Theoretical investigation of the repulsive part of the nucleon–nucleon potential

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Information about the nucleon–nucleon forces and the two-particle *NN* potential at short distances between the nucleons can be extracted from two- and many-particle scattering experiments at nonrelativistic energies. The phase shifts of nucleon–nucleon scattering up to 450 MeV in the laboratory system, the off-shell two-particle scattering amplitude, and the deuteron wave function are considered. New data on the potential at short distances are obtained, and some model potentials are constructed in the framework of meson theory, these making it possible to describe experiments involving the scattering of two or three nucleons.

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INTRODUCTION

The investigation of the two-particle nucleon–nucleon interaction and its associated potential at initial energies up to a few hundred MeV is one of the basic problems of nuclear physics. Although considerable experimental material that makes it possible to study the scattering of two or several particles at low and medium energies has by now been accumulated, there is still no unified theoretical description of the nucleon-nucleon interaction nor a general method for analyzing the experimental data. Indeed, there is a steady increase in the number of theoretical papers that discuss the results of experiments from the point of view of model or phenomenological ideas about the *NN* potential and also within the framework of the meson theory of nuclear forces.

In the field-theoretical description of the *NN* interaction at nonrelativistic energies, the quantum-mechanical potential is usually regarded as the Fourier transform with respect to the momentum transfer of the sum of all possible irreducible Feynman diagrams under the assumption that the nucleons interact through a set of boson fields. Through the work of a number of authors,[1–15] it has now become fairly clear that the peripheral and central region of the *NN* potential ($r \gtrsim 0.5$ F), which is here attractive, results basically from one- and two-pion exchanges, and also from the contributions made by the resonance parts of three-pion exchanges (ω meson, η meson, etc.). To study the inner part of the potential ($r < 0.5$ F), it is necessary to take into account the higher irreducible diagrams of perturbation theory, and considerable computational difficulties arise if this is to be done accurately.

Thus, in the existing meson models of the *NN* forces we face the problem of determining the potential at short distances between the nucleons.

It follows from the phase-shift analysis of the elastic *NN* scattering amplitude made on the basis of the available experimental data on the cross sections of *np* and *pp* interactions,[16–18] and especially from the *S*-wave phase shifts as a function of the energies of the colliding particles, that the nucleon–nucleon potential is repulsive at small r.

In the field-theoretical description of the *NN* interaction, the meson models of the nucleon–nucleon potentials can be divided into three groups, depending on the manner in which the repulsive core is introduced. In the first group, we have the models in which a local core is determined phenomenologically, for example, in the form of an "infinite wall" ($V_c = \infty$ for $0 \leq r \leq r_c$) or in some other equivalent manner.[19] In the models of the second group, a nonlocal repulsive core is specified through a separable representation of the core in momentum space.[10,20] Finally, the third group consists of potentials in which a regularization procedure is carried out on the basis of physical assumptions. For example, Green *et al.*[2–4] do not use the "bare" nucleon–meson interaction vertex in the irreducible diagrams but add a meson form factor, which decreases with increasing momentum transfer. In this case, calculation of the Fourier transform with respect to the momentum transfer of the diagrams containing single-boson exchanges does not lead to any singularities in the potential at $r = 0$. In these investigations, the meson form factor is introduced phenomenologically. Of greater interest from our point of view is the work of Jackson and Woloshyn,[13] who discussed the possibility of calculating diagrams with many-meson exchanges in the framework of the eikonal approximation. This procedure for regularizing the *NN* potential is more physical than the introduction of a phenomenological form factor, though it requires experimental confirmation. This diversity of approaches and models for describing the *NN* interaction is a clear indication that we have inadequate experimental information about this phenomenon and makes us consider new ways of extracting data on the nucleon–nucleon forces from the experiments.

In this review, we consider new information about the repulsive core of the *NN* potential that can be extracted from two- and three-nucleon scattering experiments. To this end, we consider some characteristics of the scattering of a small number of nucleons, in particular the off-shell two-particle scattering amplitude and the deuteron wave function, which are more sensitive to the form of the potential than the two-nucleon elastic phase shifts.

Methods for analyzing the off-shell behavior of the

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two-particle scattering amplitude and the deuteron wave function with a view to obtaining more exact information about the nucleon–nucleon potentials were proposed in Ref. 20.

1. NUCLEAR POTENTIAL AND NUCLEON-NUCLEON PHASE SHIFTS

Restrictions on the Heights and Radii of the Repulsive Core. The main requirement at present imposed on models of the *NN* potential is that it must describe the two-nucleon elastic phase shifts in a certain range of energies. If the partial-wave phase shift $\delta_l(E)$ were known in the energy range $0<E<\infty$ of the colliding particles, a potential $V_l(r)$ that reproduces this phase shift could be determined by the methods of the inverse problem of scattering theory.[21] However, because of the difficulties associated with determining the phase shifts from the experimental *NN* scattering cross sections, and also because of relativistic inelastic effects, the phase shifts are determined (with a certain accuracy) only in the energy region $0 \leq E_{lab} \lesssim 450$ MeV,[16] and this means that, even in the framework of meson models, one can use a large class of model potentials, which are the so-called phase-shift-equivalent (or phase-equivalent) potentials. The very existence of strongly different models of potentials that can, with a certain error, reproduce not only an individual phase shift but also a set of several phase shifts in a given energy range indicates the difficulty in determining any absolute values of the parameters of the potential function by investigating phase-shift curves. However, as we shall show in what follows, a connection between the parameters of the *NN* potentials can be established by an analysis of these curves.

To investigate the parameters that characterize the inner part of the nucleon–nucleon potential, the singlet $^1S_0$ phase shift is the most favorable, since in this case there is no centrifugal barrier masking the behavior of the potential function at short distances. In Fig. 1, for comparison, we show calculated theoretical singlet phase shifts of *S*, *P*, and *D* scattering of protons corresponding to successively increasing values of the regularizing parameter that characterizes the inner region in one of the OBE potentials of Green *et al.*[3] It can be seen that variation of this parameter has an appreciable influence on the behavior of only the *S*-wave phase shift.

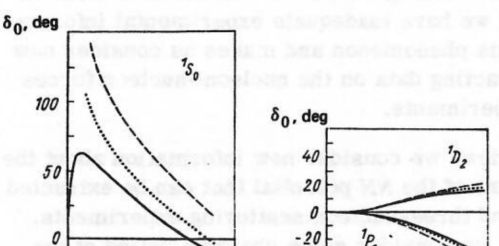

FIG. 1. Comparative change of the singlet *S*, *P*, and *D* phase shifts of *NN* scattering in the energy range $0<E_{lab}<320$ MeV in the case when one of the regularized parameters of the OBE potential of Green *et al.* takes successively increasing values.

As a tool to investigate the curve of the *S*-wave phase shift, we use the variable-phase method.[22] By means of this method, one can find many helpful restrictions on the characteristic parameters of the repulsive core (its height and radius) if the behavior of the potential is known in the peripheral and central regions.

Before we turn to the solution of the problem we have posed, we obtain an important relation that will be frequently used in what follows; namely, we consider the relation between a small change $dV(r)$ in the form of the potential and the corresponding change $d\delta_0(r,k)$ in the *S*-wave phase shift. The equation for the *S*-wave phase shift has the form[1)]

$$\partial\delta_0(r,k)/\partial r = -(V(r)/k)\sin^2[kr+\delta_0(r,k)],\quad \delta_0(0,k)=0. \tag{1}$$

Then under the assumption that the operator d of the change in the form of the potential function and the differentiation operator $\partial/\partial r$ commute, we can obtain from Eq. (1) an equation for the increment of the phase-shift function:

$$\left.\begin{aligned}&\frac{\partial}{\partial r}[d\delta_0(r,k)] = -\frac{dV(r)}{k}\sin^2[kr+\delta_0(r,k)]\\&-\frac{V(r)}{k}\sin 2[kr+\delta_0(r,k)]\,d\delta_0(r,k);\\&d\delta_0(0,k)=0,\end{aligned}\right\} \tag{2}$$

and this equation has the solution

$$d\delta_0(r,k) = -\frac{1}{k}\int_0^r \sin^2\gamma(\xi,k) \times \exp\left\{-\frac{1}{k}\int_{\xi} V(\eta)\sin 2\gamma(\eta,k)\,d\eta\right\}[dV(\xi)]\,d\xi. \tag{3}$$

For convenience, we have here introduced the notation $\gamma(r,k)=kr+\delta_0(r,k)$. The increment $d\delta_0(k)$ of the observed phase shift follows from (3) in the limit $r\to\infty$. Thus, if we consider a monotonic sequence of potential functions $V_1(r)\geq V_2(r)\geq V_3(r)\geq\ldots$, in which each potential differs from the others in at least a small region Δr, the sequence of phase-shift curves corresponding to these potentials will satisfy the system of inequalities

$$\delta_0^{(1)}(k)<\delta_0^{(2)}(k)<\delta_0^{(3)}(k)<\ldots, \tag{4}$$

which follows directly from Eq. (3).

