Color degrees of freedom in hadron physics

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Quark models including the extra color degree of freedom are reviewed. The various ways in which color can be manifested are considered. The experiments by means of which it could be established whether quark charges are integral or fractional are discussed.

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INTRODUCTION

In the present review, we shall attempt to answer two questions: 1) To what extent is it necessary to introduce into hadron physics a new, *color* degree of freedom and 2) is it already possible to say something about the properties about this new degree of freedom and propose some symmetry for it?

There is no doubt that each individual experimental fact in the field considered below could also be explained without invoking the color degree of freedom. But the naturalness with which many phenomena can be explained on its basis gives confidence in the existence of the color degree of freedom.

The entire following treatment will be based on the quark model: Mesons are regarded as bound states of a quark and antiquark ($q\bar{q}$) and baryons as bound states of three quarks (qqq).

We first give the theoretical motivation for proposing the new—color—degree of freedom for quarks. We then consider various color models that differ basically from one another in assuming either fractional or integral values of the quark charges. In models of the first type, the color degrees of freedom are not manifested explicitly in the form of high excited states. One then uses the expression *hidden color*. In models of the second type, the existence of a large number of new heavy resonances, including narrow ones, is predicted. Here, the expression *hidden color* is used at energies *below* the threshold for production of these particles (or, briefly, *below the color threshold*), and the expression *manifest* color is used at energies *above* this threshold. In the literature the following terms are also used: *freezing*, *unfreezing*, and brilliance of color degrees of freedom.

We shall also discuss the various phenomena in which color degrees of freedom can be observed. Particular attention is drawn to the possibility of testing models with integral and fractional quark charges on the basis of experiments made below the color threshold. It will be seen that such a possibility exists, but the experimental data currently available do not permit a unique choice between these two types of model.

Since color models with integral quark charges predicted a large number of new heavy resonances, which must be very narrow on account of conservation of the color quantum numbers, it appeared natural to identify some of them with the unexpectedly discovered narrow heavy vector mesons $J/\psi(3100)$ (Refs. 1 and 2) and $\psi'(3700)$ (Ref. 3). The color interpretation of these particles will therefore be given. More recently, an alternative explanation of these mesons had been favored: in the model with a fourth—*charmed*—quark (see the reviews Refs. 4 and 5). Nevertheless, interest still attaches to a discussion of the color interpretation of the new mesons. The possibility cannot be excluded that color resonances will be discovered at still higher energies, and then the properties ascribed in the color interpretation to J/ψ and ψ' will be transferred to the new resonances. We should however immediately draw attention to the existence of a different point of view that regards color symmetry as completely exact and hidden. It has been developed in the so-called *chromodynamics*. In this review, we shall only touch on this attractive possibility, since it requires its own extensive analysis. The review is rather at the phenomenological level.

At the present time it is difficult to judge what will be the direction of further development of the theory based on the introduction of color degrees of freedom and how strongly our ideas about the world of elementary particles will be changed as a result. I have merely attempted to characterize the present state of the theory.

There are several excellent reviews[6–12] on this subject. I have obtained much information from them and have attempted to emphasize what could serve as basis for further development of the theory. The present review can be regarded as an introduction to this new field.

1. MOTIVATION FOR INTRODUCING COLOR DEGREES OF FREEDOM

The problem of reconciling the spin and statistics of quarks

The brilliant hypothesis of Gell-Mann[13] and Zweig[14] about the existence of quarks—particles more fundamental than the ordinary hadrons, of which the latter are made—explained from a unified point of view, admittedly only qualitatively, a vast number of experimental facts, and then was fruitfully developed in the form of the quark model of hadrons.[15] But from the very start a certain contradiction was inherent in the hypothesis: The quarks were assumed to be particles with spin $\frac{1}{2}$ and must satisfy the Pauli principle, but in order to construct baryons out of quarks it was necessary to have three identical quarks in one and the same

TABLE I. Symmetry of baryon $SU(6)$ multiplets.

Multiplet	Reduction $SU(6) \Rightarrow SU(3) \times SU(2)$ (spin)	Young pattern
20	$(1,3/2)+(8,1/2)$	
56	$(8,1/2)+(10,3/2)$	
70	$(1,1/2)+(8,1/2)+(8,3/2)+(10,1/2)$	

state. For example, the resonance $\Delta^{++}_{S_z=3/2}=|p\uparrow, p\uparrow, p\uparrow\rangle$ consists of three p quarks with spins directed upward along the z axis. It is natural to expect that the ground state of a quark within a baryon is an s state. But then the three quarks are in one and the same space–spin state.

In general form, this problem can be formulated as follows. Consider a fundamental spinor of the group $SU(6)$ in which the components are determined by the quark states p, n, λ and the spin projections of the quark onto the z axis indicated by the arrows:

$$\xi=(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)=(p\uparrow, n\uparrow, \lambda\uparrow, p\downarrow, n\downarrow, \lambda\downarrow). \quad (1)$$

Baryon states belong to one of the irreducible representations of $SU(6)$ into which the direct product of the three fundamental spinors decomposes:

$$6_1\times 6_2\times 6_3=20+56+70+70. \quad (2)$$

Each of the multiplets on the right-hand side is obtained by a definite Young symmetrization with respect to the spaces 1, 2, 3 to which the three fundamental spinors correspond. The symmetries of the multiplets and their occurrence in the reduction $SU(6)\Rightarrow SU(3)\times SU(2)$ (spin) are given in Table I. All the low-lying baryon states with spins 1/2 ($\mathscr{P}$, $\mathscr{N}$, Σ, Ξ, Λ) and 3/2 (Δ, Σ^*, Ξ^*, Ω^-) fit beautifully into the 56-plet, which is a symmetric representation. However, the total wave function of three fermion quarks must be antisymmetric. Three ways were proposed for overcoming this difficulty:

1) the quark ground state within a baryon is a p, and not s, state.[16–20] Then the spatial part of the wave function can be antisymmetric. An objection to this is that the nucleon form factor for an antisymmetric function should have zeros, and this is not observed experimentally.[16,18,19] However, examples were given of antisymmetric wave functions for which the form factors have no zeros.[20] Another objection was that the resonances corresponding to the 70-plet could not be regarded in this case as an L excitation of the gound state.[10] Whatever the truth, one cannot regard the assumption of a ground p state of the quark as natural;

2) quarks satisfy para-Fermi statistics of third rank, which permits three parafermions to be in one state[21];

3) quarks have an additional degree of freedom, with respect to which one can antisymmetrize.[22–24]

It soon transpired that the application to quarks of para-Fermi statistics is equivalent to application to them of ordinary Fermi statistics if the quarks have an additional internal degree of freedom.[25,26] Thus, the second and third assumptions were seen to be equivalent in a certain sense.

Thus, the symmetry of the baryon 56-plet gave the first indication of the existence of a new internal degree of freedom. Gell-Mann *et al.*,[27] used for it the word *color*, which has now been widely accepted.[1)] It is postulated that each of the quarks can be in one of three new states, these being designated by a particular color, for example, *red*, *yellow*, and *blue*.

In what follows, it will be convenient to denote these colors simply by the numbers $i=1, 2, 3$. The problem of the quark statistics is then solved by antisymmetrization with respect to the color degree of freedom. For example, the Δ^{++} resonance corresponds to the state

$$|\Delta^{++}_{S_z=3/2}\rangle=\frac{1}{\sqrt{6}}\sum_{i,j,k=1}^{3}\varepsilon_{ijk}|p_i\uparrow, p_j\uparrow, p_k\uparrow\rangle, \quad (3)$$

where ε_{ijk} is the antisymmetric tensor.

Finally, the postulate that there exist three color quark states provides a simple solution to the problem of saturation: In nature, three-quark states—baryons—are realized and there are no stable states with a larger number of quarks, for example, $qqqq$, etc.

The rule $\Delta I = 1/2$

Besides reconciling the spin and statistics of the quarks, the introduction of the color degree of freedom can help to solve one further problem[28–32] presented by the existence of the hitherto unexplained rule $\Delta I=\frac{1}{2}$ (I is the isospin) for nonleptonic strangeness-changing hadron decays, for example, $K_S\to\pi^+\pi^-$. Empirically, such decays take place preferentially with $\Delta I=\frac{1}{2}$, whereas decays with $\Delta I=\frac{3}{2}$ take place, roughly speaking, 500 times less often. But the ordinary theory of weak interactions of hadrons predicts approximately the same probability for such transitions.

Let us suppose that the weak interaction is described by a Hamiltonian representable in the current×current product form, the currents being taken at the same point. In the quark model, they have the form

$$J_\mu=\bar{p}\gamma_\mu a n_\theta+\text{h.c.}, \quad (4)$$

where $n_\theta=n\cos\theta_c+\lambda\sin\theta_c$ is the "Cabibbo quark"; $a=1-\gamma_5$. The transitions mentioned above are determined by the cross terms

$$H_{\text{int}}=\sin\theta_c\cos\theta_c(\bar{\lambda}\gamma_\mu ap)(\bar{p}\gamma_\mu an)+\text{h.c.} \quad (5)$$

In the expression (5), one can interchange the second

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1)It is interesting that the analogous expression *color groups* has long been used in crystallography to classify magnetic symmetries (M. Hamermesh, Group Theory and its Application to Physical Problems, Addison Wesley, Reading, Mass. (1962); Russian translation published by Mir, Moscow (1966)).

(p) and fourth (n) operators, using the Fierz transformation.[33] The sign change accompanying the transformation is compensated by the fact that Fermi fields anticommute. One can write

$$H_{\rm int}=\sin\theta_c\cos\theta_c\,[(\bar\lambda\gamma_\mu ap)(\bar p\gamma_\mu an)+(\bar\lambda\gamma_\mu an)(\bar p\gamma_\mu ap)]/2+{\rm h.c.} \quad (6)$$

The combination $pn+np$ is an isovector, and $\overline{\lambda p}$ is an isospinor. Therefore, the Hamiltonian (6) contains the isospin change 3/2.

In the presence of a color degree of freedom, one can write the Hamiltonian of the weak interaction in such a way that it changes the isospin only by $\frac{1}{2}$ (Ref. 12):

$$H_{\rm int}=\sin\theta_c\cos\theta_c\,[(\bar\lambda_i\gamma_\mu ap_{i'})(\bar p_j\gamma_\mu an_{j'})+{\rm h.c.}]\,\varepsilon_{ijk}\varepsilon_{i'j'k}. \quad (7)$$

The Fierz transformation is now accompanied by the interchange $i'\rightleftarrows j'$, which leads to a change of sign, and we have

$$H_{\rm int}=\sin\theta_c\cos\theta_c\,[(\bar\lambda_i\gamma_\mu ap_{i'})(\bar p_j\gamma_\mu an_{j'}) - (\bar\lambda_i\gamma_\mu an_{i'})(\bar p_j\gamma_\mu ap_{j'})]\,\varepsilon_{ijk}\varepsilon_{i'j'k}/2. \quad (8)$$

Since the combination $pn-np$ is an isoscalar, the Hamiltonian (8) only changes the isospin by $\frac{1}{2}$. Note that the Hamiltonian (8) is a color invariant, so that it acts between ordinary hadron states, which, as we shall see below, are color singlets. In fact, in color models one can explain not only the rule $\Delta I=\frac{1}{2}$ but also a more general rule, the so-called octet enhancement.[12,28–32]

2. COLOR MODELS

Quark charges and two versions of color symmetry

In the original Gell-Mann–Zweig quark scheme[13,14] the charges of the quarks must be fractional:

$$e_p=2/3;\quad e_n=e_\lambda=-1/3. \quad (9)$$

The color scheme permits much greater freedom in the choice of the quark charges.

Thus, suppose that there are nine color quarks $q_{\alpha i}$. The subscript α refers to the $SU(3)$ quark state: $\alpha=p$, n, λ; the subscript i is the color number: $i=1, 2, 3$. We denote the as yet arbitrary quark charges by $\varepsilon_{\alpha i}$.

In nature the baryon decaplet is realized, the quark composition of the members being

$$\begin{array}{cccc} & & \Omega^-(\lambda\lambda\lambda) & \\ & \Xi^{0*}(p\lambda\lambda) & & \Xi^{-*}(n\lambda\lambda) \\ \Sigma^{+*}(pp\lambda) & & \Sigma^{0*}(pn\lambda) & \Sigma^{-*}(nn\lambda) \\ \Delta^{++}(ppp) & \Delta^{+}(ppn) & \Delta^{0}(pnn) & \Delta^{-}(nnn). \end{array}$$

This decaplet belongs to the symmetric representation of $SU(3)$. Since quarks are fermions and the total wave function of three quarks forming baryons must be antisymmetric, so must the color part of the wave function. Therefore, the color numbers of the quarks in each member of this decaplet must be different: $\Omega^-(\lambda_1, \lambda_2, \lambda_3)$, and so forth. Equating now the sum of the charges of such quarks to the hyperon charge, we obtain nine conditions (not all of them independent). Solving these conditions, we arrive at the result that the charges of the nine quarks are determined by three charges:

$$\left.\begin{array}{l} e_{pi},\quad i=1, 2, 3, \\ e_{ni}=e_{\lambda i}=e_{pi}-1. \end{array}\right\} \quad (10)$$

The condition

$$e_{p1}+e_{p2}+e_{p3}=2 \quad (11)$$

must also be satisfied. It is interesting that this condition is identical with the condition of neutrality of the quark world, which also holds for the ordinary Gell-Mann–Zweig quarks: The sum of the charges of all quarks is zero. The antiquarks have opposite charges.

We shall see in what follows that many results do not depend on the particular values of the charges of the color quarks but are different for models of ordinary and color quarks. Other results do depend on the choice of the quark charges. We shall consider two versions of color symmetry: with fractional and integral quark charges.

Model with hidden color and fractional quark charges[21,27]

In this model, which derives from Greenberg's hypothesis[21] about application of para-Fermi statistics to quarks, it is assumed that quarks differing only by their color enter all interactions completely symmetrically. Consider as the hadron symmetry group the group

$$G=SU(3)\times SU(3)^c, \quad (12)$$

consisting of the direct product of the ordinary $SU(3)$ group acting on the unitary space (p, n, λ) and the color group $SU(3)^c$ acting on the color unitary space (1, 2, 3). The ordinary hadrons (mesons and baryons) are assumed to be *singlets of the color group* $SU(3)^c$. It is important to note that it is an antisymmetric combination of three quarks [see (3)] that is an $SU(3)^c$ singlet, which makes it possible to have a singlet state together with complete antisymmetry of the quark wave function of the baryons.

