K^0_L–K^0_S transmission regeneration in hydrogen

I. A. Savin

Joint Institute for Nuclear Research, Dubna
Fiz. Élem. Chastits At. Yadra. **8**, 28–72 (January–February 1977)

The existing experimental data on K^0_L–K^0_S regeneration in hydrogen at high energies are compared with the predictions based on complex-angular-momentum theory and dispersion relations. The review opens with a brief description of the fundamentals of the theory and the methods of observation of regeneration.

PACS numbers: 14.40.Fw, 13.75.Jz

INTRODUCTION

Because of their unique properties, in which all modern ideas about elementary particles and their fundamental interactions meet, neutral kaons have been the subject of intense investigation for more than 20 years. Let us briefly recall these properties.

Neutral kaons form, on the one hand, a system of two states: K^0 and $\overline{K}^0$, which are eigenfunctions of the Hamiltonians of the strong, H_s, and the electromagnetic H_e, interactions with definite strangeness values +1 and −1, respectively. They are related to each other by the equation $CPT(K^0)=\overline{K}^0$, where CPT denotes the combined action of the operators of charge conjugation C, space reflection P, and time reversal T. Because of this, the eigenvalues of the functions K^0 and $\overline{K}^0$ with respect to H_s+H_e, which are the particle masses, are equal, i.e., $M_{K^0}=M_{\overline{K}^0}$.

On the other hand, from the point of view of weak interactions the system of neutral kaons also has two states with definite (different) masses and lifetimes. These states were introduced hypothetically by Gell-Mann and Pais[1] in 1955 and called K^0_1 and K^0_2. They are related to K^0 and $\overline{K}^0$ by quantum-mechanical "particle-mixing" relations:

$$K^0_1=(K^0+\overline{K}^0)\,2^{0.5};\quad K^0_2=(K^0-\overline{K}^0)/2^{0.5}. \tag{1}$$

The states K^0_1 and K^0_2 are eigenstates of the combined parity operator CP with the eigenvalues +1 and −1, respectively: $CP(K^0_1)=K^0_1$; $CP(K^0_2)=-K^0_2$.

Until 1964, it was assumed that the weak interactions are invariant under CP. Therefore K^0_1 and K^0_2 should decay through different channels in which the CP parity of the initial and final states is conserved. The allowed decays in this case are the two-pion decays $K^0_1: K^0_1\to\pi^+\pi^-(\pi^0\pi^0)$, where, as one can show, the pions are in a state with $CP=+1$. Accordingly, for K^0_2 there are allowed the three-pion decays $K^0_2\to\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0(K^0_{\pi 3})$, where the pions form a state with $CP=-1$. The kaon K^0_2 can also decay through semileptonic channels: $K^0_2\to\mu^\pm\pi^\pm\nu\ (K^0_{\mu 3})$ and $K^0_2\to e^\mp\pi^\pm\nu\ (K^0_{e3})$. The hypothesis of CP parity conservation forbids, for example, two-pion decays of K^0_2.

The decay rates of the states K^0_1 and K^0_2 are different. From the point of view of phase-space volume, decay into two pions must be much more common than, for example, decay into three pions. Therefore, the state K^0_1 is said to be short lived ($\tau_1=0.89\cdot10^{-10}$ sec) and the state K^0_2 long lived ($\tau_2=5.18\cdot10^{-8}$ sec).

The relations (1) were put forward at a time when the long lived K^0_2 meson had not yet been discovered. Its discovery in 1956 (Ref. 2) was the first brilliant confirmation of the particle-mixing hypothesis.

As Pais and Piccioni found,[3] the existence of two lifetimes leads to a small mass difference of K^0_1 and K^0_2: $|M_{K1}-M_{K2}|=\Delta M$ or $\delta=\Delta Mc^2\tau_1/\hbar$. As the measurements[4] showed, $\delta\approx0.5$, and $\Delta Mc^2\approx10^{-5}$ eV.

The superposition of the states K^0 and $\overline{K}^0$ in K^0_1 and K^0_2, the different lifetime of the second pair, and the small mass difference led to the discovery of one of the most remarkable phenomena in elementary particle physics—regeneration of one state by another. The essence of this phenomenon is as follows: Suppose that a pure K^0_2 wave is incident on a sample of matter. Before the interaction, the states K^0 and $\overline{K}^0$ are in accordance with (1) in equilibrium in the incident wave. However, after transmission through the target this equilibrium is destroyed by the difference between the amplitudes for the interaction of K^0 and $\overline{K}^0$ with matter:

$$\left.\begin{aligned}\psi_{\text{in}}&=K^0_2=(K^0-\overline{K}^0)/2^{0.5};\\ \psi_{\text{out}}&=(fK^0-\overline{f}\overline{K}^0)/2^{0.5}=(1/2)(f-\overline{f})K^0_1+(1/2)(f+\overline{f})K^0_2.\end{aligned}\right\} \tag{2}$$

As a result, the beam downstream from the sample contains in addition to the attenuated K^0_2 wave the regenerated K^0_1 wave. The amplitudes f and $\overline{f}$ characterize the change in the K^0 and $\overline{K}^0$ states, and the amplitudes

$$f_{21}\equiv(f-\overline{f})/2\quad\text{and}\quad f_{22}\equiv(f+\overline{f})/2 \tag{3}$$

are called the regeneration and scattering amplitudes, respectively.

The regeneration phenomenon was first predicted in Ref. 3 and studied theoretically in detail in Refs. 5–7. Different types of regeneration were distinguished and analytic expressions obtained for the intensity of the regenerated kaons and their angular distribution. The experimental investigation of regeneration proceeded in parallel, primarily in order to verify the particle-mixing hypothesis. Then, when the decay of long lived kaons into two pions was discovered in 1964 (Ref. 8), regeneration was used to study the problem of the breaking of CP invariance. In this connection, the states K^0_1 and K^0_2 introduced by the relations (1) are only states with definite CP parity. The short and long lived kaons came to be denoted by K_S^0 and K_L^0, respectively. They are related to K^0 and $\overline{K}^0$ by expressions more complicated than Eq. (1): linear relations with complex coefficients.[9–15] However, the change in the structure of the states of the neutral kaons did not lead to a change in the fundamental formulas describing regeneration, and the conclusions of Refs. 5–7 remained valid.

The discovery of CP breaking made it possible for the experimentalists to study problems of the strong interactions of kaons in regeneration experiments and, in particular, the asymptotic behavior of their interaction cross sections. These questions are considered in the present paper. In Sec. 1, we give a brief review of the main types of regeneration and the expressions that describe the angular distribution and intensity of the regenerated kaons; Sec. 2 is a review of the various theoretical models by means of which one can calculate the amplitude of transmission regeneration and its energy dependence; Sec. 3 gives a description of the apparatus and methods of observation of regeneration, and Sec. 4 the existing experimental data on $K_L^0-K_S^0$ regeneration in hydrogen. In the course of the exposition, we draw attention to the intimate connection between experiments on regeneration of neutral kaons and experiments to measure total interaction cross sections of charged kaons, carried out at Serpukhov.

1. FUNDAMENTAL FORMULAS OF REGENERATION

There are available many original investigations[7,11,16–18] with a rigorous mathematical description of regeneration processes. Below, we give the modern classification of the types of regeneration and the main expressions obtained in the quoted references.

Types of regeneration. In considering the essence of regeneration, we did not specify the nature of the interaction between the incident wave and the matter. However, depending on the particular processes that take place in the target when the K_L^0 beam passes through it, one can distinguish several types of regeneration, the main ones of which are the following: transmission (or coherent), elastic (or diffraction), inelastic, and electromagnetic. To each of them there corresponds a definite form of the regeneration amplitude f_{21}.

Transmission regeneration, or regeneration in the transmitted beam, occurs when the complete block of matter (or its atoms) interact as a whole with the incident K_L^0 wave and make a contribution to the amplitude of regenerated K_S^0 mesons. Pais and Piccioni,[3] for the idealized example of matter that completely absorbs $\overline{K}^0$ and completely transmits K^0, showed that this type of regeneration is a coherent process, as a result of which the regenerated K_S^0 mesons have the same energy as the incident K_L^0's, and the matter does not acquire a recoil. Subsequently, it was shown[6] that in ordinary matter too this process is coherent. Therefore, such regeneration is frequently called coherent regeneration.

Elastic regeneration of K_S^0's is a process in which the individual nucleons of a nucleus on which a K_L^0 is scattered make a coherent contribution to the amplitude of

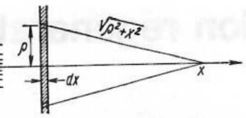

FIG. 1. Illustration of calculation of the variation of the wave function in a thin layer of matter.

scattering at a given angle. Other nuclei contribute to the intensity. In elastic regeneration, the nucleus as a whole receives hardly any recoil, and is not excited.

Inelastic regeneration of K_S^0's is characterized by the fact that as a result of the interaction between the incident K_L^0 wave and a nucleus the latter is excited or breaks up.

Electromagnetic regeneration[19,20] can occur if the K_L^0 mesons have an electromagnetic structure, with the result that K^0 and $\overline{K}^0$ interact differently with electrons (have, for example, opposite signs in front of the amplitudes).

All these types of regeneration except electromagnetic have been observed experimentally. In what follows, we shall discuss only transmission regeneration.

Transmission regeneration. We estimate the width of the angular distribution of the transmission-regenerated K_S^0's as follows. Consider a slab of matter and a layer dx thick in it (Fig. 1). The contribution to the scattered wave at point x from element $d\rho$ at distance ρ from the beam axis will be coherent only if $\rho - x \ll \lambda/2$, from which it follows that

$$\theta \ll (\lambda/\rho)^{0.5}. \tag{4}$$

If the kaons have momentum $p \geq 1$ GeV/c and $\rho \approx 1$ cm, it can be shown that θ is less than 10^{-7} rad. Thus, all the K_S^0's regenerated coherently by the slab are emitted in practically the same direction as the incident K_L^0, and their angular distribution will be determined by the resolution of the instrument by means of which they are observed.

The intensity of K_S^0 transmission regeneration can be calculated by solving the system of equations[5,6,17,18] that determine the changes in the states of K_L^0 and K_S^0 when they pass through matter. It is necessary to remember that each of them undergoes scattering, decay, and regeneration and propagates in space with a definite velocity.

In the case when the originally pure state is K_L^0 and one can ignore processes of secondary regeneration of K_L^0 by the regenerated K_S^0, the intensity of transmission regeneration (per incident K_L^0) when the beam leaves the target (regenerator) is determined by the complex regeneration coefficient $\rho'(p)$:

$$\rho'(p) \equiv |\rho(p)| \exp(i\Phi_\rho) \exp(-N\sigma_t L/2); \tag{5}$$

$$I_{K_S^0}(p) \approx |\rho'(p)|^2, \tag{6}$$

where

$$\rho(p) = 2\pi N\Lambda_S \frac{i f_{21}^0(p)}{k} \frac{\exp(-l/2) - \exp(-i\delta l)}{-(i\delta - 1/2)}. \tag{7}$$

Note the structure of the coefficient $\rho(p)$, which depends on the number of atoms N per unit volume, the $K_S^0 - \Lambda_S$ decay length, the amplitude of regeneration $f^0_{21}(p) \equiv |f^0_{21}(p)| \exp[i\varphi^0_{21}(p)]$ on the given nucleus, the wave number $k = p/\hbar$, and a factor related to the length of the target ($l = L/\Lambda_S$) and the K_L^0 and K_S^0 mass difference. For short lengths of the target, the last factor is equal to l.

The phase of the regeneration coefficient, as can be seen from (7), is a sum of two terms:

$$\Phi_\rho(p) = \arg i\,[f^0_{21}(p)] + \arg\frac{\exp(-l/2) - \exp(-i\delta l)}{-(i\delta - 1/2)}, \qquad (8)$$

the first of which is the phase of the transmission-regeneration amplitude, or the phase difference between the amplitudes of elastic scattering of K^0 and $\overline{K}^0$ in the given material through zero angle, and the second is completely calculated. For small l, the phase of the regeneration coefficient is completely determined by the first term.

The appearance in Eq. (7) of the factor $2\pi N i f^0_{21}(p)/k$ can be demonstrated in the following simple way. We write the change in the wave function due to scattering on the passage through a thin layer dx in the slab of matter in the form

$$d\psi = N\,dx \int_0^\infty 2\pi\rho\,d\rho\,\frac{\exp(ik\sqrt{\rho^2+x^2})}{\sqrt{\rho^2+x^2}}\,f(\theta) \qquad (9)$$

(see Fig. 1), where $f(\theta)$ is the scattering (regeneration) amplitude. Because of the presence in the integrand of a term that oscillates rapidly for large ρ and the narrow width of the angular distribution of the transmission-regenerated K_S^0 mesons, the effective change of the wave function is determined by

$$d\psi = 2\pi N i f^0\,dx/k, \qquad (10)$$

where $f^0 \equiv f(0)$ is the amplitude of scattering through zero angle.

The intensity of K_S^0 transmission regeneration is, in accordance with (6), determined by

$$I_{K_S^0}(p) = N^2 \Lambda_S^2 \pi^2 \frac{|f^0_{21}(p)|^2}{k^2}\,\frac{1 - \exp(-l) - 2\exp(-l/2)\cos(\delta l)}{\delta^2 + \frac{1}{4}} \times \exp(-N\sigma_t L). \qquad (11)$$

It depends on N^2 and also the length of the target. This dependence, for a given momentum of the particles, shows (Fig. 2) that the optimal yield of K_S^0 is achieved for $l \approx 3.5$, which for kaons with momentum 10 GeV/c corresponds to a length of a liquid-hydrogen target of about 3.5 m. The oscillations in the K_S^0 intensity depend on the term containing, besides l, the K_L^0 and K_S^0 mass difference.

Interference formula for two-pion decays of kaons. Experimental study of the properties and interactions of neutral kaons is possible only through the observation of the intensity of their decays as a function of the time elapsing from the time of production or interaction to the time of decay.

