# Quasiparticle-phonon model of the nucleus I. Basic propositions

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**The general assumptions of the quasiparticle-phonon model of complex nuclei are given. The choice of the model Hamiltonian as an average field and residual forces is discussed. The phonon description and quasiparticlephonon interaction are presented. The system of basic equations and their approximate solutions are obtained. The approximation is chosen so as to obtain the most correct description of few-quasiparticle components rather than of the whole wave function. The method of strength functions is presented, which plays a decisive role in the practical realization of the quasiparticle-phonon model for the description of some properties of complex nuclei. It is shown that the quasiparticle-phonon nuclear model can be applied to the few-quasiparticle components of the wave functions at low, intermediate, and high excitation energies averaged in certain energy intervals.**

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## INTRODUCTION

In the framework of the semimicroscopic theory of the nucleus a quasiparticle-phonon model has been constructed to describe the few-quasiparticle components of the wave functions of complex nuclei at low, intermediate, and high excitation energies. The quasiparticle-phonon model is formulated by describing the low lying states of nuclei as quasiparticle[1] and single-phonon[2] states, and this involves a generalization of phonons and the interaction of quasiparticles with phonons.[3] Interactions between quasiparticles and phonons are important when the energies and wave functions of nonrotational states of odd nuclei are calculated.[4-6] Methods of describing low lying states of nuclei were generalized and used to study the increasingly complex structure of states at higher excitation energies[7] and to study the structure of states at intermediate and high excitation energies. The analysis made in Ref. 8 showed that a unified description of the excited states of nuclei is possible.

The quasiparticle-phonon model is based on the following propositions[7,9-11]:

1) two-quasiparticle and vibrational states are regarded as single-phonon states;

2) the coupling between the single-particle and collective motions is described as the interaction of quasiparticles with phonons;

3) the basic approximation is chosen so as to obtain the most accurate description of only the few-quasiparticle components and not of the complete wave function.

The wave functions of highly excited states of complex nuclei consist of a few million components. It is a very complicated problem to find every such wave function. This was demonstrated, for example, in Ref. 12 for light nuclei, where matrices of very high order were diagonalized to calculate the energies and the wave functions. Investigations in the framework of the approach based on an operator form of the wave function[13-15] have demonstrated that characteristics of highly excited states such as the total photoexcitation cross sections, the spectroscopic factors of single-nucleon transfer reactions, the neutron strength functions, the partial radiation strength functions for direct transitions to low lying states, and a number of other characteristics are determined by the few-quasiparticle components of the wave functions of the states. The problem is considerably simplified if it is necessary to obtain a good description of only the few-quasiparticle parts of the wave functions of the states in a certain energy interval. In this case, one must use a formalism with a different type of strength function.

At the present stage of development of the quasiparticle-phonon model, it is possible to calculate the fragmentation (distribution of the strength) of the one-quasiparticle and one-phonon states and the quasiparticle-plus-phonon states over many nuclear levels. This makes it possible to study a large number of nuclear processes and the properties of complex nuclei in a wide range of excitation energies.

In this review, we present the basic propositions and results of the quasiparticle-phonon model of complex (medium and heavy) nuclei. For the sake of definiteness, the expressions are given for deformed nuclei, although the material applies equally to spherical nuclei. The main results of the calculations of the properties of spherical and deformed nuclei will be given in following papers.

## 1. HARTREE-FOCK-BOGOLYUBOV METHOD AND SEMIMICROSCOPIC DESCRIPTION

1. The Hartree-Fock-Bogolyubov method is the most fundamental and widely used method for solving the nuclear many-body problem. The majority of the equations solved in nuclear theory are special cases of the basic equations obtained with this method. The Hartree-Fock-Bogolyubov method enables one to express the higher correlation functions in terms of the lower functions in a definite manner. As a result, the equations of motion can be written in closed form. The basic stages formulated above make it possible to present the logical thread of the solution of the nuclear many-

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body problem.

We write the Hamiltonian of the system in the general form

$$H = \sum_{f, f'} T'(f, f') a_f^+ a_{f'} - \frac{1}{4} \sum_{f_1, f_2, f_2', f_1'} G(f_1, f_2, f_2', f_1') a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'}, \quad (1)$$

where $f$ are the quantum numbers that characterize the state of a nucleon. The nucleon absorption and creation operators $a_f$ and $a_f^+$ satisfy the commutation relations

$$a_{f_1}^+ a_{f_2} + a_{f_2} a_{f_1}^+ = \delta_{f_1, f_2}; \quad a_{f_1} a_{f_2} + a_{f_2} a_{f_1} = 0,$$

and, further,

$$T'(f, f') = T(f, f') - \lambda \delta_{f, f'},$$

where $\lambda$ is the chemical potential.

We introduce the correlation function

$$\Phi(f_1, f_2) = \langle | a_{f_1} a_{f_2} | \rangle \quad (2)$$

and the density function

$$\rho(f_1, f_2) = \langle | a_{f_1}^+ a_{f_2} | \rangle, \quad (3)$$

with

$$\Phi(f_1, f_2) = -\Phi(f_2, f_1); \qquad \rho^*(f_2, f_1) = \rho(f_1, f_2);$$

here, the expectation value is taken with respect to an arbitrary state $|\rangle$.

We consider the amplitudes $a_f$ in the Heisenberg picture, in which $a_f(t)$ depend explicitly on the time. We introduce the time-dependent functions $\rho_t(f_1, f_2)$, $\Phi_t(f_1, f_2)$. The equations of motion yield the exact relations

$$i \frac{\partial \rho_t(f_1, f_2)}{\partial t} = \langle | [a_{f_1}^+(t) a_{f_2}(t), H] | \rangle; \quad (4)$$

$$i \frac{\partial \Phi_t(f_1, f_2)}{\partial t} = \langle | [a_{f_1}(t) a_{f_2}(t), H] | \rangle. \quad (5)$$

Further, we must write down equations for $\langle | a_{f1}^+(t) a_{f2}^+(t) a_{f'2}(t) a_{f'1}(t) | \rangle$ and $\langle | a_{f1}^+(t) a_{f2}(t) a_{f'2}(t) a_{f'1}(t) | \rangle$ and express them in terms of the distribution functions of higher order, etc. A closed system of approximate equations can be obtained by some approximation which expresses the higher correlation functions in terms of the lower ones.

In nuclear theory, the Hartree-Fock-Bogolyubov method is used to obtain approximate equations by representing the higher correlation functions in terms of the lower ones. For example, the function $\langle | a_{f'1}^+ a_{f'2}^+ a_{f1} a_{f2} | \rangle$ can be expressed as follows in terms of $\rho$ and $\Phi$:

$$\langle | a_{f_1'}^+ a_{f_2'}^+ a_{f_1} a_{f_2} | \rangle = \rho(f_1', f_2) \rho(f_2', f_1) - \rho(f_1', f_1) \rho(f_2', f_2) + \Phi^*(f_2', f_1') \Phi(f_1, f_2), \quad (6)$$

where the right-hand side contains only the first term in the Hartree approximation. Fock added the second term, which takes into account the antisymmetrization, while Bogolyubov introduced the third term, which made it possible to describe pairing correlations of superconducting type. If an approximation of the type (6) is used, Eqs. (4) and (5) are closed. Symbolically, we write these equations as

$$i \frac{\partial \rho_t(f_1, f_2)}{\partial t} = \mathfrak{B}(f_1, f_2); \quad (7)$$

$$i \frac{\partial \Phi_t(f_1, f_2)}{\partial t} = \mathfrak{A}(f_1, f_2). \quad (8)$$

In the stationary case, they are as follows:

$$\mathfrak{B}(f_1, f_2) = 0; \quad (7')$$

$$\mathfrak{A}(f_1, f_2) = 0. \quad (8')$$

The explicit form of these functions is given in Ref. 16. The development of the Hartree-Fock-Bogolyubov method was made possible by Bogolyubov's introduction of quasiaverages in a rigorous mathematical formulation.[17]

2. To arrive at the next stage of transformations, we separate from the set $f$ of quantum numbers the quantum number $\sigma = \pm 1$ such that the states $f = q\sigma$ differing by the sign of $\sigma$ are conjugate under the time reversal operation. For example, $\sigma$ can be the sign of projection of the angular momentum onto the symmetry axis of the nucleus.

For virtually any interaction between the nucleons, one can find a linear unitary transformation that simultaneously reduces the function $\rho(f, f')$ to diagonal form and $\Phi(f, f')$ to canonical form, i.e.,

$$\Phi(f, f') = \Phi(f) \delta_{f, -f'} = \Phi(\sigma q) \delta_{q, q'} \delta_{\sigma, -\sigma'}; \quad \rho(f, f') = \rho(f) \delta_{f, f'} = \rho(q) \delta_{q, q'} \delta_{\sigma, \sigma'}. \quad (9)$$

Then the functions $\Phi(q)$ and $\rho(q)$ are related by

$$\rho(q) = \rho^2(q) + \Phi^*(q) \Phi(q). \quad (10)$$

In this representation, the expectation value of the energy operator has the form

$$\langle | H | \rangle = \sum_f \left\{ T'(f) - \frac{1}{2} \sum_{f'} G(f, f'; f', f) \rho(f') \right\} \rho(f) - \sum_{q, q'} G(q^+, q^-; q'^-, q'^+) \Phi^*(q) \Phi(q'). \quad (11)$$

The basic equations are written as

$$2[E(q) - \lambda] \Phi(q) - [1 - 2\rho(q)] \sum_{q'} G(q^+, q^-; q'^-, q'^+) \Phi(q') = 0; \quad (12)$$

$$N = 2 \sum_q \rho(q). \quad (13)$$

Here, $N$ is the number of neutrons or protons. Equation (12) contains the energies of the single-particle levels of the average field, which have the form

$$E(q) = T(q) - \sum_{q'\sigma'} G(q+, q'\sigma'; q'\sigma', q+) \rho(q'). \quad (14)$$

In the framework of the microscopic approach, expressions of the form (14) are calculated on the basis of experimental nucleon-nucleon scattering data.

Thus, from the general form of the potential describing the interactions between the nucleons the average field of the nucleus and the interactions leading to pairing correlations of superconducting type have been separated out. In nuclear theory one uses the postulate that the average field of the nucleus corresponds to a representation in which the density matrix $\rho(f, f')$ is diagonal for the ground states of even-even nuclei in the $\beta$-stability region. In this representation, the residual forces reduce entirely to interactions that lead to pairing correlations of superconducting type. It is therefore unnecessary to take into account any other residual interactions.

The possibility of separating the average field of the nucleus is not a mathematical device but a reflection of fundamental properties of the nucleus. It is due, first, to the effect of the Pauli principle and, second, to the ratio of the Fermi-surface momentum to the momentum of the repulsive core of the nucleon-nucleon potential. The presence of the average field of the nucleus or the nuclear shells is responsible for numerous nuclear properties. Therefore, the nucleus cannot be regarded as a fragment of nuclear matter of a particular size; rather, it is necessary to study the structure of each nucleus separately. This brings out a fundamental difference from, for example, a crystal, in which it is meaningful to study the structure of crystals of the same kind but of different sizes.

It is possible that the average field phenomenon will not apply to the hypothetical superdense state of a nucleus, and the tremendous variety of nuclear properties will probably be replaced in this state by undifferentiated nuclei representing different-sized pieces of nuclear matter.

From Eqs. (9)–(14) we obtain the basic equations of the theory of pairing correlations of superconducting type. It is assumed that the function $G(q^+, q^-; q'^-, q'^+)$ does not depend on $q$ and $q'$ and varies from nucleus to nucleus as $A^{-1}$. These assumptions are well satisfied, and two constants $G_N$ and $G_Z$, which are determined from the experimental data on the pairing energies, are used in the theory. We introduce the functions $u_q$ and $v_q$:

$$\rho(q) = v_q^2, \quad \Phi(q) = \Phi^*(q) = u_q v_q; \tag{15}$$

then the condition (10) takes the form

$$1 = v_q^2 + u_q^2, \tag{16}$$

and Eqs. (12) and (13) become

$$2[E(q) - \lambda] u_q v_q - (u_q^2 - v_q^2) G \sum_{q'} u_{q'} v_{q'} = 0;$$

$$N = 2 \sum_q v_q^2.$$

We introduce the correlation function

$$C = G \sum_{q'} u_{q'} v_{q'} \tag{17}$$

and after simple transformations we obtain

$$1 = \frac{G}{2} \sum_q \frac{1}{\sqrt{C^2 + \{E(q) - \lambda\}^2}}; \tag{18}$$

$$N = \sum_q \left\{1 - \frac{E(q) - \lambda}{\varepsilon(q)}\right\}; \tag{19}$$

$$v_q^2 = \frac{1}{2}\left\{1 - \frac{E(q) - \lambda}{\varepsilon(q)}\right\}; \quad \varepsilon(q) = \sqrt{C^2 + \{E(q) - \lambda\}^2}. \tag{20}$$

3. There exist two types of excited states (besides the rotational states). For one of them, the conditions (9) are satisfied; these are quasiparticle excitations. For the other, the conditions (9) are not satisfied; these are vibrational states. It was shown in Ref. 18 that the vibrational states are associated with the nondiagonal parts of the density matrix; nondiagonal increments $\delta\rho(q\sigma, q'\sigma')$ and $\delta\Phi(q\sigma, q'\sigma')$ were introduced and equations obtained for them. In Ref. 19, the average field was explicitly separated and the equations were reduced to the form

$$[\varepsilon(q_1) + \varepsilon(q_2)] Z^{(\pm)}(q_1, q_2) - \omega Z^{(\mp)}(q_1, q_2)$$
$$- \sum_{q_1', q_2'} G^{\xi}(q_1, q_2; q_2', q_1') v^{(\mp)}_{q_1 q_2} v^{(\mp)}_{q_1', q_2'} Z^{(\pm)}(q_1', q_2')$$
$$-2 \sum_{q_1', q_2'} G^{\omega}(q_1, q_2; q_2', q_1') u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} Z^{(\pm)}(q_1', q_2') = 0, \tag{21}$$

where

$$u^{(\pm)}_{q_1, q_2} = u_{q_1} v_{q_2} \pm u_{q_2} v_{q_1}; \quad v^{(\pm)}_{q_1 q_2} = u_{q_1} u_{q_2} \pm v_{q_1} v_{q_2}. \tag{22}$$

When studying excited states of complex nuclei, one must bear in mind that the interaction $G$ is manifested in two channels. Collective effects associated with quadrupole, octupole, and other vibrations and also with giant multipole resonances are generated by interactions in the particle-hole channel, which are denoted by $G^{\omega}(q_1 q_2; q_2' q_1')$. Interactions in the particle-particle channel are denoted by $G^{\xi}(q_1 q_2; q_2' q_1')$. Interactions of such type with total angular momentum equal to zero generate pairing correlations of superconducting type. In a number of cases, interactions in the particle-particle channel with nonvanishing total angular momentum are taken into account. The terms of Eqs. (21) containing $u^{\pm}_{q_1 q_2}$ describe the interactions in the particle-hole channel; the terms containing $v^{\pm}_{q_1 q_2}$ describe the interactions in the particle-particle channel.

