Semimicroscopic description of the density of states of complex nuclei

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The article describes a semimicroscopic method of calculation of the density of nuclear states which permits taking into account quasiparticle, vibrational, and rotational forms of nuclear excitations. A brief discussion of other nonstatistical methods of density calculation is given. A discussion is given of the main results obtained by the semimicroscopic method: the dependence of the density on excitation energy, spin, and parity of the states; the nonstatistical behavior of the density of levels of certain nuclei at intermediate excitation energies; the role of collective motions in the correct description of the density. The experimental and theoretical values of the level density at the neutron binding energy are compared for a large number of nuclei from the region $50 < A < 250$. A comparison is made with statistical calculations.

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INTRODUCTION

It is unnecessary to demonstrate how important it is both for theoretical and experimental nuclear physics to be able to calculate correctly the density of excited levels of nuclei at various excitation energies. This problem has been worked on over a period of more than thirty years, and interest in it has not weakened. The reason for such attention must be sought in the fact that a large number of problems associated with calculation of nuclear level density have remained unsolved until recently. Another and no less basic cause lies in the constant attempt to check various nuclear models in their description of excitations at various energies, and to create a nuclear-structure model which would describe the properties of nuclear excitations over the widest possible range of energy. The principal test of such a model is undoubtedly the correct description of nuclear level density. An additional stimulus to creation of models which describe the structure of the high-lying part of the nuclear spectrum in more detail than the statistical model is the fact that with their aid it may be possible to search for specific causes of the numerous experimentally observed nonstationary phenomena in the properties of nuclei at high excitation energies.

The statistical approach to description of the properties of highly excited nuclei has certainly made possible explanation of many experimental facts during the decades of its existence.[1,2] It still remains the main theoretical instrument for analysis and interpretation of experimental data. Nevertheless the necessity of tracing in more direct form the relation between the characteristics of elementary nuclear excitations and the structure of the high-lying part of the nuclear spectrum is felt all the more urgently. It is just this situation which permits us to carry out (approximately, of course) the existing nonstatistical methods of calculation of nuclear level density. In a certain sense, these methods are more fundamental than the statistical methods, but for that reason they are also much more laborious.

In the current stage of development of nuclear theory both the statistical and nonstatistical methods of calculation of nuclear level density are based on identical concepts regarding the nucleus. For all methods the nucleus is a system of nucleons moving in an average field and interacting by means of certain effective forces. All approaches utilize practically identical model nuclear Hamiltonians. However, in the statistical model there are many additional assumptions and simplifications between the initial physical postulates and the final result. Some of these are of a physical nature—they are related to the possibility of application of the methods of statistical physics to a system of a finite (and not always large) number of particles. Other assumptions or simplifications are mathematical, for example, calculation of the integrals arising in the theory by the method of steepest descent. In nonstatistical combinatorial calculations this chain of intermediate simplifications is absent. In fact, if we know how to solve the problem of the spectrum of nuclear excitations, then the means of calculation of the level density by simple scaling of the number in some energy interval suggests itself. Such a calculation is simply difficult to carry out for purely technical reasons. Nevertheless, combinatorial calculations of the density appeared already at the beginning of the fifties.[3] It goes without saying that with the level of development of computational technique at that time these calculations were only of methodological and illustrative interest.

With the development of nuclear models, methods of calculation of the level density were also improved. After the exceptional importance of pairing in determination of nuclear properties became known[4] and it was possible to take into account its effect on the density of states in terms of the statistical model,[5] it was not long before the appearance of combinatorial calculations which used the same nuclear Hamiltonian. Thus, Kluger[6] attempted by direct calculation of the levels to obtain empirically corrections to the statistical formula for the energy dependence of the level density at low excitation energies. However, combinatorial calculations have been carried out in the most complete form by Hillman and Grover.[7] Their calculations were based on a nuclear Hamiltonian including the mean field (in the form of an oscillator potential of infinite depth) and a superfluid pairing interaction (here it was assumed

that $G_N = G_Z$). The densities of levels with various spins were calculated for various nuclei and excitation energies. Reasonable agreement with experiment was found. However, this aspect of the calculations is not of interest here. The main value of the work of Hillman and Grover is in comparison of the results of statistical and combinatorial calculations under identical initial physical assumptions. It was established that there is a rather substantial difference between the results of these two methods. The differences appear particularly in the dependence of the density on excitation energy and spin.

