Photoneutron reactions near threshold and the optical model of nuclei

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A semimicroscopic approach to the description of nuclear reactions with the excitation of giant resonances in spherical nuclei on the basis of the shell model is described. The valence mechanism of the γn reaction near threshold is considered in detail. The characteristics of the mean cross sections are expressed in terms of the shell model and optical model. A brief investigation is made of the damping of the giant dipole resonance and also the influence of the latter on El-photoneutron reactions near threshold.

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INTRODUCTION

Nuclear reactions at low energies are a powerful tool for investigating nuclear structure. The transition to a detailed investigation of highly excited states of nuclei of necessity means that one must go over to studying nuclear reactions, and in the first place resonance reactions. The diversity of the phenomena—different types of resonances, different decay channels of the resonances of given type—make a unified approach to the description of the structure of nuclear states and reaction mechanisms very desirable. For not too high excitation energies, such an approach can in principle be based on the shell model. In the framework of the modern model of nuclear shells, many phenomena associated with the properties of the ground state and low-lying excited states of nuclei have been explained.[1] The reason for this is well known.[2] A nucleus with a large number of nucleons at excitation energies lower than the Fermi energy can be regarded as a gas of interacting quasiparticles. Since the number of quasiparticles at such excitation energies is less than the total number of particles, it is sufficient to take into account only the binary interactions of the quasiparticles.

There are two possible approaches to an investigation of highly excited states of nuclei and the corresponding resonance nuclear reactions on the basis of the shell model. One approach, which from the point of view of the shell model may be termed microscopic, is based on direct diagonalization of the Hamiltonian of the shell model on a fairly broad basis. Work in this direction is carried on intensively at Dubna.[3] The other approach, which is semimicroscopic, is based on the existence at sufficiently high excitation energies of a "hierarchy" of states: simple and complex configurations. To isolated simple configurations, the so-called doorway states, there correspond giant resonances in the reaction cross sections. To the systematically encountered giant resonances there correspond single-particle resonances, giant multipole resonances, and isobaric analog resonances. The coupling between the simple configurations and the more complex ones leads to the appearance of fine structure in the giant resonances. The semimicroscopic approach aims solely to describe the properties of the giant resonances averaged over the fine-structure components. In the framework of this approach, the shell model is used to describe the simple configurations. At the same time, the coupling between the simple and the complex configurations is taken into account by means of the optical model. It is important that the optical model is not introduced *ad hoc* but appears as a result of averaging of the "microscopic" equations. Thus, the semimicroscopic approach is fairly economic in its choice of tools and fairly productive in describing different properties of various giant resonances.

In the present paper, the semimicroscopic approach is used to analyze the valence mechanism of radiative capture of resonance neutrons by spherical nuclei (or the inverse reaction) and (to a lesser extent) giant multipole resonances. In Sec. 1, the mean amplitudes and cross sections of elastic scattering of neutrons and the γn reaction are parametrized in the case when the elastic neutron channel is the main decay channel of the resonances of the compound nucleus. In Sec. 2, the valence mechanism is analyzed semiquantitatively on the basis of the theory of doorway states. Convenient expressions are obtained for the neutron strength function and the partial radiative strength functions, and also for the background cross section of the γn reaction. In Sec. 3, the transition to the optical model in the expressions for the mean amplitudes of the reactions considered is made by the methods of many-body theory. The expressions for the strength functions and the mean cross sections are written in terms of the shell and optical models. These expressions are analyzed qualitatively in Sec. 4. The results of calculations are given for nuclei near s and p shape resonances, and they are compared with the results of the K-matrix approach. In Sec. 5, the damping of giant multipole resonances is considered and an expression is derived for the response function—the basic quantity in the theory of collective vibrations. Section 6 gives a systematic description of the giant dipole resonance and takes into account the influence of the giant dipole resonance on the characteristics of $E1$-photoneutron reactions near the threshold.

1. MEAN CROSS SECTIONS AND STRENGTH FUNCTIONS

In this section, we parametrize the elements of the S matrix corresponding to elastic scattering of neutrons

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and the γn reactions (S_{nn} and $S_{\gamma n}$), and we then average S_{nn}, $S_{\gamma n}$, and $|S_{\gamma n}|^2$ over an energy interval containing many neutron resonances. Here and in what follows, we consider the practically important case when the elastic neutron channel is the main channel for decay of neutron resonances, so that $|S_{nn}|^2=1$, and the photoabsorption cross section is equal to the cross section of the γn reaction.

We shall assume that simple poles of the S matrix correspond to the neutron resonances. In an energy interval near one of the nonoverlapping neutron resonances, S_{nn} and $S_{\gamma n}$ can be represented in accordance with this assumption in the form

$$S_{nn}(E)=\exp(2i\xi)\,[1-i\gamma_{nc}/(E-E_c+i\gamma_{nc}/2)]; \quad (1)$$

$$S_{\gamma n}(E)=\exp(i\psi)\,\{|S_{\gamma n}^{bg}|-\exp(i\phi)\,i\gamma_{\gamma c}^{1/2}\gamma_{nc}^{1/2}/[E-E_c+i\gamma_{nc}/2]\}, \quad (2)$$

where E is the neutron energy; $\gamma_{nc}^{1/2}$ and $\gamma_{\gamma c}^{1/2}$ are the amplitudes of the neutron width of the resonance and the partial radiative width of the resonance; $|S_{\gamma n}^{bg}|^2=\sigma_{\gamma n}^{bg}/g\pi\lambda_\gamma^2$ determines the nonresonance part of the γn reaction cross section. When the neutron strength functions $S_n=\gamma_n/d$ (γ_n is the mean neutron width; d is the mean energy interval between resonances with definite spin and parity) are small, the expressions (1) and (2) can be used to calculate the mean values $\overline{S}_{nn}$ and $\overline{S}_{\gamma n}$:

$$\overline{S}_{nn}=\exp(2i\xi)\,(1-\pi S_n); \quad (3)$$

$$\overline{S}_{\gamma n}=\exp(i\psi)\,\{|S_{\gamma n}^{bg}|-\pi\exp(i\phi)\,S_n^{1/2}(S_\gamma^{(1)})^{1/2}\}; \quad (4)$$

where $S_\gamma^{(1)}=\gamma_\gamma^{(1)}/d$; the mean amplitude of the radiative width $(\gamma_\gamma^{(1)})^{1/2}$ is defined by $\overline{\gamma_{\gamma c}^{1/2}\,\gamma_{nc}^{1/2}}=(\gamma_\gamma^{(1)})^{1/2}\gamma_n^{1/2}$. Thus, the width $\gamma_\gamma^{(1)}$ is the part of the mean partial radiative width correlated to the neutron width. On the basis of (2), we can also define $\overline{|S_{\gamma n}|^2}$:

$$\overline{|S_{\gamma n}|^2}=|S_{\gamma n}^{bg}|^2-2\pi\cos\phi\,|S_{\gamma n}^{bg}|\,S_n^{1/2}(S_\gamma^{(1)})^{1/2}+2\pi S_\gamma. \quad (5)$$

Here, $S_\gamma=S_\gamma^{(1)}+S_\gamma^{(2)}=\gamma_\gamma/d$ is the radiative strength function for the partial transition; $\gamma_\gamma=\gamma_\gamma^{(1)}+\gamma_\gamma^{(2)}$ and $\gamma_\gamma^{(2)}$ is the part of the mean radiative width that is not correlated to the neutron width.

The mean cross sections of elastic neutron scattering and the γn reaction can be represented in the form of sums of the corresponding optical and fluctuation cross sections. The optical cross sections are determined by the mean elements of the S matrix, while the fluctuation cross sections are determined in accordance with (3)–(5) by the strength functions:

$$\overline{\sigma}_{nn}=\sigma_{nn}^{\rm opt}+\sigma_{nn}^{\rm fl};\quad \sigma_{nn}^{\rm fl}/g\pi\lambda_n^2=1-|\overline{S}_{nn}|^2\approx 2\pi S_n; \quad (6)$$

$$\overline{\sigma}_{\gamma n}=\sigma_{\gamma n}^{\rm opt}+\sigma_{\gamma n}^{\rm fl};\quad \sigma_{\gamma n}^{\rm fl}/g\pi\lambda_\gamma^2=\overline{|S_{\gamma n}|^2}-|\overline{S}_{\gamma n}|^2\approx 2\pi S_\gamma. \quad (7)$$

As will be shown below, if there is a definite relationship between the parameters characterizing the damping of a single-particle state, the neutron strength functions S_n are not small. In order to generalize Eqs. (3)–(7) to this case, we use the model of equidistant resonances. Having in mind a subsequent averaging, we represent the diagonal element of the S matrix in the form

$$S_{nn}(E)=\exp(2i\xi)\,[a-ib\gamma_n\sum_c(E-E_c+i\gamma_n/2)^{-1}],$$

where a and b are smooth functions of the energy which we determine from the condition $|S_{nn}(E)|^2=1$. Using also the relation $\sum_{c'}(E-E_{c'}+i\gamma/2)^{-1}=(\pi/d)\cot(\pi/d)(E-E_c+i\gamma/2)$, where E_c is the energy of the resonance nearest E, we find

$$S_{nn}(E)=\exp(2i\xi)\,\{1-2i\tanh\eta/[\tan(\pi/d)(E-E_c)+i\tanh\eta]\};$$
$$\eta=\pi S_n/2. \quad (8)$$

In the limit $\eta\ll 1$, $|E-E_c|\ll d$, this expression goes over into Eq. (1). Averaging the relation (8) over the energy interval $|E-E_c|<d/2$, we obtain a generalization of the expression (3):

$$\overline{S_{nn}}=\exp(2i\xi-2\eta). \quad (9)$$

Therefore, the transmission coefficients T, which in accordance with Eq. (6) determine the fluctuation cross section of elastic scattering, satisfy for arbitrary values of the neutron strength function

$$T=1-|\overline{S}_{nn}|^2=1-\exp(-2\pi S_n). \quad (10)$$

Equations (9) and (10) agree with the results of Ref. 4.

