Physical and astrophysical constants and their dimensional and dimensionless combinations

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A review of the fundamental physical and astrophysical constants and their dimensional and dimensionless combinations is given.

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INTRODUCTION

The study of the laws of nature is based on measurements as a result of which the data of definite experiments or observations are associated with numbers. In turn, knowledge of numerical values is necessary in order to have the possibility of applying the most powerful tool of scientific investigation—the mathematical methods of theoretical physics. Any calculations remain incomplete if numbers are not substituted into the final formulas. Therefore it is necessary to have at hand a table of frequently used quantities arranged in a convenient compact form, in which one can easily look up the value of the necessary quantity.

It is well known that the number of fundamental constants which can be measured exceeds the number of independent constants, and in order to obtain consistent values it is necessary to use a definite statistical fitting procedure. That work is carried out by a special international working group on fundamental physical constants, and at the present time the recommended consistent values of the fundamental physical constants published in the 1973 review of Cohen and Taylor[1] are considered to be the best.

In the present review we have presented systematic data on the fundamental physical and astrophysical constants and their frequently encountered combinations; they are arranged so as to emphasize the unity and interrelation of the different physical quantities for all mass values and length and time intervals studied up to the present time (see Fig. 1).

For this purpose we have used mainly the periodically published reviews[1–3] which present exhaustive information on the huge quantity of experimental and theoretical studies of the fundamental physical and astrophysical constants.

In individual cases which require refinement, we have given references to original sources. In addition, a special section is devoted to review of empirical constants which do not follow from any theory known at the present time but which have simplicity and high accuracy.

1. ASTROPHYSICAL CONSTANTS

It should be noted that the values of the astrophysical and physical quantities are continually being improved. For example, the value of the Hubble constant accepted at the present time is almost an order of magnitude smaller than the estimate made by Hubble himself twenty years ago. Generally speaking, most of the data referring to the Metagalaxy may contain an uncertainty from one to two orders of magnitude. We have listed the basic parameters of our Galaxy, the Sun, the Earth, and the planets of the solar system. The great progress in study of the solar system in recent years is due to advances in space science. Appropriate corrections will obviously be made in the near future to the data existing at the present time. For example, the distance from the Earth to the Moon will soon be measured more accurately by means of the time of flight of a laser pulse reflected by special reflectors placed on the Moon by the Soviet "Lunokhod" and American astronauts.

More detailed data on astrophysical quantities can be found in the book by C. W. Allen.[3] (Tables I–VII)

2. ELEMENTARY PARTICLES

In this section we have listed the fundamental constants and parameters of the elementary particles. Fundamental constants is the designation usually given to quantities which determine the nature of quantum phenomena: the quantum of action or Planck's constant, the elementary electrical charge, and so forth. In re-

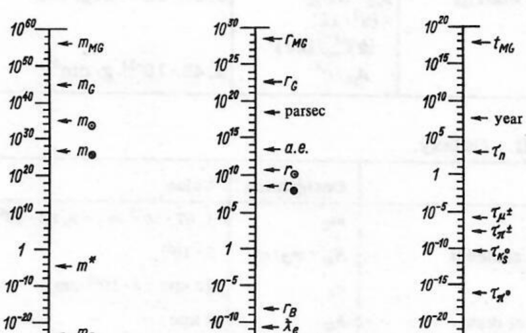

FIG. 1. Characteristic mass, length, and lifetime of various physical and astrophysical objects: m_{MG}—mass of Metagalaxy, m_G—Galaxy, $m_\odot$—Sun, $m_\oplus$—Earth, m^*—Planck particle, m_p—proton, m_e—electron; r_{MG}—radius of Metagalaxy, r_G—of Galaxy, $r_\odot$—Sun, $r_\oplus$—Earth, r_B—Bohr orbit, λ_e—electron Compton wavelength; t_{MG}—lifetime of Metagalaxy, τ_n—of neutron, τ_μ—muon, $\tau_{\pi^\pm}$—charged pions, $\tau_{K_S^0}$—short-lived neutral kaon, τ_{π^0}—neutral pion.

TABLE I. General constants.

Quantity	Designation	Value
Velocity of light in vacuum*	c	$2.997924580(12)\cdot 10^{10}$ cm/sec
Gravitational constant	G G/c G/c^2	$6.6720(41)\cdot 10^{-8}$ cm^3/(g-sec^2) $2.225\cdot 10^{-18}$ cm^2/(g-sec) $7.424\cdot 10^{-29}$ cm/g
Sidereal year	1 y	365.256 days $=3.1558\cdot 10^7$ sec $\approx(\pi\cdot 10^7$ sec)
Light year	1 l.y.	$9.460\cdot 10^{17}$ cm
Parsec	1 pc	$3.086\cdot 10^{18}$ cm $=3.261$ l.y.
Astronomical unit of length—average distance from the Earth to the Sun	1 a.u.	$1.495985(5)\cdot 10^{13}$ cm

*According to Blaney *et al.* [12] a more accurate value of the velocity of light in vacuum is $c=2.997924590(8)\times 10^{10}$ cm/sec.

TABLE II. Metagalaxy.

