Polarization phenomena in nuclear reactions

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The main recent developments in experimental and theoretical investigations into polarization phenomena in nuclear reactions at low and medium energies are reviewed. Their role in establishing the spin dependence of the nuclear interactions, the spectroscopic information they provide, and the models used to describe polarization phenomena are discussed.

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INTRODUCTION

The historical starting point in investigation into polarization phenomena in the interaction of fast particles with nuclei was probably Schwinger's paper[1] in 1946, in which he proposed a method for polarizing fast neutrons by means of the spin–orbit interaction when these particles are scattered on nuclei. In Ref. 2, he calculated the polarization of neutrons scattered on ^{4}He resulting from the interference of resonances split with respect to the total angular momentum. Seagrave's phase-shift analysis[3] was confirmed by Levintov's polarization experiments,[4] which removed the ambiguity in the data of the analysis.

In 1952, the first measurements were made of polarization in proton scattering on nuclei. These showed that when protons are scattered elastically on ^{12}C the resulting polarization is appreciably greater than the polarization in the case of nucleon–nucleon scattering. This result at first occasioned surprise, since it was assumed that the averaging over the spins of the nucleons in the nuclei would significantly reduce the polarization. But in Ref. 5 the large polarizations were explained by the suppression in elastic scattering of spin flip by the nucleon that scatters the proton in the nucleus. This group of questions is discussed in detail in Bethe's review.[6] Polarization of protons in the reaction ^{2}H$(d,p)^3$H was measured by Bishop[7] in 1952. The polarization was analyzed by a second scattering on helium. In 1953, polarization of neutrons from the reaction ^{2}H$(d,n)^3$He was observed by analyzing them by scattering on carbon.[8] These experiments were the start of a complete new direction in nuclear physics—the investigation of polarization phenomena. The information obtained at that time in the polarization experiments was analyzed in several reviews.[9]

The main current directions in the experimental and theoretical investigation of polarization effects are as follows. The interaction of polarized nucleons with polarized nucleons and few-nucleon systems is still being studied in connection with testing of the field aspects of theory and the fulfillment of P and T invariance. However, consideration of this subject would go beyond the framework of the present review (see Ref. 10).

Calculations are made of the polarization in three-nucleon systems with spin dependence of the two-particle amplitudes. Such calculations are now feasible, and confrontation of them with experiment makes it possible to estimate the applicability of the nucleon–nucleon amplitudes used, their off-shell behavior, and the role of three-nucleon interactions. A system of equations of Faddeev type has also been derived for four-nucleon systems, but their solution is presently at the limit of the computationally feasible.

In nuclear reactions at low excitation energies, for which the concept of isolated or overlapping resonances applies, measurement of the polarization greatly facilitates determination of the quantum numbers of compound-nucleus levels. Frequently, such analysis is impossible without a measurement of the polarization. Information on the polarization is very effective for identifying resonances of the type of intermediate structures, in particular isobaric analog states. Frequently, the mere measurement of the sign of the polarization makes it possible to decide whether an observed resonance belongs to a particular isotopic multiplet.

At fairly high excitation energies, the optical model formulated originally by Thomas, Porter, and Weisskopf as a model of strong absorption is used to describe the elastic channel. Although the optical model was developed quite a long time ago, investigations of elastic scattering of nucleons on nuclei still continue. This is to clarify the role of the spin–spin terms in the optical potential, to exploit the optical model at small A, to investigate the role of nonlocality, etc.

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Polarization effects in the elastic scattering of deuterons, ^3H, and ^3He on nuclei are investigated to test the applicability of the so-called folding model, in which, for example, the deuteron–nucleus potential is represented as a sum of the nucleon–nuclear potentials averaged over the deuteron ground state. This model, which is completely satisfactory for describing cross sections, predicts a spin–orbit interaction that is half the value observed in polarization experiments. In addition, the optical potential for deuterons must contain invariants that are not present in the case of particles with spin 1/2.

Besides purely spectroscopic information, polarization effects in direct nuclear reactions enable one to investigate the limits of applicability of the distorted-wave Born approximation, the parameters of the Migdal interaction[11] in the initial and final states, the importance of the deuteron–nucleus tensor interaction, and the relative importance of compound processes, especially in the resonance region. It is particularly important to take into account the contribution of square graphs,[12] which were discussed for the first time in connection with polarization phenomena in a series of papers (Ref. 13).

In elastic scattering and reactions, considerable importance frequently attaches to the coupling to other open or closed channels and also the coupling to real or virtual levels of the compound nucleus or the target nucleus. In elastic scattering, the excitation of isobaric analog states, whose presence is not taken into account by the phenomenological optical potential, is important. Such coupling to isobaric analogs was observed by Romanovskiĭ *et al.*[14] at the Nuclear Physics Institute at Moscow State University.

Finally, polarization effects are very effective when one is studying the recently discovered new giant multipole resonances in nuclei. Study of the radiative capture of polarized protons or of polarization effects in inelastic scattering enables one to determine not only the multipolarity of resonances but also to find the amplitudes of processes.

1. GENERAL RELATIONS AND DEFINITIONS

The density matrix of an ensemble of polarized particles is parametrized by means of an expansion with respect to a complete set of mutually orthogonal matrices with real matrix elements $\tau^{kq}_{mm'} = \sqrt{2j+1}\,(-)^{j-m'}(jmj\,-m'\,|\,kq)$. Then the expansion coefficients—the statistical tensors (spin tensors) t_{kq}—transform on the transition from the coordinate system K in which the density matrix is determined to some new system K' in accordance with an irreducible representation of the rotation group:

$$t_{kq}(K) = \sum_{q'} D^{k}_{qq'}(K \to K')\, t_{kq'}(K'). \tag{1}$$

The explicit form of the transformation matrices $D^{k}_{qq'}$ is given, for example, in Ref. 15. In applications, it is sometimes convenient to expand the density matrix with respect to a different complete set $\mathscr{P}_\nu$, where the index ν denotes 0, x, y, z, xx, xy, ..., xxx, xxy, xxz, etc. For example, for spin-$\frac{1}{2}$ particles,

$$\mathscr{P}_0 = 1;\ \mathscr{P}_x = \sigma_x;\ \mathscr{P}_y = \sigma_y;\ \mathscr{P}_z = \sigma_z. \tag{2}$$

For spin-1 particles, it is usual to take the set $\mathscr{P}_\nu$ as follows:

$$\left.\begin{aligned}&\mathscr{P}_0 = 1;\ \mathscr{P}_\alpha = S_\alpha\quad (\alpha = x,\, y,\, z);\\ &\mathscr{P}_{\alpha\beta} = 3S_\alpha S_\beta\ (\alpha \neq \beta);\ \mathscr{P}_{\alpha\alpha} = 3S_\alpha S_\alpha - 2.\end{aligned}\right\} \tag{3}$$

This set is overdetermined, since $\sum_\alpha \mathscr{P}_{\alpha\alpha} = 0$.

The coefficients of the expansion of the density matrix with respect to $\mathscr{P}_\nu$ are denoted by p_ν. The laws of transformation of the p_ν are less convenient than those for t_{kq}. However, the Madison Convention (Ref. 16, p. xxv) puts the Cartesian notation on an equal footing with the spherical notation.

In the reaction $1+2 \to 3+4$, the density matrix of the initial state ρ_i is transformed to the density matrix ρ_f of the final state by means of the scattering matrix $\hat{S}$ as follows:

$$\rho_f = (\hat{S}\rho_i\hat{S}^+)\,(\mathrm{Tr}\,\hat{S}\rho_i\hat{S}^+)^{-1}, \tag{4}$$

where the denominator $\sigma(\vartheta, \varphi) = \mathrm{Tr}\,\hat{S}\rho_i\hat{S}^+$ is the differential cross section of the process and ensures that the density matrix ρ_f is normalized to unity. The density matrix of the initial state is the direct product of the density matrices of the ensembles of particles 1 and 2:

$$\rho_i = \sum_{J_1M_1} t_{J_1M_1}\tau^{J_1M_1} \sum_{J_2M_2} t_{J_2M_2}\tau^{J_2M_2}. \tag{5}$$

This is because the conditions of the experiment enable one to prepare the states of particles 1 and 2 independently. With regard to the density matrix of the final state, it can be expanded with respect to the direct product of matrices $\tau^{J_3M_3}\tau^{J_4M_4}$ with certain coefficients $t_{J_3M_3J_4M_4}$, where $t_{J_3M_300}$ describes the polarization of particles 3 when the polarization of particles 4 is not measured, and $t_{00J_4M_1}$ describes the reverse situation. Using the orthogonality of the sets τ^{kq}, we can readily obtain expressions for $t_{J_3M_3J_4M_4}$ and σ (Ref. 17):

$$\left.\begin{aligned}&t_{J_3M_3J_4M_4}\sigma = (2j_1+1)^{-1}(2j_2+1)^{-1}\sum_{J_1M_1J_2M_2} t_{J_1M_1}t_{J_2M_2}\\ &\qquad \times A\begin{pmatrix}J_1J_2J_3J_4\\ M_1M_2M_3M_4\end{pmatrix};\\ &\sigma = (2j_1+1)^{-1}(2j_2+1)^{-1}\sum_{J_1M_1J_2M_2} t_{J_1M_1}t_{J_2M_2}\,A\begin{pmatrix}J_1\ J_2 00\\ M_1M_2 00\end{pmatrix};\\ &A\begin{pmatrix}J_1\ J_2\ J_3\ J_4\\ M_1M_2M_3M_4\end{pmatrix} = \mathrm{Tr}\,\hat{S}\tau^{J_1M_1}\tau^{J_2M_2}\hat{S}^+\tau^{J_3M_3}\tau^{J_4M_4}.\end{aligned}\right| \tag{6}$$

The quantity σ gives the cross section of the process for polarized particles 1 and 2, and the quantities proportional to

$$A\begin{pmatrix}J_1 & J_2 & 00\\ M_1 & M_2 & 00\end{pmatrix},$$

are called the analyzing powers for $J_1 \neq 0$ or $J_2 \neq 0$ and are simply the cross section σ_0 for unpolarized incident particles 1 and target nuclei 2 for $J_1 = J_2 = 0$:

$$\sigma_0 = (2j_1+1)^{-1}(2j_2+1)^{-1} A \begin{pmatrix} 0000 \\ 0000 \end{pmatrix}. \tag{7}$$

The polarization of the emitted particles 4 is expressed in terms of

$$A \begin{pmatrix} 000 & J_4 \\ 000 & M_4 \end{pmatrix}:$$

$$t_{J_4M_4}\sigma_0 = (2j_1+1)^{-1}(2j_2+1)^{-1} A \begin{pmatrix} 000\, J_4 \\ 000 M_4 \end{pmatrix}, \tag{8}$$

and the cross section of the process for polarized incident particles 1 is

$$\sigma = (2j_1+1)^{-1}(2j_2+1)^{-1} \sum_{J_1M_1} t_{J_1M_1} A \begin{pmatrix} J_1 000 \\ M_1 000 \end{pmatrix}. \tag{9}$$

The polarization transfer, i.e., the polarization of the emitted particles when the incident particles are polarized, is expressed in terms of

$$A \begin{pmatrix} J_1 & 00 & J_4 \\ M_1 & 00 & M_4 \end{pmatrix}.$$

We have also assumed that all the spin tensors and the scattering matrix are referred to one single coordinate system, which determines the quantization axis. Then the transition to the helicity representation[18] is achieved by a rotation of the system given by Eq. (1).

The expression for A after the matrix $\hat{S}$ has been expanded with respect to the multipoles is rather complicated and in the general theory in the channel-spin representation has the form[19]

$$\begin{aligned} A \begin{pmatrix} J_1\, J_2\, J_3\, J_4 \\ M_1 M_2 M_3 M_4 \end{pmatrix} = \frac{\pi}{K_i^2} \sum_{a_i b_i \alpha_i \beta_i l s J \bar{l} \bar{s}} (-)^{J_1+J_2+J_3+J_4+M_3+M_4} \\ \times (-)^{J+J'-s-s'+\bar{s}-\bar{s}'+l+\bar{l}'+a_1} \hat{j}_1 \hat{j}_2 \hat{j}_3 \hat{j}_4 \hat{s} \hat{s}' \hat{\bar{s}} \hat{\bar{s}}' \hat{l} \hat{l}' \hat{\bar{l}} \hat{\bar{l}}' \hat{J}^2 \hat{J}'^2 \\ \times \hat{J}_1 \hat{J}_2 \hat{J}_3 \hat{J}_4 \hat{a}_1 \hat{a}_2^2 \hat{a}_3^2 \hat{a}_4^2 \hat{a}_5 \begin{Bmatrix} j_1\, j_2 s \\ J_1 J_2 a_1 \\ j_1\, j_2 s' \end{Bmatrix} \begin{Bmatrix} j_3\, j_4 \bar{s} \\ J_3 J_4 a_5 \\ j_3 j_4 \bar{s}' \end{Bmatrix} \\ \times \begin{Bmatrix} s a_1 s' \\ a J l \end{Bmatrix} \begin{Bmatrix} a_2 s' J \\ J' a_3 l' \end{Bmatrix} \begin{Bmatrix} a_4 J' \bar{s} \\ \bar{s}' a_5 \bar{l}' \end{Bmatrix} \begin{Bmatrix} l a_1 a_2 \\ a_3 l' b_1 \end{Bmatrix} \\ \times \begin{Bmatrix} \bar{l}\, a_3 a_4 \\ a_5 \bar{l}' b_2 \end{Bmatrix} \begin{Bmatrix} b_1 a_1 a_3 \\ a_5 b_2 b_3 \end{Bmatrix} (J_1 M_1 J_2 M_2 \,|\, a_1 - \alpha_1)(J_3 M_3 J_4 M_4 \,|\, a_5 - \alpha_5) \\ \times (a_1 \alpha_1 a_5 - \alpha_5 \,|\, b_3 - \beta_3)(l0l'0 \,|\, b_1 0)(\bar{l} 0 \bar{l}' 0 \,|\, b_2 0)(\delta_{s\bar{s}} \delta_{l\bar{l}} \\ - \hat{S}^J_{sl,\, \bar{s}\bar{l}})(\delta_{s'\bar{s}'} \delta_{l'\bar{l}'} - \hat{S}^{J'*}_{s'l',\, \bar{s}'\bar{l}'}) \mathcal{Y}_{b_3\beta_3}(b_1 \mathbf{k}_i,\, b_2 \mathbf{k}_f). \end{aligned} \tag{10}$$

