Effects of ρ^0–ω mixing and dynamics of vector meson production

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Effects due to electromagnetic mixing of ρ^0 and ω mesons are reviewed. Intensive experimental investigations of these effects have recently become possible. The existing data on ρ^0–ω mixing and ρ^0–ω interference in various reactions are reviewed. The main attention is devoted to effects of ρ^0–ω interference in the reactions $\pi^\pm N \to \omega(N,\Delta)$, $\gamma N \to \pi^+\pi^-\Delta$, and $e^+e^- \to V\pi \to \pi^+\pi^-\pi^0$ and the dynamics of these processes. For the amplitudes of the reaction $\pi^\pm N \to \omega(N,\Delta)$, an appreciable violation of isotopic invariance is predicted and in the reactions $\gamma N \to \pi^+\pi^-\Delta$ and $e^+e^- \to \pi^+\pi^-\pi^0$ the existence is predicted of much more appreciable effects of ρ^0–ω interference in the $\pi^+\pi^-$ mass spectra than in reactions already studied.

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INTRODUCTION

Recently, more and more attention is being devoted to experiments with high statistics at low and medium energies. Undoubtedly, this tendency will persist. It is not merely that accurate data always have great value. The cross sections of many interesting processes, if not the majority, decrease rapidly with increasing energy, so that they can actually be investigated only if the accuracy of the experiment is increased. When qualitative experiments are designed, it is very important to know beforehand what new nontrivial information could be obtained in them. Usually, the aim is to elucidate various fine details of a reaction mechanism known hitherto only in broad features. In such investigations, a variety of interference experiments play an important role. For example, the phase of the forward scattering amplitude of strongly interacting particles is measured by means of the interference with the known Coulomb interaction.

In this review we consider vector-meson production reactions, in which one can observe interference effects due to the quantum-mechanical mixing of state accompanying electromagnetic interaction in the two-level system of the ρ^0 and ω mesons.

The phenomena of ρ^0–ω interference are currently under intensive experimental investigation. The investigation of these phenomena makes it possible to obtain qualitatively new information on the dynamics of various processes in which vector mesons participate. This information is very valuable for testing and making more precise our existing theoretical ideas.

The literature contains some detailed reviews with a description of experiments on ρ^0–ω interference and comparison of the results with various theoretical models.[1-6] Therefore, in Sec. 1 we first consider the actual phenomenon of electromagnetic ρ^0–ω mixing and then give merely the main experimental data on ρ^0–ω interference. After that, we concentrate attention on particularly interesting reactions in which dynamical considerations suggest that there must be enhancement of the effects of ρ^0–ω interference. We shall consider the processes $\pi^\pm N \to \omega(N,\Delta)$, $\gamma N \to \pi^+\pi^-\Delta$ and $e^+e^- \to V\pi \to \pi^+\pi^-\pi^0$. The study of some of them has already begun, and that of others is to be expected soon. Some of the questions touched on here are discussed for the first time.

In Secs. 2 and 3, we consider the reactions $\pi^\pm N \to \omega\Delta$ and $\pi^\pm N \to \omega N$. Until recently, effects of the breaking of isotopic symmetry in interactions of ρ^0 and ω mesons with hadrons due to electromagnetic ρ^0–ω mixing had been observed only in $\pi^+\pi^-$ mass spectra. However, as early as 1970 it was noted that in the reactions $\pi^\pm N \to \omega(N,\Delta)$ one must expect appreciable changes in the amplitudes of ω production as a result of ρ^0–ω mixing.[1,7,8] These changes must be manifested experimentally as anomalous breaking of isotopic invariance. We first describe the qualitative side of the effect and give some initial quantitative estimates. We consider here both reactions $\pi N \to \omega N$ and $\pi N \to \omega\Delta$, which have a complicated dynamics at high energies. However, the approximate numbers that characterize the effect apply primarily to the reactions $\pi^\pm N \to \omega\Delta$. The point is that hitherto the most complete information was on the reactions $\pi^+ p \to \rho^0\Delta^{++}$ and $\pi^+ p \to \omega\Delta^{++}$, and also on ρ^0–ω interference in the $\pi^+\pi^-$ mass spectrum in the reaction $\pi^+ p \to \pi^+\pi^-\Delta^{++}$, obtained by Goldhaber *et al.*[9,10] The direct investigation of this possibility of appreciable breaking of isotopic symmetry in the amplitudes of ω production in $\pi^\pm N \to \omega(N,\Delta)$ due to admixture of the amplitudes of ρ^0 production in $\pi^\pm N \to \rho^0(N,\Delta)$ through an electromagnetic ρ^0—ω transition has become possible only recently. During the last few years, groups from the United States and Canada have prepared an experiment on the Argonne synchrotron with high statistics on the reactions $\pi^- p \to \omega n$ and $\pi^+ n \to \omega p$. In 1976, they published the hitherto best data on the reaction $\pi^- p \to \omega n$ at energy 6 GeV and preliminary data on the effect of ρ^0–ω interference with allowance for the charge-symmetric reaction $\pi^+ n \to \omega p$.[11] This has revealed a somewhat unexpected but very interesting circumstance (which we shall go into in detail in Sec. 3). It has been found that the new data on the reaction $\pi^- p \to \omega n$ cannot be understood

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if only the known Regge trajectories are used to describe them. In the experiment $\rho_{00}d\sigma/dt$ (ρ_{ij} are the elements of the spin density matrix[1)] of the ω meson) does not vanish at very small momentum transfers, $-t \lesssim 0.02$ (GeV/c)^2, contradicting the theoretical expectations (exchange of a Regge B pole). This has been dubbed the Z effect[11] because of the proposal[12-14] to explain it by the introduction of a new Regge trajectory Z with quantum numbers $(\tau, P, G, I) = (+1, -1, +1, 1)$, where τ is the signature. However, it should be emphasized that an explanation by means of simple Regge poles is impossible unless the Z trajectory conspires with its daughter trajectory $Z_d(-1, +1, +1, 1)$ (Refs. 15 and 16), which escaped notice in Refs. 12–14. Thus, we need two new trajectories. However, the existence of such trajectories must be doubted since heavy particles with $I^G(J^P) = 1^+(2^-)$ and $1^+(1^+)$ lying on the Z and Z_d trajectories, respectively, are unknown. We propose an alternative explanation of this phenomenon by means of two-reggeon cuts.[17] The selection of the necessary cuts is considered in detail. Estimates show that the main contribution needed to explain the experimental data is made by the πA_2 Regge cut. We predict a dip as $t \to 0$ in $\rho_{00}d\sigma/dt$ with increasing energy. We then discuss once more the effect of ρ^0–ω mixing in $\rho_{00}d\sigma/dt$ for the reactions $\pi^\pm N \to \omega N$ and predict a more pronounced manifestation of ρ^0–ω interference with increasing energy.

In Sec. 4, we consider the reactions $\gamma N \to (\rho^0, \omega)\Delta \to \pi^+\pi^-\Delta$ and $e^+e^- \to (\rho, \omega)\pi \to \pi^+\pi^-\pi^0$. In these reactions, considerations based on SU(3) symmetry lead one to expect anomalous enhancement of ρ^0–ω interference in the $\pi^+\pi^-$ mass spectrum as compared with the reactions already investigated experimentally.[18-21] The study of this effect in the reaction $\gamma N \to \pi^+\pi^-\Delta$ may serve as a good test of the assumption of octet dominance of Regge exchanges in the t channel. Its investigation in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ makes it possible to elucidate the SU(3) structure of the hadron electromagnetic current in the timelike region, $q^2 \gtrsim 1$ GeV^2. It would be very interesting to investigate the effect of ρ^0–ω interference in the region of the presumed new vector resonances ρ' and ω', $e^+e^- \to (\rho', \omega') \to \pi^+\pi^-\pi^0$, where its influence could be particularly large. About half the $e^+e^- \to 3\pi$ reaction events in this region could be due to ρ^0–ω interference.[20,21]

1. ELECTROMAGNETIC ρ^0-ω MIXING AND RELATED INTERFERENCE EFFECTS

As early as 1961, Glashow[22] pointed out the possibility of appreciable mixing of the ρ^0 and ω resonances as a result of electromagnetic interaction, and this has recently been observed experimentally. A considerable number of theoretical investigations have already been made of this subject (see, for example, Refs. 23–28).

In the presence of the electromagnetic interaction, the physical states $|\rho^0\rangle$ and $|\omega\rangle$ must be regarded as superpositions of eigenstates $|\rho_0^0\rangle$ and $|\omega_0\rangle$ of isospin I and G parity:

$$|\rho^0\rangle = |\rho_0^0\rangle - \varepsilon\,|\omega_0\rangle; \quad |\omega\rangle = |\omega_0\rangle + \varepsilon\,|\rho_0^0\rangle, \tag{1}$$

with the complex mixing parameter

$$\varepsilon = \delta/(m_\rho - i\Gamma_\rho/2 - m_\omega + i\Gamma_\omega/2) \approx i2\delta/\Gamma_\rho; \tag{2}$$

$|\varepsilon| \ll 1$; $\delta = \langle\rho_0^0|M|\omega_0\rangle$ is the amplitude ("mass") of the electromagnetic $\rho^0 \leftrightarrow \omega$ transition; $\varepsilon \approx i2\delta/\Gamma_\rho$ since $|m_\rho - m_\omega| \approx \Gamma_\omega \ll \Gamma_\rho$.

It follows from theoretical considerations[24-27] that δ must be an almost purely real quantity, equal in order of magnitude to the electromagnetic mass difference $2\delta \approx \alpha m_\rho \approx 5.6$ MeV of the vector mesons. In fact, in all variants of broken SU(3) or SU(6) symmetry considered in the literature rather similar, albeit approximate, values have been obtained[24-26,29,30] for δ:

$$|\delta| \approx 2.5 - 5\,\text{MeV}. \tag{3}$$

To the transition $\rho^0 \leftrightarrow \omega$ there correspond electromagnetic self-energy type diagrams (Fig. 1), which in the region of the ρ^0 and ω mesons have, of course, a nonzero imaginary part. The estimates given above refer only to the real part of such diagrams, which corresponds to the contributions to δ of virtual intermediate states. Note that the contribution of the single-photon intermediate state is small. An upper limit for the imaginary part of the matrix element $\langle\rho_0^0|M|\omega_0\rangle$ can be estimated by considering the transitions of ρ^0 and ω into real 2π, 3π, $\pi\gamma$, $\eta\gamma$, etc, intermediate states.[6,25,27] The upper bound is small:

$$|\operatorname{Im}\delta| \approx \alpha\,(\Gamma_\rho + \Gamma_\omega)/2 + (\Gamma_{\rho\pi\gamma}\Gamma_{\omega\pi\gamma})^{1/2}/2 + \ldots \leqslant 0.6 - 0.8\ \text{MeV}.$$

The main contribution is made by the 2π and $\pi\gamma$ states. Thus, the theoretical estimates indicate that δ has a small imaginary part. Therefore, the phase of the mixing parameter ϵ for the states of the ρ^0 and ω mesons is $\varphi_\varepsilon \approx \pi/2$ (or $-\pi/2$) [see (2)]. We note right away that this circumstance, and also the estimate (3) for $|\delta|$, are confirmed in virtually all experiments on ρ^0–ω interference in the $\pi^+\pi^-$ mass spectra.

Let us estimate the width of the decay $\omega \to \pi^+\pi^-$, which takes place in accordance with the diagrams in Fig. 2. The simplest estimate of the order of magnitude for the width of the direct decay $\omega_0 \to \pi^+\pi^-$ (see Fig. 2a)

$$\Gamma_{\omega\pi^+\pi^-} \approx \alpha^2\Gamma_\rho \approx 8\ \text{keV}$$

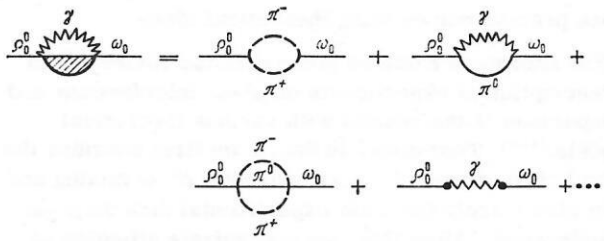

FIG. 1. Diagrams of the electromagnetic $\rho^0 \leftrightarrow \omega$ transition.

[1)] $^*\rho_{ij} = \sum_n A_{ni}A_{nj}^*/\sum_{n,m}|A_{nm}|^2$, where A_{ni} are the reaction amplitudes; i is the helicity of the vector meson; n is the set of helicities of the other particles participating in the reaction.