In the given model, the color symmetry $SU(3)^c$ is assumed exact and to be manifested solely through the presence of one further internal degree of freedom. The quantum numbers of quarks that differ by their color are the same. Hence, since $e_{p1}=e_{p2}=e_{p3}$, it follows from (11) that the quark charges are fractional:

$$\left.\begin{array}{l} e_{p1}=e_{p2}=e_{p3}=2/3; \\ e_{n1}=e_{n2}=e_{n3}=e_{\lambda1}=e_{\lambda2}=e_{\lambda3}=-1/3. \end{array}\right\} \quad (13)$$

The usual Gell-Mann–Nishijima formula $Q=I_3+Y/2$ holds. The electromagnetic current of the quarks has the structure

$$J_\mu^{\rm em}=\sum_{i=1,2,3}[(2/3)\,\bar p_i\gamma_\mu p_i-(1/3)\,\bar n_i\gamma_\mu n_i-(1/3)\,\bar\lambda_i\gamma_\mu\lambda_i]=J_\mu^{8,1c} \quad (14)$$

and is an $SU(3)$ octet and $SU(3)^c$ singlet. Because of this, it cannot excite color degrees of freedom. The current (14) is simply the electromagnetic current of ordinary quarks multiplied by three. Thus, from the point of view of this model we are always in a singlet color world. Into this model one can also readily incorporate a fourth—charmed—quark and consider the group $SU(4)\times SU(3)^c$.

The assumption that the group $SU(3)^c$ is absolutely exact enables one to regard it as a gauge (non-Abelian) group. Corresponding to the presence in it of eight generators, there arise eight massless gauge vector fields forming the color octet 8^c. With respect to the ordinary group $SU(3)$, they must all be singlets. Thus, besides the ordinary photon responsible for electromagnetic interactions, there occur eight Yang–Mills *color photons*, which are responsible for strong interactions and are called *gluons*. In recent years, it has been shown that non-Abelian gauge groups have the property of *asymptotic freedom*—the color charge disappears at short distances but has finite values at large distances. The connection between the charges is given by the following formula (see the reviews Refs. 36 and 37):

$$e^2(r_0)=e^2(r)/[1+\beta' e^2(r)\ln(r/r_0)],\quad \beta'>0. \tag{15}$$

If $r_0\to 0$, then for any finite charge $e(r)$ we have $e(r_0)\to 0$. One can expect that when quarks which exchange color gluons approach one another the interaction between them decreases, and in the limit of short distances they will behave as free particles. To explain the parton picture of deep inelastic scattering of electrons, muons, and neutrinos on nucleons it is extremely attractive if at large distances ~1 F between the quarks, or partons, the interaction between them is strong, while at the short distances characteristic of deep inelastic scattering the interaction is much weaker.

The inverse of the relation (15) shows that with increasing r (fixed r_0) there comes a time when $e^2(r)\to\infty$. The quarks begin to interact so strongly that they can never come apart and therefore cannot exist in the free state. Nor can the massless color gluons exist in the free state since they carry color charge. This phenomenon has been called infrared slavery.[38,39] Recently, however, serious objections have been advanced against this proof since the infrared divergences on which it is based can be eliminated.[40]

It must be said that practical conclusions from theories of this kind must be regarded with caution. The connection between the charges (15) is in a certain sense opposite to the corresponding connection for Abelian gauge groups (for example, in electrodynamics): $\beta<0$. And in this case the zero-charge problem already has existed for more than 20 years, although the physical charge of particles is nonzero!

The hypothesis that there exist eight massless gluons can be used in a practical direction. Assuming a definite gluon–quark coupling constant α' analogous to the fine structure constant α, one can undertake calculations of quark levels and their fine structure in complete analogy with electrodynamics. This direction in the theory of color gluons is now called *chromodynamics*. It has yielded interesting results for the mass spectrum of ordinary hadrons.[5] For example, by introducing an interaction analogous to the Fermi–Breit interaction for charged Dirac particles, one can explain the Σ–Λ mass splitting, etc. New mass relations have been obtained, and these are well satisfied.

Model with manifest color and integral quark charges

Bogolyubov, Struminskii, Tavkhelidze *et al.*,[22] Han and Nambu,[23] and Miyamoto[24] pointed out that the arbitrariness in the specification of the quark charges (e_{pi}, $i=1, 2, 3$) can be used to make all quark charges integral. There are two variants of color models with integral quark charges depending or whether or not the breaking of color symmetry is strong or only electromagnetic. Let us consider these models.

The Han–Nambu Model. In the original Han–Nambu model[23] the group

$$G=SU(3')\times SU(3'') \tag{16}$$

is postulated as the total hadron symmetry group. It is assumed further that:

1. The physical $SU(3)$ symmetry is a diagonal subgroup of G. The $SU(3)$ generators are the direct sums of the $SU(3')$ and $SU(3'')$ generators. For example, the isospin and hypercharge are defined as the sums

$$I_3=I_3'+I_3'';\quad Y=Y'+Y''. \tag{17}$$

The Gell-Mann–Nishijima formula takes the form

$$Q=I_3+Y/2=I_3'+Y'/2+I_3''+Y''/2=Q'+Q''. \tag{18}$$

2. Quarks belong to the (3, 3*) representation of G. In accordance with (17), the quantum numbers of the quark nonet can be represented by the Han–Nambu diagram shown in Fig. 1. In this diagram, p_i, n_i, λ_i form triplets (triangles) of quark $SU(3')$ states with centers at the points 1, 2, 3, which form an $SU(3'')$ antitriplet (inverted triangle). As can be seen from the Han–Nambu diagram, the quark nonet decomposes into a singlet and

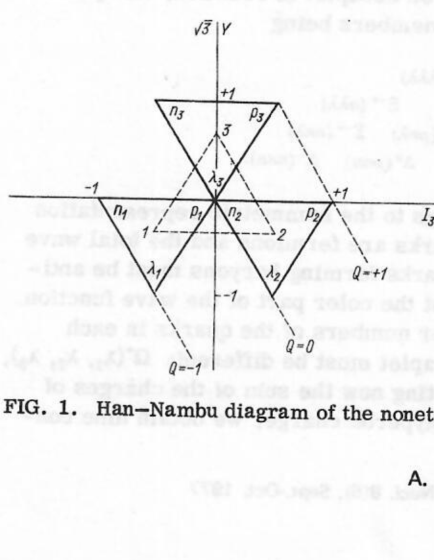


FIG. 1. Han–Nambu diagram of the nonet of quark states.

TABLE II. Quantum numbers of the quark nonet in the Han–Nambu model.

Parameter	p_1	n_1	λ_1	p_2	n_2	λ_2	p_3	n_3	λ_3
I_3	0	−1	−1/2	1	0	1/2	1/2	−1/2	0
Y	0	0	−1	0	0	−1	1	1	0
Q	0	−1	−1	1	0	0	1	0	0

octet with respect to the physical $SU(3)$ symmetry.[2)] From (17) and (18) we obtain the quantum numbers of the quark nonet given in Table II. Thus, on the basis of assumptions 1 and 2, the charges and hypercharges of the quarks come out to be integral automatically. They correspond to the following choice in (10):

$$e_{p1}=0;\ e_{p2}=e_{p3}=1. \tag{19}$$

3. There exists a superstrong interaction that has $SU(3'')$ symmetry and thus determines the classification of the energy levels with respect to the irreducible $SU(3'')$ representations. The ordinary hadrons are *$SU(3'')$ singlets.*

To justify assumption 3, Nambu put forward the following argument.[41,42] He proposed the introduction of an octet of gauge fields G_μ ($\mu=1,\ldots,8$), i.e., gluons interacting with the infinitesimal $SU(3'')$ generators (currents) of the triplets α''_μ with strength g. For a system containing N quarks, the exchange of such fields between quark pairs leads to the interaction energy

$$V_G=g^2\sum_{n>m=1}^{N}\alpha''^{(n)}_\mu\alpha''^{(m)}_\mu=g^2\Big[\sum_{n=1}^{N}\alpha''^{(n)}_\mu\Big]\Big[\sum_{m=1}^{N}\alpha''^{(m)}_\mu\Big]/2 - g^2\sum_{n=1}^{N}\alpha''^{(n)}_\mu\alpha''^{(n)}_\mu/2=g^2(C_2-NC_{20})/2, \tag{20}$$

where $\alpha''^{(n)}_\mu$ refers to the n-th quark; C_2 is the quadratic Casimir operator:

$$C_2=\Big[\sum_{n=1}^{N}\alpha''^{(n)}_\mu\Big]\Big[\sum_{m=1}^{N}\alpha''^{(m)}_\mu\Big]. \tag{21}$$

For the representation $\mathscr{D}(l_2,\ l_2)$, it has the value

$$C_2(l_1,\ l_2)=(l_1^2+l_1l_2+l_2^2)/3+(l_1+l_2). \tag{22}$$

(For reference: $\{3\}=\mathscr{D}(1,\ 0):C_{20}=4/3$; $\{1\}=\mathscr{D}(0,\ 0):C_2=0$; $\{6\}=\mathscr{D}(2,\ 0):C_2=10/3$; $\{8\}=\mathscr{D}(1,\ 1):C_2=3$; $\{10\}=\mathscr{D}(3,\ 0):C_2=6$; $\{27\}=\mathscr{D}(2,\ 2):C_2=8$.)

It is assumed that the gluons have a large mass and the dependence of the interaction energy on the coordinates in (20) can be ignored.

The expression for the total energy is obtained by adding to V_G the rest mass:

$$E=(M-C_{20}g^2/2)\,N+g^2C_2/2=N\mu_0+g^2C_2(l_1,\ l_2)/2, \tag{23}$$

where μ_0 is the effective mass of a quark within a hadron:

$$\mu_0=M-C_{20}g^2/2=M-2g^2/3. \tag{24}$$

It follows from (23) that singlet $SU(3'')$ representations are ground states. Moreover, the condition of stability of the bound state of N quarks has the form

$$V_G<0. \tag{25}$$

For $N=3$ it follows from (20) that only singlet states—ordinary baryons—satisfy this condition. All other three-quark states corresponding to higher representations must be unstable against decay into free quarks.[41–43] (Certain exotic states may also be stable: biquarks qq, biquarks–biantiquarks $qq\overline{q}\overline{q}$, etc.; these are listed in Ref. 42.) It may happen, however, that allowance for the coordinate dependence of the quark interaction potential leads to the presence of van der Waals type forces, which could ensure stability of some of the higher states.

On the basis of the data on the cross section of e^+e^- annihilation (see Sec. 3), Nambu and Han[42] suggested that the threshold for the production of a quark–antiquark pair lies in the region 3–4 GeV and that the quark mass is

$$M=1.5-2.0\ \text{GeV}. \tag{26}$$

Assuming the effective quark mass $\mu_0=0.3$ GeV, they obtained

$$g^2=1.8-2.6\ \text{GeV}. \tag{27}$$

What now happens with the free quarks themselves, which now have integral electrical charges and hypercharges? If they have fractional baryon charges (1/3), they must be stable. But then why are they not found in nature? One can, however, like Han and Nambu, assume that the baryon charges are integral. For example, one can postulate that the baryon charge is defined as[23,42]

$$B=1/3+Y''\quad\text{or}\quad B=1/3-2Y''. \tag{28}$$

Then for the quark triplets

$$B=(0,\ 0,\ 1)\quad\text{or}\quad B=(1,\ 1,\ -1). \tag{29}$$

In the first case, quarks with zero baryon charges could decay into hadrons, emitting an odd number of leptons (because of the quark spin $\frac{1}{2}$). They would then have to have a lepton quantum number (for example, from Ref. 42, $L=(1,\ -1,\ 0)=-2I_3'')$. The third quark with $B=1$ could go over into a baryon. With the second choice in (29), quarks with $B=+1$ could go over into baryons, and those with $B=-1$ into antibaryons.

The electromagnetic quark current has the structure

$$J^{\text{em}}_\mu=\overline{p}_2\gamma_\mu p_2+\overline{p}_3\gamma_\mu p_3-\overline{n}_1\gamma_\mu n_1-\overline{\lambda}_1\gamma_\mu\lambda_1, \tag{30}$$

2)To avoid misunderstanding, we emphasize once more that the states p, n, λ refer to the $SU(3')$ space, and not $SU(3)$, which is a diagonal subgroup of G.

and it can be represented in the form of a decomposition into irreducible $SU(3')\times SU(3'')$ representations:

$$J_\mu^{\rm em}=J_\mu(8,1)+J_\mu(1,8). \tag{31}$$

Explicitly,

$$J_\mu(8,1)=J_\mu^{(3,0)}+J_\mu^{(8,0)}/\sqrt{3}; \tag{32}$$
$$J_\mu(1,8)=-J_\mu^{(0,3)}-J_\mu^{(0,8)}/\sqrt{3}, \tag{33}$$

where

$$J_\mu^{(m,n)}=\bar{q}_{\alpha i}\gamma_\mu\lambda_{\alpha\beta}^{(m)}\lambda_{ij}^{(n)}q_{\beta j}/2; \tag{34}$$

$\lambda^{(m,n)}$ are the Gell-Mann matrices for m, $n=3$, 8 or the identity matrix for m, $n=0$. Thus, in the Han–Nambu model the electromagnetic current contains a color octet part, and the photon can therefore excite color degrees of freedom. The current $J(8,1)$ is called the current of the valence quarks and $J(1,8)$ is called the color current.

In this model, the gluons are also charged, and a definite boson part of the electromagnetic current corresponds to them. Its presence must be manifested in a breaking of the Callan–Gross relation in deep inelastic scattering of leptons on nucleons at high energies.

