_\perp$) but only a single variable (effective Q).

Dependence on A of the Inclusive Cross Section for the Production of Cumulative Particles. The experimental data of Refs. 19 and 20, which are given in Figs. 10 and 11, on the inclusive cross section of the process

$$p+A\to\pi^{\pm}(180°)+X$$

as a function of the atomic weight of the target nucleus at fixed pion energy were fitted by an analytic dependence of the form

$$E\,(d\sigma/d\bar{p})=cA^{n(T_\pi)}. \quad (35)$$

Figure 20 shows the values of the exponent n as a function of the pion kinetic energy.

It can be seen from Fig. 20 that at high pion energies in the region of cumulative ($Q>1$, upper scale) pion production the A dependence has a volume nature,[19] i.e., the pion yield is proportional to the volume of the fragmenting nucleus.[4)]

This experimental fact is in agreement with the prediction of the V model of the cumulative nuclear effect [the relation (17)]. The obtained A dependence of the particle production emphasizes the principal features of the cumulative interaction—its locality, the fact that the nuclear-density fluctuations are independent of the coordinate in the volume of the nucleus—and shows the irrelevance for cumulative production (in a first approximation) of effects of absorption in the matter of the nucleus of the initial wave and the produced particles.

Thus, the inclusive cross section for the production of cumulative mesons in the reaction

$$p+A\to\pi\,(180°)+M \quad (36)$$

in the arrangement of the experiment in which the mesons are fragments of nucleus A increases linearly in a first approximation with increasing atomic weight A_F of the fragmenting nucleus:

$$E\,(d\sigma/d\bar{p})=G_1\,(P_c,\ \Theta_c)\,A_F. \quad (37)$$

The change in the inclusive cross section of the reaction (36) resulting from the replacement of bombarding protons by a wide range of nuclear exciters (A_1) was studied experimentally with the accelerator at Berkeley.[35] The experiment was made in the coordinate system in which the exciting nucleus A_1 (Be, C, Cu, Pb) is at rest and the fragmenting nucleus (A_F = He) moves and has energy 2.1 GeV per nucleon (Fig. 21). The observed fragments (negative pions) are emitted in the direction of motion of the helium nucleus, i.e., the arrangement of the experiment is identical to the one with accelerated deuterons in the discovery of the cumulative effect.[7] Varying the pion energy, one can identify either single-nucleon or many-nucleon fragmentation of the helium nucleus and study the dependence of these processes on the atomic weight of the exciting nucleus.

It can be seen from Fig. 21 that for a fragmenting helium nucleus in the region of cumulative pion production (momentum greater than 1.9 GeV/c) the inclusive cross section of pion production depends weakly on the atomic weight of the exciting nucleus:

$$E\,(d\sigma/dp)=G_2\,(P_c,\ \Theta_c)\,A_1^{1/3}. \quad (38)$$

Such a weak dependence of the inclusive cross section on the atomic weight of the exciting nucleus is observed only in the region of cumulative production. At small momenta of the produced pions, the inclusive cross section is proportional to $A^{2/3}$, i.e., it is proportional to the surface area of the nucleus. The change in the regime in the dependence of the inclusive cross section of meson production on the atomic weight of the exciting nucleus $A_1^{n(P_F)}$ leads to a more rapid de-

---

4) Later, a similar dependence on the atomic weight of the target nucleus for the inclusive production of hadrons by high-energy (300–400 GeV/c) protons was obtained in Cronin's experiments[34] at large transverse momenta ($\lesssim 6$ GeV/c).

crease in the cross section of fragmentation of the nucleus A_F with increasing pion momentum[35] compared with the "pure" reaction, when the exciting nucleus is a nucleon.

Note that a weak A dependence of the type (38) was also observed in the investigation of Ref. 12 into the direct knock-out of clusters [see Fig. 4b].

Combining the previously found Q dependence (33) with the A dependence on the atomic weight of the fragmenting nucleus (A_F) and the A dependence on the exciting nucleus, we obtain

$$E\,(d\sigma/d\bar{p}) = cA_F A_1^{1/3} \exp\{-Q'/\langle Q\rangle\}, \tag{39}$$

where Q' is the cumulative number determined by Eqs. (32) and (22), $\langle Q\rangle$ is the mean cumulative number [the relations (30) and (34)], and c is a normalization constant.

The relation (39) gives a complete description of the nuclear inclusive process: the dependence on the emission angle of the inclusive particle and its energy (the Q dependence) and the dependence on the atomic weight of the fragmenting nucleus (A_F) and the atomic weight of the exciting nucleus (A_1).

Multiplicity of Cumulatively Produced Mesons. At present, the literature contains no direct experimental data on the multiplicity of produced pions for a single-event interaction in which the kinematics corresponds to interaction of the initial particle with a many-nucleon mass of the target. Moreover, it is hard to set up an experiment in which the fact of interaction with a many-nucleon intranuclear target could be unambiguously separated from preceding or succeeding interactions with individual nucleons of the nucleus. However, qualitative features of the phenomenon in which we are interested can be deduced from the experimental data of the Serpukhov investigation of Ref. 36, which are currently the only data available.

An experimental set-up with a spark chamber in a magnetic field with film readout of information was used to investigate the inclusive reaction

$$p + A \to n\pi^- + X \tag{40}$$

with 7-GeV/c initial protons. The targets were Be, C, Al, Cu, Cd, and Pb nuclei.

Figure 22 shows the mean multiplicity $\langle n_-\rangle$ of the negative pions produced in the backward hemisphere ($\theta_{\pi^-} > 90°$) as a function of the atomic weight of the fragmenting nucleus. The mean number of produced pions

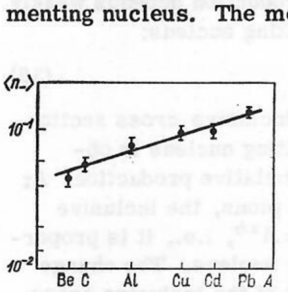

FIG. 22. Mean multiplicity of negative pions as a function of the atomic weight of the fragmenting nucleus. The mean pion momentum was ~200 MeV/c.

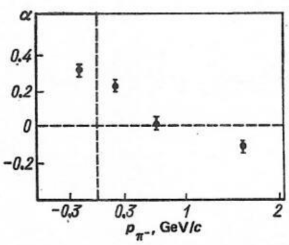

FIG. 23. Dependence of the power α on the pion momentum.

is

$$\langle n_-\rangle = cA^{\alpha}, \tag{41}$$

where $\alpha = 0.305 \pm 0.036$.

For pions emitted in the forward direction ($\theta_{\pi^-} < 90°$), the exponent α in (41) decreases with increasing pion momentum (Fig. 23).

It can be seen from the data we have given that at pion momentum 0.7 GeV/c the mean multiplicity of the produced pions does not depend at all on the atomic weight of the fragmenting nucleus and at large momenta decreases with increasing atomic weight of the fragmenting nucleus. The strongest dependence of the mean multiplicity on A is observed for low-energy pions, for which absorption within the nucleus [in the region of the (3, 3) resonance] is most important. It follows from the volume nature of the A dependence of the cumulative pion production that there are no grounds for assuming a shorter effective range for pions emitted in the backward direction. Thus, the change of regime in the A dependence on the transition from low to high energies cannot be explained by absorption of mesons in the nucleus.

