Backward peaks of the cross section in nuclear reactions and exchange processes

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A review is made of the experimental data on the large-angle behavior of the cross sections of nuclear reactions $A(x,y)B$ on light nuclei at energies $\gtrsim 15$ MeV of the bombarding particles. Peaks in the cross section around 180° (backward peaks) are observed in many reactions induced by complex particles, but the backward peaks for reactions induced by α particles on light nuclei are the most characteristic. The various characteristics of the backward peaks and their dependence on the type of reaction, the energy of the particles, and the structure of the nuclei and their mass number are considered. The possible physical reasons for the appearance of the backward peaks and the ways in which they are described are discussed. The possibility of explaining the backward peaks by exchange processes associated with breakup of the target nucleus is analyzed in detail.

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INTRODUCTION

The existing experimental data on the angular distributions of the products of nuclear reactions show that for many reactions in a wide range of energies of the bombarding particles the differential cross section peaks in the region of angles near 180°. This backward peak in the cross section is frequently called anomalous since it is unexpectedly large from the point of view of traditional methods of describing nuclear reactions.

In recent years, there has been a considerable growth of interest in the investigation of the backward peaks of the cross sections. This is due to the hope that their study will make it possible to obtain new and valuable information on the various mechanisms of the nuclear reactions and states of the nuclei with specific structure (for example, quasimolecular states). The main efforts of the experimenters and theoreticians have been directed toward obtaining data and explaining the effect in the elastic scattering of different particles and nuclei, the most numerous investigations being devoted to the elastic scattering of α particles, where the effect is most clearly manifested. At the same time, to interpret the backward peaks completely and correctly and obtain on this basis reliable information about the reaction mechanisms and the nuclear structure it is necessary to make combined investigations of different reaction channels, both elastic and inelastic. Unfortunately, the experimental data on the behavior of the reaction cross section in the region of large angles are as yet sparse, since measurements in many investigations were made only in the forward hemisphere.

In the present review, we consider the present state of the question of backward peaks in nuclear reactions; we do not give a detailed analysis of the elastic scattering data since several review papers[1,2] have been devoted to anomalous backward scattering. The aim of the present review is, on the one hand, to generalize the existing experimental data on backward peaks and possible ways of explaining this effect, and, on the other, to draw the attention of experimenters and theoreticians to a more detailed investigation of the effect in all its aspects.

First of all, it should be emphasized that one can only speak about the anamalous peaking of the cross section in the backward direction in the region of energies in which the contribution of the mechanism of compound-nucleus formation is small. If this mechanism makes an appreciable contribution, the backward peaks of the cross sections can be described either in the framework of the Hauser–Feshback theory or on the basis of statistical fluctuations of Ericson type. In addition, at below-barrier energies the Coulomb interaction between the particle and the nucleus also leads to a certain growth of the backward cross section. Therefore, as a rule, we shall consider reactions at incident-particle energies for which the number of decay channels of the compound nucleus is fairly large and the effects associated with the formation of a compound nucleus do not have a significant effect. For reactions induced by particles on light nuclei, this corresponds to α-particle energies greater than about 15 MeV.

1. REVIEW OF EXPERIMENTAL DATA

Dependence of the backwards peaks on the type of reaction. The first question to be answered is the following: In what reactions is anomalous behavior of the cross section in the region of large angles observed? Analysis of the existing experimental data provides the following answer. This effect is most clearly manifested in reactions involving reactions induced by α particles on light nuclei; on heavy nuclei it is much smaller. In deuteron stripping reactions, peaking of the cross section at large angles is seldom found and its magnitude is small compared with the cross-section peak in the forward hemisphere (see, for example, Ref. 3). In reactions in which tritium nuclei and ^{3}He participate a backward peak is not always observed, though it is more common than in deuteron stripping reactions and its magnitude is much greater. The angular distributions obtained for various

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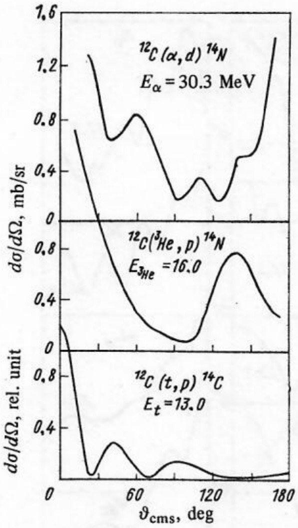

FIG. 1. Angular distributions of particles from reactions on ^{12}C with transfer of two nucleons and formation of the final nuclei ^{14}N and ^{14}C in the ground state.[4-6]

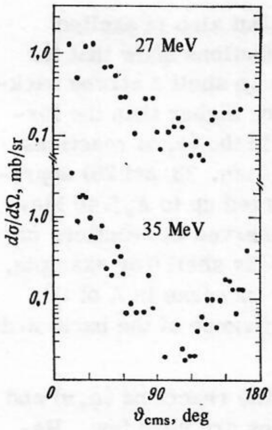

FIG. 3. Angular distributions of ^{15}N ions from the reaction $^{11}B(^{16}O, ^{15}N)^{12}C$ at different energies of ^{16}O.[8]

reactions on the ^{12}C nucleus with transfer of two nucleons and the formation of ^{14}C and ^{14}N final nuclei in the ground state are shown in Fig. 1. The experimental results were selected from Refs. 4–6 at energies of the bombarding particles such that the excitation energies of the compound nuclei and also the momentum transfer in the region of large angles should be as nearly equal as possible. As can be seen from Fig. 1, the angular distribution of the protons from the (t,p) reaction has the form characteristic of the stripping process without an appreciable increase in the cross section at 180°. In the $(^3He,p)$ reaction, there is a peak in the cross section at about 140° that is appreciably lower than the small-angle peak. In the (α,d) reaction, one observes a clear backward peak that is higher than the forward maximum.

With regard to reactions in which particles heavier than α particles participate (lithium nuclei, heavy ions), backward peaks are also observed but they are much less significant than in reactions induced by α particles (this last remark does not apply to the cases when nuclei with nearly equal masses interact, since in such a case which angles are large depends on which of the nuclei is accelerated and which serves as target). However, this conclusion is based on sparse experimental material since investigations with lithium nuclei and heavy ions in which the angular distributions have been measured over fairly large angles are as yet few. As an example, in Fig. 2 we give the angular distributions for two groups of α particles from the reaction $^{12}C(^6Li,\alpha)^{14}N$ at energies 12 and 14 MeV of the lithium ions[7]; the corresponding energies of the α particles for the inverse reactions are 21.6 and 23.3 MeV. When averaging is performed over the energy range 9–14 MeV, the angular distribution of the particles from this reaction becomes almost symmetric about 90°. In Fig. 3, we show the experimental data obtained in Ref. 8 for the $(^{16}O, ^{15}N)$ reaction on the ^{11}B nucleus at energies 27 and 35 MeV of the ^{16}O ions.

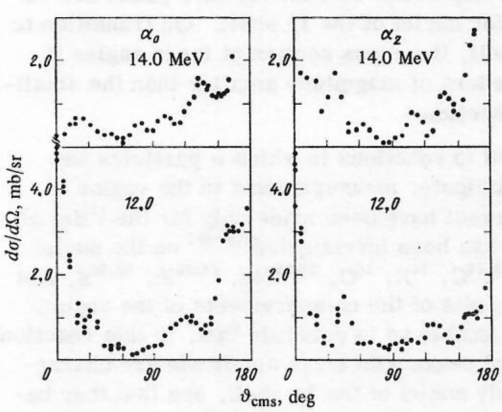

FIG. 2. Angular distributions of two groups of α particles from the reaction $^{12}C(^6Li,\alpha)^{14}N$ corresponding to the ground state and the second excited state (3.945 MeV) of the ^{14}N nucleus at different energies of lithium ions.[7]

Since the greatest number of existing experimental data on the behavior of cross sections in the region of backward angles refer to reactions with the participation of α particles, i.e., to reactions in which the backward peaks are most clearly manifested, we shall in what follows restrict ourselves basically to these reactions.

Backward peaks in reactions induced by α particles. We make a brief review of the experimental data available in the literature on the angular distributions in α-particle reactions measured at sufficiently large angles (> 150°), and, as we have already mentioned, we restrict ourselves to α-particle energies $E_\alpha \gtrsim 15$ MeV and corresponding energies of the other particles if we are talking about the inverse reactions.

The (α,p) reaction has been investigated in a wide range of the $1p$ shell: $^{6,7}Li$, $^{10,11}B$, ^{12}C, and ^{14}N (Refs. 9–17), on some nuclei of the $1d$–$2s$ shell: ^{19}F, ^{23}Na, ^{24}Mg, ^{27}Al, ^{28}Si, ^{31}P, ^{32}S, and ^{40}Ca (Refs. 14 and 18–20), and on some individual heavier nuclei: ^{45}Sc, ^{55}Mn, ^{56}Fe, and ^{59}Co (Ref. 14). In some investigations, the inverse reaction (p,α) has been studied. Measurements of the angular distributions up to large angles have been made on the nuclei ^9Be, ^{11}B, $^{12,13,14}C$, ^{14}N, $^{16,18}O$, ^{19}F, ^{26}Mg, ^{27}Al, ^{31}P, and ^{37}Cl (Refs. 21–32). In the majority of these investigations, the angular distributions were studied for several groups of particles produced in the reactions, corresponding to final nuclei in not only the ground state but also in excited states. The experimental distributions show that for all the investigated nuclei of the $1p$ shell a strong backward peak is observed, this being higher than the forward peak in individual cases. In the (p, α) reactions on the nuclei ^{9}Be, ^{11}B, and ^{12}C (Refs. 23 and 25) appreciable backward peaks are observed up to $E_p \lesssim 40$ MeV. The backward peaks are still observed in reactions on nuclei at the beginning of the $1d$–$2s$ shell (for example, ^{19}F and ^{23}Na), but with a further increase in A of the target nucleus the relative significance of the backward peaks becomes less and less.

Data on the cross sections of the reactions (α, n) and (n, α) in the region of large angles are very few. Reliable measurements have been made only for the reaction ^{9}Be$(n, \alpha)^{6}$He at $E_n = 14$ MeV (Ref. 33) and for the (α, n) reaction on ^{9}Be and ^{13}C nuclei in the range of α-particle energies from 14 to 23 MeV (Ref. 34). In the first and third of these reactions, large backward peaks were observed, but in the (α, n) reaction on the ^{9}Be nucleus a backward peak was observed only at certain energies.

The (α, d) reaction has been investigated almost exclusively on only nuclei of the $1p$ shell: ^{6,7}Li, ^{9}Be, ^{10,11}B, ^{12,13}C, and ^{14}N (Refs. 10 and 35–41). For nuclei of the $1d$–$2s$ shell the angular distributions of the deuterons for this reaction have been obtained for three nuclei (^{23}N, ^{27}Al, and ^{31}P) at $E_\alpha = 18.7$ MeV (Refs. 42–44). The inverse (d, α) reaction has been studied for a much greater number of nuclei, including many nuclei of the $1p$ shell: ^{6}Li, ^{9}Be, ^{10,11}B, ^{12,13}C, ^{14,15}N, and ^{16}O (Refs. 4, 41, and 45–57) and of the $1d$–$2s$ shell: ^{18}O, ^{19}F, ^{20,22}Ne, ^{23}Na, ^{24,25}Mg, ^{27}Al, ^{28,29,30}Si, ^{31}P, ^{32}S, ^{40}Ar, ^{39}K, and ^{40}Ca (Refs. 4, 53, 55, 58–66). The following reactions have been studied to the highest energies: ^{12}C$(\alpha, d)^{14}$N ($E_\alpha = 14.7$ MeV; Ref. 41) and ^{19}F$(d, \alpha)^{17}$O ($E_d = 20.9$ MeV; Ref. 56), which corresponds to α-particle energy 35.8 MeV for the inverse reaction. The experimental data show that in the (α, d) reaction, as in the (α, p) reaction, the backward peaks are most clearly expressed in the region of nuclei of the $1p$ shell. Backward peaks are also observed in reactions on nuclei at the beginning of the $1d$–$2s$ shell (for example, on ^{17}O at $E_\alpha = 18.6$ MeV (Ref. 52), and on ^{23}Na at $E_\alpha = 18.7$ and 21.3 MeV (Refs. 42 and 53). For heavier nuclei of this shell, an enhancement of the backward scattering has been found only in individual cases, these, as a rule, occurring at α-particle energies lower than 15 MeV, and the angular distributions frequently have a symmetric shape about 90° (see, for example, Refs. 64 and 65). For nuclei heavier than calcium at angles greater than 150° there are virtually no data, and the existing angular distributions indicate a rapid decrease of the cross section with increasing angle.[67] It is interesting to note that in the reaction ^{10}B$(\alpha, d)^{12}$C a backward peak is observed in the entire investigated energy range from 3 to 25 MeV, and that the amplitude of the backward peak relative to the forward peak decreases with increasing energy by about a factor five.