There is considerable arbitrariness with regard to the choice of the weak quark current in the framework of the Han–Nambu model. Many authors[29, 44–48] used this arbitrariness to eliminate neutral strangeness-changing weak currents (the selection rule $K_L^0\to\mu^+\mu^-$, $K\to\pi\nu\bar{\nu}$, $K\to e^+e^-\pi$). This was done most consistently in Refs. 45, 47, and 48, in which it was assumed that the weak hadron charges, by analogy with the lepton charges, form a closed $SU(2)$ algebra:

$$[W^+,W^-]=2W^0;\ [W^0,W^\pm]=\pm W^\pm, \tag{35}$$

where $W^\pm=(1/2)\int d^3xJ_0^\pm$ is the integral of the charged weak current $J_\mu^\pm=\bar{t}\gamma_\mu(1-\gamma_5)M^\pm t$. Here, t is a nine-component spinor with components q_i; $q=p$, n, λ; $i=1$, 2, 3 and M is a 9×9 matrix with nonzero elements corresponding to transitions with $\Delta Q=\pm 1$ in accordance with Table II of the quark charges. From the algebra (35) there follow restrictions on the transition matrix:

$$[[M^+,M^-],M^\pm]=\pm 2M^\pm, \tag{36}$$

and also an expression for the neutral current:

$$J_\mu^0=\bar{t}\gamma_\mu aM^0t/2;\ a=1-\gamma_5. \tag{37}$$

The conditions (36) are such that the weak neutral current does not contain components with $\Delta S=\pm 1$:

$$\bar{J}_\mu^0=-\bar{n}_1\gamma_\mu an_1-\bar{\lambda}_1\gamma_\mu a\lambda_1+\bar{p}_2\gamma_\mu ap_2+\bar{p}_3\gamma_\mu ap_3. \tag{38}$$

It is interesting that it has the same structure as the electro-magnetic current (30).

At the present time, a more popular explanation for the absence of strange neutral currents is the proposal made by Glashow, Iliopoulos, and Maiani[49] that there exists a fourth quark with charge $Q=2/3$, baryon charge $B=1/3$, isospin $I=0$, hypercharge $Y=-1/3$, and a new quantum number—charm $C=1$, which for ordinary quarks is zero. Here, I should merely like to point out that the Han–Nambu model could overcome this difficulty by itself! But in it there is a new difficulty connected with the "threat" of $SU(9)$ symmetry.[50] If one proceeds from a unified gauge theory of weak and electromagnetic $SU(2_W)$ interactions, renormalizability of the theory requires the Lagrangian of the strong interaction to be invariant under $SU(2_W)$. But of the weak currents have octet parts with respect to both $SU(3')$ and $SU(3'')$, invariance of the strong Lagrangian with respect to $SU(2_W)$ and $SU(3')\times SU(3'')$ requires that it be invariant under the very large chiral group $SU(9_L)\times SU(9_R)$. One could attempt to distinguish in a representation of this group a singlet with respect to $SU(3')\times SU(3'')$, which must not be an $SU(9)$ singlet. But this cannot be done in a simple manner.[50] The presence of the very large symmetry group $SU(9)$ for the hadron spectrum, with moreover $SU(9)$ breaking at the level of $SU(3)$ breaking, does not correspond to reality.

We note that CP nonconservation in the weak interaction can be explained naturally in the framework of the three-triplet model of Ref. 51. In this model, we have the selection rules $(\Delta Y,\Delta Q'')=(\pm 1,0)$ and $(0,\pm 1)$ and for ordinary hadrons CP nonconservation will occur in strangeness-changing nonleptonic processes.

As an example of a weak charged current, we give the expression chosen by Greenberg and Nelson[10]:

$$j_\mu^{(+)}=\Big[\sum_{i=1,2,3}\bar{p}_i\gamma_\mu an_i-\sum_{\alpha=p,n,\lambda}\bar{\alpha}_2\gamma_\mu a\alpha_1\Big]\times\cos\theta_c+\Big[\sum_{i=1,2,3}\bar{p}_i\gamma_\mu a\lambda_i-\sum_{\alpha=p,n,\lambda}\bar{\alpha}_3\gamma_\mu a\alpha_1\Big]\sin\theta_c, \tag{39}$$

where θ_c is the Cabibbo angle. This current transforms as $(8,1)+(1,8)$ with respect to $SU(3')\times SU(3'')$, but leads to neutral currents containing terms with $\Delta Q=0$, $|\Delta S|=1$.

To end our discussion of the Han–Nambu model, let us consider the meson and baryon states that occur in it. The mesons formed by a bound quark and antiquark form an $9\times 9=81$-plet. The baryons, formed by three bound quarks, form a $9\times 9\times 9=729$-plet. These multiplets decompose into the following irreducible $SU(3')\times SU(3'')$ representation:

$$(3.3^*)\times(3.3^*)=(8.1)+(1.1)+(1.8)+(8.8); \tag{40}$$
$$(3.3)\times(3.3)\times(3.3)=(1.1)+2(8.1)+(10.1)+2(1.8)+(1.10^*)+2(8.10^*)+2(10.8)+4(8.8)+(10.10^*). \tag{41}$$

Let us consider in more detail the meson states. The ordinary mesons correspond to an $SU(3'')$ singlet and have a symmetric triplet structure:

$$|1''\rangle=(q_1\bar{q}_1'+q_2\bar{q}_2'+q_3\bar{q}_3')/\sqrt{3}. \tag{42}$$

Here, q_i or q_i' take any value in the given $SU(3')$ triplet: p_i, n_i, λ_i. In reality, it is necessary to take combinations in such a way as to form a definite meson state, for example

$$\rho^0 = \sum_{i=1,2,3} (p_i\bar{p}_i - n_i\bar{n}_i)/\sqrt{6}; \quad \rho^+ = \sum_{i=1,2,3} p_i\bar{n}_i/\sqrt{3} \text{ etc.} \tag{43}$$

It is important to note that if the treatment is restricted to ordinary hadrons—$SU(3'')$ singlets—then the physical $SU(3)$ symmetry can be assumed to coincide with the $SU(3')$ component of G, and the color degree of freedom merely triplicates each quark state (in the case of baryons, it gives antisymmetrization with respect to the color index). One can show directly, using Table II, that states of the type (43) really do have the necessary quantum numbers I, I_3, Y, Q, although the quantum numbers of the quarks are "extraordinary" since the quarks form an $SU(3)$ nonet. We emphasize that the electromagnetic current (30) and the weak current (39) remain extraordinary, and this may be reflected (see Sec. 3) in certain experiments below the color threshold.

For the 72 *new* mesonic states corresponding to the $SU(3'')$ octet we have the following triplet structures: color isotriplet

$$|I''=1,\ I''_3=0,\ Y''=0\rangle = (q_1\bar{q}_1 - q_2\bar{q}_2)/\sqrt{2}; \tag{44}$$

$$|I''=1,\ I''_3=-1,\ Y''=0\rangle = q_1\bar{q}_2; \tag{45}$$

$$|I''=1,\ I''_3=1,\ Y''=0\rangle = q_2\bar{q}_1; \tag{46}$$

color isosinglet

$$|I''=0,\ Y''=0\rangle = (q_1\bar{q}_1 + q_2\bar{q}_2 - 2q_3\bar{q}_3)/\sqrt{6}; \tag{47}$$

two color isodoublets

$$|I''=1/2,\ I''_3=-1/2,\ Y''=-1\rangle = q_1\bar{q}_3; \tag{48}$$

$$|I''=1/2,\ I''_3=1/2,\ Y''=-1\rangle = q_2\bar{q}_3 \tag{49}$$

and

$$|I''=1/2,\ I''_3=1/2,\ Y''=1\rangle = q_3\bar{q}_1; \tag{50}$$

$$|I''=1/2,\ I''_3=-1/2,\ Y''=1\rangle = q_3\bar{q}_2. \tag{51}$$

We emphasize once more that each of these states is a nonet with respect to $SU(3')$. The symmetry $SU(3')$ is identical with the ordinary $SU(3)$ symmetry only for $SU(3'')$ singlet states. For $SU(3'')$ octet states, the ordinary $SU(3)$ symmetry is a diagonal subgroup of the group $SU(3')\times SU(3'')$.

For example, we have five new neutral vector mesons corresponding to the $SU(3'')$ octet:

$$\omega_{18}(I'=0,\ I''=0) = \sum_{\alpha=p,n,\lambda} (\alpha_1\bar{\alpha}_1 + \alpha_2\bar{\alpha}_2 - 2\alpha_3\bar{\alpha}_3)/3\sqrt{2}; \tag{52}$$

$$\omega_{88}(I'=0,\ I''=0) = [(p_1\bar{p}_1 + p_2\bar{p}_2 - 2p_3\bar{p}_3) + (n_1\bar{n}_1 + n_2\bar{n}_2 - 2n_3\bar{n}_3) - 2(\lambda_1\bar{\lambda}_1 + \lambda_2\bar{\lambda}_2 - 2\lambda_3\bar{\lambda}_3)]/6; \tag{53}$$

$$\rho^0_{18}(I'=0,\ I''=1) = \sum_{\alpha=p,n,\lambda} (\alpha_1\bar{\alpha}_1 - \alpha_2\bar{\alpha}_2)/\sqrt{6}; \tag{54}$$

$$\rho^0_{88}(I'=0,\ I''=1) = [(p_1\bar{p}_1 - p_2\bar{p}_2) + (n_1\bar{n}_1 - n_2\bar{n}_2) - 2(\lambda_1\bar{\lambda}_1 - \lambda_2\bar{\lambda}_2)]/2\sqrt{3}; \tag{55}$$

$$\rho^0_{88}(I'=1,\ I''=0) = [(p_1\bar{p}_1 + p_2\bar{p}_2 - 2p_3\bar{p}_3) - (n_1\bar{n}_1 + n_2\bar{n}_2 - 2n_3\bar{n}_3)]/2\sqrt{3}. \tag{56}$$

The symbols ω and ρ are used here to denote states with respect to ordinary $SU(3)$ symmetry. The subscripts 1 and 8 indicate whether they belong to the singlet or octet $SU(3')$ or $SU(3'')$ representations. In some papers, use is made of a different nomenclature (α', α'') of the states, in which α' and α'' take the symbolic values ω, φ, ρ, etc., to designate the states with respect to $SU(3')$ and $SU(3'')$, for example, ρ^0_{18} ($I'=0$, $I''=1$) $\equiv(\omega_1, \rho^0)$. Instead of (52)–(56), we shall use linear combinations that are $SU(3)$ singlet–octet superpositions analogous to the ω and φ mesons:

$$\left.\begin{aligned}\tilde{V}_\omega &= \cos\varphi V_{18} + \sin\varphi V_{88};\\ \tilde{V}_\varphi &= -\sin\varphi V_{18} + \cos\varphi V_{88},\end{aligned}\right\} \tag{57}$$

where φ is the angle of $SU(3)$ singlet–octet mixing and is, in general, different for different mesons. We denote the new vector mesons as follows: the tilde indicates that they belong to the $SU(3'')$ octet; the symbols ω, φ, and ρ, their $SU(3)$ states; the indices B, C, and D, the three different $\tilde{\rho}$ mesons that occur in the given scheme (the index A is ascribed to the ordinary ρ meson: $\rho_A\equiv\rho$). On the right, we also write the (α', α'') nomenclature. Thus, we have five new neutral vector mesons[44]:

$$\left.\begin{aligned}\tilde{\omega} &= \cos\varphi_\omega\omega_{18} + \sin\varphi_\omega\omega_{88} = (\omega,\ \omega_8);\\ \tilde{\varphi} &= -\sin\varphi_\omega\omega_{18} + \cos\varphi_\omega\omega_{88} = (\varphi,\ \omega_8);\\ \tilde{\rho}_B\,(I''=0) &= \rho^0_{88}\,(I'=1,\ I''=0) = (\rho^0,\ \omega_8);\\ \tilde{\rho}_C\,(I''=1) &= \cos\varphi_\rho\rho^0_{18} + \sin\varphi_\rho\rho^0_{88}\,(I''=1) = (\omega,\ \rho^0);\\ \tilde{\rho}_D\,(I''=1) &= -\sin\varphi_\rho\rho^0_{18} + \cos\varphi_\rho\rho^0_{88}\,(I''=1) = (\varphi,\ \rho^0).\end{aligned}\right| \tag{58}$$

On the basis of (23) and the estimate (27), we obtain for the new mesons a mass 3.3–4.5 GeV, which is very close to the masses of the J/ψ family of resonances observed in e^+e^- annihilation. The color interpretation of the J/ψ mesons will be considered in Sec. 4. Finally, we note that in the Han–Nambu model doubly charged heavy mesons of the type $p_2\bar{n}_1$ occur.

In their model in Ref. 23, Han and Nambu made one further assumption.

4. There exists a medium-strong interaction that breaks $SU(3')$ and $SU(3'')$ but in such a way that the diagonal $SU(3)$ subgroup is not broken. The presence of such an interaction would have the consequence that heavy states corresponding to high $SU(3'')$ representations would be unstable and that, even if they did not decay into quarks, they would decay into lower $SU(3'')$-singlet states—ordinary hadrons.

Modification of the Han–Nambu model. Many authors suggested[44, 51–56] that the $SU(3')$ and $SU(3'')$ breakings in the Han–Nambu model should occur in such a way that the $SU(2'_I)\times U'_Y$ and $SU(2''_I)\times U''_Y$ subgroups remain unbroken and I', Y', I'', and Y'' stay as good quantum numbers. Therefore, states corresponding to higher $SU(3'')$ representations with nonzero color isospin I'' and hypercharge Y'' would be quasistable against strong decays into ordinary hadrons. But the states with $I''=0$ and $Y''=0$ would again be unstable and decay into ordinary hadrons through the strong interaction. Since color isospin in this modification of the Han–Nambu model is a good quantum number, models of this type are called I^c-type models. By analogy with ordinary $SU(3)$ symmetry, mass formulas were derived under the assumption of octet dominance of the $SU(3')$ and $SU(3'')$

TABLE III. Quantum numbers of the quarks in the U^c-type model.

Parameter	p_1	n_1	λ_1	p_2	n_2	λ_2	p_3	n_3	λ_3
I_3	1/2	−1/2	0	1/2	−1/2	0	1/2	−1/2	0
Y	1/2	1/2	−2/3	1/3	1/3	−2/3	1/3	1/3	−2/3
I_3^c	−1/2	−1/2	−1/2	1/2	1/2	1/2	0	0	0
Y^c	−1/3	−1/3	−1/3	−1/3	−1/3	−1/3	2/3	−2/3	2/3
Q	0	−1	−1	1	0	0	1	0	0

symmetry breaking.[44,51–56] The properties of the heavy vector mesons in this modification of the Han–Nambu model will be considered in detail in Sec. 4.