Here, $\hat{j} \equiv \sqrt{2j+1}$; s and $\bar{s}$ are the channel spins of the ingoing and outgoing channels; l and $\bar{l}$ are the corresponding orbital angular momenta; J are the total angular momenta. The generalized spherical functions are defined as follows:

$$\mathcal{Y}_{jm}(l_1\mathbf{k}_i,\, l_2\mathbf{k}_f) = \sum_{m_1 m_2} (l_1 m_1 l_2 m_2 \,|\, jm)\, Y_{l_1 m_1}(\mathbf{k}_i)\, Y_{l_2 m_2}(\mathbf{k}_f). \tag{11}$$

This expression can be transformed into a working formula only in the region of low energies, where the transmissions restrict the orbital angular momenta that participate in the reaction, or in the region of isolated resonances, in which only a restricted number of angular momenta with a Breit–Wigner dispersion law of the interfering elements of the reaction matrix are important. To calculate polarization effects, it is necessary to make certain model assumptions about the elements of the reaction matrix. The quantity

$$A \begin{pmatrix} J_1 & J_2 & J_3 & J_4 \\ M_1 & M_2 & M_3 & M_4 \end{pmatrix}$$

was calculated for direct deuteron stripping and pickup nuclear reactions in the DWBA and for two-nucleon stripping in Ref. 20. Interference of the direct and the compound processes and interference between a resonance and Coulomb stripping were considered in Refs. 21 and 22. The quantity

$$A \begin{pmatrix} J_i \\ M_i \end{pmatrix}$$

given above gave the observed quantities expressed in any one coordinate system. In applications, it is convenient to use the helicity representation, in which the polarizations of the incident and emitted particles are specified in the coordinate systems K and K' associated with them. The corresponding expressions naturally have a structure analogous to the quantity

$$A \begin{pmatrix} J_i \\ M_i \end{pmatrix}$$

given above. The observed parameters can be conveniently expressed in this case in terms of p_α and $p_{\alpha\beta}$ referred to the corresponding system.

Let us consider, for example, a reaction with the spin structure $1/2 + A \to B + 1/2$. This means that we shall be not interested in the polarization of the target nuclei A or the final nuclei B, but we shall only calculate the polarization of the emitted particles, the analyzing power, and the polarization transfer from the incident polarized particle with spin $\frac{1}{2}$ to the final particle with spin $\frac{1}{2}$. Then the density matrix of the initial state will have the form

$$\rho_i = (1 + \sum_i p_i \sigma_i)/2, \tag{12}$$

the density matrix of the final state

$$\rho_f = \hat{S}\hat{S}^+/2 + \sum_i p_i \hat{S} \sigma_i \hat{S}^+/2, \tag{13}$$

and the cross section of the process

$$I = \mathrm{Tr}\, \rho_f = I_0 (1 + \sum_i p_i A_i), \tag{14}$$

with the analyzing powers

$$A_i = (\mathrm{Tr}\, \hat{S} \sigma_i \hat{S}^+)(\mathrm{Tr}\, \hat{S}\hat{S}^+)^{-1}. \tag{15}$$

The polarization of the emitted particles is

$$p_{i'} I = I_0 (p_{0i'} + \sum_j p_j K_j^{i'}), \tag{16}$$

where $p_{0i'} = (\mathrm{Tr}\, \hat{S}\hat{S}^+ \sigma_{i'})(\mathrm{Tr}\, \hat{S}\hat{S}^+)^{-1}$ is the polarization of the emitted particles for unpolarized incident particles; $K_j^{i'} = (\mathrm{Tr}\,\hat{S}\sigma_j\hat{S}^+\sigma_i)\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}$ is the polarization transfer coefficient relating component i' of the polarization of the emitted particle to component j of the incident particle's polarization. To a very high degree of accuracy, P and T invariance are satisfied in nuclear reactions. For certain questions, one can add here isotopic invariance as well, although Coulomb effects in the nucleus are not small in all cases. Conservation of spatial parity imposes the following restrictions on the first-order observable quantities[23]:

$$t_{kq} = (-)^k t_{kq}^*. \tag{17}$$

A similar expression holds for the analyzing power. Thus, the spin tensors of even rank are real, and those of odd rank are purely imaginary. In Cartesian notation, this is equivalent to $A_x = A_z = p_x = p_z = 0$. A similar rule holds for the coefficients that describe the correlation of the polarizations. They are real if the sum of the ranks of the spin tensors is even and purely imaginary if the sum is odd. In the Cartesian notation, all coefficients with odd $n_x + n_z$ (n_x and n_z are the number of indices x and z of a coefficient) vanish. In addition, the same parity relations give the following rule: Observable quantities are even functions of the scattering angle ϑ if $n_x + n_z$ is even and odd functions if it is odd.

For the reaction $1/2 + A \to B + 1/2$ considered above, the the cross section and polarization of the emitted particles are expressed[24] by the following expressions ($z \| \mathbf{k}_i$; $z' \| \mathbf{k}_f$; $y(y') \| [\mathbf{k}_i \times \mathbf{k}_f]$):

$$\left.\begin{aligned} &\sigma(\vartheta) = \sigma_0(\vartheta)\,[1 + p_y A_y(\vartheta)];\ p_{y'}(\vartheta)\,\sigma(\vartheta) = \\ &= \sigma_0(\vartheta)\,[p_{0y'}(\vartheta) + p_y K_y^{y'}(\vartheta)]; \\ &p_{x'}(\vartheta)\,\sigma(\vartheta) = \sigma_0(\vartheta)\,[p_x K_x^{x'}(\vartheta) + p_z K_z^{x'}(\vartheta)]; \\ &p_{z'}(\vartheta)\,\sigma(\vartheta) = \sigma_0(\vartheta)\,[p_x K_x^{z'}(\vartheta) + p_z K_z^{z'}(\vartheta)]. \end{aligned}\right\} \tag{18}$$

Here, $K_x^{x'}$, $K_y^{y'}$, and $K_z^{z'}$ are even functions of the scattering angle ϑ and do not disappear when $\vartheta = 0$.

One sometimes uses the Wolfenstein notation for the polarization transfer coefficients: the depolarization parameter $D = K_y^{y'}$ and the polarization rotation parameters $R \equiv K_x^{x'}$, $A \equiv K_z^{x'}$, $R' \equiv K_x^{z'}$, $A' \equiv K_z^{z'}$. This notation is used only for reactions $1/2 + A \to B + 1/2$ in which the parity does not change. The observable polarization quantities p_i and $K_i^{j'}$ take values in the range from -1 to $+1$.

For reactions with spin structure $1/2 + 0 \to 0 + 1/2$, the connection between the polarization and the analyzing power has the form $p_y = \pm A_y$, where + sign corresponds to the situation when the intrinsic parity of the colliding particles does not change and the − sign to the situation when it does.

For reactions with spin structure $1 + A \to B + 1/2$, one needs to consider not only the polarization of the emitted particles for unpolarized incident particles, $p_{0y'} = (\mathrm{Tr}\,\hat{S}\hat{S}^+\sigma_{y'})\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}$, but also two further analyzing powers: the vector $A_\alpha = (\mathrm{Tr}\,\hat{S}\mathcal{P}_\alpha\hat{S}^+)\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}$ and the tensor $A_{\alpha\beta} = (\mathrm{Tr}\,\hat{S}\mathcal{P}_{\alpha\beta}\hat{S}^+)\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}$, and two polarization transfer coefficients: from vector polarization of the incident particle to vector polarization of the emitted particle, and from tensor polarization of the incident particle to vector polarization of the emitted particle. These polarization transfer coefficients are defined as follows:

$$\left.\begin{aligned} &K_j^{i'} = (\mathrm{Tr}\,\hat{S}\mathcal{P}_j\hat{S}^+\sigma_{i'})\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}; \\ &K_{ij}^{i'} = (\mathrm{Tr}\,\hat{S}\mathcal{P}_{ij}\hat{S}^+\sigma_{i'})\,(\mathrm{Tr}\,\hat{S}\hat{S}^+)^{-1}. \end{aligned}\right\} \tag{19}$$

The differential cross section σ of the process and the polarization components $p_{i'}$ of the emitted particle can be expressed with allowance for (19) as follows:

$$\left.\begin{aligned} &\sigma = \sigma_0\,(1 + 3p_y A_y/2 + 2p_{xz}A_{xz}/3 \\ &+ p_{xx}A_{xx}/3 + p_{yy}A_{yy}/3 + p_{zz}A_{zz}/3); \\ &p_{x'}\sigma = \sigma_0\,(3p_x K_x^{x'}/2 + 3p_z K_z^{x'}/2 \\ &+ 2p_{xy}K_{xy}^{x'}/3 + 2p_{yz}K_{yz}^{x'}/3); \\ &p_{y'}\sigma = \sigma_0\,(p_{0y'} + 3p_y K_y^{y'}/2 + 2p_{xz}K_{xz}^{y'}/3 \\ &+ p_{xx}K_{xx}^{y'}/3 + p_{yy}K_{yy}^{y'}/3 + p_{zz}K_{zz}^{y'}/3); \\ &p_{z'}\sigma = \sigma_0\,(3p_x K_x^{z'}/2 + 3p_z K_z^{z'}/2 \\ &+ 2p_{xy}K_{xy}^{z'}/3 + 2p_{yz}K_{yz}^{z'}/3). \end{aligned}\right\} \tag{20}$$

The quantization axis z is along the incident beam, the y axis is along $[\mathbf{k}_{\text{in}} \times \mathbf{k}_{\text{out}}]$, and the x axis is in the reaction plane, so that (x, y, z) form a right-handed system. Although the incident beam may contain all three components p_x, p_y, p_z of the polarization vector, the cross section is sensitive to only the component p_y. The normalization is chosen here such that the first-order quantities vary from -1 to $+1$. The second-order quantities p_{xy}, p_{yz}, and p_{xz} vary from $-3/2$ to $+3/2$, and the components p_{xx}, p_{yy}, p_{zz}, A_{xx}, A_{yy}, and A_{zz} vary from -2 to $+1$. The quantities $p_{\alpha\alpha}$ and $A_{\alpha\alpha}$, like the operators $\mathcal{P}_{\alpha\alpha}$, are not linearly independent, but $p_{xx} + p_{yy} + p_{zz} = 0$, and similarly for the analyzing powers $A_{xx} + A_{yy} + A_{zz} = 0$, so that only four of the five analyzing powers are independent. The quantities A_{xx}, A_{yy}, A_{zz}, $K_x^{x'}$, $K_y^{y'}$, $K_z^{z'}$, $K_{xy}^{z'}$, $K_{zz}^{y'}$ and $K_{yz}^{x'}$ are even functions of the angle ϑ and do not vanish at $\vartheta = 0$.

The analyzing powers $A_{ij}(\vartheta)$ in the Cartesian system are related as follows to the analyzing powers $T_{kq}(\vartheta)$ in the spherical coordinate system:

$$\left.\begin{aligned} &A_y = 2iT_{11}/\sqrt{3};\ A_{zz} = \sqrt{2}\,T_{20}; \\ &A_{xx} = \sqrt{3}\,T_{22} - T_{20}/\sqrt{2};\ A_{xz} = -\sqrt{3}\,T_{21}; \\ &A_{yy} = -\sqrt{3}\,T_{22} - T_{20}/\sqrt{2};\ (A_{xx} - A_{yy})/2 = \sqrt{3}\,T_{22}. \end{aligned}\right\} \tag{21}$$

The values of A_y, $p_{y'}$, and $K_y^{y'}$ lie between ±1; those of A_{xz}, $K_{xz}^{y'}$, etc, between ±3/2; and those of $K_{xx}^{y'}$, $K_{zz}^{y'}$, etc. between ± 2. Recall that we use different coordinate systems for the incident particle (a system with z axis along $\mathbf{k}_i$) and the emitted particle (a system with z' axis along $\mathbf{k}_f$). The y and y' axes coincide, and the x and x' axes are chosen to make both systems right-handed.

For reactions with spin structure $1/2 + A \to B + 1$,

$$\left.\begin{aligned} &\sigma = \sigma_0\,(1 + p_y A_y);\ p_{x'z'}\sigma = \sigma_0\,(p_{0x'z'} + p_y K_y^{x'z'}); \\ &p_{x'}\sigma = \sigma_0\,(p_x K_x^{x'} + p_z K_z^{x'});\ p_{y'z'}\sigma = \sigma_0\,(p_x K_x^{y'z'} + p_z K_z^{y'z'}); \\ &p_{y'}\sigma = \sigma_0\,(p_{0y'} + p_y K_y^{y'});\ p_{x'x'}\sigma = \sigma_0\,(p_{0x'x'} + p_y K_y^{x'x'}); \\ &p_{z'}\sigma = \sigma_0\,(p_x K_x^{z'} + p_z K_z^{z'});\ p_{y'y'}\sigma = \sigma_0\,(p_{0y'y'} + p_y K_y^{y'y'}); \\ &p_{x'y'}\sigma = \sigma_0\,(p_x K_x^{x'y'} + p_z K_z^{x'y'});\ p_{z'z'}\sigma = \sigma_0\,(p_{0z'z'} + p_y K_y^{z'z'}). \end{aligned}\right\} \tag{22}$$

The notation in these expressions is obvious. For example

$$K_{y}^{y'v'}=(\mathrm{Tr}\,\hat{S}\sigma_y\hat{S}^{+}\mathcal{P}^{+}_{y'v'})\,(\mathrm{Tr}\,\hat{S}\hat{S}^{+})^{-1}. \quad (23)$$

The quantities $K_x^{x'}$, $K_y^{y'}$, $K_z^{z'}$, $K_z^{x'y'}$, $K_x^{y'z'}$, $K_y^{x'z'}$, $p_{0x'x'}$, $p_{0y'y'}$, $p_{0z'z'}$ are even functions of the angle. The quantities $p_{i'}$, A_i, $p_{i'j}$ lie in the ranges $-1 \leq p_i, A_i \leq +1$; $-2 \leq p_{i'j'} \leq +1$.

