Fragmentation of nuclei by high-energy particles

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The present status of the study of fragmentation of nuclei by high-energy particles is reviewed.

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INTRODUCTION

The development of experimental investigations in high energy nuclear physics in recent years has led to an apparent important change in the views concerning deep inelastic interaction between fast particles and nuclei. It has become clear that the well known model which describes these interactions as taking place in two stages has restrictions as regards its applicability to many reactions. The first of the stages is characterized by the development of a cascade of two-particle collisions with the nucleons of the nucleus, treated as a Fermi gas, and the second by equilibrium decay of the nucleus that remains after the cascade has ended.

One of the best known reactions of this kind is fragmentation. The history of the investigations of this phenomenon now stretches over nearly forty years, during which period very extensive experimental material has been accumulated and we have seen the development of refined experimental methods that use the latest advances in radiochemistry, mass spectrometry, and the technology of semiconductor and track detectors. The theoretical investigations of the problem have developed both through the use of the standard model of equilibrium evaporation and by an advance beyond the framework of ordinary ideas, since these were found to be not very applicable for understanding fragmentation. Attempts were made to attribute fragmentation processes to the formation in nuclei of shock waves,[1] the possibility of "dynamic association" of nucleons in the interior of the nucleus[2] or on its surface,[3] with the formation in nuclei of collective vibrations,[4] and so forth. Some of these hypotheses have become the subject of independent investigation,[1,3] but none of them have been successful in explaining the emission of complex particles by nuclei.

Some practical aspects of modern science and technology are already intimately related to the use of or allowance for fragmentation of nuclei. The problems for which this phenomenon is of significance include, for example, some relating to the origin and composition of chemical radiation, cosmic biology and medicine, cosmochemistry, the search for new light nuclei outside the stability band, the physics of hyperfragments, and the technology and physics of electronuclear installations. Therefore, the need to study fragmentation which arises from the perfectly natural need to understand the nature of a phenomenon that lies outside the scope of the already tested theoretical schemes, also acquires applied importance.

It is possible that in the next years it will be possible to solve the principal question relating to this remarkable phenomenon—the fragmentation reaction mechanism. Hope for this is provided by the fact that during the last ten years the experiments have succeeded in revealing a number of fundamental features of the processes of emission of "slow" isotopes from nuclei bombarded by high energy particles and the theory of pre-equilibrium processes has been developed, which in principle makes it possible to overcome the difficulties of the model of equilibrium evaporation. If it should be shown that fragmentation really is a process of pre-equilibrium type, a unique opportunity is presented for studying strongly nonequilibrium states and relaxation processes in systems of nucleons and of nucleons and hyperons (hyperfragmentation). The use of polarized nuclear targets and intense π-meson beams with pion energy near $\pi-N$ resonances and nuclear beams may make it possible to investigate not only the problem of the excitation and nature of the nonequilibrium states deexcited by fragments but also estimate the relation between the de-excitation time and the time of relaxation of the excited nuclei from the sensitivity of the reaction characteristics to the original orientation of the deformed target nuclei or the sign of the pion. However, the solution of these problems requires from theoreticians intensive development of models, and from experimentalists the systematization of the experimental data into a system as universal as possible. Both are very complicated problems.

The aim of the present review is to consider the experimental data obtained mainly during the last five to seven years and discuss the general dependences that can already be regarded as well established. We ignore the main mass of data on the emission of complex particles from light nuclei, since this problem has its own methods of solution and its own very extensive literature. An account of the work done prior to 1970 and not considered in the present review can be found in the review of Ref. 5, several books,[6–8] and the paper of Lozhkin and Perfilov in the collection of Ref. 9.

FRAGMENTATION CROSS SECTIONS AND EXCITATION FUNCTIONS

The emission of fragments is observed when nuclei interact with different particles and nuclei. Until recently the majority of data referred to the production cross sections and the kinematic characteristics of fragments convenient for identification by radiochemical methods (for example, ^{22}Na, ^{24}Na, ^{18}F, ^{7}Be, ^{3}H) and nuclides amenable to identification in photoemul-

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sions (^{8}He, ^{8}Li, ^{8}Be, ^{8}B, ^{9}Li), but now advances in semiconductor detectors and magnetic analyzers have made it possible to study practically all isotope-stable fragments from the isotopes of hydrogen to the limit of medium nuclei. For all fragments, the cross sections depend very strongly on the properties of the fragment and the target nucleus, and also on the energy and species of the bombarding particles. A general feature in the excitation functions of fragmentation reactions is the rapid growth of the cross sections in the region of primary-particle energies up to 1–2 GeV and the plateau which these cross sections then reach. The rate at which the cross sections reach the plateau and the steepness of the growth of the cross sections depend strongly on the atomic number of the target nucleus and the fragment.

The behavior of the fragmentation cross sections as functions of the target-nucleus properties at given energy of the primary particle is not determined solely by the number of nucleons of the target nucleus and the fragment (Fig. 1), but satisfies more complicated laws. These laws could be established only by studying the so-called isotope effects, i.e., the different kinds of relations between the characteristics of disintegration of the nuclei of isotopes of one element.[10–15] The first measurements of the cross sections for the emission of complicated isotopes from separated isotopes bombarded by high energy particles were reported in Ref. 10, which contains the results of investigation of the emission of sodium isotopes from ^{92}Mo and ^{100}Mo nuclei bombarded by 25-GeV protons. It should be pointed out that at this very high energy of the primary beam the sodium isotopes produced by bombardment of nuclei with $A \approx 100$ may be products of asymmetric fission. Thiboult-Philippe[10] found a difference between the cross sections for the emission of sodium isotopes from ^{92}Mo and ^{100}Mo, although no general laws were found. In Refs. 11–13, appreciable fluctuations were found for the first time in the cross sections for the emission of light fragments from isotopes of medium and heavy[10,11] and light nuclei[13] bombarded by high energy protons, and the dependence of these fluctuations on the characteristics of the target nuclei and the fragments was found. The dependences are shown in Figs. 2 and 3,

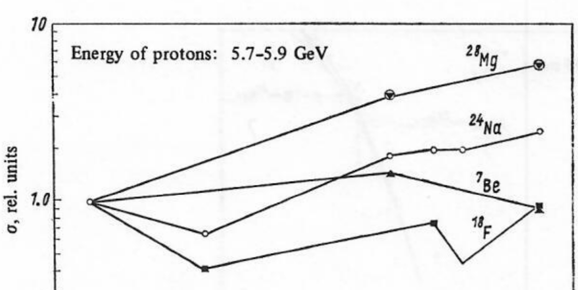

FIG. 1. Dependence of excitation functions on the masses of the fragments and target nuclei.[19]

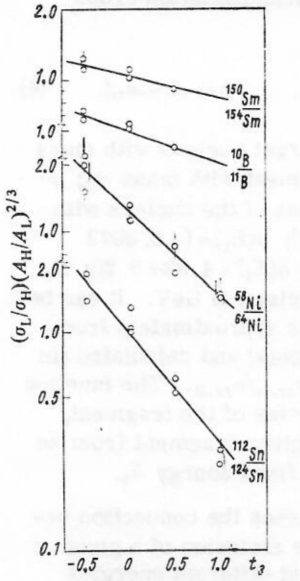

FIG. 2. Dependence of $(\sigma_L/\sigma_H)(A_H/A_L)^{2/3}$ on t_3 of the fragments.

in which the ratios of the fragmentation cross sections normalized to the total inelastic cross section for light and heavy target isotopes are shown as functions of the third projections of the isospins of the target, T_3, and fragment, t_3. The experimental data satisfy the approximate relations

$$\ln [(\sigma_L/\sigma_H)(A_H/A_L)^{2/3}] \sim t_3; \tag{1}$$

$$\ln [(\sigma_L/\sigma_H)(A_H/A_L)^{2/3}] \sim T_{3,L}/A_L^{2/3} - T_{3,H}/A_H^{2/3}, \tag{2}$$

where the subscripts L and H refer to light and heavy target isotopes, respectively.

The requirements of isospin invariance of strong interactions and the nearly equal height of the Coulomb barriers for the isotopes of one element enable one to generalize the relations (1) and (2):

$$\ln [(\sigma_L/\sigma_H)(A_H/A_L)^{2/3}] \sim t_3 (T_{3,L}/A_L^{2/3} - T_{3,H}/A_H^{2/3}). \tag{3}$$

In Ref. 16, Bogatin *et al.* proposed an empirical

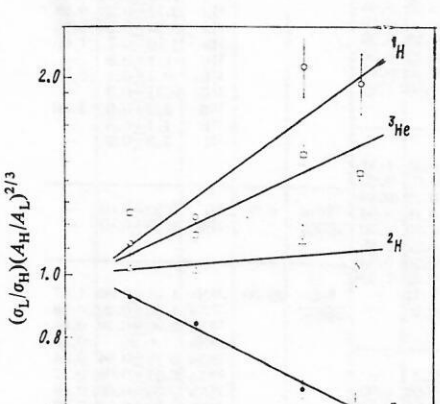

FIG. 3. Dependence of $(\sigma_L/\sigma_H)(A_H/A_L)^{2/3}$ on $\Delta(T_3/A^{2/3}) = T_{3,H}/A_H^{2/3} - T_{3,L}/A_L^{2/3}$.

formula for parametrizing the fragmentation cross sections σ_{fr}:

$$\sigma_{fr} \approx f(\mathrm{fr})\, R_0^2 \exp\{[(R_1+R_2)/r_1^3]\, \mu\, [a(E_1)+b(E_1)\, T_{3,1} t_3/m_1 m_2]\}, \qquad (4)$$

where R_0 is the radius of the target nucleus with mass m_0; R_2 is the radius of the fragment with mass m_2; $\mu = (m_0 - m)m_2/m_0$; R_1 is the radius of the nucleus with mass $m_1 = m_0 - m_2$; $r_1 = R_1/m_1^{1/3}$; $a(E_1) = (-0.0072 \pm 0.0010) - (0.432 \pm 0.0050)/E_1$; $b(E_1) = 4.32 \pm 0.20$; E_1 is the energy of the primary nucleon in GeV. It can be seen that the relation (3) follows approximately from (4). Table I gives the experimental and calculated [in accordance with (4)] values of $\sigma_{fr,L}/\sigma_{fr,H}$. The function $f(\mathrm{fr})$ depends only on the properties of the fragment, and it can be determined for a given fragment from an experiment on one nucleus at a fixed energy E_1.