At high pion momenta (>0.7 GeV/c) the $\alpha < 0$ paradox, i.e., the decrease of the mean multiplicity with increasing A, can be explained by effects of the deceleration of energetic pions produced in primary interaction events.

Note that in the considered range of momenta the A dependence of the inclusive cross section changes strongly with varying transverse component of the momentum[34]: For large transverse momenta, $\alpha \approx 1.0$.

On the basis of the general features found in the region of cumulative particle production [see Eq. (39)], we can suggest that the experimentally observed abrupt change in the regime of the A dependence is due to a change of the part played by the investigated nuclei. In the investigation of backward pions the nuclei. In the investigation of backward pions the nuclei produce pions by fragmentation, but in the investigation of the energetic forward pions the nuclei are exciting nuclei of the initial protons. Of course, this remark does not establish the actual mechanism of the process, but it does establish a group of interconnected questions.

Cumulative Production of Λ Hyperons. The production of Λ hyperons on nuclei in the region kinematically forbidden for a collision with individual nucleons of the nucleus was studied in the Serpukhov investigations of

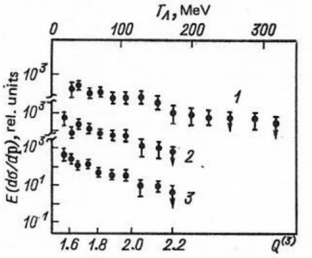

FIG. 24. Inclusive cross section for the production of cumulative Λ hyperons as a function of their kinetic energy for different emission angles: 1) $60<\theta_\Lambda<80°$, 2) $80<\theta_\Lambda<100°$, 3) $\theta_\Lambda>100°$; for curve 3 ($\theta_\Lambda\approx140°$) the scale of the cumulative numbers Q [the relation (22)] is shown.

Refs. 37 and 38; 2.9-GeV/c negative pions were used to bombard a propane-xenon chamber in a magnetic field. Inclusive Λ particles with emission angle greater than 47° in the laboratory system were selected. In the earlier investigation, 208 events were studied; in the later, 533.

The production cross section [for effective nucleus (C, Xe)] of hyperons with momentum greater than 0.2 GeV/c for emission angles 47–90° and 90–180° are, respectively, 1.83 ± 0.46 and 0.59 ± 0.15 mb.

The energy dependence of the cross sections is exponential (Fig. 24), and the following values of the temperature parameter T_0 (MeV) were found in the different angle intervals:

$$\left.\begin{array}{ll} T_0=64.6^{+7.4}_{-6.1}, & 60°<\theta_\Lambda<80°;\\ T_0=42.2^{+7.1}_{-5.2}, & 80°<\theta_\Lambda<100°;\\ T_0=36^{+4.7}_{-3.7}, & \theta_\Lambda>100°.\end{array}\right\} \quad (42)$$

Using the total cross section for hyperon production found in the investigation and given above, we find the inclusive cross section for production of hyperons with emission angle $\langle\theta_\Lambda\rangle=140°$ as a function of the kinetic energy:

$$E\,(d\sigma/d\bar{\mathbf{p}})\,(\pi^- A\to\Lambda X)=(4.5\pm1.2)\exp(-T_\Lambda/36), \quad (43)$$

this being expressed in mb·GeV^{-2}·c^3; A is the effective nucleus ($A\approx80$).

This inclusive cross section also depends exponentially on the cumulative number:

$$E\,(d\sigma/d\bar{\mathbf{p}})=C\exp\{-Q_\Lambda/\langle Q\rangle\}, \quad (44)$$

where $\langle Q\rangle=0.165^{+0.02}_{-0.017}$.

We see that the obtained mean value of the cumulative number agrees with the value for the cumulative production of pions [see Eqs. (33) and (34) and Fig. 16]:

$$E\,(d\sigma/d\bar{\mathbf{p}})\,(pA\to\pi X)\approx30A \times\exp\{-Q_\pi/0.16\}. \quad (45)$$

For the effective nucleus of this experiment on cumulative production of Λ hyperons, the pre-exponential factor in the relation (45) is 2400 in order of magnitude. With this pre-exponential coefficient at zero kinetic energy of the hyperons, when $Q_\Lambda(T_\Lambda=0)\approx1.29$, the expression (44) gives

$$E\,(d\sigma/d\bar{\mathbf{p}})\,(T_\Lambda=0)=2400\exp(-1.29/0.165)\approx1\ \text{mb}\cdot\text{GeV}^{-2}\cdot c^3$$

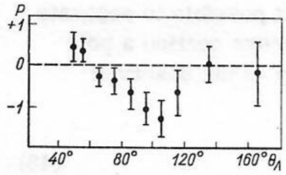

FIG. 25. Polarization of Λ hyperons as a function of the emission angle in the laboratory frame; the positive direction is taken along the vector $(\mathbf{P}_\pi\cdot\mathbf{P}_\Lambda)$.

which in order of magnitude is equal to the value found from the total cross section of hyperon production (4.5 ± 1.2).

Thus, in the framework of the hypothesis of the cumulative nuclear effect, the difference between the production probabilities of cumulative pions and hyperons (about three orders of magnitude) and also the profiles of the energy spectra can be explained naturally by the differences between the threshold values of the cumulative numbers [see Eq. (8)]. In this sense, it is obviously important to investigate the cumulative production of strange particles: Λ hyperons and kaons.

The quoted investigations measured the asymmetry in the decays of the Λ hyperons with respect to the reaction plane, which made it possible to determine their polarization (Fig. 25).

In the earlier investigation of Ref. 39, attention was drawn to the appreciable polarization of cumulative protons.

4. ORDINARY FRAGMENTS. CUMULATIVE PRODUCTION OF PROTONS, DEUTERONS, AND TRITIUM NUCLEI

Rich experimental material has now been accumulated on the cumulative production of baryon systems by the interaction of particles with different nuclei in the inclusive situation

$$a+A\to \overset{p}{\underset{t}{d}}+X, \quad (46)$$

where the initial particle is a γ ray, pion, proton, or nucleus accelerated to energies of several GeV per nucleon.

Reactions in which baryonic systems are produced differ from those in which pions, kaons, etc., are produced through the simple fact that in a state with zero momentum in the rest frame of the fragmenting nucleus such systems are more or less "ready-made." We are referring to the smallness of the binding energy of the constituent parts compared with the energy of the colliding particles. This leads to a high probability of stripping and pick-up. These processes bear no relation to Yang's limiting fragmentation extended to the interaction of relativistic nuclei. As is shown in Ref. 40, stripping and pick-up can be described by means of the usual pole approximation. If the amplitude of the nuclear reaction has t-channel singularities, the amplitude of the process has the form

$$T_{ji}=\sum\frac{T_{fj}T_{ji}}{(p_F-p_1)^2-m_2^2}. \quad (47)$$

Simple transformations make it possible to separate in the relativistically invariant cross section a pole with respect to the kinetic energy of the observed fragment with mass m_1:

$$d\sigma \sim (\alpha + 2T_1/m_1)^{-2}, \tag{48}$$

where we have the parameter

$$\alpha = (m_1 + m_2 - m_F)(m_F + m_2 - m_1)/m_F m_1, \tag{49}$$

in which m_F is the mass of the fragmenting nucleus.

