Polarization phenomena in hadron collisions at low momentum transfers

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Polarization effects in the interaction of hadrons are considered with simultaneous allowance for strong and electromagnetic interactions. The topics included for discussion include polarization effects in the scattering of pions, K mesons, and baryons by protons, in the scattering of neutral and charged baryons by nuclei, and in the coherent production of resonances. It is shown that the study of polarization effects due to the interference between the strong and electromagnetic interactions leads to a new experimental possibility for investigating the spin dependence of the strong-interaction amplitudes.

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INTRODUCTION

A spin dependence is a characteristic feature of interaction of microscopic particles. The dependence of the interaction on the spins leads to polarization phenomena. Through study of polarization effects, it was possible to establish directly the fundamental fact of violation of P and C invariance in weak interactions. The reliable determination of the spins and parities of resonances—which forms the basis of symmetry schemes—is another example which must be mentioned. Here there is still one important problem which is as yet unsolved: the spin and parity of the Ω hyperon have not yet been directly determined. Finally, data on polarization effects are decisive for the fate of many approximate theoretical constructions in elementary-particle physics and for the interaction of high energy particles with nuclei. It has been pointed out more than once in the literature that various models which give similar results for the cross sections when averaged over the particle spins differ drastically in their predictions for polarization effects.

In order to make modern investigations, it is necessary to create polarized beams of baryons, photons, and leptons and use the technique of polarized targets. The acceleration of polarized particles has a promising future. Here, for strongly interacting particles there are interesting possibilities in classical accelerators as well, including storage rings. Completely new possibilities arise for the acceleration of polarized high energy particles by the collective method of acceleration.[1]

In a collision of particles with spins, for interactions of arbitrary strength, there is an increase in the number of independent amplitudes that describe the scattering or inelastic processes in states with definite values of the spins and their projections and, accordingly, of the number of independent measurable quantities. Besides the differential cross section for interaction of unpolarized particles σ_0, one can measure the polarization P_0, the polarization correlations, the quantities that describe the change and transfer of the polarization in collisions of polarized particles, and also the differential cross sections for the interaction of polarized particles. Measurement of the total interaction cross sections of a polarized beam of particles with a polarized target gives interesting information on the total interaction cross sections in states with different energies and spin orientations of the particles. A detailed description of the theory of polarization effects in strong interactions can be found, for example, in the reviews Refs. 2 and 3.

In different energy ranges of the particles, polarization experiments illuminate different problems:

At low energies we have the range of energies in which it is sufficient (and does not cease to be sufficient) to take into account the interaction of hadrons in states with minimal values of the angular momenta. Here, unambiguity and considerable precision in the determination of the scattering lengths and effective interaction ranges are important.

In the region of nucleon energies 100–1000 MeV, where the spin dependence of the interaction in nucleon–nucleon scattering is particularly complicated and rich, a model-free and unambiguous analysis leading to establishment of scattering or interaction amplitudes is impossible without a study of polarization effects. The program of a "complete experiment"[4] has here been developed to its furthest extent.

In the energy range 1–10 GeV of the particles, the main aim of the performed and planned polarization experiments is, besides determining the spins and parities of resonances, to obtain information about the scattering and interaction amplitudes free of model assumptions. In the creation of polarized beams of baryons and in measurements of their degree of polarization, interference effects at extremely small momentum transfers, which will be discussed below, can be of assistance.

At the very highest energies achieved with powerful accelerators in the new and little studied range of energies, investigations of polarization effects are the most direct and shortest and, perhaps, the most economic way to obtain new and unambiguous information on the amplitudes for which theoretical models are de-

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veloped; they enable one to test the validity of the P and T invariances.

The dispersion relations based on the principles of causality and unitarity stimulate particular interest in forward scattering amplitudes. It is in the determination of these amplitudes that interference between the strong and electromagnetic interactions in polarization effects introduces appreciable simplifications. Apart from the region of especially small momentum transfers, electromagnetic–hadronic interference is appreciable near the minima of the differential cross sections, at which the hadronic amplitudes are strongly reduced. Electromagnetic effects must be taken into account at high energies when the polarization effects due to strong interactions in elastic scattering are significantly reduced.

The present review is devoted to a discussion of polarization effects that occur in the scattering and interaction of hadrons due to the interference between electromagnetic and strong interactions. The questions discussed here were raised initially in the investigations of Schwinger,[5] Akhiezer and Pomeranchuk,[6] Primakoff,[7] Piccioni,[8] Pomeranchuk and Shmushkevich,[9] Garren,[10] Breit,[11] Bethe,[12] Solov'ev,[13] Yennie and West,[14] and Locher.[15] The major part of the review is based on investigations of Vanzha, Kopeliovich, Tarasov, and the author.[16-22]

Note that the amplitude of electromagnetic interaction of a neutral baryon with the Coulomb field can be represented in the form

$$M^{em}=\frac{i}{\theta}\frac{e^2}{M}\mu_n Z(\sigma\mathbf{n}), \qquad (1)$$

where θ is the cms scattering angle; μ_n is the magnetic moment of the neutral baryon; M is its mass; Ze is the charge of the partner; and n is the unit normal vector to the scattering plane.

Equation (1) follows directly from a consideration of the corresponding single-photon Feynman diagram if we restrict ourselves to the terms that are the most singular in the four-momentum transfer and ignore the form factors of the particles. The photon propagator gives a factor $(-t)^{-1}$. The product of the vertices of the nucleus and the neutral baryon are proportional to the magnetic moment and the factor $(-t)^{1/2}$. For the scattering of neutral baryons, the electromagnetic amplitude (1) can be simply added to the hadronic amplitude M^h, so that the total elastic scattering amplitude, in which strong and magnetic interactions are taken into account, is

$$\left.\begin{aligned} &M=M^h+\frac{i\gamma_n}{\theta}(\sigma\mathbf{n});\\ &\gamma_n=\mu_n\frac{Ze^2}{M}=-2.94Z\cdot 10^{-16}\ \text{cm}, \end{aligned}\right\} \qquad (2)$$

and the hadronic amplitude M^h in the general case has a complicated spin structure, whose investigation is the aim of many experiments.

Equations (2) reveal qualitatively the effects of taking into account not only the strong but also the electromagnetic interactions of the hadrons. Because of the singularity with respect to the momentum transfer, the contribution M^{em} is comparable with that of the amplitude M^h at small scattering angles θ_M:

$$\theta_M=|\gamma_n|/|M^h|, \qquad (3)$$

to which there corresponds the square of the momentum transfer $-t_M$. For $-t_M$, ignoring the contributions of the real parts of the hadronic amplitudes compared with the imaginary parts, we can obtain the expression

$$-t_M\approx(Ze^2/M)^2(4\pi\mu_n/\sigma_{tot})^2, \qquad (4)$$

where σ_{tot} is the total cross section for the interaction of the baryon with the particles of the target;

$$-t_M\approx 0.5\cdot 10^{-5}(\text{GeV}/c)^2.$$

Even if the hadronic amplitude does not depend on the spin—the simplest assumption made at high energies—the dependence of M^{em} on the spin in (1) leads to polarization effects. As a result, there is a change in not only the angular distributions for the scattering of unpolarized particles through small angles but also polarization effects (appreciable at small t) due to interference between the amplitudes M^{em} and M^h.

For the scattering of charged particles, the existence of the Coulomb (charge–charge) interaction that is singular with respect to the momentum transfer introduces its own modifications. In the region $-t\approx -t_M$, which is appreciably less than $-t=-t_C$, at which the contributions of the Coulomb and strong interactions to the cross section are comparable:

$$-t_C\approx 8\pi Ze^2/\sigma_{tot}\approx 2\cdot 10^{-3}\ (\text{GeV}/c)^2, \qquad (5)$$

the Coulomb interaction predominates in the scattering cross section of charged baryons; the polarization effects in this region are strongly reduced compared with the scattering process in which one of the baryons is neutral.

For the scattering of charged baryons on one another, the question arises of the relative phase of the hadronic and electromagnetic amplitudes. This has a particularly significant effect in the interference of M^{em} with the real part of M^h.

1. SCATTERING AMPLITUDE AT SMALL MOMENTUM TRANSFERS

Scattering of Mesons by Protons and of Baryons by Nuclei. To describe the scattering of spinless mesons (pions, K mesons) by nucleons, and also the scattering of spin-$\frac{1}{2}$ baryons by spinless nuclei at arbitrary energies, it is convenient to go over from the ordinary invariant amplitudes A and B in the expression for the transition matrix

$$\bar{u}(p')\,Tu(p)=\bar{u}(p')\left[A-\tfrac{1}{2}B\hat{Q}\right]u(p) \qquad (6)$$

to the amplitudes B and A', where

$$A = A' - \left(\frac{s-u}{4M}\right)\frac{B}{(1-t/4M^2)};\qquad(7)$$

M is the nucleon mass; $Q = q + q'$; q and q' are the four-momenta of the bosons and before and after the collision; p and p' are the same for the nucleons; and s, t, and u are Mandelstam's kinematic variables.

For the differential cross section for scattering of unpolarized particles $d\sigma_0/dt$ and the polarization P_0 we have the well known expressions

$$\frac{d\sigma_0}{dt} = \frac{1}{\pi s}\left(\frac{M}{q_c^4}\right)^2\left[(1-t/4M^2)\,|A'|^2 - \frac{t}{4M^2}\frac{|B|^2(p_L^2+ts/4M^2)}{(1-t/4M^2)}\right]\qquad(8)$$

and

$$\frac{d\sigma_0}{dt}P_0 = -\sin\theta\,\frac{\mathrm{Im}\,(A'B^*)}{16\pi\sqrt{s}}.\qquad(9)$$

Here, q_c is the modulus of the cms three-momentum; θ is the cms scattering angle; $sq_c^2 = M^2p_L^2$.

If we introduce electromagnetic form factors of the spinless hadron F, and magnetic, G_M, and electric, G_E, form factors of the baryon: $G_M = F_1 + F_2$, $G_E = F_1 + t/4M^2F_2$, then the single-photon exchange diagram gives

$$\left.\begin{aligned} A^{em} &= \frac{4\pi e^2}{t}\left(\frac{s-u}{2M}\right)F\left(\frac{G_M-G_E}{1-t/4M^2}\right);\\ B^{em} &= -\frac{4\pi e^2}{t}2FG_M;\\ A'^{em} &= -\frac{4\pi e^2}{t}\left(\frac{s-u}{2M}\right)FG_E. \end{aligned}\right\}\qquad(10)$$

For meson–nucleus scattering $(s-u)/4M = \omega_L + t/4M$, where ω_L is the total energy of the meson in the nucleon rest frame before the collision.

For scattering of baryons by nuclei in the rest frame of the nucleus before the collision $(s-u)/4m = E_L + t/4m$ (m is the mass of the nucleus) and

$$\left.\begin{aligned} A'^{em} &= \frac{8\pi Ze^2}{t}\frac{m}{M}(E_L+t/4m)FG_E;\\ B^{em} &= 8\pi Ze^2FG_M/t. \end{aligned}\right\}\qquad(11)$$

When considering the effects that are singular with respect to t, it is important to remember that $G_M(0) = \mu$ (where μ is the magnetic moment of the baryon), $G_E(0) = \pm 1$ for charged baryons, and $G_E(0) = 0$ for neutral baryons.

If we represent the cms scattering amplitude in the form

$$\tilde{M} = M_0 + M_1(\sigma n),\qquad(12)$$

then, going over to the scattering angles of the baryon, for baryon–nucleus scattering we obtain

$$M_0^{em} = \frac{Ze^2}{t\sqrt{s}}2mE_LF\left(G_E - \frac{t}{4M^2}G_M\right),\qquad(13)$$

where we have retained only the terms which are most singular in t and

$$M_1^{em} = -\mathrm{i}(E-M)\sin\theta\,\frac{Ze^2}{t\sqrt{s}}F\left\{(\sqrt{s}+M)G_M + \frac{m}{M}(E_L+t/4m)\frac{G_M-G_E}{1-t/4M^2}\right\};\qquad(14)$$

here, E is the total cms energy of the baryon.

At low energies, when $E_L + t/4m \approx M$, $\sqrt{s} + M \approx m$,

$$M_{1NR}^{em} = -\mathrm{i}(E-M)\sin\theta\,\frac{2Ze^2}{t}F\left(G_M - \frac{G_E}{2}\right).\qquad(15)$$

For the scattering of charged baryons, Eq. (15) contains the so-called Thomas half. At high energies, as follows from (14), $M_{1R}^{em} \sim (\mu - 1)$.