Thus, the relation (4) establishes the connection between the cross sections for the emission of a given fragment from different nuclei at different energies (from 0.5 GeV upward) and under the condition that $m_2 \leqslant \frac{1}{3} m_0$. Figures 4 and 5 show the behavior of the fragmentation cross sections calculated in accordance with formula (4).

In Ref. 15, the existence of a logarithmic dependence of $\sigma_{fr}/\pi R_0^2$ on T_3/m_0 was confirmed for the example of fragmentation of nuclei from Ti to U at $E_1 = 1.0$ GeV.

TABLE I. Ratio of cross sections of fragment production in light (L) and heavy (H) isotopes.

Isotope pair	E_1, GeV	Fragment	σ_L/σ_H Experiment	σ_L/σ_H Calculated from Eq. (4)
$^{10}B/^{11}B$	0.66	^1H	1.23±0.04	0.95
		^2H	1.02±0.04	0.96
		^3H	0.84±0.04	0.80
		^3He	1.15±0.03	1.17
		^4He	1.07±0.02	0.98
$^{58}Ni/^{54}Ni$	0.66	^1H	2.00±0.20	—
		^2H	1.14±0.05	0.95
		^3H	0.66±0.03	0.69
		^3He	1.52±0.08	1.31
		^4He	1.17±0.03	0.95
		^6He	0.5	0.49
		^6Li	1.34±0.13	0.98
		^7Li	0.87±0.12	0.70
		^8Li	0.52±0.10	0.49
		^7Be	2.20±0.50	1.40
	1.0	^3He	1.09±0.15	1.31
		^4He	0.90±0.07	0.95
		^6He	0.29±0.12	0.49
		^6Li	0.90±0.16	0.98
		^7Li	0.53±0.13	0.70
		^8Li	0.45±0.22	0.49
		^9Li	0.40±0.20	0.33
		^7Be	1.34±0.22	1.40
		^9Be	0.60±0.12	—
		^{10}Be	0.30±0.20	—
$^{112}Sn/^{124}Sn$	0.66	^1H	1.9±0.2	—
		^2H	1.24±0.08	0.95
		^3H	0.75±0.04	0.61
		^3He	1.84±0.03	1.49
		^4He	1.61±0.04	0.97
$^{112}Sn/^{124}Sn$		^6He	0.3	0.38
		^6Li	1.26±0.04	0.99
		^7Li	0.65±0.04	0.62
		^8Li	0.36±0.05	0.38
	1.0	^3He	1.76±0.16	1.49
		^4He	1.65±0.12	0.97
		^6He	0.41±0.16	0.38
		^6Li	1.66±0.21	0.99
		^7Li	1.05±0.13	0.62
		^8Li	0.72±0.22	0.38
		^9Li	0.30±0.20	0.23
		^7Be	2.70±0.7	1.60
		^9Be	1.1±0.3	—
		^{10}Be	0.7±0.1	0.37
		^{11}Be	0.37±0.3	—
		^{10}B	2.2±0.8	1.00
		^{11}B	1.1±0.3	—
		^{12}C	2.3±0.8	—
$^{150}Sm/^{154}Sm$	0.66	^3H	0.92±0.05	—
		^3He	1.25±0.07	—
$^{92}Mo/^{100}Mo$	25.00	^{21}Na	1.36±0.30	1.47
		^{22}Na	1.00±0.20	0.98
		^{23}Na	0.90±0.10	0.64
		^{24}Na	0.95	0.42
		^{25}Na	0.78±0.04	0.24
		^{26}Na	0.61±0.05	0.17
		^{27}Na	0.34±0.06	0.16
		^{28}Na	0.28±0.13	0.10
		^{29}Na	0.13±0.16	0.07

Note. The experimental data are taken from Refs. 10 and 12–15, and the calculations from Ref. 16.

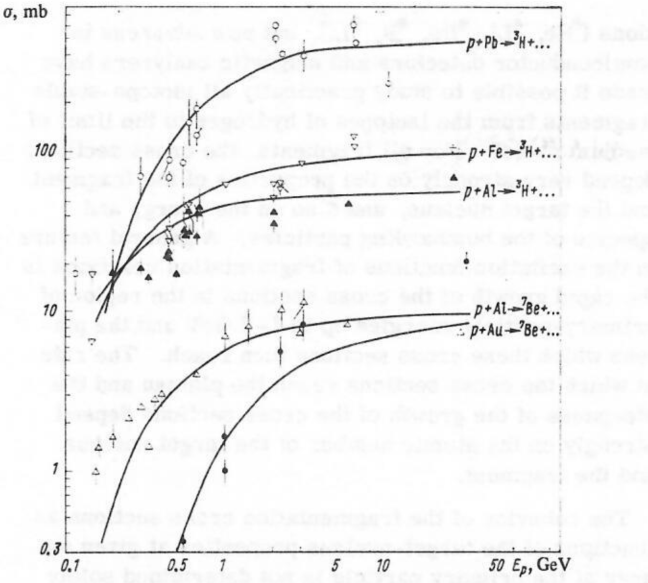

FIG. 4. Excitation functions of the fragmentation reaction $p+A\to {}^3H+\ldots$, $p+A\to {}^7Be+\ldots$.

It was already shown in Ref. 12 that the experimental data on the ratios σ_L/σ_H lie near the curve $\exp\{\alpha(Q_L - Q_H)\}$, where Q_i is the energy of separation of the fragment from the target nucleus i (or the nucleus that remains once the cascade has ended), and α is of order 0.1. Figure 6 shows the corresponding dependences. Note that the exponential dependence of σ_L/σ_H on $Q_L - Q_H$ does not contradict formula (3). This follows, for example, from von Weizsäcker's formula for the nuclear masses. However, for the cross sections σ_{fr} this dependence requires σ_{fr} to be proportional to

$$\sigma_{fr} \sim \exp\{\alpha Q\}. \qquad (5)$$

The relation (5) recalls the well known relation for reactions of ions with nuclei[17] and contradicts formula (4).

In Ref. 16 it was noted that formula (4) does not take into account fully the relation (5) and is therefore of a restricted nature. We shall return to this question below in a combined analysis of the cross sections and

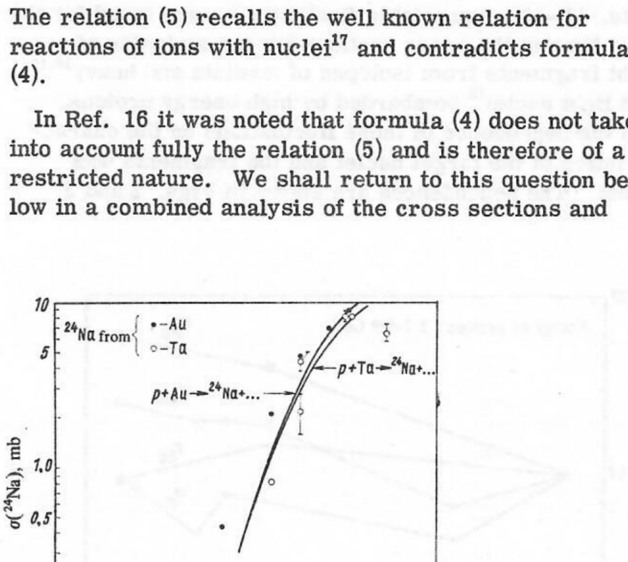

FIG. 5. Excitation functions of the fragmentation reaction $p+A\to {}^{24}Na+\ldots$.

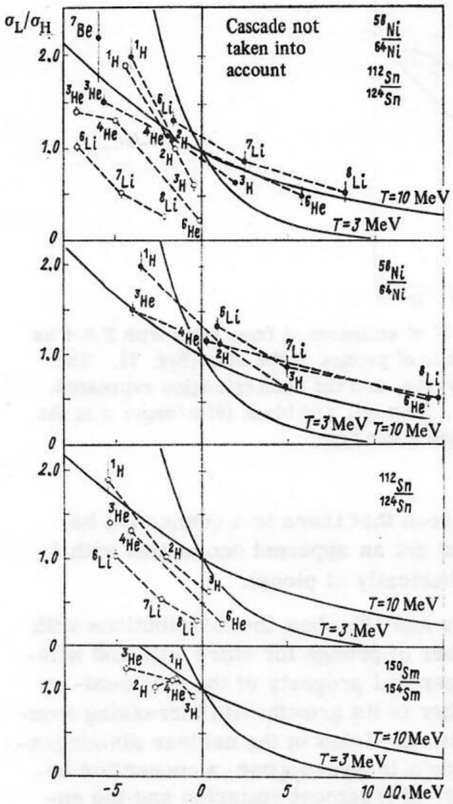

FIG. 6. Dependence of σ_L/σ_H on $Q_L - Q_H = \Delta Q$ (Ref. 12).

