Absolute electromagnetic transition probabilities in odd deformed nuclei

W. Andrejtscheff (V., Andreĭchev)

Institute of Nuclear Research and Nuclear Power, Bulgarian Academy of Sciences, Sofia
Fiz. Elem. Chastits At. Yadra 7, 1039–1079 (October–December 1976)

A review is given of some of the most recent investigations of absolute electromagnetic transition probabilities in deformed odd nuclei ($150 < A < 190$). The experimental study of isomers in the nanosecond region in nuclear reactions is discussed. The experimental data are analyzed on the basis of the nonadiabatic model. A discussion is given of the effect of Coriolis mixing, the different deformations of one-quasiparticle states, and the collective motion of the core on the transition matrix elements.

PACS numbers: 23.20.Fp

INTRODUCTION

The most valuable information on the fine details of nuclear structure is obtained from study of the probability of γ transitions, which for an energy $E\gamma$ and multipolarity σL can be determined experimentally from the lifetime of the nuclear level (isomer) being de-excited. The reduced probability $B(\sigma L)$ of an electromagnetic transition depends on the square of the transition matrix element between two states—the initial state $|i\rangle$ and the final state $|f\rangle$:

$$B(\sigma L) \sim |\langle f|M(\sigma L)|i\rangle|^2,$$

where $M(\sigma L)$ is the multipole operator. Other quantities such as the statistical moments and energies characterize only a single level. Since the dependence on the structure of the two states is quadratic, the transition probability is highly sensitive to the details of the nuclear wave functions. Therefore the values of $B(\sigma L)$ have great importance for checking and further refinement of nuclear models. In addition, they provide information on the intrinsic quadrupole moments Q_0 and the gyromagnetic ratios g_K and g_R. Determination of γ-ray transition probabilities by the methods used at the present time as a rule, requires, complicated experiments and assumes a knowledge of the level scheme of a given nucleus. In order to calculate a transition probability it is necessary to know the wave functions of the participating states, at least to a certain approximation. This means that the initial theoretical calculations of the energy must be terminated before the transition probabilities enter into the calculations. As a result it is possible to obtain and analyze experimental data on nuclear isomers only in the case when definite information is available on the structure of the given nucleus. Therefore the accumulation of experimental absolute transition probabilities and model calculations for their interpretation occurs in practice appreciably more slowly than the accumulation of data on energy levels.

For a complete understanding of the structure of the nucleus, complicated studies of the nuclear properties are necessary. Here we have in mind the fundamental difference between the uncertainties in the energies and the wave functions of the states in quantum mechanics in parametrization of the Hamiltonian. If the value of the parameter β, on which the Hamiltonian depends, is taken different from the true value β_0 by $\delta\beta$, then $\beta = \beta_0 + \delta\beta$. The deviation of the erroneous wave function ψ from the true wave function is evident from the following expansion:

$$\psi \approx \psi(\beta_0) + \delta\beta \left.\frac{\partial\psi}{\partial\beta}\right|_{\beta_0}.$$

For an energy E such an expansion has the form

$$E(\beta) \approx E(\beta_0) + \delta\beta \left.\frac{\partial E}{\partial\beta}\right|_{\beta_0} + \frac{1}{2}(\delta\beta)^2 \left.\frac{\partial^2 E}{\partial\beta^2}\right|_{\beta_0}.$$

Here the linear term disappears, since for $\beta = \beta_0$ the energy has a minimum. Thus, $\Delta\psi \sim \delta\beta$ and $\Delta E \sim (\delta\beta)^2$. Consequently, the energy will be calculated more accurately than the wave function and the quantities depending on it, for example, the γ-ray transition probability.

Odd deformed nuclei are interesting for investigation of the transition probability, since for these nuclei intensive experimentation and development of model representations have already led to significant progress in description of the energetic and other characteristics of the low-lying states, which have a primarily single-particle nature.[1]

Recently we have noted some progress in the refinement of the deformed single-particle potentials of Nilsson[2] and Woods and Saxon[3] by introduction of additional terms and of deformations of higher order.[4–7] In addition, there is interest in the theoretical studies of V.G. Solov'ev and his colleagues[8,9] who propose to take into account the individual deformation of each one-quasiparticle configuration in calculations of nuclear properties. Inclusion of a multipole-multipole term in the total Hamiltonian permits the quasiparticle-phonon interaction to be taken into account.[1]

In recent years, in addition to further experimental study of intrinsic states in radioactive decay,[10] considerable attention has been devoted to studies of the rotational motion of nuclei.[11] For understanding the observed energy and spin sequences of many rotational bands it has turned out to be necessary to take into account Coriolis mixing of a number of single-particle states.[11,12] Such nonadiabatic effects have been observed particularly clearly in nuclei with an odd number of neutrons, where the anomalous rotational band of positive parity is explained by mixing of all deformed orbitals from the spherical $i_{13/2}$ shell.[13] Systematic analysis of absolute probabilities of γ transitions is a further necessary step for checking and refinement of the idea of strong mixing of bands. Here there is great interest in those structural components which have practically no effect on the energy of the level but appear only in the highly sensitive transition matrix elements. The delayed-coincidence method is widely used for experimental study of the absolute probabilities of γ-ray transitions. Several years ago such experiments were carried out mainly in radioactive decay.

The spectroscopy of nuclear reactions in a particle beam with use of high-resolution semiconductor detectors has opened up new possibilities for study of a large number of excited states. A method of obtaining time signals from a detector of the Ge(Li) type has been developed for study of isomers in the nanosecond region: In addition to the good energy resolution of these detectors it is necessary also to obtain satisfactory time resolution. Experiments carried out in recent years in particle beams and in radioactive decay have provided important information on the probability of single-particle transitions of multipolarity $E1$, $E2$, and $M1$ in nuclei with stable quadrupole deformation. The problems of the systematics and interpretation of absolute γ-transition probabilities in deformed nuclei have been discussed in several review articles, particularly Refs. 14–16. Questions relating to electromagnetic transitions in odd-odd deformed nuclei have been discussed in articles by the author.[17,18] In the present article we review some of the latest results in the region of nuclei odd in mass number A. Many data presented here are the result of collabration of the author and P. Manfrass.

1. TIME MEASUREMENTS IN THE NANOSECOND REGION IN A PARTICLE BEAM WITH Ge(Li) TYPE DETECTORS

Measurement of the lifetime of excited nuclear levels and of the probability of radiative transitions is carried out by direct and indirect methods. By these means it is possible to study states with half-lives from 10^{-18} sec to several thousand years, each method covering a definite range of half-life (Fig. 1). The principles of the various methods and the region of their applicability are described in detail elsewhere.[16,19] From the statistics on the quantitative contribution of the individual methods[16] it follows that the principal method is the delayed-coincidence method, which can be used in the nanosecond region. This method is used to measure directly the time interval between two events: the excitation (start) and decay (stop) of a given level (Fig. 2). By means of a time-to-pulse-height converter the start-stop time interval is converted to a pulse whose amplitude corresponds to the length of the interval. The spectrum of pulse heights is analyzed in an ordinary multichannel analyzer. This method is used in experiments on nuclear reactions in an accelerator or reactor beam. On excitation of nuclear levels in a reaction, complex spectra with a large number of γ transitions are observed. At the present time such spectra can be analyzed successfully by semiconductor detectors of the Ge(Li) type, which have a high energy resolution. However, in comparison with scintillation counters the time properties of these detectors are unsatisfactory as the result of the large spread in the rise time of the output pulse. The time resolution has been improved by compensation of the time spread.[20,21] In experiments at the Central Institute for Nuclear Research at Rossendorf the method of timing from the leading edge[22] was used at first, and later a method involving a following threshold.[23] In addition, in some of the measurements an additional selection of the output signals was made on the basis of the optimal shape of the leading edge of the pulse.[24] An example of the measurement of the lifetime of one level ($T_{1/2}=0.5$ nsec) in detection of γ rays with energy 193 keV is shown in Fig. 3.

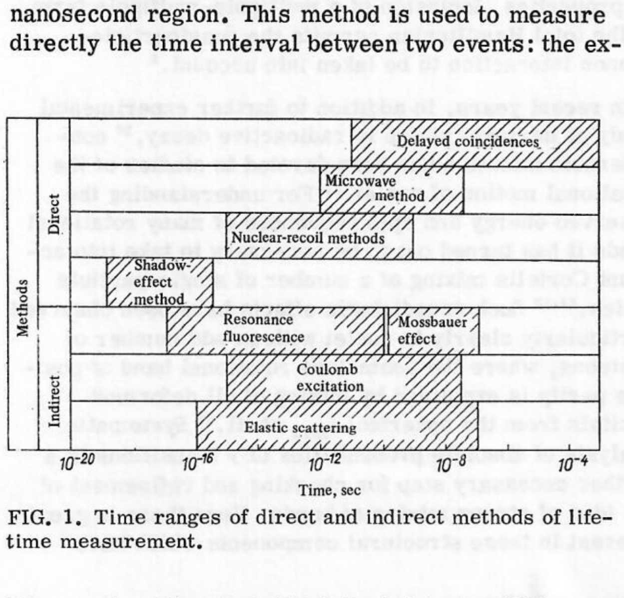

FIG. 1. Time ranges of direct and indirect methods of lifetime measurement.

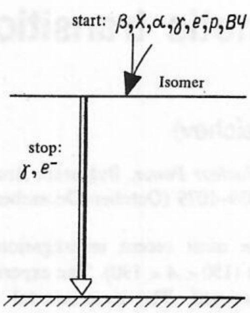

FIG. 2. Start and stop events in lifetime measurements by the delayed-coincidence method. The final particle in nuclear reactions is designated p. The start signal can also come from the high-frequency oscillator.

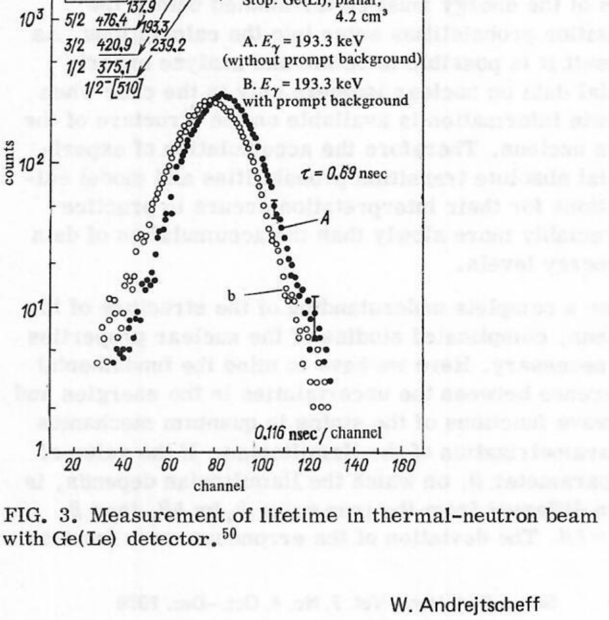

FIG. 3. Measurement of lifetime in thermal-neutron beam with Ge(Le) detector.[50]

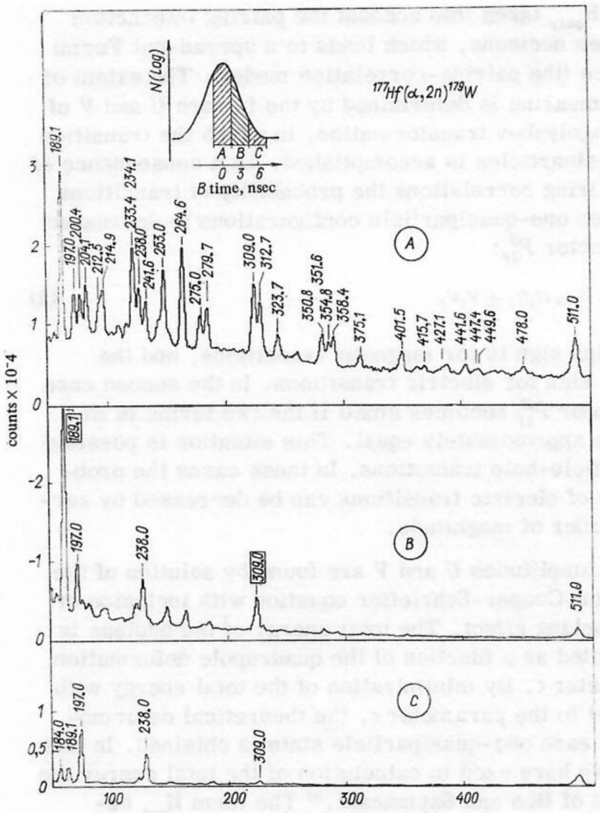

FIG. 4. Spectrum of delayed γ rays. Transitions with $E_\gamma = 189.1$ and 309.0 keV de-excite a level in ^{179}W. $T_{1/2} = 1.5$ nsec.[50]