The principal deficiency of the combinatorial calculations is their great laboriousness. If a more realistic nuclear Hamiltonian is used, they become still more laborious. We can therefore welcome any attempt to avoid this laboriousness which at the same time preserves, if only in part, the detailed nature of the combinatorial calculations.

One of the methods developed with this aim is the method of spectral distributions.[8] In contrast to traditional spectroscopic calculations, in which the eigenfunctions of the nuclear Hamiltonian H are expanded in some basis and diagonalization is carried out, in this method the inverse problem is considered, i.e., the basis wave functions are expanded in the eigenfunctions of H. The squares of the moduli of the coefficients of this expansion give the spectral distributions associated with the energy moments, which are expressed in terms of expectation values of powers of the Hamiltonian in the chosen basis and which can be calculated without knowledge of the eigen wave functions.

In this general formulation, this approach is completely equivalent to the procedure of diagonalization of the Hamiltonian in the space of states of high dimensionality, and is in no respect easier. Therefore in the method of spectral distributions instead of the distributions of individual states one considers distributions averaged over a group of levels. If the spectral distributions are very complicated, then it is practically impossible to calculate a sufficient number of moments for their description. The basic assumption of this approach is that the averaged spectral distributions are extremely close to a Gaussian dependence and therefore in the calculations it is sufficient to calculate only the first four energy moments. In this case the level density $\rho(\mathscr{E}, M)$ with a fixed value of angular momentum projection M at excitation energy $\mathscr{E}$ is given approximately by the following expression:

$$\rho(\mathscr{E}, M) = \frac{\Omega(M)}{\sqrt{2\pi}\,\sigma(M)} \left\{ 1 + \frac{\beta_3(M)}{3!} H_3(z) + \frac{\beta_4(M)}{4!} H_4(z) \right\} \exp(-z^2/2),$$

where $z = [\mathscr{E} - \mathscr{E}(M)]/\sigma(M)$; $\Omega(M)$ is the total number of states with angular momentum projection M; $\mathscr{E}(M)$, $\sigma(M)$, $\beta_3(M)$, $\beta_4(M)$ are expressed in terms of the first four energy moments of the Hamiltonian; $H_3(z)$ and $H_4(z)$ are Hermite polynomials. The expressions for these quantities, which relate them to the matrix elements of the effective interaction, and also a description of the method of calculation, can be found in Ref. 8 and 9.

The results of calculations in terms of the spectral distribution method are in good agreement with results

obtained by diagonalization on the basis of the shell model for (s, d)- and (f, p)-shell nuclei. Because of algebraic difficulties, it is not possible to construct directly a distribution with a fixed value of angular momentum by the method of spectral distributions. Therefore the functional dependence $\rho(I)$ obtained here is the same as in the statistical models, but in contrast to the latter in calculation of $\rho(\mathscr{E})$ and σ^2 the long-range part of the residual nuclear forces is also taken into account and in addition a completely different calculation technique is used.

The method described has been used for calculation of the level density and of the dependence of ρ on the spin and parity of the states and the excitation energy for several (s, d)- and (f, p)-shell nuclei. Deformed nuclei have not been studied by this method. In addition the relation between the spectral distribution method and the traditional statistical methods has not been established. The method and its possibilities are discussed in more detail in Refs. 8 and 9. In these articles there has been an attempt to approach in a unified manner the description of both low-lying nuclear states and highly excited states with a rather general form of residual nuclear forces. In combinatorial methods, as also in the statistical model, the starting point has been a nuclear Hamiltonian which at the present level of development of nuclear theory cannot be considered satisfactory for description of weakly excited levels. For this reason in both methods there is a substantial gap between the levels of theoretical understanding of the structure of the low-lying and high-lying parts of the nuclear spectrum.

