Formalism of the microscopic theory of nuclear reactions with nucleons

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A number of formal theories of nuclear reactions with nucleons of low and medium energies are systematized within the framework of a unified microscopic approach based on the integral Lippmann-Schwinger equation and the method of projection operators. A study is made of the different representations of transition amplitudes with direct processes, doorway states, and compound-nucleus resonances. The practical use of the general results of reaction theory for analyzing the detailed structure of nucleon-nucleus interaction cross sections in a wide range of energies is discussed.

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INTRODUCTION

Nuclear reactions with nucleons of low and medium energies constitute the basic and most studied branch of the physics of nuclear reactions. Here, in the last 10—15 years, significant progress has been achieved in the development of general physical ideas on the dynamics of nucleus transformation in a reaction; this has been due to the obtaining of high-quality experimental information on details of the energy and angular dependence of the cross sections and the obvious successes in the theoretical interpretation of the data on the basis of very general microscopic schemes of analysis.[1–10] These schemes have much in common with the ones generally used in model microscopic calculations of bound states of nuclei. When considering a reaction, the main task is to allow for the states of the continuum, which determine the open reaction channels.[11–15] In principle, one can give a unified description of low-lying levels of nuclei and highly excited states revealed directly in reactions on the basis of a single appropriate nuclear model.

In the shell approach, which has been widely used in recent years for the analysis of single-nucleon $[(p,p'), (p,n), (n,n')]$ reactions, the formation of different states of the system consisting of the nucleon and the target nucleus is regarded as the result of successive two-nucleon collisions with the excitation of ever more complicated configurations of the shell model. The observed features of the energy and angular dependence of the cross sections of these reactions can be directly related to definite transitions in the shell model with a pairing residual interaction.[1–14] Transitions between continuum states, for example, reflect the asymmetry characteristic of direct processes in the angular distribution of the reaction products.[16–20] Intermediate structures in reactions with protons can be interpreted in some cases as doorway analog states corresponding to simple excitations in the model of the two-particle—one-hole type.[2,3,6,21] More complicated configurations with total energy exceeding the threshold for the emission of a nucleon from the nucleus, the so-called quasibound states, are typical of resonances of the compound nucleus.[1,4,21–23] The shell approach also gives the most general justification of the complex-potential model applied to the analysis of average cross sections.[1–9] Thus, all the basic features of the structure of the cross sections of single-nucleon reactions can be obtained in a unified microscopic calculation.[4–7] However, this problem is in practice very complicated because of the need to allow for the residual interaction between all nucleons of the system and also because of some other fundamental difficulties relating to the application of the shell model to nuclear reactions.[4] To analyze experimental data, one usually employs simplified schemes in which one takes into account transitions for only a few distinguished nucleons, and the others, which form a core, are described by phenomenological nuclear models.[4–7]

The shell approach is widely used to interpret analog resonances,[21–24] to analyze the fine structure of giant dipole resonances in photonuclear reactions,[10,25–27] and to investigate preequilibrium processes.[19,20,28] The main attention is devoted to calculations of the matrix elements of transitions between definite states of the shell model in order to identify observed features in the energy dependence of the cross sections. In this review, we shall consider a somewhat different consequence of this approach: the possibility of constructing a general formalism of the theory of single-nucleon reactions with allowance for the dynamics of the transformation of the nucleus. Note that many characteristic features of the structure of the cross sections of single-nucleon reactions—the background of direct processes, intermediate structures, coupling of intermediate structures to compound-nucleus resonances—also appear in reactions with complex particles,[19,29] in the fisson cross section,[30] and in photonuclear reactions.[10] This indicates the existence of certain general fundamental features of the reaction process. The exposition of the formal theory given in the review is somewhat more general than is needed to describe single-nucleon reactions, which serve mainly as a convenient illustration of the physical meaning of the formal results.

The main part of the review is devoted to finding practical schemes for parametrizing the detailed energy dependence of the reaction cross sections in a wide range of nucleon energies, the clarification of the physical meaning of the parameters in the shell approach, and the establishment of the connection between the different schemes. The systematization of the numerous formalisms employed to analyze cross sections of nuclear reactions at low and medium energies has been considered in many specialized papers from different

points of view. We mention here the review of Lane and Thomas,[31] the papers of Bloch,[32] Lane and Robson,[33] and Lynn[5] based on R-matrix theory, and also the investigations in the unified theory of reactions set forth in detail in the general studies of Feshbach[6] and MacDonald,[7] and the applications to the analysis of cross sections in the framework of the shell approach.[4,34–36] In the present review, the systematization of the different reaction theories is based on the formalism of the Lippmann—Schwinger integral equation and the method of projection operators.[37–39] The results of the different theories and the connection between them are presented here in a very compact and general form. This makes it possible to obtain not only the well known formal results of reaction theory in the framework of a unified mathematical formalism but also opens up possibilities for constructing new schemes and methods of the theory.

The well known methods of many-level parametrization of the resonance structure of cross sections in wide energy ranges are formulated in the framework of the so-called effective-interaction method (see Sec. 2), in which the problem of determining the parameters of reaction theory from experimental data is reduced in practice to the solution of a corresponding system of coupled algebraic equations. In the determination of these parameters in model microscopic schemes, one can particularize the formalism, distinguishing doorway states of the model and excitations of the next higher complexity. The corresponding schemes for parametrizing the fine structure of nucleon cross sections near individual doorway (analog) resonances are given in Sec. 3.

In the region of overlapping resonances of a compound nucleus, integral equations for the transition amplitudes are more convenient. Different representations for these equations follow from the unified approach to the description of reactions with nucleons considered here (see Sec. 4). The transition from rigorous results to the complex-potential model and the methods of averging of cross sections with respect to the energy are discussed in Sec. 5. The problem of parametrizing the detailed structure of cross sections of photonuclear reactions near the energy of a giant dipole resonance on the basis of the shell approach is considered separately in Sec. 6.

1. SOME RESULTS OF FORMAL COLLISION THEORY

Many problems of the physics of quantum-mechanical interactions, including nuclear reactions with nucleons of low and medium energies, can be treated in a unified manner on the basis of formal collision theory. The principles of construction and the mathematical formalism of the theory have been set forth systematically in the well known monographs Refs. 37—40. We give here some results needed for the applications considered below to nuclear reactions with nucleons.

Let us consider the definition of the reaction channels, which characterize the observable states of the system consisting of the nucleon and the target nucleus in the continuum. In collision theory, channels correspond to different divisions of the system into individual fragments that do not interact with one another.[37] The channels are described by quantum numbers that refer to individual fragments. They are their isotopic composition, the internal energy state (we denote the set of corresponding quantum numbers by the subscript α), the spin of fragment i in the given energy state $I_{\alpha i}$, and one of the spin components $M_{\alpha i}$. It is convenient here to include as well the quantum numbers describing the relative motion.

In reactions with the formation of only a pair of fragments (nucleon and residual nucleus in single-nucleon reactions) there are two possibilities for doing this. One uses the plane-wave representation, in which the states are characterized by an ordinary wave vector $\mathbf{k}$. In this case, to identify a channel one takes the set of quantum numbers $\{\alpha(I_1I_2M_1M_2)\mathbf{k}\}$ or $\{\alpha(I_1I_2)s\nu\mathbf{k}\}$, where $s=|\mathbf{I}_1+\mathbf{I}_2|$ is the total spin of the channel with projection ν.[31] The other possibility is to use the representation in which the states have a definite angular momentum l of the relative motion and projection m_l and the states include the complete interval of angles. In this case, a channel is characterized by the set $\{\alpha(I_1I_2)s\nu lm_l\}$.

In some applications, it is convenient to use a representation in which the angular momenta are summed to form the total angular momentum of the system $J=|\mathbf{s}+\mathbf{l}|$ with components $M-\{\alpha(I_1I_2)slJM\}$. In the shell approach, to describe the relative motion of the nucleon and the nucleus one introduces the quantum numbers l, $j=l\pm1/2$, and m_j (Refs. 4—8, 11), which corresponds to representing the channel as $\{\alpha(I_1M_1)ljm_j\}$ or $\{\alpha(I_1)ljJM\}$.

The connection between the different representations is established by the well known schemes of collision kinematics.[4,8,31,41] The presentation of the general results of collision theory is not in principle bound to a particular channel definition. However, for perspicuity of the physical interpretation of the conclusions we shall hold to the schemes and definitions used in the shell approach.

We assume that the channel wave functions $\chi_c(\varepsilon_c)$, where the subscript c denotes the set of quantum numbers, are eigenfunctions of the Hamiltonian $K(K\chi_c=\varepsilon_c\chi_c)$ that describes the free motion of the various fragments α. We denote the total energy of the internal excitation of the fragments by ε_α and the energy of the relative motion by ε. Then $\varepsilon_c=\varepsilon_\alpha+\varepsilon$. Further, we shall assume that the total Hamiltonian H of the system can be represented as the sum $H=K+\mathcal{U}$, where K includes the total energy E; $\mathcal{U}$ describes all possible interactions between the different fragments. Then the solution of the wave equation $(E-H)\Psi=0$ or $(E-K)\Psi=\mathcal{U}\Psi$ satisfying boundary conditions corresponding to a plane wave or component thereof with definite l in channel c and outgoing spherical waves in all channels (we denote this solution by Ψ_c^+) can be formally represented as the solution of the equivalent integral Lippmann—Schwinger equation[37–40]:

$$\Psi_c^+(E)=\chi_c^+(E)+(E-K+i\eta)^{-1}\mathcal{U}\Psi_c^+(E), \tag{1}$$

where the small imaginary addition $i\eta$ in the integral operator (the Green's function of the problem) is needed

to distinguish states of outgoing waves in the solutions.[37-39] Using the operator identity

$$\frac{1}{A}\equiv\frac{1}{B}+\frac{1}{B}(B-A)\frac{1}{A}\equiv\frac{1}{B}+\frac{1}{A}(B-A)\frac{1}{B}, \quad (2)$$

we obtain a formal solution of Eq. (1) in the form

$$\Psi_c^+(E)=\chi_c(E)+(E-H+i\eta)^{-1}\mathcal{U}\chi_c(E). \quad (3)$$

Similarly, we define integral relations for the function $\Psi_c^-(E)$, which is the solution of the general wave equation for the case of ingoing waves in all channels and an outgoing plane wave in channel c after the reaction:

$$\Psi_c^-(E)=\chi_c(E)+(E-H-i\eta)^{-1}\mathcal{U}\Psi_c^-(E). \quad (4)$$

From the potential $\mathcal{U}$ we separate a certain model potential $U=\mathcal{U}-V$, where V is the residual interaction, assuming the known eigenfunctions of the equation

$$(E-H_0)\varphi=(E-K-U)\varphi=0. \quad (5)$$

To construct the general formalism of collision theory one imposes stringent requirements on the choice of the potential U; in practice, these are realized only in the shell description of single-nucleon reactions (see Sec. 2). The potential U must have finite depth, and then the eigenfunctions of Eq. (5) are defined in the region of the discrete and continuous spectra of eigenvalues. Solutions in the continuum must correspond to observed reaction channels, i.e., the potential U does not mix the reaction channels (does not contain polarizing interactions). The set of functions of the discrete spectrum must be sufficiently large, corresponding qualitatively to the number of resonance structures observed in the cross sections, and the energy eigenvalues must lie in the same region as the continuum energies, the so-called quasibound states "submerged" in the continuum.[4] We assume that the chosen potential U satisfies these requirements. Then the eigenfunctions of Eq. (5) in the continuum, $\varphi_c^\pm(E)$, which for appropriate boundary conditions satisfy the integral equation

$$\varphi_c^\pm(E)=\chi_c(E)+(E-K\pm i\eta)^{-1}U\varphi_c^\pm(E), \quad (6)$$

can be chosen as new channel functions with the same quantum numbers as χ_c, where the relative motion is described, not by plane waves, but by the waves distorted by the potential U. Expressing $\chi_c(E)$ in Eqs. (3) and (4) in terms of $\varphi_c^\pm(E)$ in accordance with Eq. (6), one can eliminate from the definition of $\psi_c^\pm$ the plane-wave states. Using the operator identity (2), we obtain (see Ref. 38, p. 204)

$$\Psi_c^\pm(E)=\varphi_c^\pm(E)+(E-H\pm i\eta)^{-1}V\varphi_c^\pm(E). \quad (7)$$

The channel wavefunction $\varphi_c^+(\varepsilon_c)$ is constructed as the product of the antisymmetrized function of the internal state of the fragment, $\psi_\alpha(\varepsilon_\alpha)$, and the solution of the problem of elastic scattering on the chosen potential U in channel c. Here, one can distinguish the spin–angle part $\Phi_c(\mathbf{r}/r)$ and the radial solution $u_{c\varepsilon}^+(r)$. The latter is a definite combination of two linearly independent solutions of the radial equation corresponding to asymptotically ingoing and outgoing waves. Regarding the functions $\varphi_c^\pm$ in what follows as a basis for expanding the exact solution in the continuum, it is also convenient to introduce real combinations of the solutions $u_{c\varepsilon}(r)$ (Ref. 7). The asymptotic behavior of these solutions for scattering on a centrally symmetric (shell) potential with allowance for the Coulomb interaction of the fragments corresponds to standing waves[44]:

$$u_{c\varepsilon}(r)\to\sin(kr+\omega_c-\pi l/2+\eta_l-h\ln 2kr)/r,$$

where ω_c is the elastic-scattering phase shift; $h=e_1e_2/\hbar v$ and $\eta_l=\arg\Gamma(l+1+ih)$ are the parameters of the Coulomb interaction.[31] We define an orthonormal set of real channel functions

$$|c\varepsilon_c\rangle=\psi_\alpha(\varepsilon_\alpha)\Phi_c(\mathbf{r}/r)u_{c\varepsilon}(r),\quad\langle c'E'|cE\rangle=\delta_{c'c}\delta(E'-E). \quad (8)$$

The corresponding normalized functions $\varphi_c^\pm(\varepsilon_c)$ can be expressed in terms of $|c\varepsilon_c\rangle$ (Refs. 42, 7, and 4):

$$\varphi_c^+(\varepsilon_c)=|c\varepsilon_c\rangle\exp(i\omega_c),$$
$$\varphi_c^-(\varepsilon_c)=|c\varepsilon_c\rangle\exp(-i\omega_c). \quad (9)$$

Thus, when the exact continuum solution is expanded with respect to channel eigenfunctions one can use any of the sets φ_c^+, φ_c^-, and $|c\varepsilon_c\rangle$.