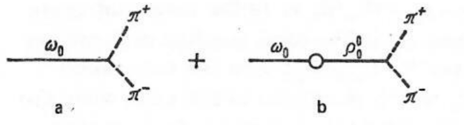

FIG. 2. Diagrams of the direct decay $\omega \to \pi^+\pi^-$ (a) and the decay $\omega \to \pi^+\pi^-$ due to $\rho^0-\omega$ mixing (b).

is appreciably smaller than the experimental value[1-6]

$$\Gamma^{\rm exp}_{\omega\pi^+\pi^-} \approx 0.1 - 0.5 \text{ MeV}. \tag{4}$$

Therefore, the contribution of the diagram in Fig. 2a is usually ignored. Allowance for $\rho^0-\omega$ mixing gives a natural explanation of the value of $\Gamma_{\omega\pi^+\pi^-}$. In accordance with (1) and Fig. 2b

$$\Gamma_{\omega\pi^+\pi^-} \approx |\varepsilon|^2 \Gamma_\rho \approx 4|\delta|^2 \Gamma_\rho/\Gamma_\rho^2$$
$$\approx \alpha^2\Gamma_\rho (m_\rho/\Gamma_\rho)^2 \approx 30\alpha^2\Gamma_\rho \approx 0.25 \text{ MeV}. \tag{5}$$

The principal reason for the significant enhancement of the decay $\omega \to \pi^+\pi^-$ through $\rho^0-\omega$ mixing is obviously [see (2) and Fig. 2b] the nearly equal masses of the ρ^0 and ω resonances, which has the consequence that the mixing parameter is

$$|\varepsilon| \approx 2|\delta|/\Gamma_\rho \approx \alpha m_\rho/\Gamma_\rho > \alpha.$$

Processes with production of $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ in the region of the ρ^0 and ω resonances with electromagnetic transitions $\omega \to 2\pi$ and $\rho^0 \to 3\pi$ due to $\rho^0-\omega$ mixing are described by the diagrams of Fig. 3. The $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ production amplitudes with allowance for $\rho^0-\omega$ mixing in the region of the resonances can be written in the form

$$T(\pi^+\pi^-) = \frac{g_{\rho\pi\pi} A_\rho}{m_\rho - m - i\Gamma_\rho/2}\left(1 + \frac{A_\omega}{A_\rho}\frac{\delta}{m_\omega - m - i\Gamma_\omega/2}\right); \tag{6}$$

$$T(\pi^+\pi^-\pi^0) = \frac{g_{\omega 3\pi} A_\omega}{m_\omega - m - i\Gamma_\omega/2}\left(1 + \frac{A_\rho}{A_\omega}\frac{\delta}{m_\rho - m - i\Gamma_\rho/2}\right), \tag{7}$$

or

$$T(\pi^+\pi^-) = \frac{g_{\rho\pi\pi}(A_\rho - \varepsilon A_\omega)}{m_\rho - m - i\Gamma_\rho/2} + \frac{\varepsilon g_{\rho\pi\pi} A_\omega}{m_\omega - m - i\Gamma_\omega/2}, \tag{8}$$

$$T(\pi^+\pi^-\pi^0) = \frac{g_{\omega 3\pi}(A_\omega + \varepsilon A_\rho)}{m_\omega - m - i\Gamma_\omega/2} - \frac{\varepsilon g_{\omega 3\pi} A_\rho}{m_\rho - m - i\Gamma_\rho/2}. \tag{9}$$

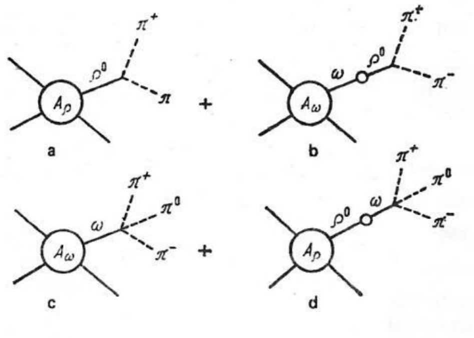

FIG. 3. Processes for the production of $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ in the region of the ρ^0 and ω resonances with allowance for $\rho^0-\omega$ mixing.

Here, A_v are the amplitudes of ρ^0 and ω production ($v = \rho^0, \omega$); m is the invariant mass of the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ systems in Eqs. (6), (8) and (7), (9), respectively. We ignore[24,25] the direct transitions $\omega_0 \to 2\pi$ and $\rho_0^0 \to 3\pi$. It can be seen from (8) and (9) that the electromagnetic $\rho^0-\omega$ mixing leads to effects of two kinds: First, it changes the mass spectrum of the decay pions; second, it changes the vector-meson production amplitudes [see the terms proportional to ε in the brackets in Eqs. (8) and (9)]. For reactions of the type (8), the change in the mass spectrum due to the narrowness ($\Gamma_\rho/\Gamma_\omega \approx 15$) of the ω resonance is the most important. For reactions of the type (9), in contrast, the change in the mass spectrum is unimportant since it is difficult to detect the low and broad ρ^0 resonance on the background of the narrow and high ω peak. For these reactions, there can be an appreciable change in the amplitude of ω production in the cases when the amplitude of ρ^0 production is appreciably greater than the ω-production amplitude. Such a situation obtains for the reactions $\pi^\pm N \to \omega(N, \Delta)$ (see Sec. 2).

In recent years, the effects of $\rho^0-\omega$ interference in the $\pi^+\pi^-$ mass spectra have been investigated in about 30 different experiments on the reactions

$$\begin{array}{ll} e^+e^- \to \pi^+\pi^-; & \pi^\pm N \to \pi^+\pi^- N; \\ \gamma A \to \pi^+\pi^- A; & \pi^\pm N \to \pi^+\pi^-\Delta; \\ K^-p \to \pi^+\pi^-(\Lambda, \Sigma^0); & N\bar{N} \to 3\pi, 4\pi, 5\pi, 6\pi \end{array} \tag{10}$$

(see, for example, the reviews Refs. 1–6, 31); new data continue to appear[32-39] with ever higher statistics.

The most attractive aspect of these investigations, apart from the determination of the $\rho^0-\omega$ mixing parameters, is the possibility of determining the relative phases of the ρ^0 and ω production amplitudes, on which the interference effect depends strongly. These new data are extremely helpful for testing and making more precise our theoretical ideas about the interaction mechanisms of vector mesons.

For determining the magnitude and phase of the $\omega \to \pi^+\pi^-$ decay amplitude, the most convenient reaction[31,32] is $e^+e^- \to \pi^+\pi^-$ because of the purely electromagnetic mechanism of the ρ^0 and ω meson production: $e^+e^- \to \gamma \to (\rho^0, \omega) \to \pi^+\pi^-$.

Data on $d\sigma/dm_{\pi\pi}$ for the reactions $\pi^-p \to (\rho^0, \omega)n \to \pi^+\pi^-$ and $\pi^+n \to (\rho^0, \omega) \to \pi^+\pi^- p$ at 4 GeV obtained by Argonne[36,37] illustrate the influence of $\rho^0-\omega$ interference on the $\pi^+\pi^-$ mass spectra in the region of the ρ^0 resonance (Fig. 4); namely, the breaking of charge symmetry in reactions with the participation of strongly interacting particles (see Sec. 2 for more detail on this). It should be said that for the majority of reactions the broad features of the $\rho^0-\omega$ interference in the $\pi^+\pi^-$ mass spectra agree qualitatively with the theoretical predictions on $\rho^0-\omega$ mixing and on the mechanisms of the ρ^0 and ω production reactions themselves (see, for example, Ref. 39, in which an experiment on the reaction $K^-p \to \pi^+\pi^-(\Lambda, \Sigma^0)$ at 13 GeV is described, and also Refs. 1, 2, 24, 40, 41). Of course, many data [for example, on the reaction $e^+e^- \to (\rho^0, \omega) \to \pi^+\pi^-$ (Ref. 32)] must be made more precise. In Table I we give the main results on ρ^0–ω interference obtained in the experiments of Refs. 1, 9–11, 31–39, and 42–58 on the reactions (10).

As a commentary on Table I, let us say a few words about the evaluation of the experimental data (details can be found in the review Ref. 1 and in Refs. 9, 33, 42–44).

For the $\pi^+\pi^-$ mass spectrum in the region of the ρ^0 and ω resonances one usually uses the simple phenomenological expression

$$\frac{dN}{dm}=\frac{1}{2\pi}\left\{\frac{N_\rho\Gamma_\rho}{|D_\rho(m)|^2}+\frac{BN_\omega\Gamma_\omega}{|D_\omega(m)|^2}+2\xi\,\mathrm{Re}\,\frac{\exp(i\varphi)(BN_\rho\Gamma_\rho N_\omega\Gamma_\omega)^{1/2}}{D_\rho(m)D_\omega^*(m)}\right\} \qquad (11)$$

where $D_v(m)=m-m_v+i\Gamma_v/2$; m is the invariant mass of the $\pi^+\pi^-$ system; N_v is the total number of v-meson production events; $B=\Gamma_{\omega 2\pi}/\Gamma_\omega$; ξ is the coherence factor $(0\leqslant\xi\leqslant 1)$, which is needed in the case when the ρ^0 and ω production processes must be described by several independent amplitudes; φ is the total relative phase of the ρ^0 and ω contributions. The coherence factor ξ and the phase φ have the form

$$\xi=\Big|\sum_i A_i^\rho A_i^\omega\Big|\Big/\Big[\Big(\sum_i|A_i^\rho|^2\Big)\Big(\sum_i|A_i^\omega|^2\Big)\Big]^{1/2}; \qquad (12)$$

$$\varphi=\varphi_{\mathrm{decay}}+\varphi_{\mathrm{prod}}=\varphi_{\mathrm{decay}}+\tan^{-1}\Big(\sum_i|A_i^\rho||A_i^\omega|\sin\varphi_i\Big)\Big/\Big(\sum_i|A_i^\rho||A_i^\omega|\cos\varphi_i\Big), \qquad (13)$$

TABLE I. Experimental data on ρ^0–ω interference.

Reaction	E_{lab}, GeV	$B=\Gamma_{\omega\pi\pi}/\Gamma_\omega$, %	φ, deg	φ_{prod}, deg	$\varphi_{\mathrm{prod}}^{\mathrm{theor}}$, deg	Assumption about ξ	Literature
$e^+e^-\to\pi^+\pi^-$	$s\approx m_\rho^2$	3.6 ± 1.9	85.7 ± 15.3	-16.3 ± 15.3	0	1	[31, 32]
$\gamma A\to\pi^+\pi^- A$ (A=H, C, Pb)	5–7	$>1.22\pm0.30$	96 ± 15	-5 ± 15	0	1	[47]
$\gamma C\to\pi^+\pi^- C$ (see Fig. 11)	4.2	$>0.8^{+0.28}_{-0.22}$	104 ± 5.1	3 ± 5.1	0	1	[46]
$\gamma A\to\pi^+\pi^- A$ (A=C, Pb, Al)	9.4	$>2.8\pm0.6$	92.4 ± 5.0	-8.6 ± 5.0	0	1	[48]
$\gamma p\to\pi^+\pi^- p$	2.8; 4.7	$>1.3^{+1.2}_{-0.9}$	≈ 92	-9	0	1	[49]
$\pi^+p\to\pi^+\pi^-\Delta^{++}$	3.7–4.0 5,45 3.4–4.0 7.1	>1.5 >1.2 >1.5 —	192 ± 17 182 ± 31 ≈ 180 —	91 ± 17 81 ± 31 79 —	90	1	[9, 10] [50] [51] [52]
$\pi^+n\to\pi^+\pi^- p$	1.5	4 ± 2	158^{+46}_{-28}	57^{+46}_{-28}	90	1	[57]
$\pi^-p\to\pi^+\pi^-(\pi^-p)$	3.9	>1.1	≈ 0	-101	-90	1	[53]
$\pi^-p\to\pi^+\pi^-n$	2,3 3–5 1,67	$>0.36\pm0.1$ — —	-15 ± 30 ≈ 0 -4 ± 20	-116 ± 30 -101 -105 ± 20	-90	1	[54] [55] [56]
$pp\to 2\pi^+2\pi^-$	1.26–1.65 1.63–2.20 0.65–1.10	>1.4 >1.9 >1.3	60–90 ≈ 100 ≈ 100	$-(41-11)$ -1 -1	0	1	[43] [44] [34]
$\bar{p}n\to(3\pi)$, (4π), (5π), (6π)	Annihilation at rest	<4.3	Incoherent production of ω and ρ mesons	—	—	1	[45]
$K^-p\to\pi^+\pi^-(\Lambda,\ \Sigma)$	1.5–2.6 13	>0.2 3.6 ± 1.9	— 99 ± 11	— -2 ± 11	0	1	[42] [39]
$\pi^-p\to\pi^+\pi^-n$ $\rho_{11}(m)d\sigma/dm$	15	$>2.1^{+2.8}_{-0.9}$	-80 ± 25	-181 ± 25	—	$0.6<\xi<$ <1	[33]
$\pi\pm N\to\pi^+\pi^-N$ * (see Fig. 4) $\pi^-p\to\pi^+\pi^-n$ $\rho_{ij}(m,\ t)\,d^2\sigma/dm\,dt$, $0<\|t\|<0{,}4$ (GeV/c)^2	3. 4. 6 17	>1 —	$\varphi(t)$ $\varphi(t)$	— —	— —	1 —	[36, 37] [38]
$\pi\pm N\to\omega N$ **	6 4	$R=\rho_{00}d\sigma/dt\,(\pi^-p\to\omega n)/\rho_{00}d\sigma/dt\,(\pi^+n\to\omega p)$ $R\approx 0.8$ (preliminary) $0<\|t\|<0.1$ (GeV/c)^2 $R=0.63\pm0.20$, $0.05<\|t\|<0.15$ (GeV/c)^2					[11] [58]

*In Refs. 36, 37, 41 and 38, 40 data are given with high statistics on ρ^0–ω interference in the different amplitudes of $\pi^+\pi^-$ production. The ratios $|A_i^\rho/A_i^\omega|$ and the phases φ_i are determined as functions of t and a comparison with theoretical models is made.