Baryon–Baryon Scattering. The spin structure of the elastic scattering amplitude of two spin-$\frac{1}{2}$ particles in the center-of-mass system has, as is well known, the form

$$\begin{aligned} T &= a + b(\sigma_1 n)(\sigma_2 n) + c[(\sigma_1 n) + (\sigma_2 n)]\\ &+ d[(\sigma_1 n) - (\sigma_2 n)] + e(\sigma_1 l)(\sigma_2 l) + f(\sigma_1 m)(\sigma_2 m), \end{aligned}\qquad(16)$$

where the unit vectors $\mathbf{l} \sim \hat{\mathbf{k}}' + \hat{\mathbf{k}}$, $\mathbf{m} \sim \hat{\mathbf{k}}' - \hat{\mathbf{k}}$, and $\mathbf{n} = \mathbf{l} \times \mathbf{m}$ are constructed by means of the unit vectors along the directions of the momenta in the initial, $\hat{\mathbf{k}} = \mathbf{p}/|\mathbf{p}|$, and final, $\hat{\mathbf{k}}' = \mathbf{p}'/|\mathbf{p}'|$, states.

As a result of consideration of the single-photon exchange diagram, we can obtain the following expressions for the contribution of the electromagnetic interaction to the amplitudes a, b, c, d, e, f:

$$\left.\begin{aligned} a^{em} &= e^2G_{1E}G_{2E}(s-m_1^2-m_2^2)/t\sqrt{s};\\ b^{em} &\approx 0;\\ c^{em} + d^{em} &= \frac{\mathrm{i}}{\theta}\frac{e^2}{m_1^2}G_{2E}\left[G_{1M} - G_{1E}\frac{s-m_1^2-m_2^2}{2\sqrt{s}(E_1+m_1)}\right];\\ c^{em} - d^{em} &= \frac{\mathrm{i}}{\theta}\frac{e^2}{m_2^2}G_{1E}\left[G_{2M} - G_{2E}\frac{s-m_1^2-m_2^2}{2\sqrt{s}(E_2+m_2)}\right];\\ f^{em} &\approx 0;\\ e^{em} &\approx 0, \end{aligned}\right\}\qquad(17)$$

if we retain only the terms that are singular in t. Here, $G_{1E,2E}$ and $G_{1M,2M}$ are the form factors of the fermions; $E_{1,2}$ are the total energy of each of them in the center-of-mass system; θ is the cms scattering angle. The symbol ≈ 0 means that b^{em}, f^{em}, and e^{em} vanish if allowance is made for only the expressions which are singular in t.

Note that $c^{em} \pm d^{em}$ are proportional to $(\mu_{1,2} - \frac{1}{4})$ at low energies [instead of $\mu - \frac{1}{2}$ in (15)] and to $(\mu_{1,2} - 1)$ at high energies.

The amplitudes in (17) are normalized in such a way that for unpolarized particles

$$\frac{d\sigma_0}{do} \equiv \sigma_0 = |a|^2 + |b|^2 + |c+d|^2 + |c-d|^2 + |e|^2 + |f|^2.\qquad(18)$$

The expression (17) is valid, for example, for scattering of $\Sigma^{\pm}$ hyperons on protons. It simplifies for the scattering of neutral baryons on charged baryons. In such a case,

$$\begin{aligned} c^{em} - d^{em} &\approx 0;\\ c^{em} + d^{em} &= \frac{1}{\theta}\frac{e^4}{m_1}G_{1M}G_{2E} \approx \frac{\mathrm{i}}{\theta}\frac{e^2}{m_1}\mu_1, \end{aligned}\qquad(19)$$

where μ_1 and m_1 are the magnetic moment and mass of the neutral baryon.

Thus, for the scattering of neutrons and neutral hyperons of arbitrary energies (in the region of applicability of the Born approximation) on particles and nuclei of spin $\frac{1}{2}$ the electromagnetic contribution to the amplitude reduces to

$$M_1^{em} = \frac{i}{\theta} G_{1M} G_{2E} \frac{e^2}{m_1} (\sigma_1 n). \quad (20)$$

This result is preserved for the scattering of neutral baryons on nuclei of arbitrary spin since the interaction of the magnetic moment of the neutral baryon with the magnetic and higher moments of the charged particles does not lead to a contribution to the scattering amplitude which is singular in t.

The total amplitude of np scattering with allowance for electromagnetic effects reduces to the expression

$$M = M^h + M^{em} = M^h + i\gamma_{np} (\sigma_1 n)/\theta, \quad (21)$$

where

$$\gamma_{np} = e^2 \mu_n / M = -2.94 \cdot 10^{-16} \text{ cm}.$$

By virtue of isotopic invariance $d^h = 0$ in (16) for np scattering, as for the scattering of identical baryons.

With allowance for (21), we have in the spin structure of the total amplitude of np scattering

$$c = c^h + i\gamma_{np}/2\theta; \quad d = i\gamma_{np}/2\theta. \quad (22)$$

The presence in (16) of the nonzero amplitude d leads to singlet–triplet transitions and to a corresponding deviation for the np system from isotopic invariance, and to a difference between the polarizations of scattered neutrons and protons.

The hadronic part of the pp scattering amplitude can be represented in the form (16) with $d = 0$. The vanishing of the amplitude d for the scattering of identical particles in the case of parity conservation follows from the Pauli principle and leads to conservation of the total spin in the elastic scattering of identical baryons.

It is easy to see that in the single-photon approximation

$$\left.\begin{aligned} a^{em} &= -2\alpha E/|t|; \\ b^{em} &\approx 0; \\ c^{em} &\approx \frac{i\alpha}{|t|^{1/2}} \gamma_{PP} = \frac{i\alpha}{|t|^{1/2}} \{E(\mu - 1)/M + 1\}; \\ e^{em} &\approx 0; \\ f^{em} &\approx 0. \end{aligned}\right\} \quad (23)$$

Here, M is the mass and E is the energy of one proton in the center-of-mass system.

Note that the expression (5) for t_c is obtained if one bears in mind that in the region of Coulomb interference

$$|a^{em}| \sim \text{Im}\, a^h = E\sigma_{tot}/4\pi.$$

Coherent Production of Resonances. The contribution of single-photon exchange is of interest for not only elastic scattering but also a number of inelastic processes. Below, we shall discuss the contribution of the electromagnetic interaction to the processes of coherent production of unstable particles on spinless nuclei.

In the study of the electromagnetic contribution to the cross section of processes of the form

$$\left.\begin{aligned} &\gamma + Z \to \pi^0,\ \eta^0 + Z; \quad \pi^\pm (K^\pm) + Z \to \rho^\pm (K^{*\pm}) + Z; \\ &N + Z \to N^* + Z; \quad Y + Z \to Y^* + Z \end{aligned}\right\} \quad (24)$$

one can obtain data on the widths of the decays

$$\left.\begin{aligned} &\pi^0,\ \eta^0 \to 2\gamma; \quad \rho^\pm \to \pi^\pm + \gamma; \quad K^{*\pm} \to K^\pm + \gamma; \\ &N^* \to N + \gamma; \quad Y^* \to Y + \gamma. \end{aligned}\right\} \quad (25)$$

We represent the general scheme of the processes (25) as

$$2 \to 1 + \gamma, \quad (26)$$

and of the processes (24) as

$$1 + Z \to 2 + Z. \quad (27)$$

The single-photon exchange contribution to the cross section of the processes (24) averaged over the spins can be expressed[23] in terms of the decay widths $\Gamma(2 \to 1 + \gamma)$:

$$\frac{d\sigma_0^{em}}{dt'} = \frac{2s_2 + 1}{2s_1 + 1} \frac{Z^2 \alpha 8\pi \Gamma (2 \to 1 + \gamma)}{(m_2^2 - m_1^2)^3} \frac{|t'|\, m_2^3}{(t' + t_m)^2}, \quad (28)$$

where $-t' = -(t - t_m) = (k \sin\theta)^2$; $-t_m = [(m_2^2 - m_1^2/2k)]^2$; k is the momentum of the projectile; $m_{1,2}$ are the masses of particles 1 and 2 and $s_{1,2}$ are their spins.

The expression (28) attains a maximum at $-t' = -t_m$. The value of $d\sigma_0/dt'$ at the maximum is inversely proportional to $-t_m$ and increases quadratically with increasing energy. The total cross section of the Primakoff effect with the production of one resonance is proportional to the logarithm of the energy. At the particle energies attainable with modern accelerators, it is important to take into account the electromagnetic and nuclear (hadronic) parts of the amplitude.

The expression for the electromagnetic part of the amplitude of the process (27) with the variation of the baryon mass at low transfers reduces with good accuracy to a spin-dependent term of the form

$$F_{em}(q) = \frac{i(\sigma \cdot (k \times q))}{k(\Delta^2 + q^2)} \left[\frac{8\pi Z^2 \alpha \Gamma (2 \to 1 + \gamma)\, m_2^3}{(m_2^2 - m_1^2)^3} \right]^{1/2} S_{em}(q). \quad (29)$$

Here, $-q^2 = t$; $-\Delta^2 = t_m$; the electromagnetic "form factor" $S_{em}(q)$ in the approximation in which absorption effects are approximated by an absolutely black nucleus of radius R is determined by the expression

$$S_{em}(q) = \frac{\Delta}{q} (\Delta^2 + q^2) \int_R^\infty b\, db J_1(qb) K_1(\Delta b) \exp [2iZ\alpha\varepsilon \ln b/R]. \quad (30)$$

In Eq. (30), $\varepsilon = 0, \pm 1$ are the charges of particles 1 and 2 (in units of e); $J_1(x)$ and $K_1(x)$ are Bessel functions.

For a clearer understanding of the structure of the expression (29), note that the vertex for the emission of a γ with a baryon of mass m_1 and spin $\frac{1}{2}$ and its transition into a baryon of spin $\frac{1}{2}$ with mass $m_2 \neq m_1$:

$$I_\mu = \bar{u}_2 [F_1\gamma_\mu + F_2\sigma_{\mu\nu}q_\nu + F_3 q_\mu] u_1$$

differs effectively from the expression for $\bar{N}\gamma_\mu N$—the elastic vertex for emission of a γ without change in the baryon mass.

The condition $\partial_\mu J_\mu = 0$ of gauge invariance has the consequence that $(m_2 - m_1)F_1 + q^2F_3 = 0$. For the elastic vertex, when $m_1 = m_2$, $F_3(t)$ vanishes because of this and at small momentum transfers the form factors $F_1(t)$ and $F_2(t)$ are decisive. For the inelastic process, $F_1(t)$ is proportional to q^2, so that with allowance for the terms singular in t we arrive at the expression (29).

2. RELATIVE PHASE OF THE HADRONIC AND ELECTROMAGNETIC AMPLITUDES

When one considers the mutual influence of the strong and electromagnetic interactions on the expression for the transition amplitude, one is forced to consider the relative phase of the nuclear (hadronic) and electromagnetic amplitudes since the strong interaction affects the electromagnetic interaction. There is also the electromagnetic influence on the hadronic amplitude, which does not reduce to a relative phase. So far as I know, a general solution for the problem of the mutual influence of the strong and electromagnetic effects for arbitrary energy and momentum transfer is not known. For hadron–hadron interaction at low energies (in S states) this problem was solved by Landau and Smorodinskiĭ,[24] and Breit *et al.*[25]

In the region of high energies, this question was considered for the first time in 1945 by Akhiezer and Pomeranchuk[6] for diffraction of spinless charged particles on a black nucleus. The further investigations[12-15] showed that for the spinless amplitudes at high energies, when the t dependence of the hadronic amplitudes can be represented in the form $\exp(At)$, the expression for the relative phase coincides with the result obtained by Akhiezer, Pomeranchuk, and Bethe.