If we wish to consider a reaction with the spin structure $1+A \to B+1$ in the Cartesian coordinate system, we need, in addition to the analyzing powers A_i and A_{ij} of the polarizations for unpolarized incident particles $p_{0i'}$ and the polarization transfer coefficients (vector → tensor) and (tensor → vector), also polarization transfer coefficients from tensor polarization of the incident particle to tensor polarization of the emitted particle:

$$K_{ij}^{i'j'}=(\mathrm{Tr}\,\hat{S}\mathcal{P}_{ij}\hat{S}^{+}\mathcal{P}^{+}_{i'j'})\,(\mathrm{Tr}\hat{S}\hat{S}^{+})^{-1}. \quad (24)$$

The differential cross sections and the polarizations expressed in terms of the analyzing powers, the polarizations for nonpolarized incident particles, and the polarization transfer coefficients are given in Refs. 24.

For reactions with the spin structure $1/2+1/2 \to A+B$, the cross section is expressed in terms of the analyzing powers (in order to distinguish the incident particle and the target, we shall identify the latter by appending an index T) and the polarization correlation coefficients:

$$C_{ij}=(\mathrm{Tr}\,\hat{S}\sigma_i\sigma_j^T\hat{S}^{+})\,(\mathrm{Tr}\,\hat{S}\hat{S}^{+})^{-1}. \quad (25)$$

Then the cross section of the process with polarized incident particle and polarized target can be expressed in terms of the analyzing powers and the polarization correlations as follows:

$$\sigma=\sigma_0\,(1+p_yA_y+p_y^TA_y^T+p_xp_x^TC_{xx}+p_yp_y^TC_{yy}+p_zp_z^TC_{zz}+p_zp_x^TC_{zx}+p_xp_z^TC_{xz}). \quad (26)$$

In the last expression, C_{xx}, C_{yy}, and C_{zz} are even functions of the angles. The values of C_{ij} vary in the range from -1 to $+1$.

For the analyzing power of $A(a,b)b'$ reactions, in which the final particles b and b' are either identical or belong to an isospin doublet, symmetry relations give $A_{kq}(\vartheta)=(-)^q A_{kq}(\pi-\vartheta)$. This relation is exact if $b=b'$ and is broken by the Coulomb interaction, which leads to isospin mixing.[25]

Invariance under the operation of time reversal leads to the following relation[23] between Eqs. (6) and the quantities

$$A\begin{pmatrix} J_1 & J_2 & J_3 & J_4 \\ M_1 & M_2 & M_3 & M_4 \end{pmatrix}:$$

$$A^{(\mathrm{direct})}(\mathbf{k}_i,\ \mathbf{k}_f)=(-)^{J_1+M_1+J_2+M_2+J_3+M_3+J_4+M_4}A^{(\mathrm{inverse})}\ (-\mathbf{k}_f,\ -\mathbf{k}_i).$$

From this there follow different relations for the observable polarization quantities, in particular: A_y(direct) $=p_y$(inverse). In addition, for elastic scattering, K_x^z(direct) $=-K_x^x$(inverse).

Let us now consider the determination of the S matrix from polarization experiments. The problem is posed as follows. What polarization experiments must be done in order to determine, at given angle and energy, all the $N=(2j_1+1)\,(2j_2+1)\,(2j_3+1)\,(2j_4+1)$ complex elements of the S matrix (Ref. 26)? In Ref. 27, Goldstein obtained sufficient conditions for determining the set of polarization experiments.

At low energies, and, especially, in the resonance region when only a small number of matrix elements is important in the multipole expansion, special cases of Eq. (10) are used. For the observable quantities of first order the expansion of $t_{kq}(\vartheta)\sigma_0(\vartheta)$ or the analyzing power with respect to Legendre polynomials $P_L(\vartheta)$ enables one to separate the products $S_1S_2^*$ corresponding to one parity of the matrix elements from products corresponding to different parities. If in the region of an isolated resonance only one matrix element is important, the ratio $a_{kq}(L)/a_{00}(0)\equiv d_{kq}(L)$ (here, $a_{00}(0)$ is the integrated reaction cross section) of the expansion coefficients does not depend on S at all, and the $d_{kq}(L)$ satisfy very simple relations. For example, for $L=2$

$$d_{20}(2)=-\sqrt{6}\,d_{21}(2)/2=\sqrt{6}\,d_{22}(2). \quad (27)$$

These relations are satisfied to a very good accuracy, for example, for the reaction ^{3}H$(\mathbf{d},n)^4$He in the region of the $3/2^+$ resonance at 107 keV. Some cases of more complicated situations have been considered by Seiler.[28]

2. METHODOLOGICAL ASPECTS OF INVESTIGATING POLARIZATION PHENOMENA IN NUCLEAR REACTIONS

Experiments to study the polarization of particles use various schemes based on determination of the asymmetries in the angular distributions of the products produced in nuclear reactions with polarized particles. In this section, we consider methodological aspects of experiments with particles of medium energies (1–50 MeV).

Polarization investigations require double and triple scattering experiments. The successes in obtaining and accelerating polarized particles and creating polarized targets have opened up new possibilities for studying polarization phenomena in nuclear reactions.

Double scattering. From Bishop's first experiments[7] in 1952 to the present day the overwhelming majority of polarization investigations on accelerators with unpolarized particle beams is made in accordance with the scheme shown in Fig. 1. The initial unpolarized beam of particles strikes the first target T. As a result of a nuclear reaction (or elastic scattering) polarized particles emerge from the target, and to determine the degree of polarization of these particles one generally uses elastic scattering on the second target, which is called the analyzer. Two detectors placed at the same angles ϑ with respect to the direction of emission of the particles from the second target record different numbers of particles. This difference in the intensities can be used to determine the polarization $p(\vartheta)$.

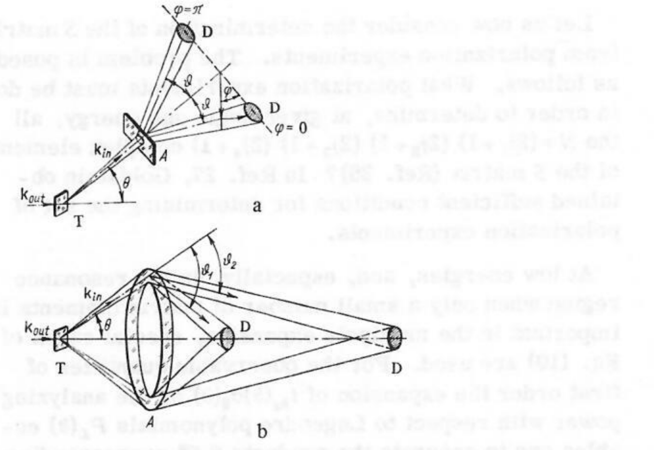

FIG. 1. Arrangement of polarization experiments. a) Measurement of left–right asymmetry; b) polarization experiment in ring geometry. T is the target, A the analyzer, and D the detector (the other notation is explained in the text).

If the beam of particles emitted from the first target at angle Θ has polarization p, then the differential cross section of the second reaction (scattering) depends on the angles ϑ and φ. The polarization of a beam of spin-$\frac{1}{2}$ particles is analyzed on the basis of the fact that the intensity of the particles scattered at the angles ϑ and φ is determined by Eq. (18) and has the form

$$I(\vartheta,\ \varphi)=I_0(\vartheta)\,[1+p_yA_y(\vartheta)]=I_0(\vartheta)\,[1+A(\vartheta)\,(\mathbf{pn})], \tag{28}$$

where the polarization (analyzing) power of the analyzer is known for certain angles in the so-called right-left geometry and in the whole range of angles for the ring geometry (see below). In accordance with the Basle Convention[29] the positive direction of polarization of the particles is taken to be the direction of the unit vector

$$\mathbf{n}=[\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{out}}]\,|\,[\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{out}}]\,|^{-1}. \tag{29}$$

This vector is perpendicular to the plane in which the scattering occurs. We denote the angle between the vector **n** and the polarization **p**, which we need to determine, by φ: $\mathbf{p}\cdot\mathbf{n}=|p|\cos\varphi$ (the angle φ is measured from the y axis, and not from the x axis, as usual!). We denote by L the intensity at $\varphi=0°$ (scattering to the left) and by R the intensity at $\varphi=180°$ (scattering to the right): $L=I_0(1+pA)$, $R=I_0(1-pA)$. Then the asymmetry of the scattering is

$$\varepsilon=pA=(L-R)/(L+R). \tag{30}$$

Dividing the asymmetry ε by the analyzing power A, which is known for the angle ϑ, we obtain the required value p of the beam polarization. The arrangement of a double-scattering experiment is shown in Fig. 1a.

One of the main problems of polarization-measurement experiments is the intensity. This problem is effectively solved by arranging the polarization experiment with a ring analyzer, which makes it possible to increase the intensity of the particles after the second scattering by 40–50 times compared with the left–right asymmetry polarimeter. In this geometry (see Fig. 1b) the polarization is determined by comparing the intensities N_1 and N_2 measured by two (or several) detectors placed at the same azimuthal angle φ but at different angles of second scattering ϑ_1 and ϑ_2.

For the ring geometry,

$$\varepsilon=(N_1-GN_2)/(N_2+GN_2); \tag{31}$$

$$\varepsilon=p\,(\Theta)\,[A\,(\vartheta_1)-A\,(\vartheta_2)]/\{p\,(\Theta)\,[A\,(\vartheta_1)+A\,(\vartheta_2)]-2\}, \tag{32}$$

where G is a normalization factor that takes into account the difference between the geometrical positions of the detectors. A polarization experiment with ring geometry was performed for the first time at Dubna.[30]

Note that the expressions (30)–(32) make it possible to obtain polarization data in two ways:

a) by investigating the characteristics of the first scattering or reaction using the experimental data L and R (or N_1 or N_2), one can determine the asymmetry ε and then, using the known analyzing power $A(\vartheta)$ of the second target, calculate the polarization $p(\Theta)$;

b) by studying the scattering on the second target and using the known polarization $p(\Theta)$ and the experimentally determined asymmetry ε, one can obtain information about the analyzing power $A(\vartheta)$.

Varying the angles Θ and ϑ, one can investigate $p(\Theta)$ and $A(\vartheta)$ in detail.

To obtain data from the averaged angular dependences of the polarization, the highly efficient method proposed by Romanovskiĭ[31] based on analysis of the spectra of backward elastic scattering of polarized protons by thick targets is of interest. If one is working with an unpolarized beam of fixed energy, the thick-target method makes it possible to obtain data on the averaged energy dependences of the polarization for elastic scattering of protons by nuclei.

Determinization of the polarization of spin-1 particles. This is more complicated than for spin-$\frac{1}{2}$ particles.[32] The intensity of the particles scattered to angles ϑ and φ is given by Eq. (20).

In addition to the coordinate system x, y, z (the z axis is along the incident beam, the y axis along the vector $[\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{out}}]$, and the x axis lies in the reaction plane in such a way that x, y, z form a right-handed system), we introduce the system X, Y, Z (the Z axis is taken as before along the beam, and the Y axis in such a way that the polarization vector lies in the YZ plane).

We denote by φ the angle between the yz and YZ planes, and append the indices X, Y, Z to the observable quantities in this new coordinate system. Then the intensity at angles ϑ and φ is

$$\begin{aligned}I(\vartheta,\ \varphi)=I_0(\vartheta)\,\{1&+3\,(p_x\sin\varphi+p_y\cos\varphi)\,A_y(\vartheta)/2\\&+2\,(p_{Xz}\cos\varphi-p_{Yz}\sin\varphi)\,A_{xz}(\vartheta)/3\\&+[(p_{XX}-p_{XX})\cos 2\varphi-2p_{XY}\sin 2\varphi]\\&\times[A_{xx}(\vartheta)-A_{yy}(\vartheta)]/6+P_{ZZ}A_{zz}(\vartheta)/2\}.\end{aligned} \tag{33}$$

The analyzing powers $A_y(\vartheta)$, $A_{xz}(\vartheta)$, $A_{xx}(\vartheta)-A_{yy}(\vartheta)$, $A_{zz}(\vartheta)$ are assumed known.

The intensities measured at one and the same scattering angle ϑ but at different azimuthal angles $\varphi = 0$, 180, 270, and 90° (to the left, to the right, upward, and downward) will be denoted by L, R, U, and D, respectively. One can then obtain five asymmetries for determining p_X, p_Y, and p_{IK}:

$$\left.\begin{array}{l} A_1=(L-R)(L+R)^{-1};\; A_2=(U-D)(U+D)^{-1};\\ A_3=\dfrac{2(L-R)}{L+R+U+D};\; A_4=\dfrac{2(U-D)}{L+R+U+D};\\ A_5=\dfrac{(L+R)-(U+D)}{L+R+U+D}. \end{array}\right\} \quad (34)$$

If, using Eq. (33), we express the asymmetries in terms of the polarizations and the analyzing powers, we find that this expression does not contain p_{XY}. This is due to the special choice of the azimuthal angles φ. If instead of the set $\varphi = 0$, 180, 270, and 90° we make measurements at $\varphi = 45$, 225, 315, and 135° (L', R', U', D'), then instead of A_5 we obtain the asymmetry A_5', which does not depend on p_{XY}:

$$A_5'=\frac{-p_{XY}(A_{xx}-A_{yy})/3}{1+p_{ZZ}A_{zz}/2} \quad (35)$$

Of course, only four of these five asymmetries are independent, and one set of measurements alone is insufficient to determine the unknown polarizations p_X, p_Y, p_{XY}, p_{YZ}, p_{XZ}, $(p_{XX}-p_{YY})$, p_{ZZ}. It is therefore necessary to make one further experiment in which the analyzing beam is first rotated without depolarization by an external electric or magnetic field.

