# Asymptotic power-law scaling of large-angle hadron-hadron scattering

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Some results obtained in the framework of the quasipotential approach for large-angle high-energy hadron scattering are reviewed.

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## INTRODUCTION

One of the currently most important problems in high energy physics is the study of the asymptotic power-law scaling of large-angle scattering cross sections. In the present paper, the phenomenon of power-law decrease of the differential cross sections at large angles will be treated in the framework of the Logunov–Tavkhelidze potential approach.

In the quasipotential theory of scattering of high energy particles[1] it is an important assumption that the local quasipotential is smooth.[2] The smoothness hypothesis played an important role in our understanding of the general laws of the interaction of particles at high energies and bounded momentum transfers. However, in the case of large-angle scattering smooth quasipotentials of finite range lead to an exponential decrease of the differential cross sections with increasing momentum transfer. This is due to the fact that such a quasipotential, a typical example of which is the complex Gaussian quasipotential,[1] corresponds to the interaction of, putting it pictorially, "crumbly" extended systems that at large momentum transfers break up with almost unit probability. The probability of large-angle elastic scattering is therefore exponentially small. This behavior is typical of the majority of models in which hadrons are regarded as complex extended objects with internal degrees of freedom, such as, for example, Yang's so-called drop model.

A power-law decrease of the differential cross sections in the region of large angles[3]:

$$d\sigma/dt \sim (1/s^n)\, f(t/s) \tag{1}$$

follows from the scaling principle,[4] i.e., the assumption that there are no essential dimensional parameters that determine the interaction dynamics at short distances. Using dimensional analysis and ideas about the composite nature of particles, one can relate the exponent $n$ in (1) to the number of elementary constituents of the hadrons. For example, for the case of the binary reaction $a+b\to a+b$ we have $n=2(n_a+n_b-1)$, where $n_a$ and $n_b$ are the numbers of elementary constituents (quarks) of hadrons $a$ and $b$, respectively. In Ref. 5, the quark diagrams for the amplitudes of two-particle scattering are given a dynamical interpretation; this results in a generalization of the quark counting rules, and an explicit expression is found for the angular dependence of the differential cross sections of large-angle scattering at high energies for different processes.

These results prompted[6–8] the following question: What structure of the local two-particle quasipotential leads to the expression (1) in the case of large-angle two-particle scattering. We give here a review of the results that were obtained, treating the scattering, in contrast to Refs. 6–8, in a unified manner for both spinless particles and particles with spin for quasipotentials that satisfy the representation

$$\hat{g}(E;\Delta^2)=g(E)\int_0^\infty dx\,\hat{\rho}(E;x)\exp(-x\Delta^2);\quad t=-\Delta^2 \tag{2}$$

and are analytic functions of $t$ in the half-plane $\mathrm{Re}\,t\leq 0$. In Eq. (2), $\hat{\rho}(E;x)$ is some matrix that determines the nature of the interaction at high energies; its rank depends on the spin of the particles. The main energy dependence is separated in Eq. (2) in the form of the factor $\hat{g}(E)$, which depends as a power on $E$. It is assumed that for fixed $x$ the matrix $\hat{\rho}(E;x)$ is a slowly varying function of the energy. With regard to the density $\hat{\rho}(E;x)$, we also assume the existence of the weak limit

$$\lim_{s\to\infty} s^N\hat{\rho}(E;x=\eta/s)=\hat{\psi}(\eta),\qquad 0<\eta<\infty;\ N>0. \tag{3}$$

Note that to describe the interaction of particles with spin we use the requirement of $\gamma_5$ invariance of the interaction at high energies and large momentum transfers.[9]

We shall say that quasipotentials which satisfy the condition (3) are analytic. For them, in the framework of the quasipotential method, we shall find a formal representation for the amplitude of large-angle elastic scattering of high energy particles with arbitrary spin, and we shall then investigate definite scattering processes and show that the asymptotic behavior of the differential cross section of large-angle scattering of high energy particles has the form

$$d\sigma/dt\sim|\exp[2i\chi(0)]|^2(1/s^M)\,f(\Delta^2/s),$$

where $\chi(0)$ is the eikonal phase shift for zero impact parameter. We recall that the eikonal function $\chi(b)$ characterizes the behavior of the small-angle scattering amplitude. Thus, our result establishes a certain correlation between the asymptotic behaviors of the scattering amplitude at small and large angles.

In the review, we shall also compare the results which

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are obtained with the experimental data on large-angle scattering.

## 1. LARGE-ANGLE SCATTERING OF ANALYTIC QUASIPOTENTIALS

We consider the quasipotential equation that describes the interaction of particles with spins; we write it in the general form

$$\hat{G}(E;\mathbf{p},\mathbf{k})=\hat{g}(E;\mathbf{p}-\mathbf{k})+\int d^3\mathbf{q}\,\hat{g}(E;\mathbf{p}-\mathbf{q})\frac{\hat{A}(E;\mathbf{q})}{E^2(\mathbf{q})-E^2-i0}\hat{G}(E;\mathbf{q},\mathbf{k}),\tag{4}$$

where $E=\sqrt{m_1^2+\mathbf{p}^2}+\sqrt{m_2^2+\mathbf{p}^2}$ is the total cms energy of the particles; $E(\mathbf{q})=\sqrt{m_1^2+\mathbf{q}^2}+\sqrt{m_2^2+\mathbf{q}^2}$; $m_1$ and $m_2$ are the masses of the first and the second particle, respectively; $\hat{A}(E;\mathbf{q})$ is some matrix whose form depends on the spin of the interacting particles. The explicit form of $\hat{A}(E;\mathbf{q})$ is here unimportant. For the scattering of spinless particles and particles with spins 0, 1/2, and 1/2, 1/2 we shall write out this matrix in the following section.