Being interested in the interference of strong and electromagnetic interactions in polarization phenomena, we shall restrict ourselves in our study of the relative phase in the presence of a spin dependence in M^h to high energies and small momentum transfers, when the t dependence of the amplitudes reduces to a dependence on the square of the transverse momentum transfer q^2. We represent the total scattering amplitude in the laboratory system in the form

$$F_T(\mathbf{q}) = \frac{iq_L}{2\pi}\int \Gamma_T(\mathbf{b}) \exp(i\mathbf{qb})\, d^2\mathbf{b}, \tag{31}$$

and

$$\Gamma_T(\mathbf{b}) = \frac{1}{2\pi i q_L}\int F_T(\mathbf{q}) \exp(-i\mathbf{qb})\, d^2\mathbf{q}. \tag{31a}$$

It is generally true that

$$\Gamma_T(\mathbf{b}) = 1 - \exp\left[-i\int_{-\infty}^{\infty} V_T(\mathbf{b}, z)\, dz\right]. \tag{32}$$

Since in our case the total interaction operator V_T is made up of the hadronic V_h and electromagnetic V_{em}:

$$V_T = V_h + V_{em}, \tag{33}$$

we have

$$\Gamma_T(\mathbf{b}) = \Gamma_{em}(\mathbf{b}) + \Gamma_h(\mathbf{b}) - \Gamma_{em}(\mathbf{b})\Gamma_h(\mathbf{b}). \tag{34}$$

Because V_{em} is real,

$$\Gamma_{em}(\mathbf{b}) + \Gamma_{em}^+(\mathbf{b}) = \Gamma_{em}(\mathbf{b})\Gamma_{em}^+(\mathbf{b}). \tag{35}$$

We represent the hadronic amplitude in the form

$$F_h(\mathbf{q}) = q_L f_1(\mathbf{q}^2) + i[\sigma\cdot(\mathbf{q}_L\times\mathbf{q})/2M] f_2(\mathbf{q}^2), \tag{36}$$

and

$$f_{1,2}(\mathbf{q}^2) = f_{1,2}(0)\exp(-a_{1,2}\mathbf{q}^2/2). \tag{37}$$

We introduce a finite photon mass $\sqrt{\lambda^2}$ and consider the effects which are singular in λ^2. In the Born approximation, ignoring the form factors,

$$F_{em}^B(\mathbf{q}) = \frac{2\alpha}{\mathbf{q}^2+\lambda^2}[q_L\varepsilon e + i\mu(\sigma\cdot(\mathbf{q}_L\times\mathbf{q}))/2M], \tag{38}$$

where $\varepsilon = 0$, $\mu = -1.79$ for scattering on a neutron and $\varepsilon = +1$, $\mu = 2.79$ for scattering on a proton. From (31) and (38)

$$\Gamma_{em}^B(\mathbf{b}) = \frac{\alpha}{i}\int\left[\varepsilon e + \mu\frac{\sigma\cdot(\mathbf{q}_L\times\mathbf{b})}{4Mq_L x}\right]\exp(-x\lambda^2 - \mathbf{b}^2/4x)\, dx/x. \tag{39}$$

It follows from (31), (36), and (37) that

$$i\Gamma_h(\mathbf{b}) = a_1^{-1}f_1(0)\exp(-\mathbf{b}^2/2a_1) + a_2^{-2}\frac{\sigma\cdot(\mathbf{q}_L\times\mathbf{b})}{2Mq_L}f_2(0)\exp(-\mathbf{b}^2/2a_2). \tag{40}$$

Using (39) and (40), we can represent the expression for $F_{ch}(q)$ in the form

$$F_{ch}(\mathbf{q}) \approx -\frac{iq_L}{2\pi}\int \Gamma_{em}^B(\mathbf{b})\Gamma_h(\mathbf{b})\exp(i\mathbf{qb})\, d^2\mathbf{b}.$$

Bearing in mind that in the interference region $a_{1,2}\mathbf{q}^2 \ll 1$, we obtain

$$\begin{aligned} &F_{ch}(0) + F_h(0) \\ &\approx q_L\left[f_1(0) + \frac{i\alpha\mu}{4M^2a_2}f_2(0)\right]\exp\left[-i\alpha\varepsilon e\ln\left(\frac{a_1\lambda^2}{2}+C\right)\right] \\ &+\frac{i(\sigma\cdot(\mathbf{q}_L\times\mathbf{q}))}{2M}[f_2(0) + i\alpha\mu f_1(0)]\exp\{-i\alpha\varepsilon e[\ln(a_2\lambda^2/2) + C + 1]\}, \end{aligned} \tag{41}$$

where C is Euler's constant.

To determine the singular part of the phase of the amplitude F_{em}, we use the representation (31) and the unitarity condition (35):

$$F_{em}(\mathbf{q}) - F_{em}^+(\mathbf{q}) \approx \frac{iq_L}{2\pi}\int \Gamma_{em}^B(\mathbf{b})\Gamma_{em}^B(\mathbf{b})\exp(i\mathbf{qb})\, d^2\mathbf{b}. \tag{42}$$

Substituting (39) in (42) and ignoring the contribution proportional to μ^2, we obtain after integration with respect to b

$$\begin{aligned} F_{em}(\mathbf{q}) - F_{em}^+(\mathbf{q}) &= \frac{iq_L}{2\pi}\alpha^2\int_0^\infty dx\, dy\frac{1}{xy}\frac{4\pi xy}{(x+y)}\left[e^2 + ie\mu\frac{\sigma\cdot(\mathbf{q}_L\times\mathbf{q})}{2Mq_L}\right] \\ &\times\exp\left[-(x+y)\lambda^2 - \mathbf{q}^2\frac{xy}{x+y}\right]. \end{aligned} \tag{43}$$

Introducing new variables u and τ in accordance with

$$2x = u(1+\tau); \quad 2y = u(1-\tau); \quad 2dx\,dy = u\,du\,d\tau,$$

and taking into account (38), we obtain

$$F_{em}(\mathbf{q}) - F^{+}_{em}(\mathbf{q}) = i\alpha^2 \varepsilon q_L \left\{ e + i\mu \frac{\sigma\cdot(\mathbf{q}_L\times\mathbf{q})}{2Mq_L} \right\} \int\limits_0^\infty du \int\limits_{-1}^{1} d\tau \exp\left[-u\lambda^2 - \frac{u\mathbf{q}^2}{4}(1-\tau^2)\right] = 2i\varepsilon\alpha F^B_{em}(\mathbf{q}) \ln \mathbf{q}^2/\lambda^2. \tag{44}$$

Therefore,

$$F_{em}(\mathbf{q}) = F^B_{em}(\mathbf{q}) \exp(i\alpha\varepsilon \ln \mathbf{q}^2/\lambda^2). \tag{45}$$

Considering the region of small $\mathbf{q}^2$ and taking into account (45), (41), (34), and (31), we finally obtain

$$F_{\tau}(\mathbf{q}) = F_{em} + F_h + F_{ch} = \exp\{i\varepsilon\alpha \ln \mathbf{q}^2/\lambda^2\} \Big\{ F^B_{em}(\mathbf{q}) + \Big[q_L f_1(0) + \frac{i\alpha\mu f_2(0)}{4M^2 a_2} \Big] \exp\Big[-i\varepsilon\alpha\Big(\ln\frac{a_1\mathbf{q}^2}{2} + C\Big)\Big] + \frac{i\sigma\cdot(\mathbf{q}_L\times\mathbf{q})}{2M}[f_2(0) + i\alpha\mu f_1(0)] \exp\Big[-i\varepsilon\alpha\Big(\ln\frac{a_2\mathbf{q}^2}{2} + C + 1\Big)\Big]\Big\}. \tag{46}$$

Thus, for the spinless amplitude we arrive at an expression for the relative phase that is equal to the one obtained previously in the literature, the contribution of the radii of the particles being added to a_1 when allowance is made for the form factors. The expression for the relative phase of the spin–orbit amplitude differs numerically (C is replaced by $C+1$ in addition to the difference between a_2 and a_1). Note that the relative phase vanishes for scattering on a neutral baryon, at least in the approximation we consider.

The vanishing of the relative phase for scattering on a neutral baryon can be directly demonstrated by going over to the unitarity condition for the S matrix in the partial waves at low energies, when the strong interaction is effective only in the S state. This follows from the fact that the partial-wave expansion of the spin–orbit amplitude begins with the p wave.

As follows from (46), the electromagnetic effects do not reduce to phase factors. Additional corrections of order $Z\alpha$ occur in the hadron amplitudes because of the electromagnetic effects, and this leads to appreciable differences in the case of scattering of charged baryons by nuclei.

In order to obtain the necessary expressions for the process of coherent production of resonances, let us consider the expression (30) for $S_{em}(\mathbf{q})$. We consider energies so high that $\Delta R \ll 1$ and scattering angles so small that $\mathbf{q} \approx \Delta$. Then the lower limit of the integral in (30) can be replaced by zero. The resulting expression is related to the hypergeometric function $F(2+iZ\alpha\varepsilon, 1+iZ\alpha\varepsilon, 2, -\mathbf{q}^2/\Delta^2)$. Using the symmetry properties of the hypergeometric function and the well known integral representation for it, we can reduce (30) to an expression which is convenient for analysis ($n = Z\alpha$):

$$S_{em}(\mathbf{q}) = \exp\Big\{-in\varepsilon \ln \frac{(\Delta^2+\mathbf{q}^2)R^2}{4} + 2\,\mathrm{Arg}\,\Gamma(1+in\varepsilon)\Big\} (1+i\varepsilon n) \int\limits_0^1 \Big[\frac{u(1+yu)}{1-u}\Big]^{in\varepsilon} du + O\Big(\frac{\Delta^2R^2}{4}, \frac{\mathbf{q}^2R^2}{4}\Big), \tag{47}$$

where $y = \mathbf{q}^2/(\mathbf{q}^2+\Delta^2)$.

In the first approximation in $Z\alpha$ at small transfers the "form factor" $S_{em}(\mathbf{q})$ reduces to a phase factor:

$$S_{em}(\mathbf{q}) = \exp i\varphi(\mathbf{q}, Z\alpha\varepsilon) + O(Z^2\alpha^2/2, \Delta^2R^2/4, \mathbf{q}^2R^2/4), \tag{48}$$

where

$$\varphi(\mathbf{q}, Z\alpha\varepsilon) = Z\alpha\varepsilon\{-\ln(\Delta^2R^2/4) + 2C + (\Delta^2/\mathbf{q}^2 - 1)\ln[(\mathbf{q}^2+\Delta^2)/\Delta^2]\}. \tag{49}$$

The presence for charged baryons of the nonzero phase φ in (48) and (49) has an appreciable effect on polarization phenomena.

3. OBSERVABLE QUANTITIES AT SMALL MOMENTUM TRANSFERS

Scattering of Neutrons on Nuclei. We begin our discussion of the various manifestations of the interference of strong and electromagnetic interaction in polarization phenomena with the elastic scattering of neutrons on (spinless) nuclei. In this case,

$$M^h = A^h + c^h(\sigma\mathbf{n}), \tag{50}$$

and the differential cross section for the scattering of unpolarized neutrons without allowance for the electromagnetic interaction is

$$\frac{d\sigma^h_0}{do} = \sigma^h_0 = |A^h|^2 + |c^h|^2. \tag{51}$$

As we have discussed above, the total scattering amplitude can be represented in the form (2). Then for the differential cross section σ_0 the asymmetry in the cross section σ_p for scattering of polarized neutrons (degree of polarization P_b) by nuclei P_0 (the value of P_0 is equal by virtue of P invariance to the polarization of the baryons that arises after the scattering of the originally unpolarized beam) and the polarization of the scattered beam after the interaction of the originally polarized beam we have

$$\sigma_0 = \sigma^h_0 + \gamma_n^2/\theta^2 + (2\gamma_n/\theta)\,\mathrm{Im}\,c^h; \tag{52}$$

$$\sigma_0\mathbf{P}_0 = \sigma_0 P_0\mathbf{n} = [\sigma^h_0 P^h_0 + (2\gamma_n/\theta)\,\mathrm{Im}\,A^h]\,\mathbf{n}; \tag{53}$$

$$\sigma_p = \sigma_0[1 + P_0(\mathbf{P}_b\mathbf{n})]; \tag{54}$$

$$\sigma_p\mathbf{P} = \sigma^h_p\mathbf{P}^h + [\gamma_n^2/\theta^2 + (2\gamma_n/\theta)\,\mathrm{Im}\,c^h][2\mathbf{n}(\mathbf{P}_b\mathbf{n}) - \mathbf{P}_b] + (2\gamma_n/\theta)\{\mathrm{Im}\,A^h\mathbf{n} - \mathrm{Re}\,A^h[\mathbf{n}\times\mathbf{P}_b]\}. \tag{55}$$

It follows from (53) and (55) that neutrons scattered elastically through small angles are strongly polarized. This can be used to create beams of high energy neutrons with a high degree of polarization if one is able to select elastically scattered neutrons. The admixture of inelastically scattered neutrons may reduce the effective polarization of the scattered particles.