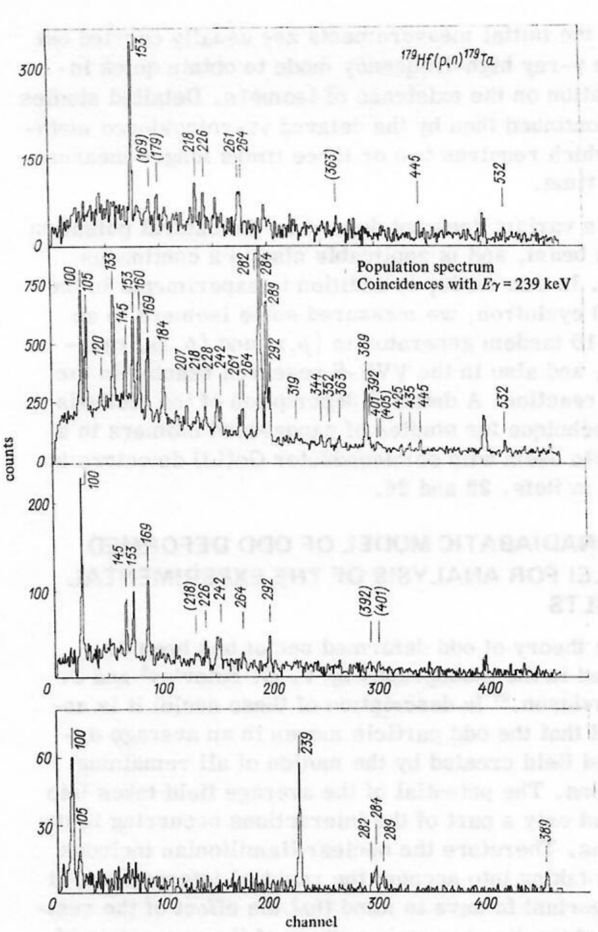

FIG. 5. Spectra of γ rays populating and de-exciting isomers in the nucleus ^{179}Ta, measured in the reaction ^{179}Hf$(p,n)^{179}$Ta.[64]

Thus, the time resolution of the measuring systems, including Ge(Li) detectors, lies within the resolution of systems with fast scintillation counters and permits experiments to be carried out successfully in the nanosecond region in a particle beam.

In study of nanosecond isomers in nuclear reactions, two variants of the delayed-coincidence method are used: The first, proposed by Yamazaki and Ewan,[25] utilizes the natural pulsing of the cyclotron beam. The beam is made up of individual packets of charged particles. Each of them bombards the target for a period of 4–5 nsec. The interval t between the packets depends on the frequency of the cyclotron oscillator (from the frequency 11 MHz of the U-120 cyclotron it follows that the interval is 90 nsec). During the bombardment, nuclear levels are excited. Some of them are de-excited promptly, and others, isomeric, with some delay. The delayed γ rays are recorded in the intervals between the packets of particles. This method is carried out technically by means of an ordinary fast-slow coincidence circuit. The start signal for the time-to-pulse-height converter is provided by a germanium detector. The stop signal is taken from the cyclotron oscillator and is strictly correlated with the frequency of the particle packets. A variation of the start and stop signals is also possible (Fig. 4). Time spectra for various energy combinations and also γ-ray energy spectra taken in coincidence with different windows in the time spectrum are measured (see Figs 4 and 5). The principal advantage of this variant is in its high efficiency, since here the efficiency is determined by one detector. In several hours of measurement in a beam it is possible to obtain quick information on the existence of isomers excited in a given reaction. This variant, known as the γ-ray high-frequency variant, has been used successfully in several laboratories.

However, in this case the principle of the measurement has also certain limitations. On the upper side, the time range of the measurements depends on the interval t between the beam bunches; for example, for $t \approx 90$ nsec measurements of lifetimes $T_{1/2} \leq 100$ nsec are possible. The length and shape of the beam pulses limit the time range on the low side ($T_{1/2} \geq 1-2$ nsec). In addition, it is difficult to take into account the effect of other isomers by which the level studied is sometimes excited; in this case the integral time spectrum is measured for a low-lying isometric state. In view of these limitations the γ-ray high-frequency variant was realized in the particle beam at Rossendorf by use of the delayed-coincidence method in its classical form,[22] using two detectors—a scintillation counter, usually NaI(Tl), and a semiconductor Ge(Li) detector. This permitted expansion of the time range to 0.5–1000 nsec. A very important advantage of this variant is the possibility of measuring the spectra of γ rays populating the isomer studied (see Fig. 5). Therefore in our experiments at the cyclotron with $(\alpha, 2n)$ and $(d, 2n)$ reactions the initial measurements are usually carried out in the γ-ray high-frequency mode to obtain quick information on the existence of isomers. Detailed studies are continued then by the delayed $\gamma\gamma$-coincidence method, which requires two or three times longer measurement time.

This variant does not depend on the natural pulsation of the beam, and is applicable also in a continuous beam. In this mode, in addition to experiments in the U-120 cyclotron, we measured some isomers in an ÉGP-10 tandem generator in (p,n) and $(p,2n)$ reactions, and also in the VVR-S research reactor in the (n,γ) reaction. A detailed description of the methods and technique for studies of nanosecond isomers in a particle beam with semiconductor Ge(Li) detectors is given in Refs. 22 and 26.

2. NONADIABATIC MODEL OF ODD DEFORMED NUCLEI FOR ANALYSIS OF THE EXPERIMENTAL RESULTS

The theory of odd deformed nuclei has been described in the monographs by V. G. Solov'ev[1] and J. P. Davidson.[27] In description of these nuclei it is assumed that the odd particle moves in an average deformed field created by the motion of all remaining nucleons. The potential of the average field takes into account only a part of the interactions occurring in the nucleus. Therefore the nuclear Hamiltonian includes terms taking into account the residual interactions. It is important to have in mind that the effect of the residual-interaction terms in a study of the properties of nuclei is sensitive to the shape of the average-field potential. In order to describe the entire set of nuclear characteristics, it is necessary to use realistic contemporary representations of the average field. In this approach to understanding of nuclear structure, residual interactions are taken into account on the basis of a single-particle Woods-Saxon potential.[1,7] To obtain physical information from the experimental results on low-lying states, a simpler model is used, employing the Nilsson single-particle potential.[11,13] This model has been used to analyze the experimental probabilities and their mutual relation to the structural features of the nuclei.[28,29] The Hamiltonian of the model consists of the following terms:

$$\mathbf{H}=\mathbf{H}_N+\mathbf{H}_{\text{pair}}+\mathbf{H}_{\text{rot}}+\mathbf{H}_{\text{Cor}}. \qquad (1)$$

The anisotropic harmonic-oscillator potential $\mathbf{H}_N$ proposed by Nilsson[2] was used for description of the average field of the nucleon motion. This potential is due to the motion of the odd particle outside the even-even core. Single-particle states are characterized by the projection Ω of the particle spin j on the deformation axis, the parity π, and the asymptotic quantum numbers N, n_z, and Λ. In the ground states of nuclei with odd mass number the particle-spin projection Λ is equal to the projection K of the nuclear spin I on the deformation axis $(K=\Omega)$.

We shall designate the single-particle states either $K^\pi[Nn_z\Lambda]$ or $2KNn_z\Lambda$ (for example, $1/2^-[521]$ or 1521). We shall use the modified form of the single-particle Hamiltonian from the work of Gustafson *et al.*[4] The term $\mathbf{H}_{\text{pair}}$ takes into account the pairing interaction between nucleons, which leads to a spread-out Fermi surface (the pairing-correlation model). The extent of this smearing is determined by the factors U and V of the Bogolyubov transformation, in which the transition to quasiparticles is accomplished. As a consequence of the pairing correlations the probability of transitions between one-quasiparticle configurations is decreased by a factor P_{if}^2:

$$P_{if}^{E,M}\approx U_iU_f\pm V_iV_f. \qquad (2)$$

The plus sign is for magnetic transitions, and the minus sign for electric transitions. In the second case the factor P_{if}^E becomes small if the two terms in Eq. (2) are approximately equal. This situation is possible in particle-hole transitions. In these cases the probability of electric transitions can be decreased by several order of magnitude.

The amplitudes U and V are found by solution of the Bardeen-Cooper-Schrieffer equation with inclusion of the blocking effect. The total energy of the nucleus is calculated as a function of the quadrupole deformation parameter ϵ. By minimization of the total energy with respect to the parameter ϵ, the theoretical deformation of each one-quasiparticle state is obtained. In this work we have used in calculation of the total energy the method of Bes and Szymanski.[30] The term $\mathbf{H}_{\text{rot}}$ describes the adiabatic rotational motion; $\mathbf{H}_{\text{Cor}}$ accomplishes mixing of the different one-quasiparticle configurations with $\Delta K=1$, i.e., it expresses the nonadiabaticity of the model discussed. The Coriolis interaction ($\mathbf{H}_{\text{Cor}}$) is important in highly deformed nuclei at energies near the Fermi surface. The term $\mathbf{H}_{\text{Cor}}$ relates states which differ in the values of the quantum number K by $\Delta K=1$. Through several intermediate states with $\Delta K=1$ there is a mixing of quasiparticle configurations with $\Delta K>1$ which are neighboring in energy ($\Delta E\lesssim 20$ keV).[11] For description of the experimental energies of the levels it is often necessary to further modify the matrix elements of the Coriolis interaction calculated in the Nilsson model by correction factors $R_{K,K+1}$ (attenuation factors), which here are introduced phenomenologically.

In some cases[31] it turns out to be important to take into account mixing of states which differ by two units in the principal quantum number N. This is true, for example, for the orbitals $1/2^+[660]$ and $1/2^+[400]$, and also for $3/2^+[402]$ and $3/2^+$ [651], which in the single-particle model quasi-intersect at definite deformations. In the present calculations for the nucleus ^{165}Er we have used the matrix elements of the $\Delta N=2$ interaction obtained in analysis of the experimental (d,p) and (d,t) reactions and the energy differences of states with $K^\pi=1/2^+$ and $3/2^+$.[28]

Since the deformation parameter ϵ is calculated in the model, in calculations with a Hamiltonian $\mathbf{H}_N+\mathbf{H}_{\text{pair}}$ there enter only four free parameters: the constants of Nilsson's model $\varkappa$ and μ and the pairing-interaction constants G_p and G_n. These parameters have been determined for the entire region $150<A<190$ as a function of the mass number A (numerical values are given in Ref. 32).

As the result of subsequent diagonalization of the complete Hamiltonian H (1), mixed wave functions ψ_{mix} are obtained which are superpositions of the deformed single-particle wave functions mixed by the Coriolis interaction H_{Cor}:

$$\psi_{\text{mix}} = \sum_{c_k} | N n_z \Lambda \Sigma \rangle_k .$$

Here c_k are the mixing coefficients; $|Nn_z\Lambda\Sigma\rangle_k$ are single-particle states with different quantum numbers K included in the mixing procedure. In this procedure the correction factors $R_{K,K+1}$, the rotational constants A_K, and the decoupling parameters a for bands with $K=1/2$ are fitted to obtain the least average deviation from the experimental energies. Thus, the quantities $R_{K,K+1}$, A_K, and a are free parameters which are varied within reasonable limits in the fitting process.

Comparison of the experimental and theoretical energies is not very useful for a check of the model at this point. For such a check it is necessary to calculate other nuclear properties with these mixed wave functions, for example, the cross sections for a nucleon-transfer reaction or the probability of electromagnetic transitions.

The reduced probability $B(\sigma L)$ of the electromagnetic transition between states $|i\rangle$ and $|f\rangle$ mixed in K is the product of the constant $C(\sigma L)$ and the matrix element M:

$$B(\sigma L) = C(\sigma L)\, M^2, \tag{3}$$

where $M = \sum_{i,f} M_{if}$.