TABLE II. Cross sections of fragment production induced by different particles with nearly equal E_1.

Projectile	E_1, GeV	Fragment	Target nucleus	σ, mb	Literature	Projectile	E_1, GeV	Fragment	Target nucleus	σ, mb	Literature
π^+	0.080	$Z \geqslant 4$	AgBr	1.2 ± 0.5	[6]	α	0.70	^{24}Na	Ag	0.227	[19]
p	0.075	$Z \geqslant 4$	AgBr	0.3 ± 0.1	[7]	p	0.70	^{24}Na	Au	0.135	[19]
p	0.10	$Z \geqslant 4$	AgBr	1.0 ± 0.3	[7]	α	0.70	^{24}Na	Au	0.308	[19]
p	0.10	$Z \geqslant 4$	AgBr	0.81 ± 0.29	[7]	p	0.70	^{24}Na	U	0.230	[19]
π^+	0.280	$Z \geqslant 4$	AgBr	1.4 ± 0.5	[6]	α	0.70	^{24}Na	U	0.502	[19]
p	0.20	$Z \geqslant 4$	AgBr	2.5 ± 0.5	[7]	p	0.70	^{28}Mg	Ag	0.012	[19]
p	0.30	$Z \geqslant 4$	AgBr	2.6 ± 0.5	[7]	α	0.70	^{28}Mg	Ag	0.026	[19]
π^-	7.2	$Z \geqslant 4$	AgBr	70 ± 15	[6]	p	0.70	^{28}Mg	Au	0.054	[19]
p	6.0	$Z \geqslant 4$	AgBr	90 ± 17	[7]	α	0.70	^{28}Mg	Au	0.102	[19]
p	9.0	$Z \geqslant 4$	AgBr	88 ± 17	[7]	p	0.70	^{28}Mg	U	0.115	[19]
p	9.0	$Z \geqslant 4$	AgBr	100 ± 30	[7]	α	0.70	^{28}Mg	U	0.502	[19]
p	0.70	^{24}Na	Ag	0.100	[19]						

the profiles of the fragment spectra.

The dependence of the fragmentation cross sections on the species of bombarding particle has been poorly studied. There is some information about the yield of ^{8}Li from a number of nuclei at energy 190 MeV of primary deuterons,[18] heavy fragments (^{24}Na, ^{28}Mg) from bombardment of medium and heavy nuclei by α particles with energy up to 0.88 GeV (Ref. 19), and light and medium fragments (up to Be) from bombardment with energy 2.1 GeV per nucleon.[20] Tables II and III contain some of the data of Refs. 6, 7, 19, 20, and 45. In all cases, at comparable energies of the bombarding nuclei and protons the cross sections for the emission of fragments in the case of bombardment by nuclei are greater than for bombardment by protons. Unfortunately, the experimental material on fragmentation due to bombardment by nuclei is not yet sufficiently extensive to reveal clear quantitative laws, but the extension of this field of investigations at Dubna and Berkeley promises to produce very interesting results.

2. DEPENDENCE OF THE PROBABILITY OF FRAGMENT EMISSION ON THE NATURE OF THE NUCLEAR DISINTEGRATION

In the preceding section, we have given experimental data suggesting that the properties of the target nucleus and the fragment and also the species of bombarding particle have a strong influence on characteristics of the fragmentation such as the fragment-emission cross section. The strong connection between the fragmentation cross section and the properties of the nuclei participating in the reaction suggested that the fragmentation reaction is sensitive to very fine details of the nuclear structure.[11,12] In favor of this assumption, one can bring forward an argument based on the fact that if fragmentation is a fast, nonequilibrium process, then the initial conditions under which the reaction takes place must affect its characteristics. That nonequilibrium processes of fragmentation are important was proved in Refs. 21–23 by analyzing data on the angular distributions and spectra of isotopes of helium and hydrogen and ^{8}Li and ^{24}Na. In Ref. 12 it was shown that the attempt to interpret the results on the ratio σ_L/σ_H in the framework of the model of equilibrium evaporation with parameters that are taken from its most perfected variant (proposed by Barashenkov and Toneev and collaborators; see, for example, Ref. 7), leads to a dependence of the form

$$\sigma_L/\sigma_H \sim \exp\{-(Q_L - Q_H)\,\alpha_1\}.$$

But the parameter $\alpha_1 = 1/T$, where T is the temperature of the nucleus, is so large that the corresponding calculated curves simply intersect the region in which the experimental data lie (see Fig. 6). Thus, the following question arises: With what stage in the development of the reaction between the particle and the nucleus is the fragmentation phenomenon associated? In proofs that the fragmentation is associated with the equilibrium stage, reference is usually made to the dependence of its characteristics on the kind of disintegration in which

TABLE III. Ratios of cross sections of fragment production induced by relativistic nuclei and protons (region of proton energies E_1 where the cross sections reach the "plateau").

Target nucleus	Fragment	$\frac{\sigma(E_1, A)}{\sigma(E_1, p)}$	Target nucleus	Fragment	$\frac{\sigma(E_1, A)}{\sigma(E_1, p)}$
U (Ref. 20) for $\frac{E_1; A}{E_1; p} = \frac{4.2;\ d}{4.9;\ p}$	^{4}He	1.4		^{8}Li	4.2
	^{6}He	1.5		^{9}Li	3.8
	^{6}Li	1.3		^{7}Be	4.3
	^{7}Li	1.5		^{9}Be	3.4
	^{8}Li	1.2		^{10}Be	4.0
	^{9}Li	1.5	Ag (Ref. 45) for $\frac{E_1; A}{E_1; p} = \frac{25.2;\ ^{12}\mathrm{C}}{300.0;\ p}$	^{7}Be	2.44 ± 0.12
	^{7}Be	1.5		^{24}Na	1.66 ± 0.06
	^{9}Be	1.3		^{28}Mg	1.73 ± 0.12
	^{10}Be	1.6			
U (Ref. 20) for $\frac{E_1; A}{E_1; p} = \frac{8.4;\ \alpha}{4.9;\ p}$	^{4}He	2.9			
	^{6}He	3.1			
	^{6}Li	3.8	Ag (Ref. 45) for $\frac{E_1; A}{E_1; p} = \frac{34.0;\ \mathrm{O}}{2.1;\ p}$	$Z = 12$	3
	^{7}Li	3.8		$Z = 8$	25

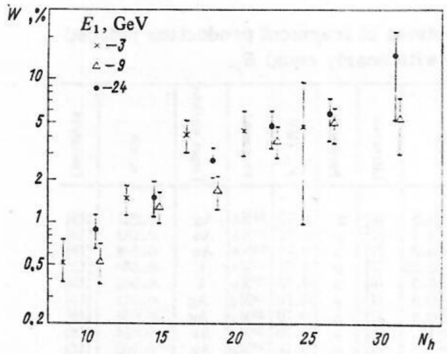

FIG. 7. Probability W of production of ^{8}Li fragment as a function of the number of heavy particles N_h in the reaction.[7]

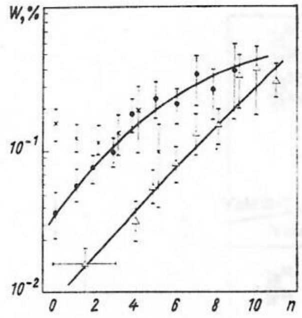

FIG. 8. Probability W of emission of fragments with $Z \geq 4$ as a function of the number of prongs in the star (Ref. 7). The crosses, the solid circles, and the open triangles represent, respectively, thin (s), grey (g), and black (b) prongs; n is the number of prongs of the given type.

the fragments are produced. In our opinion, it would therefore be helpful to consider the bases for this argument in more detail.

Investigations of fragmentation by the photoemulsion method make it possible to establish a connection between some of the characteristics of the "star" in which the fragment is emitted and the probability of emission of a fragment. The most frequently investigated characteristic, which is associated with the probability of fragment production, is the number of so-called h prongs, (N_h), i. e., the number of tracks of heavy charged particles emitted in the given disintegration. In the analysis of the experimental data in the framework of equilibrium models, this characteristic is used because it is related to the energy transferred to the nucleus by the cascade,[6] while in an analysis in the framework of ideas about fast fragmentation processes it is related to the number N_g of fast cascade particles.[24,25] In Fig. 7 and 8 and in Table IV we give examples of the dependence of the probability W of fragment production on the number of heavy particles N_h, the number of fast cascade particles N_g, the number of slow particles N_b, and the number of relativistic particles N_s. It can be seen that there is a connection between N_b and N_g, but not an apparent connection with N_s (which is made up basically of pions).

Figures 9 and 10a and 10b show the distributions with respect to the number of prongs for stars with and without fragments. A general property of the fragment-production probability is its growth with increasing number of nonrelativistic particles in the nuclear disintegration. This would seem to presuppose a connection between the probability of fragment emission and the energy transferred to the nucleus. However, such a conclusion must be drawn with care. Notice first that, as was first shown by Obukhov,[26] estimates of the energy transferred to the nucleus from the number of b prongs may give an error of order 100%. That N_b is not a quantity directly related to the energy transferred to the nuclei can also be judged from the data of Table I. Indeed, since the majority of the charged particles emitted in the disintegration consist of p, d, t, ^{3}He, ^{4}He (mainly p and ^{4}He), one must observe a strong isotope effect in the distribution with respect to N_b unless, of course, one assumes unusual differences in σ_{in} or the develop-

TABLE IV. Probability W of production of ^{8}Li fragment as a function of the number N_h of particles from AgBr at different E_1 (Ref. 46).