The situation simplifies significantly in the following practically important cases:

a) when one wishes to determine the polarization parameters of a beam produced in a source of polarized particles for which the direction of the polarizing magnetic field is known but the effective populations N_{+1}, N_0, N_{-1} of the sublevels, which are related to the beam polarization by

$$\left.\begin{array}{l} p_Z=(N_{+1}-N_{-1})/(N_{+1}+N_0+N_{-1});\\ p_{ZZ}=(N_{+1}-2N_0+N_{-1})/(N_{+1}+N_0+N_{-1}) \end{array}\right\} \quad (36)$$

(the Z axis is along the field), are not known exactly;

b) when the polarization parameters of the beam of particles from the source are known together with the direction of the orienting field, and one wishes to determine the unknown analyzing powers;

c) when the analyzed beam is produced in a reaction initiated by an unpolarized beam and the direction of the polarization vector is definitely perpendicular to the reaction plane.

For cases a) and b), denoting the angle between the direction of the beam and the orienting field by α, we obtain the following expressions for the asymmetries:

$$A_1=\frac{3p_Z\sin\alpha A_y/2}{1+p_{ZZ}(\sin^2\alpha A_{yy}+\cos^2\alpha A_{zz})/2};$$

$$A_2=\frac{p_{ZZ}\sin\alpha\cos\alpha A_{xz}}{1+p_{ZZ}(\sin^2\alpha A_{xx}+\cos^2\alpha A_{zz})/2};$$

$$A_3=\frac{3p_Z\sin\alpha A_y/2}{1+p_{ZZ}(3\cos^2\alpha-1)A_{zz}/4};$$

$$A_4=\frac{p_{ZZ}\sin\alpha\cos\alpha A_{xz}}{1+p_{ZZ}(3\cos^2\alpha-1)A_{zz}/4};$$

$$A_5=\frac{-p_{ZZ}\sin^2\alpha(A_{xx}-A_{yy})/4}{1+p_{ZZ}(3\cos^2\alpha-1)A_{zz}/4};$$

$$A_5'=0. \quad (37)$$

If the direction of the magnetic field coincides with the direction of the beam, the analysis is particularly simple since the cross sections depend only on p_{ZZ}:

$$L=R=U=D=I_0(1+p_{ZZ}A_{zz}/2). \quad (38)$$

In practice, the second-rank tensors in the polarization of deuterons at low energies can be determined by using ^{3}He as an analyzer. For the $3/2^+$ resonance at 430 keV in the reaction ^{3}He(d, p)^{4}He, the analyzing powers are[33]

$$\left.\begin{array}{l} A_y=0;\\ A_{xz}=-3\varkappa\sin\vartheta\cos\vartheta/2;\\ (A_{xx}-A_{yy})/2=-3\varkappa\sin^2\vartheta/4;\\ A_{zz}=-\varkappa(3\cos^2\vartheta-1)/2,\ \varkappa\approx 0.90. \end{array}\right\} \quad (39)$$

The arrangement of an experiment to analyze the polarization of particles with spin 1 using ^{3}He as an analyzer of the deuteron tensor polarization is shown in Fig. 2. The scattering angle ϑ is taken to be 54.7°. For $\vartheta = 0$, we have $A_{zz} = -\varkappa$; if $\vartheta = 54.7°$, then $A_{zz} = 0$. The ratio of the yield at $\vartheta = 0$ to the sum of the yields of the remaining four detectors determines p_{ZZ}, while p_{XZ}, p_{YZ}, and $p_{XX}-p_{YY}$ are determined from the asymmetries. Finally, rotating the detectors through 45°, we obtain p_{XY}.

Triple scattering. A further development of double scattering is an experiment with triple scattering, which enables one to extract valuable information about more complicated polarization parameters (depolarization, polarization transfer coefficients, etc). The first scattering is used to obtain a beam with known polarization p_1. In the second scattering, the interaction of the polarized beam with the target is investigated, this interaction leading to some new polarization p_2, which is analyzed in the third scattering. Double and triple scattering experiments present considerable methodological difficulties. The use of polarized beams and targets makes it possible to reduce by one the number of scatterings and thus provide a real possibility, on the one hand, for extending the existing programs of polarization investigation and, on the other, making possible

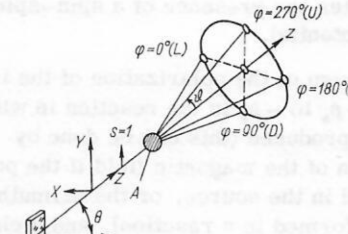

FIG. 2. Arrangement of experiment to analyze the polarization of spin-1 particles. T is the target and A the analyzer (the other notation is described in the text).

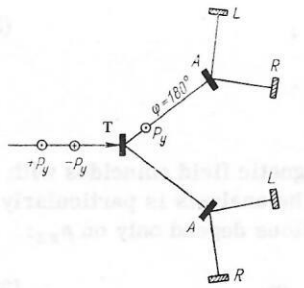

FIG. 3. Arrangement of experiment to measure polarization transfer in double scattering. T is the target, A the analyzer, and L and R are the detectors.

for the first time certain further polarization experiments.

It is worth pointing out that the reduction of triple scattering experiments to double scattering experiments by using an accelerated polarized beam means that the further development of the double-scattering method remains among the list of important tasks. A good example of such a development is Balashko's suggestion[34] to use the method of the polarization experiment in the ring geometry[30] to study polarization transfer, in particular in the (d, p) reaction.

Polarization-transfer measurements in double scattering. Let us consider the situation when spin-$\frac{1}{2}$ particles produced in some reaction have only a polarization component p_y along the y axis (Fig. 3). Then in accordance with Eqs. (22), the cross section and the $p_{y'}$ polarization component of the particles emitted in the reaction $1/2 + A \to B + 1/2$ (first scattering) can be expressed in terms of the analyzing power $A_y(\vartheta)$, the depolarization parameter $K_y^{y'}(\vartheta)$, and the polarization $p_{0y'}(\vartheta)$ for unpolarized incident particles as follows:

$$\sigma(\vartheta) = \sigma_0 [1 + p_y A_y(\vartheta)]; \quad (40)$$

$$p_{y'}\sigma = \sigma_0 [p_{0y'}(\vartheta) + p_y K_y^{y'}(\vartheta)]. \quad (41)$$

We assume the analyzing power of the second scattering known and equal to A_{2y}; the cross section σ_2 is known (the angle of the second scattering is constant, and will be omitted in the subsequent expressions). Then the intensities measured after the second scattering are given by

$$\begin{aligned} L &= I I_2 [1 + p_y A_y + (p_{0y'} + p_y K_y^{y'}) A_{2y}], \\ R &= I I_2 [1 + p_y A_y - (p_{0y'} + p_y K_y^{y'}) A_{2y}]. \end{aligned} \quad (42)$$

Note that for the elastic scattering of spin-$\frac{1}{2}$ particles on spin-0 nuclei, $K_y^{y'} = 1$. Therefore, the deviation of $K_y^{y'}$ from unity indicates the presence of a spin–spin term in the optical potential.

The change in the sign of the polarization of the incident particles from $+p_y$ to $-p_y$ in the reaction in which the incident beam is produced (this can be done by changing the direction of the magnetic field if the polarized beam is obtained in the source, or the azimuthal angle if the beam is formed in a reaction), and a change of the azimuthal scattering angle at the target from $\varphi = 0$ (a "+" experiment) to $\varphi = 180°$ (a "−" experiment) gives the intensities L_{++}, R_{++}, R_{-+}, L_{-+}, R_{+-}, L_{+-}, R_{--}, L_{--}. From these intensities, the depolarization $K_y^{y'} \equiv D$ is found.

Measurement of spin–spin correlations in the initial state. The geometry of this experiment for $1/2 \leftrightarrow 1/2$ correlations does not differ from the geometry of an ordinary double-scattering experiment, and it can be analyzed by means of Eq. (26):

$$I = I_0(\vartheta) [1 + p_y A_y(\vartheta) - p_y^T A_y^T(\vartheta) + p_y p_y^T C_{yy}(\vartheta)], \quad (43)$$

where the superscript T indicates that the corresponding quantity refers to the target. The polarization vectors of the beam and the target are directed along the y axis. For the right and left scattering intensities, we obtain

$$\left.\begin{aligned} L &\sim 1 + p_y A_y + p_y^T A_y^T + p_y p_y^T C_{yy}; \\ R &\sim 1 - p_y A_y - p_y^T A_y^T + p_y p_y^T C_{yy}, \end{aligned}\right\} \quad (44)$$

which enables us, if we know the polarization of the particles and the target nuclei, to determine the polarization correlation coefficient C_{yy}.

Polarization Analyzers. Besides having to measure the asymmetry ε with the smallest error possible, depending on the type of the experiment, one must also have detailed and accurate data on the analyzing power A or the polarization p [see Eqs. (30)–(32)]. The choice of the scatterer, the polarization analyzer, is dictated by the conditions of the experiment. Usually, nuclei with spin 0, for which the phase-shift analysis of the scattering can be made fairly fully and for which the analyzing power is known, are used as analyzers. One can determine A_y from double scattering, and also by the method of inverse reactions. To find the polarization, helium and carbon nuclei are most frequently used, for which data on the analyzing power are obtained from double-scattering experiments in wide energy and angular ranges.

Spurious assymetry. When one is investigating polarization effects, it is important to know the contribution due to the so-called spurious symmetry. In real experiments, one determines an asymmetry that is made up of the true and the spurious asymmetry. For example, in a determination of polarization by a measurement of asymmetry after left-right scattering, sources of spurious asymmetry can arise from inaccurate preparation of the polarimetric block and the setting up of the analyzer, error in the regulation of the polarimeter with respect to the direction of the ion beam, a displacement of the centers of the detectors with respect to the axes of the channels, differences between the efficiencies of the detectors used to detect the particles, and so forth. It is therefore necessary to make control and calibration experiments. A description of ways to eliminate or take into account rigorously the contribution of various sources to spurious asymmetry can be found, for example, in Refs. 30 and 35.

Beams of polarized particles. The extraction from double and triple scattering experiments of exceptionally important information entails the overcoming of serious methodological difficulties due to the low intensity of the detected particles. On top of this, the observed effect

TABLE I. Parameters of accelerators with beams of polarized ions.

Laboratory, first year with polarized beam	Type of accelerator	Type of particle	Degree of polarization of beam (exp)	Intensity of beam from accelerator, nA	Beam energy, MeV
Carnegie (USA), 1962	Van de Graaff accelerator	H^+	~0.35	7–10	0.35–3
Berkeley (USA), 1969	Cyclotron	H^+ D^+	0.8 0.54; 0.8	200–300 200–300	9–55 12–65
Stanford (USA), 1970	Van de Graaff tandem	H^- D^-	0.67 0.47	40–100 —	2–18 —
Heidelberg (West Germany), 1973	The same	$^6Li^-$ $^{23}Na^-$	>0.46 –	30–180 2	4–24 6–36
Villigen (Switzerland), 1975	Two-stage cyclotron	H^+	0.65	30–200	10–75; 590
		D^+	–	30–200	10–65
TRIUMF (Canada), 1975	Cyclotron	H^-	0.80	<200	200–500
Birmingham (England), 1975	"	$^3He^+$	0.38	0.15	33.4
Los Alamos (USA), 1975	Van de Graaff tandem	H^- T^-	0.85 0.85	100–200 50–150	2–18 –

is small because the beam is rarely polarized by 100% in the first scattering, and the last scattering is not a perfect analyzer.

It is necessary to have precise geometry since the reaction cross section may depend strongly on the angle. A source of error can arise from insufficient accuracy in the data on the polarizing (analyzing) properties of the targets. To a large extent, these difficulties can be overcome in two ways: by using a polarized accelerated beam or a target of polarized nuclei.

In 1956, it was suggested that a beam of polarized particles could be obtained by means of a source of polarized ions set up in an accelerator,[36] and in 1960 polarized protons were obtained for the first time.[29] This was the start of a new chapter in the study of polarization in nuclear reactions. The proceedings of the symposia on polarization phenomena[16,29,37,38] clearly illustrate the main stages in the development of the methods and technique for obtaining polarized beams. In the present paper, we shall say a few words about the achievements in obtaining polarized beams. A detailed discussion of the obtaining and accelerating of beams of polarized particles can be found in special reviews.[39]

There are two types of source of polarized ions: An ionization source of an atomic beam and a source based on the Lamb method. In the first type, in which the components of the hyperfine splitting of the hydrogen-atom ground state are separated in an inhomogeneous magnetic field, the atoms of the beam are ionized by electron impact. In various laboratories two variants of this physical principle of the source of polarized ions are used in working accelerators: ionization in weak[40] or strong[41] magnetic fields.

In 1950, Lamb[42] proposed the idea of using a metastable state of the hydrogen atom in one of the stages of obtaining polarized hydrogen ions, and in 1964 the possibility of technical realization of a source of negative ions with polarized protons and deuterons was proved.[43]

At the present time, more than 40 accelerators provide beams of polarized protons and deuterons.[38] Recently, polarized beams of tritons, ^3He, ^6Li, and ^{23}Na have been obtained. Data on the installations that, judged by the parameters of the polarized beams, are the best for a given type of accelerator are given in Table I.

It can be seen from this table that polarization effects in nuclear reactions can now be studied in a wide range of energies: from 0.35 to 590 MeV. The development of sources of polarized ^6Li and ^7Li ions will rapidly make it possible to obtain ion beams of other alkali metals: ^{23}Na, $^{39,41}K$, ^{85}Rb, and ^{133}Cs.