We now turn to the study of the $^1S_0$ phase shift of *NN* scattering and attempt to determine the boundary of the repulsive part of the *NN* interaction. To this end, we consider Eq. (1) at the energy $k_0^2>0$ of the colliding particles at which the $^1S_0$ phase shift passes through zero ($k_0^2=3.4$ F^{-2}), since it is precisely the presence of the repulsive core in the nuclear potential that leads to a change in the sign of the phase-shift curve; this follows, in particular, from Eqs. (3) and (4). However, we must first define what we mean by the core radius r_c.

[1)]We measure the energy in units of F^{-2}. The transition from MeV to F^{-2} is made by means of the relation $(k^2)_{F^{-2}} = (M_N/\hbar^2)(E_{c.m.s.})_{MeV}$, where $M_N/\hbar^2 = 41.47$ F^{-2}/MeV.

The scattering potential obtained from the meson theory of nuclear forces can be regarded formally as a sum in which each term corresponds to the nucleons exchanging one or several mesons of different species, so that each term of the sum is characterized by a corresponding short range.

It has been established that the interaction of nucleons at distances 0.5 F and greater is due mainly to the exchange of one or two π mesons and also the contributions from the exchange of the vector mesons ρ, ω, η, etc., which can be regarded as resonance parts of the two- and three-pion exchanges. In addition, it is necessary to take into account certain additional mechanisms, for example, the inclusion of nucleon resonances in the intermediate states of the two-pion exchange diagrams. Essentially, all potentials in which the idea of two-pion exchange is used[10-15] are constructed in accordance with this principle, and they differ only by the manner and extent of the allowance for various short-range effects, and also in the choice of the coupling constants for the vector mesons, which are determined with large errors.

Thus, the "true" nucleon–nucleon potential $V(r)$ can be represented in the form of a sum of two parts:

$$V(r) = -V_p(r) + V_c(r),$$

where $V_p(r)$ is the outer part of the potential, which can be determined with accuracy and minimal arbitrariness in the framework of a model, and $V_c(r)$ is the inner part, which contains numerous phenomenological parameters. The potential $-V_p(r)$ is basically attractive, i.e., $V_p(r) > 0$.[12-15] Then by the core radius r_c we shall mean the nucleon–nucleon separation at which the behavior of $V(r)$ begins to differ from that of $-V_p(r)$ by, say, more than 10%. It is obvious that the core radius defined in this manner will depend on the accuracy with which we have succeeded in taking into account all the peripheral mechanisms in the NN interaction. In fact, at the present time only the one-pion exchange potential (OPEP) is a candidate for the part of $V_p(r)$.

Thus, suppose that the potential $V(r)$ of nucleon–nucleon scattering in the S-wave state is purely attractive for $r > r_c$ and determined by the function $V_p(r)$, while for $r < r_c$ it has a structure whose form is as yet of no concern to us, it being known only that this part is mainly determined by finite repulsive nucleon–nucleon forces. We construct a potential $\tilde{V}(r)$ that has an infinitely high repulsive core with radius r_c and for $r > r_c$ is described by the same function $V_p(r)$. We denote the S-wave phase function in such a potential by $\tilde{\delta}_0(r, k)$, so that $\tilde{\delta}_0(k) \equiv \tilde{\delta}_0(\infty, k)$, where $\tilde{\delta}_0(k)$ is the S-wave phase shift. Now suppose that $\tilde{\delta}_0(k)$ vanishes at $k = k_0$. It is an obvious consequence of the inequalities (4) that the phase shift $\delta_0(k)$ of two nucleons in the potential $V(r)$ will be greater than zero at the point k_0. To satisfy the condition $\delta_0(k_0) = 0$, it is necessary to decrease the repulsive region by increasing r_c. In this sense, the potential with infinitely high core has the minimal core radius r_{c0} among all the potentials for which the outer part is given by the function $V_p(r)$ and $\delta_0(k)$ vanishes at k_0. The radius r_{c0} can be determined from the equation

$$r_{c0} = \frac{1}{k_0^2} \int_{r_{c0}}^{\infty} V_p(r) \sin^2[k_0 r + \tilde{\delta}_0(r, k_0; r_{c0})]\, dr. \tag{5}$$

For "realistic" potentials that have an arbitrary structure within the region $r_s < r_c$,

$$r_c \geqslant r_{c0}, \tag{6}$$

and this inequality is remarkable in that the value of r_{c0} is determined solely by the form of the known function $V_p(r)$.

The complicated transcendental equation (5) can be somewhat simplified by remembering that $\tilde{\delta}_0(r, k_0) < 0$ for $r > r_{c0}$, i.e., $k_0 r + \tilde{\delta}(r, k_0) < k_0 r$. Then in the ranges of variation of r in which the inequality

$$\pi/2 + \pi n \leqslant k_0 r + \tilde{\delta}_0(r, k_0) < k_0 r \leqslant \pi(n+1) \tag{7}$$

holds,

$$\sin^2[k_0 r + \tilde{\delta}_0(r, k_0)] > \sin^2 k_0 r.$$

Since usually the range of the NN potential does not exceed 3–4 F, it is sufficient to consider the inequality (7) for $n = 0$. In this case, it follows from (5) that

$$r_{c0} > \frac{1}{k_0^2} \int_{r_1}^{r_2} V_p(r) \sin^2 k_0 r\, dr, \tag{8}$$

where $r_2 = \pi/k_0$, and $k_0 r_1 + \tilde{\delta}_0(r_1, k_0) = \pi/2$. Since $\tilde{\delta}_0(r_1, k_0) > -k_0 r_{c0}$, we have $r_1 < r_{c0} + \pi/2k_0$. Substituting the estimates we have obtained for r_1 and r_2 in Eq. (8), we finally obtain

$$r_{c0} > \frac{1}{k_0^2} \int_{r_{c0}+\pi/2k_0}^{\pi/k_0} V_p(r) \sin^2 k_0 r\, dr. \tag{9}$$

Similarly, we can find a lower bound for the characteristic height of the repulsive core, defining it as $V_c = \max V(r)$. For this, we consider the set of potentials which are described by the function $-V_p(r)$ for $r > r_c$ and make the phase shift vanish at $k = k_0$. Once again, on the basis of the inequalities (4), it is obvious that among these potentials the repulsive potential of rectangular shape for $0 < r < r_c$ has the minimal height V_{c0}. In this case, the value of V_{c0} is determined by the equation[2)]

$$-k_0 r_c + \mathrm{arctg}\left(\frac{k_0}{x_0} \mathrm{tg}\, x_0 r_c\right) + \frac{1}{k_0} \int_{r_c}^{\infty} V_p(r) \sin^2[k_0 r + \tilde{\delta}_0(r, k_0; r_c, V_{c0})]\, dr = 0, \tag{10}$$

where $x_0 = \sqrt{k_0^2 - V_{c0}}$ and $\tilde{\delta}_0(r, k_0; r_c, V_{c0})$, is the solution of Eq. (1) with the potential

$$\tilde{V}(r) = \begin{cases} +V_{c0} & \text{for} \quad 0 \leqslant r \leqslant r_c; \\ -V_p(r) & \text{for} \quad r > r_c. \end{cases}$$

Formally, V_{c0} can be regarded in accordance with Eq. (10) as a function of the parameter r_c; then $V_{c0}(r_c)$ will be a nondecreasing function of r_c. We show an example of the graph of $V_{c0}(r_c)$ for some potential $V_p(r)$ in Fig. 2. The values of the possible heights V_c of the "real" po-

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2) *Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic trigonometric functions, etc., is retained here and throughout the article in the displayed equations.