Assuming that the quasipotential $\hat{g}(E;\mathbf{p}-\mathbf{k})$ is specified by the representation (2), we shall seek a solution of Eq. (4) by iteration:

$$\left.\begin{aligned}&\hat{G}(E;\mathbf{p},\mathbf{k})=\sum_{n=0}^{\infty}\hat{G}_{n+1}(E;\mathbf{p},\mathbf{k});\\&\hat{G}_1(E;\mathbf{p},\mathbf{k})=\hat{g}(E;\mathbf{p}-\mathbf{k});\\&\hat{G}_{n+1}(E;\mathbf{p},\mathbf{k})=\int\ldots\int d^3\mathbf{q}_1\ldots d^3\mathbf{q}_n\hat{g}(E;\mathbf{p}-\mathbf{q}_1)\\&\times\hat{A}(E;\mathbf{q}_1)\hat{g}(E;\mathbf{q}_1-\mathbf{q}_2)\hat{A}(E;\mathbf{q}_2)\ldots\hat{A}(E;\mathbf{q}_n)\\&\times\hat{g}(E;\mathbf{q}_n-\mathbf{k})\prod_{i=1}^{n}\frac{1}{E^2(\mathbf{q}_i)-E^2-i0}.\end{aligned}\right\}\tag{5}$$

After the substitution

$$\mathbf{q}_i\to\Delta_i+\lambda_i;\quad\lambda_i=\frac{\mathbf{p}+\mathbf{k}}{2}+\left[1-2\sum_{l=1}^{i}\frac{1}{x_l}\Big/\sum_{l=1}^{n+1}\frac{1}{x_l}\right]\frac{\mathbf{p}-\mathbf{k}}{2}$$

we can transform (5) to the form[6]

$$\hat{G}_{n+1}(E;\mathbf{p},\mathbf{k})=\int\ldots\int dx_1\ldots dx_{n+1}\exp\left\{t\Big/\sum_{i=1}^{n+1}\frac{1}{x_i}\right\}\hat{J}_n(x_1\ldots x_{n+1}),\tag{6}$$

where

$$\begin{aligned}\hat{J}_n(x_1\ldots x_{n+1})&=[g(E)]^{n+1}\\&\times\int\ldots\int d^3\Delta_1\ldots d^3\Delta_n\frac{\exp\{-C_{ij}\Delta_i\Delta_j\}}{\prod_{i=1}^{n}[E^2(\Delta_i+\lambda_i)-E^2-i0]}\\&\times\hat{\rho}(E;x_1)\hat{A}(E;\Delta_1+\lambda_1)\hat{\rho}(E;x_2)\ldots\\&\ldots\hat{\rho}(E;x_n)\hat{A}(E;\Delta_n+\lambda_n)\hat{\rho}(E;x_{n+1});\end{aligned}\tag{7}$$

and

$$C_{ij}\Delta_i\Delta_j=\sum_{h=1}^{n+1}(\Delta_h-\Delta_{h-1})^2x_h,\quad\Delta_0=\Delta_{n+1}=0.$$

By means of the respresentation (6), we study the behavior of the large-angle scattering amplitude at high energies, i.e., in the region

$$s\to\infty,\ |t|/s=(1-z)/2\ \text{fixed}.\tag{8}$$

In this limit, the main contribution to the asymptotic behavior of the integral (6) is made by the region $1/\sum(1/x_i)\sim0$, since otherwise we have the decreasing exponential $\exp[-|t|/\sum(1/x_i)]$. One can show that the main contribution to the asymptotic behavior of the integral (6) when the condition (3) is satisfied is made by the region for which only one $x_i\approx0$. At the same time,[1)]

$$\hat{G}_{n+1}(E;\mathbf{p},\mathbf{k})=\sum_{l=1}^{n+1}\frac{(i\hat{\chi}(0)\hat{B}(\mathbf{p}))^{l-1}}{(l-1)!}\hat{g}(E;\mathbf{p}-\mathbf{k})\frac{(i\hat{B}(\mathbf{k})\hat{\chi}(0))^{n-l+1}}{(n-l+1)!},\tag{9}$$

where we have used the notation

$$\begin{aligned}&\hat{B}(\mathbf{p})=\hat{A}(E;\mathbf{p})g(E)/16|\mathbf{p}|;\\&\hat{\chi}(0)=\int_{-\infty}^{\infty}dz\,\hat{g}(E;r=\sqrt{z^2+\mathbf{b}^2})/g(E)|_{\mathbf{b}=0},\end{aligned}\tag{10}$$

and $\hat{g}(E;r)$ is the Fourier transform of the quasipotential (2). Summing over $n$, we obtain the following formal expression for the scattering amplitude:

$$\hat{G}(E;\mathbf{p},\mathbf{k})=\exp[i\hat{\chi}(0)\hat{B}(\mathbf{p})]\hat{g}(E;\mathbf{p}-\mathbf{k})\exp[i\hat{B}(\mathbf{k})\hat{\chi}(0)].\tag{11}$$

We now consider the contribution of exchange forces in the scattering of nonidentical particles.[2)] We write the quasipotential in the form[10]

$$\hat{V}(E;\mathbf{p},\mathbf{k})=\hat{g}(E;\mathbf{p},\mathbf{k})+\hat{h}(E;\mathbf{p},\mathbf{k}).\tag{12}$$

We shall say that the quasipotential $\hat{g}$ and $\hat{h}$ are the direct and the exchange parts of the quasipotential $\hat{V}$; for $\hat{g}$ we assume the representation (2), and

$$\hat{h}(E;\mathbf{p},\mathbf{k})=h(E)\int_0^{\infty}dy\,\hat{\sigma}(E;y)\exp[-y(\mathbf{p}+\mathbf{k})^2].\tag{13}$$

We shall also assume that the densities $\hat{\rho}$ and $\hat{\sigma}$ satisfy weak limits of the type (3) with the same $N$.

We represent the scattering amplitude $\hat{T}$ as a sum of two terms:

$$\hat{T}=\hat{G}+\hat{H}.\tag{14}$$

Substituting (12) and (14) in (4), we obtain the following system of quasipotential equations:

$$\hat{G}=\hat{g}+\hat{g}\otimes\hat{G}+\hat{h}\otimes\hat{H};\tag{15}$$

$$\hat{H}=\hat{h}+\hat{h}\otimes\hat{G}+\hat{g}\otimes\hat{H}.\tag{16}$$

For meson–nucleon scattering, the backward-scattering peak is suppressed, i.e.,

$$\left|\frac{\hat{h}(E;\Delta_u\sim0)}{\hat{g}(E;\Delta_t\sim0)}\right|\to0\quad\text{as}\quad E\to\infty.$$

Therefore, in the case when the quasipotentials $\hat{g}$ and $\hat{h}$

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1) The detailed calculation of $\hat{G}_{n+1}$ is given in the Appendix.