Quantity	Designation	Value
Mass	m_{MG}	10^{56} g
Number of nucleons (Eddington number)	$N_E=m_{MG}/m_p$	10^{80}
Hubble constant	H_0	50 km/(sec-Mpc) $=1.62\cdot 10^{-18}$ sec^{-1}
Hubble age	$t_{MG}=1/H_0$	$0.62\cdot 10^{18}$ sec $=19.7\cdot 10^9$ y
Radius	$r_{MG}=c/H_0$	$1.86\cdot 10^{28}$ cm $=6000$ Mpc
Gravitational radius	Gm_{MG}/c^2	$0.74\cdot 10^{28}$ cm
Kerr angular momentum	Gm_{MG}^2/c	$2.2\cdot 10^{94}$ g-cm^2/sec
Action for Metagalaxy	$m_{MG}c^2/H_0$	$5.6\cdot 10^{94}$ g-cm^2/sec
Critical density	$\rho_c=3H_0^2/8\pi G$	$0.5\cdot 10^{-29}$ g/cm^3
Effective black-body-radiation temperature	$T_{\gamma 0}$	2.68(10) °K
Wavelength of Planck distribution maximum	$\lambda_{max}=$ Wien constant$/T_{\gamma 0}$	0.18 cm
Radiation energy density	$\rho_{\gamma 0}=aT_{\gamma 0}^4=(\pi^2/15)(kT_{\gamma 0}^4/(\pi c)^3$ $\rho_{\gamma 0}/c^2$	$3.67\cdot 10^{-13}$ erg/cm^3 $4.42\cdot 10^{-34}$ g/cm^3

TABLE III. Galaxy.

Quantity	Designation	Value
Mass	m_G	$1.67\cdot 10^{11}\, m_\odot=3.32\cdot 10^{44}$ g
Number of nucleons	$N_G=m_G/m_p$	$2\cdot 10^{68}$
Radius	r_G	20 kpc $=6\cdot 10^{22}$ cm
Thickness of disk	h_G	1 kpc
Gravitational radius	Gm_G/c^2	$2.46\cdot 10^{16}$ cm
Intrinsic angular momentum	$\mathcal{J}_G$	$1.8\cdot 10^{74}$ g-cm^2/sec
Kerr angular momentum	Gm_G^2/c	$2.4\cdot 10^{71}$ g-cm^2/sec
Radius of nucleus	r_G	10 $pc=3\cdot 10^{19}$ cm
Average density of matter	ρ_G	$7\cdot 10^{-24}$ g/cm$^3=0.1\, m_\odot/nc^3$
Average magnetic field strength in the disk	$\mathcal{H}_G$	$2\cdot 10^{-6}$ G
Optical luminosity	L_G^{opt}	$4\cdot 10^{43}$ erg/sec
Age		$7\cdot 10^9$ y

TABLE IV. Sun.

Quantity	Designation	Value
Mass	$m_\odot$	$1.989\cdot 10^{33}$ g
Number of nucleons	$N_\odot=m_\odot/m_p$	$1.2\cdot 10^{57}$
Radius	$r_\odot$	$6.960\cdot 10^{10}$ cm
Gravitational radius	$Gm_\odot/c^2$	1.475 km
Intrinsic angular momentum	$\mathcal{J}_\odot$	$1.8\cdot 10^{48}$ g-cm^2/sec
Kerr angular momentum	Gm^2/c	$8.7\cdot 10^{48}$ g-cm^2/sec
Distance to center of Galaxy		10 kpc $=3\cdot 10^{22}$ cm
Rate of rotation about Galactic center	$v_\odot$	220 km/sec
Average magnetic field strength on surface	$\bar{\mathcal{H}}_\odot$	1.5 G
Magnetic moment of equivalent dipole	$\mu_\odot$	$0.35\cdot 10^{33}$ G-cm^3
Total luminosity	$L_\odot$	$3.9\cdot 10^{33}$ erg/sec

TABLE V. Earth.

Quantity	Designation	Value
Mass	$m_\oplus$	$5.977\cdot 10^{27}$ g
Number of nucleons	$N_\oplus=m_\oplus/m_p$	$3.58\cdot 10^{51}$
Equatorial radius	$r_\oplus$	$6.378\cdot 10^8$ cm
Gravitational radius	$Gm_\oplus/c^2$	0.444 cm
Intrinsic angular momentum	$\mathcal{J}_\oplus$	$5.91\cdot 10^{40}$ g-cm^2/sec
Kerr angular momentum	$Gm_\oplus^2/c$	$7.95\cdot 10^{36}$ g-cm^2/sec
Average orbital velocity	$v_\oplus$	29.78 km/sec
Orbital angular momentum	$\mathcal{J}_\oplus^{orb}$	$2.672\cdot 10^{47}$ g-cm^2/sec
Magnetic field strength at the poles and equator	$\mathcal{H}_\oplus^p$ $\mathcal{H}_\oplus^e$	0.6 G 0.3 G
Magnetic moment of equivalent dipole	$\mu_\oplus$	$8.06\cdot 10^{25}$ G-cm^3
Age		$4.6(1)\cdot 10^9$ y

TABLE VI. Moon.

Quantity	Designation	Value
Mass	$m_☽$	$7.347\cdot 10^{25}$ g $=m_\oplus/81.35$
Radius	$r_☽$	1737.63 km
Average Earth-Moon distance	$r_{\oplus☽}$	$3.844\cdot 10^{10}$ cm
Upper limit of dipole magnetic moment	$\mu_☽$	$<4\cdot 10^{18}$ G-cm^3
Axial rotation period	$t_☽$	29.531 days

cent years considerable progress in determination of the numerical values of the fundamental constants has been achieved, in particular, as the result of use of the Josephson effect. In addition, the parameters of elementary particles such as mass, lifetime, magnetic moment, and so forth are continually being improved. The most complete data in this field are found in the Review of Particle Properties by the Particle Data Group.[2]

In the present section, in addition to data on elementary particles, we have listed also various dimensional

TABLE VII. Planets.