In the derivation of Eqs. (21), the interaction between the quasiparticles is taken in the most general form. It was shown in Ref. 19 that all equations in nonphenomenological theories used to describe vibrations of the nucleus are special cases of Eqs. (21).

Note that equations analogous to (21) were obtained in Ref. 20. In Ref. 19, Eqs. (21) were generalized to the case of an external field and the equations of the theory of finite Fermi systems[21] were obtained from them.

In Ref. 22, Eqs. (21) were derived from the equations of the theory of finite Fermi systems.

The quasiparticle-phonon model is constructed in the framework of the semimicroscopic theory of the nucleus. In semimicroscopic theories, the average field and the residual or effective interactions are not calculated but are specified in a definite form deduced from our overall knowledge of the structure of the nucleus. Therefore, in such theories one calculates relative and not absolute values. For example, one calculates the excitation energies and not the total energies of the nucleus in the ground and excited states, or one determines the variations of the energy of the nucleus with increasing deformation parameter, etc.

There are many different variants of the semimicroscopic description. We use one of them. The basic propositions of the semimicroscopic description of nuclear structure can be formulated as follows[23]:

1) the Hartree-Fock-Bogolyubov method is used to obtain a closed system of equations for the density functions and correlation functions. This is the principal approximation in the nuclear many-body problem;

2) a representation is chosen in which the density

matrix is diagonal and the correlation function has canonical form. In this representation, all the interactions between the nucleons in the nucleus reduce to an average field and to interactions which lead to pairing;

3) the average field is separated out and described by the Woods-Saxon potential, and it is postulated that the choice of the average field corresponds to the representation mentioned above for even-even nuclei in the $\beta$-stability region. The average field determines many properties of the nucleus directly and makes it possible to elucidate the effect of the residual forces;

4) excited states are determined as one-, two-, three-, etc., quasiparticle states;

5) the low lying vibrational states are associated with the nondiagonal elements of the density matrix, and multipole-multipole and spin-multipole–spin-multipole forces are introduced to describe them. The mathematical treatment is based on various forms of the method of approximate second quantization developed by Bogolyubov[24];

6) the rotational, quasiparticle, and phonon excited states are coupled through the Coriolis interaction and the interaction of the quasiparticles with the phonons.

## 2. HAMILTONIAN OF THE MODEL

1. In semimicroscopic nuclear theory the Hamiltonian describing different types of nuclear motions has the form

$$H=H_{\text{av}}+H_{\text{pair}}+T_{\text{rot}}+H_{\text{cor}}+H_Q+H_{\sigma Q}+H', \tag{23}$$

where $H_{\text{av}}$ is the average field of the neutron and proton systems; $H_{\text{pair}}$ are the interactions leading to superconducting pairing correlations; $T_{\text{rot}}$ is the rotational kinetic energy; $H_{\text{cor}}$ is the Coriolis interaction, which describes the coupling between the intrinsic motion and the rotation; $H_Q$ is the multipole-multipole interaction; $H_{\sigma Q}$ is the spin-multipole–spin-multipole interaction; $H'$ are other interactions, including, for example, an interaction of Gamow-Teller type.

To describe the states whose structure is associated with the nondiagonal parts of the density matrix, residual interactions are introduced. We write the central residual interaction in the form

$$\begin{gathered}V(|\mathbf{r}_1-\mathbf{r}_2|)+V_\sigma(|\mathbf{r}_1-\mathbf{r}_2|)(\sigma^{(1)}\sigma^{(2)})\\+\{V_\tau(|\mathbf{r}_1-\mathbf{r}_2|)+V_{\tau\sigma}(|\mathbf{r}_1-\mathbf{r}_2)(\sigma^{(1)}\sigma^{(2)})\}(\tau^{(1)}\tau^{(2)})\end{gathered} \tag{24}$$

and expand it in a series in spherical functions, obtaining

$$\begin{gathered}V(|\mathbf{r}_1-\mathbf{r}_2|)=\sum_{\lambda=0}R_\lambda(r_1,r_2)\frac{4\pi}{2\lambda+1}\sum_{\mu=-\lambda}^{\lambda}(-1)^\mu\\\times Y_{\lambda\mu}(\theta_1,\varphi_1)Y_{\lambda-\mu}(\theta_2,\varphi_2);\end{gathered} \tag{25}$$

$$\begin{gathered}V_\sigma(|\mathbf{r}_1-\mathbf{r}_2|)(\sigma^{(1)}\sigma^{(2)})=\sum_{s=0}^{\infty}R_s^\sigma(r_1,r_2)\frac{4\pi}{2s+1}\sum_{\lambda=s,\,s\pm1}(-1)^{s+1-\lambda}\\\times\sum_{\mu=-\lambda}^{\lambda}(-1)^\mu\{\sigma^{(1)}Y_s(\theta_1,\varphi_1)\}_{\lambda\mu}\{\sigma^{(2)}\mathbf{Y}_s(\theta_2,\varphi_2)\}_{\lambda,-\mu}.\end{gathered} \tag{26}$$

The expansions of the functions $V_\tau$ and $V_{\tau\sigma}$ have a similar form. Here

$$\{\sigma\mathbf{Y}_s(\theta_1,\varphi_1)\}_{\lambda\mu}=\sum_{\rho=0,\pm1}\sum_{p=-s}^{s}(1\rho sp|\lambda\mu)\sigma_\rho Y_{sp}(\theta_1,\varphi_1), \tag{27}$$

where $r_i,\theta_i,\varphi_i,\sigma^{(i)}$ determine the position and spin of the particle; the functions $R_\lambda(r_1,r_2)$ and $R_s^\sigma(r_1,r_2)$ describe the radial dependence. Thus, the most general form of the central potential is given by series in multipoles and spin-multipoles.

The existence of static quadrupole deformation of the nuclei in the region of the rare earths and of the actinides indicates that the quadrupole-quadrupole interaction plays an important part. The part of this interaction that does not reduce to an average field must describe the interaction between quasiparticles. Thus, the part $H''$ of the residual interaction can be approximated by terms of multipole-multipole and spin-multipole–spin-multipole interactions.

The radial part of the interaction is chosen differently. To obtain a simple secular equation rather than diagonalize a matrix of high order, it is necessary to take the functions $R_\lambda(r_1,r_2)$ and $R_s^\sigma(r_1,r_2)$ in factorized form:

$$R_\lambda(r_1,r_2)=\varkappa_0^{(\lambda)}R(r_1)R(r_2);\quad R_s^\sigma(r_1,r_2)=\varkappa_{\sigma0}^{(s)}R(r_1)R(r_2).$$

It is very common to choose these functions in the form

$$\begin{aligned}R_\lambda(r_1,r_2)&=\varkappa_0^{(\lambda)}r_1^\lambda r_2^\lambda;\\R_s^\sigma(r_1,r_2)&=\varkappa_{\sigma0}^{(s)}r_1^s r_2^s.\end{aligned} \tag{28}$$

The corresponding expansions of the functions $V_\tau(|\mathbf{r}_1-\mathbf{r}_2|)$ and $V_{\tau\sigma}(|\mathbf{r}_1-\mathbf{r}_2|)$ contain the constants $\varkappa_1^{(\lambda)}$ and $\varkappa_{\sigma1}^{(s)}$. The neutron-neutron $\varkappa_{nn}$, proton-proton $\varkappa_{pp}$, and neutron-proton $\varkappa_{np}$ constants are related to the isoscalar $\varkappa_0$ and isovector $\varkappa_1$ constants by

$$\begin{aligned}\varkappa_{nn}=\varkappa_{pp}&=\varkappa_0+\varkappa_1;\\\varkappa_{np}&=\varkappa_0-\varkappa_1.\end{aligned} \tag{29}$$

The interaction (28) is strongest when both particles are near the surface of the nucleus; for $r>R_0$ the corresponding single-particle matrix elements decrease very rapidly with increasing $r$ and the strength of the interaction decreases rapidly. Within the nucleus, the strength of the interaction decreases steadily. For the study of coherent effects, one frequently has $r_1=r_2$ and the radial part takes the form $R_\lambda=\varkappa^{(\lambda)}r^{2\lambda}$.

Thus, the interaction (28) has its greatest strength near the surface of the nucleus. Therefore, results of calculations with the interaction (28) of low lying vibrational states are close to the results of calculations with a surface $\delta$-function interaction,[25] and for calculations of giant multipole resonances they are close to the results of calculations with the new Skyrme interaction.[26]

Calculations of nuclear characteristics made with different residual forces give similar results. This indicates that for the calculation of matrix elements the detailed radial dependence is not manifested strongly because of the effect of the single-particle wave functions. It was shown in Ref. 27 that the two-particle transition density acts as a filter, retaining definite Fourier components of the effective forces. It was demonstrated in Ref. 27 that residual interactions used in a certain restricted configuration space have the same manifestations if their Fourier components are nearly equal in a comparatively narrow range of momentum transfers. It may be concluded that the use of the radial dependence of the residual forces in the form (28) is justified. There are no convincing arguments for using any other definite radial dependence.

2. We now turn to the construction of the Hamiltonian of the quasiparticle-phonon nuclear model intended to calculate the few-quasiparticle components of the wave functions of spherical and deformed complex nuclei at low, intermediate, and high excitation energies. For definiteness, the expressions are derived for deformed nuclei, though they can be rewritten unambiguously for spherical nuclei as well.

It is well known that rotational motion and its coupling to quasiparticle and phonon excitations play an important part in nuclei. Rotation, especially with large angular momenta, has been described in detail in, for example, Refs. 28–30. In the quasiparticle-phonon model, the coupling to the rotation is ignored in the majority of cases, and the rotation itself is described crudely. This is so because one does not consider high states with very large angular momentum. For the study of low lying states, the coupling to the rotation can be taken into account in each concrete case. In states with low angular momenta at intermediate and high excitation energies, the coupling to the rotation does not lead to a significant redistribution of the strength of the few-quasiparticle components of the wave functions. Therefore, we do not introduce the rotational kinetic energy or the Coriolis interaction explicitly into the model Hamiltonian; they can be added when necessary.

To construct the general form of the model Hamiltonian, we use the expressions given in Ref. 16. From the Hamiltonian (23), we choose the necessary terms and express them in the form

$$H_M = H_p + H_Q + H_{\sigma Q}, \tag{30}$$

where

$$H_p = H_0 + H_{pp} \tag{31}$$

includes the interactions which lead to pairing correlations of superconducting type and interactions in the particle-particle channel with nonzero angular momentum;

$$H_0 = H_0(n) + H_0(p); \tag{32}$$

$$H_0(n) = \sum_s \varepsilon(s) B(s, s) + H_0^\beta(n) + H_0''(n); \tag{33}$$

here (see Ref. 16, p. 207 of the Russian edition)

$$H_0^\beta(n) = -\frac{G_N}{2} \sum_{s, s'} [u_s^2 A^+(s, s) - v_s^2 A(s, s)] \times [u_{s'}^2 A(s', s') - v_s^2 A^+(s', s')]; \tag{34}$$

$$H_0''(n) = -\frac{G_N}{\sqrt{2}} \sum_{s, s'} (u_s^2 - v_s^2) u_{s'} v_{s'} [A^+(s, s) B(s', s') + B(s', s') A(s, s)]. \tag{35}$$

We use the notation

$$A(q, q') = \frac{1}{\sqrt{2}} \sum_\sigma \sigma \alpha_{q'\sigma} \alpha_{q-\sigma} \quad \text{or} \quad \frac{1}{\sqrt{2}} \sum_\sigma \alpha_{q\sigma} \alpha_{q'\sigma}; \tag{36}$$

$$B(q, q') = \sum_\sigma \alpha_{q\sigma}^+ \alpha_{q'\sigma} \quad \text{or} \quad \sum_\sigma \sigma \alpha_{q-\sigma}^+ \alpha_{q'\sigma}, \tag{37}$$

where $\alpha_{q\sigma}^+$ is the quasiparticle creation operator; we characterize the single-particle state by the quantum numbers $s\sigma$ for the neutron system, $r\sigma$ for the proton system, and $q\sigma$ for both systems ($\sigma = \pm 1$).

We write the interaction in the particle-particle channel with nonzero angular momentum in the form

$$H_{pp} = -\sum_\lambda \frac{G_\lambda}{2} \sum_\mu P_{\lambda\mu}^+ P_{\lambda\mu}; \tag{38}$$

$$P_{\lambda\mu} = \sum_{\substack{qq'\\ \sigma\sigma'}} \langle q\sigma | \tilde{\tilde{f}}^{\lambda\mu} | q'\sigma' \rangle a_{q'\sigma'}^+ a_{q\sigma}^+ = \sum_{qq'} \tilde{\tilde{f}}^{\lambda\mu}(q, q') \left\{ \frac{1}{\sqrt{2}} (A^+(qq') + A(qq')) v_{qq'}^{(-)} + \frac{1}{\sqrt{2}} (A^+(qq') - A(qq')) v_{qq'}^{(+)} - B(qq') (u_{qq'}^{(+)} + u_{qq'}^{(-)}) \right\}. \tag{39}$$

The multipole-multipole interaction has the form

$$H_Q = -\frac{1}{2} \sum_\lambda \sum_{\mu \geqslant 0} \{(\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)}) [Q_{\lambda\mu}^+(n) Q_{\lambda\mu}(n) + Q_{\lambda\mu}^+(p) Q_{\lambda\mu}(p)] + (\varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)}) [Q_{\lambda\mu}^+(n) Q_{\lambda\mu}(p) + Q_{\lambda\mu}^+(p) Q_{\lambda\mu}(n)]\}, \tag{40}$$

where

$$Q_{\lambda\mu}(n) = \sum_{\substack{ss'\\ \sigma\sigma'}} \langle s\sigma | f^{\lambda\mu} | s'\sigma' \rangle a_{s\sigma}^+ a_{s'\sigma'} \approx \sum_{ss'} f^{\lambda\mu}(ss') \left\{ u_{ss'}^{(+)} (A(ss') + A^+(ss')) \frac{1}{\sqrt{2}} + v_{ss'}^{(-)} B(ss') \right\}. \tag{41}$$

Here

$$f^{\lambda\mu} = \frac{r^\lambda}{\sqrt{2(1+\delta_{\mu 0})}} (Y_{\lambda\mu} + (-1)^\mu Y_{\lambda-\mu})$$

and, in contrast to Ref. 16, we do not distinguish between the matrix elements $f^{\lambda\mu}(q, q')$ and $\overline{f}^{\lambda\mu}(qq')$.

