Nuclear reactions with large momentum transfer and the hypothesis of fluctons in nuclei

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Nuclear reactions with a single feature—large momentum transfer—are considered. It is shown that they can be understood qualitatively and interpreted quantitatively by assuming that there are fluctuations in the nuclear density and that the quark-parton mechanism applies. The "fluctons" are given a physical interpretation of many-quark configurations in the nuclei. The probability of their existence is estimated in the quark bag model.

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Nuclear reactions with large momentum transfer are of interest in problems of relativistic nuclear physics,[1] the science at the frontier between nuclear physics and elementary-particle physics. Investigation of these reactions has made it necessary to introduce new ideas in ordinary nuclear physics, the quark–parton ideas, and apply them not only to the structure of nucleons but also to the structure of the nucleus itself. In the present paper, we shall develop this thesis and emphasize the possible existence of "fluctons"[2]—many-baryon configurations—in nuclei.

1. BASIC REACTIONS

There are currently known several classes of nuclear reactions with large ($q > 1$ GeV/c) momentum transfer (Table I). If such a momentum is to be transferred to the nucleus, there must be an object capable of receiving it. It follows from the uncertainty relation that its characteristic radius is of the order of the radius of the hard core of the nuclear forces:

$$r \gtrsim q_m^{-1} \approx 0.1\text{-}0.5 \text{ F}. \tag{1}$$

It is then natural to ask why we encounter the nucleus here; for at these distances only a nucleon could, in the best case, participate in the reaction. The answer is provided by reactions of cumulative type.

2. THE CUMULATIVE EFFECT

The principal result of the study of a large class of nuclear reactions with backward emission of secondary particles (type 4 in Table I) is as follows:

1. Such particles cannot be produced in an elementary event on individual nucleons. Thus, for the corresponding process $pp \to p + \ldots$ the kinematics completely forbids backward emission of protons. For the other reaction $pp \to \pi(180°) + \ldots$ the kinematics gives, for example, $T_\pi^0 = 0.26$ GeV for an energy $E_p = 8.4$ GeV of the incident protons, whereas pions with energy $T_\pi \gtrsim 1$ GeV are observed from a nuclear target. The excess is by a factor ≈ 4. The necessary energy can be obtained by increasing the "target mass" to $M_k = km$ (m is the nucleon mass). In this case, $k \approx 4$.

Other important features of such reactions are the following.

2. Exponential decrease of the invariant cross sections as functions of the kinetic energy T of the secondary particles:

$$E\frac{d\sigma}{d\mathbf{p}} \equiv \sigma_{inv} \approx C \exp(-T/T_0). \tag{2}$$

The "temperature" T_0 and the "power" C are almost independent of the energy and species of incident particle.

3. A dependence on the atomic number A of the target nucleus is manifested in (2) only as

$$C \sim A^n, \tag{3}$$

where the parameter n is close to 1 for the discussed reactions (π and p, backward angles) and may increase somewhat (by ~20–30%) for secondary particles measured at angles $\theta \neq 180°$.

Conclusion. In the reaction, k nucleons of the nucleus participate together. This is the explanation of the adjective "cumulative."[1] It is now clear that the role of the nucleus is to supply the nucleons. However, it remains unclear how these k nucleons can be manifested simultaneously and coherently in a volume of the order of only one nucleon.

3. NUCLEAR METHODS

It might seem that these reactions (of type 4 in Table I) can be explained without the introduction of new ideas on the basis of the ordinary methods of nuclear physics. Let us consider the most typical of these attempts.

Allowance for the Fermi Motion of the Nucleons in the Nucleus. A nucleon is in the average nuclear field produced by the remaining $A-1$ nucleons, and its po-

TABLE I.

Type	Process	q_{max} GeV/c	q_{max} F^{-1}	Literature
1	eA scattering	1	5	Ref. 3
2	ed scattering	2.5	12	Ref. 4
3	$(p,2p)$, etc., quasielastic	1.5	7	Ref. 5
4	$pA \to \pi + \ldots$ $pA \to p + \ldots$ of cumulative type	2	10	Refs. 1,6

tential is $V \approx -45$ MeV. This field imparts a mean momentum $p_F = \sqrt{2m(E^* - V)}$ to the nucleon. Ignoring the binding energy $E^* \approx 7$ MeV, we find $p_F \approx 0.3$ GeV/c. Clearly, this is small compared with what is observed. Nevertheless, let us calculate the reaction in the impulse approximation in accordance with the diagram in Fig. 1. At the upper vertex we have the experimental cross section of pion production on the nucleon. The momentum distribution of the nucleon is specified in accordance with its wave function in the potential of the average field of the nucleus. Then[7]

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi}(pA \to \pi + \ldots) = \int \Re E_\pi \frac{d\sigma}{d\mathbf{p}_\pi}(pp_F \to \pi + \ldots)\, \omega_1(\mathbf{p}_F) \frac{d\mathbf{p}_F}{(2\pi)^3}. \quad (4)$$

$$\omega_1(\mathbf{p}_F) = \sum_n \left| \int \psi_n(\mathbf{r}) \exp(i\mathbf{p}_F\mathbf{r})\, d\mathbf{r} \right|^2. \quad (5)$$

The factor $\Re$ takes into account off-shell effects and the redefinition of the particle flux.[8] The result of the calculation is shown in Fig. 2 (curve 2). It can be seen from the figure that the discrepancy with the experiment is very appreciable.

Relativization of the Nuclear Function. At large $p_F \approx 0.3$ GeV/c, the velocity of the intranuclear nucleon is large and a nonrelativistic equation and potential should not be used to find its wave function. However, to study this effect, one can use the so-called *relativization of the ordinary function* ψ. In accordance with Ref. 9, to do this we go over from the ordinary momentum representation, realized by a Fourier–Bessel transformation, to the relativistic configuration space. Then instead of a plane wave in (5) we use the functions from Ref. 10 (in the notation of Ref. 9):

$$\exp(i\mathbf{p}\mathbf{r}) \to \xi(\mathbf{p}, \mathbf{r}) = \left(\frac{\sqrt{\mathbf{p}^2 + m^2} - \mathbf{p}\mathbf{n}}{m}\right)^{-1-irm}; \quad \mathbf{r} = r\mathbf{n}; \quad n^2 = 1. \quad (6)$$

For the s wave, we have accordingly

$$\omega_1 = \left| 4\pi \int \frac{\sin(\chi r m)}{pr} \psi_s(r)\, r^2\, dr \right|^2, \quad (7)$$

where $\sinh \chi = p/m$.

It can be seen from Fig. 2 (curve 3) that the relativization effect is small in this case.[7]

Multiple Scattering. We consider the most favorable case: the incident proton is scattered n times on the nuclear nucleons and is then emitted backward at angle θ. The maximal momentum p_n and energy E_n after n-fold scattering are obtained if in each rescattering $\theta_n = \theta/n$. One can show that

$$p_n \equiv \frac{2m\xi_0 z_n^n}{1 - \xi_0^2 z_n^{2n}} = \frac{p_0 z_n^n}{(1 - (1 - z_n^{2n})\,\xi_0\,(p_0/2m)}; \quad (8)$$

$$E_n = \frac{E_0 - (1 - z_n^{2n})\,\xi_0\,(p_0/2m)}{1 - (1 - z_n^{2n})\,\xi_0\,(p_0/2m)}, \quad (9)$$

where

$$\xi_0 = p_0/(E_0 + m_0); \quad z_n = \cos(\vartheta/n); \quad (10)$$

p_0, E_0, and m are the momentum, energy, and mass of the nucleon in the laboratory system. In the limit $n \to \infty$, we have $z_n \to 1$, $p_n \to E_0$, as expected. The maximal number of collisions in the nucleus depends on its size. Introducing the effective radius $r_0 \approx 1.2$ F of a nucleon in the nucleus, we estimate n_{max} by means of Fig. 3:

$$R = a + r_0 = \frac{r_0}{\sin(\theta/2n)} + r_0 \approx r_0 A^{1/3}; \quad (11)$$

$$n_{max} = \theta/[2 \arcsin(A^{1/3} - 1)^{-1}]. \quad (12)$$

Hence, for example, for the Ta nucleus ($A = 154$) at $\theta = 180°$ we have $n_{max} \approx 7$, i.e., for $p_0 = 1.5$ GeV/c we obtain $p_{max} \approx 0.5$ GeV/c; similarly, for Cu ($A = 64$), $\theta = 135°$, $p_0 = 4.5$ GeV/c we have $n_{max} = 3$, $p_{max} \approx 0.6$ GeV/c. But experimentally protons with momentum $p \gtrsim 1$ GeV/c are observed.[11] Thus:

1) Allowance for the Fermi motion and multiple scattering gives a "correction" to the momenta of approximately 0.2–0.3 GeV/c; the relativization effect is small. At large momenta, there remains a qualitative discrepancy with experiment.

2) It is necessary to assume that the particles are produced on a "bunch" of nucleons of mass $M_k = km$, and then the threshold is shifted by approximately a factor k.

3) We must ask what are these "bunches," what is the mechanism of "cooperation" of the nucleons, and how all phenomena with large momentum transfer are to be unified. We shall consider these questions in what follows.

4. THE IDEA OF FLUCTONS IN NUCLEI

The idea that several particles of the nucleus could enter into a reaction simultaneously arose 20 years

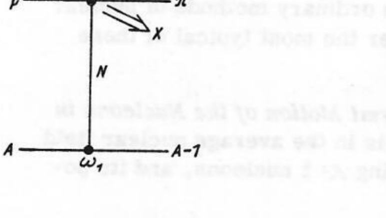


FIG. 1. Diagram of the impulse approximation for the reaction of pion production of a nuclear nucleon.

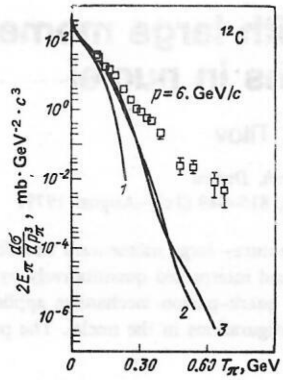


FIG. 2. Calculation of the cross section of pion production in the reaction $p^{12}C \to \pi + \ldots$ (Ref. 1). 1) Production on fixed nucleons; 2) with allowance for Fermi motion; 3) with allowance for relativistic effects.