2) The scattering of identical particles will be considered in the following section.

satisfy the conditions (2), (13), and (3), the last term in Eq. (15) can be ignored. The system of equations (15)–(16) then decouples. Note that the contribution of $\hat{g}$ has already been considered, so that we shall study only the contribution of the exchange part $\hat{h}$ of the quasipotential. Solving Eq. (16) iteratively, we represent $\hat{H}$ in the form

$$\left.\begin{aligned}&\hat{H}=\sum_{n=0}^{\infty}\sum_{k=1}^{n+1}\hat{H}_{nk};\\&\hat{H}_{nk}=\underbrace{\hat{g}\otimes\hat{g}\otimes\ldots\otimes\hat{g}}_{k-1}\otimes\hat{h}\otimes\underbrace{\hat{g}\otimes\hat{g}\otimes\ldots\otimes\hat{g}}_{n-k+1}\\&=\int\ldots\int\frac{d^3q_1\ldots d^3q_n}{\prod_{i=1}^{n}[E^2(q_i)-E^2-i0]}\hat{g}(E;\,\mathbf{p}-\mathbf{q}_1)\hat{A}(E;\,\mathbf{q}_1)\\&\ldots\hat{g}(E;\,\mathbf{q}_{k-2}-\mathbf{q}_{k-1})\hat{A}(E;\,\hat{\mathbf{q}}_{k-1})\hat{h}(E;\,\mathbf{q}_{k-1}\\&+\mathbf{q}_k)\hat{A}(E;\,\hat{\mathbf{q}}_k)\hat{g}(E;\,\hat{\mathbf{q}}_k-\mathbf{q}_{k+1})\\&\ldots\hat{A}(E;\,\mathbf{q}_n)\hat{g}(E;\,\mathbf{q}_n-\mathbf{k}).\end{aligned}\right\}\tag{17}$$

Taking into account (2) and (13) and making the change of variables[7]

$$\mathbf{q}_i=\Delta_i+\lambda_i;$$

$$\lambda_i=\begin{cases}\mathbf{l}+\mathbf{r}\left(1-2\dfrac{\sum_{m=1}^{i}1/x_m}{\sum_{\substack{m=1\\(m\neq k)}}^{n+1}1/x_m+1/y_k}\right)&(1\leqslant i\leqslant k-1);\\[2ex]-\mathbf{l}+\mathbf{r}\left(1-2\dfrac{\sum_{m=i+1}^{n+1}1/x_m}{\sum_{\substack{m=1\\(m\neq k)}}^{n+1}1/x_m+1/y_k}\right)&(k\leqslant i\leqslant n).\end{cases}$$

where $\mathbf{l}=(\mathbf{p}-\mathbf{k})/2$ and $\mathbf{r}=(\mathbf{p}+\mathbf{k})/2$, we transform the amplitude $\hat{H}_{n,k}$ to the form

$$\hat{H}_{n,\,k}=\int\ldots\int dx_1\ldots dy_k\ldots dx_{n+1}$$

$$\times\exp\left\{-\frac{(\mathbf{p}+\mathbf{k})^2}{\sum_{\substack{m=1\\(m\neq k)}}^{n+1}1/x_m+1/y_k}\right\}\hat{J}_{n,\,k}(x_1\ldots y_k\ldots x_{n+1}),$$

where

$$\hat{J}_{n,\,k}(x_1\ldots y_k\ldots x_{n+1})$$

$$=[g(E)]^n h(E)\int\frac{d^3\Delta_1\ldots d^3\Delta_n\exp\{-C'_{ij}\Delta_i\Delta_j\}}{\prod_{i=1}^{n}(E^2(\Delta_i+\lambda_i)-E^2-i0)}$$

$$\times\rho(E;\,x_1)\hat{A}(E;\,\Delta_1+\lambda_1)\ldots\hat{A}(E;\,\Delta_{k-1}+\lambda_{k-1})\,\sigma(E;\,y_k)\hat{A}(E;\,\Delta_k+\lambda_k)\ldots\hat{A}(E;\,\Delta_n+\lambda_n)\,\rho(E;\,x_{n+1});$$

$$C'_{ij}\Delta_i\Delta_j=\sum_{\substack{m=1\\(m\neq k)}}^{n+1}(\Delta_m-\Delta_{m-1})^2x_m+(\Delta_k+\Delta_{k-1})^2y_k;$$

$$\Delta_0=\Delta_{n+1}=0.$$

Using the method developed above, we can show that the main contribution to the asymptotic behavior of $\hat{H}_{n,k}$ at large $|u|=(\mathbf{p}+\mathbf{k})^2\sim s$ is made by the region $y\sim 0$. Then

$$\hat{H}^{y\sim 0}_{n,\,k}\approx\exp[i\hat{\chi}(0)\hat{B}(\mathbf{p})]\,\hat{h}(E;\,\mathbf{p}+\mathbf{k})\exp[i\hat{B}(\mathbf{k})\,\hat{\chi}(0)],$$

where $\hat{B}(\mathbf{p})$ and $\hat{\chi}(0)$ are given in (10). Note that a contribution to the asymptotic behavior of $\hat{H}_{n,k}$ is also made by the region $x_i\sim 0$, but in this case

$$\hat{H}^{x\sim 0}_{n,\,k}\approx\hat{H}^{y\sim 0}_{n,\,k}\left(\frac{\int dz\,\hat{h}\,(E;\,r=\sqrt{z^2+\mathbf{b}^2})}{\int dz\,\hat{g}\,(E;\,r=\sqrt{z^2+\mathbf{b}^2})}\right)\Bigg|_{\mathbf{b}=0}\ll\hat{H}^{y\sim 0}_{n,\,k},$$

which is due to the suppression of the backward-scattering peak.

Thus, taking into account (11) we obtain the following expression for the total large-angle scattering amplitude in the case of scattering of nonidentical particles[7]:

$$\hat{T}(E;\,\mathbf{p},\,\mathbf{k})=\exp[i\hat{\chi}(0)\hat{B}(\mathbf{p})][\hat{g}(E;\,\mathbf{p}-\mathbf{k})+\hat{h}(E;\,\mathbf{p}+\mathbf{k})]\exp[i\hat{B}(\mathbf{k})\,\hat{\chi}(0)].\tag{18}$$

For our chosen class of analytic quasipotentials satisfying the condition (3), we can write

$$g(E)\int_0^\infty dx\exp(-|t|\,x)\,\hat{\rho}(E;\,x)\to\frac{g(E)}{s^{N+1}}\int_0^\infty d\eta\,\hat{\psi}(\eta)\exp(-|t|\,\eta/s),$$

i.e., the scattering amplitude has the asymptotic form

$$\hat{T}(E;\,\mathbf{p},\,\mathbf{k})\sim\frac{1}{s^{N+1}}\exp[i\hat{\chi}(0)\hat{B}(\mathbf{p})]\times\Big[g(E)\int_0^\infty d\eta\,\hat{\psi}(\eta)\exp(-|t|\,\eta/s)+h(E)\int_0^\infty d\xi\,\hat{\varphi}(\xi)\exp(-|u|\,\xi/s)\Big]\exp[i\hat{B}(\mathbf{k})\,\hat{\chi}(0)],\tag{19}$$

which leads to a power-law decrease of the differential scattering cross section at large angles provided

$$\lim_{p\to\infty}\hat{B}(\mathbf{p})=\hat{B}(\mathbf{p}/|\mathbf{p}|),$$

and this condition is satisfied for the scattering of scalar particles and particles with spin.

