Relativistic nucleons in nuclei

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The structure of nuclear wave functions at relativistic values of the nucleon momenta is described. The problem of separating the non-nucleon components of the nuclear wave function (isobar admixtures, meson exchange currents, etc.) is discussed. The present state of the problem of nuclear reactions with large momemtum transfer is considered. The directions for further investigations that are, in the opinion of the authors, the most informative, are pointed out.

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INTRODUCTION

During the last few years, investigations have been made in nuclear physics directed toward the study of the properties of nuclei at short internucleon distances. Ideas about the nucleus have gone beyond the canonical nonrelativistic theory, which treats the nucleus as a system of nucleons interacting through a potential and described by the Schrödinger equation. This has come about because experiments with large momenta transferred to the nucleus now "feel" internucleon distances that correspond to momenta of the nucleons of the order of their mass. In this region, the nonrelativistic approach certainly breaks down.

On the other hand, the increased accuracy with which traditional nonrelativistic quantities such as the magnetic and quadrupole moments are now measured has also provided a considerable stimulus for the theoretical investigation of short internucleon distances. In light nuclei, the nucleons spend a very short time at these distances, which leads to small deviations from the predictions of nonrelativistic theory. But if such deviations are firmly established, they give valuable information about nuclear structure at short distances.

These investigations open up a completely new field of phenomena, and one may hope that their results will lead to fruitful advances in nuclear physics as well as in elementary-particle physics. It appears probable that at subnuclear distances all hadrons are in principle "constructed" from their constituents in the same way. If this is so, then study of the relativistic part of the nuclear wave function will also be of direct interest for elementary-particle physics.

On the transition to relativistic nucleon momenta, two basic theoretical problems arise simultaneously: 1) It is necessary to find an adequate formalism to describe bound systems consisting of relativistic particles; 2) in the relativistic region, it is necessary to take into account not only the nucleons but also the presence of other particles (nucleon isobars, pions, etc.) It is necessary to describe these admixtures quantitatively.

In what follows, we shall mainly discuss only the simplest of all nuclei—the deuteron. For the deuteron, the theoretical investigations in this direction are concentrated on studying the effects associated with the relativistic nature of the motion of the particles within the deuteron, elucidating the isobar admixtures in the deuteron, and on the role of meson exchange currents. We shall describe these phenomena in comparative detail, but without pretending to completeness of the exposition. It should however be noted that, despite the undoubted value of the numerous calculations being currently made, which are essentially of a "sounding" nature, some fundamental questions in the description of composite quantum relativistic systems frequently remain unanswered. This sometimes forces us to doubt the correctness of the proposed interpretation of the experimental facts despite the approximate agreement between the theoretical and experimental data.

The situation here can be compared with the initial epoch of the great geographical discoveries when searches for the western route to China led to the discovery of northern Canada, which however was assumed for some time to be the sought-after China. To some extent, one gets the same impression from the interpretation of *ed* scattering experiments in terms of a six-quark structure of the deuteron. One difference is that the actual existence of China was not at that time in doubt, which cannot yet be said of quarks.

One of the principal problems in the theory of compositie relativistic systems is that the Bethe-Salpeter functions,[1] which contain all the necessary information about the bound system, do not have such a perspicuous physical meaning as the nonrelativistic wave functions, and are parametrized quite differently. The Feynman form factors for the nuclear vertices also cannot be directly interpreted in a probability manner and, in addition, they reflect, because of crossing symmetry—a very valuable property in field theory—the presence in the system of not only particles but also antiparticles, it being impossible to separate these contributions in a relativistically invariant manner.

Anticipating, we point out that the most adequate description of bound states for the purposes of relativistic nuclear physics is obtained by means of a Fock column. For example, for the deuteron the Fock column has the form

$$\Psi_d = \begin{pmatrix} \psi(NN) \\ \psi(NN^*) \\ \psi(\Delta\Delta) \\ \psi(NN\pi) \\ \psi(NN\pi\pi) \\ \cdots\cdots \end{pmatrix},$$

where the rows of the column describe the distributions of the corresponding particles in the deuteron,

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and the moduli of the squares of these rows are probability densities. The possibility of a probability interpretation guides the physical intuition and gives the formalism great heuristic force.

In the first place, it is necessary to establish the number and meaning of the independent variables of the relativistic wave function, and also some of its necessary properties (for example, the behavior in boundary regions of the values of the variables) that follow from the general relativistic equations of motion and the nature of the interactions.

We emphasize here that, in our opinion, it is very important to achieve in the theory of composite relativistic systems a level of understanding at least adequate for us to be able, without knowledge of the details of the dynamics, to predict reliably the main quantitative properties of a relativistic nucleus in the same way as we can for the nonrelativistic deuteron on the basis of only the most general characteristics of the nuclear interaction potential (behavior of the potential at the origin, its range, asymptotic properties, spin structure, etc). The possibility of such a phenomenological approach is particularly important because hoping for a derivation in the relativistic case of an exact dynamical equation for the nuclear interaction of nucleons from first principles is even less justified than in canonical nuclear theory. At the same time, knowledge of the general properties of the relativistic wave function would provide the basis for its correct parametrization, without which theoretical interpretation of experimental data is impossible.

Before we turn to the exposition of the results hitherto obtained with a bearing on relativistically invariant wave functions, which are the rows of the Fock column, and also of the computational formalism that enables one to express the amplitudes of processes in terms of these wave functions, we discuss other existing approaches to the relativization of wave functions and the results already found relating to the structure of the deuteron at short distances and to the mechanisms of processes with large momentum transfers, together with the available experimental data.

In Sec. 1 of the review, we discuss the main methods to be found in the literature for describing bound relativistic systems and, in particular, the large-momentum components of the deuteron wave function. In Sec. 2, we briefly summarize the results relating to isobar admixtures in the deuteron. The role of meson exchange currents and their influence on the cross section of elastic *ed* scattering and electro- and photodisintegration of the deuteron are described in Sec. 3. In Sec. 4, we discuss process with large momentum transfers: elastic and deep inelastic *ed* scattering, πd and pd backward scattering, the reaction $\pi^- d \to p\Delta^-$, and cumulative meson production. In Sec. 5, we describe the wave functions that are the components of the Fock column defined on a "light front". Their parametrization is considered in detail. It should be noted that the Feynman diagram technique does not permit one to express the amplitudes of processes in terms of wave functions. Section 6 provides a computational formalism (diagram technique) that arises in the three-dimensional formulation of field theory on a light front and can be used to express the amplitude in terms of the wave functions introduced in Sec. 5.

1. RELATIVISTIC WAVE FUNCTIONS AND EQUATIONS

Relativistic bound systems are now described in numerous ways. The "family" of wave functions and computational schemes can be represented together with their relationships in the form of a "geneological tree" (Fig. 1). The boxes with heavy border indicate the direction that will be discussed in Sec. 5 and 6.

Here, we discuss the wave functions that arise when the Bethe–Salpeter equation[1] is transformed to three-dimensional form (quasipotential wave functions). In addition, we consider briefly the so-called relativistic "quantum-mechanical" approach, which does not directly follow from quantum field theory and presupposes that the wave functions describe a system consisting of a fixed number of relativistic particles. Basically, we discuss only those wave functions that have been used for calculations in nuclear physics. However, before fixing their place among other quasipotential wave functions and equations, we describe the general scheme of derivation of quasipotential equations and their classification.

Quasipotential Wave Functions. We describe the transition from the Bethe–Salpeter equation for the two-particle amplitude to a three-dimensional equation. The wave functions are defined as the residues with respect to the energy in the Green's function and satisfy corresponding homogeneous equations. For these wave functions and equations we shall use the adjective quasipotential, although its original meaning was somewhat narrower. In our exposition, we shall follow Refs. 2 and 3. The point of departure for deriving the quasipotential equations is the Bethe–Salpeter equation[1] for the two-particle amplitude:

$$T_{pq}(s) = V_{pq} + \int d^4k V_{pk} G_k(s) T_{kq}(s), \tag{1}$$

where

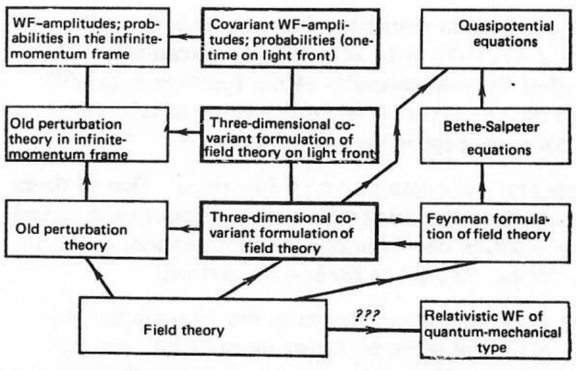

FIG. 1. Table of relativistic wave functions (WF) and the computational formalisms of field theory corresponding to them.

$$G_k(s) = \mathrm{i}\,[(2\pi)^4 (k_1^2 - m^2 + \mathrm{i}0)(k_2^2 - m^2 + \mathrm{i}0)]^{-1};$$
$$k_1 + k_2 = p;\quad k_1 - k_2 = 2k;\quad s = p^2.$$

It is necessary to rearrange this equation in such a way that it acquires a three-dimensional nature, i.e., the integration over d^4k must be replaced by d^3k, and the amplitude satisfying the equation must be manifestly two-particle unitary. We write down the two-particle unitarity condition:

$$T_{pq}(s) - T^*_{pq}(s) = \frac{\mathrm{i}}{(2\pi)^2} \int d^4k T_{pk}(s) \left[\delta^+ (k_1^2 - m^2)\, \delta^+ (k_2^2 - m^2)\right] T^*_{kq}(s), \qquad (2)$$

where

$$\delta^+ (k^2 - m^2) = \theta(k_0)\, \delta(k^2 - m^2).$$

From Eq. (1) in the case of a real kernel we obtain

$$T_{pq}(s) - T^*_{pq}(s) = \int d^4k T_{pk}(s) \left[G_k(s) - G^*_k(s)\right] T^*_{kq}(s). \qquad (3)$$

The transition from Eq. (1) to the quasipotential equation is made by replacing in it the Green's function $G_k(s)$ by $g_k(s)$, the discontinuity $g_k(s)$ being equal to

$$g_k(s) - g^*_k(s) = \Delta(s) = \mathrm{i}\,(2\pi)^2 \delta^+ (k_1^2 - m^2)\, \delta^+ (k_2^2 - m^2), \qquad (4)$$

which occurs in the unitarity condition. The fulfillment of the unitary condition (2) is then guaranteed.

At the same time as we modify the Green's function we modify the kernel of the equation in such a way as to leave the on-shell amplitude unchanged. It is readily seen that Eq. (1) is equivalent to the system of two equations

$$T = K + KgT; \qquad (5)$$
$$K = V + V(G - g)\, K. \qquad (6)$$

The kernel K, which is formally determined from Eq. (6),

$$K = [1 - V(G - g)]^{-1}\, V \qquad (7)$$

is called the quasipotential.

The solution of Eq. (4) for $g_k(s)$ is determined with a large amount of nonuniqueness. We represent it in the most convenient form for the following discussion:

$$g_k(s) = \frac{1}{(2\pi)^3} \int\limits_{4m^2}^{\infty} \frac{ds'}{s' - s - \mathrm{i}0} f(s', s)\, \delta^+ \{[(\sqrt{s'/s}\,(1-\alpha) + \alpha)\, p/2 - k]^2 - m^2\}\, \delta^+ \{[(\sqrt{s'/s}\,(1+\alpha) - \alpha)\, p/2 - k]^2 - m^2\}, \qquad (8)$$

where $f(s', s)$ is an arbitrary function satisfying the condition $f(s, s) = 1$; α is an arbitrary parameter. It is obvious that the discontinuity of the function $g_k(s)$ determined by Eq. (8) is equal to the value of $\Delta(s)$ from (4) and does not depend on $f(s', s)$ or α.

The integral (8) contains two δ functions. One of them disappears after the integration with respect to ds', and the other enables one to integrate with respect to dk_0 in Eq. (5). Thus, Eq. (5) is three-dimensional.

All the equations encountered in the literature, obtained in different ways by different authors, can be classified by specifying the manner in which the parameter α (it is equal to 0 or 1) and the function $f(s', s)$ in the Green's function $g_k(s)$ are chosen. For example, the Logunov–Tavkhelidze equation[4] is obtained for $\alpha = 0$ and $f(s', s) = 1$. The Green's function $g_k(s)$ for $\alpha = 0$ and 1 in the center-of-mass system $[p = (\sqrt{s}, 0, 0, 0)]$ is determined by the following equations, respectively:

$$g_k(s) = \delta(k_0)\, f(4\varepsilon^2(\mathbf{k}), s)/[4(2\pi)^3 \varepsilon(\mathbf{k})(\varepsilon^2(\mathbf{k}) - s/4 - \mathrm{i}0)]; \qquad (9)$$

$$g_k(s) = \delta(k_0 + \sqrt{s}/2 - \varepsilon(\mathbf{k})) \times f(4\varepsilon^2(\mathbf{k}), s)/[4(2\pi)^3 \varepsilon(\mathbf{k})(\varepsilon^2(\mathbf{k}) - s/4 - \mathrm{i}0)]. \qquad (10)$$

Setting $f(4\varepsilon^2(\mathbf{k}), s) = 1$ in (9) and writing out the operator equation (5), we arrive at the Logunov–Tavkhelidze equation

$$T_{pq}(s) = K_{pq}(s) + \frac{1}{(2\pi)^3} \int \frac{d^3k}{\sqrt{m^2 + \mathbf{k}^2}} \frac{K_{pk}(s)}{4(\mathbf{k}^2 + m^2) - s - \mathrm{i}0} T_{kq}(s), \qquad 11)$$

which was obtained in Ref. 4 by considering the one-time Green's functions. It coincides with the Blankenbecler–Sugar equation.[3]

In Table I, we give three-dimensional quasipotential equations, which can be reduced to a single normalization by fixing the unitarity condition (2). Table I is based on the list of quasipotential equations described by Brown and Jackson.[2] Table I does not include the equation of Ref. 9 because the expression for $f(s', s)$ is rather cumbersome.

We emphasize that all quasipotential equations with different functions $g_k(s)$ and corresponding quasipotentials K determined by (7) are in principle equivalent to one another and determine an amplitude which is equal on the mass shell to the Bethe–Salpeter amplitude. However, it is in practice difficult to find the kernel K explicitly, and usually a model is used for the quasipotential. The result then depends on the type of equation and the dynamical model, and it is hard to give convincing estimates for the accuracy of the approximations.

The problem of expressing the form factors of a bound system in terms of a quasipotential wave function was solved by Faustov.[10] The form factor is represented in the form of an integral containing three-dimensional wave functions and a three-dimensional generalized vertex part, which arises in the quasipotential approach. For this last, one can write down a series in the interaction, this being in nature close to the series for the quasipotential that follows from Eq. (7). Questions relating to the speed of convergence of this series and the

TABLE I. Three-dimensional quasipotential equations[2]

Equation	α	$f(s', s)$	Literature
Logunov-Tavkhelidze, Blankenbecler-Sugar	0	$f(s', s) = 1$	Ref. 4 Ref. 3
Thompson	0	$f(s', s) = \frac{\sqrt{s'} + \sqrt{s}}{2\sqrt{s'}}$	Ref. 5
Kadyshevskii	1	$f(s', s) = \frac{\sqrt{s'} + \sqrt{s}}{2\sqrt{s'}}$	Ref. 6
Gross	1	$f(s', s) = \frac{\sqrt{s} + \sqrt{s'}}{2\sqrt{s}}$	Ref. 7
Holinde *et al.*	1	$f(s', s) = 1$	Ref. 8

physical interpretation of its terms remain as yet open, which makes it difficult to use these expressions in calculations in which a small parameter is absent.

A three-particle relativistic equation analogous to the nonrelativistic Faddeev equation was also considered in Ref. 3. This equation was studied in Refs. 11 and 12 for the pole two-particle amplitude. After expansion with respect to partial waves, a one-dimensional equation is obtained. The three-particle variant of the equation of Refs. 3 and 4 was used by Weber to calculate the isobar admixtures in the deuteron wave function.[13] We shall consider the results in Sec. 2. Quasipotential equations find a further application in nuclear physics in the relativistic corrections to the deuteron wave function by means of Gross's equation.[7] We shall describe in more detail this formalism and the results to which it leads.[7,14-16]

Gross's equation can be formally obtained by integrating in the Bethe–Salpeter equation with respect to dk_0, ignoring the singularities of the kernel and the amplitude, and taking into account only the singularity of the propagator corresponding to the nucleon. Graphically, Gross's equation is given in Fig. 2. It is an equation for the vertex part $d \to NN$ with one of the nucleons on the mass shell. The vertex part is then related to the quasipotential wave function of the deuteron.