We have the following definitions:

$$M_{if}^{sp} = P_{if} c_i c_f T_{if}^{sp}\,; \tag{4}$$

$$T_{if}^{sp} = [C_{I_i K_i L K_f - K_i}^{I_f K_f} + b_{\sigma L} (-1)^{I_f + K_f} C_{I_i K_i L - K_f - K_i}^{I_f - K_f}]\, G_{\sigma L}^{sp}(i, f). \tag{5}$$

Here c_i and c_f are the admixture coefficients of single-particle configurations in the initial and final states; P_{if} is the pairing factor (2). The terms $b_{\sigma L}$ and $G_{\sigma L}$ which occur in the expression for the single-particle matrix element T_{if}^{sp} have been defined by Nilsson [see Eqs. 35(a) and (b) and 36(a) and (b) in Ref. 2].

Electric-dipole transitions connect states of opposite parity, whose wave functions are always orthogonal. Here in many cases for $E1$ transitions one uses different values of the deformation parameters ($\epsilon_i \neq \epsilon_f$), which are taken from calculations of equilibrium deformations. In the case of $M1$ and $M2$ multipolarity the same deformation value is taken for the two states, for example, the value for the ground state.

In the adiabatic approximation the probability $B(E2)$ for transitions within the rotational band depends on the intrinsic quadrupole moment Q_0:

$$B(E2)_{\text{rot}} = (5/16\pi)\, [C_{I_i K 20}^{I_f K}]^2 Q_0^2 e^2. \tag{6}$$

In the case of $E2$ transitions between different rotational bands, in addition to the matrix element of the M transition one takes into account the collective terms T_{if}^{coll}, which are proportional to the quadrupole moment Q_0[33]:

$$\left.\begin{aligned} G_{\text{coll}}(E2) &= 49{,}41\,(1 - 1/2\epsilon^2 - 2/27\epsilon^3)^{-1/3} Q_0 A^{-1/3};\\ M_{\text{coll}}(E2) &= c_i c_f T_{if}^{\text{coll}}\, \delta_{if};\ \delta_{if} = \begin{cases} 0 & \text{for } i \neq f;\\ 1 & \text{for } i = f. \end{cases} \end{aligned}\right\} \tag{7}$$

The values of the quadrupole moment Q_0 are related to the theoretical deformation value ϵ on the basis of the work of Löbner *et al.*[34]

In the model, which is described here schematically, vibrational impurities in the wave functions are not taken into account. Piepenbring,[35] Faessler *et al.*,[36] and Bernthal and Rasmussen[37] have discussed the interaction of quasiparticles with octupole phonons and its influence on $E1$ transitions with $\Delta K = 0$. The interaction of quasiparticles with quadrupole phonons and its influence on electromagnetic transitions has been taken into account in the model used by Weller.[38] Below we present some new experimental indications of the quasiparticle-phonon interaction.

A systematic investigation of the probability of γ transitions in a nuclear model in which all residual interactions are considered at the contemporary level is one of the urgent problems of the theory at the present time.

Comparison of the experimental and theoretical transition probabilities is usually carried out by means of hindrance factors

$$F = B(\sigma L)_{\text{theo}} / B(\sigma L)_{\text{exp}}.$$

Subscripts are used to indicate the terms of the complete Hamiltonian H(1) included in the calculations: $F_N(H_N)$; $F_N^p(H_N \leqslant H_{\text{pair}})$, $F_N^p = F_N \times P_{if}^2$; $F_N^{pK}(H)$. In comparison with the Weisskopf single-particle estimate the hindrance factors are designated F_W. The factors F_W were calculated from the formulas given in Ref. 39 without taking into account the statistical factor S.

In the adiabatic approximation the spin projection K is a good quantum number. Then for a transition of multipole order L we obtain the selection rule $L \geqslant |K_f - K_i|$. If it is not satisfied, the transition is K-forbidden with a degree of hindrance $|K_f - K_i| - L$. As the result of mixing of single-particle configurations (see Chapter 4) a K-forbidden transition usually occurs, but with a reduced probability. With increase of the degree of hindrance by unity, an increase in the value of F_W by about a factor of 100 is observed.[40] On the basis of a simple nonadiabatic model we shall investigate in what follows the effect of the average field and the residual interactions in formation of the matrix elements of electromagnetic transitions.

3. EFFECT OF AVERAGE FIELD AND PAIRING CORRELATIONS

To investigate the role of the single-particle potential and the pairing effect, electric-dipole transitions turn out to be most appropriate. Most of the experimentally known probabilities of $E1$ transitions are forbidden by asymptotic quantum numbers, for which the following combinations are permitted in Nilsson's model:

ΔK	ΔN	Δn_z	$\Delta\Lambda$
1	± 1	0	1
0	± 1	± 1	0

(8)

TABLE I. Hindrance factors of the transition $5/2^- [512] \longleftrightarrow 7/2^+ [633]$. Here $5.1(-3)$ means 5.1×10^{-3}. F_N was calculated for various versions of the Nilsson model.

Nucleus	F_W	F_N [2]	F_N [4]	F_N^p	Nucleus	F_W	F_N [2]	F_N [4]	F_N^p
${}^{165}_{66}Dy_{99}$	5.1 (3)	13	4.6	1.5	${}^{171}_{70}Yb_{101}$	1.2 (5)	180	131	0.9
${}^{167}_{68}Er_{99}$	3.1 (4)	52	28	11	${}^{173}_{70}Yb_{103}$	7.5 (5)	880	587	168
${}^{169}_{68}Er_{101}$	8.9 (5)	1070	722	4.6	${}^{173}_{72}Hf_{101}$	2.6 (5)	237	141	2.3
${}^{169}_{70}Yb_{99}$	2.7 (4)	36	21	8	${}^{175}_{72}Hf_{103}$	1.6 (4)	18	10.2	4.3

Violation of the rule (8) leads to representation of the single-particle transition matrix elements $T^{sp}_{if}(5)$ as the sum of approximately equal amplitudes with different phases. As a result an exceptional sensitivity of the $E1$ matrix elements to the details of the single-particle potential arises. This sensitivity is demonstrated by the hindrance factors F_N calculated by us previously[41] with use of various modifications of the Nilsson model for the transition $5/2^- [512] \longleftrightarrow 7/2^+ [633]$ $(\Delta K = 1, \Delta n_z = 2)$ in nuclei with an odd number of neutrons (Table I). Inclusion of the term $\langle l^2 \rangle = N(N+3)/2$ and renormalization of the constants $\varkappa$ and μ in the Hamiltonian leads to a change in the theoretical probabilities by almost a factor of two.

It is well known[42] that the properties of the single-particle potential generated by the average field significantly affect the accuracy of calculations taking into account residual interactions. In this sense for description of the transitions it is more successful to use the realistic Woods-Saxon potential with a smeared edge, which has been investigated in detail by V. G. Solov'ev and his co-workers.[7] Calculations of the electromagnetic transition probabilities carried out with this potential for $E1$ transitions[43] and $M1$ transitions[44] show certain differences in comparison with calculations in the Nilsson model. In spite of this, it turns out that for transitions between low-lying states the anisotropic-harmonic-oscillator model gives similar results. The modification of Nilsson's model proposed by Andersen[6] approaches in its properties a potential with a smeared edge, for example, by turning on the $\Delta N = 2$ interaction. On the basis of this more accurate modification of the model it has been shown[45] that in some cases the theoretical probability exceeds by an order of magnitude or more the results of the initial Nilsson model and, for example, for the transition $7/2^+ [404] \longleftrightarrow 7/2^- [523]$ the results of the Woods-Saxon model are approached. On the basis of the variant of the Nilsson model proposed by Lamm,[46] Boisson and Piepenbring[47] have calculated some matrix elements for comparison with the results of the initial model. It turned out that one can expect large variations (by one or two orders of magnitude) in $E2$ transitions with $\Delta K = 0$. Pairing correlations lead to a decrease in the γ-transition probability in comparison with the results of the single-particle model. In magnetic transitions this effect is not important, since the corresponding factor P^M_{if} is of the order of unity. In electric-quadrupole transitions a strong effect of collective motion of the rotational and vibrational types is added to the effect of pairing. The influence of pairing is observed more distinctly in $E1$ transitions. For example, several authors[42,48,49] point out the decisive value of the average field and the rapid change of the calculated probabilities on taking into account the correction factor of the pairing-correlation model P^E_{if} in some cases (if $P^E_{if} \ll 1$). It is possible to distinguish some cases of $E1$ transitions in which other important interactions—Coriolis mixing and quasiparticle-phonon interaction (see Section 6)—give a weak contribution to the matrix elements. Then the pairing effect is traced more clearly. We shall give some examples from the experimental data accumulated in recent years. However, in these cases the accuracy of the calculations is low.

TABLE II. Hindrance factors for the transition $3/2^- [521] \longleftrightarrow 5/2^+ [642]$ in nuclei with an odd number of neutrons.[50]

Nucleus	F_W	F_N	F_N^p
${}^{157}_{64}Gd_{93}$	3.4 (5)	433	4.4
${}^{159}_{64}Gd_{95}$	1.4 (4)	22	4.5
${}^{159}_{66}Dy_{93}$	2.8 (5)	233	3.8
${}^{161}_{66}Dy_{95}$	9.2 (3)	20	4.1
${}^{165}_{68}Er_{97}$	1.4 (5)	286	150

The pairing factor for electric transitions $P^E_{if} = U_i U_f - V_i V_f$ becomes small for transitions between particle and hole states (see Section 2). In the systematics of $E1$ transitions between identical Nilsson states in nuclei with a different number of nucleons, appreciable jumps are observed in some cases. The cause of these anomalies may be the pairing effect, i.e., in these nuclei the initial and final states may turn out to be particle and hole, which leads to a small pairing factor P^E_{if}. In this case an additional hindrance appears in the transitions, which is not sensitive to the average field. Some of the investigated transitions where the effect of pairing correlations is clearly revealed are given in Tables I and II. The large single-particle hindrance factors F_N for the transition $5/2^- [512] \longleftrightarrow 7/2^+ [633]$ in nuclei with a number of neutrons $N = 101$, and also for the transition $5/2^+ [642] \longleftrightarrow 3/2^- [521]$ in nuclei with $N = 93$, are smoothed by the pairing factors P^E_{if}. In the tables we have also given examples (${}^{173}Yb$ and ${}^{165}Er$) in which the pairing correlations are not responsible for a large hindrance. In these cases, as in those where the pairing factor destroys the apparent good agreement with Nilsson's theory (this is observed particularly for $E1$ transitions with $\Delta K = 0$; see Section 6), it is necessary to look for other reasons for the hindrance or enhancement of the transitions.

The examples given here show that the model of pairing correlations $(\mathbf{H}_N + \mathbf{H}_{\text{pair}})$ correctly describes electromagnetic transitions between pure one-quasiparticle configurations. On the basis of this assumption we will discuss below the effect of configureation admixtures. As a rule, in comparison of the theoretical and experimental probabilities, it is assumed that a possible disagreement indicates the need of improving model wave functions, it being assumed that the expressions for the multipole operators are valid. We shall give an example of how the experimental study of γ transitions has led to the necessity of changing the structure operator. For the transition $5/2^+ [402] \longleftrightarrow 7/2^+ [404]$ in various nuclei anomalous jumps are observed in the experimental probability $B(M1)$. Since the inclusion of pairing correlations and Coriolis mixing in the calculations did not lead to satisfactory results, Wahlborn and Blomquist[52] have proposed a modification of the magnetic-dipole operator by an additional tensor term $k_2\mathbf{O}$ arising as the result of spin polarization and pion-exchange contributions. An upper limit $k_2 \leqslant 0.4$ has been obtained[53] for the empirical constant k_2. Here we must also point out the interesting work of Feifrlik *et al.*,[54,55] who succeeded in explaining quantitatively a number of $E1$ transitions in terms of the theory of finite Fermi systems with inclusion of the interactions between all quasiparticles.

Fellah and Hammann[51] have considered the effect of projection of a Bardeen function on the space with a given number of particles for the case of $E1$ transitions with $\Delta K = 1$. It turns out that here some results can change by an order of magnitude.