Projectile momentum, GeV/c	1.5 K^-	3.0 K^-	5.0 K^-	17.2 π^-	14.0 p	25.0 p
$W(^{18}\mathrm{Li})$ $N_h>6$	$(0.70\pm0.05)\cdot10^{-2}$	$(1.47\pm0.15)\cdot10^{-2}$	$(2.07\pm0.14)\cdot10^{-2}$	$(1.41\pm0.30)\cdot10^{-2}$	—	$(1.36\pm0.30)\cdot10^{-2}$
$W(^{28}\mathrm{Li})$ $N_h>6$	$0.75\cdot10^{-4}$	$2.3\cdot10^{-4}$	$3.4\cdot10^{-4}$	$1.9\cdot10^{-4}$	—	$2.5\cdot10^{-4}$
$W^2(^{18}\mathrm{Li})$	$0.56\cdot10^{-4}$	$2.16\cdot10^{-4}$	$4.3\cdot10^{-4}$	$2\cdot10^{-4}$	—	$1.85\cdot10^{-4}$
Mean value of N_h for stars without fragments	9.92 ± 0.18	12.1 ± 0.19	13.16 ± 0.19	—	—	—
Mean value of N_h for stars with one ^{8}Li	11.63 ± 0.25	15.97 ± 0.29	18.58 ± 0.25	17.9 ± 0.4	20.0 ± 0.4	18.2 ± 0.4
Mean value of N_h for stars with two ^{8}Li	12.0 ± 3.5	22.3 ± 1.4	20.4 ± 1.5	24.8 ± 3.1	24.5 ± 4.5	20.3 ± 1.0

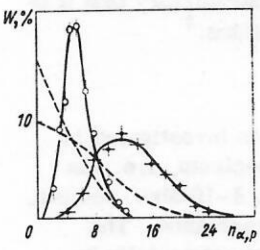

FIG. 9. Distribution of $n_{\alpha,p}$ reactions of Ag and Br nuclei with fragments $Z \geq 4$ with respect to the number of slow protons and α particles at proton energies $E_1 = 0.66$ and 9.0 GeV. $E_1 = 0.66$ GeV, open circles; $E_1 = 9.0$ GeV, solid circles; the dashed curves are the distributions of $n_{\alpha,p}$ reactions without fragments.[9,28]

ment of a nuclear cascade. Thus, although the energy transferred to the nuclei by the cascade cannot differ appreciably, the N_b values are very different. To verify the conclusion that there is a difference between the mean values of N_b for isotopes of one element, we made an experiment, using the method of sandwiches of foil and BR-2 photographic emulsion layers, to study disintegration of ^{112}Sn and ^{124}Sn by 610-MeV protons. It was found that for $N_h \geqslant 3$, $N_{b,av}(^{112}Sn)/N_{b,av}(^{124}Sn) = 2.0 \pm 0.2$ on one side of the foil. At the same time, the absence of a correlation between the fragment-emission probability and N_s can serve as an indication of a connection between this probability and the extent to which the cascade develops.[7] This is also indicated by the existence of a correlation between W and N_g. Therefore, it is very probable that there is a correlation between W and N_b, on the one hand, and N_g on the other. Analyzing experimental facts relating to the dependence of W on $N_{h,b,g}$, Barashenkov and Toneev concluded that there is a significant connection between the character $W(N_{h,b,g})$ and the degree of branching of the cascade in the nucleus. They especially emphasized the role of an effect which they studied in detail and called the "etching effect" i.e., the influence of perturbation of the density of the nucleus during the development of the cascade; this has the consequence that already in the range of energies $E_1 > 3$ GeV of the primary protons the number of cascade collisions is saturated, which also leads to saturation of the fragmentation cross sections. In principle, this also enables one to understand qualitatively why the cross sections of fragmentation caused by bombardment with nuclei are larger than those resulting from bombardment by nucleons.

Since the experimental data on the connection between the probability of emission of fragments and the disintegration characteristics are available only for fragmentation of the nuclei Ag and Br, it would clearly be unjustified to draw far reaching conclusions. We have already seen that even a slight change in the nucleon composition of the nuclei can give rise to a strong variation of the mean values of N_b. Thus, it only remains to assert that the fragment-emission probability W is related to the degree of branching of the cascade process. From Fig. 7, which gives data on the dependence W on $N_h = N_b + N_g$ at different energies E_1, one can see that for $E_1 \geqslant 3$ GeV the probability W depends[7] only on the number of cascade particles, and not on E_1.

With regard to events with two or more fragments in one disintegration, a rule has been established for them according to which the probability of emission of N fragments $P(N)$ satisfies the relation[27,28]

$$P(N) \approx P^N(1). \qquad (6)$$

3. ANGULAR DISTRIBUTIONS OF FRAGMENTS

The angular distributions of fragments have a strong anisotropy in the laboratory coordinate system. The anisotropy depends on the energy and species of the bombarding particles, and, in fact, in such a way that with increasing energy of the primary particle of a given species the anisotropy decreases. In Tables V and VI and in Figs. 11a and 11b we have collected together the experimental data on the ratio of the number of fragments emitted in the forward direction in the laboratory coordinate system, F, to the number B emitted backward.[9,20,29–31] Of course, these data are not complete. They are obtained by different methods, including ones in which the angular distribution of the fragments is not directly measured but deduced on the basis of various arguments, the most widely used being approximation of the angular distribution by the function

$$a + b\cos\theta. \qquad (7)$$

Nevertheless, it can be seen from Table V that at a given energy and for a given species of the primary particle the ratio F/B for different fragments can be different. As a rule, F/B is appreciably larger for

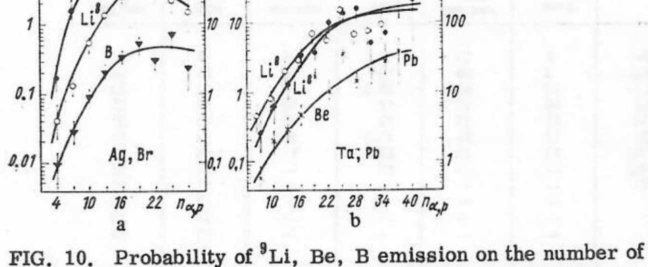

FIG. 10. Probability of ^{9}Li, Be, B emission on the number of slow protons and α particles $n_{\alpha,p}$ off Ag, Br, Ta, Pb (Ref. 9).

TABLE V. Ratio F/B for different fragments off Ag nuclei[29] and U nuclei[30,31] bombarded by 5.5-GeV protons.

Ag	Fragment	^6Li	^7Li	^8Li	^7Be	^9Be	^{10}Be	^3He	^4He	^6He	—
	F/B	1.3	1.38	1.50	1.42	1.39	1.38	1.23	1.16	1.36	—
U	Fragment	^6Li	^7Li	^8Li	^9Li	^7Be	^9Be	^{10}Be	^{10}B	^{11}B	—
	F/B	1.39	1.33	1.42	1.46	1.81	1.43	1.57	1.61	1.56	—
	Fragment	^{12}B	^{13}B	^{11}C	^{12}C	^{13}C	^{14}C	^3He	^4He	^6He	^8He
	F/B	1.66	1.70	2.16	1.58	1.56	1.69	2.1	1.24	1.33	1.46

TABLE VI. Ratio of $(F/B)_\alpha$ values obtained from bombardment of U nuclei by 8.4-GeV α particles to the $(F/B)_p$ values obtained from bombardment of U nuclei with 4.9-GeV protons.[20]

Fragment	^{4}He	^{6}He	^{6}Li	^{7}Li	^{7}Be	^{9}Be	^{10}Be
$(F/B)_\alpha$	1.41	1.48	1.75	1.67	1.7	1.64	1.67
$\frac{(F/B)_\alpha}{(F/B)_p}$	1.13	1.13	1.14	1.12	1.1	1.22	1.1

neutron-deficient fragments than for other fragments. For the heavy fragments ^{24}Na and ^{28}Mg in disintegrations of the nuclei ^{197}Au and ^{238}U by protons with energy up to 300 GeV, it was found in Ref. 32 that F/B has a maximum at $E_p = 3$ GeV ($F/B \approx 3$) and that there is a smooth decrease to $F/B = 1.3$ at $E_p = 300$ GeV. It is noted in Ref. 32 that this behavior of the anisotropy, together with the fact that the mean range of the fragments at $E_p = 300$ GeV is 25% less than at $E_p = 3$ GeV, is similar to the behavior of neutron-deficient fragments (^{131}Ba) of uranium fission.

An important feature of the angular distributions of the fragments is that for a sufficiently heavy target nucleus one can choose an energy E_1 for which the kinematically allowed range of velocities v of systems in which one can have symmetric emission of fragments about the direction at right angles to the beam is insufficient to explain the experimentally observed anisotropy of the angular distribution of the fragments. Such conditions were realized in Ref. 21 for the emission of ^{2}H, ^{3}H, ^{3}He, and ^{4}He resulting from the bombardment of heavy nuclei by 156-MeV protons, in Ref. 22 for the emission of ^{8}Li from ^{197}Au nuclei resulting from bombardment with 660-MeV protons, and in Ref. 23 for the emission of ^{24}Na from ^{209}Bi nuclei bombarded by 2.9-GeV protons. In these investigations, it was also shown that there exists no value of v that would enable one, assuming isotropic emission of fragments in the decaying system, to describe simultaneously the profile of the fragment spectra at different angles and the angular distribution. These results are a model-independent proof that the fragmentation mechanism differs from equilibrium decay (into two or more bodies).[21–24]

With regard to the dependence of F/B on the number of prongs in different types of disintegrations, the experimental data are here still so contradictory that it is difficult to draw any definite conclusions.[7]

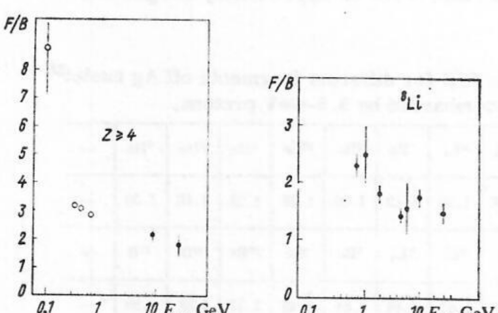

FIG. 11. Anisotropy of angular distribution of fragments with $Z \geq 4$ and ^{8}Li as a function of the energy of the projectile protons E_p (Refs. 9 and 28).