In the calculations of the vertex part Γ for the kernel, one chooses a sum of Feynman diagrams determined by meson exchanges, and not the quasipotential K from (7). This leads to a loss of some of the relativistic corrections.

The main thrust of Gross's work is in the allowance for the spin of the nucleons in the vertex part $d \to NN$ in such a way that in the zeroth approximation in the relativistic corrections the wave functions corresponding to S and D waves are explicitly separated. The spin structure of the form factor with one on-shell nucleon, which occurs in Gross's equations (see Fig. 2), has the form[17]

$$\Gamma^{\alpha}(p_2)\,\xi_\alpha = F(u)\,\gamma\xi + G(u)\,\frac{(p-p_1)}{2m}\,\xi + \frac{\hat{p}_1 - m}{m}\left[H(u)\,\xi\gamma + I(u)\,\frac{p-p_1}{2m}\,\xi\right]. \tag{12}$$

where $u = p_1^2$; m is the nucleon mass; γ are the Dirac matrices; ξ_α is a spinor corresponding to spin 1. The Γ_α are matrices with respect to the spin indices: $\Gamma_\alpha \equiv (\Gamma_\alpha)_{\delta\gamma}$.

The form factor is determined by the four invariant functions $F(u)$, $G(u)$, $H(u)$, and $I(u)$. In the nonrelativistic limit, there remain only two independent invariant functions of the S and D waves. They are expressed in terms of combinations of F, G, H, and I. It is convenient to write down the form factor directly in terms of the functions that remain in the nonrelativistic limit, and in terms of those that disappear. In addition, instead of the matrix $\Gamma_\alpha \xi_\alpha$ we consider its matrix elements:

$$\psi^{+}_{s_2 s_1}(k_1) = \frac{m}{[2M_d(2\pi)^3]^{1/2}}\,\frac{\bar{u}(p_1, s_1)\,\Gamma_d U_c \bar{u}^T(p_2, s_2)}{E_1(2E_1 - M_d)}; \tag{13}$$

$$\psi^{-}_{s_2 s_1}(k_1) = -\frac{m}{[2M_d(2\pi)^3]^{1/2}}\,\frac{v(-p_1, s_1)\,\Gamma_d U_c \bar{u}^T(p_2, s_2)}{E_1 M_d}, \tag{14}$$

where $k_1 = (p - p_1)/2$; $E_1 = \sqrt{m^2 + \mathbf{k}_1^2}$; u and v are the spinors corresponding to the nucleon and antinucleon; U_c is the charge conjugation matrix. It is knowledge of these matrix elements that is needed in the calculations. Expressing the Dirac matrices in terms of the Pauli matrices and going over to two-component spinors, we obtain in the deuteron rest frame

$$\psi^{+}_{s_2 s_1}(k_1) = \frac{\sqrt{4\pi}}{(2\pi)^{3/2}}\,\chi^{+}(s_1)\Big[u_0(k_1)\,\sigma\xi + \frac{w_2(k_2)}{\sqrt{2}}\Big(\frac{3(\sigma\mathbf{k}_1)(\xi\mathbf{k}_1)}{\mathbf{k}_1^2} - \sigma\xi\Big)\Big]\frac{i\sigma_y}{\sqrt{2}}\,\chi^{*}(s_2); \tag{15}$$

$$\psi^{-}_{s_1 s_2}(k_1) = -\frac{\sqrt{3}\,\sqrt{4\pi}}{(2\pi)^{3/2}}\,\chi^{+}(s_1)\Big[\frac{v_{t1}(k_1)}{\sqrt{2}}\,i\,\frac{\sigma[\mathbf{k}_1 \times \xi]}{|\mathbf{k}_1|} + v_{s1}(k_1)\,\frac{\xi\mathbf{k}_1}{|\mathbf{k}_1|}\Big]\frac{i\sigma_y}{\sqrt{2}}\,\chi^{*}(s'). \tag{16}$$

The invariant functions $u_0(k_1)$, $w_2(k_2)$, and $v_{t1}(k_1)$, $v_{s1}(k_1)$ can be expressed in terms of four invariant form factors from Eq. (12) (Ref. 15). The function ψ^- is intepreted by Gross as a wave function with negative energy. It contains singlet and triplet P waves and vanishes in the nonrelativistic limit. The functions $u_0(k_1)$ and $w_2(k_2)$ correspond to the S and D deuteron waves.

The normalization condition is obtained as follows. We note that $T^{-1} = K^{-1} - g$ follows from (5). We write down the identity

$$T(K^{-1} - g)\,T \equiv T. \tag{17}$$

Bearing in mind that near the pole $s = s_0$

$$T(s) \sim \Gamma^{*}\Gamma/(s \equiv s_0),$$

we equate the residues on the two sides of Eq. (17). We then arrive at a normalization condition which for an energy-independent quasipotential has in terms of the functions u, w, and v the form

$$\frac{2}{\pi}\int_0^\infty p^2\,dp\,[u^2(p) + w^2(p) + v_t^2(p) + v_s^2(p)] = 1. \tag{18}$$

Going over to the coordinate space in accordance with

$$f(r) = \frac{2}{\pi}\,r\int_0^\infty p^2\,dp\,j_l(pr)\,f(p), \tag{19}$$

where $l = 0$ for u, $l = 1$ for v, and $l = 2$ for w, we obtain

$$\int_0^\infty dr\,[u^2(r) + w^2(r) + v_t^2(r) + v_s^2(r)] = 1. \tag{20}$$

Although the square integrability of the functions we have introduced and the normalization conditions (20) make it convenient to describe the relative values of the functions as if they were probabilities of the corresponding components in the deuteron, we must emphasize that in reality these functions do not have a probability interpretation since they are related to a form factor and not to the deuteron state vector. If the wave functions are to have a probability interpretation, they must be coefficients in the expansion of the state

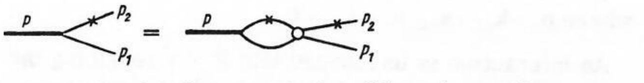

FIG. 2. Graphical representation of Gross's equation.

vector with respect to states with quantum numbers in whose probability we are interested. The components of the Fock column are quantities of this kind. For the functions ψ^+ and ψ^-, we obtain an approximate system of coupled equations in the coordinate space:

$$(\hat{T}-\varepsilon)\psi^+ = -V^{++}\psi^+ - V^{+-}\psi^-; \quad (21)$$
$$-2m\psi^- = -V^{-+}\psi^+ - V^{--}\psi^-. \quad (22)$$

Eliminating ψ^- from Eq. (1) to terms V/M inclusively, we obtain

$$[\hat{T} + V^{++} + (V^{+-})^2/2m]\,\psi^+ = \varepsilon\psi^+. \quad (23)$$

The term $(V^{+-})^2/2m$ in (23) has a relativistic origin. It is positive and at short distances leads to a repulsive core irrespective of the meson species exchanged.

In Ref. 15, the deuteron wave functions ψ^+ and ψ^- were found numerically with allowance for the exchange of only π and σ mesons, which lead to attraction. However, because of the term $(V^{+-})^2/2m$ the wave functions were close to Reid's wave functions with a soft core.[18] The following probabilities of D and P states were obtained: $P(^3D_1) = 5.10 - 6.31\%$; $P(^3P_1) = 0.70 - 2.00\%$; $P(^1P_1) = 0.01 - 0.11\%$. In Ref. 16, these wave functions were used to calculate the cross sections of radiative capture of a neutron near the $np \to d\gamma$ threshold. A problem arose because the nonrelativistic calculation of the diagram in Fig. 3 did not completely explain the experimental value of the cross section: $\sigma^{\exp} = 334.2 \pm 0.5$ mb (Ref. 19), $\sigma^{\text{teor}} = 302.5 \pm 4.0$ mb (Refs. 20 and 21). There was a discrepancy of 9.5% between theory and experiment. As was shown in Ref. 22 (see also Refs. 23–25), allowance for meson exchange currents, and, in particular, "pair creation" (Fig. 4), eliminates this discrepancy (see Sec. 3 for more details).

In Ref. 16, the contribution of pair creation was taken into account automatically by including states ψ^- with negative energy in the deuteron wave function. The contribution of pair creation calculated in this manner was close to but somewhat smaller than the value found earlier (1.86% in the amplitude compared with the 3.72% of Ref. 23) and improves the agreement with experiment.

Relativistic "Quantum-Mechanical" Systems. We now turn to the description of relativistic bound systems consisting of a fixed number of particles. Essentially, this concept is not based on that of a quantized field. Indeed, in a certain sense it contradicts it since the Hamiltonian of any system of interacting quantized fields does not commute with the particle number operator and, therefore, cannot be diagonalized simultaneously with it. Wave functions of quantum-mechanical type were first considered by Dirac.[26]

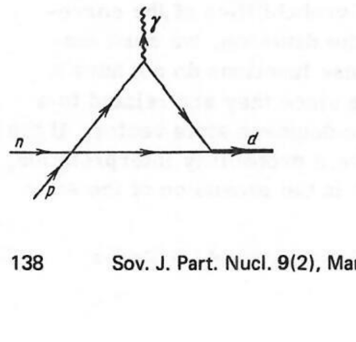

FIG. 3. Diagram corresponding to the process $np \to d\gamma$.

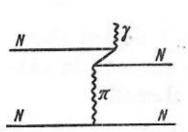

FIG. 4. Meson exchange current with pair creation.

In such schemes, the dynamical variables are not field operators but coordinates of the particles. In order to obtain a relativistically invariant theory, it is necessary to construct the generators P_μ and $M_{\mu\lambda}$ of the Poincaré group from these dynamical variables in such a way as to satisfy the commutation relations of the group:

$$\left.\begin{aligned} &[P_\mu, P_\nu] = 0; \ \frac{1}{i}[P_\mu, M_{\varkappa\lambda}] = g_{\mu\lambda}P_\varkappa - g_{\mu\varkappa}P_\lambda; \\ &\frac{1}{i}[M_{\mu\nu}, M_{\varkappa\lambda}] = g_{\mu\varkappa}M_{\nu\lambda} - g_{\nu\varkappa}M_{\mu\lambda} + g_{\nu\lambda}M_{\mu\varkappa} - g_{\mu\lambda}M_{\nu\varkappa}. \end{aligned}\right\} \quad (24)$$

For example, for one free particle the generators P_μ and $M_{\mu\nu}$ can be chosen in the form

$$\left.\begin{aligned} &P_r = p_r; \ P_0 = \sqrt{\mathbf{p}^2 + m^2}; \\ &M_{rs} = q_r p_s - q_s p_r; \ M_{r0} = q_r(\mathbf{p}^2 + m^2)^{1/2}, \end{aligned}\right\} \quad (25)$$

where $[q_r, p_s] = i\delta_{rs}$; $r, s = 1, 2, 3$. In the nonrelativistic Galileo-invariant theory of n particles, this formulation of the problem leads to the generators

$$\mathbf{P} = \sum_1^n \mathbf{p}_a, \ \mathbf{M} = \sum_1^n \mathbf{r}_a \times \mathbf{p}_a; \ H = \sum_1^n (\mathbf{p}_a^2/2m_a) + V,$$

where V is a scalar function of the internal coordinates $\mathbf{r}_a - \mathbf{R}$; $\mathbf{R} = \sum_1^n [(m_a/M)\mathbf{r}_a]$; $M = \sum_1^n m_a$. The problem of constructing representations of the group with such generators leads, in its turn, to the Schrödinger equation and nonrelativistic quantum mechanics.

From this point of view, the derivation of the explicit form of the generators in field theory on the basis of the Lagrangian and in accordance with Noether's theorem is the most direct and simplest way of constructing these generators. In a theory with a fixed number of particles [for example, two; the corresponding dynamical variables are x_1, x_2, $x = (t, \mathbf{x})$] in the absence of interaction one can also construct fairly easily the generators P_μ and $M_{\mu\nu}$ from x_1, x_2 and p_1, p_2. The introduction of operators that take into account interaction only in the generators P_ν (as in the nonrelativistic approximation) violates the commutation relations in the relativistic case. It is necessary to introduce interaction operators in the generators $M_{\mu\nu}$ as well. However, a fairly complicated algebra then arises. It simplifies if one formulates the theory on a light front.[26]

This approach was used in Refs. 27 and 28 to obtain a dynamical basis of the quark model. The properties of relativistic quantum-mechanical systems were studied in Ref. 28 for the example of a covariant harmonic oscillator. Replacing t and z by the "coordinates" $x^{\pm}_{1,2} = (z_{1,2} \pm t_{1,2})/\sqrt{2}$ and $\mathbf{x}_{\perp 1,2} = (x_{1,2}, y_{1,2})$ and introducing the corresponding canonically conjugate momenta $k_{\pm 1,2}\mathbf{k}_{\perp 1,2}$, we find that, for example, the generator $\hat{P}_+$ takes the form

$$\hat{P}_+ = -(m^2 + \mathbf{p}_\perp^2)/2p_-,$$

where $\mathbf{p}_\perp = \mathbf{k}_{1\perp} + \mathbf{k}_{2\perp}$; $p_- = k_{1-} + k_{2-}$.

An interaction is introduced into $\hat{P}_+$ by replacing the mass m^2 by the mass operator $\hat{M}^2$, which depends on the relative "coordinates" and "momenta". The operator $\hat{M}^2$ must be introduced simultaneously into the generators $\hat{M}_{\mu\nu}$.

The wave functions must transform in accordance with a representation of the Poincaré group. The problem of finding these representations leads to the following equation for the wave function:

$$\hat{M}^2\varphi = \mu^2\varphi. \tag{26}$$

As was shown by Terent'ev,[27] the wave function depends on only the single variable

$$M_0^2 = (m^2 + \mathbf{q}_\perp^2)/(1/4 - \xi^2),$$

where $\xi = k_-/p_-$; k_- is the relative "coordinate" $k = (k_1 - k_2)/2$; $\mathbf{q}_\perp = \mathbf{k}_\perp - \xi\mathbf{p}_\perp$.

We emphasize that the coordinates $x_{1,2}$ and momenta $k_{1,2}$ in this approach do commute "correctly" with one another despite their being operators, but after the introduction of the interaction operator they no longer have the meaning of true particle coordinates in the four-dimensional configuration space. This can be seen from the fact that they do not satisfy the commutation relations which any four-vector must satisfy:

$$[x_\mu, M_{\varkappa\lambda}]/i = g_{\mu\lambda}x_\varkappa - g_{\mu\varkappa}x_\lambda. \tag{27}$$

The interaction operator introduced in $M_{\mu\nu}$ precludes fulfillment of the commutation relation (27). Because of the impossibility of separating the center-of-mass motion in a relativistic system of interacting particles, the two-particle relativistic wave function depends on two physical variables (see Sec. 5) and a function with true particle coordinates has not been found.

In nuclear physics, such an approach was used in Ref. 30 to find the relativistic corrections to the deuteron wave function. The algebra was considered, not on a light front, but in the plane $t=0$, and it was required that the commutation relations should be satisfied to terms $\sim 1/c^2$. Expressions for the generators with accuracy to $1/c^2$ were found earlier in Ref. 31; in Ref. 30, the deuteron was assumed to consist of only two nucleons at all nucleon momenta (both small and large). Other degrees of freedom were not introduced. As interaction operator in the generators, an ordinary nonrelativistic potential was chosen. An investigation was made of the magnitude of the relativistic corrections to the deuteron wave function, to the electromagnetic form factors of the deuteron, and to polarization in ed scattering. The corrections are of order $\sim q^2/m^2$ (q is the momentum transferred to the deuteron). Additional degrees of freedom must be added in order to take into account exchange currents and isobars in such a theory. But for the description of many-component systems, a more adequate formalism is field theory, in which the deuteron state vector at short distances includes organically the non-nucleon components.

2. ISOBARS IN THE DEUTERON

Because virtual transitions of the type $NN \to \Delta\Delta$, $NN \to NN^*$, etc, are possible in an interacting two-nucleon system, the deuteron always contains an admixture of isobars. In the deuteron, these virtual transitions take place most intensively at large relative momentum of the nucleons, and therefore the isobar admixtures in the deuteron are mainly concentrated at relatively short distances. Although they make a small contribution to the normalization integral (~1%), in reactions with large momentum transfers the isobar admixtures may exceed the nucleon component, and in some cases (for example, in reactions such as isobar knockout) even completely determine the phenomenon.