4. EFFECT OF CONFIGURATION MIXING

A. General remarks

Low-lying states of odd deformed nuclei have preferentially a one-quasiparticle nature. Mixing of configurations in the general case includes other one-quasiparticle components and collective admixtures of the rotational and vibrational type. In this section we discuss, in terms of the model described in Chapter 2, the effect of mixing of one-quasiparticle configurations by the Coriolis interaction. For $E1$ transitions collective admixtures of the rotational type are also taken into account. It turns out that inclusion of these mixing effects facilitates to a considerable degree the satisfactory description of electromagnetic transitions. It has been experimentally established that it is necessary to take into account also the interaction of quasiparticles with vibrational phonons (see below).

There is a fundamental difference between taking into account the effect of configuration admixtures on calculation of the energies of levels and on the probability of electromagnetic transitions. Small admixtures (2–3%) make almost no change in the excitation energies and therefore components with such amplitudes are ordinarily neglected. As we have already pointed out, the reduced probability $B(\sigma L)$ is proportional to the sum of terms, each of which consists of four factors:

$$B(\sigma L) \sim M^2 = \Big|\sum_{i,f} M_{if}\Big|^2 = \Big|\sum_{i,f} c_i c_f P_{if} T_{if}^{\mathrm{sp(coll)}}\Big|^2.$$

If one of these factors is small, then the term M_{if} will also be small. Thus, M_{if} for certain weak components may become dominant for large pairing factors P_{if} and matrix elements T_{if}. It is clear that in calculations of the transition probability it is necessary to include from the beginning the maximum possible number of components, and then to find among them the most important. This idea has been carried out for nuclei in the region of Er and W. Thus, in calculations of the probability of transitions from the isomer $11/2^-$ [505] ~100 components were taken into account, of which, as a rule, no more than ten turned out to be important.

Among the most interesting results of recent years in the field of spectroscopy of deformed nuclei is the observation of strongly mixed rotational bands of positive parity. In studies of the isotopes Gd, Dy, Er, and Yb in (α, xn) reactions in a particle beam, rotational bands have been observed whose ground state often is determined as the configuration $5/2^+$ [642] from the $i_{13/2}$ spherical shell. The highly perturbed sequence of energies and spins of this band, which was observed for the first time in ^{161,163,165}Er, indicated a substantial admixture of some state with $K = 1/2$ and a large decoupling parameter. Hjorth and Ryde and others[13] have explained the structure of these bands by a Coriolis interaction of single-particle states from the $i_{13/2}$ shell and have included the configuration $1/2^+$ [660] with a decoupling parameter $a \approx 6$. The ground state of this band, which in the cases known up to this time has a lifetime in the nanosecond region, is de-excited by $E1$ transitions to the levels of the ground-state band of negative parity (for example, $3/2^-$ [521] or $5/2^-$ [523]). At an energy of several hundred keV the isomer $11/2^-$ [505] is observed; this isomer is de-excited by K-forbidden transitions to the levels of the perturbed positive-parity band and the ground-state band. In some nuclei, in addition to the perturbed band, individual quasiparticle excitations with positive parity are also observed, for example $1/2^+$ [660] and $1/2^+$ [400], and also $3/2^+$ [651] and $3/2^+$ [402]. These configurations with $\Delta N = 2$ interact with each other: $3/2^+$ [651] and $3/2^+$ [402] (see for example the levels 507.3 and 745.2 keV in ^{165}Er). The situation described is typical for neutron nuclei of the beginning of the rare earth region ($A \approx 160$) and is illustrated in the example of the ^{165}Er level scheme (Fig. 6).

For the comprehensive study of the energy and probability of transitions, extensive calculations have been carried out of the mixing of configurations in the isotopes ^{161,163,165}Er. For the positive-parity levels all seven states of the $i_{13/2}$ shell were included, and for the nucleus ^{165}Er also the configurations $1/2^+$ [400] and $3/2^+$ [402], i.e., all states with $1/2^+ \leqslant K^\pi \leqslant 13/2^+$ were taken into account. For the negative-parity levels ten states were taken into account, including $11/2^-$ [505] with $1/2^- \leqslant K^\pi \leqslant 11/2^-$. The average discrepancy obtained in these calculations[28,29] between the calculated

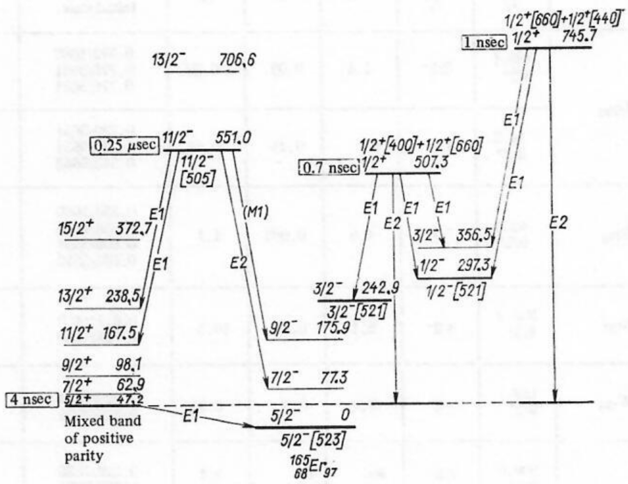

FIG. 6. Part of the level scheme of ^{165}Er.[28]

TABLE III. Hindrance factors of $E1$ transitions between the bands $9/2^-$ [514] and $7/2^+$ [404].[56]

Nucleus	I_i	I_f	F_N	F_N^p	F_N^{pC}	Nucleus	I_i	I_f	F_N	F_N^p	F_N^{pC}
^{175}Lu	$9/2^-$	$7/2^+$	157	20	11	^{177}Ta	$9/2^-$	$7/2^+$	98	1.7	2.8
	$9/2^-$	$9/2^+$	18.5	2.3	4.9	^{179}Ta	$9/2^-$	$7/2^+$	76	2.0	3.5
	$9/2^-$	$11/2^+$	3.9	0.5	2.2	^{181}Ta	$9/2^-$	$7/2^+$	25	1.0	1.6
^{177}Lu	$9/2^-$	$7/2^+$	230	31	27	^{183}Ta	$9/2^-$	$7/2^+$	27	1.2	1.8

and experimental energy values are in the range of 1–6 keV. Let us further consider the study of the probability of electromagnetic transitions for investigation of mixed wave functions. The effect of mixing of one-quasiparticle configurations on the absolute transition probabilities (mainly $E1$ transitions) is analyzed here on the basis of the values of the matrix elements M_{if}.

B. Electric dipole transitions allowed by the quantum number K

In the nonadiabatic model the angular-momentum projection K is not a good quantum number. However, the concept of K-allowed or K-forbidden transitions can be retained with respect to the principal components of the corresponding states.

Cases of transitions between states in which a definite one-quasiparticle component is dominant over the remaining components (percentage content in the wave function $\gtrsim 60\%$) are considered first. This situation is frequently encountered in $E1$ transitions in odd proton nuclei, in which mixing of orbitals with different K values within a single oscillator shell is observed. This refers to the transitions $5/2^-$ [532] $\leftrightarrow$ $3/2^+$ [411], $5/2^-$ [532] $\leftrightarrow$ $5/2^+$ [413], $7/2^+$ [404] $\leftrightarrow$ $7/2^-$ [523], $9/2^-$ [514] $\leftrightarrow$ $7/2^+$ [404], and also $1/2^+$ [411] $\leftrightarrow$ $1/2^-$ [541].

As a typical example let us discuss in more detail the $E1$ transitions $9/2^-$ [514] $\leftrightarrow$ $7/2^+$ [404] in the isotopes of Lu and Ta.[56] The experimental probabilities of these transitions are described satisfactorily by the Nilsson model when pairing correlations are taken into account (Table III). The Coriolis interaction is expressed, first of all, as the contribution of the components $7/2^-$ [523] with matrix elements ⟨7523/5413⟩ and ⟨7523/7404⟩. In these cases taking into account the Coriolis effect leads to an unimportant change in the theoretical result (comparison of the factors F_N^p and F_N^{pC} in Table III). Similar conclusions have been reached by Baznat and Pyatov,[57] who used a nonadiabatic model[12] to calculate the contribution of impurities to the transition probability up to 20% for $E1$ transitions with $\Delta K = 0$ and up to 80% for transitions with $\Delta K = 1$.

A more appreciable influence of configuration admixtures on the transition probability is expected in strongly perturbed states, for example, in the cases of orbitals from the $i_{13/2}$ shell in neutron nuclei. Typical examples for nuclei of different mass numbers, in which $E1$ transitions between principal components are allowed by K, are given in Table IV. Nevertheless the configuration admixtures give contributions M_{if} to the total transition matrix element M which are comparable with the contributions of the principal components. Thus, analysis of the 45.6-keV transition in ^{161}Er shows that the contribution of the principal components $5/2^+$ [642] and $3/2^-$ [521] are significantly smaller than the contribution of several other components and play practical-

TABLE V. Energy and half-life of the isomer $11/2^-$ [505] in deformed neutron nuclei with a number of nucleons N and Z.[58]

Z \ N	89	91	93	95	97
$_{62}$Sm	260 1.4 (–6)	98,4 1.1 (–2)			
$_{64}$Gd	171.2 7,9 (–5)	121.5 3.1 (–2)	425 1.7 (–5)		
$_{66}$Dy	233.4 6.0 (–6)	199.2 1.92 (–2)	352.9 1.15 (–4)		
$_{68}$Er		429.4 5.5 (–7)	396.4 7.5 (–6)	443.8 5.8 (–7)	551.0 2.5 (–7)

TABLE IV. Hindrance factors and largest components of $E1$ transition matrix elements.

Nucleus	E_γ, keV I_i^π	I_f^π	F_N	F_N^p	F_N^{pC}	Largest components: Initial state	Final state	Matrix elements M_{if}
$^{157}_{66}$Dy$_{91}$	188.1 $5/2^+$	$3/2^-$	1.4	0.02	0.20	0.320/1660 0,776/3651 0.776/3651	0.127/1530 0.127/1530 0,992/3521	–0,120 (–02) –0.192 (–02) –0.136 (–02)
	14.2 $9/2^+$	$7/2^-$	15	0,23	0.84	0.720/3651 0.720/3651 0.513/5642	0.243/1530 0.955/3521 0.955/3521	–0.312 (–02) –0,148 (–02) +0.495 (–02)
$^{161}_{68}$Er$_{93}$	45.6 $9/2^+$	$7/2^-$	4.6	0.005	1.1	0.356/1660 0.356/1660 0.609/3651 0.695/5642	0.099/1530 0.955/3521 0.955/3521 0.955/3521	–0.117 (–02) –0.127 (–02) –0.544 (–02) –0.468 (–03)
$^{165}_{68}$Er$_{97}$	389.2 $1/2^+$	$3/2^-$	8.1	0.26	10.3	0.611/1400 0.791/1660	0,996/1521 0.996/1521	–0.779 (–02) –0.110 (–02)
$^{179}_{74}$W$_{105}$	189.1 $9/2^+$	$9/2^-$	2.6	0.5	0.84	0.529/7633 0.827/9624	0.990/7514 0.990/7514	–0.254 (–02) –0.414 (–02)
	309.0 $9/2^+$	$7/2^-$	80	15	5.1	0.529/7633 0.827/9624	1,0/7514 1.0/7514	+0.154 (–02) –0.877 (–02)

ly no role. A similar situation is encountered in the 389.2-keV transition in ^{165}Er (the interaction between the configurations $1/2^+$ [400] and $1/2^+$ [660] with $\Delta N=2$). In comparison with this, in ^{179}W the principal components of the states give an even larger contribution to the matrix element.

C. Electric-dipole transitions forbidden by the quantum number K

A quantitative description of K-forbidden transitions is possible only in terms of nonadiabatic models. Such transitions are allowed only by configuration admixtures, which sometimes in the initial or final state are less than 1%. Description of the experimental probabilities for such transitions is an extremely rigorous verification of the mixed wave functions. This is true particularly for the highly sensitive $E1$ transitions.