A fraction with the denominator (48) has a pole with respect to the variable T_1. This is due to the smallness of the parameter α or the smallness of the factor $(m_1 + m_2 - m_F)$ (stripping reaction) or $(m_F + m_2 - m_1)$ (pick-up). It is easy to show that the kinetic energy T of the fragment in the rest frame of the fragmenting nucleus m_F can be expressed in terms of relativistic invariants in the form

$$T_1 = b_{F1} = (p_F \cdot p_1)/m_F - m_1. \tag{50}$$

Thus, one can propose the following classification of collisions of relativistic nuclei.[5] The group of reaction products for $b \geqslant \varepsilon$, where ε is the binding energy of the nucleus, are to be regarded as "fragments" from the breakup of the nucleus excited to relatively high energies and regarded as a weakly bound system. At large values of the parameter b, i.e., large fragment energies, the introduced parameter determines the smallest possible cumulative number (expressed in the number m of nucleon masses), i.e., the cumulative number

$$Q \approx A_1 + (T_1 - p_1 \cos\theta_1)/m, \tag{50'}$$

where θ_1 is the angle of emission of the fragment relative to the direction of motion of the projectile (exciting nucleus), and A_1 is the atomic weight of the fragment. The relation (50′) can be readily obtained from Eq. (22) for $T_c \ll T_\mu$, where T_μ is the projectile kinetic energy. For the analysis of experimental data at finite energies ($T_c \sim T_\mu$), the exact equation (22) should be used.

As in the case of the production of specific fragments (pions, kaons, etc.), we can assert that if

$$(Q - A_1) \sim 1,$$

then the fragments (A_1) observed in the nuclear reaction correspond to the process of fusion of a group of nucleons into a single entity. Moreover, this assertion is based solely on the energy and momentum conservation laws.

Energy Spectra of Cumulative Protons, Deuterons, and Tritons. Proton-nucleus interactions:

$$p + A \to \overset{p}{d} + X. \tag{51}$$

The simplest cumulative nuclear reaction of this type is the process of production of protons and deuterons emitted in the backward direction when protons bombard the lightest nuclei, deuterium and helium. Figure 26 shows the experimental data[25] on the reaction (51) for initial 8.9-GeV/c protons. The secondary particles (p, d) were detected at 180° to the direction of motion

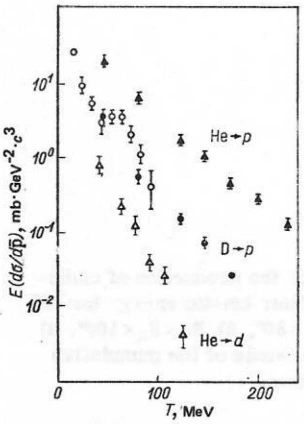

FIG. 26. Inclusive cross section for production of cumulative protons and deuterons produced in the interaction of protons with light nuclei. The open circles are the (D→p) cross sections at 1.66 GeV/c; the remainder are at 8.9 GeV/c.

of the bombarding protons; the momentum resolution for the fragment detection was ± 6%. In the same figure we have plotted the experimental data[26] on the inclusive production of protons on deuterium nuclei (averaged over the interval 132–180° of proton emission angles) for initial 1.66-GeV/c protons.

As can be seen from the figure, all the cross sections have the exponential dependence (28) in a first approximation, and the parameter T_0 (the temperature, MeV) is

$$\left.\begin{aligned} &T_0(\mathrm{He}\to p) = 38 \pm 1.5 \\ &T_0(\mathrm{He}\to d) = 17 \pm 1 \\ &T_0(\mathrm{D}\to p) = 29 \pm 1.8 \end{aligned}\right\} p_0 = 8.9\ \mathrm{GeV}/c;$$
$$T_0(\mathrm{D}\to p) = 29 \pm 2.7,\ p_0 = 1.66\ \mathrm{GeV}/c.$$

Note that the cross sections $E(d\sigma/d\bar{p})(D\to p)$ for 8.9- and 1.66-GeV/c initial protons almost coincide in their dependence on the kinetic energy of the secondary protons. The cross sections at $T = 95$ and 175 MeV for $p_0 = 1.66$ and 8.9 GeV/c correspond to the phase-space limit of the reaction, i.e., to the cumulative number $Q = 2$. The smallest possible mass of the "target" in the reaction He→d at kinetic energy 126 MeV of the deuterons is more than three nucleon masses.

These experimental data rule out the mechanism of

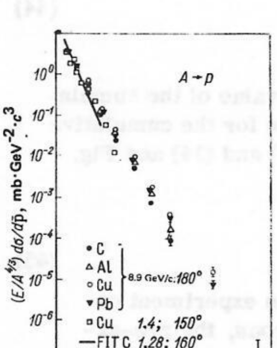

FIG. 27. Inclusive cross section (normalized to $A^{4/3}$) for proton production as a function of the kinetic energy of the fragments.

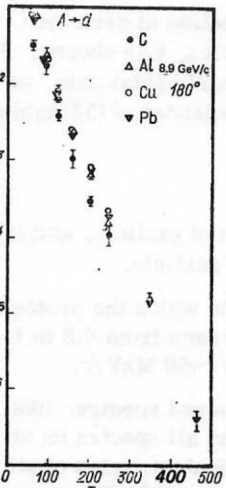

FIG. 28. Inclusive cross section (normalized to $A^{5/3}$) for production of deuterons as a function of the kinetic energy of the fragments.

"intranuclear rescattering" on independent centers (nucleons) distributed uniformly over the volume of these nuclei, since the smallest possible masses of the targets are of order (or are equal to) the masses of the fragmenting nuclei.

Figures 27, 28, and 29 show the experimental data of Ref. 41 on the dependence on the fragment kinetic energy of the inclusive cross sections for the production of protons, deuterons, and tritons, respectively, when C, Al, Cu, and Pb nuclei were bombarded with 8.9-GeV/c protons. The fragments were detected at 180° to the initial protons. It can be seen that all the spectra are exponential. For the protons (see Fig. 27), this is true for variation of the inclusive cross section over eight orders of magnitude. Figure 27 also shows data for copper[30] and carbon[42] nuclei at momenta of the initial protons smaller than 1.5 GeV/c. It can be seen that, as in the case of fragmentation of deuterium nuclei (see Fig. 26), the absolute values of the cross sections at low energies are virtually equal to the cross sections at high energies (8.9 GeV/c) for fragmenting C and Cu nuclei.

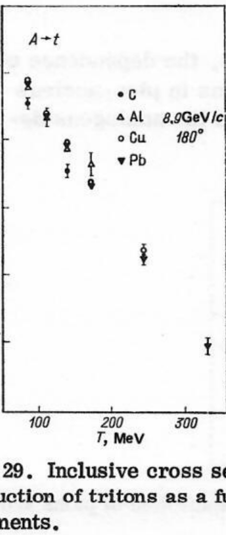

FIG. 29. Inclusive cross section (normalized to A^2) for the production of tritons as a function of the kinetic energy of the fragments.

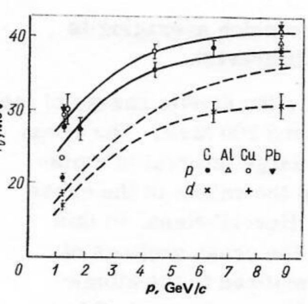

FIG. 30. Dependence of the parameter T_0 of the energy spectra of protons and deuterons as a function of the momentum of the initial protons and the atomic weight of the fragmenting nucleus.