We obtain the expression describing the intensity of

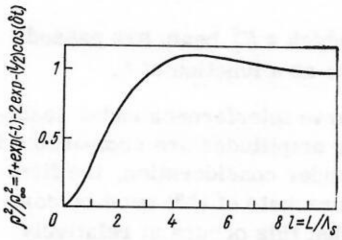

FIG. 2. Regeneration coefficient as a function of the target length.

two-pion decays of kaons as follows. Suppose a slab of matter is placed in a beam of pure K_L^0 mesons. In accordance with what we have said earlier, downstream from the slab we have not only the attenuated K_L^0 wave but in the same direction the coherent regenerated K_S^0 wave with amplitude $\rho(p)$ determined by the expression (7). If they can both decay into $\pi^+\pi^-$ pairs, the wave function of this state downstream from the regenerator can be written in the form

$$\psi_{+-}(p, t) = \psi_L(p, t) A_L(\pi^+\pi^-) + \rho(p)\,\psi_S(p, t) A_S(\pi^+\pi^-), \qquad (12)$$

where $\psi_L(p, t) = \exp[-(\Gamma_L/2 + iM_L)t]$, $\psi_S(p, t) = \exp[-(\Gamma_S/2 + iM_S)t]$ are the K_L^0 and K_S^0 propagation wave functions (Refs. 9–12) and A_L and A_S are their decay amplitudes. Substituting into (12) the expressions for the propagation functions, taking A_S outside the brackets, and using $\eta_{+-} = A_L/A_S \equiv |\eta_{+-}| \exp(i\Phi_{+-})$, we obtain

$$I_{+-}(p, t) = |\psi_{+-}(p, t)|^2 = \Gamma_{S+-}\Big[\,|\eta_{+-}|^2 \exp(-\Gamma_L t) + |\rho(p)|^2 \exp(-\Gamma_S t) + 2\,|\rho(p)\,\eta_{+-}| \exp\Big(-\frac{\Gamma_L + \Gamma_S}{2}\,t\Big)\cos(\delta t + \Phi_\rho(p) - \Phi_{+-})\Big], \qquad (13)$$

where Γ_L and Γ_S are the K_L^0 and K_S^0 decay rates, respectively; Γ_{S+-} is the partial decay rate of K_S^0 into $\pi^+\pi^-$ pairs; t is the time that elapses in the kaon rest frame from the time of interaction to the time of decay. Since the point (time) of interaction is usually unknown, in the expression (13) the time origin is chosen such that $t = 0$ on the exit side of the regenerator slab.

The number of observed decays of kaons with momenta from p to $p + \Delta p$ during the time interval from t to Δt depends on the total intensity of the K_L^0 mesons that have passed through the slab (M), the efficiency of detection $\varepsilon(p, t)$ of the decays that take place, and the fraction of kaons in the given range of momenta, i.e., the K_L^0 momentum spectrum $S(p)$:

$$\frac{d^2N}{dp\,dt}(p, t)\,\Delta p\,\Delta t = MS(p)\,\varepsilon(p, t)\,I_{+-}(p, t)\,\Delta p\,\Delta t. \qquad (14)$$

The efficiency of detection of K^0 mesons by an experimental apparatus can be calculated by the Monte Carlo method, and M and $S(p)$ can be determined experimentally in, for example, an investigation of three-particle decays of K_L^0's detected by the same experimental apparatus.

As can be seen from (13), the change in the intensity of two-pion decays of kaons in the space downstream from the target through which a K_L^0 beam has passed has an interference nature as a function of t.

It is convenient to observe interference under conditions when the interfering amplitudes are comparable in magnitude. In the case under consideration, the first two terms in the square brackets of (13) must be comparable. One can show that this occurs at relatively large distances from the regenerator, when

$$t \sim 2\tau_S \ln|\rho/\eta_{+-}|. \quad (15)$$

Estimates show that for heavy regenerators $\ln|\rho/\eta_{+-}| \sim 3-4$ and $t \sim (6-8)\tau_S$. In the case of light regenerators, for example, liquid hydrogen, $\ln|\rho/\eta_{+-}| \sim 0.5-1$ and $t \sim \tau_S$, i.e., the contribution of the interference term will be appreciable near the target.

2. REGENERATION AND ASYMPTOTIC BEHAVIOR OF THE SCATTERING AMPLITUDES OF NEUTRAL KAONS

The study of the energy dependence of the amplitude of $K_L^0-K_S^0$ transmission regeneration on protons makes it possible to obtain information about the value, and behavior as a function of the energy, of the corresponding difference between the kaon and antikaon forward scattering amplitudes, or data on the value and energy dependence of the real $\mathrm{Re}[f^0(K^0p)-\bar{f}^0(\bar{K}^0p)] = \mathrm{Re}[2f_{21}^0]$ and imaginary $\mathrm{Im}[f^0(K^0p)-\bar{f}^0(\bar{K}^0p)] = \mathrm{Im}[2f_{21}^0]$ parts of it since, by definition, $f_{21}^0 = [f^0(K^0p)-\bar{f}^0(\bar{K}^0p)]/2$.

There are other ways of measuring the magnitude and studying the energy dependence of the difference between the elastic scattering amplitudes of kaons and antikaons on hydrogen. For example, measuring the total cross sections for the interaction of K^+ and K^- mesons with hydrogen and using the optical theorem, one can obtain data on the behavior of the difference between the forward scattering amplitudes. The real part of the same difference can be determined by studying the interference between the nuclear and Coulomb interactions in the elastic scattering of K^+ and K^- mesons on hydrogen with subsequent extrapolation to zero scattering angles.

Compared with these experiments, regeneration experiments have two important advantages: 1) In them, one directly measures the difference between the zero-angle scattering amplitudes, whereas in charged beams these quantities are obtained by extrapolating data obtained at nonzero scattering angles; 2) in a single experiment one can study the behavior of both the imaginary and the real part of the difference of the scattering amplitudes, whereas at least four experiments are needed in the case of charged kaon beams.

These advantages of regeneration experiments are particularly clear if one bears in mind that there are available for them clear cut predictions based on Pomeranchuk's theorem, dispersion relations, and various models of the Regge-pole type or the model of complex angular momenta. All these predictions indicate that the difference between the scattering amplitudes tends to zero as the energy tends to infinity. Therefore, to obtain a physical result at high energies it is necessary to eliminate possible systematic errors associated with the performing of several experiments and the extrapolation procedures.

Interaction of K mesons and Pomeranchuk's theorem. In 1958, Pomeranchuk[21] proved a theorem which states that at high energies (rigorously as $E\to\infty$) there exist certain limiting (asymptotic) relations between the cross sections for the interaction of particles and antiparticles with nucleons and antinucleons.

Applied to K mesons, the relations of the theorem predict, in particular, at sufficiently high energies equality of the total cross sections for the interaction of kaons and antikaons with one and the same nucleon. Since this equality was not satisfied at the energies available at the time the theorem was derived, and the cross sections for negative particles were appreciably greater than for positive, the theorem indicated that the corresponding differences between the total cross sections would decrease and tend to zero with increasing available energies. For example

$$\Delta\sigma(E) \equiv [\sigma_t(\bar{K}^0p)-\sigma_t(K^0p)]_{E\to\infty}\to 0; \quad [\sigma_t(K^-n)-\sigma_t(K^+n)]_{E\to\infty}\to 0. \quad (16)$$

In the proof of his theorem, Pomeranchuk used fairly general theoretical assumptions and experimental facts: isotopic invariance, dispersion relations, crossing symmetry of the wave function, small charge-exchange cross section, large number of open channels in inelastic interactions at high energies, etc. Therefore, Pomeranchuk's theorem is the basis of modern ideas about scattering processes at high energies, though this does not preclude the desire for an experimental verification of it, especially with the attainment of a new energy region through the construction of new accelerators.

The need to discuss a possible failure of the theorem arose, for example, quite recently on the basis of the following experimental facts. In 1969 there were published[22] (Fig. 3) measurements of total interaction cross sections of negative particles with protons and deuterons that did not correspond to the generally accepted

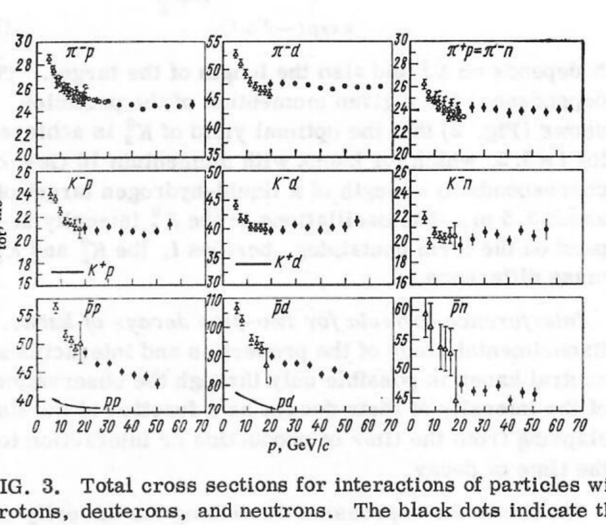

FIG. 3. Total cross sections for interactions of particles with protons, deuterons, and neutrons. The black dots indicate the results of Ref. 22.

predictions[23]; these later became known as the Serpukhov effect. A striking feature of these data is the constancy of the total cross sections for the interaction of K^- mesons in the momentum range from 20 to 55 GeV/c. According to the results[24] obtained in Brookhaven, the total cross sections of K^+ mesons in the range from 6 to 20 GeV/c are also constant. Thus, the data of Refs. 22 and 24 taken together do not contradict the assertion that the total cross sections for interactions of K^+ and K^- mesons with nucleons reach an asymptotic limit, the former at 6 GeV and the latter at 20 GeV, and that as $E \to \infty$ their difference is constant and nonzero. If this assertion were confirmed by direct measurements of the total cross sections for K^+ in the energy range up to 60 GeV, this would be a serious indication of failure of Pomeranchuk's theorem, according to which all the $\Delta\sigma(\infty)$'s must be equal to zero.

From the experimental point of view, it is obvious that the assertion made above is incorrect, since within the error limits one can approximate all the data of Refs. 22 and 24 by not merely a constant but also by a function that decreases with the energy and even has a minimum. Moreover, it was shown in Ref. 25 that if one allows a systematic error of about 4% in the data of Ref. 22 or 24 the complete set of data could be explained in the framework of ordinary models.

However, the theoretical discussion of the possible failure of Pomeranchuk's theorem and the consequences that flow from this for theory and experiment was necessary and helpful. Theoretical investigations[26–33] showed that failure of the theorem would lead to a very complicated picture in scattering processes at high energies such as could hardly be realized in nature and would perhaps entail violation of fundamental principles such as analyticity and (or) crossing symmetry. Various experiments casting light on this situation were therefore proposed in the literature.

These included, in the first place, direct measurements of the total cross sections of interactions of K^+ mesons with nucleons, measurements of $K_L^0-K_S^0$ regeneration on nucleons and nuclei, and measurements of charge-exchange cross sections.

As we have seen, regeneration of neutral kaons is an ideal reaction for experimental verification of Pomeranchuk's theorem and the assertion made above about the values of $\Delta\sigma$ in the asymptotic region. Indeed, applying the optical theorem, we obtain the following relation between $\Delta\sigma$ and the imaginary part of the regeneration amplitude:

$$\Delta\sigma(p) = (4\pi/k)\,\mathrm{Im}\,[2f_{21}^0(p)] = 4\pi\,[2\,|f_{21}^0(p)|/k]\sin\varphi_{21}^0(p). \tag{17}$$

It follows from isotopic invariance and the data of Refs. 22 and 24 that

$$\sigma_t(K^-n) - \sigma_t(K^+n) = \sigma_t(\bar{K}^0p) - \sigma_t(K^0p) = 2.5 \pm 1.5\ \mathrm{mb}\ . \tag{18}$$

Substituting (18) into (17), we obtain a prediction for the imaginary part of the amplitude of regeneration on protons:

$$2\,\mathrm{Im}\,f_{21}^0(p)/k = -(0.2 \pm 0.12)\ \mathrm{mb}\ . \tag{19}$$

On the other hand, it is shown in Ref. 31 that if Pomeranchuk's theorem[21] is satisfied, the ratio of $\mathrm{Re}\,f_{21}^0$ to $\mathrm{Im}\,f_{21}^0$ with constant difference of the corresponding cross sections must increase logarithmically with the energy, i.e.,

$$\alpha(E) = \mathrm{Re}\,f_{21}^0(E)/\mathrm{Im}\,f_{21}^0(E) = -2\ln E/\pi. \tag{20}$$

Since experiments at low energies on nuclei had shown[34] that $\alpha \sim 1$, Eq. (20) means that the real part must change sign with increasing energy. Rough estimates showed[31] that this can occur already at kaon energies 20–40 GeV if Eq. (18) holds.

The importance of setting up experiments to study transmission regeneration of neutral kaons in connection with the verification of a possible failure of Pomeranchuk's theorem has been noted many times.[26–33, 35–39]

Regeneration amplitude in the theory of complex angular momenta. In the theory of complex angular momenta, the scattering amplitudes are usually described in the form of a sum of partial-wave amplitudes, each of which corresponds to the exchange of a pole with definite quantum number between particles. The poles P, P', ω, ρ, A_2, and others contribute to the kaon–nucleon forward scattering amplitudes. It is easy to show[40] that if the additional contribution of branch points is ignored, the amplitude of $K_L^0-K_S^0$ regeneration on protons is determined by the two poles ω and ρ:

$$f_{21}^0 \equiv M(K_L^0 + p \to K_S^0 + p) = M_\omega - M_\rho, \tag{21}$$

where M_a ($a = \omega, \rho$) are the corresponding partial-wave amplitudes of the model. Each of the amplitudes has the structure

$$M_a(t, E) = B_a(t)\,\xi_-(\alpha_a(t))\,(1/E)^{1-\alpha_a(t)}, \tag{22}$$

where $B_a(t)$ is the residue of the amplitude in the cross channel; E is the kaon energy in the laboratory system; ξ is the signature of the pole:

$$\xi_-(\alpha_a(t)) = \frac{-1-\exp[-i\pi\alpha_a(t)]}{\sin\pi\alpha_a(t)} = -\left(i + \tan\frac{\pi\alpha_a(t)}{2}\right); \tag{23}$$

$\alpha_a(t) = \alpha_a(0) + \alpha_a'(0)t + \ldots$ is the trajectory of the pole.

For transmission regeneration, the structure of the partial-wave amplitudes is simplified since, by definition, it must be given for $t = 0$.