Comparing the resonance terms in the expressions (1) and (8), we can see how we must generalize Eq. (2) to the case of arbitrary values of S_n:

$$S_{\gamma n}(E)=\exp(i\psi)\,\{|S_{\gamma n}^{bg}|-\exp(i\phi)\,(S_\gamma/S_n)^{1/2}$$
$$\times 2i\tanh\eta/[\tan(\pi/d)(E-E_c)+i\tanh\eta]\} \quad (11)$$

Averaging of this expression, and also $|S_{\gamma n}|^2$ defined by Eq. (11), leads to the result

$$\overline{S}_{\gamma n}=\exp(i\psi)\,\{|S_{\gamma n}^{bg}|-\exp(i\phi)\,[1-\exp(-2\eta)]\,(S_\gamma^{(1)}/S_n)^{1/2}\}; \quad (12)$$

$$\overline{|S_{\gamma n}|^2}=|S_{\gamma n}^{bg}|^2+2\,[1-\exp(2\eta)]\,[(S_\gamma/S_n)-\cos\phi\,|S_{\gamma n}^{bg}|\,(S_\gamma^{(1)}/S_n)^{1/2}]. \quad (13)$$

Using these formulas, we find an expression for the fluctuation cross section of the γn reaction for arbitrary values of S_n:

$$\sigma_{\gamma n}^{\rm fl}/g\pi\lambda_\gamma^2=[1-\exp(-4\eta)]\,(S_\gamma^{(1)}/S_n)+2\,[1-\exp(-2\eta)]\,(S_\gamma^{(2)}/S_n). \quad (14)$$

Equations (9), (10), (12)–(14) relate observable quantities to the mean values $\overline{S}_{nn}$, $\overline{S}_{\gamma n}$, $\overline{|S_{\gamma n}|^2}$. The following exposition is devoted to the theoretical analysis of the mean values. In Secs. 2–4 we describe, on the basis of the shell model, the valence mechanism of radiative capture of neutrons (or the reverse reaction), in accordance with which the nondiagonal elements of the $S_{\gamma n}$ scattering matrix are determined solely by the transition of the valence neutron. In this approximation, $S_\gamma^{(2)}\to 0$, $S_\gamma^{(1)}\to(S_\gamma)_{sp}$, $\sigma_{\gamma n}\to(\sigma_{\gamma n})_{sp}$, etc. In Secs. 2–4, the subscript sp will be dropped for brevity.

2. QUASIDISCRETE LEVEL AS DOORWAY STATE

Before we propose a quantitative interpretation of the valence mechanism of radiative capture of neutrons in terms of the shell and optical models, we give a semiquantitative solution of this problem based in fact on treating a single-particle state (quasidiscrete level) as a doorway state. The perspicuity of the results and the possibility of establishing a connection with the theory of doorway states justifies the interest in such an undertaking.

At excitation energies that are greater than or of the order of the nucleon binding energy, there are, besides single-particle configurations, many-particle configurations as well: two-particle–one-hole, three-particle–two-hole, etc., (to be specific, we consider odd compound nuclei). As a rule, the matrix elements of the interaction of the many-particle configurations exceed the energy intervals between them. It is therefore convenient to introduce the so-called "unrenormalized" levels $|\lambda\rangle$ of the compound nucleus, which are determined by diagonalizing the Hamiltonian of the shell model on the basis of the many-particle configurations. The coupling of the single-particle states to the levels $|\lambda\rangle$ by the "residual" interaction H' leads to resonance scattering of the nucleons. The diagonal element of the S matrix is related as follows to the corresponding element of the T matrix:

$$S_{nn}(E)=\exp(2i\delta_p)-2\pi i\langle E|T|E\rangle, \tag{15}$$

where δ_p are the phase shifts of the nucleon scattering on the shell potential. We find the expression for the matrix element $\langle E|T|E\rangle$ by summing the perturbation series:

$$\langle E|T|E\rangle=\sum_{\lambda}\frac{\langle E|H'|\lambda\rangle\langle\lambda|H'|E\rangle}{E-E_\lambda}$$
$$+\sum_{\lambda\lambda'E'}\frac{\langle E|H'|\lambda\rangle\langle\lambda|H'|E'\rangle\langle E'|H'|\lambda'\rangle\langle\lambda'|H'|E\rangle}{(E-E_\lambda)(E-E'+i\varepsilon)(E-E_{\lambda'})}+\ldots \tag{16}$$

This expression in fact presupposes that we have restricted the treatment to an energy interval near the quasidiscrete level, since (16) does not take into account virtual excitation of one-hole configurations with the considered values of the angular momentum and parity. In this energy range, the following approximate expressions hold for the S matrix of potential scattering, $\exp(2i\delta_p)$, for the radial wave function of the continuum normalized to a δ function of the energy, $|E\rangle\to r^{-1}\chi_{0E}^{(+)}(r)=\exp(i\delta_p)r^{-1}\chi_{0E}(r)$, and for the Green's function $G_0^{(+)}(r,r';E)$ of the radial Schrödinger equation (see Ref. 5):

$$\left.\begin{aligned}&\exp(2i\delta_p)\approx\exp(2i\delta_0)\{1-i\Gamma^{\uparrow}(E-E_a+i\Gamma^{\uparrow}/2)^{-1}\};\\&\chi_{0E}\approx\exp(i\delta_0)(\Gamma^{\uparrow}/2\pi)^{1/2}(E-E_a+i\Gamma^{\uparrow}/2)^{-1}\chi_a(r);\quad(r<R);\end{aligned}\right\} \tag{17}$$

$$\begin{aligned}G_0^{(+)}(r,r';E)&=\sum_{E'}\chi_{0E'}(r)\chi_{0E'}(r')(E-E'+i\varepsilon)^{-1}\\&\approx\chi_a(r)\chi_a(r')(E-E_a+i\Gamma^{\uparrow}/2)^{-1},\ (r<R).\end{aligned} \tag{18}$$

Here, δ_0 is the nonresonant part of the potential-scattering phase shift: $\Gamma\uparrow$ is the width for the decay of the quasidiscrete level to the continuum; $r^{-1}\chi_a(r)$ is a regular solution of the radial Schrödinger equation normalized by the condition $\int_0^R\chi_a^2(r)\,dr=1$ (R is the radius of the nucleus). Introducing the definition $\langle\lambda|H'|E\rangle=\exp(i\delta_0)\langle\lambda|H'|a\rangle\equiv\exp(i\delta_0)g_\lambda^{(a)}=\langle E|H'|\lambda\rangle$, on the basis of (16)–(18) we obtain the following expression for the diagonal element of the scattering matrix:

$$S_{nn}(E)=\exp(2i\delta_0)\left\{1-\frac{i\Gamma^{\uparrow}}{E-E_a+i\Gamma^{\uparrow}/2-\sum_\lambda(g_\lambda^{(a)})^2(E-E_\lambda)^{-1}}\right\}. \tag{19}$$

Similarly, we can find the nondiagonal element of the S matrix $S_{\gamma n}=-2\pi iM_{\gamma n}$ corresponding to radiative transition of a neutron from the bound state $|b\rangle$ to the continuum. Summation of the perturbation series for the γn reaction amplitude

$$M_{\gamma n}(E)=\langle b|H_\gamma|E\rangle+\sum_{E'\lambda}\frac{\langle b|H_\gamma|E'\rangle\langle E'|H'|\lambda\rangle\langle\lambda|H'|E\rangle}{(E-E'+i\varepsilon)(E-E_\lambda)}$$
$$\ldots+(E=E_\gamma+E_b)$$

using Eqs. (17) and (18) leads to the following expression for $S_{\gamma n}$:

$$S_{\gamma n}(E)=-\exp(i\delta_0)\frac{i\Gamma_\gamma^{1/2}(\Gamma^{\uparrow})^{1/2}}{E-E_a+i\Gamma^{\uparrow}/2-\sum_\lambda(g_\lambda^{(a)})^2(E-E_\lambda)^{-1}}. \tag{20}$$

Here, $\Gamma_\gamma=2\pi|\langle b|H_\gamma|a\rangle|^2$ is the radiative width of the quasidiscrete level corresponding to the valence transition.

We establish further the connection between $\cos\phi$ and $|S_{\gamma n}^{bg}|$ in Eq. (2). In accordance with (19) and (20),

$$S_{nn}(E)\equiv\exp(2i\delta_0)\{1+S'(E)\};$$
$$S'(E)=\exp(-i\delta_0)(\Gamma^{\uparrow}/\Gamma_\gamma)^{1/2}S_{\gamma n}(E).$$

It follows from this formula and the unitarity condition $|S_{nn}(E)|^2=1$, that

$$|S'(E)|^2=-2\,\mathrm{Re}\,S'(E). \tag{21}$$

Using the representation (2) for $S_{\gamma n}$ and setting $\gamma_{\gamma c}^{1/2}/\gamma_{nc}^{1/2}=(\Gamma_\gamma/\Gamma\uparrow)^{1/2}=S_\gamma/S_n$ (these relations, which in fact follow from Eq. (20), will be confirmed below), and taking into account (21), we find

$$(S_n/S_\gamma)^{1/2}|S_{\gamma n}^{bg}|=2\cos\phi. \tag{22}$$

In accordance with Eqs. (12)–(14), this equation enables us to find the three quantities S_γ/S_n, $|S_{\gamma n}^{bg}|$ and $\cos\phi$ if we know the mean values $\overline{S}_{\gamma n}$ and $\overline{|S_{\gamma n}|^2}$.

The procedure for averaging the amplitude for scattering of a nucleon (or γ ray) on the nucleus, $f(E)$, is based on the analytic properties of the amplitude. The assumption that simple poles of the function $f(E)$ correspond to physical levels of the compound nucleus leads to the well known relation (see, for example, the monograph Ref. 6)

$$\overline{f}(E)=f(E+iI), \tag{23}$$

where I is an averaging interval such that $I/d\gg1$. Using Eqs. (19), (20), and (23), we find expressions for the mean values $\overline{S}_{nn}$ and $\overline{S}_{\gamma n}$:

$$\overline{S}_{nn}(E)=\exp(2i\delta_0)\{1-i\Gamma^{\uparrow}/[E-E_a+i(\Gamma^{\uparrow}+\Gamma^{\downarrow})/2]\}; \tag{24}$$

$$\overline{S}_{\gamma n}(E)=-i\exp(i\delta_0)\Gamma_\gamma^{1/2}(\Gamma^{\uparrow})^{1/2}/[E-E_a+i(\Gamma^{\uparrow}+\Gamma^{\downarrow})/2]; \tag{25}$$

where the width $\Gamma\downarrow=-2\,\mathrm{Im}\sum_\lambda(g_\lambda^{(a)})^2(E-E_\lambda+iI)^{-1}$

$=2\pi (g_\lambda^{(a)})^2/d$ is the width for the decay of the single-particle state $|a\rangle$ to levels of complex nature (we ignore the shift of the quasidiscrete level). In accordance with Eqs. (10) and (24), the transmission coefficients and the neutron strength functions near the shape resonance are determined as follows by the parameters of the potential problem and the width $\Gamma\downarrow$:

$$T(E)=\frac{\Gamma^\uparrow\Gamma^\downarrow}{(E-E_a)^2+(\Gamma^\uparrow+\Gamma^\downarrow)^2/4};$$
$$2\pi S_n=\ln\left[1+\frac{\Gamma^\uparrow\Gamma^\downarrow}{(E-E_a)^2+(\Gamma^\uparrow-\Gamma^\downarrow)^2/4}\right]. \quad (26)$$

In the limiting cases $\Gamma\downarrow \gg \Gamma\uparrow$, $\Gamma\downarrow \ll \Gamma\uparrow$ we arrive at the theory of doorway states (see, for example, Ref. 6):

$$2\pi S_n=\Gamma^\uparrow\Gamma^\downarrow[(E-E_a)^2+(1/4)\max^2\{\Gamma^\uparrow,\ \Gamma^\downarrow\}]^{-1}. \quad (27)$$

In these cases $S_n \ll 1$. When $\Gamma\uparrow \sim \Gamma\downarrow$, in the immediate neighborhood of the shape resonance $|E-E_a|\sim\Gamma\uparrow\sim\Gamma\downarrow$ the strength functions S_n are not small, as follows from Eq. (26).