4. SPECTRA OF FRAGMENTS

The spectra of fragments have been investigated in fair detail in the main part of the spectrum, i.e., in the range of fragment energies up to 8–10 MeV/nucleon, and not at all well for high energy fragments. The fragment spectra are investigated experimentally by track detectors, semiconductor telescopes, and magnetic analyzers. For slow fragments, these investigations have given results that agree fairly well with one another, and these results can be discussed in detail. The fast-fragment data cannot yet be reliably systematized, and we shall restrict ourselves to a very general description.

In the region of fragment energies ε near the maximum of the spectrum, it is customary to approximate the profile of the spectrum by the function

$$(\varepsilon - kB)\exp\{-\varepsilon/\tau\}, \tag{8}$$

where B is the Coulomb barrier of the nucleus for the fragment; k is the so-called barrier-lowering coefficient, which is considered in detail, for example, in the book, Ref. 7; τ is a parameter of the spectral profile, which has been called the "temperature" since the time when attempts were made to describe fragmentation by the model of equilibrium evaporation.

For the hard part of the spectra, one can, as is shown in Refs. 9 and 28, also use the approximation (8), but now with a larger (about two times) value of the parameter τ. The values of τ are given in Table VII or some fragments (for different parts of the spectra).

The spectra of fragments emitted at different angles θ to the primary beam are different. As θ decreases, the spectrum becomes broader (Fig. 12). The change in the profile of the spectrum with a change in θ is particularly noticeable for the hard part of the fragment spectrum.

In Ref. 33, the fragment spectra were considered as functions of θ and A in the reaction $p(A, {}^8\mathrm{Li})\ldots$ for the nuclei Al, V, Ag, Au, Th. Figure 13 shows the results of Ref. 33 for the θ and A dependence of the mean en-

TABLE VII. Parameters of fragment spectral profiles (τ, MeV) for different sections of the spectrum.

Fragment	p + U (E_1 = 4.9 GeV) (Ref. 20)		α + U (E_1 = 8.4 GeV) (Ref. 20)		p + U (E_1 = 5.5 GeV) (Ref. 30)	
	Near peak	High energies	Near peak	High energies	Near peak	High energies
^{4}He	6	19	6.5	16	6	20
^{6}He	9	16	10	18	10	20
^{6}Li	13	18	13.5	19	10	20
^{7}Li	10	15	12	19	15	23
^{7}Be	17	19	19	22	12	20
^{9}Be	12	13	13.5	19	13	19
^{10}Be	13	15	13.5	17	13	15
^{11}Be	—	—	15	—	13	14
^{10}B	—	—	—	—	13	14
C	—	—	—	—	13	13
N	—	—	—	—	13	13
O	—	—	—	—	13	13

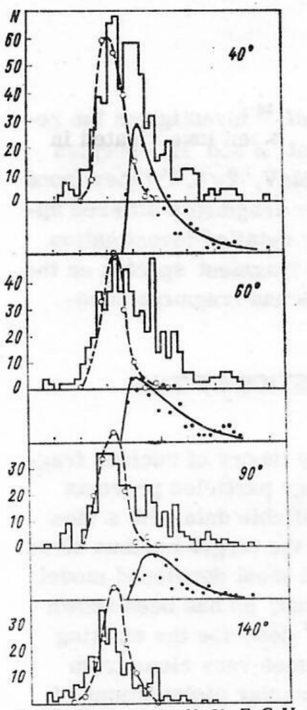

FIG. 12. Differential spectra of ^{8}Li from ^{232}Th at $E_p = 660$ MeV. The dashed curves were calculated in accordance with the evaporation model[33]; the solid curves are the difference between the experimental spectra and the calculated spectra normalized relative to the former at the peak.[33]

ergy $\bar{E}$ and the standard deviation of the spectrum $\sigma(\sigma^2 = \bar{E^2} - \bar{E}^2)$. One can see that E increases with θ and also A (Z) of the target nucleus. The value of σ for different θ (for example, 30° and 90°) is different; it is larger for 30° than for 90°, but weakly dependent on A.

An interesting feature of the fragment spectra is the existence of a certain correlation between τ and the properties of the target nucleus and the fragment. In Refs. 29–31 it was noted that τ for ^{7}Be is larger than for other fragments. Consulting Tables VII and VIII, in which we have collected data on τ for different fragments, we see that τ fluctuates as a function of the nucleon composition of the target nucleus and the fragment. These fluctuations are usually small, but they are manifested systematically.

In Refs. 12, 13, 15 it was found that the profile of the spectra of fragments emitted from different isotopic targets was different. Figure 14 shows the ratios of the differential cross sections for a number of isotopes emitted from ^{58}Ni and ^{64}Ni and ^{112}Sn and ^{124}Sn bombarded by 660-MeV protons.[12] A general property of these ratios is that the differences between them decrease

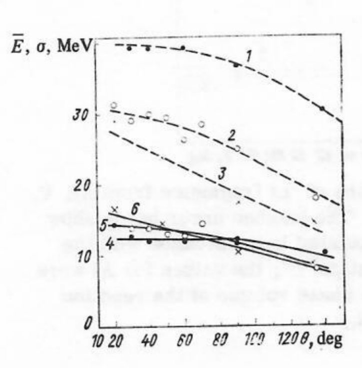

FIG. 13. Mean energy $\bar{E}$ (curves 1–3) and standard deviation σ of the spectrum (curves 4–6) as a function of the angle. 1) and 4) for Th; 2) and 5) for Ag; 3) and 6) for V (Ref. 33).

TABLE VIII. Parameters τ of the spectral profile for different fragments and target nuclei bombarded by protons with energy $E_1 = 1.0$ (Ref. 31) and 5.5 GeV (Ref. 29).

Fragment	Target nucleus	E_1, GeV	τ, MeV	Fragment	Target nucleus	E_1, GeV	τ, MeV
^{3}He	Ag	1.0	11.2±0.2	^{8}Li	Au	1.0	7.5±0.2
^{3}He	Ag	5.5	8	^{8}Li	U	1.0	8.7±0.2
^{4}He	Ag	1.0	4.62±0.01	^{9}Li	Ag	1.0	9.6±1.2
^{4}He	Ag	5.5	6	^{9}Li	Au	1.0	8.2±0.7
^{4}He	U	1.0	5.14±0.01	^{9}Li	U	1.0	7.8±0.3
^{6}He	Ag	1.0	6.9±0.3	^{7}Be	Ag	1.0	10.0±0.3
^{6}He	Au	1.0	6.0±0.2	^{7}Be	Ag	5.5	11
^{6}He	U	1.0	6.8±0.1	^{7}Be	Au	1.0	11.1±0.5
^{6}Li	Ag	1.0	8.5±0.1	^{9}Be	Ag	1.0	7.7±0.2
^{6}Li	Ag	5.5	11	^{9}Be	Ag	5.5	10
^{6}Li	Au	1.0	6.0±0.2	^{9}Be	Au	1.0	6.5±0.2
^{6}Li	U	1.0	9.9±0.2	^{10}Be	Ag	1.0	8.8±0.4
^{7}Li	Ag	1.0	8.2±0.1	^{10}Be	Ag	5.5	11
^{7}Li	Ag	5.5	11	^{10}Be	Au	1.0	7.0±0.2
^{7}Li	Au	1.0	7.1±0.1	^{10}Be	U	1.0	7.9
^{7}Li	U	1.0	8.7±0.1	^{10}B	Ag	1.0	8.7±0.3
^{8}Li	Ag	1.0	8.7±0.4	^{10}B	Au	1.0	8.5±0.4
^{8}Li	Ag	5.5	11	^{10}B	U	1.0	9.1±0.5

with increasing energy of the fragment and disappear to the accuracy of the ratio $(A_L/A_H)^{2/3}$. If the spectra are characterized by the parameter τ, then the sign of $\tau_L^{-1} - \tau_H^{-1} = \Delta$ follows the sign of the function

$$t_3 (T_{3,L}/A_L - T_{3,H}/A_H). \qquad (9)$$

One can therefore assert that the profile of the spectra of slow fragments is modulated by the isospins of the target nucleus and the fragment. Note, however, that for the fragment ^{4}He, whose isospin is zero, the same differences in the spectral profiles are found. It is highly probable that this fragment can evaporate from the nuclei that remain after the completion of the fast stage of the reaction, but it may be a product knocked out of the surface layer of a nucleus of α clusters,[3,6,7] this resulting in its well known distinguished properties among the other isotopes.