Planet	Mass, g	Radius, km	Average distance from Sun, cm	Axial rotation period	Sidereal rotation period, y	Orbital velocity, km/sec	Angular momentum g-cm²/sec		Dipole magnetic moment, G-cm³
							Intrinsic	Orbital	
Mercury ☿	$0.32 \cdot 10^{27}$	2437	$5.8 \cdot 10^{12}$	58.646 days	0.241	47.90	—	$8.890 \cdot 10^{45}$	$\sim -10^{21}$
Venus ♀	$4.78 \cdot 10^{27}$	6052.5 ± 2.5	$1.08 \cdot 10^{13}$	244.3 days	0.615	35.05	—	$1.809 \cdot 10^{47}$	~ 0
Earth ⊕	$5.977 \cdot 10^{27}$	6378	$1.50 \cdot 10^{13}$	23 h 56 min 4 sec	1.000	29.80	$5.91 \cdot 10^{40}$	$2.672 \cdot 10^{47}$	$-8.06 \cdot 10^{25}$
Mars ♂	$6.423 \cdot 10^{26}$	3386 ± 6	$2.28 \cdot 10^{13}$	24 h 37 min 23 sec	1.881	24.14	—	$3.535 \cdot 10^{46}$	$+2.47 \cdot 10^{22}$
Jupiter ♃	$1.899 \cdot 10^{30}$	71600	$7.78 \cdot 10^{13}$	9 h 50 min	11.86	13.06	$4.43 \cdot 10^{45}$	$1.929 \cdot 10^{50}$	$+1.3 \cdot 10^{30}$
Saturn ♄	$5.685 \cdot 10^{29}$	60250	$1.43 \cdot 10^{14}$	10 h 14 min	29.46	9.65	$7.9 \cdot 10^{44}$	$7.813 \cdot 10^{49}$	—
Uranus ♅	$0.87 \cdot 10^{29}$	25400	$2.87 \cdot 10^{14}$	10 h 7 min	84.01	6.80	$1.9 \cdot 10^{43}$	$1.700 \cdot 10^{49}$	—
Neptune ♆	$1.00 \cdot 10^{29}$	24750	$4.50 \cdot 10^{14}$	15 h 8 min	164.7	5.43	—	$2.444 \cdot 10^{49}$	—
Pluto ♇	$5.38 \cdot 10^{27}$	3200	$5.93 \cdot 10^{14}$	6 days 9 h 16 min 54 sec ± 26 sec	248.9	4.74	—	$1.5 \cdot 10^{47}$	—

Note: 1. Total mass of planets $m_{tot} = 2.68 \times 10^{30}$ g $= 447.9\ m_{\oplus}$.
2. Total angular momentum of solar system $\mathcal{J}_{tot} = 3.15 \times 10^{50}$ g-cm²/sec.
3. Dipole magnetic moments included in parentheses were obtained on the basis of Blackett's empirical formula (P. M. S. Blackett, Russian translation in Usp. Fiz. Nauk **33**, 52 (1947)): $\mu = \sqrt{G}\ \mathcal{J}/c \approx 10^{-15}\ \mathcal{J}$.

TABLE VIII. General constants.

Quantity	Designation	Value
Planck's constant	h	$6.626176(36) \cdot 10^{-27}$ erg-sec
	$\hbar = h/2\pi$	$1.0545887(57) \cdot 10^{-27}$ erg-sec $= 6.582173(17) \cdot 10^{-22}$ MeV-sec
	$\hbar c$	$3.16129 \cdot 10^{-17}$ erg-cm $= 197.32858(51)$ MeV-fermi
Elementary charge	e	$4.803242(14) \cdot 10^{-10}$ CGSE $= 1.6021892(46) \cdot 10^{-19}$ C
Fine-structure constant*	$\alpha = e^2/\hbar c$	$1/137.03604(11)$ $= 0.0072973506(60)$
Frequency of photon with energy 1 eV	eV/h	$2.4179696(63) \cdot 10^{14}$ Hz
Wavelength of photon with energy 1 eV	hc/eV	$1.2398520(32) \cdot 10^{-4}$ cm
Fermi	1 fermi	10^{-13} cm
Barn	1 barn	10^{-24} cm²
Conversion factors	1 MeV	$1.6021892(46) \cdot 10^{-6}$ erg
	1 MeV/c	$5.344 \cdot 10^{-17}$ g-cm/sec
	MeV/c^2	$1.783 \cdot 10^{-27}$ g

*The most accurate value of the fine-structure constant[23] is $\alpha = 1/137.035987(29)$.

TABLE IX. Deuteron.

Quantity	Designation	Value
Mass	m_d	1875.628(5) MeV/c^2
Mass defect	$\Delta m_d = m_d - (m_p + m_n)$	$-2.22462(150)$ MeV/c^2
Binding energy	$E_B = \Delta m_d c^2$	$-2.22462(150)$ MeV
Radius	r_d	4.3157(5) fermi
Magnetic moment	μ_d	$0.18579612\ \mu_N$
	$\mu_d - (\mu_p + \mu_n)$	$0.0224\ \mu_N$
Electric quadrupole moment	Q	2.88 mb = 0.288 fermi²

TABLE X. Muon.