The spin-multipole–spin-multipole interaction has the form

$$H_{\sigma Q} = -\frac{1}{2} \sum_\lambda \sum_{\mu=-\lambda}^{\lambda} \sum_{s=\lambda,\ \lambda\pm 1} \{(\varkappa_{\sigma 0}^{(s)} + \varkappa_{\sigma 1}^{(s)}) [T_{\lambda\mu s}^+(n) T_{\lambda\mu s}(n) + T_{\lambda\mu s}^+(p) T_{\lambda\mu s}(p)] + (\varkappa_{\sigma 0}^{(s)} - \varkappa_{\sigma 1}^{(s)}) [T_{\lambda\mu s}^+(n) T_{\lambda\mu s}(p) + T_{\lambda\mu s}^+(p) T_{\lambda\mu s}(n)]\}. \tag{42}$$

where

$$T_{\lambda\mu s}(n) = \sum_{s_2 s_2'} \langle s_2\sigma | r^s [\{\sigma Y_s\}_{\lambda\mu} + (-1)^\mu \{\sigma Y_s\}_{\lambda, -\mu}] | s_2'\sigma' \rangle a_{s_2\sigma}^+ a_{s_2'\sigma'} \approx \sum_{s_2 s_2'} f_s^{\lambda\mu}(s_2 s_2') \left\{ \frac{1}{\sqrt{2}} u_{s_2 s_2'}^{(-)} [\mathfrak{A}(s_2 s_2') + \mathfrak{A}^+(s_2 s_2')] + v_{s_2 s_2'}^{(+)} \mathfrak{B}(s_2, s_2') \right\}. \tag{43}$$

Here

$$\mathfrak{A}(ss') = \frac{1}{\sqrt{2}} \sum_\sigma \alpha_{s'\sigma} \alpha_{s-\sigma} \quad \text{or} \quad \frac{1}{\sqrt{2}} \sum_\sigma \sigma \alpha_{s\sigma} \alpha_{s'\sigma}; \tag{44}$$

$$\mathfrak{B}(ss') = \sum_\sigma \sigma \alpha_{s+\sigma}^+ \alpha_{s'-\sigma} \quad \text{or} \quad \sum_\sigma \alpha_{s-\sigma}^+ \alpha_{s'\sigma}. \tag{45}$$

## 3. SINGLE-PHONON STATES

1. We consider the single-phonon states generated by the multipole-multipole forces in the particle-hole channel. We take the following part of the model Hamiltonian:

$$H_v = \sum_q \varepsilon(q) B(q, q) + H_0^\beta + H_Q^v, \tag{46}$$

where $H_Q^v$ is the part of $H_Q$ (40) not containing terms describing the interactions between the quasiparticles and the phonons.

We introduce the phonon creation operator

$$Q_t^+=\frac{1}{2}\sum_{qq'}\{\psi_{qq'}^t A^+(qq')-\varphi_{qq'}^t A(qq')\};\qquad(47)$$

here $t=\lambda\mu i$ and $i$ is the number of the state with the given $\lambda\mu$. After simple transformations,[16] we obtain

$$H_v=\sum_q \varepsilon(q)B(qq)-\frac{1}{4}\delta_{\lambda\mu,20}\sum_{ii'}\Big\{G_N\sum_{ss'}[(u_s^2-v_s^2)(u_{s'}^2-v_{s'}^2)g_{ss}^{20i}\times g_{s's'}^{20i}+w_{ss}^{20i}w_{s's'}^{20i}]+G_z\sum_{rr'}[(u_r^2-v_r^2)(u_{r'}^2-v_{r'}^2)g_{rr}^{20i}g_{r'r'}^{20i}+w_{rr}^{20i}w_{r'r'}^{20i}]\Big\}Q_{20i}^+Q_{20i}-\frac{1}{2}\sum_{tt'}\Big\{(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})\times\Big[\sum_{\substack{ss'\\s_2s_2'}}u_{ss'}^{(+)}u_{s_2s_2'}^{(+)}f^t(ss')g_{ss'}^t f^{t'}(s_2s_2')g_{s_2s_2'}^{t'}+\sum_{\substack{rr'\\r_2r_2'}}u_{rr'}^{(+)}u_{r_2r_2'}^{(+)}f_{(rr')}^t g_{rr'}^t f_{(r_2r_2')}^{t'}g_{r_2r_2'}^{t'}\Big]+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})\sum_{\substack{ss'\\rr'}}u_{ss'}^{(+)}u_{rr'}^{(+)}\times[f_{(ss')}^t g_{ss'}^t f_{(rr')}^{t'}g_{rr'}^{t'}+f_{(ss')}^{t'}g_{ss'}^{t'}f_{(rr')}^t g_{rr'}^t]\Big\}Q_t^+Q_{t'},\qquad(48)$$

where $g_{qq'}^t=\psi_{qq'}^t+\varphi_{qq'}^t$; $w_{qq'}^t=\psi_{qq'}^t-\varphi_{qq'}^t$; the matrix element $f^t(qq')\equiv f^{\lambda\mu}(qq')$ does not depend on $i$.

The wave function of the single-phonon state has the form

$$Q_t^+\Psi_0;\qquad(49)$$

here, the wave function $\Psi_0$ of the ground state of an even-even nucleus is the phonon vacuum:

$$Q_t\Psi_0=0.\qquad(50)$$

The normalization condition (49) is written as

$$\sum_{qq'}g_{qq'}^t w_{qq'}^{t'}=2\delta_{tt'}.\qquad(51)$$

Following Ref. 16, we find the energies $\omega_t$ of single-phonon states with fixed $\lambda\mu$ or $K^\pi$ by means of the variational principle

$$\delta\{\langle Q_t H_v Q_t^+\rangle-\omega_t[\sum_{qq'}g_{qq'}^t w_{qq'}^t-2]/2\}=0.\qquad(52)$$

As a result of transformations, we obtain a secular equation in the form

$$\begin{vmatrix}(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X^t(n)-1 & (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X^t(n)\\(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X^t(p) & (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X^t(p)-1\end{vmatrix}=0,\qquad(53)$$

which coincides with (8.134) of Ref. 16. Here

$$X^t(n)=2\sum_{ss'}\frac{f^t(ss')\tilde{f}^t(ss')u_{ss'}^2\varepsilon(ss')}{\varepsilon^2(ss')-\omega_g^2},\qquad(54)$$

where

$$\tilde{f}^t(ss')=f^t(ss')-\frac{\Gamma_n^t(s)}{\gamma_n^g}\delta_{s,s'};\ \varepsilon(ss')=\varepsilon(s)+\varepsilon(s');\qquad(55)$$

$$\gamma_n^t=\sum_{ss'}\frac{4C_n^2-\omega_t^2+4\epsilon(s)\epsilon(s')}{\varepsilon(s)(4\varepsilon^2(s)-\omega_t^2)\varepsilon(s')(4\varepsilon^2(s')-\omega_t^2)};\qquad(56)$$

$$\Gamma_n^t(s)=\sum_{s_2's_2'}\frac{f^t(s_2s_2')[4C_n^2-\omega_t^2+4\epsilon(s_2)\epsilon(s_2')-4\epsilon(s)\epsilon(s_2)+4\epsilon(s)\epsilon(s_2')]}{\varepsilon(s_2)[4\varepsilon^2(s_2)-\omega_t^2]\varepsilon(s_2')[4\varepsilon^2(s_2')-\omega_t^2]};\qquad(57)$$

$\epsilon(s)=E(s)-\lambda_n$. It can be seen from (55) that if $\lambda\mu\neq 20$ then $\bar{f}^t(ss')$ is equal to $f^t(ss')$.

Equation (53) can be written in the form

$$\mathcal{F}(\omega)=\varkappa_0^{(\lambda)}\varkappa_1^{(\lambda)}(X^t(n)-X^t(p))^2-(1-\varkappa_0^{(\lambda)}X^t)(1-\varkappa_1^{(\lambda)}X^t)=0,\qquad(58)$$

where $X^t=X^t(n)+X^t(p)$.

Note that the influence of the constant $\varkappa_1^{(\lambda)}$ on the first single-phonon states was investigated in Ref. 31, in which it was shown that the introduction of $\varkappa_1^{(\lambda)}$ renormalizes the constant $\varkappa_0^{(\lambda)}$ without significantly changing the structure of the state.

To find the functions $g_{qq'}^t$ and $w_{qq'}^t$, we use the normalization condition (51) and obtain after lengthy calculations[32]

$$g_{rr'}^t=\sqrt{\frac{2}{Y_t}}\,y_p^t\frac{\tilde{f}^t(rr')u_{rr'}^{(+)}\varepsilon(rr')}{\varepsilon^2(rr')-\omega_t^2};\qquad(59)$$

$$w_{rr'}^t=\sqrt{\frac{2}{Y_t}}\,y_p^t\left\{\frac{\tilde{f}^t(rr')u_{rr'}^{(+)}\omega_t}{\varepsilon^2(rr')-\omega_t^2}-\delta_{r,r'}\frac{C_p\Xi_p^t}{\varepsilon(r)\omega_t\gamma_p^t}\right\},\qquad(60)$$

where

$$y_p^t=\frac{(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})X^t(n)}{1-(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})X^t(p)};\qquad(61)$$

$$\Xi_p^t=\sum_{rr'}\frac{f^t(rr')}{\varepsilon(r)(4\varepsilon^2(r)-\omega_t^2)}\frac{4C_p^2-\omega_t^2+4\epsilon(r)\epsilon(r')}{\varepsilon(r')(4\varepsilon^2(r')-\omega_t^2)}.\qquad(62)$$

The expressions for $g_{ss'}^t$, $w_{ss'}^t$ have an analogous form with $C_p$, $\Xi_p^t$, $\gamma_p^t$, $y_p^t$ replaced by the corresponding quantities for neutrons: $C_n$, $\Xi_n^t$, $\gamma_n^t$, $y_n^t\equiv 1$. Here

$$Y_t=Y_t(n)+(y_p^t)^2Y_t(p)=\frac{1}{4}\frac{y_p^t}{\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}}\frac{\partial\mathcal{F}(\omega)}{\partial\omega};\qquad(63)$$

$$Y_t(n)=\sum_{ss'}\frac{(\tilde{f}^t(ss')u_{ss'}^{(+)})^2\varepsilon(ss')\omega_t}{[\varepsilon^2(ss')-\omega_t^2]^2}=\frac{1}{4}\frac{\partial X^t(n)}{\partial\omega};\qquad(64)$$

the expression for $Y_t(p)$ is similar to (64). If the isovector component of the forces is absent ($\varkappa_1^{(\lambda)}=0$), then $y_p^t\equiv 1$ and all the expressions (59)–(64) take the form given in Chap. 8 of Ref. 16, while the secular equation is

$$1-\varkappa_0^{(\lambda)}X^t(\omega_i)=0.\qquad(65)$$

2. To consider spin-multipole phonons of degree $s$, we take the Hamiltonian of the system in the form

$$H_v^s=\sum_q\varepsilon(q)B(qq)=-\frac{1}{2}\sum_{tt'}\Big\{(\varkappa_{\sigma0}^{(s)}+\varkappa_{\sigma1}^{(s)})\Big[\sum_{\substack{s_1s_1'\\s_2s_2'}}u_{s_1s_1'}^{(-)}u_{s_2s_2'}^{(-)}\times f_s^t(s_1s_1')f_s^{t'}(s_2s_2')g_{ss'}^t g_{s_2s_2'}^{t'}+\sum_{\substack{rr'\\r_2r_2'}}u_{rr'}^{(-)}u_{r_2r_2'}^{(-)}f_s^t(rr')f_s^{t'}(r_2r_2')g_{rr'}^t g_{r_2r_2'}^{t'}\Big]+(\varkappa_{\sigma0}^{(s)}-\varkappa_{\sigma1}^{(s)})\sum_{\substack{s_1s_1'\\rr'}}u_{s_1s_1'}^{(-)}u_{rr'}^{(-)}[f_s^t(s_1s_1')g_{s_1s_1'}^t f_s^{t'}(rr')g_{rr'}^{t'}+f_s^{t'}(s_1s_1')g_{s_1s_1'}^{t'}f_s^t(rr')g_{rr'}^t]\Big\}Q_t^+Q_{t'}.\qquad(66)$$

In this case, the secular equation is

$$\varkappa_{\sigma0}^{(s)}\varkappa_{\sigma1}^{(s)}[S^t(n)-S^t(p)]^2-(1-\varkappa_{\sigma0}^{(s)}S^t)(1-\varkappa_{\sigma1}^{(s)}S^t)=0,\qquad(67)$$

where

$$S^t=S^t(n)+S^t(p);\qquad(68)$$

$$S^t(n)=2\sum_{ss'}\{(f^t(ss')u_{ss'}^{(-)})^2\varepsilon(ss')/[\varepsilon^2(ss')-\omega_t^2]\}.\qquad(69)$$

The expressions for $g_{qq'}^t$, $w_{qq'}^t$, and $Y_t^s$ are obtained from (59), (60), and (63) by replacing $u_{qq'}^{(+)}$ and $f^t(qq')$ by $u_{qq'}^{(-)}$ and $f_s^t(qq')$; the expressions for $y_p^{ts}$ are obtained by replacing $X^t(n)$ in (61) by $S^t(n)$ and $X^t(p)$ by $S^t(p)$.