Model with electromagnetic breaking of color symmetry. The ordinary physical $SU(3)$ symmetry is identified with the $SU(3'')$ symmetry; as in the model with hidden color (see above), the total hadron symmetry is taken to be[57–67]

$$G = SU(3) \times SU(3^c). \qquad (59)$$

Quarks correspond to the (3, 3*) representation. (In Refs. 57 and 58, the possibility that the quarks belong to the (3, 3) representation was discussed.) But in contrast to the model with hidden color, the Gell-Mann–Nishijima formula is generalized in the same way as in the Han–Nambu model:

$$Q = I_3 + Y/2 + I_3^c + Y^c/2. \qquad (60)$$

The quark charges are integral. But, in contrast to the Han–Nambu model, the quarks in this model form an $SU(3)$ triplet, and not a nonet.[3)] The quantum numbers of the quarks are given in Table III.

The electromagnetic quark current has the form (30) and the transformation properties with respect to $SU(3)$ and $SU(3^c)$ given in Eq. (31). Thus, as in the Han–Nambu model, it has an octet color part, and the photon can therefore excite color degrees of freedom.

Further, one again assumes the existence of a superstrong interaction having $SU(3^c)$ symmetry, which thus determines the classification of the energy levels with respect to the irreducible $SU(3^c)$ representations. The ordinary hadrons are also assumed to be $SU(3^c)$ singlets.

But now the important assumption of this model is that the *color* $SU(3^c)$ *symmetry is broken only at the level of the electromagnetic interaction.*[59–67] Therefore, all the higher color states are quasistable and decay into the low-lying states only through the electromagnetic or the weak interaction.

In this modification of the Han–Nambu model, the states are determined by the quantum number of not the color isospin I^c, but the color spin U^c. This is called a U^c-type model.

The eigenstates of the spin U^c are superpositions of the states (44) and (47). For example, for vector mesons

$$\tilde{V} = \sqrt{3}\, V(I^c = 1)/2 + V(I^c = 0)/2; \qquad (61)$$

$$\tilde{\tilde{V}} = -V(I^c = 1)/2 + \sqrt{3}\, V(I^c = 0)/2, \qquad (62)$$

where the symbol V stands for any state ω, φ, or ρ^0 with respect to the ordinary $SU(3)$ symmetry. Both states (61) and (62) are degenerate with respect to the mass, and the degeneracy is lifted only by the electromagnetic breaking.

Model with discrete S_3^c symmetry. The model is similar to the one considered in Refs. 68 and 69. However, a discrete symmetry—the group of permutations S_3^c of the color indices—is taken as color symmetry, and the total hadron symmetry group is taken to be

$$G = SU(3) \times S_3^c. \qquad (63)$$

It is interesting that in this model one can formulate a *selection principle* for color states, and greatly reduce their number[68]:

1) electrical charges are integral and the same as in the Han–Nambu model;

2) the physical states must be vectors of irreducible S_3^c representations;

3) simultaneously, they must be charge eigenstates.

From this follows a condition determining the allowed physical states:

$$[Q, \mathcal{P}]\,|\,\text{physical state} = 0, \qquad (64)$$

where $\mathcal{P}$ is any permutation in S_3^c. It is easy to see that (64) is not satisfied for the single-quark state. For example, for the symmetric p quark

$$Q\mathcal{P}_{1\rightleftarrows 2}\,|\,p_1 + p_2 + p_3\rangle = |\,p_2 + p_3\rangle$$
$$\neq \mathcal{P}_{1\rightleftarrows 2}Q\,|\,p_1 + p_2 + p_3\rangle = |\,p_1 + p_2\rangle.$$

It can be shown that the condition (64) selects only states with quark number that is a multiple of three and any number of quark–antiquark pairs. The allowed combinations for mesons and baryons must have the form

$$\sum_{i=1,2,3} y_i q_i \bar{q}_i; \qquad (65)$$

$$\sum_{i\neq j\neq k=1}^{3} y_{ijk} q_i q_j q_k, \qquad (66)$$

where the coefficients y_i and y_{ijk} are chosen in such a way as to obtain irreducible S_3^c representations. For the states (65) and (66), $I_3^c = 0$ and $Y^c = 0$, and the Gell-Mann–Nishijima formula is satisfied. From this, for example, it also follows that doubly charged meson states cannot exist. The model has not been developed very far as regards its experimental consequences.

Model with orthogonal color group. Tati[70] suggested that as hadron symmetry group one should consider

$$G = SU(3) \times SO(3). \qquad (67)$$

[3)]The states p, n, λ now correspond to the unitary space of the ordinary $SU(3)$ symmetry; see footnote 2.

The Gell-Mann–Nishijima formula is generalized to

$$Q=I_3+Y/2+L_3, \tag{68}$$

where L_3 is an $SO(3)$ generator and takes the three values 1, 0, −1. In this model, the quarks have fractional charges:

$$\left.\begin{array}{l} q_1:(5/3,\ 2/3,\ 2/3); \\ q_2:(2/3,\ -1/3,\ -1/3); \\ q_3:(-1/3,\ -4/3,\ -4/3). \end{array}\right\} \tag{69}$$

The Pati–Salam model with four quarks. Pati and Salam[71,72] noted the possible existence of a deep symmetry of all fundamental fermions—quarks and leptons. They proposed a scheme with four quarks p, n, λ, p', in which the fourth, charmed quark p' is an isosinglet, but, in contrast to the ordinary scheme with fractional charges, they ascribed to all quarks integral charges, using three color degrees of freedom. The fourth color degree of freedom was taken to be the lepton quantum number. Then all fundamental fermions can be combined as follows:

$$\left[\begin{array}{cccc} p_1^0 & n_1^- & \lambda_1^- & p_1'^0 \\ p_2^+ & n_2^0 & \lambda_2^0 & p_2'^+ \\ p_3^+ & n_3^0 & \lambda_3^0 & p_3'^+ \\ p_4=\nu_e & n_4=e^- & \lambda_4=\mu^- & p_4'=\nu_\mu \end{array}\right]. \tag{70}$$

The fundamental symmetry group is

$$G=SU(4)\times SU(4^c). \tag{71}$$

The fundamental fermions belong to the (4, 4*) representation.

On the basis of this symmetry, Pati and Salam attempt to create a unified theory of strong, electromagnetic, and weak interactions. In such a scheme, the baryon quantum number is not conserved, and the quarks can decay into leptons. However, the proton is unstable only in the sixth order in the constant of the quark decay, and therefore has a long life. In their model, Pati and Salam also introduce massive gluons, which could be observed (see Sec. 4).

2. EXPERIMENTAL VERIFICATION OF THE EXISTENCE OF COLOR DEGREES OF FREEDOM. MEASUREMENTS OF THE QUARK CHARGES

General comments

Above, we have considered the theoretical arguments for introducing color degrees of freedom. Let us now consider the experiments. A rigorous confirmation would be the direct observation of the predicted excited color states. At the present time, we do not have such a direct confirmation (see Sec. 4). We shall therefore now discuss experiments in which the presence of color degrees of freedom could appear indirectly even if the experiments are made at energies below the color threshold.

Of course, the quark charges are manifested in electromagnetic effects. We shall classify these effects on the basis of the following proposition: If in a given process that takes place *below* the color threshold only one photon participates, this process can reveal no more than the actual presence of the color degree of freedom and cannot distinguish between models with integral and fractional quark charges.[73] Processes involving at least two photons are needed to make the distinction.

Suppose that the initial $|i\rangle$ and final $|f\rangle$ states correspond to the ordinary world of hadrons and leptons, i.e., are singlets with respect to the color group $SU(3)^c$. In models with integral quark charges, the electromagnetic current contains color-singlet and color-octet parts:

$$J^{\mathrm{em}}=J_{8,1^c}+J_{1,8^c}. \tag{72}$$

If only a single photon participates in the process, the transition is determined by the matrix element

$$\langle f|J^{\mathrm{em}}|i\rangle. \tag{73}$$

But then only the color-singlet—valence—part of the electromagnetic current (72) contributes to it, and this is the same for all color models and is simply the threefold current of ordinary quarks.

But if two photons participate in the process, the transition is determined by a matrix element of the form

$$\langle f|J^{\mathrm{em}}\times J^{\mathrm{em}}|i\rangle=\langle f|(J_{8,1^c}+J_{1,8^c})(J_{8,1^c}+J_{1,8^c})|i\rangle. \tag{74}$$

Since the decomposition of $J_{1,8^c}\times J_{1,8^c}$ into irreducible representations of $SU(3^c)$ contains the singlet representation (see the Appendix), both parts of the electromagnetic current (72) will contribute to the matrix element (74). The color-octet part in models with fractional quark charges is absent. However this does not mean that in all processes involving two photons the difference will be manifested (see the discussion of the processes $\pi^0\to 2\gamma$ and $\eta\to 2\gamma$ below).

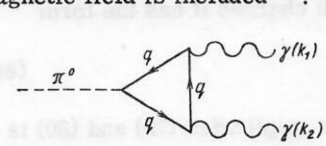
Being guided by this rule, let us now consider definite processes.

The decays $\pi^0\to 2\gamma$, $\eta\to 2\gamma$, $\eta'\to 2\gamma$

The calculation of the probability of the decay

$$\pi^0\to 2\gamma \tag{75}$$

provided one of the first and most serious arguments for the existence of a color degree of freedom in quarks.[74,75] The calculation of the probability of the decay (75) is based on the triangle diagram, which leads to the so-called *Adler anomaly* in the PCAC (partial conservation of axial current) relation when an electromagnetic field is included[74].



Adler and Bardean showed[76] that this simple triangle diagram is the only one that contributes to the process (75) in any finite order of renormalizable perturbation theory, and therefore the calculation made on the basis of it in the PCAC approximation is exact.

For the width of the decay (75) we obtain the expression

$$\Gamma(\pi^0 \to 2\gamma) = \alpha^2 [\mu^3/(16\pi^3 F_\pi^2)] S^2, \tag{76}$$

where $\alpha = e^2/4\pi \approx 1/137$; μ is the mass of the π^0 meson; F_π is a constant in the PCAC relation and is determined by the decay $\pi^+ \to \mu^+\nu$ ($|F_\pi| \approx 95$ MeV). The value of S for the π meson, which consists of p and n quarks (and antiquarks) is given by

$$S = (e_p^2 - e_n^2)/2. \tag{77}$$

In the model of ordinary quarks ($e_p = 2/3$, $e_n = -1/3$) we have $S = 1/6$. In any three-triplet model it is a consequence of (10) and (11) that

$$S = \sum_{i=1,2,3} (e_{pi}^2 - e_{ni}^2)/2 = \sum_{i=1,2,3} e_{pi} - 3/2 = 1/2. \tag{78}$$

Thus, the presence of three color states of the quarks increases the width of the decay (75) by a factor 9 compared with the width obtained in the ordinary quark model. The numerical values of the width are

$$\Gamma(\pi^0 \to 2\gamma) = \begin{cases} 0.87 \text{ eV for ordinary quark model;} \\ 7.84 \text{ eV for color models.} \end{cases}$$

Comparing the values obtained here with the experimental value 7.92 ± 0.42 eV (Ref. 77), we see brilliant agreement between it and the theoretical prediction of the color model. Note that this agreement indicates the fact that there are precisely three color states. However, one should bear in mind Drell's skeptical remark about this calculation[78] to the effect that the extrapolation used in the derivation of the equation (76) from zero pion mass to its physical mass (the "soft-pion approximation") can readily lead to the loss of a factor ~3, and then there is no need to introduce color degrees of freedom in order to reconcile the predictions of the ordinary quark model and the experimental data.

Why is it that the process (75), which includes two photons, does not distinguish between the color models with fractional and integral quark charges? In the model with fractional quark charges, the amplitude of the process has the form

$$\langle \pi^0 | J_{8,1^c} \times J_{8,1^c} | 0 \rangle, \tag{79}$$

where $J_{8,1^c}$ is the electromagnetic current (14), and in models with integral quark charges it has the form

$$\langle \pi^0 | (J_{8,1^c} + J_{1,8^c})^2 | 0 \rangle. \tag{80}$$

The difference between the amplitudes (79) and (80) is

$$\langle \pi^0 | J_{8,1^c} J_{1,8^c} + J_{1,8^c} J_{8,1^c} + (J_{1,8^c})^2 | 0 \rangle. \tag{81}$$

But π^0 is a member of the $(8, 1^c)$ multiplet. Further, $J_{8,1^c} \times J_{1,8^c} = (8, 8^c)$ does not have a projection onto to the color singlet; similarly, neither does $J_{1,8^c} \times J_{1,8^c}$; $(J_{1,8^c})^2$ does not have a projection onto the $SU(3)$ octet. Therefore, (81) vanishes and the amplitudes (79) and (80) are equal.[11]

A different result is obtained for decay of the $SU(3)$ singlet: $\eta_1 \to 2\gamma$ (Refs. 75 and 79). In this case, the decay amplitude in the model with integral quark charges:

$$\langle 1;\ 1^c | (J_{8,1^c} + J_{1,8^c})^2 | 0 \rangle = \langle 1;\ 1^c | J_{8,1^c}^2 + J_{1,8^c}^2 | 0 \rangle = 2 \langle 1;\ 1^c | J_{8,1^c}^2 | 0 \rangle \tag{82}$$

is twice the amplitude in the model with fractional quark charges.[11] This must affect the widths of the decays into two gamma rays of the η and η' mesons, which are singlet–octet mixtures:

$$\left. \begin{aligned} \eta &= \cos\theta\eta_8 - \sin\theta\eta_1; \\ \eta' &= \sin\theta\eta_8 + \cos\theta\eta_1. \end{aligned} \right\} \tag{83}$$

However, there is an uncertainty associated with the uncertainty in the ratio F_8/F_1 (F_8 and F_1 are constants analogous to F_π and occur in the PCAC relation for η_8 and η_1, respectively) and the mixing angle θ. To eliminate these uncertainties, Chanowitz[79] suggested that one should consider simultaneously the decays

$$\eta,\ \eta' \to 2\gamma \tag{84}$$

and

$$\eta,\ \eta' \to \pi^+\pi^-\gamma. \tag{85}$$

One can calculate such processes in the soft-pion approximation and from the experimentally determined ratio[77]

$$\Gamma(\eta \to 2\gamma)/\Gamma(\eta \to \pi^+\pi^-\gamma) = 7.60 \pm 0.25 \tag{86}$$

determine[79]

$$(F_8/F_1)\tan\theta = \begin{cases} -0.12 \pm 0.016 \text{ for fractional quark charges;} \\ -0.05 \pm 0.007 \text{ for integral quark charges.} \end{cases}$$