Quantity	Designation	Value
Mass	m_μ	105.65948(35) MeV/c^2 $= 1.883566(11) \cdot 10^{-25}$ g
Ratio of muon and electron masses	m_μ/m_e	206.76865(47)
Magnetic moment*	μ_μ	$1.00116616(31)\ e\hbar/2m_\mu c$ $= 4.490474(18) \cdot 10^{-23}$ G-cm³
Ratio of muon and proton magnetic moments	μ_μ/μ_p	3.1833402(72)
Mean life	τ_μ	$2.19711(8) \cdot 10^{-6}$ sec

*According to Bailey *et al.*[13] an improved value is $\mu_\mu = 1.0011658350(22)\ e\hbar/2m_\mu c$.

TABLE XI. Proton.

Quantity	Designation	Value
Mass	m_p	938.2796(27) MeV/c^2 $= 1.6726485(86) \cdot 10^{-24}$ g $= 1836.15152(70) m_e$ $= 6.72270(31) m_{\pi^\pm}$
Radius	$\langle r_p^2 \rangle^{1/2}$	0.85(6) fermi
Compton wavelength	$\lambda_p = \hbar/m_p c$	$0.21030892(36) \cdot 10^{-13}$ cm
	$\lambda_p/c = \hbar/m_p c^2$	$0.7015 \cdot 10^{-24}$ sec
Nuclear magneton	$\mu_N = e\hbar/2m_p c$	$0.5050824(20) \cdot 10^{-23}$ G-cm³ $= 3.1524515(53) \cdot 10^{-18}$ MeV/G
Magnetic moment	μ_p	$2.7928456(11) \mu_N$ $= 1.4106171(55) \cdot 10^{-23}$ G-cm³
Ratio of electron and proton magnetic moments	μ_e/μ_p	$-658.2106880(66)$
Ratio of neutron and proton magnetic moments	μ_n/μ_p	-0.6850
Gravitational radius	Gm_p/c^2	$1.242 \cdot 10^{-52}$ cm
Kerr angular momentum	Gm_p^2/c	$3.723 \cdot 10^{-66}$ g-cm²/sec

TABLE XII. Pion.

Quantity	Designation	Value
Charged-pion mass	$m_{\pi^\pm}$	139.5688(64) MeV/c^2 $=2.488\cdot 10^{-25}$ g $=273.1\ m_e$
Neutral-pion mass	$m_{\pi 0}$	134.9645(74) MeV/c^2 $=2.407\cdot 10^{-25}$ g
Radius	$\langle r^2\rangle^{1/2}_{\pi^\pm}$	$0.78^{+0.09}_{-0.10}$ fermi
Compton wavelength	$\lambda\!\!\!^{-}_{\pi^\pm}=\hbar/m_{\pi^\pm}c$ $\lambda\!\!\!^{-}_{\pi 0}=\hbar/m_{\pi 0}c$	1.4135 fermi 1.4615 fermi
Mean life	$\tau_{\pi^\pm}$ $\tau_{\pi 0}$	$2.6030(23)\cdot 10^{-8}$ sec $8.21(43)\cdot 10^{-17}$ sec
Average mass	$m_\pi=(m_{\pi^\pm}+m_{\pi 0})/2$	137.26665 MeV/c^2

TABLE XIII. Electron.

Quantity	Designation	Value
Mass	m_e	0.5110034(14) MeV/c^2 $=0.9109534(47)\cdot 10^{-27}$ g
Classical radius	$r_e=e^2/m_e c^2=\alpha\lambda\!\!\!^{-}_e$	$2.8179380(70)\cdot 10^{-13}$ cm
Compton wavelength	$\lambda\!\!\!^{-}_e=\hbar/m_e c$ $\lambda\!\!\!^{-}_e/c$	$3.8615905(64)\cdot 10^{-11}$ cm $1.2881\cdot 10^{-21}$ sec
Radius of first Bohr orbit	$r_B=\hbar^2/m_e e^2$ $=\alpha^{-1}\lambda\!\!\!^{-}_e$	$0.52917706(44)\cdot 10^{-8}$ cm
Velocity in first Bohr orbit	$v_B=e^2/\hbar=\alpha c$	$2.1877\cdot 10^{8}$ cm/sec
Momentum in first Bohr orbit	$P_B=me^2/\hbar=\alpha m_e c$	$1.9929\cdot 10^{-19}$ g-cm/sec
Bohr magneton*	$\mu_B=e\hbar/2m_e c$	$0.9274078(36)\cdot 10^{-20}$ G-cm^3 $=0.57883785(95)\cdot 10^{-14}$ MeV/G
Magnetic moment	μ_e	$-1.0011596567(35)\mu_B$ $=-0.9284832(3.6)\cdot 10^{-20}$ G-cm^3
Frequency and amplitude of Schrödinger vibration in the rest system	$\nu=2m_e c^2/\hbar$ $\hbar/2m_e c$	$1.553\cdot 10^{21}$ Hz 193.08 fermi
Gravitational radius	Gm_e/c^2	$6.763\cdot 10^{-56}$ cm
Kerr angular momentum	Gm_e^2/c	$1.846\cdot 10^{-72}$ g-cm^2/sec

*Since $[G]=[g^{1/2}/(cm^{1/2}\ sec)]$, we have G-cm$^3\equiv$ erg/G.

and dimensionless combinations of the fundamental constants, which play an important role in the physics of electromagnetic, strong, and weak interactions.