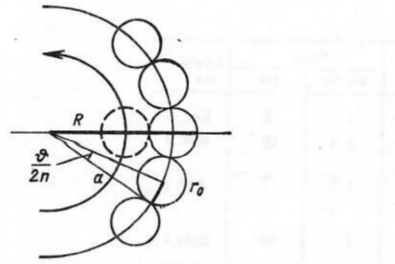


FIG. 3. Trajectory of "maximal momentum transfer" in nucleus A.

ago[2] in connection with the interpretation of experiments on the quasielastic scattering of protons by nuclei[12]; in the energy distributions, peaks were observed corresponding kinematically to scattering on "intranuclear deuterons." It was suggested[2] that this effect is actually a density fluctuation of nuclear matter [in a small correlation volume $V_\xi \approx (4/3)\pi r_\xi^3$($r_\xi$ is the correlation range)]. Then the proton could interact with the fluctuation as if it were a single object of mass $M_k = km$ (the flucton mass). Using the classical theory of fluctuations of an ideal gas, one can estimate the probability of such an event (normalized to A nucleons of the nucleus):

$$\beta_k^A = \binom{A}{k}(V_\xi/V_0)^{k-1}A^{1-k} \underset{\text{for } A\gg k}{\approx} A \gg k\,(A/k!)\,(V_\xi/V_0)^{k-1}. \tag{13}$$

Here, AV_0 is the volume of the complete nucleus, $V_0 = (4/3)\pi r_0^3$ is the volume of the nucleon, and r_0 is its effective radius (≈ 1.2 F). Such a fluctuation has a lifetime τ_k that is small, for example, compared with the period t^A of the motion of the nucleon in orbit. One must have $\tau_k/t^A \approx \beta_k^A$.

The reaction cross section and the form factors for scattering by the nucleus A can now be written in the form

$$\sigma = \Sigma_k \beta_k^A \sigma_k;\quad F = \Sigma_k \beta_k^A F_k, \tag{14}$$

where σ_k and Fk are the cross sections and form factors for interaction of the incident particle with a k-flucton (k is the number of nucleons in the flucton). Their form at large momentum transfers can be obtained in the framework of the quark–parton model of interaction of elementary particles using the quark counting rules.[13, 14]

The fluctons in the nucleus are not at rest. Therefore, (14) must be averaged over the momentum distribution of the fluctons in the nucleus. This distribution is determined by the wave function of the motion of their center of gravity. It can be found[15] by calculating the overlap of the wave function χ_k of the internal state of the flucton and the nuclear wave function, which is constructed, for example, in the form of a product of single-particle nucleon functions:

$$\Psi_k(\mathbf{R}) = c_k \int \chi_k(\xi)\,\phi_1(\mathbf{r}_1)\,\ldots\,\phi_k(\mathbf{r}_k)\,\{d\xi\}_{1\ldots(k-1)};\quad \xi_{ij} = \mathbf{r}_i - \mathbf{r}_j. \tag{15}$$

By definition, $\xi \approx r\xi \ll R$ (R is the radius of the nucleus); hence $r_i \approx r_j \approx R$, and then

$$\Psi_k(\mathbf{R}) \approx c_k V_\xi^{(k-1)/2} \phi^k(\mathbf{R}) \approx c_k V_\xi^{(k-1)/2} n^{k/2}(R), \tag{16}$$

where $n(R) = |\phi(R)|^2$ is the density distribution of the nuclear matter. From the normalization conditions

$$\int |\chi_k(\xi)|^2 \{d\xi\}_{1\ldots(k-1)} = 1;\quad \int n(R)\,d\mathbf{R} = 1;$$
$$\int |\Psi_k(\mathbf{R})|^2 d\mathbf{R} = \beta_k^A \tag{17}$$

we find the factor $|c_k|^2 = \binom{A}{k}$. Thus, the probability of finding a flucton of mass km with momentum $\mathbf{P}$ in the nucleus is

$$W_k(\mathbf{p}) = \left|\int \Psi_k(\mathbf{R}) \exp(i\mathbf{pR})\,d\mathbf{R}\right|^2 = \beta_k^A \omega_k(\mathbf{p}), \tag{18}$$

where the probability of the momentum distribution is

$$\omega_k(\mathbf{p}) = (V_0 A)^{k-1} \left|\int n^{k/2}(R) \exp(i\mathbf{pR})\,d\mathbf{R}\right|^2, \tag{19}$$

and it is normalized as

$$\int \omega_k(\mathbf{p})\,d\mathbf{p}/(2\pi)^3 = 1. \tag{20}$$

Thus, the idea of fluctons in nuclei makes it possible to calculate the cross sections and form factors (14). For this, it is necessary to know the probabilities β_k^A and to be able to calculate σ_k and F_k. With regard to β_k^A, we shall for the time being specify them in the form (13), regarding the correlation range r_ξ as a parameter. Below, we shall consider especially the nature of the fluctons. The cross sections σ_k and form factors F_k for scattering on fluctons can be calculated in two ways: The first is phenomenological, these quantities being found by analyzing independent experiments; the second is microscopic and based on quark–parton ideas about the mechanism of hadron– and lepton–flucton interaction.

5. PHENOMENOLOGY OF THE CUMULATIVE REACTIONS $pA \to \pi(180°) + \ldots$ AND $pA \to p(180°) + \ldots$

We calculate first the cross section of pion production. It is determined by the sum of the cross sections (14) of pion production on k-fluctons, which exist in the nucleus with the probability β_k^A (13). In addition, we take into account the motion of the fluctons in the nucleus, integrating for this purpose (14) over the momentum distribution (19). Thus,[15]

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi}(pA \to \pi + \ldots) = \sum_k \beta_k^A \int \Re E_\pi \frac{d\sigma}{d\mathbf{p}_\pi}(pk(\mathbf{p}_F) \to \pi)\,\omega_k(\mathbf{p}_F)\,\frac{d\mathbf{p}_F}{(2\pi)^3} \tag{21}$$

[the factor $\Re$ is the same as in (4)]. As cross section for the proton–flucton interaction, we take the cross section, parameterized in the variables $x^\pi = p^\pi/p_{0\max}$, of the reaction $pp \to \pi + \ldots$, making in it the substitution $x^\pi \to x_k^\pi = x^\pi/k$. Note that the elementary $pp \to \pi + \ldots$ cross section decreases rapidly with increasing x and, in principle, admits a different form of parameterization (we are interested in the case $p_\perp \approx 0$, since $\theta = 180°$). Below, in the calculations we use the parametrization with respect to x^π in the form of the product of a polynomial and an exponential function as given in Ref. 8. The results of the comparison with experiment[16] are shown in Fig. 4, which shows the following.

1. To explain the experimental data, it is sufficient to introduce fluctons with $k = 1, 2, 3, 4$, i.e., up to $k_{\max} = 4$. At the same time, in the region of the kinematic limit ($x^\pi \approx k$) for the k-flucton, the following $(k+1)$-fluc-flucton contributes to the cross section in the region of intermediate values of its variable $x_{k+1}^\pi \approx x^\pi/(k+1)$. Thus, the *asymptotic* behavior of the parametrized $pp \to \pi + \ldots$ cross section need be known only for the last value $k_{\max} = \max k$. In general, this may be $k = A$.

2. In all cases (different k and A) the correlation range r_ξ in the flucton is in the interval 0.5–0.7 F,

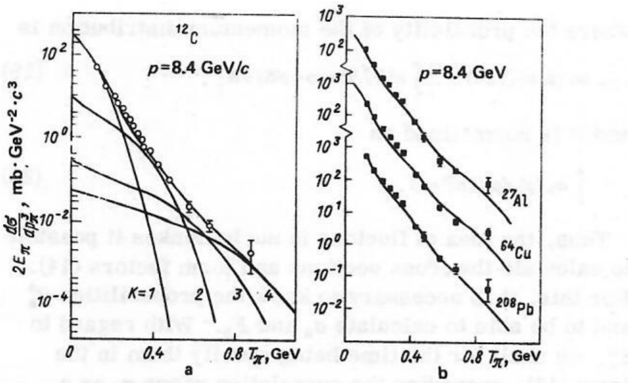


FIG. 4. Contribution to the cross section of pion production from individual fluctons with mass $M_k = km$ (a) and comparison of the theoretical cross sections with experiment for a series of nuclei (b).

i.e., it is of the order of the radius of the hard core of the NN forces.

3. A dependence of the cross section on the atomic number A of the target nucleus occurs in β_k^A. In accordance with (13),

$$\beta_k^A \sim A \quad \text{for} \quad A \gg k. \tag{22}$$

At low energies T_π of the emitted pions, it is necessary to take into account a factor for their absorption in the nucleus, which is $\sim A^{-1/3}$ in the limit $T_\pi \to 0$ (Ref. 2). Thus, parametrizing the cross section in the form $\sigma_{\text{inv}} \sim A^n$, we obtain a result corresponding to what is observed experimentally (Fig. 5).[16] Note that when the secondary particles are measured in a kinematic region different from the one considered in Ref. 16 ($\theta = 180°$), the A dependence of the cross section may differ somewhat from the one given here. This is due to the change in the reaction mechanism and will be considered below.

Now a few words on the phenomenology of the $pA \to p(180°) + \ldots$ reaction. A particular feature of this reaction is that a contribution to it from pp collisions is forbidden by the kinematics, so that the sum over the fluctons in the cross section (14) begins with $k = 2$. Further, it is necessary to parametrize the elementary cross section $\sigma(pp \to p + \ldots; p_\perp \approx 0)$ as a function of $x^p = p_\parallel^* / p_{\parallel\,\text{max}}^*$. Experiment shows that in the interval $0 \leqslant x^p \leqslant 1$ it is, up to a factor ≈ 3, a constant, the main change occurring in the region of the bump at $x^p \approx 1$. This is evidently the contribution of diffraction processes at small scattering angles. It is difficult to "shave off" these processes, since the mechanism of the elementary reaction must be particularized; however, for qualitative estimates it is sufficient to take

$$\sigma_{\text{inv}}^p = E_p \frac{d\sigma^p}{d\mathbf{p}_p} (pp \to p + \ldots,\ p_\perp \approx 0,\ x^p) \approx \sigma_0^p \theta (1 - x^p), \tag{23}$$

where $\sigma_0^p \approx 3$–15 mb $(\text{GeV}/c)^2$ (Ref. 17). This cross section differs strongly from the corresponding $pp \to \pi\ (180/) + \ldots$ cross section, which decreases rapidly, for example, as (see the parametrization of Ref. 17)

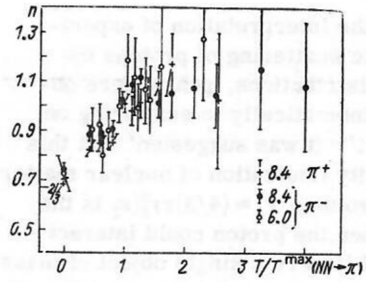


FIG. 5. The A dependence of the cross section of pion production with different energies of the emitted pions.

$$\sigma_{\text{inv}}^\pi = E_\pi \frac{d\sigma^\pi}{dp_\pi} (pp \to \pi + \ldots,\ p_\perp \approx 0,\ x^\pi) \approx \sigma_0^\pi (1 - x^\pi)^{6.45}, \tag{24}$$

where $\sigma_0^\pi \approx 75$ mb$(\text{GeV}/c)^2$. We now see the validity of the flucton mechanism at the qualitative level, considering the ratio of the cross sections of cumulative reactions with emission of p and π. It is convenient to choose their momenta such that the reaction on one flucton makes the main contribution to the cross section. Then (without allowance for Fermi motion in the nucleus) we have

$$\frac{\sigma_{\text{inv}}^p (pA + p\,(180°) + \ldots)}{\sigma_{\text{inv}}^\pi (pA \to \pi\,(180°) + \ldots)} \approx \frac{\beta_k^A \sigma_{\text{inv}}^p (pp \to p + \ldots, x_k^p)}{\beta_k^A \sigma_{\text{inv}}^\pi (pp \to \pi + \ldots, x_k^\pi)} \approx \frac{\sigma_{\text{inv}}^p (x_k^p)}{\sigma_{\text{inv}}^\pi (x_k^\pi)}. \tag{25}$$

For example, for the reaction on a heavy nucleus at $E_p = 8.4$ GeV and $\theta = 180°$ at momentum ≈ 0.5 GeV of the detected particles it is mainly fluctons with $k = 2$ that contribute to both cross sections (respectively, $x_2^\pi = 0.64$ and $x_2^p = 0.86$). From (23)–(25) we then obtain

$$\frac{\sigma_{\text{inv}}^p (pA \to p\,(180°) + \ldots)}{\sigma_{\text{inv}}^\pi (pA \to \pi\,(180°) + \ldots)} \approx 30 - 150, \tag{26}$$

which is in qualitative agreement with experiment.[1,6,16] More rigorous calculations, especially of the cross sections of the $pA \to p + \ldots$ reaction, require particularization of the mechanism. This will be done below on the basis of quark–parton ideas about the nucleon–flucton interactions.