Figure 30 gives the values of the parameter T_0 (the temperature) in the exponential dependence for different fragmenting nuclei (C, Al, Cu, and Pb) as a function of the momentum of the initial protons: 1.22 GeV/c (180°) (Ref. 43); 1.28 GeV/c (140°) (Ref. 42); 1.39 GeV/c (150°) (Ref. 30); 1.86, 4.50, and 6.57 GeV/c (137°) (Refs. 13 and 39); 8.9 GeV/c (180°) (Ref. 41). The dependences have a regular nature, except for a possibly large difference between the data of Ref. 43 and Ref. 42.

It follows from the dependences in Fig. 30 that the parameter T_0:

a) increases with increasing energy of the initial protons[42];

b) increases with increasing atomic weight of the fragmenting nucleus.

It can also be noted that the dependence on the energy of the initial protons becomes weaker with increasing energy, and the dependence on the atomic weight of the fragmenting nucleus is in this sense universal.

Pion-nucleus interactions. Figure 31 shows experimental data on the emission of protons in pion-nucleus interactions obtained using bubble chambers (propane and xenon) bombarded by pions of different momenta (GeV/c): 3.34 (π^-Xe, 160–180°) (Ref. 44); 2.34 (π^+Xe, 143–180°) (Ref. 45); 3.5 (π^-Xe, 143–180°) (Ref. 46); 40 (π^-C, 160–180°) (Ref. 47) (the interval

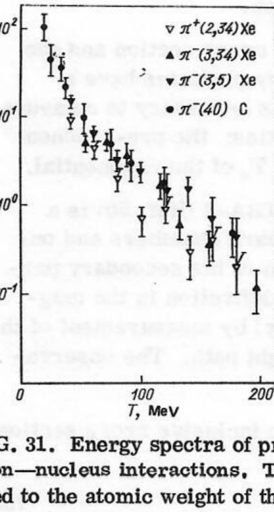

FIG. 31. Energy spectra of protons (emission angle 160°) in pion—nucleus interactions. The cross sections are normalized to the atomic weight of the fragmenting nucleus (C, Xe).

of proton emission angles over which averaging is performed is indicated in the brackets).

As can be seen from Fig. 31, the kinetic energy of the detected protons does not exceed 200 MeV. The cross sections are averaged over a large interval of emission angles, and the errors in the values of the cross sections are large (20–50%). Nevertheless, to this accuracy one can say that all the cross sections of πC and πXe interactions, normalized to the atomic weight of the fragmenting nucleus, are equal, this being so in a wide range of initial-pion energies from 3 to 40 GeV/c. A difference is noted[48] only in the slope parameters (in the temperature, MeV) for xenon nuclei at pion momenta 2.34 and 3.50 GeV/c ($30 < T_p < 200$ MeV):

$$T_0(2.34) = 28.3 \pm 3;$$
$$T_0(3.50) = 37.8 \pm 4.6.$$

Note that the "temperature" of the proton spectrum, i.e., the profile of the spectrum in the pion-nucleon interactions, is close to the corresponding values in the proton-nucleus interactions. This circumstance was first pointed out by Leksin *et al.*[44] In the same paper they noted that similar proton (and deuteron) spectra were obtained in the experiment of Ref. 49 in the interaction of photons with nuclei. These experimental facts enabled Leksin to formulate an empirical rule, a hypothesis (of nuclear scaling) for the spectra of protons, deuterons, and other baryonic systems:

a) the profile of the spectrum (the temperature, its reciprocal value) does not depend on the energy or species of projectile;

b) the absolute value (normalization of the spectrum) is proportional to the total cross section of the corresponding interaction, viz., hadron-nucleus, electromagnetic, and so on.

Thus, the inclusive cross section, normalized to the total cross section, is a universal function that does not depend on the type of interaction or the energy of the interacting particles. The hypothesis of nuclear scaling has been investigated in many studies at Serpukhov, especially for pion-nucleus interactions.

Note that a) and b) above are special consequences of Yang's limiting fragmentation.

Since the invariant inclusive cross section and the energy spectra of the secondary particles have a simple exponential nature, it is necessary to measure experimentally just two quantities: the pre-exponential factor C and the argument T_0 of the exponential.

The experimental facility ISTRA-2 (Ref. 50) is a magnetic spectrometer with spark chambers and on-line computer. The momentum of the secondary particles is determined from the deflection in the magnetic field and (sometimes only) by measurement of the time of flight over a known flight path. The observation angle is 162°.

As regression function of the inclusive cross sections, the dependence

$$E\, d\sigma/d\bar{p} = C \exp\{-Bp^2\}, \qquad (52)$$

where C is the inclusive cross section at zero momentum p of the secondary particles, was chosen. For convenience of comparing the experimental data, we convert the coefficient B in the dependence (52) into a temperature:

$$T_0 = (1 - \langle T\rangle/2m)/2mB, \qquad (53)$$

where m is the mass of the detected particle, and $\langle T\rangle$ is the mean kinetic energy of the particle.

For the quoted investigations, in which the protons were detected in the momentum range from 0.2 to 1 GeV/c, the mean momentum was ~450 MeV/c.

Assuming that each of the measured spectra, like the spectrum obtained by summing all spectra on all nuclei at all energies of the incident particles of all species, is described by the relation (52), the logarithm was taken of the ratio of the number of events in the given interval of squares of momenta of the secondary protons to the number of events in the same p^2 interval of the spectrum for all data. It can be shown that the logarithm of this ratio satisfies

$$\ln R = \text{const} + \Delta B p^2,$$

where ΔB is the deviation of the coefficient B for the given series from the mean value $\langle B\rangle$ of the coefficient for all data.

Figure 32 gives the experimental data of Ref. 51 on the values of ΔB for different targets and momenta: π^- mesons (1.55, 3.10, 4.13, 5.16, and 6.2 GeV/c) and initial protons (6.2, 7.5, and 9 GeV/c).

It can be seen that to an accuracy of 10–15% all the coefficients B in the dependence (52) are the same.

While this conclusion is also true, for the given range of initial proton momenta and the indicated accuracy, for proton-nucleus interactions (see Fig. 30), the constancy of the temperature of the proton spectra in the pion-nucleon interactions up to pion momenta of order 1.5 GeV/c is a specific result. Later, the same facility INSTRA-2 was used to obtain the absolute values of the parameters B (reciprocal temperature) for C, Cu, and Pb nuclei in the range of pion momenta from 1.5 to 6 GeV/c.[52] Figure 33 shows the experimental values of the parameter T_0.

As can be seen from this figure, the dependence of T_0 on the energy of the initial pions in pion-nucleus interactions differs strongly from the analogous de-

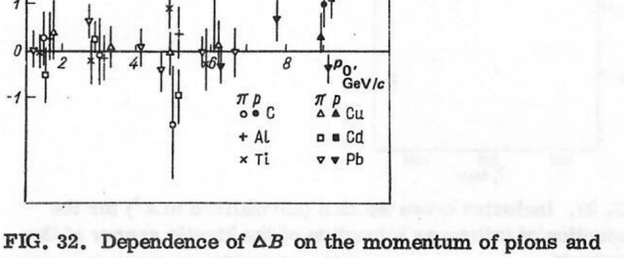

FIG. 32. Dependence of ΔB on the momentum of pions and protons incident on different nuclei.