## 2. STUDY OF DEFINITE SCATTERING PROCESSES

We now calculate the scattering cross sections for definite processes. We first consider the scattering of two identical spinless particles. The quasipotential, which in this case is a scalar function, can be written with allowance for exchange forces[10] in the form (12), and the scattering amplitude can be represented by the expression (14). Taking into account crossing, we obtain

$$g(E;\,\mathbf{p}-\mathbf{k})=h(E;\,\mathbf{p}-\mathbf{k});$$
$$G(E;\,\mathbf{p},\,-\mathbf{k})=H(E;\,\mathbf{p},\,\mathbf{k}),$$

whence for the total amplitude

$$T(E;\,\mathbf{p},\,\mathbf{k})=(1+\hat{P})\,G(E;\,\mathbf{p},\,\mathbf{k}),\tag{20}$$

where $\hat{P}$ is the operator of reflection of the relative coordinate in the final state. When applied to the on-shell amplitude $G$, the operator $\hat{P}$ makes the substitution $t\rightleftarrows u$.

The amplitude $G(E;\mathbf{p},\,\mathbf{k})$ satisfies the Logunov–Tavkdelidze quasipotential equation, which can be written in the form (4). At the same time, $A(E;\mathbf{q})=4/\sqrt{m^2+\mathbf{q}^2}$; $g(E)=E^2=s$. Using (10), we find that $B(\mathbf{p})\to 1$ in the limit $p\to\infty$, and $\chi(0)$ is equal to the eikonal phase shift of elastic scattering (see, for example, Ref. 1).

Taking into account (11) and (20), we find

$$\left.\begin{aligned}&T(E;\,\mathbf{p},\,\mathbf{k})=\exp[2i\chi(0)]\,(g(E;\,t)+g(E;\,u));\\&d\sigma/dt\approx(1/s^2)\,|\exp[2i\chi(0)]|^2\,|g(E;\,t)+g(E;\,u)|^2.\end{aligned}\right\}\tag{21}$$

We now consider the scattering of a scalar particle on a spinor particle. The quasipotential equation that describes this system of particles is obtained in Ref. 11, and it can be represented in the form (4) with

$$\hat{A}(E;\mathbf{q})=\{\gamma_0 E-(1+\sqrt{\mu^2+\mathbf{q}^2}/\sqrt{M^2+\mathbf{q}^2})(\gamma\mathbf{q}-M)\}/\sqrt{\mu^2+\mathbf{q}^2}. \tag{22}$$

In this case, the quasipotential (2) is a 4×4 matrix. Below, we shall consider a simple form of a $\gamma_5$-invariant quasipotential with density

$$\left.\begin{array}{c}\hat{\rho}(E;x)=\gamma_0\rho(E;x);\ \hat{\sigma}(E;x)=\gamma_0\sigma(E;x);\\ g(E)=2E.\end{array}\right\} \tag{23}$$

Using (22) and (23), we obtain

$$\hat{B}(\mathbf{p})\xrightarrow[p\to\infty]{}(\gamma_0-\gamma\mathbf{p}/|\mathbf{p}|)/2=\hat{n}(\mathbf{p})/2, \tag{24}$$

whence for the total amplitude

$$\hat{T}(E;\mathbf{p},\mathbf{k})=[g(E;\mathbf{p}-\mathbf{k})+h(E;\mathbf{p}+\mathbf{k})]\times[1+\gamma_0\hat{n}(\mathbf{p})F][1+\gamma_0\hat{n}(\mathbf{k})F]\gamma_0,$$

where

$$F=\{\exp[i\chi(0)]-1\}/2; \tag{25}$$

$\chi$ is the eikonal phase of meson–nucleon scattering[12];

$$2i\chi(\mathbf{b})=-\int_{-\infty}^{\infty}dz\,g(E;r=\sqrt{z^2+\mathbf{b}^2})/2ip.$$

Going over to the differential scattering cross section, we find[7]

$$d\sigma/dt\approx(1/s)\exp[2i\chi(0)]^2(1+z)|g(E;\mathbf{p}-\mathbf{k})+h(E;\mathbf{p}+\mathbf{k})|^2. \tag{26}$$

We now study the scattering of two spinor particles. The quasipotential equations that describe the interaction of two particles with spin 1/2 were obtained in Refs. 13 and 14. Here, we shall use the equation from Ref. 14, which enables us to apply the method developed here without modification to study of the asymptotic behavior of the nucleon–nucleon scattering amplitudes. The equation can be written in the form (4), and

$$\hat{A}(E;\mathbf{q})=\left[\frac{E^2-E^2(\mathbf{q})/2}{E}+\hat{H}_1(\mathbf{q})+\hat{H}_2(-\mathbf{q})+\frac{2}{E}\hat{H}_1(\mathbf{q})\hat{H}_2(-\mathbf{q})\right],$$

where $\hat{H}_{1,2}(\mathbf{q})$ are the energy operators of the first and the second particle, respectively:

$$\hat{H}_{1,2}(\mathbf{q})=m\gamma_0^{(1,2)}+\gamma_0^{(1,2)}\gamma^{(1,2)}\mathbf{q}.$$

The quasipotential $\hat{g}(E;\mathbf{p},\mathbf{k})$ can be represented in the form of a 16×16 matrix with $g(E)\sim$const, which for convenience we take equal to 4. For the scattering amplitude in this case we obtain the expression (11), in which

$$\hat{B}(\mathbf{p})\xrightarrow[p\to\infty]{}\hat{n}_1(\mathbf{p})\hat{n}_2(-\mathbf{p})/4, \tag{27}$$

and the operators $\hat{n}(\mathbf{p})$ are defined in (24).