Data on the behavior of the cross sections at large

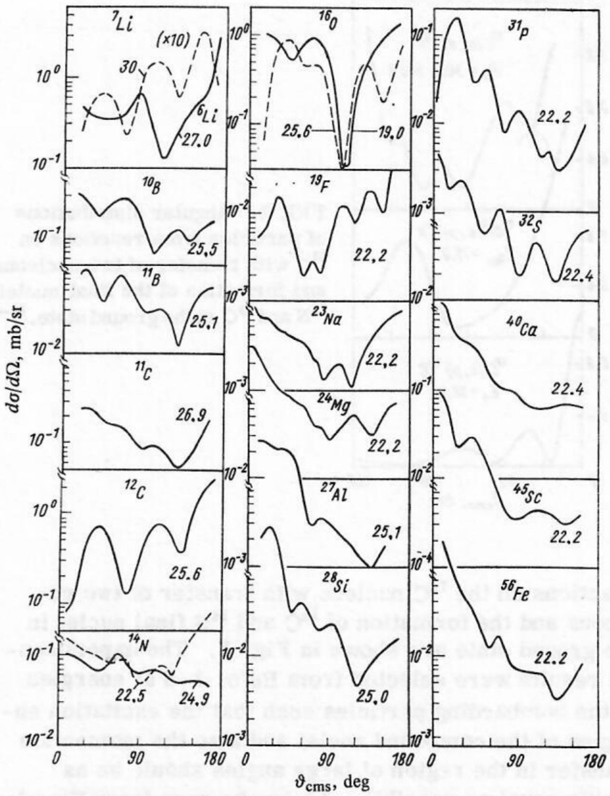

FIG. 4. Angular distributions of protons from (α, p) reaction with formation of final nuclei in the ground state. The α-particle energies are given in MeV; the data are taken from the literature as follows: ^{6}Li (Ref. 21), ^{7}Li (Ref. 10), ^{10}B (Ref. 11), ^{11}B (Ref. 12), ^{11}C (Ref. 28), ^{12}C (Ref. 13), ^{14}N (Ref. 17), ^{16}O (Refs. 29 and 30), ^{19}F (Ref. 18), ^{27}Al (Ref. 19), ^{28}Si (Ref. 20), ^{23}Na, ^{24}Mg, ^{31}P, ^{32}S, ^{40}Ca, ^{45}Sc, ^{56}Fe (Ref. 14).

angles in the (α, t) reaction are sparse. In the region of nuclei of the $1p$ shell, measurements have been made[10,68] for target nuclei ^{7}Li, ^{11}B, and ^{13}C. For heavier nuclei, measurements have been made[69,70] of the angular distributions at one or two α-particle energies on the targets ^{19}F, ^{27}Al, ^{51}V, and ^{59}Co. The inverse (t, α) reaction was investigated[71–77] at triton energies around 10 MeV on the nuclei ^{9}Be, ^{10}B, ^{12}C, ^{16}O, ^{40}Ca, ^{144}Sm, ^{208}Pb, and ^{209}Bi. In these reactions, backward peaks comparable with the forward peaks are observed only for nuclei of the $1p$ shell. On transition to the $1d$–$2s$ shell, the cross section at large angles is one or two orders of magnitude smaller than the small-angle cross section.

With regard to reactions in which α particles and ^{3}He ions participate, measurements in the region of angles of interest have been made only for the $(^{3}\text{He}, \alpha)$ reaction. It has been investigated[78–89] on the nuclei ^{7}Li, ^{10}B, ^{12,13,14}C, ^{14}N, ^{16}O, ^{24,26}Mg, ^{28,30}Si, ^{30,32}S, and ^{58}Ni. The results of the measurements of the angular distributions enable us to conclude that, in this reaction too, backward peaks with large amplitude are characteristic of only nuclei of the $1p$ shell, and that they become comparatively small on transition to the next shell.

This can be illustrated by experimental results (Figs. 4–8). In these figures, we have collected together the

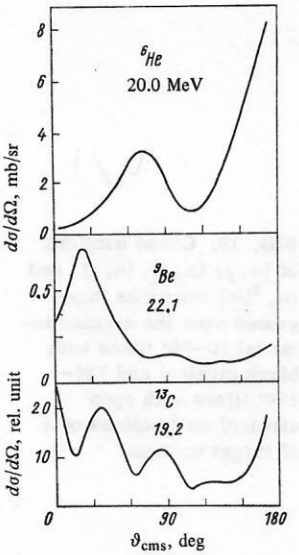

FIG. 5. Angular distributions of neutrons from (α, n) reactions with formation of the final nuclei in the ground state. The data are taken from the literature as follows: ^{6}He (Ref. 33), ^{9}Be, ^{13}C (Ref. 34). The energies of the incident α particles are given.

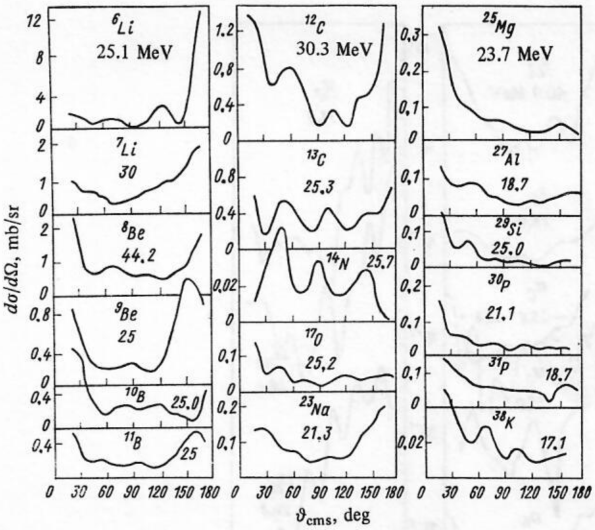

FIG. 6. Angular distributions of deuterons from (α, d) reactions with the formation of the final nuclei in the ground state. The data are taken from the literature as follows: ^{6}Li (Ref. 35), ^{7}Li (Ref. 10), ^{8}Be (Ref. 57), ^{9}Be, ^{10}B, ^{11}B (Ref. 38), ^{12}C (Ref. 4), ^{13}C (Ref. 40), ^{14}N (Ref. 51), ^{17}O (Ref. 52), ^{23}Na (Ref. 53), ^{25}Mg (Ref. 63), ^{27}Al (Ref. 43), ^{29}Si, ^{30}P (Ref. 60), ^{31}P (Ref. 44), ^{38}K (Ref. 66). The energies of the incident α particles are given.

angular distributions of particles from the reactions (α, p), (α, n), (α, d), (α, t), and $(\alpha, ^3He)$ or from the inverse reactions[1)] with the formation of the final nuclei in the ground state. About one third of these results were obtained at the Institute of Nuclear Physics of Moscow State University; the remainder have been taken from different papers, no selection with regard to the shape of the angular distributions having been made; rather, data at α-particle energy around 25 MeV or at the energy nearest to this value were used (for some of the reactions the distributions given are the only ones measured).

As we have already noted, backward peaks are observed not only for ground states but also for excited states of final nuclei, in every case at excitation energies within a few MeV. As an example, Fig. 9 shows the angular distributions of α particles from the reaction $^{19}F(d, \alpha)^{17}O$ for five energy states of the ^{17}O nucleus.[59] It can be seen that backward peaks are observed for all the investigated transitions, although the angular position of the backward peaks and their amplitude compared with the forward peaks are different.

Some idea about the dependence of the backward peaks on the mass of the target nucleus can be obtained from Fig. 10, in which for reactions with emission of p, d, t, and ^{3}He we give cross sections obtained from experimental data and integrated over the region of the backward peaks from 130 to 170° (integrated intensity of the backward peaks)[2)] and over the range of angles from 10 to 90°. The results given in the figure refer to the ground states of the final nuclei, but the qualitative nature of the dependences is also preserved for the excited states. It follows from Fig. 10 that the integrated intensity of the backward peaks, like the integrated cross section in the forward hemisphere, decreases on the average with increasing A. This de-

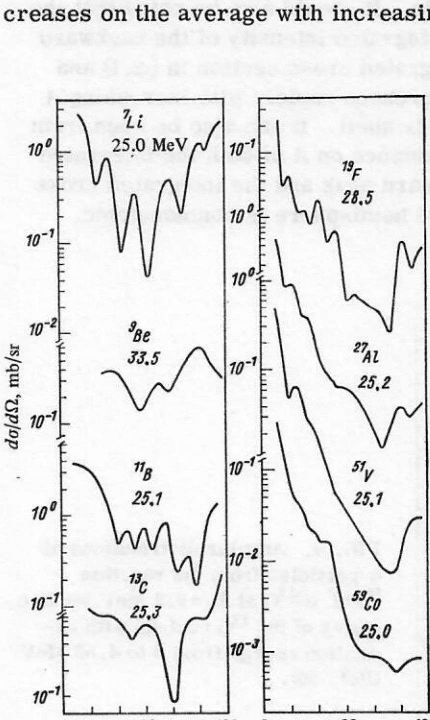

FIG. 7. Angular distribution of tritons from (α, t) reactions with the formation of final nuclei in the ground state. The data are taken from the literature as follows: ^{7}Li, ^{11}B, ^{13}C (Ref. 68), ^{9}Be (Ref. 71), ^{19}F (Ref. 69), ^{27}Al, ^{51}V, ^{59}Co (Ref. 70). The energies of the incident α particles are given.

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1) Here and in what follows, if we speak of transitions to the ground state of the final nucleus, the inverse reactions (y, α) will, for unity of notation, be represented in the form (α, y) with specification of the corresponding energy of the α particle.

2) In this way, the strength of the backward peaks can be estimated only qualitatively since the isotropic part of the angular distributions makes an appreciable contribution to the integral.

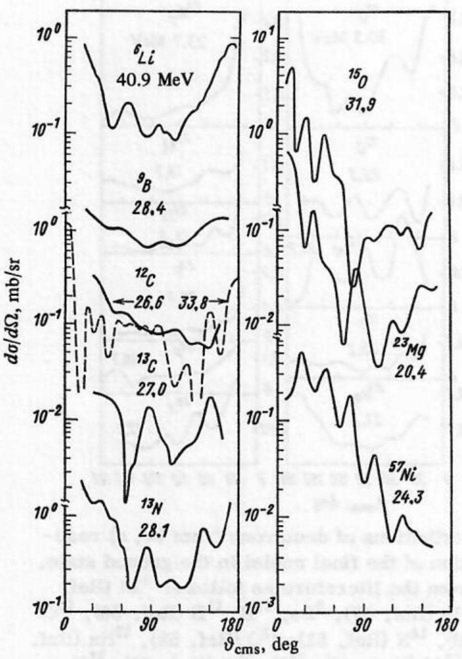

FIG. 8. Angular distributions of ^{3}He ions from (α, ^{3}He) reactions with the formation of the final nuclei in the ground state. The data are taken from the literature as follows: ^{6}Li (Ref. 78), ^{8}B (Ref. 79), ^{12}C (Refs. 82 and 83), ^{13}C (Ref. 84), ^{13}N (Ref. 85), ^{15}O (Ref. 81), ^{23}Mg (Ref. 87), ^{57}Ni (Ref. 89). The energies of the incident α particles are given.

crease takes place the faster, the greater is the mass of the emitted particle. It should also be noted that the contribution of the integrated intensity of the backward peak to the total integrated cross section in (α, t) and (α, ^{3}He) reactions decreases rapidly with increasing A starting with the $1d$–$2s$ shell. It can also be seen from Fig. 10 that the dependence on A of both the integrated intensity of the backward peak and the integrated cross section in the forward hemisphere is nonmonotonic.

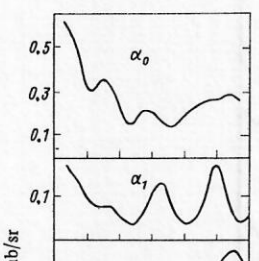

FIG. 9. Angular distributions of α particles from the reaction ^{19}F$(d, \alpha)^{17}$O at $E_d = 9.2$ MeV for five states of the ^{17}O nucleus with excitation energy from 0 to 4.55 MeV (Ref. 59).

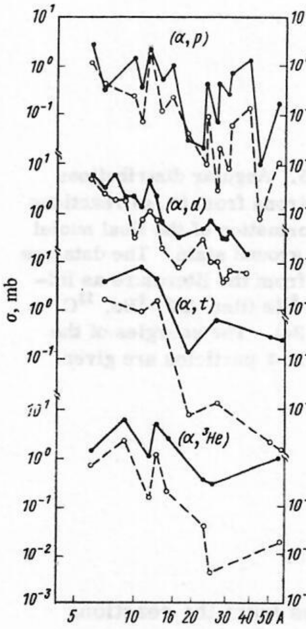

FIG. 10. Cross sections of (α, p) (α, d), (α, t), and (α, ^{3}He) reactions integrated over the angular interval 10–90° (lines with black circles) and 130–170° (lines with open circles) as functions of A of target nucleus.

This may be attributed to the pronounced individuality in the structure of the lightest nuclei.

Thus, virtually all reactions with the participation of α particles on nuclei of the $1p$ shell, and (α, p) and (α, d) reactions on nuclei at the beginning of the $1d$–$2s$ shell as well, exhibit backward peaks in the angular distributions that are comparable with the forward peaks. As a rule, the peaks are at 180°, but in individual cases they are also observed at smaller angles (140–160°). For target nuclei with mass number greater than 20–25, backward peaks, although manifested in some reactions, become much smaller than the forward peaks. The decrease in the backward peaks with increasing A takes place most rapidly in reactions with the emission of t and ^{3}He, more slowly in reactions with production of d, and slowest of all in reactions with emission of p. The absolute values of the backward peaks vary appreciably from nucleus to nucleus, and also in the isotopes of a given element. However, it is impossible to discover a definite dependence on the structure of the nuclei.

We must dwell separately on reactions in which the target nuclei are lithium isotopes. The experimental angular distributions for the different reactions, for elastic scattering of p, d, ^{3}He, and α particles on the ^{6}Li nucleus, and for reactions with the participation of α particles on the ^{7}Li nucleus are given in Fig. 11. Among all these distributions, only those for elastic scattering of protons and for the (d, p) reaction do not exhibit pronounced backward peaks; in all the remaining reactions there are peaks, these being most clearly manifested in the reactions induced by α particles. It may be said that in reactions on lithium nuclei the backward peaks are largest and are observed for more reactions and in a wider energy range than on other heavier nuclei.