We note that improvement of the fundamental constants, in addition to a fundamental purely scientific interest, also has great practical value for establishment of pre-

TABLE XIV. Electromagnetic interactions.

Quantity	Designation	Value
Characteristic time	$r_B/v_B=\hbar^3/m_e e^4$	$2.4189\cdot 10^{-17}$ sec
Thomson cross section	$\sigma^0_{\gamma e}=8\pi r_e^2/3$	0.6652448(33) barn
Rydberg constant	$R_\infty=(\alpha^2/2)\ m_e c^2=m_e e^4/2\hbar^2$ $R_\infty/\hbar$	13.605804(36) eV $3.28984200(25)\cdot 10^{15}$ Hz
Quantum of magnetic flux	$\Phi_0=h/2e$	$2.0678506(54)\cdot 10^{-7}$ G-cm^2
Quantum of circulation	$h/2m_e$	3.6369455(60) cm^2/sec
Electric potential of proton at first Bohr orbit	$e/r_B=m_e e^3/\hbar^2$	27.210 V
Critical field intensities	$\mathscr{E}_{\rm crit}=m_e^2c^3/e\hbar$ $\mathscr{H}_{\rm crit}=m_e^2c^3/e\hbar$	— $4.414\cdot 10^{13}$ G
Lamb shift[19] in hydrogen	1057.90(6) MHz (experiment)	1057.911(12) MHz (theory)
Hyperfine splitting frequency (experiment):		
in hydrogen ($\lambda\approx 21$ cm)	ν_{Hhfs}	1420405751.7662(3) Hz
in muonium, μ^+e^- (Ref. 23)	ν_{Mhfs}	4463302.2 (1.4) kHz
in positronium, e^+e^- (Ref. 20)	ν_{Phfs}	203387.0 (1.6) MHz
Decay rate[21,22]		
parapositronium ($^1S_0$)	$1/\tau_{2\gamma}=\alpha^5 m_e c^2/2\hbar$ $\times\left[1-\frac{\alpha}{\pi}(5-(1/4)\pi^2)\right]$ $=0.79854\cdot 10^{10}$ sec^{-1} (theory)	$0.799\ (11)\cdot 10^{10}$ sec^{-1} (experiment)
orthopositronium ($^3S_1$)	$1/\tau_{3\gamma}=2\alpha^6 m_e c^2(\pi^2-9)/$ $9\pi\hbar\left[1+\frac{\alpha}{\pi}(1.86\pm 0.45)\right]$ $=0.7242(8)\times 10^7$ sec^{-1} (theory)	$0.7275\ (15)\cdot 10^7$ sec^{-1} (experiment)

TABLE XV. Weak interactions.

Quantity	Designation	Value
Renormalized Fermi constant for μ decay	G_μ	$1.4320(11)\cdot 10^{-49}$ erg-cm^3 $=1.024\cdot 10^{-5}\,\hbar^3/m_p^2c$
Renormalized Fermi vector-interaction constant for neutron β decay	G_V	$1.4149(22)\cdot 10^{-49}$ erg-cm^3 $=1.012\cdot 10^{-5}\,\hbar^3/m_p^2c$
Ratio of axial-vector and vector constants for neutron β decay	$\left\lvert\frac{G_A}{G_V}\right\rvert$	1.23(2)
Cabibbo angle	ϑ_c $\cos\vartheta_c$ $\sin\vartheta_c$	13°20′ 0.973(3) 0.230(3)
Dimensionless fine-structure constant for Fermi interaction	$\frac{G_V^2}{(\hbar c)^2}\left(\frac{m_e c}{\hbar}\right)^2\left(\frac{m_p c}{\hbar}\right)^2 \equiv \left(\frac{G_V m_e m_p c}{\hbar^3}\right)^2$	$1.085\cdot 10^{-17}$
Characteristic time	$\left(\frac{G_V m_e m_p c}{\hbar^3}\right)^{-2}\frac{\hbar}{m_e c^2} \equiv \frac{\hbar^7}{G_V^2 m_e^3 m_p^2 c^4}$	$1.187\cdot 10^{-4}$ sec
Heisenberg fundamental length	$\sqrt{\frac{G_V}{\hbar c}}$	$6.690\cdot 10^{-17}$ cm
Characteristic cross section	$\left(\frac{G_V m_e m_p c}{\hbar^3}\right)^2\left(\frac{\hbar}{m_e c}\right)^2 \equiv \frac{G_V^2 m_p^2}{\hbar^4}$	$4.528\cdot 10^{-38}$ cm^2
Charged-pion decay constant (Goldberger–Treiman ratio $g_{\pi NN} f_\pi = \sqrt{2}\, m_p\, G_A/G_V$)	f_π	$0.97\ m_\pi$

TABLE XVI. Stable and metastable elementary particles.