In the exposition of the general results of formal collision theory we take the original definition of the S matrix to be[37]

$$S_{c'c}=\langle\Psi_{c'}^-,\Psi_c^+\rangle. \quad (10)$$

We use the integral relations for the solutions Ψ_c^+ and Ψ_c^-, the identical property (2) of the operators, and the rules for going to the limit $\eta\to 0$. Then the expression (10) for $S_{c'c}$ can be represented in the form (see Ref. 38, p. 207)

$$S_{c'c}=\langle\varphi_{c'}^-(E')\varphi_c^+(E)\rangle$$
$$-2\pi i\delta(E'-E)\left\langle\varphi_{c'}^-(E')\left|V+V\frac{1}{E-H+i\eta}V\right|\varphi_c^+(E)\right\rangle \quad (11)$$

(the δ function reflects the law of conservation of energy in the reaction).

We introduce a transition operator τ satisfying the operator equation

$$\tau=V+V\frac{1}{E-H+i\eta}V=V+V\frac{1}{E-H_0+i\eta}\tau. \quad (12)$$

The matrix elements of this operator on the channel wave functions define the transition amplitudes

$$t_{c'c}(E',E)=\langle\varphi_{c'}^-(E')\tau\varphi_c^+(E)\rangle$$

and

$$T_{c'c}(E',E)=\langle c'E'|\tau|cE\rangle. \quad (13)$$

Using these definitions and Eqs. (9), we represent $S_{c'c}$ in (11) as

$$S_{c'c}(E',E)=[\exp(2i\omega_c)\delta_{c'c}-2\pi i t_{c'c}(E',E)]\delta(E'-E), \quad (14a)$$

or

$$S_{c'c}(E',E)=\exp(i\omega_{c'})[\delta_{c'c}-2\pi i T_{c'c}(E',E)]\exp(i\omega_c)\delta(E'-E). \quad (14b)$$

The transition amplitudes calculated for one and the same total energy E of the channels (on the "energy shell") form the matrices $t(E)$ and $T(E)$. The corresponding collision matrix $S(E)$ can be expressed as

$$S(E)=\exp(2i\omega)-2\pi i t(E)=\exp(i\omega)[1-2\pi i T(E)]\exp(i\omega), \quad (15)$$

where $\exp(i\omega)$ is a diagonal matrix with the elements $\exp(i\omega_c)$. The determination of the collision matrix is the main part of the problem of constructing different interaction cross sections in terms of the elements $S_{c'c}(E)$ in the well known schemes of the kinematics of nuclear reactions.[41]

The investigation of the structural details of the energy dependence of the matrix elements of the operator τ entails a further particularization of Eq. (12). For this, we introduce projection operators P and Q corresponding to solutions of Eq. (5) in the region of the discrete spectrum, $|\lambda\rangle$, and the continuum, $|c\varepsilon_c\rangle$:

$$P=\sum_c P_c;\ P_c=\int |c\varepsilon_c\rangle\langle c\varepsilon_c|\,d\varepsilon_c,\ Q=\sum_\lambda |\lambda\rangle\langle\lambda|, \tag{16}$$

which satisfy the relations $P+Q=1$; $P_c^2=P_c$; $P_cP_{c'}=P_c\delta_{c'c}$; $P^2=P$; $PP_c=P_c$; $P_cQ=0$; $Q^2=Q$. We write Eq. (12) in the form

$$\tau=V+V\frac{1}{E-H_0+i\eta}P\tau+V\frac{1}{E-H_0}Q\tau. \tag{17}$$

In the last term, which contains an operator of projection onto states of the discrete spectrum, the small addition $i\eta$ needed to distinguish outgoing waves in the solutions for the continuum can always be omitted. We use the definition (as $\eta\to 0$)[37]

$$\frac{1}{E-H_0+i\eta}P=\sum_c\int\frac{|c\varepsilon_c\rangle\langle c\varepsilon_c|}{E-\varepsilon_c+i\eta}d\varepsilon_c=\frac{1}{E-H_0}P-i\pi P\delta(E-H_0)P, \tag{18}$$

where the first term on the right-hand side corresponds to the principal value of the integral, and $P\delta(E-H_0)P=\sum_c|cE\rangle\langle cE|$. Then Eq. (17) can be represented in the form

$$\tau=V+V\frac{1}{E-H_0}\tau-i\pi P\delta(E-H_0)P. \tag{19}$$

This form is convenient for the transition to the real operator $\mathscr{K}$, associated with τ by the so-called Heitler operator equation[37-39]:

$$\tau=\mathscr{K}-i\pi\mathscr{K}P\delta(E-H_0)P\tau. \tag{20}$$

Substituting τ into (19), we obtain an equation for the operator $\mathscr{K}$:

$$\mathscr{K}=V+V\frac{1}{E-H_0}\mathscr{K}=V+V\frac{1}{E-H_0}P\mathscr{K}+V\frac{1}{E-H_0}Q\mathscr{K}. \tag{21}$$

The matrix elements of the operator $\mathscr{K}$ on the channel wave functions $|cE\rangle$ for one and the same total energy E of the channels:

$$K_{c'c}(E)=\langle c'E|\mathscr{K}|cE\rangle,$$

form a real and symmetric matrix $K(E)$ which is related to the matrix $T(E)$ by Eq. (20):

$$K(E)=T(E)+i\pi K(E)T(E)$$

or

$$T=(1+i\pi K)^{-1}K. \tag{22}$$

At the same time, the matrix $S(E)$ in (15) can be represented in the form

$$S=\exp(i\omega)(1+i\pi K)^{-1}(1-i\pi K)\exp(i\omega), \tag{23}$$

where fundamental properties of the S matrix such as unitarity ($SS^+=1$) and symmetry ($S_{c'c}=S_{cc'}$) (Ref. 43) are manifested explicitly.

2. EFFECTIVE-INTERACTION METHOD

The operator relations for $\mathscr{K}$ (21) and τ (17) define corresponding systems of coupled integral equations for the matrix elements of these operators on all possible solutions of the model Hamiltonian H_0. In the construction of solutions, wide use is made of the effective-interaction method, which enables one to go over from the system of integral equations to a system of algebraic equations.[4,7] We define the operator of the effective interaction by[7]

$$V^{\text{eff}}=V+V\frac{1}{E-H_0}PV^{\text{eff}}$$

or

$$V^{\text{eff}}=\left(1-VP\frac{1}{E-H_0}\right)^{-1}V, \tag{24}$$

and then the operator equation for $\mathscr{K}$ (21) can be transformed to the form

$$\mathscr{K}=V^{\text{eff}}+V^{\text{eff}}\frac{1}{E-H_0}Q\mathscr{K}$$
$$=V^{\text{eff}}+V^{\text{eff}}Q\frac{1}{E-H_0-QV^{\text{eff}}Q}QV^{\text{eff}}. \tag{25}$$

The expression on the right, which represents a formal solution of the operator equation, is obtained by means of the identity (2). In this case, the matrix elements of $\mathscr{K}$ can be represented in the form[44]

$$K_{c'c}(E)=K^0_{c'c}(E)+K^1_{c'c}(E)=K^0_{c'c}(E)+\frac{1}{2\pi}\sum_{\lambda,\mu}\gamma_{c'\lambda}(A^{-1})_{\lambda\mu}\gamma_{\mu c}, \tag{26}$$

where

$$K^0_{c'c}(E)=\langle c'E|V^{\text{eff}}|cE\rangle;\ \gamma_{\lambda c}=\sqrt{2\pi}\langle\mu|V^{\text{eff}}|cE\rangle; \tag{27}$$

$(A^{-1})_{\lambda\mu}$ are the elements of the matrix reciprocal to

$$A_{\lambda\mu}=(E-E_\lambda)\delta_{\lambda\mu}-\langle\lambda|V^{\text{eff}}|\mu\rangle. \tag{28}$$

In applications to the parametrization of the energy dependence of cross sections in a resonance region, one usually uses a somewhat different expression for the elements $K^1_{c'c}$ with a diagonalized level matrix $\tilde{A}$. The diagonalization procedure is associated with the definition of a matrix of orthogonal transformation $\Omega(\Omega\Omega^+=1)$ such that $\Omega^+A\Omega=\tilde{A}$, $A=\Omega\tilde{A}\Omega^+$, where $\tilde{A}$ is the diagonal matrix with the elements $\tilde{A}_k=E-\tilde{E}_k$. Noting that

$$(A^{-1})_{\lambda\mu}=\sum_k\Omega_{\lambda k}\frac{1}{E-\tilde{E}_k}\Omega_{\mu k}, \tag{29}$$

we arrive at an expression for the elements $K_{c'c}$ typical of the R-matrix formalism[31]:

$$K_{c'c}(E)=K^0_{c'c}(E)+\frac{1}{2\pi}\sum_k\tilde{\gamma}_{c'k}\frac{1}{E-\tilde{E}_k}\tilde{\gamma}_{kc}, \tag{30}$$

where

$$\tilde{\gamma}_{c'k}=\sum_k\gamma_{c'\lambda}\Omega_{\lambda k}.$$

Note, however, that the R and K matrices coincide only as regards the form of the parametrization, since

the structure and physical meaning of the parameters are here fundamentally different.[2,4-7,31] The matrix K^0 corresponds to transitions to the continuum without excitation of bound states, and it can be identified in the scheme considered here with direct processes. The remaining part K^1 corresponds to resonances of the compound nucleus that decay to the continuum because of the nonzero matrix elements $\langle c'E|V^{\text{eff}}|\lambda\rangle$.

This separation of the direct processes can also be made in the T matrix. The corresponding representation for the transition operator follows from Eq. (17):

$$\tau=\tau_0+\tau_0\frac{1}{E-H_0}Q\tau=\tau_0+\tau_0Q\frac{1}{E-H_0-Q\tau_0Q}Q\tau_0, \tag{31}$$

where τ_0 is introduced by the relation

$$\tau_0=\left(1-VP\frac{1}{E-H_0+i\eta}\right)^{-1}V$$

or

$$\tau_0=V^{\text{eff}}-i\pi\tau_0P\delta(E-H_0)PV^{\text{eff}}. \tag{32}$$

The elements of the T matrix can be represented in this case in the form

$$T_{c'c}(E)=T^0_{c'c}(E)+\frac{1}{2\pi}\sum_{\lambda\mu}\Gamma^{1/2}_{c'\lambda}(B^{-1})_{\lambda\mu}\Gamma^{1/2}_{\mu c}, \tag{33}$$

where

$$T^{0*}_{c'c}(E)=\langle c'E|\tau_0|cE\rangle;\ \Gamma^{1/2}_{\mu c}=\sqrt{2\pi}\,\langle\mu|\tau_0|cE\rangle;$$
$$B_{\lambda\mu}=(E-E_\lambda)\delta_{\lambda\mu}-\langle\lambda|\tau_0|\mu\rangle. \tag{34}$$