Using Eqs. (21)–(23) for $t = 0$, we obtain the following transmission-regeneration amplitude:

$$f_{21}^0 = B_\rho(0)\left[\tan\frac{\pi\alpha_\rho(0)}{2} + i\right]\left(\frac{1}{E}\right)^{1-\alpha_\rho(0)} - B_\omega(0)\left[\tan\frac{\pi\alpha_\omega(0)}{2} + i\right]\left(\frac{1}{E}\right)^{1-\alpha_\omega(0)}. \tag{24}$$

The values of the constants in (24) can be obtained from the currently known experimental data on the total interaction cross sections, the elastic-scattering cross sections, the charge-exchange cross sections, etc. In particular, in Table I we give the set of these constants

TABLE I. Parameters characterizing the contribution of the ρ and ω poles to the scattering amplitude.

Pole	$\alpha_a(0)$	$\alpha'_a(0)$ $(GeV/c)^{-2}$	$B_a(0)$, mb, for $\Delta\sigma(\infty)=0$	$B_a(0)$, mb, for $\Delta\sigma(\infty)$ = 0.55 mb	Literature
ω	0.45	1.0	8.96±0.26	7.69±0.55	[41]
	0.43±0.01	—	7.95±0.13	—	[42]
ρ	0.49	0.7	2.74±0.26	2.61±0.26	[41]
	0.57±0.01	—	1.31±0.13	—	[42]

that satisfies best the experimental data up to 1972 and also their values with allowance for one of the latest (as of 1975) phenomenological analyses.[42] In contrast to Ref. 41, branch points were not taken into account in the analysis of the data in the framework of complex angular momenta in Ref. 42. If it is assumed that $\alpha_\rho(0) \approx \alpha_\omega(0) \equiv \alpha(0)$, and we write $B_\omega(0) - B_\rho(0) = \sigma_{12}$, then the form of the amplitude f^0_{21} simplifies, especially if we remember that $\alpha_a(0) \approx 0.5$:

$$f^0_{21} \approx -\sigma_{12}\left[\tan\frac{\pi\alpha(0)}{2} + i\right] E^{-0.5} \approx -\sigma_{12}[1+i]\, E^{-0.5} \qquad (25)$$

From this there follow simplified predictions of the theory of complex angular momenta for the transmission-regeneration amplitude:

1) the tangent of the phase of the amplitude ($\varphi^0_{21} = \arg f^0_{21}$) is determined by the relation

$$\tan\varphi^0_{21} = \operatorname{Im} f^0_{21} / \operatorname{Re} f^0_{21} = \cot(\pi\alpha(0)/2); \qquad (26)$$

since $\alpha(0) \approx 0.5$, and the real and the imaginary parts of the amplitude are negative, it follows that $\varphi^0_{21} \approx -135°$;

2) the differential regeneration cross section

$$(d\sigma/dt)_{t=0} = |f^0_{21}|^2/16\pi = \sigma^2_{12}/8\pi E \sim p^{2\alpha(0)-2} \qquad (27)$$

decreases in inverse proportion to the energy of the incident kaon.

As is shown in Refs. 41 and 43, more accurate allowance for the values of the parameters of the poles and also for branch points of the ρ and ω poles does not significantly alter these predictions. For example, the phase is changed by not more than 2°.

In some theoretical studies,[37,41,43] predictions of the theory of complex angular momenta have been obtained under the assumption of failure of Pomeranchuk's theorem in kaon–nucleon interactions. The simplest way of violating this theorem in the theory of complex angular momenta is by the artificial introduction into the amplitude of an additional term ΔM that increases logarithmically with the energy. The presence of such a term distorts the asymptotic form of the amplitude and means, as a consequence, that the difference between the total cross sections of K^+ and K^- interactions with nucleons at infinite energy is not zero. With allowance for the additional term, the transmission-regeneration amplitude in the case of failure of Pomeranchuk's theorem can have the form[41]

$$f^{0\prime}_{21} = -\frac{\sigma_{12}}{\sqrt{E}}\left(\tan\frac{\pi\alpha(0)}{2} + i\right) + \frac{\Delta\sigma(\infty)}{2}\left(\frac{2}{\pi}\ln\frac{E}{E_0} - i\right) \equiv R - iI. \qquad (28)$$

Approximation of the total-cross-section data of Refs. 22 and 24 under the assumption of failure of Pomeranchuk's theorem gave $\Delta\sigma(\infty) = 0.55$ mb (Ref. 41). The constant E_0 is not determined by these data.

The predictions for the regeneration amplitude (Fig. 4) depend strongly on the values of the constants $\Delta\sigma(\infty)$ and E_0. This is particularly so in the case when E_0 = 1 GeV. With increasing energy, the logarithmic term in (28) begins to dominate. At the same time, the momentum dependence of the cross section in logarithmic scale is strongly nonlinear, and the phase of the regeneration amplitude decreases in absolute magnitude because R tends to zero. Thus, the vector $f^{0\prime}_{21}$ at low energies in the third quadrant of the complex plane ($\operatorname{Re} f^{0\prime}_{21}$, $\operatorname{Im} f^{0\prime}_{21}$) must rotate with increasing energy in the anticlockwise sense, passing through −90° at some energy that depends on $\Delta\sigma(\infty)$.

If $E_0 \gg 1$, the picture is not so clear and strong changes in the regeneration cross section in a restricted energy region do not occur. However, (see Fig. 4)

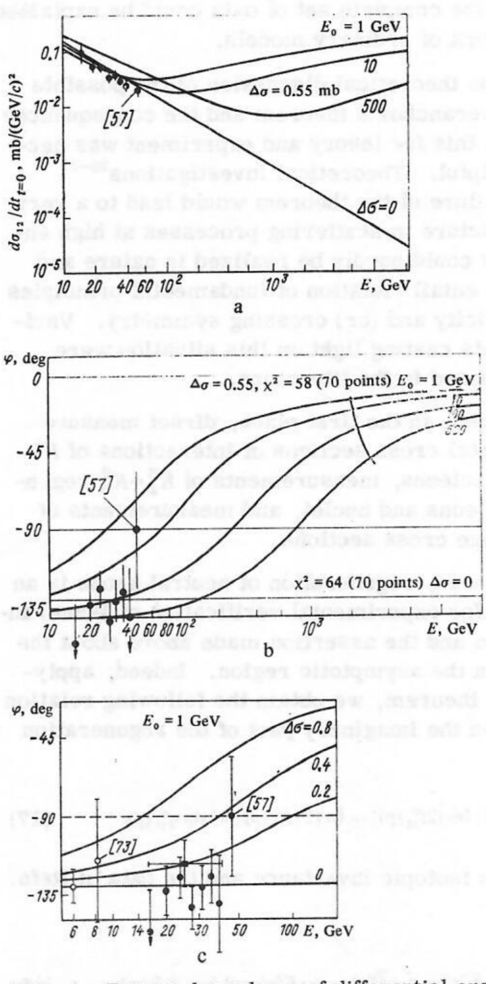

FIG. 4. Energy dependence of differential cross section (a) and phase of the amplitude (b, c) of transmission regeneration in hydrogen calculated on the basis of the model of complex angular momenta when Pomeranchuk's theorem is fulfilled ($\Delta\sigma = 0$) and is not fulfilled ($\Delta\sigma \neq 0$).

measurement of the phase of the regeneration amplitude in the energy range 10–70 GeV with accuracy of about 5% would already enable one in this case too to establish whether Pomeranchuk's theorem is violated in kaon–nucleon processes.

A failure of Pomeranchuk's theorem is not the only way of explaining the results of the measurements of the total interaction cross sections of K^- mesons in the Serpukhov energy range. It has been pointed out[37–39] that they can be explained under the assumption that in the interval from 20 to 60 GeV the total cross sections for interaction of K^+ mesons with nucleons increase, i.e., the cross section of regeneration of K_S^0 mesons on nucleons decreases with increasing energy.

In Ref. 37, various complicated models of complex angular momenta are analyzed. These models, which, besides the usual poles, take into account:

a) strong cuts associated with the Pomeranchuk pole,[38, 44–46]

b) dipoles with violation of Pomeranchuk's theorem,[29, 33, 37, 41]

c) dipoles with asymptotically increasing cross sections,[37] and

d) complex-conjugate poles,[47]

give a fairly good description of the dependence of the total cross sections. However, the data on the total cross sections in the range up to 70 GeV are inadequate alone to choose between these models. As follows from Refs. 37, experiments on the regeneration of neutral kaons enable one to restrict the possible class of these models.

Regeneration and dispersion relations. There have been several studies devoted to the use of different types of dispersion relations for calculating the ratio of the real to the imaginary part of the regeneration amplitude[48–55]; we review these below briefly. In the majority of them, the behavior of the total cross sections at infinitely high energies is estimated on the basis of the theory of complex angular momenta. In this case, the dispersion-relation predictions hardly differ from those of the theory of complex angular momenta.

On the basis of dispersion relations, Lusignoli *et al.*[48] calculated the phase of the regeneration amplitude as a function of the momentum in the region 1.5–5.0 GeV/c. However, because of the large uncertainties in the estimate of the contribution of the pole terms associated with hyperon production and also in allowing for the low-energy scattering region, where the experimental data contain large errors, the accuracy in the prediction of the phase is 50–100%.

In the framework of dispersion relations, Aznauryan and Solov'ev[50] considered the problem that arises from the constancy of the total cross sections for K^-p interactions at energies 20–60 GeV and showed that if Pomeranchuk's theorem is violated and the experimental data are parametrized in a definite manner, the ratio of the real part of the regeneration amplitude to the imag-

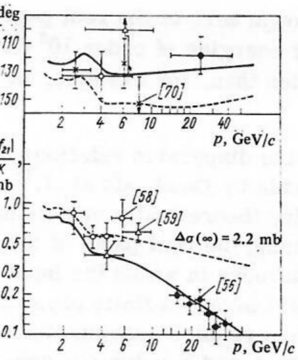

FIG. 5. Energy dependence of the modulus $2|f_{21}^0|/k$ and phase φ_{21}^0 of the amplitude of transmission regeneration in hydrogen calculated[51] on the basis of dispersion relations. The continuous curve is for the case of fulfillment of Pomeranchuk's theorem and uses in the asymptotic limit the model of complex angular momenta that takes into account the contribution of poles and branch points to the amplitude; the dashed curve corresponds to the case when Pomeranchuk's theorem is violated.

inary decreases with increasing energy from about unity to zero at energies 20–130 GeV. At the same time, the behavior of the regeneration cross section at angle 0° at energies greater than 50 GeV and below 10 GeV differs strongly from the behavior predicted by the theory of complex angular momenta.

Using dispersion relations, Vishnevskiĭ *et al.*[51] calculated the real parts of the $K^\pm N$ amplitudes, and on the basis of these the modulus and phase of the transmission-regeneration amplitude under three assumptions about the behavior of the total cross sections of $K^\pm N$ interactions as $E \to \infty$: 1) The asymptotic behavior is determined by the model of complex angular momenta in which only poles contribute to the scattering amplitudes; 2) the differences between the total cross sections are $\Delta\sigma(\infty)(K^\pm n) = 2.2$ mb and $\Delta\sigma(\infty)(K^\pm p) = 3.2$ mb, i.e., not equal to zero at infinite energy, which corresponds to violation of Pomeranchuk's theorem; 3) the asymptotic behavior is determined by a model of complex angular momenta with contribution by poles and branch points.[38] In the case of failure of Pomeranchuk's theorem, they used the dispersion relations of Ref. 52 with a subtraction at energy 10^3 GeV. The results of the calculation under the last two assumptions are shown in Fig. 5. It can be seen that the phases of the amplitude of regeneration on protons depend strongly on the type of calculation: In the region accessible to measurements (10–50 GeV/c), assumptions 2 and 3 differ in the φ_{21}^0 value by 20°! The moduli of the regeneration amplitude in the cases when the calculations are made with and without violation of Pomeranchuk's theorem also differ strongly from one another. This is particularly pronounced at energies around 20–50 GeV, where the difference reaches 200% and more. In this respect, the data of the calculation differ from other calculations, which in the energy range 10–50 GeV do not give such a pronounced difference in the $|f_{21}^0(p)|/k$ values for the case of validity and invalidity of the theorem. On the other hand, in the present model one obtains a weak dependence of the phase of the regeneration amplitude on the energy: The transition through zero of the real part of the amplitude is expected at energies of order 10^3 GeV, i.e., at much higher energies than, for example, in Refs. 31, 41, and 50.

Careful investigations of the dispersion relations for the KN system have been made by Dumbrajs *et al.*[53] They developed an interesting theoretically model-independent method for determining the real parts of the $K^{\pm}p$ forward scattering amplitudes in which the input data are experimental results in only a finite physical energy range and one does not require assumptions about the contributions of the Λ and Σ poles nor concerning the amplitudes in the unphysical region or the asymptotic region. The phase of the amplitude of regeneration on protons calculated on the basis of this method under the assumption of failure of Pomeranchuk's theorem must pass through the value $-90°$ at energies around 500 GeV.

Thus, the predictions for the modulus and phase of the amplitude of forward regeneration obtained on the basis of dispersion relations differ strongly because of the great uncertainty in the input data and are largely of qualitative nature. If Pomeranchuk's theorem is violated, they indicate that the real part of the regeneration amplitude changes sign as a function of the energy, but the value of the energy at which this occurs lies in the range 25–1000 GeV.

In Ref. 55, Nguen Van Hieu, using a method developed by Logunov *et al.*,[54] investigated transmission regeneration from the point of view of the general requirements of quantum field theory on the scattering amplitude. He showed that if the imaginary part of the function $f^0_{21}(s)/s$ has logarithmic behavior, i.e.,

$$\operatorname{Im} f^0_{21}(s) \sim s(\ln s)^{\beta}. \tag{29}$$

then

$$\lim_{s\to\infty} |\operatorname{Re} f^0_{21}(s)/\operatorname{Im} f^0_{21}(s)| \geqslant \text{const} \ln s. \tag{30}$$

But if the asymptotic expression for the amplitude has, for example, the form

$$f^0_{21}(s) \sim s^{\alpha}(\ln s)^{\beta}, \quad \alpha < 1, \tag{31}$$

then

$$\tan \varphi^0_{21} = \operatorname{Im} f^0_{21}(s)/\operatorname{Re} f^0_{21}(s) \xrightarrow[s\to\infty]{} \cot(\pi\alpha/2), \tag{32}$$

i.e., the phase of the regeneration amplitude is asymptotically constant. These alternatives can be verified experimentally in the accessible energy region.