Particle	$I^G(\mathcal{J}^P)^{C_n}$	Mass m, MeV/c^2	Mean life τ, sec
γ	$0.1\,(1^-)^-$	$0\ (<3.5\cdot 10^{-48}$ g)	Stable
$\nu_e,\ \bar\nu_e$	$\mathcal{J}=1/2$	$0\ (<60$ eV/c^2)	"
$\nu_\mu,\ \bar\nu_\mu$	$\mathcal{J}=1/2$	$0\ (<1.2$ MeV/c^2)	"
$e^\mp$	$\mathcal{J}=1/2$	0.5110034 (14)	"
$\mu^\mp$	$\mathcal{J}=1/2$	105.65948 (35)	$2.19711\ (8)\cdot 10^{-6}$
π^0	$1^-\,(0^-)^+$	134.9645 (74)	$8.21\ (43)\cdot 10^{-17}$
$\pi^\mp$	$1^-\,(0^-)$	139.5688 (64)	$2.6030\ (23)\cdot 10^{-8}$
$K^\mp$	$1/2\,(0^-)$	493.707 (37)	$1.2371\ (26)\cdot 10^{-8}$
$K^0,\ \bar K^0$	$1/2\,(0^-)$	497.70 (13)	50% K_S^0, 50% K_L^0
K_S^0	$1/2\,(0^-)$	$m_{K_L^0}-m_{K_S^0}$	$0.886\ (7)\cdot 10^{-10}$
K_L^0	$1/2\,(0^-)$	$=3.5564\cdot 10^{-12}$	$5.179\ (40)\cdot 10^{-8}$
η	$0^+\,(0^-)^+$	548.8 (6)	$7.74\ (1.09)\cdot 10^{-19}$
p	$1/2\,(1/2^+)$	938.2796 (27)	Stable $(>2\cdot 10^{30}$ y)
n	$1/2\,(1/2^+)$	939.5731 (27)	918 (14)
Λ	$0\,(1/2^+)$	1115.60 (5)	$2.578\ (21)\cdot 10^{-10}$
Σ^+	$1\,(1/2^+)$	1189.37 (6)	$0.800\ (6)\cdot 10^{-10}$
Σ^0	$1\,(1/2^+)$	1192.48 (8)	$\leqslant 10^{-14}$
Σ^-	$1\,(1/2^+)$	1197.35 (6)	$1.482\ (17)\cdot 10^{-10}$
Ξ^0	$1/2\,(1/2^+)$	1314.9 (6)	$2.96\ (12)\cdot 10^{-10}$
Ξ^-	$1/2\,(1/2^+)$	1321.29 (14)	$1.652\ (23)\cdot 10^{-10}$
Ω	$0\,(3/2^+)$	1672.2 (4)	$1.3^{+0.3}_{-0.2}\cdot 10^{-10}$
$\mathcal{J}/\psi$ (3098)	$0^-\,(1^-)^-$	3095 (5)	$9.54\ (2.07)\cdot 10^{-21}$
ψ' (3684)	$0^-\,(1^-)^-$	3684	$2.93\ (73)\cdot 10^{-21}$

Note. The table is composed on the basis of the review of the Particle Data Group,[2] except for the mean life of the particles $\mu^\pm$, π^0, η, $\psi(3095)$, and $\mathcal{J}/\psi(3684)$.[18]

cise units and measurement standards on the basis of the methods of quantum metrology.[4] (Tables VIII–XVII)

3. PHYSICAL-CHEMICAL CONSTANTS

New methods have recently been proposed for measurement of Faraday's constant and Avogadro's number, which have permitted improvement of their values. However, these values are not included among those recommended in the review of Cohen and Taylor.[1] (Table XVIII)

TABLE XVII. Strong interactions.

Quantity	Designation	Value
Characteristic time	$\hbar/m_\pi c^2$	$4.8\cdot 10^{-24}$ sec
Characteristic cross section	$\sigma=\pi(\hbar/m_\pi c)^2$	65 mb $=6.5\cdot 10^{-26}$ cm^2
Renormalized πN-interaction constant	$g^2_{\pi NN}/4\pi$	14.5
Renormalized $\rho\pi$-interaction constant	$f^2_{\rho\pi\pi}/4\pi$	2.6 (3)
Average value of transverse momentum of particles produced in hadron-hadron collisions	$\langle P_\perp\rangle$	0.4 (1) GeV/c
Hagedorn limiting temperature	T_H KT_H	$2.56\cdot 10^{12}$ °K 160 MeV

TABLE XVIII. Physico-chemical constants.

Quantity	Designation	Value
Avogadro's number*	N_A	$6.022045(31)\cdot 10^{23}$ mole^{-1}
Gas constant	R	$8.31441(26)\cdot 10^{7}$ erg/(mole °K) $=1.98719(6)$ cal/(mole °K)
Volume of 1 mole of ideal gas at $T_0=273.15$ °K, $P_0=1$ atm	$V_m=RT_0/P_0$	$22.41383(70)\cdot 10^{3}$ cm^3/mole
Loschmidt's number	$L_0=N_A/V_m$	$2.686754(84)\cdot 10^{19}$ cm^{-3}
Faraday's constant	$F=N_{Ae}$	$2.8925342(82)\cdot 10^{14}$ CGSE/mole $=9.6484456(27)$ C/mole
Boltzmann constant	$k=R/N_A$	$1.380662(44)\cdot 10^{-16}$ erg/°K $=8.61735(27)\cdot 10^{-11}$ MeV/°K $=1/11604.50(36)$ eV/°K
Stefan constant (σT^4 is the density of energy radiated per unit time by a unit surface of an absolute black body)	$\sigma=\pi^2k^4/60\hbar^3c^2$	$5.67032(71)\cdot 10^{-5}$ erg/(sec-cm^2-°K^4)
	$a=4\sigma/c=\pi^2k^4/15c^3\hbar^3$	$7.5657\cdot 10^{-15}$ erg/(cm^3-°K^4)
Wien's constant		0.289 cm-°K

*Hänsch *et al.*[15] obtained a more accurate value of Avogadro's number $N_A=6.0220943(61)\cdot 10^{23}$ mole^{-1}.