**In these reactions, a study is made of the influence of ρ^0–ω mixing on the ω production amplitudes (see Secs. 2 and 3).

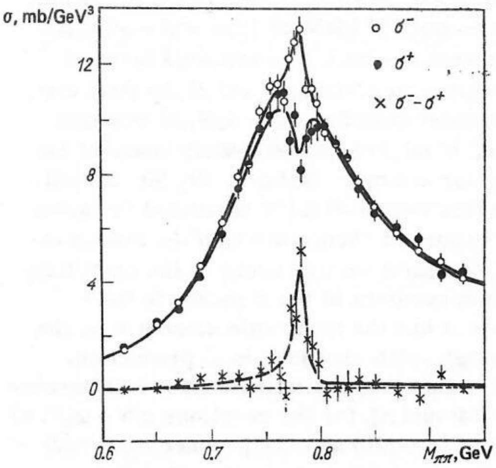

FIG. 4. $\sigma^{\pm}=d^2\sigma/dt\,dm\,(\pi^{\pm}N\to\pi^+\pi^-N)$ for $E_{lab}=4$ GeV and $0.08 \leqslant |t| \leqslant 0.2$ $(\text{GeV}/c)^2$. The curves are obtained with allowance for the ρ^0 and ω contributions to the $\pi^+\pi^-$ mass spectra.[37]

where A_i^v is the amplitude for production of a v meson with definite configuration of the spins of the particles participating in the reaction; φ_i is the relative phase between the amplitudes A_i^ρ and A_i^ω;

$$N_v \sim \sum_i |A_i^v|^2. \tag{14}$$

Since the $\omega\to\pi^+\pi^-$ decay amplitude has in accordance with (1) the form

$$g_{\omega\pi\pi}=g_{\omega_0\pi\pi}+\varepsilon g_{\rho_0\pi\pi}, \tag{15}$$

we can use the following expression for its phase φ_{decay} in various estimates [see (2)]:

$$\varphi_{decay}\approx \tan^{-1}\Gamma_\rho/2(m_\rho-m_\omega)\approx 101^\circ, \tag{16}$$

corresponding to the fact that the decay $\omega\to\pi^+\pi^-$ proceeds predominantly through ρ^0–ω mixing ($g_{\omega 0\pi\pi} \approx \alpha g_{\rho 0\pi\pi} \ll \epsilon g_{\rho 0\pi\pi}$), and the amplitude δ of the $\rho^0 \rightleftharpoons \omega$ transition is real.[24-27]

In Table I, we give the values of the relative phase of ρ^0 and ω meson production φ_{prod} obtained by means of (13) and (16). For comparison, we give the values of φ_{prod}^{teor} which follow from simple theoretical models of the ρ^0 and ω production mechanisms (see, for example, Refs. 1, 24, 40, 41).

The contribution of the background is usually taken into account in the form of an additive correction to the expression (11), this being a smooth function of m in the region of the resonances. The coherence factor and the phase φ can be determined from the fitting simultaneously in only a number of definite cases[1,33,43] if only N_ρ is known. In order to find $B=\Gamma_{\omega\pi\pi}/\Gamma_\omega$ as well, it is necessary to know N_ρ and N_ω, i.e., have data on two reactions, for example, $\pi^+p\to\rho^0\Delta^{++}$ and $\pi^+p\to\omega\Delta^{++}$. In addition, since the term in (11) quadratic in δ plays an unimportant role in all the above reactions (it has no additional enhancement in the reactions (10) compared with the interference term because the ω production amplitudes are less than the ρ^0 production amplitudes by about a factor 3 in the reactions $e^+e^-\to\omega\to\pi^+\pi^-\pi^0$, $e^+e^-\to\pi^+\pi^-$ and $\gamma A\to\omega A$, $\gamma A\to\rho^0 A$ and by more than a factor 3 in $\pi^{\pm}N\to\omega(N,\Delta)$, $\pi^{\pm}N\to\rho^0(N,\Delta)$ or, in the best case, are the same, as, for example, in the reactions $K^-p\to\omega(\Lambda,\Sigma^0)$, $K^-p\to\rho^0(\Lambda,\Sigma^0)$,[31,1,2,39,41] it is impossible to determined simultaneously ξ and B, which occur as a product in the fitting of data. Frequently, ξ is simply set equal to 1 and a lower limit is obtained for B. An upper limit can be obtained for B in reactions with incoherent production of ρ^0 and ω mesons ($\xi=0$), for example, in annihilations at rest $\bar{p}n\to(\rho\pi)$, $(\omega\pi)\to 3\pi$.[45] It should be noted that the phase φ is the quantity most reliably determined by the fitting.

We give one further expression for the $\pi^+\pi^-$ mass spectrum, which is also frequently used in fittings.[1,9] It is obtained directly from (6):

$$\frac{dN}{dm}=\frac{\Gamma_\rho N_\rho}{2\pi\,|D_\rho(m)|^2}\left(1+2\xi\cos\varphi\,\frac{N_\omega^{1/2}}{N_\rho^{1/2}}\left|\frac{\delta}{D_\omega(m)}\right|+\frac{N_\omega}{N_\rho}\left|\frac{\delta}{D_\omega(m)}\right|^2\right); \tag{17}$$

$$\varphi=\varphi_\delta+\tan^{-1}[\Gamma_\omega/2(m_\omega-m)]+\varphi_{prod}, \tag{18}$$

where φ_δ is the phase of δ; φ_{prod} and the coherence factor ξ are determined by Eqs. (12) and (13).

If we know the ρ^0–ω mixing parameters, for example, from the reaction $e^+e^-\to\rho^0$, $\omega\to\pi^+\pi^-$ (Ref. 32) or if we use the theoretical estimates[24-26] for them, we can, using the interference effects, estimate the relative phase of the ρ^0 and ω production amplitudes and their relative values $|A_\omega/A_\rho|=(N_\omega/N_\rho)^{1/2}$. In virtually all the current experiments involving reactions with ρ^0 and ω production, the experimentalists attempt to include effects of ρ^0–ω interference in order to obtain new information.

We now turn to concrete reactions of vector meson production.

2. ON THE POSSIBILITY OF "STRONG" VIOLATION OF ISOTOPIC INVARIANCE IN THE REACTIONS $\pi^{\pm}N\to\omega\Delta$ AND $\pi^{\pm}N\to\omega N$ DUE TO ELECTROMAGNETIC ρ^0-MIXING

Electromagnetic ρ^0–ω mixing must be manifested as a breaking of isotopic symmetry in the strong interactions of ρ^0 and ω mesons. Below, we consider its influence on the ω production amplitudes in the reactions $\pi^{\pm}N\to\omega(N,\Delta)$.

The influence of ρ^0–ω mixing in these reactions was studied in detail in Refs. 7 and 8. Here, we describe the qualitative side of the effect, giving briefly initial quantitative estimates and discussing the predictions of the simplest Regge pole models concerning the nature of ρ^0–interference

Qualitative Analysis of ρ^0–ω Interference. We rewrite (9), taking into account only the change in the amplitude of ω production:

$$T(\pi^+\pi^-\pi^0) = g_{\omega 3\pi}(A_\omega + \varepsilon A_\rho)/(m_\omega - m_{3\pi} - i\Gamma_\omega/2). \tag{19}$$

For simplicity, we do not write out the spin indices.

It can be seen from (19) that the effect of $\rho^0-\omega$ mixing depends on the relative magnitude and relative phase of the ρ^0 and ω production amplitudes in the strong interactions and is maximal when the ρ^0 and ω contributions to (19) are coherent, i.e., their relative phase is $\varphi_{3\pi} = \varphi_\rho + \varphi_\varepsilon - \varphi_\omega \approx 0$ or π. In addition, the effect may be enhanced dynamically if the amplitude of ρ^0 production is greater than the amplitude of ω production: $|A_\rho| > |A_\omega|$. What is the real situation with regard to the reactions $\pi N \to \omega(N, \Delta)$ and $\pi N \to \rho^0(N, \Delta)$?

Information about the phases can be obtained from experiments to determine the interference patterns in the $\pi^+\pi^-$ mass spectra. In these spectra, in the region of the ω resonance in the reactions $\pi^+N \to \pi^+\pi^-(N, \Delta)$ at energies 2–10 GeV and $|t| \lesssim 0.2$ (GeV/c)^2 a dip is observed[1, 9, 10, 36, 37, 50] (destructive $\rho^0-\omega$ interference; see, for example, Fig. 4), i.e., the relative phase of the ρ^0 and ω contributions to (8) is $\varphi_{2\pi} \approx \varphi_\omega + \varphi_\varepsilon - \varphi_\rho \approx \pi$. In the reactions $\pi^-N \to \pi^+\pi^-(N, \Delta)$ at $m_{\pi\pi} \approx m_\omega$ a peak is observed[1, 36, 37, 53, 54] (constructive $\rho^0-\omega$ interference, so that $\varphi_{2\pi} \approx 0$; see, for example, Fig. 4) since the amplitude of ρ^0 production has changed sign. This is a consequence of isotopic invariance for the reactions $\pi^\pm N \to \rho^0 N$. For the reactions $\pi^\pm N \to \rho^0\Delta$, this also follows from isotopic invariance if the contribution of the t-channel exotic exchanges with isospin 2 is negligibly small, which is independently confirmed by other experiments. Therefore, $\varphi_{3\pi} \approx 2\varphi_\varepsilon - \pi$ for the reactions $\pi^+N \to \omega(N, \Delta)$, and $\varphi_{3\pi} \approx 2\varphi_\varepsilon$ for the reactions $\pi^-N \to \omega(N, \Delta)$. But, as we have already said, it follows from the experimental data and theoretical considerations that the phase is $\varphi_\varepsilon \approx \pi/2$ (or $-\pi/2$).[1, 2, 24-27, 31]

We conclude from this that the cross sections of the reactions $\pi^+N \to \omega(N, \Delta)$ are increased maximally by the term linear in $|\varepsilon|$:

$$d\sigma^\omega/dt = |A_\omega|^2 + 2|\varepsilon||A_\omega||A_\rho| + |\varepsilon|^2|A_\rho|^2, \tag{20}$$

while the $\pi^-N \to \omega(N, \Delta)$ cross sections are maximally reduced:

$$d\sigma^\omega/dt = |A_\omega|^2 - 2|\varepsilon||A_\omega||A_\rho| + |\varepsilon|^2|A_\rho|^2. \tag{21}$$

Note that deviation of φ_ε from $\pi/2$ (or $-\pi/2$) by $\Delta\varphi$ leads to a decrease of the term linear in $|\epsilon|$ by a factor $\cos 2\Delta\varphi$. Therefore, not too large values of $\Delta\varphi$ do not significantly affect the results. For example, if $\Delta\varphi = 0.25$ (15°), the interference term in (20) and (21) is multiplied by 0.88. Currently, the value of $|\varepsilon|$ is insufficiently well determined by the experiments. On the basis of all the various experiments and the methods of determination, and also on the basis of theoretical estimates, we may assume that $0.03 \lesssim |\varepsilon| \lesssim 0.06$.[1-6, 25] Below, to estimate the effect we shall use $|\varepsilon| = 0.05$. Such a value of $|\varepsilon|$ was obtained in the "cleanest" experiment on $\rho^0-\omega$ interference in the reaction $e^+e^- \to \pi^+\pi^-$ made at Orsay[31, 32] ($\varphi_\varepsilon^{\text{Orsay}} \approx 90°$).