Energy dependence of the backward peaks. The question of the dependence of the backward peaks on

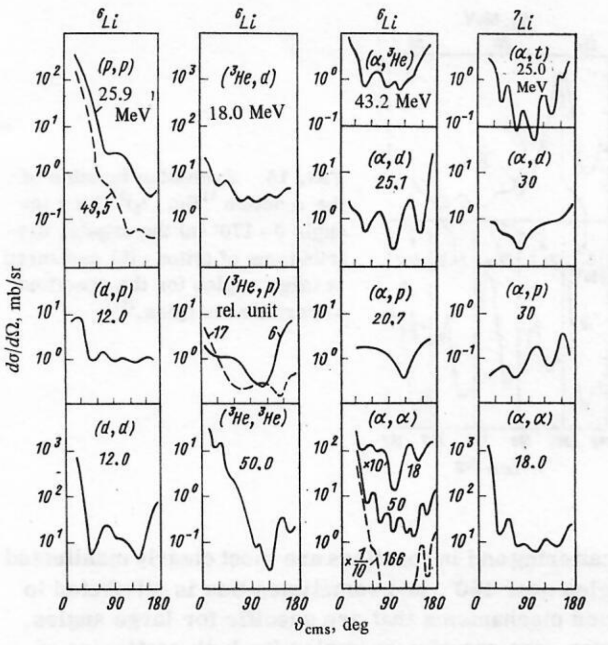

FIG. 11. Angular distributions for different reactions on lithium isotopes with formation of final nuclei in the ground state.[10,22,35,68,78,90–97] The numbers are the energies of the incident particles in MeV.

the energy E of the bombarding particles actually divides into two parts. The first of them is the following: On the average, how does the shape of the angular distributions in the region of large angles change when there is an appreciable change in the energy, and what then happens to the backward peaks? The experimental data reveal that a backward peak is an effect which is "stable" against variation of the energy and is observed, not at individual energies, but in a wide range of energies; for example, in the reaction $^{12}C(\alpha, p)^{15}N$ from 15 to 30 MeV (Refs. 13 and 98), in the reaction $^{10}B(\alpha, d)^{12}C$ from 3 to 25 MeV (Refs. 38 and 39), etc. As an example, in Fig. 12 we give the angular distributions of protons from the (α, p) reaction on several light nuclei at different energies. In the reaction $^{6}Li(\alpha, p)^{9}Be$, the shape of the angular distribution does not change qualitatively from 15 to 70 MeV. For nuclei heavier than lithium, the backward peak decreases with increasing energy much faster than the forward peak, so that, beginning at an energy aroun 40 MeV, the enhancement of the backward cross section becomes small, and it is sometimes difficult to say at all that there is any peaking at large angles. This is also true for reactions of other types. For example, in the (α, d) reaction on the ^{12}C nucleus at $E_\alpha = 41.7$ MeV (Ref. 41) and on the ^{17}O nucleus at $E_\alpha = 35.8$ MeV (Ref. 56) backward peaks are observed but they are much smaller than at lower energies.[52,59,61] Unfortunately, the majority of the experimental data refer to the region $E_\alpha \lesssim 25$ MeV; at higher energies, measurements are lacking for the majority of reactions, and in the individual cases when they are available the results have been obtained at only one or two energies. Therefore, it is as yet hard to decide what are the energies at which the backward peaks in the various reactions disappear and the rate at which the "damping" occurs. With regard to the width of the backward peaks, one can only make the general comment that it apparently decreases somewhat with increasing energy.

The second part of our question relating to the energy dependence of the backward peaks is this: How do the excitation functions behave at angles near 180° and how do the height and shape of the backward peaks change when there are relatively small changes in the energy? In the majority of reactions, though not in all, the excitation functions behave extremely irregularly, exhibiting resonances of different shapes, widths, and amplitudes. Such behavior of the excitation functions is very characteristic of nuclei in the middle and at the end of the $1p$ shell.[13,17,38,50] In reactions on lithium nuclei, the excitation functions are almost smooth.[22,35,78] The width of the resonances changes from hundreds of keV to several MeV. It is possible that the lower limit of the observed width is sometimes attributable to the inadequate monochromaticity of the bombarding particle, although in some investigations made with good energy resolution[86,101–103] a fine structure was not observed. For $E \gtrsim 25$–30 MeV the excitation functions are smoother and the width of the resonances increases, as, for example, in the case of the reaction $^{12}C(\alpha, p_0)^{15}N$ (Fig. 13). The smoothing of the excitation functions at higher energies of the projectiles was also noted in Ref. 102.

Comparison of the angular distributions measured at different energies shows that there is no apparent unique connection between the backward peaks and the resonance structure of the excitation functions. In some cases, in particular in the reactions $^{12}C(\alpha, p_0)^{15}N$ and $^{10}B(\alpha, d_0)^{12}C$ (see Figs. 13 and 14), the cross section at all investigated energies has a peak at 180°, but its height increases at the positions of the resonances and decreases between them. However, in the reaction $^{11}B(\alpha, t_0)^{12}C$, for example, the situation is different. As can be seen from Fig. 15, at the resonances of the excitation function the maximum of the cross section is at 180°, while between the resonances it is shifted to smaller angles. Because there are not enough determinations of the excitation functions and angular distributions in a wide range of energies and angles, it is at present impossible to establish with confidence a qualitative correspondence between the resonances of

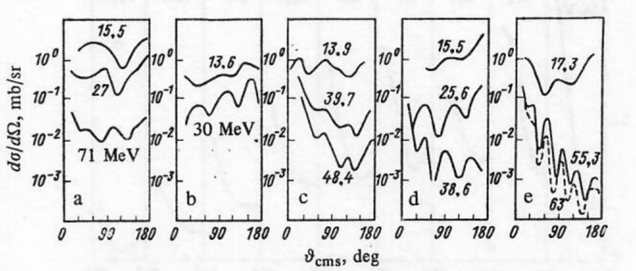

FIG. 12. Energy dependence of angular distributions of protons from the following reactions: a) $^{6}Li(\alpha, p)^{9}Be$ (Refs. 21–23); b) $^{7}Li(\alpha, p)^{10}Be$ (Refs. 10 and 99); c) $^{9}B(\alpha, p)^{12}C$ (Refs. 21, 25, and 100); d) $^{12}C(\alpha, p)^{15}N$ (Refs. 13, 15, and 98); e) $^{16}O(\alpha, p)^{19}F$ (Refs. 25, 29, and 100).

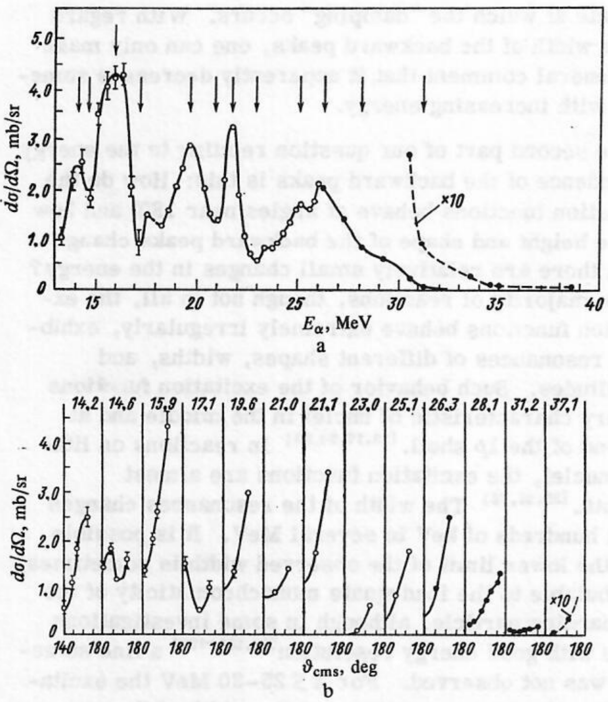

FIG. 13. Excitation function of the reaction $^{12}C(\alpha, p_0)^{15}N$ for the angle $\vartheta = 160°$ (a) and angular distributions of protons (b) measured at large angles for specified α-particle energies.[13,15,98]

the excitation functions, on the one hand, and the amplitude and position of the backward peaks, on the other.

From the point of view of clarifying the nature of the resonance structure of the excitation functions and its connection with the backward peaks, it is important to establish to what extent such structure is characteristic of large angles. It has been noted in some papers[86,104,105] that resonances in the excitation functions

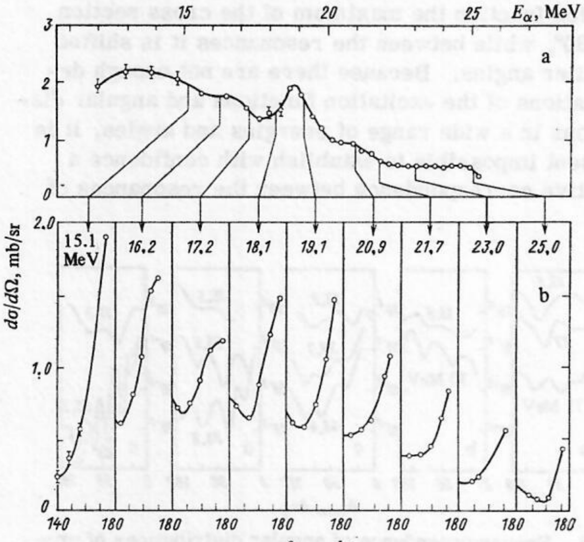

FIG. 14. Excitation function of the reaction $^{10}B(\alpha, d_0)^{12}C$ for the angle $\vartheta = 170°$ (a) and angular distributions of deuterons (b) measured at large angles for the specified α-particle energies.[38,48]

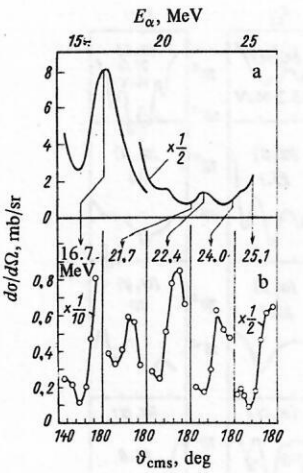

FIG. 15. Excitation function of the reaction $^{11}B(\alpha, t_0)^{12}C$ for the angle $\vartheta = 170°$ (a) the angular distributions of tritons (b) measured at large angles for the specified α-particle energies.[68]

for scattering and in reactions are most clearly manifested at angles near 180°, and sometimes this is attributed to reaction mechanisms that are specific for large angles. However, one can give examples for both scattering of α particles[40,106] and for reactions induced by α particles on light nuclei[13,17,40] which show that the amplitude, frequency, and width of the peaks in the excitation functions measured at different angles do not differ from one another in any pronounced way, although detailed investigations of this matter have not been made. Thus, the question posed above remains as yet open. This also applies to the question of whether there exists a correlation between the position of the resonances in the excitation functions for different reaction channels. In some papers[101,107,108] it has been noted that such a correlation is observed, but no investigation has yet been made to establish whether the correlation is characteristic of only large angles or whether it is observed at all angles. As an example of the correlation between the position of the resonances in the excitation functions measured at different angles and for different reaction channels, we show in Fig. 16 the arrangement of the peaks in the excitation functions for several reactions in which the ^{17}O nucleus is an intermediate nucleus.[40,101,109] It can be seen from Fig. 16 that many peaks appear in different reactions. In particular, this

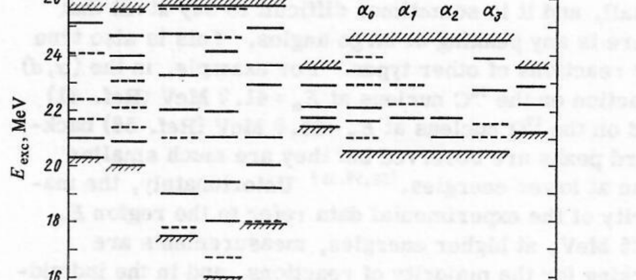

FIG. 16. Positions of peaks in the excitation functions of reactions with intermediate ^{17}O nucleus.[40,101,109] The dashed lines indicate peaks that are weakly manifested.

applies to the peaks whose position corresponds to states of the ^{17}O nucleus with excitation energy between 22 and 23 MeV.

Comparison of backward peaks in reactions and in elastic scattering. It is natural to compare the basic features of the backward peaks in reactions with anomalous backward scattering, which, as we have already said, has been studied in considerably more detail. Above all, this is important in order to understand whether the physical reasons behind the appearance of the backward peaks in scattering and in reactions are the same or whether in elastic scattering many features of the anomalous backward scattering are determined by processes characteristic of scattering alone.

We mention first of all the basic features that characterize the backward peaks and are common to elastic scattering and reactions. The principal one is that the backward peaks are most clearly manifested in processes in which α particles participate. It may be concluded from this that the appearance of the backward peaks is intimately related to the unique dense "packing" of the α particles, and also to effects of the α-cluster structure of nuclei. To a lesser extent, backward peaks are also observed in scattering and reactions in which other complex particles, for example ^3He, participate. Another feature common to scattering and reactions is the decrease in the absolute and relative size of the backward peaks with increasing mass of the target nucleus and with increasing energy of the bombarding particles.