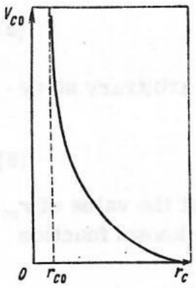


FIG. 2. Example of the dependence $V_{c0}(r_c)$. The parameters and form of the attractive part of the potential are fixed.

tentials will lie above the curve $V_{c0}(r_c)$, which can be regarded as universal for a definite model of the peripheral nucleon–nucleon interaction.

Thus, the connection between the parameters of the attractive part of the nucleon–nucleon potential and the possible heights and radii of the repulsive core determines the amount of information about the core that can be extracted by studying the behavior of the $^1S_0$ phase shift of two-nucleon scattering in the range of energies up to 450 MeV in the laboratory system. The information is so meager above all because of the relatively small range of energies accessible for determining the phase shifts. Of course, if one could extend the phase-shift analysis to 1 GeV, for example, then one could at least qualitatively determine the "strength" of the core (Fig. 3). On the other hand, as is shown in Ref. 17, to describe the $^1S_0$ phase shift at energies of the interacting particles up to 450 MeV it is sufficient to have a potential function with five parameters chosen appropriately. It follows that even if we specify an attractive potential of a quite definite form we can still construct a set of models of the repulsive core that, taken in conjunction with the attractive part, reproduce the experimentally observed energy dependence of the $^1S_0$ phase shift.

Comparative Analysis of Nucleon–Nucleon Phase Shifts. As a rule, when the form of the *NN* potential is reconstructed, the investigation is based on one separately taken phase-shift curve of two-nucleon scattering. It is, however, also of interest to compare phase-shift curves, for example, the $^1S_0$ phase shifts of *np* and *pp* scattering or the $^1S_0$ and $^3S_0$ phase shifts of *pp* scattering, etc. Since the difference in the behavior of the first group of curves is intimately related to the problem of isospin invariance of the nucleon–nucleon forces, it is

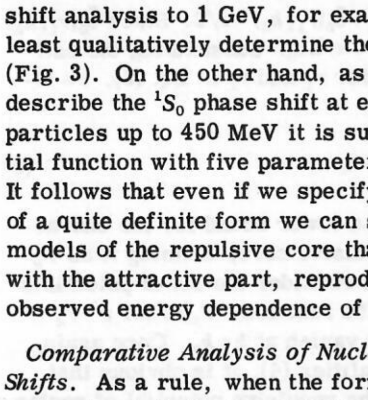


FIG. 3. The *S* phase-shift curves for different potentials in the region of energies E_{1ab} up to 4 GeV. 1) Hamada–Johnston potential; 2) Reid potential with hard core; 3) Reid potential with soft core; 4) one of the Sprung–Srivastava potentials; 5) Gogny–Pires–de Tourreil potential.

to be expected that a comparative investigation of them will yield additional information about the nucleon–nucleon potential and, in particular, its repulsive core.

The hypothesis of isospin invariance of the *NN* interaction requires that the potentials of $^1S_0$ scattering in the *np* and the *pp* systems be equal when Coulomb forces are ignored. It follows from Ref. 17 that the difference between the singlet $^1S_0$ phase shifts of *np* and *pp* scattering is at least non-negative up to an energy $E_{1ab} \approx 450$ MeV of the colliding particles and at this energy is

$$\Delta\delta_0 \equiv \delta_{np} - \delta_{pp} \approx 0.22 \pm 0.07. \tag{11}$$

This phase difference is due to the interference between the nuclear and Coulomb interactions, so that the phase shift of two charged nuclear particles can be represented in the form

$$\delta_0(k) = \delta_0^{\text{Coul}}(k) + \delta_0^{\text{nuc}}(k) - \Delta\delta_0(k), \tag{12}$$

where $\delta_0^{\text{Coul}}(k)$ is the phase shift of scattering by the Coulomb potential, and $\delta_0^{\text{nuc}}(k)$ is the phase shift for the "purely" nuclear potential. To calculate the correction $\Delta\delta_0(k)$ at large momenta k of the relative motion it is usual to expand the phase shift $\delta_0(k)$ in a series in the parameter $\nu = \alpha/k$, where $\alpha = M_N e^2/2\hbar^2 = 1.74 \cdot 10^{-2}$ F^{-1} is the nucleon Coulomb constant.

Thus, the variable-phase equation (1) in the case of scattering of charged nucleons on an isospin-invariant potential takes the form

$$\begin{gathered}\partial\delta_0(r, k; \alpha)/\partial r = -(1/k)\,[V(r) + 2\alpha/r]\sin^2[kr + \delta_0(r, k; \alpha)],\\ \delta_0(0, k; \alpha) = 0.\end{gathered} \tag{13}$$

For convenience, we also introduce the function $\gamma(r, k; \alpha)$, which is related to the variable-phase function $\delta_0(r, k; \alpha)$ by $\gamma(r, k; \alpha) = kr + \delta_0(r, k; \alpha)$. We make a formal expansion of $\gamma(r, k; \alpha)$ in a series in the parameter ν under the condition $|\nu| \ll 1$:

$$\gamma(r, k; \alpha) = \sum_{n=0}^{\infty} \nu^n \gamma_n(r, k). \tag{14}$$

Then Eq. (13), rewritten for the function $\gamma(r, k; \alpha)$, decomposes into a system of first-order differential equations for the coefficients $\gamma_n(r, k)$ in Eq. (14):

$$\left.\begin{array}{l}\partial\gamma_0(r, k)/\partial r = k - (V(r)/k)\sin^2\gamma_0(r, k), \quad \gamma_0(0, k) = 0;\\ \partial\gamma_1(r, k)/\partial r = -(2/r)\sin^2\gamma_0(r, k)\\ -(V(r)/k)\,\gamma_1(r, k)\sin 2\gamma_0(r, k), \quad \gamma_1(0, k) = 0;\\ \dots\dots\dots\dots\dots\dots\end{array}\right\} \tag{15}$$

The equations containing $\gamma_i(r, k)$ with $i = 1, 2, \ldots$ can be readily integrated and expressed in terms of the function $\gamma_0(r, k)$ of scattering by the "purely" nuclear potential $V(r)$. Performing this integration, we can readily show that

$$\begin{gathered}\delta_0(r, k; \alpha) = \delta_0(r, k; 0)\\ -\int_0^r d\xi F(\xi, k; \nu)\exp\left[-\frac{1}{k}\int_\xi^r \sin 2\gamma_0(\eta, k)\,V(\eta)\,d\eta\right].\end{gathered} \tag{16}$$

Here $F(\xi, k; \nu) = \sum_{n=1}^{\infty} \nu^n g_n(\xi, k)$, and the functions $g_n(\xi, k)$ can be expressed in terms of $\gamma_n(\xi, k)$ as follows:

$$\begin{gathered}g_1(\xi, k) = (2/\xi)\sin^2\gamma_0(\xi, k);\\ g_2(\xi, k) = \gamma_1(\xi, k)\,[(2/\xi)\sin 2\gamma_0(\xi, k) + (V(\xi)/k)\,\gamma_1(\xi, k)\cos 2\gamma_0(\xi, k)];\\ g_3(\xi, k) = [(2/\xi)\,\gamma_2(\xi, k) - (2/3)\,\gamma_1^3(\xi, k)\,(V(\xi)/k)]\sin 2\gamma_0(\xi, k)\\ + 2\gamma_1(\xi, k)\,[(1/\xi)\,\gamma_1(\xi, k) + (V(\xi)/k)\,\gamma_2(\xi, k)]\cos 2\gamma_0(\xi, k), \quad \text{etc.}\\ \gamma_n(r, k) = -\int_0^r g_n(\xi, k)\exp\left[-\frac{1}{k}\int_\xi^r \sin 2\gamma_0(\eta, k)\,V(\eta)\,d\eta\right].\end{gathered}$$

In order to obtain from (16) the observed phase shift of charged particles, we must let r tend to infinity. Then the integral will diverge logarithmically. We determine the phase-shift function of S-wave scattering by the potential $V(r,\alpha)=V(r)+2\alpha/r$ as $r\to\infty$ as follows:

$$\delta_0(r,k;\alpha)_{r\to\infty}\approx\delta_0(k,\alpha)-\nu\ln 2kr + (1/2i)\ln[\Gamma(1+i\nu)/\Gamma(1-i\nu)]. \quad (17)$$