TABLE XIX. Particles and gravitation.

Quantity	Designation	Value
Planck mass	$m^*\sqrt{\hbar c/G}$	$2.177\cdot 10^{-5}$ g
Planck radius	$r^*=\hbar/m^*c=\sqrt{\hbar G/c^3}$	$1.616\cdot 10^{-33}$ cm
Planck time	$t^*=r^*/c=\sqrt{\hbar G/c^5}$	$5.390\cdot 10^{-44}$ sec
Planck density	$\rho^*=m^*/r^{*3}=c^5/G^2\hbar$	$5.159\cdot 10^{93}$ g/cm^3
Planck energy	$E^*=m^*c^2=\sqrt{\hbar c^5/G}$	$1.221\cdot 10^{28}$ eV
Planck magneton	$\mu^*=e\hbar/m^*c$	$0.776\cdot 10^{-42}$ G-cm^3
Gravitational charge of electron	$\sqrt{G}m_e$	$2.353\cdot 10^{-31}$ g$^{1/2}$-cm$^{3/2}$/sec
Gravitational charge of proton	$\sqrt{G}m_p$	$4.320\cdot 10^{-28}$ g$^{1/2}$-cm$^{3/2}$ sec
Ratio of electric charge to gravitational charge:		
for electron	$e/\sqrt{G}\,m_e$	$2.041\cdot 10^{21}$
for proton	$e/\sqrt{G}\,m_p$	$1.112\cdot 10^{18}$
Dimensionless gravitational ratios:		
for electron	$\hbar c/Gm_e^2$	$5.710\cdot 10^{44}$
for proton	$\hbar c/Gm_p^2$	$1.694\cdot 10^{38}$
for pion	$\hbar c/Gm_\pi^2$	$0.793\cdot 10^{40}$

TABLE XX. Ratios between masses of elementary particles.

Ratio	Reference	Ratio	Reference
$m_p/m_e=6\pi^5$ $(1836.1515\approx 1836.1181)$	[24]	$m_\eta=2m_{\pi^\pm}+2m_{\pi^0}$	[28]
$m_e/m_\mu=2\alpha/3$	[25]	$m_p:m_\Lambda:m_\Sigma:m_\Xi=1:\sqrt[4]{2}:\sqrt[3]{2}:\sqrt{2}$	[29]
$m_e/m_{\pi^\pm}=\alpha/2$			
$m_e/m_\mu=3\alpha\ln 2/\pi$ $[(3\alpha\ln 2/\pi)^{-1}m_e=105{,}7943$ MeV$]$	[26]	$m_{\psi'}(3684)/m_{J/\psi}(3098)=\sqrt[4]{2}$	
$(m_n-m_p)/(m_\Lambda-m_p)=\alpha$	[27]	$m_\mu=\sqrt{3/G}\,e\alpha^9$	[30]

TABLE XXI. Empirical ratios of MacGregor[17] between metastable-particle lifetimes.

Particle	Experimental lifetime τ, sec	Ratio of τ to neutron lifetime (experiment)	According to MacGregor
n	$918(14)$	1	1
$\mu^\pm$	$2.19711(8)\cdot 10^{-6}$	$2.4\cdot 10^{-9}$	α^4
K_L^0	$5.179(40)\cdot 10^{-8}$	$5.6\cdot 10^{-11}$	$\alpha^5\cdot 2$
$\pi^\pm$	$2.6030(23)\cdot 10^{-8}$	$2.8\cdot 10^{-11}$	α^5
$K^\pm$	$1.2371(26)\cdot 10^{-8}$	$1.3\cdot 10^{-11}$	$\alpha^5\cdot 1/2$
Ξ^0	$2.96(12)\cdot 10^{-10}$	$3.2\cdot 10^{-13}$	$\alpha^6\cdot 2$
Λ	$2.578(21)\cdot 10^{-10}$	$2.8\cdot 10^{-13}$	$\alpha^6\cdot 2$
Ξ^-	$1.652(23)\cdot 10^{-10}$	$1.8\cdot 10^{-13}$	$\alpha^6\cdot 2$
Σ^-	$1.482(17)\cdot 10^{-10}$	$1.6\cdot 10^{-13}$	α^6
Ω^-	$1.3^{+0.3}_{-0.2}\cdot 10^{-10}$	$1.4\cdot 10^{-13}$	α^6
K_S^0	$0.886(7)\cdot 10^{-10}$	$0.9\cdot 10^{-13}$	$\alpha^6\cdot 1/2$
Σ^+	$0.800(6)\cdot 10^{-10}$	$0.9\cdot 10^{-13}$	$\alpha^6\cdot 1/2$
Σ^0	$\leqslant 1.0\cdot 10^{-14}$	$\leqslant 1.1\cdot 10^{-17}$	α^8
π^0	$8.21(43)\cdot 10^{-17}$	$8.9\cdot 10^{-20}$	α^9
η	$7.74(1.09)\cdot 10^{-19}$	$8.4\cdot 10^{-22}$	α^{10}
J/ψ (3098)	$9.54(2.07)\cdot 10^{-21}$	$1\cdot 10^{-23}$	$\alpha^{11}\cdot 2$
ψ' (3684)	$2.93(73)\cdot 10^{-21}$	$3.2\cdot 10^{-24}$	α^{11}

Note. Values of α^n for n from 1 to 11 are given in Fig. 2.