The expression for the polarization

$$P_0=\frac{\sigma_0^h P_0^h+(2\gamma_n/\theta)\,\mathrm{Im}\,A^h}{\sigma_0^h+\gamma_n^2/\theta^2+(2\gamma_n/\theta)\,\mathrm{Im}\,c^h} \tag{56}$$

is frequently discussed in the case when $P_0^h=0$ and $c_0^h=0$. This corresponds to the range of energies in which the strong interaction is effective only in the s state, or, under simplified assumptions, in the high-energy region. At the same time, (56) can be represented in the form

$$P_0^{\mathrm{int}}=-\frac{\mathrm{Im}\,A^h}{|A^h|}\,\frac{2y}{1+y^2}, \tag{57}$$

if we introduce $y=\theta/\theta_M$ [θ_M is introduced in (3)]. The sign of the polarization P_0 at small angles, where $\mathrm{Im}A^h(t)>0$, is determined by the sign of the magnetic moment of the neutron. For neutral hyperons, this observation may be of interest in order to obtain more accurate information about the magnetic moments.

The polarization P_0 attains a maximum (in absolute magnitude) at $y=1$:

$$[P_0^{\mathrm{int}}(\theta_M)]_{\max}=-\mathrm{Im}\,A^h/|A^h|. \tag{58}$$

The value of $\mathrm{Im}A^h/|A^h|$ is 0.7–0.8 at moderate neutron energies[26] and may approach –1 at high energies, where usually $\mathrm{Re}A^h\ll\mathrm{Im}A^h$. The small value of $|t_M|$ makes it significantly harder to perform experiments in the region of momentum transfers, $P_0^{\mathrm{int}}\approx 1$. But, as can be seen from (53), (56), and (58), P_0 decreases comparatively slowly $[(-t)^{-1/2}]$ from $P_0^{\max}$ with increasing t in the region of small scattering angles. Thus, at $|t|=100|t_M|$ the value of P_0 reaches 10%, which is still appreciable. From an experimental determination of $P_0(\theta_M)$ one can find $|\mathrm{Re}A^h|$.

Electromagnetic effects lead to an interesting phenomenon in polarization rotation. As can be seen from (55), the effect is sensitive to $\mathrm{Re}A^h$. If from a measurement of σ_0 and P_0 one can obtain data on the imaginary part of the scattering amplitude and on the modulus of its real part, then results of measurement of the polarization rotation provide one with the possibility of determining the real part of the hadronic scattering amplitude together with its sign.

As can be seen from (54), the significant values of P_0 at small scattering angles makes it possible to measure the degree of polarization of high energy neutrons, for which it is hard to find other ways for analyzing polarization of the beam.

Note also that (1)–(4) hold for scattering of neutrons and neutral baryons by nuclei of arbitrary spin, since the expression for M^{em} reduces to (1) for scattering on a nucleus of arbitrary spin if the contributions which are singular in the momentum transfer are retained in the expression for the amplitude.

The interference of the electromagnetic and strong interactions in the polarization phenomena lead to appreciable effects not only in the region of small scattering angles but also near diffraction minima in the cross section, where M^h is strongly reduced in magnitude and the interference becomes maximal.

Scattering of Neutrons by Protons. In this case, the hadronic amplitude depends in a complicated manner on the spins of both particles. The amplitude contains five independent scalar amplitudes. However, by virtue of isotopic invariance the spin operators of both particles occur symmetrically in M^h, in the same way as in collisions of identical particles. The difference between the electromagnetic properties of the proton and the neutron break this symmetry because the interaction of the magnetic moment of the neutron with the charge of the proton has an influence in the approximation in which allowance is made for only the terms in the amplitude M^{em} that are singular in the momentum transfer. The magnetic moment of the proton does not interact with the neutron since the charge form factor of the neutron is zero at small t. Thus, the electromagnetic interaction introduces into the scattering amplitude only a dependence on the neutron spin.

This leads directly to some interesting effects. Using (21), we obtain the expressions

$$\left.\begin{aligned}\sigma_0&=\sigma_0^h+(2\gamma_{np}/\theta)\,\mathrm{Im}\,c^h+\gamma_{np}^2/\theta^2;\\ \sigma_0^h&=|a^h|^2+|b^h|^2+2|c^h|^2+|e^h|^2+|f^h|^2\end{aligned}\right\} \tag{59}$$

for the differential cross section for scattering of unpolarized particles, and

$$\left.\begin{aligned}\sigma_0P_{0n}&=\sigma_0^hP_0^h+(2\gamma_{np}/\theta)\,\mathrm{Im}\,a^h;\\ \sigma_0P_{0p}&=\sigma_0^hP_0^h+(2\gamma_{np}/\theta)\,\mathrm{Im}\,b^h\end{aligned}\right\} \tag{60}$$

for the polarizations of the scattered neutron P_{0n} and the recoil proton P_{0p}. Since the amplitude c^h is proportional to the scattering angle at small angles, the second term in (59) leads to a term which depends weakly on the scattering angle. As follows from (60), as a result of the presence of the single–triplet transitions the total spin is not conserved ($d\neq 0$) in neutron-proton interactions, and the neutron and proton polarizations are different. By virtue of isotopic invariance,

$$P_{0n}^h=P_{0p}^h=P_0^h. \tag{61}$$

Electromagnetic effects introduce deviations from (61) that are appreciable at small scattering angles. As can be seen from (60),

$$(\sigma_0P_{0p}-\sigma_0^hP_0^h)/(\sigma_0P_{0n}-\sigma_0^hP_0^h)=\mathrm{Im}\,b^h/\mathrm{Im}\,a^h.$$

The value of $\mathrm{Im}\,a^h$ at small angles is determined by the total cross section for the interaction of unpolarized particles. The unitarity of the S matrix has the consequence that[3]

$$\mathrm{Im}\,b^h(0)=(k/4\pi)(\sigma_0^t-\sigma^s)/4, \tag{62}$$

where σ^s is the total cross section in the singlet states; σ_m^t is the same for the initial triplet states with projections $m=0$, ± 1 of the spin of the system onto the direction of the incident beam.

Thus, measurement of the difference between the neutron and proton polarizations makes it possible to determine directly

$$\mathrm{Im}\,[a^h(0)-b^h(0)]=(k/4\pi)\,(\sigma^s+\sigma_+^t)/2. \tag{63}$$

This is of interest for the region of nucleon energies of a few hundred MeV, in which the spin dependence of the amplitude (16) is particularly complicated. Without allowance for the interference between M^h and M^{em} experiments with polarized beam and target are necessary for this.

In the limiting case when M^h does not depend on the spin, it follows from (60) that the neutrons after np scattering are strongly polarized in agreement with (57), while the protons remain unpolarized. From this point of view, larger effects will be observed in the scattering of polarized neutrons by unpolarized protons than in the scattering of unpolarized neutrons by polarized protons.

We recall that $P_{0p}(P_{0n})$ determines the asymmetry in the differential cross section for scattering of unpolarized (respectively, polarized) neutrons on polarized (unpolarized) protons. With allowance for interference effects, we obtain the following expression for the differential cross section for scattering of polarized neutrons (with polarization $\mathbf{P}_1$) by polarized protons (polarization $\mathbf{P}_2$):

$$\sigma_{P_1P_2}=\sigma^h_{P_1P_2}+\frac{2\gamma_{np}}{\theta}[(1+(\mathbf{P}_1\mathbf{n})(\mathbf{P}_2\mathbf{n}))\,\mathrm{Im}\,c^h+(\mathbf{P}_1\mathbf{n})\,\mathrm{Im}\,a^h$$
$$+(\mathbf{P}_2\mathbf{n})\,\mathrm{Im}\,b^h+(\mathbf{P}_1\mathbf{m})(\mathbf{P}_2\mathbf{l})\,\mathrm{Re}\,f^h+(\mathbf{P}_1\mathbf{l})(\mathbf{P}_2\mathbf{m})\,\mathrm{Re}\,e^h]+\gamma_{np}^2/\theta^2. \tag{64}$$

As follows from (64), if a transversely polarized neutron beam is scattered by an unpolarized proton target, one can determine $\mathrm{Im}c^h+(\mathbf{P}_1\mathbf{n})\mathrm{Im}a^h$, and if unpolarized neutrons are scattered on a polarized proton target $\mathrm{Im}c^h+(\mathbf{P}_2\mathbf{n})\mathrm{Im}b^h$. To determine $\mathrm{Re}\,e^h$ and $\mathrm{Re}f^h$, one needs experiments with polarized beam and polarized target in which both $\mathbf{P}_1$ and $\mathbf{P}_2$ are in the scattering plane.

In Ref. 19, other polarization effects were considered: polarization rotation, polarization correlation, etc. In Ref. 27, the electromagnetic effect was considered at energies of hundreds of MeV in the framework of a numerical solution of the Schrödinger equation. The treatment in the Born approximation was confirmed with good accuracy.

All that we have said above about the scattering of neutrons by protons can be directly transferred to the scattering of neutral hyperons by protons. Since the hyperon–nucleon interaction has been little studied, the interest in such investigation is considerable. The experimental difficulties here are great but a circumstance which does facilitate experiments is that the decay of hyperons is a good natural analyzer of the polarization.

Scattering of Protons on Nuclei. For the amplitude of elastic scattering of charged baryons by spinless nuclei, (50) is replaced by

$$M=A^h\exp(i\varphi_A)+a_c/\theta^2+[c^h\exp(i\varphi_C)+i\gamma_p/\theta](\sigma n) \tag{65}$$

(if we ignore the corrections to A^h and c^h that do not reduce to phases). Here, φ_A and φ_C are the relative phases determined in (46); a_c/θ^2 is the Coulomb amplitude, and the factor

$$\gamma_p=\frac{Ze^2}{Mc^2}\mu(E)=\frac{Ze^2}{Mc^2}\left[\mu-\frac{s-M^2-m^2}{2\sqrt{s}(E+M)}\right], \tag{66}$$

where E is the energy of one nucleon in the center-of-mass system, reduces to

$$\gamma_p^{NR}\approx\frac{Ze^2}{Mc^2}(\mu-1/2)=3.5Z\cdot10^{-16}\ \mathrm{cm}.$$

at low energies and to

$$\gamma_p^{R}\approx\frac{Ze^2}{Mc^2}(\mu-1)=2.75Z\cdot10^{-16}\ \mathrm{cm}.$$

at high energies; at the same time,

$$a_c=-\frac{2}{\beta_L^2}\frac{Ze^2}{Mc^2}=-\frac{Z}{\beta_L^2}3.06\cdot10^{-16}\ \mathrm{cm}.$$

Using (65) and (46) and taking into account the smallness of the phases φ_A and φ_C, we obtain

$$\frac{d\sigma_0}{do}=\sigma_0\approx\sigma_0^h+(a_c/\theta^2)^2+(2a_c/\theta)(\mathrm{Re}\,A^h-\varphi_A\,\mathrm{Im}\,A^h)$$
$$+\gamma_p^2/\theta^2+(2\gamma_p/\theta)(\mathrm{Im}\,C^h+\varphi_C\,\mathrm{Re}\,c^h); \tag{67}$$

$$\sigma_0P_{0p}\approx\sigma_0^hP_{0p}^h-2(\varphi_C-\varphi_A)\,\mathrm{Im}\,(A^{h*}c^h)$$
$$+2[\gamma_p\,\mathrm{Im}\,A^h+(a_c/\theta)\,\mathrm{Re}\,c^h]/\theta; \tag{68}$$

$$\sigma_0^h=|A^h|^2+|c^h|^2;\quad \sigma_0^hP_0^h=2\,\mathrm{Re}\,(A^{h*}c^h). \tag{69}$$