Before we substitute (17) in (16), we note that

$$\ln 2kr = 2\int_0^r \frac{d\xi}{\xi}\sin^2 k\xi + \mathrm{Ci}(2kr) - C,$$

where C, Euler's constant, is equal to 0.577..., and the cosine integral satisfies $\mathrm{Ci}(2kr)\to 0$ as $r\to\infty$. If we now take into account (12) and also expand the Coulomb phase shift $(1/2i)\ln[\Gamma(1+i\nu)/\Gamma(1-i\nu)]$ in a series in the parameter ν, we obtain the following expression for the interference correction from (16):

$$\Delta\delta_0(k) = \int_0^\infty d\xi\left[F(\xi,k;\nu)\exp\left(-\frac{1}{k}\int_0^\infty \sin 2\gamma_0(\eta,k)V(\eta)\,d\eta\right)-2\nu\frac{\sin^2 k\xi}{\xi}\right] - \sum_{m=1}^{\infty}(-1)^m\nu^{2m}\zeta(2m+1). \quad (18)$$

All the integrals in the expression (18) exist, and therefore to estimate $\Delta\delta_0(k)$ under the condition $|\nu|\ll 1$ it is meaninfgul to restrict ourselves to the first term of the expansion in ν:

$$\Delta\delta_0(k)\approx 2\nu\int_0^\infty \frac{d\xi}{\xi}\left[\sin^2\gamma_0(\xi,k)\times\exp\left(-\frac{1}{k}\int_\xi^\infty \sin 2\gamma_0(\eta,k)V(\eta)\,d\eta\right)-\sin^2 k\xi\right]. \quad (19)$$

Note that the condition $|\nu|\ll 1$ in the case of scattering of protons is well satisfied already at the interaction energy $E_{\mathrm{lab}}\gtrsim 2$ MeV.

Before we apply Eq. (19) to investigate the difference (11) between the phase shifts, we must discuss the justification for such a comparison. The problem is that at c.m.s. energy of order 200 MeV one must take into account relativistic corrections to the phase shifts, the influence of the electromagnetic form factor of the nucleons (which modifies the Coulomb law at short distances), the effect of vacuum polarization, etc. However, for the estimate of the difference between the phase shifts of np and pp scattering, the relativistic corrections are proportional to $\nu(\lambda k)^2$, where $\lambda=\hbar/M_N c = 0.2$ F is the nucleon Compton wavelength, and the relativistic corrections can be ignored in comparison with $\Delta\delta_0$ in (11). Further, in this analysis one can ignore the influence of vacuum polarization on the difference between the phase shifts at the given energy, since this effect plays an appreciable role[28] at energies $E_{\mathrm{lab}}\lesssim 20$ MeV. We shall consider allowance for direct and indirect electromagnetic effects somewhat later.

We now investigate the expression (19). Assuming that the value (11) of $\Delta\delta_0$ occurs on the left-hand side of (19), we can simplify the expression (19) by taking into account the range R of the nuclear potential outside which one can in practice assume $V(r)=0$ (usually, R does not exceed 3–4 F). In this case

$$\Delta\delta_0(k)=2\nu\int_0^R \frac{d\xi}{\xi}\sin^2\gamma_0(\xi,k)\exp\left(-\frac{1}{k}\int_\xi^R \sin 2\gamma_0(\eta,k)V(\eta)\,d\eta\right) + H, \quad (20)$$

where

$$H = 2\nu\left\{-\int_0^R \frac{d\xi}{\xi}\sin^2 k\xi + \int_R^\infty \frac{d\xi}{\xi}\left[\sin^2(k\xi+\delta_{np}^{\exp}(k))-\sin^2 k\xi\right]\right\}$$

$$=\nu\left[\cos 2\delta_{np}^{\exp}(k)\,\mathrm{Ci}(2kR)-\sin 2\delta_{np}^{\exp}(k)\,\mathrm{Si}(2kR)-C-\ln(2kR)\right].$$

At an energy $E_{\mathrm{lab}}\approx 450$ MeV ($k=2.35\ \mathrm{F}^{-1}$), H is negative and its modulus has a value of order 0.01–0.03 for $0.5<R<5$ F, so that the second term in (20) can be ignored. Thus,

$$\Delta\delta_0(k)\approx 2\nu\int_0^R \frac{d\xi}{\xi}\sin^2\gamma_0(\xi,k)\exp\left(-\frac{1}{k}\int_\xi^R \sin 2\gamma_0(\eta,k)V(\eta)\,d\eta\right). \quad (21)$$

Using the expression (21), we shall show that the value of the investigated phase-shift difference $\Delta\delta_0(k)$ given by (11) basically determines the behavior of the potential in the region of the core, i.e., we shall show that the so-called meson tail of the NN potential does not make an appreciable contribution to the difference between the phase shifts of np and pp scattering. By the meson tail we mean the peripheral attractive part of the potential, where the function $V(r)$ increases monotonically to zero. Suppose that beyond the point r_p the function $V(r)$ satisfies $V(r)<0$, and $V'(r)>0$ for $r>r_p$. Note that

$$J_0^2(r,k)=\sin^2\gamma_0(r,k)\exp\left[-\frac{1}{k}\int_r^\infty \sin 2\gamma_0(\eta,k)V(\eta)\,d\eta\right]$$

satisfies the integral equation

$$J_0^2(r,k)=\sin^2\gamma_0(r,k)\left[1+\frac{1}{k^2}\int_r^\infty V'(\xi)J_0^2(\xi,k)\,d\xi\right]\times\left[1-\frac{V(r)}{k^2}\sin^2\gamma_0(r,k)\right]^{-1}. \quad (22)$$

Suppose that for $r=r_m>r_p$ the function $J_0^2(r,k)$ reaches its largest value M in the given range of variation of r. It then follows from (22) that

$$M\leqslant\sin^2\gamma_0(r_m,k)\left[1+(M/k^2)|V(r_m)|\right]\left[1+\frac{|V(r_m)|}{k^2}\sin^2\gamma_0(r_m,k)\right]^{-1},$$

whence

$$M\leqslant\sin^2\gamma_0(r_m,k)\leqslant 1. \quad (23)$$

Substituting the estimate (23) in Eq. (21), we obtain

$$\Delta\delta_0(k)\leqslant 2\nu\int_0^{r_p}\frac{d\xi}{\xi}J_0^2(\xi,k)+2\nu\ln\frac{R}{r_p}. \quad (24)$$

In the usually employed models of phase-equivalent potentials, the ratio R/r_p does not exceed 10, i.e., $2\nu\ln(R/r_p)\leqslant 0.04$, and this quantity can be ignored in comparison with $\Delta\delta_0$ in accordance with (11). For the same reason, some models of potentials with an infinite hard core, in which $r_p\equiv r_c$ is the core radius, cannot describe the observed difference between the phase shifts, since in this case the first term in (24) is zero.

We now consider the first term in Eq. (24). A direct calculation of the value of this term using the well-known phenomenological phase-equivalent potentials of Reid[24] and Bressel, Kerman, and Rouben[29] showed that the left-hand side of Eq. (24) is about an order of magnitude greater than the right-hand side. It the region of strong repulsion of the potential extends right down to $r=0$, then to all appearances the integral term in (24) is of order unity. The smallness of the wave function $J_0(r,k)$ in the region of the core makes such a conclusion possible. To see this, consider the identity

$$2\int_0^R J_0^2(\xi,k)\,d\xi=\left[R+\frac{\partial\delta_{np}(k)}{\partial k}-\frac{\sin 2(kR+\delta_{np}(k))}{2k}\right].$$

Substituting here the values of $\delta_{np}(k)$ and $\partial\delta_{np}(k)/\partial k$ taken from the experiment at $k\approx 2.4\ \mathrm{F}^{-2}$, we find that

$$\frac{2}{R}\int_0^R J_0^2(\xi,k)\,d\xi\approx 1. \tag{25}$$

Using the mean-value theorem and taking into account (25), we obtain from (21)

$$\Delta\delta_0(k)\approx 2\nu\int_{\bar\xi}^{R}\frac{d\xi}{\xi}J_0^2(\xi,k)=\frac{2\nu}{\bar\xi}\int_0^R J_0^2(\xi,k)\,d\xi\approx\nu R/\bar\xi. \tag{26}$$

Substituting in (26) the numerical values at $R\approx 3$-4 F of the quantities in the equation, we find the restriction $\bar\xi\lesssim 0.2$ F on $\bar\xi$, i.e., $\bar\xi\ll R$. At the same time, from (26) we obtain

$$\int_0^{\bar\xi} J_0^2(\xi,k)\left(\frac{1}{\xi}-\frac{1}{\bar\xi}\right)d\xi=\int_{\bar\xi}^{R} J_0^2(\xi,k)\left(\frac{1}{\bar\xi}-\frac{1}{\xi}\right). \tag{27}$$

On the left-hand side of Eq. (27) we have a quantity that is nearly zero, since the wave function is exponentially small in the region of strong repulsion, but in this case we must have $\bar\xi\sim R$ if the right-hand side of (27) is also to be near zero. This contradiction enables us to conclude that model potentials in which attraction is then followed by strong repulsion (high core) do not describe the observed difference between the phase shifts in the indicated energy range.