The approach to the description of K-forbidden transitions in terms of the model used here and the role of configuration admixtures are discussed for the example of transitions from the isomer $11/2^-$ [505] to the levels of the strongly perturbed rotational band of positive parity. The experimental data on this isomeric state are given in Table V. The configurations $11/2^-$ [615] and $13/2^+$ [606] of the $i_{13/2}$ shell, which are located 3–5 MeV from the Fermi surface, take part very weakly in mixing of positive-parity levels at low spins and have practically no significance for the energy. For this reason they are usually neglected.[13] However, electric-dipole transitions between the states $11/2^-$ [505] with these configurations are asymptotically allowed [see Eq. (8)], which leads to matrix elements T^{sp}_{if} with amplitudes significantly higher (2–3 orders of magnitude) than for asymptotically forbidden transitions. Therefore they cannot be neglected in discussion of the transition probabilities. In our calculations the attenuation factors of the matrix elements of the Coriolis interaction between configurations with $K^\pi=9/2^+$, $11/2^+$, and $13/2^+$ were determined at first for values $R_{K,K+1}=1$, in order to have fewer free parameters. For the nucleus ^{165}Er the theoretical single-particle energies E^{sp} were decreased by 15% on the basis of the experimental data on states with $K^\pi=1/2^-$. In this case the mixed wave functions obtained gave a good quantitative description of K-forbidden transitions from the isomer $K^\pi=11/2^-$. However, for the nuclei ^{161,163}Er, where unmodified model single-particle energies were used, the wave functions led to complete disagreement with the experimental $B(E1)$ values.

TABLE VI. Hindrance factors F^{pC}_N of $E1$ transitions from the isomer $11/2^-$ [505] for different sets of attenuation factors $R_{K,K+1}$. The spin values of the final states are shown ($I^\pi_f=9/2^+$, $11/2^+$, and $13/2^+$).

Nucleus	$R_{K,K+1}$			F^{pC}_N		
	7/2–9/2	9/2–11/2	11/2–13/2	9/2+	11/2+	13/2+
^{161}Er	0.73 0.4	1 0.4	1 0.4	292 39	216 13	159 5.6
^{163}Er	1 0.4	1 0.4	1 0.4	173 3.4	412 5.0	82 0.82
^{165}Er	0.6	1	1	—	3.6	1,1

Analysis showed that the amplitudes of the matrix elements $|M_{if}|$ were too large. The principal cause of this turned out to be the admixture coefficients of configurations with $K^\pi=9/2^+$, $11/2^+$, and $13/2^+$. (The admixtures in the almost pure initial state $11/2^-$ [505] are assumed to be less important.) The mixing coefficients of states with $K^\pi \geqslant 9/2^+$ become smaller if the corresponding attenuation factors decrease, for example, to a value $R_{K,K+1}\approx 0.4$ and the fit is repeated. Here the other free parameters are almost unchanged, and the average discrepancy between the experimental and theoretical energies ($\overline{\chi}\approx 1-2$ keV) is no greater. Thus, transition matrix elements $M=\sum_{i,f} M_{if}$, are obtained which are very close to the experimental values (Table VI).

We can expect that similar agreement, i.e., roughly the same wave functions, will be obtained by modification of the single-particle energies, as was shown for ^{165}Er. The hindrance factors F^{pC}_N in the corresponding transition matrix elements are given in Table VII. The small contributions of the components $11/2^+$ [615] and $13/2^+$ [606] are noteworthy (for example, $c_f=0.0009$ represents an admixture of less than 10^{-4}%!).

Of course, in this method the calculations involve some uncertainty. Nevertheless, the possibility has been demonstrated in principle of describing quantitatively such critical quantities as K-forbidden $E1$ transitions and definite correlations are revealed between the free parameters of the mixing procedure.

The sensitivity of the transition matrix elements to small mixing amplitudes, which have no importance for the energies, characterizes the investigation of electromagnetic transitions as an effective method for the study of configuration mixing.

D. Magnetic dipole transitions

In K-allowed magnetic transitions the principal components of the wave functions give the greatest contribution to the transition matrix element. On the other hand, the pairing factor $P^M_{if}=U_iU_f+V_iV_f$ is of the order of unity. Therefore the pairing correlations and the Coriolis interaction have a significantly weaker effect on the probability of $M1$ transitions than for electric multipoles. The agreement between the theoretical and experimental values is satisfactory and the discrepancy rarely exceeds an order of magnitude.[28,50,59]

TABLE VII. Characteristics of $E1$ transitions from the isomer $11/2^-$ [505] to highly mixed states with $I^\pi=13/2^+$.

I^π_f	F^{pC}_N	Largest components		
		Initial state	Final state	M^{sp}_{if}
$13/2^+$	0.8	^{163}Er 0.053/9514	0.213/7633 0.021/9624	−0.794 (−4) 0.114 (−3)
		0.999/11505	0.0009/11615 0.00001/13606	0.110 (−3) 0.102 (−4)
$13/2^+$	1.1	^{165}Er 0.100/9514	0.490/7633 0.078/9624	−0.527 (−3) −0.199 (−3)
		0.995/11505	0.078/9624 0.008/11615 0.0003/13606	0,200 (−3) 0.540 (−3) 0.127 (−3)

TABLE VIII. Examples of K-forbidden transitions in odd neutron nuclei.[59] (The values of the gyromagnetic factors are $g_R = 0.3$ and $g_{s\,\mathrm{ef}}/g_{s\,\mathrm{free}} = 0.7$).

Nucleus	Initial state	Final state	Degree of K-forbiddenness	F_N^{pC}
^{161}Er	11/2 11/2⁻ [505]	9/2 3/2⁻ [521]	3	7.1
^{163}Er	11/2 11/2⁻ [505]	11/2 5/2⁻ [523]	2	0.94
		9/2 5/2⁻ [523]	2	2.0
^{183}W	7/2 7/2⁻ [503]	5/2 1/2⁻ [510]	2	17
		7/2 1/2⁻ [510]	2	2
		9/2 1/2⁻ [510]	2	1.7

K-forbidden $M1$ transitions are also explained in terms of the nonadiabatic model used here. This refers to transitions de-exciting the isomers 11/2⁻ [505] in the Er isotopes and 7/2⁻ [503] in the W isotopes (Table VIII). The problems associated with description of electric-quadrupole transitions are discussed below.

5. EFFECT OF QUADRUPOLE DEFORMATIONS

Calculations of the wave functions of the initial and final states involve the quadrupole-deformation parameter ϵ, which is usually chosen identical for the two states. As a rule, the value of ϵ used is usually that known from experiment for the ground state. It has turned out that variation of this parameter within the experimental error (~15%) does not lead to a substantial change in the theoretical transition probabilities (see for example Ref. 43). In comparison with this, individual empirical studies showed that in some cases an insignificant difference in the deformation of the initial and final states ($\Delta\epsilon/\epsilon \approx 10\%$) strongly changes the theoretical probability of an $E1$ transition.[15,60] However, there was no theoretical justification for such arbitrary changes of the parameters.

In analysis of the experimental probabilities of $E1$ transitions between the states 1/2⁻ [541] and 1/2⁺ [411] a substantial hindrance was observed[22,63] in comparison with the predictions of the Nilsson model ($F_N \approx 10^2, \ldots, 10^4$). Such a hindrance is especially unusual for transitions with $\Delta K = 0$ (see also Section 6). In the work of V. G. Solov'ev[8] on the theoretical possibility of different values of deformations of one-quasiparticle states of the same nucleus it was noted, in particular, that for the 1/2⁻ [541] state the quadrupole deformation ϵ was found larger by $\Delta\epsilon \approx 0.02$ than for the 1/2⁺ [411] state. Preliminary investigations of the effect of this difference[61,22] showed a high sensitivity of the theoretical transition matrix elements to small values of $\Delta\epsilon$ (Fig. 7).

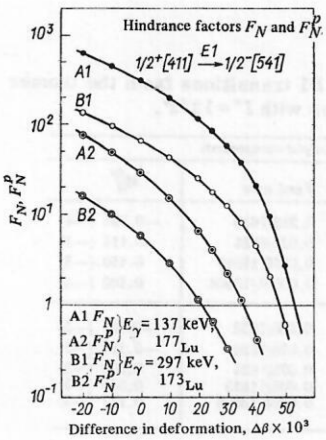

FIG. 7. Hindrance factors as a function of the difference in deformation of the initial and final states; $\Delta\beta = \beta(1541) - \beta(1411)$.[22]

For a systematic study of this phenomenon, the measurements of the absolute probabilities of the transition discussed, carried out at first for the nuclei ^{171,173}Lu, were later extended to ^{175,177}Lu and ^{179}Ta.[62,64] The experimental data existing at the present time permit better justified conclusions on the value of the quadrupole deformation.[32,65,66]

The effect of deformation can be evaluated correctly only after correct inclusion of the other known interactions. Below we systematically analyze the individual effects which affect the formation of the matrix elements of the $E1$ transitions 1/2⁻ [411] ⟷ 1/2⁻ [541]: pairing, the Coriolis interaction, and change in the quadrupole deformations. Löbner *et al.*[67] have given a proof that the effect of octupole vibrations in this case is weak and they need not be taken into account. This assumption is supported also by the systematics of $E1$ transitions with $\Delta K = 0$ (see Section 6 and Fig. 13), from which it follows that for $A \sim 175$ the octupole admixtures in the matrix elements are significantly weaker than for lower values of A.

In the isotopes of Lu and Ta discussed, the $E1$ transition is of the particle-hole type with small factors $P_{if}^E \ll 1$. The location of the levels 1/2⁺ [411] and 1/2⁻ [541] relative to the Fermi surface is correctly reflected by the model calculations, except for the nucleus ^{177}Lu, where the factor P_{if}^E is found theoretically to be very large (Table IX). This nucleus differs from the neighboring isotopes of Lu in the fact that in it the 1/2⁺ [411] level lies lower than the 1/2⁻ [541] state (Fig. 8). This change in the relative location of the two levels is not predicted by the model, and therefore the factor P_{if}^E is calculated with a large error in this model. In

TABLE IX. Hindrance factors of $E1$ transitions between the rotational bands 1/2⁺ [411] and 1/2⁻ [541].[66]

Nucleus	I_i	I_f	F_N	F_N^p	F_N^{pC} $\varepsilon_i = \varepsilon_f$	F_N^{pC} $\varepsilon_i \neq \varepsilon_f$
^{171}Lu	1/2⁺	1/2⁻	305	12	15	2.0
	1/2⁺	1/2⁻	92	3.3	3.9	0.56
	1/2⁺	3/2⁻	50	1.8	4.4	1.0
^{173}Lu	3/2⁺	5/2⁻	129	4.5	7.6	1.5
	3/2⁺	3/2⁻	7.3	0.26	0.26	0.045
	3/2⁺	1/2⁻	53	1.9	1.7	0.14
^{175}Lu	1/2⁺	1/2⁻	740	45	48	6,5
^{177}Lu {	5/2⁻	3/2⁺	3100	500	870	84
	5/2⁻	5/2⁺	8300	460	190	5.2
^{175}Ta	1/2⁺	1/2⁻	2430	0.018	0.32	14
	1/2⁺	3/2⁻	5700	0.071	30	1.3
^{177}Ta	1/2⁺	1/2⁻	11700	8.9	13.2	3.5
	1/2⁺	3/2⁻	216	0.16	2.5	0.23
^{179}Ta	5/2⁻	3/2⁺	660	2.9	6.9	0.81

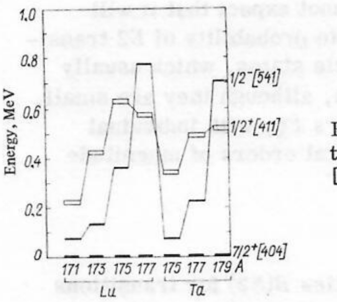

FIG. 8. Experimental excitation energies of the states 1/2⁻ [541] and 1/2⁺ [411].

calculation of the coefficients U and V it is possible to use also an approximate formula involving the experimental values of the quasiparticle energies E_{exp} and the gap Δ[68] on the assumption that the ground state 7/2⁺ [404] lies near the Fermi surface,

$$V^2 = [1 \mp \sqrt{E_{\text{exp}}^2 + 2\Delta E_{\text{exp}}} / (\Delta + E_{\text{exp}})]/2. \qquad (9)$$

In this way we obtain for ^{177}Lu values of P_{if}^E (experimental) which are significantly smaller than the model values. Thus, for a gap value $\Delta = 800$ keV we obtain $|P_{if}^E(\text{exp})|^2 = 0.0018$ and $F_N^b = 5.6$ in comparison with $|P_{if}^E|^2 = 0.16$ and $F_N^b = 500$ in Table X. If $\Delta = 700$ keV, we obtain $|P_{if}^E|^2 = 0.00175$ and $F_N^b = 5.4$. For the other investigated isotopes of Lu with $A = 171-175$ the values of U and V obtained from Eq. (9) are in good agreement with the predictions of the model used here.[56] The influence of configuration mixing by the Coriolis interaction is discussed on the basis of studying the contribution of the various components to the complete transition matrix element. According to the data, in Table X the intensity of the $E1$ transition between the one-quasiparticle states 1/2 1/2⁺ [411] and 1/2 1/2⁻ [541] is determined primarily by the principal components. The contributions of other configurations, for example, 1/2⁺ [420] and 1/2⁻ [530], are significantly smaller and can be neglected. The same is true also for the transitions 3/2 1/2⁺ ⟶ 1/2 1/2⁻. A greater importance is acquired by the contributions of the components 1/2⁻ [530] and 3/2⁻ [532] in formation of the matrix elements of transitions involving rotational levels with $I^\pi = 3/2^-$ and $5/2^-$. These contributions are comparable with the contributions of the principal components. Since the phases of the contributions in the cases discussed with certain exceptions are identical, configuration mixing leads to enhancement of the $E1$ transitions, i.e., to increase of the $B(E1)$ values with increasing spin. This tendency is actually observed experimentally in ^{171}Lu, where the $B(E1)$ values increase from $0.055(-6)\ e^2b$ at $I_i = 1/2$ to $2.7(-6)\ e^2b$ at $I_i = 19/2$.[69] However, we should not expect that such an increase occurs in all bands. The phase and amplitude of the pairing factors P_{if}^E and mixing factors c_{if} vary, with significant fluctuation of the reduced probabilities, as appears experimentally in some cases.[69] Pairing correlations and, to a certain degree, Coriolis mixing are taken into account in this model with the necessary correctness.