The second term in (68) is small at small $\theta(c^h(\theta)\sim\theta)$, and the interference effect in the polarization is proportional to

$$\frac{2}{\theta}[\gamma_p\,\mathrm{Im}\,A^h+\frac{a_c}{\theta}\mathrm{Re}\,c^h(\theta)], \tag{70}$$

and

$$P_{0p}\approx\left\{\sigma_0^hP_0^h+\frac{2}{\theta}\left[\gamma_p\,\mathrm{Im}\,A^h+\frac{a_c}{\theta}\,\mathrm{Re}\,c^h\right]\right\}$$
$$\times\left\{\sigma_0^h+\left(\frac{a_c}{\theta_c}\right)^2+\frac{2a_c(\mathrm{Re}\,A^h-\varphi_A\,\mathrm{Im}\,A^h)+\gamma_p^2}{\theta^2}\right.$$
$$\left.+\frac{2\gamma_p}{\theta}(\mathrm{Im}\,c^h+\varphi_C\,\mathrm{Re}\,c^h)\right\}^{-1}. \tag{71}$$

Thus, for $-t\approx-t_M\ll-t_c$ because of the dominant contribution of Coulomb scattering

$$P_{0p}(-t\ll-t_c)\approx P_{0p}^h\frac{\sigma_0^h}{(a_c/\theta^2)^2}-$$
$$-\frac{\theta^3\beta_L^2}{2}\left(\frac{Mc^2}{Ze^2}\right)\left[1-\frac{2}{\beta_L^2}\frac{\mathrm{Re}\,c^h}{\theta\,\mathrm{Im}\,c^h}\right]\mathrm{Im}\,A^h;$$
$$|P_{0p}(-t_M)|\ll|P_{0n}(-t_M)|. \tag{72}$$

However, in the region of $-t$ values in which the contribution of the Coulomb interaction can be ignored, the values of P_{0n} and P_{0p} are comparable, differing in sign from the signs of the magnetic moments of the neutron and proton; the differences are determined by the values of the magnetic moments and the total interaction cross sections. If we ignore the spin dependence of M^h

and the contribution of the real part of the spin amplitude (assumptions from the region of very high energies), then

$$P_{0p} \approx (2/\theta)\,\gamma_p\,\mathrm{Im}\,A^h\,(\sigma_0^h + a_c^2/\theta^4)^{-1} \tag{73}$$

will have a maximum at $-t_p = -\sqrt{3}t_c$, where $-t_c$ is given in (5), and the proton polarization is

$$P_{0p}(t_p) \approx \frac{\sqrt{3}}{4}\,\frac{\mathrm{Im}\,A^h}{|A^h|}\,\mu(E)\,\frac{\sqrt{-t_p}}{M} = 0.045\,\frac{\mathrm{Im}\,A^h}{|A^h|}, \tag{74}$$

if we ignore the weak dependence of $-t_p^{pA}$ on Z and A of the nucleus, which at high energies can be represented in the form

$$|t_p^{pA}| \approx \frac{Z\sigma_{\mathrm{tot}}^{pP}}{\sigma_{\mathrm{tot}}^{PA}}\,|t_p^{Pp}| \approx \frac{Z}{A^{2/3}}\,|t_p^{PP}|.$$

Altogether, because of the Coulomb interaction, the 100% interference effects that hold for neutrons disappear for protons, but in the region $-t \gtrsim -t_c$ and somewhat higher, effects of order 5% remain. The main interest in their experimental study is as before. The investigation of polarization effects at small scattering angles due to hadron–electromagnetic interference makes it possible to determine the spin structure of the hadronic amplitude by simpler experiments than when the programs of a complete experiment are implemented.

Scattering of Mesons on Protons. It is here appropriate to mention one further manifestation of the electromagnetic contribution to the spin–orbit amplitude [M_1^{em} in (14), c^{em} and d^{em} in (22), and c^{em} in (23)].

In the general case, when this amplitude contains both hadronic, $c^h(t)$, and electromagnetic, $c^{em}(t)$, contributions, the polarization $P^0(t)$ has a specific dependence, decreasing slowly to zero with decreasing t.

Since

$$c^h(t) = \sqrt{-t}\,\varphi(t) \approx \sqrt{-t}\,\varphi(0);\quad \varphi(0) \neq 0,$$

and

$$c^{em}(t) \approx c^{em}(0)/\sqrt{-t} = \gamma k/\sqrt{-t},$$

the decrease of $c^h(t)$ at small $-t$ can be compensated by an increase in $c^{em}(t)$. This is reflected in the fact that the total polarization (71) in a certain region of small $-t \approx 0.1$ (GeV/c)^2 does not decrease at high energies with decreasing $-t$. The value of $P_0(t)$ begins to decrease appreciably at $-t < -t_s$, values which are so small that the decrease of $c^h(t)$ with decreasing $-t$ is not compensated by the increase in $c^{em}(t)$. The contributions $c^h(t)$ and $c^{em}(t)$ are comparable at

$$-t_s = \gamma k/\varphi(0). \tag{75}$$

A model estimate[17] for π^+p scattering leads to the value $-t_s^{\pi P} \approx 0.01(\mathrm{GeV}/c)^2$. A similar effect occurs also in other elastic scattering processes involving hadrons.

If we ignore the existence of the spin dependence in M^h and the contribution ReA^h, we obtain for $P_0^{\mathrm{int}}(t)$ in pion–proton scattering an expression that is equal to (73) and (74) [and (82) below]. For the maximal value of $P_0(t_p)$ in the region of small t, we have

$$[P_0^{\mathrm{int}}(t_p)]^{\pi\pm p} \approx \pm 5\ 2\%\,\mathrm{Im}\,A^h/|A^h|,$$

where the signs + and − refer to π^+p and π^-p scattering, respectively.

As can be seen from (74) and (5), P_0^{int} for Kp scattering is related to P_0^{int} for πp scattering by

$$\sqrt{\sigma_{\mathrm{tot}}^{\pi P}}\,P_0^{\pi p}(t_p^{\pi p}) = \sqrt{\sigma_{\mathrm{tot}}^{Kp}}\,P_0^{Kp}(t_p^{Kp}).$$

Scattering of Protons on Protons. The electromagnetic effects we have been considering are of interest from various points of view in different energy ranges of the particles. The appreciable increase in the accuracy with which proton–proton scattering can be studied in the proton beams of meson factories makes it possible to investigate the interference of strong and electromagnetic interactions in order to obtain more precise information about the structure of the hadronic amplitude. As estimates based on the available phase-shift analysis of the data on pp scattering show, the electromagnetic contribution $c^{em}(t)$ is comparable with $c^h(t)$ at 500–600 MeV in the range of cms scattering angles less than 5 or 6°.

If to estimate $-t_s^{pp}$ at 6 GeV/c we use the representation for $c^h(t)$ in the form

$$c^h(t) \approx 2.3\sqrt{-t}\exp(6.3t)\ (\mathrm{mb/GeV})^{1/2},$$

obtained in Ref. 28, then $-t_s^{pp} \approx 0.01(\mathrm{GeV}/c)^2$. In the region $-t < -t_s$, we can expect a pronounced decrease in the amount of polarization compared with the region $-t > -t_s$.

At high energies (even in experiments with colliding beams) the investigation of the polarization P_0 at small t gives interesting information on the hadronic amplitude. With allowance for (23) and (46), the cross section for scattering of unpolarized particles (in the absence of a spin dependence in M^h) is

$$\sigma_0 = \sigma_0^h + 4\alpha^2E^2/t^2 - (4\alpha E/|t|)\,\mathrm{Im}\,a^h(t) \times \{\rho\cos\varphi_a + \sin\varphi_a - 2(\mu-1)^2\pi\alpha/M^2\sigma_{\mathrm{tot}}^{pp}\}, \tag{76}$$

where the last term takes into account the contribution of the interaction with the magnetic moment, and $\rho = \mathrm{Re}\,a^h(t)/\mathrm{Im}\,a^h(t)$. The general expression for the polarization P_0

$$\sigma_0P_0 = 2\,\mathrm{Re}\,c^*(a+b) \tag{77}$$

shows that a measurement of P_0 at small angles makes it possible to determine the imaginary part of the amplitude $a^h + b^h$. Indeed, ignoring the small Coulomb phases, which are approximately equal to $3\cdot10^{-3}$, we obtain

$$\sigma_0P_0 = \sigma_0^hP_0^h + [2\gamma_{pp}/|t|^{1/2}]\,\mathrm{Im}\,(a^h+b^h) \tag{78}$$

[γ_{pp} is given in (23)].

From the generalized optical theorem

$$\mathrm{Im}\,[a^h(0)+b^h(0)]=\frac{k}{4\pi}\frac{1}{2}(\sigma_0^t+\sigma_+^t) \qquad (79)$$

and, therefore, in pp scattering the electromagnetic amplitude interferes with the amplitude that characterizes the interactions in the triplet states. Ignoring the contribution of $2\gamma_{pp}|t|^{-1/2}$ to (78), we obtain

$$P_0^{\mathrm{int}}(t)=2\gamma_{pp}|t|^{3/2}\,\mathrm{Im}\,a^h\,[t^2|a^h|^2+4\alpha^2E^2]^{-1}. \qquad (80)$$

Thus, the polarization P_0 attains a maximum at $-t_p=-\sqrt{3}t_c$, and the value of the polarization at the maximum is

$$P_0^{\mathrm{int}}(t_p)\approx\frac{\sqrt{3}}{4}\frac{\mathrm{Im}\,a^h}{|a^h|}(\mu-1)\frac{\sqrt{-t_p}}{M}\approx 4.5\%. \qquad (81)$$

Deviation of the measured value of P_0 from that predicted in (81) would be a direct indication of a spin dependence of the hadronic amplitude, since the contribution of the real part of the spinless amplitude at high energies (where $\rho\approx 0.1$) is small.

In experiments with a stationary target, one has the problem of determining the polarization of the recoil protons with a kinetic energy around

$$T_p^0=|t_p|/2M=1.5\ \mathrm{MeV}$$

or (which appears more attractive) of measuring the symmetry in the cross section of scattering on a polarized target at the same energies of the recoil protons.

Introducing $z=|t|/|t_p|=T_p/T_p^0$, we can rewrite (80) in the form

$$P_0^{\mathrm{int}}(t)=P_0^{\mathrm{int}}(t_p)\,4z^{3/2}/(1+3z^2), \qquad (82)$$

from which it can be seen that the decrease of $P_0(t)$ with increasing z takes place comparatively slowly.

It is well known that the pp scattering matrix contains at $t=0$ the three complex amplitudes $a^h(0)$, $b^h(0)=e^h(0)$, and $f^h(0)$:

$$M^h(0)=a^h(0)+b^h(0)(\sigma_1\sigma_2)+[f^h(0)-b^h(0)](\sigma_1\hat{\mathbf{k}})(\sigma_2\hat{\mathbf{k}}). \qquad (83)$$

It is well known[3] that the imaginary parts of the amplitudes $b^h(0)$ and $f^h(0)-b^h(0)$ are related to the total cross sections of the pp interaction in pure spin states:

$$\mathrm{Im}\,[f^h(0)-b^h(0)]=\frac{p_c}{4\pi}\frac{1}{2}(\sigma_+^t-\sigma_0^t). \qquad (84)$$

The investigation in the region of hadron–electromagnetic interference of spin effects more complicated than polarization makes it possible to obtain data on the imaginary and real parts of the amplitude (83).