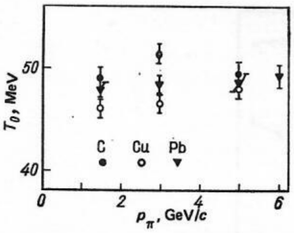

FIG. 33. Temperature of the proton spectra in pion—nucleus interactions as a function of the momentum of the initial pions.

pendence in proton-nucleus interactions (see Fig. 30). The parameter T_0 is virtually independent of the pion momentum, and its absolute value exceeds by about 10 MeV the temperature (40 MeV) of the proton spectra in the proton-nucleus interactions (see Fig. 30).

In Ref. 53, measurements were made of the slope parameters B for C, Cu, and Pb nuclei for the deuteron spectra in pion-nucleus interactions in the range of pion momenta from 1.5 to 6.0 GeV/c. It follows from the data that the parameter T_0 for the deuteron spectra is almost identical for Cu and Pb nuclei and varies from 34 ± 1.3 to 37 ± 1.3 MeV when the momenta of the initial pions vary from 1.5 to 6.0 GeV/c. Thus, in the case of the deuteron spectra as well the parameter T_0 hardly changes with changing pion energy, and in absolute magnitude it exceeds the temperature of the deuteron spectra in proton-nucleus interactions.

In Refs. 54, 48, and 55, a very interesting and important experimental fact was established, namely, that the parameter T_0 of the proton spectra in pion-nucleon interactions is virtually independent of the multiplicity of the protons produced in the interaction event.

In the quoted investigations, complete studies were made of the inclusive pion-nucleus cross sections for proton production under the assumption that they are described by the very simple function (52). Studies were made of the dependences of the pre-exponential coefficient C (or, more precisely, the total cross section of cumulative production of protons) on the energy of the initial pions and on the atomic weight of the fragmenting nucleus. Figure 34 shows the ratio of the proton production cross sections, integrated over the emission angles 160–164° and momenta 0.4–1.0 GeV/c, to the total inelastic interaction cross section, as a function of the energy of the initial pions for different fragmenting nuclei (C, Al, Ti, Cu, Cd, Pb).[51] The black points correspond to proton-nucleus interactions (data obtained

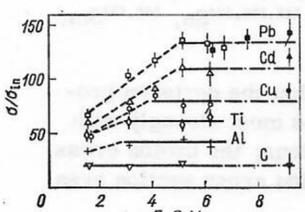

FIG. 34. Dependence of the ratio $\sigma/\sigma_{\text{in}}$ on the energy of the initial particle for different fragmenting nuclei.

TABLE II. Values of the parameter T_0 (MeV) as a function of E_γ and A.

A	E_γ, GeV		
	2.0	3.0	4.5
C	51.2±4.3	44.7±4.1	43±2
Cu	46.3±3	41.5±4	—
Pb	48.1±4.5	44.2±5	—

with the same experimental facility). The characteristic dependence is clearly observed: The larger the atomic weight of the fragmenting nucleus, the higher the energy at which the transition to a constant value is observed.

Photon-nucleus interactions. There are now available systematized experimental data on inclusive proton spectra obtained with the Erevan accelerator by the group of Egiyan[56] for bremsstrahlung photons with maximal energies from 2 to 4.5 GeV. The emission angles of the protons are 60, 90, and 150°. The kinetic energy of the secondary protons is 65–280 MeV. The fragmenting nuclei are C, Cu, and Pb.

The observed proton spectra are exponential. The values of the parameter T_0 are given in Table II. The proton emission angle is 150° for 2- and 3-GeV photons and 137° for 4.5 GeV.

It can be seen from Table II that for photon-nucleus interactions the parameter T_0 decreases with increasing energy of the initial photons and is virtually independent of the atomic weight of the fragmenting nucleus.

Dependences of the Inclusive Cross Sections on the Emission Angles of Cumulative Protons, Deuterons, and Tritons. Figure 35 shows the inclusive cross section for production of protons (and deuterons) with momentum 500 MeV/c as a function of the emission angles θ_p and θ_d; for proton-nucleus interactions at momentum 8.9 (Ref. 24) and 1.39 (Ref. 30) GeV/c of the initial protons; for pion-nucleus interactions at momentum 3.5 GeV/c of the pions[46]; and for photon-nucleus interactions at maximal energy 4.5 GeV.[56]

It can be seen from Fig. 35 that all the angular de-

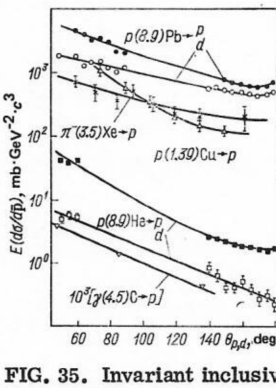

FIG. 35. Invariant inclusive cross sections as a function of the emission angle of protons and deuterons with momentum 500 MeV/c for different types of interactions and fragmenting nuclei.

pendences are similar. Note especially the similarity of the dependences of the fragmentation cross sections of He and Pb nuclei with emission of protons and deuterons in the case of 8.9-GeV/c initial protons.

According to the hypothesis of the cumulative nuclear effect [see Eq. (8)] and its obvious consequence—the Q dependence [see Eq. (33)]—the variation of the inclusive cross section with variation of the proton emission angle must be

$$E\frac{d\sigma}{d\bar{p}}=G_1 \times\exp\left\{\frac{p_\mu p_p\cos\theta_p}{\langle Q\rangle\, m\,(T_\mu-T_p)}\left(\frac{R_0}{R_0+\lambda_\mu}\right)^3\right\} \quad (54)$$

[we have used the relation (22) for the connection between the cumulative number and the emission angle θ_p of the secondary particles and the momentum p_p], i.e., at fixed momenta of the inclusive proton and momentum p_μ of the initial particle (and also its kinetic energy T_μ) this relation has the simple form

$$E\frac{d\sigma}{d\bar{p}}(\cos\theta_p) = G_1\exp\{B(p_\mu, \langle Q\rangle, R_0)\cos\theta_p\}, \quad (55)$$

where B for the data in Fig. 35 is a constant which varies weakly with the momentum p_μ. Thus, from the point of view of the Q dependence, the similarity of the angular dependences (see Fig. 35) can be explained simply. It is natural to expect that in a definite model of the cumulative effect without a sharp cutoff at the smallest possible cumulative number higher powers in $\cos\theta_p$ will arise, but the dominant contribution will be due to an expression of the type (55).

In Fig. 36, we have plotted experimental data on the angular dependences of the production of cumulative particles with momentum $640Z_c$ MeV/c (Z_c is the charge of the cumulative particle) in the case of interaction of 8.9-GeV/c deuterons with lead nuclei. In the interval 150–180° of emission angles a well-defined structure is observed. The error in the values of the cross sections for protons, deuterons, and tritons does not exceed the size of the points in Fig. 36.

Dependence on A of the Inclusive Cross Section for the Production of Cumulative Protons, Deuterons, and

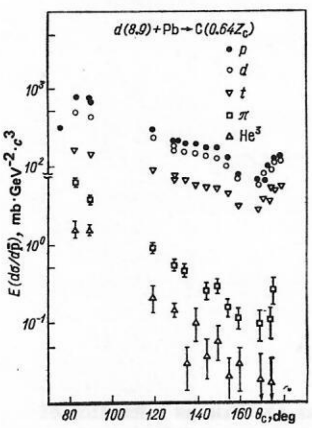

FIG. 36. Inclusive cross sections for production of cumulative particles as a function of the emission angle in dPb interactions.