3. APPARATUS AND METHOD OF PERFORMING K^0_L–K^0_S REGENERATION EXPERIMENTS

Experiments to measure K^0_L–K^0_S regeneration in hydrogen are categorized as "difficult." They make great demands on the apparatus and could not be performed at all until a certain level of development of methods and experimental technique had been achieved.

As is clear from the above, the amplitude of transmission regeneration in hydrogen is determined by studying interference in $K^0_{\pi 2}$ decays of K^0_L and K^0_S mesons. This interference takes place in the space behind the hydrogen target placed in the K^0_L beam. Observing the distribution of the intensity of two-pion decays in this space and approximating it by expressions of the type (13) and (14), one can determine $|\rho(p)|$ and $\Phi_\rho(p)$ if all the other quantities are known or can be determined. Then, using Eqs. (7) and (8), one can find $|f^0_{21}(p)|$ and $\varphi^0_{21}(p) \equiv \arg f^0_{21}(p)$. In accordance with (15), interference is observed over a distance of a few K^0_S decay lengths. Since the K^0_S decay length is fairly long at high energies, the apparatus for detecting the $K^0_{L,S}$ decays must have a fairly high and relatively uniform efficiency over a base length of several meters.

The smallness of the regeneration cross section and the dependence of the optimal yield of regenerated K^0_S mesons on the target length (see Fig. 2) make it necessary to construct a liquid-hydrogen target several meters long.

At high energies, simulation by the Monte Carlo method shows that the efficiencies of detection of different modes of K^0 meson decays are fairly similar, and it is difficult to choose conditions under which the $K^0_{\mu 3}$, K^0_{e3}, and $K^0_{\pi 3}$ decays are strongly suppressed compared with the $K^0_{\pi 2}$ decays. Therefore, to separate out the transmission regeneration the apparatus must have good angular and momentum resolution, and also contain lepton detectors, by means of which the $K^0_{\mu 3}$ and K^0_{e3} decays can be sufficiently well identified by analyzing the information obtained.

The physical problem requires one to study regeneration as a function of the energy right up to that maximally possible with the given accelerator. For this, it is necessary to have a particle beam oriented onto an interior target of the accelerator at a small angle, for which the high-energy K^0_L yield is sufficient for performing an experiment. However, at small angle there is a strong increase in the intensity of neutrons, whose interactions in hydrogen may overload the apparatus. To facilitate the problem of arranging a trigger signal, hodoscope counters are usually included in the apparatus.

Finally, the required accuracy of the measurements and also the impossibility of prior selection of the investigated decay type force one to detect as many events as possible over a cycle of the accelerator operation, and this can be achieved only if there is automatic reception of information by means of a computer.

The parts of the apparatus and its arangement in the majority of regeneration experiments[56–60] performed electronically are more or less similar. All these experiments have been carried out by means of magnetic spark spectrometers. For example, let us consider the apparatus used to perform the experiments of Refs. 56 and 57 at Serpukhov (Fig. 6), pointing out its differences from other similar set ups.

The experiments of Refs. 56 and 57 were made by means of a filmless spark spectrometer[61] on line with a BÉSM-3M computer used to receive, control, and re-

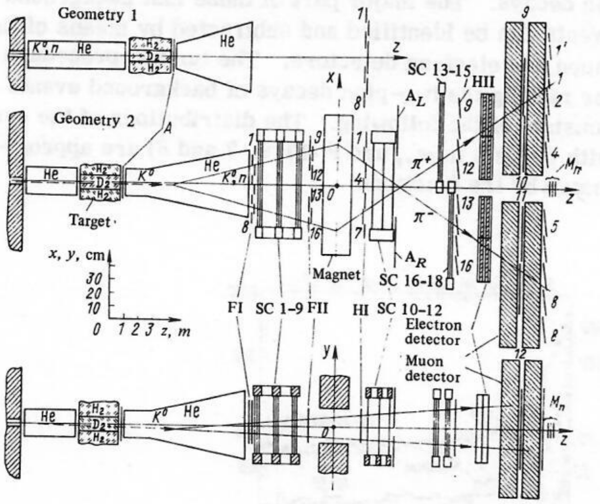

FIG. 6. Arrangement of the filmless spark spectrometer (Joint Institute of Nuclear Research). FI, FII, HI, HII are scintillation counter hodoscopes for triggering the spectrometer; SC 1–18 are spark chambers with magnetostrictive extraction of information; A, A_L, A_R are anticoincidence counters; M_n are monitor counters. The functions of the remaining elements are indicated in the figure.

cord information on magnetic tape. The apparatus detected known types of decays of K^0 mesons into two charged particles. The point of the decay was determined from the paths of the decay particles observed by means of spark chambers set up in front of and behind the magnet. From the deflection of the particles in the magnet, their momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ were determined, and then $m_{\pi\pi}$, the invariant mass of the two charged particles $[m_{\pi\pi}^2=(E_1+E_2)^2-(\mathbf{p}_1+\mathbf{p}_2)^2,\ E_{1,2}^2=(p_{1,2}^2+m_\pi^2)]$, and the angle θ between the direction of the incident kaon and the total momentum vector of the two decay particles. The direction of the incident kaon was determined from the known coordinates of the target in the accelerator chamber and the coordinates of the decay point. Examination of the distributions of all events with respect to the variables $m_{\pi\pi}$ and θ makes it possible to distinguish $K_{2\pi}^0$ decays in the background of the large number of all the remaining types of kaon decays (see below).

In the filmless spark spectrometer apparatus they used a spectrometric magnet with effective length of the pole tips of about 200 cm, width 100 cm, and gap height 25 cm. In other setups, magnets of considerably greater size were used to increase the efficiency at low energies.

The experiments of Refs. 56, 57, and 59 used the so called crossed geometry, in which before the magnet one of the decay particles passes to the left of the decay axis and the other to the right and their paths cross behind the magnet. The crossed geometry guarantees good resolution of the apparatus for momentum and the effective mass without significant loss of uniformity of the efficiency of detection of the kaon decays along the decay volume, which is needed both in order to obtain the distributions (14) and to distinguish the two-pion decays from all other types of decays.

The experiments of Refs. 58 and 60 used "parallel geometry" in which the two decay particles have paths almost parallel to the spectrometer axis behind the magnet.

In both geometries, use is made of the kinematic properties of two-particle decays, for which the momentum of the secondary particles in the center-of-mass system is constant and their angular distribution is isotropic.

In all experiments, selection of the decay configurations and triggering of the spark chambers were accomplished by means of scintillation counters set up in hodoscopic planes in front of and behind the magnet. The arrangement of the counters and the logic of the spectrometer triggering depended on the energy range, but in each case one required simultaneous responses from at least several counters along the paths of the decay particles and absence of signals from the anticoincidence counter set up immediately behind the regenerator.

For the identification of the three-particle $K_{\mu 3}^0$ and K_{e3}^0 decays muon and electron detectors were used in all the setups. The muon detector consisted of one or two series of scintillation counters and sections of a steel filter in front of them. Strongly interacting particles were absorbed in the filter.

In the experiments of Refs. 56, 57, and 59, the electrons were detected by total-absorption sandwich counters, in which electron–photon cascades developed and were detected. In the experiments of Refs. 58 and 60, gas Čerenkov counters were used.

The information from the lepton detectors was used to identify the decay types. In the event of a signal from a counter of the muon detector along one of the paths of the decay particles, the event was classified as a potential $K_{\mu 3}^0$. If the signal from the electron detector exceeded a definite level and one of the paths of the decay particles passed through the counter, the event was classified as a potential K_{e3}^0.

A three-meter liquid-hydrogen target was used as regenerator in the filmless-spark-spectrometer experiment. In the other experiments, the target was shorter ($\lesssim 1$ m).

The arrangement of the elements of the apparatus, the triggering logic, and the choice of the magnetic field were usually optimized by Monte Carlo calculations. The integrated geometric efficiency of such an apparatus depends on the investigated region of K_L^0-meson momenta. The efficiency of the filmless-spark-spectrometer experiment for $K_{\pi 2}^0$ decays of different momenta in the range 10–50 GeV/c was about 5–20%. In other experiments, it was much less.

The information recorded on the primary magnetic tapes during the experiments was analyzed in the following two main stages: 1) geometric reconstruction of events; 2) statistical evaluation of the reconstructed events, choice of selection criteria, and selection of events from the decay channels; 3) calculation of the efficiency of the apparatus for decays through different channels; 4) approximation of the intensity of the ob-

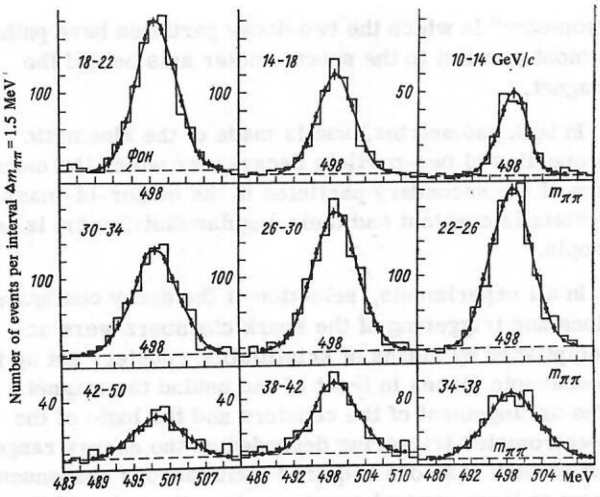

FIG. 7. Examples of histograms of the distributions of events with respect to the invariant two-pion mass.[57] In each range of momenta (indicated to the left of the peak) events for which the angle θ is near zero have been taken as candidates for $K^0_{\pi 2}$ decays. The continuous curves approximate the distributions by means of the dependence (35).

served $K^0_{\pi 2}$ decays by the theoretical expression (14) and determination of the physical parameters.

After the geometrical reconstruction, the events were usually recorded on secondary magnetic tapes for statistical analysis, during the course of which the geometrical and kinematic characteristics of the events were studied, the events classified and sorted according to decay modes by means of the muon and electron detectors, the background events and their nature analyzed, and an investigation made of the resolution of the spectrometer with respect to the invariant mass $m_{\pi\pi}$ and the angle θ as a function of the momentum of the K^0 meson. Analysis showed that in the filmless-spark-spectrometer experiment the resolution of the spectrometer with respect to the mass and the angle for the momentum range 10–50 GeV/c was given by

$$\sigma_m(p) = (2.06 + 0.058p)\ \text{MeV}/c^2; \quad (33)$$

$$\sigma_{\theta^2}(p) = (0.0172 + 9.14p^{-2})\ \text{mrad}^2 \quad (34)$$

where p is the momentum of the decaying kaon in GeV/c. In other experiments, the resolutions were somewhat less good. For example, in the apparatus described in Ref. 59, $\sigma_m = \pm 7$ MeV/c^2 and $\sigma_{\theta^2} = 3$ mrad^2 were obtained in the momentum range 3–10 GeV/c.

The two-pion decays of kaons are usually distinguished from all other events recorded on the secondary magnetic tapes by means of three main criteria: 1) The decay particles must not be leptons; 2) the invariant mass $m_{\pi\pi}$ must be approximately equal to the kaon mass; and 3) the angle θ must be near zero. After these criteria have been applied, there remains a certain number of background events in the selected $K^0_{\pi 2}$ decays. Their presence can be seen in the distributions of the events with respect to the invariant mass and the angle θ. This background is basically due to inelastic interactions of neutrons of the beam in the target and other elements of the apparatus and to $K^0_{\mu 3}$ and K^0_{e3} three-particle decays. The major part of these last background events can be identified and subtracted by means of the muon and electron detectors. The further procedure for ridding the two-pion decays of background events consists of the following. The distributions of the events with respect to $m_{\pi\pi}$ and θ (Figs. 7 and 8) are approximated by the functions

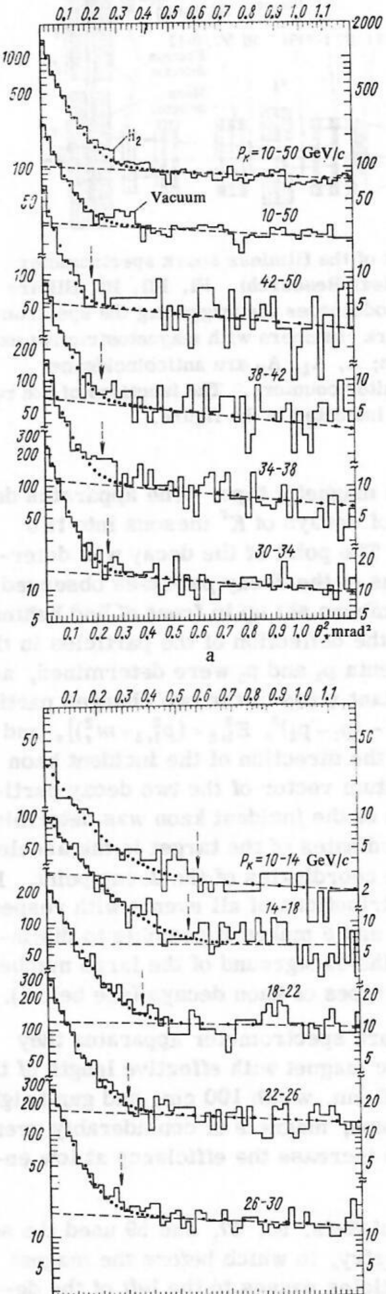

FIG. 8. Examples of histograms of the distributions with respect to the angle θ between the directions of the incident and the outgoing kaon.[57] In each range of momenta (indicated above the distribution) events for which the invariant two-pion mass is near the kaon mass have been taken as candidates for $K^0_{\pi 2}$ decays. The points approximate the distributions by means of the dependence (36). The vacuum events are those obtained in measurements without hydrogen.

$$N(m_{\pi\pi}) = A_1(p) + A_2(p)\exp[-(m_{\pi\pi} - \overline{m}_{\pi\pi})^2/2\sigma_m^2(p)], \quad (35)$$

$$N(\theta^2) = B_1(p) + B_2(p)\theta^2 + B_3(p)\exp[-B_4(p)\theta^2], \quad (36)$$

where $A_{1,2}$ and B_{1-4} are constants for the given momentum range. In these expressions, the last terms determine the expected distribution of two-pion events with respect to the corresponding variable, and the first terms the background. Extrapolating the corresponding θ^2 distribution (36) from the region of large θ^2 values, where only background events are present, into the region of the peak, where the $K^0_{\pi 2}$ decays are concentrated, one can subtract the background and determine the number of pure $K^0_{\pi 2}$ decays at the given momentum.