At the same time, some of the features of the backward peaks in elastic scattering and in reactions are different. In particular, in elastic (and inelastic) scattering anomalies in the region of large angles are observed right up to the calcium isotopes, whereas in reactions the backward peaks are insignificant from about $A = 20-25$ onward. It should also be noted that the shape and width of the backward peaks in nuclear reactions and inelastic scattering are very different. As a rule, the angular distributions of the elastically scattered particles have the form of regular oscillations with width of the last maximum (near 180°) less than 20°, but the angular distributions of particles from reactions frequently have a more complicated, sometimes irregular nature, and the backward peaks are considerably wider, being on the average about 50° (from the point at which the cross section begins to rise).

The dependence of the backward peaks on the structure of the target nuclei is a more complicated question. In elastic scattering, such a dependence is observed fairly clearly, an isotope effect is observed, and definite regularities are found. In reactions, no clear connection between the backward peaks and the structure of the nuclei participating in the reaction has yet been established but it is very probable that the reason for this is to be sought in the insufficiency of the experimental data.

In some reactions of α particles with light nuclei backward peaks are not observed. However, since in these cases measurements up to large angles have been

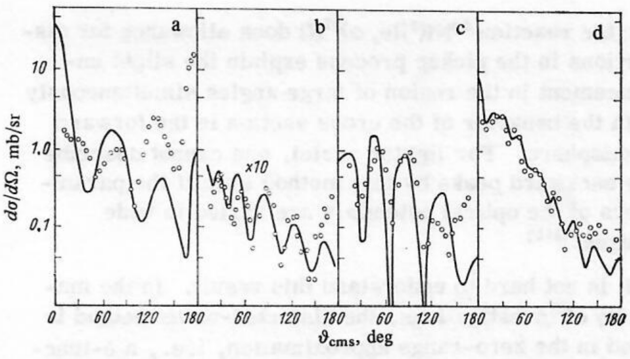

FIG. 17. Comparison of experimental angular distributions (open circles) with the theoretical distributions (continuous curves) calculated in the distorted-wave method under the assumption of the stripping or pickup mechanisms. a) $^6Li(\alpha, d)^8Be$, $E_\alpha = 25.1$ MeV (Ref. 35); b) $^{19}F(d, \alpha_1)^{17}O$, $E_d = 20.9$ MeV (Refs. 56 and 102); c) $^{30}Si(^3He, \alpha)^{29}Si$, $E_{^3He} = 8$ MeV (Ref. 88); d) $^{58}Ni(^3He, \alpha)^{57}Ni$, $E_{^3He} = 18$ MeV (Ref. 89).

made at only one energy of the bombarding particles, it cannot be asserted that these reactions are characterized by an absence of backward peaks. Backward peaks may be present at other energies.

Thus, there is a fairly large class of reactions in which, as is shown by numerous experiments, a backward peak is observed, and this is undoubtedly a characteristic feature of many reactions. As will be shown in the following section, the backward peaks cannot be described in the framework of the ordinary, traditional methods of describing nuclear reactions.

2. METHODS OF DESCRIBING THE BACKWARD PEAKS IN REACTIONS

In the overwhelming majority of studies of backward peaks, the attempt is made to describe them either by means of the distorted-wave method for the mechanism that is assumed to be the main one in the forward hemisphere (usually stripping or pickup), or alternatively processes peculiar to the backward peaks are invoked (stripping of a heavy particle and replacement of nucleon clusters). Let us consider this in more detail.

Stripping (Pickup). The distorted-wave method (DWBA) with zero interaction range[110] for stripping or pickup processes, which is used in almost all investigations, can, it is true, produce an enhancement of the cross section in the region of large angles if the parameters of the optical potentials are appropriately chosen, but it cannot explain the backward peaks observed, for example, in α-particles reactions on light nuclei. This assertion must not be understood literally. Indeed, using the distorted-wave method one can readily obtain a maximum of the cross section in the backward direction even in reactions induced by α particles. The problem is to achieve general agreement between the profile of the calculated angular distribution and the experimental profile, i.e., to describe small and large angles simultaneously. In Fig. 17, for several reactions we give the experimental angular distributions and those calculated by the distorted wave-method for stripping or pickup. It follows from Fig. 17 that only for the reaction $^{58}Ni(^3He, \alpha)^{57}Ni$ does allowance for distortions in the pickup process explain the slight enhancement in the region of large angles simultaneously with the behavior of the cross section in the forward hemisphere. For lighter nuclei, one cannot describe the backward peaks by this method even if the parameters of the optical potentials are varied in wide ranges.[111]

It is not hard to understand this result. In the majority of investigations, the distorted-wave method is used in the zero-range approximation, i.e., a δ-function nature of the interaction responsible for the rearrangement of the particles is assumed. Since the zero-range approximation is valid only in the case of small momentum transfers,[110] then in the region of large angles, where the momentum transfers are certainly not small, this approximation ceases to justified and cannot pretend to describe the experimental distributions. Giving up the zero-range approximation and taking into account the finite range of the interaction are not fundamental matters, but they entail considerable computational difficulties. The finite-range distorted-wave method[112] holds for all momentum transfers, and it is therefore more correct to use it to analyze the angular distributions of the products of reactions with complex particles in the complete range of angles than it is to use the ordinary distorted wave method.

So far, very few calculations have been made of stripping cross sections in a wide range of angles using the finite-range distorted wave method.[113,114] Nevertheless, on the basis of these one can conclude that this method does not essentially change the profile of the angular distributions in the region of large angles from the one obtained with the ordinary distorted wave method,[114] and one cannot therefore hope to describe the backward peaks by means of this method. Thus, the backward cannot be explained by the stripping mechanism alone, i.e., by a process in which the initial nucleus is regarded as a strongly absorbing system, in other words, as an inert potential core that does not undergo change during the reaction process (for the pickup mechanism, this applies to the final nucleus). Therefore, to explain the backward peaks many authors have invoked other mechanisms, in particular, ones associated with exchange interactions of the particles participating in the reaction.

Stripping of a heavy particle. This process (for brevity, we shall call it heavy stripping), whose essence consists of the transfer of a heavy cluster from the target nucleus to the incident particle with formation of the final nucleus, was proposed more than 20 years ago by Owen and Madansky[115,116] with the very intention of explaining the peaks in the cross section at large angles. By virtue of its kinematic properties (the momentum transfer is minimal at the largest angles) the cross section of heavy stripping has maximal value at large angles. The Japanese physicists[117–119] obtained expressions for calculating the angular distributions of heavy stripping in the plane-wave approximation and used them to analyze some data at large angles

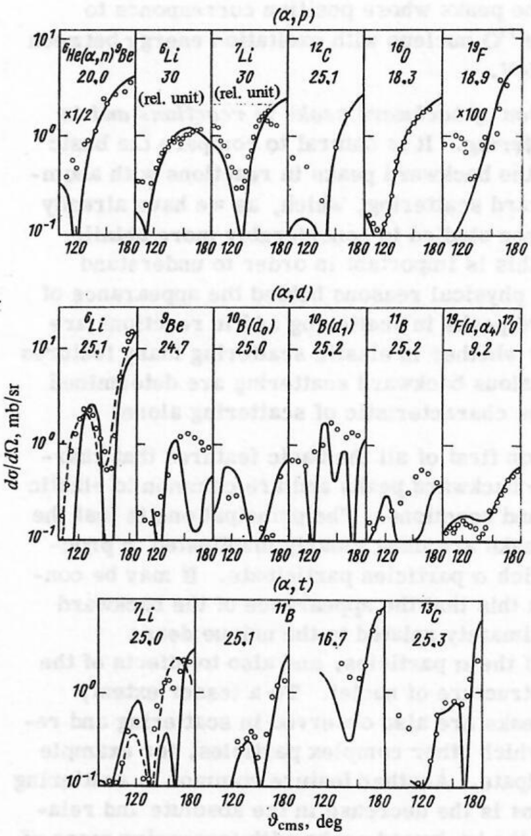

FIG. 18. Comparison of experimental angular distributions of protons, neutrons, deuterons, and tritons in reactions induced by α particles for light nuclei[9,15,24,33,35,38,59,68,125–128] and theoretical distributions calculated in the plane-wave approximation for the heavy-stripping mechanism (continuous curves)[33,35,38,59,68,117,118,126–129] and in the framework of the peripherical model (dashed curves).[124]

for scattering and reactions. In Refs. 120 and 121, an investigation was made of the interference between different states of the intermediate nucleus during heavy stripping. Noble and Coelho[122] introduced for this process qualitative allowance for distortions and used a more realistic form of the vertex functions. All these investigations showed that the heavy stripping mechanism enables one in many cases to provide a perfectly satisfactory description of the backward peaks even in the plane-wave approximation. In Fig. 18, for a number of reactions, we compare the angular distributions calculated for heavy stripping with the experimental data. It can be seen that in almost all cases when a peak is observed at 180° the calculated curves describe the experiment qualitatively in the region of angles $\vartheta > 140°$, and, moreover, the cutoff radii that occur in the calculation as variable parameters do not change too strongly with varying atomic number of the target nucleus or energy of the incident particles. This conclusion has been drawn on the basis of calculations made in the plane-wave approximation (the continuous curves in Fig. 18), but it also remains true in the peripheral model of nuclear reactions[123] (the dashed curves of Ref. 124).

However, in some reactions the heavy-stripping

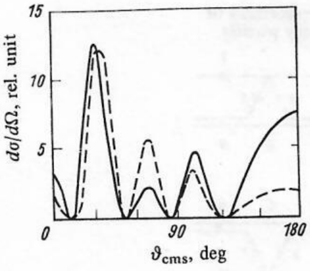

FIG. 19. Angular distributions of protons from the reaction $^{19}F(\alpha, p_0)^{22}Ne$ calculated in the plane-wave approximation for the replacement mechanism with allowance for anisotropy of x-y scattering (continuous curve)[132] and under the assumption of isotropy of x-y scattering (dashed curve) for $E_\alpha = 22.2$ MeV.

mechanism cannot describe the position of the peaks even at large angles. This applies above all to the cases when the peak is displaced from 180° and observed at smaller angles. In addition, when the plane-wave approximation is used for the heavy-stripping mechanism one cannot correctly reproduce the relative magnitudes of the backward peaks for different target nuclei, different energies of the projectiles, and different states of the residual nuclei. Estimates show[124] that this conclusion also remains true when qualitative allowance is made for distortions in the plane waves of the initial and final particles (in the eikonal approximation).

In some investigations, in the analysis of concrete experimental data, people have succeeded in describing them satisfactorily in the complete range of angles by summing the cross sections of stripping (pickup) and heavy stripping (see, for example, Refs. 59 and 129). However, the agreement with experiment of the profile of the angular distributions for individual reactions without analysis of the relative values of the cross section (including the region of large angles) at different energies and for different nuclei creates only the illusion of a good description of the experimental results and cannot provide a sound foundation for the assertion that these processes alone are predominant.

Replacement of nuclear clusters. To describe the backward peaks, recourse is sometimes had to one further mechanism associated with the exchange interaction of light particles—replacement of clusters (the knock-out mechanism). The possible occurrence of this mechanism in nuclear reactions was first noted by Holmgren and Wolicki,[130] who pointed out that the cross section of α particle reactions on light nuclei "permit" the replacement of particle y in the initial nucleus by the incident particle x. An expression for the cross section of the replacement mechanism was first obtained for nucleon scattering in the plane-wave approximation in Ref. 131. In this paper, it was shown that if a δ-function interaction of particles x and y is used, which corresponds to an isotropic angular distribution of x-y scattering, the replacement mechanism leads to an angular distribution whose profile almost coincides with the angular distribution of ordinary cluster stripping. The momentum transfer in these two processes depends in the same way on the angle of emission of the final particle. However, allowance for angular anisotropy in the amplitude of x-y scattering changes the situation considerably, and in the angular distribution there appears a maximum at large angles due to the interaction of particles x and y. This feature of the replacement mechanism enables one to use it to describe the backward peaks.

The first calculation of the angular distributions in the complete range of angles for the displacement mechanism was made in the plane-wave approximation by Tanifuji,[132] who proposed a fairly convenient qualitative method for taking into account the anisotropy of the x-y scattering. His calculations for the (α, p) reaction on the nuclei ^{19}F and ^{31}P showed that allowance for anisotropy increases the cross section at large angles. The results obtained by Tanifuji for the reaction $^{19}F(\alpha, p)^{22}Ne$ both with and without allowance for the anisotropy of the x-y scattering are given in Fig. 19. It can be seen that dependence of the amplitude of x-y scattering on the angle leads to a pronounced increase in the cross section at large angles, although Tanifuji did not succeed in obtaining quantitative agreement with experiment. Analogous calculations made for α-particle reactions on nuclei of the $1p$ shell[133,134] also produced a peak at large angles. But these calculations, too, showed that if only the replacement mechanism is used, especially in the plane-wave approximation, it is impossible to describe either the magnitude or the structure of the backward peaks. It should be noted that, since the peak at large angles in the cross section of the replacement mechanism is related to the amplitude of x-y scattering, its angular width must vary little with varying atomic number of the target nucleus and energy of the incident particles.

The replacement mechanism, like the heavy-stripping mechanism, can be used to describe the backward peaks in not only reactions but also in scattering. The currently available calculations of the angular distribution of elastically scattered α particles on ^{40}Ca for the replacement mechanism using the distorted-wave method[135,136] show that this mechanism can explain the backward peaks in elastic scattering on fairly heavy nuclei. In the region of light nuclei, inclusion of only the replacement mechanism for scattering, as also for reactions, does not lead to a satisfactory description of the experimental data at large angles.[137]

Some general comments. The most important assumption on which the above mechanisms are based—stripping of a heavy particle and the replacement of nucleon clusters—is that it is the target nucleus, and not the incident particle (as happens for the stripping mechanism), which suffers dissociation. It was shown above that both mechanisms lead to the appearance of a maximum in the differential cross section in the region of large angles. However, allowance for such processes in the description of nuclear reactions is dictated not only by the need to explain the backward peaks but also by more general considerations.