The existence of isobar admixtures in the deuteron was first proposed by Kerman and Kisslinger[32] in 1969. Since then, many papers have been devoted to this question. Since the detailed reviews Refs. 33–35 (see also Ref. 36) on this subject are available, we shall restrict ourselves here to a brief exposition of the main results.[1)]

The isobar admixtures in the deuteron wave function are calculated as follows. To be specific, we shall consider the $\Delta\Delta$ admixture. We consider the two-channel system of equations describing the transitions $NN \to NN$, $NN \to \Delta\Delta$, $\Delta\Delta \to \Delta\Delta$. Since the transition $NN \to \Delta\Delta$ leads to a small isobar admixture, it can be taken into account by perturbation theory.

The $\Delta\Delta \to \Delta\Delta$ potential is in fact unknown. Part of the $\Delta\Delta$ interaction is due to the virtual decay of Δ and to $N\Delta$ interaction (Fig. 5b). As is shown in Ref. 37, this "induced" interaction is much smaller than the $N\Delta$ interaction, which, in its turn, is weaker than the NN interaction. In addition, one can imagine "direct" exchange of bosons between isobars, this being identical in its physical nature to the NN interaction. Estimates based on SU(6) symmetry give a "direct" $\Delta\Delta$ interaction of the same order as the "exchange" interaction due to the process $\Delta \to N\pi$. The diagrams of Figs. 5a, 5b, and 5c correspond to the direct, induced, and exchange $\Delta\Delta$ interactions.

For the exchange interaction, the $\Delta\Delta \to NN$ transition amplitude (Fig. 6) is an irreducible block, and this is frequently called the "transition potential" $V_{\Delta N}$. Apart from the normalization, this is equal to the amplitude $F_{N\Delta}$, which, in its turn, can be expressed in terms of the amplitudes $\Gamma_{\Delta N\pi}$ of $\Delta \to N\pi$ decay. As a result,

$$V_{\Delta N}(\mathbf{q}) = -\Gamma_{\Delta N\pi}(\mathbf{q})\,\Gamma_{\Delta N\pi}(\mathbf{q})/[\mathbf{q}^2 + \mu^2 - (M_\Delta - m - i\Gamma/2)^2].$$

If we ignore the $\Delta\Delta \to \Delta\Delta$ potential, then for the vertex part $d \to \Delta\Delta$ we obtain

$$\Gamma_{\Delta\Delta}(\mathbf{p}) = -(2\pi)^{-3/2}\int d^3k V_{\Delta N}(\mathbf{p}-\mathbf{k})\,\psi(\mathbf{k}), \tag{28}$$

where $\psi(\mathbf{k})$ is the two-nucleon wave function.

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1) After this paper had been written, we became acquainted with the new reviews Refs. 106–108 devoted to isobars in nuclei.

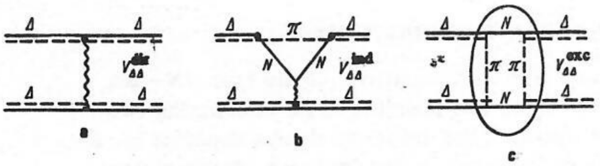

FIG. 5. Potentials of ΔΔ interaction. a) is the direct interaction, b) the induced, and c) the exchange interaction.

In the deuteron wave function, the following ΔΔ states are possible: $^3S_1$, $^3D_1$, $^7D_1$, and $^7G_1$. Of these, the most important are $^3S_1$ and $^7D_1$. In Fig. 7, which is taken from Markushin's paper Ref. 38, we give the squares of the moduli of the $^3S_1$ and $^7D_1$ vertex functions together with the S-wave nucleon–nucleon vertex part. To describe the $NN\to\Delta\Delta$ transition, the one-pion exchange potential shown in Fig. 6 was used. The nucleon–nucleon wave function corresponded to the Hamada–Johnston potenial.[39] At the vertex $\Delta\to N\pi$, a cutoff was introduced by means of the form factor

$$F(q^2)=(c^2+q_0^2)/(c^2+q^2), \qquad (29)$$

where $q_0=231$ MeV/c, $c=3\mu$, and μ is the pion mass.

The relative weight of the various ΔΔ states in the deuteron wave function is shown in Fig. 8, taken from Ref. 38. The probabilities of admixtures of different ΔΔ states with the Hamada–Johnston nucleon wave function and with cutoff parameter $c=3\mu$ are as follows: $P(^3S_1)=0.66\%$, $P(^3D_1)=0.08\%$, $P(^7D_1)=1.64\%$, $P(^7G_1)=0.14\%$. The total probability is $P(\Delta\Delta)=2.52\%$. The results are very sensitive to the choice of the nucleon-nucleon potential and the cutoff parameter c, and the probabilities $P(\Delta\Delta)$ vary by about a factor of two when these vary in reasonable limits.

However, introduction into the $NN\to\Delta\Delta$ exchange potential of a term corresponding to ρ-meson exchange leads to a certain cutoff at short distances, and the sensitivity to the parameter c and the nucleon–nucleon potential becomes weaker.[40]

Estimates of $NN^*(1520)$ isobar admixtures in the deuteron in the framework of relativistic equations were made by Weber.[13] As we have already noted in Sec. 1, the point of departure for these estimates was the three-particle quasipotential equations for the πNN system.[11,12] It was assumed that the πN amplitude in them has a resonance nature. The $NN\to NN^*$ and $NN^*\to NN^*$ interactions were described by π-meson exchange. Calculation with the McGee wave function leads to a total probability of the $NN^*(1520)$ admixture of order 1%. A nonrelativistic estimate under the same original assumptions gives the much smaller value ≈0.057%.

A direct confirmation of the presence of isobars in the deuteron would be observation of their knockout from the deuteron. In Ref. 41 an experimental search was made for isobars in the deuteron. The estimates of these papers for the isobar admixtures in the deuteron have a certain spread, but in order of magnitude are in the region 1%. For unambiguous interpretation of the isobar knockout experiments, one must be sure that the observed isobars were contained in the deuteron and not produced in the collision. Questions relating to the identification of the mechanisms of processes with large momentum transfers have only just begun. We mention in this connection Kolybasov's paper Ref. 42, in which he shows that study of the polarization of the isobars in the final state is very informative from the point of view of identifying the reaction mechanism. Since the polarization of the isobars is deduced from the angular distribution of their decay products, and these are detected in experiments, to study the polarization of the isobars it is only necessary to accumulate statistics.

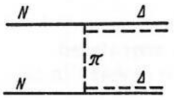

FIG. 6. Potential of the transition $NN\to\Delta\Delta$.

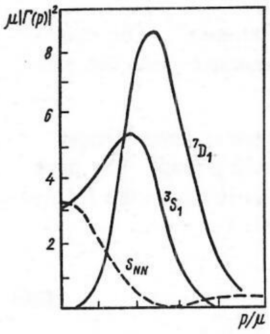

FIG. 7. The ΔΔ vertex functions in the deuteron for the $^3S_1$ and $^7D_1$ states. Continuous curves for the $^3S_1$ and $^7D_1$ states; dashed curve for the S-wave NN vertex function.[38]

Besides isobar knockout phenomena, admixtures of resonances in the deuteron wave function lead to appreciable effects in the magnetic and quadrupole moments of the deuteron (see Sec. 3), in pd backward scattering, in photodisintegration, and in other processes.

It must be borne in mind that, as was shown by Markushin,[43] allowance for the finite lifetime of the isobar has the consequence that the knockout processes involve the $d\to\Delta\Delta$ vertex function averaged over the mass of the isobar within the width of Δ, and the electromagnetic form factors contain the vertex function for a complex mass $M_\Delta - i\Gamma/2$. The vertex function corresponding to Δ knockout is appreciably larger than the vertex function that determines the form factors. It would be unjustified to ignore this circumstance when comparing the isobar admixtures extracted from isobar

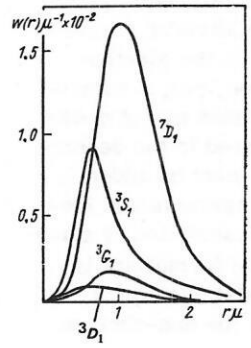

FIG. 8. Probability densities of different ΔΔ states in the deuteron.

knockout data and from the form factors.

Note that the question of the influence of isobar admixtures on the electromagnetic form factors of the deuteron at large momentum transfers has not yet been completely elucidated. A number of papers have been devoted to this question (see, for example, Refs. 44 and 45). In the literature, one can find contradictory assertions about the influence of Δ admixtures on the deuteron form factors. In one of the latest papers [45] it is shown that a $\Delta\Delta$ admixture of 0.7–1.5% only slightly changes the form factor right up to momentum transfer $q^2=100\ F^{-2}$ (4 GeV^2) and that its influence on the form factor is less than the influence of meson exchange currents. The calculations were made in the framework of the nonrelativistic formalism and with a real mass of the isobar.

Despite the existing uncertainties, on the basis of all the existing calculations and the experimental data we may conclude that a probability of isobar admixtures in the deuteron at the 1% level is very likely. Further theoretical and experimental investigation of isobar admixtures in nuclei is of the greatest interest.

3. MESON EXCHANGE CURRENTS

Mesons are present in a system of interacting nucleons because the nuclear forces arise from meson exchange. In the deuteron, the nonrelativistic nucleons move slowly, and the interaction between them is therefore retarded to a small degree. In other words, this means that the effect of the presence of the mesons in the nucleus is manifested weakly because of the short time that a meson spends between its emission by one nucleon and its absorption by the other. However, even this small admixture of mesons has an appreciable influence on the magnetic and quadrupole moments of the deuteron, appreciably exceeding the experimental errors with which these quantities are known. The influence of the meson exchange currents on the deuteron form factors at large momentum transfer q^2, which corresponds to short internucleon distances where meson exchanges already occur very intensively, becomes even more pronounced. At large q^2, the ed scattering amplitude has a contribution from not only the scattering of the electron on the nucleons in the deuteron but also from scattering on the mesons in the deuteron.

Here, we consider the influence of the meson exchange currents on the static properties of the deuteron (magnetic and quadrupole moments), on the electromagnetic form factors of the deuteron at large momentum transfers, and on the cross sections of the processes of electrodisintegration of the deuteron, $ed\to enp$, and radiative capture of a neutron, $np\to d\gamma$.

Before we discuss the influence of meson exchange currents on the properties of the deuteron, let us compare the predictions of the canonical theory of the deuteron (which neglects the contribution of isobars and the meson exchange currents) for the magnetic and quadrupole moments μ_d and Q of the deuteron with the experimental values of μ_d and Q. To calculate μ_d and Q, it is necessary to average the operators of the magnetic and quadrupole moments of the deuteron over the wave function. As a result, we arrive at the expressions

$$\mu_d=(\mu_p+\mu_n)-(3/2)P_D(\mu_p+\mu_n-1/2); \quad (30)$$

$$Q=\frac{1}{5\sqrt{2}}\int_0^\infty\left(uw-\frac{w^2}{2\sqrt{2}}\right)r^2\,dr, \quad (31)$$

where μ_p and μ_n are the magnetic moments of the proton and the neutron; u and w are the S- and D-wave functions of the deuteron; P_D is the D-wave admixture in the deuteron: $P_D=\int_0^\infty w^2(r)dr$.

The results of calculations with different wave functions are given in Table II, which was compiled on the basis of Refs. 46 and 47. None of the wave functions in Table II gives a simultaneous description of the magnetic and quadrupole moments. Note that the experimental data are systematically larger than the theoretical predictions. Whereas the quadrupole moment and a number of other deuteron data agree with a value

TABLE II. Comparison with experiment of the calculated values of the magnetic μ_D and quadrupole Q moments of the deuteron for different variants of the wave function

Wave function	P_D, %	Predicted μ_D	Deviation from experiment ($\mu_{exp}-\mu_{theor}$)	Predicted Q, F^2	Deviation $Q_{exp}-Q_{theor}$	Core or boundary condition, F
Hamada-Johnston	6.96	0.840	0.01547	0.281	0.001	0.485
Bressel	6.49	0.842	0.01347	0.281	0.001	0.686
Feshbach-Lomon	4.31	0.854	0.00347	0.268	0.014	0.734
Bethe-Reid (hard core)	6.50	0.842	0.01347	0.277	0.005	0.548
Bethe-Reid (soft core)	6.47	0.842	0.01347	0.280	0.002	0.057
Hulthén (without core)	4.00	0.856	0.00147	0.271	0.011	0.000
Hulthén	4.00	0.856	0.00147	0.271	0.011	0.432
Hulthén	4.00	0.856	0.00147	0.271	0.011	0.561
Bryan-Scott	5.44	0.8485	0.00897	0.2589	0.023	—
Ueda-Green I	5.47	0.8483	0.00917	0.2811	0.001	—
Ueda-Green II	6.01	0.8452	0.01227	0.2797	0.002	—
Ueda-Green III	4.93	0.8512	0.00627	0.2605	0.0022	—
		$\mu_D^{exp}=0.85747\pm0.00008$	±0.00008	$Q_{exp}=0.282\pm0.002$	±0.002	

equal to $P_D = 5.5 \pm 0.7\%$ for the D wave in the deuteron, to describe the magnetic moment it is necessary to assume a smaller admixture of the D wave: $P_D = 3.90 \pm 0.06\%$. It is natural to attribute this discrepancy to the ignored contribution of the isobars and the meson exchange currents. Allowance for these effects has the consequence that the theoretical values of μ_D and Q are shifted in the correct direction. However, the calculations contain significant uncertainties, which arise from the inaccuracies in the meson–nucleon constants and the nonuniqueness in the choice of the most important meson configurations and isobars in the deuteron wave function. In addition, the calculations are sensitive to the behavior of the deuteron wave function at short distances, which contains appreciable uncertainties and is assumed to be nonrelativistic.

We give the very typical results of Ref. 47, in which allowance was made for the contributions of isobars and also of the meson exchanges shown in Figs. 4 and 9. Here, B and B' are the mesons that contribute to the OBEP potential. The results are given in Table III. It can be seen that the isobars and meson exchange currents give a value somewhat larger than is needed for reconciling the experimental data. The calculations have a semi-quantitative nature and indicate that a more careful investigation of these phenomena is required.

Pokrovskiĭ[48] succeeded in reconciling the μ_D and Q values without introducing isobars or meson exchange currents but at the price of introducing momentum-dependent terms (spin-orbit terms and terms quadratic in the momenta) into the phenomenological NN potential. The operator of the interaction with the electromagnetic field was obtained by the substitution $p \to p - (e/c)A$. Further terms were added to the expression for the magnetic moment (30). One can find a potential that describes the NN scattering phase shifts at energies up to the pion production threshold and the binding energy and the quadrupole and magnetic moments of the deuteron. This also indicates that the question of isobar admixtures and meson exchange currents and their influence on the magnetic and quadrupole moments of the deuteron cannot be regarded as definitively settled.

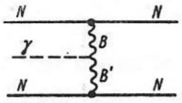

FIG. 9. Meson exchange current. B and B' are meson exchanges.

We now turn to a consideration of the influence of meson exchange currents on the electromagnetic form factors of the deuteron. The deuteron has three electromagnetic form factors: charge, magnetic, and quadrupole. Their number is equal to the number of invariant amplitudes of the matrix element of the electromagnetic current $p'\lambda' | J\mu(0)|p\lambda\rangle$ between states with spin 1. A decomposition of the matrix element $\langle p'\lambda' | J^\mu(0)| p\lambda\rangle$ into invariant amplitudes is given in Ref. 49, and an expression for the electromagnetic form factors in terms of them in Ref. 50. The review Ref. 51 is devoted to the electromagnetic properties of the deuteron.

We give the expression for the cross section of elastic ed scattering:

$$d\sigma/d\Omega = (d\sigma/d\Omega)_{\rm Mott}\, [A(q^2) + B(q^2)\tan^2(\theta/2)], \tag{32}$$

where $(d\sigma/d\Omega)_{\rm Mott}$ is the cross section of scattering on a point particle. The invariant functions $A(q^2)$ and $B(Q^2)$ can be expressed in terms of the deuteron form factors:

$$A(q^2) = F^2_{\rm ch}(q^2) + \frac{8}{9}\eta^2 F^2_Q(q^2) + \frac{2}{3}\eta(1+\eta) F^2_{\rm mag}(q^2); \tag{33}$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)^2 F^2_{\rm mag}(q^2), \tag{34}$$

where $\eta = q^2/4M_d^2$.