Hitherto, we have considered a Coulomb potential of the form $V_{\mathrm{Coul}}(\xi)=2\alpha/\xi$ right down to short distances. However, the estimates made above are not significantly changed if allowance is made for the proton and neutron electromagnetic form factors.[30] Moreover, in this case the Coulomb potential becomes regular at $\xi\sim 0$, and the order of magnitude of its value will be determined by the same constant α.

Thus, our investigation of the difference between the phase shifts of np and pp scattering in the region $E_{\mathrm{lab}}\approx 450$ MeV shows that the nucleon–nucleon potential must have a complicated form at short distances between the particles or else that there is a breaking of the isospin invariance of the nuclear forces. It is also very probable that indirect electromagnetic effects associated with the difference between the masses of the mesons exchanged by the interacting nucleons also make no significant contribution to the investigated difference between the phase shifts. Indeed, as the estimates of Ref. 31 show, indirect electromagnetic effects make up 2–3% of the OBE potential. Substituting the value $\Delta V(\xi)\sim 0.03V(\xi)$ in Eq. (2) instead of $2\alpha/r$ and remembering that at $E_{\mathrm{lab}}\approx 450$ MeV

$$\exp\left[-\frac{1}{k}\int_{\xi}^{\infty}V_{\mathrm{OBE}}(\eta)\sin 2\gamma_0(\eta,k)\,d\eta\right]\sim 1,$$

we find that the phase-shift difference due to indirect electromagnetic effects is also an order of magnitude smaller than the experimental value, although these effects can in principle explain the difference between the np and nn scattering lengths.[32]

Note that, generally speaking, the experimental value of the difference between the $^1S_0$ phase shifts of np and pp scattering can be explained if one assumes the existence in the two-nucleon system of a quasistationary state, even if this has a very short lifetime. This is possible if the nuclear potential takes the form of an attractive part followed by strong repulsion, with weak repulsion or attraction near the origin. The increase in the calculated value of $\Delta\delta_0(k)$ in this case is due to the increase in the wave function $J_0(r,k)$ as the energy of the interacting particles approaches the energy of the quasistationary state, or, more precisely, its real part.[33] Unfortunately, from the available experimental phase shifts one cannot uniquely determine the parameters of the putative quasistationary state nor give preference to any particular NN interaction at small r.

2. EXTRACTION OF INFORMATION ABOUT THE NUCLEON-NUCLEON INTERACTION FROM THREE-PARTICLE SCATTERING

We now turn to the investigation of experiments with three nucleons, with a view to establishing the nature of the NN forces, assuming that at nonrelativistic energies only the two-body interactions between the particles are important. It is well known[34] that the three-nucleon scattering amplitude depends strongly on the off-shell characteristics of two-particle scattering, namely, the two-particle scattering amplitude off the energy shell and, in the case of nucleon scattering on deuterons, the deuteron wave function. Since these characteristics are determined by the form of the NN potential, it is to be expected that investigation of the three-nucleon scattering cross section in definite kinematic regions will yield more detailed information about the region of the repulsive core than a phase-shift analysis of two-particle processes.[35] Indeed, let us consider, for example, the asymptotic behavior with respect to the variable k of the function $\delta_0(k)$ and the off-shell S partial-wave amplitude $t_0(k,k';k^2)$ at fixed k'. In the first case, the asymptotic form is determined by

$$\delta_0(k)\approx-\frac{1}{2k}\int_0^{\infty}V(r)\,dr, \tag{28}$$

and in the second case by the approximate formula[36]

$$t_0(k,k';k^2)\approx V_0(k,k')/[1+ikV_0(k,k)]\approx-2V'(0)/k^4, \tag{29}$$

which is meaningful for potentials that are sufficiently smooth in the region of small r. Comparing (28) and (29), we readily see that the asymptotic form of the off-shell amplitude depends directly on the value of the potential at $r=0$ [(28) and (29) are obtained under the assumption that the potential function is bounded], whereas the asymptotic behavior of the phase shift is determined by the form of the potential in its complete range.

Nuclear Potential and Differential Cross Section of Elastic pd Scattering. We show now that the structure of the repulsive core of the nucleon–nucleon interaction can be investigated by analyzing the angular distributions of protons scattered elastically on deuterons through large angles at proton energies 150–200 MeV in the laboratory system. For this, we define some functions that characterize the scattering and bound state of two nucleons and are needed for the analysis. We specify the deuteron wave function $G_d(\mathbf{p})$ in the momentum representation:

$$G_d(\mathbf{p}) = (1/4\pi)\,[u_S(p) + (T_{12}(\mathbf{p})/\sqrt{8})\,u_D(p)]. \qquad (30)$$

Here, $u_S(p)$ and $u_D(p)$ are the scalar wave functions of the pn system in the S and D states, and

$$T_{12}(\mathbf{p}) = \left[\frac{3(\sigma_1\mathbf{p})(\sigma_2\mathbf{p})}{p^2} - (\sigma_1\sigma_2)\right]$$

is the tensor operator. The wave function $G_d(\mathbf{p})$ is normalized by the condition

$$\int_0^\infty [u_S^2(p) + u_D^2(p)]\,dp = 1. \qquad (31)$$

Besides the deuteron wave function, we also consider the deuteron form factor

$$\Phi(\mathbf{k}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} G_d(\mathbf{p}) \frac{-iM}{\mathbf{p}^2+\varkappa^2} \frac{-iM}{(\mathbf{p}-\mathbf{k})^2+\varkappa^2} G_d(\mathbf{p}-\mathbf{k}). \qquad (32)$$

Here, $\varkappa^2 = ME_d$, the deuteron binding energy, is equal to $(0.23)^2\ \mathrm{F}^{-2}$.

We define the off-shell two-nucleon scattering amplitude $t(\mathbf{k}, \mathbf{k}'; ME)$, which can be found by solving the Lippmann–Schwinger equation:

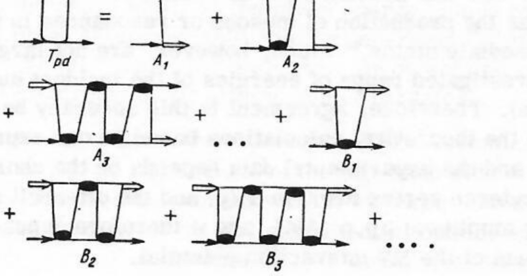
$$t(k, k'; ME) = V(k, k') + 4\pi \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{V(\mathbf{k}, \mathbf{p})\,t(\mathbf{p}, \mathbf{k}'; ME)}{ME - \mathbf{p}^2 + i\tau}. \qquad (33)$$

Here,

$$V(k, k') = \frac{1}{4\pi} \int \exp[i(\mathbf{k}-\mathbf{k}')\mathbf{r}]\,V(\mathbf{r})\,dr^3, \qquad (34)$$

with $\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2$, $\mathbf{k}' = (\mathbf{k}_1' - \mathbf{k}_2')2$, $E = \varepsilon_1 + \varepsilon_2 - (\mathbf{k}_1 + \mathbf{k}_2)^2/4M$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the first and the second nucleon, respectively, in the initial state, $\mathbf{k}_1'$ and $\mathbf{k}_2'$ are the corresponding momenta in the final states, and ε_1, ε_2 and ε_1', ε_2' are the corresponding energies in the same states.