It is to be expected that the reaction mechanism depends strongly on the structure of the nuclei—both the target and the projectile—participating in the reaction. Because of the low binding energy of the deuteron and the ^{6}Li nucleus, it is natural to expect that, for example, in the (d,p) and (^{6}Li, α) reactions the process of nucleon or cluster stripping will predominate. This is confirmed by a large number of experimental data and spectroscopic information obtained by means of such reactions. One may think that for reactions induced by other particles, whose breakup is energetically less advantageous, processes associated with dissociation of the target nuclei will acquire importance. In the first place, this applies to reactions induced by α particles, whose binding energy appreciably exceeds the binding energy of the majority of clusters in light nuclei. Such conclusions are confirmed by the experimental data that we have given, i. e., by the fact that it is in reactions with the participation of α particles that the backward peaks are manifested most clearly and are largest.

Another case when one can expect a large contribution by the processes of heavy stripping and replacement is in reactions on target nuclei in which the binding energy of the corresponding clusters is low. The most favorable reactions in this sense are those on lithium isotopes, in particular those in which the breakup $^6\mathrm{Li}\to\alpha+d$ and $^7\mathrm{Li}\to\alpha+t$ must occur. Such a conclusion is also well confirmed (see Fig. 11), and backward peaks are above all characteristic in reactions on lithium isotopes with the participation of α particles, i. e., when the projectile is strongly bound but the target weakly bound. In these cases, backward peaks are manifested in a wide range of energies; for example, an appreciable increase in the backward cross section of elastic scattering of α particles on ^{6}Li is observed[96] even at the energy 166 MeV, and in the reaction $^6\mathrm{Li}(\alpha,p)^9\mathrm{Be}$ up to 70 MeV (Ref. 23). The available experimental data show that the profile of the angular distribution is stable when the energy of the projectile particles for these reactions is changed. One can attempt to explain these experimental features of the reactions on lithium nuclei by invoking only processes associated with dissociation of the target nuclei. As will be shown below, the results of these calculations agree with the experimental data.

Of course, the qualitative arguments given above about the importance of processes associated with breakup of the target nucleus for explaining the backward peaks in reactions on different nuclei must be confirmed by more detailed analysis and calculations.

3. DESCRIPTION OF BACKWARD PEAKS BY MEANS OF EXCHANGE PROCESSES

Classification of the simplest direct and exchange processes. Exact calculation of the amplitude of any nuclear reaction $A(x,y)B$ (in what follows we shall everywhere assume that $x\geqslant y$) requires the solution of a many-body problem, but in some limiting cases the problem can be simplified and reduced to a three-body problem. This can be done in two cases: 1) The incident particle x dissociates in the field of nucleus A into

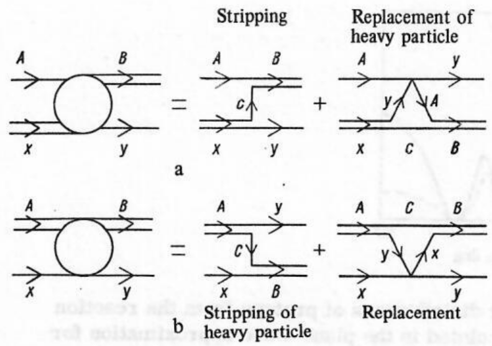

FIG. 20. Graphical representation of the main direct (a) and exchange (b) processes for the reaction $A(x,y)B$.

two clusters, $x\to c+y$, and nucleus A remains unchanged; 2) particle x remains unchanged and nucleus A decays into two parts, $A\to C+y$, one of which is the final particle. Processes corresponding to the first case and associated with breakup of the incident particle may be called *direct processes*, and processes corresponding to the second case and associated with dissociation of the target nucleus *exchange processes*. In these two cases, an exact analytic expression can be obtained for the reaction amplitude.[138] The matrix element of the direct processes can be written in the form

$$M_{if}^{\mathrm{dir}}=\langle\Phi_f\,|\,V_{yc}\,|\,\chi_i^{(+)}\rangle+\langle\chi_f^{(-)}\,|\,T_{yA}\,|\,\chi_i^{(+)}\rangle, \tag{1}$$

where V_{yc} is the interaction between the final particle y and the particle c transferred to the nucleus; Φ_f is the plane wave describing the motion of the final particles; T_{yA} is the integral operator of the scattering of particle y on the intial nucleus A, and it is determined by the equation $T_{yA}\Phi_i=V_{yA}\Psi_i^{(+)}$; $\Psi_i^{(+)}$ is an exact solution of the Schrödinger equation; $\chi_i^{(+)}$ and $\chi_f^{(-)}$ a solution of the Schrödinger equation with truncated Hamiltonian without the interaction V_{yA} (Ref. 138).

Let us elucidate the physical meaning of the two terms in Eq. (1). The first of them corresponds to direct capture of particle c, i.e., it is the matrix element of ordinary cluster stripping. The second term describes the scattering of particles y and A with allowance for interaction of nucleus A and particle c in the final nucleus B and of particles y and c in the initial projectile x. This mechanism is usually called heavy replacement. Graphically, the matrix element (1) for the direct processes is shown in Fig. 20a, the pole graph corresponding to the mechanism of cluster stripping and the triange graph to the mechanism of heavy replacement.

The matrix element of the reaction in the case exchange processes can be written down similarly:

$$M_{if}^{\mathrm{exc}}=\langle\Phi_f\,|\,V_{Cy}\,|\,\chi_i^{(+)}\rangle+\langle\chi_f^{(-)}\,|\,T_{xy}\chi_i^{(+)}\rangle. \tag{2}$$

The first term in the expression (2) corresponds to the interaction of the final particle y and the intermediate nucleus C in the initial nucleus A, i.e., it corresponds to the stripping of a heavy particle (pole graph in Fig. 20b). The second term, which describes the scattering of particles x and y in the field of the intermediate nucleus C, corresponds to the ordinary replacement of clusters (triangle graph in Fig. 20b). Thus, the mechanisms considered above of stripping of a heavy particle and replacement of nucleon clusters are combined in the framework of the three-body problem into a single group of exchange processes.

Analysis of the expressions (1) and (2) shows that the matrix element of the exchange processes has a more complicated structure than the matrix element of the direct processes. This complication is due to the fact that the matrix elements of each of the exchange mechanisms contain a coherent sum over all possible states, i.e., states allowed by the laws of conservation of the total angular momentum, of the virtual intermediate nucleus C. In its turn, this circumstance means that the amplitudes of the reduced widths of the nucleon clusters y and x in the initial and final nuclei A and B (Ref. 139), on which the matrix element (2) depends, occur in the sum over the states of the nucleus C, and therefore the corresponding spectroscopic factors cannot be separated in the form of a factor in the expression for the reaction cross section. In this connection, one must consider how important is the role of coherent summation of the contribution of the various states of particle C and whether one could not dispense with allowance for all these states but retain in the sum only a single term. Analysis of the amplitudes of the mechanisms of replacement[139] and heavy stripping[120,121] showed that the interference between the states of the intermediate nucleus is very important. For example, for the replacement mechanism[139] inclusion of all terms of the sum enables one to reproduce correctly the intensities of the individual peaks of the angular distribution in the forward hemisphere and the relative values of the cross sections (in particular, abrupt changes of them) on the transition from nucleus to nucleus. For the heavy stripping mechanism,[121] the interference between the states of the intermediate nucleus leads to a narrowing of the backward peak in the angular distributions (without allowance for this interference, the width of the peak for the heavy stripping mechanism appreciably exceeds the experimentally observed values). In addition, allowance for the coherent contribution of the different states of nucleus C for heavy stripping considerably increases the amplitude of the peak at backward angles.

It follows from the expressions (1) and (2) that in each group of processes the individual mechanisms must be added coherently and it is necessary to take into account their interference. Direct and exchange processes in the framework of the three-body problem are not coupled to each other, and the question of their interference does not arise. In a more complicated formalism (for example, in the case of a four-body problem) all four mechanisms may make a coherent contribution to the total matrix element of the reaction. However, it should be noted that in the case of such a complication of the formalism the mechanisms themselves will not have such a simple form as is shown in Fig. 20. In addition, there are new "unrenormalized" mechanisms associated with the introduction of additional interactions. But if we restrict ourselves to the approximation of the three-body problem, we must take into account the coherent addition of only those mechanisms that correspond to the same manner of breakup of the particles that participate in the reaction. Note that the problem of the interference of the individual mechanisms in the three-body problem would be less important if the four-prong vertices in the triangle diagrams could be assumed constants, since the cross sections of the mechanisms corresponding to the pole and triangle diagrams (see Fig. 20) reach maxima in different ranges of angles (for example, the angular distribution of the stripping mechanism has a maximum in the forward hemisphere; that of heavy replacement, in the backward). However the presence of angular anisotropy in the four-prong vertex of the triangle diagram (see Sec. 2) considerably changes the situation and the interference between the pole diagram and the triangle diagram is important. Allowance for this interference has a strong influence on the calculated profile of the angular distribution in the complete range of emission angles of the final particle.

In order to describe quantitatively the angular distributions, it is also necessary to take into account as accurately as possible the distortion of the plane waves in the initial and final channels. At the present time, this problem is solved approximately by means of the optical model, i.e., by reduction of the many-body problem to a two-body problem. The validity of such a procedure remains uncertain as yet, and it is known[140] that allowance for many-body corrections, at least in the framework of the three-body problem, must influence the characteristics of the optical potential. Nevertheless, the optical model is as yet the most developed and convenient tool for describing the interaction of particles with nuclei in the initial and final states. It is therefore used for quantitative calculations of the matrix elements (1) and (2). However, one cannot hope for success if these calculations are made by means of the distorted-wave method in the zero-range approximation. The point is that for mechanisms such as heavy stripping and heavy replacement the momentum transfers are certainly not small and the zero-range approximation is fundamentally unsuitable for calculating the amplitudes of these mechanisms. Therefore, quantitative agreement with the experimental data can be obtained only if the finite-range distorted-wave method is used. The corresponding computational expressions for the cross sections of direct and exchange processes with allowance for interference between the individual mechanisms (and for exchange processes, with allowance for interference between the different states of the intermediate nucleus C) for $A(x, y)B$ reactions on light nuclei are given in the Appendix.[141]

In a real situation, a reaction proceeds with dissociation of both the projectile nucleus and the target nucleus, and the probability of dissociation of each of them with formation of a cluster y is determined by the structural features of the two of them and the corre-

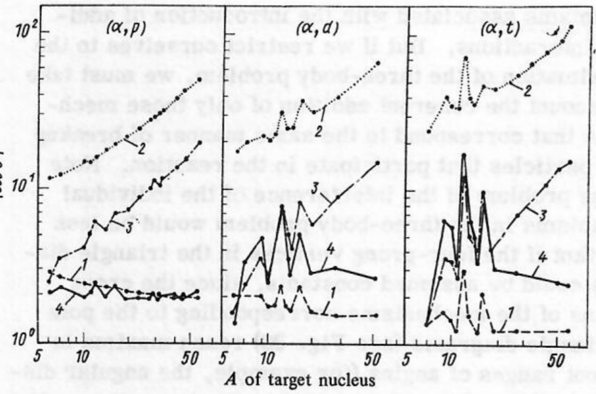

FIG. 21. The A dependence of $|\cos\vartheta|$, which characterizes the position of the singularities, for the (α,p), (α,d), and (α,t) reactions with the formation of ground states of the final nuclei at $E_\alpha = 25$ MeV: 1) stripping, 2) heavy replacement; 3) stripping of heavy particle; 4) replacement.

sponding binding energies. However, it is impossible to take into account simultaneously all the processes if one remains in the framework of the three-body problem. Therefore, the only possibility of describing reactions in this approximation consists of using, in the calculation of the cross sections, the incoherent sum of the contributions of the direct and the exchange processes, treating the coefficient that determines the relative value of their contributions as a variable parameter. We emphasize once more that such a procedure is inconsistent and can be regarded only as a compromise made necessary by our restricting ourselves to the three-body approximation.

Qualitative estimates of the contribution of individual mechanisms. It is in practice impossible to determine in general form the contribution of the individual mechanisms to the cross section. However, one can put forward a number of arguments and use them to estimate the importance of the individual mechanisms, the dependence of their contribution to the cross section on the masses of the nuclei, the energy of the incident particles, and the other characteristics of the reaction.

Let us consider first the factors that may influence the contribution of individual coherent mechanisms. For direct and exchange processes, the two terms of the matrix elements (1) and (2) have the same order of magnitude and contain the same spectroscopic factors. Therefore, for each pair of coherent mechanisms, the contribution of individual mechanisms will be basically determined by kinematic factors whose values depend on the distance the singularity of the corresponding diagrams is removed from the boundary of the physical region. The most convenient way of characterizing this distance is by using the cosine of the scattering angle ϑ at the singularity since the physical region of $\cos\vartheta$ is restricted to the interval from −1 to +1.