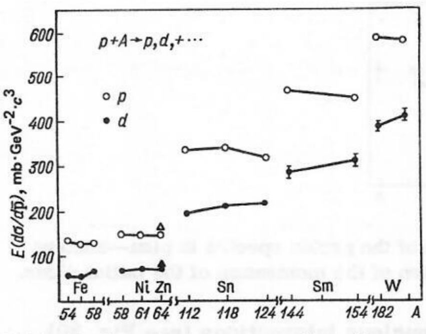

FIG. 37. Invariant cross sections for production of protons and deuterons as a function of the neutron excess in the nucleus.

Tritons. Proton-nucleus interactions. Qualitatively, we have already considered this question in our analysis of the energy dependences. In Fig. 27, the inclusive cross sections for proton production are normalized to $A^{4/3}$, and it can be seen that the dependence on the atomic weight of the nucleus is even stronger for light nuclei ($A<27$). Similarly, Fig. 28 illustrates the growth of the inclusive cross section of deuteron production with increasing atomic weight as $A^{5/3}$ and, correspondingly, A^2 for tritium nuclei (see Fig. 29). Thus

$$\left.\begin{aligned} &E\,d\sigma/d\bar{p}\,(pA\to p)\sim A^{4/3};\\ &E\,d\sigma/d\bar{p}\,(pA\to d)\sim A^{5/3};\\ &E\,d\sigma/d\bar{p}\,(pA\to t)\sim A^{2}. \end{aligned}\right\} \quad (56)$$

However, detailed investigations of the A dependence for separated nuclei of the isotopes[22,41] showed that the invariant cross section for the production of protons and deuterons at angle 180° and momentum 500 MeV/c in the case of bombardment by 8.9-GeV/c protons does not depend on the neutron excess in the nucleus. Figure 37 shows the obtained experimental data.

It can be seen from the figure that the yield of protons and deuterons from the isotopes ^{54,56,58}Fe, ^{58,61,64}Ni, ^{112,118,124}Sn, ^{144,154}Sm and ^{182,186}W does not change with increasing number of neutrons in the fragmenting nucleus. Thus, it is meaningful to speak, not of the A dependence of the inclusive cross sections, but of the Z dependence, i.e., the dependence on the charge of the fragmenting nucleus. Indeed, the dependence of the invariant cross sections on the charge of the nucleus is more regular. Figure 38 shows experimental data on the inclusive cross sections[41] for production of particles with momentum 500 MeV/c (protons, deuterons, and pions) as a function of the charge of the fragmenting nucleus. The targets were the following nuclei: ^{6,7}Li, Be, C, Al, Si, ^{54,56,58}Fe, ^{58,61,64}Ni, Cu, ^{64}Zn, ^{112,118,124}Sn, ^{144,154}Sm, ^{182,186}W, Pb, U.

It can be seen from Fig. 38 that the deuteron production cross section increases most strongly with the changing charge of the nucleus; the proton cross section changes less and the pion cross section even less.

According to the χ^2 test, the Z dependence for protons cannot be satisfactorily described by either the func-

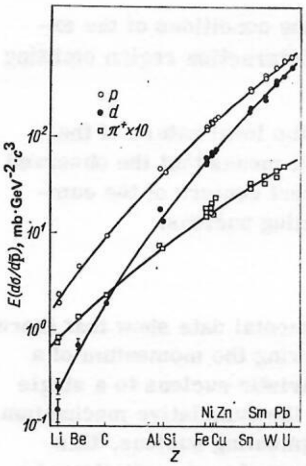

FIG. 38. Z dependence of the invariant cross sections for production of pions, protons, and deuterons with 8.9-GeV/c initial protons.

tion

$$E\frac{d\sigma}{d\bar{p}}=\sum_{0}^{3} a_n Z^n,$$

or the function

$$E\,(d\sigma/d\bar{p})=a_1 Z^{n_1}+a_2 Z^{n_2}.$$

Figure 39 shows the Z dependence for protons, deuterons, and tritons with momentum 800 MeV/c for 8.9-GeV-c initial protons.

It can be seen that the dependence of the yield of tritons increases with increasing Z faster than the deuteron and proton contributions.

Pion-nucleus interactions. The experimental data on pion-nucleus interactions obtained by means of bubble chambers (C, Xe) give an indication (see Fig. 31) that the inclusive cross section for proton production is proportional to the atomic weight of the fragmenting nucleus:

$$E\,d\sigma/d\bar{p}\,(\pi A\to p)\sim A.$$

Systematic investigations into the A dependence of pion-nucleus interactions were made by a Serpukhov

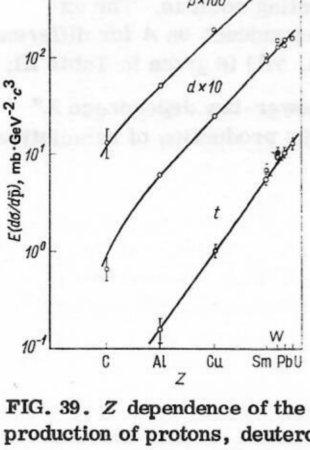

FIG. 39. Z dependence of the invariant cross sections for the production of protons, deuterons, and tritons.

group. In Ref. 57, under the assumption that the coefficient B is constant [see the relation (52)], the dependence of the pre-exponential coefficient C on the atomic weight of the fragmenting nucleus was found for the emission of protons at angle 162° in the momentum interval 400-1000 MeV/c; the momentum of the initial pions was 3.1 GeV/c and the fragmenting nuclei were C, Al, Cu, Cd, and Pb.

It was found that the dependence of C on A can be well approximated by the function

$$C=a_0+a_1A^n; \tag{57}$$

the value of the exponent is $n=1.03\pm0.06$.

A linear dependence of C on the atomic weight of the fragmenting nucleus was then obtained with the same facility under the same assumptions in a wide range of momenta of initial pions from 1.5 to 6.0 GeV/c.[58]

In Ref. 53, again under the assumption of constancy of the parameter B in the relation (52), the A dependences of the yields of deuterons for pions with momentum from 1.5 to 5.0 GeV/c were studied. Secondary deuterons with momenta greater than 500 MeV/c, were detected. The fragmenting nuclei were C, Cu, and Pb. It was found that the ratio of the deuteron to the proton yield increases as $A^{1/3}$ for 5.0-GeV/c initial pions. At lower momenta, the power is somewhat smaller than 1/3. Thus, it can be said that the dependence of the inclusive cross section for the production of deuterons in the pion-nucleon interaction is not stronger than $A^{4/3}$, i.e.,

$$\left.\begin{array}{l} E\,d\sigma/d\bar{p}\,(\pi A\to p)\sim A;\\ E\,d\sigma/d\bar{p}\,(\pi A\to d)\sim A^{4/3}\end{array}\right\} \tag{58}$$

or, comparing this with (56), is weaker by $A^{1/3}$ than in proton-nucleus interactions.