As we noted earlier, to study the energy dependence of $f^0_{21}(p)$ it is necessary to obtain distributions of the type (14). For this, all the observed two-pion decays must be distributed over (p_i, t_j) intervals. To approximate these distributions and obtain data on $|\rho(p)|$ and $\Phi_\rho(p)$, it is necessary to know, as we can see from the right-hand side of Eq. (14), $S(p)$, $\varepsilon(p, t)$, and M_H. The profile of the kaon momentum spectrum $S(p)$ can be determined from the $K^0_{\mu 3}$, $K^0_{\pi 3}$, or K^0_{e3} decays detected by the apparatus simultaneously with the $K^0_{\pi 2}$ decays. The spectrum is recovered by the well known method of comparing the experimental and Monte Carlo generated distributions of "true" and "spurious" energies of the decay particles.[62,63]

The efficiency $\varepsilon(p, t)$ of event detection is usually calculated by the Monte Carlo method. In the calculations, allowance is made for the experimental errors in the measurements of the track coordinates in the spark chambers, the efficiency of the spark chambers, multiple Coulomb scattering of particles in the material of the apparatus, and the efficiency of the program of geometric reconstruction.

The expression (14) contains the product $K_H = M_H \cdot \Gamma_{S+-}|\eta_{+-}|^2$. It can be determined in two ways. The first consists of calculating M_H from the experimentally observed number of $K^0_{\mu 3}$, K^0_{e3}, and (or) $K^0_{\pi 3}$ decays (or all together) and the calculated efficiency of detection of these modes by the apparatus, while the values of Γ_{S+-} and $|\eta_{+-}|$ are taken from the tables of particle properties (Ref. 4). The second method enables one to determine K_H entirely in the framework of the given experiment. For this, one uses the decays $K^0_L \to \pi^+\pi^-$ detected in measurements with a dummy target. The number of these decays is determined by the expression

$$N_{2\pi} = M_V \Gamma_{S+-} |\eta_{+-}|^2 \int\int \exp(-\Gamma_L t)\, S(p)\, \varepsilon(p, t)\, dp\, dt, \quad (37)$$

which contains the product $K_V = M_V\,\Gamma_{S+-}|\eta_{+-}|^2$. The transition from K_V to K_H can be made by using the fact that the ratio of the total number of three-particle decays observed in the measurements with and without the hydrogen target is equal to the ratio M_H/M_V of the monitors.

The experimentally observed intensities of the two-pion decays are approximated by formula (14) using the method of least squares. As free parameters, one takes either the three quantitites K_H, $R = |\rho(p)/\eta_{+-}|$, and

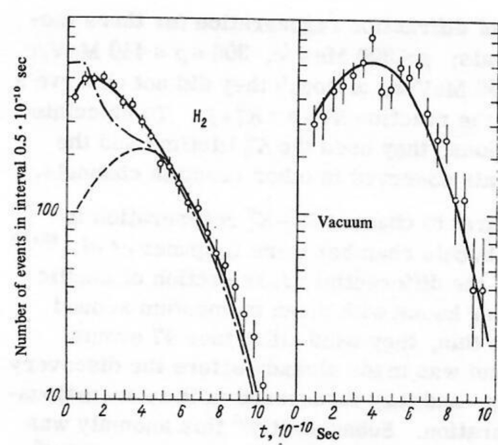

FIG. 9. Examples of distributions of the number of two-pion decays of K^0_L and K^0_S as a function of the time to decay in the K^0 rest frame observed in measurements with hydrogen (on the left) and without hydrogen (on the right).

$\varphi^0_{21}(p)$, or the last two of them. The parameter K_H may remain free and common to all the momentum intervals. In the case of three free parameters and unavoidable correlations between them, the errors of the physical quantities determined as a result of the approximation procedure will be greater than in the case when there are only two free parameters. But the reliability of the data obtained in the first case will be greater, since the total errors include not only the statistical ones but also systematic uncertainties associated with inaccuracies of the experimental parameters of the weak interactions, the momentum spectrum of the incident kaons, and the monitoring of these last. The values of the coefficient K_H in each momentum interval must be equal within the error limits to one another and to the value determined by the above method in the measurements with the dummy target.

The experimental data of Ref. 57 on the time dependence of the intensity of the two-pion decays of K^0_L and K^0_S and the results of their approximation by Eq. (14) are shown in Fig. 9 (continuous curve, $\chi^2 = 51$ for 93 degrees of freedom). To determine the contribution of the interference term, the same distributions were approximated by formula (14) under the assumption that there is no interference. As can be seen from Fig. 9 (the dot-dash-dot curve, $\chi^2 = 307$), this hypothesis has a low confidence level. The same figure shows the efficiency of the spectrometer[61] as a function of t for the decays $K^0_L \to \pi^+\pi^-$ (dashed curve).

4. RESULTS OF EXPERIMENTS ON K^0_L–K^0_S REGENERATION IN HYDROGEN AND THEIR ANALYSIS

1. Before the start of the K^0_L–K^0_S regeneration experiments in hydrogen at Serpukhov, only a few studies had been made in the world which reported observation of such regeneration or information deduced about it indirectly.

Luers *et al.*[64] studied the interaction of K^0_2 mesons in a 20-inch hydrogen bubble chamber and determined the cross section of diffraction regeneration for three momentum intervals: $p < 300$ MeV/c, $300 \leqslant p \leqslant 440$ MeV/c, and $400 < p \leqslant 650$ MeV/c, although they did not observe a single case of the reaction $K_2^0 + p \to K_1^0 + p$. To calculate the cross sections, they used the K_2^0 lifetime and the number of events observed in other reaction channels.

Among the first to observe K_2^0–K_1^0 regeneration in hydrogen in a bubble chamber were Leipuner *et al.*,[65,66] who measured the differential cross section of elastic regeneration for kaons with mean momentum around 1 GeV/c. For this, they used altogether 47 events. This experiment was made already before the discovery of CP breaking, and they discovered in it an anomalously high regeneration. Subsequently[67] this anomaly was explained by interference in the two-pion decays of K_L^0 and K_S^0 and by possible statistical fluctuations.

The first observation of transmission regeneration in hydrogen was made by Christenson *et al.*[18] using a spark spectrometer for K_L^0's with momentum 1.1 ± 0.1 GeV/c. They obtained 48 ± 10 events, from which they estimated the transmission-regeneration amplitude at 0.35 ± 0.18 F $< |f_{21}^0| < 1.6 \pm 0.2$ F. They did not observe interference between K_L^0 and K_S^0 decays into two pions and only obtained an indication of its existence.

Firestone *et al.*[68] measured regeneration in hydrogen in an 80-inch hydrogen chamber. The chamber was bombarded with K_L^0 mesons whose spectrum extended from 1 to 7 GeV/c with peak at 5 GeV/c. Only events with momentum greater than 2 GeV/c were used. In the experiment, they determined the differential cross section of elastic regeneration, described by the exponential dependence

$$d\sigma/dt = (77^{+29}_{-24}) \exp[-(2{,}9^{+1.3}_{-1.1})\,t]\ \mu\text{b}/(\text{GeV}/c)^2\,, \tag{38}$$

whose integration gave the total cross section 46 ± 10 μb of elastic regeneration. On the other hand, this experiment too did not reveal interference between decays of K_L^0 and K_S^0 into pion pairs, on the basis of which it was suggested that the phase of the amplitude of regeneration in hydrogen is $\varphi_{21}^0 = -90°$.

Hawkins[67] studied the interaction of K_L^0's with momentum 590 ± 50 MeV/c in a liquid-hydrogen chamber and observed 22 cases of the reaction $K_L^0 + p \to K_S^0 + p$, from which he determined the total cross section of elastic regeneration, 0.09 ± 0.24 mb, and the differential cross section of regeneration at zero angle—about 0.4 mb/sr.

Thus, we see that in each of these experiments the statistics of the events consisted of altogether only 20–50 cases, on the basis of which it is difficult to obtain any reliable data. This fact emphasizes once more the complexity of regeneration experiments in hydrogen.

2. The K_L^0–K_S^0 regeneration experiments in hydrogen in Serpukhov were made in several stages. Already in the first stage[69] it proved possible to detect about 600 cases of coherent regeneration in the momentum range 14–42 GeV/c and establish the existence of interference in the decays of K_L^0 and K_S^0 into two pions in the same way as had been done in cases with solid regenerators.[34] For the first time, measurements were made of the modulus and phase of the transmission-regeneration amplitude in hydrogen and, on this basis, the difference between the total cross sections of interactions of K^0 and $\bar{K}^0$ with protons was calculated (Fig. 10). As was noted in Allaby's report[69] at the International Conference on High Energy Physics at Kiev, these results indicated that the difference between the total cross sections of K^0p and $\bar{K}^0p$ interactions, and therefore of K^+n and K^-n interactions as well, continues to decrease with increasing momentum, which must lead to an increase in the total cross sections of K^+n interactions since $\sigma_t(K^-n)$ remains constant in this energy range. Thus, of the two possibilities for explaining the behavior of the total cross sections at Serpukhov energies—violation of Pomeranchuk's theorem and growth of the total cross sections for positive particles—the latter remained more probable. True, this conclusion could not yet be

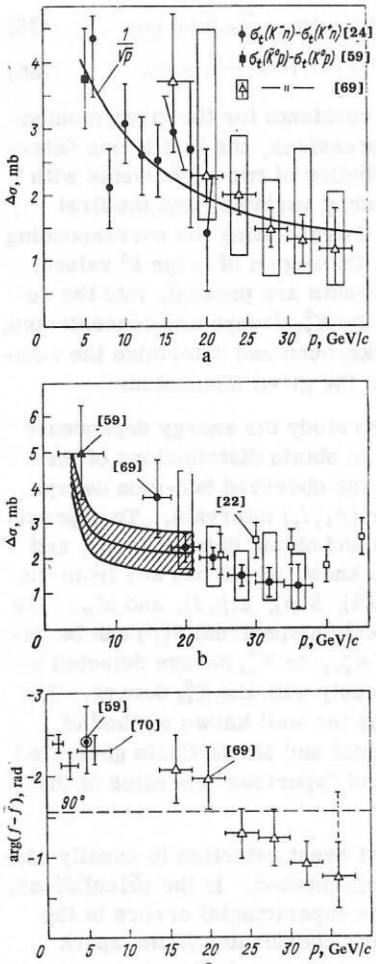

FIG. 10. Difference between total interaction cross sections of kaons (a), the same dependences compared with the existing data on the total K^+n and K^-n interaction cross sections (b), and phase of the $K_L^0 - K_S^0$ regeneration amplitude in hydrogen (c). The dashed line in Fig. c indicates the possible systematic displacement of all points.[69] The continuous curves in (b) and the hatched region represent $\Delta\sigma(p)$ and the corresponding error corridor determined in earlier experiments.

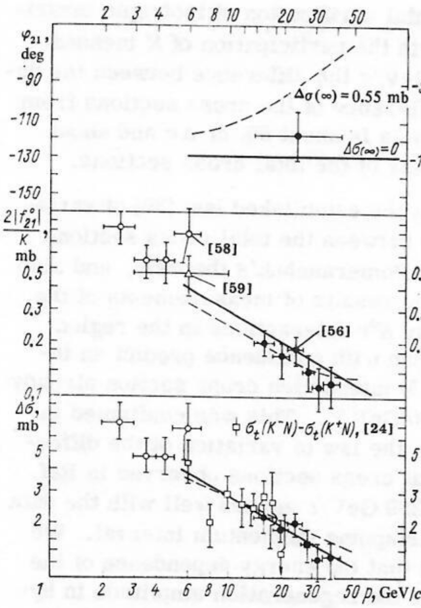

FIG. 11. Results of measurements of the phase and modulus of the $K^0_L - K^0_S$ transmission-regeneration amplitude in hydrogen and comparison of them with calculations made with the theory of complex angular momenta. The solid line corresponds to fulfillment of Pomeranchuk's theorem and the dashed curve to the case when it is not fulfilled. Below, the difference of the total cross sections, $\Delta\sigma(p) = \sigma_t(K^0p) - \sigma_t(K^0_p)$, is shown.

confirmed by measurements of the phase of the regeneration amplitude because of the large statistical and possible systematic uncertainties.

The experiments on kaon regeneration at Serpukhov stimulated similar experiments on other accelerators. At the Kiev conference in 1970, the results were given of other measurements of transmission regeneration[58,59,70] in hydrogen made at energies up to 10 GeV and based on statistics of about 500 cases in each. The combination of all the then existing data on regeneration in hydrogen and the cross-section differences $\sigma_t(\overline{K^0}p) - \sigma_t(K^0p)$ and $\sigma_t(K^-n) - \sigma_t(K^+n)$ calculated on the basis of them (see Fig. 10) did not contradict the law of variation $\Delta\sigma \sim p^{-0.5}$, which follows from the predictions of the model of complex angular momenta for the exchange of ω and ρ poles. This is also an indication in favor of Pomeranchuk's theorem being valid.

3. After the completion of the second stage of the experiment with the Serpukhov accelerator, about 2000 cases of $K^0_L - K^0_S$ regeneration in hydrogen were observed.[56] In Figs. 5 and 11, the data are compared with calculations by dispersion relations[51] and the theory of complex angular momenta.[41] It can be seen that a strong violation of Pomeranchuk's theorem, in which $\Delta\sigma(\infty) = 2$ mb and even $\Delta\sigma(\infty) = 0.55$ mb, is precluded both by the measurements of the modulus and of the phase of the regeneration amplitude. Strictly speaking, there remains the possibility of only a small violation of the theorem, for which $\Delta\sigma(\infty) < 0.2$ mb.