In order to determine the partial radiative strength function we must, in accordance with (14), find not only $\overline{S}_{\gamma n}$ but also $\overline{|S_{\gamma n}|^2}$. We find the latter by means of Eqs. (21) and (25):

$$\overline{|S_{\gamma n}|^2}=\Gamma_\gamma(\Gamma^\downarrow+\Gamma^\uparrow)/[(E-E_a)^2+(\Gamma^\downarrow+\Gamma^\uparrow)^2/4]. \quad (28)$$

Therefore, in accordance with (14), (25), (26), and (28), we obtain for the ratio S_γ/S_n the expression

$$S_\gamma/S_n=\Gamma_\gamma/\Gamma^\uparrow. \quad (29)$$

This conclusion agrees with the result of the valence model.[7]

The ratio $|\overline{S}_{\gamma n}|^2/\overline{|S_{\gamma n}|^2}=1-\sigma_{\gamma n}^{fl}/\overline{\sigma}_{\gamma n}$ characterizes the relative contribution of the optical cross section of the γn reaction to the corresponding mean cross section. In accordance with (25) and (28), this ratio is

$$\sigma_{\gamma n}^{\text{opt}}/\overline{\sigma}_{\gamma n}=\Gamma^\uparrow/(\Gamma^\uparrow+\Gamma^\downarrow). \quad (30)$$

In the limiting case $\Gamma\downarrow \gg \Gamma\uparrow$, when the fluctuation cross section is in fact equal to the mean cross section, this last determines the strength function S_γ. A simple expression for the background cross section $\sigma_{\gamma n}^{bg}$ can be obtained by means of Eqs. (13), (22), and (26)–(28) in limiting cases:

$$\frac{\sigma_{\gamma n}^{bg}}{g\pi\lambda_\gamma^2}=\begin{cases}\dfrac{\Gamma_\gamma\Gamma^\uparrow}{(E-E_a)^2+\frac{1}{4}(\Gamma^\uparrow)^2} & (\Gamma^\uparrow\gg\Gamma^\downarrow);\\[2ex] \dfrac{\Gamma_\gamma\Gamma^\uparrow(E-E_a)^2}{\left[(E-E_a)^2+\frac{1}{4}(\Gamma^\downarrow)^2\right]^2} & (\Gamma^\uparrow\ll\Gamma^\downarrow).\end{cases} \quad (31)$$

The treatment in this section shows that the single-particle state (quasidiscrete level) can be regarded as a doorway state characterized by the energy E_a and the widths $\Gamma\uparrow$, Γ_γ, $\Gamma\downarrow$. The first three quantities are the potential characteristics of this state that also exist in the absence of coupling with complex configurations and are determined by the potential of the shell model; The last quantity characterizes the strength of the coupling and is a phenomenological parameter of the theory.

Resonances of the compound nucleus and, therefore, partial radiative strength functions also exist in the subthreshold region when the neutron decay channel is closed. (These resonances are excited, for example, in γ-ray scattering). In this case, there is no need for recourse to diagonalization on the basis of the states $|a\rangle$ and $|\lambda\rangle$; the necessary relations can be found by going to the limit $\Gamma\uparrow\to 0$, $\delta_0\to 0$, $\xi\to 0$ in the equations given above. Thus, in accordance with (17) and (29),

$$2\pi S_\gamma=\Gamma_\gamma\Gamma^\downarrow[(E-E_a)^2+(\Gamma^\downarrow)^2/4]^{-1}. \quad (32)$$

In the above limit, the ratios $\gamma_n/\Gamma\uparrow=\gamma_\gamma/\Gamma_\gamma=(2\pi)^{-1}d\Gamma\downarrow \times[(E-E_a)+(\Gamma\downarrow)^2/4]^{-1}\equiv w^{(a)}(E)$ are obviously the mean probability of finding the single-particle state in the physical levels of the compound nucleus. In the same limit, the ratio $\mathrm{i}(S_{nn}-1)\Gamma\uparrow\equiv\mathcal{G}(E)$ is the Green's function for the given diagonalization problem. An explicit expression for this quantity can be obtained by means of Eqs. (8), (19), and (27):

$$\mathcal{G}(E)=[E-E_a-\sum_\lambda (g_\lambda^{(a)})^2(E-E_\lambda)^{-1}]^{-1}=\sum w^{(a)}(E_c)/(E-E_c). \quad (33)$$

We shall use this expression subsequently.

The description of the quasidiscrete level as a doorway state contains a number of restrictions: 1) the restriction to the energy interval $|E-E_a|\ll D$ (D is the energy interval between neighboring single-particle levels with the considered values of the angular momentum and the parity); 2) the inaccuracy in the quantitative determination of the widths $\Gamma\uparrow$ and Γ_γ associated with the approximate nature of Eqs. (14) and (18). The following section is devoted to the calculation of the mean values $\overline{S}_{nn}$, $\overline{S}_{\gamma n}$, and $\overline{|S_{\gamma n}|^2}$ by the methods of many-body theory in a way which is free of these restrictions.

3. TRANSITION TO OPTICAL MODEL AND VALENCE MECHANISM OF THE γn REACTION

Physically, the width $\Gamma\downarrow$ is intimately related to the imaginary part of the optical potential. It is convenient to make the transition to the optical model of elastic scattering of nucleons by the methods of the theory of finite Fermi systems.[2] The single-particle Green's function $G_0^F(r, r'; E)$ for the Fermi system corresponding to motion of a nucleon in the average field of the shell model has the form (here and below the angular variables have been separated off):

$$G_0^F(r,\ r';\ E)=\sum_{E'}\chi_{0E'}(r)\,\chi_{0E'}(r')\,G_{0E'}^F(E);$$
$$G_{0E'}^F(E)=[E-E'+\mathrm{i}\varepsilon\,\mathrm{Sgn}\,(E'-\mu)]^{-1}, \quad (34)$$

where $r^{-1}\chi_{0E}(E)$ are the radial wave functions of the nucleon corresponding to definite values of the energy, angular momentum, and parity: $(h_0-E)\chi_{0E}=0$, $h_0(E)$ is the Hamiltonian of the shell model; μ is the chemical potential. After the substitution $E\to E+\mathrm{i}I$, the Green's function (34) goes over into the radial Schrödinger equation $G_0^{(+)}(r, r'; E)=G_0^F(r, r'; E+\mathrm{i}I)$:

$$(h_0(r)-E)\,G_0^{(+)}(r,\,r';\,E)=-\delta(r-r'). \quad (35)$$

The single-particle Green's function $G^F(r,r';E)$ corresponding to motion of a nucleon in the nuclear medium satisfies Dyson's equation[8]:

$$G^F(r,\,r';\,E)=G_0^F(r,\,r';\,E) + \int G_0^F(r,\,r_1;\,E)\,T^0(r_1,\,r_2;\,E)\,G^F(r_2,\,r';\,E)\,dr_1\,dr_2 \quad (36)$$

or

$$G_{E'}^F(E)=G_{0E'}^F(E)+\sum_{E''} G_{0E'}^F(E)\,T_{E'E''}^0(E)\,G_{E''}^F(E),$$

where $T^0(E)$, the irreducible self-energy part, depends in a complicated manner on the energy because of the virtual excitation of complex configurations. After the substitution $E\to E+\mathrm{i}I$, the Green's function (36) goes over into the Green's function $G^F(r,r';E+\mathrm{i}I) = G^{(+)}(r,r';E)$ of the radial Schrödinger equation with the Hamiltonian of the optical model:

$$\begin{aligned}&(h(r)-E)\,G^{(+)}(r,\,r';\,E)=-\delta(r-r');\\ &T^{(0)}(r,\,r';\,E+\mathrm{i}I)=\Delta h(r;\,E)\,\delta(r-r'),\end{aligned} \quad (37)$$

where $\Delta h(r;E)=-\mathrm{i}\,w(r;E)+\Delta(r;E)$ is the difference between the Hamiltonian of the optical model and the Hamiltonian of the shell model: $h=h_0+\Delta h$ (we restrict ourselves here to the case of a local potential of the optical model). By means of Eqs. (35)–(37) the problem of going over to the optical model in the description of the average properties of the single-particle excitations of nuclei is in fact solved. By definition, the T matrix, in the expression (15) is the reducible self-energy part, which is related as follows to T^0:

$$\int T(r,\,r';\,E)\,G_0^F(r,\,r';\,E)\,dr'=\int T^0(r,\,r';\,E)\,G^F(r,\,r';\,E)\,dr' \quad (38)$$

If, further, we define the functions $\chi_E^{(+)}$ by

$$\int T^0(r,\,r';\,E+\mathrm{i}I)\,\chi_E^{(+)}(r')\,dr'=\int T(r,\,r';\,E+\mathrm{i}I)\,\chi_{0E}^{(+)}(r')\,dr', \quad (39)$$

then on the basis of (36)–(39) we may conclude that these functions satisfy the equation

$$\chi_E^{(+)}(r)=\chi_{0E}^{(+)}(r)+\int G_0^{(+)}(r,\,r';\,E)\,\Delta h(r';\,E)\,\chi_E^{(+)}(r')\,dr', \quad (40)$$

i.e., they are eigenfunctions of the Hamiltonian of the optical model: $(h-E)\chi_E^{(+)}=0$.

We represent the expression for the mean S matrix in accordance with (15), (37), and (38) in the form

$$\begin{aligned}&\overline{S}_{nn}(E)=S_{nn}(E+\mathrm{i}I)=\exp(2\mathrm{i}\delta_p)\\ &-2\pi\mathrm{i}\int \chi_{0E}^{(+)}(r)\,\Delta h(r;\,E)\,\chi_E^{(+)}(r)\,dr.\end{aligned} \quad (41)$$

If, using a representation of the Green's function $G_0^{(+)}$ of the form (64), we go over in Eq. (40) to the limit $r\to\infty$, then, using (41), we find[9]

$$\overline{S}_{nn}=\exp(2\mathrm{i}\delta);\quad \delta=\xi+\mathrm{i}\eta, \quad (42)$$

where ξ and η are the real and imaginary parts of the phase shift of nucleon scattering on the optical potential.