For simultaneous allowance for multipole-multipole and spin-multipole–spin-multipole forces in the particle-hole channel in the case $\lambda=s$ for $\varkappa_1^{(\lambda)}=\varkappa_{\sigma1}^{(s)}=0$ we have

$$(1-\varkappa_0^{(\lambda)}X^t)(1-\varkappa_{\sigma 0}^{(s)}S^t)=\varkappa_0^{(\lambda)}\varkappa_{\sigma 0}^{(s)}(W^t)^2, \quad . \tag{70}$$

where

$$W^t=2\sum_{q,q'}[f_s^t(qq')f^t(qq')u_{qq'}^{(+)}u_{qq'}^{(-)}\omega_t/(\varepsilon^2(qq')-\omega_t^2)]. \tag{71}$$

3. We consider single-phonon states for simultaneous allowance for the multipole-multipole forces in the particle-particle and particle-hole channels. We introduce phonon operators and write the corresponding part of the Hamiltonian (34) for $G_\lambda \equiv G_\lambda(n) = G_\lambda(p) = G_\lambda(pn)$ in the form

$$H_v^{pp}=-\frac{G_\lambda}{2}\sum_i\left\{\left(\sum_{qq'}\tilde{\tilde{f}}^{\lambda\mu}(qq')g_{qq'}^{\lambda\mu i}v_{qq'}^{(-)}\right)^2+\left(\sum_{qq'}\tilde{\tilde{f}}^{\lambda\mu}(qq')w_{qq'}^{\lambda\mu i}v_{qq'}^{(+)}\right)^2\right\}Q_i^+Q_i. \tag{72}$$

In the case of simultaneous allowance for multipole-multipole forces in the particle-particle and particle-hole channels the secular equation is

$$\begin{vmatrix} X^{\lambda\mu i}-\frac{1}{\varkappa^{(\lambda)}} & \mathcal{L}_1^{\lambda\mu i} & \mathcal{L}_2^{\lambda\mu i} \\ \mathcal{L}_1^{\lambda\mu i} & M_{\lambda\mu i}^{(-)}-\frac{1}{G_\lambda} & M_{\lambda\mu i} \\ \mathcal{L}_2^{\lambda\mu i} & M_{\lambda\mu i} & M_{\lambda\mu i}^{(+)}-\frac{1}{G_\lambda} \end{vmatrix}=0, \tag{73}$$

where

$$M_{\lambda\mu i}^{(-)}=2\sum_{q_1q_2}\{(\tilde{\tilde{f}}^{\lambda\mu}(q_1q_2)v_{q_1q_2}^{(-)})^2\varepsilon(q_1q_2)/[\varepsilon^2(q_1q_2)-\omega_i^2]\}; \tag{74}$$

$$M_{\lambda\mu i}^{(+)}=2\sum_{q_1q_2}\{(\tilde{\tilde{f}}^{\lambda\mu}(q_1q_2)v_{q_1q_2}^{(+)})^2\varepsilon(q_1q_2)/[\varepsilon^2(q_1q_2)-\omega_i^2]\}; \tag{75}$$

$$M_{\lambda\mu i}=2\sum_{q_1q_2}\{(\tilde{\tilde{f}}^{\lambda\mu}(q_1q_2))^2v_{q_1q_2}^{(-)}v_{q_1q_2}^{(+)}\omega_i/[\varepsilon^2(q_1q_2)-\omega_i^2]\}; \tag{76}$$

$$\mathcal{L}_1^{\lambda\mu i}=2\sum_{q_1q_2}\{f^{\lambda\mu}(q_1q_2)u_{q_1q_2}^{(+)}\tilde{\tilde{f}}^{\lambda\mu}(q_1q_2)v_{q_1q_2}^{(-)}\varepsilon(q_1q_2)/[\varepsilon^2(q_1q_2)-\omega_i^2]\}; \tag{77}$$

$$\mathcal{L}_2^{\lambda\mu i}=2\sum_{q_1q_2}\{f^{\lambda\mu}(q_1q_2)u_{q_1q_2}^{(+)}\tilde{\tilde{f}}^{\lambda\mu}v_{q_1q_2}^{(+)}\omega_i/[\varepsilon^2(q_1q_2)-\omega_i^2]\}. \tag{78}$$

Equation (73) can be readily obtained from Eqs. (21) by replacing $G^\omega$ and $G^\xi$ by the expressions for $H_v$ from (48) and $H_{pp}^v$ from (72). Analogous equations for spherical nuclei are obtained in Ref. 33.

## 4. PHONON DESCRIPTION

1. The secular equation determining the energies of single-phonon states with fixed $K^\pi$ ($K^\pi \neq 0^+$) takes the following simple form for multipole-multipole forces:

$$2\varkappa_0^{(\lambda)}\sum_{qq'}\frac{(f^{\lambda\mu}(qq')u_{qq'}^{(+)})^2\varepsilon(qq')}{\varepsilon^2(qq')-\omega_i^2}=1. \tag{79}$$

For each solution $\omega_i$ of Eq. (79), the wave function has the form (49). The number of roots $\omega_i$ of this equation is equal to the number of two-quasiparticle states with the same values of $K^\pi$ in the neutron and proton systems. The energies of the two-quasiparticle states are the poles of Eq. (79). If the root $\omega_i$ is far from the corresponding pole, the state is collective. As the root approaches the pole, the state becomes a two-quasiparticle state. In the majority of cases, the roots $\omega_i$ are fairly close to the poles $\varepsilon(qq')$, and the states are weakly collective.

In the secular equation (79), the interaction between the quasiparticles in the particle-hole channel is taken into account. If $q$ is a particle state and $q'$ a hole state, then $(u_{qq'}^{(\pm)})^2 \gtrsim 0.5$; in the majority of cases, $(u_{qq'}^{(\pm)})^2$ is near unity. If both single-particle states $q$ and $q'$ are either particle or hole states, then $(u_{qq'}^{(\pm)})^2$ are very small and such states appear as pure two-quasiparticle states. These states do not in fact influence the collective properties of nuclei. To take into account the part played such states, it is necessary to introduce interactions in the particle-particle channel.

Thus, the roots of the secular equation (79) and the corresponding single-phonon wave functions describe the complete system of states with given $K^\pi$. They include collective, weakly collective, and two-quasiparticle states.

Secular equations of the type (79) are widely used to calculate the energies of the first quadrupole and octupole collective states. The isoscalar constants $\varkappa_0^{(\lambda)}$ are fixed by requiring that the calculated and experimental energies of the first states with the corresponding $K^\pi$ and $I^\pi$ be nearly equal. In deformed nuclei, the same constant $\varkappa_0^{(\lambda)}$ is used to describe the single-phonon states of all nuclei in each $A$ zone.[34] Study of the low lying states makes it possible to fix the parameters of the Woods-Saxon potential, the pairing constants, the isoscalar constants of the quadrupole-quadrupole, $\varkappa_0^{(2)}$, and the octupole-octupole, $\varkappa_0^{(3)}$, interactions. The isovector constants $\varkappa_1^{(\lambda)}$ are determined from the energies of the isovector resonances. The same ratio $\varkappa_0^{(\lambda)}/\varkappa_1^{(\lambda)}$ can be used for large groups of deformed and spherical nuclei.

2. In the quasiparticle-phonon nuclear model the definition of phonons is generalized. Collective or weakly collective single-phonon states as well as two-quasiparticle states are described in the language of phonons. The generalization is in two directions: 1) all (and not only the first) roots of secular equations of the type (79) are calculated and their wave functions are regarded as single-phonon functions; 2) to describe single-phonon states with all $K^\pi$ values in deformed nuclei and all $I^\pi$ in spherical nuclei, one introduces multipole-multipole and spin-multipole-spin-multipole forces with any $\lambda$ and $s$, including large multipolarities. The treatment of all states with fixed $K^\pi$ or $I^\pi$ as single-phonon states does not introduce any difficulties if the constants $\varkappa_0^{(\lambda)}, \varkappa_1^{(\lambda)}$ in Eq. (53) or the constants $\varkappa_{\sigma 0}^{(s)}, \varkappa_{\sigma 1}^{(s)}$ in Eq. (67) are fixed. In deformed nuclei, there is no apparent need to take into account interactions in the particle-particle channel apart from the description of states with $K^\pi = 0^+$. In spherical nuclei, to describe $0^+$ and a number of $2^+$ states it is necessary to take into account the interactions in the particle-particle channel and therefore to solve a secular equation of the type (73) (see Ref. 33).

The use of equations of the type (53) and (67) and the wave functions (49) to describe states in deformed nuclei with $K^\pi = 1^+, 3^+, 4^+, 5^+$ and $4^-, 5^-, 6^-, \ldots$ in spherical nuclei with the corresponding values of $I^\pi$ requires the introduction of the new constants $\varkappa_0^{(\lambda)}, \varkappa_1^{(\lambda)}, \varkappa_{\sigma 0}^{(s)}, \varkappa_{\sigma 1}^{(s)}$. There is a considerable arbitrariness in the fixing of these constants,[35] due to the fact that nonrotational states of high multipolarity have been very poorly studied experimentally. We do not know whether there are strongly collective states of high multipolarity and, if there are, where they are situated. It should be noted that if such states exist they will be hard to detect experimentally. Undoubtedly, there is an upper limit for the constants $\varkappa_0^{(\lambda)}, \varkappa_{\sigma 0}^{(s)}$. In even-even nuclei, there are many low lying states with $K^\pi$ or $I^\pi$ equal to $1^+, 3^+, 4^-, \ldots$, and the choice of $\varkappa_0^{(\lambda)}, \varkappa_{\sigma 0}^{(s)}$ is restricted by the requirement that such states must not sink too strongly.

In the study of states with high multipolarity, it must be borne in mind that besides the maximum at the energy where the bands corresponding to the matrix elements with $\lambda = \Delta N$ predominate there must also be maxima at lower energies where the bands corresponding to the matrix elements with $\Delta N < \lambda$ predominate. For example, in Ref. 36 low energy octupole resonances with energy 5–10 MeV were found experimentally in a number of spherical and deformed nuclei, matrix elements with $\Delta N = 1$ predominating in their wave functions. It is interesting to note that the calculations made in Refs. 37 and 38 without any parameter fitting confirmed the existence of low energy octupole resonances in spherical and deformed nuclei.

For deformed nuclei, it is necessary to take into account the following circumstances. Multipole-multipole interactions with large $\lambda$ describe phonons not only with $\lambda = K$ but also with $K < \lambda$. Therefore, phonons with fixed $K$ are determined by multipole-multipole forces with $\lambda = K, K+2, K+4\ldots$. The constants $\varkappa_0^{(\lambda)}$ for large $\lambda$ must be chosen in such a way that they do not strongly change states with $K < \lambda$, which are determined by interaction with lower $\lambda$. For example, in studying $K^\pi = 2^+$ states, one can additionally take into account multipole-multipole forces with $\lambda = 4, 6, \ldots$. The constants $\varkappa_0^{(4)}, \varkappa_0^{(6)}$ must be taken such that they do not strongly change the energy and structure of the lowest $K^\pi = 2^+$ states determined by forces with $\lambda = 2$. There are no experimental indications of simultaneous influence of forces with different $\lambda$ values on single-phonon states. Because of the arbitrariness associated with the introduction of interactions in nuclei, we can stipulate that in the study of states with definite $K^\pi$ we use multipole-multipole (or spin-multipole-spin-multipole) forces with only one value of $\lambda$. To calculate quantities of the type $B(E\lambda)$, it is necessary to take into account such transitions to rotational states with $I = \lambda$ and $K < \lambda$, since they make a coherent contribution.

If multipole and spin-multipole forces are taken into account simultaneously, the secular equation has the more complicated form (70). Investigations have shown that the influence of spin-multipole forces on the first quadrupole and octupole states in deformed and especially spherical nuclei is small. There are no experimental indications that spin-multipole forces are manifested in these states. Because of the arbitrariness in the choice of the interaction, it can be assumed that there is no need to take into account simultaneously multipole and spin-multipole forces to calculate the characteristics of single-phonon states.

It may be asserted that to find the energies of single-phonon states there is no need to solve the secular equations (21); it is sufficient to solve equations of the type (53) and (67), and in particular cases Eq. (73).

3. In the construction of the model Hamiltonian, there is considerable arbitrariness, associated with both the shape of the potential of the average field as well as with the form of the residual forces. One can therefore introduce some restrictions on the description of single-phonon states. Later, if relevant experimental data or weighty theoretical considerations appear, some of these restrictions can be lifted.

We formulate the following rules for describing single-phonon states with fixed $K^\pi$ in deformed nuclei and fixed $I^\pi$ in spherical nuclei.

1. To find the energies, we solve: a) the secular equations (53) with multipole-multipole forces with minimal value of $\lambda$, and b) the secular equations (67) with spin-multipole–spin-multipole forces with minimal $s$ if there are no corresponding multipole forces or if they are of higher multipolarity.

2. Forces of different multipolarity or multipole and spin-multipole forces are not taken into account simultaneously.

3. In deformed nuclei, for the calculation of $B(E, \lambda)$, the spectroscopic factors, and other functions, transitions to rotational states are calculated, i.e., to all states with $I = \lambda$ and different values of $K$.

4. The interaction in the particle-hole channel is taken into account. The interaction in the particle-particle channel is taken into account in the calculation of: a) $0^+$ states in all nuclei, and b) $2^+$ states in individual spherical nuclei [equation of the type (73)].

5. The isoscalar constants $\varkappa_0^{(\lambda)}$ for $\lambda < 4$ are determined from the energy of the first corresponding state; for $\lambda \geq 4$ they are taken sufficiently small to prevent the corresponding first states sinking too low and to prevent them becoming strongly collective.

6. The ratio $\varkappa_1^{(\lambda)}/\varkappa_0^{(\lambda)}$ is determined from: a) the position of the corresponding isovector resonance, and b) phenomenological estimates.

In individual cases, the secular equations can be complicated in order to eliminate resulting ghost states. The completeness of the phonon space is confirmed by the good agreement between the calculated density of nuclear states[39] and the experimental data at the neutron binding energy $B_n$.

## 5. INTERACTION BETWEEN QUASIPARTICLES AND PHONONS

1. In the quasiparticle-phonon nuclear model, all two-quasiparticle and vibrational states are represented in terms of phonon operators. In the absence of interaction between the phonons, the complete set of nonrotational states of an even-even nucleus is described as a series of one-, two-, and $n$-phonon states. The set of nonrotational states of an odd nucleus is repre-

sented as a series of one-quasiparticle states, quasiparticle-plus-phonon states, quasiparticle-plus-two-phonon states, etc.

The nonrotational states of odd-odd nuclei consist of several states with proton and neutron quasiparticles, to which are added one, two, and more phonons in each. This picture of excited states was used in Ref. 39 to calculate the density of excited states at different energies right up to the neutron binding energy $B_n$, and good agreement with the experiments was obtained.

A set of noninteracting quasiparticles and phonons does not give a correct picture of the excited states of nuclei. The correct wave functions of excited nuclear states are described as superpositions of components with different numbers of phonons. Wave function components differing by one phonon are coupled by the quasiparticle-phonon interaction. If the phonons are fixed, the corresponding parts of the multipole-multipole and spin-multipole–spin-multipole forces describing the quasiparticle-phonon interactions are uniquely determined. If the secular equations for the phonons are solved, all the parameters of the model are fixed. The interaction between quasiparticles and phonons that couples, for example, the one-quasiparticle and quasiparticle-plus-phonon states is the stronger, the more collective is the phonon.