A general property of the parameter τ as a function of E_1 is the increase of τ with increasing E_1 followed by

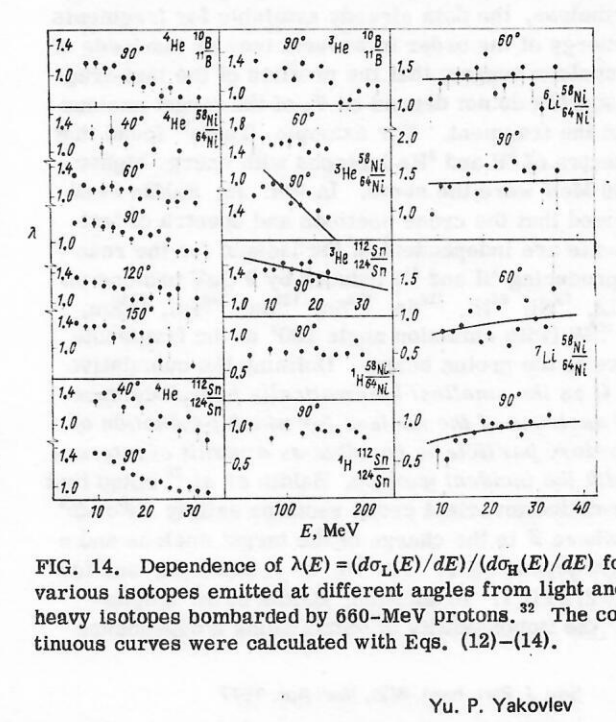

FIG. 14. Dependence of $\lambda(E) = (d\sigma_L(E)/dE)/(d\sigma_H(E)/dE)$ for various isotopes emitted at different angles from light and heavy isotopes bombarded by 660-MeV protons.[32] The continuous curves were calculated with Eqs. (12)–(14).

TABLE IX. Mean energies of ^{24}Na fragment from Ag, Au, and U nuclei bombarded by protons and α particles.[19]

$\bar{E}$, GeV	Ag	Au	U
$E_\alpha = 0.88$	32.7	77.0	89.6
$E_p = 0.70$	26.6	80.3	85.4
$E_p = 3.0$	22.9	63.0	78.4

the virtual cessation of this growth in the region $E_1 > 3$ GeV, i.e., where the fragmentation excitation functions reach the plateau. As a function of the properties of the primary particle, the parameter τ has not been well studied. Poskanzer's group has published[20] the most important results on fragmentation induced by relativistic nuclei with energy 2.1 GeV/nucleon (see Table VII). When the projectiles are complex nuclei, the parameter τ increases with the mass of the projectile (the spectrum broadens) if the energy per nucleon of the projectile is the same (data on d and α) and even if it is less than the energy of a proton (data on p and α).

The differences between the spectra of fragments emitted after bombardment of heavy nuclei by protons and α particles were also known earlier from the data on the mean energy $\bar{E}$ of the spectra of the fragment ^{24}Na emitted from Cu, Ag, Au, and U (Ref. 19). The corresponding data, collected in Table IX, illustrate the differences. It can be seen that there is also a difference between the values of $\bar{E}$ when the bombarding particles have similar energies.

Thus, the experimental data on the dependence of the profiles of fragment spectra on the projectile species reveal a sensitivity of the spectra to the species of the bombarding isotopes, although the nature of this sensitivity is not yet clear.

Unfortunately, there has been practically no systematic study of the profiles of the fast-fragment spectra. Nevertheless, the data already available for fragments with energy of the order of several tens or hundreds of MeV/nucleon suggest that the profiles of the fast-fragment spectra do not depend on T_3 of the target nucleus or t_3 of the fragment. For example, Yasin[34] found that the spectra of ^3H and ^3He isotopes with energy higher than 80 MeV were the same. In Ref. 35, Baldin *et al.* confirmed that the cross sections and spectra of fast fragments are independent of the isospin for the reactions producing ^2H and ^3H induced by 9 GeV protons on ^6Li, ^7Li, ^{58}Ni, ^{64}Ni, ^{112}Sn, ^{118}Sn, ^{124}Sn, ^{144}Sm, ^{154}Sm, ^{182}W, ^{186}W (with emission angle 180° of the fragments relative to the proton beam). Defining the cumulative order Q as *the smallest kinematically permitted number of nucleons of the nucleus for which production of a secondary particle is possible as a result of interaction with the incident nucleon,* Baldin *et al.*[35] noted that the so-called invariant cross sections satisfy $Ed^3\sigma/dp^3 \sim Z^n$, where Z is the charge of the target nucleus and n does not depend on Q. For ^2H, $n = 1.82 \pm 0.04$, and for ^3H, $n = 2.2 \pm 0.1$. In addition, Baldin *et al.* emphasized[35] the impossibility of establishing a dependence of the form

$$Ed^3\sigma/dp^3 \sim A^n.$$

However, when Komarov *et al.*[36] investigated the reactions $^{12}C + p \to {}^3H, {}^3He, + \ldots$ at ^3H and ^3He energies higher than 400 MeV ($E_1 = 660$ MeV, $\theta = 5.5°$) they found that the spectra of these mirror fragments differed appreciably. We see that further detailed investigation into the dependence of the fast-fragment spectra on the properties of the target nucleus and fragment is required.

5. INCLUSIVE CHARACTERISTICS OF THE FRAGMENTATION PROCESS

The absence of a satisfactory theory of nuclear fragmentation induced by high energy particles prevents our analyzing the currently available data with a view to establishing the influence of the target-nucleus structure on the fragmentation. The most developed model of equilibrium evaporation cannot, as has been shown in many papers,[9,12,15,21-24,28-33] describe the existing experimental facts. This becomes very clear from Fig. 15, which compares the angular distributions of the fragment ^8Li with the angular distributions predicted by the model of equilibrium evaporation. We have already seen (in Figs. 5 and 12) that the model is also incapable of describing characteristics such as the spectra and isotope ratios of the fragment production cross sections.

Equilibrium evaporation fails to explain the fragmentation process because it presupposes a complete redis-

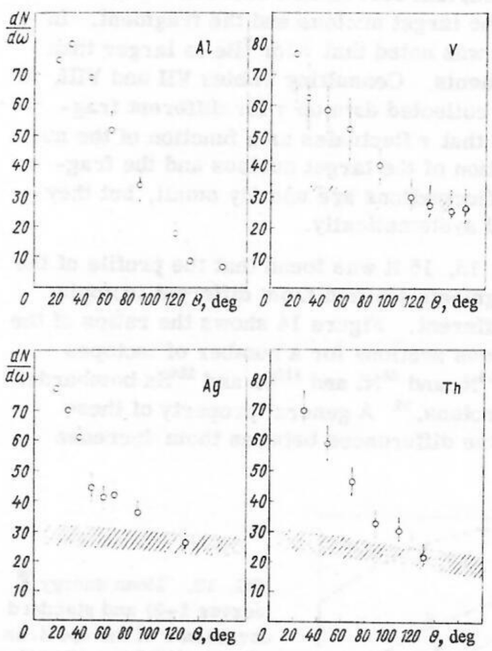

FIG. 15. Angular distributions of ^8Li fragments from Al, V, Ag, and Tb at $E_p = 660$ MeV. The hatched error bands show tha angular distributions calculated in accordance with the model of equilibrium evaporation[28,33]; the values for Al were calculated on the basis of the phase volume of the reaction $^{27}Al + p \to 2p + n + {}^4He + {}^8Li + {}^{13}N$.

tribution of the quantities that characterize the initial state of the system over all possible degrees of freedom. This assumption can be given up, either by using the idea of direct-fragmentation reactions (for example, interaction of cascade particles with clusters[6,7,24,28]), or by going over the so-called pre-equilibrium models.[7] In these, it is assumed that the reaction takes place in a short time (compared with the time for equilibrium to be established in the nucleus). Then an appreciable energy can be concentrated in a relatively small number of degrees of freedom of the excited nucleus and one can have a more direct relationship between the characteristics of the fragments and the manner in which the intranuclear cascade develops. Qualitatively, this connection must lead to a broadening of the fragment spectrum beyond the predictions of the equilibrium model, a greater anisotropy of the fragments, and a correlation between the kinematics of fragments emitted at different angles and the kinematics of the cascade particles emitted at these angles. These are the effects that are observed.[6,7,24] But unless one adopts the model of quasielastic knockout of clusters from nuclei, whose applicability for heavy fragments is disputed, one cannot treat nonequilibrium and direct models mathematically. Nevertheless, using ideas about fast fragmentation processes, one can systematize quite a lot of the experimental facts. An example is the formula (4) obtained in Ref. 16. The approach developed in Ref. 16 was improved in Refs. 38 and 39, in which the basic properties of fast reactions induced by high energy particles were taken into account more fully. In the second of these, Ref. 39, the following properties of fast reactions were used.

1. The probability of excitation of nuclei to fragmentation states is determined by the extent to which a cascade develops.

2. Nucleons of the nucleus are excited to fragmentation states with probability determined solely by the composition of the nucleus (cluster effects are here ignored).

3. Fragmentation states, as in nonequilibrium reactions between ions and nuclei,[40,41] can be described approximately as quasimolecules formed by the fragment and the target nucleus.

4. The fragments are emitted so rapidly that the nucleus cannot relax nor the quasifragment interact with the unperturbed nuclear density. The fragment-emission process itself is regarded as nonequilibrium decay into two bodies.

What we have said presupposes the possibility of writing down for $\sigma_k(\varepsilon)$, the fragment-emission cross section (in excitation state $\{k\}$),

$$\sigma_k(\varepsilon) = \mu\varepsilon(2S_k+1)\,\eta_1(E_1)\,\eta_2(E_1,\varepsilon)\,\eta_3(\varepsilon), \tag{10}$$

where $\mu = m_2(m_0 - m_2)/m_0$; S_k is the spin of state k of the fragment; $\eta_1(E_1)$ is a factor that describes the change in the number of cascade collisions leading to the excitation of fragmentation states with varying energy of the primary particle E_1; $\eta_2(E_1, \varepsilon)$ describes the probability that in these collisions a quasifragment with given m_2, t_3, and kinetic energy ε will be excited; $\eta_3(\varepsilon)$ describes the influence of the interaction with the nucleus on the emission of the fragment.