6. ELASTIC eD AND eA SCATTERING

We begin with the deuteron.[18] In accordance with (14), we represent the form factor as

$$F_d (q^2) = F_1^d (q^2) + \beta_d F_2 (q^2), \tag{27}$$

where F_1^d is the "nonrelativistic" deuteron form factor; F_2 corresponds to scattering by a deuteron in a state of fluctuation contraction. For $F_2(q^2)$, we can determine the asymptotic behavior in accordance with the quark counting rules[13,14]:

$$(q^2)^{n-1} F \to \text{const} \quad \text{as} \quad q^2 \to \infty, \tag{28}$$

where n is the number of quarks in the system. For the deuteron, $n = 6$. To analyze the experiment of Ref. 4 in the region of large and medium values of q^2 it is necessary to parametrize F_2. For example,[4,19]

$$F_2^{\text{I}} (q^2) = (1 + q^2/36 m_Q^2)^{-5}; \quad m_Q = 0.28\ \text{GeV}; \tag{29}$$

$$F_2^{\text{II}} (q^2) = (1 + q^2/m_0^2)\, F_N^2 (q^2/4); \quad m_0 = 0.28\ \text{GeV}, \tag{30}$$

where F_N is the nucleon form factor. Note that (29) and (30) satisfy (28) at very large q^2.

The admixture of the state of fluctuation contraction is determined at large q^2, where the nonrelativistic form factor is not important. Then

$$F_d^{\exp}/F_2=\beta_d. \quad (31)$$

This ratio, taken from Refs. 4 and 19, is given in Fig. 6. It can be seen that β_d depends on the parametrization of F_2. For $F_2^{\rm I}$ (parametrization for large q^2) we have $\beta_d\approx 2$–4%, and for $F_2^{\rm II}$ (medium and large q^2) $\beta_d\approx 12$–15%. Thus, the problem of finding β_d reduces to the following: Where does the asymptotic behavior begin and how does it "join on" to the low-energy behavior of the form factor? It is interesting that the data on the cumulative effect, which, as we have noted above, are more sensitive to the choice of the corresponding proton–flucton cross section in the region of intermediate values of the momentum transfer, give the ratio $r_\xi/r_0\approx 0.4$–0.6, which leads to $\beta\approx 3$–11%, i.e., is within the accuracy of a determination from *ed* scattering. Here, we have used the fact that the probability of the deuteron, which occupies the volume $2V_0$, being in the volume V_ξ is

$$\beta_d=V_\xi/2V_0=(r_\xi/r_0)^{3/2}.$$

Analysis of the data in Fig. 6 shows that the problem of finding contraction fluctuations of nuclear matter can be most clearly posed only in the region of momentum transfers $q^2>1.5$–$2\ ({\rm GeV}/c)^2$. Unfortunately, at these values of q^2 the form factors of other nuclei (besides *d*) have not been measured. Therefore, we shall treat the nuclear form factors qualitatively. Representing, as usual, the form factor as a product of the form factor proper of the flucton, $F_k(q^2)$, and a form factor of the shape, i.e., the Fourier transform of the square of the modulus of the wave function of the motion of its center of gravity in the nucleus, we write

$$F^A(q^2)=\sum_{k=1}^{A}F_k(q^2)\int|\Psi_k(R)|^2\exp(i\mathbf{qR})\,d\mathbf{R}. \quad (32)$$

Substituting the expression (16) in (32), we find

$$F^A(q^2)=F_1\overline{F}_1+\frac{3}{4}\beta_2^AF_2\overline{F}_2+\sum_{k=3}\beta_k^AB_kF_k(q^2)\overline{F}_k(q^2), \quad (33)$$

where

$$\overline{F}_k(q^2)=(V_0A)^{k-1}\int n^k(\mathbf{R})\exp(i\mathbf{qR})\,d\mathbf{R} \quad (34)$$

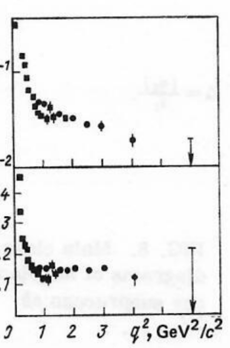


FIG. 6. Ratio of the experimental deutron form factor to $F_2^{\rm I}$ (above) and $F_2^{\rm II}$ (below).

is the form factor of the shape of the motion of the k-flucton in the nucleus, normalized to unity as $q^2\to 0$; a general normalization of F^A is not made, since in the limit $q^2\to 0$ it is assumed that all terms in it apart from the first are small; $\overline{F}_1$ is the usual form factor of the nucleus, calculated with nonrelativistic single-particle functions. For F_1 and F_2, the proton and deuteron form factors are taken. The coefficient B_k takes into account the isotopic composition of the flucton, which contains k_p protons and k_n neutrons. For light nuclei,

$$B_k=\binom{N}{k_n}\binom{Z}{k_z}\binom{A}{k}^{-1}\frac{1}{Z}. \quad (35)$$

The factor $1/Z$ comes in because of the normalization of all the form factors to unity as $q^2\to 0$. Especially for a flucton of deuteron type, allowance has here been made for the statistical weight 3/4 of the spin state I among all spin combinations of n and p. The flucton form factors F_k proper can be taken in accordance with the quark counting rules (28).

Figure 7 shows the calculated form factor of the nucleus ^{12}C in accordance with (33), in which allowance was made for the first two terms with $k=1$ and 2, the form (30) was chosen as the deuteron form factor, and the parameter β_2^A was calculated for r_ξ equal to 0.75 F. It can be seen that the contribution of the fluctuation correction begins to be felt at $q^2\approx 0.4\ ({\rm GeV}/c)^2$ if the symmetrized Fermi density[3] is taken as $n(R)$.

Thus, we can take it that the relevant problem now is the experimental investigation of nuclear scattering of electrons in the region $q^2>1\ ({\rm GeV}/c)^2$.

7. PARTON MODEL AND FLUCTONS

The microscopic approach, in contrast to the phenomenology, particularizes the mechanism of interaction of the incident particle and a flucton. For this, it is necessary to generalize the parton models of interaction of elementary particles and regard the flucton as a collection of partons. This provides a general basis for the construction of the production mechanism of any kind of cumulative particle in collisions of various particles and nuclei.

The notion of partons—the constituents of colliding objects—is used when the energies of these objects are large compared with their intrinsic mass. It is therefore natural to treat the problem in the coordinate system of colliding beams, in which the velocities of the incident particle and the target are equal and opposite, and then, by the condition of the problem, $E_{\rm inc}\gg m_{\rm inc}$

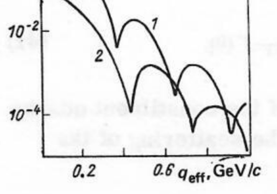


FIG. 7. Elastic form factor of ^{12}C calculated with nonrelativistic nuclear functions (1); contribution of admixture (2) from a $k=2$ flucton.

and $E_{targ} \gg m_{targ}$. In this case, the partons themselves appear as noninteracting parts of the object; they may be quarks q, antiquarks $\bar{q}$, gluons, mesons, and so forth. We shall now ignore the corrections for the Fermi motion of the fluctons, which produces a "smearing" of the momenta of the secondary particles within a range of about 0.15 GeV/c. Then the nucleus is merely the supplier of free fluctons, which play the part of the target. Both the incident particles and the fluctons can now be regarded as systems of noninteracting partons. The momentum distribution of the latter in each of the objects can be found by integrating over the phase space of the remaining partons with allowance for the law of conservation of the total four-momentum. Then[20]

$$G_{i/N}(x) \approx A_N^i(x)(1-x)^{\delta\gamma_{i/N}} \quad (36)$$

$$G_{j/k}(x) \approx A_k^j(x)(1-x_k)^{\delta\gamma_{j/k}}, \quad (37)$$

where $x_k = x/k$; the functions A_N^i and A_k^j determine the behavior at small x and therefore will not be important for what follows; $\gamma_{i/N}$ and $\gamma_{j/k}$ are parameters of the parton model, and $\gamma_{j/k} = \gamma_{i/N} + 6(k-1)$. For $\delta = 1$, a power-law decrease of (36) and (37) with increasing $x_k \to 1$ satisfies the quark counting rules. The parameter $\delta < 1$ is introduced here in order to parametrize the data for the corresponding elementary reactions in the region of intermediate values $x < 1$ at insufficiently large initial collision energies, since it is precisely this region, as we have seen above, that makes the most important contribution to the reaction on nuclei.

Below, we give some relations that follow from the quark counting rules.[13,14] Thus, the values of the parameters γ are determined in accordance with the "decay vertex" by the formula

$$\gamma_{a/b} = 2(N_b - N_a) - 1; \quad (38)$$

where N_a and N_b are the numbers of quarks constituting particles a and b. If as partons in nucleon N we have a quark q, a pion π, and an antiquark $\bar{q}$, then

$$\gamma_{q/N} = 3; \quad \gamma_{\pi/N} = 5; \quad \gamma_{\bar{q}/N} = 7\ldots \quad (39)$$

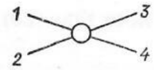
Further, in the diagrams of the impulse approximation corresponding to the processes of nucleon–flucton interaction, the main elementary subprocesses will now be the interaction of the partons of the incident particle with those of the flucton. Thus, the cross section for scattering of partons with small momentum transfer (t fixed and small; s and u are large) is the well-known cross section of Reggeon exchange:

$$d\sigma/dt \approx s^{2\alpha(t)-2}F(t). \quad (40)$$

At small t, $\alpha \to \alpha(0) = 1$, and the dependence on s becomes unimportant. Conversely, at large s, t and u, the cross section for scattering of partons through a fixed angle[13,14,21] is

$$\frac{d\sigma}{dt} \approx \frac{1}{s^{n_{act}-2}} f(\theta) \approx \frac{1}{(p_\perp^2 + m^2)^{n_{act}-2}} f'(\theta), \quad (41)$$

where $n_{act} = \sum_{i=1}^4 N_i$ is the sum of the constituent quarks at the vertex corresponding to the scattering of the given particles 1, 2, 3, 4;



For example, for $q\pi \to q\pi$ scattering $n_{act} = 6$; for $qq \to qq$ scattering, $n_{act} = 4$, etc.