If polarized protons are accelerated in storage rings or in experiments with polarized beams and target it is possible to study the polarization correlations at small angles. It is easy to see that

$$\sigma_0^h(0)=|a^h|^2+2|b^h|^2+|f^h|^2. \qquad (85)$$

The hadronic contribution to the parameter C_{nn} at $t=0$ is given by

$$\sigma_0^h(0)\,C_{nn}^h(0)=2\,\mathrm{Re}\,[b^{h*}(a^h-f^h)]. \qquad (86)$$

If $\mathrm{Re}\,b^h/\mathrm{Im}\,b^h\ll 1$, then measurement of $C_{nn}^h(0)$ makes it possible to determine $\mathrm{Im}f^h(0)$. The value of $\mathrm{Re}\,b^h/\mathrm{Im}\,b^h$ can be found by measuring the contribution of the interference to C_{nn} since

$$\sigma_0C_{nn}^{\mathrm{int}}=\sigma_0(C_{nn}-C_{nn}^h-C_{nn}^{em})=\frac{4\alpha E}{|t|}\mathrm{Re}\,b^h. \qquad (87)$$

The value of C_{nn}^{int} reaches a maximum at $-t=-t_c$, and

$$C_{nn}^{\mathrm{int}}(t_c)/C_{nn}^h(0)=\mathrm{Re}\,b^h/\mathrm{Im}\,b^h. \qquad (88)$$

Similarly,

$$\sigma_0C_{ll}^{\mathrm{int}}=\frac{4\alpha E}{|t|}\mathrm{Re}\,f^h \qquad (89)$$

and

$$C_{ll}^{\mathrm{int}}(t_c)/C_{ll}^h(0)=\mathrm{Re}\,f^h/\mathrm{Im}\,f^h. \qquad (90)$$

Since $C_{lm}^h(0)=0$,

$$\sigma_0C_{lm}^{\mathrm{int}}=-\frac{2\gamma_{pp}}{|t|^{1/2}}\mathrm{Re}\,(b^h-f^h)$$

and

$$\frac{C_{lm}^{\mathrm{int}}(t)}{P_0^{\mathrm{int}}(t)}=-\mathrm{Re}\,(b^h-f^h)/\mathrm{Im}\,(a^h+b^h). \qquad (91)$$

Coherent Production of Resonances. To elucidate the different features of polarization at small t, let us consider in more detail the processes

$$N+Z\to N(1470)+Z; \qquad (92)$$

$$\pi+Z\to A_1+Z. \qquad (93)$$

We consider the spin-independent nuclear part of the amplitude of the process (92). It can be expressed in terms of the spin-independent part of the amplitude $f_h(\mathbf{q})$ of the process $N+N\to N^*+N$ as follows:

$$M^h(\mathbf{q})=f_h(0)\,S_h(\mathbf{q}). \qquad (94)$$

The nuclear form factor $S_h(\mathbf{q})$ has the form

$$S_h(\mathbf{q})=4\pi\int bdbJ_0(\mathbf{qb})\,[\exp(-\sigma_1T(\mathbf{b})/2)-\exp(-\sigma_2T(\mathbf{b})/2)]/(\sigma_1-\sigma_2), \qquad (95)$$

where $\sigma_{1,2}$ are the total cross sections for interaction of particles 1 and 2 with the nucleon; $J_0(x)$ is the Bessel function;

$$T(\mathbf{b})=\int_{-\infty}^{+\infty}\rho(\mathbf{b},z)\,dz \qquad (96)$$

for

$$\int T(\mathbf{b})\,d^2\mathbf{b}=A. \qquad (97)$$

We now consider the observable quantities. To calculate the polarization P_0, we shall assume that M^h is purely imaginary since the coherent production of resonances is a diffraction process. Then

$$P_0^{\text{int}} = 2\left[\frac{d\sigma^{em}}{dt}\Big/\frac{d\sigma^h}{dt}\right]^{1/2}\left[1+\frac{d\sigma^{em}}{dt}\Big/\frac{d\sigma^h}{dt}\right]^{-1}\cos\varphi, \tag{98}$$

where $d\sigma^{em}/dT$ is given in (28), φ in (49), and

$$d\sigma^h/dt = |M^h|^2. \tag{99}$$

As can be seen from (98) and (99), the interference of the strong and electromagnetic interactions leads to an appreciable asymmetry in the differential cross section of the process with a polarized beam. Even if the contribution of the Primakoff effect to the interaction cross section of unpolarized particles is a few percent, its contribution to the polarization may be tens of percent.

The presence of a nonzero phase φ of the electromagnetic amplitude reduces P_0, and sometimes appreciably. For example, in the process

$$N + \text{Pb} \to N(1470) + \text{Pb}, \tag{100}$$

($R = 6.7$ F, $Z = 82$) at $E_N = 40$ GeV, $\Delta = 1.5 \cdot 10^{-2}\,(\text{GeV}/c)^2$, and $q = \Delta$ the phase $\varphi(\Delta = q) = \pi/2$, and P_0 vanishes. For the same reaction on copper nuclei ($R = 4.5$ F; $Z = 28$) $\varphi(q = \Delta) = 0.16$ and

$$P_0 \approx 2\left[\frac{d\sigma^{em}}{dt}\Big/\frac{d\sigma^h}{dt}\right]^{1/2}. \tag{101}$$

Thus, the maximal value of P_0 on all nuclei in processes of the form (92) and

$$Y + Z \to Y^* + Z \tag{102}$$

is attained in experiments with neutrons and neutral hyperons.

In other cases, the existence of an appreciable Coulomb phase leads to significant polarization effects. To see this, let us consider the expression for the polarization of baryons in processes of the type (92) and (102) with a polarized beam of incident baryons (degree of polarization $\mathbf{P}_b$):

$$\begin{aligned}\sigma_p\mathbf{P} = {} & [\sigma_0 P_0 + 2(\mathbf{P}_B\cdot\mathbf{n})|F^{em}|^2]\,\mathbf{n} + \mathbf{P}_B[|M^h|^2 - |F^{em}|^2] \\ & + i(F^{em}M^{h*} - F^{em*}M^h)[\mathbf{n}\times\mathbf{P}_B].\end{aligned} \tag{103}$$

Here,

$$\sigma_p = \sigma_0(1 + \mathbf{P}_B\mathbf{P}_0). \tag{104}$$

The value of σ_0 is given in (99), of P_0 in (98), and of F^{em} in (29).

If the experiment is made with a longitudinally polarized beam, i.e., if $\mathbf{P}_B = P_B\hat{\mathbf{k}}$, then it follows from (103) that

$$\sigma_0\mathbf{P} = \sigma_0 P_0\mathbf{n} + P_B\mathbf{k}_0[|M^h|^2 - |F^{em}|^2] + i(F^{em}M^{h*} - F^{em*}M^h)[\mathbf{n}\times\hat{\mathbf{k}}]. \tag{105}$$

It can be seen from (105) that at small scattering angles new information is associated with the occurrence of a post-scattering transverse (in the scattering plane) polarization component:

$$\begin{aligned}\sigma_0\mathbf{P} = {} & \mathbf{P}_B\cdot[\mathbf{n}\times\hat{\mathbf{k}}]\,2\frac{q\sin\theta}{(q^2+\Delta^2)}\left[\frac{2s_2+1}{2s_1+1}\right. \\ & \times\left.\frac{8\pi Z^2\alpha m_2^3\Gamma(2\to 1+\gamma)}{(m_2^2-m_1^2)^3}\right]^{1/2}(\text{Im}\,M^h\sin\varphi - \text{Re}\,M^h\cos\varphi).\end{aligned} \tag{106}$$

Investigation of diffraction processes of resonance production at high energies is of interest from the point of view of studying the structure of the vacuum singularity in strong interactions. In a theory with a Pomeranchuk trajectory, when the value of the pomeron trajectory at $t = 0$ is $\alpha(0) = 1$, the total cross sections are asymptotically constant and the cross sections of coherent resonance production decrease slowly, as $(R^2 + \alpha' \ln s/s_0)^{-1}$, with the energy.

In a theory based on exchange of Pomeranchuk trajectories with $\alpha(0) > 1$, when the total cross sections asymptotically satisfy the Froissart bound, the cross sections of coherent production of resonances increase asymptotically with the energy in proportion to the logarithm of the energy.[29]

Study of the interference of the strong and electromagnetic contributions to the resonance-production cross section when vacuum exchange is allowed in strong interactions is necessary at various energies. The investigation of the interference noted above between the strong and electromagnetic interactions in polarization phenomena makes it possible to obtain data on the moduli and phases of amplitudes due to vacuum exchange.

In processes for which the hadronic amplitude is determined by vector exchanges, the electromagnetic contribution begins to predominate with increasing energy. It is of interest here to study the angular distributions at small t'. In this region, and also near the diffraction minima, the electromagnetic contribution will be noticeable.

4. INTERFERENCE OF THE STRONG AND ELECTROMAGNETIC INTERACTIONS IN THE POLARIZATION AT MOMENTUM TRANSFERS THAT ARE NOT SMALL

Scattering of Baryons on Nuclei. The contribution of the interference of the spin-dependent amplitude of the magnetic interaction with the hadronic amplitude can be manifested not only at small t but also at larger momentum transfers near the positions of the diffraction minima of the differential cross sections. At superhigh energies, when pomeron exchange predominates, the spin-dependent hadronic amplitude of baryon–nucleon scattering appears to become small.[1)] Therefore, the possibility cannot be ruled out that polarization phenomena in baryon–nucleus (B–A) scattering will be to a large extent determined by the interference between M^{em} and the spin-independent M^h.

It is not entirely correct to treat the electromagnetic effects for B–A scattering in the first order in $Z\alpha$. It is also necessary to take into account the t dependence of the nuclear amplitudes.

We use the Glauber representation for the amplitude of scattering of baryons by a spinless nucleus with

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[1)]Although data on the polarization right up to 100 GeV/c suggest the existence of a spin dependence of M_{pp}^h.

mass number A:

$$F(\mathbf{q}) = \frac{ik}{2\pi} \int d\mathbf{b} \exp(i\mathbf{q}\mathbf{b}) \left\{ 1 - \left(1 - \frac{1}{A} \Gamma(\mathbf{b}) \right)^A \right\}$$
$$= \frac{ik}{2\pi} \int d\mathbf{b} \exp(i\mathbf{q}\mathbf{b}) \{1 - \exp[-\Gamma(\mathbf{b})]\} + O(A^{-1}), \quad (107)$$

where $\Gamma(\mathbf{b})$ is the two-dimensional Fourier transform of the B–A scattering amplitude.

We represent the BN scattering amplitude in the laboratory system averaged over the spin of one of the nucleons in the form

$$f(\mathbf{q}) = f^h(\mathbf{q}) + f^{em}(\mathbf{q}), \quad (108)$$

where

$$f^h(\mathbf{q}) = \frac{ik}{4\pi} \widetilde{\sigma} [1 + i(\mathbf{c}\mathbf{q}) \widetilde{\nu}] G_h(\mathbf{q}); \quad (109)$$
$$f^{em}(\mathbf{q}) = -\frac{2\alpha E}{\mathbf{q}^2} [\varepsilon e + i(\mathbf{c}\mathbf{q}) \widetilde{\mu}] G_{em}(\mathbf{q}); \quad (110)$$
$$\mathbf{c} = [\sigma \times \mathbf{k}]/2ME; \; \widetilde{\sigma} = \sigma_{tot} [1 - i \operatorname{Re} f^h(0)/\operatorname{Im} f^h(0)];$$
$$\widetilde{\mu} = \mu - \varepsilon E/(E+M); \; \widetilde{\nu} = c^h/a^h;$$

E, k, and M are the energy, momentum and mass of the baryon; $\varepsilon = 0, \pm 1$ are its charge in fractions of e; μ is the baryon magnetic moment; $G_{em}(\mathbf{q})$ is the product of the baryon and nucleon form factors;

$$G_h(\mathbf{q}) = \left\{ \frac{d\sigma_{BN}(q)}{dt} \Big/ \frac{d\sigma_{BN}(0)}{dt} \right\}^{1/2}. \quad (111)$$

If allowance is made for the spin-flip amplitudes of BN scattering in the first approximation, then from (107)–(110) we obtain for the total amplitude of B–A scattering

$$F(\mathbf{q}) = f^h(\mathbf{q}) G_h^{-1}(\mathbf{q}) S_h(\mathbf{q}) + f^{em}(\mathbf{q}) G_{em}(\mathbf{q}) S_{em}(\mathbf{q}), \quad (112)$$

where

$$S_h(\mathbf{q}) = -2\pi \int \frac{bdb}{\mathbf{q}} J_1(\mathbf{q}b) \frac{dT(b)}{db} \exp\left[-\frac{\widetilde{\sigma}}{2} T + i\varepsilon\chi \right]; \quad (113)$$
$$S_{em}(\mathbf{q}) = -\mathbf{q} \int bdb J_1(\mathbf{q}b) \frac{d\chi}{db} \exp\left[-\frac{\widetilde{\sigma}}{2} T + i\varepsilon\chi \right]; \quad (114)$$

$$\left. \begin{aligned} T(b) &= \frac{1}{2\pi} \int S(q) G_h(q) J_0(qb)\, q dq; \\ \chi(b) &= 2Z\alpha \int \frac{S(q) G_{em}(q)}{q^2 + R_{sc}^{-2}} J_0(qb)\, q dq; \\ S(q) &= \int \rho(\mathbf{r}) \exp(i\mathbf{q}\mathbf{r})\, d\mathbf{r}; \quad S(0) = A; \end{aligned} \right\} \quad (115)$$

$J_n(x)$ is a Bessel function; $\rho(r)$ is the matter density in the nucleus; R_{sc} is the radius of the screening of the Coulomb field of the nucleus by the atomic electrons.