TABLE X. Components and matrix elements of the transitions 1/2 1/2⁺ [411] and 1/2 1/2⁻ [541].[56]

Nucleus	Initial state	Final state	M_{if}	
			$\varepsilon_i = \varepsilon_f$	$\varepsilon_i \neq \varepsilon_f$
^{173}Lu	0.999/1411 0.999/1411 0.0078/1420	0.999/1541 0.0213/1530 0.999/1541	—10.94 —0.58 —0.441	—3.614 —0.479 —0.32
^{175}Lu	1/1411 1/1411	0,998/1541 0.030/1530	—14.82 —0.737	—5.076 —0.700
^{177}Ta	1/1411 1/1411	0,999/1541 0.031/1530	—1.716 —0.373	1.400 —0.327

To take into account the individual quadrupole deformation of each one-quasiparticle configuration, a minimization was carried out of the total energy of the nucleus with respect to the deformation parameter ϵ with blocking of the one-quasiparticle state being investigated (see Sec. 1). The ϵ values obtained in this way are in good agreement in most cases with the known experimental values for the ground states[34] and with calculations by Strutinsky's method.[9] The results of calculations for one-quasiparticle configurations included in the mixing procedure in the case of ^{171,173}Lu are given in Table XI. With these theoretical values of the individual quadrupole deformation of each configuration, we calculated the transition probabilities [see the factors F_N^{bC} ($\epsilon_i \neq \epsilon_f$) in Table IX]. It can be seen from Table IX that in spite of some exceptions the best agreement between theory and experiment is achieved when the individual deformations are taken into account. An analysis shows that in those rare cases where inclusion of individual deformations leads to poorer agreement with experiment it is necessary to calculate more accurately the mixing amplitudes, which can play a decisive role, especially for transitions between rotational states.

The difference of the quadrupole deformations for the transitions discussed is shown in Fig. 9 for several typical examples. The agreement with experiment is improved by an order of magnitude. In order to explain the relative fluctuations between the hindrance factors of individual nuclei, it is necessary to draw on effects which have not been taken into account, for example, hexadecapole deformation and vibrational admixtures in the wave functions. The total matrix element of a γ transition is the product of the overlap interval of the core wave functions and the single-particle matrix element. In the calculations of matrix elements of transitions with different deformations of the initial and final states, the overlap integral is taken equal to unity, although it is actually smaller. Evaluation of the overlap integral for $A = 170$, carried out with anisotropic-harmonic-oscillator wave functions,[32] gave a value ~0.8, which confirms the validity of the as-

TABLE XI. Calculated deformations for states in ^{171,173}Lu. (From interpolation of experimental data on neighboring nuclei[34] we can expect $\varepsilon \approx 0.27$ for the ground states of both isotopes.)

Nucleus	State					
	7/2+ [404] ground	1/2+ [411]	1/2+ [420]	1/2− [541]	1/2− [530]	3/2− [532]
$^{171}_{71}$Lu$_{100}$	0.268	0.264	0.252	0.288	0.285	0.283
$^{173}_{71}$Lu$_{102}$	0.267	0.261	0.247	0.286	0.282	0.282

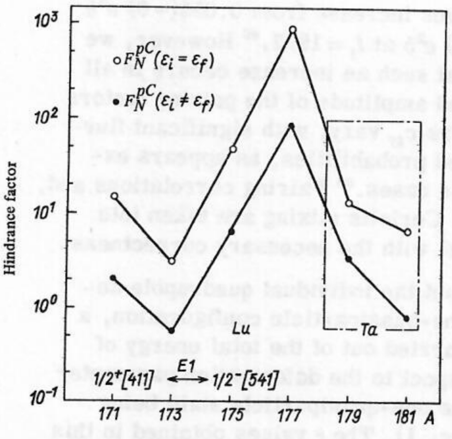

FIG. 9. Hindrance factors for identical ($\varepsilon_i \neq \varepsilon_f$) and different ($\varepsilon_i \neq \varepsilon_F$) deformations of the initial and final states for transitions between ground states of bands.

sumption that this integral is not significantly different from unity. It is known the matrix elements of certain $E1$ transitions are very sensitive to the hexadecapole deformation of the nucleus. In the Ho—Tm region a considerable change in $B(E1)_{\exp}$ values by several orders of magnitude is observed for the transition $7/2^+$ [404] $\longleftrightarrow 7/2^-$ [523]. In Ref. 80 the connection between this change in the probability $B(E1)$ and the monotonic behavior of the parameter ε_4 has been demonstrated for the nuclei under consideration. The results presented show how the close connection between the experimental and theoretical investigations leads to further refinement of the model calculations.

6. EFFECT OF COLLECTIVE MOTION OF THE CORE

A. General remarks

The effect of collective motion occurs very clearly in electric-quadrupole transitions. Therefore we shall discuss them here in more detail. In even-even nuclei for rotational transitions (within a single rotational band) the experimental values $B(E2)_{\rm rot}$ are of the order 1 e^2b^2 ($F_W \approx 10^2, \ldots, 10^{-3}$).

In comparison with this, in de-excitation of β and γ vibrational states the reduced probabilities observed are $B(E2_{\rm vib} \approx 10^{-2}, \ldots, 10^{-3}$ e^2b^2 ($F_W \approx 0.1, \ldots, 1$) (see Ref. 72), which are close to the experimental values of those transitions in odd nuclei which are practically single-particle and which have probabilities of the order $B(E2)_{\exp\,\rm sp} \approx 10^{-5}$ e^2b^2 (see Figs. 11 and 12). Physically this fact is explained by the following property of nuclear vibrations.[70] Description of a vibration involves two-quasiparticle components (in odd nuclei, three-quasiparticle), the number of which is not very large and which includes only the outer nucleons. Consequently, this effect has a nature less collective than, for example, nuclear rotation, in which all nucleons take part. It follows from the above that insignificant collective admixtures to the wave functions (1% or less, see Ref. 7), as a rule, appreciably change the value of the $E2$-transition matrix elements (for $E1$ transitions see below). The part of the total Hamiltonian (1) $\mathrm{H}_N + \mathrm{H}_{\rm pair}$ does not include the collective motion of the core and therefore we cannot expect that it will correctly describe the absolute probability of $E2$ transitions between one-quasiparticle states, which usually contain collective components, although they are small. The resulting hindrance factors F_N^p, with individual exceptions, are actually several orders of magnitude less than unity.

B. $E2$ transitions with $\Delta K = 1$

The experimental probabilities $B(E2)$ for transitions with ΔK=1 nuclei odd in mass number A are given in Fig. 10. The values lie in the range $10^{-3}, \ldots, 1$ e^2b^2 ($10 > F_W > 10^{-2}$). The dominant effect of configuration admixtures which is expected in transitions with $\Delta K=1$ between states with identical parity is due to the Coriolis mixing of the initial and final states, and it leads to appearance in the matrix elements of collective components of the rotational type. The region of values of $B(E2)$ for transitions with $\Delta K=1$ lies closer to the rotational values than for transitions with $\Delta K=2$ (see Figs. 10 and 11). This is explained by the stronger rotational admixtures.

Thus, the $B(E2)$ values for the region $A \approx 185$ in Fig. 10 characterize transitions between the configurations $1/2^-$ [510] and $3/2^-$ [512], whose mixing in the isotopes of W is well known.[59,71] In some nuclei[28] we have also studied the Coriolis mixing for the other configurations shown in Fig. 10. In many cases, taking this interaction into account actually leads to a quantitative explanation of the experimental results. Typical examples are given in Table XII. Other examples can be found in Ref. 72. However, in some cases the agreement with experiment is unsatisfactory (for example, for the transition 5/2 $3/2^-$ [512] $\to$ 1/2 $1/2^-$ [510] in ^{183}W, $F_N^{pC} = 6.3 \times 10^{-2}$). On the basis of Refs. 38 and 73 the descriptions of nuclear properties, including $E2$ transitions with $\Delta K=1$, take into account interaction between single-particle rotational and vibrational motion (β and γ vibrations). Recently interest has been aroused by the work of N. I. Pyatov *et al.*[74] on the mixing of one-quasiparticle configurations of an odd nucleus with vibrational core excitations with $K^\pi = 1^+$. With such mixing, for each nucleus a renormalization of the effective charges of the single-particle matrix elements of $E2$ transitions with $\Delta K=1$ is carried out. In the

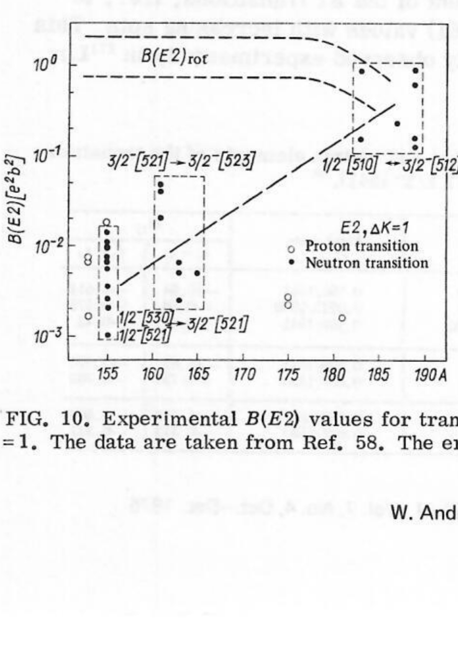

FIG. 10. Experimental $B(E2)$ values for transitions with ΔK =1. The data are taken from Ref. 58. The error is ~25%.

TABLE XII. Electric-quadrupole transitions.

Nucleus	State: Initial	State: Final	F_N	F_N^p	F_H^{nK}	Main components: Initial state	Main components: Final state	Main components: M_{if}^{sp} K-forb
^{165}Er	3/2 3/2⁻ [521]	7/2 5/2⁻ [523]	0.005	$\Delta K=1$ 0.002	5.8	0.997/3521 −0.144/1510	0.130/3521 0.999/1510	0.477 (1) 0.356 (1)
^{183}W	3/2 3/2⁻ [512]	1/2 1/2⁻ [510]	0.027	0.013	0.18	0.989/3512	0.899/1510	−0.125 (1)
^{165}Er	3/2 1/2⁻ [521]	5/2 5/2⁻ [523]	0.86	$\Delta K=2$ 0.04	0.13	0.996/1521 0.038/3521	0.996/5523 0.084/3521	0.339 (0) 0.156 (0)
	1/2 1400/1600	5/2 5/2⁺ [642]	0.0001	0.00007	3.2	0.791/1400 −0.611/1660	−0.009/1400 0.051/1660 0.986/5642	−0.411 (0) −0.154 (1) −0.734 (0)
^{161}Er	11/2 11/2⁻ [505]	7/2 3/2⁻ [521]	2K-forb.	K-forb.	30	−0.003/3521	0.957/3521	−0.103 (0)
^{165}Er	11/2 11/2⁻ [505]	7/2 5/2⁻ [523]	1K-forb.		3.8	0.100/9514	0.989/5523	0.323 (−1)

nucleus ^{235}U this correct inclusion of single particle contributions led to a change in the probability $B(E2)$ by 20–30% in comparison with calculations carried out with inclusion only of Coriolis mixing.