We now consider what are the relative values of the amplitudes A_ρ and A_ω. It is well known experimentally that at medium energies and $|t| \lesssim 0.2$ [here and below, t is measured in units of (GeV/c)^2] the $\pi^\pm N \to \rho^0(N, \Delta)$ reaction cross sections have a pronounced forward peak due to single pion exchange, and at the peak are 10–20 times greater than the $\pi^\pm N \to \omega(N, \Delta)$ reaction cross sections, which are approximately constant for $|t| \lesssim 0.2$ (see, for example, Refs. 9, 10, 50, 59–62). Therefore, in this region of $|t|$ it is natural to expect that the most important change due to $\rho^0-\omega$ mixing in the $\pi^\pm N \to \omega(N, \Delta)$ reactions will occur in the amplitude with the quantum numbers of the B meson in the t channel because it has the same spin structure as the amplitude of single-pion exchange in ρ^0 production. Experimentally, this must be manifested in a difference between $\rho_{00}^\omega d\sigma^\omega/dt$ and ρ_{00}^ω for the reactions $\pi^+N \to \omega(N, \Delta)$ and $\pi^-N \to \omega(N, \Delta)$ [see (20) and (21)]. Since $\rho_{11}^\omega d\sigma^\omega/dt \approx \rho_{11}^\rho d\sigma^\rho/dt$ in the considered range of t, the other ω production amplitudes should not be appreciably changed by $\rho^0-\omega$ mixing.

Initial Quantitative Estimates of the Effect. We make some approximate estimates that characterize the violation of isotopic invariance in these reactions.

Let us consider, for example, the reactions $\pi^+p \to (\omega, \rho^0)\Delta^{++}$ at 3.7 GeV.[9, 10] To a good approximation, $\rho_{00}^\omega d\sigma^\omega/dt = \text{const}$, and $(\rho_{00}^\rho d\sigma^\rho/dt)/(\rho_{00}^\omega d\sigma^\omega/dt) \approx 10 \exp(12t)$ for $|t| \lesssim 0.14$.[10] Then using (20) and (21) for $|\varepsilon| = 0.05$ and $|t| \approx 0$, we obtain

$$\left.\frac{\rho_{00}^\omega\, d\sigma^\omega/dt\,(\pi^+p \to \omega\Delta^{++})}{\rho_{00}^\omega\, d\sigma^\omega/dt\,(\pi^-n \to \omega\Delta^-)}\right|_{t=0} \approx 2.2. \tag{22}$$

The integral effect is also large. The ratio of the cross sections in the interval $0 \leqslant |t| \leqslant 0.14$ for these two reactions is

$$R = \frac{\sigma_{00}^\omega(\pi^+p \to \omega\Delta^{++})}{\sigma_{00}^\omega(\pi^-n \to \omega\Delta^-)} \approx 1.7 \quad \left(\sigma_{00}^\omega = \int_0^{0.14} \rho_{00}^\omega \frac{d\sigma^\omega}{d|t|}\, d|t|\right). \tag{23}$$

In the absence of $\rho^0-\omega$ mixing, the ratios (22) and (23) must be equal to unity because of isotopic invariance.

Similar predictions can be obtained for the reactions $\pi^+n \to \omega p$ and $\pi^-p \to \omega n$ when $0.02 \leqslant |t| \leqslant 0.02$.[7, 8, 41, 63-66]

Note that the change in the cross section of ω-meson production in, for example, the reaction $\pi^+p \to \omega\Delta^{++}$ due to $\rho^0-\omega$ mixing can be estimated directly on the basis of the number of events taken out of the two-pion mass spectrum in the reaction $\pi^+p \to \pi^+\pi^-\Delta^{++}$ as a result of the destructive $\rho^0-\omega$ interference, because in this case, with $\varphi_\varepsilon \approx \pm\pi/2$, a similar number of events is added to the reaction $\pi^+p \to \omega\Delta^{++}$. An analysis of this kind was made in Ref. 8 using the data of Goldhaber *et al.*[9, 10] It was assumed that the destructive $\rho^0-\omega$ interference observed in the reaction $\pi^+p \to \pi^+\pi^-\Delta^{++}$ takes place primarily between the amplitudes of ρ^0 and ω production with longitudinal polarization, which have π- and B-meson quantum numbers in the t channel. The predictions obtained for $\rho_{00}^\omega d\sigma^\omega/dt(\pi^-n \to \omega\Delta^-)$ are close to the estimates (22) and (23); they are given in Fig. 5.

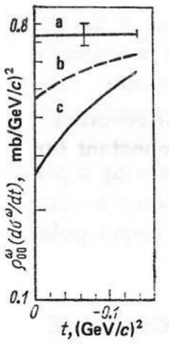

FIG. 5. Influence of $\rho^0-\omega$ mixing on $\rho_{00}\, d\sigma/dt(\pi^{\pm}N \to \omega\Delta)$ (Ref. 8): a) for $\rho_{00}d\sigma/dt(\pi^+p \to \omega\Delta^{++})$ in the interval $0 \leq |t| \leq 0.14$ the mean value obtained in accordance with the data of Ref. 10 was used; b) prediction for the reaction $\pi^- n \to \omega\Delta^-$ calculated in accordance with Eqs. (20) and (21) using the information[9,10] on $\rho^0-\omega$ interference in the reaction $\pi^+p \to \pi^+\pi^-\Delta^{++}$. The amplitude of ρ^0 production is taken from Ref. 10 in accordance with $\rho^0_{00}d\sigma^{\rho}/dt$; c) $\rho^{\omega}_{00}d\sigma^{\omega}/dt$ in the absence of $\rho^0-\omega$ mixing $R = \sigma_{00}(\pi^+p \to \omega\Delta^{++})/\sigma_{00}(\pi^- n \to \omega\Delta^-) \approx 2$; $|\varepsilon| \approx 0.06 (\varphi_\varepsilon \approx \pi/2)$.

In 1973, an experiment was made[36,37] on $\rho^0-\omega$ interference in the reactions $\pi^{\pm}N \to \pi^+\pi^-N$, and this experiment measured directly the mass spectrum $\rho^0_{00}(m_{\pi\pi})d\sigma^{\rho}/dm_{\pi\pi}$. An estimate of the change in the cross section of ω production with longitudinal polarization ($\rho^{\omega}_{00}d\sigma^{\omega}/dt$) from the number of events lost from $\rho^0_{00}(m_{\pi\pi})d\sigma^{\rho}/dt\,(\pi^+n \to \pi^+\pi^-p)$ by destructive $\rho^0-\omega$ interference agrees with the picture described above.

Until recently, a direct verification of breaking of isotopic symmetry in the reactions $\pi^{\pm}N \to \omega(N, \Delta)$ was impossible since there were no good data on the reactions $\pi^- N \to \omega(N, \Delta)$ at small $|t|$.

In 1973, it was announced that experiments were planned at Argonne with high statistics for the reactions $\pi^- p \to \omega n$ and $\pi^+ n \to \omega p$ at 6 GeV with a view to making a detailed study of the influence of $\rho^0-\omega$ mixing on the ω production amplitude.[66] This program has already been partly completed.[11] We shall discuss the results in Sec. 3. Various groups at CERN[58,67] are now also studying $\rho^0-\omega$ interference in the reactions $\pi^{\pm}N \to \omega N$.

Let us briefly discuss some aspects of the analysis. Since there are several independent spin amplitudes describing the reactions, the interference term in (20) and (21), and also the interference term in the total two-pion mass spectrum (11), must, in general, be multiplied by the coherence factor $\xi (0 \leq \xi \leq 1)$. If the $\rho^0-\omega$ interference takes place primarily through the longitudinal amplitudes of ρ^0 and ω production with π- and B-meson quantum numbers in the t channel, the coherence factor is $\xi \approx (\rho^{\omega}_{00}\rho^{\rho}_{00})^{1/2}$, and the phase φ [see (13)] in the expression for the total $\pi^+\pi^-$ mass spectrum (11) acquires the simple meaning of the relative phase of the two interfering amplitudes.

We ignore the amplitudes of the reactions $\pi N \to (\rho^0, \omega)\Delta$ with the π- and B-meson quantum numbers that change the t-channel helicity at the baryon vertex. For small $|t|$, this is justified. For the reactions $\pi N \to (\rho^0, \omega)N$, such an estimate of ξ under the assumption made about the $\rho^0-\omega$ interference is all the more valid because here there is in each case just one amplitude with zero helicity (of ρ^0 and ω) and the π- and B-meson quantum numbers in the t channel. We calculate the mean coherence factor in the region $0 \leq |t| \leq 0.14$ on the basis of the data of Refs. 9 and 10 for the reaction $\pi^+p \to \pi^+\pi^-\Delta^{++}$:

$$\langle\xi\rangle \approx (\langle\rho^{\omega}_{00}\rangle \langle\rho^{\rho}_{00}\rangle)^{1/2} \approx (0.56 \cdot 0.74)^{1/2} \approx 0.64.$$

It is interesting that analysis of the data of Ref. 9 with allowance for the coherence factor[54,68] gives 0.6 for the lower limit of $\langle\xi\rangle$.

To estimate $|\varepsilon|$, it is important to know in which spin amplitudes the interference occurs. Evaluation of the data of Ref. 9 with $\langle\xi\rangle = 1$ gives a lower limit[9] of $|\varepsilon| \approx 0.034$. The value $\langle\xi\rangle = 0.64$ corresponds to $|\varepsilon| \approx 0.053$. Such a value of $|\varepsilon|$ does not contradict the existing theoretical estimates[1,24-26] and agrees with the results obtained at Orsay on colliding e^+e^- beams.[32] It should be noted that the estimate made for $\langle\xi\rangle$ is in itself too large since the $\rho^0-\omega$ mixing increases the value of $\langle\rho^{\omega}_{00}(\pi^+p \to \omega\Delta^{++})\rangle$. In this case, a simultaneous evaluation of the data on the reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to \omega\Delta^{++}$ with allowance for $\rho^0-\omega$ interference is in all respects helpful.

Regge Pole Models for the Reactions $\pi N \to V^0(N, \Delta)$. Hitherto, we have not had recourse to concrete theoretical models for the amplitudes of the ρ^0 and ω production reactions. It is therefore now helpful to consider the predictions of the simplest Regge pole models concerning the nature of the $\rho^0-\omega$ interference and the assumptions which are usually made in the theoretical estimates.

The Regge pole model for the amplitudes of the reactions $\pi^{\pm}N \to (\rho^0, \omega)\Delta$ and $\pi^{\pm}N \to (\rho^0, \omega)N$ with the π- and B-meson quantum numbers in the t channel gives the relative phase of the amplitudes of ρ^0 and ω production with zero helicity in the t channel needed for destructive (or constructive, depending on the reaction) $\rho^0-\omega$ interference.[24] Let us show this. The ratio of the amplitudes A_ω and A_ρ with exchange of B and π poles, respectively, has the form

$$\frac{A_\rho}{A_\omega} = \pm\, \beta i \exp\left[i\pi\,(\alpha_B(t) - \alpha_\pi(t))/2\right] \frac{\cos \pi\alpha_B(t)/2}{\sin \pi\alpha_\pi(t)/2}, \tag{24}$$

where β is the ratio of the residues of the π and B poles; $\alpha_i(t)$ is the trajectory of the i-th Regge pole; the $\pm$ refer to (ρ^0, ω) production in π^+N and π^-N collisions, respectively. It follows from the Chew-Frautschi graph that the π and B trajectories are close to one another: $\alpha_\pi(t) \approx \alpha_B(t)$. The model of π and B Regge poles with similar trajectories is of course the simplest acceptable variant. From the theoretical point of view this model is good in that under the assumption of exchange degeneracy[69] the π and B trajectories must in fact coincide. Then the relative phase of the π and B contributions, which is determined by the ratio of the signature factors [see (24)], is $\pi/2$ (or $-\pi/2$). Therefore, in (19) we have $\varphi_{3\pi} \approx \varphi_\varepsilon \pm \pi/2 \approx 0$ (or

π), and we arrive at the expressions (20) and (21).

In the limit of so-called strong exchange degeneracy[41] it follows that, besides $\alpha_\pi(t) = \alpha_B(t)$, the value of β in (24) cannot depend on t and is equal to unity. Then

$$A_\rho/A_\omega = \pm\, i \cot(\pi\alpha_\pi(t)/2)\,|_{|t|\leqslant 0.2} \approx \pm\, 2i/\pi\alpha'(t-\mu^2), \tag{25}$$

where $\alpha_\pi(t) = \alpha'(t-\mu^2)$; α' is the slope of the Regge trajectory; μ is the π-meson mass. This prediction is based on duality and SU(3) symmetry.[24,50] Let us consider, for example, the reactions $\pi N \to (\rho^0, \omega, \varphi)N$ and $KN \to K^*N$. If we assume that in these reactions t-channel exchanges with quantum numbers of an SU(3) octet are predominant and there is "ideal" φ–ω mixing, i.e., $\Gamma(\varphi\to\rho\pi) = \Gamma(B\to\varphi\pi) = 0$, then the amplitudes of these reactions can be expressed in terms of the two amplitudes A_+ and A_-, which correspond to exchange with G parity $+1$ and -1:

$$\left.\begin{array}{l} |A(\pi^+n\to\rho^0p)| = |A_-|;\ |A(\pi^+n\to\omega p)| = |A_+|; \\ |A(K^+n\to K^{*0}p)| = |A_+ + A_-|/2; \\ |A(K^-p\to K^{*0}n)| = |A_+ - A_-|/2. \end{array}\right\} \tag{26}$$

The absence of resonances in the reactions $K^+n \to K^{*0}p$ at low energies implies on the basis of duality that the imaginary part of the amplitude of this reaction at high energies is zero:

$$\operatorname{Im} A(K^+n\to K^{*0}p) = \operatorname{Im}(A_+ + A_-)/2 = \operatorname{Im}(\pi + B)_{\text{exch}}/2 = 0.$$

This is possible for $\alpha_\pi(t) = \alpha_B(t)$ and $\beta = 1$ in (24).