For $\theta = -11°$, the value which follows from the quadratic mass formulas,[77] one obtains[79]

$$F_8/F_1 = \begin{cases} 0.62 \pm 0.083 \text{ for fractional quark charges;} \\ 0.257 \pm 0.036 \text{ for integral quark charges} \end{cases}.$$

and for the decay widths

$$\Gamma(\eta \to 2\gamma) = \begin{cases} 283 \pm 21 \text{ eV for fractional quark charges;} \\ 258 \pm 16 \text{ eV for integral quark charges} \end{cases}$$

and

$$\Gamma(\eta \to \pi^+\pi^-\gamma) = \begin{cases} 38 \pm 1 \text{ eV for fractional quark charges;} \\ 32 \pm 0.6 \text{ eV for integral quark charges.} \end{cases}$$

The experimental data are[80]

$$\Gamma(\eta \to 2\gamma) = 324 \pm 46 \text{ eV};\quad \Gamma(\eta \to \pi^+\pi^-\gamma) = 43 \pm 6 \text{ eV}.$$

The last value is obtained by means of (86). The agreement between Chanowitz's theoretical predictions[79] and the experimental data is excellent. But at the same time, the difference between the predictions of the models with integral and fractional quark charges is so slight that on the basis of them it is difficult to give preference to one of these models over the other. Chanowitz notes, however, that the value of F_8/F_1 obtained above for the model with fractional quark charges is closer to the expected ratio $F_8/F_1 \sim 1$ than the same value for the model with integral quark charges. Thus, if one takes $F_8 = F_1 = F_\pi$ and $\theta = -11°$, for the widths one obtains the values[79]

$$\Gamma(\eta \to 2\gamma) = \begin{cases} 379 \text{ eV for fractional quark charges;} \\ 696 \text{ eV for integral quark charges} \end{cases}$$

and

$$\Gamma(\eta \to \pi^+\pi^-\gamma) = 45 \text{ eV for both models.}$$

A significantly greater difference between the predictions of the models with fractional and integral quark charges must occur for the decays $\eta' \to 2\gamma$. Chanowitz estimated the ratio for $\theta = -11$ (Ref. 79):

$$\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to 2\gamma) = \begin{cases} 5.4 \pm 0.2 \text{ for fractional quark charges;} \\ 0.5 \pm 0.1 \text{ for integral quark charges.} \end{cases}$$

Experimentally, it has not been well measured but the preliminary value of this ratio lies in the range 9.1 ± 2.2 to 17 ± 3, which favors rather the model with fractional quark charges.

The question of the possibility of extrapolating the physical masses of the η and η' mesons, which are greater than the π^0 meson mass, to zero has an important bearing on all this.

Annihilation of e^+e^- into hadrons

On the colliding rings of SPEAR at Stanford the famous branching ratio[81, 82]

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^-e^- \to \mu^+\mu^-) \tag{87}$$

has now been measured in a fairly large range of energies.[83] Its behavior as a function of $s = (E_+ + E_-)^2$ exhibits the following remarkable features: In the interval $2 \leqslant s \leqslant 10$ GeV^2 it is approximately constant and equal to 2.5 ± 0.4; for $s \gtrsim 10$ GeV^2 there commences a "new resonance region" which may be related to the production of ψ'', ψ''' etc., resonances; for $s \gtrsim 25$ GeV^2, a new constant value 5.2 ± 0.8 is established.

Theoretically, the annihilation $e^+e^- \to$ hadrons is assumed to proceed through a single-photon intermediate state[4)]:

$$e^+e^- \to \gamma^* \to \text{hadrons.} \tag{88}$$

At asymptotic values $s \to \infty$ in the parton model the intermediate photon goes over as a result of a point interaction into a noninteracting parton–antiparton pair, which then "pulls out" out of the vacuum other parton–antiparton pairs and is transformed as a result of recombination with them into the final hadrons. The contributions from the different originally formed parton pairs are assumed to be incoherent. This assumption is equivalent to restricting onself to single-loop diagrams when calculating the cross section of the process (88) in terms of the absorptive part of the photon self-mass diagram.

Under these assumptions and at asymptotic values, the following simple constant value is obtained for R:

$$R_\infty = \lim_{s\to\infty} \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_j \left(\frac{1}{4} + \frac{3}{2}\sigma_j\right) e_j^2, \tag{89}$$

where e_j is the charge of parton j and σ_j is its spin (0 or $\frac{1}{2}$).

Making the Bjorken–Paschos hypothesis that partons are identical to quarks, we obtain simply

$$R_\infty = \sum_j e_j^2. \tag{90}$$

The color models give

$$R_\infty = \sum_{i=1,2,3} [e_{pi}^2 + 2(e_{pi} - 1)^2] = 3 \sum_{i=1,2,3} e_{pi}^2 - 2. \tag{91}$$

For the different models we have:

$$R_\infty = \begin{cases} 2/3 \text{ for ordinary quarks} = 10/9\ (\ldots + \text{charmed quark}); \\ 2 \text{ for color quarks with fractional charges} = 10/3\ (\ldots + \text{charmed quark}); \\ \text{the same for color quarks with integral charges below the color threshold;} \\ 4 \text{ for color quarks with integral charges above the color threshold (the Han-Nambu model)} = 6\ (\ldots + \text{charmed quark, Pati-Salam model}); \\ 8 \text{ for color quarks with fractional charges in the Tati model above the color threshold.} \end{cases} \tag{92}$$

Note that this is a single-photon process and therefore the model with integral quark charges below the color threshold leads to the same results as the model with fractional quark charges.[73] Let us compare the predictions of the different models with the experimental $R(s)$ behavior. One might think that the constancy $R \approx 2.5$ in the region up to the "new resonance region" is due to the establishment of a "first asymptotic regime" in the multiple production of ordinary hadrons. We then obtain clear evidence in favor of the existence of the color degree of freedom. Further, if it is assumed that the color threshold lies around $s = 20$–25 GeV^2, the model with integral quark charges explains the rise of R intriguingly and naturally as the establishment of a "second asymptotic regime" when the new, color hadrons are also produced multiply. The difference between the predicted asymptotic value $R_\infty = 4$ and the experimental value $R \approx 5.2$ can be explained by

4) For a detailed review of the process $e^+e^- \to$ hadrons, see Ref. 84.

the production in e^+e^- annihilation of heavy leptons with mass 1.6–2 GeV (Ref. 85), which can then decay into ordinary hadrons + one lepton (for example, a new neutrino). The Pati–Salam model does not require the last hypothesis: For it, $R_\infty = 6$. It is assumed that in this energy range quarks with integral charges are themselves produced.[86]

The alternative model with fractional quark charges does not give the two-step $R(s)$ behavior since in it the photon cannot excite higher color states. It gives the simple doubling of R_∞ obtained in the ordinary quark model. The second step in the $R(s)$ behavior is explained by "new physics": production of charmed particles ($R_\infty = 10/3$) and heavy leptons.[87]

Deep inelastic processes

Deep inelastic electron–nucleon scattering. The process

$$e + N \to e' + \text{hadrons} \tag{93}$$

in the parton model is treated in the infinite-momentum frame of the nucleon as a process of elastic scattering of a virtual photon on each parton, a point particle, with subsequent recombination of the partons into hadrons.[88] In this model, the well-known structure functions, which determine the cross section of the process (93) and are measurable quantities, scale in the asymptotic region (Bjorken scaling):

$$\left.\begin{array}{l} \nu W_2(Q^2, \nu)/M_N = F_2(x) \\ W_1(Q^2, \nu) = F_1(x) \end{array}\right\} \begin{array}{l}\text{for } Q^2, \nu \to \infty \text{ and fixed} \\ x = Q^2/(2M_N\nu), \end{array} \tag{94}$$

where M_N is the nucleon mass; $\nu = E - E'$ is the energy (in the laboratory system) of the virtual photon; $Q^2 = 4EE' \sin^2\theta/2$ is its mass (θ is the scattering angle of the electron). For partons or quarks, particles with spin $\frac{1}{2}$, the Callan–Gross relation holds,

$$F_2(x) = 2xF_1(x) \tag{95}$$

and we have the formula

$$F_2(x) = \sum_j e_j^2 x f_j(x), \tag{96}$$

where $f_j(x)$ is the density of the probability that parton j carries away the fraction x of the nucleon momentum in the infinite-momentum frame of the nucleon; e_j is the parton charge.

We write $u_j(x) = xf_j(x)$. On the basis of isotopic invariance for the partons or quarks in the proton and neutron we deduce

$$u_p^{\mathscr{P}}(x) = u_n^{\mathscr{N}}(x); \quad u_n^{\mathscr{P}}(x) = u_p^{\mathscr{N}}(x); \quad u_\lambda^{\mathscr{P}}(x) = u_\lambda^{\mathscr{N}}(x). \tag{97}$$

For the proton and neutron, the relation (96) has the form

$$\left.\begin{array}{l} F_2^{\gamma\mathscr{P}}(x) = e_p^2[u_p(x) + u_{\bar{p}}(x)] + e_n^2[u_n(x) + u_{\bar{n}}(x)] \\ \qquad + e_\lambda^2[u_\lambda(x) + u_{\bar{\lambda}}(x)]; \\ F_2^{\gamma\mathscr{N}}(x) = e_p^2[u_n(x) + u_{\bar{n}}(x)] + e_n^2[u_p(x) + u_{\bar{p}}(x)] \\ \qquad + e_\lambda^2[u_\lambda(x) + u_{\bar{\lambda}}(x)]. \end{array}\right\} \tag{98}$$

It is easy to obtain a lower bound for the ratio:

$$F_2^{\gamma\mathscr{N}}(x)/F_2^{\gamma\mathscr{P}}(x) \geqslant e_n^2/e_p^2. \tag{99}$$

In color models, it is necessary to take the sums over all quarks of a given species, and we then obtain

$$F_2^{\gamma\mathscr{P}}(x) = \sum_{i=1,2,3} \{e_{pi}^2[u_{pi}(x) + u_{\bar{p}i}(x)] + e_{ni}^2[u_{ni}(x) + u_{\bar{n}i}(x)] + e_{\lambda i}^2[u_{\lambda i}(x) + u_{\bar{\lambda} i}(x)]\}. \tag{100}$$

In nucleons, which are color singlets, all colors of a given quark occur with the same weight 1/3, and therefore (100) takes the form

$$F_2^{\gamma\mathscr{P}}(x) = [\sum_{i=1,2,3} e_{pi}^2/3][u_p(x) + u_{\bar{p}}(x)] + [\sum_{i=1,2,3} e_{ni}^2/3][u_n(x) + u_{\bar{n}}(x)] + [\sum_{i=1,2,3} e_{\lambda i}^2/3][u_\lambda(x) + u_{\bar{\lambda}}(x)]. \tag{101}$$

Instead of (99), we obtain

$$F_2^{\gamma\mathscr{N}}(x)/F_2^{\gamma\mathscr{P}}(x) \geqslant \sum_{i=1,2,3} e_{ni}^2 / \sum_{i=1,2,3} e_{pi}^2. \tag{102}$$

It follows from (98) and (101) that the ordinary quark model, the model of color quarks with fractional charges, and the model of color quarks with integral charges, but below the color threshold, lead to the same structure function

$$F_2^{\gamma\mathscr{P}}(x) = (4/9)[u_p(x) + u_{\bar{p}}(x)] + (1/9)[u_n(x) + u_{\bar{n}}(x) + u_\lambda(x) + u_{\bar{\lambda}}(x)], \tag{103}$$

and (99) has the form[89]

$$F_2^{\gamma\mathscr{N}}(x)/F_2^{\gamma\mathscr{P}}(x) \geqslant 1/4. \tag{104}$$

For the model of color quarks with integral charges above the color threshold, the structure function increases[10]:

$$F_2^{\gamma\mathscr{P}}(x) = 2[u_p(x) + u_{\bar{p}}(x)]/3 + [u_n(x) + u_{\bar{n}}(x) + u_\lambda(x) + u_{\bar{\lambda}}(x)]/3 \tag{105}$$

and

$$F_2^{\gamma\mathscr{N}}(x)/F_2^{\gamma\mathscr{P}}(x) \geqslant 1/2. \tag{106}$$

The experiments made at SLAC in a wide range of Q^2: $2 \lesssim Q^2 \lesssim 14$ GeV^2, do not reveal any significant increase of the structure functions $F_2^{\gamma\mathscr{N}}(x)$ and $F_2^{\gamma\mathscr{P}}(x)$ as Q^2 increases, as should occur in accordance with the model with integral quark charges above the color threshold. However, at small x appreciable deviation from Bjorken scaling was found.[90] Further, the ratio (104) determined on the basis of the experimental data[91] was found to be a decreasing function of x: from 1 at $x=0$ to 0.25 at $x=1$. Around $x \approx 0.5$ it becomes less than $\frac{1}{2}$. Thus, the deep inelastic electron–nucleon scattering experiments would seem to be against the model with integral quark charges.

It is, however, necessary to take into account the kinematic restrictions. If the color degrees of freedom are to be excited, we must have

$$(P+q)^2=M_N^2+2M_N\nu-Q^2\geqslant M_c^2, \quad (107)$$

where P and q are the four momenta of the proton and virtual photon ($q^2\equiv-Q^2$); M_c is the mass characterizing the color threshold. Manifestation of color degrees of freedom can be expected only for x values satisfying the condition

$$x\leqslant[1+(M_c^2-M_N^2)/Q^2]^{-1}. \quad (108)$$

For $M_c^2\approx 20$ GeV^2 (see above) and $Q^2\approx 15$ GeV^2, we obtain $x\leqslant 0.45$. Thus, we can expect excitation of color degrees of freedom only for small x, where Bjorken scaling is violated and the simple parton model is probably untenable.

In Ref. 92, kinematic restrictions were introduced into the parton model under the assumption that when new (for example, color) degrees of freedom are excited partons with large mass m_c are produced. Then the Bjorken variable must be replaced by

$$z=x+m_c^2/(2M_N Ey), \text{ where } y=\nu/E. \quad (109)$$

Finally, we mention that Pati and Salam[93] proposed a color model in which the contributions from excitation of the color degrees of freedom by the photon and a heavy color gluon interfere and cancel one another in the asymptotic region. On the basis of the unified gauge theory of weak and electromagnetic interactions, they assumed

1) leptons and color singlets;

2) the valence and color currents [see (31)] are gauge independent.

The first is associated with the weak interaction, the second with the strong.