8. CUMULATIVE $pA \to \pi(180°) + \ldots$ REACTION

To this reaction there contribute many diagrams, in which the pion, as a parton of the incident particle or target, is scattered on partons of the oncoming object (for example, $i\pi \to i\pi$, $\pi\pi \to \pi\pi, \ldots$). Processes of the type $q\bar{q} \to \pi\bar{\pi}$ are also possible. A calculation was made in Ref. 22 in this manner. Below, we shall give the more perspicuous treatment of Ref. 23, in which one dominant class of diagrams is separated, namely, the pion π as parton of flucton k is scattered on the partons i of the incident nucleon with small momentum transfer and is thus detected at small backward angles (Fig. 8). In the system of the colliding beams, parton i has momentum $p_i = yp_1$ $(0 < y < 1)$, where p_1 is the nucleon momentum, and the parton π has $p_\pi = xkp_2$, where kp_2 is the flucton momentum; x and y are the fractions of the parton momentum in the total momentum of the corresponding object. The variables s, t, and u of the given diagram are related in the relativistic limit ($E \gg m$) to the s_1, t_1, and u_1 of the elementary $p_1p_2 \to \pi$ process by

$$\left.\begin{aligned} s &= (yP_1 + kxP_2)^2 \approx kxys_1 \quad (\mathbf{p}_1\uparrow\downarrow\mathbf{p}_2);\\ t &= (kxP_2 - P_\pi)^2 \approx kxu_1 \quad (\mathbf{p}_2\uparrow\uparrow\mathbf{p}_\pi);\\ u &= (yP_1 - P_\pi)^2 \approx yt_1 \quad (\mathbf{p}_1\uparrow\downarrow\mathbf{p}_\pi). \end{aligned}\right\} \quad (42)$$

In the elementary process s_1 and t_1 are large and u_1 is small, and therefore in the nuclear reaction s and u are large and t is small; hence, we can take (40) as the cross section of the subprocess.

Thus, the invariant cross section of the $pA \to \pi(180°) + \ldots$ process in the impulse approximation is expressed as a product of the cross section (40) for interaction of the corresponding partons and the probabilities (36) and (37) of finding them in the incident nucleon and the flucton with the given momenta and the probability (13) for the existence of a flucton in the nucleus, i.e.,

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi} = \sum_{k,i} \beta_k^A \int dx\, dy G_{\pi/k}(x) G_{i/N}(y) \frac{1}{\pi} F(t) s\delta(s+t+u). \quad (43)$$

We integrate (43) over dy, using the δ function:

$$\delta(s+t+u) = \frac{1}{ks_1} \frac{\delta(y - y(x))}{x - x_\Delta}, \quad (44)$$

where

$$y(x) = x\Delta/(x - x_\Delta); \quad (45)$$

$$x_\Delta = x_k(1-\Delta); \quad x_k = \frac{|t_1|}{ks_1}\frac{1}{1-\Delta}; \quad \Delta = \frac{|u_1|}{s_1}. \quad (46)$$

Then

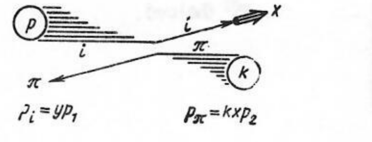


FIG. 8. Main class of diagrams of the elementary subprocess $pk \to \pi + \ldots$.

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi} = \sum_{k,\,i} \beta_k^A \int_{x_k^\pi}^{1} dx\,(1+\delta\gamma_{\pi/k})\,\varphi_k^\pi(x,\ i)\,(1-x)^{\delta\gamma_{\pi/k}}, \tag{47}$$

where

$$\varphi_k^\pi(x,\ i) = \frac{1}{\pi}\,\frac{x^2\Delta}{(x-x_\Delta)^2}\,F(kxu_1)\,G_{i/N}(y(x))\,A_k^\pi(x)\,(1+\delta\gamma_{\pi/k})^{-1}. \tag{48}$$

This function increases rapidly from zero to an almost constant value in a small range of variation of x around x_k. At the same time, the function $(1-x)^{\delta\gamma_{\pi/k}}$ decreases rapidly with increasing x. Since $\Delta \ll 1$, one can show that the main contribution to the integral is made by the region around the point $x \approx x_k(1+a)$, where

$$a \approx 3\,\frac{1-x_k}{x_k(\gamma_{\pi/k}+\gamma_0)};\quad \gamma_0 = 3 + \frac{(\gamma_{\pi/k}+2)\ln 2}{\ln[1-x_k\Delta/(1-x_k)]}. \tag{49}$$

Then, taking out the function $\varphi_k^\pi(x)$ in (47) at the point $\tilde{x} \approx x_k(a \ll 1$, since $\Delta \ll 1)$ and integrating the remainder, we find

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi} = \sum_{k,\,i} \beta_k^A \varphi_k^\pi(x_k,\ i)\,(1-x_k)^{1+\delta\gamma_{\pi/k}}. \tag{50}$$

Hence, for $k=1$ we obtain the cross section of the elementary reaction $pp \to \pi(180°) + \ldots (\beta_1^A \Rightarrow 1)$:

$$E_\pi \frac{d\sigma^N}{d\mathbf{p}_\pi} = \sum_i \varphi_1^\pi(x_1,\ i)\,(1-x_1)^{1+\delta\gamma_{\pi/N}}. \tag{51}$$

And since $\gamma_{\pi/k} = \gamma_{\pi/N} + 6(k-1)$, we write finally, instead of (50),

$$E_\pi \frac{d\sigma}{d\mathbf{p}_\pi} \approx C \sum_k \beta_k^A (1-x_k)^{6(k-1)\delta}\, E_\pi \frac{d\sigma^N}{d\mathbf{p}_\pi}(pp \to \pi(180°) + \ldots,\ x_k), \tag{52}$$

where indeed the constant $C \approx \sum_i \varphi_k^\pi(x_k i)/\sum_i \varphi_1^\pi(x_k,\ i)$ depends weakly on x_k and k $(C \sim [\gamma_{\pi/N} + 6(k-1)]^{-1})$.

The results of calculations[22,23] of the elementary cross section and the cumulative nuclear reaction on the ^{12}C nucleus[1)] are given in Figs. 9 and 10. It can be seen that at high energy (300 GeV) $\delta \approx 1$, i.e., the asymptotic conditions are satisfied for the choice of (36) and (37). At energies around 10 GeV, it is necessary to take the parameter value $\delta = 0.5$. The calculation of the cumulative reaction ($p_0 = 8.4$ GeV/c) also agrees with experiment at $\delta = 0.5$.

Thus:

1. The expression (52) obtained for the cross section of the cumulative reaction is close to the phenomenological expression (14). It really is proportional to the elementary cross section with the substitution $x \to x_k = x/k$ made in it (the cumulative effect!).

2. The cross section includes an additional "quenching factor" $(1-x_k)^{6(k-1)\delta}$, which depends on the number of nucleons forming the flucton.

3. As is shown by comparison of the theory and experiments for a group of nuclei, the parameter β_k^A is stabilized because of this factor. It now turns out that

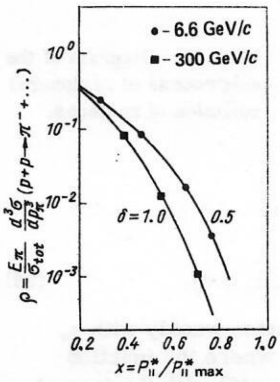


FIG. 9. Fitting of the cross section of the $pp \to \pi^- + \ldots$ reaction by means of the parameter δ (ρ in relative units).

for all nuclei and k-fluctons the correlation range is the same and is equal to $r_\xi = 0.75$ F (when the effective radius of the nucleon in the nucleus is taken to be $r_0 = 1.2$ F).

4. The parameter δ, which takes into account the deviation of the momentum distributions of the partons of the colliding particles from the asymptotic distributions (36) and (37), is 0.5 at energies around 10 GeV and increases to the value $\delta = 1$ at $E \approx 300$ GeV.

9. CUMULATIVE $pA \to p(180°) + \ldots$ REACTION

The main contribution to the cross section is made by the following subprocesses:

a) peripheral (with small momentum transfer) collision of a nucleon in a flucton with the incident nucleon;

b) fragmentation of the partons of the flucton with emission of a proton.

Subprocess a) is a special case of the fragmentation subprocesses b), but for methodological reasons we shall find it convenient to consider it separately; moreover, it is a) that makes the main contribution. The corresponding kinematic variables in the relativistic limit have the form

$$\left.\begin{array}{ll} s \approx kxs_1 & \text{is large,} \\ t \approx kxu_1 & \text{is small,} \\ u \approx t_1 & \text{is large.} \end{array}\right\} \tag{53}$$

The diagram of process a) (Fig. 11) is treated in the same way as in Sec. 8 for the $pA \to \pi(180°) + \ldots$ reaction. The results are

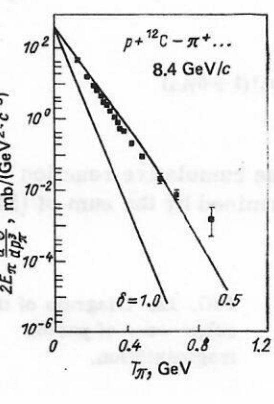


FIG. 10. Comparison of calculations of reaction on the basis of the microscopic approach with the experimental data of Ref. 1.

1)In the calculation of Refs. 22 and 23, a method of direct summation of the principal diagrams equivalent to the method presented here was used and corrections $m/E \ll 1$ were taken into account.

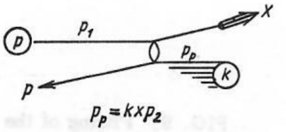


FIG. 11. Diagram of the subprocess of peripheral collision of nucleons.

$$E_p \frac{d\sigma}{d\mathbf{p}_p} \approx C \sum_i \beta_k^A (1-x_k)^{6(k-1)\delta} \times E_p \frac{d\sigma^N}{d\mathbf{p}_p} \Big(pp \to p + \ldots;\ x_k = \frac{x}{k};\ x_\perp \approx 0 \Big). \qquad (54)$$

Here, we have a constant that varies weakly with x_k and k, namely $C \approx \varphi_k^p(x_k)/\varphi_1^p(x_1)$, where the function $\varphi_k^p = \varphi_k^p(x, p)$ has the same form as (48) with the formal substitution $\gamma_{p/k} = \gamma_{p/N} + 6(k-1)$. It should be noted that at the same time the elementary $pp \to p + \ldots$ cross section has the form

$$E_p \frac{d\sigma}{d\mathbf{p}_p} \approx \varphi_1^p(x_1)(1-x_1)^{1+\delta\gamma_{I/N}}. \qquad (55)$$

Indeed, this cross section does not exhaust everything that is observed experimentally, and it must be augmented by other diagrams.