Comparing Eqs. (42) and (9), we see that the nonresonant part of the S matrix and the neutron strength function are determined, respectively, by the real and imaginary parts of the optical scattering phase shift:

$$S_{nn}^{bg}=\exp(2\mathrm{i}\xi);\quad \eta=\pi S_n/2. \quad (43)$$

In accordance with Eqs. (39) and (40), the mean amplitude $\overline{M}_{\gamma n}$ of the γn reaction due to he valence transition can be calculated by means of the optical model. In the case of an $E1$ transition,

$$\overline{M}_{\gamma n}(E)=(n_b K_{ab}/2\pi)^{1/2}\int \chi_E^{(+)} r\,\chi_b\,dr, \quad (44)$$

where K_{ab} is a kinematic factor whose explicit expression will be given below; $r^{-1}\chi_b(r)$ is the radial wave function corresponding to the bound state of the valence neutron: $\chi_b\equiv\chi_{0Eb}$; $E=E_b+E_\gamma$, E_γ is the energy of the γ ray; the index a characterizes the angular momentum and parity of the nucleon in the continuum (for brevity, this index is omitted from the continuum wave function); n_b is a spectroscopic factor. Thus, the optical cross section of the γn reaction

$$\sigma_{\gamma n}^{\text{opt}}/g\pi\lambda_\gamma^2=|\overline{S}_{\gamma n}|^2=2\pi n_b K_{ab}\left|\int \chi_E^{(+)} r\chi_b\,dr\right|^2 \quad (45)$$

can be expressed in terms of the shell model and optical model.

In the case considered here when the deuteron channel is the main channel for decay of the resonances of the compound nucleus, the cross section of the γn reaction is equal to the photoabsorption cross section σ_c. The part of the mean cross section of dipole photoabsorption due to the single-particle transition can be expressed as follows in terms of the shell and optical models.[10] The cross section σ_{cE1} is proportional to the imaginary part of the dipole polarizability of the nucleus (see, for example, the monograph Ref. 2): $\sigma_{cE1}\sim \mathrm{Im}\,\mathcal{P}(E_\gamma)$. The dipole polarizability due to transition of the valence neutron from the bound state to the continuum is, to within a geometrical factor,

$$\mathcal{P}(E_\gamma)\sim\sum_{E'} Z_{E'E_b}(E_\gamma)\left(\int \chi_{0E'}(r)\,r\chi_b(r)\,dr\right)^2. \quad (46)$$

Here, $Z_{E'E_b}$ is the so-called response function, which is the change in the density matrix induced by unit field:

$$Z_{E'E_b}(E_\gamma)=\int \exp(-\mathrm{i}\varepsilon\tau)\,G_{E'}^F(\varepsilon)\,G_{0E_b}^F(\varepsilon-E_\gamma)\,d\varepsilon/2\pi\mathrm{i}. \quad (47)$$

In this expression, we have used the "free" Green's function (34) for the states of the valence neutron near the Fermi limit.

Further we use Lehmann's expansion for the Green's function (36) (see, for example, Ref. 2):

$$G_E^F(\varepsilon)=\sum_c \frac{b_c(E)}{\varepsilon-\mu-E_c+i\delta}+\sum_{c'}\frac{\overline{b}_{c'}(E)}{\varepsilon-\mu+E_{c'}-i\delta}\equiv G_E^{(+)}(\varepsilon)+G_E^{(-)}(\varepsilon). \quad (48)$$

The calculation of the response function (47) in accordance with (35) and (48) leads to the expression

$$Z_{E'E_b}(E_\gamma)=n_b G_{E'}^F(E=E_b+E_\gamma)-G_{E'}^{(-)}(E=E_b+E_\gamma). \quad (49)$$

In accordance with the expansion (48), the function $G_E^{(-)}(E=E_b+E_\gamma)$ does not have poles at positive values of $E_b+E_\gamma-\mu$ therefore $\mathrm{Im}\,G_E^{(-)}(E=E_b+E_\gamma)=0$. Thus, in accordance with (36) and (49),

$$\mathrm{Im}\,\mathscr{P}(E_\gamma)\sim n_b\,\mathrm{Im}\int \chi_b(r)\,rG_a^F(r,r';E=E_b+E_\gamma)\,r'\chi_b(r')\,dr\,dr'. \quad (50)$$

After the substitution $E\to E+iI$ in this expression and with allowance for (37), we obtain the following expression for the mean cross section of dipole photoabsorption due to the valence transition:

$$\overline{\sigma}_{cE1}/g\pi\lambda_\gamma^2=\overline{|S_{\gamma n}|^2}=n_bK_{ab}\left\{-2\,\mathrm{Im}\int \chi_b rG_a^{(+)}(r,r';E)\,r'\chi_b\,dr\,dr'\right\}. \quad (51)$$

The kinematic factor is here the same as in Eq. (45) since in the absence of coupling of the single-particle states to complex configurations ($\Delta h\to 0$) the optical cross section of the γn reaction is equal to the photoabsorption cross section. Equations (14), (43), (45), and (51) enable one to express the single-particle $E1$-radiative strength function for the partial transition in terms of the shell and optical models[11]:

$$S_\gamma/S_n=n_bK_{ab}\,[1-\exp(-4\eta)]^{-1}\times\left\{-2\,\mathrm{Im}\int \chi_b rG_a^{(+)}(r,r';E)\,r'\chi_b\,dr\,dr'-2\pi\left|\int \chi_E^{(+)}r\chi_b\,dr\right|^2\right\}. \quad (52)$$

We now consider the determination of $S_{\gamma n}^{bg}$ and $\cos\phi$ in the expression (26) for the S matrix $S_{\gamma n}$ by means of the values found for $\overline{S}_{\gamma n}$ and $\overline{|S_{\gamma n}|^2}$. As the missing connection between these quantities, we use Eq. (22), which was obtained by regarding the quasidiscrete level as a doorway state. Note that Eq. (22) does not explicitly contain the quantum numbers of the doorway state and is expressed solely in terms of observable quantities. We may therefore hope that the region of applicability of this equation is not restricted to the definite assumptions made for its derivation. Having this remark in mind, using the expression (52) found for S_γ, and also Eqs. (13) and (51) [or (12) and (45)], we find[11]

$$\sigma_{\gamma n}^{bg}/g\pi\lambda_\gamma^2=|S_{\gamma n}^{bg}|^2=n_bK_{ab}\exp(2\eta)\times\left\{2\pi(1+\tanh\eta)\left|\int \chi_E^{(+)}r\chi_b\,dr\right|^2+2\tanh\eta\,\mathrm{Im}\int \chi_b rG_a^{(+)}(r,r';E)\,r'\chi_b\,dr\,dr'\right\}. \quad (53)$$

Equations (43), (52), and (53) in conjunction with (22) solve the problem of expressing the parameters of the partial cross sections of photoneutron reactions near threshold due to an $E1$-valence transition in terms of the shell and optical models. In the subthreshold region, when $\sigma_{\gamma n}^{opt}=\sigma_{\gamma n}^{bg}=0$, the value of $2\pi S_\gamma$ is determined by Eq. (51), in which the Green's function $G_a^{(+)}$ corresponds to negative energies E.

Let us consider further the derivation of approximate expressions for S_γ and $\sigma_{\gamma n}^{bg}$ in the case when the absorption intensity is fairly strong and these formulas can be obtained using only the mean amplitude (44) of the γn reaction and without recourse to the fluctuation cross sections. In accordance with the approximate expressions (31) and (27), we have in the limiting case $\Gamma{\downarrow}\gg\Gamma{\uparrow}$ the estimates $|S_{\gamma n}^{bg}|\sim(\Gamma_\gamma\Gamma{\uparrow})^{1/2}/\Gamma{\downarrow}$; $\cos\phi\sim\eta\sim\Gamma{\uparrow}\Gamma{\downarrow}$. Therefore, in this limiting case and to terms of order $\Gamma{\uparrow}\Gamma{\downarrow}$ inclusively, the expression for the mean nondiagonal element of the S matrix (4) has the form

$$\overline{S}_{\gamma n}\approx\exp(i\psi)\{|S_{\gamma n}^{bg}|-i\pi(S_nS_\gamma)^{1/2}\}. \quad (4')$$

Comparing further the expressions (1) and (2) for $S_{nn}(E)$ and $S_{\gamma n}(E)$ in an energy interval near one of the neutron resonances and taking into account the estimates made above, we conclude that $\exp(i\xi)=i\exp(i\Psi)$. Using this relation, we find on the basis of (4′) and (44) the following approximate expressions for S_γ and $\sigma_{\gamma n}^{bg}$:

$$S_\gamma/S_n=\pi n_bK_{ab}\left(-\eta^{-1}\,\mathrm{Im}\exp(-i\xi)\int \chi_E^{(+)}r\chi_b\,dr\right)^2\Big/2; \quad (52')$$

$$\sigma_{\gamma n}^{bg}/g\pi\lambda_\gamma^2=2\pi n_bK_{ab}\left(\mathrm{Re}\exp(-i\xi)\int \chi_E^{(+)}r\chi_b\,dr\right)^2. \quad (53')$$

In connection with these approximate expressions, it is appropriate to consider the expression for S_n, S_γ, $S_{\gamma n}^{bg}$ obtained in Ref. 12 by a transition to the optical model in the expression for the mean K matrix using only the mean amplitudes of the nn and γn reactions. In the notation adopted in the present paper, these equations have the form

$$\pi S_n^K/2=(\mathrm{Im}\tan\delta)\,[1+(\mathrm{Re}\tan\delta)^2]^{-1}; \quad (54)$$

$$S_\gamma^K/S_n^K=(S_\gamma'/S_n')\left\{1+\left[(S_n'/S_\gamma')^{1/2}(\pi n_bK_{ab}/2)^{1/2}\times\mathrm{Re}\left(\exp(-i\delta)\cos^{-1}\delta\int \chi_E^{(+)}r\chi_b\,dr\right)-\mathrm{Re}\,\mathrm{tg}\,\delta\right]^2\right\};$$

$$(S_\gamma'/S_n')^{1/2}=-(\pi n_bK_{ab}/2)^{1/2}\,\mathrm{Im}\exp(-i\delta)\cos^{-1}\delta\times\int \chi_E^{(+)}r\chi_b\,dr\,(\mathrm{Im}\tan\delta)^{-1}; \quad (55)$$

$$\sigma_{\gamma n}^{bg}/g\pi\lambda_\gamma^2=2\pi n_bK_{ab}\left[\mathrm{Re}\exp(-i\delta)\cos^{-1}\delta\int \chi_E^{(+)}r\chi_b\,dr\right]^2[1+(\mathrm{Re}\tan\delta)]^{-1}. \quad (56)$$

Qualitative and quantitative analysis of the relations obtained on the basis of the shell approach and also by the K-matrix method will be made in the following section.[13]