With a view to elucidating the behavior of the deuteron wave function, we can study the angular distributions of protons in the reaction $p + d \to p + d$ in the relevant energy range of the primary particles (150–200 MeV in the laboratory system) on the basis of the integral equations of the nonrelativistic problem of three-body scattering.[34] The differential cross section of Nd scattering is calculated in this case on the basis of a representation of the $N + d \to N + d$ reaction amplitude as a sum of the infinite series of diagrams



(35)

formed by the successive iterations of the integral equation for determining the Nd elastic scattering amplitude. Here, $\mathbf{k}_0$ is the momentum of the incident nucleon, and $\mathbf{k}$ is the momentum of the scattered nucleon in the center of mass system. It was shown in Ref. 37 that at energies of the primary nucleons that appreciably exceed the binding energy of the nucleons in the deuteron, i.e., for $k_0 \gg \varkappa$, one can obtain a condition of convergence of the infinite series of contributions from the diagrams of (35) and separate the leading contributions among them in a definite kinematic region. An example of such an analysis can be found in Ref. 38, in which the amplitude of elastic Nd scattering through zero angle is considered.

The method of Ref. 37 is based on taking the ratio $\varkappa/k_0$ as the convergence parameter of the series (35), the contributions of the diagrams A_2, B_2, A_3, etc. being proportional to $(\varkappa/k_0)^2$ and higher powers of this parameter in the complete range of angles θ between the vectors $\mathbf{k}_0$ and $\mathbf{k}$. Thus, if $(\varkappa/k_0)^2 \ll 1$, only the diagrams A_1 and B_1 "survive" in the expansion (35), and the contributions of the remaining diagrams are small corrections to them. In particular, for particle scattering through 0° such a picture agrees as a whole with the Glauber model of nucleon scattering on deuterons at high energies.[39]

We consider here the case of backward elastic scatter of protons through ~180° in the center of mass system. For this, we calculate the contributions from the diagrams (35) in order to separate the most important of them in the considered kinematic region, and we also take into account diagrams proportional to the second order of the convergence parameter of the series, since the initial energies of the colliding particles are still not too large. We calculate the contributions from the diagrams A_2, A_3, B_1, etc., under the following assumptions:

a) nucleons with large momentum in the intermediate state are scattered predominantly, like free nucleons, through angles ~0°;

b) nucleons with small relative momentum in the intermediate state can form a virtual deuteron;

c) a change of nucleon momentum in a rescattering process is possible only through addition with the relative momentum β of the nucleons in the deuteron.[3)]

To estimate the value of $\beta = |\boldsymbol{\beta}|$, we calculate the mean square β_0^2 of the nucleon momentum in the deuteron; for this, because the contribution of the D state is small, it is sufficient to use the deuteron wave function in the S state. Figure 4 shows the S-wave deuteron wave functions $u_S(r)$ in configuration space for different phase-equivalent triplet potentials. It follows from Fig. 4 that these functions differ little from the point of view of estimating the mean value of β_0^2, and therefore we shall use the Hulthén deuteron wave function

[3)]This model of the structure of the intermediate states is based on an estimate of the integrals[37] corresponding to the contributions from diagrams that take into account different intermediate states in the given reaction.

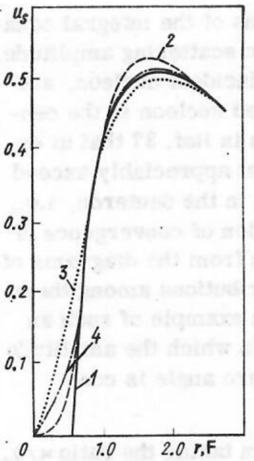


FIG. 4. Wave function of the deuteron in the S state as a function of the internucleon distance. 1) Hamada–Johnston potential; 2) Bressel–Kerman–Rouben potential; 3) Jackson's meson potential; 4) Reid potential with soft core.

$$\psi(\mathbf{r}) = u_S(r)/r$$
$$= (\sqrt{\varkappa\mu(\varkappa+\mu)}/\sqrt{2\pi}(\mu-\varkappa))(1/r)[\exp(-\varkappa r) - \exp(-\mu r)], \quad (36)$$

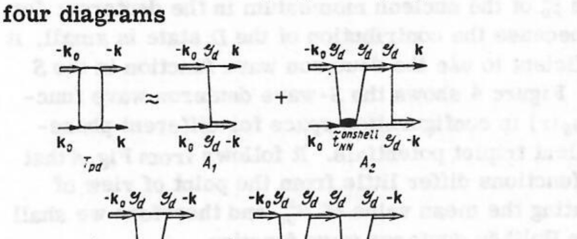
where $\mu \approx 5\varkappa$. Then

$$\beta_0^2 = \int d^3r\psi(\mathbf{r})(-\nabla^2)\psi(\mathbf{r}) \approx \varkappa\mu \approx 5\varkappa^2, \quad (37)$$

i.e., as one would expect, $\beta_0 \sim \varkappa$.

As we have already noted, in such an approach the ratio β_0/k_0 is the convergence parameter of the infinite series (35). At energy 150–200 MeV of the primary protons in the laboratory system, $\beta_0/k_0 \approx 0.25$–0.30. Therefore, to calculate the reaction amplitude to accuracy 10%, one can ignore the diagrams in (35) whose contributions are proportional to $(\beta/k_0)^3$ and higher powers of this parameter.

An estimate of the contributions from the diagrams in Eq. (35) by the method given in Ref. 37 showed that to calculate the amplitude of elastic pd scattering through large angles with the given accuracy it is sufficient to take the contributions from the diagrams A_1, A_2, B_1, B_2 under the condition that in A_2 and B_2 the first intermediate interaction is scattering of nucleons with large relative energy predominantly through 0° and the second intermediate interaction corresponds to the formation of a virtual deuteron. Allowance for other intermediate states gives a higher order than the second in the parameter β_0/k_0. Thus, the amplitude of elastic scattering of protons on deuterons through large angles should be represented graphically by the sum of the four diagrams



(38)

Under the above assumptions, the contributions from the diagrams in the expression (38) can be written in the form

$$A_1 = G_d\left(\frac{\mathbf{k}_0}{2}+\mathbf{k}\right)\frac{-iM}{(\mathbf{k}_0/2+\mathbf{k})^2+\varkappa^2}G_d\left(\frac{\mathbf{k}}{2}+\mathbf{k}_0\right); \quad (39)$$

$$A_2 = \frac{1}{(2\pi)^6}\int\int d^3\mathbf{p}d^3\mathbf{q}G_d(\mathbf{p})\frac{-iM}{\mathbf{p}^2+\varkappa^2}$$
$$\times \tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \frac{3}{4}\mathbf{k}_0; M\varepsilon\right)\frac{-iM}{\mathbf{p}^2+\varkappa^2}G_d(\mathbf{p})\frac{iM}{\frac{3}{4}(\mathbf{k}_0^2-\mathbf{q}^2)+i\tau}$$
$$\times G_d\left(\frac{\mathbf{k}_0}{2}+\mathbf{k}\right)\frac{-iM}{(\mathbf{k}_0/2+\mathbf{k})^2+\varkappa^2}G_d\left(\mathbf{k}_0+\frac{\mathbf{k}}{2}\right)$$
$$= \frac{M}{12\pi}\frac{\beta_0^2}{k_0}\Phi(0)\tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \frac{3}{4}\mathbf{k}_0; M\varepsilon\right)A_1; \quad (40)$$

$$B_1 = \frac{1}{(2\pi)^3}\int d^3\mathbf{p}_d G_d(\mathbf{p})\frac{-iM}{\mathbf{p}^2+\varkappa^2}\tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \mathbf{k}+\frac{\mathbf{k}_0}{4}; M\varepsilon\right)$$
$$\times \frac{-iM}{[(\mathbf{k}_0+\mathbf{k})/2-\mathbf{p}]^2+\varkappa^2}G_d\left(\frac{\mathbf{k}_0+\mathbf{k}}{2}-p\right)$$
$$= \tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \mathbf{k}+\frac{\mathbf{k}_0}{4}; M\varepsilon\right)\Phi\left(\frac{\mathbf{k}_0+\mathbf{k}}{2}\right); \quad (41)$$