Polarized Targets. Besides polarized beams, the use of targets with polarized nuclei provides another effective means of overcoming the experimental difficulties with multiple scattering.

Some successes have now been achieved in producing polarized targets to study nuclear reactions at low energies: proton, deuteron, ^3He, and ^6Li targets. One of the methods of obtaining polarized targets is based on the use of dynamical polarization in a solid diamagnet with paramagnetic impurities. This method has achieved polarization of protons (~50%) in a single crystal of lanthanum magnesium double nitrate with 1% neodymium impurity. The results of the first experiments to investigate nuclear reactions at low energies with proton targets are given in Ref. 44. The method of dynamical polarization was used to create a polarized deuteron target, on which scattering of polarized low-energy neutrons was investigated, which made it possible to choose between two sets of scattering lengths.[45]

To produce a polarized ^{3}He target, one can use the method of optical pumping, by means of which one can achieve a 40% polarization of ^{3}He nuclei. Examples of the successful obtaining of such targets and their use for solving problems in nuclear physics can be found in the papers of Ref. 46. In Ref. 47, a polarized ^{6}Li target obtained by using an atomic beam of polarized ^{6}Li atoms is described.

The obtaining of polarized targets in the majority of methods is governed from the experimental point of view by the development of the technique for producing ultralow temperatures. These complicated developments have been transformed into an independent field of activity that straddles nuclear physics and solid-state physics. The methods used to polarize nuclei and the arrangement of experiments with polarized targets are discussed with an extensive bibliography in Refs. 9, 16, 37, and 48.

To polarize heavy nuclei, equilibrium thermal methods would seem to be the most promising. In them, one either polarizes directly the nuclear spins by a strong magnetic field under deep cooling conditions —this is the brute force method—or one aligns the electron spins to take advantage of subsequent transitions between hyperfine splitting levels. Targets with polarized ^{59}Co, ^{115}In, and ^{165}Ho nuclei are described in Refs. 49–51, but these targets have not yet been used in polarization experiments.

3. FEW-NUCLEON SYSTEMS

Three-nucleon systems. Three-nucleon systems have been studied in the $\mathbf{p}+d$, $\mathbf{d}+p$, $p+\mathbf{d}$, and $\mathbf{n}+d$ elastic-scattering reactions.[52–58] The differential cross sections σ, the vector and tensor analyzing powers iT_{11} and T_{2q}, the vector and tensor polarizations p and p_{ik} of the reaction products, the spin–spin correlation coefficients C_{xx} and C_{yy}, and the polarization transfer coefficients $K_y^{y'}$ were measured.

The interest in three-nucleon systems is largely due to the hope of obtaining new information about the nucleon–nucleon forces and the forces that act between three nucleons. This hope is based on the fact that modern computers make it possible to solve the Faddeev equations for three bodies with the necessary accuracy for any nucleon–nucleon interaction.

In the majority of investigations,[53,55,59–62] solutions of the equations for various nonlocal separable nucleon–nucleon interactions have been sought. Different numbers of orbital angular momenta (up to $l=2$) and tensor forces have been used. In some cases, the effect of the higher angular momenta has been taken into account approximately. The results of the calculations[59,60,63,64] for the differential cross section and the various polarization parameters describe well qualitatively, and in the majority of cases quantitatively as well, the experimental data at an energy of order 10 MeV in the center-of-mass system. Appreciable qualitative differences are observed for the parameters p and iT_{11}.

It should be pointed out that the tensor forces employed do not enable one to obtain simultaneously the correct values for the phase shifts $\delta(^3D_1)$ of the nucleon–nucleon interaction and the mixing parameter ε_1 (Ref. 60). In Refs. 63 and 65, local potentials were used instead of a separable potential, and these reproduce $\delta(^3D_1)$ and ε_1. However, the deviation of the calculated values from the experimental is of the same kind as for separable potentials. In none of these calculations are free parameters used. The task of further investigations is to establish the connection between the existing discrepancies and the uncertainties in the determination of the *NN* phase shifts.

Study of the three-nucleon systems has shown that whereas it is sufficient to take into account only *S* waves in the nucleon–nucleon potential to describe the differential cross sections, the tensor components of the polarization can be described only with allowance for tensor forces. And in order to describe the vector polarization, it is necessary to take into account simultaneously *S*, *P*, and *D* waves and tensor forces.

Measurements of the differential cross sections, the polarizations, and the analyzing powers of reactions make it possible to determine much more precisely the necessary approximation for the nucleon–nucleon potential which is introduced. However, these measurements do not give sufficiently accurate information on the doublet *ND* amplitude, which, in its turn, is sensitive to the short-range part of the nucleon–nucleon potential. Such information can be obtained only by investigating the spin–spin correlations C_{xx} and C_{yy}. Such experiments have been begun,[56,57,63] and there are grounds for hoping that they will give important information about nucleon–nucleon forces.

In addition to the study of nucleon–nucleon forces in three-nucleon systems, considerable attention is devoted to the investigation of the excited state of the ^{3}He nucleus at excitation energy $E^*=14$ MeV. Information about this state is obtained from detailed phase-shift analysis of the experimental results of elastic proton scattering on deuterons. The information obtained is as yet ambiguous; Two values, $1/2^-$ or $3/2^-$, are ascribed to the total angular momentum.[55] Further investigations are required to clarify this question.

Recently, work has begun on study of the polarization characteristics of deuteron disintegration initiated by protons.

Four-nucleon systems. Polarization effects in systems consisting of four nucleons have been investigated in the reactions $\mathbf{p}+T$, $\mathbf{n}+^3$He, $\mathbf{p}+^3$He, $\mathbf{d}+d$, and ^{3}He$+p$ (Refs. 66–71). In these reactions, the differential cross sections, the polarization, and the analyzing powers were measured, and in some cases the polarization transfer coefficients were studied in detail (Table II).

The interest in the study of four-nucleon systems is due, on the one hand, to the fact that these nuclei are the lightest in which excited states are observed, and on the other hand, to the development in recent years of effective methods for calculating such systems.[72] It has not yet proved possible to calculate a four-nucleon

TABLE II. Basic parameters investigated in four-nucleon systems.

Type of reaction (literature)	Energy range, MeV	Measured parameters	Phase-shift analysis
$p+T$ [66, 68]	1–15	σ, iT_{11}	Up to E_p=7MeV
$p+{}^3He$ [69]	2–15	σ, iT_{11}, $K_x^{x'}$, $K_y^{y'}$, $K_z^{z'}$	—
$p+{}^3He$ [71]	32	σ, iT_{11}	—
$n+{}^3He$ [67]	8–17	σ, iT_{11}	Up to E_n=25MeV
$d+d$ [70]	0,1–15	σ, iT_{11}, T_{20}, T_{21}, T_{22}, $K_y^{y'}$, $K_x^{x'}$, $K_z^{z'}$, K_{yz}^{x}, $K_{yy}^{y'}$, K_{xz}^{y}, K_{zz}^{y}	For E_d < 1 MeV

system by means of the Faddeev equations. The theoretical investigations therefore use less rigorous approaches, and a considerable number of calculations have been made of excited states of these nuclei in the framework of the shell model with allowance for residual particle–hole interaction.[66,73]

In Ref. 74, microscopic calculations were made by resonating-group method with allowance for channel coupling. In these calculations, not only the level positions but also quantities directly measured in reactions were calculated. In none of the calculations did it prove possible to reproduce correctly the sequence of all excited states found experimentally (negative-parity states with isospin $T=0$ and 1), although the majority of states have positions qualitatively in agreement with experiment.

Except for the ground state of the ^4He nucleus, all levels of the four-nucleon systems lie in the continuum and have larger (single-particle) widths. This circumstance renders their experimental detection extremely difficult. Only a phase-shift analysis of all the experimental quantities and parametrization of the phase shifts by means of the equations of the R-matrix theory make it possible to establish the existence of the levels. In order to carry out the phase-shift analysis, it is necessary to measure a definite number of parameters (cross section, polarization, polarization transfer coefficients, etc) in a wide range of energies. Currently, information is being gathered rapidly. It can be seen from Tables I and II that experiments are already being made with polarized beams of protons, deuterons, tritons, and ^3He ions in a wide energy range.

In Ref. 75, a phase-shift analysis was made of the experimental data for p^3He and n^3H scattering, namely, the differential cross sections, the polarizations, and the asymmetry. The analysis did not lead to a unique result, and several sets of phase shifts were found. All sets are characterized by the presence of broad levels of the nuclei ^4Li and ^4H in states formed with $l=1$, and the absence of levels in states with $l=0$. The isospin of the ^4Li and ^4H levels is naturally 1. The phase-shift analysis for these four-nucleon systems is simpler than for ^4He, since for them fewer channels are open in the investigated energy range. In Ref. 76, a phase-shift analysis of p^3He scattering was made with allowance also for experimental data on spin correlation, which made it possible to reduce significantly the previously found uncertainties.

Phase-shift analysis of interactions leading to the compound nucleus ^4He proved to be a more complicated task. A phase-shift analysis of pT scattering[66] and $n\,^3He$ scattering[67] was made. However, the analysis used only the experimental data on the differential cross section, the polarization, and the total reaction cross section. For this reason, the number of parameters that had to be determined was appreciably greater than the number of independent quantities that could be extracted from the experiment. The results of these phase-shift analyses are ambiguous and cannot be regarded as reliable.

For the ^4He nucleus an important advance in the analysis was achieved by using the principle of charge invariance in the inner region of the nucleus. This made it possible to reduce appreciably the number of independent parameters determining the elements of the S matrix, and to describe simultaneously three interactions, namely $^3He(n,n)\,^3He$, $^3H(p,n)\,^3He$, and $^3H(p,p)\,^3H$, by means of a single set of parameters. A charge-invariant phase-shift analysis[77] produced a unique solution and reliably established the existence of three excited states of the ^4He nucleus with isospin $T=0$ ($0^+, 0^-, 2^-$). However, the last result does not remove the problem of analyzing the experimental data without this additional assumption. It is also necessary to solve the problem of analyzing the excited states of the ^4He nucleus at high excitation energies.

In the investigation of the four-nucleon system, considerable attention has been devoted to testing charge symmetry. Conclusions about this question can be drawn by comparing the values of the polarization p or the analyzing power A of the reaction for $^2H(d,p)\,^3H$ and $^2H(d,n)\,^3He$ or $^3H(p,p)\,^3H$ and $^3He(n,n)\,^3He$. In studies made in recent years, it has been established that the values of p (or A) for these reactions are equal (Fig. 4).[67] It has been shown that the previously observed difference was due to the fact that the Coulomb energy in the outgoing reaction channel was not taken into account.

Note that in the investigation of the four-nucleon systems the contribution of different types of nucleon–nucleon potential to the description of this system has not yet been analyzed.

Five-nucleon systems. Systems consisting of five nucleons have been investigated[78–88] in the following

TABLE III. Basic parameters investigated in five-nucleon systems.

Type of reaction (literature)	Energy range, MeV	Measured parameters	Phase-shift analysis
$p+{}^4He$ [78, 79]	1–18	σ, iT_{11}	Up to E_p=18 MeV
$d+{}^3He$ [80–82]	1–16	σ, iT_{11}, T_{20}, T_{21}, T_{22}, $K_x^{x'}$, $K_y^{y'}$	—
$d+{}^3He$ [84]	3,7–12	σ, iT_{11}, T_{20}, T_{21}, T_{22}, C_{yy}, C_{xy}, C_{zx}, $C_{yy,y}$	—
$d+{}^3He$ [83]	27, 33	σ, iT_{11}	—
$d+{}^3H$ [85, 86]	1–12	σ, iT_{11}, T_{20}, T_{21}, T_{22}	—
$d+{}^3H$ [87]	10,5	σ, iT_{11}, $K_y^{y'}$	—

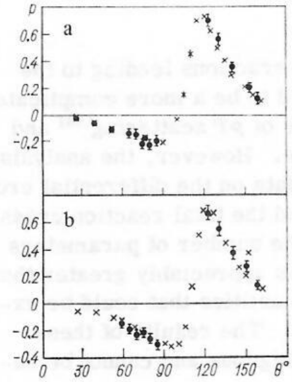

FIG. 4. Comparison of angular dependences of the polarization p for various reactions: a) the black circles, ${}^3\mathrm{He}(n, n)\,{}^3\mathrm{He}$, at 8.0 MeV; the crosses, ${}^3\mathrm{H}(p, p)\,{}^3\mathrm{H}$ at 9.4 MeV; b) the black circles, ${}^3\mathrm{He}(n, n)\,{}^3\mathrm{He}$ at 12.0 MeV; the crosses, ${}^3\mathrm{H}(p, p)\,{}^3\mathrm{H}$ at 13.4 MeV.

nuclear reactions with polarized particles: $\mathbf{p}+{}^4\mathrm{He}$, $\mathrm{d}+{}^3\mathrm{He}$, $d+{}^3\mathbf{He}$, $\mathbf{d}+{}^3\mathrm{H}$, $d+{}^3\mathbf{H}$, $\mathbf{d}+{}^3\mathbf{He}$. Interest in the study of five-nucleon systems was stimulated by the possibility of obtaining information about the structure of the ${}^5\mathrm{Li}$ and ${}^5\mathrm{He}$ nuclei and comparing the results of phase-shift analysis of the experimental data with theoretical calculations (Table III). The most complete theoretical study of the $d+T$ and $d+{}^3\mathrm{He}$ systems was made in Ref. 88 using the single-channel resonating-group method with phenomenological imaginary potentials. The phase shifts were calculated right up to $l=7$; an R-matrix analysis of the phase shifts made in Ref. 88 indicates a large number of levels with resonance energies below 12 MeV. This question requires a more complete investigation.

It can be seen from Table III that the five-nucleon systems have been studied experimentally very intensively. However, because of the complexity of these systems, insufficient data for making an accurate phase-shift analysis of the interaction of the deuteron with ${}^3\mathrm{He}$ nuclei or T have so far been obtained. The variants of experiments needed for analysis of the $d+{}^3\mathrm{He}$ system were considered in detail in Ref. 82.