The errors in the determination of the phase $\varphi^0_{21}(p)$ for each momentum interval were large (at this stage of the experiment) and did not permit one to draw a definite conclusion about its energy dependence. However, these data did not contradict the hypothesis that the phase does not depend on the energy, and its mean value $-118° \pm 13°$ agrees to within one standard deviation with the value $-130°$ predicted in Ref. 41 for the case when Pomeranchuk's theorem is satisfied.

The data given in Fig. 11 show that $2|f^0_{21}(p)|/k$ and $\Delta\sigma$ decrease with increasing momentum in accordance with a law close to $p^{-0.5}$. The data on the total cross sections of K^-n and K^+n interactions[24] available at that time in the investigated energy region agreed with the regeneration data but had significantly larger errors. Decrease of $\Delta\sigma(p)$ with increasing kaon momenta in conjunction with the known constancy of the K^-n interaction cross sections implies that the total K^+n interaction cross sections in this momentum range must increase. This conclusion agreed with the data of direct measurements of the total cross sections for K^+ mesons made at Serpukhov and reported simultaneously with the regeneration data at the International Conference in Amsterdam.[71] Later, they were also published in Refs. 56 and 72.

4. The principal result of the last stage of the Serpukhov experiments, the results of which are presented in Fig. 12, was the establishment of the fact that the phase of the regeneration amplitude is independent of the energy in the range 10–50 GeV.[57] If one takes the complete set of existing regeneration data, one can see that the phase φ^0_{21} depends weakly on the energy even in the interval 1–50 GeV/c. This is indicated by the following figures. The mean value of the phase determined in the experiment of Ref. 73, which provided the most accurate data in the momentum range 1.5–10 GeV/c, is equal to $-133.9° \pm 4°$. The mean value of the phase determined in Ref. 57 in the range 14–50 GeV/c is $-132.3° \pm 5.7°$. Recently, there has appeared one further paper[60] in which results are given of measurements of the modulus and phase of the regeneration amplitude in hydrogen in the momentum range 3.5–10.5 GeV/c. According to the data of Ref. 60, the mean value of the phase is $-157.4° \pm 8°$ (if one uses $\Phi_{+-} = 42°$ to calculate it). It can be seen that the results of Refs. 73 and 60 do not agree within the limits of the standard deviations. However, if we ignore this and calculate the mean value of φ^0_{21} from the two experiments, we then find that to within the experimental errors the phase of the regeneration amplitude in hydrogen is constant in the momentum range 1–50 GeV/c.

The establishment that the phase of the regeneration amplitude is independent of the energy at high energies seems to have dispelled the last doubts concerning the fulfillment of Pomeranchuk's theorem in KN interactions at energies right up to 50 GeV.

In order to have a quantitative criterion for estimating the fulfillment of this theorem, the $|f^0_{21}(p)|$ and $\Delta\sigma(p)$ data were approximated in Ref. 57 by p-independent constants. As can be seen from Fig. 12, these assumptions have a low confidence level. The data in no way agree with the prediction (18), which follows from assuming constancy of the total K^-n and K^+n interaction cross sections.

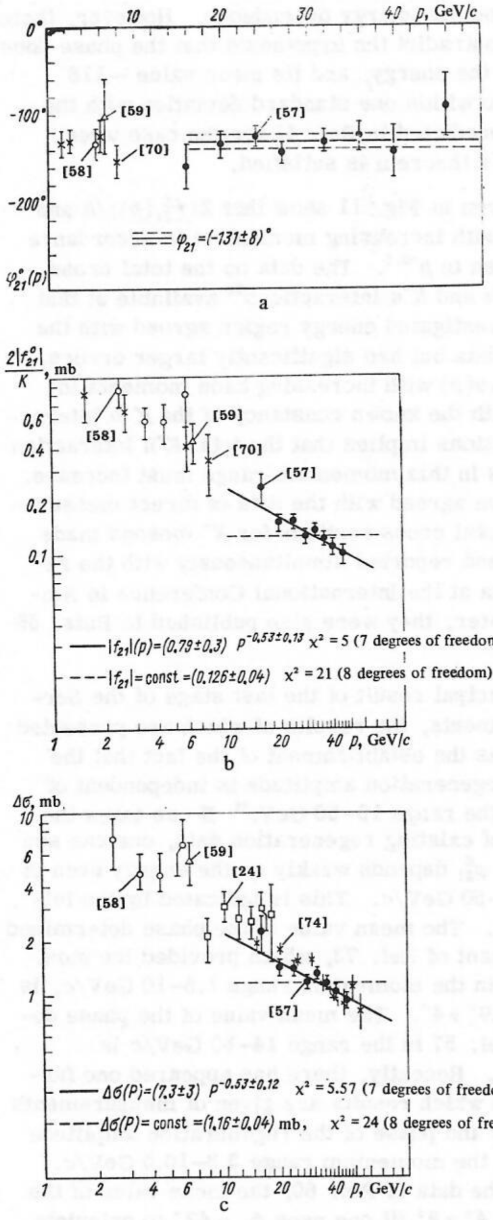

FIG. 12. Energy dependences of the phase $\varphi^0_{21}(p)$ (a) and the modulus $2|f^0_{21}(p)|/k$ (b) of the $K^0_L-K^0_S$ regeneration amplitude in hydrogen, and also the difference $\Delta\sigma$ of the $\bar{K}^0p$ and K^0p total interaction cross sections calculated on the basis of these dependences (c).

A no less important conclusion of the final stage of the Serpukhov experiments was the establishment of agreement between the energy dependence of the difference between the total interaction cross sections of charged and neutral kaons. It follows from Refs. 57 and 74 that

$$\sigma_t(\bar{K}^0p)-\sigma_t(K^0p)=(7.3\pm3)\,p^{-0.53\pm0.13}\ \text{mb}\ ; \qquad (39)$$

$$\sigma_t(K^-n)-\sigma_t(K^+n)=(12.3\pm3.9)\,p^{-0.65\pm0.09}\ \text{mb}\qquad (p,\text{GeV}/c), \qquad (40)$$

from which it can be seen that within one standard deviation the two parameters describing the momentum dependence of the difference between the cross sections agree with one another.

The degree of agreement of these data is simultaneously an experimental verification of isotropic invariance for reactions with the participation of K mesons. For example, at 50 GeV/c the difference between the determination of the difference of the cross sections from one or the other formula is about 6% of $\Delta\sigma$ and about 0.3% of the value of any of the total cross sections.

Taking into account the established law (39) of variation of the difference between the total cross sections and the fulfillment of Pomeranchuk's theorem, and also taking into account the results of measurements of the total cross sections of $K^{\pm}n$ interactions in the region up to 55 GeV/c, we can with confidence predict an increase of the total K^-n interaction cross section already in the region 100–200 GeV/c. This was confirmed in Ref. 75. In its turn, the law of variation of the difference between the total cross sections observed in Ref. 75 in the region 23–280 GeV/c agrees well with the data of Ref. 57 in the overlapping momentum interval. We can therefore expect that the energy dependence of the phase and modulus of the regeneration amplitude in hydrogen established in the region 14–50 GeV/c will also hold in a wider energy range, at least to 300 GeV/c.

One can show that the measurements of the phase of the $K^0_L-K^0_S$ transmission-regeneration amplitude in hydrogen[56–59,73] agree with the values of the phase $\varphi^0_{21}(p)$ calculated from measurements of the difference between the K^+n and K^-n total interaction cross sections[74] and the measurements of the zero-angle differential cross section of regeneration. In accordance with the optical theorem, $(d\sigma/dt)_{\text{opt}}\sim[\Delta\sigma_t(K^{\pm}n)]^2\sim(\text{Im}f^0_{21})^2$, and the zero-angle regeneration cross section is by definition $(d\sigma/dt)_0\sim(\text{Re}f^0_{21})^2+(\text{Im}f^0_{21})^2$. Therefore, $\text{Re}f^0_{21}/\text{Im}f^0_{21}\sim[(d\sigma/dt)_0/(d\sigma/dt)_{\text{opt}}-1]^{0.5}$. The phase φ^0_{21} calculated by the same method is shown by the solid line in Fig. 13a. The difference between the coefficients of the parametrization of the cross sections obtained in Refs. 57 and 74 by the law $\sim p^{-n}$ gives a small change of the phase with energy.

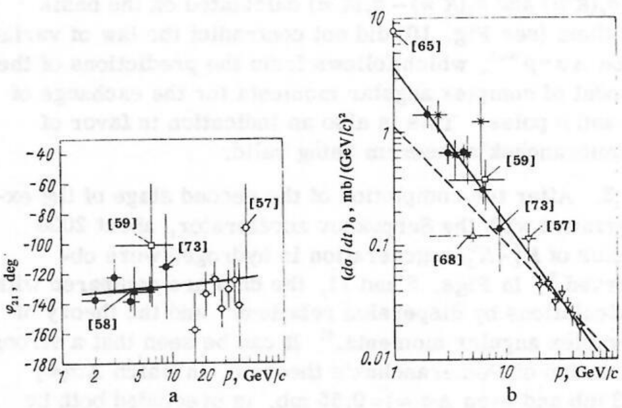

FIG. 13. Verification of the compatibility of the data obtained from measurements of the phase of the regeneration amplitude and the K^+n and K^-n total interaction cross sections (a); cross section of $K^0_L-K^0_S$ transmission regeneration in hydrogen as a function of the momentum K_L. The continuous line is the approximation of all data by the dependence $(d\sigma/dt)_0=Ap^{-n}$; the dashed line is the approximation of the results of Ref. 57 by this same dependence (b).

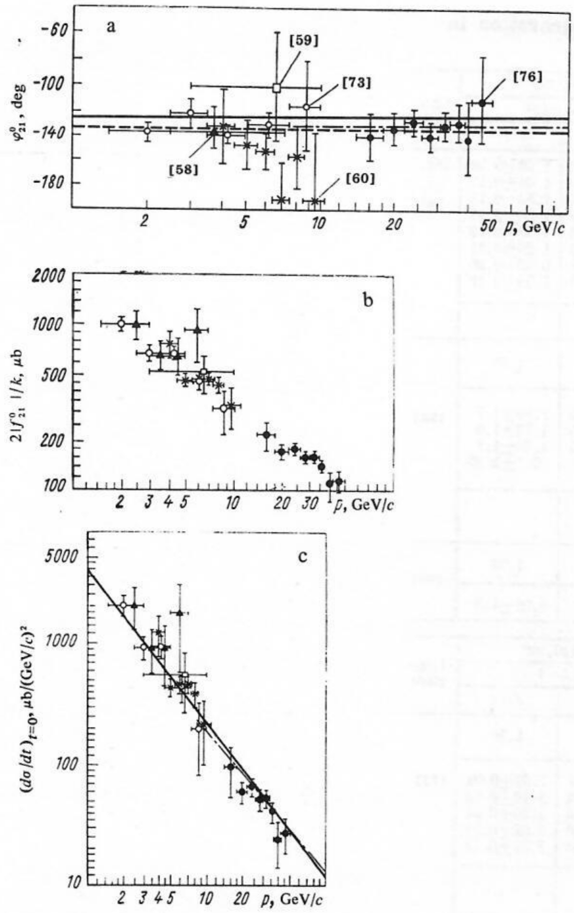

FIG. 14. Total results of measurements of the phase (a) and modulus (b) of the amplitude of $K_L^0 - K_S^0$ transmission regeneration in hydrogen and the differential regeneration cross sections at zero four-momentum transfer (c) calculated on the basis of them. The dashed line is the mean value of the phase, $\varphi^0_{21} = -133°$; the continuous line is the result of calculation in the model of complex angular momenta using Eq. (24) and the parameters of the ω and ρ trajectories from Ref. 42; the dot-dash-dot line gives the results of the same calculations but with the trajectory parameters and allowance for the cuts from Ref. 41.

5. Since the publication of the results of Ref. 57, new experimental data have been published that characterize the neutral-kaon system: δ, τ_S, $|\eta_{+-}|$, Φ_{+-}, some of which contradict the measurements made up to 1972. In this connection, the experimental data given in Refs. 56 and 57 have been reanalyzed. The resulting weighted mean values of $R = |\rho(p)/\eta_{+-}|$ and φ^0_{21} are given in Table II of Ref. 76. For the calculation, the following parameter values were used:

$$\Phi^0_{+-} = 42°;\ \tau^0_S = 0.895 \cdot 10^{-10}\ \text{sec},\ \delta^0 = 0.54 \cdot 10^{10} \cdot \hbar\ \text{sec}^{-1}. \quad (41)$$

The weighted mean values of R and φ^0_{21} yielded the values of $2|f^0_{21}(p)|/k$, $[d\sigma/dt]_{t=0}$ and also the difference between the total cross sections. The results of the calculations and all the other data currently available are represented in Figs. 14 and 15 and in Table II. Since the values of $2|f^0_{21}(p)|/k$ are usually expressed in mb, and those of $d\sigma/dt$ in mb/(GeV/c^2), the conversion in Table II is made in accordance with (27) by means of the factor $(\hbar c)^2 = (0.624)^2$ GeV$^2\cdot$ mb:

$$[2|f^0_{21}(p)|/k]^2\ (\text{mb})^2 = (0{,}624)^2 (4/\pi)\,(d\sigma/dt)_0\ \text{mb}/(\text{GeV}/c)^2.$$

From the experimental data obtained without hydrogen, the parameter $|\eta_{+-}| = (2.14 \pm 0.15)10^{-3}$ was determined in Ref. 76; its value is close to the weighted mean world value in 1974 (Ref. 4), but differs from the results of the latest experiments.[77,78] Since the discrepancy between the data on this parameter in the world has not yet been eliminated, in the corresponding places in Tables II and III we have given the results of the calculation of the regeneration amplitude for two values of $|\eta_{+-}|$.