In Fig. 21, we give the values of $|\cos\vartheta|$ calculated at the singularities of all four mechanisms for the reactions (α,p), (α,d), and (α,t) in the region of mass numbers $6 \leqslant A \leqslant 60$ at $E_\alpha = 25$ MeV with the formation of the residual nuclei in the ground state. Several conclusions can be drawn from Fig. 21. Above all, the contribution of the heavy replacement mechanism to the cross section of the direct processes is evidently unimportant for all nuclei except the lightest, since its singularities are much further from the boundary of the physical region than the singularities of the stripping mechanism. Further, it is necessary to point out the considerable difference between the dependences of $|\cos\vartheta|$ on A for the (α,p) reaction, on the one hand, and for the (α,d) and (α,t) reactions, on the other. In the case of the (α,p) reaction, the positions of the singularities of all the mechanisms vary from nucleus to nucleus without significant abrupt changes, approaching slowly with increasing A the boundary of the physical region for stripping and replacement and increasing rapidly for heavy stripping. It is therefore to be expected that in the (α,p) reaction, beginning already with nuclei at the end of the $1p$ shell, the heavy stripping contribution to the cross section of exchange processes will not be important and the cross section of the exchange processes must be basically determined by the replacement mechanism.

A different picture is observed for the (α,d) and (α,t) reactions. In the region $A \lesssim 20$, the positions of the singularities of all mechanisms for these reactions changes appreciably on the transition from one nucleus to another, this being most strongly so in the (α,t) reaction. Therefore, the cross section of such reactions, including the cross section of exchange processes, must vary as a function of A much more strongly than for the (α,p) reaction. In the same region $A \lesssim 20$, the positions of the singularities of both the exchange mechanisms—replacement and heavy stripping—are at approximately the same distance from the boundary of the physical region, so that in the (α,d) and (α,t) reactions for $A \lesssim 20$ both mechanisms must contribute significantly to the cross section of the exchange processes.

Let us now consider how the relative contribution of pairs of incoherent processes varies with increasing A. In the forefront we here encounter the structural characteristics of the nuclei participating in the reaction. For direct processes, they are determined by the spectroscopic factor $S = |\Theta^B_{cA}|^2$, where Θ^B_{cA} is the amplitude of the reduced width of cluster c in the final nucleus B relative to the initial nucleus A. For exchange processes, we have seen above that it is not possible to separate the spectroscopic factors as a common factor. However, to estimate the contribution of the various processes to the reaction cross section, one can use $|\Sigma\Theta^B_{Cx}\Theta^A_{Cy}|^2$, where the summation is over all allowed states of the intermediate nucleus C. This may be called the "spectroscopic factor" S for the exchange processes. In Table I, we give the results of approximate estimates of the ratios of the spectroscopic factors S for direct and exchange processes in the (α,p), (α,d), and (α,t) reactions on nuclei of the $1d$–$2s$ and $1f$–$3s$ shells to their values in the same reactions on nuclei of the $1p$ shell.[134] It can be seen from Table I that with increasing A the contribution of the exchange processes is, as a rule, smaller than that of the direct

TABLE I. Ratios of spectroscopic factors for reactions on nuclei of different shells.[134]

Reaction	Direct processes		Exchange processes	
	S_{2s}/S_{1p}	S_{3s}/S_{1p}	S_{2s}/S_{1p}	S_{3s}/S_{1p}
(α, p)	$1.4\cdot10^{-1}$	$3.0\cdot10^{-2}$	$4.9\cdot10^{-1}$	$4.5\cdot10^{-2}$
(α, d)	$4.2\cdot10^{-1}$	$2.0\cdot10^{-1}$	$2.0\cdot10^{-1}$	$8.9\cdot10^{-3}$
(α, t)	1	1	$7.0\cdot10^{-2}$	$1.4\cdot10^{-3}$

processes (or rather, of the stripping mechanism). This is most pronounced in the case of the reaction (α, t), in which the damping of the exchange processes through the spectroscopic factors is large already for nuclei of the $1d$–$2s$ shell. On the other hand, in (α, p) reactions, the exchange processes (principally the replacement mechanism) are hardly damped at all and may contribute to the cross section on other than only light nuclei.

One can also consider the dependence of the contribution of the four simplest mechanisms to the reaction cross section on the energy E of the incident particles. We note first of all that with increasing energy the singularities of heavy stripping and heavy replacement move away from, and the singularities of replacement and stripping move toward, the boundary of the physical region. It follows that the cross sections of the heavy stripping and heavy replacement mechanisms decrease with increasing E. In addition, with increasing E the cross section of all four mechanisms decreases because of the energy dependence of the vertex factors of the corresponding matrix elements. However, this dependence is different for different mechanisms. The amplitude of the heavy-stripping mechanism is proportional to the product of the form factors of the two nuclear vertices, and both these form factors decrease rapidly with increasing E. The amplitude of the mechanism of ordinary stripping contains only one nuclear form factor, and its decrease with increasing energy is therefore much weaker. With regard to the mechanisms of replacement and heavy replacement, with increasing E their contribution must decrease because of the reduction in the overlapping of the form factors of the nuclear vertices and the amplitudes of x-y and y-A scattering, respectively.[134] This effect is manifested the more strongly, the greater is the difference between the x and y masses for one mechanism and y and A masses for the other. Therefore, the cross section of the replacement mechanism will decrease most rapidly for the (α, p) and (α, n) reactions and least rapidly for (α, α) scattering. In addition, the amplitude of x-y (y-A) scattering will itself decrease with increasing energy.

Therefore, we may expect that with increasing energy of the incident particles the contribution of the exchange processes and the mechanism of heavy replacement will decrease faster than the contribution of the mechanism of ordinary stripping, and among the exchange processes the cross section of the heavy-stripping mechanism must decrease fastest. Thus, with increasing E the mechanisms of heavy replacement and heavy stripping are the first to die out, followed by the replacement mechanism, so that at sufficiently high energies the stripping mechanism must predominate.

Thus, qualitative estimates of the contribution of the individual mechanisms show that the exchange processes must make an appreciable contribution to the cross section of (α, d) and (α, t) reactions up to nuclei with $A \approx 20$ and for (α, p) reactions on nuclei of the $1d$–$2s$ shell as well. With increasing energy of the incident particles, the exchange processes "die out" faster than the mechanism of cluster stripping. These conclusions about the behavior of the exchange processes agree well with the experimentally observed features of the backward peaks described in Sec. 1, which directly indicates an important role of exchange processes in the formation of the backward peaks.

Results of calculations of the angular distributions for the simplest direct and exchange processes. We have already said that in an "exact" calculation of the reaction cross section it is necessary to take into account the incoherent contribution of the direct and the exchange processes and that their amplitudes must be calculated by means of the finite-range distorted-wave method. Since allowance for the finite range of the interaction in the distorted-wave method entails serious computational difficulties, calculations of the cross sections of nuclear reactions on the basis of the finite-range distorted-wave method have been made only during the last three years. Therefore, the majority of calculations of the contribution of exchange processes were made in the plane-wave approximation, and they can pretend to only a qualitative description of an experiment.

The first attempt to explain the angular distribution of a reaction in the complete range of emission angles of the final particles with allowance for the contributions of different mechanisms was undertaken by Japanese physicists.[119] In Ref. 119, however, in the calculation of the angular distribution of α particles from the reaction $^{12}C(t, \alpha)^{11}B$ the square of the amplitude of heavy stripping was added to either the square of the replacement amplitude or the square of the pickup amplitude, i.e., the interference between the coherent processes was not taken into account. Italian physicists (Gambarini *et al.*[25]) were the first to use all four processes to analyze experimental data on the (p, α) reactions at $E_p = 38$ MeV on nuclei of the $1p$ shell. However, they used too drastic simplifications (besides the plane-wave approximation), and therefore on the basis of their paper one cannot say how well the experimental angular distributions are described. In some papers,[10,23,142] the coherent contribution of different mechanisms to the total amplitude has been taken into account in an analysis of reactions induced by α particles on lithium isotopes in the plane-wave approximation. However, in these papers the heavy-stripping amplitude was added either the replacement-mechanism amplitude or the ordinary-stripping amplitude. In addition, the interaction between particles x and y at the four-prong vertex of the triangle diagram (see Fig. 20b) was assumed to be δ-functional. Finally, the calculation included not only the usual cutoff radii charac-

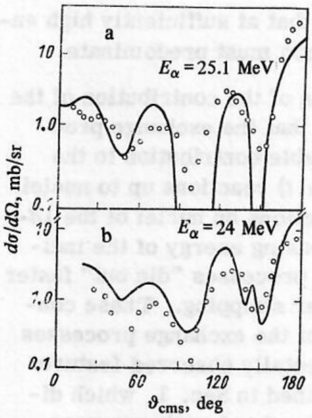

FIG. 22. Comparison of experimental and theoretical angular distributions of deuterons from the $^6Li(\alpha, d_0)^8Be$ reaction. The open circles are from the experiments of Refs. 35 and 36; a) calculation in the plane-wave approximation for exchange processes[35]; b) calculation by the finite-range distorted-wave method for the four main processes.[36]

teristic of the plane-wave approximation but also some further adjustable parameters (the amplitude of the reduced width on the surface of the nucleus and the amplitude of the δ-functional interaction at the x-y scattering vertex). The most systematic allowance for exchange processes in the plane-wave approximation without the approximations made in Refs. 10, 23, and 142 was made in Ref. 35 in a calculation of the angular distribution of deuterons from the reaction $^6Li(\alpha, d)^8Be$. The angular distribution obtained at $E_\alpha = 25$ MeV is shown in Fig. 22a. It can be seen that the calculation reproduces the qualitative features of the angular distribution in the complete range of angles. In particular, there is a correct description of not only the backward peaks but also of the entire structure of the angular distribution at intermediate angles (the number of peaks, their position, and relative magnitude). Comparison of the theoretical angular distributions of deuterons from the reaction $^6Li(\alpha, d)^8Be$ calculated for exchange processes (see Fig. 20a) and for the ordinary-stripping process using the distorted-wave method (see Fig. 17) shows that only exchange processes enable one to obtain qualitative agreement with experiment for all deuteron emission angles. Thus, many calculations have shown that exchange processes can serve as a good approximation for describing the angular distributions in the complete range of emission angles of the final particle. But this conclusion was only qualitative in nature, since all calculations were made in the plane-wave approximation. In recent years, the conclusion that exchange processes are important in nuclear reactions has been confirmed quantitatively, above all by the cycle of investigations made by Edwards and his collaborators.[36,90,113,143] The angular distributions were calculated by them for the first time on the basis of the finite-range distorted-wave method with allowance for all four possible (the FANLU2 program).[144] In their investigations, two aspects may give rise to objection. One of them is the fact that in such an approach allowance is made not only for interference of the mechanisms that correspond to one way of breaking up the nuclei participating in the reaction, but also for interference between direct and exchange mechanisms that cannot occur in the framework of the three-body problem. It is, however, true that, for example, for the reaction $^6Li(\alpha, d)^8Be$ the interference term between the direct and the exchange processes vanishes because of the structural features of the nuclei participating in the reaction, and the approach used is correct.

The second point is that in Edwards's program the intermediate nucleus C (see Fig. 20b) is assumed structureless, i.e., not to have excited states, which seriously restricts the class of reactions for which the program of Ref. 144 can be used. Therefore, Edwards and his collaborators consider only those reactions in which the intermediate nucleus C is an α particle, i.e., for the reaction $^6Li(\alpha, d)^8Be$ the second objection is also removed.

The results of calculations by means of the program FANLU2 of the angular distribution of deuterons from the reaction $^6Li(\alpha, d)^8Be$ at $E_\alpha = 24$ MeV are shown in Fig. 22b (Ref. 36). Here, the adjustable parameters are the coefficients characterizing the relative contribution of the direct and exchange processes. Comparison with experiment showed that the contribution of the direct processes is about 0.5%, i.e., very small. It can be seen from comparison of the curves in Figs. 22a and 22b that allowance for distortions of the plane waves of the initial and final particles does not essentially change the theoretical results but merely "smears" the deep minima and somewhat alters the relative intensities of the peaks. This shows that if the matrix element of the reaction is correctly expressed even a plane-wave calculation can reproduce qualitive features of the angular distributions.

The relative contributions of the two exchange mechanisms, replacement and heavy stripping, calculated by the program FANLU2[113] for the reaction $^7Li(d, t)^6Li$ is shown in Fig. 23. Because of the specific features (both kinematic and structural) of this reaction, it is atypical in the sense of the relationships between the individual mechanisms but, unfortunately, it is only for this reaction that the angular distributions are given separately for each mechanism.

Phenomenological methods of describing the backward peaks and comparison with the microscopic ap-

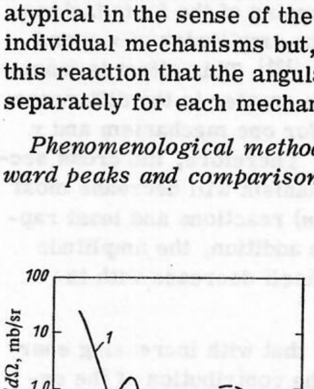

FIG. 23. Angular distributions of tritons from the reaction $^7Li(d, t_0)^6Li$ calculated on the basis of the finite-range distorted-wave method.[13] 1) Replacement; 2) heavy stripping.