A phase-shift analysis for the $p+{}^4\mathrm{He}$ system has been made with sufficient reliability in the energy range up to 30 MeV (Figs. 5 and 6). It has been shown that the S, P, D, and F phase shifts are important. Study of the five-nucleon systems has already made it possible to obtain information about levels of the nuclei ${}^5\mathrm{Li}$ and ${}^5\mathrm{He}$.

4. RESONANCE NUCLEAR REACTIONS

Reactions of this type have mainly been studied with a view to obtaining information about the quantum numbers of excited states of compound nuclei. Information about such levels is obtained by evaluating experimental data on the polarization and cross section by the method of phase-shift analysis. In Refs. 89–92, in the framework of the S-matrix formalism in the channel-spin representation, expressions were obtained that make it possible to obtain the quantum numbers of the levels of the investigated nuclei from experimental data on the polarization and cross section.

The $A(p,p)A$ reactions on spin-0 nuclei. Under favorable conditions, one can obtain all the elements of the S matrix from measurements of the cross section and polarization, and for resonances one can obtain the quantum numbers of the levels of the nucleus and the parameters of R-matrix theory. It is convenient to describe the elastic scattering of protons on spin-0 nuclei, not by specifying the S matrix as a function of the energy, but by means of a complete set of phase shifts, which can then be used to recover the S matrix. At relatively low energies, the number of phase shifts that must be taken into account is small, which makes it possible to carry out the phase-shift analysis.

The differential cross section and the polarization can be expressed[9] in this case in terms of the amplitude of coherent and incoherent scattering $A(\vartheta)+B(\vartheta)[\sigma\times\mathbf{n}]$ as follows:

$$\mathrm{p}(\vartheta)=\frac{2\,\mathrm{Im}\,(A^*B)}{|A|^2+|B|^2}\,\mathbf{n}; \tag{45}$$

$$\sigma(\vartheta)=\lambda^2\left(|A|^2+|B|^2\right). \tag{46}$$

The quantities $A(\vartheta)$ and $B(\vartheta)$ are given by

$$A(\vartheta)=-\frac{1}{2}\eta\,\mathrm{cosec}^2\,\vartheta/2\exp\left(i\eta\ln\mathrm{cosec}^2\,\vartheta/2\right)$$
$$+\sum_{l=0}^{\infty}\left[(l+1)f_l^++lf_l^-\right]\exp\left(i\alpha_l\right)P_l(\cos\vartheta); \tag{47}$$

$$B(\vartheta)=\sin\vartheta\sum_{l=1}^{\infty}\left(f_l^+-f_l^-\right)\exp\left(i\alpha_l\right)P_l^1(\cos\vartheta), \tag{48}$$

where

$$f_l^{\pm}=\left(S^{\pm}-1\right)/2i; \tag{49}$$

α_l is the phase shift due to Coulomb scattering; Z is the charge of the target nucleus; v is the velocity of relative motion; and $P_l(\cos\vartheta)$ and $P_l^1(\cos\vartheta)$ are Legendre polynomials and associated Legendre functions.

The diagonal elements of the scattering matrix $S_l^{\pm}$ can be determined from R-matrix theory. In the single-channel single-resonance approximation, the elements have the particularly simple form

$$S_l^{\pm}=\exp\left(2i\delta_l^{\pm}\right), \tag{50}$$

and the amplitudes are

$$f_l^{\pm}=\exp\left(i\delta_l^{\pm}\right)\sin\delta_l^{\pm}. \tag{51}$$

Here, $\delta_l^{\pm}$ are the phase shifts corresponding to the partial wave with orbital angular momentum l and total angular momentum $J=l\pm1/2$. The phase shift is

$$\delta_l^{\pm}=\Phi_l+\beta_l^{\pm}, \tag{52}$$

where Φ_l is the phase shift of potential scattering: $\beta_l^{\pm}$ is the phase shift of resonance scattering expressed in terms of the partial width $\Gamma_{\lambda l}$ of the level and the resonance energy E_{RC}. The phase shift is

$$\beta_l^{\pm}=\tan^{-1}\left[\Gamma_{\lambda l}/(E_{RC}-E)\right]. \tag{53}$$

In the region of an isolated resonance, the phase shift Φ_l of potential scattering varies slowly with the energy,

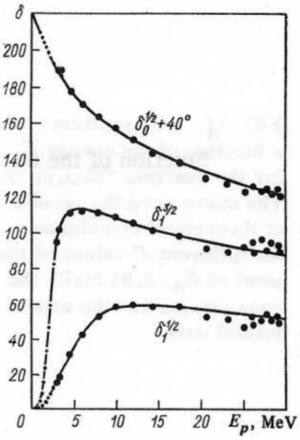

FIG. 5. Phase shifts δ as functions of the proton energy E_p for the reaction $^4He(p,p)^4He$.

and the resonance phase shift $\beta_l^{\pm}$ corresponding to an excited state of a nucleus with total angular momentum $J=l\pm1/2$ varies rapidly with the energy in accordance with the law given by (53). Since $B(\vartheta)$ contains the amplitude difference f^+-f^-, the polarization differs in sign for resonances with the same l but different J, which makes it much easier to determine the spins of the excited states of the nuclei.

As an example, Fig. 7 shows the experimental and theoretical results[93] for investigation of the polarization in the elastic scattering of protons on ^{40}Ca nuclei. The results of the measurements are compared with the theoretical curves calculated under the assumption that the resonance energy is $E_R=2720$ keV, $\Gamma=12$ keV, and that the spin and parity J^π are 1/2⁻ or 3/2⁻. It can be seen from the comparison that one must ascribe $J^\pi=1/2^-$ and not $3/2^-$ to the excited state of ^{41}Sc. From measurements of only the differential cross section for this state, it was possible to determine only the orbital angular momentum. The resonance approximation makes it possible to analyze the polarization $p(\vartheta,E)$ and the cross section $\sigma(\vartheta,E)$ also for two or more interfering resonances. The analysis for several well separated levels with different J^π can be made in accordance with Eqs. (46)–(49), but with allowance not only for the phase

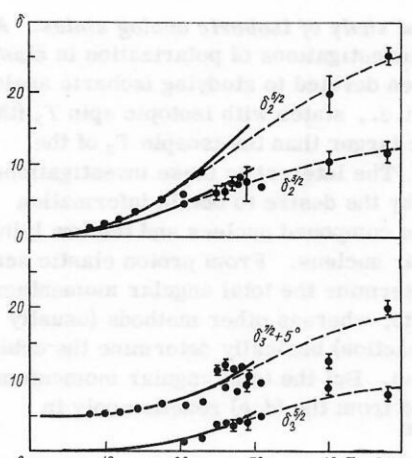

FIG. 6. Phase shifts δ as functions of the proton energy E_p for the reaction $^4He(p,p)^4He$.

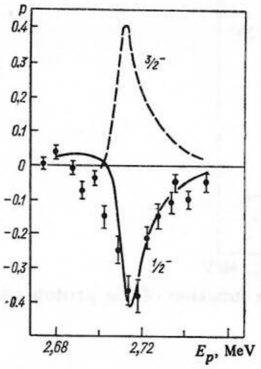

FIG. 7. Polarization of protons for the reaction $^{40}Ca(p,p)^{40}Ca$ as a function of the energy. The points are the experimental results; the continuous curve, calculation for $J^\pi=1/2^-$ and the dashed curve for $J^\pi=3\ 2^-$; $\theta_{c.m}=60°$.

shifts for each level but also for the difference between these phase shifts. The possibilities of such an analysis are illustrated in Fig. 8. In the range $E=1.5-3.0$ MeV of energies of the incident protons, we see three resonances corresponding to excited states of the nucleus ^{29}P with $J^\pi=3/2^-$, $1/2^-$, and $1/2^+$. The experimental results are compared with the theoretical curve calculated under the assumption that the parameters of these levels are, respectively, $E=1652$ keV and $\Gamma=48$ keV, $E=2088$ keV and $\Gamma=15.5$ keV, and also $E=2880$ keV and $\Gamma=30$ keV. As a result of the analysis, the phase shifts δ_0^+, δ_1^+, and δ_1^- were determined as functions of the energy, and a polarization map was plotted.

If two or three levels are situated close to one another and their total angular momenta are different, such a situation can be analyzed by means of the equations given above. The results of the analysis for elastic scattering of protons on ^{12}C at $\vartheta_{c.m}$ are given in Fig. 9. Two closely spaced states with total angular momenta $J=3/2$ and $5/2$ are excited[94] in the compound nucleus ^{13}N. In the cases when the width of the resonances is greater than the distances between them, polarization measurements can decide whether there is one broad resonance or two overlapping resonances. For one broad resonance, the energy dependence of the phase shifts for the ^{16}O nucleus[95] is shown in Fig. 10a. Here, there is interference between the potential $S_{1/2}$ scattering and the resonance $P_{1/2}$ scattering. Figure 10b gives the energy dependence of the phase shifts for the case of overlapping $P_{3/2}$ and $D_{5/2}$ resonances for elastic scattering of protons on ^{12}C nuclei.[96] The polarization when two resonances interfere is given by

$$p\sim\sin(\delta_{l_1}-\delta_{l_2})\sin\delta_{l_1}\sin\delta_{l_2}. \quad (54)$$

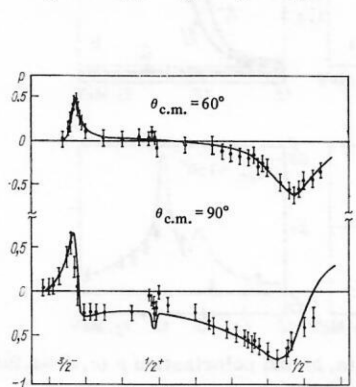

FIG. 8. Proton polarization p as a function of the proton energy E_p for the reaction $^{28}Si(p,p)^{28}Si$.

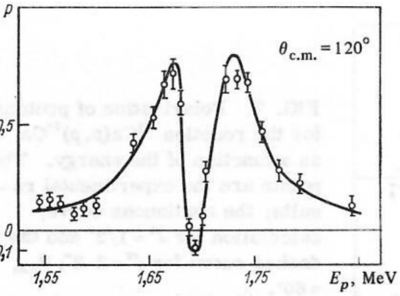

FIG. 9. Proton polarization p as a function of the proton energy E_p for the reaction $^{12}C(p,p)^{12}C$.

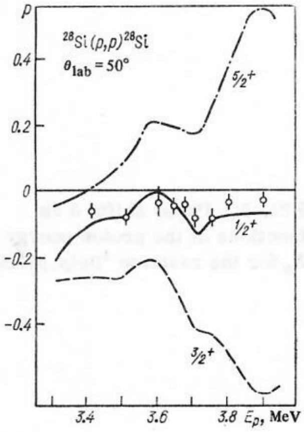

FIG. 11. Polarization p as a function of the energy E_p for the reaction $^{28}Si(p,p)^{28}Si$. The curves are the results of theoretical calculation for different J^π values of the level at $E_R = 3.98$ MeV; the open circles are the experimental data.

It follows from the expression (54) and Figs. 10a and 10b that for overlapping levels the polarization must not change sign as the energy varies near the resonance, whereas the polarization *does* change sign if potential and resonance scattering interfere. Indeed, it can be seen from the results[94,96] shown in Figs. 10c and 10d that for overlapping ($p + {}^{12}C$) resonances the sign does not change, whereas it does for the single resonance ($p + {}^{16}O$).

The single-channel approximation of R-matrix theory which we have considered is not always valid. If other reaction channels are open as well as the elastic-scattering channel, the single-channel approximation is replaced by the many-channel approximation. In this case, the diagonal matrix elements are

$$S_l = \exp(2i\Phi_l) + \exp(2i\Phi_l) \times \frac{i\Gamma_{\lambda p}}{E_R - E - i\Gamma_\lambda/2}, \tag{55}$$

and the amplitude is

$$f_l^{\mp} = \exp(i\Phi_l)\sin\Phi_l + (\Gamma_{\lambda p}/\Gamma_\lambda)\sin\beta_\lambda^{\pm}\exp(i\beta_l^{\pm} + 2i\Phi_l). \tag{56}$$

The subscript λ is used to denote a definite level that has total width Γ_λ and partial elastic-scattering width $\Gamma_{\lambda p}$. The total width is $\Gamma_\lambda = \sum_s \Gamma_{\lambda s}$, where the subscript s denotes the different possible decay channels of the compound nucleus. An example of the use of the many-channel approximation of R-matrix theory to analyze the results of polarization measurements is Ref. 97, in which the total angular momenta of the excited states of the ^{29}P nucleus in elastic scattering of protons on ^{28}Si are determined in the region of incident-proton energies for which the inelastic channel is open. It was known from analysis of the energy dependence of the differential cross section at 90° that the parity of the state observed at $E_p = 3.98$ MeV is positive and that J^π can be equal to one of the three values $1/2^+$, $3/2^+$, and $5/2^+$. Comparison of the experimental values of the polarization with the theoretical curves calculated in the many-channel approximation for $J^\pi = 1/2^+$, $3/2^+$, and $5/2^+$. shows convincingly that $J^\pi = 1/2^+$ (Fig. 11). A similar analysis is made in Ref. 98 for the reaction $^{20}Ne(p,p)^{20}Ne$.

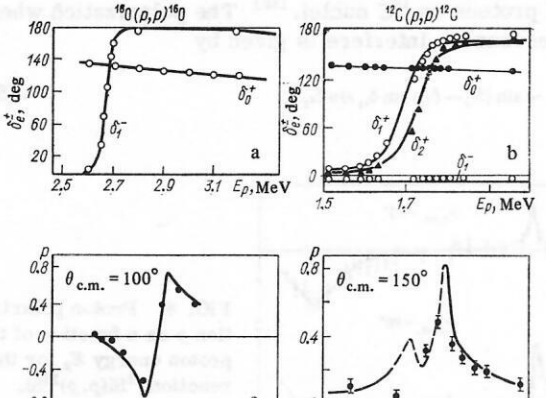

FIG. 10. Phase shifts δ (a, b) and polarization p (c, d) as functions of the proton energy E_p.