One of the problems unsolved up to the present time in the theory of the density of nuclear states has been the question of the effect of collective excitations. It is known that collective excitations play an important role in formation of the structure of the low-lying part of the nuclear spectrum. The long-range components of nuclear forces, which are responsible for the existence of these excitations, determine many qualitative features of the spectrum also at high energies (for example, the giant quadrupole resonance). In order to take into account the effect of these forces on the level density, it is necessary to use a nuclear Hamiltonian which includes the long-range effective interaction. This problem has been solved by purely nonstatistical methods of calculation of the density. One of these has just been discussed, and another, proposed by Solov'ev and Malov,[10] is based on an extended variant of the superfluid model of the nucleus[4] and has been used repeatedly to study a wide range of questions associated with nuclear level density.[11-17]

The present review summarizes the results of these studies. In addition, a comparison is made with the results of the statistical model and, where it appeared to be useful, with calculations carried out by other nonstatistical methods.

1. DESCRIPTION OF THE SEMIMICROSCOPIC METHOD OF CALCULATION OF THE DENSITY OF EXCITED STATES

The method of calculating the density of excited nu-

clear levels proposed in Ref. 10 has received the name semimicroscopic, since it is based substantially on the picture of nuclear excitations which arises in terms of the semimicroscopic superfluid model of the nucleus.[4] We briefly list the main postulates of this model.

The nuclear Hamiltonian in the superfluid model consists of three parts:

$$H = H_{sp} + H_{pair} + H_Q, \qquad (1)$$

where H_{sp} is the single-particle nuclear potential describing the mean field for the proton and neutron system; H_{pair} is the pairing interaction in the particle—particle channel, which leads to pairing correlations of the superconducting type; H_Q is the long-range residual interaction in the particle-hole channel. It leads to appearance of collective vibrational excitations in the nuclear spectrum. The ordinary form of this interaction consists of factorized quadrupole—quadrupole and octupole—octupole forces.

The Hamiltonian (1) permits in principle a description of vibrational and quasiparticle excitations in spherical and deformed nuclei. However, for description of the rotational properties of deformed nuclei it is necessary to include additional terms in Eq. (1).

If we neglect the long-range forces H_Q, the excitation pattern of even—even nuclei turns out to be extremely simple: two-quasiparticle states, four-quasiparticle states, and so forth. The long-range quadrupole—quadrupole and octupole—octupole forces lead to a strong interaction of pairs of quasiparticles coupled to angular momenta 2^+ or 3^- (in deformed nuclei with quantum numbers $K^\pi = 0^+$, 2^+, 0^-, 1^-, 2^-, 3^-). As a result there appear strongly collectivized low-lying states with corresponding quantum numbers. They are interpreted as vibrational. Now the correlated states with $I^\pi = 2^+$, 3^- (or the corresponding K^π) are described in the phonon language and are considered as bosons excitations. The energy of single-phonon states $\omega_{\lambda i}$ is found by solution of well known equations by the method of approximate second quantization:

$$(2\lambda + 1)/\varkappa^\lambda = \sum_{j'j} [f^\lambda(j'j) u_{j'j}^{(+)}]^2 \, \varepsilon_{j'j}/(\varepsilon_{j'j}^2 - \omega_{\lambda i}^2). \qquad (2)$$

Here $\varkappa^\lambda$ is the constant of the multipole interaction; $\varepsilon_{j'j}$ is the energy of the two-quasiparticle state $(j'j)_\lambda$; $f^\lambda(j'j)$ is the single-particle matrix element of the multipole operator; $u_{j'j}^{(+)} = u_j v_{j'} + u_{j'} v_j$, where u_j and v_j are the coefficients of the Bogolyubov transformation.

States with $I^\pi = 0^+$ in spherical nuclei are described as pairing vibrations generated by the pairing interaction H_{pair}. In deformed nuclei the structure of the 0^+ levels is more complex, since in addition to H_{pair} it is determined also by the quadrupole—quadrupole interaction. The corresponding equations are rather cumbersome, and we will not write them out here (for example, see the book by Solov'ev[4]).