4. ANALYSIS OF THE EXPRESSIONS DESCRIBING THE VALENCE MECHANISM OF THE γn REACTION

When the imaginary part of the optical potential is small ($w\ll D$), the approximate expressions for the functions $\exp(2i\delta)$, $\chi_E^{(+)}(r)$, $G^{(+)}(r,r';E)$ can be obtained from Eqs. (17) and (18) by the substitution $E_a\to E_a-iw$, where $w=\int_0^R w(r)\,\chi_a^2(r)\,dr$ in the case of volume absorption. The expressions found in this way (for future reference we shall label them (57) and (58), respectively) enable us to obtain approximate expressions for S_n (43), S_γ/S_n (52), (52′), $\sigma_{\gamma n}^{bg}$ (53), (53′); and $\sigma_{\gamma n}^{opt}$ (45) in the neighborhood of the shape resonance. Direct calculation leads, as one would expect, to the already known expressions (26), (29), (31), and (25), respectively, in which

$$\Gamma^{\downarrow}=2w; \quad \Gamma_{\gamma}=n_b K_{ab}\left(\int_0^R \chi_a r \chi_b \, dr\right)^2. \tag{59}$$

Using Eq. (59) for the single-particle $E1$-radiative width Γ_γ, one can find an explicit expression for the kinematic factor (see, for example, the monograph Ref. 14):

$$K_{ab}=\bar{e}^2 k_\gamma^3 \mathscr{K}_{ab}, \tag{60}$$

where $\bar{e}=-Ze/A$ is the kinematic effective charge of the neutron for $E1$ transitions; k_γ is the wave vector of the γ ray; $\mathscr{K}_{ab}$ is a geometric factor. In the cases of neutron capture by an even–even spherical nucleus,

$$\mathscr{K}_{ab}=\frac{4}{3}(2l_a+1)(2j_b+1) \times (l_a 100 \mid l_b 0)^2 W^2\left(l_a j_a, \ l_b j_b; \ \tfrac{1}{2}\ 1\right). \tag{61}$$

We turn to the qualitative analysis of Eqs. (54)–(56). The approximate expression for the neutron strength function S_n^K has in accordance with (54) and (57) the form ($\Delta_a \equiv E - E_a$)

$$2\pi S_n^K = \frac{2w\Gamma^{\uparrow}\cos^{-2}\delta_0\,[(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)^2+w^2]}{[(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)^2+w^2]^2+[(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)(\Delta_a\tan\delta_0-\Gamma^{\uparrow}/2)+w^2\tan^2\delta_0]^2}. \tag{62}$$

The realistic values of the phase shift δ_0 are small. Therefore, when the neutron strength functions are small, Eqs. (27) and (62) lead to practically the same results except in the immediate neighborhood of the shape resonance: $(\Delta_a+\Gamma^{\uparrow}\tan\delta_0)^2 \sim w^2 < (\Gamma^{\uparrow}/2)^2$, where the ratio S_n^K/S_n may differ appreciably from unity. Such a difference can be expected in the case of s shape resonances, for which the values of $\Gamma^{\uparrow}$ and δ_0 are maximal. The approximate expression for the strength-function ratio S_γ^K/S_n^K is, in accordance with (55) and (57), equal to the ratio S_γ/S_n (29). Therefore, the ratio S_γ^K/S_γ may differ appreciably from unity in such a neighborhood of the shape resonance. The approximate expression (56) for the background cross section $(\sigma_{\gamma n}^{bg})^K$ has the form

$$\frac{(\sigma_{\gamma n}^{bg})^K}{g\pi\lambda\!\!\!^{-}{}_\gamma^2} = \frac{\Gamma_\gamma\Gamma^{\uparrow}\cos^{-2}\delta_0\,(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)^2}{[(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)^2+w^2]^2+[(\Delta_a^2+w^2-(\Gamma^{\uparrow})^2/4)\tan\delta_0-\Delta_a\Gamma^{\uparrow}(1-\tan^2\delta_0)/2]^2}. \tag{63}$$

In accordance with (31) and (63), the ratio $(\sigma_{\gamma n}^{bg})^K/\sigma_{\gamma n}^{bg}$ may be appreciably different from unity in the following neighborhood of the shape resonance $(\Delta_a+\Gamma^{\uparrow}\tan\delta_0/2)^2 < w^2$,

We now turn to a quantitative analysis of the expressions for the single-particle quantities. These are determined by the parameters of the shell and optical models for neutrons, and also by the spectroscopic factor n_b for a neutron in a bound state. The wave functions $\chi_b(r)$ needed to calculate the single-particle quantities were found by numerical integration of the Schrödinger equation with the Hamiltonian $h_0(r)$ of the shell model and the boundary conditions ($\hbar=1$)

$$\chi_b(r) \underset{r\to 0}{\sim} r^{l_b+1}; \quad \chi_b(r) \underset{r\to\infty}{\sim} \exp[-(2m\,|E_b|)^{1/2} r],$$

where m is the nucleon mass. The continuum wave functions $\chi_E^{(+)}(r)=\exp(i\delta)(2mk_n/\pi)^{1/2}\,u_E(r)$ and $v_E(r)$ were found by numerical integration of the radial Schrödinger equation with the Hamiltonian $h=h_0(r) - iw(r)$ of the optical model and the boundary conditions

$$u_E(r) \underset{r\to 0}{\sim} r^{l_a+1}; \quad u_E(r) \underset{r\to\infty}{\to} k_n^{-1}\sin(k_n r - \pi l_a/2+\delta_a);$$
$$v_E(r) \underset{r\to 0}{\sim} r^{-l_a}; \quad v_E(r) \underset{r\to\infty}{\to} \exp[i(k_n r-\pi l_a/2+\delta_a)].$$

The expression for the optical-model Green's function $G_a^{(+)}(r, r'; E)$ is a direct generalization of the expression for the Green's function $G_{0a}^{(+)}(r, r'; E)$ (Ref. 5):

$$G^{(+)}(r, r'; E)=2mu_E(r_>)v_E(r_<), \quad r_>=\max\{r, r'\}; \quad r_<=\min\{r, r'\}. \tag{64}$$

The potentials of the shell and optical models for neutrons are chosen in the standard form:

$$\left.\begin{array}{l} U(r)=-U_0[1-\alpha(N-Z)A^{-1}]f(r, R, a) \\ \qquad +U_{SO}(\sigma l)\Lambda^2 r^{-1}\,df/dr; \\ f(r, R, a)=[1+\exp(r-R)a^{-1}]^{-1}; \quad R=r_0 A^{1/3}; \\ w_s(r)=-4a_w w_s\,df(r, R, a_w)/dr; \quad w_v(r)=w_v f(r, R, a_w). \end{array}\right\} \tag{65}$$

The parameters of the shell potential are chosen in such a way as to reproduce the experimental binding energies of the last neutron in a wide range of atomic masses[15]:

$$U_0=52\ \text{MeV}; \ r_0=1.245\ \text{F}; \ a=0.63\ \text{F}; \ \alpha=0.53;$$
$$U_{SO}=7.5\ \text{MeV}; \ \Lambda=1.41\ \text{F}; \ a_w=0.70\ \text{F}. \tag{65'}$$

The values of the absorption intensities $w_{s,v}$ were varied. Numerical calculations of S_n, S_γ, $\sigma_{\gamma n}^{bg}$ were made for the nuclei ^{53}Cr ($s_{1/2}\to p_{3/2}$), ^{57}Fe($s_{1/2}\to p_{1/2}$)—shape resonance for s wave; ^{91}Zr ($p_{3/2}\to d_{5/2}$), ^{115}Sn ($p_{1/2}\to s_{1/2}$)—shape resonance for $p_{3/2}$ and $p_{1/2}$ waves. The results of the calculations of the single-particle quantities for these nuclei as a function of the intensity w_s of surface absorption are given in Figs. 1–7. Use of volume absorption of the form (65) leads to virtually the same results, and therefore the corresponding dependences on w_v are not given. Everywhere in the calculations, unless otherwise stated, we have taken $n_b=1$, $E=100$ keV. This range of energies is investigated experimentally (Sec. 6).

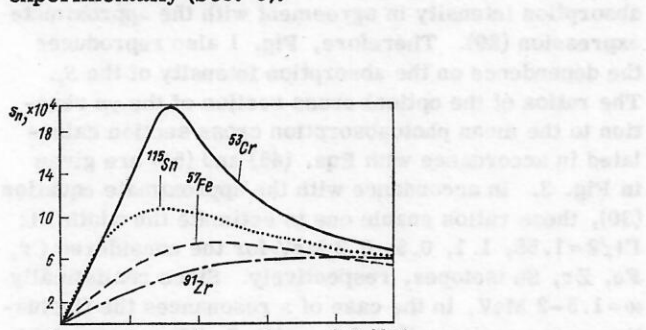

FIG. 1. Dependence of reduced neutron strength functions on the absorption intensity.

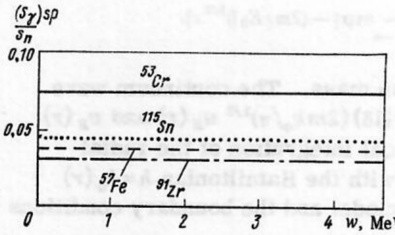

FIG. 2. The ratio $(S_\gamma)_{sp}/S_n$ as function of the absorption intensity.

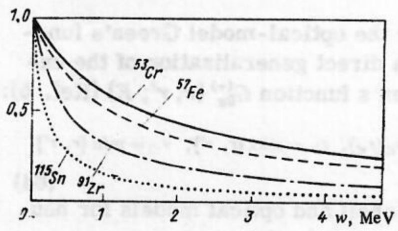

FIG. 3. The cross section ratio $(\sigma^{opt}_{\gamma n})_{sp}/(\bar{\sigma}_{\gamma n})_{sp}$ as function of the absorption intensity.

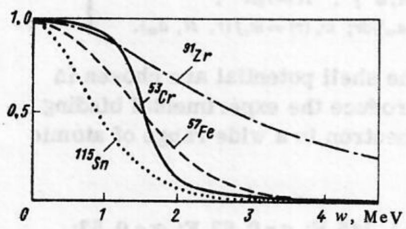

FIG. 4. The ratio $(\sigma^{bg}_{\gamma n})_{sp}/(\sigma^{bg}_{\gamma n}(w=0))_{sp}$ as function of the absorption intensity.