Because the deuteron has zero isospin, only isoscalar exchange currents contribute to elastic ed scat-

TABLE III. Contribution of isobar configurations to the deuteron wave function and of meson exchange currents to the magnetic and quadrupole moments of the deuteron

NN potential	P_D, %	$P_{\rm isobar}$,%	μ_D^{NN*}	$\mu^{NN+IC+MEC*2}$	$\frac{\mu^{MEC}}{(\mu^{IC}+\mu^{MEC})}$	$\mu^{\rm exp}-\mu^{\rm theor}_{\rm tot}$	Q^{NN*3}, F	$Q^{NN+IC+MEC*4}$	$Q^{\rm exp}-Q^{\rm theor}_{\rm tot}$
Bryan-Scott	5.44	1.51	0.8485	0.8893	22	--0.03183	0.2589	0.2555	0.026
Ueda-Green I	5.47	1.38	0.8483	0.8812	17	−0.02373	0.2811	0.2778	0.004
Ueda-Green II	6.01	1.35	0.8452	0.8766	17	−0.01913	0.2797	0.2765	0.005
Ueda-Green III	4.93	1.59	0.8512	0.8931	16	−0.03563±0.00008	0.2605	0.2567	0.025
				$\mu^{\rm exp}$ =0.85747±0.00008				$Q^{\rm exp}$ =0.282±0.002	

*μ_D without allowance for isobar admixtures and meson exchange currents.
*2μ_D with allowance for these corrections.
*3Without allowance for admixture of isobars and meson exchange currents.
*4With allowance for these corrections.

tering. We consider here the results of one of the latest investigations,[52] in which the contribution of several isoscalar meson exchange currents is calculated. Earlier, the contributions of different currents were studied separately. In Ref. 53, Adler and Drell (see also Ref. 54) took into account the contribution of the $\rho\pi\gamma$ process shown in Fig. 9 (with B and B' mesons corresponding to the ρ and π mesons). They used a value $g_{\rho\pi\gamma}$ of the $\rho\pi\gamma$ vertex equal to unity, which follows from the quark model. The latest measurements[55] gave the value $g_{\rho\pi\gamma}=0.38$. In Ref. 56, the contribution of the $\rho\pi\gamma$ current for the value $g_{\rho\pi\gamma}=1$ was also calculated. In addition, in Ref. 56 allowance was made for the $\omega\sigma\gamma$ process. Blankenbecler and Gunion[57] took into account the meson configurations shown in Fig. 10. In Refs. 44 and 58, the contribution of "pair creation" (see Fig. 4) was considered. The papers Refs. 59–61, and also Ref. 62, are devoted to calculations of the contributions of the meson exchange currents to the electromagnetic form factors of the deuteron.

In Ref. 52, Gari and Hyuga took into account the contributions of the $\rho\pi\gamma$ exchange current (see Fig. 9) and pair creation (see Fig. 4) with intermediate π, ρ, and ω exchanges. In order to separate the contribution of the meson exchange currents, they ignored the contributions of the isobars and the relativistic corrections to the deuteron wave function. They investigated the sensitivity to the deuteron wave function, to the form factors at the meson–nucleon vertices, and to the electromagnetic form factors of the nucleons. It proved to be very great. The contribution of pair creation with π-meson exchange (see Fig. 4) and $\rho\pi\gamma$ exchange current (see Fig. 9) predominated over the exchange of ρ and ω mesons (Fig. 4). The charge, magnetic, and quadrupole form factors of the deuteron were calculated up to $q^2 \leqslant 200\ F^{-2}$ (8 GeV^2). The results of the calculation of the function $A(q^2)$ with the Hamada–Johnston wave function and the parametrization of the nucleon form factors from Ref. 63 together with the experimental data of Arnold *et al.*[64] are given in Fig. 11. There is satisfactory agreement between the calculations with allowance for the exchange currents and the experimental data. We shall consider below what significance should be attributed to this agreement.

Experimental data on the function $B(q^2)$, which is expressed solely in terms of the deuteron magnetic form factor in accordance with (34), are at present available up to $q^2=25F^{-2}$ (1 GeV^2).[65] They are shown in Fig. 12 and also agree with the calculations that take into account the exchange currents.

We now discuss the contribution of the meson exchange currents to the cross section of deuteron electrodisintegration, $ed\to enp$. We shall consider only the paper of Simon *et al.*[66] (see also Ref. 67 and the references given there). Deuteron disintegration processes are interesting in that isovector currents which are not manifested in elastic ed scattering, contribute to them. One of these is the $2\pi\gamma$ isovector current (see Fig. 9, in which B and B' are π mesons). In Ref. 66, the cross section of the process $ed\to enp$ was measured and calculated. The calculation included isobar admixtures in the deuteron wave function and meson exchange currents. The results of the comparison between theory and experiment are given in Fig. 13. The calculations were made with the Hamada–Johnston potential The isovector pion exchange current makes the maximal contribution, allowance for it increasing the cross section around the threshold by 60% with a consequent good description of the experimental data. The isobar contribution is about 20% that of the pion exchange current. The large contribution of the isovector pion exchange current is due to the small mass of the pion. Near the threshold in the final NN state the $^1S_0$ resonance with isospin 1 is predominant, and it is this that singles out the contribution of the isovector current.

The contribution of exchange currents for the cross section of neutron radiative capture $np\to d\gamma$ was first studied by Riska and Brown.[22] As we have already

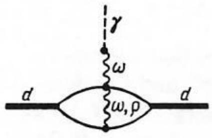

FIG. 10. Meson exchange current considered in Ref. 57.

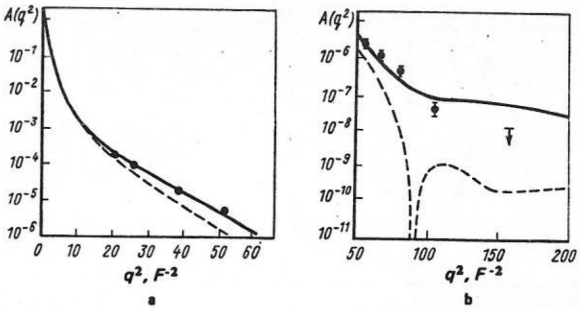

FIG. 11. Comparison of the theoretical calculations of *ed* scattering in Ref. 52 with the Hamada–Johnston wave function and the experimental data of Ref. 64. The continuous curve is with allowance for meson exchange currents; the dashed curve is without allowance for them. The nucleon form factors were parametrized in accordance with Ref. 63.

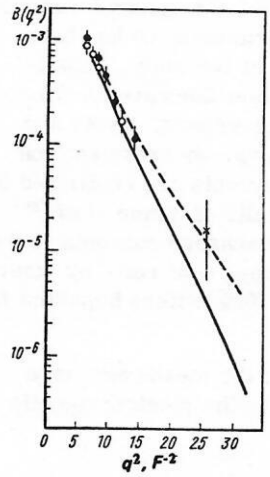

FIG. 12. The function $B(q^2)$ from Ref. 65. The continuous curve is the calculation with the contribution of the exchange currents (Ref. 58); the dashed curve is without allowance for them. The wave function was taken in Reid's form.[18] The nucleon form factors were parametrized in accordance with Ref. 63.

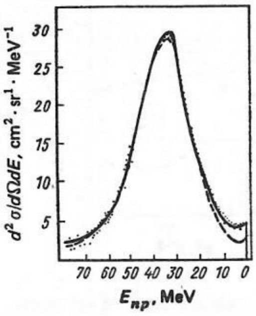

FIG. 13. Doubly differential cross section of the process $ed \to enp$ at the angle 90° as a function of the relative energy E_{np}. The continuous curve is the calculation with allowance for the exchange currents; the dashed curve is without allowance for them.[66]

said above, allowance for the meson exchange currents leads to an explanation of the 9.5% discrepancy between the experimental data[19] on the $np \to d\gamma$ reaction and the theoretical calculations[20, 21] without exchange currents. The contribution of meson exchange currents to this process was studied in Refs. 23–25 as well as in Ref. 22. In the latter, a study was made of the contribution of pair creation with pion exchange (see Fig. 4) and the intermediate formation of the Δ isobar (Fig. 14). The first process gives a 3.28% correction to the amplitude, and the second a 1.45% correction. The total contribution to the amplitude is 4.73%, and to the cross section 9.7%, which eliminates the discrepancy. The contribution of heavier mesons is slight since it is determined by shorter distances, where the wave function is small. Thus, allowance for a diagram of the type of Fig. 9 with intermediate ω and π mesons ($\omega\pi\gamma$ configuration) gives a 0.15% correction.

Concerning the inverse reaction $\gamma d \to pn$, we point out that the polarization of the final protons and neutrons is sensitive to meson exchange currents.[68]

On the basis of what we have said in this section, we draw the following conclusions. A contribution of the isovector meson exchange currents to neutron radiative capture and deuteron electrodisintegration has been established with a high degree of certainty. This is because one does not need to know the wave function at very short distances in these processes. Riska and Brown's calculations[22] of the $np \to d\gamma$ cross section with allowance for exchange currents are confirmed by Refs. 23–25. Although the results of Simon *et al.*[66] relating to the contribution of exchange currents to the process $ed \to enp$ have not yet been confirmed by other authors, the size of the effect (60%) offers hope that the result will not disappear.

Qualitatively, it is clear that the meson exchange currents must also contribute to the electromagnetic

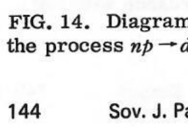

FIG. 14. Diagram with intermediate Δ isobar contributing to the process $np \to d\gamma$.

form factors and to the deuteron magnetic and quadrupole moments. However, the quantitative side of the matter is not clear since the uncertainties in the form factors associated with the deuteron wave function at short distances are comparable with the effect of the meson currents. The magnetic and quadrupole moments of the deuteron in different calculations can be satisfactorily described both with and without allowance for isobars and exchange currents. The elucidation of the effects of isobars and meson exchange currents, which is of considerable independent interest, is also necessary for finding the nucleon component of the wave function at short distances.

4. PROCESSES WITH LARGE MOMENTUM TRANSFERS

The structure of the deuteron wave function at short distances is most fully manifested in processes with large momentum transfers. To extract information about the wave function from experimental data on reactions with large momentum transfers, it is necessary to know the mechanisms of these reactions. Of course, there can also be mechanisms in which, despite the large transfer of momentum to the nucleus as a whole, the effective momenta of the nucleons are not particularly large. In this section, we shall discuss a number of processes with large momentum transfers from the point of view of their mechanisms and the light they shed on the wave function at short distances. We consider the following processes: elastic and deep inelastic ed scattering, πd and pd backward scattering, the reaction $\pi^- d \to p\Delta^-$, and cumulative meson production. Some of these questions have been considered in the review Ref. 69

Elastic ed Scattering at Large Momentum Transfers. The contribution of isobars and meson exchange currents to elastic ed scattering at large momentum transfers has already been discussed in Sec. 2 and 3. Here, we discuss the interpretation of the latest experimental data[64] in the framework of the quark model and the canonical nonrelativistic theory of the deuteron. The papers of Ref. 70 are devoted to studying the behavior of the form factors of heavier nuclei at large momentum transfers.

It was noted in Ref. 64 by Arnold *et al.* that the experimental data on ed scattering approach the predictions of the quark model. Let us consider to what extent this is true. As was shown in Refs. 71, and also Ref. 72, the form factor of a system consisting of n constituents has the asymptotic behavior $F(t) \sim 1/t^{n-1}$ as $t \to \infty$. Using this formula, we obtain the following predictions for the asymptotic behavior of the form factors:

electron $(n=1)$: $F_e(t) \sim 1$ (point);
meson $(n=2)$: $F_\pi(t) \sim 1/t$ (pole);
nucleon $(n=3)$: $F_N(t) \sim 1/t^2$ (dipole).

If we assume that at the short distances that correspond to large $|t|$ the nucleons "overlap" and the deuteron consists of six quarks, we then obtain the following law of decrease of the deuteron form factor; $F_d(t)$

$\sim 1/t^5 = 1/q^{10}$ and for the function $A(q^2)$ this corresponds to $A(q^2) \sim 1/q^{20}$. It was pointed out in Ref. 64 that the observed decrease of the function $\sqrt{A(q^2)}$ approaches the law $1/q^{10}$, and is also well described by the function [73, 74]

$$F_d(q^2) \sim F_p^2(q^2/4)(1+q^2/m^2)^{-1}, \quad (35)$$

where

$$F_p(q^2) = (1+q^2/0.71)^{-2}; \quad (36)$$

$F_p(q^2)$ is the proton form factor.

The function $6.3 \cdot 10^{-2} F_d(q^2)$, and also the functions $1.6 \cdot 10^{-2}/q^6$ and $4.9 \cdot 10^{-2}/q^{10}$ together with the experimental points [64] for $\sqrt{A(q^2)}$ are shown in Fig. 15. If the experimental data are approximated by a power law, then in the investigated region this law is $1/q^6$ and not $1/q^{10}$. The function (35) also gives a good description of the experimental data. It indeed has the asymptotic behavior $1/q^{10}$, but in the region $q^2 \leqslant 6$ GeV^2 the asymptotic behavior has not yet been reached. It should be noted that in the quark model one can obtain a prediction not only for the asymptotic behavior of the form factors but also for the order of magnitude of the critical momentum transfer q^2_{cr} from which onward the form factor is described approximately by the asymptotic formula. For a system consisting of n constituents, $q^2_{cr} = n^2 q_0^2$ (see Ref. 75), where q_0 is a characteristic momentum that can be determined from data on the nucleon form factor. For the proton ($n=3$), formula (36) coincides with the power-law behavior $1/q^4$ (with an accuracy of about 25%) from $q^2_{cr} = 5$ GeV$^2/c^2$ onward. Hence, for the deuteron ($n=6$) one must expect $q^2_{cr} = 20$ GeV$/c^2$. This means that a comparison of the predictions of the quark model with the existing experimental data ($q^2 \leqslant 6$ GeV$^2/c^2$) is premature.

We now consider the results obtained by describing the deuteron form factor at large q^2 by means of the nonrelativistic wave functions. It can be seen from Fig. 11a that the form factor calculated in the nonrelativistic model with the Hamada–Johnston wave function differs, even without allowance for isobars and exchange currents, from the experimental data in the interval $q^2 \leqslant 2$ GeV^2 by less than a factor of two as $A(q^2)$ decreases by five orders of magnitude. As is shown in Ref. 64, the Feshbach–Lomon wave function gives a

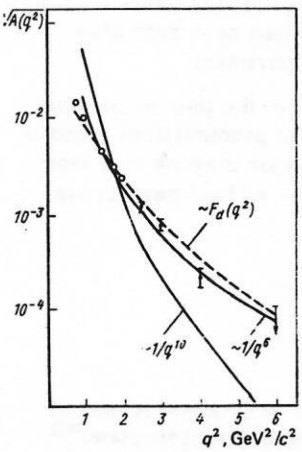

FIG. 15. Comparison of the power laws $1/q^6$ and $1/q^{10}$ for the function $\sqrt{A(q^2)}$ with the experimental data.[64] The dashed curve is the parametrization in accordance with (35) proposed in Refs. 73 and 74.

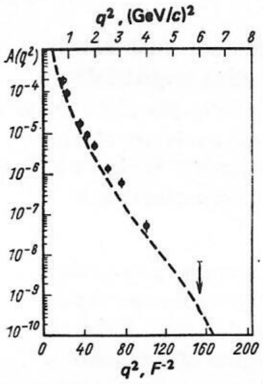

FIG. 16. Comparison of nonrelativistic calculation of the function $A(q^2)$ with the Feshbach–Lomon wave function and the experimental data of Ref. 64.

satisfactory description of $A(q^2)$ to $q^2 \leqslant 6$ GeV^2 (Fig. 16). A power law decrease of the form factor is readily obtained in the nonrelativistic model. If the S-wave function $u(r)$ behaves at the origin as $u(r) \sim r^n$, then ignoring the D waves we obtain in the limit $q^2 \to \infty$

$$\sqrt{A(q^2)} \sim F_p(q^2)/q^{2n+2} \sim 1/q^{2n+6}$$

The $1/q^{10}$ law is obtained, for example, if for $u(r)$ we choose the so-called "second Moravcsik function". It is hardly possible to take seriously the agreement between the nonrelativistic description and the experimental data in the region of large transfers since extrapolation of the nonrelativistic wave functions to the region $r \sim 1/m$ is physically unjustified (one can always find wave functions that give a satisfactory description of the deuteron properties in the nonrelativistic region and at large q^2 lead to a strong difference between the calculated and measured form factor; see, for example, Fig. 11b). In the analysis of experimental data at large momentum transfers, it is above all necessary to establish the basic principles which the wave function must satisfy at short distances.