Note that at high energies and small momentum transfers the spin-flip amplitudes are small compared with the amplitudes without spin flip.[15] This condition is satisfied, for example, by the eikonal phase shift[12]

$$\hat{\chi}(\mathbf{b})=\gamma_0^{(1)}\gamma_0^{(2)}\chi(\mathbf{b}). \tag{28}$$

Substituting (27) and (28) into (11), we obtain

$$\hat{G}(E;\mathbf{p},\mathbf{k})=[1+\gamma_0^{(1)}\hat{n}^{(1)}(\mathbf{p})\gamma_0^{(2)}\hat{n}^{(2)}(-\mathbf{p})F/2]\times\hat{g}(s,t)[1+\gamma_0^{(1)}\hat{n}^{(1)}(-\mathbf{k})\gamma_0^{(2)}\hat{n}^{(2)}(\mathbf{k})F/2];$$

where $F$ is determined by the expression (25), in which $\chi$ is the eikonal phase shift of nucleon–nucleon scattering[12]:

$$2i\chi(\mathbf{b})=-\frac{1}{2i}\int_{-\infty}^{\infty}dz\,g(E;r=\sqrt{z^2+\mathbf{b}^2}).$$

We choose the quasipotential $\hat{g}(s,t)$ in the $\gamma_5$-invariant form

$$\hat{g}(s,t)=\gamma_\mu^{(1)}\gamma^{(2)\mu}C(s,t)+\gamma_5^{(1)}\gamma_5^{(2)}\gamma_\mu^{(1)}\gamma^{(2)\mu}D(s,t), \tag{29}$$

where $C(s,t)$ and $D(s,t)$ are the quasipotentials that describe the vector–vector and axial–axial interaction. Going over to the differential scattering cross section

$$d\sigma/dt\sim\sum_{\text{spin}}M(\mathbf{p},\mathbf{k})M^+(\mathbf{p},\mathbf{k}),$$

where

$$M(\mathbf{p},\mathbf{k})=\langle\bar{\psi}_1^{\sigma_1'}(\mathbf{p})\bar{\psi}_2^{\sigma_2'}(-\mathbf{p})|\hat{G}(E;\mathbf{p},\mathbf{k})|\psi_1^{\sigma_1}(\mathbf{k})\times\psi_2^{\sigma_2}(-\mathbf{k})\rangle-\langle\bar{\psi}_1^{\sigma_1'}(\mathbf{p})\bar{\psi}_2^{\sigma_2'}(-\mathbf{p})|\hat{G}(E;\mathbf{p},-\mathbf{k})|\psi_1^{\sigma_2}(-\mathbf{k})\psi_2^{\sigma_1}(\mathbf{k})\rangle,$$

we obtain in the limit (8) the expression[8]

$$d\sigma/dt\approx|\exp[2i\chi(0)]|^2\{4|C(s,t)+D(s,t)+C(s,u)+D(s,u)|^2+(1+z)^2|C(s,t)-D(s,t)|^2+(1-z)^2|C(s,u)-D(s,u)|^2\}. \tag{30}$$

## 3. COMPARISON WITH EXPERIMENT

We give some simple examples of the use of our expressions. Let us first consider the scattering of scalar particles. As the quasipotential $g(E;\Delta)$, we choose a Gaussian quasipotential with a small addition satisfying the condition (3):

$$\rho(E;x)=ig\delta(x-a)+(h/s)x^{m-1}\exp(-bx)/s^{M-1}. \tag{31}$$

For small-angle scattering at high energies, the main contribution is made by the region $x\approx a$, and we obtain the eikonal representation for the amplitude of scattering on the Gaussian quasipotential. The eikonal phase shift is determined by the expression[1,6]

$$2i\chi(\mathbf{b})=-4\pi^2g\exp(-\mathbf{b}^2/4a)/a;\ 2i\chi(0)=-4\pi^2g/a.$$

For scattering through fixed angles in the limit $s\to\infty$, the region $xs\sim\eta$ fixed is predominant. At the same time

$$\psi(\eta) = \lim_{s\to\infty} s^N \rho(E; x = \eta/s) = h\eta^{m-1}; \quad (N = m + M - 1) \tag{32}$$

and from (21) we obtain

$$d\sigma/dt \approx (1/s^{2N+2}) |\exp(-4\pi^2 g/a)|^2 [1/(1-z)^m + 1/(1+z)^m]^2. \tag{33}$$

Thus, our quasipotential (31) leads to a power-law decrease of the differential cross section, and the dependence on the cosine $z$ of the scattering angle is such that there is the same growth for scattering near the forward and backward peaks.

Let us now consider the scattering of particles with spins 0 and 1/2. Suppose the function $\psi(\eta)$ corresponds to the density $\rho(E; x)$ and the function $\varphi(\xi)$ to the density $\sigma(E; y)$ as in (23) with the form (32):

$$\psi(\eta) = \text{const}\, \eta^{m-1}; \quad \varphi(\zeta) = \text{const}\, \zeta^{k-1}. \tag{34}$$

Then for the large-angle differential scattering cross section we obtain from (26)

$$\frac{d\sigma}{dt} \approx \frac{(1+z)}{s^{2N+2}} \left|\exp[2i\chi(0)]\right|^2 \left(\frac{A}{(1-z)^m} + \frac{B}{(1+z)^k}\right)^2, \tag{35}$$

from which it follows that the differential cross section of large-angle meson–nucleon scattering will exhibit peaks at both $\theta \sim 0$ and $\theta \sim 180°$.

We now consider the scattering of spinor particles. We shall assume that both quasipotentials $C(s, t)$ and $D(s, t)$ in (29) are given by the representation (2) and satisfy the condition (3), the functions $\psi_{C,D}(\eta)$ being determined by the expression (32); then

$$C(s, t) = \alpha/[s^{N+1}(1-z)^m]; \quad D(s, t) = \beta'[s^{N+1}(1-z)^m].$$

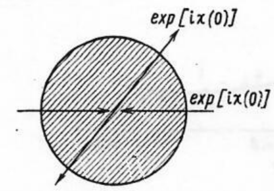
In this case, for the differential cross section we obtain from (30)

$$\frac{d\sigma}{dt} \sim \frac{|\exp[2i\chi(0)]|^2}{s^{2N+2}(1-z^2)^{2m}} \{|\alpha+\beta|^2 [(1+z)^{2m+2} + (1-z)^{2m+2} + 4((1+z)^m + (1-z)^m)^2] - 2(\alpha\beta^* + \beta\alpha^*)[(1+z)^{2m+2} + (1-z)^{2m+2}]\}. \tag{36}$$

Thus, the chosen class of analytic $\gamma_5$-invariant quasipotentials leads to power-law scaling asymptotic behaviors for the differential cross sections of large-angle scattering of high energy particles. Note that the appearance of the factor $(1+z)$ in (26) and the factors $(1 \pm z)^2$ in (30) is a direct consequence of the $\gamma_5$-invariance of the interaction for $s$, $|t|$, $|u| \gg m^2$.