In Fig. 1 we give the dependences of the s and p neutron strength functions (reduced to 1 eV) calculated in accordance with Eqs. (42) and (43):

$$s^0_{n1/2} = (2/\pi)\eta^0_{1/2}(1\,\mathrm{eV}/E)^{1/2};$$
$$s^1_{n1/2,\,3/2} = (2/\pi)\eta^1_{1/2,\,3/2}\left[1+(k_nR)^{-2}\right](1\,\mathrm{eV}/E)^{1/2}. \qquad (66)$$

The nonmonotonic dependence of $s_n(w)$ is due to the shape resonance in agreement with the approximate expression (26). The strength function ratios S_γ/s_n calculated in accordance with Eqs. (42) and (52) are given in Fig. 2. These ratios are almost independent of the absorption intensity in agreement with the approximate expression (29). Therefore, Fig. 1 also reproduces the dependence on the absorption intensity of the S_γ. The ratios of the optical cross section of the γn reaction to the mean photoabsorption cross section calculated in accordance with Eqs. (45) and (51) are given in Fig. 3. In accordance with the approximate equation (30), these ratios enable one to estimate the width $\Gamma\uparrow$: $\Gamma\uparrow/2 = 1.55$, 1.1, 0.5, 0.2 MeV for the considered Cr, Fe, Zr, Sn isotopes, respectively. Since realistically $w = 1.5$–2 MeV, in the case of s resonances the fluctuation cross sections that determine S_γ differ appreciably from the corresponding mean cross sections. The values found for the width $\Gamma\uparrow$ make it possible, by comparing the calculated values of S_n (see Fig. 1) with the

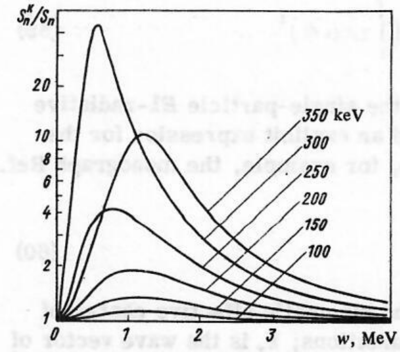

FIG. 5. The ratio S^K_n/S_n as a function of the absorption intensity at neutron energies $E = 350$–100 keV (at intervals of 50 ieV) for the nucleus ^{57}Fe.

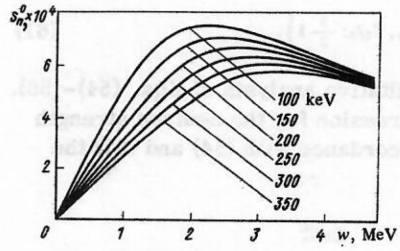

FIG. 6. Dependence of the reduced neutron strength function s^0_n on the absorption intensity at different energies E of the neutron for the nucleus ^{57}Fe.

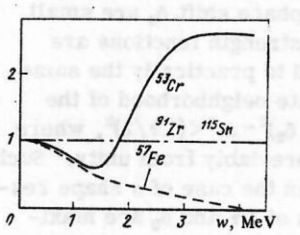

FIG. 7. The cross section ratio $(\sigma^{bg}_{\gamma n})^K_{sp}/(\sigma^{bg}_{\gamma n})_{sp}$ as a function of the absorption intensity.

approximate equation (27), to estimate $|E - E_a| \equiv |\Delta_a|$: $|\Delta_a| = 0.42$, 1.5, 4.5, 1.35 MeV for the given nuclei, respectively. Thus, the nuclei ^{53}Cr and ^{57}Fe (for the chosen set of parameters of the shell potential) correspond to the direct neighborhood of the s shape resonance since $|\Delta_a| \gtrsim \Gamma\uparrow/2$ for these nuclei. The cross-section ratios $\sigma^{bg}_{\gamma n}(w)/\sigma_{\gamma n}(0)$ calculated in accordance with (53) are shown in Fig. 4. The decrease of this ratio with increasing absorption agrees with the approximation equation (31). In accordance with (22) and the results of the calculations of the ratios S_γ/S_n (see Fig. 2), the curves in Fig. 4 also describe the dependence on the absorption of the ratios $\cos^2\phi(w)/\cos^2\phi(0)$, with $\cos^2\phi(0) = 0.96$, 0.33, 0.012, 0.021 for the given nuclei, respectively. The approximate expression $\cos^2\phi(0)0 = (\Gamma\uparrow/2)^2[\Delta^2_a + (\Gamma\uparrow/2)^2]^{-1}$, which follows from (22), (29), and (31), agrees with the values found for $\cos^2\phi(0)$, $|\Delta_a|$, $\Gamma\uparrow/2$.

We now turn to a comparative analysis of the results of calculating S^K_n, S^K_γ, $(\sigma^{bg}_{\gamma n})^K$. In accordance with the approximate expressions (26) and (62), the ratios of the neutron strength functions S^K_n/S_n calculated in accordance with (42), (43), and (54) hardly differ from unity except in the immediate neighborhood of the shape resonance. Thus, for the nucleus ^{57}Fe the ratio depends strongly on the neutron energy in the range $E = 100-350$ keV and reaches $\sim 10^2$ (Fig. 5). At the same time, s_n^0 changes by not more than a factor two in the same energy range (Fig. 6). The calculations made in accordance with Eq. (55) show that the strength function ratios S_γ^K/S_n^K are hardly dependent on the absorption,[16] like the ratio S_γ/S_n (see Fig. 2). Therefore, the ratios S_γ^K/S_γ of the partial radiative strength functions are virtually equal to the ratios of the corresponding neutron strength functions S_n^K/S_n discussed above. The ratios of the background cross sections $(\sigma_{\gamma n}^{bg})^K/\sigma_{\gamma n}^{bg}$ calculated in accordance with (53) and (56) are given in Fig. 7. In accordance with the approximate equations (31) and (63), these ratios (like the ratios $\cos^2\phi^K/\cos^2\phi$) differ appreciably from unity for nuclei corresponding to the direct neighborhood of a shape resonance. For the considered p shape resonances, the realistic values of w exceed $\Gamma\uparrow/2$. Therefore, calculation of S_γ and $\sigma_{\gamma n}^{bg}$ in accordance with Eqs. (52) and (52′), and (53) and (53′), respectively, leads to similar results, like the calculation in accordance with (52) and (55), and (53) and (56).

5. DAMPING OF GIANT MULTIPOLE RESONANCES

Hitherto, we have not taken into account the influence of the giant dipole resonance on the $E1$-radiative partial strength functions, nor on the cross section of the γn reaction. In order to take into account this influence, it is necessary to consider the problem of damping of giant multipole resonances; this is of independent interest also. The microscopic origin of the giant multipole resonances is fairly well known. Basically, they correspond to isoscalar or isovector excitations of particle–hole type. The traditional approach to the study of the giant multipole resonances is based on diagonalization of the shell-model Hamiltonian on the basis of particle–hole configurations in the random-phase approximation (Ref. 17 is one of the latest reviews of this subject). In the framework of such an approach, one can obtain the mean energy of the resonance and various sum rules but, as is well known, one cannot obtain the width of the giant multipole resonances. Their width arises from the coupling of the collective states to both the continuum ($\Gamma\uparrow$) and complex configurations ($\Gamma\downarrow$). In the framework of the semimicroscopic approach adopted here, there is the possibility of quantitative interpretation of the width of the giant multipole resonances. So far, this possibility has been partly realized in Ref. 18, which gives an interpretation of the width $\Gamma\downarrow$ of the giant dipole resonance. Below, we give a corresponding derivation (in a form somewhat different from that of Ref. 18) in order to use the result in a systematic form in what follows.

The basic quantity in the theory of collective excitations is the change in the density matrix induced by a weak effective periodic field $V(\mathbf{r})$ (Ref. 2):

$$\left.\begin{aligned}\rho(\mathbf{r}) = \sum_{ik} \rho_{ik}(\omega)\,\varphi_i^*(\mathbf{r})\,\varphi_k(\mathbf{r}) \equiv \int V(\mathbf{r}')\,Z(\mathbf{r},\mathbf{r}';\omega)\,d\mathbf{r}';\\ \rho_{ik}(\omega) = Z_{ik}(\omega)\,V_{ki},\end{aligned}\right\} \qquad (67)$$

where ω is the field frequency; $\{\varphi_i(\mathbf{r})\}$ is the system of single-particle (shell) functions; $Z_{ik}(\omega)$ is the response function discussed above ($\hbar = 1$)

$$Z_{ik}(\omega) = \int G_i^F(\varepsilon)\,G_k^F(\varepsilon-\omega)\,d\varepsilon/2\pi i. \qquad (68)$$

Here $G^F(\mathbf{r},\mathbf{r}';\varepsilon) = \sum_i \varphi_i^*(\mathbf{r})\,\varphi_i(\mathbf{r}')\,G_i^F(\varepsilon)$ is the single-particle Green's function for the Fermi system defined with allowance for damping of the quasiparticles, and it satisfies an equation of the form (36). Calculation of the response function (68) by means of the expansion (48) leads to

$$Z_{ik} = \sum_{cc'} \frac{b_c^{(i)}\tilde{b}_{c'}^{(k)}}{\omega - E_c - E_{c'}} - \sum \frac{b_c^{(k)}\tilde{b}_{c'}^{(i)}}{\omega + E_c + E_{c'}} \equiv Z_{ik}^{(+)} + Z_{ik}^{(-)}. \qquad (69)$$

If there is no coupling between the single-particle (single-hole) states and complex configurations, when $G_i^F(\varepsilon) \to G_{0i}^F(\varepsilon)$, i.e., in accordance with (34) $b_c^{(i)} \to (1 - n_i)\,\delta(E_c - E_i + \mu)$, $b_{c'}^{(k)} \to n_k\,\delta(E_{c'} - \mu + E_k)$, the function (69) goes over into the well known expression

$$Z_{ik} \to Z_{ik}^{(0)} = (n_i - n_k)(E_i - E_k - \omega)^{-1}. \qquad (70)$$

Equations (67) and (70) and also the expansion (18) for the Green's function $G_0^{(+)}(\mathbf{r},\mathbf{r}';\varepsilon)$ of the form enable one in principle to determine the width $\Gamma\uparrow$ of the giant multipole resonance. Indeed, as the kernel of the integral relation (67) one can use an expression that takes into account exactly the contribution of the continuum[19]:

$$\begin{aligned}Z(\mathbf{r},\mathbf{r}';\omega) \to \sum_k n_k\varphi_k^*(\mathbf{r})\,\varphi_k(\mathbf{r}')\,G_0^{(+)}(\mathbf{r},\mathbf{r}';E = E_k + \omega)\\ + \sum_i n_i\varphi_i^*(\mathbf{r})\,\varphi_i(\mathbf{r}')\,G_0^{(+)}(\mathbf{r},\mathbf{r}';E = E_i - \omega).\end{aligned} \qquad (71)$$