We now consider diagrams of type b) (Fig. 12). The corresponding cross section is

$$E_p \frac{d\sigma}{d\mathbf{p}_p} = \sum_{k,\, i} \beta_k^A \int dx G_{i/k}(x) E_p \frac{d\sigma^i}{d\mathbf{p}_p} \quad (ip \to p + \ldots;\ x'_k), \qquad (56)$$

where $x'_k = -t/s = |t_1|/kxs = (x_1/kx)$ is the fraction of the momentum transfer at the pi scattering vertex. It is assumed that the corresponding elementary cross section of the reaction on a parton has the form

$$E_p \frac{d\sigma^i}{d\mathbf{p}_p} \approx \Theta(1-x') f_i(x'), \qquad (57)$$

where $f_i(x')$ is a weak function of x'. Then

$$E_p \frac{d\sigma}{d\mathbf{p}_p} = \sum_{k,\, i} \beta_k^A \int_{x_1/k}^{1} G_{i/k} f_i\Big(\frac{x_1}{kx}\Big) \approx \sum_{k,\, i} \beta_k^N \frac{1}{1+\gamma_{i/k}\delta} A_k^i(x_k)(1-x_k)^{1+\delta\gamma_{i/k}} f_i\Big(\frac{x_1}{k}\Big). \qquad (58)$$

Or, introducing again the elementary cross section ($\beta_1^A \to 1$)

$$E_p \frac{d\sigma^N}{d\mathbf{p}_p} \approx \sum_i A_1^i(x_1) \frac{f_i(x_1)}{1+\delta\gamma_{i/N}} (1-x_1)^{1+\delta\gamma_{i/N}}, \qquad (59)$$

we find

$$E_p \frac{d\sigma}{d\mathbf{p}_p} \approx C' \sum_k \beta_k^A (1-x_k)^{6(k-1)\delta} E_p \frac{d\sigma^N}{d\mathbf{p}_p} (pp \to p + \ldots;\ x_k = x/k), \qquad (60)$$

where

$$C' \approx \sum_i A_k^i(x_k)(1+\delta\gamma_{i/N}) \Big/ \sum_i A_1^i(x_k)(1+\delta\gamma_{i/k})$$

is a weak function of x_k and k.

Thus, the cross section of the cumulative reaction with proton production is determined by the sum of (54) and (60). The form of each term is the same, the constants C and C' basically determining the absolute value of the cross sections, in which we are here not interested (an estimate is given in Sec. 5). They are calculated and compared with experiment in Fig. 13 (Refs. 22 and 23).

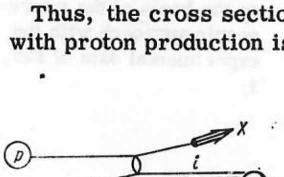


FIG. 12. Diagram of the subprocess of parton fragmentation.

Contributions to the cross section are made by the terms with $k = 2, 3, 4$; the value of β_k^A was calculated for a correlation range $r_t = 0.75$ F in the flucton. The Θ-like nature of the elementary cross section (23) is manifested in the fact that the cross section on the nucleus has a form with more prounounced relief than the reaction with the emission of pions. It is smoothed when allowance is made for the Fermi motion of the fluctons. As in the $pA \to \pi + \ldots$ reaction, we have here taken the parameter value $\delta = 0.5$. The elementary cross section of the $pp \to p + \ldots$ reaction was not analyzed for the reasons given above.

Thus, the structure of the $pA \to p + \ldots$ and $pA \to \pi + \ldots$ cross sections is the same. To describe both these reactions, we must introduce the same parameters $r_t = 0.75$ F and $\delta = 0.5$. The mechanisms of these reactions have much in common.

10. NUCLEAR REACTIONS $pA \to c + \ldots$ WITH LARGE MOMENTUM TRANSFER (THE EXPERIMENTS OF CRONIN *et al.*)

A feature of the experiments of Ref. 24 was that the measurements of the $pA \to c + \ldots$ cross sections were made: a) at high energy ($p_0 = 200$–400 GeV/c), so that we can take $\delta = 1$ in the distributions (36) and (37); b) at angles of detection of the secondary particles—partons, kaons, and antiprotons—in the C system of the NN collision $\theta^C = 90°(\mathbf{p}_c^* \perp \mathbf{p}_1)$, or $\theta^{\text{lab}} = 5.6°$; c) in a region kinematically accessible for the elementary reaction $p \to c + \ldots$, i.e., the contribution of the following orders in the cumulative effect—the reactions on fluctons with $k = 2$ and more—can be regarded as an admixture. Measurements were made of the dependence of the cross sections on the perpendicular component of the momentum or on $x_\perp = (|u| + |t|)/s$. To such processes there correspond diagrams of hard collisions of the type shown in Fig. 14.

The corresponding cross section has the form

$$E_c \frac{d\sigma}{d\mathbf{p}_c} = \sum_{kij} \beta_k^A \frac{1}{\pi} \int dx\, dy G_{i/N}(y) G_{j/k}(x) \frac{d\sigma}{dt} s\delta(s+t+u). \qquad (61)$$

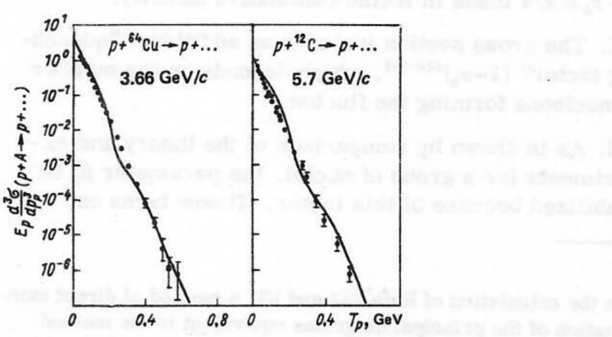


FIG. 13. Comparison with the experiment of Ref. 6. of calculations of the $pA \to p + \ldots$ reaction.

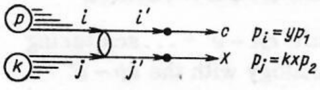


FIG. 14. Diagram of hard parton collisions in the reaction $pk \to c+$.

The invariant variables in the relativistic limit, expressed in terms of the momenta of the pp collision in the C system, have the form

$$\left.\begin{aligned} s&=(P_i^*+P_j^*)^2\approx 4kxyp_1^*p_2^*=kxys_1 \quad (\mathbf{p}_i^*\uparrow\downarrow\mathbf{p}_j^*);\\ t&=(P_i^*-P_c^*)^2\approx -2yp_1^*p_c^*=yt_1 \quad (\mathbf{p}_c^*\perp\mathbf{p}_i^*);\\ u&=(P_j^*-P_c^*)^2\approx -2kxp_2^*p_c^*=kxu_1 \quad (\mathbf{p}_j^*\perp\mathbf{p}_c^*).\end{aligned}\right\} \tag{62}$$

Substituting the expressions (36), (37), and (44) in (61), and integrating over dy, we obtain

$$E_c\frac{d\sigma}{d\mathbf{p}_c}\approx\sum_{hij}\beta_h^A\frac{1}{\pi}\int_{x_h}^1 dx A_N^i(y(x))A_h^i(x)(1-x)^{\gamma_{j/h}} \times(1-y(x))^{\gamma_{i/N}}s(x)\frac{d\sigma^{ij}}{dt}\frac{1}{ks_1}\frac{1}{x-x_h(1-\Delta)}, \tag{63}$$

where

$$\left.\begin{aligned} &y(x)=x\Delta/[x-x_h(1-\Delta)];\quad \Delta=|u_1|/s_1=x_\perp/2;\\ &x_h=|t_1|/(ks_1-|u_1|)=(x_\perp/2k)(1-x_\perp/2)^{-1}.\end{aligned}\right\} \tag{64}$$

In contrast to Secs. 8 and 9, here s_1, t_1, and u_1 are large and $u_1\approx t_1$. Thus, in (64) we cannot ignore the dependence on u_1, and therefore the method of integration in (63) differs from that used earlier. We make the substitution

$$x=x_h+(1-x_h)z. \tag{65}$$

This gives

$$(1-x)=(1-x_h)(1-z);\qquad (1-y)=(1-\Delta)bz/(1+bz), \tag{66}$$

where[2)]

$$b=\frac{1-x_h}{x_h\Delta}=\frac{1-(k+1)x_\perp/2k}{1-x_\perp/2}=\frac{2[k(2-x_\perp)-x_\perp]}{x_\perp^2} \tag{68}$$

As a result, (63) is transformed into

$$E_c\frac{d\sigma}{d\mathbf{p}_c}=\sum_{ijh}\beta_h^A\int_0^1 dz\varphi_{N,h}^{i,j}(z)(1-\Delta)^{\gamma_{i/N}}b^{\gamma_{i/N}}(1-x_h)^{1+\gamma_{j/h}} \times\frac{d\sigma}{dt}z^{\gamma_{i/N}}(1+bz)^{-\gamma_{i/N}}(1-z)^{\gamma_{j/h}}, \tag{69}$$

where

$$\varphi_{N,h}^{i,j}(z)=\frac{1}{\pi}A_N^iA_h^j\frac{xy}{x-x_h(1-\Delta)} \tag{70}$$

is a function that depends relatively weakly on z and can therefore be taken out of the integral at the point $z\sim 1/b$ of the main contribution. The integral itself is

$$\int_0^1 dz\frac{(1-z)^{\gamma_{j/h}}z^{\gamma_{i/N}}}{(1+bz)^{\gamma_{i/N}}}\approx\frac{1}{1+\gamma_{j/h}}\frac{1}{b^{\gamma_{i/N}}}[1+O(1/b)]. \tag{71}$$

Then from (69) we obtain the final result

$$E_c\frac{d\sigma}{d\mathbf{p}_c}=\sum_{ijh}\beta_h^A\varphi_{Nh}^{ij}\frac{d\sigma^{ij}}{dt}\frac{1}{1+\gamma_{j/h}}\frac{(1-(k+1)x_\perp/2k)^{1+\gamma_{j/h}}}{(1-x_\perp/2)^{1+\gamma_{j/h}-\gamma_{i/N}}}. \tag{72}$$

The case $k=1$ corresponds to the reaction of nucleon-nucleon interaction ($\beta_k^A\to 1$):

$$E_c\frac{d\sigma^N}{d\mathbf{p}_c}=\sum_{ij}\varphi_{NN}^{ij}\frac{d\sigma^{ij}}{dt}\frac{1}{1+\gamma_{i/N}}\frac{(1-x_\perp)^{1+\gamma_{j/N}}}{(1-x_\perp/2)^{1+\gamma_{j/N}-\gamma_{i/N}}}. \tag{73}$$