The factor $|\exp[2i\chi(0)]|^2$, which appears in the expressions for the differential cross sections of large-angle scattering (33), (35), and (36), characterizes the extent to which the particles are "transparent" at small impact parameters, and the factor is determined by the parameters that describe the small-angle scattering.

The following physical ideas about the scattering process correspond to our results. At the start of the process there are multiple scatterings through small angles, leading to an advance of the eikonal phase; there is then a single scattering through a large angle described by the corresponding Born term, after which the process of multiple scatterings through small angles is repeated:




For the validity of the scaling asymptotic behavior it is necessary that the eikonal phase should not depend on $s$. The presence of the logarithmic dependence on the energy invariant leads to correlations to the quark counting rules. Thus, from the eikonal representation in the case of a Gaussian quasipotential, we obtain

$$\sigma_{tot} = 8\pi a I(x); \quad I(x) = \int_0^x dy\,(1 - \exp(-y))/y;$$
$$x = -2i\chi(0) = 4\pi^2 g/a,$$

whence $\sigma_{tot} \geqslant a \ln x$ as $x \to \infty$.

On the other hand, it can be seen from (33) that if the power-law decrease of the scattering amplitude at large angles is to be preserved the growth of $x$ must not exceed $\ln s$. If it is assumed that the small-angle scattering is described by a Gaussian quasipotential with range $a \sim \ln s$, then in our model we obtain a bound on the growth of the total cross section:

$$\sigma_{tot} \leqslant \ln s\,(\ln \ln s). \tag{37}$$

This is an example of the correlation mentioned in the introduction between the large- and small-angle scattering.

It is interesting to compare our results with experimental data, which we give for the case of large-angle $\pi^{\pm}p$ and $pp$ scattering at energies $p_L > 7$ GeV/$c$ (Ref. 16). Thus, for $\pi^+p$ scattering the best description of the data is obtained for the following values of the parameters [see (35)]:

$$A = 79.1 \pm 6; \quad B = 84 \pm 16; \quad m = 2.3 \pm 0{,}08; \quad k = 2.35 \pm 0.16.$$

Using $s \rightleftarrows u$ crossing, we can find an expression for the differential cross section of $\pi^-p$ scattering. The corresponding curves are shown in Figs. 1 and 2 and il-

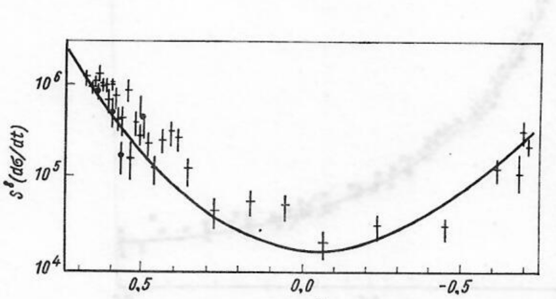



FIG. 1. $s^8(d\sigma/dt)$ for $\pi^+p$ scattering for $8 < p_L < 10$ GeV/$c$.

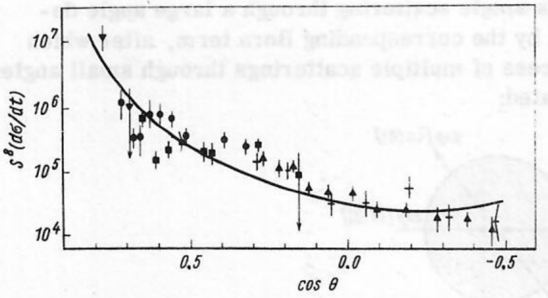



FIG. 2. $s^8(d\sigma/dt)$ for $\pi^- p$ scattering for $7 < p_L < 14$ GeV/$c$.

lustrate the agreement with the experimental data (Refs. 17 and 18) ($\chi^2/\overline{\chi}^2 = 3.0$).

In the case of $pp$ scattering (Fig. 3) [see, for example, (36)] for $m = 2.51 \pm 0.02$ and $\alpha = \beta = 14011 \pm 109$, a description of the experimental data of Ref. 19 is obtained ($\chi^2/\overline{\chi}^2 = 7.2$). The introduction of a logarithmic energy dependence of the parameters makes it possible to improve considerably the quality of the description:

$$m = 2.42 \pm 0.01;\quad \chi^2/\overline{\chi}^2 = 2.7;$$
$$\alpha = \beta = (15718 \pm 144) - (5658 \pm 850)\ln^2 s/(17.3 \pm 1).$$

Thus, the best description of the $pp$ scattering data is achieved when there is equality of the vector–vector and axial–axial interactions.

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## APPENDIX

To investigate the contributions of the different regions in which the parameters $x_i$ vanish, we represent each of the integrals in (6) in the form

$$\int_0^\infty dx_i = \int_0^\varepsilon dx_i + \int_\varepsilon^\infty dx_i,$$

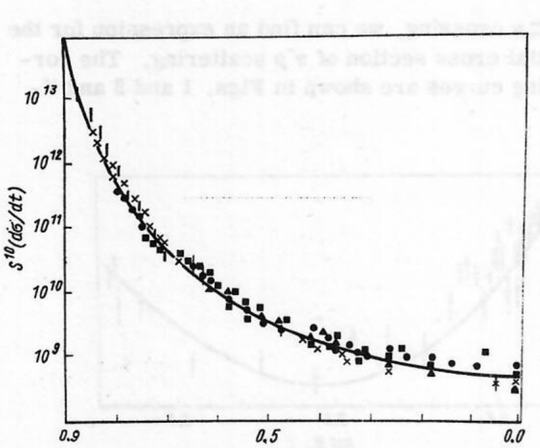


FIG. 3. $s^{10}(d\sigma/dt)$ for $pp$ scattering for $7 < p_L < 22$ GeV/$c$.