Deep Inelastic Electroproduction on the Deuteron. Recently, experimental data have been published on deep inelastic ed scattering in a region forbidden for the reaction on the nucleon.[76] Measurements of the deuteron structure function $F_{2D}(x)$ were made to a value $x = -q^2/2M_d \nu = 0.7$ (ν is the energy transferred to the electron in the laboratory system; q^2 is the square of the four-momentum transfer). The variable x defined in this manner can reach a maximal value of 0.5 for electroproduction on the nucleon. Theoretically, these experimental data have been described by Strikman and Frankfurt[77] (see also West's paper, Ref. 78). In Ref. 77, the structure function of deep inelastic ed scattering is expressed in the impulse approximation in terms of the nucleon structure functions F_{2p} and F_{2n} and the deuteron wave function by the formula

$$F_{2D}(x) = \int d^3k \psi^2(\mathbf{k}) [F_{2p}(x/\alpha) + F_{2n}(x/\alpha)], \quad (37)$$

where $\alpha = (1 + k_z/\sqrt{m^2 + \mathbf{k}^2})/2$.

There is a more or less satisfactory agreement between the calculated and experimental data when the nonrelativistic wave functions $\psi(\mathbf{k})$ are used to describe the deuteron.[2)] However, in our opinion, such calculations can pretend to nothing more than a qualitative (in order of magnitude) description of the phenomenon at effective momenta of the intranuclear nucleons of order m and are quite unsuitable for significantly larger momenta and values of the variable x corresponding to them.

As was pointed out in Ref. 77, momenta in the range $\sim 0.7-1.3$ GeV/c are important in the deuteron wave function at $x=0.7$. Therefore, extrapolation of the nonrelativistic wave function to this region of momenta is invalid and leads to large indeterminacies depending on the type of wave function employed. In this respect, the situation is completely analogous to the one in elastic *ed* scattering. In addition, in Ref. 77 Strikman and Frankfurt ignored the contribution of isobar and meson exchange currents to the deuteron wave function. As can be seen from the discussion in Secs. 2 and 3, this contribution is very important for $k \approx 1$ GeV/c. Note also that the nucleon structure functions in the integral (37) are also found off the energy shell, which was ignored in Ref. 77. (We recall that in the formalism of the old perturbation theory used here, i.e., noncovariant Schrödinger representation, all particles are on the mass shell, but the energies of the initial state and the virtual state are not equal. For more detail on this see Secs. 5 and 6.)

It was pointed out in Ref. 77 that if the deep inelastic *ed* scattering is described by the Feynman technique, and not in the old perturbation theory, then a formula analogous to the one used by Lobov *et al.*[79] to describe cumulative meson production (see the end of this section) lowers the results by up to two orders of magnitude. In this connection, we note that the Feynman diagram of the impulse approximation in the scaling limit also leads to (37) with the substitution $\alpha \to \tilde{\alpha} = 1 - (\varepsilon(\mathbf{k})/2m)(1-k_z/\varepsilon(\mathbf{k}))$. For $\mathbf{k}^2 \ll m^2$, we have $\tilde{\alpha} = (1+k_z/m)/2$ and to terms $\mathbf{k}^2/m^2$ it is equal to α. A difference between the two calculations arises when α differs appreciably from $\tilde{\alpha}$. But, for the reasons given above, it is precisely in this region that the accuracy of both formulas is such that it becomes meaningless to distinguish between them.

In order to solve theoretically the problem of deep inelastic *ed* scattering, as well as to describe other such phenomena, we need above all an understanding of the deuteron structure at short (relativistic) internucleon distances. This requires some model or, perhaps, on the basis of general propositions of relativistic theory, a "natural" idea for direct parametrization of the wave function in the region of large velocities of the relative motion of the nucleons. We assume also that the non-nucleon component of the deuteron (isobar and other admixtures) can make a comparable contribution to the amplitudes of processes in which the intranuclear nucleons can take part only as relativistic particles. Finally, to achieve qualitative agreement between theory and experiment at large but not asymptotically large momentum transfers, one cannot avoid solving the problem of the parametrization of the amplitudes of the elementary processes, i.e., of electroproduction on nucleons or other constituents, off the energy (or mass) shell.

It is fairly clear that in the absence of a quantum-field theory of nuclear interactions that is derived from first principles and is at the same time realistic, one can hardly hope to obtain the necessary information to settle the above questions solely on the basis of data (even if very exact) on the inclusive reaction of deep inelastic *ed* scattering. Given the present situation, a realistic approach in our view is a composite description of different processes with the participation of relativistic constituents of the nucleus, i.e., a combined explanation of reactions characterized by different mechanisms and therefore corresponding to amplitudes in which the nuclear wave functions and elementary amplitudes enter in functionally different ways. This, in its turn, requires experiments in which the reaction mechanism can be reliably identified. In this respect, inclusive experiments are not sufficiently informative.

Elastic πd Backward Scattering. Successful attempts to describe elastic πd backward scattering at large momentum transfers were made by Kondratyuk and Lev.[80] They proposed a mechanism of double scattering with intermediate production of a resonance (Fig. 17). At pion energies close to the resonance production threshold, the diagram of Fig. 17 gives a peak in the πd scattering which arises because the propagator $(\mathbf{k}^2 - 2\mu\varepsilon + i\mu\Gamma)^{-1}$ is maximal; this is because the particle resonance near the threshold is almost on the mass shell. This circumstance renders the calculations much more definite (the problem does not arise of describing the $\pi N \to \eta(\rho, \ldots)N$ amplitude off the mass shell). The η-meson production threshold is at 0.71 GeV/c in the laboratory system. It is at this value of the pion momentum that the sharp peak is observed[81] in the πd scattering cross section at 180° (Fig. 18). The theory predicts the position, width, and height of the peak with good accuracy. The contribution of the mechanism in Fig. 17 with an intermediate η meson is 11.5 ± 2.2 μb/sr at the threshold value of the pion momentum (the experimental value is 12 ± 1 μb/sr, and the contribution of the background is about 2 μb/sr). The predicted angular distributions near 180° also agree satisfactorily with the experiment.

The shoulder noted in Fig. 18 at the pion momentum $k \approx 1$ GeV/c can be related to the production of ρ and ω mesons. The production of heavier mesons may lead to the appearance of peaks in the πd backward cross

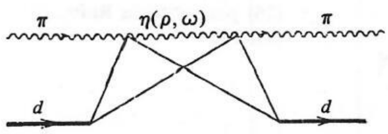

FIG. 17. Diagram for amplitude of πd backward scattering with production of resonances in the intermediate state.[80]

[2)]The calculations were made with the Hamada–Johnston wave function.

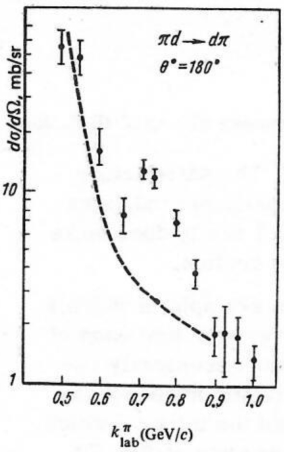

FIG. 18. Cross section of πd backward scattering.[81] The dashed curve is the extrapolation of the nonresonance background.

section at larger momenta. The positions of these peaks are given in Table IV. Note that the choice of the deuteron wave function variant determines only the shape of the resonance peak. The value of the cross section at the peak is determined by $\langle r^{-2}\rangle$, which does not depend too strongly on the details of the wave-function behavior. Measurement of the πd backward scattering cross section and the angular distributions at momenta $k \geqslant 1$ GeV/c are important for understanding the mechanisms of reactions with large momentum transfers.

Elastic pd Backward Scattering. At the present time experimental data are available on the cross section of pd backward scattering in a fairly wide range of proton energies in the laboratory system from a few tens of MeV to 2.5 GeV.[82] Here, we shall briefly discuss the main theoretical ideas about the mechanism of the $pd \to dp$ reaction, beginning by considering the pole diagram (Fig. 19).

The angular distributions and energy dependence of the cross section calculated in the pole model are compared with the experimental data in Ref. 69. The pole diagram gives an order of magnitude of the cross section close to the experimental value. For example, at proton energy 1 GeV in the laboratory system and cms scattering angle 160° the calculation gives 17 μb/sr. However, the calculations are very sensitive to the wave function (it occurs to the fourth power in the expression for the cross section). At the given energies and angle the wave function occurs at momentum 450 MeV/c. At proton energy 2 GeV in the laboratory system and cms scattering angle 135° the argument of the wave function is 1 GeV/c, and calculations using the third Moravcsik function and the Hamada–Johnston wave function differ by three orders of magnitude. To relativize the wave function, Karmanov[83] used the representation of the wave function proposed in Ref. 84

TABLE IV. Values of the pion momentum in the laboratory system at which one can expect a maximum in the cross section of πd backward scattering due to the production of resonances in the intermediate state

Resonances	η(549)	ρ(765)	ω(784)	A_1(1070)	f(1260)
k^{π}_{lab} (GeV/c)	0.71	1.1	1.15	1.74	2.40

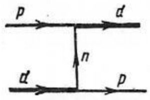

FIG. 19. Pole diagram for amplitude of pd backward scattering.

by Shapiro in the relativistic coordinate space that is the Fourier transform of the rapidity space (see Sec. 5 for more detail about the relativistic coordinate space). The relativity effect is greater than the experimental errors and comparable with the difference between the different types of nonrelativistic wave functions. This indicates that pd backward scattering may be an effective means for probing the deuteron structure at short distances and at the same time that the question of relativistic nucleons in nuclei has been "put on the agenda" by the modern experimental possibilities.

Some investigations have been devoted to the pole model with intermediate exchange of an isobar contained in the deuteron. This model was first proposed in Ref. 32. It gave an increase in the cross section (compared with nucleon exchange).

Kolybasov and Smorodinskaya,[85] and also Craigie and Wilkin[86] proposed a "triangle" mechanism corresponding to the diagram in Fig. 20. Because of the resonance energy dependence of the $pp \to \pi^+ d$ reaction amplitude in this diagram, this mechanism predicts a peak in the $pd \to dp$ reaction cross section at 660 MeV, and this was indeed subsequently observed experimentally.

Kondratyuk and Lev[87] considered the mechanism of double scattering with intermediate production of a Δ isobar (Fig. 21) analogous to the mechanism with an intermediate resonance in πd backward scattering (see Fig. 17). This mechanism in conjunction with the pole mechanism leads to a satisfactory description of the experimental data at energies 0.4–1 GeV and predicts an increase in the slope of the cross section toward the angle 180°, which may provide a test of the model.

In Ref. 88, Gurvitz and Rinat described pd scattering in the impulse approximation (Fig. 22). In their opinion, the justification for its use is that the NN scattering amplitude in the expression for the diagram in Fig. 22 increases as the angle 180° is approached. The amplitude of double scattering will be determined by the elementary amplitude at an angle near 90° and is therefore smaller. Here too a satisfactory description of the experimental data was obtained, and the deuteron form factor thus found–it is determined by the triangle diagram in Fig. 22–agrees with the form factor obtained from ed scattering.[64]

Thus, at the present time, there are numerous mechanisms that have been invoked to describe pd backward scattering. It is possible that each of them works only

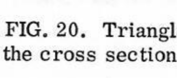

FIG. 20. Triangle diagram leading to resonance behavior of the cross section of pd backward scattering.[85,86]

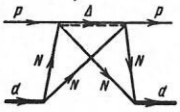

FIG. 21. Diagram for amplitude of pd backward scattering with production of Δ isobar in the intermediate state.[87]

in a restricted range (for example, an important contribution of the mechanism of Fig. 20 seems very probable in the region of energies around 660 MeV), but, as a whole, the question remains open. Here, we come right up against the problem of identifying the reaction mechanism at large momentum transfers. Its solution includes the specification of the set of distinctive "signs" of each given mechanism, i.e., the existence of a theoretical program of experiments needed to identify the mechanism of the processes. A program of this kind for a number of direct nuclear reactions of ordinary type (with small momentum transfer) was proposed in theoretical investigations and then successfully implemented experimentally (in this connection, see the review Ref. 89). For reactions with large momentum transfers, an analogous detailed consideration of the problem is as yet lacking, and this is one of the most important open questions. We do not have the possibility of considering this problem in more detail here. We merely mention that polarization experiments would to a considerable extent help in the identification of the pd backward scattering mechanism at high energies (see, in particular, Ref. 85).

The Reaction $\pi^- d \to p\Delta^-$. In Ref. 90, a mechanism of the reaction $\pi^- d \to p\Delta^-$ with emission of Δ^- backward was proposed, this corresponding to the backward scattering of a fast π^- meson with transfer to it of a large fraction of the energy and the subsequent production of the Δ^- isobar in the $\pi^- n$ interaction (Fig. 23). In the framework of this mechanism, the differential cross section of the reaction $\pi^- d \to p\Delta^-$ is given by the expression

$$d\sigma_d/du = F(u)\, d\sigma_p/du, \tag{38}$$

where $d\sigma_d/du$ is the cross section of $\pi^- p$ scattering. The function $F(u)$ is determined by the triangle diagram in Fig. 23. The background from quasielastic scattering (the process in which the π^- meson, without interacting with the neutron, arrives after scattering on the proton in the region of masses of the $\pi^- n$ system in the vicinity of the Δ isobar) can be suppressed by a special selection of events with emission of the π^- meson in the forward hemisphere in the laboratory system. This mechanism predicts that the energy dependence of the cross section $d\sigma_d/du$ at a fixed value of the invariant $u = u_0$ must repeat the energy dependence of the differential cross section of $\pi^- p$ scattering. In Fig. 24, we compare the predictions of this model with the experi-

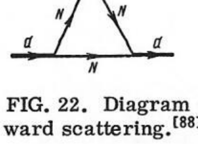

FIG. 22. Diagram of the impulse approximation for pd backward scattering.[88]

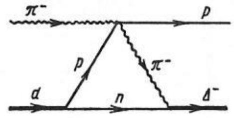

FIG. 23. Triangle diagram for the process $\pi^- d \to p\Delta^-$ (Ref. 90).

mental data of Abramov *et al.*[91] The satisfactory agreement between theory and experiment indicates that the triangle diagram of Fig. 23 really does make the main contribution to the cross section.

In this reaction we again find an example in which a two-particle reaction at high energy with emission of one of the particles backward is not necessarily related to the participation of relativistic nucleons, as might appear at the first glance (in the integral which determines the amplitude of the diagram of Fig. 23 the main contribution is made by momenta of order $1/R$ of the intranuclear nucleons; for this reason, different deuteron wave functions differing in their behavior at short distances give approximately the same results for the cross section).

A reaction mechanism that is to a large extent related to relativistic effects was proposed in Ref. 92. To describe the reaction $\pi^- d \to p\Delta^-$, Nath *et al* considered a pole mechanism determined by a ΔΔ admixture (Fig. 25). It predicts a different energy dependence of the cross section. In the investigated region, the invariant u is nearly zero, and the isobar Δ leaves the mass shell by an amount of the order of its mass. This renders the calculations of the diagram of Fig. 25 less reliable. As was noted in Ref. 35, great interest attaches to study of this reaction at pion momentum 2.1 GeV/c where the cross section of $\pi^- p$ backward scattering has a sharp minimum. At this pion momentum, mechanisms different from those of the diagram of Fig. 23 would be manifested in full measure. In addition, to identify the mechanism it is important to have the angular distributions of the pions in the center of mass of the $\pi^- n$ system.

Cumulative Meson Production. In Refs. 93 and 94, an experimental investigation was made of the inclusive reactions

$$d + p \to \pi^-(0^0) + X; \tag{39}$$
$$d + A \to \pi^-(0^0) + X \tag{40}$$

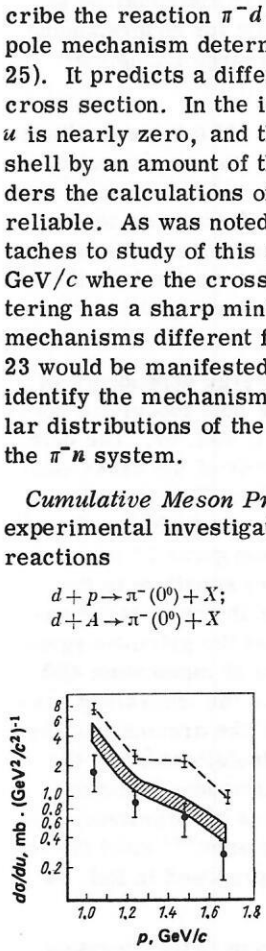

FIG. 24. Comparison of the results of theoretical calculation of the $\pi^- d \to p\Delta^-$ reaction cross section[90] with the experiment of Ref. 91. The lower series of points are the experimental values; the hatched region corresponds to calculation on the basis of the triangle mechanism of Fig. 23; the upper series of points joined by the dashed line is the cross section of elastic $\pi^- p$ backward scattering.