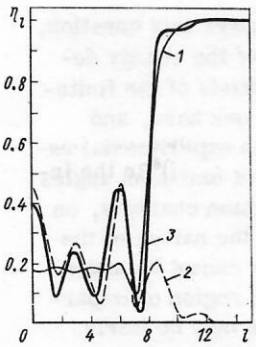

FIG. 24. Reflection coefficient η_l for elastic scattering of α particles on ^{12}C at $E_\alpha = 25$ MeV. 1) Calculation by optical model; 2) calculation for exchange processes; 3) their coherent sum.

proach. All explanations of backward peaks in either scattering or reactions must be based on physical ideas that reject the concept of strong absorption of particles on the edge of the nucleus. [1] The majority of models in which nuclear absorption is reduced near the surface are based on the phenomenological introduction of an additional dependence of the S matrix on the orbital angular momentum l in the neighborhood of $l = l_0 \sim kR$, where k is the wave number and R the radius of the nucleus, and therefore all these phenomenological models apply only to elastic scattering. Nevertheless, one can compare the phenomenological models of the backward peaks and the microscopic approach based on allowance for the exchange processes, since these last describe reactions and elastic scattering in a unified manner and enhance the backward cross section not only in reactions but also in scattering. [119,122,135] Moreover, the exchange processes are particularly important in scattering because in this case there are no direct processes associated with dissociation of the incident particle (such as, for example, stripping).

The phenomenological models used to explain the backward peaks in elastic scattering are analyzed in detail in the review of Ref. 1. We consider here only the most fully developed of them: the model of Regge poles, [145] models with l-dependent[146] and l-splitting[147] imaginary part of the optical potential. In the Regge pole model, the weakening of the nuclear absorption at the edge of the nucleus is achieved by introducing a resonance dependence of the S matrix on l. In the model with l-dependent imaginary part of the optical potential, the large-angle enhancement of the cross section is attributed to an inflection in the region of increase of the S matrix for partial waves with $l \sim l_0$. In the model with l-splitting imaginary part of the optical potential, the absorption is reduced by a correction of variable sign to the imaginary part of the S matrix. This correction has phase $(-1)^l$ and produces a wave-like dependence of the S matrix on l. In other words, the phenomenological models intended to explain the backward peaks lead to a nonmonotonic dependence of the S matrix on l. It is easy to see that allowance for exchange processes in elastic scattering leads to a similar nonmonoticity. For by definition the exchange processes are associated with interaction of the incident particle with individual clusters in the target nucleus. Therefore, the probability of these processes is small for both small and large r, since the overlapping of the wave functions of the bound cluster and the incident particle is small (at small r—in the interior of the nucleus—because of the "dying out" of the particle wave function; at large r—at large distances—because of the strong decrease of the cluster wave functions). Therefore, the addition to the S matrix of elastic scattering associated with exchange processes must have a maximum (or maxima) on the surface of the nucleus. As a result, the dependence of the total S matrix on l may be nonmonotonic, i.e., the microscopic approach with allowance for exchange processes is equivalent in this sense to phenomenological models that introduce a nonmonotonic dependence of the S matrix on the orbital angular momentum.

It was shown in Ref. 137 for the example of elastic scattering of α particles on the ^{12}C nucleus that such equivalence does indeed exist, i.e., the addition of the exchange processes to the scattering potential potential permits one to explain the backward peaks in the elastic scattering of α particles on ^{12}C and to obtain an angular distribution in the complete range of angles in qualitative agreement with the experiments, and gives a fairly complicated dependence of the S matrix on l. The reflection coefficient $\eta_l = |S_l|$ calculated in Ref. 137 is shown in Fig. 24. It can be seen that the behavior of η_l with allowance for the exchange processes is much more complicated than in any phenomenological model. The exchange processes lead to sharp peaks in η_l, which are in fact taken into account in the Regge pole model, and to the appearance of an inflection in the region $l \sim l_0$ (model with l-dependent imaginary part of the potential), and to a correction of variable sign with phase $(-1)^l$ to the imaginary part of the optical S matrix (l-splitting imaginary part of the potential). Therefore, while the various phenomenological models describe only individual aspects of the complete physical behavior of the S matrix, the microscopic approach based on allowance for exchange processes yields the entire behavior.

It should be noted that for the analysis of anomalous backward scattering one can use microscopic approaches that cannot be directly transferred to nuclear reactions. This applies above all to the hypothesis of quasimolecular states[148] formed during the interaction of a particle with a nucleus. In Ref. 149, Fuller has shown that quasimolecular resonances are equivalent to phenomenological models of the Regge pole type, i.e., they must also lead to the appearance of backward peaks in elastic scattering. The quasimolecular resonances must clearly be due, in the first place, to potential scattering. But at the same time it is obvious that if there exist mechanisms leading to the appearance of backward peaks in elastic scattering (related, for example, to some extent with potential scattering), they may also be manifested indirectly in nuclear reactions either through an optical potential, whose parameters must take into account such mechanisms or through channel coupling.

CONCLUSIONS

Our review of experimental investigations in this paper shows that for many reactions backward peaks,

i. e., an enhancement of the cross section near 180°, are observed that are frequently comparable with the small-angle cross section. Backward peaks are typical of a large class of reactions, above all those of α particles on light nuclei, in a wide range of projectile energies. This clearly indicates that the backward peaks reflect fairly general features of the reactions. In model parlance, this means that the backward peaks are a characteristic feature of specific reaction mechanisms. The qualitative analysis and the semiquantitative and first quantitative calculations discussed in the review permit the assertion that the backward peaks, at least basically, are due to exchange processes, in other words, processes in which the target nucleus dissociates. The simplest of these processes are the mechanisms of replacement of nuclear clusters and heavy-particle stripping.

The direct connection between the backward peaks and exchange processes enables us to use data on the behavior of reaction cross sections at large emission angles of the produced particles to investigate clustering effects in different states of nuclei and study structural characteristics of the nuclei. It is important to note that exchange processes, in contrast to direct processes (for example, cluster stripping), enable one to obtain information about not only the reduced widths but also their signs, since the cross section of exchange processes contains a coherent sum over states of the intermediate nucleus, this sum containing the amplitudes of the reduced widths of the individual clusters. Therefore, study of exchange processes may appreciably augment spectroscopic information obtained in other ways.

However, the use of backward peaks as a tool for investigating nuclear structure is possible only after a thorough theoretical analysis of the reaction cross sections in the complete range of emission angles of the final particle and in a fairly wide range of projectile energies. For this it is necessary to make lengthy calculations of the angular and energy dependences of the cross sections on the basis of the finite-range distorted-wave method for reactions of different type with allowance for the contribution of direct and exchange processes. Only such calculations permit one to establish quantitatively the extent to which exchange processes depend on the energy. It would also be worthwhile developing the finite-range distorted-wave method in the framework of the three-body problem by taking into account the three-body corrections to the optical potential, since these corrections can also enhance the cross section at large angles.

It should be emphasized that an important element of uncertainty about the nature of the backward peaks is introduced by the nonmonotonic, resonance nature of the excitation functions at large angles. This aspect of the problem has not been at all well investigated either from the experimental side or from the point of view of explaining the observed resonances. At present, it is impossible to give a definite answer to the question of the extent to which the resonance behavior of the cross section is related to the processes responsible for the backward peaks. To answer this question, it is necessary to make calculations of the energy dependence of the cross section on the basis of the finite-range distorted-wave method, on the one hand, and carry out a detailed comparison of the experimental excitation functions in different ranges of emission angles of the final particle for different reaction channels, on the other. Without this information, the nature of the resonances in the excitation functions cannot be understood. It is to be expected that in the region of α-particle energies below 20–25 MeV there may be fairly broad (hundreds of keV of more) resonances associated, not with direct or exchange processes, but with the formation of comparatively simple configurations of the intermediate nucleus, the so-called doorway states. With increasing energy, the number of such states must decrease.

There has also been inadequate investigation of the amplitudes and other characteristics of the backward peaks as functions of the mass A of the target nuclei and the energy E of the projectile particles. In particular, there is not complete clarity with regard to the question of in which reactions the backward peaks decrease (nor how fast) with increasing A and E. If the backward peaks are basically due to exchange processes, then with increasing A the backward peaks in the (α, t) reaction must decrease the fastest; with increasing E, it must be those in the (α, p) reaction. Evidently, this is confirmed by the currently available experimental data, but they are too few to draw a final conclusion.

It is of interest to investigate reactions for which the exchange processes must make the main contribution to the cross section. As an example we may take the elastic and inelastic scattering of α particles when direct processes are absent and also reactions with the formation of states of the residual nuclei to which transitions are difficult for direct processes (for example, $^6Li(\alpha, d)^8Be$ and $^7Li(\alpha, t)^8Be$ with the formation of a 4^+ state or $^{10}B(\alpha, t)^{11}C$ with formation of a lower $1/2^-$ state).

The results of the experimental investigation of backward peaks for light nuclei show that there are strong differences between the amplitudes of the peaks even for neighboring nuclei. Nevertheless, the detailed connection between the backward peaks and the structure of the nuclei participating in the reaction remains unclear. Theoretical estimates show that the cross section of the exchange processes must depend on the structure of the nuclei, but it is not possible to find a unique correspondence between this and the experimental dependence.

Highly excited states of nuclei have so far remained outside the investigation of backward peaks. With increasing excitation energy, the kinematic singularities of all the simplest mechanisms approach the boundary of the physical region, so that the relative contribution of each of them is basically determined by the reduced width of the corresponding cluster. The reduced width must be close to the Wigner limit in the region of the threshold of breakup of the final nucleus through the given channel. Since direct and exchange processes are related to the reduced width of the various clusters in the final nucleus, direct processes must predominate near one threshold and exchange processes near another.

Additional information about the reaction mechanism when the final nuclei are formed in excited states can be provided by study of the particle–particle angular correlations if the states are above the breakup threshold, or the particle–γ-ray correlations if the excitations of the final nuclei are small.

Thus, investigation of the backward peaks is important both for deeper understanding of reaction mechanisms and to obtain new data about the structure and spectroscopic characteristics of nuclei.

APPENDIX

We consider the derivation of the basic formulas of the finite-range distorted-wave method for calculation of the differential cross sections of nuclear reactions. These formulas are valid if the amplitude of the reaction $A(x,y)B$ is treated in the three-body approximation, i.e., it is assumed that the reaction $A(x,y)B$ takes place in one of the two possible ways:

$$\left.\begin{array}{l} A+x\to A+(y+c)\to (A+c)+y\to B+y-\ \text{(direct processes)};\\ A+x\to (C+y)+x\to (C+x)+y\to B+y-\ \text{(exchange processes)}.\end{array}\right\} \quad \text{(A.1)}$$

In what follows, we shall use this notation: $\mathbf{J}_i$, $\mathbf{L}_i$, and $\mathbf{S}_i$ are the total and orbital angular momenta of particle i and its spin (by i we mean the particles x, y, c, as well as the nuclei A, B, C); $\mathbf{j}=\mathbf{L}_A+\mathbf{S}_B$ is the spin of the channel; $\mathbf{l}=\mathbf{L}_B-\mathbf{L}_A$ is the orbital angular momentum transfer; $\mathbf{s}=\mathbf{S}_B-\mathbf{S}_A$ is the spin transfer; R_i are the center-of-mass coordinates of the corresponding particles; $\mathbf{r}_{in}=\mathbf{R}_i-\mathbf{R}_n$ is the relative coordinate of particles i and n; the relative coordinates $\mathbf{r}_{xA}$ and $\mathbf{r}_{yB}$ will for convenience be denoted simply $\mathbf{r}_x$ and $\mathbf{r}_y$; m_i is the mass of the corresponding particle; $\mu_{in}=m_i m_n/(m_i+m_n)$ is the reduced mass of particles i and n.

We introduce the structure factor Θ_{lsj}, which determines the decay of the corresponding nuclei. For direct processes, $\Theta^{\text{d.p}}_{lsj}$ is the reduced width of decay of nucleus B into $A+C$; for exchange processes, $\Theta^{\text{e.p}}_{slj}$ is determined by the product of the reduced width of decay of nucleus A into $C+y$ and of nucleus B into $C+x$. Then, irrespective of the mechanism, the matrix element of the reaction can be represented in the form

$$M_{if}=\sum_{lsjm_lm_sm_j}\langle J_AM_Asm_s\,|\,jm_j\rangle\langle lm_ljm_j\,|\,J_BM_B\rangle\,\Theta_{lsj}\,\langle s_y\sigma_ysm_s\,|\,s_x\sigma_x\rangle \times\int\chi^*_{ky}(\mathbf{r}_y)\,\mathbf{f}_{lsjm_l}\chi_{kx}(\mathbf{r}_x)\,d\mathbf{r}_x\,d\mathbf{r}_y. \quad \text{(A.2)}$$

In accordance with the splitting of the total amplitude, the form factor f_{lsjm_l} is a sum of two terms of which contains the product of the interaction potential and the wave functions of the relative motion of the particles. For direct processes $f^{\text{d.p}}_{lsjm_l}$ contains[112] the wave function $\Psi_{lm_l}(r_{cA})$ of the relative motion of particle c and nucleus A in the given state of the final nucleus B and the wave function $\varphi(r_{cy})$ of the relative motion of particles y and c in the initial projectile x. For the stripping process, $f^{\text{d.p}}_{lsjm_l}$ contains the potential $V(r_{yc})$ of the interaction of particles y and c (see Fig. 20a). For heavy replacement, the form of the potential is obviously changed; namely, in this case it is necessary to take into account the interaction $V(r_{yA})$ between the initial nucleus A and particle y (see Fig. 20a). As a result

$$\mathbf{f}^{\text{d.p.}}_{lsjm_l}=\Psi_{lm_l}(\mathbf{r}_{cA})\{V(\mathbf{r}_{cy})+V(\mathbf{r}_{yA})\}\,\varphi(\mathbf{r}_{cy}). \quad \text{(A.3)}$$

For exchange processes, the form factor $\mathbf{f}^{\text{e.p}}_{lsjm_l}$ contains the wave functions $\Psi_{\Lambda_x\mu_x}(\mathbf{r}_{xC})$ and $\Psi_{\Lambda_y\mu_y}(\mathbf{r}_{yC})$, which describe the motion of particles y and x in the initial and final nuclei relative to the intermediate nucleus C. As a result, the form factor $\mathbf{f}^{\text{e.p}}_{lsjm_l}$ for the exchange processes depends on the orbital angular momenta Λ_y and Λ_x, which determine the decay of the initial (final) nucleus into the final (incident) particle and the intermediate nucleus C. For the replacement mechanism, the form factor will also contain the potential of the interaction of particles x and y (see Fig. 20b); for the heavy-stripping mechanism, it will contain the potential of the interaction of particle y and the intermediate nucleus C. As a result, for the exchange processes the form factor $f^{\text{e.p}}_{lsjm_l}$ has the form

$$\mathbf{f}^{\text{e.p.}}_{lsjm_l}\equiv\mathbf{f}^{\text{e.p.}}_{lsjm_l\Lambda_x\Lambda_y}=\sum_{\mu_x\mu_y}(-1)^{\mu_y}\langle\Lambda_x\mu_x\Lambda_y-\mu_y\,|\,lm_l\rangle \times\Psi^*_{\Lambda_x\mu_x}(\mathbf{r}_{xC})\{V(\mathbf{r}_{xy})+V(\mathbf{r}_{yC})\}\,\Psi_{\Lambda_y\mu_y}(\mathbf{r}_{yC}). \quad \text{(A.4)}$$

In what follows, we shall ignore the spin–orbit interaction of the particles. Of course, when the polarization of the particles is calculated this is certainly unjustified. However, to calculate the cross sections, the excitation spectra of the final nuclei, the angular correlations of the reaction products, and the other characteristics it is perfectly justified to ignore the spin dependence of the potential. Since the form factors $\mathbf{f}_{lsjm_l}$ do not depend on s or j for central potentials, these indices can be omitted, i.e., we can write $\mathbf{f}_{lsjm_l}=\mathbf{f}_{lm_l}$.