where $\varepsilon \sim 1/s^{1-\alpha}$; $0 < \alpha < 1$. As a result, we obtain the representation[6]

$$\hat{G}_{n+1} = \sum_{m=0}^{n+1} \hat{G}_{n+1}^{(m)}, \tag{A.1}$$

where $\hat{G}_{n+1}^{(m)}$ is given by Eq. (6), in which the integration with respect to any $m$ parameters $x_i$ is in the region of zero, and with respect to the remainder is from $\varepsilon$ to $\infty$. It is obvious that $\hat{G}_{n+1}^{(0)}$ is exponentially small in the limit (8). For $\hat{G}_{n+1}^{(1)}$, we have the expression

$$\hat{G}_{n+1}^{(1)} = \sum_{l=1}^{n+1} \int_\varepsilon^\infty dx_1 \ldots \int_0^\varepsilon dx_l \ldots \int_\varepsilon^\infty dx_{n+1} \exp(-x_l|t|) \times \tilde{J}_n(x_1 \ldots x_{n+1})\Big|_{x_l=0}, \tag{A.2}$$

where

$$\tilde{J}_n(x_1 \ldots x_{n+1})\Big|_{x_l=0} = (g(E))^{n+1} \times \int \ldots \int \frac{d^3\Delta_1 \ldots d^3\Delta_n \exp\{-\tilde{C}_{ij}\Delta_i\Delta_j\}}{\prod_{i=1}^{l-1}[E^2(\Delta_i + \mathbf{p}) - E^2 - i0] \prod_{i=l}^{n}[E^2(\Delta_i + \mathbf{k}) - E^2 - i0]} \times \prod_{i=1}^{l-1}[\hat{\rho}(E; x_i)\hat{A}(E; \mathbf{p} + \Delta_i)]\hat{\rho}(E; x_l) \times \prod_{i=l+1}^{n}[\hat{A}(E; \mathbf{k} + \Delta_i)\hat{\rho}(E; x_i)]; \tag{A.3}$$

here

$$\tilde{C}_{ij}\Delta_i\Delta_j = \sum_{\substack{i=1\\ i\neq l}}^{n+1}(\Delta_i - \Delta_{i-1})^2 x_i;\quad \Delta_0 = \Delta_{n+1} = 0.$$

The main contribution to (A.3) is made by the region $|\Delta_i| \lesssim 1/\sqrt{\varepsilon} \ll |\mathbf{p}|$. Therefore, in (A.3) we may retain only the principal terms of the expansion in $1/p$:

$$\tilde{J}_n(x_1 \ldots x_{n+1})\Big|_{x_l=0} \approx \frac{(g(E))^{n+1}}{(8p)^n} \times \prod_{i=1}^{l-1}[\hat{\rho}(E; x_i)\hat{A}(E; \mathbf{p})]\hat{\rho}(E; x_l) \times \prod_{i=l+1}^{n+1}[\hat{A}(E; \mathbf{k})\hat{\rho}(E; x_i)] f_n(x_1 \ldots x_{n+1}), \tag{A.4}$$

where

$$f_n(x_1 \ldots x_{n+1}) = \int \ldots \int d^3\Delta_1 \ldots d^3\Delta_n \times \exp\{-\tilde{C}_{ij}\Delta_i\Delta_j\} / \prod_{i=1}^{l-1}(\mathbf{n}_p\Delta_i - i0) \prod_{i=l}^{n}(\mathbf{n}_k\Delta_i - i0), \tag{A.5}$$

and $\mathbf{n}_p$ and $\mathbf{n}_k$ are unit vectors along the momenta $\mathbf{p}$ and $\mathbf{k}$, respectively. It is easy to see that the integral (A.5) can be represented in the form of the product

$$f_n(x_1 \ldots x_{n+1}) = \tilde{f}_{l-1}\tilde{f}_{n-l+1},$$

where

$$\tilde{f}_{l-1} = \int \ldots \int d^3\Delta_1 \ldots d^3\Delta_{l-1} \times \exp\{-\sum_{i=1}^{l} x_i(\Delta_i - \Delta_{i-1})^2\} / \prod_{i=1}^{l-1}(\mathbf{n}_p\Delta_i - i0); \tag{A.6}$$

$$\tilde{f}_{n-l+1} = \int \ldots \int d^3\Delta_l \ldots d^3\Delta_n \times \exp\{-\sum_{i=l+1}^{n+1} x_i(\Delta_i - \Delta_{i-1})^2\} / \prod_{i=l}^{n}(\mathbf{n}_k\Delta_i - i0). \tag{A.7}$$

To calculate $\tilde{f}_{l-1}$ and $\tilde{f}_{n-l+1}$, it is convenient to direct the $z$ axis along $\mathbf{n}_p$ and $\mathbf{n}_k$ and use the representation

$$\exp[-x_i(\Delta_i-\Delta_{i-1})_z^2]=(1/2\sqrt{\pi x_i})\times\int_{-\infty}^{\infty}dz_i\exp[iz_i(\Delta_i-\Delta_{i-1})_z]-z_i^2/4x_i,$$

which makes it possible to integrate with respect to $\Delta$. For example, for $\tilde{f}_{l-1}$ we obtain

$$\tilde{f}_{l-1}=(i\pi^{3/2})^{l-1}\frac{1}{(x_1\ldots x_{l-1})^{3/2}}\times\int_0^{\infty}\ldots\int_0^{\infty}dz_1\ldots dz_{l-1}\theta(z_1-z_2)\times\theta(z_2-z_3)\ldots\theta(z_l)\exp\left\{-\sum_{i=1}^{l-1}(z_i^2/4x_i)\right\}.$$

An analogous expression also holds for $\tilde{f}_{n-l+1}$. Using these representations, we substitute (A. 4) in (A. 2) and integrate with respect to $x_i$, remembering that

$$\pi^{3/2}\int_{\varepsilon}^{\infty}dx\frac{\hat{\rho}(E;x)}{x^{3/2}}\exp(-z^2/4x)\xrightarrow[\varepsilon\to 0]{}\frac{\hat{g}(E;r=\sqrt{z^2+b^2})}{g(E)}\Bigg|_{b=0},$$

where $\hat{g}(E,r)$ is the Fourier transform of the quasipotential. We also introduce the notation

$$i\int_{-\infty}^{\infty}dz\frac{\hat{g}(E;r)}{g(E)}\Bigg|_{b=0}=i\hat{\chi}(0).$$

Symmetrizing the resulting integrands, we arrive at the representation

$$\hat{G}_{n+1}^{(1)}=\sum_{l=1}^{n+1}\frac{(i\hat{\chi}(0)\hat{B}(\mathbf{p}))^{l-1}}{(l-1)!}\left[g(E)\int_0^{\varepsilon}dx\hat{\rho}(E;x)\exp(xt)\right]\times\frac{(\hat{B}(\mathbf{k})i\hat{\chi}(0))^{n-l+1}}{(n-l+1)};\qquad\text{(A. 8)}$$

here

$$\hat{B}(\mathbf{p})=\hat{A}(E;\mathbf{p})g(E)/16|\mathbf{p}|.$$

Since $|t|\to\infty$, the main contribution to (A. 8) is made by the region of small $x\sim 1/s$. Thus, the substitution

$$g(E)\int_0^{\varepsilon}dx\hat{\rho}(E,x)\exp(xt)\to g(E)\int_0^{\infty}dx\hat{\rho}(E;x)\exp(xt)=\hat{g}(E;t)$$

does not change the asymptotic behavior (A. 6). Thus, we find that $\hat{G}_{n+1}^{(1)}$ has the form (9).