C. $E2$ transitions with $\Delta K = 2$

The experimental probabilities $B(E2)$ in transitions with $\Delta K=2$ are shown in Fig. 11 as a function of mass number A.[75] The following features are observed: a) The values lie in the range $10^{-6}, \ldots, 10^{-2}$ e^2b^2; b) a characteristic shape with a minimum at $A\approx 175$ is traced out. An attempt is made to interpret quantitatively the experimental $B(E2)$ values with allowance for rotational components. In some cases a substantial improvement of the agreement with experiment is achieved Typical examples are shown in Table XIII (sic). In transitions with $\Delta K=2$ the amplitudes of the rotational admixtures in the participating states, as a rule, are less than in transitions with $\Delta K=1$, since the mixing is accomplished by intermediate states (see Sec. 1). Therefore the contributions M_{ij}^{coll} have frequently a comparable amplitude but, as the result of the sign of the mixing coefficients c_i and c_f, a different phase in comparison with the single-particle admixtures M_{if}^{sp}. In some cases the calculations disagree substantially with the experimental values. Thus, it turned out to be impossible to describe even qualitatively the probability of $E2$ transitions from the isomer 7/2⁻ [503] to rotational states of the configuration 3/2⁻ [512] in the isotopes ^{183,185,187}W.[59] Thus, the dependence of $B(E2)$ on mass number for transitions with $\Delta K=2$ cannot be understood only by means of rotational admixtures. It turns out to be necessary to take into account also other forms of collective motion.

In many deformed even-even nuclei, states $K^\pi=2^+$ are observed with energy ~1 MeV, which are de-excited by $E2$ transitions with $\Delta K=2$ (γ vibrations). The corresponding $B(E2)$ values have an order of magnitude 5×10^{-2} e^2b^2. An admixture of this one-phonon excitation of the core, for example, in the initial state of a given transition in an odd nucleus, which appears as the result of the quasiparticle-phonon interaction, will change the absolute intensity of the $E2$ multipolarity. V. G. Solov'ev[1] has systematized the energy E_{2^+} and the structure of these states. The dependence of their excitation energies on mass number corresponds in a definite way (Fig. 12) to the dependence found for the $B(E2)$ on mass number for transitions with $\Delta K=2$ cannot this energy amounts to 800–900 keV, and for $A\approx175$ one obtains values of about 1600 keV. From this systematics we can suppose that as a consequence of the higher energies of the vibrational excitations a weakening is expected in the interaction between the quasiparticle configurations of the odd nucleus and the vibrational states of the core for $A\approx175$. As a result the $E2$ matrix elements in this region contain less vibrational admixtures and consequently their amplitudes will be smaller.

In this sense the following fact turns out to be very interesting. The lowest value of the probability $B(E2)$ in Fig. 11 is found for the transition 1/2 1/2⁻ [521] →5/2 5/2⁻ [512] in the nucleus ^{173}Yb: $B(E2)=1.79(-6)$ e^2b^2, $F_W=3200$. In the core nucleus ^{172}Yb the 2⁺ state is found at 1486 keV. This state forms an exception[1] and does not have a collective nature, but consists 98.8% of the two-quasiparticle configuration n1/2⁻ [521] n5/2⁻ [512].

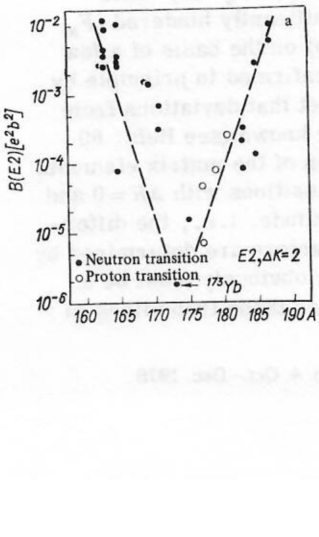

FIG. 11. Experimental $B(E2)$ values for transitions with $\Delta K=2$. The data are taken from Ref. 28. The error is ~25%.

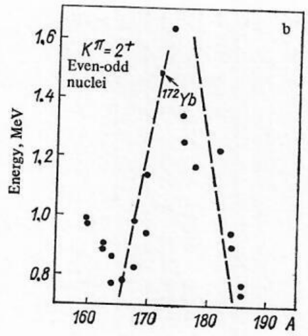

FIG. 12. Experimental values of the energies E_{2^+} states[1] of even-even nuclei which are the cores of the nuclei represented in Fig. 11.

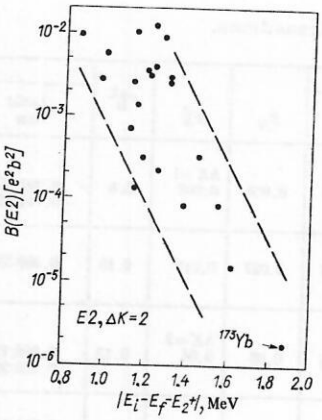

FIG. 13. Values of $B(E2)$ from Fig. 11 as a function of the energy difference of the final state and the quadrupole-vibrational excitation of the initial one-quasiparticle configuration.

We can conclude that the two states in ^{173}Yb do not have appreciable phonon admixtures (Fig. 13). Thus, the low value of the probability $B(E2)$ in ^{173}Yb becomes clear, and the transition has a purely single-particle nature and is described reasonably in terms of the pairing-correlation model ($F_N = 10$, $F_N^p = 3.5$). Faessler *et al.*[36] have calculated the octupole admixtures in $E1$ transitions with $\Delta K = 0$, which are discussed in the present section. Proceeding from their calculation, we can suggest that in an odd nucleus the initial state $|i\rangle$ of a given $E2$ transition with $\Delta K = 2$ (energy E_i, spin I) interacts with the electric-quadrupole excitation of the final state $|f\rangle$ (energy E_f, spin $I \pm 2$) and vice versa. For the energy of this vibrational state with spin I we can take the value $E_f + E_{2^+}$, where E_{2^+} is taken from the spectrum of the even-even core nucleus. According to first-order perturbation theory the admixture coefficient of quadrupole vibration in the initial state $|i\rangle$ will be proportional to $1/|E_i - (E_f + E_{2^+})|$ and in the final state—to the value $1/|E_f - (E_i + E_{2^+})|$. Consequently the reduced probability $B(E2)$ of a transition between the two states will depend to a certain degree also on this energy difference. The behavior of the experimental data confirms the conclusions drawn, as is shown in Fig. 13, where the parameter $|E_f - (E_i + E_{2^+})|$ is plotted as the abscissa.

Thus, the dependence of $B(E2)$ on the mass number A for transitions with $\Delta K = 2$ indicates directly the interaction of quasiparticles with quadrupole phonons. Practical calculations including this interaction have been carried out by Weller,[38] although he discussed only two examples of such transitions.

D. Effect of octupole vibrations on $E1$ transitions

In the first comparisons of absolute probabilities of $E1$ transitions with the predictions of the Nilsson model it was established by Vergnes[76] that transitions with $\Delta K = 0$ are described satisfactorily ($F_N \approx 1$), while transitions with $\Delta K = 1$ are significantly hindered ($F_N \gg 1$). This division, carried out on the basis of a few experimental data, has been confirmed in principle by later results, in spite of the fact that deviations from this tendency have also become known (see Refs. 60 and 67 and Table X). The values of the matrix elements in the Nilsson model for $E1$ transitions with $\Delta K = 0$ and 1 do not differ in order of magnitude, i.e., the differences given for the hindrance factors are determined by the experimental values. There obviously must be a structural effect which influences these transitions in different ways and does not "sense" the single-particle model. A vibrational band of the ground state with $K^\pi = 0^-$ is expected as the lowest band of octupole vibrations in deformed even-even nuclei. The state of this band with $I^\pi = 1^-$ is de-excited by a collective $E1$ transition to the ground state. An estimate shows that the dipole strength of such collective transitions is about 100 times that of single-particle transitions.[78] The suggestion has been made[48] that small configuration admixtures of the band with $K^\pi = 0^-$ of the core strongly affect the probability of $E1$ transitions with $\Delta K = 0$ in an odd nucleus. Specific calculations with allowance for such admixtures have been carried out by Piepenbring[35] and by Faessler *et al.*[36] and have confirmed the validity of this assumption. In the interesting investigation of Bernthal and Rasmussen[37] in order to explain quantitatively the absolute probabilities of $E1$ transitions in ^{177}Hf on the basis of Ref. 36, the Coriolis interaction was also taken into account. However, the phases of the octupole admixtures were determined empirically from the best agreement with experiment.

For $E1$ transitions with $\Delta K = 1$ a similar effect could arise as the result of admixtures of the octupole band with $K^\pi = 1^-$. However, octupole excitations with $K = 1$ in even-even deformed nuclei are located higher than levels with $K^\pi = 0^-$, and therefore their effect on $E1$ transitions with $\Delta K = 1$ is significantly weaker, as is confirmed experimentally (Fig. 14).

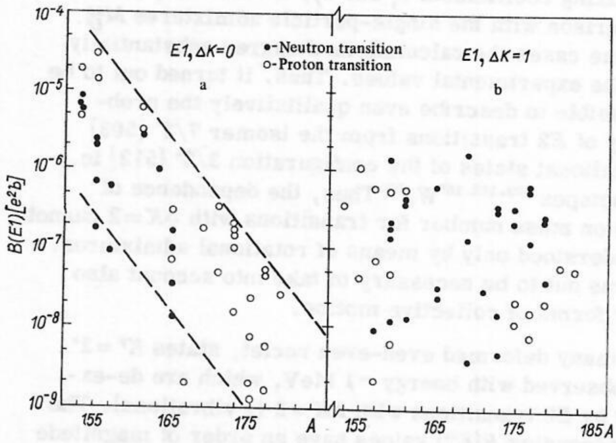

FIG. 14. Experimental values of $B(E1)$ in odd nuclei.[58] The error is $\sim 25\%$.

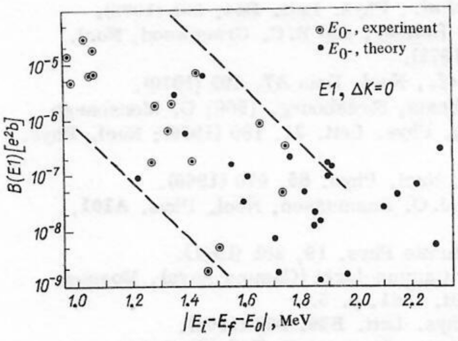

FIG. 15. Values of $B(E1)$ from Fig. 14a as a function of the energy difference of the final state and the octupole-vibrational excitation of the initial one-quasiparticle configuration.

The experimental data available at the present time permit a systematic discussion of the probabilities of K-allowed $E1$ transitions for the entire mass range $150 < A < 190$. The experimental $B(E1)$ values in Fig. 14 show characteristic differences for transitions with $\Delta K = 0$ and 1 as a function of the mass number. In spite of fluctuations due to the internal structure of the states, the $B(E1)$ values for transitions with $\Delta K = 0$ decrease by about two orders of magnitude in the mass region considered. At the same time no systematic behavior is observed for transitions with $\Delta K = 1$.

The behavior of the experimental probabilities for transitions with $\Delta K = 0$ is interpreted as the result of interaction of quasiparticles with octupole phonons. In Fig. 15, which is constructed in the same way as Fig. 13, a correlation is also observed between the $B(E1)$ values and the energy difference of one-quasiparticle states and quadrupole excitations. Since there are few experimental data on the energies E_{0^-}, we have used also theoretical values from Neergard and Vogel.[79] The vibrational admixtures in the one-quasiparticle wave functions are calculated for a number of deformed nuclei on the basis of the Woods-Saxon model with inclusion of the multipole-multipole interaction.[7]

CONCLUSION

In the present work we describe several results obtained recently in investigation and systematization of the absolute γ-transition probabilities in odd deformed nuclei. In Chapter 2 we discuss the promising use of the delayed-coincidence method in a particle beam with Ge(Li) detectors for measurement of the lifetime of excited nuclear states in nuclear reactions.