The Regge pole model for the amplitudes of ρ^0 and ω production with transverse polarizations in the t channel (A_2 and π Regge poles for ρ^0 production and ρ and B Regge poles for ω production) also allows in principle the correct relative phase needed for destructive (constructive) interference. However, the contributions of all Regge poles to these amplitudes for the reactions $\pi N \to V^0(N, \Delta)$ vanish at small $|t|$ and the A_2P, πP, ρP, and BP Regge cuts come into play (P is the Pomeranchuk pole). The picture becomes complicated. In this situation, it is hard to expect the "correct" relative phase of the ρ^0 and ω production amplitudes. In addition, the interference terms between the contributions with the quantum numbers of the π and B mesons may partly cancel with the interference terms between the contributions with the A_2- and ρ-meson quantum numbers to the amplitudes of ρ^0 and ω production with transverse polarizations. This can occur[50] for $|t| \lesssim 0.5$; for in the framework of strong exchange degeneracy for ρ and A_2 exchanges a relation analogous to (25) for B and π exchanges is obtained. Then the relative sign of the ρ^0–ω interference between the contributions with the "natural" parity (ρ and A_2 Regge poles) and between the contributions with "unnatural" parity (B and π Regge poles) is fixed by the sign of the ratio.

$$[A_\rho(A_2)/A_\omega(\rho)]/[A_\rho(\pi)/A_\omega(B)] = [\cot\pi\alpha_\rho(t)/2]/[\cot\pi\alpha_\pi(t)/2] \tag{27}$$

[in the curly brackets on the left-hand side of (27) we have specified the type of t-channel exchange], which is negative for $|t| \lesssim 0.5$ since $\alpha_\pi(t) < 0$ for $t < 0$ and $\alpha_\rho(t) > 0$ for $t \gtrsim 0.5$.

The predictions of Regge pole models undoubtedly play an important role in our understanding of the experimental situation. Of course, they are not always fulfilled in a pure form (we recall, for example, the important role of the πP Regge cut in the description of the reaction $\pi N \to \rho^0 N$). In fact, in the following section we shall have the possibility of considering a polarization phenomenon observed in the reaction $\pi^-p \to \omega n$ that cannot be accommodated in the usual Regge pole models.

3. EXPERIMENTS WITH HIGH STATISTICS IN THE REACTIONS $\pi N \to V^0 N$

A New Look at the Problem of Describing the Reaction $\pi N \to \omega N$. The Role of Two-Reggeon Cuts. In Fig. 6, we give data for $\rho_{00}d\sigma/dt(\pi^-p \to \omega n)$ obtained recently at Argonne at $q_L = 6$ GeV/c in an experiment with high statistics.[11] The data refer to the center-of-mass system of the s channel. As we already noted in the introduction, these data cannot be understood on the basis of known Regge trajectories. In the experiment, $\rho_{00}d\sigma/dt$ does not vanish at very small transfers, $-t \lesssim 0.02$, contradicting the theoretical expectations (exchange of a B Regge pole):

$$\rho_{00}\, d\sigma/dt = s^{-2}\{|A^\omega_{0\,1/2-1/2}|^2 + |A^\omega_{0\,1/2\,1/2}|^2\}, \tag{28}$$

where $A^\omega_{\lambda_\omega\lambda_n\lambda_p}$ are the helicity s-channel amplitudes; λ_ω, λ_n, and λ_p are the helicities of the ω meson, the neutron, and the proton, respectively. According to the quantum numbers, B exchange makes a contribution to the amplitude $A^\omega_{01/2-1/2}$ proportional to $(-t)^{1/2}$ as $t \to 0$ (we ignore $|t|_{\min}$).

Before this experiment, data on the reactions $\pi^\pm N \to \omega N$ at small $|t|$ had not made it possible to discern a clear absence of a dip in $\rho_{00}d\sigma/dt$ as $t \to 0$ (see, for example, Refs. 58, 60, 61, 70, 71). The observed effect means that the helicity-conserving amplitude in the center-of-mass system, $A^\omega_{01/21/2}$, is important.

This amplitude is described at high energies by exchange in the t channel with the quantum numbers $(\tau, P, G, I) = (+1, -1, +1, 1)$, where τ is the signature. In principle, the observed effect can be explained by Z exchange.[12-14] It should, however, be emphasized that an explanation by means of simple Regge poles is possible only if the Z trajectory conspires with its

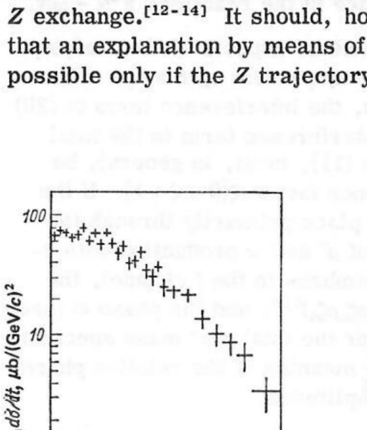

FIG. 6. The dependence $\rho_{00}d\sigma/dt(\pi^-p\to\omega n)$ at $E_{\text{lab}} = 6$ GeV.[11]

daughter trajectory $Z_d(-1,+1,+1,1)$.[15,16] Otherwise, it follows from the analyticity of the invariant amplitudes that $A^{\omega}_{0\,1/2\,1/2} \sim t$. A similar conspiracy for the A_1 and A_{1d} trajectories in the reaction $\pi N \to \rho N$ was considered in Ref. 72.

Thus, we need two new trajectories with unnatural parity. However, one must question the existence of such hypothetical trajectories since we do not know any heavy particles with $I^G(J^P) = 1^+(2^-)$ and $1^+(1^+)$ that lie on the Z and Z_d trajectories, respectively; $m_z^2 \approx m_{z_d}^2 \gtrsim 2$ (GeV)^2.

Here, we consider an alternative explanation by means of two-reggeon cuts.[2)]

We predict the appearance of a dip in $\rho_{00}d\sigma/dt$ at small transfers, $|t| \to 0$, with increasing energy.

We then discuss once more the effects of $\rho^0-\omega$ mixing in $\rho_{00}d\sigma/dt$ for the reactions $\pi^{\pm}N \to \omega N$ and predict a more pronounced manifestation of $\rho^0-\omega$ interference with increasing energy.

First of all, we establish what cuts contribute to the amplitude $A^{\omega}_{0\,1/2\,1/2}$. It was shown in Ref. 73 that the signature of the Regge cut is equal to the product of the signatures of the Regge poles forming the cut. Therefore, only two-reggeon cuts from poles with the same signature are important:

$$\tau_{\text{cut}} = \tau_1\tau_2 = +1. \tag{29}$$

One can show that for $t=0$ only cuts from poles with natural ($\tau P = +1$) and unnatural ($\tau P = -1$) parities contribute to the amplitude $A^{\omega}_{0\,1/2\,1/2}$; for conservation of P parity gives

$$A^{\omega}_{0\,1/2\,1/2} = -A^{\omega}_{0\,-1/2\,-1/2}. \tag{30}$$

The contribution of the cut generated by two-reggeon exchange (Fig. 7) to the amplitudes is

$$A^{\omega}_{0\pm 1/2 \pm 1/2} \sim \sum_{\lambda} \left\langle \pm\tfrac{1}{2} \right| R_2(\mathbf{q}-\mathbf{k}_\perp) \left| \lambda \right\rangle \left\langle \lambda \right| R_1(\mathbf{k}_\perp) \left| \pm\tfrac{1}{2} \right\rangle; \tag{31}$$

$$q = p_3 - p_1;\ t = -\mathbf{q}^2.$$

Here, λ are the helicities of the intermediate state; in addition, summation is assumed over all intermediate states as well as integration with respect to $\mathbf{k}_\perp$ and $s_1 = (k+p_1)^2$ along the right-hand (or left-hand, since these integrals are equal) cut of the amplitude for reggeon production.[73-75] For simplicity, we have also omitted the upper block in Fig. 7, which is common to both amplitudes, since it is unimportant at this juncture for our arguments. It follows obviously from conservation of P parity that

$$A^{\omega}_{0\,-1/2\,-1/2} \sim \sum_{\lambda} \left\langle -\tfrac{1}{2} \right| R_2(\mathbf{q}-\mathbf{k}_\perp) \left| \lambda \right\rangle \left\langle \lambda \right| R_1(\mathbf{k}_\perp) \left| -\tfrac{1}{2} \right\rangle$$
$$= (\tau_1 P_1)(\tau_2 P_2) \sum_{\lambda} \left\langle \tfrac{1}{2} \right| R_2(\mathbf{q}-\mathbf{k}_\perp) \left| \lambda \right\rangle \left\langle \lambda \right| R_1(\mathbf{k}_\perp) \left| \tfrac{1}{2} \right\rangle. \tag{32}$$

We conclude from (30) that the observed effect can

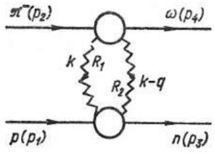

FIG. 7. Diagram of two-reggeon exchange in the reaction $\pi^- p \to \omega n$.

explain the cuts formed by poles with

$$(\tau_1 P_1)(\tau_2 P_2) = -1. \tag{33}$$

Among the Regge poles known to us, the pairs πA_2, $B\rho$, $A_1\omega$ satisfy the conditions (29) and (33). All these cuts give approximately the same energy behavior: $A^{\omega}_{0\,1/2\,1/2} \sim s^{-1/2}$. We shall not consider the cut $A_1\omega$. Since the A_1 Regge pole is not manifested anywhere, we can hope that the contributions due to it are unimportant. In addition, we shall not consider the contributions from the exchange of two strange Regge poles. Their contribution to the amplitude is $A^{\omega}_{0\,1/2\,1/2} \sim s^{-1}$.

The contribution from the πA_2 cut is the most important because the π exchange has a pole at $t = \mu^2$. We shall restrict ourselves to single-particle states in the reggeon-production amplitudes, i.e., essentially we shall operate within the framework of the absorption-model approximations.[76,77] Putting it more precisely, we shall use the technique developed in Ref. 75 and carry out the integration with respect to s_1 and $s_2 = (p_2 - k)^2$ in (32) along the right-hand cut of the amplitudes for the production of the reggeons $R_1 = \pi$, $R_2 = A_2$. We take into account the $\rho^0 n$ and $\rho^- p$ intermediate states: $\pi^- p \to (\rho^0 n,\ \rho^- p) \to \omega n$. It follows from isotopic invariance that these contributions are equal, so that the result is doubled. At the A_2NN vertex, we have taken into account only the spin-flipping residue since it is this one that contributes to $A^{\omega}_{0\,1/2\,1/2}$. We have taken into the account the amplitude of $\pi N \to \rho N \to \omega N$ only with longitudinal polarization of the ρ meson. The contribution of the amplitudes with transverse polarization is 6–7 times smaller. Using the experimental and theoretical information available on the residues of the Regge poles, we have concluded that the contribution of the intermediate states $\rho\Delta$: $\pi^- p \to (\rho^0\Delta^0,\ \rho^-\Delta^+) \to \omega n$ is 5–6 times smaller than the contribution of the ρN intermediate states. At the same time, for the $A_2N\Delta$ vertices we have used the model of Ref. 78 and $\rho - A_2$ strong exchange degeneracy.

The $B\rho$ cut is evidently 6–7 times smaller than the πA_2 cut since the amplitude with B exchange does not have a pole at $t = \mu^2$.