Using the relation (32), we obtain an expression for T^0 in terms of K^0:

$$T^0=K^0-i\pi K^0T^0 \quad \text{or} \quad T^0=(1+i\pi K^0)^{-1}K^0. \tag{35}$$

In the general case, the quantities $\Gamma^{1/2}_{\mu c}$ are complex. It is convenient to represent their relation to the real elements $K^0_{c'c}(E)$ and $\gamma_{\mu c}$ (27) in matrix form. We introduce a rectangular matrix $\Gamma^{1/2}$ with elements $\Gamma^{1/2}_{c\lambda}$; then, using the relation for τ_0 (32), we obtain

$$\Gamma^{1/2}=\gamma-i\pi K^0\Gamma^{1/2} \quad \text{or} \quad \Gamma^{1/2}=(1+i\pi K^0)^{-1}\gamma, \tag{36}$$

where γ is a real rectangular matrix with the elements $\gamma_{c\lambda}$. Finally, the elements $\langle\lambda|\tau_0|\mu\rangle$ in the definition of $B_{\lambda\mu}$ (34) can also be expressed in terms of the matrix elements of the operator V^{eff}:

$$\langle\lambda|\tau_0|\mu\rangle=\langle\lambda|V^{\text{eff}}|\mu\rangle-\Delta_{\lambda\mu}-i\Gamma_{\lambda\mu}/2, \tag{37}$$

where

$$\Delta_{\lambda\mu}=\frac{\pi}{2}\sum_{c'c}\Gamma^{1/2*}_{\lambda c}K^0_{cc'}\Gamma^{1/2}_{c'\mu},\quad \Gamma_{\lambda\mu}=\sum_c\Gamma^{1/2*}_{\lambda c}\Gamma^{1/2}_{c\mu} \tag{38}$$

are real quantities. The parameters $\Gamma_{\mu c}$ and $E'_\lambda=E_\lambda+\langle\lambda|V^{\text{eff}}|\lambda\rangle$ are usually associated with the width and energy of the observed resonances in the cross sections. In practical applications, it is frequently convenient to use the expression for $T_{c'c}$ (33) with diagonalized level matrix $\tilde{E}$. The diagonalization procedure is here analogous to the one considered for $K_{c'c}$ (30), although the matrices Ω are matrices of a complex orthogonal transformation.[45] As a result, we obtain

$$T_{c'c}(E)=T^0_{c'c}(E)+\frac{1}{2\pi}\sum_k\tilde{\Gamma}^{1/2}_{c'k}\frac{1}{E-\tilde{E}_k}\tilde{\Gamma}^{1/2}_{kc}, \tag{39}$$

where $\tilde{\Gamma}^{1/2}_{kc}=\sum_\mu\Omega_{k\mu}\Gamma^{1/2}_{\mu c}$ and $\tilde{E}_k$ are complex. The corresponding schemes for parametrizing the energy structure of the cross sections in the resonance region are typical of the so-called S-matrix formalism.[46-48]

The relations given here for the energy dependence of the elements of the matrices $K(E)$ (Eqs. (26) and (30)) and $T(E)$ (Eqs. (33) and (39)) are very similar in form to the corresponding results of R-matrix theory.[31] Therefore, in applications to the parametrization of cross sections in the resonance region many of the practical analysis schemes constructed earlier can be applied. As a new feature we can identify the immediate separation of the direct process, for an estimate of which the methods of the theory of direct interactions are used.[16-20] In addition, under certain assumptions about the structure of the effective-interaction operator one can estimate the width of the resonances and in some cases associate observed features in the cross sections with definite transition matrix elements (see Sec. 3). If however we restrict ourselves to a phenomenological description of the energy dependence of the cross sections in the resonance region, the interpretation of the results in this scheme remains the same as in R-matrix theory.

First of all, we must mention the methods of single- and many-level analysis that enable one to obtain the set of resonance parameters describing the main features of the energy structure of the cross sections in the region of resolved resonances,[49-53] and then we shall consider the methods of mathematical statistics of random variables used to systematize the parameters. The relevant distributions are the Porter–Thomas distribution, which describes the fluctuations of the resonance widths of individual levels[60,61] and the Wigner distribution for the fluctuations of the distances between neighboring resonances.[62] In the region of overlapping resonances, these fluctuations are manifested in the energy dependence of the cross sections as the so-called Ericson fluctuations of the cross sections.[63] The average cross sections are analyzed by the same schemes as in R-matrix theory, using the well known concepts of the compound nucleus.[31,64] It is important that in the shell approach there appears directly the single-particle structure of the average resonance widths (strength functions), this being determined by the energy dependence of the solutions for the channel wave functions $|c\varepsilon_c\rangle$ (8). This leads to a simple physical interpretation of the complex-potential model when cross sections averaged over the resonances are analyzed[4,7,9] (see Sec. 5).

In the practical analysis of experimental data one uses different parametrization schemes, the particular choice being dictated by the aim of the analysis, the quality of the experimental information, and by other, partly subjective factors. Another of the requirements on the choice of the parametrization scheme is that of the most detailed reproduction of the energy dependence of the cross sections in definite ranges of variation of the energy of the incident nucleons, given a minimal number of experimentally determined parameters. This requirement, besides its obvious meaning for the confirmation of definite assumptions about the nature of the

interaction, has important practical value, for example, for the introduction of detailed information about the structure of neutron cross sections into reactor calculations.[65] This need for exact reproduction of the structure of the cross sections in wide energy ranges by means of a self-consistent set of physical parameters makes the problem of the further refinement of the formalism of the theory of nuclear reactions with nucleons very important.

3. SEPARATION OF STATES OF VARIOUS KINDS

The general results of the formal theory presented in Secs. 1 and 2 use in reality an expansion of the exact wavefunction of the problem with respect to a complete set of eigenfunctions of the model Hamiltonian H_0 (Ref. 7). The separation in this set of the bound states P and in the continuum states Q corresponding to asymptotically observed (physical) reaction channels makes it possible to represent the amplitude as a sum of the amplitudes of the direct processes and the resonance part. In applications to the parametrization of the cross sections this possibility provides definite advantages compared with the results of the R-matrix theory in the physical interpretation of the background (or part of the background) in the resonance region. The main criterion of a direct process must here be taken to be stable asymmetry of the angular distribution of the reaction products observed in the cross sections averaged over the resonances in a wide range of variation of the nucleon energies.[20]

The use of the effective-interaction method in this approach makes possible a further particularization of the formalism and separation from the bound states of a basis set of solutions of different kinds. For example, one can have doorway states, which are coupled directly to the continuum, or more complicated states, which decay only because of their coupling to the doorway states.[2-4] To construct the corresponding schemes of the formalism, we represent the projection operator Q in the form of the sum $Q=Q_0+Q_1+\cdots+Q_n$, where each of the terms refers to a definite type of state. We consider the structure of the expression for the amplitude $T_{c'c}(E)$ in the case when $Q=Q_0+Q_1$. In Eq. (31) for the transition operator

$$\tau=\tau_0+\tau_0\frac{1}{E-H_0}Q_0\tau+\tau_0\frac{1}{E-H_0}Q_1\tau$$

we transfer to the left-hand side the term containing Q_0. Applying then the operator $[1-\tau_0Q_0(1/E-H_0)]^{-1}$, to both sides of the equation, we obtain

$$\tau=\tau_1+\tau_1\frac{1}{E-H_0}Q_1\tau=\tau_1+\tau_1Q_1\frac{1}{E-H_0-Q_1\tau_1Q_1}Q_1\tau_1, \tag{40}$$

where

$$\tau_1=\left(1-\tau_0Q_0\frac{1}{E-H_0}\right)^{-1}\tau_0=\tau_0+\tau_0Q_0\frac{1}{E-H_0-Q_0\tau_0Q_0}Q_0\tau_0.$$

The corresponding expression for the transition amplitude $T_{c'c}(E)$ can be represented in the form[66]

$$T_{c'c}=T^0_{c'c}+\frac{1}{2\pi}\sum_{\lambda_0\mu_0}\Gamma^{1/2}_{c'\lambda_0}(B_0^{-1})_{\lambda_0\mu_0}\Gamma^{1/2}_{\mu_0c}+\frac{1}{2\pi}\sum_{\lambda_1\mu_1}\beta^{1/2}_{c'\lambda_1}(M^{-1})_{\lambda_1\mu_1}\beta^{1/2}_{\mu_1c}, \tag{41}$$

where B_0 is the block of the matrix B (34) containing only the states $Q_0(\lambda_0,\mu_0)$:

$$\left.\begin{aligned}\beta^{1/2}_{c'\lambda_1}&=\sqrt{2\pi}\,\langle c'E\,|\,\tau_1\,|\,\lambda_1\rangle=\Gamma^{1/2}_{c'\lambda_1}\\&+\sum_{\lambda_0\mu_0}\Gamma^{1/2}_{c'\lambda_0}(B_0^{-1})_{\lambda_0\mu_0}\langle\mu_0\,|\,\tau_0\,|\,\lambda_1\rangle;\\M_{\lambda_1\mu_1}&=(E-E_{\lambda_1})\,\delta_{\lambda_1\mu_1}-\langle\lambda_1\,|\,\tau_0\,|\,\mu_1\rangle\\&-\sum_{\lambda_0\mu_0}\langle\lambda_1\,|\,\tau_0\,|\,\lambda_0\rangle(B_0^{-1})_{\lambda_0\mu_0}\langle\mu_0\,|\,\tau_0\,|\,\mu_1\rangle.\end{aligned}\right\} \tag{42}$$

The matrix elements $\langle\lambda_1\,|\,\tau_0\,|\,\mu_1\rangle$, $\langle\lambda_1\,|\,\tau_0\,|\,\lambda_0\rangle$, and also $\langle\lambda_0\,|\,\tau_0\,|\,\mu_0\rangle$ in $(B_0)_{\lambda_0\mu_0}$ (34) can be expressed in terms of the elements of the effective-interaction operator by equations of the type (37).

In its extreme formulation, the hypothesis of doorway states presupposes that for one of the types of state the matrix elements of transitions to the continuum are zero. Suppose, for example, $PV^{\rm eff}Q_1=0$ $[\Gamma^{1/2}_{c'\lambda_1}=0$ (36)]; then

$$\left.\begin{aligned}B^{1/2}_{c'\lambda_1}&=\sum_{\lambda_0\mu_1}\Gamma^{1/2}_{c'\lambda_0}(B_0^{-1})_{\lambda_0\mu_0}\langle\mu_0\,|\,V^{\rm eff}\,|\,\lambda_1\rangle;\\M_{\lambda_1\mu_1}&=(E-E_{\lambda_1})\,\delta_{\lambda_1\mu_1}-\langle\lambda_1\,|\,V^{\rm eff}\,|\,\mu_1\rangle\\&-\sum_{\lambda_0\mu_0}\langle\lambda_1\,|\,V^{\rm eff}\,|\,\lambda_0\rangle(B_0^{-1})_{\lambda_0\mu_0}\langle\mu_0\,|\,V^{\rm eff}\,|\,\mu_1\rangle.\end{aligned}\right\} \tag{43}$$

It is obvious that in this case the last term in $T_{c'c}$ (41) is nonzero only because of the residual interaction between the states of different kind ($\langle\mu_0\,|\,V^{\rm eff}\,|\,\lambda_1\rangle\neq0$). Note that the definitions of the coefficients $\beta^{1/2}_{c'\lambda}$ [see Eqs. (42) and (43)] contain the energy denominator $(B_0^{-1})_{\lambda_0\mu_0}$, which leads to a resonance dependence of the widths averaged over many levels.[4-7,22,66]

An equivalent representation for the transition amplitudes can be obtained from the general expression (41) by setting $PV^{\rm eff}Q_0=0$ ($\Gamma^{1/2}_{c'\lambda_0}=0$). Then $\beta^{1/2}_{c'\lambda_1}=\Gamma^{1/2}_{c'\lambda_1}$, and the second term in (41) is equal to zero:

$$T_{c'c}=T^0_{c'c}+\frac{1}{2\pi}\sum_{\lambda_1\mu_1}\Gamma^{1/2}_{c'\lambda_1}\times\left\langle\lambda_1\left|\frac{1}{E-H_0-Q_1\tau_0Q_1-Q_1V^{\rm eff}Q_0\dfrac{1}{E-H_0-Q_0V^{\rm eff}Q_0}Q_0V^{\rm eff}Q_1}\right|\mu_1\right\rangle\Gamma^{1/2}_{\mu_1c}. \tag{44}$$

This formula clearly reflects the qualitative features of the energy dependence of the cross sections associated with the doorway-state hypothesis. For example, at energies corresponding to levels not coupled to the continuum the resonance part of the amplitude vanishes. To illustrate this, we diagonalize in the expression (44) for $T_{c'c}$ with respect to the residual interaction separately in each of the groups of levels. The new states $|\tilde\lambda_0\rangle$ and $|\tilde\lambda_1\rangle$ satisfy the equations

$$(E-H_0-Q_0V^{\rm eff}Q_0)\,|\,\tilde\lambda_0\rangle=0;\quad(E-H_0-Q_1V^{\rm eff}Q_1)\,|\,\tilde\lambda_1\rangle=0,$$

and for the projection operators $Q_0=\sum\tilde\lambda_0\,|\lambda_0\rangle\langle\lambda_0|$ and $Q_1=\sum\tilde\lambda_1\,|\tilde\lambda_1\rangle\langle\tilde\lambda_1|$ it is assumed that $\tilde Q_0=Q_0$, $\tilde Q_1=Q_1$ (Refs. 4 and 7). Then the expression (44) with allowance for the explicit form (37) of the elements $Q_1\tau_0Q_1$ (37) can be represented as

$$T_{c'c}=T^0_{c'c}+\frac{1}{2\pi}\sum_{\tilde\lambda_1\tilde\mu_1}\Gamma^{1/2}_{c'\tilde\lambda_1}\times\left\langle\tilde\lambda_1\left|\frac{1}{E-\tilde H_0+\Delta+i\Gamma/2-\tilde Q_1\sum\limits_{\lambda_0}V^{\rm eff}\,|\,\tilde\lambda_0\rangle\dfrac{1}{E-E_{\tilde\lambda_0}}\langle\tilde\lambda_0\,|\,V^{\rm eff}\tilde Q_1}\right|\tilde\mu_1\right\rangle\Gamma^{1/2}_{\tilde\mu_1c}. \tag{45}$$

Obviously, for $E=E_{\lambda_0}$ the resonance part of (45) is zero.