Photon-nucleus interactions. The A dependence of the inclusive cross section for proton production was investigated by the Erevan group.[56] The fragmenting nuclei were C, Al, Cu, Sn, and Pb, the energies of the secondary protons were in the range 65–280 MeV, and the emission angle was 150°. It was found that the proton yield can be well approximated by the power-law dependence A^n, with $n\sim1.1$. Thus, taking all the experimental data on protons, we have the following values for the exponent n:

for proton-nucleus interactions $n\sim4/3$;

for pion-nucleus interactions $n\sim1$;

for photon-nucleus interactions $n\sim1$.

Multiplicity of Cumulatively Produced Protons. The A dependence of the mean multiplicity of cumulatively produced protons in proton-nucleus interactions was investigated in Ref. 36 with 7-GeV/c initial protons. The fragmenting nuclei were Be, C, Al, Cu, Cd, and Pb. The mean multiplicity of the protons satisfied

$$\langle n_p\rangle\sim A^{\alpha}.$$

For protons emitted in the backward hemisphere ($\theta_p\geqslant90°$) with momentum greater than 300 MeV/c, $\alpha=0.62\pm0.04$, i.e., the mean multiplicity of the produced protons in the proton-nucleus interactions increases with increasing atomic weight of the fragmenting nucleus in accordance with the law

$$\langle n_p\rangle \sim A^{2/3}. \tag{59}$$

For protons emitted in the forward hemisphere ($\theta_p > 90°$), the following values were obtained:

$$\alpha\,(p < 300\,\text{MeV}/c) = 0.39 \pm 0.02;$$
$$\alpha\,(300 < p < 1000\,\text{MeV}/c) = 0.3 \pm 0.02;$$
$$\alpha\,(p > 1000\,\text{MeV}/c) = -0.04 \pm 0.05.$$

Study of the mean multiplicity of the particle production makes it easier to understand the physical significance of the A dependence of the inclusive cross sections. A sum rule gives

$$\frac{1}{\sigma_t}\int E\frac{d\sigma}{d\bar{p}}\frac{d\bar{p}}{E} = \langle n\rangle. \tag{60}$$

Since the inclusive particle production cross section $Ed\sigma/d\bar{p}$ has, as we have seen, a universal exponential dependence in virtually the whole of the phase space and the actual value of the cross section on the boundary of the phase space is small, it can be assumed that the A dependence of the invariant cross section is proportional to the product of the A dependence of the total interaction cross section σ_t and the A dependence of the mean multiplicity.

The A dependence of the mean multiplicity of cumulative protons in pion-nucleus interactions was studied in Ref. 59 in the semi-inclusive set-up. For the experiment with spark chamber and negative 3.7-GeV/c pions as projectiles, the trigger was a positive particle emitted forward with momentum greater than 700 MeV/c. The fragmenting nuclei were C, Al, Cu, and Pb.

The obtained data do not contradict a dependence of the type

$$\langle n\rangle \sim A^{1/3}. \tag{61}$$

Thus, the A dependence of the mean multiplicity of cumulative protons can be described by the dependence for proton-nucleus interactions ($A^{2/3}$) and pion-nucleus interactions ($A^{1/3}$).

Correlation Effects in the Production of Cumulative Protons. Correlations in the production of cumulative protons with energy higher than 30 MeV emitted in the backward hemisphere were studied in a semi-inclusive experiment by a Serpukhov group.[55] The nuclei Al, Cu, and Pb were bombarded with negative 3.7-GeV/c mesons.

If the radius of the region from which the protons are emitted is R, then two protons with momentum difference $q < \hbar/R$ cannot have parallel spins, i.e., the correlation must reduce the number of proton pairs with small q by a factor 2 compared with what would be expected without the Pauli principle.[60] A similar effect can be obtained by investigating the effective mass distributions for two protons.

Correlations were not observed in the effective-mass distributions in Ref. 55. However, if one assumes that they do exist in the region of effective masses inaccessible under the conditions of the experiment, the radius of the interaction region emitting the protons exceeds 2.5 F.

From the point of view of the local nature of the cumulative effect, this result means that the observed protons correspond to different centers of the cumulative effect in the fragmenting nucleus.

CONCLUSIONS

1. The considered experimental data show that there is a mechanism for transferring the momentum of a group of nucleons in a relativistic nucleus to a single inclusive particle, which is the cumulative mechanism. In the rest frame of the fragmenting nucleus, this mechanism corresponds to transfer of excitation of a group of nucleons to a single secondary particle. The traditional mechanisms of Fermi motion and rescattering do not play a significant part on the scale of the observed cross sections and the characteristic dependences.

2. The observed spectrum of cumulative particles satisfies the principle of scale invariance of the interactions in accordance with the hypothesis of the cumulative nuclear effect. Thus, there has been found a law of scaling of the interactions of relativistic nuclei in the region of kinematic variables of the inclusive particles corresponding to interaction with a group of nucleons of the nucleus; this is the cumulative nuclear effect.

3. The scale-invariant interaction of relativistic nuclei is characterized by the following new features:

a) the spectrum of the secondary particles is exponential:

$$E\,d\sigma/d\bar{p} = C\exp(-T/T_0),$$

which is a consequence of the model of the cumulative nuclear effect; this model gives (as we have shown for meson production) a universal Q dependence and the dependences on the energy, emission angles, and mass of the produced particles;

b) the inclusive cross section for production of cumulative particles (π, p, d, t) in the interaction of relativistic nuclei depends weakly on the atomic weight of the exciting nucleus and in a specific manner on the atomic weight of the fragmenting nucleus. The exponent n in the power-law dependence on A for different types of interaction (pA, πA, γA) is given in Table III;

c) the exponent α in the power-law dependence A^α of the mean multiplicity of the production of cumulative

TABLE III. Values of the exponent n for different types of interaction.

Particle	n		
	pA	πA	γA
(π)	1	—	—
(p)	4/3	1	1
d	5/3	4/3	—
t	2	—	—

particles (π, p) for different interactions (pA, πA) is

$$\alpha_\pi = 1/3 \text{ for } pA;$$
$$\alpha_p = \begin{cases} 2/3 & \text{for } pA; \\ 1/3 & \text{for } \pi A; \end{cases}$$

d) the production cross sections for positive and negative cumulative mesons are virtually equal;

e) the cumulative protons and Λ hyperons are strongly polarized;

f) the yield of cumulative protons and deuterons in proton-nucleus interactions does not depend on the neutron excess of the fragmenting nucleus.