Using the experimental information on π exchange in the reaction $\pi N \to \rho N$,[59] on the spin-flipping A_2NN residue from the reaction $\pi N \to \eta N$,[79] and the results of analysis of the total cross section of hadron photoproduction on nucleons[80] to estimate the $\rho N \to \omega N$ amplitudes, we found that at $q_L = 6$ GeV/c the πA_2 cut gives

$$\rho_{00}\, d\sigma/dt\, (\pi^- p \to \omega n)\,|_{t=0} \approx 45.5\ \mu\text{b}/(\text{GeV}/c)^2. \tag{34}$$

The difference between the estimate and the experimental value [70 μb/(GeV/c)^2] can be explained by assuming that the contributions of the other intermed-

[2)]The results given below were obtained in collaboration with A. A. Kozhevnikov.[17]

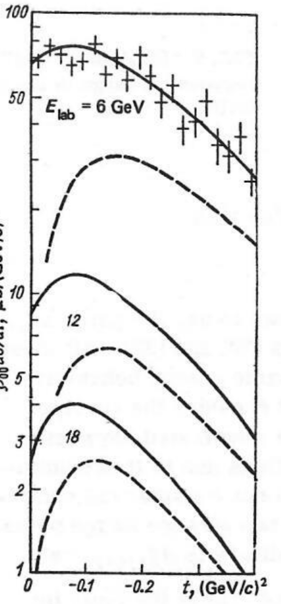

FIG. 8. Dependence $\rho_{00}d\sigma/dt(\pi^-p\to\omega n)$ obtained with allowance for the πA_2 cut and the B Regge pole (continuous curves); the dashed curves show the contribution of only the B pole.

iate states and also, possibly, the contributions of the other cuts ($B\rho$, etc) must be 24%. It is difficult to judge whether this difference is to be attributed to the contributions of other intermediate states or to inaccurate calculation of the main contribution, especially the amplitude of A_2 exchange in $\rho N\to\omega N$. In Fig. 8, our result for $\rho_{00}d\sigma/dt$ is compared with the experiment; we give predictions for different energies. We fixed $\rho_{00}d\sigma/dt=70\ \mu b/(GeV/c)^2$ at $t=0$. The contribution of the B Regge pole was chosen in accordance with strong π–B exchange degeneracy. With increasing energy, the cut contribution becomes small compared with B exchange, and we predict the appearance of a dip in $\rho_{00}d\sigma/dt$ at $t\approx 0$ (see Fig. 8).

A fuller exposition of the questions touched upon here and details of the calculations can be found in Ref. 17.

Estimates of the Effect of ρ^0–ω Interference in the Reactions $\pi^\pm N\to\omega N$. According to the preliminary data obtained at Argonne[11] $\rho_{00}d\sigma/dt(\pi^+n\to\omega p)$ is 20% larger than $\rho_{00}d\sigma/dt(\pi^-p\to\omega n)$ for $-t<0.1$ at $q_L=6$ GeV/c, i.e., the effect of ρ^0–ω interference in $\pi^\pm N\to\omega N$ (Refs. 1, 7, 8) is much smaller than predicted in Refs. 7 and 8 (see Sec. 2). We shall see below that this can be explained by the contributions of the πA_2 cut.

We write down $\rho_{00}d\sigma/dt$ for $\pi^\pm N\to\omega N$ with allowance for the admixture of $\pi^\pm N\to\rho^0 N$ reaction amplitudes [see (19) and (28)]:

$$\rho_{00}\frac{d\sigma}{dt}(\pi^\pm N\to\omega N)=s^{-2}\{|A^\omega_{0\,1/2-1/2}|^2(1\pm|\varepsilon|2\beta_1/\pi\alpha'(\mu^2-t))^2+|A^\omega_{0\,1/2\,1/2}|^2(1\pm|\varepsilon|\beta_2/2)^2\}. \quad (35)$$

The signs + and − in (35) correspond to the reactions $\pi^+n\to\omega p$ and $\pi^-p\to\omega n$. In the ω production amplitudes we have taken into account the B pole and the πA_2 cut, and in the ρ^0 production amplitudes the π pole and the $\pi\rho$ cut. We also used Eqs. (25) and (27) and, as usual, assumed that the mixing parameter ε is almost purely imaginary. The coefficients β_1 and β_2 take into account the deviation from the predictions of strong exchange degeneracy (which, naturally, cannot be absolutely exact) for the π, B, and A_2, ρ Regge poles, respectively. The amplitude $A^\rho_{0\,1/2\,1/2}$ of the process $\pi N\to\rho^0 N$ has the quantum numbers of the A_1 meson in the t channel. We describe it by the $\pi\rho$ Regge cut. In calculating the πA_2 cut, we took into account two intermediate states $\rho^0 n$ and $\rho^- p$, $\pi^-p\to(\rho^0 n,\ \rho^-p)\to\omega n$. In contrast, in the reaction $\pi^-p\to\rho^0 n$ there is one analogous intermediate state: $\pi^-p\to\rho^-p\to\rho^0 n$. Therefore, for $\beta_2=1$ the amplitude $A^\omega_{0\,1/2\,1/2}(\pi A_2)$ is twice the amplitude $A^\rho_{0\,1/2-1/2}(\pi\rho)$ [see (35)].

The influence of ρ^0–ω interference for $0\lesssim|t|\lesssim 0.1$ on the amplitude $A^\omega_{0\,1/2-1/2}(\pi^\pm N\to\omega N)$ is large because of the strong enhancement due to the exchange of the π Regge pole [see the first term in (35)]. For $A^\omega_{0\,1/2\,1/2}$ there is no such enhancement. On the other hand, this amplitude is important in the region of small $|t|$. In this connection, the influence of ρ^0–ω interference on $\rho_{00}d\sigma/dt(\pi^\pm N\to\omega N)$ is smaller for $q_L=6$ GeV/c than predicted earlier.[7,8,41] However, with increasing energy the πA_2 and $\pi\rho$ cuts die out in the background of B and π exchanges, and we predict an enhancement of the effect of ρ^0–ω interference as the energy increases (Fig. 9). Whereas the integrated effect at $q_L=6$ GeV/c for $|t|<0.1$ is 20%, as the preliminary data of Ref. 11 indicate, at 12 GeV/c it will be [in accordance with (35)] 30%, and at 18 GeV/c, 40% (see Fig. 9). Note that the integrated effect of ρ^0–ω interference in $|A^\omega_{0\,1/2-1/2}|^2$ for $|t|<0.1$ is at the same time about 70%, i.e., about the same as predicted in Refs. 7 and 8.

What can we say about the parameter ε if we wish to obtain a $\sigma_{00}(\pi^+n\to\omega p)$ that is 20% larger than $\sigma_{00}(\pi^-p\to\omega n)$ for $|t|<0.1$ and $q_L=6$ GeV/c? Note that in the model under consideration both the interference terms, which are proportional to $|\varepsilon|$, have the same signs in (35). If as usual[31,32] we take $|\varepsilon|=0.05$, then we obtain a 10% isotopic symmetry violation solely due to the

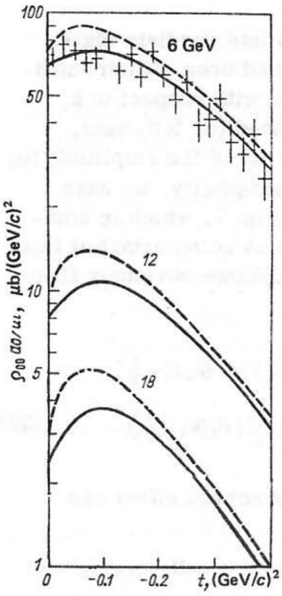

FIG. 9. Influence of ρ^0–ω mixing on $\rho_{00}d\sigma/dt(\pi^\pm N\to\omega N)$ at different energies: the continuous curves correspond to the reaction $\pi^-p\to\omega n$, the dashed curves to $\pi^+n\to\omega p$.

amplitude $A^{\omega}_{01/21/2}$, assuming $\beta_2=1$. Altogether, $\sigma_{00}(\pi^+n\to\omega p)$ is approximately 1.45 times greater than $\sigma_{00}(\pi^-p\to\omega n)$ for $\beta_1=\beta_2=1$. Note that it follows from simultaneous fitting of the data on $\rho_{00}d\sigma/dt$ for $\pi^-p\to\rho^0 n$ (Ref. 59) and $\pi^-p\to\omega n$ for $q_L=6$ GeV/c (Ref. 11) that π–B strong exchange degeneracy is satisfied fairly well, i.e., $\beta_1\approx 1$. A 20% effect is obtained for $|\varepsilon|\approx 0.03$ ($|\varepsilon|\approx 2|\delta|/\Gamma_\rho$, $|\delta|\approx 2.25$ MeV). It is probable that we shall soon have data with high statistics on the reaction $\pi^+n\to\omega p$ at 6 GeV/c (Ref. 11) and will be able to make a more detailed analysis. In addition, experiments are also being made on the reactions $\pi^\pm N\to\omega N$ at $q_L=8.5$ GeV/c.[11] An analysis of data with lower statistics made in Ref. 58 shows that $\sigma_{00}(\pi^+n\to\omega p)/\sigma_{00}(\pi^-p\to\omega n)\approx 1.5\pm 0.5$ for $0.05\leqslant|t|\leqslant 0.15$ and $3\lesssim q_L\lesssim 6$ GeV/c.

Let us make a short comment about the reactions $\pi^\pm N\to\omega\Delta$. The amplitudes with longitudinal polarization of the ω meson have the B- and Z-meson quantum numbers in the t channel, as in the reactions $\pi^\pm N\to\omega N$. However, the B Regge pole does not vanish in $\rho_{00}d\sigma/dt(\pi^\pm N\to\omega\Delta)$ as $t\to 0$.[9,10] Therefore, the contribution of the πA_2 cut may not be so pronounced here. But, on the other hand, the quantum numbers of the B and Z mesons in the reactions $\pi N\to\omega\Delta$ contribute to the same helicity amplitudes and may interfere. In contrast to the $\pi N\to\omega N$ reactions, the πA_2 cut may occur in the expression for $\rho_{00}d\sigma/dt(\pi N\to\omega\Delta)$ linearly. This can also affect the ρ^0–ω interference. Of course, these are preliminary considerations and require a detailed theoretical analysis. In any case, the investigations of ρ^0–ω interference and searches for contributions to $\rho_{00}d\sigma/dt$ for the reactions $\pi^\pm N\to\omega N$ over and above that of the B pole are of undoubted interest.

4. EFFECTS OF ρ^0-ω INTERFERENCE IN THE $\pi^+\pi^-$ MASS SPECTRA

The Reaction $\gamma N\to\pi^+\pi^-\Delta$ and SU(3) Symmetry. Quasi-two-particle channels of photoproduction of multipion systems on nucleons and complex nuclei are currently under intensive study in many experimental centers. Besides the well known experiments on the reactions $\gamma N\to\pi N$, $\gamma N\to(\rho^0,\ \omega,\ \varphi)$, the more complicated photoproduction processess of two resonances such as, for example, $\gamma N\to\rho\Delta\to 3\pi N$ and $\gamma N\to\omega\Delta\to 4\pi N$ are also being investigated.[81-86] Of these, the as yet least studied reaction is $\gamma N\to\rho^0\Delta\to\pi^+\pi^-\Delta$. But, from our point of view, this reaction is very interesting in connection with the large ρ^0–ω interference effect to be expected in the $\pi^+\pi^-$ mass spectrum.[18]

To determine what ρ^0–ω interference effect can be expected in $\gamma N\to\pi^+\pi^-\Delta$ reactions at high energies, let us attempt to establish the relative magnitudes and relative phases of the $\rho^0\Delta$ and $\omega\Delta$ photoproduction amplitudes. For this, let us consider the mechanisms of the $\gamma N\to\rho^0\Delta$ and $\gamma N\to\omega\Delta$ processes. The $\gamma N\to\omega\Delta$ reaction amplitudes are due to exchanges in the t channel with the following quantum numbers: isospin $I=1$, charge parity $C=+1$, and parity $P=\pm 1$. In the reaction $\gamma N\to\rho^0\Delta$, besides exchanges in the t channel with the same quantum numbers, one can also have $I=2$ exchange. At high energies, it is natural to assume that exchange with exotic quantum numbers ($I=2$) is asymptotically small compared with the $I=1$ contribution. In addition, interference between the amplitudes with $I=2$ and $I=1$ can be eliminated by considering the sum of the $\gamma p\to\rho^0\Delta^+$ and $\gamma n\to\rho^0\Delta^0$ reaction cross sections. In what follows, we shall ignore $I=2$ exchange in the t channel. Then the mechanisms of $\rho^0\Delta$ and $\omega\Delta$ photoproduction are entirely due to exchange of identical t-channel quantum numbers.