The idea of separating in Q the model states coupled to the continuum can be further developed for the construction of the transition amplitude in the so-called hierarchy scheme, in which complicated states are excited by successive transitions between simpler states. We represent Q in the form of the sum $Q=Q_0+Q_1+\ldots+Q_n$. Suppose $PV^{\text{eff}}Q_0\neq 0$, but for all the remainder $PV^{\text{eff}}Q_i=0$ $(i=1,2,\ldots,n)$. Thus, doorway states are distinguished. Suppose that $Q_0V^{\text{eff}}Q_1\neq 0$ but for the remaining $j=2,3,\ldots,n$ we have $Q_0V^{\text{eff}}Q_j=0$ (the states 1 correspond to "corridor" states[3,4,67,68]). We then set $Q_1V^{\text{eff}}Q_2\neq 0$ but $Q_1V^{\text{eff}}Q_k=0$ $(k=3,4,\ldots,n)$, etc. Denoting $Q'=Q_1+Q_2+\ldots+Q_n$, we transform the equation for the transition operator τ (31) to the form

$$\tau=\tau'+\tau'\frac{1}{E-H_0}Q_0\tau=\tau'+\tau'Q_0\frac{1}{E-H_0-Q_0\tau'Q_0}Q_0\tau', \tag{46}$$

where

$$\tau'=\tau_0+\tau_0\frac{1}{E-H_0}Q'\tau'. \tag{47}$$

In principle, to determine the transition amplitudes we need only the operator $P\tau P$. Under our assumptions, using the definition (32) for τ_0, from which $P\tau_0Q'=0$ follows, we obtain

$$P\tau P=P\tau_0P+P\tau_0Q_0\frac{1}{E-H_0-Q_0\tau'Q_0}Q_0\tau_0P. \tag{48}$$

We now split the operator Q' into two parts: $Q'=Q_1+Q''$, where $Q''=Q_2+\ldots+Q_n$, and we transform Eq. (47):

$$\tau'=\tau''+\tau''\frac{1}{E-H_0}Q_1\tau'=\tau''+\tau''Q_1\frac{1}{E-H_0-Q_1\tau''Q_1}Q_1\tau'', \tag{49}$$

where

$$\tau''=\tau_0+\tau_0\frac{1}{E-H_0}Q''\tau''. \tag{50}$$

Then the operator $Q_0\tau'Q_0$ in (46) can be represented in the form

$$Q_0\tau'Q_0=Q_0\tau_0Q_0+Q_0V^{\text{eff}}Q_1\frac{1}{E-H_0-Q_1\tau''Q_1}Q_1V^{\text{eff}}Q_0, \tag{51}$$

with allowance for the fact that $Q_0\tau''Q''=0$. The next step is associated with the definition of the operator $Q_1\tau''Q_1$, for which one can use a similar scheme, writing $Q''=Q_2+Q'''$. Continuing this procedure to the $(n-1)$th division $Q^{n-1}=Q_{n-1}+Q_n$, we arrive as a result at the representation of $P\tau P$ (48) in the form of a continued fraction.[69] In a basis diagonalized with respect to the residual interaction in each of the groups of coupled model states, expression for $P\tau P$ [and accordingly for the elements $T_{c'c}(E)$] can be written in the operator form

$$\left.\begin{aligned}
&P\tau P=P\tau_0P+P\tau_0\widetilde{Q}_0\frac{1}{E-\widetilde{H}_{00}+\Delta+i\Gamma/2-N_1}\widetilde{Q}_0\tau_0P;\\
&N_1=\widetilde{Q}_0V^{\text{eff}}\widetilde{Q}_1\frac{1}{E-\widetilde{H}_{01}-N_2}\widetilde{Q}_1V^{\text{eff}}\widetilde{Q}_0;\\
&N_2=\widetilde{Q}_1V^{\text{eff}}\widetilde{Q}_2\frac{1}{E-\widetilde{H}_{02}-N_3}\widetilde{Q}_2V^{\text{eff}}\widetilde{Q}_1;\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&N_n=\widetilde{Q}_{n-1}V^{\text{eff}}\widetilde{Q}_n\frac{1}{E-\widetilde{H}_{0n}}\widetilde{Q}_nV^{\text{eff}}\widetilde{Q}_{n-1},
\end{aligned}\right\} \tag{52}$$

where $\widetilde{H}_{0m}=H_0+Q_mV^{\text{eff}}Q_m$. The expressions for the transition amplitudes in the hierarchy scheme[69] can be reduced to a similar form.

These results for the energy dependence of the transition amplitude (the operator $P\tau P$) correspond to the limiting situation in which the complicated states decay to the continuum only through the successive excitations of simpler and simpler states. Such a treatment can also be given for the more general case when the decay of a complicated state to the continuum proceeds in accordance with the hierarchy scheme directly. In this case, a detailed analysis of the energy structure of the cross sections in individual resonance regions can give information about the predominance of any particular reaction mechanism. As an example, we may take the investigation of the fine structure of doorway analog states[21–23] and the structure of giant dipole resonances in photonuclear reactions.[25–27] Here allowance is made for only one level in Q_0, and the Q_1 states are usually chosen in a diagonalized basis, which considerably simplifies the general expressions for the amplitudes $T_{c'c}(E)$.

The hypothesis of doorway states and the hierarchy scheme are associated with definite physical assumptions of the model, which have been most fully studied at the present time in the shell approach.[3–10] In a consistent scheme of this approach, the total Hamiltonian of the system of nucleons $H(1,2,\ldots,A)$, where the numbers denote the set of spatial, spin, and isospin variables of the individual nucleons, is split into the sum

$$H=\mathcal{H}_0+\mathcal{V}.$$

Here, $\mathcal{H}_0$ is the Hamiltonian of the shell model:

$$\mathcal{H}_0=\sum_{i=1}^{A}h(i)=\sum_i[t(i)+v_0(i)];$$

$t_i=(\hbar^2/2m_i)\Delta_i$ is the kinetic energy operator of nucleon i; $v_0(i)$ is the potential of the shell model; and $\mathcal{V}$ is the residual interaction, which in the approximation of pairing forces is defined as

$$\mathcal{V}=\sum_{i<j}v(i,j)-\sum_i v_0(i). \tag{53}$$

This representation of the total Hamiltonian is analogous to the one usually employed in calculations of bound states of nuclei. The only specific feature is the choice for v_0 of a potential of finite depth. The basis set is formed by all possible antisymmetrized products of the single-particle functions of all the nucleons of the system that are eigenfunctions of the Hamiltonian $\mathcal{H}_0$ of the shell model. Here, one can distinguish several types of solutions corresponding to different physical situations: 1) All nucleons of the system are in bound states (however, their total energy may also exceed the threshold for the emission of a nucleon into the continuum); 2) $A-1$ nucleons are in bound states and one is in the continuum; 3) $A-2$ nucleons are in bound states and two are in the continuum, etc. In the theory of single-nucleon reactions, we restrict ourselves to solutions of the first two types. This results in certain limitations of the approach associated with the incomplete basis, but at the same time it enables one to avoid serious complications of the formalism and does not

affect a number of fundamental questions of the theory of many-particle processes.[37–42] Functions of the first type determine the operator

$$Q=\sum_{\lambda}|\lambda\rangle\langle\lambda|,$$

where

$$\langle \mathbf{r}_1 \dots \mathbf{r}_A|\lambda\rangle=\mathcal{A}\prod_{i=1}^{A}u_{\lambda i}(\mathbf{r}_i); \tag{54}$$

$u_{\lambda i}(\mathbf{r}_i)$ are the single-particle solutions in the shell potential $v_0(i)$ and they are orthonormalized and damped exponentially at infinity; $\mathcal{A}$ is the operator of antisymmetrization with respect to all the nucleons of the system. The total energy $E_\lambda(\hat{\mathcal{H}}_0|\lambda\rangle=E_\lambda|\lambda\rangle)$ of a state is made up of the energies of the individual nucleons. States with E_λ exceeding the energy of emission of a nucleon from the system are quasibound. This includes various particle–hole configurations of the shell model, the number of which per unit interval of excitation energy of the system (the density of states) increases rapidly with increasing energy, approximately as the density of the observed resonances.[70–71]

Solutions of the second type $|\beta jl\varepsilon\rangle$ $(\hat{\mathcal{H}}_0|\beta jl\varepsilon\rangle=(\varepsilon_\beta+\varepsilon)|\beta jl\varepsilon\rangle)$ can be written in similar form[4,7]:

$$\begin{aligned}\langle \mathbf{r}_1 \dots \mathbf{r}_A|\beta jl\varepsilon\rangle&=\mathcal{A}\prod_{i=1}^{A-1}u_{\beta i}(\mathbf{r}_i)\,u_{jl\varepsilon}(\mathbf{r}_A)\\&=\mathcal{A}'(\mathbf{r}_1\dots\mathbf{r}_{A-1}|\beta)\,u_{jl\varepsilon}(\mathbf{r}_A),\end{aligned} \tag{55}$$

where the function $u_{jl\varepsilon}(\mathbf{r}_A)$ refers to the continuum; $(\mathbf{r}_1\cdots\mathbf{r}_{A-1}|\beta)$ is the antisymmetrized product of the wavefunctions of bound states; $\mathcal{A}'$ is the operator of antisymmetrization of a nucleon in the continuum with the remaining nucleons. The orthonormalized solutions $u_{jl\varepsilon}(\mathbf{r})$ are chosen to be real with the asymptotic behavior

$$\begin{aligned}u_{jl\varepsilon}(\mathbf{r})\to&\left(\frac{2m}{\pi\hbar^2k}\right)^{1/2}\frac{1}{r}\sin\left(kr+\omega_{jl}-\frac{\pi l}{2}\right.\\&\left.+\eta_l-h\ln 2kr\right)\Phi_{jl}\left(\frac{\mathbf{r}}{r}\right).\end{aligned}$$

The solutions $|\beta jl\varepsilon\rangle$ in the systematic shell approach determine the projection operator

$$P=\sum_{\beta jl}\int d\varepsilon\,|\beta jl\varepsilon\rangle\langle\beta jl\varepsilon|.$$

However, the corresponding reaction channels are model channels since the solution for bound nucleons corresponds to a definite "pure" configuration of the shell model. Transition to the physical channels requires diagonalization of the Hamiltonian of the system of $A-1$ bound nucleons, this leading to real states of the target nucleus (the residual nucleus), i.e., to solutions in the continuum $|c\varepsilon_c\rangle$ (8), where c is the set of quantum numbers of the channel $\{\overline{\beta}(I)ljJM\}$, $\varepsilon_c=\varepsilon_{\overline{\beta}}+\varepsilon_c$. Thus, requiring correspondence between the solutions in the continuum and the physical reaction channels, we arrive at the Hamiltonian H_0 (5), in which the solution in the continuum $|c\varepsilon_c\rangle$ is constructed as a product of the wavefunction of a real state of the target nucleus and a single-particle solution in the potential v_0.