The presence of a number of interfering resonances considerably complicates the analysis, and in such a case investigations of the polarization are particularly necessary. This is because the polarization is proportional to $\sin(\delta_1 - \delta_2)$, while the cross section is proportional to $\sin^2(\delta_1 - \delta_2)$, so that, if there is a small difference between the phase shifts δ_i, measurement of the cross section does not reveal the difference, whereas the difference can be noted in the polarization.

Polarization and study of isobaric analog states. A large number of investigations of polarization in elastic scattering has been devoted to studying isobaric analog states of nuclei, i.e., states with isotopic spin $T_>$ (the value of $T_>$ is one larger than the isospin T_0 of the ground state).[99] The interest in these investigations can be explained by the desire to obtain information about levels of the compound nucleus and the low lying levels of its isobar nucleus. From proton elastic scattering one can determine the total angular momentum of the excited state, whereas other methods (usually using the (d,p) reaction) basically determine the orbital angular momentum. But the total angular momentum can be determined from the (d,p) reaction only in special cases.[100]

To analyze the results of investigation into isobaric analog states, one starts from the fact that these are ob-

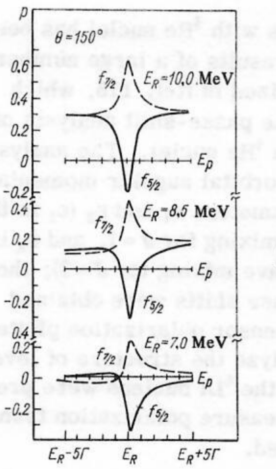

FIG. 12. Energy dependence of the polarization p for the reaction $^{90}Zr(p,p)^{90}Zr$ calculated in Ref. 102.

served on the background of states with $T_< \equiv T_0$. The isobaric analog state averaged over the fine structure is analyzed in terms of the many-channel approximation since other reaction channels are open. The contribution of all background levels with $T = T_<$ is taken into account in nonresonance complex phase shifts. The diagonal matrix elements are[101]

$$S_l = \exp(2i\lambda_l)\left[\exp(-2\mu_l^{\pm}) + \exp(2i\Phi_m)\frac{i\Gamma_p}{E_R - E + \frac{i}{2}\Gamma}\right], \quad (57)$$

where $\lambda_l^{\pm} + i\mu_l^{\pm}$ are the nonresonance phase shifts; Φ_m is the difference between the resonance and nonresonance phase shifts.

It follows from the analysis of Eqs. (46)–(50) with allowance for the expression for the matrix elements (57) that the energy dependence of the differential cross section in the region of an isobaric analog state, especially when $\Gamma_p/\Gamma < 1$, differs little from that of Rutherford scattering. The difference in the cross sections of elastic scattering is also small in the regions of resonances corresponding to states with the same l but different $J = l \pm 1/2$. In addition, the presence in the excitation function of fluctuations due to the interference of neighboring states with $T_<$ makes it difficult to analyze experiments studying isobaric analog states.

In a polarization investigation[102] the picture is changed considerably. Polarization is an interference effect, and at energies below the Coulomb barrier is entirely due to interference between the resonance and nonresonance parts of the scattering amplitude. Therefore at resonance the polarization is much greater than far from resonance, and for two resonances with the same l but different $J = l \pm 1/2$ the polarizations differ in sign. Therefore, even for states with small ratio Γ_p/Γ the polarization can be readily measured and is observed on a small background; the determination of the total angular momentum is also unambiguous. The picture is somewhat complicated when the energy of the incident protons is increased. The results of calculations of the dependence $p(E)$ for the reaction $^{90}Zr(p,p)^{90}Zr$ at different resonance energies of the protons[102] are shown in Fig. 12. It can be seen that, because of nonresonance effects, the background values of the polarization increase with increasing energy. However, even at high energies the value of the polarization at the resonance is comparable with its value outside it.

Investigations of polarization with a view to determining J^{π} of isobaric analog states are currently being made for a large number of nuclei. As an example, Fig. 13 gives the results[101] of an investigation of isobaric analog states in the nucleus ^{139}La. Investigation of the polarization in the region of isobaric analog resonances has unquestionable advantages not only in the determination of the level quantum numbers but also in the clarification of various features of the process mechanism such as, for example, in the study of internal isospin mixing.[103] In Ref. 104, a study of the elastic scattering of polarized protons on ^{40}Ca nuclei provided an experimental confirmation of the theoretically predicted[105] splitting of the single-particle $2P_{1/2}$ state in the ^{41}Sc nucleus into a multiplet of five components, and the investigation in Ref. 106 of isobaric analog states yielded the spin and parity of the ground state of the ^{63}Ni nucleus, these confirming the calculations[107] by means of the shell model with a surface δ interaction.

It is much harder to analyze the polarization for elastic scattering of protons on nuclei whose spin is nonzero. However, the general rules for using phase-shift analysis and R-matrix theory are similar to the case considered above. Here the need for polarization experiments is even greater.

Polarization in inelastic (p,p') scattering. Besides the study of polarization in elastic scattering, investigations are also made of polarization phenomena in inelastic proton scattering. Investigation by means of a polarized proton beam of the vector analyzing power iT_{11} and the polarization p_y of inelastically scattered protons makes it possible to obtain information about highly excited states of the compound nucleus and low-lying states of the target nucleus.[108–112] The analyzing

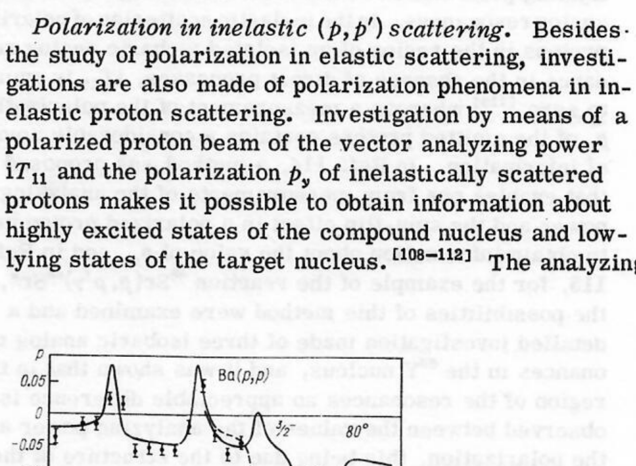

FIG. 13. Polarization p as a function of the proton energy E_p for the reaction $^{138}Ba(p,p)^{138}Ba$. The points are the experimental data; the curves are the results of theoretical calculation. The orbital angular momenta are plotted along the abscissa (the angles are in the center-of-mass system).

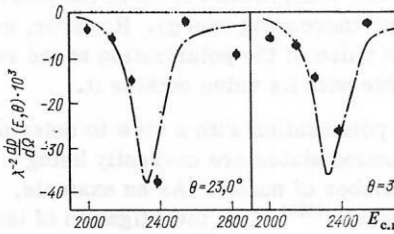

FIG. 14. Comparison of experimental (points) and theoretical (dot-dash-dot curves) values of the differential polarization $\lambda(dp/d\Omega)$ in the $^{12}C(^3He, p)^{14}N$ reaction.[121] (Fitting to three levels: 2.18 MeV, $1/2^+$; 2.29 MeV, $7/2^-$; 2.33 MeV, $5/2^+$; E is the energy of the ^3He ions and θ is the angle in the center-of-mass system).

power of the reactions depends strongly on the interference of several amplitudes in the ingoing channel, and therefore its investigation also helps in a detailed study of the process mechanism. In particular, iT_{11} is very sensitive to different nonresonance effects, which makes it possible[108,109] to determine the contribution of the nonresonance part of the amplitude to the reaction cross section. It is hard to obtain such information by measurement of only the differential cross sections, since the contribution of the interference terms to the cross section of the process is small.

Very interesting information can be obtained by investigating polarization in inelastic scattering near isobaric analog resonances. In the inelastic scattering of polarized protons in the region of an isolated isobaric analog resonance in the absence of direct processes, iT_{11} is equal to zero,[113] whereas a measurement of the polarization p_y of the emitted protons contains a considerable amount of information. In Ref. 114, a method was proposed that enables one from measurements of the analyzing power and the spin-flip effect in a polarized proton beam to obtain information about the value of p_y, and in Ref. 115, for the example of the reaction $^{88}Sr(p, p'\gamma)^{88}Sr^*$, the possibilities of this method were examined and a detailed investigation made of three isobaric analog resonances in the ^{89}Y nucleus, and it was shown that in the region of the resonances an appreciable difference is observed between the values of the analyzing power and the polarization, this being due to the structure of the isobaric analog resonances. This method will undoubtedly be used widely in investigations of different inelastic processes, since it enables one to study not only resonance nuclear reactions, but also the contribution of direct processes.

Polarization in reactions induced by deuterons. The quantum numbers of excited nuclear states have also been studied in resonance nuclear reactions induced by deuterons. Penetration of a deuteron into a nucleus is always accompanied by significant excitation of the compound system that is formed. Therefore, resonance nuclear reactions that take place with excitation of individual levels of the compound nucleus occur only when low-energy deuterons interact with light nuclei ($A < 20$), when the excitation energy and, accordingly, the level density and the number of open channels are still small.

The interaction of deuterons with ^4He nuclei has been the most fully studied. The results of a large number of investigations are summarized in Ref. 116, which gives data on a fairly complete phase-shift analysis of deuteron elastic scattering on ^4He nuclei. The analysis was made with allowance for orbital angular momenta $l \leqslant 4$ and with two mixing parameters ε_1 and ε_2 (ε_1 is the parameter of S- and D-wave mixing for $J=1$, and ε_2 is the parameter of P- and F-wave mixing for $J=2$); the energy dependences of all phase shifts were obtained and charts of the vector and tensor polarization plotted. The results were used to analyze the structure of levels; previously unknown levels in the ^6Li nucleus were predicted; and experiments to measure polarization transfer coefficients were discussed.

Detailed investigations of this kind with deuterons have been made only with ^4He nuclei. Polarization effects in the interaction of deuterons with other nuclei: ^6Li (Ref. 117), ^{12}C (Ref. 118), ^{16}O (Ref. 119), in which levels of the compound nuclei can be investigated, have been studied in only a restricted energy range. In addition, in the majority of investigations, in addition to the differential cross sections, only individual polarization components were measured, which makes it much harder to carry out a complete phase-shift analysis.

Polarization in reactions induced by ^3He *and* T *ions.* In the first experiments[30,120] to study polarization in two-nucleon transfer reactions, the $(^3He, p)$ reaction on ^{10}B and ^{12}C nuclei was studied at low energies. It followed from the data on the cross sections and polarization of the protons in these reactions that they take place preferentially through a stage in which a compound nucleus is formed. Analysis of the experimental data on the polarization of protons in the reaction $^{12}C(^3He, p)^{14}N$, obtained in Ref. 121 on the basis of R-matrix theory under the assumption that isolated levels of the compound nucleus are formed, made it possible to establish reliably the existence in the compound nucleus ^{15}O of a new level with spin and parity $7/2^-$ at energy 2.29 MeV (Fig. 14) and to completely resolve the contradictions found earlier when the cross section of this reaction was analyzed.

In 1967, a start was made on investigating the polarization of neutrons in $(^3He, n)$ reactions on light nuclei, and so far results have been published[16] of polarization measurements on ^3H, ^9Be, ^{11}B, ^{12}C, and ^{24}Mg nuclei. The experimental data on the polarization of neutrons in these reactions were analyzed in the DWBA.

The theoretical interpretation of experimental data on nuclear reactions initiated by ^3He and T ions, including data on the polarization of the products from such reactions, has not been sufficiently developed. One of the serious difficulties is that two-nucleon transfer reactions on light nuclei can take place either through the formation of a compound nucleus or through direct interaction processes. In Ref. 22, Vysotskii and Chegoryan studied the interference of the direct and compound processes in the polarization of particles in two-nucleon transfer reactions. Assuming that the interference does make an appreciable contribution, they succeeded in describing the broad features of the experimental data[30] on the (^{3}He, p) reaction on ^{12}C.

5. POLARIZATION PHENOMENA AND THE OPTICAL MODEL

The optical model is used to analyze elastic scattering data. Fairly complete data on optical potentials that describe elastic scattering of nucleons on nuclei are given in Ref. 122. The optical potentials have been studied much less well for interaction of deuterons, T, ^{3}He, and ^{6}Li with nuclei.

Polarized deuteron beams are used to investigate the tensor terms in the deuteron–nuclear potential describing the elastic scattering of deuterons on nuclei. At the present time, there is insufficient information to construct the phenomenological dependence of the potentials on the energy. The tensor terms are most sensitive to the tensor part of the analyzing power, which is small (as a rule, less than 0.1) and which leads to an uncertainty in the potential of from 10 to 30%.