TABLE XXII. Cosmological relations.

Quantity	Formula	Reference
Stewart ratio $Gm_e^2/r_e=\alpha^2\hbar H_0$	$m_e=\alpha(\hbar^2H_0/Gc)^{1/3}$	[31]
Weinberg ratio $Gm_\pi^2/\lambda_\pi=\hbar H_0$	$m_\pi=(\hbar^2H_0/Gc)^{1/3}$	[32]
Dirac ratio	$m_{MG}=m_p(\hbar c/Gm_p^2)^2$	[33]
Typical mass of galaxies	$10^{11}\,m_\odot=m_\pi(\hbar c/Gm_\pi^2)^{7/4}$	[34]
Typical mass of stars	$3.8\cdot 10^{33}$ g $=m_p(\hbar c/Gm_p^2)^{3/2}$	—
Mass of Planck particle (by definition)	$m^*=m_p(\hbar c/Gm_p^2)^{1/2}$	—
Eddington number $N_E=m_{MG}/m_p$	$N_E=\frac{3}{2}\cdot 2^{16^2}\cdot 136\approx 10^{79}$	[35]
Ratio of action for Metagalaxy to elementary action	$m_{MG}c^2/\hbar H_0\approx 10^{120}\approx N_E^{3/2}$	—
Ratio of parameters of Metagalaxy and Planck particle	$m_{MG}/m^*=r_{MG}/r^*=t_{MG}/t^*\approx 10^{60}$	—
Fine-structure constant	$\alpha=1/\ln 10^{60}\approx 1/\frac{3}{4}\ln N_E$	—
Hubble age	$t_{MG}=\hbar/\alpha^{18}m_ec^2$	[36]

TABLE XXIII. Titius-Bode law of planetary distances.*

Planet	☿	♀	⊕	♂	Asteroids	♃	♄	♅	♆	♇
Calculated distance, a.u.	0.55	0.70	1.0	1.60	2,80	5.20	10	19.6	38,8	77.2
True distance, a.u.	0.39	0.72	1.0	1.52	—	5.20	9.54	19.2	30.1	39.4

* *Note.* The distance of the n-th planet from the Sun in astronomical units is given by the empirical Titius-Bode formula: $a_n = 0.4 + 0.075 \times 2^n$, $n = 1, 2, 3, \ldots$

4. PARTICLES AND GRAVITATION

Here we have given values of certain combinations containing the gravitational constant and the fundamental constants. (Table XVIX)

5. EMPIRICAL RELATIONS

There are several relations between various quantities which do not follow from theories existing at the present time, and therefore we must consider them as numerological coincidences. Several such ratios are given in this section, together with references to primary sources. (Tables XX–XXIII)

Some of these ratios are satisfied with exceptionally high accuracy. For example, the ratio of the masses of the recently discovered particles J/ψ and ψ' is $m_{\psi'}(3684)/m_{J/\psi}(3098) = 1.1891542$, while $\sqrt[4]{2} = 1.1892071$. With a similar high degree of accuracy we have the ratio $m_\Lambda/m_p = \sqrt[4]{2}$. We can expect that some of the ratios given here are not accidental and will soon be explained theoretically.

6. NUMERICAL CONSTANTS

Here we have listed certain mathematical constants and also the powers of the fine-structure constant (Fig. 2).

$$\pi = 3.1415927 = 1 : 0.3183099$$
$$\sqrt{\pi} = 1.7724539 = 1 : 0.5641896$$
$$4\pi/3 = 4.1887902 = 1 : 0.2387324$$
$$e = 2.7182818 = 1 : 0.3678794$$
$$\lg e = 0.434945 = 1 : \ln 10 = 1 : 2.3025851$$
$$\lg 2 = 0.3010300$$
$$\ln 2 = 0.6931472$$
$$\sqrt{2} = 1.4142136 \qquad \sqrt{3} = 1.7320508 \qquad \sqrt{5} = 2.2360680$$
$$\sqrt[3]{2} = 1.2599211 \qquad \sqrt[3]{3} = 1.4422496 \qquad \sqrt[3]{5} = 1.7099760$$
$$\sqrt[4]{2} = 1.1892071 \qquad \sqrt[4]{3} = 1.3160740$$
$$\sqrt[5]{2} = 1.1486984$$
$$1\ \text{rad} = 180^\circ : \pi = 57^\circ 17' 44'',8 = 57^\circ.2957795$$
$$1^\circ = \pi : 180^\circ = 0.017453\ \text{rad},\ 1'' = \pi : 648000 = 4.8481 \cdot 10^{-6}\ \text{rad}$$
$$\alpha = 7.2973506\,(60) \cdot 10^{-3} = 1 : 137.03604\,(11)$$
$$\alpha^2 = 5.325 \cdot 10^{-5} \qquad \alpha^6 = 1.510 \cdot 10^{-13} \qquad \alpha^9 = 5.86 \cdot 10^{-20}$$
$$\alpha^3 = 3.885 \cdot 10^{-7} \qquad \alpha^7 = 1.102 \cdot 10^{-15} \qquad \alpha^{10} = 4.282 \cdot 10^{-22}$$
$$\alpha^4 = 2.836 \cdot 10^{-9} \qquad \alpha^8 = 8.041 \cdot 10^{-18} \qquad \alpha^{11} = 3.125 \cdot 10^{-24}$$
$$\alpha^5 = 2.069 \cdot 10^{-11}$$

FIG. 2. Numerical constants.

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