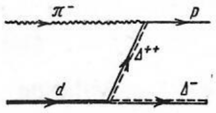

FIG. 25. Mechanism of the process $\pi^- p \to p\Delta^-$ determined by the $\Delta\Delta$ admixture in the deuteron wave function.[92]

at a pion momentum beyond the limit of the nucleon–nucleon kinematics. In the papers of Lobov *et al*,[79] these reactions were analyzed in the impulse approximation of Fig. 26. The following expression was obtained for the invariant cross section:

$$\frac{1}{p_\pi}\frac{d^2\sigma}{d\Omega\, dp_\pi}=\frac{4}{\pi}\sigma_{pp}\int\int (|a(Q)|^2 + |b(Q)|^2)\frac{I_{Np}}{I_{dp}}\frac{E_N}{E_{N'}} \times \rho_p(x_{Np}, p_{\pi\perp})\, dzp'^2\, dp', \qquad (41)$$

where σ_{pp} is the total cross section of the pp interaction; I_{Np}/I_{dp} is the ratio of the fluxes in the nucleon–nucleon and deuteron–nucleon collisions; a(Q) and b(Q) are the Fourier components of the S– and D-wave functions of the deuteron; $\rho(x, p_{\pi\perp}) = (E/\sigma_{pp}p_\pi^2)(d^2\sigma/d\Omega dp_\pi)$ is the cross section of inclusive production of pions in a nucleon–nuclear (or nucleon–nucleon) collision; $Q = [(p_d p')^2/m_d^2 - m^2]^{1/2}$.

Figure 27 shows the experimental data of Ref. 94 for ^{12}C and the theoretical calculations of Ref. 79. It can be seen from the comparison that the experimental data on cumulative meson production can be explained in order of magnitude in the framework of the impulse approximation under the usual assumptions about the deuteron wave function. Bertocchi and Treleani[95] proceeded from a more complicated model, which takes into account rescattering of both the nucleons of the deuteron in the nucleus in the framework of the Glauber approximation. They showed that there is a mutual cancellation of the contributions of different terms, and the result reduces to the impulse approximation (41). Note that the applicability of the Glauber approximation in the investigated region is very doubtful.

The characteristic values of the argument Q for which the deuteron wave function occurs in the expression (41) are comparatively large: $Q \approx 0.4$–0.6 GeV/c. If further investigations confirm the dominance of the impulse approximation, then cumulative meson production may give helpful information about the deuteron wave function at short distances.

Note that Gerasimov and Giordenescu[109] and Burov and Titov[110] assert that the intranuclear motion of the nucleons cannot explain the cumulative effect even in order of magnitude. After this present review had been written, Baldin's review Ref. 111 was published, in which he repeats this assertion on the basis of Refs. 109 and 110. These assertions contradict the results

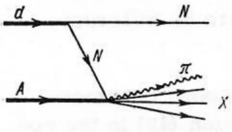

FIG. 26. Diagram of the impulse approximation for the process $d + A \to \pi^- + \cdots$.

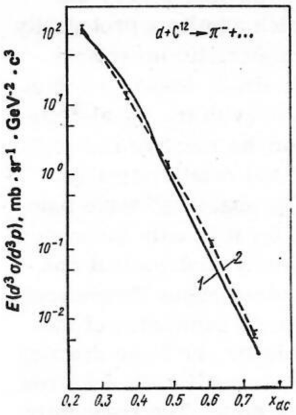

FIG. 27. Invariant cross section of the process $d + {}^{12}C \to \pi^-(0^0) + \cdots$. 1) Theoretical calculation in the impulse approximation of Fig. 26 (Ref. 79) with the Hamada–Johnston wave function[39]; 2) curve drawn for convenience through the experimental points of Ref. 94; the abscissa variable is $x_{dc} = p_\pi^{\parallel}(p_{\pi\,\max})pc$; $p_\pi^{\parallel}$ is the component of the pion momentum along the incident beam; $(p_{\pi\,\max})pc$ is the maximally possible pion momentum in the reaction $p + {}^{12}C \to \pi^- + \cdots$.

of Refs. 79, 94, and 95, and also our point of view.

In some investigations, including the review Ref. 111, fluctuations in the density of nuclear matter are invoked to explain the cumulative effect. This idea, which was first put forward by Blokhintsev,[112] corresponds to the presence of internucleon correlations in the wave function of a complex nucleus, the existence of which is not in doubt. In accordance with the uncertainty principle, this corresponds in the momentum space to the presence of nucleons in nuclear matter with momenta of the order of the inverse correlation lengths. Thus, the explanation of the cumulative effect by fluctuations in the density of nuclear matter is equivalent physically to taking into account the intranuclear motion of the nucleons. Everything depends on the correlation length. In Refs. 79, 94, and 95, it is asserted that exotically short correlation lengths are not required to explain the experimental data on cumulative pion production; in the language of density fluctuations, this means that one does not require the too frequent appearance of nucleons at short mutual distances. We emphasize that calculations made in the impulse approximation can pretend, as was noted in Ref. 79, only to a description of the order of magnitude of the cross sections. The true reaction mechanism is of course more complicated.

To identify the reaction mechanism with large momentum transfer one needs a set of experiments analogous to the programs developed earlier for "ordinary" direct nuclear reactions (see Ref. 89).

5. INVARIANT FOCK COLUMN

The above analysis of phenomena associated with large intranuclear momenta shows that despite the progress already achieved the theory of these phenomena is in its infancy. Very often, attempts are made to describe theoretically concrete reactions by a dubious extrapolation of the nonrelativistic wave functions to the relativistic region. On the transition to the relativistic region, we encounter in nuclear theory the need to have a wave function which admits a probability interpretation and depends on relativistic invariant variables. In the nonrelativistic limit, these variables must coincide with the usual ones (with the equal-time distance between the particles or the relative momentum). The choice of the correct and relativistically natural parametrization of the "probability" wave function is exceptionally important, for it is only for such a choice of the wave function and the variables that one can hope to obtain a physically perspicuous theoretical formalism in which even incomplete knowledge of the relativistic dynamics does not hinder one from drawing correct conclusions about the main qualitative features of the investigated composite systems. The frequently used, or rather, frequently mentioned Bethe–Salpeter formalism is good when the dynamics is completely determined, i.e., when the kernel of the corresponding integrodifferential or integral equation is exactly known. But for heuristic conclusions and working hypotheses based on correct basic principles the Bethe–Salpeter approach is frequently of little use because of the very indirect connection between the theoretical and the observed quantities.

In this section, we consider relativistic wave functions whose properties are analogous to those of the nonrelativistic theory. These wave functions are probability amplitudes, are relativistically invariant, and admit representation in a three-dimensional relativistic "coordinate space". The set of them together determines the quantum-field state vector of a bound system of particles, and they themselves are components of a Fock column defined on a light front. Results relating to these functions in the momentum space were obtained by one of the authors in Ref. 96.

Before we consider the Fock column defined on the light front, we recall some properties of the "one-time" Fock column in the Schrödinger representation on the hyperplane $t=0$. The state vector $\Phi(p)$ of a system with four-momentum p in the Fock representation has in the coordinate space the form

$$\Phi(p)=\begin{Vmatrix} \psi_2(\mathbf{x}_1,\mathbf{x}_2) \\ \psi_3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) \\ \cdots\cdots \\ \psi_n(\mathbf{x}_1,\ldots,\mathbf{x}_n) \\ \cdots\cdots \end{Vmatrix}. \qquad (42)$$

The wave functions $\psi_n(\mathbf{x}_1,\ldots,\mathbf{x}_n)$ have the meaning of a probability amplitude, the square of the amplitude being the probability density for finding n particles in the system. Such an interpretation follows from the fact that the functions $\psi_n(\mathbf{x}_1,\ldots,\mathbf{x}_n)$ are coefficients of the expansion

$$\Phi(p)=\sum_n\int\psi_n(\mathbf{x}_1,\ldots,\mathbf{x}_n)\,\varphi^{(+)}(\mathbf{x}_1)\ldots\varphi^{(+)}(\mathbf{x}_n)\,|0\rangle\,d^3x_1\ldots d^3x_n, \qquad (43)$$

where $\varphi^{(+)}(\mathbf{x})$ is the positive-frequency part of the field operator $\varphi(\mathbf{x})$ in the Schrödinger representation[3)]:

$$\varphi(\mathbf{x})=\varphi^{(-)}(\mathbf{x})+\varphi^{(+)}(\mathbf{x})=\frac{1}{(2\pi)^{3/2}}\int[a(\mathbf{k})\exp(i\mathbf{kx}) + a^+(\mathbf{k})\exp(-i\mathbf{kx})]\frac{d^3k}{\sqrt{2\varepsilon_k}}. \qquad (44)$$

Here, a and a^+ are, as usual, operators of annihilation and creation of particles, and they satisfy the commutation relation

$$[a(\mathbf{k}),\ a^+(\mathbf{k}')]=\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

(for simplicity, we shall assume that all the particles are spinless bosons).

In momentum space, the expansion (43) has the form

$$\Phi(p)=Z^{1/2}\sum_n C_n(\mathbf{k}_1,\ldots,\mathbf{k}_n,\mathbf{p})\,a^+(\mathbf{k}_1)\ldots a^+(\mathbf{k}_n)\,|0\rangle \times\delta^{(3)}(\mathbf{k}_1+\ldots+\mathbf{k}_n-\mathbf{p})\frac{d^3k_1}{\sqrt{2\varepsilon_1}}\ldots\frac{d^3k_n}{\sqrt{2\varepsilon_n}}, \qquad (45)$$

and the functions C_n are related to ψ_n by

$$Z^{1/2}C_n(\mathbf{k}_1,\ldots,\mathbf{k}_n,\mathbf{p})\,\delta^{(3)}(\mathbf{k}_1+\ldots+\mathbf{k}_n-\mathbf{p}) =\int\psi_n(\mathbf{x}_1,\ldots,\mathbf{x}_n)\exp\left(-i\sum_1^n\mathbf{k}_i\mathbf{x}_i\right)\frac{d^3x_1}{(2\pi)^{3/2}}\ldots\frac{d^3x_n}{(2\pi)^{3/2}}. \qquad (46)$$

For convenience, we have introduced the normalization factor Z.

The wave function C_n is the probability amplitude for finding n particles with momentum distribution determined by the square of C_n in the bound system. Note that the momenta of all the particles in (46) are on the mass shells: $k_i^2=m_i^2$.

A shortcoming of the wave functions ψ_n in (43) and the C_n in (45) is that they are not relativistically invariant. The obvious reason for this is that these wave functions arise on the decomposition of the one-time state vector $\Phi(p)$, which is defined on the hyperplane $t=0$, which is specified in some fixed frame of reference. Let us elucidate this in more detail.

Under infinitesimally small Lorentz transformations $x\to x'=x+\delta x$, $\delta x_i=\delta\omega_{ik}x_k$, the state vector $\Phi(p)$ transforms as

$$\Phi(p)\to\Phi'(p')=\hat{U}\Phi(p)=(1+\delta\hat{U})\,\Phi(p);\ \delta\hat{U}=i/2\hat{M}_{ik}\delta\omega_{ik}, \qquad (47)$$

where $\hat{M}_{ik}$ are the generators of the Lorentz group. Since the operators $\hat{M}_{0u}$ which realize the Lorentz transformation contain the interaction (this can be readily seen from the expression for $\hat{M}_{ik}$ in terms of the Lagrangian), which changes the number of particles, the component C_n is transformed under the operation (47) not only in terms of itself but also in terms of the other components. Therefore, for the components C_n there does not exist a group law of the transformation and they are not Lorentz covariant quantities. For wave functions defined in this way, it then follows, in particular, that the probability of isobar admixture in the deuteron:

$$W(\mathbf{p})=\int\left|C_{\Delta\Delta}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p})\right|^2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\frac{d^3k_1}{2\varepsilon_1}\frac{d^3k_2}{2\varepsilon_2}$$

will depend explicitly on the momentum with which the deuteron is moving, and will therefore be different in different frames of reference.

The physical meaning of this pehomenon becomes clearest if we consider the Fock column (42) in the coordinate space. In a frame B moving with respect to the

[3)]We shall omit the factorial factors due to the identity of the particles. In each concrete case, they can be readily restored.

system A, the Fock column (42) will consist of wave functions that are one-time functions in the system B. Two events that are simultaneous in the system B and are described by the wave function $\psi^B(\mathbf{x}_1, \mathbf{x}_2)$ are not simultaneous in the system A. Therefore, to these events there does not correspond a probability amplitude $\psi^A(\mathbf{x}_1, \mathbf{x}_2)$ that is a one-time amplitude in the system A, and therefore ψ^B cannot be expressed directly in terms of ψ^A. To express a function that is not a one-time function in the system A in terms of a function which is defined at one time in this system, we must in principle know the law of development of the system in time, which is determined by the Hamiltonian. In the nonrelativistic limit, the position of the particles cannot change significantly during the time difference between the events, and therefore the wave function ψ_n can be expressed in terms of itself, in accordance with the Galileo transformations.

These well known questions are discussed here in such detail because, as will be shown below, it is precisely these properties of the relativistic wave functions that lead to a significant modification of their parametrization compared with a nonrelativistic wave function. In particular, the relativistic wave functions will depend on a new additional variable. This result can be readily obtained already from what we have said above. However, we shall obtain it together with a general expression for the state vector $\Phi(p)$ in terms of invariant Fock rows. In this way, we shall arrive at the most convenient parametrization of the wave functions with a direct relation to the nonrelativistic one.

To find a manifestly covariant relativistic wave function, we consider, following Tomonaga's paper Ref. 97, a state vector defined on a spacelike hypersurface. We restrict ourselves to a hyperplane defined by the equation $\lambda x = 0$, where $\lambda = (\lambda_0, \boldsymbol{\lambda})$ is a four-vector with the properties $\lambda^2 = 1$, $\lambda_0 > 0$. We represent the state vector $\Phi(p)$ in the form of the expansion [cf (43)].

$$\Phi(p) = \sum_n \int \psi_n(x_1, \ldots, x_n, \lambda)\, \varphi^{(+)}(x_1) \ldots \varphi^{(+)}(x_n)\,|0\rangle \times \delta(\lambda x_1)\, d^4x_1 \ldots \delta(\lambda x_n)\, d^4x_n. \quad (48)$$

The state $|n\rangle = \varphi^{(+)}(x_1) \ldots \varphi^{(+)}(x_n)|0\rangle$ for $x_i \lambda = 0$ ($i = 1, \ldots, n$) describes n free particles on the hypersurface $\lambda x = 0$ at the points with coordinates $x_1, \ldots, x_n$, and $\psi_n(x_1, \ldots, x_n)$ is the amplitude of the probability with which the state $|n\rangle$ is present in the state $\Phi(p)$. The functions $\psi_n(x_1, \ldots, x_n, \lambda)$ are manifestly covariant. They do not depend on the frame of reference but they do depend on the plane characterized by the four-vector λ on which they are defined. This circumstance will lead in what follows to an argument that is not contained in the nonrelativistic wave functions. To determine the parametrization of the wave functions in the momentum space, we shall establish the properties of $\psi_n(x_1, \ldots, x_n, \lambda)$ under the translations $x \to x + a$.

Just as the wave functions $\psi_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ in (43) do not depend on t, the wave functions $\psi_n(x_1, \ldots, x_n, \lambda)$ do not change under a shift of the argument along the direction of λ. Under a translation along a direction μ orthogonal to λ ($\mu\lambda = 0$), the wave functions transform in accordance with the law

$$\psi(x_1, x_2, \lambda) \to \psi(x_1 - \mu, x_2 - \mu, \lambda) = \exp(ip\mu)\, \psi(x_1, x_2, \lambda) \quad (49)$$

(for brevity, we shall write out only the two-particle component). Thus, under an arbitrary shift $x \to x + a$ the wave functions transform in accordance with the law (49), where $\mu = a - \lambda(\lambda a)$.

Formally, the transformation law (49) is obtained as follows. The operator of translation in the direction μ is $U = \exp(i\hat{P}\mu)$, where $\hat{P}$ is the operator of the four-momentum. The state vector $\Phi(p)$ transforms in accordance with the law

$$\Phi(p) \to U\Phi(p) = \exp(i\hat{P}\mu)\, \Phi(p). \quad (50)$$

The operator $\hat{P}$, expressed in terms of the fields in the plane $\lambda x = 0$, has the form[96]

$$\hat{P}_i = \hat{P}_i^0 + \lambda_i H_{\lambda x}, \quad (51)$$

where $\hat{P}_i^0$ is the free operator of the four-momentum:

$$\hat{P}_i^0 = \int a^+(\mathbf{k})\, a(\mathbf{k})\, k_i d^3k; \quad (52)$$

$$H_{\lambda x} = \int H_{\rm int}(x)\, \delta(\lambda x)\, d^4x; \quad (53)$$

$H_{\rm int}(x)$ is the interaction Hamiltonian.