The wave functions of the single-phonon states are a superposition of two-quasiparticle components. Thus, vibrational motion arises as the result of correlations in the motion of quasi-particles. The lowest root of Eq. (2) turns out, as a rule, to be the most collective, i. e., many two-quasiparticle states contribute appre-

ciably to its wave function. The higher single-phonon states in most cases are not so collective and their structure receives contributions only from a few and sometimes only one two-quasiparticle component. With decrease of the constant $\varkappa^\lambda$ the collective states disappear,[4] i. e., as $\varkappa^\lambda \to 0$ the wave functions of the single-phonon states reduce to one of their two-quasiparticle components, and the photon energy $\omega_{\lambda i} \to \varepsilon_{j'j}$.

Thus, in the spectrum of an even—even nucleus in addition to two-quasiparticle, four-quasiparticle, and so forth, excitations there are now also single-phonon, two-phonon, etc., excitations. It is also possible to have mixed states consisting of various numbers of quasiparticles and phonons. In an odd nucleus in addition to all these excitations there is added an odd quasiparticle, and in an odd—odd nucleus—neutron and proton quasiparticles.

All of the excitations of an even—even nucleus can be described by means of phonons. Here a fundamental fact is that the wave function of a two-quasiparticle state can be considered as a limiting case of the wave function of a one-photon state. For conversion to photon language which it is sufficient to include in the Hamiltonian long-range forces which will generate phonons with given λ^π (or K^π). As such forces we can choose (for example see Refs. 11—15) in deformed nuclei multipole—multipole forces with $\lambda > 3$, and in spherical nuclei in addition to these forces also (spin-multipole)—(spin-multipole) forces. The latter are necessary in order to obtain phonons with $I = \lambda$ and $\pi = (-1)^{\lambda+1}$, which in spherical nuclei cannot be generated by multipole—multipole interaction. Introduction of such components of residual forces can be justified by multipole expansion of the residual central interaction between quasiparticles.[4] Here it remains an open question how to calculate their constants. The existing studies[18] have not yet led to completely unambiguous results. However, this question is not very important for us at this time. Taking the constants of the new forces to be small, we shall consider their inclusion in the Hamiltonian as a *formal* procedure. Limitations arising here and due to the use of phonon language in construction of the wave functions will be discussed in Section 2.

The picture which we have drawn of the spectrum of excited states of a nucleus is highly idealized. Actually the elementary nuclear excitations interact with each other. There is an interaction of quasiparticles with phonons, which leads to an appreciable change in the properties of the nuclear levels in comparison with the ideal picture We note that the strength of this interaction is calculated microscopically without introduction of additional parameters into the Hamiltonian (1). The interaction of quasiparticles with phonons is imposed on the Hamiltonian structure. Interaction of the various types of nuclear excitations leads to complication of the structure of the states and this complication increases rapidly as we move up the energy scale. In order to describe correctly the structure of states with increasing excitation energy $\mathcal{E}$, it is necessary to take into account an ever increasing number of quasiparticle and phonon excitations. Here it turns out to be useful to use the phonon language for description of all many-quasi-

particle excitations (with an even number of quasiparticles). In some earlier articles,[10,19] for description of states with intermediate and high excitation energies, we have formulated a model which defines the general principles of the semimicroscopic approach to understanding nuclear properties at these energies.[20] The basic equations of this model have been obtained and, since their exact solution is an extremely difficult computational problem, effective approximate methods of solution of these equations have been developed.[10,21]

Studies[10] of the basic equations of the model taking into account interaction of quasiparticles with phonons have shown that the number of roots of these equations in some energy interval $\Delta\mathscr{E}$ is equal to the number of basis multiphonon states in the same interval. It can be stated that the interaction of quasiparticles with photons has an insignificant effect on such an integrated characteristic as the level density. This fact is one of the bases of the semimicroscopic method of calculation of nuclear level densities.

We note that in all combinatorial calculations there is implicit use of the assumption that the discarded residual interactions insignificantly affect the density. These calculations utilize simplified nuclear models whose wave functions do not pretend to describe correctly the structure of highly excited states. The density calculations proceed from the fact that the total number of levels in these models should be the same as for a correct description. However, a more accurate treatment necessarily leads to a redistribution of the levels of the simplified model. The problem is to take into account the most important part of the interactions without having the more rigorous treatment affect the energies of the states substantially. It is apparently possible to achieve this in the proposed semimicroscopic method of calculation of the density of nuclear excitations.