The transition to the model Hamiltonian H_0 extends the possibility of the microscopic description of single-nucleon reactions. Here, one can use the results of the collective nuclear models in which V corresponds to the interaction between the single-particle and collective degrees of freedom of the nucleus. This is a very complicated and laborious problem.[12–14] At the same time, detailed investigations of the structure of the wavefunctions of real states and numerous results obtained for reactions with nucleons of low and medium energies indicate the possibility of physical interpretation of the data on the basis of the shell model. Simple modifications of the shell approach, in which one or several nucleons of the real state of the target are in definite shells, and the remainder are not excited during the reaction process, enable one to estimate the matrix elements of transitions due to the pairing forces, and relate these estimates to the observed parameters of the doorway states in the cross sections and the following excitations in the hierarchy.[2–10,21–27,67] In shell calculations of the matrix elements, one usually ignores the effect of rescattering of nucleons in the continuum ($V^{\text{eff}}=V$). At the same time, in the matrix elements $\langle cE|V^{\text{eff}}|V\lambda\rangle$ (27) the final state differs from $|\lambda\rangle$ only by the transition of two nucleons to new shells (one nucleon to a bound state and the other into the continuum). In the same way one has transitions between the bound states, $\langle\lambda|V^{\text{eff}}|\mu\rangle$, leading in the extreme scheme of the shell approach to a hierarchy of successive excitations. It is interesting that for the description of the resonance structure observed in the cross sections of single-nucleon reactions there is no need to consider rather complicated particle–hole configurations. In some nuclei, the density of observed resonances corresponds qualitatively to the number of possible states with the excitation of not more than three or four nucleons.[70,71] Similar qualitative considerations follow from comparison of the resonance widths with those obtained in shell calculations.[21–23]

4. INTEGRAL EQUATIONS FOR THE TRANSITION AMPLITUDES

In the effective-interaction method, the construction of the reaction amplitudes reduces in practice to the solution of a corresponding system of linear algebraic equations with coefficients determined by the matrix elements of the operator V^{eff} on the eigenfunctions of the model Hamiltonian H_0. In reality, however, the expressions for these elements are integral equations for the matrix elements of the operator of the residual interaction (24). Therefore, in model approaches in which it is important to separate effects associated with a definite choice of the residual interaction it is more convenient to use a different method in which one considers explicitly processes of scattering of nucleons in the continuum. This method consists of formulating integral equations for the transition amplitudes obtained in the construction of the solution $P\Psi_c^+$ in the continuum.[6] The general form of this solution follows from the integral Lippmann–Schwinger equation (7):

$$P\Psi_c^+=\varphi_c^+\frac{1}{E-H_0+i\eta}PVP\Psi_c^+\frac{1}{E-H_0+i\eta}PVQ\Psi_c^+. \tag{56a}$$

Defining

$$Q\Psi_c^+ = \frac{1}{E-H_0} QVQ\Psi_c^+ + \frac{1}{E-H_0} QVP\Psi_c^+ = \frac{1}{E-H_0-QVQ} QVP\Psi_c^+$$

and substituting $Q\Psi_c^+$ into (56a), we arrive at the equation[6]

$$P\Psi_c^+ = \varphi_c^+ + \frac{1}{E-H_0+i\eta} P\bar{V}P\Psi_c^+, \tag{56b}$$

where

$$\bar{V} = V + VQ\frac{1}{E-H_0-QVQ} QV = V + V\frac{1}{E-H_0} Q\bar{V}. \tag{57}$$

Further transformation of Eq. (56b) by means of the operator identity (2) enables us to define the transition operator

$$P\Psi_c^+ = \varphi_c^+ + \frac{1}{E-H_0+i\eta} P\tau P\varphi_c^+,$$

where

$$\begin{aligned} P\tau P &= P\bar{V}P + P\bar{V}P\frac{1}{E-H_0-P\bar{V}P+i\eta} P\bar{V}P \\ &= P\bar{V}P + P\bar{V}P\frac{1}{E-H_0+i\eta} P\tau P. \end{aligned} \tag{58}$$

It is this operator which is needed to determine the amplitudes $T_{c'c}(E)$.

The relation (58) can be obtained directly from the equation (17) for τ by transferring the term containing Q in (17) to the left-hand side and defining $\bar{V}$ in accordance with Eq. (57).

The operator equation (58) corresponds to a system of integral equations for the transition amplitudes

$$\begin{aligned} &\langle c'E' | \tau | cE\rangle = \langle c'E' | \bar{V} | cE\rangle \\ &+ \sum_{c''} \int dE'' \frac{\langle c'E' | \bar{V} | c''E''\rangle \langle c''E'' | \tau | cE\rangle}{E-E''+i\eta}, \end{aligned} \tag{59a}$$

where the last term is associated with effects of rescattering on and off the energy shell. Using the definition (18), we can write down a similar equation for the matrix elements of the operator $\mathscr{K}$ (20):

$$\begin{aligned} &\langle c'E' | \mathscr{K} | cE\rangle = \langle c'E' | \bar{V} | cE\rangle \\ &+ \sum_{c''} \int dE'' \frac{\langle c'E' | \bar{V} | c''E''\rangle \langle c''E'' | \mathscr{K} | cE\rangle}{E-E''}, \end{aligned} \tag{59b}$$

where the integral is understood in the principal-value sense, i.e., off the energy shell. It is obvious that if we neglect rescattering for $E'' \neq E$ we arrive at the results of the effective-interaction method (see Secs. 2 and 3) but with $V^{\text{eff}} = V$ [this follows directly from the definition of $\bar{V}$ (57)].

To the integral equation (56) there corresponds a wavefunction of the form

$$(E - H_0 - P\bar{V}P)\, P\Psi(E) = 0,$$

which was used by Feshbach[6] to construct the formalism of a unified theory of reactions.[1)] Thus, the conclusions of this theory, like the schemes of the effective-interaction method, can be regarded as consequences of different representations of the equation for τ (17). This circumstance is convenient for systematizing the formalisms in a unified approach and establishing the connection between the parameters and the definitions in the different schemes for analyzing the cross sections. Considering different forms of representation of Eq. (17), one can also formulate new schemes useful in applications to definite reactions. For example, investigating the reaction associated with the definite transition $c \to c'$, one can separate these two channels in Eq. (17) and include the remainder in the effective-interaction operator. We then obtain a representation of Eq. (17) in the form

$$\tau = \tilde{\bar{V}} + \tilde{\bar{V}}\frac{1}{E-H_0+i\eta} P^0\tau, \tag{60}$$

where

$$\tilde{\bar{V}} = \left(1 - \bar{V}\frac{1}{E-H_0+i\eta} P''\right)^{-1} \bar{V} = \bar{V} + \bar{V}\frac{1}{E-H_0+i\eta} P''\tilde{\bar{V}} \tag{61}$$

($P^0 = P_c + P_{c'}$, $P'' = P - P^0$). The corresponding integral equations for the amplitudes $T_{c'c}$ contain here the matrix elements of the operator $\tilde{\bar{V}}$:

$$\begin{aligned} &\langle c'E' | \tau | cE\rangle = \langle c'E' | \tilde{\bar{V}} | cE\rangle \\ &+ \sum_{c''\neq c,c'} \int dE'' \frac{\langle c'E' | \tilde{\bar{V}} | c''E''\rangle \langle c''E'' | \tau | cE\rangle}{E-E''+i\eta}. \end{aligned} \tag{62}$$

The equations for the elements $\langle c'E' | \mathscr{K} | cE\rangle$ are determined similarly.

Ignoring rescattering processes in the channels c' and c, we arrive directly at the results of the effective-interaction method, in which V^{eff} must be replaced by the operator V''^{eff}:

$$V''^{\text{eff}} = \left(1 - V\frac{1}{E-H_0} P''\right)^{-1} V = V + V\frac{1}{E-H_0} P''V''^{\text{eff}}.$$

A different variant of the construction of the equation for the transition amplitude corresponds to elastic scattering in channel c. Distinguishing in Eq. (17) only this channel, we obtain the representation

$$\tau = \bar{V}_c + \bar{V}_c\frac{1}{E-H_0+i\eta} P_c\tau, \tag{63}$$

with

$$\bar{V}_c = \left(1 - \bar{V}\frac{1}{E-H_0+i\eta} P'\right)^{-1} \bar{V} = \bar{V} + \bar{V}\frac{1}{E-H_0+i\eta} P'\bar{V}_c \tag{64}$$

($P' = P - P_c$). The integral equation for the amplitude of elastic scattering in channel c can be written in the simple form

$$\begin{aligned} &\langle cE' | \tau | cE\rangle = \langle cE' | \bar{V}_c | cE\rangle \\ &+ \int dE'' \frac{\langle cE' | \bar{V}_c | cE''\rangle \langle cE'' | \tau | cE\rangle}{E-E''+i\eta}, \end{aligned} \tag{65}$$

although it is obvious that this is only a formal expression for the complicated system of integral equations that arise when the matrix elements of the operator $\bar{V}_c$ are determined through the elements of the residual interaction V (64).

The operator form of the representation of the equa-

[1)] The operators P and Q differ from those used in Ref. 6, in which P is constructed as the sum of the states of the target nucleus excited at the given energy E, and Q is the sum over the remainder. Although the physical interpretation of the results and the computational schemes are different in this case, the general structure of the expressions for the transition amplitudes remains the same.[6,7]

tion for τ (60) corresponds to a transformation of the wave equation for $P^0\Psi$ to the form

$$(E-H_0-P^0\widetilde{\overline{V}}P^0)\,P^0\Psi(E)=0, \tag{66a}$$

and the representation (63) follows from the formal solution of the single-channel equation

$$(E-\widetilde{H}_0^c)\,P_c\Psi(E)=(E-H_0-P_c\widetilde{\overline{V}}_cP_c)\,P_c\Psi(E)=0, \tag{66b}$$

which describes exactly the process of elastic scattering in the individual channel.

As we have already noted, neglect of rescattering effects in the continuum off the energy shell leads to the results of the effective-interaction method with a redefined potential V^{eff}. Therefore, the main advantage of the integral equations is the possibility of describing processes of transition from one channel to another under definite assumptions about the structure of the kernel of the equation (the matrix elements $\langle c'E'|\overline{V}|cE\rangle$). Assuming that the potential $\overline{V}$ is given, we can directly determine the amplitudes (59). In physical problems of reaction theory wide use is made of a representation of the kernel of the equation in separable form. In this case, a solution of the integral equation can be found analytically.[72,73,76] A different way of finding solutions of the integral equations (59) is the method of successive approximations leading to the Born series.[37-39] The choice for $\overline{V}$ of some nonresonance potential averaged over the bound states makes it possible to construct an integral scattering theory, which is usually used at relatively high energies.[16-18] In the region of low and medium energies, these results are in practice needed to estimate the contribution to the amplitudes of direct processes. This follows directly from our definition of the amplitude of the direct τ_0 (32):

$$P\tau_0P=PVP+PVP\frac{1}{E-H_0+i\eta}P\tau_0P,$$

which has the saume form as the definition of $P\tau P$ by Eq. (58). All the considered variants of the integral equations for $T_{c'c}$ can be formulated similarly for $T^0_{c'c}$ as well.

The different nature of the solutions for $T_{c'c}$ and $T^0_{c'c}$ is due solely to the difference between the interaction potentials V and $\overline{V}$ (57). The resonance part of the potential $\overline{V}$ leads to resonances in the amplitudes, but if one analyzes the amplitudes $\overline{T}_{cc}$ averaged over many levels, this part is usually transformed into a smooth complex addition to the potential U (Ref. 4). In this case, the form of the potential $\overline{V}$ averaged over the energy is quite similar to the potential V, which leads to a very similar nature of the solutions for the amplitudes T^0_{cc} and $\overline{T}_{cc}$ describing elastic scattering in this example. Therefore, using data on the total cross sections averaged over a wide range and corresponding angular distributions of elastically scattered nucleons, it is rather difficult, on the basis of the existing experimental information, to estimate the relative contribution of the direct process, which is determined here basically by the matrix elements of the potential U (53). It is more perspicuous to separate the direct process in the cross sections of inelastic reactions, in which the nondiagonal elements of U in the shell approach are zero. The spectra of nucleons formed in reactions are interpreted in this approach as the sum of individual intense lines corresponding to direct transitions for states with similar shell structure and a set of weak lines associated with transitions through the stage of excitation of complicated configurations of the compound nucleus. As a result, the reaction cross sections averaged over a large number of different transitions can be represented as a sum of cross sections of direct processes with characteristic angular distributions of the reaction products and resonance processes with symmetric angular distribution and energy dependence of the spectrum of the reaction nucleons typical of the evaporative model.[19,20] The relation between the symmetric and antisymmetric parts of the angular distribution can here serve as a qualitative estimate for the contribution of the direct process.

In investigations of nuclear reactions, induced by nucleons of low and medium energies, integral equations are widely used to estimate the influence of rescattering effects off the energy shell on the matrix elements $\langle c'E'|\mathscr{K}^0|cE\rangle$ of direct transitions:

$$\langle c'E'|\mathscr{K}^0|cE\rangle=\langle c'E'|V|cE\rangle+\sum_{c''}\int dE''\,\frac{\langle c'E'|V|c''E''\rangle\langle c''E''|\mathscr{K}^0|cE\rangle}{E-E''}.$$

As direct calculations show, rescattering effects are important only in the cases when the matrix elements $\langle c'E'|V|cE\rangle$ are small.[72-74,76] In the shell approach, this obviously follows from the selection rules for transitions between definite particle—hole configurations for a pairing residual interaction.[4,11]

5. AVERAGING WITH RESPECT TO THE ENERGY

The results we have considered have a rigorous formal nature. Their application in practical problems entails certain approximations that permit one to simplify the analysis of given experimental data. In other words, one must pass from the general conclusions of the formal theory to practical schemes and methods of analysis adequate for the experimental information on the structure of interaction cross sections in different energy regions. In their most general form, the results of the effective-interaction method are used at the present time for many-level parametrization of the energy dependence of cross sections in the region of resolved resonances. However, here too in the analysis one takes into account a small number of bound states corresonding to the number of resonances observed in the considered interval, and the influence of the remaining states and the contribution of the direct processes are usually taken into account approximately.[51-59]

In the region of unresolved and overlapping resonances, the fine structure of the cross sections is smoothed by the energy averaging. If the averaging interval I is large compared with the mean distance between the resonances of the compound nucleus but at the same time small compared with the widths of the doorway states, the averaged cross sections will reveal only the corresponding intermediate structures and very broad optical resonances associated with the distortion of the channel wavefunctions $|cE\rangle$ by the model potential U. In this case, the procedure for averaging the cross sections entails rather cumbersome and at the same

time approximate transformations. The main difficulty resides in the need to average quantities proportional to $|S_{c'c}(E)|^2$. The only experimental characteristic of the interaction determined by the amplitude itself (linear in S) is the total cross section of all processes:

$$\sigma \sim \sum_c (1 - \mathrm{Re}\, S_{cc}).$$

Therefore, the averaging considered below of the amplitude t_{cc} corresponds in practice only to the average total cross section.