On the other hand, bearing in mind that $\hat{P}\mu = \hat{P}^0\mu$, we obtain

$$\begin{aligned}
&\exp(i\hat{P}\mu) \int \psi(x_1, x_2, \lambda)\, \varphi^{(+)}(x_1)\, \varphi^{(+)}(x_2)\,|0\rangle\, d\sigma_1 d\sigma_2 \\
&= \int \psi(x_1, x_2, \lambda) \exp(i\hat{P}^0\mu)\, \varphi^{(+)}(x_1) \exp(-i\hat{P}^0\mu) \\
&\times \exp(i\hat{P}^0\mu)\, \varphi^{(+)}(x_2) \exp(-i\hat{P}^0\mu)\,|0\rangle\, d\sigma_1 d\sigma_2 \\
&= \int \psi(x_1, x_2, \lambda)\, \varphi^{(+)}(x_1 + \mu)\, \varphi^{(+)}(x_2 + \mu)\,|0\rangle\, d\sigma_1 d\sigma_2 \\
&= \int \psi(x_1 - \mu, x_2 - \mu, \lambda)\, \varphi^{(+)}(x_1)\, \varphi^{(+)}(x_2)\,|0\rangle\, d\sigma_1 d\sigma_2,
\end{aligned} \quad (54)$$

where $d\sigma = \delta(x\lambda) d^4x$. In deriving the relation (54), we have remembered that the free operator $\exp(i\hat{P}^0\mu)$ realizes a translation of the Schrödinger operator $\varphi^{(+)}(x)$ in the hyperplane $\lambda x = 0$. From comparison of Eqs. (50) and (54) we obtain (49).

We go over in the expansion (48) to the momentum space:

$$\Phi(p) = \sum_n \int \tilde{C}_n(k_1, \ldots, k_n, p, \lambda)\, a^+(\mathbf{k}_1) \ldots a^+(\mathbf{k}_n)\,|0\rangle \times \frac{d^3k_1}{\sqrt{2\varepsilon_1}} \ldots \frac{d^3k_n}{\sqrt{2\varepsilon_n}}. \quad (55)$$

The function $\tilde{C}_2$ is related to ψ_2 by

$$\tilde{C}(k_1, k_2, p, \lambda) = \int \psi(x_1, x_2, \lambda) \exp(ik_1x_1 + ik_2x_2)\, \delta(\lambda x_1) \frac{d^4x_1}{(2\pi)^{3/2}}\, \delta(\lambda x_2) \frac{d^4x_2}{(2\pi)^{3/2}}; \quad (56)$$

$$\psi(x_1, x_2, \lambda) = \int \tilde{C}(k_1, k_2, p, \lambda) \exp[-i(k_1x_1 - (k_1\lambda)(x_1\lambda))] \times \exp[-i(k_2x_2 - (k_2\lambda)(k_2\lambda))] \frac{(k_1\lambda)}{(2\pi)^{3/2}} \frac{d^3k_1}{2\varepsilon_1} \frac{(k_2\lambda)}{(2\pi)^{3/2}} \frac{d^3k_2}{2\varepsilon_2}. \quad (57)$$

All the four-vectors in $\tilde{C}(k_1, k_2, \ldots, k_n, p, \lambda)$ are on the mass shells: $k_i^2 = m^2$, $p^2 = M^2$.

Under a shift of the argument $x \to x - \mu$ in $\psi(x_1, x_2, \lambda)$, the factor $\exp(ik_1\mu + ik_2\mu)$ appears in the integrand in Eq. (57). If, in accordance with Eq. (49), it is to be equal to $\exp(ip\mu)$, it is necessary that $\tilde{C}(k_1, k_2, p, \lambda)$ contain a δ function to make the projections onto the plane $\lambda x = 0$ of the sum $k_1 + k_2$ and the momentum p equal.

A function $\tilde{C}$ with such a property can be conveniently represented in the form

$$\tilde{C}(k_1, k_2, p, \lambda) = Z^{1/2} \int_{-\infty}^{+\infty} C(k_1, k_2, p, \lambda\tau)\, \delta^{(4)}(k_1 + k_2 - p - \lambda\tau)\, d\tau, \quad (58)$$

where τ is a scalar parameter.

Thus, the expansion (55) takes the form

$$\Phi(p) = Z^{1/2} \sum_n \int C_n(k_1, \ldots, k_n, p, \lambda\tau)\, a^+(\mathbf{k}_1) \ldots a^+(\mathbf{k}_n) \,|\, 0\rangle \times \delta^{(4)}(k_1 + \ldots + k_n - p - \lambda\tau) \frac{d^3k_1}{\sqrt{2\varepsilon_1}} \ldots \frac{d^3k_n}{\sqrt{2\varepsilon_n}} d\tau. \quad (59)$$

We shall call the four-momentum $\lambda\tau$ in C_n the "spurion" momentum.[4)]

The state vector in the hyperplane $t = 0$ is obtained from (59) under the condition $\lambda = 0$, $\lambda_0 = 1$. After integration with respect to $d\tau$, we return to the expansion (43).

The functions $C_n(k_1, \ldots, k_n, p, \lambda\tau)$ in (59) are the probability amplitudes for finding n particles on the hypersurface $\lambda x = 0$ with the four-momenta $k_1, \ldots, k_n$. For what follows, instead of an arbitrary hyperplane $\lambda x = 0$ it is convenient to choose the "light front" $\omega x = 0$, where $\omega = (\omega_0, \boldsymbol{\omega})$, $\omega^2 = 0$, $\omega_0 > 0$.

When considering the decomposition of the translation vector a into parallel $(a_\parallel \| \omega)$ and orthogonal $(\mu\omega = 0)$ parts, care must be taken since the vector ω in this sense is at once parallel and orthogonal to itself $(\omega \cdot \omega = 0)$. One can go over to ω gradually by the introduction of an intermediate four-vector λ_δ, $\lambda_\delta^2 = \delta^2$, letting δ tend to zero. After this we find that the formulas with $\omega^2 = 0$ have the same form as with $\lambda^2 = 1$ with the direct substitution $\lambda \to \omega$.

We consider the parametrization of the wave functions for the example of the two-particle component:

$$C_2 = C_2(k_1, k_2, p, \omega\tau), \quad k_1 + k_2 = p + \omega\tau. \quad (61)$$

In what follows, we shall consider bound systems with vanishing total angular momentum. The wave functions (61) can be represented in the form of a four-leg diagram (Fig. 28). It is clear from this that, like any other four-leg diagram, i.e., $1 + 2 \to 3 + 4$ reaction amplitude, the function $C(k_1, k_2, p, \omega\tau)$ can be parametrized by means of any pair of Mandelstam variables:

$$\left.\begin{aligned} s &= (k_1 + k_2)^2 = M^2 + 2\tau(\omega p); \quad t = (p - k_1)^2 = m^2 - 2\tau(\omega k_2); \\ u &= (p - k_2)^2 = m^2 - 2\tau(\omega k_1); \quad s + t + u = 2m^2 + M^2. \end{aligned}\right\} \quad (62)$$

The physical region of the variables s, t, and u is determined as usual, by Kibble's equation.

The problem of finding the two-particle Fock component now reduces to determining a function of two variables: $C = C(s, t)$. For the practical solution of this problem it is more convenient to introduce other variables corresponding to the modulus of the momentum of one of the final particles in the center-of-mass system

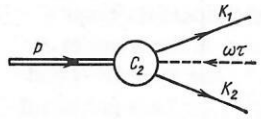

FIG. 28. Graphical representation of the two-particle wave function. The vertex part is represented by the same diagram.

of the reaction in Fig. 28 and the cms direction of the momentum of one of the incident particles. As the latter, we choose the spurion. This parametrization, which is the closest to the nonrelativistic, enables us, in addition, to introduce a relativistic coordinate space.[98] For this purpose, we shall use the operation of a shift on an hyperboloid. For two vectors $\mathbf{k}$ and $\mathbf{p}$ belonging to an hyperboloid of mass m, it is defined as follows (see Ref. 98)·

$$\left.\begin{aligned} \mathbf{f} &= L(\mathbf{p})\,\mathbf{k} \equiv \mathbf{k}(-)\,\mathbf{p} \equiv \mathbf{k} - \frac{\mathbf{p}}{m}\left[\varepsilon(\mathbf{k}) - \frac{\mathbf{kp}}{m + \varepsilon(\mathbf{p})}\right]; \\ \varepsilon(\mathbf{k}) &= \sqrt{\mathbf{k}^2 + m^2}; \quad \varepsilon(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}. \end{aligned}\right\} \quad (63)$$

The operation (63) is a Lorentz transformation $L(\mathbf{p})$ on the vector $\mathbf{k}$ resulting from transition to a system moving with velocity $\mathbf{v} = \mathbf{p}/\varepsilon(\mathbf{p})$. In the nonrelativistic limit, we obviously obtain the vector difference $\mathbf{k} - \mathbf{p}$. Note that

$$\left.\begin{aligned} (\mathbf{k}(-)\,\mathbf{p})^2 &= m^{-2}(kp)^2 - m^2; \\ (\mathbf{k}_1(-)\,\mathbf{p})(\mathbf{k}_2(-)\,\mathbf{p}) &= m^{-2}(k_1p)(k_2p) - (k_1k_2). \end{aligned}\right\} \quad (64)$$

Introducing the notation

$$Q = p + \omega\tau, \quad \text{where} \quad Q^2 = s, \quad (65)$$

we define the variables

$$\mathbf{q}_1 = \mathbf{k}_1(-)\frac{m}{\sqrt{Q^2}}\mathbf{Q} = \mathbf{k}_1 - \frac{\mathbf{Q}}{\sqrt{Q^2}}\left[\varepsilon(\mathbf{k}_1) - \frac{(\mathbf{k}_1\mathbf{Q})}{\sqrt{Q^2} + Q_0}\right]; \quad (66)$$

$$\mathbf{q}_2 = \mathbf{k}_2(-)\frac{m}{\sqrt{Q^2}}\mathbf{Q} = -\mathbf{q}_1; \quad (67)$$

$$\boldsymbol{\omega}' = \boldsymbol{\omega} - \frac{\mathbf{Q}}{\sqrt{Q^2}}\left[\omega_0 - \frac{\boldsymbol{\omega}\mathbf{Q}}{\sqrt{Q^2} + Q_0}\right]; \quad (68)$$

$$\mathbf{n} = \boldsymbol{\omega}'/|\boldsymbol{\omega}'| = \sqrt{Q^2}\,\boldsymbol{\omega}'/(\omega p). \quad (69)$$

When a Lorentz transformation is applied to the four-vectors k_1, k_2, p, and ω, the vectors $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{n}$ are only rotated, by virtue of Eqs. (64), and the angles between them are not changed.

Leaving aside group questions, we note that the vector $\mathbf{f}$ [Eq. (63)] transforms under the Lorentz transformation Λ as follows:

$$\mathbf{f}' = R(\Lambda)\,\mathbf{f}; \quad R(\Lambda) = L(\Lambda\mathbf{p})\,\Lambda L^{-1}(\mathbf{p}) \quad (70)$$

with $R(\Lambda_1\Lambda_2) = R(\Lambda_1)R(\Lambda_2)$.

The wave function $C(k_1, k_2, p, \omega\tau)$ can be assumed to depend on the two vectors $\mathbf{q} \equiv \mathbf{q}_1$ and $\mathbf{n}$:

$$C = C(\mathbf{q}^2, \mathbf{qn}). \quad (71)$$

Indeed $|\mathbf{n}| = 1$, and by virtue of Eqs. (64)

$$\mathbf{q}^2 = s/4 - m^2; \quad (72)$$

$$(\mathbf{nq}) = (u - t)\sqrt{s}/2\,(s - M^2). \quad (73)$$

Therefore, the parametrization in the variables $\mathbf{q}^2$ and $\mathbf{n} \cdot \mathbf{q}$ is equivalent to parametrization in the variables s and t.

[4)]In Ref. 96, the expression (58) was obtained in the derivation of the general form of the solution of the equation

$$\hat{P}\,\Phi(p) = p\,\Phi(p) \quad (60)$$

in the Schrödinger representation on the hyperplane $\lambda x = 0$.

Thus, the two-particle wave function of the relativistic system depends on not only $\mathbf{q}^2$ but also $\mathbf{q}\cdot\mathbf{n}$. The anisotropy of the function $C(q^2, \mathbf{n}\cdot\mathbf{q})$ does not reduce to a Lorentz contraction of the longitudinal scale. Indeed, this effect is separated out from the function (71). The wave function of N particles is parametrized in the same way as the amplitude of the process $a+b\to 1+2+\ldots+N$. The change of its parametrization compared with the nonrelativistic case also reduces to the appearance of an additional vector argument $\mathbf{n}$, which corresponds to two independent scalar parameters (two angles fixing the vector $\mathbf{n}$ in the system of vectors $\mathbf{q}_1, \ldots, \mathbf{q}_{N-1}$). In the nonrelativistic limit, the dependence on $\mathbf{n}$ vanishes, and the variable $\mathbf{q}$ in (66) goes over into $\mathbf{k}_1 - \mathbf{p}m/M$.

The state vector $\Phi(p)$ is normalized by the condition

$$(\Phi(p), \Phi(p')) = \int_{-\infty}^{+\infty} \delta^{(4)}(p-p'-\omega\tau)\, d\tau, \tag{74}$$

which for the wave function gives

$$\sum_n \int |C_n(k_1, \ldots, k_n, p, \omega\tau)|^2 \frac{d^3k_1}{2\varepsilon_1} \ldots \frac{d^3k_n}{2\varepsilon_n}\, d\tau\, \delta(k_1+\ldots+k_n-p-\omega\tau) \equiv \sum_n I_n = Z^{-1}. \tag{75}$$

In the variables $\mathbf{q}$ and $\mathbf{n}$, the normalization condition (75) for $C(k_1, k_2, p, \omega\tau)$ has the form

$$I_2 = \frac{1}{(\omega p)} \int |C(\mathbf{q}^2, \mathbf{nq})|^2 \frac{d^3q}{2\varepsilon(\mathbf{q})}. \tag{76}$$

In accordance with the paper Ref. 98 of Kadyshevskii *et al*, one can introduce the relativistic analog of the three-dimensional coordinate space by means of expansion with respect to functions that form a unitary irreducible infinite-dimensional representation of the Lorentz group. These functions were obtained by one of the authors in Ref. 99:

$$\xi(\mathbf{q}, \boldsymbol{\rho}) = ([\varepsilon(\mathbf{q}) - \boldsymbol{\rho}\mathbf{q}/\rho]/m)^{-1-i\rho m}. \tag{77}$$

The functions $\xi(\mathbf{p}, \boldsymbol{\rho})$ are orthogonal and complete.

Expanding $C(\mathbf{q}^2, \mathbf{n}\cdot\mathbf{q})$ with respect to the functions (77), we obtain a representation of the wave functions in ρ space:

$$C(\boldsymbol{\rho}, \mathbf{n}\boldsymbol{\rho}) = \frac{1}{(2\pi)^{3/2}} \int C(\mathbf{q}^2, \mathbf{nq})\, \xi(\mathbf{q}, \boldsymbol{\rho}) \frac{m\,d^3q}{\varepsilon(\mathbf{q})}; \tag{78}$$

$$C(\mathbf{q}^2, \mathbf{nq}) = \frac{1}{(2\pi)^{3/2}} \int C(\boldsymbol{\rho}, \mathbf{n}\boldsymbol{\rho})\, \xi^*(\mathbf{q}, \boldsymbol{\rho})\, d^3\rho. \tag{79}$$

The normalization integral (76) takes the form

$$I_2 = \frac{1}{2m\,(\omega p)} \int |C(\boldsymbol{\rho}, \mathbf{n}\boldsymbol{\rho})|^2 d^3\rho. \tag{80}$$

The parametrization of the wave functions in the ρ space has the consequence that in the momentum space the wave function is characterized by a smooth dependence, not on the momentum $\mathbf{q}$ itself, but on the rapidity corresponding to it:

$$y = \ln[(\varepsilon(\mathbf{q}) - q)/(\varepsilon(\mathbf{q}) + q)]/2,$$

and in some of the simplest cases the ρ space is related to the rapidity space by a Fourier transformation.