$$B_2 = \frac{1}{(2\pi)^9}\int d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{q}G_d(\mathbf{p}_1)\frac{-iM}{\mathbf{p}_1^2+\varkappa^2}$$
$$\times \tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \frac{3}{4}\mathbf{k}_0; M\varepsilon\right)\frac{-iM}{\mathbf{p}_1^2+\varkappa^2}G_d(\mathbf{p}_1)\frac{iM}{\frac{3}{4}(\mathbf{k}_0^2-\mathbf{q}^2)+i\tau}$$
$$\times G_d(\mathbf{p}_2)\frac{-iM}{\mathbf{p}_2^2+\varkappa^2}\tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \mathbf{k}+\frac{\mathbf{k}_0}{4}; M\varepsilon\right)\frac{-iM}{[(\mathbf{k}_0+\mathbf{k})/2-\mathbf{p}_2]^2+\varkappa^2}$$
$$\times G_d\left(\frac{\mathbf{k}_0+\mathbf{k}}{2}-\mathbf{p}_2\right) = \frac{M}{12\pi}\frac{\beta_0^2}{k_0}\Phi(0)\tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \frac{3}{3}\mathbf{k}_0; M\varepsilon\right)B_1. \quad (42)$$

In Eqs. (39)–(42), $\tilde{t}_{NN} = t_{np} + t_{pp}$; and $M\varepsilon = (3\mathbf{k}_0/4)^2$. Note also that the integral

$$I = \frac{1}{(2\pi)^3}\int d^3q\frac{iM}{\frac{3}{4}(\mathbf{k}_0^2-\mathbf{q}^2)+i\tau}$$

in Eqs. (40) and (42) is equal to $(M/12\pi)\beta_0^2/k_0$, since in accordance with the assumptions made above $\mathbf{q} = \mathbf{k}_0 + \boldsymbol{\beta}$. Adding Eqs. (39)–(42), we obtain the amplitude of elastic pd scattering through large angles:

$$T_{pd} \approx \left[1 + \frac{M}{12\pi}\frac{\beta_0^2}{k_0}\Phi(0)\tilde{t}_{NN}\left(\frac{3}{4}\mathbf{k}_0, \frac{3}{4}\mathbf{k}_0; M\varepsilon\right)\right](A_1 + B_1). \quad (43)$$

The amplitude of pd elastic scattering was also calculated in Refs. 40 and 41. The expression (43) for the amplitude of elastic scattering through large angles differs from the expression given in Ref. 40 by our allowance here for processes of scattering of the nucleon through 0° in the initial state, i.e., the contributions from the diagrams A_2 and B_2 in Eq. (38). As a whole, allowance for these diagrams reduces the cross section by 20–25%. In Ref. 41, the amplitude of elastic pd scattering was obtained with allowance for interaction processes in the initial state, although the contribution from the process of direct pd scattering [diagrams B_1 and B_2 in Eq. (38)], which gives a correction ~10% to the cross section, was omitted.

Thus, we have obtained a scheme for calculating the cross section of nucleon elastic scattering on deuterons through large angles that makes it possible to take into account with an accuracy of ~10% all processes of particle rescattering in the considered reaction (here, we do not take into account various relativistic corrections such as the production of mesons or resonances in the intermediate states,[42] which, however, are not large in the investigated range of energies of the incident nucleons). Therefore, agreement to this accuracy between the theoretical calculations based on our expressions and the experimental data depends on the choice of the deuteron vertex function $G_d(\mathbf{p})$ and the off-shell scattering amplitude $t(\mathbf{p}, \mathbf{p}'; M\varepsilon)$, and it therefore depends on the form of the NN interaction potential.

We have calculated the angular distributions of protons scattered elastically on deuterons through 130–180° in the center-of-mass system at initial proton energies 150 and 185 MeV in the laboratory system on the basis of Eq. (43). To calculate the deuteron vertex function and the off-shell NN scattering amplitude, we used the phenomenological phase-equivalent potentials of Hamada and Johnston,[23] Reid with soft core,[24] Bressel, Kerman, and Rouben,[29] and one of Jackson's meson potentials.[13] In Fig. 5, we show the results of comparison of calculations of the differential cross section in accordance with Eq. (43) and the experimental data on elastic pd scattering.[43] It follows from Fig. 5 that meson potential of Ref. 13 gives the best agreement with the experiment.

Analyzing the dependence of the calculated differential cross sections of elastic pd scattering for different forms of the two-particle NN potential and the behavior of the deuteron wave function in the S state in configuration space (see Fig. 4), we can see that the core region of the NN potential affects these cross sections. It follows from Fig. 4 that the greatest difference in the dependence of the wave functions occurs in the region up to $r \sim 0.7$ F.

In this connection, the fact that the differential cross section corresponding to the meson potential of Ref. 13 agrees well with the experiment indicates that it was a fruitful idea of Jackson and Woloshyn[13] to describe the inner region of the NN interaction in terms of exchanges of neutral vector mesons. In such an approach, the sum of the irreducible diagrams containing an arbitrary number of meson exchanges can be calculated in the framework of the eikonal model.

Thus, study of the angular distributions of fast protons scattered elastically on deuterons through large angles can be one of the possible methods for analyzing the inner region of the nucleon–nucleon forces, and this is so even at comparatively low energies of the incident protons, when relativistic effects can be ignored.

Nuclear Potential and Off-Shell Two-Particle Scattering Amplitude. Another way of obtaining more detailed information about the nucleon–nucleon interaction than is possible by phase-shift analysis of NN elastic scattering cross sections is to separate the off-shell amplitude by analyzing experiments involving the scattering of three or more nucleons. It is well known that the amplitudes of these reactions can be determined on the basis of the integral equations of scattering theory. The kernels of these equations contain the off-shell two-particle scattering amplitude. Therefore, one can arrange an experiment to investigate nuclear reactions with three and more nucleons in such a way that the behavior of the differential cross sections in a certain range of variation of the energies and momenta of the incident particles is determined by the off-shell scattering amplitude of one of the pairs of particles.

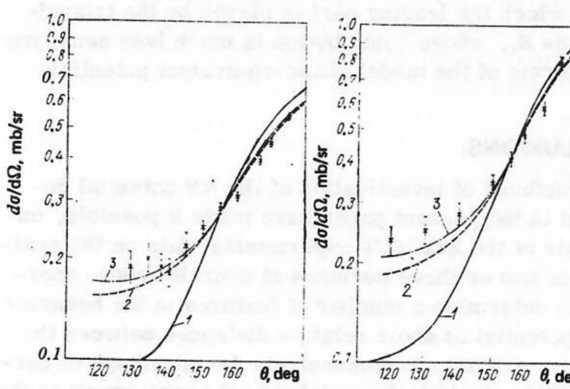


FIG. 5. Differential cross sections of backward pd scattering at initial proton energies $E_{1ab} = 185$ (a) and 150 MeV (b). 1) Hamada–Johnston potential; 2) Bressel–Kerman–Rouben potential; 3) meson potential of Chemtob, Jackson, *et al*.

We consider the basic possibility of extracting information about the nuclear potential from experimental data on the behavior of the off-shell two-particle amplitude for the example of the S-wave partial amplitude. We recall that the S partial-wave amplitude $t_0(k, k'; ME)$ is calculated in accordance with the formula

$$t_0(k, k'; ME) = \frac{1}{2}\int_{-1}^{1} t(\mathbf{k}, \mathbf{k}'; ME)\, d\xi,$$

where $\xi = \cos(\widehat{\mathbf{k}\mathbf{k}'})$, and the Lipmann–Schwinger equation (33) is used to determine $t(\mathbf{k}, \mathbf{k}'; ME)$. If one of the relative momenta, for example, k, is related to the relative energy E by the usual dispersion law $ME = k^2$, then the so-called half-on-shell amplitude $t_0(k, k'; k^2)$ satisfies the relation[44]

$$t_0(k, k'; k^2) = (g_0(k, k')/g_0(kk))\, t_0(k), \tag{44}$$

where $t_0(k) \equiv t_0(k, k; k^2)$ is the on-shell scattering amplitude, and

$$g_0(k, k') = \frac{1}{k'}\int_0^{\infty} \sin k'\xi J_0(\xi, k)\, V(\xi)\, d\xi.$$

Here, $J_0(\xi, k)$ is the wave function of the two-nucleon system. Some properties of the function $J_0(\xi, k)$ were considered in Sec. 1 of the present review.