The main complexity in transforming the factor arises from the need to go over to the variables r_x and r_y, since the distorted optical wave functions over which the matrix element (A.2) is averaged depend on precisely these variables. According to Ref. 112, this problem is solved as follows.

1. The form factor f_{lm_l} is expanded with respect to the partial waves of the ingoing, L_x, and outgoing, L_y, channels by means of the relation

$$F_{L_xL_yl}(r_x,r_y)=\sum_{\mu_x\mu_y}\langle L_xM_xL_yM_y\,|\,lm_l\rangle\int d\vartheta_x\,d\vartheta_y\,\mathbf{f}_{lm_l}Y_{L_xM_x}(\mathbf{r}_x)\,Y_{L_yM_y}(\mathbf{r}_y). \quad \text{(A.5)}$$

2. The coordinates r_{in} are expressed in terms of r_x and r_y. For direct processes

$$\left.\begin{array}{l} \mathbf{r}_{cA}\equiv\mathbf{r}_1=\alpha(\mathbf{r}_x-\gamma\mathbf{r}_y);\quad m_M\equiv m_x+m_A\equiv m_y+m_B;\\ \mathbf{r}_{cy}\equiv\mathbf{r}_2=\alpha(\mathbf{r}_y-\delta\mathbf{r}_x);\quad \alpha=m_xm_B/m_cm_M;\\ \mathbf{r}_{yA}\equiv\mathbf{r}_3=(m_x\mathbf{r}_x+m_B\mathbf{r}_y)/m_M;\quad \gamma=m_y/m_x;\quad \delta=m_A/m_B.\end{array}\right\} \quad \text{(A.6)}$$

For exchange processes, we have the analogous expressions

$$\left.\begin{aligned} \mathbf{r}_{xC} &\equiv \mathbf{r}_1' = \alpha'(\mathbf{r}_x + \gamma'\mathbf{r}_y); \quad \alpha' = m_A m_B/m_C m_M; \\ \mathbf{r}_{yC} &\equiv \mathbf{r}_2' = \alpha'(\mathbf{r}_y + \delta'\mathbf{r}_x); \quad \gamma' = m_y/m_A; \quad \delta' = m_x/m_B; \\ \mathbf{r}_{xy} &\equiv \mathbf{r}_3' = m_A\mathbf{r}_x/m_M - m_B\mathbf{r}_y/m_M. \end{aligned}\right\} \tag{A.7}$$

After corresponding expression of the coordinates, the vector spherical harmonics $r^l Y_{lm}(\mathbf{r})$, where $\mathbf{r} = s\mathbf{r}_x + t\mathbf{r}_y$, are expanded in a series in accordance with[150]:

$$r^l Y_{lm}(\mathbf{r}) = \sum_{\lambda\mu} \sqrt{\frac{4\pi}{2\lambda+1}} \binom{2l+1}{2\lambda}^{1/2} (sr_x)^{l-\lambda}$$
$$\times (tr_y)^{\lambda} \langle l-\lambda\ m-\mu\ \lambda\mu \mid lm\rangle Y_{l-\lambda\ m-\mu}(\mathbf{r}_x) Y_{\lambda\mu}(\mathbf{r}_y). \tag{A.8}$$

The expression (A.8) enables one to factorize the angular dependences of the form factors (A.3) and (A.4) and calculate with respect to the angles the integral in the invariant form factor (A.5).

3\. The remaining scalar part [for stripping this is the product of the radial functions of the relative motion (without r^l) and the potential of the y-c interaction] is expanded in a series in Legendre polynomials of the cosine of the angle between r_x and r_y (in what follows, we write $\cos(r_x r_y) = \mu$). For stripping, this expansion has the form

$$r_1^{-l} R_l(r_1) V(r_2) \varphi(r_2) = \sum_{K=0}^{\infty} (K+1/2)\, g_K^{\mathrm{st}}(r_x, r_y) P_K(\mu), \tag{A.9}$$

where

$$g_K^{\mathrm{st}}(r_x, r_y) = \int_{-1}^{+1} d\mu r_1^{-l} R_l(r_1) \varphi(r_2) V(r_2) P_K(\mu). \tag{A.10}$$

One can write down similarly expressions $g_K(r_x, r_y)$ for the other mechanisms

$$g_K^{h.\,\mathrm{rep}}(r_x, r_y) = \int_{-1}^{+1} d\mu r_1^{-l} R_l(r_1) V(r_3) \varphi(r_2) P_K(\mu); \tag{A.11}$$

$$g_K^{\mathrm{rep}}(r_x, r_y) = \int_{-1}^{+1} d\mu r_1'^{-\Lambda_x} R_{\Lambda_x}(r_1') V(r_3') r_2'^{-\Lambda_y} R_{\Lambda_y}(r_2') P_K(\mu); \tag{A.12}$$

$$g_K^{h.\,\mathrm{st}}(r_x, r_y) = \int_{-1}^{+1} d\mu r_1'^{-\Lambda_x} R_{\Lambda_x}(r_1') V(r_3') r_2'^{-\Lambda_y} R_{\Lambda_y}(r_2') P_K(\mu). \tag{A.13}$$

After these transformations, the form factor (A.5) can be calculated analytically. For direct processes, it can be represented in the form

$$F^{\mathrm{d.p.}}_{L_x L_y l}(r_x, r_y) = \alpha^l \sqrt{\pi(2l+1)} \sum_{K\lambda} (-1)^K (2K+1) \binom{2l}{2\lambda}^{1/2}$$
$$\times \langle \lambda O K O \mid L_x O\rangle \langle l-\lambda O K O \mid L_y O\rangle r_x^{\lambda} (-y r_y/x)^{l-\lambda}$$
$$\times W(L_x \lambda L_y l-\lambda : Kl) \{g_K^{\mathrm{st}} + g_K^{h.\,\mathrm{rep}}\}, \tag{A.14}$$

and for exchange processes in the form

$$F^{\mathrm{e.p.}}_{L_x L_y l}(r_x, r_y) \equiv F^{\mathrm{e.p.}}_{L_x L_y l \Lambda_x \Lambda_y}(r_x, r_y) = 1/2 \sum_{\lambda_x \lambda_y K} (\alpha')^{\Lambda_x + \Lambda_y}$$
$$\times (r_x)^{\Lambda_x - \lambda_x} (\delta' r_x)^{\lambda_y} (r_y)^{\Lambda_y - \lambda_y} (\gamma' r_y)^{\lambda_x} (-1)^K (2K+1)$$
$$\times \binom{2\Lambda_x}{2\lambda_x}^{1/2} \binom{2\Lambda_y}{2\lambda_y}^{1/2} (2\Lambda_x+1)(2\Lambda_y+1) \{g_K^{\mathrm{rep}} + g_K^{h.\,\mathrm{rep}}\}$$
$$\times \sum_{l_x l_y} \sqrt{(2l_x+1)(2l_y+1)} \langle \Lambda_x - \lambda_x O \lambda_y O \mid l_x O\rangle$$
$$\times \langle l_x O K O \mid L_x O\rangle \langle \lambda_x O \Lambda_y - \lambda_y O \mid l_y O\rangle \langle l_y O K O \mid L_y O\rangle$$
$$\times W(l_y L_y l_x L_x : Kl) \begin{Bmatrix} \Lambda_x - \lambda_x & \lambda_x & \Lambda_x \\ \lambda_y & \Lambda_y - \lambda_y & \Lambda_y \\ l_x & l_y & l \end{Bmatrix}. \tag{A.15}$$

Using partial-wave expansions of the distorted waves:

$$\chi_k(r) = \sum_{LM} \frac{4\pi}{kr} i^L \chi_L(kr) Y_{LM}(r) Y^*_{LM}(k),$$

we can write the differential cross section of the direct processes as follows:

$$\frac{d\sigma^{\mathrm{d.p.}}}{d\Omega} = \frac{\mu_{xA}\mu_{yB}}{(2\pi\hbar^2)^2} \frac{k_y}{k_x} \frac{2J_B+1}{2J_A+1} \sum_{ls} \frac{S_{ls}}{2s+1} \sum_{m_l} |\beta^{\mathrm{d.p.}}_{lm_l}(\vartheta)|^2, \tag{A.16}$$

where

$$\beta^{\mathrm{d.p.}}_{lm_l}(\vartheta) = \frac{4\sqrt{2}\,\pi}{k_x k_y} \sum_{L_x L_y} i^{L_x + L_y - l} \langle L_y m_l l - m_l \mid L_x O\rangle I^{\mathrm{d.p.}}_{L_x L_y l} \overline{P}^{m_l}_{L_y}(\vartheta) (-1)^{m_l}, \tag{A.17}$$

in which $\overline{P}^{m}_{L}(\vartheta)$ are normalized associated Legendre polynomials; $S_{ls} = \sum_j \Theta^2_{lsj}$ are the spectroscopic factors of particle c; $I_{L_x L_y l}$ are radial integrals associated with the invariant form factors (A.14) and (A.15):

$$I_{L_x L_y l} = \int_0^\infty r_x\, dr_x \int_0^\infty r_y\, dr_y \chi_{L_y}(k_y r_y) F_{L_x L_y l}(r_x, r_y) \chi_{L_x}(k_x r_x). \tag{A.18}$$

Since the invariant form factor for the exchange processes depends on Λ_x and Λ_y, the expression for the differential cross section in this case has a different form since $\beta^{\mathrm{e.p}}_{lm_l}(\vartheta)$ also depends on Λ_x and Λ_y. As a result,

$$\frac{d\sigma^{\mathrm{e.p.}}}{d\Omega} = \frac{\mu_{xA}\mu_{yB}}{(2\pi\hbar^2)^2} \frac{k_y}{k_x} \frac{2J_B+1}{2J_A+1} \sum_{lsjm_l} \Big| \sum_{\Lambda_x \Lambda_y} \Theta^{\mathrm{e.p.}}_{lsj} \beta^{\mathrm{e.p.}}_{lm_l}(\vartheta) \Big|^2, \tag{A.19}$$

where

$$\beta^{\mathrm{e.p.}}_{lm_l}(\vartheta) \equiv \beta^{\mathrm{e.p.}}_{lm_l \Lambda_x \Lambda_y}(\vartheta) = \frac{4\sqrt{2}\pi}{k_x k_y} \sum_{L_x L_y} i^{L_x + L_y - l}$$
$$\times \langle L_y m_l l - m_l \mid L_x 0\rangle I^{\mathrm{e.p.}}_{L_x L_y l \Lambda_x \Lambda_y} \overline{P}^{m_l}_{L_y}(\vartheta) (-1)^{m_l}. \tag{A.20}$$

For completeness, we also give the expression for $\Theta^{\mathrm{e.p}}_{lsj}$ which are determined by the amplitudes of the reduced widths $\Theta_{\Lambda_i s_i j_i}$ of the decay of nucleus A (respectively, B) into the final (respectively, initial) particle and the intermediate nucleus C in the state with total angular momentum J_C. These quantities can be written in the form

$$\Theta^{\mathrm{e.p.}}_{lsj} \equiv \Theta^{\mathrm{e.p.}}_{lsj\Lambda_x \Lambda_y} = \sum_{j_x j_y J_C} \Theta_{\Lambda_y s_y j_y} \Theta_{\Lambda_x s_x j_x} (-1)^{J_C} \sqrt{\frac{2J_A+1}{2J_C+1}}$$
$$\times u(j_x s_x j_y s_y : J_C s)\, u(\Lambda_y J_A j_x s : j_y j)\, u(j \Lambda_y J_B \Lambda_x : j_x l). \tag{A.21}$$

The expressions (A.16)–(A.21) enable one to calculate the differential cross section of the reaction $A(x, y)B$ by means of the finite-range distorted-wave method in the three-body problem.

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