In the nonrelativistic limit $|\mathbf{q}|/m \to 0$ and $m\rho \to \infty$, the function (77) goes over into a plane wave, and the transformations (78) and (79) into ordinary Fourier integrals.

As is noted in Ref. 100, this fact is very useful for understanding how one should parametrize the nuclear wave function in the relativistic region using what we know about its behavior at sufficiently large nonrelativistic distances. It is to be expected that it is in precisely the ρ space or in the rapidity space that the wave functions will have their simplest form. This group of questions is discussed in Ref. 101.

The wave functions $\psi(x_1, \ldots, x_n, \lambda)$ considered above on the plane $\lambda x = 0$ are equal to the Fock components in (43) defined in a system moving with the four-velocity λ, where the plane $\lambda x = 0$ coincides with the plane $t = 0$. For the plane $\omega x = 0$, the infinite-momentum frame is such a system, and therefore the function $C(k_1, k_2, p, \omega\tau)$ is obviously equal to $C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p})$ defined in the infinite-momentum frame. The variables s, t, and u are related to the well known variables $\mathbf{R}_\perp, x_1$, and x_2 $(x_1 + x_2 = 1)$ in the infinite-momentum frame by[96]

$$\begin{aligned} x_1 &= (m^2-u)/(s-M^2);\\ x_2 &= (m^2-t)/(s-M^2);\\ \mathbf{R}_\perp^2 &= s(m^2-t)(m^2-u)/(s-M^2)^2 - m^2. \end{aligned} \tag{81}$$

We can also relate the variables $\mathbf{q}$ and $\mathbf{n}$ to the variables in the infinite-momentum frame:

$$\begin{aligned} \mathbf{R}_\perp^2 = \mathbf{q}^2 - (\mathbf{qn})^2,\quad x_1 &= [1 - \mathbf{nq}/\varepsilon(\mathbf{q})]/2;\\ x_2 &= [1 + \mathbf{nq}/\varepsilon(\mathbf{q})]/2. \end{aligned} \tag{82}$$

Note that in Ref. 102 Garsevanishvili *et al.* made the transition to light front variables in the framework of the quasipotential approach.

Thus, from the point of view of the theory of noninvariant Fock components, we have finally arrived at the well known wave functions in the infinite-momentum frame, which are widely used in modern parton models. However, the approach based on studying the state vector on the light front makes it possible to approach the parametrization and investigation of the wave functions from the most general point of view without restricting oneself in advance to the narrow framework of the variables introduced in the infinite-momentum frame or the noncovariant computational formalism of the "old perturbation theory". The variables $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$ in (71) are especially convenient for describing the currently most interesting region of not too large but already relativistic values of the intranuclear momenta since the appearance in the wave functions of a dependence on $\mathbf{n}\cdot\mathbf{q}$ when $\mathbf{q}^2$ increases is similar to the appearance of $l \neq 0$ partial waves in the scattering amplitude. Just as the scattering amplitude at low energies is described by the S wave is spherically symmetric, the wave function at nonrelativistic values does not depend on the vector $\mathbf{n}$. At higher energies, it is necessary to add higher partial waves to the scattering amplitude. Similarly, at large $\mathbf{q}^2$ it is necessary to introduce terms containing $\mathbf{n}\cdot\mathbf{q}$ in the wave function. Evidently, there also exists a parameter analogous to the number of partial waves $l \sim kR$ in the scattering amplitude. It is to be expected that this number is determined by the ratio $|\mathbf{q}|^2/m^2$.

Hitherto, we have considered systems of spinless particles. The method of introduction of spin variables is almost obvious: It reduces to the separation of invariant amplitudes of a two-particle reaction in which one of the initial "particles" is scalar (to it there corresponds the four-momentum $\omega\tau$); the other (the four-momentum p) is the considered bound state. Under the assumption that the Fock component of the state vector depends on only a single variable, the wave function of a system of two relativistic particles has been constructed for particles with spin in the framework of the "quantum-mechanical" approach (see Sec. 1) by Terent'ev.[27] The same wave function for three particles has been constructed by Berestetskii and Terent'ev.[28]

6. EXPRESSION OF RELATIVISTIC AMPLITUDES IN TERMS OF WAVE FUNCTIONS

One of the main problems of any theory of interacting particles is to express the amplitudes of scattering on a bound system in terms of wave functions and amplitudes of elementary events. For the wave functions considered above, the solution of this problem is based on the diagram technique developed by Kadyshevskii.[6,103] It can be obtained by perturbative solution of an equation of the Tomonga–Schwinger type for the state vector on the hyper plane $\lambda x = \sigma$:

$$i\,\partial\Phi(\sigma)/\partial\sigma = H(\sigma)\,\Phi(\sigma), \tag{83}$$

where

$$H(\sigma) = \int \delta(\lambda x - \sigma)\, H_{\text{int}}(x)\, d^4x.$$

We formulate the rules of this technique for a light front[96] and the theory of a scalar field with the Lagrangian $\mathcal{L} = g\varphi$.[3] To find the matrix element $F\delta^{(4)}(\cdots)$ of n-th order, we must[6,96,103]:

1) label the vertices arbitrarily in the Feynman graph of n-th order. Join the vertices in the order of increasing number by a directed dashed line (we shall call it a spurion line). Orient the continuous lines (the lines of particles) in the direction from a larger to a smaller number. Diagrams in which there are vertices with three ingoing or outgoing continuous lines (vacuum vertices, Figs. 29a and 29b) need not be considered (they vanish). To each continuous lines there is assigned a corresponding four-momentum, and to each i-th dashed line the four-momentum $\omega\tau_i$;

2) with every internal continuous line with four-momentum k associate the propagator $\theta(\omega k)\delta(k^2 - m^2)$; with every spurion line, the factor $1/2\pi(\tau - i0)$;

3) with every vertex associate $(2\pi)^{-1/2}g\delta^{(4)}(\cdots)$, where the δ function takes into account conservation of the four-momenta at the vertex, including the spurion momentum;

4) integrate over all the four-momenta of the internal particles and with respect to $d\tau$ for the spurion lines within infinite limits;

5) repeat the procedure described under 2)–4) for all $n!$ possible labelings of the vertices.

The normalization of the matrix elements then obtained is clear from the connection between the amplitude F of the $1 + 2 \to 3 + \ldots + n$ process and the cross section:

$$d\sigma = \frac{(2\pi)^2}{4j\varepsilon_1\varepsilon_2}|F|^2 \frac{d^3k_3}{2\varepsilon_3} \ldots \frac{d^3k_n}{2\varepsilon_n}\, \delta^{(4)}(k_1 + k_2 - k_3 - \ldots - k_n), \tag{85}$$

where j is the flux density of the incident particles.

As in Sec. 5, we omit here the factorial factors that arise because of the identity of the particles and that depend on the variant of the theory. Because the propagator in this technique is the expression $\theta(\omega k)\delta(k^2 - m^2)$, the momenta of all the particles are on the mass shells. However, there does exist the concept of amplitudes off the energy shell. These are amplitudes containing external spurion lines. They are internal parts of the diagrams.

The vacuum vertices (see Figs. 29a and 29b) vanish because it is impossible to satisfy the conservation laws at them. The old perturbation theory in the infinite-momentum frame has a similar property of vanishing of the vacuum diagrams. The casuality and unitarity of field theory formulated on a light front were investigated in Ref. 104.

The vertex parts of the diagram technique we have formulated depend on the same variables as the wave functions with the corresponding number of particles, and are represented by diagrams of the type shown in Fig. 28. Between the vertex parts and wave functions there exists a connection analogous to the one in the nonrelativistic diagram technique. The relationship between the wave function and the vertex part Γ_n is[96]

$$C_n(k_1, \ldots, k_n, p, \omega\tau) = (1/2\pi)\,\Gamma_n(k_1, \ldots, k_n, p, \omega\tau)/(s - M^2), \tag{86}$$

where $s = (k_1 + \ldots + k_n)^2$, and the four-momenta are related by the conservation law $p + \omega\tau = k_1 + \ldots + k_n$.

Equation (86) makes it possible to eliminate the vertex parts from the expressions for the diagrams and express the amplitudes in terms of the wave functions. Here, there arise two problems, which we shall illustrate by considering an amplitude of the ed scattering type. When the amplitude of ed scattering is calculated in accordance with rules 1)–5) there arises a complete series of diagrams differing by the labeling of the vertices. Some of them are shown in Figs. 30a–d. The diagram of Fig. 30a contains at vertices 1 and 4 vertex parts of the type in Fig. 28, and this diagram can be expressed in terms of a wave function. The diagram of Fig. 30b at vertex 2 contains a quantity that cannot

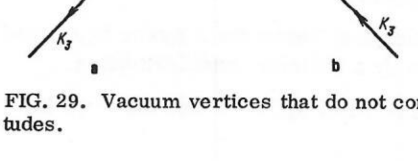

FIG. 29. Vacuum vertices that do not contribute to the amplitudes.

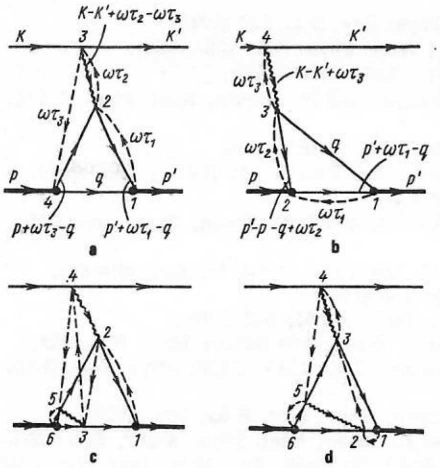

FIG. 30. Examples of diagrams for the process of elastic *ed* scattering.

be reduced to a vertex part and expressed in terms of a wave function. By vertices 1 and 4 in Fig. 30a and 1 and 2 in Fig. 30b, we understand blocks containing the sum of all possible meson exchanges. Simultaneously with these blocks, it is necessary to take into account processes of the type shown in Fig. 30c, in which the meson is emitted from vertex 5 before the emission of the γ ray from vertex 4 (as also in vertex 4 of Fig. 30a), and is absorbed at vertex 3 before the absorption of the γ at vertex 2.

Such a diagram is also obviously not contained on the left-hand side of the vertex considered above, and cannot be expressed in terms of a wave function. In addition, because of the spurion momenta, each of the diagrams depends on the four-vector ω. Only the sum of all the diagrams in each order is independent of ω and equal to the expression calculated in the Feynman technique. If we were to ignore all the diagrams that cannot be expressed in terms of wave functions, then we would not obtain a unique result for the diagram of Fig. 30a since it would contain the free parameter ω. Thus, it is not for all diagrams that the amplitudes can be expressed in terms of wave functions and, moreover, the resulting amplitudes depend on the four-vector ω. It is necessary to find a principle determining the best choice of ω. In the language of the old perturbation theory in the infinite-momentum frame this means that although the transition to that frame eliminates the vacuum diagrams the direction of the frame must be chosen in some manner relative to the momenta of the particles participating in the reaction in order to obtain a unique result.

These two problems are inter-related, and solution of one of them automatically leads to the solution of the other. Here, we shall briefly explain the main results obtained by one of the authors.

By virtue of a relativistic invariance and the invariance of the theory under stretching of $\omega(\omega \to \omega' = \alpha\omega$, where α is a number), the amplitudes depend only on the ratios of the scalar products of ω with the four-vectors of the external particles participating in the reaction. The number of such additional independent invariant variables is in the general case equal to two. Since the sum of all diagrams obtained from a given Feynman diagram by "entangling" a spurion line does not depend on ω nor, therefore, on these additional invariants, we can change the relative contribution of the considered diagrams by changing the values of these variables. It can be shown that the values of the invariants can be chosen in such a way that all of the diagrams of Figs. 30b and 30c, which cannot be expressed in terms of wave functions, vanish, and the amplitude can be expressed solely in terms of the diagram of Fig. 30a, i.e., in terms of a wave function. The diagram of Fig. 30d gives a nonvanishing contribution, but it is obviously contained in the amplitude that arises from the presence of the $NN\pi$ component in the Fock column of the deuteron. If the contribution of this component is important, it must be taken into account together with the two-nucleon component. For *ed* scattering when the diagrams have the simplest topology, to eliminate the contribution of the diagrams of Figs. 30b and 30c it is sufficient to fix the single variable $\omega p'/\omega p$, setting it equal to unity. The amplitude for the diagram of Fig. 30a does not depend on the second variable. After this, the expression for the form factor of the scalar deuteron calculated in accordance with rules 1)–5) and transformed to the infinite-momentum frame variables (81) and (82) takes the form

$$F(t)=\int C(\mathbf{R}_\perp^2, x)\, C((\mathbf{R}_\perp + x\mathbf{Q}_\perp)^2, x)\frac{d^2R_\perp\, dx}{2x(1-x)^2}, \qquad (87)$$

where

$$\mathbf{Q}_\perp^2=-t;\quad t=(p-p')^2.$$

The expression (87) can be obtained in the old perturbation theory in the infinite-momentum frame by choosing the direction of the infinite momentum orthogonal to the spatial part of the momentum transfer (see, for example, the review Ref. 105 of Kogut and Susskind). One can show that in the nonrelativistic limit Eq. (87) goes over into the ordinary nonrelativistic expression

$$F(t)=\int \psi(\mathbf{q})\,\psi(\mathbf{q}+\mathbf{Q}/2)\,d^3q. \qquad (88)$$

Although the expression (87) for the form factor can also be obtained without the diagram technique set forth under 1)–5) by means of the old perturbation theory, the diagram technique presented here enables one to express the amplitudes comparatively easily in terms of wave functions also for diagrams of general form, provided that this can be done at all.

For the general case, we elucidate the method by the example of the diagram of double scattering (Fig. 31). The amplitude for the diagram in Fig. 31 depends on the scalar variables

$$\left.\begin{array}{ll} y_1=\omega p/[\omega(p+k)]; & y_2=\omega k/[\omega(p+k)];\\ y_3=\omega p'/[\omega(p+k)]; & y_4=\omega k'/[\omega(p+k)]. \end{array}\right\} \qquad (89)$$

By virtue of the relations $y_1+y_2=1$ and $y_3+y_4=1$, there are two independent variables, for which we choose y_1 and y_3. If vertex 4 (or 1) in Fig. 31 is to contain only outgoing (ingoing) nucleon lines and therefore be expressible in terms of a wave function, it is necessary to require fulfillment of the condition $y_1=y_3=1$. For an arbitrary diagram, it is also convenient to intro-

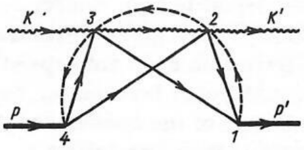

FIG. 31. Diagram of double scattering. The vertices 1 and 4 and the amplitude corresponding to the diagram can be expressed in terms of wave functions.

duce the variables $y_i = \omega k_i / \omega \sum_i k_i$, where $\sum_i k_i$ is the sum of the four-momenta of the initial particles. The two independent variables y_i for the external lines corresponding to bound states must be set equal to 1. Then the only nonvanishing diagrams are those in which the corresponding vertices have the largest or the smallest number and, therefore, contain, respectively, only outgoing or ingoing lines of particles and an ingoing or outgoing spurion line (see Fig. 31, vertices 4 and 1). It is precisely these vertices that can be expressed in terms of wave functions.

Note that values of the variables y_i equal to 1 are outside the physical range of variation of these variables (the range admitted for timelike four-vectors k_i). In practice, it may be more helpful not to go outside the boundary of the physical region but take the corresponding variables y_i equal to the maximal boundary values. When applied to the diagrams of Figs. 30a–30c, these rules lead to Eq. (87) for the form factor.

CONCLUSIONS

The conclusion to be drawn from the results discussed above is that the effects associated with relativistic motion of the nucleons in nuclei such as the presence in them of isobars and mesons are already manifested in the existing experimental data and are fully amenable to further experimental study. Moreover, without allowance for these effects one cannot seriously hope to understand how nuclei are constructed at short internucleon distances.

But on the other hand, with increasing momentum transfer the reaction mechanisms may become more complicated, whereas the wave function can in practice be effectively probed only for fairly simple mechanisms (of the type of pole and triangle diagrams). In this connection, we encounter the same problem that is acute for direct processes with small transfers: the experimental identification of the contribution of the simple diagrams. From this point of view, polarization experiments are of considerable interest. The theoretical and experimental study of the mechanisms of ordinary direct reactions[89] has shown that the identification of the mechanism definitely requires experiments of noninclusive type.

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