For large E_1 ($E_1 \geqslant 0.6$ GeV) and ε values satisfying $\varepsilon/m_2 \leqslant 10$ MeV, the following expressions were obtained for η_1 in Ref. 39:

$$\left.\begin{aligned}
&\eta_1(E_1) \approx \pi R_0^2 \exp\{(R_0/r_0^3)\,a(E_1)\};\\
&\eta_2(E_1,\varepsilon) \approx 2^{-m_2}\frac{(1-4T_3^2/m_0^2)^{m_2/2}}{\tau_0^2(E_1)}\\
&\qquad\qquad \times\left(\frac{1+2T_3/m_0}{1-2T_3/m_0}\right)^{t_3}\exp\{-\varepsilon/\tau_0(E_1)\};\\
&\eta_3(\varepsilon) \approx \left(1-\frac{B}{\varepsilon}\right)\exp\left\{-\frac{Q+\varepsilon_k}{\tau_Q(0)} - 10^{-3}\frac{\varepsilon(Q+\varepsilon_k)}{m_2}\right.\\
&\qquad \left.-\left(1-\frac{0.01\varepsilon}{m_2}\right)\beta\left(\frac{\varepsilon}{m_2}\right)\frac{R_0+R_2}{R_0^3+R_2^3}\left[T_3t_3 - \frac{R_0^3}{R_2^3}t_k(t_k+1)\right]\right\};\\
&\tau_Q^{-1}(0) \approx 0.1\sqrt{\frac{\mu}{m_2}};\ B = \frac{Z_2(Z_0-Z_2)}{R_0-R_2}.
\end{aligned}\right\} \tag{11}$$

Here, $a(E_1)$, $\tau_0(E_1)$, and $\beta(\varepsilon/m_2)$ are model parameters, which are the same for all fragments and target nuclei (medium and heavy) under the single restriction $m_2 \ll m_0$.

For the model parameters, the following linear expansions were proposed in Ref. 39:

$$a(E_1) = 0.45 - 0.55/E_1;$$
$$\tau_0(E_1) = 10 - 1.6/E_1 \text{ MeV, where } E_1 \text{ is given in GeV;}$$
$$\beta\left(\frac{\varepsilon}{m_2}\right) = \begin{cases} 3.1 - 0.3\varepsilon/m_2 & \text{for } \varepsilon/m_2 < 10 \text{ MeV},\\ 0 & \text{for } \varepsilon/m_2 > 10 \text{ MeV}.\end{cases}$$

For the total fragmentation cross section $\sigma_k = \int_B^\infty d\varepsilon\,\sigma_k(\varepsilon)$ we obtain[39]

$$\begin{aligned}
\sigma_k \approx{}& \pi R_0^2\mu(2S_k+1)\left[\frac{\tau(E_1)}{\tau_0(E_1)}\right]^2 2^{-m_2}\left(1-\frac{4T_3^2}{m_0^2}\right)^{m_2/2}\left(\frac{1+2T_3/m_0}{1-2T_3/m_0}\right)^{t_3}\\
&\times\exp\left\{\frac{R_0}{r_0^3}a(E_1) - \frac{Q+\varepsilon_k}{\tau_Q(0)} - \frac{B}{\tau(E_1)} + \beta_0\frac{R_0+R_2}{R_0^3+R_2^3}\right.\\
&\qquad\left.\times\left[T_3t_3 - \frac{R_0^3}{R_2^3}t_k(t_k+1)\right]\right\}.
\end{aligned} \tag{12}$$

If the main part of the fragment spectrum is in the region $\varepsilon/m_2 \leqslant 10$ MeV, then $\beta_0 = 3.1$. If the fragment spectrum lies in the region $\varepsilon/m_2 > 10$ MeV, one must set $\beta_0 = 0$. The parameter $\tau(E_1)$ is determined by

$$\left.\begin{aligned}
&\tau^{-1}(E_1) \approx \tau_0^{-1}(E_1) - 10^{-3}\frac{Q+\varepsilon_k}{m_2} + 0.1\frac{\beta_0}{m_2}\left(1-\frac{0.01\varepsilon_{\text{av}}}{m_2}\right)\\
&\qquad\qquad \times\frac{R_0+R_2}{R_0^3+R_2^3}\left(T_3t_3 - \frac{R_0^3}{R_2^3}t_k(t_k+1)\right);\\
&\varepsilon_{\text{av}} = B + 2\tau.
\end{aligned}\right\} \tag{13}$$

To estimate the total cross section for emission of the fragment m_2 in all bound states, $\sigma(\text{fr})$, it is necessary to use the formula $\sigma(\text{fr}) = \sum_k \sigma_k + \sum_{k'} \sigma_{k'}$, where k labels the bound states of the fragment m_2; k' labels the unbound states of a heavier fragment that decays with the emission of the fragment m_2.

Tables X and XI give examples of the use of Eqs. (11)–(13) to describe the experimental data on the fragment cross sections and the parameters of the spectrum profile, which, as is readily seen, are $\tau(E_1)$. Equations (11)–(13) enable one to describe the experimental data for ^2H and fragments heavier than ^4He at energies E_1 greater than 0.55 GeV and for variation of $\sigma(\text{fr})$ from 0.13 to 1240 mb.

It can be seen from Table XI and Fig. 14 that formula

(13) describes small isotopic differences of the profile of the fragment spectra. For all the fragments except

TABLE X. Cross sections for the production of various nuclides and parameters of the spectra.

Fragment	r_2, F	Target nucleus	Proton energy E_1, GeV	σ, mb: Calculation	σ, mb: Experiment	τ, MeV: Calculation	τ, MeV: Experiment	Literature	Notes
^{2}H	—	Ag	5.5	1000	1240	—	—	[29]	Decay $^{6}Li^* \to {}^{2}H + {}^{4}He$ allowed for.
^{3}H	1.4	Ag	5.5	425	690	—	—	[29]	Equilibrium evaporation not allowed for.
^{3}He	1.4	Ag	5.5 1.0	174 68	345 53.7±10	10.1 10	8 11.2	[29] [47]	Decays with emission of fragments.
		U	5.5 1.0	290 29	600 26.7±7.4	— —	— —	[30] [47]	Fragments ^{3}H and ^{3}He not taken into account.
^{4}He	1.3	Ag	5.5 1.0	300 60	2030 610	9.7 10.1	6 4.62	[29] [47]	Contribution of decays $^{6}Li^* \to {}^{4}He + {}^{2}H$ taken into account.
		U	5.5 1.0	550 70	3700 1070±130	— —	— —	[30] [47]	Equilibrium evaporation not taken into account.
^{6}He	1.8	Ag	5.5 1.0	14 2.6	19.2 2.7±0.5	— 9.2	— 6.9±0.3	[29] [47]	—
		U	5.5 1.0	77.5 8.3	87 17.6±2.8	— 8.2	— 6.8±0.1	[30] [47]	—
^{6}Li	1.7	Ag	5.5 1.0	55 9.7	55 8.5±1.6	— —	— —	[29] [47]	—
		U	5.5 1.0	91 8.6	73 9.8±1.6	— —	— —	[30] [47]	—
^{7}Li	1.5	Ag	5.5 1.0	64 11.7	69 9.9±1.8	10.3 8.8	11 8.2±0.1	[29] [47]	Decay $^{8}Li^* \to {}^{7}Li + n$ taken into account.
^{8}Li	1.3	Ag	5.5 1.0	12 1.9	12.8 1.5±0.3	12.7 10.4	11 8.7±0.4	[29] [47]	Decay $^{9}Li^* \to {}^{8}Li + n$ taken into account.
^{9}Li	1.55	Ag	5.5 1.0	3.9 0.6	2.6 0.29±0.08	— —	— —	[29] [47]	—
^{7}Be	1.5	Co	300.0	10	11.2	—	—	[48]	—
		Ag	5.5 1.0	13 2	17.2 2.2±0.4	11.8 9.8	11 10.0±0.3	[29] [47]	—
		Au	30.0 3.0 1.0 0.55	19 8 1.1 0.14	21.6 8.7±2.2 1.3±0.3 0.35±0.05	— — 10.5 —	— — 11.1±0.5 —	[47] [47] [47] [47]	—
		U	30.0 10.0 5.5 3.0 1.0	26 21 17 10 1.3	20.2 20.2 17.6 7 0.92±0.2	— — 13.2 — —	— — 15.0. — —	[7] [7] [30] [7] [47]	—
^{8}B	1.3	U	5.5	<0.8	0.3	—	—	[30]	—
^{12}C	1.3	U	5.5	16	25	—	—	[30]	Decays $^{12}C^* \to {}^{12}C + n$ and $^{16}O^* \to {}^{12}C + {}^{4}He$ taken into account.
^{13}N	1.34	Zn	0.94 1.84	0.14 <0.4	0.13 0.33	— —	— —	[7] [7]	Calculation for ^{64}Zn with allowance for the decay $^{14}O^* \to {}^{13}N + p$.

TABLE XI. Values of $\Delta = \tau^{-1}(^{112}Sn) - \tau^{-1}(^{124}Sn)$, MeV, for various fragments at $E_1 = 1.0$ GeV.