Then the basic Lagrangian without spontaneous breaking of the gauge invariance can be written in the form

$$L_{\text{int}}=gW_\mu(J_\mu^{\text{valence}}+J_\mu^{\text{lept}})+fV_\mu J_\mu^{\text{color}}, \quad (110)$$

where W and V are the weak and strong (gluon) gauge fields, respectively. Note that with this Lagrangian the leptons interact by W exchange only with the valence current and do not interact with the color current. As a result of spontaneous breaking of the gauge symmetry, W_μ and V_μ are mixed and lead to the appearance of a massless photon A_μ, which is a mixture of W_3 and $\sqrt{3}V_3/2+V_8/8$, and to a color massive gauge partner $\tilde{U}_\mu$ orthogonal to the photon. The Pati–Salam theorem consists of proving that the contributions from A_μ and $\tilde{U}_\mu$ to the interaction of the leptons with the color current cancel one another asymptotically. In other words, the interaction of the leptons with the color current due to the difference between the photon and gluon propagators disappears asymptotically and the quarks exhibit only their valence, i.e., fractional charges.

In any case, one can conclude that the experimental data with regard to the process (93), which is a single-photon process, do not reject models with integral quark charges.

Deep inelastic scattering of neutrinos (antineutrinos) on the nucleon. The following process of inelastic scattering of neutrinos or antineutrinos on the nucleon is analogous to the process (93):

$$\nu,\ \bar{\nu}+N\to l+\text{hadrons}, \quad (111)$$

where l is the corresponding lepton ($e^\pm$, $\mu^\pm$, ν, $\bar{\nu}$), which in the parton model is regarded as a point interaction of heavy $W^\pm$ and Z^0 bosons, which transmit the weak interaction with individual partons. One can, however, assume that the interaction of a neutrino with a parton (quark) is a four-fermion contact interaction:

$$[\bar{\mu}\gamma_\mu a\nu][\bar{p}\gamma_\mu a n_\theta]+\ldots, \quad (112)$$

where $n_\theta=\cos\theta_c n+\sin\theta_c\lambda$ is the Cabbibo quark.

Now the electromagnetic interaction of a quark is determined by its charge, and the weak interaction of the quark by its isospin state. Since the connection between these properties of quarks is different in the different models, one can attempt, on the basis of data referring to the processes (93) and (111), to determine a quark characteristic such as the charge in the framework of one and the same parton picture.

On the basis of the parton model in the asymptotic limit $Q^2\to\infty$, $\nu\to\infty$ (x fixed)[5)] we obtain the following sum rule (see, for example, Refs. 94 and 95):

$$\sigma^{\nu N}+\sigma^{\bar{\nu}N}=\frac{2}{3}\frac{G^2M_NE_{\nu,\bar{\nu}}}{\pi}\int[F_2^{\nu N}(x)+F_2^{\bar{\nu}N}(x)]\,dx, \quad (113)$$

where G^2 is the constant of the weak interaction ($G=1.0\cdot 10^{-5}$); $\sigma^{\nu,\bar{\nu}N}$ is the total cross section for interaction of neutrinos or antineutrinos with the nucleon (proton or neutron). Further, in accordance with (112), the model of ordinary Gell-Mann–Zweig quarks gives for the structure function (for simplicity we assume that the Cabibbo angle is zero, $\theta_c=0$, and we ignore the contribution of the strange partons λ)

$$\begin{aligned}F_2^{\nu\mathscr{P}}(x)=F_2^{\bar{\nu}\mathscr{N}}(x)=2[u_n(x)+u_{\bar{p}}(x)];\\ F_2^{\bar{\nu}\mathscr{P}}(x)=F_2^{\nu\mathscr{N}}(x)=2[u_p(x)+u_{\bar{n}}(x)].\end{aligned} \quad (114)$$

Therefore, instead of (113) we can write

$$\sigma^{\nu N}+\sigma^{\bar{\nu}N}=\frac{4}{3}\zeta\frac{G^2M_NE_{\nu\bar{\nu}}}{\pi}u, \quad (115)$$

where

$$u=\int[u_p(x)+u_{\bar{p}}(x)+u_n(x)+u_{\bar{n}}(x)]\,dx;\ \zeta=1. \quad (116)$$

At the same time, from (98)

$$\langle e^2\rangle u=\int F_2^{\gamma N}(x)\,dx, \quad (117)$$

5) We now have $\nu=E_\nu-E_l$, $Q^2=4E_\nu E_l\sin^2\theta/2$, where θ is the emission angle of lepton l.

where the average charge of the parton for ordinary quarks is

$$\langle e^2\rangle = (e_p^2 + e_n^2)/2 = 5/18; \tag{118}$$

$F_2^{\gamma N}$ is the average function for the nucleon:

$$F_2^{\gamma N} = (F_2^{\gamma \mathscr{P}} + F_2^{\gamma \mathscr{N}})/2. \tag{119}$$

(In the case of measurements on nuclei, this averaging occurs automatically.) From (115) and (117) there follows the relation[94,95]

$$\langle e^2\rangle \zeta^{-1} = \int F_2^{\gamma N}(x)\,dx \Big/ \left[\frac{3\pi}{4G^2 M_N} \frac{\sigma^{\nu N} + \sigma^{\bar{\nu} N}}{E_{\nu,\bar{\nu}}}\right]. \tag{120}$$

Obviously, the same result holds in the color model with fractional charges. However, in the model with integral quark charges the result is different and depends on the choice of the weak interaction of the color quarks. As an example, let us consider the choice (39) of Greenberg and Nelson.[10] Bearing in mind that each color state occurs with the same weight 1/3, we obtain for the structure functions[10]

$$\begin{aligned} F_2^{\nu \mathscr{P}}(x) &= F_2^{\bar{\nu} \mathscr{N}}(x) = 2\,[4u_n(x)/3 + u_p(x)/3 + 4u_{\bar{p}}(x)/3 + u_{\bar{n}}(x)/3]; \\ F_2^{\bar{\nu} \mathscr{P}}(x) &= F_2^{\nu \mathscr{N}}(x) = 2\,[4u_p(x)/3 + u_n(x)/3 + 4u_{\bar{n}}(x)/3 + u_{\bar{p}}(x)/3]. \end{aligned} \tag{121}$$

In the expression (115), we now have $\zeta = 5/3$. Further, the average parton charge is determined in accordance with (101) as

$$\langle e^2\rangle = \sum_{i=1,2,3} (e_{pi}^2 + e_{ni}^2)/6. \tag{122}$$

A simple calculation for the Han–Nambu model gives

$$\langle e^2\rangle \zeta^{-1} = 2/5. \tag{123}$$

The experimental data[94] are compared with the theoretical predictions in Fig. 2. The best agreement with the experiments is given by the models with fractional quark charges and average charge (118). If one simply assumes $e_p = 1$ and $e_n = 0$, then the value $\frac{1}{2}$ obtained for the left-hand side of (120) is definitely ruled out by the experiment. But one cannot reject the model with integral quark charges on this basis. In the example considered in the framework of this model the value of the left-hand side of (120) is 2/5, which lies above the experimental points. But we must again recall the kinematic restrictions discussed above. They can have the consequence that at the energies of the existing electron beams only small x contribute to the integral in the numerator of the right-hand side of (120). Effectively, this reduces the left-hand side of (120). If it is assumed that in electron–proton collisions color degrees of freedom cannot be excited at all, then for the left-hand side of (120) one can obtain the value 1/6. The experimental points lie between the two limiting values 2/5 and 1/6 given by the model with integral quark charges. Feldman and Matthews[44] obtained for the left-hand side of (120) the value 3/14 in the framework of the variant of weak color currents that they chose.

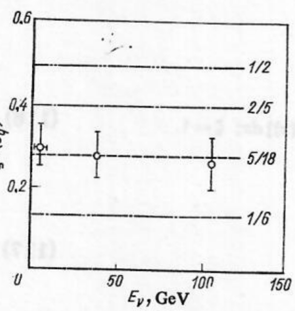


FIG. 2. Mean square charge of partons determined from comparison of the data on deep inelastic scattering of electrons, neutrinos, and antineutrinos on the nucleon.

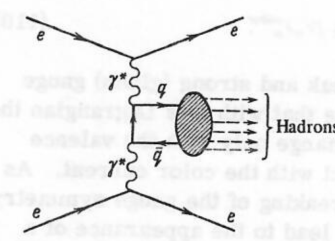
Two-photon deep inelastic processes.

Thus, a single-photon deep inelastic process does not enable one to distinguish between models with integral and fractional quark charges either below the color threshold or even above it, because of the kinematic restrictions at the energies of the existing electron beams. Of great interest therefore are processes with more than one photon in which a difference between the models must appear at energies *below* the color threshold.

Deep inelastic ee scattering.[96] In the parton model, the process

$$e + e \to e + e + \gamma^* + \gamma^* \to e + e + q + \bar{q} \to e + e + \text{hadrons} \tag{124}$$

is interpreted as the production of a parton–antiparton pair in a collision of two virtual photons with subsequent recombination of this pair into the hadron final state[97]:



The contributions to this state from the individual parton pairs are assumed to be *incoherent*. This is equivalent, in the calculation of the absorptive part of the rectangle diagram with four photons at the vertices, to allowance for only the single-loop quark diagrams (see the Appendix). The calculation gives for the asymptotic value the ratio

$$T_\infty = \lim_{\substack{s\to\infty \\ Q^2\to\infty}} \frac{\sigma_{2\gamma^*}(ee \to ee + \text{hadrons})}{\sigma_{2\gamma^*}(ee \to ee + \mu^+ + \mu^-)} = \sum_j e_j^4. \tag{125}$$

In the various models we obtain the following values for the asymptotic ratio (125):

$$T_\infty = \begin{cases} 2/9 \text{ for ordinary quarks} = 34/81\ (\ldots + \text{charmed quark}); \\ 2/3 \text{ for color quarks with fractional charges} = 34/27\ (\ldots + \text{charmed quark}); \\ 2 \text{ for color quarks with integral charges below the color threshold} = 10/3\ (\ldots + \text{charmed quark}); \\ 4 \text{ for color quarks with integral charges above the color threshold} = 6\ (\ldots + \text{charmed quark}). \end{cases} \quad (126)$$

This process has not yet been subjected to experimental investigation because of its small cross section $\sim 10^{-36}\ \mathrm{cm}^2$ at ~ 3 GeV (Ref. 97). The cross section decreases rapidly for strongly virtual photons because of the decrease of the photon propagators. However, in Ref. 98 it was shown to be possible to measure the sum of the fourth powers of the quark charges in the case when only one photon is strongly virtual and the other is near the mass shell.

Deep inelastic Compton effect. This process

$$\gamma + N \to \gamma' + \text{hadrons} \quad (127)$$

is determined in the parton picture by the diagram for the scattering of a photon on one and the same parton. Bjorken and Paschos[99] found that in the deep inelastic region the cross sections of the processes (93) and (127) are related by the simple equation

$$(d^2\sigma/d\Omega'\, dE')_{\gamma N} = (\nu^2/EE')\,(d^2\sigma/d\Omega'\, dE')_{eN}\,[\sum_j e_j^4 / \sum_j e_j^2]. \quad (128)$$

Depending on the assumptions about the significance of the statistical background of quark–antiquark pairs existing in addition to the three valence quarks, one obtains the estimate (see the Appendix)

$$\frac{\langle \sum_j e_j^4 \rangle}{\langle \sum_j e_j^2 \rangle} = \begin{cases} 1/9\text{—}4/9 & \text{for fractional quark charges;} \\ 1 & \text{for integral quark charges.} \end{cases} \quad (129)$$

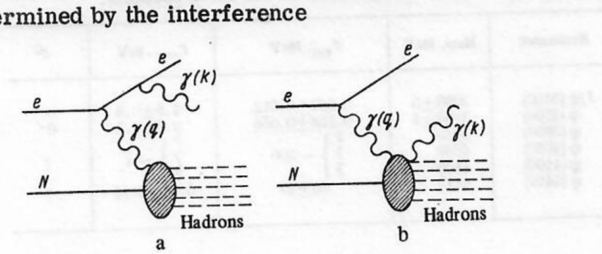
The process (127) has been investigated experimentally,[100] and it was found that single-photon events are much more copious than expected on the basis of a calculation from the data on deep inelastic eN scattering even under the assumption of integral quark charges. The most probable explanation for the excess is through the exchange subprocess $\bar{q}^{\mathcal{F}} \to qq\gamma$ (Ref. 100). But this subprocess is due probably to the proton structure and not to the photon–parton interaction.

Thus, the deep inelastic Compton effect is not suitable for determining the parton charges.

Deep inelastic bremsstrahlung. Brodsky, Gunion, and Jaffe[101] suggested a way of determining the parton charges on the basis of simultaneous investigation of the deep inelastic bremsstrahlung by electrons and positrons:

$$e^\pm + N \to e^\pm + \text{hadrons} + \gamma. \quad (130)$$

The method is based on the fact that the difference between the cross sections of the processes (130) is determined by the interference



between Bethe–Heitler (a) and Compton (b) scattering, which have opposite signs for the electron and positron. Introducing the corresponding structure factor $V(x)$ for this difference, Bodsky *et al.*[101] showed that in the parton model it is proportional to the sum of the cubes of the parton charges (see the Appendix). From this, there follow the sum rule

$$\int_0^1 V(x)\,dx = Q = 1 \text{ for the proton, } 0 \text{ for the neutron above the color threshold;} \quad (131)$$

$$\int_0^1 V(x)\,dx = (5Q + B)/9 = 7/9 \text{ for the proton, } 2/9 \text{ for the neutron below the color threshold} \quad (132)$$

in models with integral charges of the partons (or quarks), and

$$\int_0^1 V(x)\,dx = Q/3 + 2B/9 = 5/9 \text{ for the proton, } 2/9 \text{ for the neutron} \quad (133)$$

in models with fractional charges of the partons (quarks). Attempts to determine the parton charges on the basis of this sum rule have already been made,[102] but the results are as yet ambiguous.

Color interpretation of the J–ψ mesons. As we have seen in Sec. 1, the color models with integral quark charges predict a large number of heavy resonances of a completely new type corresponding to higher representations of the color symmetry. As we said in the introduction, after the discovery of the J–ψ mesons it was very natural to attempt to explain these mesons in the framework of color symmetry [note that even before the discovery the possible existence of new narrow heavy vector mesons had been proposed on the basis of color symmetry by various authors.[51,68,75] In particular, in Ref. 68 the present author gave the lower bound 1.5–2.5 GeV for the masses of these mesons on the basis of an analysis of the high-energy Compton effect using a generalized Weinberg sum rule].