It is interesting that in the limit $x_\perp\to 1$ we obtain $b\ll 1$ from (68), so that (69) does not depend on b. Then from (69) we obtain a result that generalizes (50):

$$E_c\frac{d\sigma}{d\mathbf{p}_c}\approx\sum_{ijk}\beta_k^A\tilde\varphi_{Nk}^{ij}\frac{d\sigma^{ij}}{dt}(1-x_k)^{\gamma_{i/N}+\gamma_{j/k}+1} =\sum_{ijk}\beta_k^A\tilde\varphi_{Nk}^{ij}\frac{d\sigma^{ij}}{dt}\frac{[1-(k+1)x_\perp/2k]^{\gamma_{i/N}+\gamma_{j/k}+1}}{(1-x_\perp/2)^{\gamma_{j/N}+\gamma_{i/k}+1}}. \tag{74}$$

From (74) the elementary cross section follows in the form

$$E_c\frac{d\sigma^N}{d\mathbf{p}_c}=\sum_{ij}\tilde\varphi_{N1}^{ij}\frac{d\sigma^{ij}}{dt}\left(\frac{1-x_\perp}{1-x_\perp/2}\right)^{\gamma_{i/N}+\gamma_{j/N}+1}, \tag{75}$$

which after substitution of (41) as $x_\perp\to 1$, when the variation of the denominator can be ignored, is equal to

$$E_c\frac{d\sigma^N}{d\mathbf{p}_c}\sim\sum_{ij}(1-x_\perp)^{\gamma_{i/N}+\gamma_{j/N}+1}p^{-2(n_{act}-2)}. \tag{76}$$

This expression was compared in Ref. 25 with experiment for inclusive reactions of production of π, K, and other particles in the elementary reaction, the powers $\gamma_{i/N}$ being chosen in accordance with the quark counting rules. For K mesons it was found that the main contribution is made by the channel $q\bar q\to K\bar K$, for which in accordance with what we have said in Sec. 7 we obtain $\gamma_{q/N}+\gamma_{qs/N}+1=3+7+1=11$, $n_{act}=6$. It can be seen from Fig. 15 that the obtained agreement is very good, and this gives hope of success in calculations in accordance with (72) for a nuclear reaction in which the elementary subprocesses are taken to be the same for the fluctons as for the nucleons.

As we have already said, in the experiments of Ref. 24 the nuclear reaction was studied within the range of the kinematics of the NN collision (specifically, $x_\perp=0.6$). Thus, if fluctons do contribute to the reaction, then only in the nearest order with $k=2$ (of deuteron type). Assuming that the main contribution to the nuclear cross section is made by the same subprocesses as in the elementary reaction, where, however, quarks belonging to fluctons occur as valence quarks, we obtain the possibility of separating out the elementary cross section (73) from the nuclear cross section (74):

$$E_c\frac{d\sigma^A}{d\mathbf{p}_c}\approx AE_c\frac{d\sigma^N}{d\mathbf{p}_c}[1+\zeta]=A\frac{d\sigma^N}{d\mathbf{p}_c}A^n, \tag{77}$$

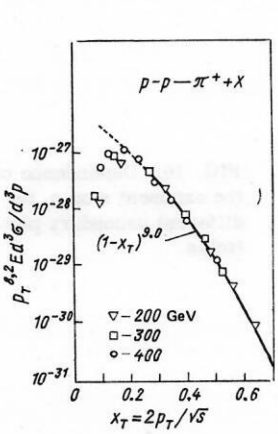


FIG. 15. Comparison of predictions of the quark-parton model $E_c d\sigma/d\mathbf{p}_c = c\,(1-x_\perp)^F p_\perp^{-n}$ with the experimental data for the $pp\to\pi+\dots$ reaction.

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2)*Editor's Note:* There is no equation (67) in the original Russian text.

where we have introduced an "effective" exponent n in the A dependence of the cross section:

$$n=1+\ln(1+\zeta)/\ln A, \tag{78}$$

and for $i,j=\pi(K)$, q we have

$$\zeta_{\pi(K)}=\frac{\beta_2^A}{A}\sum_{ij}\left\{\frac{(1-3x_\perp/4)^{\gamma_{i/2}+1}}{(1+\gamma_{i/2})(1-x_\perp/2)^{\gamma_{i/2}-\gamma_{j/N}+1}}\right\}$$
$$\times\left\{\sum_{ij}\frac{(1-x_\perp)^{\gamma_{i/N}+1}}{(1+\gamma_{i/N})(1-x_\perp/2)^{\gamma_{i/N}-\gamma_{j/N}-1}}\right\}^{-1}. \tag{79}$$

Substituting here $x_\perp=0.6$ and $\beta_2^A=0.125$, we obtain for nuclei with $A=10$ and 184 the corresponding values $n(\pi)=1.2$–1.09 and $n(K)=1.4$–1.18. It can be seen that for the given case the value averaged over A, namely $n(\pi)\approx 1.15$, is smaller for the reaction with pion production than for the reaction with kaon production: $n(K)\approx 1.3$. This corresponds qualitatively to the experiment of Ref. 24 (Fig. 16). As the kinematic boundary $x_\perp=1$ of the reaction on a free nucleon is approached, the contribution to the cross section will be determined solely by the second-order flucton with $k=2$ (the admixture of the next one with $k=3$ is still negligibly small), and then we must have $n\to 1$. With increasing x, there then again begins a small growth of n because of the $k=3$ correction, etc.

Note that in the region of single-nucleon kinematics we must, in analyzing the A^n dependence of the cross sections also take into account purely nuclear processes such as rescattering of the incident and secondary particles in the nucleus. Such an interpretation of the data of Ref. 24 on the inclusive production of protons was given in Ref. 26.

Thus, we draw the following conclusions:

1) In contrast to reactions with the backward emission of particles, in the reaction mechanism at $\theta<180°$ there is a change of regime; namely, the partons constituting the nucleon and flucton now undergo hard collisions with large momentum transfer.

2) In accordance with the flucton mechanism, one must expect in the A^n dependence of such cross sections small (≈20–30%) increases in the exponent $n=1$ in the region of the comparable contribution from fluctons with number k of nucleons differing by unity.

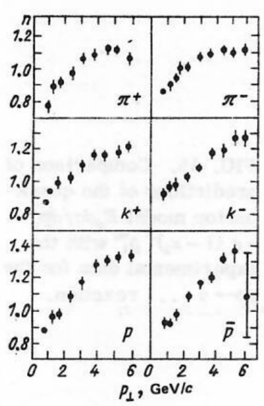

FIG. 16. Dependence of the exponent n on $p_\perp$ for different secondary particles.

11. DEEP INELASTIC eD AND eA SCATTERING

We shall treat deep inelastic $eA\to e'+\ldots$ scattering of electrons on nuclei[18] by analogy with the $ep\to e'+\ldots$ elementary process. As for nucleons, in our case the principal assumption is that the nucleon, and here the flucton, can be represented as a collection of noninteraction partons, these being distributed over the momenta in accordance with (36) and (37).

Then the main elementary subprocess is scattering of the electron by a quark of species i in a flucton with large momentum transfer $q^2=4E_eE_{e'}\sin^2(\theta/2)=-Q^2$. The cross section of this process has the form[21,22]

$$\frac{d\sigma_i}{dQ^2\,d\nu}=e_i^2\frac{d\sigma_M}{dQ^2}\frac{x_k^i}{\nu}\delta\left(\frac{x}{k}-x_k^i\right). \tag{80}$$

Here, $x_k^i=p_i/kp$ is the fraction of the quark momentum in the flucton consisting of k nucleons, p is the nucleon momentum, $x=Q^2/2M\nu$; $\nu=E_e-E_{e'}$ is the energy loss, and $d\sigma_M/dQ^2=4\pi\alpha^2/Q^4$ is the Mott cross section. Using the momentum distribution function $G_{q/k}^{(i)}$ of the partons, we write down the cross section of the deep inelastic process on the nucleus in the form

$$\frac{d\sigma^A}{dQ^2\,d\nu}=\sum_{k,\,i}\beta_k^A B_k\int dx_k^i\frac{d\sigma_i}{dQ^2\,d\nu}G_{q/k}^{(i)}(x_k^i)$$
$$=\frac{d\sigma_M}{dQ^2}\sum_{k=1}^{A}\beta_k^A B_k\frac{x}{k\nu}\sum_{i=1}^{3}G_{q_i/k}^{(i)}\left(\frac{x}{k}\right)e_i^2, \tag{81}$$

where B_k is determined by Eq. (35). Here, we have ignored the momentum distribution of the fluctons themselves, which is justified if the momentum transfers are significantly larger than the characteristic momentum of the Fermi motion of the flucton within the nucleus. The same cross section can be expressed in terms of the nuclear structure functions

$$\frac{d\sigma^A}{dQ^2\,d\nu}=\frac{d\sigma_M}{dQ^2}\frac{1}{\nu}[\nu W_{2A}(Q^2,\nu)$$
$$+2\nu W_{1A}(Q^2,\nu)\,\mathrm{tg}^2(\theta/2)](E_{e'}/E_e)\cos^2(\theta/2) \tag{82}$$

(at small scattering angles, the last term is negligibly small), in which

$$\nu W_{2A}(Q^2,\nu)=\sum_{k=1}^{A}\beta_k^A B_k\frac{x}{k}\sum_{i=1}^{3}e_i^2G_{q/k}^{(i)}(x/k). \tag{83}$$

For the following calculations, we use the explicit form of the quark momentum distribution function $G_{q/k}^{i}(x)$ in the form

$$G_{q/k}^{(i)}(x)=\frac{1}{x}a_ik^2(1-x)^{6k-3};\quad x\to 1, \tag{84}$$

in which the parameter a_i, determined from the data on elastic ep scattering, has been separated; k is the number of nucleons in the flucton. We recall that the use of $G_{q/k}^{(i)}$ in the form (84) is sensible only in the asymptotic region when $x\to 1$. On the transition to the region of intermediate values of x, it requires modification.

Note that for deep inelastic scattering on nuclei one has the characteristic possibility of going beyond the kinematic limits in the elementary $ep\to e'+\ldots$ process with respect to the variable x of the parton distribution in the nucleon, which is typical for all reactions of cumulative type.