For scattering on heavy nuclei, one can use the black-sphere approximation, which is equivalent to the following approximate relations:

$$\exp\left[-\frac{\widetilde{\sigma}}{2} T(b) \right] \approx 0 (b - \widetilde{R});$$
$$-\frac{\widetilde{\sigma}}{2} \frac{dT}{db} \exp\left[-\frac{\widetilde{\sigma}}{2} T(b) \right] \approx \delta(b - \widetilde{R}). \quad (116)$$

Here $\widetilde{R}$ is the opacity radius of the nucleus, which differs from the radius R of the nucleus by a quantity of order l^2/R^2, where l is the mean free path of a baryon in the nucleus.

In this approximation ($n = Z\alpha$)

$$S_h(q) = \frac{4\pi \widetilde{R} J_1(q\widetilde{R})}{\widetilde{\sigma} q} \exp[-2in\varepsilon \ln \widetilde{R}/R_{sc}]; \quad (117)$$
$$S_{em}(q) = Zq \int_{\widetilde{R}}^{\infty} db J_1(qb) \exp[-2in\varepsilon \ln b/R_{sc}]. \quad (118)$$

For $qR \ll 1$, $S_{em}(q)$ is transformed into the usual Coulomb phase factor multiplied by Z:

$$Z \exp\left[2in\varepsilon \left(\ln \frac{q^2 R_{sc}^2}{2} + C \right) \right] + O(q^2 R^2). \quad (119)$$

For $qR \gg 1$

$$S_{em}(q) = Z \left\{ J_0(qR) + \frac{2in\varepsilon}{qR} J_1(qR) \right\}$$
$$\times \exp[-2in\varepsilon \ln \widetilde{R}/R_{sc}]. \quad (120)$$

For scattering of neutral baryons by nuclei,

$$S_{em}(q) = Z J_0(qR). \quad (121)$$

Ignoring the spin dependence and the real part of the amplitude in M^h, we obtain for the total amplitude of scattering of neutral baryons by nuclei

$$F^{B^0 A} = \frac{ikR}{q} \left[J_1(qR) - \frac{n\widetilde{\mu}}{RM} J_0(qR)(\sigma\mathbf{n}) \right], \quad (122)$$

where $\mathbf{n}$ is the normal to the reaction plane.

We have $\delta = n\widetilde{\mu}/RM \ll 1$ and this quantity increases monotonically with increasing A. Therefore, an appreciable influence of the electromagnetic interaction in B^0–A scattering can be expected only in a small region $\Delta q = \delta/R$ near the zeros of the Bessel function $J_1(y)$.

To estimate the expected experimental effect, it must be borne in mind that in an experiment that does not distinguish separately elastic and inelastic scattering one measures, not P_0^{el}, A^{el}, R^{el} (common notation P in this section), but

$$P^{exp} = (P^{el}\sigma_0^{el} + P^{qel}\sigma_0^{qel})/(\sigma_0^{el} + \sigma_0^{qel}). \quad (123)$$

In quasielastic scattering at high energies and small momentum transfers, $P_0^{qel} = A^{qel} = 0$, $R^{qel} = 1$. The differential cross section of quasielastic scattering at $q^2 \ll H^{-1}$, where H is the slope parameter of the B–A scattering amplitude, is equal to

$$\sigma_0^{qel} = \left(\frac{k\sigma_{BN}^{tot}}{4\pi} \right)^2 (1 + \beta^2) N_1(\sigma_{BN}, A), \quad (124)$$

where

$$\beta = \operatorname{Re} f_{BN}(0)/\operatorname{Im} f_{BN}(0);$$
$$N_1 = \int T(\mathbf{b}) \exp(-\sigma_{BN}) T(\mathbf{b})\, d^2\mathbf{b}$$

is the effective number of the nucleons in the nucleus participating in the quasielastic scattering.

Introducing

$$x = RMJ_1(qR)/[\widetilde{\mu} n J_0(qR)] = J_1(qR)/[\delta J_0(qR)]; \quad (125)$$
$$u = q\sigma M \sqrt{N_1(1+\beta^2)}/[4\pi n \widetilde{\mu} J_0(qR)], \quad (126)$$

we find from (123) that

$$\left.\begin{array}{l}A=0;\\P_0=2x/(1+x^2+u^2);\\R=(1-x^2+u^2)/(1+x^2+u^2).\end{array}\right\}\qquad(127)$$

For $qR \ll 1$,

$$x=qR/2\delta;\ u=\frac{qM\sigma}{4\pi n\tilde{\mu}}\sqrt{N_1(1+\beta^2)}.$$

Introducing

$$q_M=2|\delta|(1+a)^{-1/2}/R;\ a=u^2/x^2=\sigma^2N_1(1+\beta^2)/4\pi^2R^4,$$

we obtain

$$\left.\begin{array}{l}P_0=-2(q/q_M)(1+a)^{-1/2}[1+(q/q_M)^2]^{-1};\\R=[(q/q_M)^2-1][(q/q_M)^2+1]^{-1}.\end{array}\right\}\qquad(128)$$

For all nuclear targets, $a \ll 1$. Therefore, a beam of neutral baryons scattered through an angle q_M/k is almost completely polarized.

Let us consider the neighborhood in the region of vanishing of $J_1(qR)$, i.e., $q=q_1+\Delta$, $q\sim q_1=y_1/R$, where $J_1(y_1)=0$ and $\Delta R \ll 1$. In this region,

$$\left.\begin{array}{l}J_0(qR)\approx J_0(q_1R);\ J_1(qR)\approx J_0(q_1R)\,\Delta R;\\x=\Delta R/|\delta|;\ u=u_0=q_1M\sigma\sqrt{N_1(1+\beta^2)}/[4\pi J_0(q_1R)]\end{array}\right\}\qquad(129)$$

and P_0 vanishes at $q=q_1$, changes sign above and below:

$$P_0(q_1+\Delta)=-P_0(q_1-\Delta),\qquad(130)$$

and $|p_0|$ attains a maximum if the contribution of the real parts of the amplitude is ignored, equal to

$$|P_0|=(1+u_0^2)^{-1/2},\qquad(131)$$

for

$$\Delta_M=\frac{|\delta|}{R}(1+u_0^2)^{1/2}\approx\frac{q_M}{2}(1+u_0^2)^{1/2}.\qquad(132)$$

If purely elastic scattering is separated, then $u=0$ and, ignoring the contribution of the real parts of the amplitudes, $|P_0(q_1\pm\Delta_M)|=1$. Allowance for the contribution of quasielastic scattering significantly changes this value. Since $N_1\sim A^{2/5}$, $R\sim A^{1/3}$, the maximal polarization is attained on heavy nuclei. For scattering of neutrons on lead nuclei ($A=208$, $n=0.6$, $N_1(\sigma=40$ mb$)=9$, $R=6.7$ F) $u_0\approx 3$ (in the region of the first J_1 zero $y_1=3.65$) and $|P_0^{n\,\mathrm{Pb}}(q_1\pm\Delta_M)|=0.3$. Near the second zero, $y_2=7$, $u_0\approx 7.5$, and $|P_0^{n\,\mathrm{Pb}}(q_2\pm\Delta_M)|=0.12$.

For scattering of charged baryons by nuclei near the diffraction minima of the cross section when the contribution of quasielastic scattering is ignored, the maximal values of $|P_0|$ are attained for $-t=-t_K\pm\Delta_t$, $t_K=(y_k/R)^2$, $\Delta t=4Z\alpha/R^2$, $J_1(y_k)=0$, $y_k\neq 0$ and are approximately equal to

$$|P_0|\approx\frac{y_k|\tilde{\mu}\cos\varphi_{em}(y_k)|}{2MR(1\pm\varepsilon\sin\varphi_{em}(y_k))};\qquad(133)$$

$$|A|\approx\frac{y_k|\tilde{\mu}\varepsilon\sin\varphi_{em}(y_k)|}{2MR(1\pm\varepsilon\sin\varphi_{em}(y_k))}.\qquad(134)$$

Since $y_k \gg 1$, using (120) we can obtain

$$S_{em}(y_k)\approx ZJ_0(y_k)[1-4i\varepsilon n(i\varepsilon n-1)/y_k],$$

whence approximately

$$\varphi_{em}(y_k)=\tan^{-1}[4n\varepsilon/(y_k+4n^2)].\qquad(135)$$

The signs ± in (133) and (134) correspond to constructive and destructive interference of M^h and M^{em} at the points $-t=-t_K\pm\Delta t$. Therefore, the property (130) for the scattering of charged baryons does not hold. The difference between the values of P_0 at the points $-t_K\pm\Delta t$ is maximal for scattering on heavy nuclei. For scattering on lead,

$$|P_0|=\begin{cases}0.07;\\0.17;\end{cases}\ |A|=\begin{cases}0.03, & -t=-t_1+|\Delta t|;\\0.08, & -t=-t_1-|\Delta t|.\end{cases}$$

The values of the polarization parameters decrease by approximately 25% when allowance is made for quasielastic scattering on heavy nuclei. For nuclei with $A<100$, the contribution of quasielastic scattering exceeds the contribution of elastic scattering at the minimum of the cross section, which strongly reduces the polarization effects.

The polarization P_0 vanishes not only at the zeros of the Bessel function $J_1(qR)$ but also at the zeros of $J_0(qR)$ if the approximation (122) is taken for the amplitude. Here, (130) also holds for the scattering of neutral baryons, and the value of $|P_0|$ attains a maximum

$$|P_0|=(1+v_0^2)^{-1/2},\qquad(136)$$

where

$$v_0=q_1\sigma\sqrt{N_1(1+\beta^2)}/[4\pi RJ_1(q_1R)].\qquad(137)$$

Since the zeros of $J_0(y)$ and $J_1(y)$ nowhere coincide, in the approximation (122) the dependence $P_0(t)$ will be characterized by a deep minimum at $-t=-t_M$, and then by decreasing values of $P_0(t)$ with the maxima (136) and (131) near the zeros of $J_0(y)$ and $J_1(y)$, respectively. Since the values of v and u increase with increasing q, the values of P_0 at the minima and maxima decrease with increasing scattering angle. The polarization $(P_0)_{\mathrm{int}}$ changes sign near each zero.

Scattering of Mesons and Baryons by Nucleons. At high energies, when the polarization is small, the interference between M^h and M^{em} leads to appreciable effects in polarization in collisions of mesons and nucleons also at appreciable momentum transfers.[30]

If we ignore the relative phase of the hadronic and electromagnetic amplitudes and the antisymmetrization of M^{em}, then in the one-photon approximation for pp scattering

$$(P_0)_{\mathrm{int}}^{pp}=\frac{4\sqrt{\pi}e^2[(\mu-1)/2M]F_{1p}(t)F_{2p}(t)\,\mathrm{Im}a^h(t)}{|t|^{1/2}d\sigma_0/dt},\qquad(138)$$

where

$$F_{1p}(t)=[G_E(t)-tG_M(t)/4M^2](1-t/4M^2)^{-1};\qquad(139)$$

$$F_{2p}(t)=[G_M(t)-G_E(t)](\mu-1)^{-1}(1-t/4M^2)^{-1}.\qquad(140)$$

It can be seen from (138)–(140) that study of P_0 also outside the interference region enables one to determine $\mathrm{Im}\, a^h(t)$ as a function of t, together with its sign. If we ignore the interference contribution of M^h and M^{em} in the expression for $d\sigma_0/dt$, then in the region $|t| > 0.01\ (\mathrm{GeV}/c)^2$ we obtain from (138) for the polarization

$$(P_0)^{pp}_{\mathrm{int}} = \pm \frac{2\sqrt{\pi} e^2}{M} (\mu - 1) F_{1p} F_{2p} / |t|^{1/2} \sqrt{(1+\rho^2)\, d\sigma_0^h/dt}, \tag{141}$$

where $\rho(t) = \mathrm{Re}\, a^h(t)/\mathrm{Im}\, a^h(t)$. From (141), using the experimental data on $d\sigma_0/dt$, we find[30] that $(P_0)^{pp}_{\mathrm{int}} \approx 1\%$ in a wide range of t for 100–280 GeV.