On the basis of calculations of $t_0(k, k'; k^2)/t_0(k)$ for different model potentials $V(r)$ as a function of the relative momentum k' made by a number of authors (Fig. 6) it has been shown that in a comparatively large range of energies $10 < E_{\text{c.m.s.}} < 180$ MeV ($0.5 < k < 2.1$ F^{-1}) phase-equivalent potentials give very different curves of $t_0(k, k'; k^2)/t_0(k)$ when k' varies in the region $3 < k' < 4$ F^{-1}, where these curves attain an extremal value. It is interesting that the position of this extremum for each potential is virtually independent of the momentum k, whereas the extremal value of $t_0(k, k'; k^2)/t_0(k)$ also depends on the value of the repulsive core.

As an illustration, Fig. 7 shows the value of the second extremum of the partial-wave amplitude $t_0(k, k'; k^2)/t_0(k)$ as a function of the core height V_c for two core radii r_c in the case of a potential consisting of two rectangles:

$$V(r) = \begin{cases} +V_c, & 0 \leqslant r \leqslant r_c; \\ -V_p, & r_c < r \leqslant R; \\ 0, & r > R. \end{cases} \tag{45}$$

The parameters r_c and R were fixed, while V_p and V_c

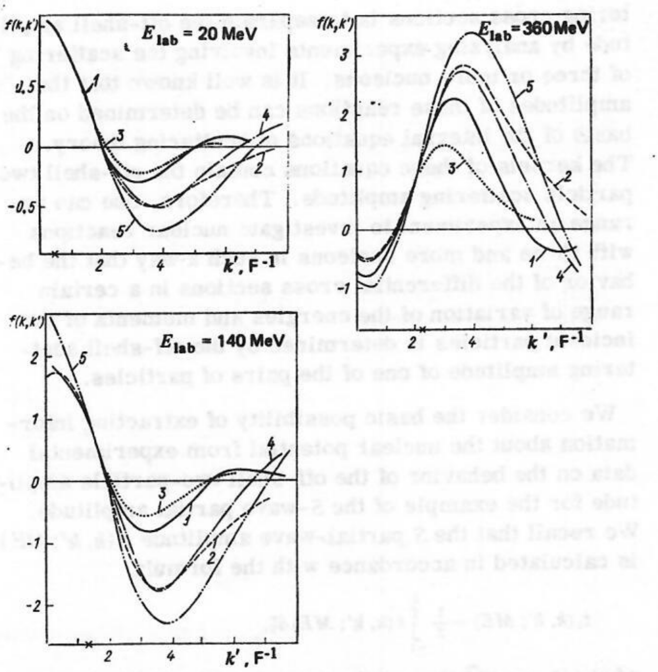
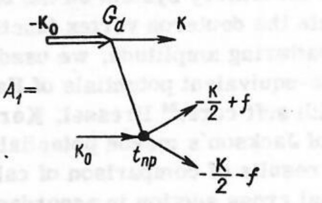


FIG. 6. Dependence of the function $f(k,k')=t_0(k,k';\,k^2)/t_0(k)$ on k' for three different values of E. 1) One of the Sprung–Srivastava potentials; 2) Reid potential with soft core; 3) Gogny–Pires–de Tourreil potential; 4) Bressel–Kerman–Rouben potential; 5) Hamada–Johnston potential.

were chosen in such a way that the potential (45) correctly describes the scattering length in the $^1S_0$ state.

It follows from Fig. 7 that the value of the amplitude at this extremum can vary by a few times depending on the height of the core and, therefore, there will be an appreciable change of the differential cross section of the reaction in a definite range of magnitudes and directions of the momenta of the final particles.

It appears important to obtain experimental data on the variation of the function $t(\mathbf{k},\mathbf{k}';ME)$ in the indicated range of variation of the variable $|\mathbf{k}|$. For this, it is expedient to study the differential cross section of inelastic scattering of protons on dueterons in the region of large scattering angles under the condition that the relative momentum of the two nucleons (proton and neutron) in the final state is small. As in the case of elastic pd scattering, to describe the inelastic reaction $p+d\to p+p+n$ we can restrict ourselves to the contributions from the lowest diagrams provided the inequality $\beta_0/k_0 \ll 1$ is satisfied.[37] In this case, the leading contribution to the reaction amplitude is the contribution from the diagram

$A_1=$  (46)

which can be written in the form

$$A_1=G_d\left(\frac{\mathbf{k}_0}{2}+\mathbf{k}\right)\frac{-iM}{(\mathbf{k}_0/2+\mathbf{k})^2+\varkappa^2}\,t_{pn}\left(\mathbf{k}_0+\frac{\mathbf{k}}{2},\,f;\,f^2\right). \qquad (47)$$

In addition, by analogy with the graphical equation (38) and Eqs. (39)–(42), we can calculate the corrections to the expression (47), etc. As was shown in Ref. 37, the calculation of the corrections in this case is identical to the calculation of the analogous corrections in the case of elastic pd scattering.

However, it is readily seen that to achieve the values of the relative momenta in which we are interested appreciably higher initial energies of the protons are required. For example, if the relative momentum of the proton and neutron is small, more precisely if $f^2/k_0^2 \ll 1$, then to study the behavior of the differential cross section for the emission of protons at 120° in the center-of-mass system under the condition that the modulus of the off-shell momentum satisfies $|\mathbf{k}_0+\mathbf{k}/2|>3$ F^{-1} initial proton energies higher than 450 MeV in the laboratory system are needed. At such energies, it is necessary to take into account the contribution from relativistic, inelastic processes, in particular from meson production.[42]

Thus, in this section we have demonstrated the possibility of obtaining information about the core region of the two-particle nuclear potential by analyzing the off-shell characteristics of NN scattering, which can be obtained explicitly from the differential cross sections for scattering of protons through large angles. We should emphasize the importance of backward scattering, since in this case the leading contribution to the amplitude is made by the pole diagrams A_1 in Eqs. (32) and (46), in contrast to small-angle forward scattering, in which the leading part is played by the triangle diagrams B_1, whose contribution is much less sensitive to the forms of the model phase-equivalent potentials.

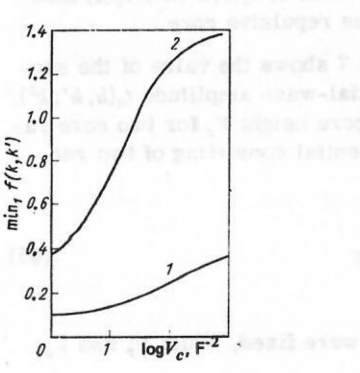


FIG. 7. Magnitude of the first extremum of the function $f(k,k')$ with respect to k' as a function of the core height V_c for the potential (45). 1) $r_c=0.2$ F; 2) $r_c=0.4$ F.

CONCLUSIONS

The methods of investigation of the NN potential developed in the present paper have made it possible, on the basis of the available experimental data on the scattering of two or three nucleons at nonrelativistic energies, to determine a number of features in the behavior of the potential at short relative distances between the nucleons, and also to demonstrate the advantage of certain model potentials formulated in the framework of the meson theory of nuclear forces. If the properties of the NN forces at short distances are to be determined more precisely, we must turn to an analysis of experimental data at higher energies, where relativistic effects become important,[45] and where the analysis must be made on the basis of the equations of relativistic interaction theory. We should like to point out that the proposed methods can be generalized to the case of relativistic energies. For example, the variable-phase functions can be analyzed similarly on the basis of quasirelativistic equations.[46]

The transition to the analysis of experimental data on the scattering of three or more nucleons at high energy obviously requires, in addition to the relativistic generalization of many-particle scattering theory developed in quantum mechanics, the introduction of additional assumptions about the nature of the scattering, namely about the important role of the many-particle forces.[47] Note that the transition to high energies in the analysis of experimental data with a view to establishing the properties of the *NN* interaction at short distances will undoubtedly make necessary a simultaneous study of the properties of ΛN, NN^*, and other interactions.[48,49]

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