[1] A. M. Baldin, Kratk. Soobshch. Fiz. No. 1, 35 (1971).
[2] A. A. Logunov, Nguen Van Hieu, and O. A. Krustalev, Problemy teoreticheskoĭ fiziki (Problems of Theoretical Physics), Nauka, Moscow (1969), p. 107.
[3] J. Benecke *et al.*, Phys. Rev. 188, 2159 (1969).
[4] A. M. Baldin, Dokl. Akad. Nauk SSSR 222, 1064 (1975) [Sov. Phys. Dokl. 20, 418 (1975)].
[5] A. M. Baldin, Fiz. Elem. Chastits At. Yadra 8, 429 (1977) [Sov. J. Part. Nucl. 8, 175 (1977)].
[6] V. S. Stavinskiĭ Soobshcheniya (Communication) R2-9528, JINR, Dubna (1976).
[7] A. M. Baldin, Preprint R1-5819 [in Russian], JINR, Dubna (1976).
[8] S. B. Gerasimov and N. Giordenescu, Preprint R2-7687 [in Russian], JINR, Dubna (1974).
[9] G. A. Leksin, Zh. Eksp. Teor. Fiz. 32, 445 (1957) [Sov. Phys. JETP 5, 378 (1957)].
[10] L. S. Azhgireĭ *et al.*, Zh. Eksp. Teor. Fiz. 33, 1185 (1957) [Sov. Phys. JETP 6, 906 (1958)].
[11] R. J. Sutter *et al.*, Phys. Rev. Lett. **19**, 1189 (1967).
[12] V. N. Komarov, Fiz. Elem. Chastits At. Yadra 5, 419 (1974) [Sov. J. Part. Nucl. 5, 168 (1974)].
[13] Yu. D. Bayukov *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. 30, 521 (1966).
[14] D. I. Blokhintsev, Zh. Eksp. Teor. Fiz. 33, 1295 (1957) [Sov. Phys. JETP 6, 995 (1958)].
[15] D. E. Dorfan *et al.*, in: Voprosy teorii élementarnykh chastits (Questions in the Theory of Elementary Particles), Isd. Akad. Nauk ArmSSR, Erevan (1966).
[16] T. E. Kalogeropoulos, Preprint 72-10, CERN (1972).
[17] A. M. Baldin *et al.*, Preprint 1-8028 [in Russian], JINR, Dubna (1974).
[18] T. V. Avericheva *et al.*, Soobshcheniya (Communication) 1-11317, JINR, Dubna (1978).
[19] A. M. Baldin *et al.*, Preprint E1-8054, JINR, Dubna (1974).
[20] A. M. Baldin *et al.*, Preprint 1-8249 [in Russian], JINR, Dubna (1974); Yad. Fiz. 21, 1008 (1975) [Sov. J. Nucl. Phys. 21, 517 (1975)].
[21] A. M. Baldin, in: Proc. Sixth Intern. Conf. on High Energy Physics and Nuclear Structure, Sante Fe (1975), p. 621.
[22] A. M. Baldin *et al.*, in: IV Mezhdunarodnyĭ seminar po problemam fiziki vysokikh énergiĭ (Fourth Intern. Seminar on Problems of High Energy Physics), D1, 2-9224, Dubna (1975).
[23] V. S. Stavinskiĭ, in: Tr. XVIII Mezhdunarodnoĭ konferentsii po fizike vysokikh énergiĭ (Proc. 18th Intern. Conf. on High Energy Physics), Vol. 1, A6-1, Tbilisi (1977).
[24] A. M. Baldin, in: Seventh Intern. Conf. on High Energy Physics and Nuclear Structure, Zürich (1977).
[25] A. M. Baldin *et al.*, Soobshcheniya (Communication) R1-11168, JINR, Dubna (1977).
[26] V. S. Alazashvili *et al.*, Preprint R1-10719 [in Russian], JINR, Dubna (1977).
[27] V. V. Burov and A. I. Titov, Preprint R2-9426 [in Russian], JINR, Dubna (1975).
[28] G. A. Lobov, Preprint ITEP-88, Moscow (1976).
[29] L. L. Frankfurt and M. I. Strikman, Phys. Lett. 65B, 51 (1976).
[30] D. R. F. Cochran *et al.*, Phys. Rev. D 6, 3085 (1972).
[31] V. V. Volkov, Fiz. Elem. Chastits At. Yadra 6, 1040 (1975) [Sov. J. Part. Nucl. 6, 420 (1975)].
[32] L. A. Kondratyuk and B. V. Kopeliovich, Pis'ma Zh. Eksp. Teor. Fiz. 21, 508 (1975) [JETP Lett. 21, 234 (1975)].
[33] I. G. Bogatskaya, S. M. Eliseev, and G. M. Zinovjev, Preprint E2-10172, JINR, Dubna (1976).
[34] I. W. Cronin *et al.*, Phys. Rev. D 11, 3105 (1975).
[35] H. H. Hekcman, L. B. L. -2053 (1973).
[36] A. V. Arefyev *et al.*, Preprint ITEP-56, Moscow (1977).
[37] I. I. Vorob'ev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 22, 390 (1975) [JETP Lett. 22, 184 (1975)].
[38] G. A. Leksin and A. V. Smirnitsky, Preprint ITEP-87, Moscow (1977).
[39] Yu. D. Bayukov *et al.*, Yad. Fiz. 5, 336 (1967).
[40] A. M. Baldin, Dokl. Akad. Nauk SSSR **222**, 1064 (1975) [Sov. Phys. Dokl. **20**, 428 (1975)].
[41] A. M. Baldin *et al.*, Preprint R1-11302 [in Russian], JINR, Dubna (1978).
[42] V. I. Komarov *et al.*, Preprint E-10573, JINR, Dubna (1977).
[43] S. Frankel *et al.*, Phys. Rev. Lett. **36**, 642 (1976).
[44] Yu. D. Bayukov *et al.*, Yad. Fiz. 19, 1266 (1974) [Sov. J. Nucl. Phys. 19, 648 (1974)].
[45] B. Slovinskii and Z. Strugal'skiĭ, Preprint R1-9614 [in Russian], JINR, Dubna (1976).
[46] L. S. Okhrimenko *et al.*, Preprint R1-9692 [in Russian], JINR, Dubna (1976).
[47] Collaboration, Yad. Fiz. 22, 5 (1975) [Sov. J. Nucl. Phys. 22, 1 (1975)]; Preprint R1-8566 [in Russian], JINR, Dubna (1975).
[48] L. S. Okhrimenko *et al.*, Preprint R1-9806 [in Russian], JINR, Dubna (1976).
[49] Yu. P. Antuf'ev *et al.*, Yad. Fiz. 13, 473 (1971) [Sov. J. Nucl. Phys. 13, 265 (1971)]; Ukr. Fiz. Zh. 14, 499 (1969).
[50] M. K. Vlasov *et al.*, Preprint ITÉF-81 [in Russian], Moscow (1976); M. K. Vlasov *et al.*, Preprint ITÉF-82 [in Russian], Moscow (1977).
[51] N. A. Burgov *et al.*, Yad. Fiz. 24, 1183 (1976) [Sov. J. Nucl. Phys. 24, 620 (1976)].
[52] N. A. Burgov *et al.* Preprint ITÉF-85 [in Russian], Moscow (1977).
[53] N. A. Burgov *et al.*, Preprint ITÉF-129 [in Russian], Moscow (1977).
[54] A. V. Aref'ev *et al.*, Preprint ITÉF-109 [in Russian], Moscow (1975).
[55] Yu. D. Bayukov *et al.*, Preprint ITÉF-103 [in Russian], Moscow (1976).
[56] M. J. Amaryan *et al.*, Yerevan 173 (19) (1976); 175 (21) (1976); 174 (20) (1976).
[57] N. A. Burgov *et al.*, Preprint ITÉF-66 [in Russian], Moscow (1975).
[58] N. A. Burgov *et al.*, Preprint ITÉF-97 [in Russian], Moscow (1975).
[59] A. V. Alef'ev *et al.*, Preprint ITÉF-56 [in Russian], Moscow (1975).
[60] G. I. Kopylov, Preprint R2-7211 [in Russian], JINR, Dubna (1973).

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