To establish the dependence of the results of measurements of the transmission-regeneration amplitude on the weak-interaction parameters of the $K_L^0 - K_S^0$ system, the data of Refs. 56 and 57 were analyzed in Ref. 76 for not only the parameter values given in (41) but also for some other values. It was found that the possible variations of the mean value of the phase of the regeneration amplitude are determined by the formula

$$\varphi^0_{21} = (-132° \pm 5°) + 70°\,(\delta^0 - \delta)/\delta^0 + 108°\,(\tau_S - \tau^0_S)/\tau^0_S + \Phi_{+-} - \Phi^0_{+-}. \quad (42)$$

Investigations were also made of the systematic errors associated with the procedure for selecting $K^0_{\pi 2}$ decays and subtracting the background. The systematic error for the mean value of φ^0_{21} does not exceed $\pm 3°$, and for $2|f^0_{21}(p)|/k$ it is about ± 8 μb.

To determine the energy dependence of $2|f^0_{21}(p)|/k$, $[d\sigma/dt]_0$, and $\Delta\sigma(p)$ the data given in Table II were approximated by expressions of the type Ap^{-n}, where p is the kaon momentum and A and n are the required parameters. The results are given in Table III together with the results of direct measurements of the difference between the total interaction cross sections of the kaons.

The analysis showed that the established laws of the energy dependence of the modulus of the regeneration amplitude and the cross-section difference do not depend on the possible experimental spread of the parameters δ, τ_S, $|\eta_{+-}|$, and Φ_{+-}. When these are varied in possible limits, there is a systematic displacement of the difference between the cross sections at all momenta that does not change n.

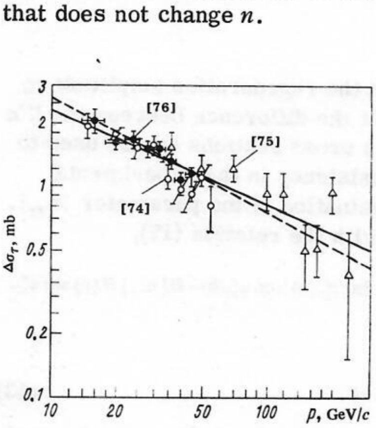

FIG. 15. Total results of measurements of the difference between the kaon and antikaon total interaction cross sections. The solid line approximates the data of Refs. 74 and 75 by means of the dependence Ap^{-n}; the dashed line does the same for Ref. 76.

TABLE II. Experimental data on the amplitude of $K_L^0-K_S^0$ transmission regeneration in hydrogen.

$P_{K_L^0}$, GeV/c	$R = \lvert\rho(p)/\eta_{+-}\rvert$	$-\varphi_{21}^0$, deg ($\Phi_{+-}=42°$)	$2\lvert f_{21}^0(p)\rvert/k$, μb; $\lvert\eta_{+-}\rvert\cdot 10^3$		$[d\sigma/dt]_0$, μb/(GeV/c)^2; $\lvert\eta_{+-}\rvert\cdot 10^3$		$\Delta\sigma(p)$, mb; $\lvert\eta_{+-}\rvert\cdot 10^3$		Literature
			2.35	2.14	2.35	2.14	2.35	2.14	
14–18	1.60±0.35	139±20	219±43	199±14	97±43	80±35	2.04±0.40	1.86±0.36	[56, 57, 76]
18–22	1.49±0.15	132±13	173±18	157±16	60±12	50±10	1.61±0.17	1.46±0.15	
22–26	1.74±0.14	127±11	181±15	165±14	66±11	55±9	1.69±0.14	1.53±0.13	
26–30	1.68±0.16	139±12	161±16	147±15	52±11	43±9	1.50±0.15	1.36±0.13	
30–34	1.80±0.15	130±12	162±14	148±13	53±9	44±8	1.51±0.13	1.38±0.12	
34–38	1.65±0.15	127±16	142±13	129±12	41±8	34±6	1,32±0.12	1.20±0.11	
38–42	1.32±0.29	142±29	109±24	99±22	24±10	20±8	1.02±0.22	0.93±0.20	
42–50	1.46±0.25	109±34	116±19	106±18	27±9	22±8	1.08±0.18	0.99±0.16	
14–50		132±5							
			2.35	1.95	2.35	1.95	2.35	1.95	
2–3	—	—	1010±190	840±160	2060±770	1423±542	9.00±1.85	7.20±1.7	[58]
3–4	—	—	675±130	560±110	920±355	633±249	6.0±1.25	4.70±1.0	
4–5	—	—	675±170	560±140	920±465	633±316	6.0±1.65	4.70±1.1	
5–7	—	—	940±315	780±260	1790±1200	1227±818	8.35±3.1	6,7±2.35	
2–7		135±17							
			2.35	1.92	2.35	1.92	2.35	1.92	
3–10		103±42	526±135	430±110	560±290	375±190	6.45±1.7	5.25±1.3	[59]
$P_{K_L^0}$, GeV/c	$R = \lvert\rho(p)/\eta_{+-}\rvert$	$-\varphi_{21}^0$, deg ($\Phi_{+-}=42°$)	$2\lvert f_{21}^0(p)\rvert/k$, μb; $\lvert\eta_{+-}\rvert\cdot 10^3$		$[d\sigma/dt]_0$, μb/(GeV/c)^2; $\lvert\eta_{+-}\rvert\cdot 10^3$		$\Delta\sigma(p)$, mb; $\lvert\eta_{+-}\rvert\cdot 10^3$		Literature
			2.35	2.14	2.35	2.14	2.35	2.14	
			2.35	1.96	2.35	1.96	2.35	1.96	
1.5–2.5	—	138±6.9	1010±95	842±83	2060±388	1425±282	9.0±1.07	7.62±0.91	[73]
2.5–3.5	—	123±11	680±71	566±59	935±195	646±134	6.05±0.74	5.12±0.63	
3.5–5.0	—	139.4±7.1	686±72	574±60	952±200	665±138	6.10±0.75	5.20±0.64	
5.0–7.5	—	131.4±9.3	466±60	389±50	440±113	305±78	4.15±0.60	3.52±0.51	
7.5–10	—	116.6±36.0	314±93	262±78	200±118	138±82	2.8±0.85	2.37±0.72	
1.5–10		134.9±4							
			2.30		2.30		2.30		
3.5–4.5	—	132±30	780±140	—	1230±440	—	6.95±1.45	—	[60]
4.5–5.5	—	147±20	470±40	—	445±76	—	4.18±0.48	—	
5.5–6.5	—	152±15	480±40	—	465±75	—	4.27±0.48	—	
6.5–7.5	—	192±25	480±40	—	465±75	—	4.27±0.48	—	
7.5–8.5	—	157±25	440±50	—	390±88	—	3.92±0.55	—	
8.5–10.5	—	192±55	330±90	—	220±120	—	2.94±0.88	—	
3.5–10.5		157±8							

Notes: 1. The table contains the original data and their conversion for $|\eta_{+-}| = 2.35\cdot 10^{-3}$.
2. Data on the magnitude of the phase φ_{21}^0 obtained in each experiment, reduced to one value of $\Phi_{+-}=42°$.
3. Values of $\Delta\sigma$ for the experiments of Ref. 60 calculated for $\varphi_{21}^0 = -135°$ instead of the the value $\varphi^{21} = -157.4°$ observed in Ref. 60.

The measurements of the regeneration amplitude in conjunction with those of the difference between the K^+n and K^-n total interaction cross sections can be used to resolve the above inconsistency in the experimental data[4,77,78] on the determination of the parameter $|\eta_{+-}|$. Indeed, in accordance with the relation (17),

$$\Delta\sigma(p) = 8\pi\,\mathrm{Im}\,f_{21}^0(p)/k = 8\pi\,|f_{21}^0(p)|\sin\varphi_{21}^0/k = B\,|\eta_{+-}|\,R(p)\sin\varphi_{21}^0,$$

whence

$$|\eta_{+-}| = \Delta\sigma(p)/BR(p)\sin\varphi_{21}^0, \qquad (43)$$

where B is a constant determined from the relation (7) for given momentum and given target length, and $R(p) = |\rho(p)/\eta_{+-}|$ is the value determined in the regeneration experiments without use of information about the particular value of $|\eta_{+-}|$.

For the calculation of $|\eta_{+-}|$ in accordance with (43), use was made of the $\Delta\sigma(p)$ data from Refs. 72 and 75 and the values of $R(p)$ and φ_{21}^0 from Ref. 76 (Table II). The data of each of the experiments of Ref. 72 and 75 were approximated separately by power-law dependences of the type $\Delta\sigma(p)=Ap^{-n}$ in the momentum ranges 15–65 GeV/c and 23–280 GeV/c, respectively. The total data of the experiments of Refs. 72 and 75 in the range 15–280 GeV/c were approximated by the same dependence. The values of the constants A and n for all three approximation cases are given in Table III. For comparison with the regeneration experiments, the total cross sections in each case were interpolated or extrapolated to the region 10–50 GeV/c, where $R(p)$ and φ_{21}^0 were obtained. Then the values of the parameters $|\eta_{+-}|_i$ were calculated for each of the eight values of the momentum p_i of Ref. 76 and averaged.

TABLE III. Results of approximation of data on $K_L^0-K_S^0$ regeneration in hydrogen and the difference between the K^+n and K^-n total interaction cross sections by a function of the type Ap^{-n} (p is the K_L^0 momentum in GeV/c).

Range of momenta, GeV/c	Function	A	n	Literature
14–50	$2\mid f_{21}^0(p)\mid/\varkappa$, mb	0.77 ± 0.38	0.49 ± 0.14 *	[76]
14–50	»	0.84 ± 0.42	0.50 ± 0.15 ²*	[76]
14–50	$[d\sigma/dt]_0$, $\mu b/(GeV/c)^2$	1234 ± 1202	1.02 ± 0.29 *	[76]
14–50	»	1474 ± 1575	1.04 ± 0.32 ²*	[76]
14–50	$\sigma_t(\overline{K}^0p)-\sigma_t(K^0p)$, mb	8.4 ± 3.2	0.55 ± 0.11 *	[76]
14–50	»	9.7 ± 3.4	0.56 ± 0.10 ²*	[76]
15–65	$\sigma_t(K^-n)-\sigma_t(K^+n)$, mb	12.1 ± 4.1	0.65 ± 0.10	[72]
23–280	»	12.1 ± 5.9	0.57 ± 0.11	[75]
15–280	»	6.8 ± 1.3	0.46 ± 0.05	[76]
1.5–10	$[d\sigma/dt]_0$, $\mu b/(GeV/c)^2$	3255 ± 988	1.33 ± 0.24	[73]
1.5–50	»	3274 ± 607	1.36 ± 0.06	[73]

*The parameters A and n are obtained for data calculated using the value $|\eta_{+-}| = 2.14\cdot10^{-3}$.
²*The same using the value $|\eta_{+-}| = 2.35\cdot10^{-3}$.

The calculations of Ref. 79 showed in particular that the total data of the experiments of Refs. 72, 75, and 76 lead to the mean value $|\overline{\eta_{+-}}| = (2.34\pm0.09)\cdot10^{-3}$. It can be seen from this that the combination of all the existing data on the measurements of the difference between the total kaon interaction cross sections and the $K_L^0-K_S^0$ regeneration amplitude at high energies agrees well with the new value $|\eta_{+-}| = (2.30\pm0.03)\cdot10^{-3}$ obtained in the experiments of Refs. 77 and 78.

6. In accordance with the simplified model of complex angular momenta, from measurements of the phase φ_{21}^0 one can find the value of the pole trajectory for zero four-momentum transfer, $\alpha(0)$, if in the given energy range only one pole or two poles with coincident values at the origin make the decisive contribution. Using Eq. (26) and the mean value $\varphi_{21}^0 = -132°\pm5°$ of the phase, we obtain $\alpha(0) = 0.47\pm0.05$.

In accordance with (27), the energy dependence of the zero-angle differential regeneration cross section is also determined by $\alpha(0)$. From the data of Ref. 57, approximated by the dependence $A\cdot p^{-n}$, it follows that $\alpha(0) = 0.48\pm0.14$. We see that the two independent methods of determining $\alpha(0)$ give the same results, which, in their turn, agree with the values $\alpha_\rho(0)$ and $\alpha_\omega(0)$ found in Ref. 74.

It is interesting to note that in Ref. 73 in the region 1.5–10 GeV/c a somewhat different energy dependence of the zero-angle differential regeneration cross section was found, compared to that in Ref. 57 (see Table III), this leading to the value $\alpha(0) = 0.30\pm0.03$, whereas it follows from the measurements of the phase in the same investigation that $\alpha(0) = 0.49\pm0.05$. Brandenburg *et al.*[73] assume that in the framework of the theory of complex angular momenta this fact indicates a departure of the energy dependence of the phase from the relation (26) or the analogous one that follows from (24). It can be seen that the data of Ref. 57 do not contain such indications. It would be more natural to assume that at energies of several GeV the contribution to the scattering amplitude of direct channels is still important. This contribution can change the energy dependence of the regeneration cross section in this region. Above 6 GeV/c, the data agree better with the dependence that does not lead to discrepancy of the $\alpha(0)$ results.

It was shown earlier (see Sec. 2) that in the framework of the theory of complex angular momenta one can explain the growth of the total cross sections of K^+ interactions in different ways.[37] Figure 16 compares the predictions of these models and the data for the phase of the regeneration amplitude and regeneration cross section at zero angle. As can be seen from the figure, the model to be preferred is the one that takes into account the contribution of the ω and ρ poles to the amplitude. Both the energy dependence of the regeneration cross section and the ratios of the real and imaginary parts of the regeneration amplitudes eliminate the more complicated models in which allowance is made for not only these two poles but also dipoles and complex conjugate poles.

It is interesting to compare the complete regeneration data with the complex-angular-momentum-theory predictions for the general form (24) of the amplitude $f_{21}^0(24)$ with parameters of the poles determined in the phenomenological analysis[42] with allowance for the most recent experimental results. These parameters are given in Table I and the results of the comparison are presented in Fig. 14 (continuous curve). Qualitatively, there is good agreement between the experimental data and the predictions of the theory in the whole of the available energy range.