One can invoke SU(3) symmetry to establish more precisely the connection between the amplitudes A^{ρ}_i and A^{ω}_i. Suppose that at high energies the intermediate states of the t channel in the reactions $\gamma N\to V^0\Delta$ are only SU(3) octets. This is true, for example, in the Regge pole model and the Regge pole model with absorption. It is also one of the consequences of the ordinary quark model. Applying now the standard SU(3) technique, in which the hadronic electromagnetic current is a number of an octet, to the two t-channel $\gamma N\to V^0\Delta$ reaction amplitudes, we obtain the relations

$$\left.\begin{aligned} A^{\rho}_i &= A^{\varphi_8}_i;\\ A^{\omega}_i &= 3^{1/2}(\sin\theta A^{\varphi_8}_i+\cos\theta A^{\varphi_0}_i);\\ A^{\varphi}_i &= 3^{1/2}(\cos\theta A^{\varphi_8}_i-\sin\theta A^{\varphi_0}_i),\end{aligned}\right\} \tag{36}$$

where φ_8 and φ_0 are the states of the φ and ω mesons before mixing; θ is the angle of φ–ω mixing. They can be represented in the form of the single equation

$$3^{1/2}A^{\rho}_i-(\cos\theta A^{\varphi}_i+\sin\theta A^{\omega}_i)=0. \tag{37}$$

Since the φ meson has virtually no coupling to nonstrange particles, the amplitudes of the reaction $\gamma N\to\varphi\Delta$ can be ignored. In addition, it is well known that $\sin\theta\approx(1/3)^{1/2}$ in the case of φ–ω mixing. Therefore,

$$A^{\rho}_i(\gamma N\to\rho^0\Delta)\approx A^{\omega}_i(\gamma N\to\omega\Delta)/3. \tag{38}$$

With allowance for (38), Eqs. (12) and (18) for the coherence factor ξ and the phase φ take the simple form

$$\xi\approx 1;\ \varphi=\varphi_\delta+\tan^{-1}[\Gamma_\omega/2(m_\omega-m)];\ \varphi_{\rm prod}=0. \tag{39}$$

The number of events satisfies

$$N_\omega(\gamma N\to\omega\Delta)\approx 9N_\rho(\gamma N\to\rho^0\Delta). \tag{40}$$

With allowance for (39) and (40), the expression (17) for the $\pi^+\pi^-$ mass spectrum takes a form analogous to that of the expression for dN/dm in the case of reactions described by a single amplitude:

$$dN/dm=(1/2\pi)\,|[\Gamma_\rho^{1/2}N_\rho^{1/2}/D_\rho(m)]\,[1+3\delta/D_\omega(m)]|^2. \tag{41}$$

Assuming $\delta\approx 3.5$ MeV ($|\varepsilon|\approx 0.05$), which is obtained in the experiment on the reaction $e^+e^-\to(\rho^0,\omega)\to\pi^+\pi^-$,[31,32] and also $\Gamma_\omega\approx 9.1$ MeV,[31] we find for $m=m_\omega$

$$dN/dm\approx(2/\pi\Gamma_\rho)\,N_\rho\,|1+i3\cdot 2\delta/\Gamma_\omega|^2\approx(2/\pi\Gamma_\rho)\,N_\rho(1+5). \tag{42}$$

Thus, the incoherent contribution of the ω meson to the $\pi^+\pi^-$ mass spectrum at $m=m_\omega$ is approximately

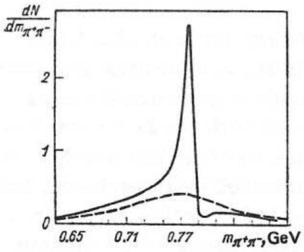

FIG. 10. Mass spectrum of $\pi^+\pi^-$ mesons in the reaction $\gamma N \to \pi^+\pi^-\Delta$ with allowance for $\rho^0-\omega$ mixing. The dashed curve shows the contribution of only the ρ^0 meson.

five times greater than the contribution of the ρ^0 resonance. Figure 10 shows the general picture of the expected $\rho^0-\omega$ interference.

We also compare the contributions of the ρ^0 and ω mesons to the total number of events:

$$N = \int \frac{dN}{dm} dm \approx N_\rho \left(1 + \frac{N_\omega}{N_\rho} \frac{\Gamma_{\omega 2\pi}}{\Gamma_\omega}\right) \approx N_\rho (1 + 0.33), \quad (43)$$
$$\Gamma_{\omega 2\pi} \approx 4 |\delta|^2/\Gamma_\rho.$$

It can be seen from (43) that a third of the $\gamma N \to \pi^+\pi^-\Delta$ reaction events in the region of the ρ^0 resonance is due to $\rho^0-\omega$ mixing. The integral containing the first power of δ in (43) is virtually equal to zero.

It is interesting to compare these predictions with the effect of $\rho^0-\omega$ interference in the well studied reactions $\gamma A \to \pi^+\pi^- A$. For these, as usual, it is assumed that exchange of the vacuum quantum numbers in the t channel is predominant, and that

$$A_1^\rho (\gamma A \to \rho^0 A) \approx 3 A_1^\omega (\gamma A \to \omega A). \quad (44)$$

Therefore, in the reactions $\gamma A \to \pi^+\pi^- A$ the distortion of the ρ^0 meson mass spectrum is small and due solely to the interference term in Eq. (17). As can be seen from (38) and (44), the interference term in (17) for $\gamma N \to \pi^+\pi^-\Delta$ is enhanced by a factor 9 compared with the interference term for $\gamma A \to \pi^+\pi^- A$, and the term quadratic in δ is enhanced accordingly by 81 times. Figure 11 illustrates the effect of $\rho^0-\omega$ interference in the reaction $\gamma C \to \pi^+\pi^- C$.[46]

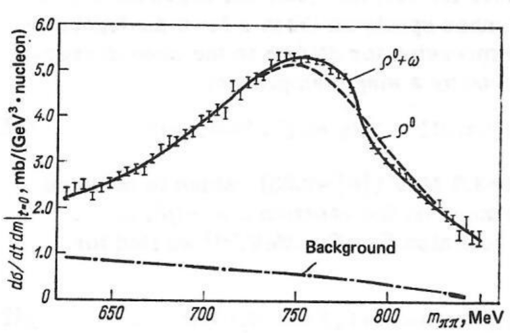

FIG. 11. Effect of $\rho^0-\omega$ interference in the reaction $\gamma C \to \pi^+\pi^- C$.[46]

Let us consider briefly the problem of separating the backgrounds (see, for example, Refs. 83 and 85). In the study of quasi-two-particle reactions the problem arises of separating the nonresonant processes. In the present case, the most important of these at high energies are evidently the nonresonant (in the region of Δ) processes $\gamma N \to (\rho^0, \omega)\pi N$, in which the πN system is in a state with $I = \frac{1}{2}$, and exchange of the vacuum quantum numbers takes place in the t channel.[83] In principle, their admixture can smooth the picture we have described of $\rho^0-\omega$ interference in the reactions $\gamma N \to \pi^+\pi^-\Delta \to \pi^+\pi^-(\pi N)$ since the amplitudes of such nonresonant processes satisfy the relation (44). If in the region of the Δ resonance we take the sum of the cross sections of the processes $\gamma p \to \pi^+\pi^-(\pi^+ n)$ and $\gamma n \to \pi^+\pi^-(\pi^- p)$, then the interference with the indicated background disappears and it can be separated as incoherent. This will occur automatically in an investigation of the reaction $\gamma d \to V^0 \Delta N$. The $\rho^0-\omega$ interference effect considered above can be used as a background-separation criterion.

For the reactions $\gamma N \to V^0\Delta$ there are as yet fairly crude data, but an increase in the statistics is planned for their further investigation.[82, 83, 85]

SU(3) Symmetry and the Reaction $e^+e^- \to V\pi \to \pi^+\pi^-\pi^0$. At the present time, single-photon annihilation into hadrons is being intensively studied in colliding e^+e^- beams at energies above 1 GeV. On many accelerators, magnetic detectors have been set up that make it possible to determine the momenta of charged particles with high accuracy. Increased luminosity and the use of the magnetic detectors provide a possibility for studying various specific properties of each particular channel.

In the reaction $e^+e^- \to (\rho, \omega)\pi \to \pi^+\pi^-\pi^0$ at $2E_e > 1$ GeV considerable interest could attach to an investigation of $\rho^0-\omega$ interference in the $\pi^+\pi^-$ mass spectrum due to electromagnetic $\rho^0-\omega$ mixing.[19-21]

At e^+e^- cms energies greater than 1 GeV one can observe ρ-meson production in the reaction $e^+e^- \to 3\pi$ in the two-pion mass spectra:

$$e^+e^- \to \gamma^* \to (\rho^+\pi^- + \rho^-\pi^+ + \rho^0\pi^0) \to \pi^+\pi^-\pi^0, \quad (45)$$

where γ^* denotes a virtual γ. It is natural to assume that this is the main process, at least for $m_{\pi\pi} \approx m_\rho$. It then follows from the usually assumed isotopic properties of the electromagnetic current that in this reaction the three-pion system is in a state with zero isospin. Then by isotopic invariance the $\pi^+\pi^-$ and $\pi^\pm\pi^0$ mass spectra must be the same. Electromagnetic $\rho^0-\omega$ mixing violates this symmetry and can appreciably affect the $\pi^+\pi^-$ mass spectrum. The process $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0$ corresponding to the contribution of the ω meson to the $\pi^+\pi^-$ mass spectrum is shown in Fig. 12. Why is this contribution important? The point is that according to the predictions of SU(3) symmetry the amplitude (A_ω) of $\omega\pi^0$ production must be approximately three times larger than the amplitude A_ρ of $\rho^0\pi^0$ production:

$$A_\rho \approx A_\omega/3. \quad (46)$$

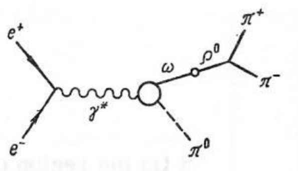

FIG. 12. Diagram of the process $e^+e^- \to \gamma^* \to \omega\pi^0 \to \pi^+\pi^-\pi^0$ due to electromagnetic $\rho^0-\omega$ mixing.

Here, as in the preceding section, we assume that the electromagnetic current in the U-scalar term of the SU(3) octet and that the sine of the $\varphi-\omega$ mixing angle is $(1/3)^{1/2}$, and we ignore the amplitude of the reaction $e^+e^- \to \varphi\pi$ since the φ meson has virtually no coupling to particles that do not contain strange quarks.

We shall consider below what we are to conclude about the approximate prediction (46) for SU(3) symmetry in the region of time-like momenta of the virtual photon; here, let us say a few words about the effect of $\rho^0-\omega$ interference.

The expected effect is analogous to the one considered in the previous section. The estimates (42) and (43) hold for comparing the contributions of the processes $e^+e^- \to \gamma^* \to \rho^0\pi^0 \to (\pi^+\pi^-)\pi^0$ and $e^+e^- \to \gamma^* \to \omega\pi^0 \to (\pi^+\pi^-)\pi^0$. The main difference is that the process $e^+e^- \to \rho\pi \to 3\pi$ proceeds through three channels [see (45)]. The channels $e^+e^- \to \rho^+\pi^- + \rho^-\pi^+ \to 3\pi$ constitute the natural background of the process $e^+e^- \to (\rho^0, \omega)\pi^0 \to (\pi^+\pi^-)\pi^0$. This, in particular, has the consequence that the contribution of the process $e^+e^- \to \omega\pi^0 \to (\pi^+\pi^-)\pi^0$ to the total number of $e^+e^- \to V\pi \to 3\pi$ reaction events is reduced compared with (43) by about 3 times and is approximately 10%.

The mass spectrum of the $\pi^+\pi^-$ mesons in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ with allowance for the contribution of the ω meson and all three charge channels of the process $e^+e^- \to \rho^0\pi^0 + \rho^+\pi^- + \rho^-\pi^+ \to 3\pi$ is given in Fig. 13 for $s \equiv 4E_e^2 = 1.5$ GeV^2. To obtain definite (but only approximate) values of $d\sigma/dm \sim dN/dm$, we use the usual vector-dominance model for the $\gamma^* \to \omega\pi^0$ and $\gamma^* \to \rho\pi$ vertices, within the framework of which the relation (46) is satisfied quite well.

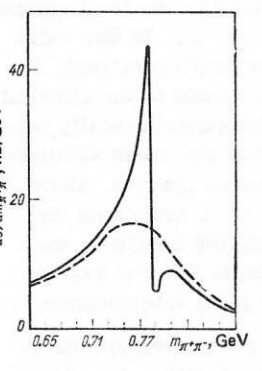

FIG. 13. The dependence $d\sigma/dm_{\pi^+\pi^-}$ for the process $e^+e^- \to (\rho^0\pi^0 + \rho^+\pi^- + \rho^-\pi^+) + \omega\pi^0 \to \pi^+\pi^-\pi^0$ with the allowance (continuous curve) and without allowance (dashed curve) for $\rho^0-\omega$ mixing (the process $e^+e^- \to \rho\pi \to \pi^+\pi^-\pi^0$).