The interaction of quasiparticles with phonons has the following advantages over the other types of effective interactions:

1) simultaneous and consistent description of quasiparticle and phonon states and their coupling;

2) unique choice of the form and the constants of the interaction;

3) applicability for the description at low, intermediate, and high excitation energies.

2. The Hamiltonian $H_Q$ (40) of the multipole-multipole interaction contains not only the part in (48) used to calculate the single-phonon states but also terms containing operators of the form $\alpha^+_{q\sigma}\alpha_{q\sigma'}(Q^+_t+Q_t)$, which describe the quasiparticle-phonon interaction. We denote the corresponding part of the Hamiltonian (40) by $H'_{vq}$ and write it in the form

$$
\begin{aligned}
H'_{vq} = -\frac{1}{2\sqrt{2}} \sum_t \Big\{ (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)}) \Big[ \sum_{ss'} f^t(ss')\, u^{(+)}_{ss'} g^t_{ss'} \sum_{s_2 s_2'} v^{(-)}_{s_2 s_2'} f^t(s_2 s_2') \\
\times ([Q^+_t+Q_t]\, B(s_2 s_2') + B(s_2 s_2')\,[Q^+_t+Q_t]) \\
+ \sum_{rr'} f^t(rr')\, u^{(+)}_{rr'} g^t_{rr'} \sum_{r_2 r_2'} f^t(r_2 r_2')\, v^{(-)}_{r_2 r_2'} ([Q^+_t+Q_t]\, B(r_2 r_2') \\
+ B(r_2 r_2')\,[Q^+_t+Q_t]) \Big] + (\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)}) \Big[ \sum_{ss'} f^t(ss')\, u^{(+)}_{ss'} g^t_{ss'} \\
\times \sum_{rr'} f^t(rr')\, v^{(-)}_{rr'} ([Q^+_t+Q_t]\, B(rr') + B(rr')\,[Q^+_t+Q_t]) \\
+ \sum_{rr'} f^t(rr')\, u^{(+)}_{rr'} g^t_{rr'} \sum_{ss'} f^t(ss')\, v^{(-)}_{ss'} ([Q^+_t+Q_t]\, B(ss') \\
+ B(ss')\,[Q^+_t+Q_t]) \Big] \Big\}. \qquad (80)
\end{aligned}
$$

We transform $H'_{vq}$ with allowance for the fact that the energies of the single-phonon states are determined from the solutions of the secular equations (53), and their wave functions are expressed in terms of $g^t_{qq'}$ and $w^t_{qq'}$ in the form (59)–(64). We obtain

$$
\begin{aligned}
H_{vq} = -\frac{1}{2} \sum_t \Big\{ \sum_{ss'} \Gamma^t_{ss'}(n)\, [B(ss')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, B(ss')] \\
+ \sum_{rr'} \Gamma^t_{rr'}(p)\, [B(rr')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, B(rr')] \Big\}, \qquad (81)
\end{aligned}
$$

where

$$
\Gamma^t_{ss'}(n) = \frac{v^{(-)}_{ss'}}{2\sqrt{Y_t}} f^t(ss'); \quad \Gamma^t_{rr'}(p) = \frac{v^{(-)}_{rr'}}{2\sqrt{Y_t}} y^t_p f^t(rr'), \qquad (82)
$$

and $Y_t$ and $y^t_p$ are determined by (63) and (61).

For the spin-multipole interaction, we must add to the Hamiltonian $H^s_v$ (66) a part corresponding to the quasiparticle-phonon interaction in the form

$$
\begin{aligned}
H^s_{vq} = \frac{1}{2} \sum_g \Big\{ \sum_{ss'} \Gamma^{tn}_{ss'}(n)\, [\mathfrak{B}(ss')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, \mathfrak{B}(ss')] \\
+ \sum_{rr'} \Gamma^{tn}_{rr'}(p)\, [\mathfrak{B}(rr')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, \mathfrak{B}(rr')] \Big\}, \qquad (83)
\end{aligned}
$$

where

$$
\Gamma^{ts}_{ss'}(n) = \frac{1}{2} \frac{v^{(+)}_{ss'}}{\sqrt{Y^s_t}} f^t_s(ss); \quad \Gamma^{ts}_{rr'}(p) = \frac{1}{2} \frac{v^{(+)}_{rr'}}{\sqrt{Y^s_t}} y^t_p f^t_s(rr'). \qquad (84)
$$

To take into account multipole-multipole forces with nonvanishing angular momentum, the corresponding part of the Hamiltonian in the particle-particle channel for the isoscalar interaction has the form

$$
\begin{aligned}
H^{pp'}_{vq} = \frac{G_\lambda}{2\sqrt{2}} \sum_i \Big( \Big\{ \sum_{qq'} \tilde{\tilde{f}}^{\lambda\mu}(qq')\, [g^t_{qq'} v^{(-)}_{qq'} + w^t_{qq'} v^{(+)}_{qq'}]\, Q^+_i \\
+ \sum_{qq'} \tilde{\tilde{f}}^{\lambda\mu}(qq')\, [g^t_{qq} v^{(-)}_{qq'} - w^t_{qq'} v^{(+)}_{qq'}]\, Q_t \Big\} \\
\times \sum_{q_2 q_2} \tilde{\tilde{f}}^{\lambda\mu}(q_2 q_2')\, (u^{(+)}_{q_2 q_2'} + u^{(-)}_{q_2 q_2'})\, B(qq') + \text{h.c.} \Big). \qquad (85)
\end{aligned}
$$

The Hamiltonian of the model with allowance for the secular equations for the phonons has the form

$$
\begin{aligned}
H_M = \sum_q \varepsilon(q)\, B(qq') - \frac{1}{2} \sum_t \frac{1}{Y_t} \\
\times \Big\{ \sum_{ss'} \frac{(f^t(ss')\, u^{(+)}_{ss'})^2\, \varepsilon(ss')}{\varepsilon^2(ss') - \omega^2_t} + y^t_p \sum_{rr'} \frac{(f^t(rr')\, u^{(+)}_{rr'})^2\, \varepsilon(rr')}{\varepsilon^2(rr') - \omega^2_t} \Big\} Q^+_t Q_t \\
- \frac{1}{2} \sum_t \sum_{qq'} \Gamma^t_{qq'} \{B(qq')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, B(qq')\} \\
- \frac{1}{2} \sum_t \frac{1}{Y^s_t} \Big\{ \sum_{ss'} \frac{(f^t_s(ss')\, u^{(-)}_{ss'})^2\, \varepsilon(ss')}{\varepsilon^2(ss') - \omega^2_t} + y^{ts}_p \sum \frac{(f^t_s(rr')\, u^{(+)}_{rr'})^2\, \varepsilon(rr')}{\varepsilon^2(rr') - \omega^2_t} \Big\} Q^+_t Q_t \\
+ \frac{1}{2} \sum_t \sum_{qq'} \Gamma^{ts}_{qq} \{\mathfrak{B}(qq')(Q^+_t+Q_t) + (Q^+_t+Q_t)\, \mathfrak{B}(qq')\}. \qquad (86)
\end{aligned}
$$

Here $\Gamma^t_{qq'}$ is equal to $\Gamma^t_{ss'}(n)$ and $\Gamma^t_{rr'}(p)$; $\Gamma^{ts}_{qq'}$ is equal to $\Gamma^{ts}_{ss'}(n)$ and $\Gamma^{ts}_{rr'}(p)$.

For additional allowance for interactions in the particle-particle channel with nonvanishing angular momentum, it is necessary to add to the Hamiltonian (86) the terms (72) and (85) transformed with allowance for the secular equations (73).

## 6. SYSTEMS OF BASIC EQUATIONS AND SOLUTIONS

1. To obtain the basic equations of the model with the Hamiltonian (86), we use the variational principle. We consider first the case of an odd deformed nucleus. We represent the wave function of the nucleus with an odd number of neutrons in the form of the expansion

$$\Psi_n(K^\pi)=\frac{1}{\sqrt{2}}\sum_\sigma\Big\{\sum_s C_s^n\alpha_{s\sigma}^+ +\sum_g D_g^n(\alpha^+Q^+)_g+\frac{1}{\sqrt{2}}\sum_G F_G^n(\alpha^+Q^+Q^+)_G+\ldots\Big\}\Psi_0, \qquad (87)$$

where $\Psi_0$ is the wave function of the ground state of the even-even nucleus with one neutron less, this being determined by the expression (50); $n$ is the number of the excited state with given $K^\pi$; $g=qt; G=qt_1t_2; t=\lambda\mu i$.

Following the adopted procedure, we find the expectation value of $H_M$ in the state (87) and, on the basis of the variational principle, obtain a hierarchy of coupled equations. It was shown in Ref. 40 that this hierarchy is equivalent to the hierarchy of coupled equations for the corresponding Green's functions. Truncating the series at some term in the wave function (87) corresponds to a definite truncation of the hierarchy of equations for the Green's functions.

A system of equations with the wave function (87) containing all terms up to the quasiparticle-plus-three-phonon terms was given in Refs. 9 and 10 and investigated in Refs. 41-43. The corresponding equations for spherical nuclei were obtained in Ref. 44.

We give a unified description of the multipole and spin-multipole interactions; for this, we introduce the notation[45]

$$\Gamma_{qg}=\begin{cases}\Gamma_{qq'}^t & \text{for multipole interactions;}\\ -\Gamma_{qq'}^{ts} & \text{for spin-multipole interactions;}\end{cases} \qquad (88)$$

$$\Gamma_{gG}=\begin{cases}\frac{1}{2}\{\Gamma_{qq_2}^{t_2}\delta_{t,t_1}+\Gamma_{qq_2}^{t_1}\delta_{t,t_2}\} & \text{for multipole interactions;}\\ -\frac{1}{2}\{\Gamma_{qq_2}^{t_2s}\delta_{t,t_1}+\Gamma_{qq_2}^{t_1s}\delta_{t,t_2}\} & \text{for spin-multipole interactions;}\end{cases} \qquad (89)$$

We take the wave function of an $N$-odd nucleus in the form

$$\Psi_n(K^\pi)=\frac{1}{\sqrt{2}}\sum_\sigma\Big\{\sum_s C_s^n\alpha_{s\sigma}^+ +\sum_g D_g^n(\alpha^+Q^+)_g+\frac{1}{\sqrt{2}}\sum_G F_G^n(\alpha^+Q^+Q^+)_G\Big\}\Psi_0 \qquad (90)$$

with the normalization condition

$$\sum_s (C_s^n)^2+\sum_g (D_g^n)^2+\sum_G (F_G^n)^2=1. \qquad (91)$$

The expectation value of $H_M$ (86) in the state (90) has the form

$$(\Psi_n^*(K^\pi)H_M\Psi_n(K^\pi))=\sum_s\varepsilon(s)(C_s^n)^2+\sum_g p(g)(D_g^n)^2+\sum_G p(G)(F_G^n)^2-2\sum_{s,g}\Gamma_{sg}C_s^nD_g^n-2\sum_{g,G}\Gamma_{gG}D_g^nF_G^n, \qquad (92)$$

with the fundamental poles $p(g)=\varepsilon(q)+\omega_t$; $p(G)=\varepsilon(q)+\omega_{t_1}+\omega_{t_2}$.

By means of the variational principle

$$\delta\{(\Psi_n^*(K^\pi)H_M\Psi_n(K^\pi))-\eta_n[(\Psi_n^*(K^\pi)\Psi_n(K^\pi))-1]\}=0 \qquad (93)$$

we obtain the system of equations

$$(p(g)-\eta_n)D_g^n-\sum_s\Gamma_{sg}C_s^n-\sum_G\Gamma_{gG}F_G^n=0; \qquad (94)$$

$$(\varepsilon(s)-\eta_n)C_s^n-\sum_g\Gamma_{sg}D_g^n=0; \qquad (95)$$

$$(p(G)-\eta_n)F_G^n-\sum_g\Gamma_{gG}D_g^n=0. \qquad (96)$$

We introduce the notation

$$K(g,g')=\sum_s\frac{\Gamma_{sg}\Gamma_{sg'}}{\varepsilon(s)-\eta_n}+\sum_G\frac{\Gamma_{gG}\Gamma_{g'G}}{p(G)-\eta_n} \qquad (97)$$

and rewrite Eq. (94) as

$$(p(g)-\eta_n)D_g^n-\sum_{g'}K(g,g')D_{g'}^n=0. \qquad (98)$$

The secular equation has the form

$$\theta(\eta_n)=\det\|\delta_{gg'}(p(g)-\eta_n)-K(g,g')\|=0. \qquad (99)$$

The rank of the determinant is equal to the number of the quasiparticle-plus-phonon components in the wave function (90). If we take a sufficiently large space of single-particle states, the rank of this determinant will be of order $10^4$–$10^5$. It was shown in Ref. 41 that the determinant of the system of equations (98) can be represented in the form

$$\prod_g(p(g)-\eta)^{-1}\theta(\eta)=1-\sum_s\frac{A_s}{\varepsilon(s)-\eta}-\sum_g\frac{A_g}{p(g)-\eta}-\sum_G\frac{A_G}{p(G)-\eta}, \qquad (100)$$

where the coefficients $A_s$, $A_g$, and $A_G$ are sums of determinants of different rank that do not depend on $\eta$. Therefore the secular equation $\theta(\eta_n)=0$ contains only simple poles.