The photoabsorption cross section can be determined by means of (71) by the standard methods.[2,17] Note that the problem of determining the width $\Gamma\uparrow$ has not yet been solved numerically. Another more important case $\Gamma\downarrow \gg \Gamma\uparrow$ was considered in Ref. 18. Restriction of the basis of shell states to discrete (and quasidiscrete) levels makes it possible to obtain approximately a closed expression for the response function (69). To this end, we consider the coefficients of the Lehmann expansion (48) for the functions $G_i^F(\varepsilon)$:

$$\left.\begin{aligned}b_c^{(i)} = |\langle c|\,a_i^+\,|0\rangle|^2;\quad \sum_c b_c^{(i)} = \langle 0|\,a_i a_i^+\,|0\rangle \approx 1 - n_i;\\ \tilde{b}_{c'}^{(k)} = |\langle c'|\,a_k\,|0\rangle|^2;\quad \sum_{c'} \tilde{b}_{c'}^{(k)} = \langle 0|\,a_k^+ a_k\,|0\rangle \approx n_k,\end{aligned}\right\} \qquad (72)$$

where a_i^+ and a_k are nucleon creation and annihilation operators. The approximate equations (72), which hold to accuracy $\tilde{E}_i = E_i - \mu \gg \Gamma\downarrow_i$, $\tilde{E}_k \equiv \mu - E_k \gg \Gamma\downarrow_k$, enable us to identify the coefficients b_c and $\tilde{b}_c$, with the probability of finding single-particle and single-hole states in the exact states $|c\rangle$ and $|c'\rangle$ of the system:

$$b_c^{(i)} = (1 - n_i)\,w_c^{(i)};\quad \tilde{b}_{c'}^{(k)} = n_k w_{c'}^{(k)}, \qquad (73)$$

and the functions $G_c^{(\pm)}$ in the expansion (48) with the Green's function of the corresponding diagonalization problem (33):

$$G_i^{(+)}(\varepsilon)\approx\sum_c\frac{(1-n_i)\,w_c^{(i)}}{\varepsilon-\mu-E_c}=\frac{1-n_i}{\varepsilon-\mu-\widetilde{E}_i-\sum\limits_{\lambda}(g_\lambda^{(i)})^2\,(\varepsilon-\mu-E_\lambda)^{-1}};$$

$$G_k^{(-)}(\varepsilon)\approx-\sum_{c'}\frac{n_k w_{c'}^{(k)}}{\mu-\varepsilon-E_{c'}}=-\frac{n_k}{\mu-\varepsilon-\widetilde{E}_k-\sum\limits_{\lambda'}(g_{\lambda'}^{(k)})^2\,(\mu-\varepsilon-E_{\lambda'})^{-1}}.$$

These relations enable us to obtain the following approximate equations for the functions $Z_{ik}^{(\pm)}(\omega)$:

$$Z_{ik}^{(+)}(\omega)\approx\sum_{c'}b_{c'}^{(k)}\frac{1-n_i}{\omega-E_{c'}-\widetilde{E}_i-\sum\limits_{\lambda}(g_\lambda^{(i)})^2\,(\omega-\widetilde{E}_k-E_\lambda)^{-1}}$$

$$=\frac{(1-n_i)\,n_k}{\omega-\widetilde{E}_i-\widetilde{E}_k-\sum\limits_{\lambda}(g_\lambda^{(i)})^2\,(\omega-\widetilde{E}_k-E_\lambda)^{-1}-\sum\limits_{\lambda'}(g_{\lambda'}^{(k)})^2\,[\omega-\widetilde{E}_i-E_{\lambda'}-\sum\limits_{\lambda}(g_\lambda^{i})^2\,(\omega-\widetilde{E}_k-E_\lambda)^{-1}]^{-1}}.\qquad(74)$$

The first of these equations has been obtained with allowance for the existence in accordance with (73) of a peak of the function $\widetilde{b}_{c'}^{(k)}$ at the energy $E_{c'}=\widetilde{E}_k$. In order to calculate the mean photoabsorption cross section (which is proportional to the imaginary part of the mean amplitude of γ-ray scattering through zero angle), it is necessary to make in Eq. (74) the subsititution $\omega\to\omega+\mathrm{i}I$ in accordance with (23). After this substitution, we obtain

$$Z_{ik}^{(+)}=\frac{(1-n_i)\,n_k}{\omega-\widetilde{E}_i-\widetilde{E}_k+\mathrm{i}\Gamma_i^{\downarrow}(\omega-\widetilde{E}_k)/2+\mathrm{i}\Gamma_k^{\downarrow}(\omega-\widetilde{E}_i)/2},\qquad(75)$$

where, as in Eqs. (24) and (59), $\Gamma\downarrow_{i,k}(E)=\int w(r;E)\,\chi_{i,k}^2(r)\,dr$ are the widths of the single-particle and single-hole states due to their coupling to complex configurations. (We ignore the "shift" of these states). In accordance with (74) and (75), the widths $\Gamma\downarrow_{i,k}(E)$ are, generally speaking, taken off the mass shell, i.e., at energy $E=\widetilde{E}_{i,k}+(\omega-\widetilde{E}_i-\widetilde{E}_k)$. In the approximation $\Gamma\downarrow_{i,k}\ll\widetilde{E}_{i,k}$, the function $Z_{ik}^{(-)}(\omega+\mathrm{i}I)$ does not, in accordance with (69), have an imaginary part, and this quantity can therefore be found with neglect of the coupling to complex configurations: $Z_{ik}^{(-)}\approx(Z_{ik}^{(-)})^{(0)}=-n_i(1-n_k)\,(\widetilde{E}_i+\widetilde{E}_k+\omega)^{-1}$. Without exceeding the accuracy, this expression can be modified in such a way that the sum $Z_{ik}(\omega+\mathrm{i}I)=Z_{ik}^{(+)}(\omega+\mathrm{i}I)+Z_{ik}^{(-)}(\omega+\mathrm{i}I)$ does not change under the substitution $i\to k$, $k\to i$, $\omega+\mathrm{i}I\to-\omega-\mathrm{i}I$, as must be so in accordance with (69). As a result, we obtain

$$Z_{ik}(\omega+\mathrm{i}I)=\frac{n_i-n_k}{E_i-E_k-\omega-\mathrm{i}\Gamma_i^{\downarrow}(\omega-\mu+E_k)/2-\mathrm{i}\Gamma_k^{\downarrow}(\omega-E_i+\mu)/2}\qquad(76)$$

(the damping of the particle and hole states is assumed to depend in the same way on the excitation energy). In Ref. 18, a satisfactory description of the cross section of dipole photoabsorption by the nucleus ^{208}Pb as a function of the γ-ray energy was obtained under the following assumptions: 1) the expression (76) is used as response function; 2) a schematic dipole–dipole effective interaction of the nucleus is used; 3) the dependence $w(r;E)$ is chosen with allowance for analysis of the elastic scattering of nucleons by means of the optical model. A further improvement in the description of the giant dipole resonance in the framework of the semimicroscopic approach consists of taking into account the continuum, using a realistic interaction of the nucleons (including velocity forces),[2,20] and taking into account the possible existence of an imaginary part of this interaction.[21]

6. INFLUENCE OF GIANT DIPOLE RESONANCE ON E1-PHOTONEUTRON REACTIONS NEAR THRESHOLD

In the absence of a quantitative formulation of the theory of dipole photoabsorption on the basis of the semimicroscopic approach, the influence of the giant dipole resonance on $E1$-photoneutron reactions near the threshold can be taken into account by means of a schematic description of the giant dipole resonance.[2,20] In the framework of this description, the resonance is regarded as a certain single-phonon state with energy E_g. In accordance with (76), this state acquires a width due to the damping of the quasiparticles forming the giant dipole resonance, and this width can be introduced into the expressions of the schematic theory by means of the substitution $\omega\to\omega+\mathrm{i}\Gamma_g(\omega)/2$. Then the dependence on the γ-ray energy E_γ of the mean cross section of dipole photoabsorption normalized to the sum rule with allowance for velocity forces has the form

$$\overline{\sigma}_{cE1}(E_\gamma)=\pi\frac{e^2\hbar}{mc}A\,(1+\varkappa)\frac{E_\gamma^2\,\Gamma_g(E_\gamma)}{(E_\gamma^2-E_g^2)^2+E_\gamma^2\,\Gamma_g^2(E_\gamma)},\qquad(77)$$

where A is the number of nucleons and $\varkappa$ is a constant which characterizes the contribution of the velocity forces to the dipole sum rule. Equation (77) can serve as a rough approximation of the dependence $\overline{\sigma}_{cE1}(E_\gamma)$ that follows from the consistent semimicroscopic approach to the description of the giant dipole resonance. This expression enables one to obtain a schematic description of the strength function $S_\gamma^{(2)}$. The existence of a mean partial $E1$-radiative width $\gamma_\gamma^{(2)}$ that is not correlated with the neutron width can be interpreted as the result of the wave function's containing compound-nucleus resonances of particle–hole 1^- configurations built on a state of the odd-neutron nucleus with angular momentum j_b, so that $\{j_b,1^-\}_{j_a}$, which is the so-called Brink hypothesis. In other words, the width $\gamma_\gamma^{(2)}$ exists to the extent that the giant dipole resonance is coupled to complex configurations and, therefore, it is proportional to the width $\Gamma_g(E_\gamma)$. Since the energy $E_\gamma\approx|E_b|$ of the γ rays is less than the energy E_g of the giant dipole resonance, $S_\gamma^{(2)}$ is determined by the low-energy "tail" of the giant dipole resonance. In accordance with (14), the strength function $S_\gamma^{(2)}$ is related as follows to the mean cross section of dipole photoabsorption:

$$S_\gamma^{(2)}=\frac{1}{3\pi^2}\,\frac{2\eta}{1-\exp(-2\eta)}\,k_\gamma^2\,\overline{\sigma}_{cE1}(E_\gamma).\qquad(78)$$

(It is assumed that the contribution of the valence transition to the integrated cross section of photoabsorption can be ignored). Combining (77) and (78), we obtain ($\eta\ll1$)

$$S_\gamma^{(2)}=\frac{1}{3\pi}\cdot\frac{1}{137mc^2}A\,(1+\varkappa)\frac{E_\gamma^4\,\Gamma_g(E_\gamma)}{(E_\gamma^2-E_g^2)^2+E_\gamma^2\,\Gamma_g^2(E_\gamma)}.\qquad(79)$$

This relation differs from the Axel–Bollinger formula[22] by allowance for the possible dependence $\Gamma_g(E_\gamma)$.

The influence of the giant dipole resonance on the valence transition consists of the appearance of the effective dynamical charge $e_{eff}(r; E_\gamma)$ due to the virtual excitation of the giant dipole resonance because of the effective interaction of the nucleons in the nucleus. The expression for the effective charge describing the difference of the effective field within and without the nucleus has in the framework of the schematic theory of the giant dipole resonance the form

$$C(E_\gamma) = (1+\varkappa)\frac{(E_g^0)^2 - (E_\gamma + i\Gamma_g(E_\gamma)/2)^2}{E_g^2 - (E_\gamma + i\Gamma_g(E_\gamma)/2)^2}; \quad E_g = E_g^0[(1+\varkappa)(1+f)]^{1/2}, \tag{80}$$

where $f \approx 1.4$ is the dimensionless intensity of the isovector part of the effective nucleon interaction; $\varkappa_{exp} \lesssim 0.4$ (we ignore the difference between the effective nucleon mass and the vacuum value); $E_g^0 \approx 44.5 A^{-1/3}$ MeV is the energy of the giant dipole resonance in the single-particle shell model, so that $E_g \approx 81 A^{-1/3}$ MeV. For γ-ray energies $E_\gamma \approx |E_b|$, the value of $|C|^2$ does not exceed 0.1 (see Table I). This is in qualitative contradiction with the analysis of the experimental partial $E1$-radiative strength functions $S_\gamma^{(1)}$ on the basis of the valence mechanism (see, for example, Ref. 16, and also the calculation made below), from which there follows the assertion that the effective charge is near unity. A possible interpretation of the appreciable weakening of the coupling between the valence transition and the giant dipole resonance consists of using instead of (80) a modified expression that takes into account explicitly the existence of polarization effects only within the nucleus:

$$e_{eff}(r; E_\gamma) = 1 + [C(E_\gamma) - 1] f(r, R, a). \tag{81}$$

The second term in (81) must be proportional to the transition density associated with the excitation of the giant dipole resonance. Since the transition density is determined by a large number of nucleons (of order $A^{2/3}$), it can be taken approximately proportional to the nuclear density. Such a modification of the effective charge (without allowance for velocity forces) is used to analyze the radiative capture of nucleons in the region of a giant dipole resonance.[23]

With allowance for the virtual excitation of the giant dipole resonance, the valence part of the partial $E1$-radiative strength function is modified: $(S_\gamma)_{sp} \to S_\gamma^{(1)}$ where $S_\gamma^{(1)}$ is determined by (52) in which the substitution $r \to r e_{eff}(r; E_\gamma)$, $rr' \to rr' e_{eff}(r; E_\gamma) e_{eff}^*(r'; E_\gamma)$ is made.