The use of averaged amplitudes in the calculation of other cross sections always entails errors of the order of the difference $\overline{|t_{c'c}|^2}$. For theoretical constructions, it is convenient to choose an averaging function of the Lorentz type:

$$\bar{t}_{c'c}(E) = (I/\pi) \int_{-\infty}^{\infty} dE' t_{c'c}(E')/[(E-E')^2 + I^2]. \qquad (67a)$$

Going over to a complex variable and closing the contour of integration above, one can calculate this integral for $E' = E + iI$:

$$\bar{t}_{c'c}(E) = t_{c'c}(E + \mathrm{i}I). \qquad (67b)$$

Here we use the fact that the poles of the amplitudes $t_{c'c}(E)$ lie in the lower half-plane [for $E = E_k - i\Gamma_k/2$ (39)]. A similar averaging for $|t_{c'c}(E)|^2$ is impossible since in this case the poles of the amplitude are within the contour of integration and it is necessary to take the sum of all the residues at these poles. The averaging in the form (67) does not take into account the energy dependence of the amplitudes of the direct processes and the resonance parameters in the interval I, which may lead to errors in the region of the reaction threshold.[4-7]

The averaging of the general results for the transition amplitudes with respect to the resonances of the fine structure considerably simplifies the scheme for parametrizing the corresponding cross sections. As an example we may take the procedure for averaging over the interval I ($\overline{D} \ll I \ll \Gamma_{\tilde{\lambda}_1}$, where $\Gamma_{\tilde{\lambda}_1}$ is the width against decay of the doorway state and $\overline{D}$ is the mean distance between the levels of the fine structure) of the expression for $T_{c'c}(E)$ (45) corresponding to the extreme concept of doorway states when none of the remaining states have a direct coupling to the continuum. Writing the expression (45) in the approximate form

$$\overline{T}_{c'c} \approx T^0_{c'c} + \frac{1}{2\pi} \sum_{\tilde{\lambda}_1} \Gamma^{1/2}_{c'\tilde{\lambda}_1} \Gamma^{1/2}_{\tilde{\lambda}_1 c} \Big\{ E - E_{\tilde{\lambda}_1} + \Delta_{\tilde{\lambda}_1} + \mathrm{i}\Gamma_{\tilde{\lambda}_1}/2 + \sum_{\tilde{\lambda}_0} \langle \tilde{\lambda}_1 | V^{\mathrm{eff}} | \tilde{\lambda}_0 \rangle^2 / (E_{\tilde{\lambda}_0} - E - \mathrm{i}I) \Big\}^{-1}, \qquad (68a)$$

where the approximation refers to the diagonalization with respect to the doorway states $\tilde{\lambda}_1$, we represent the sum over the resonances of the fine structure in the interval I as a smooth function of the energy[4]:

$$\sum_{\tilde{\lambda}_0} \langle \tilde{\lambda}_1 | V^{\mathrm{eff}} | \tilde{\lambda}_0 \rangle^2 / (E_{\tilde{\lambda}_0} - E - \mathrm{i}I) \approx \mathrm{i}\Gamma^{\downarrow}_{\tilde{\lambda}_1}/2. \qquad (68b)$$

Such a representation is introduced in R-matrix theory for resonances of the compound nucleus and is only a qualitative estimate of the sum in the limiting case of a large number of terms of a sum with statistically distributed parameters $\langle \tilde{\lambda}_1 | V^{\mathrm{eff}} | \tilde{\lambda}_0 \rangle$ and $E_{\tilde{\lambda}_0}$ (Refs. 4 and 31). With allowance for the representation (68b), the expression (68a) for $\overline{T}_{c'c}$ can be written in the form widely used in the analysis of doorway analog resonances[4,7,34]:

$$\overline{T}_{c'c}(E) = T^0_{c'c}(E) + \frac{1}{2\pi} \sum_{\tilde{\lambda}_1} \Gamma^{1/2}_{c'\tilde{\lambda}_1} \Gamma^{1/2}_{\tilde{\lambda}_1 c} / [E - E_{\tilde{\lambda}_1} + \Delta_{\tilde{\lambda}_1} + \mathrm{i}(\Gamma^{\uparrow}_{\tilde{\lambda}_1} + \Gamma^{\downarrow}_{\tilde{\lambda}_1})/2], \qquad (68c)$$

where the width of the decay of the doorway state to the continuum is denoted by $\Gamma^{\downarrow}_{\tilde{\lambda}_1} \equiv \Gamma_{\tilde{\lambda}_1}$.[3] The width $\Gamma^{\uparrow}_{\tilde{\lambda}_1}$ determines the "spreading" of the doorway state over the resonances of the fine structure that do not have a direct coupling to the continuum. The expression (68c) has a form similar to the element $T_{c'c}$ (33) in the case of a certain reaction channel corresponding to $\Gamma^{\downarrow}_{\tilde{\lambda}_1}$ in addition to one considered. This is merely a formal similarity reflecting the violation of the unitarity property in processes described by $\overline{T}_{c'c}(E)$. In the extreme concept of doorway states, Eq. (68b) establishes a sum rule for the resonances of the fine structure. In addition, using the representation of the amplitude (41), we obtain a resonance dependence of the average observed widths of the corresponding levels at energies near the energy of the doorway state (43).[3,4,7,33-35] The procedure for averaging the amplitude $T_{c'c}$ (41) in the case when all resonances have a direct exit to the continuum is more cumbersome but the scheme remains the same as in the derivation of the expression (68b).[3,4,7,34]

Averaging of the transition amplitude over a wide energy interval appreciably exceeding the distance between the doorway states is usually associated with the results of the complex-potential model. To determine this potential in the framework of formal reaction theory one uses a number of approximations and assumptions relating to the distribution of the signs and magnitudes of the matrix elements $\langle cE | V^{\mathrm{eff}} | \lambda \rangle$ in the averaging interval. In the K-matrix scheme, which is analogous to the R-matrix scheme, the matrix elements of the operator $\mathscr{K}$ (25) averaged with respect to the energy are determined in the form[31]

$$\overline{K}_{c'c}(E) \approx K^0_{c'c}(E) - \frac{\mathrm{i}}{2} \frac{\overline{\gamma}^2_c}{\overline{D}} \delta_{c'c} = \overline{K}'_{c'c} + \overline{K}''_{c'c}, \qquad (69)$$

where $\overline{K}'_{c'c} = [K^0_{c'c} - (i/2)(\overline{\gamma}^2_c/\overline{D})]\,\delta_{c'c}$ is the diagonal part of $\overline{K}_{c'c}$; $\overline{K}''_{c'c} = (1 - \delta_{c'c}) K^0_{c'c}$ is the nondiagonal part. The transition amplitude $\overline{T}_{c'c}(E)$ averaged over the resonances can be expressed in terms of $\overline{K}(E)$ in accordance with the definition (22):

$$\overline{T}_{c'c}(E) = [(1 + \mathrm{i}\pi\overline{K}(E))^{-1} \overline{K}(E)]_{c'c}, \qquad (70)$$

the different approximate forms of which have been considered[31,4-7,33] in detail in R-matrix theory.[2)]

[2)] The representation of $\overline{T}_{c'c}$ in the form (70) requires a certain care near a reaction threshold energy. If the elements of the $T(E)$ matrix always correspond to complex energies, the elements of $K(E)$ are real and their analytic continuation into the region of the complex variable $E + iI$ is not defined in the general case. If the averaging in $T_{c'c}(E + iI)$ is done correctly there may appear additional poles associated with singularities of the inverse matrix $(1 + i\pi K)^{-1}$, but these however correspond to the unphysical region.[75]

We now define a certain potential on which elastic scattering leads to the amplitudes $\overline{T}_{cc}(E)$ (70). For this, we consider the exact equation (66b) for elastic scattering in channel c, writing down its integral expression and averaging this over a wide interval I including resonances of different type. We obtain

$$P_c\overline{\Psi}_c^+(E)=\varphi_c^+(E)+\frac{1}{E-H_0+i\eta}P_c\overline{\tilde{V}}_cP_c\overline{\Psi}_c^+(E). \tag{71}$$

It is obvious that the potential $U+P_c\overline{\tilde{V}}_cP_c$ gives the scattering amplitude $\overline{T}_{cc}(E)$, which is a matrix element of the operator $P_c\overline{\tau}P_c$ on the energy shell:

$$P_c\overline{\tau}P_c=P_c\overline{\tilde{V}}P_c+P_c\overline{\tilde{V}}_cP_c\frac{1}{E-H_0+i\eta}P_c\overline{\tau}P_c.$$

Using Heitler's equation, one can introduce a corresponding operator $P_c\tilde{\mathcal{K}}P_C$ satisfying the equation

$$P_c\tilde{\mathcal{K}}P_c=P_c\overline{\tilde{V}}_cP_c+P_c\overline{\tilde{V}}_cP_c\frac{1}{E-H_0}P_c\tilde{\mathcal{K}}P_c \tag{72}$$

with matrix elements $\tilde{K}_{cc}$ on the energy shell defined by

$$(1+i\pi\tilde{K}_{cc})^{-1}\tilde{K}_{cc}=\overline{T}_{cc}\quad\text{or}\quad\tilde{K}_{cc}=(1-i\pi\overline{T}_{cc})^{-1}\overline{T}_{cc}. \tag{73}$$

An approximate expression for $P_c\overline{\tilde{V}}_cP_c$ can be obtained in the form

$$P_c\overline{\tilde{V}}_cP_c\approx P_c\tilde{\mathcal{K}}P_c$$

$$\approx\int\int d\varepsilon_1\,d\varepsilon_2\,|c\varepsilon_1\rangle\left[\langle c\varepsilon_1|\,\tilde{\mathcal{K}}^0\,|c\varepsilon_2\rangle-\frac{i}{2}\frac{\overline{\tilde{v}}_c^2}{\overline{D}}\right]\langle c\varepsilon_2|, \tag{74}$$

where the nondiagonal elements $\overline{K}''_{c'c}$ are assumed to be small compared with the diagonal elements $\overline{K}'_{cc}$ and rescattering effects off the energy shell [the integral term in (72)] are ignored.

In the general case, the potential $U+P_c\overline{\tilde{V}}_cP_c$ is nonlocal. However, in the complex-potential model it is usually taken to be local with parameters that depend slightly on the energy.[77,78] The form of the potential is close to the density distribution of the nucleons in the target nucleus, and the imaginary part is concentrated near the surface of the nucleus.[13] The effect of nonlocality is here partly compensated by the energy dependence of the parameters of the equivalent local potential and by the spin–orbit interaction.[77–80]

Choosing the potential of the model in such a way that the solutions in the individual channels $P_c\Psi_c^{\pm}(E)$ averaged over the resonances [we denote them by $\tilde{\varphi}_c^{\pm}(E)$] coincide with the model solutions, we can use these solutions to construct average transition amplitudes $\bar{t}_{c'c}(E)$. To this end, we consider the more general problem of constructing the amplitude $t_{c'c}(E)$ as the matrix element of some new transition operator τ' on the wave functions:

$$P_c\Psi_c^+(E)=\varphi_c^+(E)+\frac{1}{E-H_0+i\eta}P_c\tilde{V}_cP_c\Psi_c^+(E); \tag{75a}$$

$$P_c\Psi_c^-(E)=\varphi_c^-(E)+\frac{1}{E-H_0-i\eta}P_c\tilde{V}_c^*P_c\Psi_c^-(E). \tag{75b}$$

We determine the operator $P_{c'}\tau'P_c$ for transitions between channels c and c' by the condition

$$t_{c'c}=\langle\varphi_{c'}^-|\,\tau\,|\varphi_c^+\rangle=\langle\Psi_{c'}^-|\,P_{c'}\tau'P_c\,|\Psi_c^+\rangle.$$

Expressing φ_c^+ and $\varphi_{c'}^-$ in terms of $P_c\Psi_c^+$ and $P_{c'}\Psi_{c'}^-$ and using for τ the representation (63), we find

$$P_{c'}\tau'P_c=P_{c'}\tilde{V}_{c'}P_c-P_{c'}\tilde{V}_{c'}P_c\frac{1}{E-H_0+i\eta}P_c\tilde{V}_cP_c. \tag{76a}$$

For a concrete pair of channels c and c' it is convenient to go over from $\tilde{V}_c$ and $\tilde{V}_{c'}$ to the operator $\tilde{\tilde{V}}$ (61). Using Eqs. (64) and (76a), and also operator identities of the type (2), we can write the equation for $P_{c'}\tau'P_c$ in the form[69]

$$P_{c'}\tau'P_c=P_{c'}\tilde{\tilde{V}}P_c-P_{c'}\tau'P_c\frac{1}{E-\tilde{\tilde{H}}_0^c+i\eta}P_c\tilde{\tilde{V}}P_{c'}\frac{1}{E-\tilde{\tilde{H}}_0^{c'}+i\eta}P_{c'}\tau'P_c. \tag{76b}$$