The above system of equations can be used to study the fragmentation of single-particle states (see Refs. 46 and 47). To study the fragmentation of a definite single-particle state $s_0$, we transform it by introducing the functions

$$\tilde{C}_{\bar{s}}^n=C_{\bar{s}}^n/C_{s_0}^n;\quad \tilde{D}_g^n=D_g^n/C_{s_0}^n;\quad \tilde{F}_G^n=F_G^n/C_{s_0}^n,$$

where $\bar{s}\neq s_0$. We rewrite the system of equations (94)–(96) in the form (see Ref. 46):

$$\mathscr{F}_{s_0}(\eta_n)=\varepsilon(s_0)-\eta_n-\sum_g\Gamma_{s_0g}\tilde{D}_g^n=0; \qquad (101)$$

$$(\varepsilon(\bar{s})-\eta_n)\tilde{C}_{\bar{s}}^n-\sum_g\Gamma_{\bar{s}g}\tilde{D}_g^n=0; \qquad (102)$$

$$(p(g)-\eta_n)\tilde{D}_g^n-\sum_{\bar{s}}\Gamma_{\bar{s}g}\tilde{C}_{\bar{s}}^n-\sum_G\Gamma_{gG}\tilde{F}_G^n=\Gamma_{s_0g}; \qquad (103)$$

$$(p(G)-\eta_n)\tilde{F}_G^n-\sum_g\Gamma_{gG}\tilde{D}_g^n=0. \qquad (104)$$

We rewrite the normalization condition of the wave function (90) as

$$(C_{s_0}^n)^{-2}=1+\sum_{\bar{s}}(\tilde{C}_{\bar{s}}^n)^2+\sum_g(\tilde{D}_g^n)^2+\sum_G(\tilde{F}_G^n)^2.$$

To Eq. (98) there corresponds the equation

$$(p(g)-\eta_n)\tilde{D}_g^n-\sum_{g'}K_{s_0}(g,g')\tilde{D}_{g'}^n=\Gamma_{s_0g}, \qquad (105)$$

where

$$K_{s_0}(g,g')=\sum_{\bar{s}}\{\Gamma_{\bar{s}g}\Gamma_{\bar{s}g'}/[\varepsilon(\bar{s})-\eta_n]\}+\sum_G\{\Gamma_{gG}\Gamma_{g'G}/[p(G)-\eta_n]\}. \qquad (106)$$

The determinant of the system (105) is denoted by $\theta(s_0;\eta)$. The following relation is strictly satisfied:

$$(C_{s_0}^n)^{-2}=-\partial\mathscr{F}_{s_0}(\eta)/\partial\eta|_{\eta=\eta_n}. \qquad 107$$

The solution of Eq. (105) has the form

$$\tilde{D}_g^n=\theta_g(s_0;\eta)/\theta(s_0;\eta), \qquad (108)$$

where $\theta_g(s_0;\eta)$ is obtained from $\theta(s_0;\eta)$ by replacing the column $g$ by the free terms of (105). We substitute (108) in (101) and obtain after transformations

$$\mathscr{F}_{s_0}(\eta)=\frac{1}{\theta(s_0\eta)}\cdot$$

$$\times\begin{vmatrix} \varepsilon(s_0)-\eta & \Gamma_{s_0g_1} & \ldots & \Gamma_{s_0g_N} \\ \Gamma_{s_0g_1} & p(g_1)-\eta-K_{s_0}(g_1g_1) & \ldots & -K_{s_0}(g_1g_N) \\ \ldots & \ldots & \ldots & \ldots \\ \Gamma_{s_0g_N}-K_{s_0}(g_Ng_1) & \ldots & & p(g_N)-\eta-K_{s_0}(g_Ng_N) \end{vmatrix}, \quad (109)$$

where $N$ is the number of states $g$. We expand the determinant, regroup the terms, and obtain

$$\mathscr{F}_{s_0}(\eta)=(\varepsilon(s_0)-\eta)\,\theta(\eta)/\theta(s_0;\eta). \quad (110)$$

We substitute (108) in Eq. (102) and find

$$\widetilde{C}_s^n=\frac{1}{\varepsilon(\bar{s})-\eta_n}\sum_g \Gamma_{\bar{s}g}\widetilde{D}_g^n=\frac{1}{(\varepsilon(\bar{s})-\eta_n)\,\theta(s_0;\eta_n)}\sum_g \Gamma_{\bar{s}g}\theta_g(s_0;\eta_n).$$

We expand the determinant, regroup the terms, and obtain

$$\widetilde{C}_{\bar{s}}^n=-\frac{1}{(\varepsilon(\bar{s})-\eta_n)\,\theta(s_0;\eta_n)}$$

$$\times\begin{vmatrix} 0 & \Gamma_{\bar{s}g_1} & \ldots & \Gamma_{\bar{s}g_N} \\ \Gamma_{s_0g_1} & p(g_1)-\eta_n-K_{s_0}(g_1g_1) & \ldots & -K_{s_0}(g_1g_N) \\ \ldots & \ldots & \ldots & \ldots \\ \Gamma_{s_0g_N}-K_{s_0}(g_N,g_1) & \ldots & & p(g_N)-\eta_n-K_{s_0}(g_Ng_N) \end{vmatrix}$$

$$\equiv\frac{\Delta(s_0,\bar{s};\eta_n)}{\theta(s_0,\eta_n)}. \quad (111)$$

We derive the expressions for the simplified case when the wave function has the form

$$\Psi_n(K^\pi)=\frac{1}{\sqrt{2}}\sum_\sigma\left\{\sum_s C_s^n\alpha_{s\sigma}^+ +\sum_g D_g^n(\alpha^+Q^+)_g\right\}\Psi_0, \quad (112)$$

with the normalization

$$1=\sum_s (C_s^n)^2+\sum_g (D_g^n)^2.$$

Then

$$(\varepsilon(s)-\eta_n)\,C_s^n-\sum_g \Gamma_{sg}D_g^n=0; \quad (113)$$

$$(p(g)-\eta_n)\,D_g^n-\sum_s \Gamma_{sg}C_s^n=0. \quad (114)$$

These expressions can be rewritten in the form

$$(\varepsilon(s)-\eta_n)C_s^n-\sum_{s'} K(ss')\,C_{s'}^n=0, \quad (115)$$

where

$$K(ss')=\sum_g \{\Gamma_{sg}\Gamma_{s'g}/[p(g)-\eta_n]\}. \quad (116)$$

The secular equation has the form

$$\theta(\eta_n)=\det\|\delta_{ss'}(\varepsilon(s)-\eta_n)-K(ss')\|=0. \quad (117)$$

The rank of this determinant is equal to the number of single-quasiparticle components in the first sum in (112). In deformed nuclei it is sufficient to take 10–15 terms; in spherical nuclei, 1–3 terms.

We separate the state $s_0$ and write the equations in the form

$$\mathscr{F}_{s_0}(\eta_n)=\varepsilon(s_0)-\eta_n-\sum_g \Gamma_{s_0g}\widetilde{D}_g^n=0; \quad (118)$$

$$(\varepsilon(\bar{s})-\eta_n)\,\widetilde{C}_{\bar{s}}^n-\sum_g \Gamma_{\bar{s}g}\widetilde{D}_g^n=0; \quad (119)$$

$$(p(g)-\eta_n)\,\widetilde{D}_g^n-\sum_{\bar{s}} \Gamma_{\bar{s}g}\widetilde{C}_{\bar{s}}^n=\Gamma_{s_0g}; \quad (120)$$

$$(C_{s_0}^n)^{-2}=1+\sum_{\bar{s}}(\widetilde{C}_{\bar{s}}^n)^2+\sum_g(\widetilde{D}_g^n)^2. \quad (121)$$

We rewrite Eqs. (119) and (120) as

$$\sum_{\bar{s}'}\{(\varepsilon(\bar{s})-\eta_n)\,\delta_{\bar{s}\bar{s}'}-K(\bar{s},\bar{s}')\}\,\widetilde{C}_{\bar{s}'}^n=K(s_0,\bar{s}). \quad (122)$$

The solution of this equation can be written in the form

$$\widetilde{C}_s^n=\Theta_{s_0}(\bar{s})/\Theta_{s_0}, \quad (123)$$

where $\Theta_{s_0}$ is the determinant of the system (122), and $\Theta_{s_0}(\bar{s})$ is obtained from it by replacing the column $\bar{s}$ by the right-hand side of Eq. (122):

$$\mathscr{F}_{s_0}(\eta)=\Theta/\Theta_{s_0}. \quad (124)$$

When only one single-particle state $s_0$ with given $K^\pi$ is taken into account, the wave function is represented as

$$\Psi_n(K^\pi)=C_{s_0}^n\frac{1}{\sqrt{2}}\sum_\sigma\left\{\alpha_{s_0\sigma}^+ +\sum_g \widetilde{D}_g^n(\alpha^+Q^+)_g\right\}\Psi_0, \quad (125)$$

and the secular equation and the expression for $(C_{s_0}^n)^2$ have the form

$$\mathscr{F}_{p_0}(\eta_n)=\varepsilon(s_0)-\eta_n-\sum_g\{\Gamma_{s_0g}^2/[p(g)-\eta_n]\}=0; \quad (126)$$

$$(C_{s_0}^n)^{-2}=1+\sum_g\{\Gamma_{s_0g}^2/[p(g)-\eta_n]\}. \quad (127)$$

To find the energies of states described by the wave function (90), it is necessary to diagonalize the matrix (99) of very high rank. Mathematically, this problem is very complicated. In Refs. 41–43, approximate methods of solving equations of the type (99) were studied, and approximations were found that give a good description of the largest components of the wave functions (90). For intermediate and high excitation energies, the few-quasiparticle components of interest account for a small fraction of the normalization (91) and are poorly described. Therefore, the methods of obtaining solutions of equations of the type (99) developed in Refs. 41–43 are not suitable for studying the fragmentation of single-particle states.

To study the fragmentation of single-particle states and calculate the neutron strength functions and the spectroscopic factors of single-nucleon transfer reactions, the following approximate approach consisting of four stages is proposed.

*First stage*. We take the wave function in the form (112) and find the solution of the secular equation (117). This problem is fairly simple and is solved in Refs. 46 and 48. From the complete set of states $g$ in (99) the selection rules for $\Gamma_{sg}$ select a set of states $g'$ whose number is two orders of magnitude less than the total number of states $g$.

*Second stage*. We choose a set of states $g'$ whose number is equal to the total number of solutions of Eqs. (117) minus the number of one-quasiparticle components in the first term of (112). For each solution $\eta_n$ corresponding to the pole $p(g')$ we find $(C_{s_0}^n)^2$.

*Third stage*. We choose from the set of states $g'$ a set $g''$ for which the corresponding $(C_{s_0}^n)^2$ are larger than a definite value $C_0^2$. If we take $C_0^2=0.002$, then the number of states $g''$ is 15–20.

*Fourth stage*. In the determinant (99), we restrict ourselves to the system of states $g''$ and diagonalize. In this way we find the energies of the states and $(C_{s_0}^n)^2$. This problem can be solved on a computer for many nuclei, as it is necessary to diagonalize matrices of ranks between 10 and 100.

2. We consider the case of an even-even deformed

nucleus. We rewrite the Hamiltonian of the model (86) in the form

$$H_M=\sum_t \omega_t Q_t^+ Q_t$$

$$-\frac{1}{2}\sum_t \sum_{qq'} \Gamma_{qq'}^t \{B(qq')(Q_t^+ + Q_t)+(Q_t^+ + Q_t)B(qq')\}$$

$$+\frac{1}{2}\sum_t \sum_{qq'} \Gamma_{qq'}^{ts} \{\mathfrak{B}(q,q')(Q_t^+ + Q_t)+(Q_t^+ + Q_t)\mathfrak{B}(qq')\}. \tag{128}$$

We represent the wave function as the expansion

$$\Psi_n(K^\pi)=\Big\{\sum_t R_t^n(\lambda\mu)Q_t^+ + \frac{1}{\sqrt{2}}\sum_{t_1t_2} P_{t_1t_2}^n(\lambda\mu)Q_{t_1}^+Q_{t_2}^+ + \mathcal{L}_{t_1t_2t_3}^n(\lambda\mu)Q_{t_1}^+Q_{t_2}^+Q_{t_3}^+ + \dots\Big\}\Psi_0. \tag{129}$$

We find the expectation value of $H_M$ in the state (129) and, on the basis of the variational principle, obtain a hierarchy of coupled equations. The case in which only the terms containing one, two, three, and four phonons are taken into account in the wave function was investigated in Ref. 49, in which a system of basic equations was obtained and an approximate method for solving them was proposed. Expressions for spherical nuclei were found in Ref. 50.

We study in detail the problem with the simple wave function in the form

$$\Psi_n(K^\pi)=\Big\{\sum_i R_i^n(\lambda\mu)Q_i^+ + \frac{1}{\sqrt{2}}\sum_{t_1t_2} P_{t_1t_2}^n(\lambda\mu)Q_{t_1}^+Q_{t_2}^+\Big\}\Psi_0. \tag{130}$$

Its normalization condition is

$$\sum_i (R_i^n(\lambda\mu))^2+\sum_{t_1t_2}(P_{t_1t_2}^n(\lambda\mu))^2=1. \tag{131}$$

We find the expectation value of $H_M$ (128) in the state (130):

$$(\Psi_n^*(K^\pi)H_M\Psi_n(K^\pi))=\sum_i \omega_i(R_i^n(\lambda\mu))^2 + \sum_{t_1t_2}\omega_{t_1t_2}[P_{t_1t_2}^n(\lambda\mu)]^2 - 2\sum_{it_1t_2}U_{t_1t_2}(\lambda\mu i)R_i^n(\lambda\mu)P_{t_1t_2}^n(\lambda\mu), \tag{132}$$

where $\omega_{t_1t_2}=\omega_{t_1}+\omega_{t_2}$;

$$U_{t_1t_2}(\lambda\mu i)=\frac{1}{\sqrt{2}}\langle Q_{\lambda\mu i}H_M Q_{t_1}^+Q_{t_2}^+\rangle \equiv U_{t_1t_2}(t); \tag{133}$$

for the multipole-multipole interaction, the explicit form of $U_{t_1t_2}(\lambda\mu i)$ is given in Ref. 16 by Eq. (9.75).

We use the variational principle

$$\delta\{(\Psi_n^*(K^\pi)H_M\Psi_n(K^\pi))-\eta_n[(\Psi_n^*(K^\pi)\Psi_n(K^\pi))-1]\}=0 \tag{134}$$

and obtain the system of basic equations

$$(\omega_i-\eta_n)R_i^n-\sum_{t_1t_2}U_{t_1t_2}(t)P_{t_1t_2}^n=0; \tag{135}$$

$$(\omega_{t_1t_2}-\eta_n)P_{t_1t_2}^n-\sum_i U_{t_1t_2}(t)R_i^n=0 \tag{136}$$

or

$$(\omega_i-\eta_n)R_i^n-\sum_{i'}K_{ii'}R_{i'}^n=0, \tag{137}$$

where

$$K_{ii'}=\sum_{t_1t_2}[U_{t_1t_2}(\lambda\mu i)U_{t_1t_2}(\lambda\mu i')/(\omega_{t_1t_2}-\eta_n)]. \tag{138}$$

Therefore, the secular equation has the form

$$\theta(\eta_n)=\det\|(\omega_i-\eta_n)\delta_{ii'}-K_{ii'}\|=0, \tag{139}$$

and the rank of the determinant is equal to the number of single-phonon states included in the first sum in (130).