The values of the transition matrix elements are in practice sensitive not only to the main properties, but also to the details of nuclear models. Therefore experimental information on the absolute probabilities of electromagnetic transitions plays an important role in checking and improvement of model wave functions. This is demonstrated here by several investigations based on the semiphenomenological model described in Chapter 3. The role of the average field and of pairing correlations (Chap. 4) is discussed briefly, since a number of papers have already been devoted to these questions. For the case of K-allowed and K-forbidden transitions we have shown in Chapter V the possible effect of one-quasiparticle admixtures with different K in the systematic application of the scheme of mixing of a large number of Nilsson states by the Coriolis interaction (for example, ten levels of negative parity and nine levels of positive parity in Er nuclei). The correct description of the transition probabilities and the requirement of agreement with the experimental values impose additional conditions on the wave-function mixing scheme: for example, inclusion of states lying far from the Fermi surface, and definite correlations between the free parameters (single-particle energies and attenuation factors). In discussion of electric-dipole transitions it turns out that if one-quasiparticle configurations which asymptotically allow a given transition take part in the mixing, then very small admixtures to the wave functions ($< 10^{-3}\%$) can provide an appreciable contribution to the transition matrix element. This appears in practice in K-forbidden transitions. In the case of K-allowed transitions between weakly perturbed states the main contribution is from the principal components of the wave functions.

The influence of different deformation values of the initial and final states on the electric-dipole transitions is the subject of discussion of Chapter 6. Model calculations and comparison with a number of experimental values show that use of individual values of the theoretical quadrupole-deformation parameters for each one-quasiparticle configuration is not only correct, but is a more correct method than use of one value for all states.

Finally, at the present stage, the accuracy of the calculations presented in Chapter 5 and 6 is low as the result of a number of approximations, and the specific results refer, generally speaking, only to the model used. In spite of this, the conclusions of these studies have a more general and fundamental nature.

The interesting dependence of certain $B(E1)$ probabilities on the value of the hexadecapole deformation of the nucleus has been cited only in this review.

It is shown that in different cases the electromagnetic-transition matrix element is affected by different residual interactions. In many cases the very possibility of observing what interactions play a decisive role is a successful result of the investigations. Clarification of the role of residual interactions in each specific case is the result of a detailed analysis and it is still difficult to formulate selection rules determining which forms of residual interaction play a determining role in a given transition.

Systematic study of the reduced probabilities of $E2$ transitions with $\Delta K = 2$ and $E1$ transitions with $\Delta K = 0$ as a function of mass number indicates the effect of collective motion of the core as the result of quasiparticle-phonon interaction (Chap. 7). The qualitative discussions presented emphasize the need of taking into account interaction of quasiparticles with quadrupole and octupole phonons in calculations of transition probabilities.

A deeper understanding of nuclear structure requires that nuclear models describe the measured quantities with ever increasing accuracy. In the future refinement of model representations the role of electromagnetic-transition-probability studies will increase.

The author expresses his gratitude to L. Münchow for reading the manuscript and for critical remarks, and also to H. F. Brinkmann and F. Stary for their attention and interest in this work. He thanks the Academy of Sciences of the German Democratic Republic for creating excellent working conditions at the Central Institute for Nuclear Research (Rossendorf).

[1]V. G. Solov'ev, Teoriya slozhnykh yader. (Theory of Complex Nuclei), Moscow, Nauka, 1971.
[2]S. G. Nilsson, K. Dan Vidensk. Selsk. Mat.-Fys. Medd. 29, No. 16 (1955); in: Deformatsiya atomnykh yader (Nuclear Deformations), translated from English, Moscow, IIL, 1958, p. 232.
[3]P. É. Nemirovskiĭ and V. A. Chepurnov, Yad. Fiz. 3, 998 (1966) [Sov. J. Nucl. Phys. 3, 730 (1966)]; F. A. Gareev, S. P. Ivanova, and B. N. Kalinkin, Izv. AN SSSR, ser. Fiz. 33, 1690 (1968) [Bull. USSR Acad. Sci. Phys. Ser.].
[4]C. Gustafson et al., Ark. Fys. 36, 613 (1967).
[5]S. G. Nilsson et al., Nucl. Phys. A131, 1 (1969).
[6]B. L. Andersen, Nucl. Phys. A162, 208 (1971).
[7]F. A. Gareev et al., Fiz. Élem. Chastits At. Yadra 4, 357 (1973) [Sov. J. Part. Nucl. 4, 148 (1973)].
[8]V. G. Soloviev, Phys. Lett. 21, 311 (1966).
[9]D. A. Arseniev et al., Phys. Lett. 40B, 305 (1972).
[10]R. Arlt et al., Fiz. Élem. Chastits At. Yadra 5, 843 (1974) [Sov. J. Part. Nucl. 5, 339 (1975)]; K. Ya. Gromov et al., Fiz. Élem. Chastits At. Yadra 6, 971 (1975) [Sov. J. Part Nucl. 6, 393 (1975)].
[11]G. Winter et al., Fiz. Élem. Chastits At. Yadra 4, 895 (1973) [Sov. J. Part. Nucl. 4, 365 (1974)].
[12]M. I. Baznat et al., Fiz. Élem. Chastits At. Yadra 4, 941 (1973) [Sov. J. Part. Nucl. 4, 384 (1973)].
[13]S. A. Hjorth et al., Nucl. Phys. A144, 513 (1970).
[14]K. E. G. Löbner and S. G. Malmskog, Nucl. Phys. 80, 505 (1966).
[15]C. F. Perdrisat, Rev. Mod. Phys. 38, 41 (1966).
[16]É. E. Berlovich, S. S. Vasilenko, and Yu. E. Novikov, Leningrad, Nauka, 1972.
[17]W. Andrejtscheff, Nucl. Phys. A178, 563 (1972).
[18]W. Andrejtscheff et al., Nucl. Phys. A226, 142 (1974).
[19]R. B. Begzhanov and F. S. Akilov, Vremennaya spektroskopiya atomnykh yader (Time Spectroscopy of Nuclei), Tashkent, Branch of the Academy of Sciences, 1972.
[20]Yu. K. Akimov et al., Bystrodeĭstvuyushchaya élektronika dlya registratsii yadernykh chastits (Fast Electronics for Particle Detection), Moscow, Atomizdat, 1970.
[21]W. Meiling and F. Stary, Nanosecond Pulse Techniques, Berlin, Akademie Verlag, 1969 (Russ. transl., Moscow, Atomizdat, 1973).
[22]W. Andrejtscheff et al., Nucl. Phys. A190, 489 (1972).
[23]F. Gabriel, Preprint ZfK-228, 1972.
[24]P. Manfrasz et al., JINR Preprint D6-7094, 1973, p. 170.
[25]T. Yamazaki and G. T. Ewan, Nucl. Phys. A134, 81 (1969).
[26]P. Manfrasz et al., Preprint ZfK-255, 1973.
[27]J. P. Davidson, Collective Models of the Nucleus, N. Y.-London, Academic Press, 1968.
[28]W. Andrejtscheff et al., Nucl. Phys. A220, 438 (1974).
[29]W. Andrejtscheff et al., Izv. AN SSSR, ser. fiz. 38, 2161 (1974) [Bull. USSR Acad. Sci., Phys. Ser.].
[30]D. R. Bès and Z. Szymanski, Nucl. Phys. 28, 42 (1961); Z. Szymanski, Nucl. Phys. 28, 63 (1961).
[31]B. Andersen, Nucl. Phys. A196, 547 (1972); M. I. Chernej, F. A. Gareev et al., Phys. Lett. B27, 117 (1968); JINR Preprint E4-5877 (1971).
[32]W. Andrejtscheff et al., Phys. Lett. B44, 351 (1973).
[33]C. W. Reich, R. G. Helmer, and R. C. Greenwood, Nucl. Phys. A168, 487 (1971).
[34]K. E. G. Löbner et al., Nucl. Data A7, 495 (1970).
[35]R. Piepenbring, Thesis, Strasbourg, 1966; G. Monsonego and R. Piepenbring, Phys. Lett. 21, 180 (1966); Nucl. Phys. 78, 265 (1966).
[36]A. Faessler et al., Nucl. Phys. 85, 670 (1966).
[37]F. M. Bernthal and J. O. Rasmussen, Nucl. Phys. A101, 513 (1967).
[38]F. Weller, Fortschritte Phys. 19, 201 (1971).
[39]M. E. Voĭkhanskiĭ, Gamma-luchi (Gamma Rays), Moscow-Leningrad, AN SSSR, 1961, p. 5.
[40]K. E. G. Löbner, Phys. Lett. B26, 369 (1968).
[41]W. Andrejtscheff et al., Phys. Lett. B40, 92 (1972).
[42]O. G. Gadetskiĭ and N. I. Pyatov, JINR Preprint R-1907, 1964.
[43]F. A. Gareev et al., Yad. Fiz. 9, 308 (1969) [Sov. J. Nucl. Phys. 3, 182 (1969)].
[44]H. Schulz and H. J. Wiebicke, JINR Preprint, E4-4210 (1968).
[45]G. Winter, Private Communication.
[46]I.-L. Lamm, Nucl. Phys. A125, 504 (1969).
[47]J. P. Boisson and R. Piepenbring, Nucl. Phys. A168, 385 (1971).
[48]M. N. Vergnes and J. O. Rasmussen, Nucl. Phys. 62, 233 (1965).
[49]W. Meiling and F. Stary, Preprint ZfK-PhA 21, 1966.
[50]W. Andrejtscheff et al., Nucl. Phys. A225, 300 (1974).
[51]M. Fellah and T. F. Hammann, Phys. Rev. C10, 284 (1974).
[52]S. Wahlborn and J. Blomqvist, Nucl. Phys. A133, 50 (1969).
[53]W. Andrejtscheff, Preprint ZfK-243, 84 (1972); K. D. Schilling et al., Nucl. Phys. A208, 417 (1973).
[54]B. L. Birbrair and V. Fejfrlik, Yad. Fiz. 3, 406 (1966) [Sov. J. Nucl. Phys. 3, 293 (1966)].
[55]V. Feifrlík and J. Rízek, Nucl. Phys. A121, 153 (1968).
[56]P. Manfrasz, Private communication.
[57]M. I. Baznat and N. I. Pyatov, JINR Preprint, R4-6980, 1973.
[58]W. Andrejtscheff, K. D. Schilling, and P. Manfrass, Tables of γ-Ray Transition Probabilities, Atomic Data and Nuclear Data Tables, Vol. 16, No. 6, 1975.
[59]H. Prade et al., Preprint ZfK-260, 1973.
[60]M. Vetter, Z. Phys. 225, 336 (1969).
[61]W. Andrejtscheff and G. Winter, Preprint ZfK-223, 1971, p. 54.
[62]G. Winter et al., Nucl. Phys. A223, 320 (1974).
[63]P. Manfrass and W. Andrejtscheff, Nucl. Phys. A194, 561 (1972).
[64]P. Manfrass et al., Nucl. Phys. A226, 157 (1974).
[65]W. Andrejtscheff et al., Preprint ZfK-243, 1972, p. 138.
[66]W. Andrejtscheff et al., Preprint ZfK-262, 1973, p. 91.
[67]K. E. G. Löbner et al., Nucl. Phys. A197, 553 (1972).
[68]D. Burke et al., K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 35, 2 (1966).
[69]G. Winter et al., Nucl. Phys. A199, 1 (1973).
[70]M. E. Bunker and C. W. Reich, Rev. Mod. Phys. 43, 348 (1971).
[71]A. K. Kerman, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 30, No. 15 (1956).
[72]A. Tveter and B. Herskind, Nucl. Phys. A134, 599 (1969).
[73]A. Faessler, Nucl. Phys. 59, 177 (1964).
[74]M. I. Baznat and N. I. Pyatov, JINR Preprint R4-7907, 1974.
[75]W. Andrejtscheff and P. Manfrass, Phys. Lett. B55, 159 (1975).
[76]M. N. Vergnes, Nucl. Phys. 39, 273 (1962).
[77]T. Tamura, Nucl. Phys. A115, 193 (1968).
[78]S. G. Malmskog et al., Nucl. Phys. A103, 481 (1967).
[79]K. Neergård and P. Vogel, Nucl. Phys. A145, 33 (1970).
[80]W. Andrejtscheff et al., Phys. Lett. B60, 241 (1976).

Translated by Clark S. Robinson