We first consider the simplest case of deep inelastic scattering on a deuteron. Using the isotopic relation

$$\nu W_{2n} \approx (2/3)\,\nu W_{2p}, \tag{85}$$

which holds to within 30%, which is sufficient for a semiquantitative calculation, we obtain

$$\nu W_{2D} = \frac{5}{3}\,\nu W_{2p} + \frac{x}{2}\,\beta_d \sum_{i=1}^{3} e_i^2 G_{q/2}^{(i)}\left(\frac{x}{2}\right). \tag{86}$$

Further, assuming that the distribution of the quarks in the $k=2$ flucton, the deuteron, does not depend on the species i of the partons, we obtain

$$\nu W_{2D} = \frac{5}{3}\,\nu W_{2p} + \frac{5}{3}\,\beta_d \frac{x}{2}\, G_{q/2}\left(\frac{x}{2}\right). \tag{87}$$

Calculations of the structure function νW_{2D} in accordance with Eq. (87) and the experimental data of Ref. 28, in which, in view of the finiteness of Q^2 and ν, the scaling variable was replaced by x' (Refs. 20 and 27),

$$x \to x' \equiv (\omega')^{-1} = Q^2/(2M\nu + M^2), \tag{88}$$

are compared in Fig. 17. The parametrization of the experimental data for νW_{2p} from Ref. 29 was used in the calculation:

$$\nu W_{2p}(x') = \sum_{j=1}^{3} b_j (1 - x')^{j+2}, \tag{89}$$

where $b_1 = 1.274$, $b_2 = 0.599$, and $b_3 = -1.675$. It can be seen that in the region $\omega' \gg 1$ the main contribution to νW_{2d} is made by the first term, while in the region $\omega' \lesssim 1$ it is made by the second. Agreement with experiment is obtained for the value $a\beta_d = 0.025$. From the normalization conditions (84) for the proton and deuteron under the assumption that the main contribution to (89) is made by the first term, we can obtain the estimate $a = 1.2$. From this we find $\beta_d \approx 2\%$. The order of magnitude of this quantity is the same as for the one obtained in the analysis of *ed* scattering by means of the form factor F_2^I, which gives a good parameterization of experiment in the region of large q^2. Since the comparison in Fig. 17 for $\omega' \lesssim 1$ also corresponds to the asymptotic region $x \to 1$, which justifies the choice of $G_{q/k}^{(i)}$ in the form (84), the agreement for the β_d values obtained in these different experiments can be regarded as not fortuitous.

In Ref. 30, a different approach is proposed for the calculation of deep inelastic scattering, the collective effect being taken into account by a modification of the nonrelativistic wave function of the deuteron in the region of large q^2. Evidently, this is simply a different interpretation of one and the same mechanism. The difficulties of such an approach arise in the description of deep inelastic eA scattering with $x \gg 1$, when one must introduce phenomenologically specific many-particle residual nuclear interactions, justify them, and calculate with them the corresponding momentum distributions.

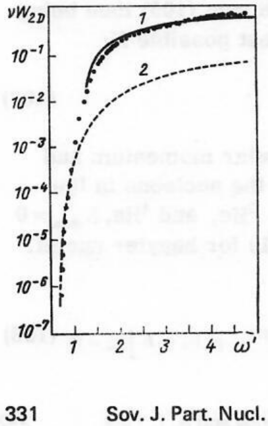

FIG. 17. Contribution to the structure function of the deuteron from deep inelastic scattering on an individual proton and individual neutron (1) and on the deuteron as a whole (2).

For an understanding of the qualitative behavior of νW_{2A} when $x \gg 1$ in reactions of deep inelastic scattering of electrons by nuclei we used the fact that terms with $k > x$ contribute to the sum (83) (since $G_{q/k}(x/k) = 0$ for $x/k > 1$). Then, noting that at large x the term with $\bar{k} \approx x + 1$ makes the main contribution for each given value, and using the approximate formulas

$$k! \approx \sqrt{2\pi}\exp[(k + 1/2)\ln k - k]; \tag{90}$$

$$\beta_k^A \approx \frac{A}{k!}\left(\frac{V_\xi}{V_0}\right)^{k-1} = \frac{A}{k!}\beta_0^{k-1} \approx \frac{\sqrt{k}A}{\sqrt{2\pi}\,\beta_0}\exp[-k(\ln k - 1 - |\ln\beta_0|)]; \tag{91}$$

$$(1 - x/\bar{k})^{6(\bar{k}-1)+\gamma} \approx (1/x)^{6x+\gamma}\exp(-6), \tag{92}$$

we obtain for the structure function the expression

$$\nu W_2(x) \approx \exp(-ax), \tag{93}$$

where the parameter a does not depend on the atomic number of the nucleus and increases logarithmically with increasing x:

$$a = 7\ln x + |\ln\beta_0| + (\gamma + 1)\ln x/x - 1. \tag{94}$$

From this there follows the main result that in reactions of deep inelastic scattering of electrons by nuclei a strong decrease of the structure function must be observed for $x \gg 1$.

Thus, the interpretation of deep inelastic *ed* scattering on the basis of the idea of fluctons gives a basis for predicting the cross section of deep inelastic scattering by nuclei; in particular, it predicts the nature of the decrease of its structure function.

12. PROBABILITY OF EXISTENCE OF FLUCTONS IN NUCLEI

As we have already noted, the idea of fluctons in nuclei arose originally as the idea of compression fluctuations of nuclear matter in small volumes, and their probability was estimated on the basis of the classical theory of the fluctuations of an ideal gas. It can now be seen that to explain the main features of the cumulative effect and the data on the elastic and deep inelastic scattering of electrons we require a deeper concept of a flucton, and we assume that 1) the reaction takes place on it as on an object in which the nucleons lose their individuality; 2) the corresponding cross sections and form factors must be calculated on the basis of quark–parton ideas about the flucton structure. As yet, the probabilities β_k^A have been used essentially as parameters. Use of these quantities to analyze the experiments has shown that the correlation range r_ξ in a flucton is everywhere constant and equal to $r_\xi = 0.75$ F, a quantity that is of the order of the hard-core radius of the *NN* forces. This suggests that there is a deep connection between the hard core as a phenomenological concept and the microscopic structure of two or more nucleons at small separation, i.e., what is now called a flucton or a quark bag in nuclei. We now calculate the probabilities β_k^A on a more realistic basis[31] than the simple model of an ideal Fermi gas. For this, we exploit the main hypothesis that the nucleons of the nucleus lose their individuality in the correlation region $r<r_\xi$ and go over into a new state—the many-quark object. This last can be calculated on the basis of one of the models of quantum chromodynamics.

Thus, we redefine β_k^A as follows:

$$\beta_k^A=b_k^A D_k, \tag{95}$$

where b_k^A is the probability of finding in nucleus A an ordinary, uncompressed cluster of k nucleons, and D_k is the probability of finding this cluster in a state of fluctuation compression. Essentially, D_k is the probability of a phase transition of the k nucleons into the state of a $3k$-quark object. We can calculate b_k^A by the usual methods of nuclear physics, and we shall not dwell on this here. The quantity D_k can be determined as an integral over the volume of the flucton:

$$D_k=\int_{V_\xi}|\psi(1\ldots k)|^2\,dV, \tag{96}$$

where ψ is the wave function determining the motion of the centers of gravity of the k nucleons in their center-of-mass system. Thus, the problem consists of finding the function ψ in a region of space of the order of the core diameter, and this requires a solution of what is here the main problem—the calculation of the many-particle repulsive nuclear potential at short distances. By its meaning, this potential is the difference between the energies of the $3k$-quark hadron bag (the true state of the hadronic matter at short distances) and the mass of the k-nucleon cluster (its state at distances outside the core):

$$V_k^c=E(3k)-km_Nc^2. \tag{97}$$

We begin with the equation for the simplest case of two-baryon states:

$$\left[\frac{d^2}{d\rho^2}-\frac{2\mu}{\hbar^2}\left(\tilde V+\Theta(R_c-\rho)V_0-\varepsilon\right)\right]\chi(\rho)=0,\quad \chi=\rho\psi. \tag{98}$$

Here, ρ is the distance between the nucleons, $\tilde V$ is the attractive potential due to meson exchange, V_2^c is the repulsive barrier, and R_c is the radius of the hard core. Following Ref. 32, we find the height of the barrier V_2^c as the difference between the energy of the 6-quark hadronic "deuteron-like" bag and the deuteron mass:

$$V_2^c=E(6)-2m_Nc^2. \tag{99}$$

To calculate $E(3k)$, we use the model of a spherical hadronic bag, the MIT bag,[33] in which the mass of the $3k$-quark system is determined by the expression

$$E(3k)=E_v+E_0+E_Q+E_M, \tag{100}$$

where E_v is the energy of the external pressure that prevents the quarks leaving the bag, E_0 is the "zero-point" energy of the quark field, E_Q is the contribution of the free energy and the kinetic energy of the quarks, and E_M is the quark interaction energy. The calculation of Refs. 12 and 5 for a six-quark system gives $V_2^c(6)=0.27$ GeV. The solution of Eq. (98) has the form

$$\chi(\rho)=\begin{cases}\chi_1(\rho)=c_1\,\mathrm{sh}(p\rho), & \rho<R_c,\quad p=\sqrt{2\mu V_2^c/\hbar};\\ \chi_2(\rho)=c_2\exp(-a\rho), & \rho\geqslant R_c,\end{cases} \tag{101}$$

where $\chi_2(\rho)$ is the "ordinary" deuteron wave function, which determines its average characteristics, i.e., the diameter, binding energy, etc., so that the parameter a is fixed: $a=1/R_d$ and $R_d=1.7$ F. The coefficients c_1 and c_2 are determined from the condition of fitting of χ_1 and χ_2 at the point $\rho=R_c$ and by the normalization. Finally, we obtain

$$D_2=\int_0^{R_c}d\rho\,|\chi_1(\rho)|^2=\frac{aR_c}{2}\left[\frac{\mathrm{ch}\,(2x)}{2x}-1\right]\frac{\exp(-2aR_c)}{\mathrm{sh}^2\,x};\quad x=pR_c. \tag{102}$$

Substituting here $R_c=0.5$ F, we find $D_2\approx 8\times10^{-2}$.