We use the normalization condition of the wave function and obtain the following expressions for its coefficients:

$$R_i^n=M_{ii}/N; \tag{140}$$

$$P_{t_1t_2}^n=\frac{1}{N}\sum_i[U_{t_1t_2}(\lambda\mu i)M_{ii}/\omega_{t_1t_2}-\eta_n], \tag{141}$$

where $M_{ii}$ is the minor of the determinant (139):

$$N=\Big(\sum_i(M_{ii})^2+\sum_{t_1t_2}\Big\{\sum_i[U_{t_1t_2}(\lambda\mu i)M_{ii}/(\omega_{t_1t_2}-\eta_n)]\Big\}^2\Big)^{1/2}. \tag{142}$$

We rewrite Eq. (137) in the form

$$\mathcal{F}_{i_0}(\eta_n)=\omega_{i_0}-\eta_n-K_{i_0i_0}-\sum_{i'}K_{i_0i'}\tilde{R}_{i'}^n=0; \tag{143}$$

$$(\omega_i-\eta_n)\tilde{R}_i^n-\sum_{i'}K_{ii'}\tilde{R}_{i'}^n=K_{i_0i'}. \tag{144}$$

It is easy to show that

$$\mathcal{F}_{i_0}(\eta)=\Theta(\eta)/M_{i_0i_0}; \tag{145}$$

$$\tilde{R}_i^n=R_i^n/R_{i_0}^n=(-1)^{i_0+i}M_{ii_0}/M_{i_0i_0}; \tag{146}$$

$$(R_{i_0}^n)^{-2}=-\frac{\partial}{\partial\eta}\{\mathcal{F}_{i_0}(\eta)\}|_{\eta=\eta_n}, \tag{147}$$

where the determinant $M_{ii}$ is obtained from the minor $M_{i_0i_0}$ by replacing column $i$ by the column of free terms in (144).

It is not difficult to solve the system (139) and to find the functions $R_i^n$ and $P_{t_1t_2}^n$. For a restricted number of states $i$ and $t_1t_2$ in deformed nuclei, the problem was solved in Ref. 51. For spherical nuclei, this problem was solved in Ref. 37, in which the reduced probabilities $B(E\lambda)$ for excitation of giant multipole resonances were also calculated. In Ref. 52, the $E1$ radiative strength functions in half-magic nuclei were calculated.

When only one single-phonon state is taken into account in (130), i.e.,

$$\Psi_n(K^\pi)=R^n(\lambda\mu)\{Q_{i_0}^+-\sum_{t_1t_2}P_{t_1t_2}^n(\lambda\mu)Q_{t_1}^+Q_{t_2}^+\}\Psi_0, \tag{148}$$

the secular equation and the expression for $(R^n(\lambda\mu))^2$ have the form

$$\omega_{t_0}-\eta_n=\frac{1}{2}\sum_{t_1t_2}\{[U_{t_1t_2}(\lambda\mu n)]^2/(\omega_{t_1t_2}-\eta_n)\}, \tag{149}$$

$$[R^n(\lambda\mu)]^{-2}=1+\frac{1}{2}\sum_{t_1t_2}\{[U_{t_1t_2}(\lambda\mu n)]^2/(\omega_{t_1t_2}-\eta_n)^2\}. \tag{150}$$

3. In the wave functions (90) and (130), there are terms containing products of two phonon creation operators. Because the phonon operators are constructed from a product of quasiparticle operators satisfying fermion commutation relations, there is a certain violation of the Pauli principle in products of two phonon operators. The problem of eliminating the terms that violate the Pauli principle has been investigated on many occasions, for example, in Refs. 53–55. We show that in the framework of the quasiparticle-phonon nuclear model the problem can be formulated without violating the Pauli principle.

We demonstrate the mathematical method for cases when there are not more than two phonons in the wave functions. We introduce operators of "true" bosons $b^+(q,q')$ and $b(q,q')$ satisfying the commutation relations

$$[b(q,q'),b^+(q_2q_2')]=\delta_{qq_2}\delta_{q'q_2'}+\delta_{qq_2'}\delta_{q'q_2};\quad [b(q,q'),b(q_2,q_2')]=0 \tag{151}$$

and the condition

$$b(q, q') = b(q', q). \tag{152}$$

Using the exact commutation relations, we express the operators $A^+(q,q')$ and $B(q,q')$ in terms of boson operators as follows:

$$B(q, q') = \sum_{q_2} b^+(q, q_2)\, b(q', q_2); \tag{153}$$

$$A^+(q, q') = b^+(q, q') + x b^+(q, q') \sum_{q_2 q_2'} b^+(q_2 q_2')\, b(q_2, q_2') + y \sum_{q_2 q_2'} b^+(q, q_2)\, b^+(q', q_2')\, b(q_2, q_2'),$$

where $x = -(3-\sqrt{6})/6$ and $y = -1\sqrt{6}$.

We introduce the phonon operators

$$\tilde{Q}_t = \frac{1}{2} \sum_{q, q'} \{\psi^t_{qq'} b(q, q') - \varphi^t_{qq'} b^+(q, q')\} \tag{155}$$

and express in terms of them the operators of the multipole moments (41). So as not to overburden the exposition, we take the Hamiltonian in the form

$$H'_M = \sum_q \varepsilon(q)\, B(q, q) - \frac{1}{2} \sum_{\lambda, \mu \geqslant 0} \varkappa_0^{(\lambda)} \{Q^+_{\lambda\mu}(n)\, Q_{\lambda\mu}(n) + Q^+_{\lambda\mu}(p)\, Q_{\lambda\mu}(p) + 2Q^+_{\lambda\mu}(n)\, Q_{\lambda\mu}(p)\}, \tag{156}$$

where $Q_{\lambda\mu}(n)$ is determined by Eq. (41). We take into account the secular equation (65), and we then obtain the Hamiltonian, expressed in terms of the new phonons $\tilde{Q}^+_t$ and $\tilde{Q}_t$, in the form

$$\begin{aligned} H'_M = {} & \sum_t \omega_t \tilde{Q}^+_t \tilde{Q}_t - \frac{1}{2} \sum_{t_2} \frac{1}{\sqrt{Y_{t_2}}} \sum_{q_2 q_2' q_3} f^{t_2}(q_2 q_2')\, v^{(-)}_{q_2 q_2'} \\ & \times \sum_{t, t'} [\psi^t_{q_2 q_3} \varphi^{t'}_{q_2' q_3} \tilde{Q}^+_{t'} \tilde{Q}^+_t \tilde{Q}_{t_2} + \varphi^t_{q_2 q_3} \psi_{q_2' q_3} \tilde{Q}^+_{t_2} \tilde{Q}_t \tilde{Q}_{t'} \\ & + (\psi^t_{q_2 q_3} \psi^{t'}_{q_2' q_3} + \varphi^t_{q_2' q_3} \varphi^{t'}_{q_2 q_3}) (\tilde{Q}^+_{t_2} \tilde{Q}^+_t \tilde{Q}_{t'} + \tilde{Q}^+_{t'} \tilde{Q}_t \tilde{Q}_{t_2})] \\ & - \frac{1}{4} \sum_{\substack{tt' \\ t_2 t_2'}} \{V^{tt'}_{t_2 t_2'} \tilde{Q}^+_{t_2'} \tilde{Q}^+_t \tilde{Q}_{t'} \tilde{Q}_{t_2} + V^{tt'}_{t_2 t_2'} \tilde{Q}^+_t \tilde{Q}^+_{t'} \tilde{Q}_{t_2} \tilde{Q}_{t_2'}\}, \end{aligned} \tag{157}$$

where the functions $V^{tt'}_{t_2 t_2'}$ and $\tilde{V}^{tt'}_{t_2 t_2'}$ consist of sums of products of matrix elements and the functions $\psi^t_{qq'}$ and $\varphi^t_{qq'}$.

We take the wave function in the form (148) and find the expectation value of $H'_M$ in this state:

$$\begin{aligned} (\Psi^*_n(K^\pi)\, H'_M \Psi_n(K^\pi)) = (R^n)^2 \{ & \omega_{t_0} + 2 \sum_{t, t'} \omega_{tt'} (P^n_{tt'})^2 \\ & - 2 \sum_{t, t'} U_{tt'}(t_0)\, P^n_{tt'} \\ & - \sum_{\substack{tt' \\ t_2 t_2}} [V^{tt'}_{t_2 t_2'} P^n_{tt_2'} P^n_{t' t_2} + \tilde{V}^{tt'}_{t_2 t_2'} P^n_{tt'} P^n_{t_2 t_2'}]\}. \end{aligned} \tag{158}$$

We use the variational principle and obtain the system of equations

$$\omega_{t_0} - \eta_n - \sum_{t, t'} U_{tt'}(t_0)\, P^n_{tt'} = 0; \tag{159}$$

$$P^n_{tt'} = \frac{1}{2} \frac{U_{tt'}(t_0)}{\omega_{tt'} - \eta_n} + \frac{1}{2} \frac{1}{\omega_{tt'} - \eta_n} \sum_{t_2 t_2'} (V^{tt_2}_{t' t_2'} + \tilde{V}^{tt'}_{t_2 t_2'})\, P^n_{t_2 t_2'}. \tag{160}$$

The transition from quasibosons to bosons has led to the appearance of the second term in (160). If this term is ignored and (160) is substituted in (159), we obtain the secular equation (149).

In (160), we separate the coherent terms and rewrite this equation in the form

$$\left\{\omega_{tt'} - \eta_n - \frac{1}{2}(V^{tt}_{t't'} + \tilde{V}^{tt'}_{tt'})\right\} P^n_{tt'} = \frac{1}{2} U_{tt'}(t_0) - \frac{1}{2} \sum_{t_2 t_2 \neq tt'} (V^{tt_2}_{t' t_2'} + \tilde{V}^{tt'}_{t_2 t_2'})\, P^n_{t_2 t_2'}. \tag{161}$$

In the first approximation, when the noncoherent terms in Eq. (161) are ignored, the transition from the quasibosons to the bosons reduces to a displacement of the energies of the two-phonon states. Using perturbation theory, one can take into account the noncoherent terms, which represent rescattering of phonons on phonons.

Thus, in the framework of the quasiparticle-phonon nuclear model one can work with true bosons and eliminate all violations of the Pauli principle. In the framework of each approximation, i.e., for a completely definite truncation of the expansions in the wave functions (87) and (129), the corresponding expression for the operators $A(qq')$ and $A^+(qq')$ is chosen, i.e., for example, one chooses the following terms in the expansion (154) and finds the system of basic equations.

## 7. METHOD OF STRENGTH FUNCTIONS

1. Wave functions of the type (90) and (130) at intermediate and high excitation energies of complex nuclei do not give a correct description of the structure of the states because they do not contain many-phonon components. Thus, to describe excited states with an energy of about 4 MeV in $^{239}U$ the wave function (87) must contain quasiparticle-plus-four-phonon components. In the formulation of the quasiparticle-phonon model, the problem of finding the correct wave functions of highly excited states was not posed. The model was formulated to obtain the most accurate description of the few-quasiparticle components of wave functions averaged over a certain energy interval.

For intermediate and high excitation energies, it is difficult to represent clearly the results of calculations of the characteristics of each state. For example, in $^{239}U$ at excitation energies 3–5 MeV in an interval of 100 keV there are 10–20 poles (and corresponding solutions) of quasiparticle-plus-phonon type. Therefore, to study the fragmentation of one-quasiparticle states, sums of the type $\sum_n (C^n_{s_0})^2$ for states lying in intervals of 200 and 400 keV were calculated in Refs. 48 and 56, and the results were represented in the form of histograms. In such calculations, the energy of each state was found, the components (many thousands of them) of the wave functions were found, and the value of only one of the components was used to calculate quantities of the type $\sum_n (C^n_{s_0})^2$. Only a small fraction of the large amount of obtained information was used. The need therefore arose for a mathematical formalism by means of which one could directly calculate the distribution of the required quantities in the chosen range of excitation energies. The method of strength functions met this requirement; it is a method of direct calculation of averaged characteristics without a detailed calculation of each state, as was done in Refs. 46 and 57.

To expound the essence of the method, we consider the fragmentation of the single-particle state described by the wave function (125) with secular equation (126) and expression $(C^n_{s_0})^2$ in the form (127). We construct the function

$$\Phi_{s_0}(\eta)=\sum_n (C^n_{s_0})^2\,\rho(\eta_n-\eta), \quad (162)$$

where

$$\rho(\eta_n-\eta)=(2\pi)^{-1}\Delta/[(\eta-\eta_n)^2+(\Delta/2)^2]. \quad (163)$$

The method of representing the results of the calculations depends on the width $\Delta$ of the averaging energy interval. Strength functions using the function $\rho(\eta_n-\eta)$ were calculated in Ref. 58. These functions were widely used in the quasiparticle-phonon-model to study the fragmentation of one-quasiparticle states, to calculate neutron strength functions, and to study giant multipole resonances.[23,32,37,38,46,47,52,59,60] When the functions $\rho(\eta_n-\eta)$ were introduced, results close to those used in the method of Green's functions were obtained.[61] To calculate the excitation probabilities of giant multipole resonances in Refs. 26 and 61 expressions that can be obtained by introducing a function $\rho(\eta_n-\eta)$ were used.