We now consider the term $G_{n+1}^{(2)}$ in the expansion (A. 1):

$$\hat{G}_{n+1}^{(2)}=\sum_{l<k=1}^{n+1}\int_{\varepsilon}^{\infty}dx_1\ldots\int_0^{\varepsilon}dx_l\int_0^{\varepsilon}dx_k\ldots\int_{\varepsilon}^{\infty}dx_{n+1}\times\exp[tx_lx_k/(x_l+x_k)]\hat{J}_n(x_1\ldots x_{n+1})\Big|_{x_lx_k\to 0}=\sum_{l<k=1}^{n+1}\hat{G}_{lk}^{(2)}.\qquad\text{(A. 9)}$$

In contrast to the calculation of $\hat{G}_{n+1}^{(1)}$, we cannot everywhere set $x_l=x_k=0$ in $\hat{J}_n$ because in that case divergences with respect to $\Delta_l$ in $J_n$ could appear. This comes about because of the vanishing in the form $C_{ij}\Delta_i\Delta_j$ of the terms quadratic in $\Delta_l$ and $\Delta_k$. Of particular danger are $l$ and $k$ values that differ by unity. It is precisely these terms that make the main contribution to the asymptotic behavior of $\hat{G}_{n+1}^{(2)}$, and therefore when we go to the limit $x_l$, $x_{l+1}\to 0$ in $J_n$ we retain the terms that ensure convergence of the integral with respect to $\Delta_l$:

$$C_{ij}\Delta_i\Delta_j|\xrightarrow[x_l,x_{l+1}\to 0]{}\sum_{i=1}^{l-1}(\Delta_i-\Delta_{i-1})^2x_i+\Delta_l^2(x_l+x_{l+1})+\sum_{i=l+2}^{n+1}(\Delta_i-\Delta_{i-1})^2x_i.$$

Then for $\hat{J}_n$ we can write down an expression analogous to (A. 4):

$$\hat{J}_n\sim\frac{(g(E))^{n+1}}{(8p)^{n-1}}\prod_{i=1}^{l-1}[\hat{\rho}(E;x_i)\hat{A}(E;\mathbf{p})]\hat{\rho}(E;x_l)\times\hat{\Phi}_n(x_1\ldots x_{n+1})\hat{\rho}(E;x_{l+1})\prod_{i=l+2}^{n+1}[\hat{A}(E;\mathbf{k})\hat{\rho}(E;x_i)],$$

where $\hat{\Phi}_n$ can be represented in the form $\hat{\Phi}_n(x_1\cdots x_{n-1})=\tilde{f}_{l-1}\tilde{f}_{n-l}\hat{\Phi}$, Eq. (2. 6) determines $\tilde{f}_{l-1}$, $\tilde{f}_{n-l}$ is obtained from (A. 7) by the substitution $l\to l+1$, and $\hat{\Phi}$ is given by

$$\hat{\Phi}=\int d^3\Delta_l\frac{\exp[-\Delta_l^2(x_l+x_{l+1})]\hat{A}(E;\Delta_l+\lambda_l)}{[E^2(\Delta_l+\lambda_l)-E^2-i0]};$$

where

$$\lambda_l=(x_l\mathbf{p}+x_{l+1}\mathbf{k})/(x_l+x_{l+1}).$$

Substituting the expressions for $\tilde{f}_{l-1}$, $\tilde{f}_{n-l}$, and $\hat{\Phi}$ in Eq. (A. 9), we obtain

$$\hat{G}_{l,l+1}=\{(g(E))^2[i\hat{\chi}(0)\hat{B}(\mathbf{p})]^{l-1}/(l-1)!\}\times\int_0^{\varepsilon}dx_l\,dx_{l+1}\hat{\rho}(E;x_l)\hat{\Phi}(x_l,x_{l+1})\hat{\rho}(E;x_{l+1})\times\exp[-|t|x_lx_{l+1}/(x_l+x_{l+1})][i\hat{B}(\mathbf{k})\hat{\chi}(0)]^{n-l}/(n-l)!$$

Making the substitution $x_l$, $x_{l+1}\to\eta_1/s$, $\eta_2/s$ and going to the limit $s\to\infty$, we obtain for our class of quasipotentials

$$\hat{G}_{l,l+1}\to\frac{g(E)}{s}\frac{1}{s^{2N}}\frac{[i\hat{\chi}(0)\hat{B}(\mathbf{p})]^{l-1}}{(l-1)!}\int_0^{s^{\alpha}}d\eta_1\,d\eta_2\hat{\psi}(\eta_1)\hat{\varphi}\hat{\psi}(\eta_2)\times\exp\left(-|t|/s\frac{\eta_1\eta_2}{\eta_1+\eta_2}\right)\frac{[i\hat{B}(\mathbf{k})\hat{\chi}(0)]^{n-l}}{(n-l)!};$$

$$\hat{\varphi}=\int d^3\Delta_l\frac{\exp[-\Delta_l^2(\eta_1+\eta_2)]}{[(\Delta_l+\tilde{\lambda}_l)^2-1/4-i0]}\hat{B}(2p(\Delta_l+\tilde{\lambda}_l));$$

$$\tilde{\lambda}_l=(\eta_1\mathbf{n}_p+\eta_2\mathbf{n}_k)/(\eta_1+\eta_2).$$

It can be seen from this that

$$\hat{G}_{l,l+1}<1/s^Ns^{(1-\alpha)N},\quad 0<\alpha<1.\qquad\text{(A. 10)}$$

Comparing (A. 10) and (A. 12), we see that the contribution to the scattering amplitude $\hat{G}$ from $\hat{G}^{(1)}$ is asymptotically greater than the contribution from $G^{(2)}$. Similarly, one can show that this is true for any amplitude $\hat{G}^{(m)}$ with $m>1$. Thus, Eq. (11) determines the complete amplitude $\hat{G}$ at high energies and large momentum transfers.


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