The main difficulty in calculating D_k for a large number of nucleons is that Eq. (98) becomes multidimensional and the variables in it do not separate. Here, however, we can use a special choice of the collective variable ρ corresponding to the transition to the flucton state:

$$\rho^2=(1/k)\sum_{i>j}(\mathbf r_i-\mathbf r_j)^2. \tag{103}$$

The remaining $3k-4$ variables are hyperangles in a $3(k-1)$-dimensional space. It follows from the definition (103) that $\langle\rho^2\rangle$ is related to the mean-square radius $\langle R_k^2\rangle$ of the system and the mean-square distance $\langle r^2\rangle$ between the nucleons:

$$\langle\rho^2\rangle=k\langle R_k^2\rangle=(k-1)\langle r^2\rangle. \tag{104}$$

We seek the wave function of the k-nucleon cluster in the form

$$\psi(1\ldots k)=\rho^{(3k-4)/2}\sum_{K\gamma}\chi_{K\gamma}(\rho)\,U_{K\gamma}(\Omega_\rho,\Omega_{s\tau}). \tag{105}$$

Here, $U_{K\gamma}$ depend on the angles in the coordinate and spin–isospin space and ensure antisymmetrization of the total wave function.[34]

$$\left[\frac{d^2}{d\rho^2}-\frac{L_K(L_K+1)}{\rho^2}-\frac{2m}{\hbar^2}(E+V_{K\gamma}^{K\gamma})\right]\chi_{K\gamma}=\Sigma' V_{K\gamma}^{K'\gamma'}\chi_{K'\gamma'}, \tag{106}$$

where $L_K=K+(3/2)(k-2)$; $V_{K\gamma}^{K'\gamma'}$ are the matrix elements of the interaction potentials. We are interested in the region of small ρ, in which the "centrifugal" potential $L(L+1)/\rho^2$ is dominant; therefore, in the method of K harmonics it is sufficient to take only the zeroth iteration with respect to the nondiagonal matrix elements $V_{K\gamma}^{K'\gamma'}$, the main contribution to the sum (105) then being given by the term with the smallest possible K:

$$K_{\min}=\sum_{i=1}^{k}(l_i+2n_i), \tag{107}$$

where l and n are the orbital angular momentum and the principal quantum number of the nucleons in the cluster. For the light nuclei ^{3}H, ^{3}He, and ^{4}He, $K_{\min}=0$ and this quantity increases rapidly for heavier nuclei. Finally, we obtain

$$\left[\frac{d^2}{d\rho^2}-\frac{L(L+1)}{\rho^2}-\frac{2m}{\hbar^2}\tilde V(\rho)+\Theta\left(R_c(k-1)^{1/2}-\rho\right)V_k^c-E\right]\chi_k=0. \tag{108}$$

As for the deuteron, we find the size of the core in the k-nucleon system by means of the relation (97). The corresponding values of V_k^c, calculated in the MIT bag model,[33] are given in Table II, and also in Fig. 18. We choose the nuclear part of the potential $\tilde{V}(\rho)$ in the form of a harmonic oscillator. We seek the solutions of Eqs. (108) in two regions:

$$\chi(\rho)=\begin{cases} c_1\sqrt{p\rho}\,I_{L+1/2}(p\rho), & \rho<\rho_c, \quad p=\sqrt{2mV_k^c/\hbar^2};\\ c_2\rho^L\exp\left(-(\rho/R_k)^2/2\right), & \end{cases} \qquad (109)$$

where $\rho_c=R_c(k=1)^{1/2}$, R_c is the radius of the nucleon cluster, and $I_\lambda(x)$ are Bessel functions of imaginary argument.

Using (96) and (109) and the normalization condition, we find for the probability

$$D_k=\int_0^{\rho_c} d\rho\,|\chi|^2=\frac{(\rho_c/R_k)^{2L+1}}{\Gamma(L+1/2)}\left(1-d^2+d\,\frac{2L+1}{p\rho_c}\right);$$
$$d=I_{L-1/2}(p\rho_c)/I_{L+1/2}(p\rho_c). \qquad (110)$$

It follows that $D_k\sim(R_c/R_0)^{3k}$, i.e., this corresponds to the phenomenological formula (13), although the meaning of this expression is much deeper. The results of calculations of the probabilities of many-baryon configurations in accordance with (110) are given in Fig. 19; throughout, $R_c=0.5$ F. It can be seen that D_k decreases strongly with increasing k. The inflection in D_k on the transition from $k=4$ (^{4}He) to $k=5$ (^{5}Li) is due to the increase of $K_{\min}$ from zero to unity, this being associated with the start of filling of a new shell with $l=1$ on the transition from ^{4}He (^{5}Li), and it forbids the simultaneous presence of all five nucleons in a small volume on account of the uncertainty principle. The probability of the two-baryon system in the deuteron, 8–9%, agrees well with the data on elastic scattering (see Sec. 6). The calculated values of D_k are of a predictive nature. However, their order of magnitude can be compared with the corresponding result of analysis of the cumulative production of particles in $pA\to c+\ldots$ reactions. The value of $D_k^{\exp}$ is given by

$$D_k^{\exp}=\beta_k^A/b_k^A;\quad b_k^A=\left(\frac{A}{k}\right)\left(\frac{kV_0}{AV_0}\right)^{k-1}, \qquad (111)$$

where β_k^A is a quantity extracted from experimental data on cumulative reactions ($r_t/r_0=0.63$). It can be seen that the calculated probabilities basically agree with the corresponding experimental values.

Thus, nucleons in a region of space of the order of the size of a nucleon itself lose their individuality, and we are then concerned with many-quark objects—"quark bags in nuclei." The consequences in the field of relativistic nuclear physics have already been discussed. However, fluctons can also be manifested in nonrelativistic nuclear physics as specific many-particle correlations of nucleons of repulsive type. Allowance for correlations of this type may be helpful, in particular, in the solution of a number of problems of few-nucleon systems, for example, in the explanation of the binding energy of the lightest nuclei such as ^{3}H, ^{3}He, and ^{4}He, the description of their electric form factors in the region of the second maximum ($q\approx 1$ GeV/c), and also in some other problems which cannot yet be solved using only two-body NN forces.

TABLE II.

k	1	3	4	5
V_k^c, GeV	0.27	0.80	0.99	1.37

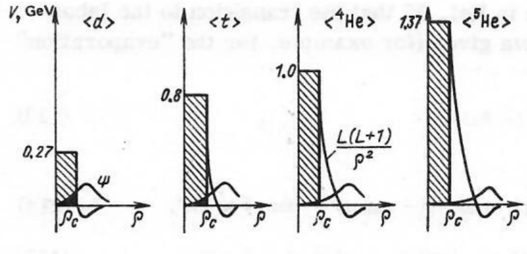

FIG. 18. Behavior of the wave function in the core.

13. OTHER APPROACHES

There have recently been published other explanations of these and other features of cumulative reactions.[35-42] We shall consider here only briefly the conceptual aspect of those of them that use notions that differ from the flucton notion. (Calculations similar to those made here were made in Ref. 35, in which the expression "correlated clusters" was used instead of the word flucton, and in Ref. 36, in which fluctuations were not introduced but it was assumed that the entire nucleus can to some extent behave as a quark object.)

In Refs. 37 and 38 there is constructed a model of a hadron cluster (fireball), this being formed in the nucleus as a single system on the first collision of the incident nucleon with one of the nucleons in the nucleus. At this time, it has mass M_0 and velocity v_0 In the next stage, as it passes through the nucleus, this fireball undergoes a number of inelastic collisions with the nuclear nucleons, as a result of which its mass increases to M_{crit} and its velocity decreases to v_{crit}. (These quantities depend on the range, the cross section of inelastic interaction in the nucleus, the atomic weight, and so forth.) Leaving the nucleus (in the third stage), the fireball has a high temperature in its associated frame and can decay (isotropically in that frame) with the emission of different particles (p,π, etc.) in accordance with the law

$$W\sim\exp(-E/\tilde{T}_0). \qquad (112)$$

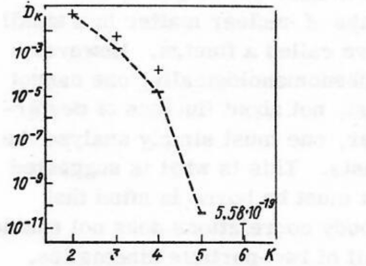

FIG. 19. Probability of existence of fluctons with k nucleons in nuclei.

It is shown in Ref. 37 that the transition to the laboratory system gives (for example, for the "evaporation" of pions)

$$W \sim \exp(-E_\pi/T_0), \tag{113}$$

where

$$T_0 = \tilde{T}_0[(1-v_{\rm crit})/(1+v_{\rm crit})]^{1/2} \quad \text{for} \quad \theta \approx 180°; \tag{114}$$

$$T_0 = \tilde{T}_0[(1+v_{\rm crit})/(1-v_{\rm crit})]^{1/2} \quad \text{for} \quad \theta \approx 0°. \tag{115}$$

Thus, the nature of the cumulative effect is attributed in such approaches to the nature and mechanism of formation and passage through the nucleus of the fireball. An attractive feature of the model is that it is also used to analyze other phenomena of nucleon–nucleus interaction, for example, to explain the effect of leading particles, to calculate multiplicities, and so forth. The difficulties relate to its statistical nature and must be manifested on the transition to light nuclei, in the analysis of correlation and polarization phenomena, and so forth.

Another approach[39] assumes that the incident hadron interacts with a group of nucleons at once in a tube. The cross section of this interaction is described in the form of the nucleon–nucleon cross section with replacement of the mass m_N of the nucleon target by the mass m_T of the tube. Thus, the cumulative effect is actually taken as the basis of the model, and the reasons for it are not discussed. The approach has proved very fruitful in the interpretation of a large body of experimental data on hadron–nucleus interactions at high energies.

At intermediate energies ($E_0 \lesssim 1$ GeV) a currently very popular approach was developed in Ref. 40, in which data on the yields of backward protons were interpreted. This is the impulse approximation, in which the wave function of the nucleon in the nucleus is parameterized. The main question is this: How can such a large momentum, capable of "knocking" the incident particle backward, arise in the nucleus? It is assumed in Ref. 40 that one of the nucleons accumulates the momentum as a result of multiple intranuclear rescattering. In other words, it is asserted that in the complex many-particle function of the nucleus a high-momentum single-particle component of this kind may be present. However, Amado and Woloshyn[41] did not succeed in obtaining it in the framework of the usual ideas about a nucleus. The model retains its validity as a felicitous parametrization of the experimental data. More realistic in such an approach would be to take into account the two-body short-range correlations of the nucleons. The nature of the hard core of the *NN* forces, as we have seen in Sec. 12, is due to the formation of a different phase of nuclear matter in a small volume V_ξ, which we have called a flucton. However, if the core is specified phenomenologically, one cannot speak of fluctons (at least, not about fluctons of deuteron type with $k=2$); rather, one must simply analyze the available data on this basis. This is what is suggested in Ref. 42. However, it must be borne in mind that allowance for only two-body correlations does not enable one to go beyond the limit of two-particle kinematics, and that is precisely where the most interesting questions arise.

CONCLUSIONS

Let us summarize the main results we have obtained by introducing the concept of fluctons in nuclei. The idea of fluctons in conjunction with parton concepts makes it possible to understand the main features of the mechanism of the hadron–nucleus and electron–nucleus interactions at high energies. This includes the interpretation of cumulative processes, and in particular the features associated with the change in the regime of the reaction mechanism on the transition from angles $\theta = 180°$ to small angles of detection of the secondary particles. At the same time, there must be manifested a correlation in the yields of cumulative particles and the accompanying recoil particles at emission angles $\varphi \approx 180°$. The same idea makes it possible to explain the large polarization of the secondary particles in cumulative reactions.[43] The main question then becomes that of the manifestation of the quark phase in nuclear matter. Fluctons are a real competitor for this part in finite nuclei. Calculations in the model of a quark bag give qualitative arguments for this. At the same time, they make it possible to relate the concept of the repulsive core in the phenomenological interaction potential of two and (which is the main thing) several nucleons at short distances to the idea of a quark structure of fluctons. A quark phase in stars has recently come under discussion.[44] All these problems form a new direction of investigation, and this is now being called quark nuclear physics.

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