The results of the calculations of the ratio $S_\gamma^{(1)}/(S_\gamma)_{sp} \equiv e_{eff}^2$ given in Table I on the basis of (52) and (81) (the choice of the parameters is clarified in Sec. 4) show that e_{eff}^2 is much nearer to unity than $|C|^2$ (Ref. 24). This change can be explained by the fact that the dipole matrix elements are determined by the region near the boundary of the nucleus, where the modification of the second term in (81) is important. As calculations show, e_{eff}^2 is almost independent of the absorption intensity. One can modify similarly the background cross section of the γn reaction (53): $(\sigma_{\gamma n}^{bg})_{sp} \to \sigma_{\gamma n}^{bg}$. As calculations show, the values of the ratio $\sigma_{\gamma n}^{bg}/(\sigma_{\gamma n}^{bg})_{sp} \equiv e_{eff}^2$ differ little from the values given in Table I.

TABLE I. Results of calculations of factorized and unfactorized effective charges ($\Gamma_g(E_\gamma) = 2$ MeV).

Nucleus	^{53}Cr	^{57}Fe	^{91}Zr	^{115}Sn
$\lvert C\rvert^2$	0,08	0,08	0,06	0,04
e_{eff}^2	0.88	0.86	0.92	0.59

An experimental determination of the valence part of the partial radiative strength function $S_\gamma^{(1)}$ is possible in the case when one has found the values of the corresponding total strength function $S_\gamma = S_\gamma^{(1)} + S_\gamma^{(2)}$ and the coefficient of correlation of the partial radiative width and the neutron width: $C(\gamma_\gamma, \gamma_n)$. For spherical nuclei, $S_\gamma^{(1)}$ has been determined only in a few cases. The set of parameters of the shell and optical potentials needed to calculate $S_\gamma^{(1)}$ must enable one, not only to reproduce the binding energy of the valence neutron, but also to describe the elastic scattering of neutrons and the neutron strength functions. Such a procedure for choosing the parameters is in particular necessary for nuclei in the immediate neighborhood of a shape resonance: $|E - E_a| \gtrsim \max\{\frac{1}{2}\Gamma\uparrow, w\}$ for which the values of $\bar{\sigma}_{nn}$ and S_n, and, therefore, $S_\gamma^{(1)}$ are sensitive to the real and imaginary parts of the optical potential. Such nuclei include ^{53}Cr and ^{57}Fe (see Sec. 4), which correspond to a $3s$ shape resonance.

As an example of the application of the relations we have obtained to analysis of experimental data, let us consider the valence $E1$ transition $p_{3/2} \to d_{5/2}$ in the nucleus ^{91}Zr. This nucleus does not belong to the direct neighborhood of the $p_{3/2}$ shape resonance, and, therefore, in this case we can confidently use the mean parameters of the shell potential (65′). In addition, the experimental determination $S_\gamma = (2.8 \pm 0.6)\cdot 10^{-5}$ and $C(\gamma_\gamma, \gamma_n) = 0.59$ for this transition[25] can be regarded as one of the currently best. To reproduce for this nucleus the experimental value of the p-neutron strength function $s_n^1 = (2/3)s_{n3/2}^1 + s_{n1/2}^1/3 = 3.3\cdot 10^{-4}$ (Ref. 25) by means of (66), it is necessary to take the absorption intensity equal to $w_s = 1.5$ MeV. Calculation in accordance with Eqs. (43), (52), (60), and (61) leads for this w_s to the value $S_\gamma^{(1)} = 1.35\cdot 10^{-5}$ (e_{eff}^2 is chosen in accordance with Table I, and for the spectroscopic factor we take $n_b = 1$). The calculated value of $S_\gamma^{(1)}$ is in satisfactory agreement with the experimental value: $(S_\gamma^{(1)})_{exp} = S_\gamma C(\gamma_\gamma, \gamma_n) = 1.65\cdot 10^{-5}$. If the experimental value of the strength function $(S_\gamma^{(2)})_{exp} = S_\gamma(1 - C(\gamma_\gamma, \gamma_n)) = 1.15\cdot 10^{-5}$ is matched with the schematic expression (79), it is necessary to choose $\Gamma_g(E_\gamma) \approx \Gamma_g(E_g)$ [$\Gamma_g(E_g) = 4.4$ MeV]. This value of $\Gamma_g(E_\gamma)$ does not contradict the inequality $(E_\gamma/E_g)^2\,\Gamma_g(E_g) < \Gamma_g(E_\gamma) < \Gamma_g(E_g)$, which is obtained by applying the relation (76) to the schematic description of the giant dipole resonance with the dependence $\Gamma\downarrow(\tilde{E}) \sim \tilde{E}^2$ chosen in the same way as in an infinite Fermi system.[2]

CONCLUSIONS

The analysis of photoneutron reactions near threshold made here shows that the semimicroscopic approach is a constructive method of investigating highly excited states of nuclei and the corresponding resonance nuclear reactions. In this connection, let us consider some questions that require further study on the basis of this approach.

1. The use of the local optical potential (37), which depends smoothly on the excitation energy, is based on the assumption that the two-particle–one-hole configurations (it is the extent of coupling to these that determines the damping of the single-particle state) are sufficiently thermalized, i.e., the widths for the decay of these configurations to more complex configurations exceeds the mean energy interval between them. When the energy interval in which we are interested contains isolated doorway states, there arises an effective nonlocal optical potential: $\Delta h \to \Delta\tilde{h}$. The difference between $\Delta\tilde{h}$ and Δh in this case may be described in terms of the shell model and an "unrenormalized" optical model. As an example, let us consider the s-neuneutron strength functions, the dependence of whose experimental values on the atomic mass is reproduced by the optical model with universal imaginary part by means of Eq. (43) only qualitatively (see, for example, the monograph Ref. 1). In this case, the isolated doorway states are the following: 1) a bound neutron in a d state + a low-lying 2^+ phonon; 2) a bound neutron in an s state + a giant isoscalar 0^+ resonance. Allowance for the first doorway state leads to the following modification of the imaginary part of the optical potential:

$$\Delta h(r) = -\mathrm{i}w(r)\,\delta(r-r') \to \Delta\tilde{h}$$

$$= \Delta h + [g(r)\,\chi_d(r)\,\chi_d(r')\,g(r')]/(E - E_d + \mathrm{i}w - E_{2+}), \qquad (82)$$

where $g(r)$ is the effective field associated with the excitation of the 2^+ state; $r^{-1}\chi_d(r)$ is the radial wave function of the d state. (Equation (58) and the approximation $E_{2+} \ll |E_b|$ have been used). Since the shape resonances for s and d neutrons are close, the modification of the imaginary part of the optical potential is important near these resonances when $\Delta \equiv |E - E_d - E_{2+}| \gtrsim w$. In the case of volume absorption, the effective imaginary part that determines the s-neutron strength functions in accordance with (82) is

$$\tilde{w} = w\,[1 + g^2/(\Delta^2 + w^2)]; \quad g = \int \chi_s(r)\,g(r)\,\chi_d(r)\,dr. \qquad (83)$$

We can estimate $g(r)$ on the basis of the data on inelastic scattering of nucleons with excitation of a 2^+ state. The modification of the imaginary part of the optical potential of the form (82)–(83) makes it possible to reconcile on the average the values of the maximum and minimum of the s-neutron strength function in spherical nuclei,[26] since g is comparable with the "unrenormalized" imaginary part $w \approx 2$ MeV. Similarly, one can modify the imaginary part of the optical potential by taking into account the coupling of the scattered s neutron to the giant isoscalar 0^+ resonances;

$$-\mathrm{i}w\delta(r-r') \to -\mathrm{i}w\delta(r-r') + \frac{G(r)\,\chi_s(r)\,\chi_s(r')\,G(r')}{E - E_s - E_g + \mathrm{i}\Gamma_g^{\downarrow}/2}(1 - n_s). \qquad (84)$$

The experimental values of the energy E_g are near the neutron binding energy, so that the energies of the initial state and the doorway state may be close for nuclei in which the single-particle s level intersects the Fermi limit (isotopes of Sn, Te and others). For these nuclei, one can expect an appreciable modification of the imaginary part of the optical potential and, in accordance with (84), of the correlation of the strength function and the spectroscopic factor n_s.

2. In connection with (52), we mention the possibility of analyzing the partial $E2$-radiative strength functions $S_{\gamma E2}^{(1)}$. An expression for this quantity can be obtained from (52) by making the substitution $r \to r^2 e_{eff}(r; E_\lambda)$, also by the well known modification of the kinematic factor. The effective charge in this case is due to the coupling of the single-particle transition to the giant isoscalar and isovector quadrupole resonances.

3. It is of interest to calculate, on the basis of Eq. (51), the partial $E1$-radiative strength functions in the subthreshold region. The mean cross section of resonance elastic scattering of γ rays in this region:

$$\bar{\sigma}_{\gamma\gamma}/g\pi\lambda\!\!\!^{-}{}_\gamma^2 = 2\pi S_\gamma^2/S$$

(S is the strength function corresponding to the total radiative width of the compound nucleus resonances) can in accordance with the approximate expression (32) exhibit peaks with width around $2w$. In this way, one can interpret the experimental data.[27]

4. Directions of investigation of giant multipole resonances on the basis of the semimicroscopic approach were considered in Sec. 5.

5. Finally, we mention a subject that goes outside the framework of the present paper—the theory of isobaric analog resonances. Analog states are a clear example of isolated doorway states since, because of approximate conservation of isospin, they have a small (compared with $2w$) width $\Gamma\!\downarrow$. The quantitative interpretation of this width in terms of the shell and optical models is an as yet open problem.[28] (For some of the results and difficulties of the semimicroscopic approach applied to analog resonances, see Refs. 9, 29, and 30).

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