The behavior of $P_0(t)$ near $-t_0 \approx 1.5\ (\mathrm{GeV}/c)^2$ requires a separate commentary; here $d\sigma_0/dt$ has a minimum and $\mathrm{Im}\, a^h(t)$ evidently vanishes. It follows from (138) that near $-t_0$

$$(P_0(t))^{pp}_{\mathrm{int}} = \pm \frac{\sqrt{\pi} e^2 (\mu - 1)}{M |t|^{1/2}} \frac{F_{1p} F_{2p}}{|R|} \frac{2w}{1+w^2}, \tag{142}$$

where

$$w = A'(t_0)(t - t_0)/|R|; \quad A'(t_0) = \frac{d}{dt} \mathrm{Im}\, a^h(t) \Big|_{t=t_0}; \tag{143}$$

$$|R|^2 = d\sigma_0(t_0)/dt \tag{144}$$

is the value of the cross section at the minimum.

If $\mathrm{Im}\, a^h(t)$ decreases with increasing t (first minimum in the cross section), then $A' < 0$ and $w \gtrless 0$ for $|t| \lessgtr |t_0|$. Then below $-t_0$ for $-t_1 = -t_0 - |R|/|A'|$ the value of $(P_0)^{pp}_{\mathrm{int}}$ attains a maximum, and above $-t_0$ for $-t_2 = -t_0 + |R|/|A'|$ a minimum, and

$$|P_0(t_1)| = \frac{\sqrt{\pi} e^2}{M} (\mu - 1) \frac{F_{1p} F_{2p}}{\sqrt{|t_1|\, d\sigma_0(t_0)/dt}} \approx \frac{|P_{0\,\mathrm{int}}|}{2} \left[\frac{d\sigma_0(t_1)/dt}{d\sigma_0(t_0)/dt}\right]^{1/2}. \tag{145}$$

For the asymmetry in pion or K-meson scattering in a polarized proton target, we have, like (141),

$$(P_0(t))^{\pi^\pm - p}_{\mathrm{int}} = \pm \frac{2\sqrt{\pi} e^2}{M} \frac{(\mu - 1) F(t) F_{2p}(t)}{[|t|[1+\rho^2(t)]\, d\sigma_0/dt]^{1/2}}, \tag{146}$$

where $F(t)$ is the form factor of the spinless meson and the sign of the interference is different for scattering of positively and negatively charged mesons on the protons. Here, it is also necessary to take into account the change in the sign of $\mathrm{Im}\, a^h(t)$ at the minima of the cross sections.

For scattering of neutrons on nuclei in the same approximation at high energies

$$(P_0(t))^{hA}_{\mathrm{int}} = \frac{2\sqrt{\pi} (Ze^2/M) \mu_n F_A(t) F_{2n}(t)\, \mathrm{Im}\, a^h(t)}{|t|^{1/2}\, d\sigma_0/dt}, \tag{147}$$

where $F_A(t)$ is the form factor of the nucleus;

$$F_{2n}(t) = [G_M(t) - G_E(t)]/[\mu_n (1 - t/4M^2)]. \tag{148}$$

For the asymmetry in the cross section for scattering of polarized neutrons by unpolarized protons, Eq. (138) is replaced by

$$(P_0(t))^{n}_{\mathrm{int}} = \frac{2\sqrt{\pi} (e^2/M) \mu_n F_{1p} F_{2n} \mathrm{Im}\, a^h(t)}{|t|^{1/2}\, d\sigma_0/dt}. \tag{149}$$

For the asymmetry in the cross section for scattering of unpolarized neutrons on a polarized-proton target we have

$$(P_0(t))^{p}_{\mathrm{int}} = \frac{2\sqrt{\pi} (e^2/M)(\mu - 1) F_{1n} F_{2p} \mathrm{Im}\, a^h(t)}{|t|^{1/2} d\sigma_0/dt}. \tag{150}$$

Thus,

$$(P_0)^{p}_{\mathrm{int}}/(P_0)^{n}_{\mathrm{int}} = (\mu_p - 1) F_{1n} F_{2p}/\mu_n F_{1p} F_{2n} \tag{151}$$

so that $|P_0^p| \ll |P_0^n|$ for small t', as we have noted above. With increasing t, as follows from (150), the difference in the polarization of the neutrons and protons persists but becomes less.

4. CONCLUSIONS

We have seen that the study of the electromagnetic–hadronic interference in polarization phenomena is a rich source of information otherwise very hard to obtain about the structure and properties of amplitudes determined by the strong interactions. As in its other manifestations, the electromagnetic field is here an effective "test body" by means of which the unknown strong interaction can be probed.

Appreciable deviations from the conclusions obtained for various processes about the interference polarization in future experiments will make it possible to obtain information about the spin dependence of the strong interactions, while agreement between the measured quantities and the predicted values will indicate that M^h has no spin dependence at high energies.

With increasing accuracy of the investigation of polarization phenomena with baryons whose energy lies in the range from a few hundred MeV to 1 GeV, where the spin structure of M^h is particularly rich, and at high energies, where the total polarization effects are small, it is necessary to take into account the electromagnetic effects in addition to the spin-independent Coulomb interaction.

In conclusion, let us mention some of the new possibilities opened up by electromagnetic–hadronic interference in polarization:

a) the possibility arises of obtaining polarized beams of baryons and measuring the polarization of beams of high energy baryons;

b) the study of the polarization rotation of neutrons makes it possible to determine $\mathrm{Re}\, a^h(t)$ together with its sign in the region of small scattering angles;

c) study of the asymmetry in the cross section of NN scattering with a polarized beam (by a polarized target) and an unpolarized target (with an unpolarized beam) and other spin effects at small t makes it possible to determine the total cross section of NN interaction in spin states;

d) a prediction of an isospin-invariance-breaking difference between the polarizations of scattered neutrons and protons in np scattering, the magnitude of the breaking being determined by the ratio $\mathrm{Im}\, b^h/\mathrm{Im}\, a^h$;

e) the presence of interference between M^{em} and M^h can in the general case significantly alter the angular dependence of $P_0(t) = P_0^h + P_0^{int}$ in the region $-t < 0.1$ $(\mathrm{GeV}/c)^2$;

f) because of the Coulomb (charge–charge) interaction, P_0^{int} in $\pi^{\pm}p, K^{\pm}p, pp, \bar{p}p, pA$ elastic collisions is smaller than the polarization of neutrons. Study of P_0^{int} near $-t_p = -\sqrt{3}t_c$, where a maximum equal to about 4.5–5% is expected, makes it possible to investigate the spin dependence of M^h at small t;

g) apart from the region of small t, interference between M^h and M^{em} is also important in the polarization parameters near the zeros of the Bessel functions $J_0(qR)$ and $J_1(qR)$ for baryon–nucleus scattering. The value of P_0^{int} vanishes at the zeros of the Bessel functions and attains a minimum and a maximum near the zeros;

h) study of P_0^{int} at high energies, where P_0 is small, makes it possible to determine $a^h(t)$ in a wide range of t;

i) investigation of P_0 in coherent production of baryon resonances makes it possible to determine more accurately and more reliably the contribution of the Primakoff effect and to deduce the associated information on resonances.

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[1]L. I. Lapidus, Yu. A. Plis, V. P. Sarantsev, and L. M. Soroko, Preprint JINR R1-5209 (in Russian), Dubna (1970).
[2]L. Wolfenstein, Ann. Rev. Nucl. Sci. 6, 43 (1956).
[3]S. M. Bilen'kii, L. I. Lapidus, and R. M. Ryndin, Usp. Fiz. Nauk 84, 243 (1964) [Sov. Phys. Usp. 7, 721 (1965)].
[4]L. D. Puzikov, R. Ryndin, and Ja. A. Smorodinskii, Zh. Éksp. Teor. Fiz. 32, 592 (1957) [Sov. Phys. JETP 5, 489 (1957)].
[5]J. Schwinger, Phys. Rev. 73, 407 (1948).
[6]A. I. Akhiezer and I. Ya. Pomeranchuk, J. Phys. USSR 9, 471 (1945); I. Ya. Pomeranchuk, Sobranie Nauchnykh Trudov (Collection of Scientific Works, Vol. 3), Nauka, Moscow (1972), p. 141.
[7]H. Primakoff, Phys. Rev. 81, 899 (1951).
[8]O. Piccioni, in: Trudy Konferentsii po Fiziki Vysokikh Énergiĭ (Proc. Conf. on High Energy Physics), Kiev (1959).
[9]I. Ya. Pomeranchuk and I. M. Shmushkevich, Nucl. Phys. 23, 452 (1961).
[10]A. Garren, Phys. Rev. 96, 1709 (1954); 101, 419 (1956).
[11]G. Breit, Phys. Rev. 99, 1581 (1955).
[12]H. A. Bethe, Ann. Phys. (N.Y.) 3, 190 (1958).
[13]L. D. Solov'ev, Zh. Éksp. Teor. Fiz. 49, 292 (1965) [Sov. Phys. JETP 22, 205 (1966)].
[14]G. B. West and P. R. Yennie, Phys. Rev. 172, 1413 (1968).
[15]M. P. Locher, Nucl. Phys. B 2, 525 (1967).
[16]L. I. Lapidus, in: Materialy 7-ĭ Shkoly LIYaF (Proc. 7th School at the Leningrad Institute of Nuclear Physics), Vol. 2 (1972); Preprint R1, 2-8529 (in Russian), JINR, Dubna (1975), p. 295.
[17]A. P. Vanzha, L. I. Lapidus, and A. V. Tarasov, Yad. Fiz. 16, 1023 (1972) [Sov. J. Nucl. Phys. 16, 565 (1973)].
[18]L. I. Lapidus, Yad. Fiz. 17, 592 (1973) [Sov. J. Nucl. Phys. 17, 307 (1973)].
[19]B. Z. Kopeliovich and L. I. Lapidus, Yad. Fiz. 19, 218 (1974) [Sov. J. Nucl. Phys. 19, 114 (1974)].
[20]B. Z. Kopeliovich and L. I. Lapidus, Yad. Fiz. 19, 340 (1974) [Sov. J. Nucl. Phys. 19, 168 (1974)].
[21]A. P. Vanzha, L. I. Lapidus, and A. V. Tarasov, Yad. Fiz. 20, 416 (1974) [Sov. J. Nucl. Phys. 20, 223 (1975)].
[22]A. P. Vanzha, L. I. Lapidus, and A. V. Tarasov, Preprint JINR R2-8623 (in Russian), Dubna (1975).
[23]A. Halprin, C. M. Anderson, and C. M. Primakoff, Phys. Rev. 152, 1295 (1966).
[24]L. D. Landau and Ya. A. Smorodinskiĭ, Zh. Éksp. Teor. Fiz. 14, 269 (1944); J. Phys. USSR 8, 154 (1944); L. D. Landau, Sobranie Trudov (Collected Works), Vol. 1, Nauka, Moscow (1969), p. 467.
[25]G. Breit, E. U. Condon, and R. D. Present, Phys. Rev. 50, 825 (1936).
[26]G. V. Gorlov, N. S. Lebedeva, and V. M. Morozov, Yad. Fiz. 8, 1086 (1968) [Sov. J. Nucl. Phys. 8, 636 (1969)]; F. T. Kuchnir *et al*, Phys. Rev. 176, 1405 (1968).
[27]W. S. Hogan and R. G. Seylor, Phys. Rev. C 1, 17 (1970).
[28]D. R. Rust *et al*, Phys. Lett. B 58, 114 (1975).
[29]V. Z. Kopeliovich and L. I. Lapidus, Zh. Éksp. Teor. Fiz. 71, 61 (1976) [Sov. Phys. JETP 44, 31 (1976)]; M. S. Dubovikov and K. A. Ter-Martirosyan, Preprint ITÉF-37 (in Russian), Institute of Theoretical and Experimental Physics, Moscow (1976); M. S. Dubovikov *et al*, Preprint JINR D2-9789, Dubna (1976); Nucl. Phys. B 123, 147 (1977).
[30]C. Bourrely and J. Soffer, Note on the Size of the Coulomb-Nuclear Interference Polarization in Hadronic Reactions at High Energy and Large Momentum Transfer; Preprint CNSR, 77/P.899, Marseilles (1977).

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