Fragment	^{3}He	^{4}He	^{6}He	^{6}Li	^{7}Be
Δ_{exp}	−0.004 ±0.003	−0.025 ±0.003	−0.011 ±0.019	−0.003 +0.005	+0.008 ±0.010
Δ_{cal}	−0.011 ±0.002 *	+0.0006	−0.010	+0.001	+0.006

*For ^{3}He the value $\Delta = \pm 0.002$ was obtained with allowance for $\varepsilon_{av}/m_2 > 10$ MeV

^{4}He this effect, too, can be reproduced. For ^{4}He, the probability of equilibrium emission when $E_1 \geqslant 1.0$ GeV is so large (see, for example, Ref. 7) that it determines almost the entire yield. For ^{3}He fragments[7] at $E = 1.0$ GeV for nuclei $m_0 \approx 100$ and $m_0 \approx 200$ the contribution of equilibrium emission is small, but it increases appreciably at $E_1 = 5.5$ GeV. It is possible that the contribution of this effect determines the difference between the calculated and experimental results at $E_1 = 5.5$ GeV. Such a possibility is also indicated by the fact that $\tau(1.0) > \tau(5.5)$ for ^{3}He (see Table X). However, a final judgement about this can be made only after the role of the cluster relaxation mechanism has been clarified and the instability in Eqs. (12)–(14) at $\varepsilon/m_2 \approx 10$ MeV eliminated (at $E_1 = 5.5$ GeV, the data of Table X for ^{3}He and ^{8}B refer to the case when this instability is important). Thus, Eqs. (11)–(13) make it possible to systematize the fragmentation cross sections and the data on the dependence of the spectral profiles on the properties of the target nucleus and the fragment. Moreover, it was shown in Ref. 39 that with increasing ε the dependence of the fragmentation characteristics on T_3, t_3 and Q becomes weaker and may even disappear. This also agrees with the experimental facts we have already discussed. Since $a(E_1) \to$ const and $\tau_0(E_1) \to$ const as E_1 increases, the prerequisites are given for realization of so-called nuclear scaling in the region of large ε, if, of course, it remains possible to represent the function $\eta_2(E_1, \varepsilon)$ as before in the form

$$\eta_2(E_1, \varepsilon) \approx 2^{-m_2}\left(1 - \frac{4T_3^2}{m_3^2}\right)^{m_2/2}\left(\frac{1+2T_3/m_0}{1-2T_3/m_0}\right)^{t_3} f(E_1, \varepsilon),$$

where $f(E_1, \varepsilon)$ does not depend on T_3, t_3, m_0, m_2.

An important property of the scheme proposed in Refs. 38 and 39 is the need to take into account unbound states of fragments heavier than the observed one. Experimentally, these unbound states may also be manifested through the production in fragmentation of groups of particles that have a small angle of divergence between them (isotope jets).

6. ISOTOPE JETS AND THE CONTRIBUTION OF FRAGMENTATION PROCESSES TO THE TOTAL INELASTIC INTERACTION CROSS SECTION

The problem of angular correlations of charged particles in nuclear fragmentation was posed some years ago on the basis on two different basic ideas. Lozhkin *et al.*[28,42] suggested that the fragments and light particles could exhibit angular correlation if they are the

FIG. 16. Scheme for observing angular correlations of parallel (p) and antiparallel (a) type.[44]

decay products of a heavier fragment. A trivial example of such a system is ^{8}Be. Podgoretskiĭ *et al*[43] predicted correlations between particles in nuclear fragmentation by analyzing the quantum-mechanical interference in systems of identical particles. Several experimental attempts were made to discover both types of correlation, but the attempts to discover decay correlation did not answer the question of whether correlations of this type exist,[42] and correlations of interference type are not the subject of this review.

In Ref. 44, a first attempt was made not only to establish the existence of isotope jets but also to compare the characteristics of reactions accompanied by the emission of isotope jets with the characteristics of reactions producing fragments and reactions without either "jet" or fragments. An investigation was made of fragmentation of Ag and Br nuclei by protons with energy 460, 610, 660, 930 MeV and by 600 MeV deuterons. To demonstrate the existence of isotope jets, events were selected in which, at angles from 80 to 100° to the projectile beam, particle pairs were emitted (slow *b* prongs) in the same solid angle formed by a cone with opening angle 20° in either the same direction or opposite directions (Fig. 16). The azimuthal symmetry of the problem presupposes that the number of particles emitted in the same direction, n_p, and the number emitted in opposite directions, n_a, must satisfy the relation $n_p/n_a = 1$.

Table XII gives the ratios n_p/n_a obtained in the experiment; their values are approximately equal to 2. It was concluded from this that isotope jets exist. It was then assumed that all pairs of particles emitted into a cone with opening angle 20° are isotope jets, and all the events with such isotope jets were separated out from the complete set of events. For these events, the distributions were then plotted that are usually used to compare disintegrations with and without fragments.

These distributions are shown in Figs. 17–19. It can be seen that in all cases the characteristics of the reactions with isotope jets are similar to those of reactions with fragments, and that the characteristics which

TABLE XII. Values of n_p/n_a (Ref. 44).

θ, deg	E_p, MeV				E_d, MeV
	160	610	660	930	600
90±10	7±3	2.3±0.2	2.31±0.47	1.63±0.42	—
All angles	2.55±0.4	3.0±0.5	2.1±0.2	1.8±0.2	2.2±0.4

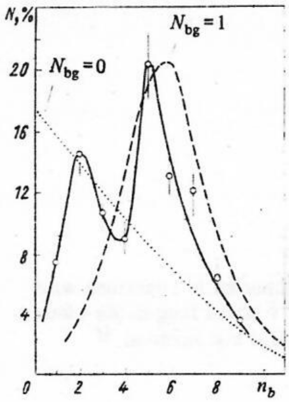

FIG. 17. Distribution of reactions from nuclei of AgBr photoemulsion with respect to the number of black prongs. The continuous curve is the distribution with isotope jets; the dots are the distribution without fragments; the dashed curve is the distribution with fragment (E_p = 610 MeV).

for reactions with fragments differ from the characteristics of ordinary disintegrations also differ from them. For reactions with two isotope jets, it was established that relation (6) holds. For example, at E_1 = 610 MeV it was found that $P(1) = 0.11 \pm 0.02$, $P(2) = 0.007 \pm 0.002$, $P^2(1) = 0.012$. Bearing in mind that $P(1)$ contains an admixture of background isotope jets, we can assume that the relation (6) holds.

In Ref. 44 it was also reported that the isotope jet events included some that can be interpreted as biproton events. On the basis of these results, it is concluded in Ref. 44 that the mechanism of production of isotope jets has much in common with the fragment-emission mechanism. At energy E_1 less than 1 GeV, the cross section for the production of fragments on Ag and Br nuclei is a few percent of the total inelastic cross section σ_{in}.

The results of Ref. 44 show that interactions of the fragmentation type do indeed form a larger fraction of σ_{in} than was previously assumed, and that these reactions cannot be ignored. Bearing in mind that all the restrictions on the opening angle of the cone for the emission of the jet components were chosen arbitrarily, one can here speak of an important contribution of fragmentation processes (in the broad sense) to σ_{in}.

CONCLUSIONS

Concluding our examination of the present status of the study of fragmentation of medium and heavy nuclei by high energy particles and nuclei, we can notice that in recent years significant progress has been made in this field. New features have been established in the behavior of the inclusive characteristics of fragmentation induced by nucleons, and these features have finally made it possible to systematize a large body of experi-

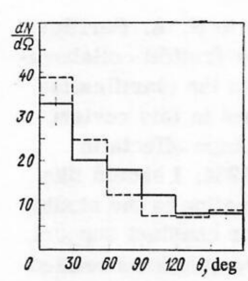

FIG. 18. Angular distribution of fragments $Z \geq 4$ with respect to the beam of projectile protons (dashed histograms) and isotope jets (solid histograms).[44]

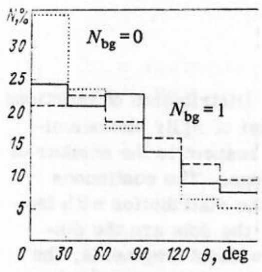

FIG. 19. Angular distribution of final nuclei in reactions with slow fragments (dashed histograms), without fragments (dotted histograms), and with isotope jets (solid histograms).[44]

mental facts. Systematic study of fragmentation induced by relativistic nuclei has begun. This is very important, since it has now become possible to create nucleon systems with unusual properties (enhanced density and energy concentration). We can be confident that investigations in fast-fragment physics will be continued and extended.

Turning to the essence of the fragmentation phenomenon, we may point out that there is now hope of construction of a reasonable model in which fragmentation appears as the decay of strongly nonequilibrium states of nuclei. If such a model proves to be correct, we may, by studying fragmentation, obtain very valuable information about the properties of nonequilibrium systems of nucleons and the excitation of nuclei by fast particles. This is all the more important in that if the conclusions drawn about the importance of fragmentation processes on the basis of the study of isotope jets are confirmed, the contribution of strongly nonequilibrium states to σ_{in} will be very appreciable. The systemization of the fragmentation data may also cast light on the role of cluster states in the ground state of nuclei.

We should like to point out the circumstance that the investigation of the characteristics of nuclear disintegrations with fragments has so far been possible in practice only for the nuclei Ag and Br of photoemulsions, while it is obviously important to obtain reliable data on the dependence of the fragmentation characteristics on the type of nuclear disintegration and the characteristics of the particles emitted with the fragment.

Thus, experimental investigations into fragmentation, which have become one of the most rapidly developing branches of nuclear physics, may, provided theory advances in step, significantly augment our ideas about nuclear structure, the behavior of nuclear matter under unusual nonequilibrium conditions, and the interaction of fast particles and nuclei with nuclei.

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