The $\rho^0-\omega$ mixing in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ has an appreciable influence on only the $\pi^+\pi^-$ mass spectrum. The mass spectra of the $\pi^+\pi^0$ and $\pi^-\pi^0$ mesons are hardly changed and are equal to one another by virtue of conservation of C parity. Therefore, if we know the $\pi^+\pi^-$ and $\pi^\pm\pi^0$ mass spectra, we can draw a virtually model-independent conclusion about the $\omega\pi^0$ production process in the reaction $e^+e^- \to \omega\pi^0 \to (\pi^+\pi^-)\pi^0$ by taking the difference of such spectra.

A fuller analysis of this effect, details of the calculations, and a discussion of the possibility of its experimental investigation can be found in Refs. 19–21. We note only that, using magnetic detectors and the already existing luminosities, an investigation of the two-pion mass spectra in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ is perfectly feasible.

The picture we have described above of influence of electromagnetic $\rho^0-\omega$ mixing on the $\pi^+\pi^-$ mass spectrum in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ would be expected immediately after $s \approx 1$ GeV^2 if there are no contributions to the electromagnetic current that significantly violate the prediction of SU(3) [Eq. (46)] for the amplitudes of the interaction $\gamma^* \to V\pi$. Let us now consider more closely the validity of the relation (46). At the present time, realistic solution of this problem is possible only in the framework of the orthodox model of vector dominance. In this model the relation (46) takes the form

$$(em_\omega^2/f_\omega)\, g_{\omega\rho\pi}/(s - m_\omega^2 + im_\omega\Gamma_\omega) + (em_\varphi^2/f_\varphi)\, g_{\varphi\rho\pi}/(s - m_\varphi^2 + im_\varphi\Gamma_\varphi) \approx 3^{-1}(em_\rho^2/f_\rho)\, g_{\omega\rho\pi}/(s - m_\rho^2 + im_\rho\Gamma_\rho) \qquad (47)$$

(em_v^2/f_v are the constants of the $\gamma^* \to V$ transitions), so that in the resonance region it is not satisfied because of the breaking of SU(3) for the particle masses and their instability. Since $g_{\varphi\rho\pi} \ll g_{\omega\rho\pi}$, since SU(3) is satisfied not too badly for the constants f_v, and since $f_\rho \approx f_\omega/3$,[31,32,87] one can hope for the relation (46) to be correct for the amplitudes A_v when $s > m_\varphi^2$, as follows, for example, from (47). For $s \approx 1$ GeV^2, the main violation of (46) is due to the contribution of the φ meson, but it decreases rapidly with increasing s since $g_{\varphi\rho\pi}/g_{\omega\rho\pi} \approx 1/15-1/20$. The effect must be observed for $s \geq 1.2$ GeV^2.[19,20]

From the magnitude of the effect and the profile of the $\pi^+\pi^-$ mass spectrum one can draw conclusions about the fulfillment of the SU(3) predictions for the relative magnitudes and phases of the matrix elements $\langle\pi V|j_\mu^v|0\rangle$ of the electromagnetic current in the time-like region. The main interest attaches to a study of the behavior with varying energy. This will give information, for example, about the s dependence of the relative phase of the $\rho\pi$ and $\omega\pi$ production amplitudes. The optimal region for observations is $1.1 \leq 4E_e^2 \cdot 1$ GeV$^{-2} \leq 2$, simply because with increasing energy the $e^+e^- \to 3\pi$ reaction cross section decreases. In addition, in this region there may be an additional enhancement of the $\rho^0-\omega$ interference effect. Let us consider this possibility more closely.

The $\rho^0-\omega$ Interference in the Region of the ρ' and ω' Resonances in the Reaction $e^+e^- \to V\pi \to \pi^+\pi^-\pi^0$. It is interesting to consider how the picture of $\rho^0-\omega$ interference that we have described is changed if in the reactions $e^+e^- \to \rho\pi$ and $e^+e^- \to \omega\pi$ not only the ordinary contributions to the electromagnetic current from the ω and ρ mesons are important, but also the contributions of the presumed heavy ω' and ρ' resonances.

There are now a number of theoretical ideas and experimental data on resonances of the type ρ' ($J^{PG} = 1^{-+}$) (see, for example, Refs. 88–92). From the reactions $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$, $\gamma p \to \omega\pi^0 p$ and $p\bar{p} \to \omega\pi^+\pi^-$ data have been obtained indicating the existence of a ρ' meson with mass $m_{\rho'} \approx 1250$ MeV and decay width $\Gamma_{\rho'} \approx 150$ MeV, the decay of ρ' into $\omega\pi$ being apparently predominant.[88-92] The cross section $\sigma(s)$ the reaction $e^+e^- \to \omega\pi^0$ at $s \approx m^2_{\rho'}$ is in the range 40–120 nb according to the estimates.[88] For comparison, we point out that the cross section of the reaction $e^+e^- \to \rho\pi$ in this region of s, with allowance for the contribution of the ω meson in the vector-dominance model, is $\approx$4 nb ($e^+e^- \to \omega \to \rho\pi$), and the cross section of the reaction $e^+e^- \to \omega\pi^0$ from the contribution of the ρ meson is $\approx$10 nb ($e^+e^- \to \rho \to \omega\pi^0$). If $\rho'(1250)$ really exists, then in accordance with the quark model there must also exist $\omega'(1250, 1^{--})$ meson with width

$$\Gamma_{\omega'} \approx 3\Gamma_{\rho'} \approx 450 \text{ MeV}, \tag{48}$$

since ω' decays through three channels: $\omega' \to \rho^0\pi^0 \to \rho^+\pi^- + \rho^-\pi^+$. It is natural to assume that SU(3) symmetry works in the same way for the new vector nonet as for the ordinary ρ, ω, φ, and K^* mesons. For example, $m_{\rho'} \approx m_{\omega'}$, $g_{\omega'\rho\pi} = g_{\rho'\omega\pi}$, $f_{\rho'} \approx f_{\omega'}/3$, etc.

Let us now consider what is the effect of $\rho^0-\omega$ interference in the reaction $e^+e^- \to 3\pi$ at $s^{1/2} \approx m_{\rho'} \approx m_{\omega'}$ with allowance for only the ω' and ρ' contributions to A_v. The relation (46) is violated in this region and has the form

$$A_\rho\,(e^+e^- \to \omega' \to \rho^0\pi^0)/A_\omega\,(e^+e^- \to \rho' \to \omega\pi^0) = (f_{\rho'}/f_{\omega'})\,(s - m^2_{\rho'} + im_{\rho'}\Gamma_{\rho'})/(s - m^2_{\omega'} + im_{\omega'}\Gamma_{\omega'}) \approx 3^{-1}\Gamma_{\rho'}/\Gamma_{\omega'} \approx 1/9, \tag{49}$$

where $f_{\rho'}/f_{\omega'} \approx 1/3$ according to SU(3) symmetry; em^2_v/F_v are the constants of the $\gamma^* \to V'$ transitions.

From the expression for the total amplitude of the processes $e^+e^- \to \rho\pi \to 3\pi$ and $e^+e^- \to \omega\pi \to 3\pi$

$$T = g_{\rho\pi\pi}A_\rho\,\{1/D_{\rho+} + 1/D_{\rho-} + (1/D_{\rho 0})\,[1 + (A_\omega/A_\rho)\,(2m_\omega\delta/D_\omega)]\} \tag{50}$$

($D_v = m^2_v - m^2_{\pi\pi} - im_v\Gamma_v$) it follows that the ratio (49) is significantly enhanced compared with (46) as a result of $\rho^0-\omega$ mixing. Now it is not 10%, but approximately half the total number of $e^+e^- \to 3\pi$ reaction events at $s^{1/2} \approx m_{\rho'}$ which must be due to the process $e^+e^- \to \rho' \to \omega\pi \to 3\pi$, which proceeds through electromagnetic $\rho^0-\omega$ mixing. This can be seen from the estimate

$$\begin{aligned}\sigma\,(e^+e^- \to 3\pi) &\approx \sigma\,(e^+e^- \to \omega' \to \rho\pi \to 3\pi) + \sigma\,(e^+e^- \to \rho' \to \omega\pi \to 3\pi)\\ &\approx \sigma\,(e^+e^- \to \omega' \to \rho\pi \to 3\pi)\,(1 + \Gamma_{\omega\pi\pi}\Gamma_{\omega'}f^2_{\omega'}/\Gamma_\omega\Gamma_{\rho'}f^2_{\rho'})\\ &\approx 2\sigma\,(e^+e^- \to \omega' \to \rho\pi \to 3\pi).\end{aligned}$$

Here, we have used Eq. (50) and for the ratio $\Gamma_{\omega\pi\pi}/\Gamma_\omega \approx 4\,|\delta\,|^2/\Gamma_\rho\Gamma_\omega$ we have taken, as before, the value 3.6%.[31] Note that the channels hardly interfere at all.

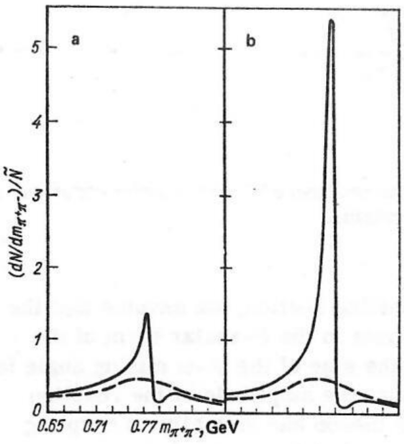

FIG. 14. Comparison of the effects of $\rho^0-\omega$ interference for the reaction $e^+e^- \to \rho\pi + \omega\pi^0 \to \pi^+\pi^-\pi^0$ outside (a) and inside (b) the region of the ρ' (1250) resonance.

It is obvious that the $e^+e^- \to \rho' \to 3\pi$ contribution must lead to a more rapid change of the $e^+e^- \to 3\pi$ reaction cross section in the region of the ω' resonance—to an effective reduction of the width $\Gamma_{\omega'}$ by 1.5–2 times.

Estimates for the cross section of the reaction $e^+e^- \to 3\pi$ in the region $1.1 \leqslant s \leqslant 2$ GeV^2 give values of order 5–15 nb.[19,20] In Ref. 20 a detailed study is made of the dependence of the $\rho^0-\omega$ interference effect in the $\pi^+\pi^-$ mass spectrum on the energy with allowance for the ω, ρ, φ, ω', and ρ' contributions to the amplitudes A_v. We shall not dwell on this general picture, but give only one example that clearly indicates a possible strong dependence of $\rho^0-\omega$ interference on the energy. Figure 14a shows the $\pi^+\pi^-$ mass spectrum outside the region of the ω' resonance, where the relation (46) predicted by SU(3) symmetry holds. The enhancement of the effect in the region of the ω' resonance (see Fig. 14b) due to the violation of (46) is due to the difference between the widths of the ρ' and ω' mesons [see (49)]. The dashed curves in the figures show the $\pi^+\pi^-$ mass spectrum without allowance for electromagnetic $\rho^0-\omega$ mixing, which corresponds to the process $e^+e^- \to \rho\pi \to \pi^+\pi^-\pi^0$. For clarity, we have plotted along the ordinate the values of $dN/dm_{\pi^+\pi^-}$ divided by the total number of events $\tilde{N}$ in the channel $e^+e^- \to \rho\pi \to 3\pi$. In this ratio the entire dependence on s is due to $\rho^0-\omega$ mixing. It should be noted that the relative change of the effect at different energies can be followed experimentally (independently of the normalization of the cross sections) by comparing the $\pi^+\pi^-$ and $\pi^\pm\pi^0$ mass spectra, since the influence of electromagnetic $\rho^0-\omega$ mixing on the latter is small. For a more complete analysis, we require a simultaneous evaluation of the $\pi^+\pi^-$ and $\pi^\pm\pi^0$ mass spectra with allowance for $\rho^0-\omega$ interference.

Experimental investigation of $\rho^0-\omega$ interference in the two-pion mass spectra in the reaction $e^+e^- \to 3\pi$ may give important information on the hadronic structure of the electromagnetic current—on what in fact the amplitudes A_ρ and A_ω are.

CONCLUSIONS

In recent years, experiments with high statistics (for example, on the reactions $\pi^{\pm}N \to (\rho^0, \omega)N$ in Refs. 11, 36–38) have made it possible to use electromagnetic $\rho^0-\omega$ mixing to obtain qualitatively new information about the dynamics of vector-meson production processes. Above, we have considered the interesting, in our opinion, reactions $\pi^{\pm}N \to \omega(N, \Delta)$, $\gamma N \to \pi^+\pi^-\Delta$, $e^+e^- \to \pi^+\pi^-\pi^0$, in which the effects of ρ^0 and ω interference can have a strong dynamic enhancement. We have considered in detail the mechanisms of these reactions and estimated the $\rho^0-\omega$ interference effects. We believe that the experimental study of these reactions at different energies will also be helpful for determining more precisely the $\rho^0-\omega$ mixing parameters and for elucidating the vector-meson production mechanisms.

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