However, it has recently become more popular to explain the $J-\psi$ mesons in the framework of an alternative scheme with charged quarks as a bound state of one of these new quarks and antiquarks giving *charmonium*. Confidence in this explanation was strengthened by *the prima facie* discovery of charmed D^0 particles from their decay into the $K\pi$ and $K\pi\pi\pi$ systems[103] and $D^\pm$ particles from their decay into $K^\mp\pi^\pm\pi^\pm$ (Ref. 104).[6)]

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6)Note, however, the attempt in Ref. 118 to interpret D mesons as diquark color states.

TABLE IV. Basic properties of the $J-\psi$ mesons.

Resonance	Mass, MeV	Γ_{tot}, MeV	$\Gamma_{e^+e^-}$, keV	I^G
J/ψ (3100)	3098 ± 3	0.067 ± 0.012	4.8 ± 0.6	0^-
ψ (3700)	3684 ± 4	0.228 ± 0.056	2.2 ± 0.3	0^-
ψ (3950)	3950	? }	? }	
ψ (4030)	4030	? } ~ 200	? } ~ 4	
ψ (4100)	4100	? }	? }	?
ψ (4410)	4415	33 ± 10	0.44 ± 0.14	?

Let us briefly consider the color interpretation of the $J-\psi$ mesons as an historical exercise.

First of all, let us list the properties of the $J-\psi$ mesons that will be required in the following discussion (for detailed references, see Ref. 77):

1) all $J-\psi$ mesons are formed in e^+e^- annihilation through a single-photon intermediate state, and their quantum numbers are $J^{PC}=1^{--}$;

2) only two narrow meson resonances have been discovered: $J/\psi(3100)$ and $\psi(3700)$. The basic properties of these mesons are given in Table IV (Refs. 105–107);

3) in e^+e^- annihilation in the energy range 4–5 GeV one observes a complicated structure.[108] We shall regard it as due to some of the broad resonances listed in Table IV;

4) there exist the strong transitions[109]

$$\psi(3700) \to J/\psi(3100) \begin{matrix} +2\pi, \\ +\eta; \end{matrix}$$

5) among the products of the decay of the broad resonances, $J/\psi(3100)$ and $\psi(3700)$ are not found in appreciable numbers;

6) in the cascade γ and 2γ transitions $\psi(3700)-J/\psi(3100)$ there is evidence of at least three intermediate levels $P_c(3300)$ or (3500), $\chi(3410)$, $\chi(3530)$.

The color models with integral charges are divided, as we have shown in Sec. 1, into two types: with strong (I^c-type) and electromagnetic (U^c-type) breaking of the color symmetry. The identification of the $J-\psi$ mesons with the vacant states of heavy vector mesons (58) and (61)–(62) in these schemes is shown in Table V (for the I^c-type, only one of the possible identifications is given).

Below, in estimates, we shall always assume that in (57) the angle of the $SU(3)$ singlet–octet mixing is

TABLE V. Identification of $J-\psi$ mesons with vacant states of vector mesons in color models with integral quark charges.

Resonance	I^c-type	U^c-type
J/ψ (3100)	$\tilde{\rho}_C^0\ (I''=1)$	$\tilde{\omega}$
ψ (3700)	$\tilde{\rho}_D^0\ (I''=1)$	$\tilde{\varphi}$
ψ (3950)	$\tilde{\rho}_B^0\ (I''=0)$	}
ψ (4030) ?	Exotic state	} Radial excitations of $\tilde{\omega}$ and $\tilde{\varphi}$; possibly doublet states
ψ (4100)	$\tilde{\omega}\ (I''=0)$	}
ψ (4410)	$\tilde{\varphi}\ (I''=0)$	}

"ideal": $\tan\varphi=1/\sqrt{2}$. Therefore, all ρ^0 and ω mesons will consist only of nonstrange p and n quarks, and all the φ mesons will consist of strange λ quarks.

The production of the vector V meson in e^+e^- annihilation through the intermediate single-photon state is determined by the matrix element

$$\langle 0|J_\mu(0)|V\rangle=\varepsilon_\mu m_V^2 f_V^{-1}, \tag{134}$$

where ε_μ is the polarization vector of the photon; $f_U^2/4\pi$ is the constant of the vector meson–photon transition.

I^c-type.[44,51–56] The electromagnetic current (30) can be written in the form

$$J^{em}=(\sqrt{6}/2)\,[(\rho^0+\omega/2-\sqrt{2}\,\varphi/3) + (-\sqrt{2/3}\,\tilde{\rho}_C^0+\sqrt{1/3}\,\tilde{\rho}_D^0-\sqrt{2}\,\tilde{\omega}/3+\tilde{\varphi}/3)]. \tag{135}$$

It follows that e^+e^- annihilation could lead to the production of four new heavy vector mesons: $\tilde{\rho}_C^0$, $\tilde{\rho}_D^0$, $\tilde{\omega}$, $\tilde{\varphi}$. Two of them, $\tilde{\rho}_C^0$ and $\tilde{\rho}_D^0$, have nonzero color isospin, and therefore in the modified Han–Nambu model (see above) must be narrow. They are identified with $J/\psi(3100)$ and $\psi(3700)$, respectively. The two broad resonances $\tilde{\omega}$ and $\tilde{\varphi}$ with zero color isospin are identified with $\psi(4100)$ and $\psi(4410)$. In the limit of exact $SU(3')\times SU(3'')$ symmetry, one would have the following relations for the transition constants:

$$f_{\tilde{\rho}_C^0}^{-1} : f_{\tilde{\rho}_D^0}^{-1} : f_{\tilde{\omega}}^{-1} : f_{\tilde{\varphi}}^{-1} = -\sqrt{\frac{2}{3}} : \sqrt{\frac{1}{3}} : -\frac{\sqrt{2}}{3} : \frac{1}{3}. \tag{136}$$

The values obtained from the experimental data are[52]

$$\left.\begin{matrix} f_{J/\psi(3100)}^2/4\pi=11.5\pm1.4;\ f_{\psi(3700)}^2/4\pi=29.7\pm4.1; \\ f_{\psi(4410)}^2/4\pi=178\pm57 \end{matrix}\right\} \tag{137}$$

and they do not contradict the relations (136). It should be recalled that the use of these relations on the mass shell of the vector mesons has restricted significance.

The fifth heavy vector meson $\tilde{\rho}_B^0$ does not occur directly in the electromagnetic current (135). But it could be that there is nevertheless a small admixture of the color singlet $1''$ to the color octet $8''$ (and vice versa!). Through this slight admixture, $\tilde{\rho}_B^0$ could be produced in e^+e^- annihilation.[44] Since this meson has zero color isospin, it must be broad.

The resonance $\psi(4030)$ is explained by Stech and Marinescu[100] on the basis of the assumed exotic state which is a $\tilde{\rho}_B^0+\pi$ bound state formed from the intermediate single-photon state through the $\tilde{\omega}$. The scheme is attractive in that it provides a natural explanation of 2) and 5) (because of conservation of the color isospin I'') in the above list of properties of the $J-\psi$ mesons.

The intermediate levels P_c and χ mentioned under 6) above can be interpreted as L excitations of $J/\psi(3100)$ and as certain new states of the type of the heavy pseudoscalar meson η_c, etc., (see, for example, Ref. 52).

The shortcomings of the scheme include its ascription of isovector properties to $J/\psi(3100)$ and $\psi(3700)$, although these last are more probably isoscalars.[107]

This difficulty could, however, be avoided as follows. Suppose[53] that the so-called direct decays of $J/\psi(3100)$ and $\psi(3700)$ into hadrons take place through the electromagnetic self-interaction ("tadpole"), which has the form $H(1, I_3)$ by analogy with $H(I_3, 1)$ for the electromagnetic decay $\eta \to 3\pi$. This mechanism would be equivalent to the decay of a fictitious heavy ω meson and would effectively lead to the correct values $I^G = 0^-$.

Without going into detail about the weak interactions of the new color mesons, let us mention that in the Feldman–Matthews model[44] there is only one long-lived heavy meson: the strange heavy meson $K_C^{*\pm}$ with color isospin $I'' = \frac{1}{2}$. The lifetime of this meson is estimated at about 10^{-13} sec, and the mass is around 3.9 GeV.

U^c-type.[59–69] In models of U^c-type, the existence of six new heavy neutral vector mesons is predicted: $\tilde{\omega}, \tilde{\tilde{\omega}}, \tilde{\varphi}, \tilde{\tilde{\varphi}}, \tilde{\rho}_0, \tilde{\tilde{\rho}}^0$, which are analogous with regard to their $SU(3)$ properties to the ordinary ω, φ, and ρ^0 mesons. The states designated by a tilde and two tildes are degenerate doublets. This degeneracy is lifted only by the electromagnetic interaction. All six of these mesons must be narrow since they can decay into ordinary hadrons only electromagnetically.

The electromagnetic current (30) can be rewritten in the form

$$J^{em} = (\sqrt{6}/2)(\rho^0 + \omega/2 - \sqrt{2}\varphi/3) - 2\sqrt{3}\,\tilde{\omega}/3 + \sqrt{6}\,\tilde{\varphi}/3. \quad (138)$$

It can be seen from this that in e^+e^- annihilation only the two narrow resonances $\tilde{\omega}$ and $\tilde{\varphi}$ can be produced. In the limit of exact $SU(3)\times SU(3^c)$ symmetry, the ratio for the vector meson–photon transition constants must be

$$f_{\tilde{\omega}}^{-1} : f_{\tilde{\varphi}}^{-1} = -\sqrt{2} : 1. \quad (139)$$

It agrees well with the values (137) obtained from the experimental data. It is therefore natural to identify $\tilde{\omega}$ with $J/\psi(3100)$ and $\tilde{\varphi}$ with $\psi(3700)$. The decays of $\tilde{\omega}$, $\tilde{\varphi}$, and $\tilde{\rho}^0$ with emission of a photon must satisfy the selection rules[59]

$$\tilde{\omega},\ \tilde{\varphi} \to \gamma + X(\{1^c\},\ I^G = 0^+),\quad X = \eta,\ \eta',\ \ldots; \quad (140)$$

$$\tilde{\rho}^0 \to \gamma + X(\{1^c\},\ I^G = 1^-),\quad X = \pi^0,\ A_1,\ A_2,\ \ldots \quad (141)$$

In the limit of exact symmetry, we must have

$$\Gamma(\tilde{\omega} \to \eta + \gamma) : \Gamma(\tilde{\varphi} \to \eta + \gamma) : \Gamma(\tilde{\omega} \to \eta' + \gamma) : \Gamma(\tilde{\varphi} \to \eta' + \gamma) = 1 : 2 : 2 : 1. \quad (142)$$

Note that in this model there is a rigorous prohibition on the experimentally unobserved decay $J/\psi(3100) \to \pi^0 + \gamma$, whereas the allowed decays (140) have been observed[111,112] and the following ratio obtained: $\Gamma(J/\psi(3100) \to \eta'\gamma) : \Gamma(J/\psi(3100) \to \eta\gamma) = 4 \pm 2.5$ (in Ref. 112) and < 5 (in Ref. 111).

Let us now list the main difficulties of this model:

a) the broad resonances (see 3) above) in this model are interpreted as radial excitations of $J/\psi(3100)$ and $\psi(3700)$. But since in this model the empirical Okubo–Zweig–Inizuki rule does not work (it works in models with charm[5]), decays of the broad resonances into $J/\psi(3100)$ and $\psi(3700)$ are in no way forbidden. The model has great difficulty in explaining 5);

b) the P_c and χ states (see 6) above) are interpreted as orbital excitations of $J/\psi(3100)$. But since in this scheme $\psi(3700) \equiv \tilde{\varphi}$ consists basically of strange quarks, the radiative transitions $\psi(3700) \to P_c + \gamma$ or χ will be strongly suppressed (they are proportional to $\sin^2\tilde{\delta}$, where $\tilde{\delta}$ is the angle of deviation from ideal $SU(3)$ singlet–octet mixing for the new mesons);

c) the main decays of $J/\psi(3100)$ and $\psi(3700)$ must be radiative transitions, which is not confirmed experimentally. One can, however, explain such suppression by the presence of form factors.[8,66,69,113] Greenberg proposed in Ref. 114 a model in which the color quark is a bound state of an ordinary fermion quark carrying $SU(3)$ quantum numbers and a boson with spin 0 carrying $SU(3^c)$ color quantum numbers. In this model, the J/ψ meson is not only an excited state of the color degrees of freedom but is also a spatial excitation. The first circumstance forbids strong transitions into ordinary hadrons (because of conservation of the color quantum numbers); the second, radiative transitions into ordinary hadrons (because of the vanishing of the dipole moment)[114];

d) the $\tilde{\rho}^0$ meson has charged analogs $\tilde{\rho}^\pm$ which can be detected in the decay

$$\psi(3700) \to \tilde{\rho}^\pm + \pi^\mp, \quad (143)$$

and whose probability is comparable with the probability of the transition $\psi(3700) \to J/\psi(3100) + 2\pi$ (Refs. 7 and 59). However, the sharp peak in the single-pion decay spectrum of $\psi(3700)$ which should be observed in this case has not been found[111];

e) $J/\psi(3100)$ and $\psi(3700)$ must be members of an $SU(3)$ octet. The unitary properties of the new mesons have not yet been established, but it is more probable that they are $SU(3)$ singlets.[115,116]

One could give up the assumption of a purely electromagnetic breaking of the color symmetry and make the present scheme closer to the foregoing one by assuming the existence of not too strong an interaction breaking the $SU(3^c)$ symmetry and leading to mixing of the U^c eigenstates (61) and (62). In this case, both such states could be formed in e^+e^- annihilation, and one obtains a very interesting prediction on the doublet splitting of each of the observed heavy resonances.[68] Does not the structure in the e^+e^- annihilation in the region of 4–5 GeV indicate such splitting?

The Pati–Salam model. Pati and Salam suggested that one of the ψ mesons should be interpreted as a massive color gluon.[72] They proposed three identification schemes:

I. $J/\psi(3100)$ = color gluon $\tilde{U}$;
$\psi(3700)$ = color excited (octet) state of a color quark and color antiquark;
$\psi(4100)$ = orthocharmonium, $p'\bar{p}'$.