A formal solution of this equation can be found by means of the relation

$$P_{c'}\tau'P_c\frac{1}{E-\tilde{\tilde{H}}_0^c+i\eta}=P_{c'}\tilde{\tilde{V}}P_c\frac{1}{E-\tilde{\tilde{H}}_0^c+i\eta},$$

where $\tilde{\tilde{H}}_0^c=H_0+P_c\tilde{\tilde{V}}P_c$; then

$$P_{c'}\tau'P_c=P_{c'}\tilde{\tilde{V}}P_c-P_{c'}\tilde{\tilde{V}}P_c\frac{1}{E-\tilde{\tilde{H}}_0^c+i\eta}P_c\tilde{\tilde{V}}P_{c'}\frac{1}{E-\tilde{\tilde{H}}_0^{c'}+i\eta}P_{c'}\tilde{\tilde{V}}P_c. \tag{76c}$$

The set of solutions $P_c\Psi_c^{\pm}$ in all the reaction channels can be regarded as a new basis in the continuum. Defining the projection operator

$$\tilde{P}_c=\int|P_c\Psi_c^+(\varepsilon)\rangle\langle\Psi_c^+(\varepsilon)\,P_c|\,d\varepsilon=\int|P_c\Psi_c^-(\varepsilon)\rangle\langle\Psi_c^-(\varepsilon)\,P_c|\,d\varepsilon,$$

which commutes with the Hamiltonian $\tilde{\tilde{H}}_0^c=H_0+P_c\tilde{V}_cP_c$, and normalizing the functions $P_c\Psi_c^{\pm}$ in such a way that $\tilde{P}_c=P_c$, we can write down as integral equation for the amplitude $t_{c'c}$ corresponding to the operator form (76b):

$$t_{c'c}(E',\,E)=\langle\Psi_{c'}^-(E')\,\tilde{P}_{c'}\tilde{\tilde{V}}\tilde{P}_c\Psi_c^+(E)\rangle$$
$$-\int\int d\varepsilon\,d\varepsilon' t_{c'c}(E',\,\varepsilon')\frac{1}{E'-\varepsilon'+i\eta}\langle\Psi_c^+(\varepsilon')\,\tilde{P}_c\tilde{\tilde{V}}\tilde{P}_{c'}\Psi_{c'}^-(\varepsilon)\rangle$$
$$\times\frac{1}{E-\varepsilon+i\eta}t_{c'c}(\varepsilon,\,E). \tag{76d}$$

A somewhat simpler equation is obtained when one uses as a basis for the continuum the solutions

$$P_c\tilde{\tilde{\Psi}}_c^{\pm}(E)=\varphi_c^{\pm}(E)+\frac{1}{E-H_0+i\eta}P_c\tilde{\tilde{V}}P_c\tilde{\tilde{\Psi}}_c^+(E),$$

which describe elastic scattering in channel c approximately.[69]

Obviously, the scheme for constructing the amplitude averaged with respect to the resonances, $\bar{t}_{c'c}(E)$, as a matrix element of the operator $\overline{\tau}'$ on the functions $\tilde{\varphi}_c^{\pm}(E)\equiv P_c\overline{\tilde{\Psi}}_c^{\pm}(E)$ is completely analogous to the scheme considered above. We represent the potential $\tilde{\tilde{V}}$ of the residual interaction by the equation [see Eqs. (57) and (61)]

$$\tilde{\tilde{V}}=V+VP''\frac{1}{E-H_0+i\eta}\tilde{\tilde{V}}+VQ\frac{1}{E-H_0}\tilde{\tilde{V}}=\tau_0''+\tau_0''Q\frac{1}{E-H_0}\tilde{\tilde{V}}, \tag{77}$$

where

$$\tau_0''=V+V\frac{1}{E-H_0+i\eta}P''\tau_0''=V''^{\text{eff}}-i\pi V''^{\text{eff}}P''\delta(E-H_0)\,P''\tau_0''$$

differs from τ_0 (32) only by the fact that we here ignore

the distinguished channels c' and c. In the case of two reaction channels, $\tau_0'' = V$.

For the averaging with respect to the energy of the matrix elements of the operator $\tilde{\tilde{V}}$ for different channels ($c' \neq c$) the use of statistical assumptions for the signs and amplitudes of the elements $\langle c | V''^{\text{eff}} | \lambda \rangle$ makes the resonance part vanish, so that the elements averaged with respect to the energy satisfy $\langle c' | \tilde{\tilde{V}} | c \rangle \approx \langle c' | \tau_0'' | c \rangle$. Writing down for $\bar{t}_{c'c}(E)$ an equation of the type (76d) and ignoring the integral term, we obtain the approximate expression

$$\bar{t}_{c'c}(E) \approx \langle \tilde{\varphi}_{c'}^{-}(E) | \tau_0'' | \tilde{\varphi}_c^{+}(E) \rangle. \tag{78a}$$

This expression in the case $\tau_0'' \approx V$, i.e., when the integral term (77) makes a small contribution, corresponds to the result of the distorted-wave Born approximation[16–18]:

$$\bar{t}_{c'c}(E) \approx \langle \tilde{\varphi}_{c'}^{-}(E) | V | \tilde{\varphi}_c^{+}(E) \rangle. \tag{78b}$$

Another result, determined by the system of equations

$$\bar{t}_{c'c}(E) \approx \langle \tilde{\varphi}_{c'}^{-}(E) | V''^{\text{eff}} | \tilde{\varphi}_c^{+}(E) \rangle - i\pi \sum_{c'' \neq c,\, c'} \langle \tilde{\varphi}_{c'}^{-}(E) | V''^{\text{eff}} | \tilde{\varphi}_{c''}^{-}(E) \rangle \bar{t}_{c''c}(E), \tag{78c}$$

corresponds to the simplest scheme of the coupled-channel approximation.[18,81] Both results (78b) and (78c) are widely used to analyze cross sections and angular distributions of inelastic processes in the region of overlapping resonances in the case of poor experimental resolution.[18] As a rule, the results of calculations in these two approximations are similar.[82–84] Only in cases when the Born matrix element of a transition is small for certain reasons (because of a selection rule in the case of a pairing residual interaction, for example) does the coupled-channel approximation lead to qualitatively new dependences.[84]

The results we have given determine the amplitudes $\bar{t}_{cc}(E)$ and $\bar{t}_{c'c}(E)$ averaged over the resonances and the related reaction cross sections. The real average cross sections of inelastic processes are determined, however, by the average value of the square of the amplitude: $\overline{|t_{c'c}(E)|^2}$, which in principle is different from $|\bar{t}_{c'c}(E)|^2$. In the effective-interaction method, the scheme for averaging the reaction cross sections has much in common with the one usually employed in R-matrix theory.[31] In the limit of a large number of resonances with elements $\langle cE | V^{\text{eff}} | \lambda \rangle$ distributed statistically in the averaging interval, an approximate expression for $\overline{|t_{c'c}(E)|^2}$ can be obtained on the basis of the general definition of $T_{c'c}(E)$ (33). We write the corresponding result in the form[31,33,64,20]

$$\overline{|t_{c'c}(E)|^2} \approx |t_{c'c}^{0}(E)|^2 + \frac{1}{2\pi} \frac{\bar{\Gamma}_{c'}(E)\bar{\Gamma}_c(E)}{\bar{D}(E)\bar{\Gamma}(E)} F, \tag{79}$$

where $\bar{\Gamma}_{c'}(E)$ and $\bar{\Gamma} = \sum_c \bar{\Gamma}_c$ are the mean resonance widths;

$$\bar{\Gamma}_{c'}(E) \approx |\langle c'E | V^{\text{eff}} | \lambda \rangle|^2; \tag{80}$$

F is a function of the statistical averaging that takes into account the difference between the mean value of the ratio $\overline{\Gamma_c \Gamma_{c'}/\Gamma}$ and the ratio of the mean values.[85] Here, we assume that the matrix elements of direct transitions in the average widths (36) are small ($\Gamma_{c'\lambda}^{1/2} \approx \langle c'E | V^{\text{eff}} | \lambda \rangle$) and, using the statistical hypothesis for the distributions of the resonance widths, we ignore the interference term between the direct and the resonance part of the transition amplitude. The average resonance widths $\bar{\Gamma}_c(E)$ (strength functions) depend implicitly on the energy in accordance with the resonance nature of the functions $|cE\rangle$ near the single-particle resonances of the continuum.[9]

A similar expression for the average reaction cross section can be constructed if one uses as basis functions in the continuum the solutions for a complex potential.[33–35] However, the scheme of the transition from the definition of $t_{c'c}$ in the form (76) to $\overline{|t_{c'c}|^2}$ is more complicated because of the need to estimate the average value of the integral term of (76d). A detailed treatment of the problem of determining the formalism of the theory of resonance reactions and different cross sections using basis functions of the complex-potential model can be found in Ref. 35. Because the model Hamiltonian is non-Hermitian there are certain complications in the use of the Lippmann–Schwinger method for constructing the transition amplitudes (for example, for the physical interpretation of the solutions $\tilde{\varphi}_{c'}^{-}$). In addition, in the complex-potential model all states decay. The main idea of the shell approach—a common basis for the bound states and reaction channels—is here violated.

From the point of view of the systematic approach considered in the present paper, the transition to a complex potential is only a convenient approximation of the data on the average cross sections and it enters the formalism phenomenologically. In this sense, for the analysis of average cross sections of inelastic processes it would be more consistent to use schemes of the shell approach that employ a real model potential.[7]

6. AMPLITUDES OF PHOTONUCLEAR REACTIONS

General schemes for parametrizing the energy dependence of the cross sections of photonuclear reactions can be formulated in much the same way as the results considered above. Although the inclusion of a new photon channel requires a redefinition of the total Hamiltonian and model Hamiltonian of the problem, and also of the projection operator P, one can formulate in the first perturbation order in the electromagnetic interaction general schemes for defining the amplitude of the transition from the photon channel (γ) to the single-nucleon channel c using expansions with respect to the same basis functions as in the case of nucleon reactions.[10,86]

Let us consider the structure of the general expression for the amplitude of photonuclear reactions in formal collision theory[38,39]:

$$t_{c\gamma}(E) = \langle \Psi_c^{-}(E) | V_\gamma | \chi_\gamma(E) \rangle, \tag{81}$$

where $\chi_\gamma(E)$ is the wavefunction of the initial state, which here corresponds to the target nucleus in the ground state and a photon in the absence of electromagnetic interaction between them; $\Psi_c^{-}(E)$ is the solution of

the exact wave equation of the problem $H\Psi = E\Psi$ with the asymptotic form of a plane wave in channel c and ingoing waves in all reaction channels; V_γ is the operator of the electromagnetic interaction.[3)] Restricting ourselves in the construction of the amplitude $t_{c\gamma}$ to the first order of perturbation theory in V_γ, we can introduce in a unified manner both nucleon and photon reaction channels.[31] In this approximation, assuming that the channels c do not contain photons, we can choose $\Psi_c^-(E)$ in the form

$$\langle \Psi_c^-(E)| = \langle \varphi_c^-(E)| \left(1 + V_n \frac{1}{E - H + i\eta}\right),$$

where $H = H_0 + V_n$ (V_n is the potential of the residual interaction between the distinguished nucleon and the target nucleus for channels c) does not contain the electromagnetic field. As a result, for the amplitude $t_{c\gamma}(E)$ (81) we obtain a relation of the type

$$t_{c\gamma}(E) \approx \langle \varphi_c^-(E)|\tau_\gamma|\chi_\gamma(E)\rangle, \tag{82}$$

where the transition operator τ_γ is determined by the equation

$$\tau_\gamma = V_\gamma + V_n \frac{1}{E - H + i\eta} V_\gamma = V_\gamma + V_n \frac{1}{E - H_0 + i\eta} \tau_\gamma. \tag{83}$$

Introducing the projection operators P and Q (16), we must also include in P the photon channel. However, the corresponding matrix elements in $t_{c\gamma}$ when the operator τ_γ (83) is represented by the equation

$$\tau_\gamma = V_\gamma + V_n \frac{1}{E - H_0 + i\eta} P\tau_\gamma + V_n \frac{1}{E - H_0} Q\tau_\gamma \tag{84}$$

vanish. This is because the operator V_n in the matrix element $\langle \varphi_c^-|V_n|\chi_\gamma\rangle$ does not contain the photon coordinates, and the parts of the solutions φ_c^- and χ_γ associated with the photon parts correspond to different physical situations and are therefore orthogonal. Thus, the projection operator P in the definition τ_γ (84) is the same as for the nucleon reactions (16). We write Eq. (84) in the form [see Eqs. (17) and (19)]

$$\tau_\gamma = V_\gamma + V_n \frac{1}{E - H_0} P\tau_\gamma + V_n \frac{1}{E - H_0} Q\tau_\gamma - i\pi V_n P\delta(E - H_0) P\tau_\gamma$$

and define the effective-interaction operator V_γ^{eff} like V^{eff} (24):

$$V_\gamma^{\text{eff}} = \left(1 - V_n P \frac{1}{E - H_0}\right)^{-1} V_\gamma = V_\gamma + V_n \frac{1}{E - H_0} PV_\gamma^{\text{eff}} \tag{85a}$$

and the operator

$$\tau_\gamma^0 = V_\gamma^{\text{eff}} - i\pi V_n^{\text{eff}} P\delta(E - H_0) P\tau_\gamma^0, \tag{85b}$$

where $V_n^{\text{eff}} \equiv V^{\text{eff}}$ (24). Then for the operator τ_γ we obtain

$$\tau_\gamma = \tau_\gamma^0 + \tau_n^0 \frac{1}{E - H_0} Q\tau_\gamma = \tau_\gamma^0 + \tau_n^0 Q \frac{1}{E - H_0 - Q\tau_n^0 Q} Q\tau_n^0 \tag{86}$$