As long as polarization data were ignored, the fitting of individual cross sections appeared successful, and this resulted in the intensive development of the folding model, in which the deuteron–nuclear potential is expressed as the sum of the nucleon potentials averaged over the ground state of the incident particle. When it was introduced, this model was justified since it eliminated to a large extent the usual discrete and continuous indeterminacies that arise when a model is compared with experiments. However, it has now been established that the folding model can be successfully used only to describe polarization effects in the elastic scattering of deuterons below the Coulomb barrier. If the folding model is applied to the deuteron, it gives the expression

$$V(r) = V_c(r) + V_{so}(r)(\mathbf{SL}) + V_T(r)[(\mathbf{Sr})^2 r^{-2} - 2/3], \quad (58)$$

where $\mathbf{S}$ is the deuteron spin, for the elastic deuteron potential. A more general expression for the potential may include not only the term

$$V_T T_T = V_T(r)[(\mathbf{Sr})^2 r^{-2} - 2/3] \quad (59)$$

taken into account in the folding model but also the invariants

$$T_p = (\mathbf{Sp})^2 - 2\mathbf{p}^2/3;$$
$$T_L = (\mathbf{SL})^2 + (\mathbf{SL})/2 - 2l(l+1)/3, \quad (60)$$

where $\mathbf{p}$ and $\mathbf{L}$ are the momentum and orbital-angular-momentum operators of the deuteron. The folding model takes into account only the first of the possible tensor terms of the optical potential. In the generalization to an n-nucleon incident particle, the central part of the potential is approximately n times greater than the single-nucleon part, while the spin–orbit part is n times smaller. For composite particles with spin $\frac{1}{2}$, for example, for the triton and ^{3}He, the tensor part of the potential naturally drops out. Preliminary investigations have given a value from 3 to 8 MeV for the spin–orbit term in the optical potential of T and ^{3}He, which appreciably exceeds the expected[123] value ≈ 2 MeV. The data used to determine the cross sections for d, T, ^{3}He, and heavier particles are discussed in the review Ref. 124.

In the original formulation, the folding model for deuterons did not contain tensor terms since the admixture of higher orbital angular momenta was ignored. So were exchange terms, perturbation of the ground-state wave function, and the decay channels. These effects were taken into account in Ref. 125.

The basic data on the real and imaginary parts of the central potential of the deuteron–nucleus interaction were obtained without allowance for the spin–orbit and tensor terms. It was found that the parameters of the optical potential with radial dependence of the Woods–Saxon type vary in the range

$$V = 105 - 120 \text{ MeV}, \; r_0 = 0.95 - 1.1 \text{ F}, \; a_0 = 0.9 \text{ F};$$
$$W = 10\text{–}20 \text{ MeV}, \; r_W = 1.3\text{–}1.6 \text{ F}, \; a_W = 0.5\text{–}0.8 \text{ F}.$$

With these values of the parameters of the potential, the differential cross sections for interaction of deuterons with nuclei can be satisfactorily described. For scattering on medium and heavy nuclei, the parameters of the optical potential for neighboring nuclei differ only slightly. For light nuclei, there is no universal set of parameters—their values differ strongly from nucleus to nucleus. For the averaged values of the parameters, it has been shown that the imaginary part iW of the potential for scattering on light nuclei is greater than for scattering on heavy nuclei, whereas the real part V of the potential changes little. Allowance for spin–orbit and tensor terms[123] somewhat improves the agreement between the theoretical calculations and the experimental results, but their influence on the differential cross section is slight. Because the spin–orbit and tensor terms are weakly manifested in the optical potential, it is difficult to obtain information about them from measurements of differential cross sections. More complete information can be deduced from polarization phenomena. Most such investigations have been made in recent years after accelerated beams of polarized ions have become available. Figure 15 shows the results of measurements of the vector polarization of deuterons scattered elastically on Ni, ^{93}Nb, Ag, In, ^{117}Sn, and ^{208}Pb nuclei at 12-MeV energy[127] and of theoretical calculations in accordance with the optical model. In Ref. 127 and some other studies,[126,128] the parameters of the spin–orbit term in the optical potential were determined: $V_{SO} = 5.5\text{–}7$ MeV, $r_{SO} = 0.75\text{–}1.0$ F, $a_{SO} = 0.38\text{–}0.4$ F. Measurements of the tensor polarization of deuterons established[125,128,129] that $V_L = 1$ MeV, $r_L = 0.5$ F, $a_L = 0.5$ F, $V_T = -2.0$ MeV, $r_T = 2.5$ F, and $a_T = 0.9$ F. The analysis of the experimental data on the vector and tensor polarization makes it possible to establish a number of general features, namely:

a) to analyze data on the vector polarization in elastic scattering of deuterons it is basically sufficient to take into account the spin–orbit interaction; the tensor terms of the potential make only a slight contribution;

b) to analyze the tensor polarization of elastically scattered deuterons one must also take into account the tensor components of the optical potential. The influence of tensor terms of the potential has been most fully studied in Ref. 130. It was established that the P_L interaction (60) plays the main role;

c) the contribution of the deuteron D state is not important in the analysis of data on the differential cross sections and the vector polarization but is important for the tensor components.[131] In Ref. 125, data on the differential cross sections and the polarization of deuterons scattered elastically on ^{40}Ca nuclei were used to find the dependence of the central and spin-orbit terms in the optical potential on the deuteron energy in the range from 5 to 34 MeV. The energy dependence of the tensor terms of the optical potential has been inadequately studied.

In recent years, a start has been made on experiments with polarized ^{3}He and ^{6}Li beams with a view to determining the optical potentials in the interaction of these particles with nuclei.[132]

6. DIRECT NUCLEAR REACTIONS

Broadly, one can identify two directions in the investigation of nuclear reactions in which the direct mechanism is predominant. The first of them, the traditional one, uses direct reactions for the purpose of nuclear spectroscopy. The experiments are generally interpreted by means of the DWBA.

The second direction is associated with various refinements of theory: allowance for coupled channels, resonances in the initial and final states, a two-step reaction mechanism, the three-particle nature of the reaction, exchange effects, etc. The last three are particularly important in two-nucleon transfer reactions. For example, the excitation of levels with anomalous parity $(-)^{J+1}$ in two-nucleon transfer can be explained by a mechanism associated with a square graph.[12,13]

In the model of stripping reactions proposed by Butler,[133] the interaction of incident deuterons and nucleons produced in the reaction of nucleons with nuclei is described by means of plane waves. In this approximation, the polarization of the emitted nucleons and the vector analyzing power of the stripping reaction are equal to zero. However, the distortion of the plane waves in the interaction of the particles with the nucleus leads to polarization. The qualitative aspect of the mechanism of the polarization in this case is described in Ref. 134. Polarization is analyzed by means of the distorted-wave Born approximation in Ref. 135. Allowance for the spin-orbit interaction of the emitted nucleons with the residual nuclei improves the description of the experimental results.[136] Besides the DWBA, the weakly-bound-projectile method[137] is now also used to describe stripping reactions.

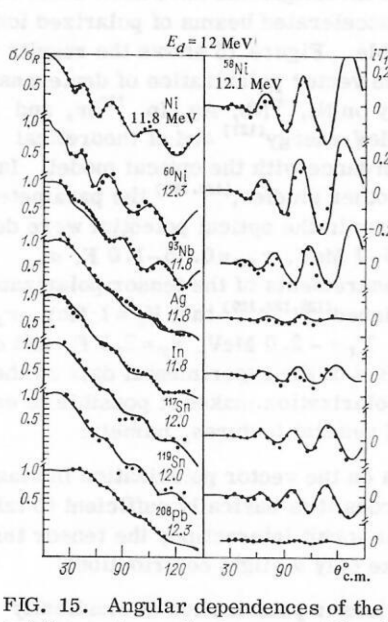

FIG. 15. Angular dependences of the cross-section ratio σ/σ_R and the vector analyzing power iT_{11} of elastic scattering of deuterons on different nuclei.

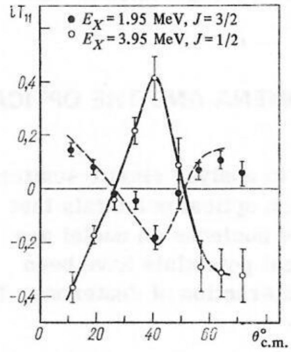

FIG. 16. Angular dependence of the vector analyzing power iT_{11} for the reaction ^{40}Ca$(d,p)^{41}$Ca at E_d = 7.0 MeV.

After it had been shown in Ref. 138 (see Fig. 16) that the vector analyzing power of a reaction has opposite signs for capture by a nucleus of nucleons with different angular momenta J but the same l values, this method was widely used in (d,p), (d,n), and $(d,{}^3\text{He})$ reactions[138,139] to determine the spins of the excited states of nuclei.

In the description of direct nuclear reactions, it is necessary to consider separately the role of polarization investigations into stripping at energies of the particles below the Coulomb barrier. In this case, for example, in (d,p) reactions, one uses functions distorted only by the Coulomb interaction; the angular dependence of the differential cross section is insensitive to the orbital angular momentum of the transferred neutron. Therefore, hopes were attached to polarization investigations. Calculations made by different authors gave contradictory results.[140,141] A detailed calculation was made in Ref. 142, in which the possibility was pointed out of the occurrence of a nonzero polarization whose sign depends on the total angular momentum of the captured nucleon. In Refs. 143, the first measurements were made of the vector and tensor analyzing powers of the reactions ^{208}Pb$(d,p)^{209}$Pb and ^{208}(d, $T)^{207}$Pb, and the results of the calculations in Ref. 142 were confirmed. It is necessary to make studies of polarization effects on lighter nuclei as well.

7. INTERMEDIATE STRUCTURES IN NUCLEI

In the classification of processes according to the time they require, one distinguishes direct nuclear reactions, reactions that proceed through a compound nucleus, and intermediate structures. In a direct reaction, a compound system exists for a very short time equal to the time of flight of the particle near the nucleus, i.e., 10^{-22} sec. Reactions that proceed through a compound nucleus are characterized by a time of order $10^{-16} - 10^{-20}$ sec. An intermediate structure corresponds to the formation of a system with lifetime $10^{-20} - 10^{-21}$ sec. In the simplest variant, such a system consists of the incident particle and the nucleus in which a particle-hole pair has been formed as a result of excitation. In this case, the intermediate system has a two-particle-one-hole structure. After a short time, such a state can either decay with the emission of particles or go over into a long-lived state.[144]

In experimental investigations, intermediate-structure resonances are manifested as broad (100–300 keV) anomalies in the excitation functions with definite resonance characteristics. However, besides being of this nature (doorway states) broad anomalies in the excitation function can be due to Ericson fluctuations and fluctuations of the level densities, so that in an analysis of only the differential cross sections it is difficult to obtain an unambiguous answer. Important information on the nature of anomalies can be obtained by investigating polarization effects. This is because the vector analyzing power of a reaction (or the polarization of the reaction products) in the region of statistical anomalies due to Ericson fluctuations must be zero, but in intermediate-structure resonances a nonzero value of iT_{11} is observed in the region of the resonance. The first investigations made with polarized beams of protons (inelastic scattering on ^{24}Mg, ^{52}Cr, ^{58}Ni nuclei[145]) and deuterons (reactions on ^{28}Si nuclei[146]) showed that iT_{11} and p have nonzero values in the region of broad anomalies. A correlation is observed between the polarization effects in different reaction channels. The widths and resonance energies were determined and an attempt made to establish their spins and parities. An analysis of the results presented at the Zurich Conference[38] indicates that polarization investigations have been widely used to study broad anomalies in the excitation functions. In our opinion, such investigations are promising.

Polarization experiments could be helpful for testing the assumptions of the statistical model. In Ref. 147, the statistical model was used to consider polarization transfer in a reaction in which two spin-1 particles are produced in the final state, and it was shown that the second-rank statistical tensors of the two particles are the same. Confirmation of this assumption would unambiguously indicate the applicability of the statistical model.

Fluctuations of the polarization characteristics have hitherto been little studied.[148] The main idea in this problem was formulated in Ref. 149 and it is that the interaction cross section of a polarized beam containing only selected magnetic quantum numbers will fluctuate with the energy more rapidly than the cross section of an unpolarized beam. The variations of the cross sections for the polarized beam are proportional to $1+p^2$, where p is the polarization (the case in which spin-$\frac{1}{2}$ particles impinge on spin-0 nuclei was considered). Study of Ericson fluctuations with a polarized beam is of twofold interest. On the one hand, the fluctuating amplitude of the compound process can be more readily separated from the amplitude of the direct processes. On the other hand, it is easier to analyze the compound amplitude—to determine the mean distance D between the levels and their widths Γ.

8. POLARIZATION EXPERIMENTS AND GIANT RESONANCES IN NUCLEI

The discovery in recent years of new giant resonances in nuclei, such as the isoscalar and isovector quadrupole resonances, has put the investigation of polarization effects in the region of these resonances on the agenda. The analyzing power is most readily investigated in radiative capture. In this case, the resonances are described by not only an expansion of the cross section with respect to Legendre polynomials,

$$\sigma(\vartheta, E) \sim 1 + \sum_k a_k(E) P_k(\cos\vartheta), \qquad (61)$$

but also by means of an expansion of the analyzing power with respect to associated Legendre functions:

$$A(\vartheta, E)\,\sigma(\vartheta, E) \sim \sum_k b_k(E) P_k^1(\cos\vartheta). \qquad (62)$$

For a resonance with multipolarity L, we have $k \leq 2L$ (k is even). For interfering resonances with multipolarities L and L', $k \leq L+L'$ and k is even if the resonances have the same parity and odd if they have opposite parity.

For a pure $E1$ or $M1$ resonance, a_2, $b_2 \neq 0$; for a pure $E2$ resonance, a_2, a_4, b_2, $b_4 \neq 0$. In the case of $E1$–$M1$ interference, a_1, $b_1 \neq 0$; for $E1$–$E2$ interference, a_1, a_3, b_1, $b_3 \neq 0$; finally, for $E1$–$M2$ or $E2$–$M1$ interference, only a_2, $b_2 \neq 0$.

For some cases, for example, radiative capture of protons by ^{15}N nuclei, the spin structure of the reaction enables one to determine[150] completely the reaction matrix elements from the expansion of the cross section and the analyzing power.

CONCLUSIONS

The analysis of the experimental and theoretical investigations of nuclear reactions at low and medium energies shows that measurement of polarization effects is an important and effective method for studying the spin dependence of the nuclear interactions, for testing nuclear models, and determining the spectroscopic characteristics of nuclei. The information obtained from the polarization measurements cannot be extracted from any indirect experiments, and it is this that justifies the considerable work involved. The progress achieved in recent years in creating sources of polarized particles, polarized targets, and effective analyzers has considerably increased the role and relative importance of polarization investigations in modern nuclear physics.

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