II. $J/\psi(3100)=\tilde{U}$;

$$\left.\begin{array}{l}\psi(3700)=\tilde{\omega}\\ \psi(4100)=\tilde{\varphi}\end{array}\right\}\begin{array}{l}=\text{color analogs of } \omega \text{ and } \varphi\\ \text{(see above).}\end{array}$$

III. $J/\psi(3100)=$ orthocharmonium;

$\psi(3700)=$ radial excitation of orthocharmoinum;

$\psi(4100)=\tilde{U}$.

We shall not discuss in detail these identification schemes (see, for example, Ref. 72) since the third state $\psi(4100)$ is in fact a complex structure. We give only the possible gluon decay channels[72].

$$\left.\begin{array}{l}\tilde{U}\to\gamma+\text{hadrons};\\ \to\text{hadrons};\\ \to e^-\mu^+,\ \mu^-\mu^+.\end{array}\right\} \qquad (144)$$

The first decay takes place through transition of the gluon into an intermediate heavy color state with subsequent transformation of this state into a photon, for example

$$\tilde{U}\to\eta'+\text{color octet.}\atop \quad\quad\quad\quad\quad\quad\quad \lfloor\!\!\to\gamma \qquad (145)$$

Because of the large mass of the color octet state (~ 3 GeV), this decay is strongly suppressed. The second decay mode is due to the effective breaking of the color symmetry of nonelectromagnetic origin resulting from the terms of fourth order in the scalar potential. This breaking is assumed small and to have the properties $H(1,8)_{I_3^c=0,\,Y^c=0}$ with respect to $SU(3)\times SU(3^c)$. The third type of decay takes place through the direct interaction of $\tilde{U}$ with leptons by virtue of its W component [see (110)]. For these reasons, the meson $J/\psi(3100)\equiv\tilde{U}$ in schemes I and II is narrow.

CONCLUSIONS

A number of theoretical arguments (quark statistics, the $\Delta I=\frac{1}{2}$ rule for weak nonleptonic decays, the introduction of color gluons) and experimental facts (behavior of the famous ratio R in e^+e^- annihilation, the decays $\pi^0\to2\gamma$, $\eta\to2\gamma$) suggest the natural hypothesis that hadrons have color degrees of freedom. It is possible that we shall soon have new facts which make this natural conjecture into a necessity, and we shall then advance significantly in our understanding of the internal symmetries on which the world of elementary particles is based. In this connection, the unified symmetry $SU(4)\times SU(4^c)$ proposed by Pati and Salam for all fundamental fermions (leptons and quarks) is very attractive.

But what could color symmetry be in reality? The most important question relates to the charges of the fundamental entities, or quarks, from which, for good or bad, we have learnt to construct hadrons. If the quark charges are fractional, then a way is opened up for theory that leads to a complete non-Abelian gauge symmetry with all the consequences that flow from it (existence of massless color gluons, asymptotically free theories, quark confinement, etc). But if the quark charges are integral, then this opens up the no less attractive prospect of the existence of quarks with integral charges in a free state. The quarks would at the same time be unstable.

As we have seen, the experimental resolution of the problem of the quark charges is not an easy matter. If the color threshold is in the region $Q^2\sim20$ GeV^2, the single-photon processes investigated at contemporary energies can hardly answer this question. One can expect a changed situation when electron or muon beams with energies greatly exceeding the color and other thresholds are available. If, however, the Pati–Salam theorem[93] to the effect that there is photon and gluon compensation is valid, then the situation is much more complicated. It would seem to be more promising to investigate two-photon processes with the existing accelerators.

Undoubtedly, the schemes with integral quark charges would be directly confirmed by the observation of the excitations of color degrees of freedom that is predicts: new hadronic states belonging to higher representations of the color symmetry or the massive gluons themselves. As we have seen, the color interpretation of the $J-\psi$ mesons has probably failed. However, it could be that different models have definite regions of applicability. One can already conceive of the following situation: The narrow $J/\psi(3100)$ and $\psi(3700)$ mesons are explained on the basis of schemes with charmed quarks, and the broad resonances in e^+e^- annihilation in the region 4–5 GeV have a color interpretation. The prediction, in the framework of the color interpretation, of doublet splitting of each resonance appears very attractive in connection with the complex structure of these resonances. In this case, we should obtain a unification in the spirit of Pati and Salam of schemes that are a present alternatives. There is no need to emphasize the tentative nature of considerations of this kind.

I should like to express my deep gratitude to Professor A. M. Baldin and P. S. Isaev for suggesting I write this review in order to systematize the currently known facts, and I am also sincerely grateful to S. B. Gerasimov for numerous discussions of the material.

APPENDIX

Structure factors for two-photon processes below the color threshold

At energies below the color threshold, the initial and final states belong to the ordinary world of hadrons and electrons and are, by hypothesis, color singlets. If two photons participate in the process, the transition is determined by a matrix element of the form

$$\langle f_{(\text{singl})}|J_\mu^{\text{em}}\times J_\nu^{\text{em}}|i_{(\text{singl})}\rangle=\langle f|J_\mu(1^c)\times J_\nu(1^c)||i\rangle + \langle f|\{J_\mu(8^c)\times J_\nu(8^c)\}_{\text{singl}}|i\rangle, \qquad (A.1)$$

where $J_\mu(1^c)$ and $J(8^c)$ are the color singlet and octet electromagnetic currents, respectively. The first term on the right-hand side of (A.1) is the same for all color models, and the color singlet current in it is given by the expression

$$J_\mu(1^c)=\sum_{i=1,2,3}(2\bar{p}_i\gamma_\mu p_i/3-\bar{n}_i\gamma_\mu n_i/3-\bar{\lambda}_i\gamma_\mu\lambda_i/3). \qquad (A.2)$$

The second term on the right-hand side of (A.1) occurs only in models with integral quark charges. The subscript "singl" indicates that only the color singlet part is to be taken from the product of the two color octet currents (to shorten the expressions, we shall omit below the here irrelevant Lorentz indices, γ matrices, etc):

$$\{J(8^c)\times J(8^c)\}_{\text{singl}}=-\frac{\sqrt{2}}{3}\{1^c\}$$

$$=-\frac{1}{3}J\begin{pmatrix}8^c\\1/2\ \ -1\end{pmatrix}_{-1/2}\times J\begin{pmatrix}8^c\\1/2\ \ 1\end{pmatrix}_{1/2}$$
$$+\frac{1}{3}J\begin{pmatrix}8^c\\1/2\ \ -1\end{pmatrix}_{-1/2}\times J\begin{pmatrix}8^c\\1/2\ \ -1\end{pmatrix}_{1/2}$$
$$-\frac{1}{3}J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_1\times J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_1+\frac{1}{6}J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_0\times J\begin{pmatrix}8^c\\2\ 0\end{pmatrix}_0$$
$$+\frac{1}{6}J\begin{pmatrix}8^c\\0\ 0\end{pmatrix}\times J\begin{pmatrix}8^c\\0\ 0\end{pmatrix} \qquad \text{(A.3)}$$

The first step in (A.3) was the separation of the singlet part from the product of octet currents determined by the expression (33). The second term, conversely, was decomposition of the color isosinglet $\{1^c\}$ into the sum of products of the color octet currents, for which the notation $J(_{I^c Y^c}^{8^c})_{I_3^c}$ was used and which in the quark model are determined as follows (with phases chosen in de Swart's manner[117]):

$$J\begin{pmatrix}8^c\\1/2\ \ -1\end{pmatrix}_{-1/2}=-\bar{\alpha}_1\alpha_3/\sqrt{2};\ J\begin{pmatrix}8^c\\1/2\ \ 1\end{pmatrix}_{1/2}$$
$$=\bar{\alpha}_3\alpha_1/\sqrt{2};\ J\begin{pmatrix}8^c\\1/2\ \ 1\end{pmatrix}_{-1/2}=\bar{\alpha}_3\alpha_2/\sqrt{2};$$
$$J\begin{pmatrix}8^c\\1/2\ \ -1\end{pmatrix}_{1/2}=\bar{\alpha}_2\alpha_3/\sqrt{2};\ J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_1 \qquad \text{(A.4)}$$
$$=\bar{\alpha}_2\alpha_1/\sqrt{2};\ J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_{-1}=\bar{\alpha}_1\alpha_2/\sqrt{2};$$
$$J\begin{pmatrix}8^c\\1\ 0\end{pmatrix}_0=(\bar{\alpha}_1\alpha_1-\bar{\alpha}_2\alpha_2)/2;\ J\begin{pmatrix}8^c\\0\ 0\end{pmatrix}=(\bar{\alpha}_1\alpha_1+\bar{\alpha}_1\alpha_1-2\bar{\alpha}_3\alpha_3)/2\sqrt{3}.$$

The symbol α can take any of the three values p, n, λ, and in fact for each current in (A.4) it is necessary to take the sum over these states. In addition, it is assumed that the currents have the form $\bar{\alpha}_i\gamma_\mu\alpha_i$.

We now recall that in the quark–parton picture of the deep inelastic interaction of photons with hadrons, the two photons interact with the same parton. Therefore, the parton lines close the same quark states that occur next to one another in the products of the currents in (A.3). Making simple arithematical calculations, we obtain

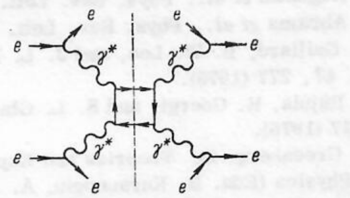
$$\{J(8^c)\times J(8^c)\}_{\text{singl}}\Rightarrow(2/9)\sum_{\substack{i=1,2,3\\ \alpha=p,n,\lambda}}\bar{\alpha}_i\alpha_i. \qquad \text{(A.5)}$$

Making a similar closure of the parton lines for the product of the color singlet currents in (A.1), we obtain the final expression

$$\langle f\,(\text{singl})\,|\,J^{\text{em}}\times J^{\text{em}}\,|\,i\,(\text{singl})\rangle\Rightarrow\sum_{i=1,2,3}\left(\frac{2}{3}\bar{p}_ip_i+\frac{1}{3}\bar{n}_in_i+\frac{1}{3}\bar{\lambda}_i\lambda_i\right). \qquad \text{(A.6)}$$

Let us consider the two-photon processes discussed in the text. Using unitarity, we can relate the cross section of the deep inelastic process to the imaginary part of the amplitude of the corresponding elastic process. Below, we give the diagrams and amplitudes of these elastic processes and the coefficients (calculated in accordance with our rules) determined by the charges and structure functions of the partons:

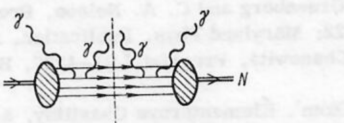
deep inelastic ee scattering



(A.7)

$$\langle 0\,|\,T(J\times J)_{\text{singl}}\,T(J\times J)_{\text{singl}}\,|\,0\rangle\Rightarrow\sum_{i=1,2,3}[(2/3)^2+(1/3)^2+(1/3)^2]=2;$$

deep inelastic Compton effect on the nucleon N



(A.8)

$$\langle N\,|\,T(J\times J)_{\text{singl}}\,T(J\times J)_{\text{singl}}\,|\,N\rangle$$
$$\Rightarrow\sum_{i=1,2,3}[(2/3)^2(u^N_{p_i}+u^N_{\bar{p}_i})+(1/3)^2(u^N_{n_i}+u^N_{\bar{n}_i})+(1/3)^2(u^N_{\lambda_i}+u^N_{\bar{\lambda}_i})]$$
$$=(4/9)[u^N_p(x)+u^N_{\bar{p}}(x)]+[u^N_n(x)+u^N_{\bar{n}}(x)]/9+[u^N_\lambda(x)+u^N_{\bar{\lambda}}(x)]/9.$$

Here, we have remembered that each quark color state occurs with the same weight 1/3. Since the expression obtained here coincides with the expression for single-photon deep inelastic electron–nucleon scattering, for the ratio (129) we obtain

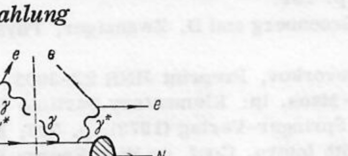
$$\left\langle\sum_j e_j^4\right\rangle\Big/\left\langle\sum_j e_j^2\right\rangle=1 \qquad \text{(A.9)}$$

in color models with integral quark charges in the entire energy range (both above and below the color threshold). In models with fractional quark charges,

$$\frac{\left\langle\sum_j e_j^4\right\rangle}{\left\langle\sum_j e_j^2\right\rangle}=\frac{16\,(u^N_p+u^N_{\bar{p}})+(u^N_n+u^N_{\bar{n}})+(u^N_\lambda+u^N_{\bar{\lambda}})}{36\,(u^N_p+u^N_{\bar{p}})+9\,(u^N_n+u^N_{\bar{n}})+9\,(u^N_\lambda+u^N_{\bar{\lambda}})}. \qquad \text{(A.10)}$$

This ratio varies in the range 1/9–4/9;

interference of Bethe–Heitler and Compton deep inelastic bremsstrahlung



$$\langle N\,|\,J_{\text{singl}}\,T(J\times J)_{\text{singl}}\,|\,N\rangle \qquad \text{(A.11)}$$
$$\Rightarrow\left\langle N\,\left|\,\sum_{i=1,2,3}\left(\frac{2}{3}\bar{p}_ip_i-\frac{1}{3}\bar{n}_in_i-\frac{1}{3}\bar{\lambda}_i\lambda_i\right)\right.\right.$$
$$\times\left.\left.\sum_{j=1,2,3}\left(\frac{2}{3}\bar{p}_ip_i+\frac{1}{3}\bar{n}_in_i+\frac{1}{3}\bar{\lambda}_i\lambda_i\right)\right|\,N\right\rangle$$
$$=\frac{4}{9}(u^N_p-u^N_{\bar{p}})-\frac{1}{9}(u^N_n-u^N_{\bar{n}})-\frac{1}{9}(u^N_\lambda-u^N_{\bar{\lambda}})$$
$$=\sum_{\alpha=p,n,\lambda}\left(\frac{5}{9}e_\alpha+\frac{2}{9}b_\alpha\right)(u^N_\alpha+u^N_{\bar{\alpha}}),$$

where $e_\alpha=(2/3,\,-1/3,\,-1/3)$ for $\alpha=p$, n, λ and $b_\alpha=1/3$. After integration with respect to x, we obtain the sum rule for the structure factor introduced by Brodsky *et al.*[101]:

$$\int_0^1 V(x)\,dx=5Q/9+2B/9. \qquad \text{(A.12)}$$

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