[$\tau_n^0 \equiv \tau^0$ (32)], which determines the general form of the amplitude of the photonuclear reaction in the effective-interaction method:

$$t_{c\gamma}(E) = \langle \varphi_c^-(E)|\tau_\gamma^0|\chi_\gamma(E)\rangle + \sum_{\lambda\lambda'} \langle \varphi_c^-(E)|\tau_n^0|\lambda\rangle (B^{-1})_{\lambda\lambda'} \langle \lambda'|\tau_\gamma^0|\chi_\gamma(E)\rangle, \tag{87}$$

where $(B^{-1})_{\lambda\lambda'}$ are the elements of the matrix that is the inverse of B (33). The first term corresponds to the amplitude of the direct transition resulting from the photon—nucleus interaction and the second to the excitation of quasibound states $|\lambda\rangle$ with their subsequent decay to the single-nucleon channels c. The matrix elements of the operator τ_γ^0 are defined in the same way as the elements of τ^0 [(34), (36)]:

$$\langle \varphi_c^-(E)|\tau_\gamma^0|\chi_\gamma(E)\rangle = \exp(i\omega_c) \sum_{c'} (1 + i\pi K^0)_{cc'}^{-1} \langle c'E|V_\gamma^{\text{eff}}|\gamma(E)\rangle \exp(i\omega_\gamma). \tag{88}$$

Here we have separated the phase $\chi_\gamma = \gamma \exp(i\omega_\gamma)$ in such a way that the matrix elements $\langle c'E|V_\gamma^{\text{eff}}|\gamma\rangle$ are real. Besides the amplitude of the direct transition, we write down an expression for the amplitudes

$$\langle \lambda|\tau_\gamma^0|\gamma\rangle = \langle \lambda|V_\gamma^{\text{eff}}|\gamma\rangle - i\pi \sum_{c'} \langle \lambda|V_n^{\text{eff}}|c'\rangle \langle c'|\tau_\gamma^0|\lambda\rangle, \tag{89}$$

which follows directly from Eq. (85b). These elements, which determine the probability of excitation of a quasibound state $|\lambda\rangle$ resulting from the electromagnetic interaction, reflect the possible realization of this excitation directly as a result of the nonzero matrix elements $\langle \lambda|V_\gamma^{\text{eff}}|\gamma\rangle$ and with a preliminary rescattering in the continuum. If direct excitation of the state $|\lambda\rangle$ is forbidden for any reasons, the second term in (89) may be important in the determination of the radiative widths of the resonances.[10]

The expression for the amplitude of a photonuclear reaction in the method of the effective interaction (87) may serve as a basis for the formulation of practical schemes of analysis of the fine structure of a dipole resonance and of reaction cross sections near the nucleon-emission threshold.[10,25–27,57,87] Usually, one uses the results for the diagonalized level matrix B corresponding to the formalism of the S matrix (39) (Ref. 10). Although comparatively simple expressions are obtained for the energy dependence of the resonance cross sections, the S-matrix approach leads to parameters that are related in a very complicated manner to the calculated matrix elements in the shell model; for example, in such an approach one can in practice estimate only the widths of individual resonances. The experimental information relating to interference effects in cross sections and angular distributions is here not very amenable to model interpretation. Therefore, to analyze detailed data on the fine structure of giant resonances in photon-in, nucleon-out reactions the K-matrix formalism has found ever wider use in recent years; it is more convenient for physical interpretation of interference effects in the cross sections.[57,86]

We define the operator $\mathscr{K}_\gamma$ by the Heitler equation (20):

$$\mathscr{K}_\gamma = \tau_\gamma + i\pi\tau_n P\delta(E - H_0) P\mathscr{K}_\gamma. \tag{90a}$$

Then for $\mathscr{K}_\gamma$ we can write down an operator equation similar to (25):

[3)] Direct application of the results of particle collision theory to the problem of the interaction of the electromagnetic field and the nucleon field requires a certain care. The definition of $t_{c\gamma}(E)$ (81) holds only in the first order of perturbation theory.[38]

$$\mathscr{K}_\gamma = V_\gamma^{\text{eff}} + V_n^{\text{eff}} \frac{1}{E - H_0} Q\mathscr{K}_\gamma = V_\gamma^{\text{eff}} + V_n^{\text{eff}} Q \frac{1}{E - H_0 - QV_n^{\text{eff}} Q} QV_\gamma^{\text{eff}}, \tag{90b}$$

and determine the structure of the matrix elements of this operator on the energy shell[86]:

$$\langle cE | \mathscr{K}_\gamma | \gamma \rangle = \langle cE | V_\gamma^{\text{eff}} | \gamma \rangle + \sum_{\lambda\lambda'} \langle cE | V_n^{\text{eff}} | \lambda \rangle (A^{-1})_{\lambda\lambda'} \langle \lambda' | V_\gamma^{\text{eff}} | \gamma \rangle, \tag{90c}$$

where $(A^{-1})_{\lambda\lambda'}$ are the elements of the matrix reciprocal to A (28). Obviously, the parameters in this case are directly the matrix elements of the operators V_γ^{eff} and V_n^{eff} on the model wave functions. The transition amplitudes (to within phase factors) are determined by Eq. (90a) in the form

$$\langle cE | \tau_\gamma | \gamma \rangle = \sum_{c'} (1 - i\pi T_n)_{cc'} \langle c'E | \mathscr{K}_\gamma | \gamma \rangle = \sum_{c'} [(1 + i\pi K_n)^{-1}]_{cc'} \langle c'E | \mathscr{K}_\gamma | \gamma \rangle, \tag{91}$$

where T_n and K_n are matrices that coincide with the matrices T and K (22) defined above. Analyzing the fine structure of the cross sections of photonucleon by means of the well known computational parametrization schemes formulated in the R-matrix theory,[51,57] we can directly determine the matrix elements occurring in $\langle cE | \mathscr{K}_\gamma | \gamma \rangle$ (90c).

The obvious possibilities of the shell approach to the interpretation of data on photonuclear reactions in the region of a giant dipole resonance make possible, in the analysis of resonance structures, wide use of the expressions for the transition amplitudes obtained in the doorway-state model and in the hierarchy scheme (see Sec. 3). They are determined for photonuclear reactions in the same way as for reactions induced by nucleons.[86] In practical applications, it is convenient to use simplified formulas in which the number of levels taken into account in the analysis and the number of possible open reaction channels are restricted and the level matrix is also diagonalized. As a rule, in the interpretation of resonance structures in photonuclear reactions one ignores direct transitions and so-called external mixing because of redistributions in the continuum.[10,25–27,57] A detailed analysis of data which are obtained with good experimental resolution, and which include the energy dependence of the cross sections in a wide range of variation of the γ-ray energies and the angular distributions of the nucleons of the reaction and their polarization on the basis of the K-matrix formalism, gives the set of parameters calculated directly in concrete model calculations.[25,26]

7. THE PRACTICAL USE OF THE RESULTS

The very general and compact method of construction of different formalisms of the microscopic theory of reactions presented in this review employs a splitting of the total Hamiltonian H of the problem into the sum of a model Hamiltonian H_0 and a residual interaction V. It is obvious that the main fundamental question in the systematic analysis of a given nuclear reaction is the choice of the model in which the solutions in the continuum contain the complete set of observable physical reaction channels and the density of bound model states is of the same order as the density of the observed resonances of the fine structure.[2–7,33,66] At the present time, this choice is restricted to the modified shell approach considered here and the case of only single-nucleon reactions. However, one can construct a general formalism of the microscopic theory using different divisions of the total Hamiltonian $H = H_0^c + V^c$ in individual channels c, where H_0^c may correspond to different nuclear models: single-particle and collective. The practical implementation of this formalism encounters the difficulties common in nuclear theory relating to the expansion of one basis set with respect to another, subject to the need to take into account in each of them the continuum states.[13,15,88,89] Applied to the analysis of the detailed energy structures of reaction cross sections with the emission of complex particles, such a microscopic approach can at present hardly give any new information about the details of the interaction compared with that obtained in the R-matrix scheme of analysis, although at relatively high energies the formalism of the microscopic theory can be successfully used to describe the process of cluster knockout from a nucleus in reactions induced by protons.[18,19,90] Therefore, considering the possibilities of the microscopic approach presented here in the practical analysis of experimental data and its advantages over phenomenological theories, we shall restrict ourselves to the case of single-nucleon nuclear reactions.

The effective-interaction method, the K-matrix formalism, has in particular been widely used in recent years to analyze the energy dependence in the cross sections of (p,p'), (n,n'), (p,n), (γ,n), (γ,p) reactions. The structures observed here in the different energy ranges at a definite experimental resolution can be related directly to definite matrix elements of transitions between states of the chosen model. In the case of averaging over a broad interval (~100 keV) in the cross sections integrated with respect to the angles, broad optical resonances appear, and the angular distributions of the reaction products in some cases clearly exhibit the asymmetry typical of direct processes. In the analysis of these data one makes more precise the model potential U used to describe the channel wavefunctions $|cE\rangle$, which depend resonantly on the energy near the single-particle levels in the continuum.[9] From the form of the angular distributions one can estimate the contribution to the cross sections of direct processes,[19,20] and also the strength functions of different reactions that proceed through resonance states.[31,5]

If the experimental resolution function has a width of the order of a few keV, the cross sections exhibit intermediate structures, which in individual cases can be determined as doorway analog states. The energy E_λ and the decay width $\Gamma_\lambda^\uparrow$ of the corresponding resonance can be found in a shell calculation of the simplest state of the two-particle–one-hole type directly coupled through the pairing residual interaction to the continuum (see Sec. 3). From the analysis of the experimental data one can also determine the total width of the "spreading" of the doorway state over more complicated states that are not directly coupled to the

continuum: $\Gamma_\lambda^\downarrow$ (69). The effective interaction method reflects in the most general form the concept of doorway states and the consequences associated with it in the energy and angular dependences of the cross sections.

Analysis of the splitting of analog resonances in reactions induced by protons and of the dipole resonance in a photonuclear reaction on the basis of states of more complicated nature, if the splitting is observed with sufficiently good experimental resolution, enables one in individual cases to draw a conclusion about the nature of the coupling of the complicated states of the continuum and the doorway state on the basis of investigations of the energy dependence of the various partial-wave cross sections in the considered region.[3,4,23–27] Thus, if there is one broad doorway state and one level of complicated nature, the analysis of the cross sections gives, beside the resonance widths of these levels, two further important parameters: the width $\Gamma_{\lambda\mu}$ (38), which characterizes the so-called external mixing in the reaction channels (interference of levels), and the value of the matrix element $\langle\lambda|V^{\text{eff}}|\mu\rangle$, the so-called internal mixing.[21–27,36]

In noting the advantages of the K-matrix formalism over the formalism of R-matrix theory in the analysis of the detailed structure of cross sections of single-nucleon reactions, the main one of which is the direct model interpretation of the resonance parameters of individual levels and their relationship to one another, we must also point out the relative value of these two schemes in the problem of parametrizing cross sections in the region of resolved resonances. Here one retains the same methods of the single- and many-level analysis as in the R-matrix theory and also the statistical assumptions deduced from comparison with experimental data about the distribution of the resonance widths and the distances between individual resonances.[31,60–62]

The integral equations obtained for the transition amplitudes in the review on the basis of the general method are more convenient in applications to the description of cross sections averaged over resonances. We have given here a systematic formulation of the complex-potential model, we have considered the relationship between this model and the effective-interaction method, and we have also formulated general distorted-wave Born approximation and coupled-channel approximation schemes in the calculation of direct transitions. The development of the method of integral equations applied to the analysis of average cross sections contains in particular a correct modification of the Hauser–Feshbach formulas (79), which are widely used in practical analysis.[64,91–93] At relatively high energies of the incident nucleons, when one can ignore the influence of the discrete spectrum, the integral equations for the transition amplitudes written down in the momentum representation go over directly into the well known equations of the many-channel dispersion theory of reactions.[17,38,76]

The connection considered here between the different schemes for parametrizing the energy dependence of the amplitudes of single-nucleon reactions makes it possible to express the various parameters in terms of the matrix elements of transitions between model states of the discrete spectrum and of the continuum for a given residual interaction. The determination of this universal set of parameters and their model interpretation is the final aim of the practical analysis and systematization of the entire enormous bulk of experimental information hitherto accumulated on the detailed energy structure of the cross sections of nuclear reactions induced by nucleons of low and medium energies. This activity stimulates the development of precision methods of measurement of nucleon cross sections in wide energy ranges and progress in the perfection of computational methods in the theory of the nucleus.

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