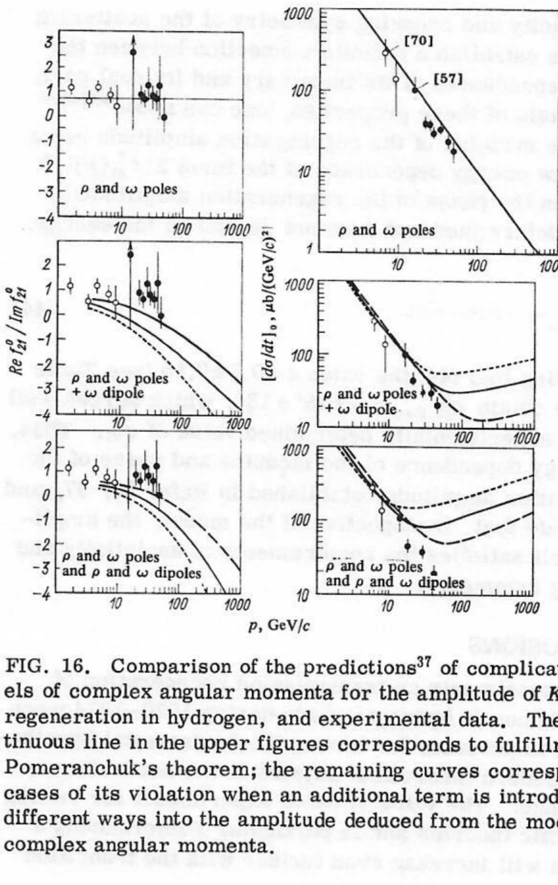

FIG. 16. Comparison of the predictions[37] of complicated models of complex angular momenta for the amplitude of $K_L^0-K_S^0$ regeneration in hydrogen, and experimental data. The continuous line in the upper figures corresponds to fulfillment of Pomeranchuk's theorem; the remaining curves correspond to cases of its violation when an additional term is introduced in different ways into the amplitude deduced from the models of complex angular momenta.

Earlier, we have discussed the calculations of the differential cross section of transmission regeneration and the phase φ^0_{21} in the model of complex angular momenta with allowance for poles and cuts[41] (see Fig. 4). These calculations are also shown in Fig. 14 (dot-dash-dot curve), from which it can be seen that, to within the achieved accuracy of the experiment, one cannot give a preference to any particular set of trajectory parameters (see Table I).

The calculations in accordance with Eq. (24) show that the phase of the regeneration amplitude depends very weakly on the energy. For example, the transition of the phase through the value $\varphi^0_{21}(p_0) = -90°$ is expected at $p_0 = 16 \cdot 10^3$ GeV/c.

7. Experiments on the regeneration of neutral kaons in which the energy behavior of the modulus and the phase of the regeneration amplitude are determined permit one to verify the most general assumptions of the theory.

In particular, in Ref. 57 the compatibility of the existing data with the predictions (29) and (31) for the behavior of the regeneration amplitude as $E \to \infty$, which follow from generalized hypotheses of quantum field theory, was tested. It was shown that the asymptotic form of the regeneration amplitude (31) satisfies better the experimental data of Ref. 57, $\chi^2 = 4$ (8 degrees of freedom) than does the form of the amplitude (29): $\chi^2 = 10$. This conclusion corresponds to the results of comparison of the data with the predictions of the model of complex angular momenta for the case when Pomeranchuk's theorem is satisfied.

Analyticity and crossing symmetry of the scattering amplitude establish a definite connection between the energy dependences of its imaginary and its real part. On the basis of these properties, one can show[54,55,80,81] that if the modulus of the regeneration amplitude has a power-law energy dependence of the form $2|f^0_{21}(p)|/k \sim p^{-n}$, then the phase of the regeneration amplitude is strictly determined and does not depend on the energy, namely

$$\varphi^0_{21\,\text{theor}} = -\pi(1 - n/2). \tag{44}$$

Substituting into (44) the value $n = 0.5 \pm 0.15$ (see Table III),[76] we obtain $\varphi^0_{21\,\text{theor}} = -135° \pm 13°$, which agrees well with the experimentally determined value of φ^0_{21}. Thus, the energy dependence of the modulus and phase of the regeneration amplitude established in Refs. 56, 57, and 76 indicate that, irrespective of the model, the amplitude itself satisfies the requirements of analyticity and crossing symmetry.

CONCLUSIONS

The experiments on transmission regeneration of neutral kaons in hydrogen made during 1970–1974 were of great importance for testing the fundamental hypotheses of modern theoretical physics in the accessible energy range. The value of these experiments for testing asymptotic theories and in particular Pomeranchuk's theorem will increase even further with the transition to energies of 100 and more GeV since the difference between the total cross sections for the interaction of particles and antiparticles with a given nucleon have reached the level of possible systematic errors of each measurement separately, and it will be very difficult to reduce this level in practice.

I should like to express my deep thanks to M. F. Likhachev and V. G. Krivokhizhin for a number of valuable comments on this work.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] K. Lande *et al.*, Phys. Rev. **103**, 1901 (1956); **105**, 1925 (1957).
[3] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).
[4] Rev. of Particles Properties.-Phys. Lett. B **50**, N 1 (1974).
[5] K. M. Case, Phys. Rev. **103**, 1449 (1956).
[6] M. L. Good, Phys. Rev. **106**, 591 (1965); **110**, 550 (1958).
[7] R. H. Good *et al.*, Phys. Rev. **124**, 1223 (1961).
[8] J. H. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).
[9] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).
[10] M. V. Terent'ev, Usp. Fiz. Nauk **86**, 231 (1965) [Sov. Phys. Usp. **8**, 445 (1965)].
[11] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966); T. D. Lee and C. S. Wu, Slabye Vzaimodeĭstviya (Weak Interactions; Russian translation), Mir, Moscow (1968), pp. 182–295.
[12] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930); 65, 18 (1930).
[13] R. G. Sachs, Ann. Phys. **22**, 239 (1963).
[14] B. Aubert, Le système K°–K̄°. Ecole Intern. Phys. des Part. Elem., Herceg Novi, Yugoslavia (1967).
[15] T. T. Wu and C. N. Yang, Phys. Rev. Lett. **13**, 380 (1964).
[16] J. S. Bell and J. Steinberger, Proc. of Oxford Intern. Conf., 1965. Rutherford High Energy Lab. (1966), p. 195.
[17] J. Steinberger, Report CERN 70-1, Jan. (1970).
[18] J. M. Christenson *et al.*, Phys. Rev. B **140**, 74 (1965).
[19] Ya. B. Zel'dovich, Zh. Éksp. Theor. Fiz. **36**, No. 5, 1381 (1959) [Sov. Phys. JETP **36**, 984 (1959)].
[20] É. O. Okonov, Preprint JINR, RI-3788 [in Russian], Dubna (1968).
[21] I. Ya. Pomeranchuk, Zh. Éksp. Teor. Fiz. **34**, 725 (1958) [Sov. Phys. JETP **34**, 499 (1958)].
[22] J. V. Allaby *et al.*, Phys. Lett. B **30**, 500 (1969).
[23] V. Barger *et al.*, Nucl. Phys. B **5**, 411 (1968).
[24] W. Galbraith *et al.*, Phys. Rev. B **138**, 913 (1965).
[25] M. Restignoli and G. Violini, Phys. Lett. B **31**, 533 (1970).
[26] V. N. Gribov *et al.*, Phys. Lett. B **32**, 129 (1970).
[27] A. A. Ansel'm *et al.*, Yad. Fiz. **11**, 896 (1970) [Sov. J. Nucl. Phys. **11**, 500 (1971)].
[28] D. Horn, Phys. Lett. B **31**, 30 (1970).
[29] R. J. Eden, Phys. Rev. D **2**, 529 (1970).
[30] S. M. Roy, Phys. Lett. B **34**, 407 (1971).
[31] S. S. Gershteĭn *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **11**, 75 (1970) [JETP Lett. **11**, 48 (1970)].
[32] J. Finkelstein and S. M. Roy, Phys. Lett. B **34**, 322 (1971).
[33] J. Finkelstein, Phys. Rev. Lett. **24**, 172 (1970).
[34] Bott-Bodenhausen *et al.*, Phys. Lett. **20**, 212 (1966).
[35] N. Cabibbo, Phys. Lett. **22**, 212 (1967).
[36] M. L. Blackmon and G. R. Goldstein, Phys. Rev. D **1**, 2707 (1970).
[37] V. Barger and R. L. Phillips, Phys. Lett. B **33**, 425 (1970); Phys. Rev. D **2**, 1871 (1970); Phys. Lett. B **31**, 643 (1970).
[38] A. I. Lendel and K. A. Ter-Martirosyan, Pis'ma Zh. Éksp. Teor. Fiz. **11**, 70 (1970) [JETP Lett. **11**, 45 (1970)].
[39] K. G. Boreskov *et al.*, Nucl. Phys. B **40**, 307 (1972).
[40] F. J. Gilman, Phys. Rev. **171**, 1453 (1968).
[41] V. Lisin *et al.*, Nucl. Phys. B **40**, 298 (1972).

[42]R. E. Hendrick *et al.*, Phys. Rev. D **11**, 536 (1975).
[43]K. A. Ter-Martirosyan, Binarnye Reaktsii Adronov pri Vysokikh Énergiyakh. Trudy Mezhdunar. Seminara (Two-Particle Hadron Reactions at High Energies. Proc. Intern. Seminar), Preprint JINR D-6004, Dubna (1972).
[44]V. Barger and R. J. N. Phillips, Phys. Rev. Lett. **24**, 291 (1970).
[45]V. N. Gribov, Zh. Éksp. Teor. Fiz. **53**, 654 (1967) [Sov. Phys. JETP **26**, 414 (1968)]; V. N. Gribov and A. A. Migdal, Yad. Fiz. **8**, 1002 (1968) [Sov. J. Nucl. Phys. **8**, 583 (1969)]; **8**, 1213 (1968) [Sov. J. Nucl. Phys. **8**, 703 (1969)].
[46]S. Frautschi and B. Margolis, Nuovo Cimento A **56**, 1155 (1968).
[47]G. F. Chew and D. R. Snider, Phys. Lett. B **31**, 75 (1970).
[48]M. Lusignoli *et al.*, Nuovo Cimento A **49**, 705 (1967); **45**, 792 (1966); Phys. Lett. B **24**, 296 (1967).
[49]N. M. Queen *et al.*, F. Phys. **17**, 467 (1969).
[50]I. G. Aznauryan and L. D. Solov'ev, Yad. Fiz. **12**, 638 (1970) [Sov. J. Nucl. Phys. **12**, 347 (1971)].
[51]M. E. Vishnevskiĭ *et al.*, Yad. Fiz. **13**, 855 (1971) [Sov. J. Nucl. Phys. **13**, 489 (1971)].
[52]P. T. Mathews and A. Salam, Phys. Rev. **110**, 565 (1958).
[53]O. V. Dumbrais and N. M. Queen, Pis'ma Zh. Éksp. Teor. Fiz. **11**, 414 (1970) [JETP Lett. **11**, 280 (1970)]; **12**, 191 (1970) [JETP Lett. **12**, 133 (1970)]; Phys. Lett. B **32**, 65 (1970); Nucl. Phys. B **26**, 497 (1971).
[54]A. A. Logunov *et al.*, Ann. Phys. (N. Y.) **31**, 203 (1965); Phys. Lett. **7**, 71 (1969).
[55]Nguen Van Hieu, Teor. Mat. Fiz. **8**, 354 (1971).
[56]V. K. Birulev *et al.*, Preprint JINR, R1-6164, Dubna (1971); Yad. Fiz. **15**, 959 (1972) [Sov. J. Nucl. Phys. **15**, 534 (1972)]; Phys. Lett. B **38**, 452 (1972).
[57]V. K. Birulev *et al.*, Preprint JINR E1-6851, Dubna (1972).
[58]P. Darriulat *et al.*, Phys. Lett. B **33**, 433 (1970).
[59]C. D. Buchanan *et al.*, Phys. Lett. B **37**, 213 (1971).
[60]D. Freytag *et al.*, Phys. Rev. Lett. **35**, 412 (1975).
[61]S. Basiladze *et al.*, Preprint JINR, R1-5261, Dubna (1970).
[62]G. G. Takhtamyshev, Preprint JINR, 2453, Dubna (1966).
[63]X. de Bouard *et al.*, Nuovo Cimento A **52**, 662 (1967).
[64]D. Luers *et al.*, Phys. Rev. B **133**, 1276 (1964).
[65]L. B. Leipuner *et al.*, Phys. Rev. **132**, 2285 (1963).
[66]R. K. Adair and L. B. Leipuner, Phys. Lett. **12**, 67 (1964).
[67]C. J. B. Hawkins, Phys. Rev. **156**, 1444 (1967).
[68]A. Firestone *et al.*, Phys. Rev. Lett. **16**, 556 (1966); **17**, 116 (1966).
[69]J. V. Allaby, Trudy XV Mezhdunar. Konf. po Fizike Vysokikh Énergii. Kiev (1970) (Proc. 15th Intern. Conf. on High Energy Physics, Kiev, 1970), Naukova Dumka, Kiev (1972), p. 11.
[70]A. D. Brody *et al.*, Phys. Rev. Lett. **26**, 1050 (1971).
[71]G. Giacomelli, Proc. of Intern. Conf. on Elementary Particles, Amsterdam, 1971. North-Holland, Amsterdam (1971), p. 1.
[72]Yu. P. Gorin *et al.*, Yad. Fiz. **14**, 998 (1971) [Sov. J. Nucl. Phys. **14**, 560 (1972)].
[73]G. W. Brandenburg *et al.*, Phys. Rev. D **9**, 1939 (1974).
[74]Yu. P. Gorin *et al.*, Yad. Fiz. **17**, 309 (1973) [Sov. J. Nucl. Phys. **17**, 157 (1973)].
[75]A. S. Carroll *et al.*, Preprint FERMILAB-PUB-75/51-Exp. (1975).
[76]V. K. Birulev *et al.*, Preprint JINR, R1-9434, Dubna (1976).
[77]R. Messner *et al.*, Phys. Rev. Lett. **30**, 876 (1973).
[78]C. Geweniger *et al.*, Phys. Lett. B **48**, 487 (1974).
[79]V. G. Krivokhozhin and I. A. Savin, Preprint JINR, E1-9394, Dubna (1976).
[80]R. J. Eden, High Energy Collisions of Elementary Particles, Cambridge University Press (1968).
[81]J. Fisher *et al.*, Preprint P-FZU 75/1, Physics Institute, Czechoslovak Academy of Sciences, Prague (1975).

Translated by Julian B. Barbour