Taking into account Eq. (108), we write the function $\Phi_{s_0}(\eta)$ in the form

$$\Phi_{s_0}(\eta)=-\sum_n (\partial\mathcal{F}_{s_0}(\eta)/\partial\eta)^{-1}_{\eta=\eta_n}\rho(\eta_n-\eta). \quad (164)$$

We use the residue theorem and express the function (164) in terms of a contour integral around the poles that are solutions of Eq. (126). We obtain

$$\Phi_{s_0}(\eta)=\frac{-1}{2\pi i}\,\frac{\Delta}{2\pi}\oint\limits_{l_p}\frac{dz}{\mathcal{F}_{s_0}(z)}\,\frac{1}{(\eta-z)^2+(\Delta/2)^2}; \quad (165)$$

the contour $l_p$ is shown in Fig. 1. Remembering that the contour integral around a circle of infinite radius in the complex $z$ plane vanishes, we go over from the integral around $l_p$ to two contour integrals $l_1$ and $l_2$ around the poles $z_1=\eta+i\Delta/2,\ z_2=\eta-i\Delta/2$:

$$\Phi_{s_0}(\eta)=\frac{1}{2\pi i}\,\frac{\Delta}{2\pi}\oint\limits_{l_1+l_2}\frac{dz}{\mathcal{F}_{s_0}(z)}\,\frac{1}{(\eta-z)^2+(\Delta/2)^2}. \quad (166)$$

We make simple calculations

$$\Phi_{s_0}(\eta)=-\frac{\Delta}{2\pi}\,\frac{1}{2(\eta-z)}\,\frac{1}{\mathcal{F}_{s_0}(z)}\Big|_{z=\eta\pm i\Delta/2}=\frac{1}{\pi}\,\mathrm{Im}\left(\frac{1}{\mathcal{F}_{s_0}(\eta+i\Delta/2)}\right) \quad (167)$$

and, using $\mathfrak{F}_{s_0}(\eta+i\Delta/2)$ in the form (126), we obtain

$$\Phi_{s_0}(\eta)=(\Delta/2\pi)\{\Gamma(\eta)/[(\varepsilon(s_0)-\gamma(\eta)-\eta)^2+(\Delta/2)^2\Gamma^2(\eta)]\}, \quad (168)$$

where

$$\Gamma(\eta)=1+\sum_g\{\Gamma^2_{s_0g}/[(p(g)-\eta)^2+(\Delta/2)^2]\}; \quad (169)$$

$$\gamma(\eta)=\sum_g\{\Gamma^2_{s_0g}(p(g)-\eta)/[(p(g)-\eta)^2+(\Delta/2)^2]\}. \quad (170)$$

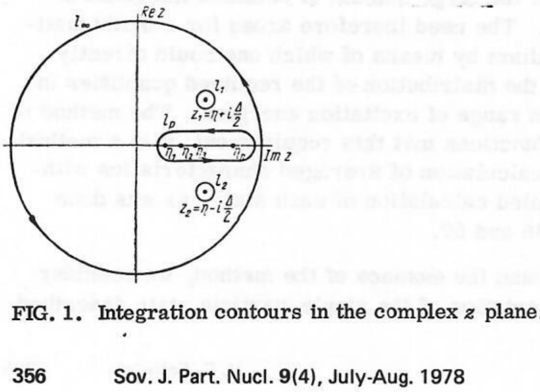

FIG. 1. Integration contours in the complex $z$ plane.

The function $\Phi_{s_0}(\eta)$ is represented in Breit-Wigner form, though the dependence of $\Gamma(\eta)$ and $\gamma(\eta)$ on the energy $\eta$ is very important, leading to a strong departure of $\Phi_{s_0}(\eta)$ from the Breit-Wigner form.[47] Usually,[62] one uses functions of the form (168) with constant $\Gamma$ and $\gamma$, which cannot be justified.

2. The method of strength functions is an integral and important part of the quasiparticle-phonon nuclear model. The method of strength functions was used in Refs. 46 and 47 to study the basic features of the fragmentation of single-particle states in odd deformed nuclei. If the wave function is taken in the form (90) and the function $\mathfrak{F}_{s_0}(\eta)$ in the form (102), and the same calculations as in Eqs. (162), (165), and (166) are made, we can obtain

$$\Phi_{s_0}(\eta)=\pi^{-1}\,\mathrm{Im}\{1/\mathcal{F}_{s_0}(\eta+i\Delta/2)\}, \quad (171)$$

i.e., the same form as (167). To calculate $\Phi_{s_0}(\eta)$, it is not necessary to diagonalize a matrix of high order $g'$ or $g''$, but only to calculate it, which is mathematically much easier.

Knowledge of the fragmentation of the single-quasiparticle states makes it possible to find the strength functions of neutron resonances and single-nucleon transfer reactions; the formulas for these contain expressions of the form

$$\Big(\sum_s a^{sK}_{lI}u_sC^n_s\Big)^2; \quad (172)$$

$$\Big(\sum_s a^{sK}_{lI}v_sC^n_s\Big)^2. \quad (173)$$

The single-particle wave functions $\varphi^K_s$ are represented in the form of an expansion with respect to a spherical basis:

$$\varphi^K_s=\sum_{NlI}a^{sK}_{NlI}\varphi_{NlI};\quad a^{sK}_{lI}=\sum_N a^{sK}_{NlI}. \quad (174)$$

In Ref. 46, the strength functions

$$S^{pK}_{lI}(\eta)=\sum_n\rho(\eta_n-\eta)\Big|\sum_s a^{sK}_{lI}u_sC^n_s\Big|^2, \quad (175)$$

$$s^{hK}_{lI}(\eta)=\sum_n\rho(\eta_n-\eta)\Big|\sum_s a^{sK}_{lI}v_sC^n_s\Big|^2 \quad (176)$$

were introduced and the following transformations performed:

$$S^{pK}_{lI}(\eta)=\sum_n\rho(\eta_n-\eta)\sum_s(a^{sK}_{lI}u_s)^2(C^n_s)^2$$
$$+\sum_n\rho(\eta_n-\eta)\sum_{\substack{s,\,s'\\ s\neq s'}}a^{sK}_{lI}a^{s'K}_{lI}u_su_{s'}(C^n_s)^2(C^n_{s'}/C^n_s)$$
$$=-\sum_n\rho(\eta_n-\eta)\sum_s(a^{sK}_{lI}u_s)^2(\partial\mathcal{F}_s(\eta_n)/\partial\eta)^{-1}-\sum_n\rho(\eta_n-\eta)$$
$$\times\sum_{\substack{s,\,s'\\ s\neq s'}}a^{sK}_{lI}a^{s'K}_{lI}u_su_{s'}(\partial\mathcal{F}_s(\eta_n)/\partial\eta)^{-1}[\Delta(s,\ s';\ \eta)/\theta(s,\ \eta)].$$

Here, we have used Eqs. (107) and (111). We carry out the same procedure as on the transition from (162) to (167), obtaining

$$S^{pK}_{lI}(\eta)=\frac{1}{\pi}\sum(a^{sK}_{lI}u_s)^2\,\mathrm{Im}\{1/\mathcal{F}_s(\eta+i\Delta/2)\}$$
$$+\frac{2}{\pi}\sum_{s>s'}a^{sK}_{lI}a^{s'K}_{lI}u_su_{s'}\,\mathrm{Im}\left\{\frac{\Delta(s',\ \eta+i\Delta/2)}{\theta(s,\ \eta+i\Delta/2)}\right\}. \quad (177)$$

The expression for $S^{hK}_{lI}(\eta)$ differs from (177) in that the functions $u_s$ and $u_{s'}$ are replaced by $v_s$ and $v_{s'}$. In Ref. 46, the completeness condition was used to obtain the following expression of the sum-rule type:

$$\sum_n \Big(\sum_s a_{iI}^{sK} U_s C_s^n\Big)^2 = \sum_s U_s^2 (a_{iI}^{sK})^2. \quad (178)$$

The right-hand side of (178) is an upper limit for the strength function (177). Calculating (177) in a definite energy interval and comparing the result with the right-hand side of (178), we can establish which part of the strength function is exhausted in this energy interval.

To calculate the strength functions, one does not diagonalize matrices of high order $g'$ or $g''$ for each state but calculates the imaginary parts of determinants of order $g'$ or $g''$ at different values of the energy $\eta$ with a step of order $\Delta$. The transition to the calculation of strength functions rather than calculations of these quantities for each state reduces the computer time by a factor $10^2$-$10^3$.

3. To study giant multipole resonances and also neutron resonances, strength functions for the reduced probabilities of $E\lambda$ transitions are widely used in calculations in the quasiparticle-phonon model.

We derive an expression for the strength function of $E\lambda$ excitation of an even-even deformed nucleus for which the excitation of the state is described by the wave function (130). The reduced probability of $E\lambda$ transition has the form

$$B(E\lambda;\ 0^+0 \to I_f^{\pi_f} K_f n) \equiv B(E\lambda;\ \eta_n) = (00\lambda\mu \mid I_f K_f)^2 \mathcal{M}_n^2; \quad (179)$$

$$\mathcal{M}_n = \frac{1}{2}\sum_i R_i^n(\lambda\mu)\,(2-\delta_{\mu,0}/Y_t)^{1/2}$$

$$\times [e_{\rm eff}^{(\lambda)}(p)\,X^t(p)\,y_p^t + e_{\rm eff}^{(\lambda)}(n)\,X^t(n)] = \sum_i R_i^n(\lambda\mu,\ \eta)\,L_i(\lambda\mu), \quad (180)$$

where $e_{\rm eff}^{(\lambda)}(p)$ and $e_{\rm eff}^{(\lambda)}(n)$ are the effective electric charges; for $E1$ transitions, they are equal to $e_{\rm eff}^{(\lambda)}(p) = Ne/A$ and $e_{\rm eff}^{(\lambda)}(n) = -ze/A$; the functions $Y_t$, $X^t$, and $y_p^t$ are determined by Eqs. (63), (54), and (61). Then

$$\mathcal{M}_n^2 = \sum_{i,i'} R_i^n(\lambda\mu)\,R_{i'}^n(\lambda\mu)\,L_i(\lambda\mu)\,L_{i'}(\lambda\mu)$$

$$= \sum_{i,i'} (R_i^n(\lambda\mu))^2\,R_{i'}^n(\lambda\mu)\,L_i(\lambda\mu)\,L_{i'}(\lambda\mu). \quad (181)$$

We introduce the strength function

$$b(E\lambda,\ \eta) = \sum_n B(E,\ \lambda;\ \eta_n)\,\rho(\eta_n - \eta), \quad (182)$$

where $\rho(\eta_n - \eta)$ is determined by Eq. (163). It is easy to show that with sufficiently good accuracy

$$\int_{\eta-\Delta/2}^{\eta+\Delta/2} b(E\lambda,\ \eta')\,d\eta' \approx \sum_{n}{}_{\Delta} B(E\lambda;\ \eta_n), \quad (183)$$

where the summation is over all states $n$ with given $K^\pi$ in the energy interval $\Delta$. We substitute (179) and (181) in (182), use (146) and (147), and obtain

$$b(E\lambda,\ \eta) = -(00\lambda\mu \mid I_f K_f)^2 \sum_n \sum_{ii'} \frac{L_i(\lambda\mu)\,L_{i'}(\lambda\mu)}{\partial\mathcal{F}_i(\eta_n)/\partial\eta_n}$$

$$\times \frac{(-1)^{i+i'} M_{ii'}}{M_{ii}} \frac{1}{(\eta_n-\eta)^2+(\Delta/2)^2} = -\frac{\Delta}{2\pi}\frac{(00\lambda\mu \mid I_f K_f)^2}{2\pi i}$$

$$\times \oint_{l_p} \sum_{ii'} \frac{L_i(\lambda\mu)\,L_{i'}(\lambda\mu)}{\mathcal{F}_i(z)} \frac{(-1)^{i+i'} M_{ii'}(z)}{M_{ii}(z)} \frac{dz}{(\eta-z)^2+(\Delta/2)^2}. \quad (184)$$

Here, the integration is around the contour given in Fig. 1. We perform the same procedure as on the transition from (165) to (167), use (145), and obtain

$$b(E\lambda,\ \eta) = \frac{1}{\pi}(00\lambda\mu \mid I_f K_f)^2 \sum_{i,i'} (-1)^{i+i'}$$

$$\times L_i(\lambda\mu)\,L_{i'}(\lambda\mu)\,{\rm Im}\left\{\frac{M_{ii'}(\eta+i\Delta/2)}{\theta(\eta+i\Delta/2)}\right\}. \quad (185)$$

To calculate the strength functions $b(E\lambda, \eta)$, it is not necessary to diagonalize the matrices $\theta$ and $M_{ii}$; instead, one calculates their imaginary parts for different values of $\eta$. This reduces the computing time by a factor $10^2$–$10^3$. The rank of the determinants $\theta$ can be chosen to be in the range 10–20 for spherical nuclei and 20–100 for deformed nuclei. From what we have said above, we can conclude that the computing time is not too long to rule out the calculation of many giant multipole resonances for a large number of nuclei.

To determine the regions in which the giant multipole resonances are localized, an important role is played by the calculation of energy weighted sum rules. The energy weighted sum has the form

$$S_\lambda(\eta) = \sum_{\mu n} \eta_n(\lambda\mu)\,B(E\lambda,\ \eta_n)\,\rho(\eta_n - \eta)$$

$$= \frac{1}{\pi}\sum_\mu (00\lambda\mu \mid I_f K_f)^2 \sum_{i,i'} (-1)^{i+i'} L_i(\lambda\mu)$$

$$\times L_{i'}(\lambda\mu)\,{\rm Im}\{(\eta+i\Delta/2)\,M_{ii}(\eta+i\Delta/2)/\theta(\eta+i\Delta/2)\}. \quad (186)$$

For spherical nuclei, there is no summation over $\mu$ or $K$.

To elucidate the completeness of the employed single-particle basis, model-free energy weighted sum rules are useful. The model-free dipole sum rule has the form

$$\sum_n B(E1,\ \eta_n)\,\eta_n = 0.18\,\frac{ZN}{A}\,e^2{\rm b}\cdot{\rm MeV}. \quad (187)$$

For $\lambda > 1$

$$\sum_n B(E\lambda,\ \eta_n)\eta_n = 4.8\lambda\,(3+\lambda)^2 \frac{Z}{A^{2/3}} B(E\lambda)_{\rm s.\,p.}\,{\rm MeV}. \quad (188)$$

## CONCLUSIONS

1. The quasiparticle-phonon nuclear model can be used to calculate many properties of complex nuclei at low, intermediate, and high excitation energies. Some of these calculations have already been made. There is no doubt that when calculations are made in the future more and more complicated variants of the model will be developed by introducing new terms in the wave functions (87) and (129), and also by taking into account new forces.

2. It should be noted that many-quasiparticle components make the main contribution to the wave functions of highly excited states. There is no doubt that in the future we shall witness the discovery of new properties of highly excited states determined by many-quasiparticle components. At present, there is no information about the values and distribution of the many-quasiparticle components of the wave functions of highly excited states. Even for lower states such as neutron resonances it has been shown[14, 15, 63] that direct experimental data on the characteristics of the many-quasiparticle components of their wave functions are absent. The contribution of the few-quasi-

particle components to the normalization of the wave functions of neutron resonances is only $10^{-4}$–$10^{-6}$.

3. With increasing excitation energy, the structure of the states becomes more and more complicated. One can expect an appreciably greater variety of the properties of the high lying states compared with the low lying states. It is hard to imagine the structure of nuclear states at very high excitation energies. Will this be a state of undifferentiated nuclear matter or something different?

4. There are no grounds for hoping that a simple and perspicuous description of complex nuclei will be found. The nucleus is a very complicated system and this complexity can be understood, the known properties described, and new properties predicted on the basis of a theory of the nucleus which exploits the tremendous possibilities of the rapidly developing computational technology.

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