The method, for which we are about to give a direct description, consists of the following. Knowing the energies of all single-phonon states of an even—even nucleus, and these we can calculate, we can determine the energy of any multiphonon state with specified angular momentum and parity (or K^π in a deformed nucleus). Now, desiring to find the density of states with given quantum numbers I^π (or K^π) at an energy $\mathscr{E}$, we calculate all possible one-phonon and multiphonon states of the type:

$$
\begin{aligned}
&\omega_{Ii}, \\
&\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2}, \\
&\omega_{\lambda_1' i_1'} + \omega_{\lambda_2' i_2'} + \omega_{\lambda_3' i_3'}, \text{etc.},
\end{aligned}
\tag{3}
$$

in some energy interval $(\mathscr{E}, \mathscr{E} + \Delta\mathscr{E})$. Of course, the angular momentum and purity of all states (3) must coincide with I^π. The number of states of the type (3) in the interval $\Delta\mathscr{E}$ we shall designate by n. Then the quantity $\rho = n/\Delta\mathscr{E}$ also gives the desired level density. In spherical nuclei in calculation of the number n it is necessary to take into account the degeneracy of multiphonon states in the intermediate angular momentum. The energies of the states (3) are reckoned from the ground state of the even—even nucleus, which is considered as a photon vacuum.

In an odd nucleus in the interval $(\mathscr{E}, \mathscr{E} + \Delta\mathscr{E})$ we count states of the type

$$
\begin{aligned}
&\varepsilon_I - \mathscr{E}_0, \\
&\varepsilon_j + \omega_{\lambda i} - \mathscr{E}_0, \\
&\varepsilon_{j'} + \omega_{\lambda' i'} + \omega_{\lambda'' i''} - \mathscr{E}_0, \text{ etc.},
\end{aligned}
\tag{4}
$$

Here $\mathscr{E}_0$ is the energy of the ground state of the odd nucleus. The phonon energies are taken the same as in the neighboring even—even nucleus with mass number smaller by unity. In an odd—odd nucleus, correspondingly, we count states of the type:

$$
\begin{aligned}
&\varepsilon_{j_n j_p} - \mathscr{E}(j_n^0 j_p^0), \\
&\varepsilon_{j_n' j_p'} + \omega_{\lambda i} - \mathscr{E}(j_n^0 j_p^0), \\
&\varepsilon_{j_n'' j_p''} + \omega_{\lambda' i'} + \omega_{\lambda'' i''} - \mathscr{E}(j_n^0 j_p^0), \text{ etc.}
\end{aligned}
\tag{5}
$$

Here $\mathscr{E}(j_n^0 j_p^0)$ is the energy of the ground state of the odd—odd nucleus. Since among the phonons there are collective ones which describe vibrations of the nuclear surface, this method automatically takes into account the effect of vibrational motion on the level density. However, in deformed nuclei there is an additional type of collective motion—rotation, which is not taken into account in this method. If we count states of the types (3)—(5) in a deformed nucleus, we obtain the density of only the *internal* states of the nucleus, characterized by the projection of the angular momentum on the symmetry axis of the nucleus, and not by the total angular momentum. At the same time the experiments measure the density of levels with a given angular momentum and therefore in our discussion it is necessary to include also the rotational motion. This can be done purely phenomenologically.[14,22] We construct on each internal state of a deformed nucleus with quantum numbers K^π a rotational band whose energy is described by the formula

$$
E_{\text{rot}} = [I(I+1) - K^2]/2T.
\tag{6}
$$

Thus, the nucleus is considered to be an ideal rotator and the mutual influence of the internal motion and the rotation is completely neglected. If we designate the density of the internal states of a deformed nucleus with quantum numbers K^π at energy $\mathscr{E}$ as $\rho(\mathscr{E}, K^\pi)$, then the density of levels of the same nucleus with spin and parity I^π can be found from the formula

$$
\rho(\mathscr{E}, I^\pi) = \sum_{K=K_0}^{I} \rho\left(\mathscr{E} - \frac{1}{2T}[I(I+1) - K^2], K^\pi\right),
\tag{7}
$$

in which $K_0 = 0$ for an even—even nucleus and $K_0 = 1/2$ for an odd nucleus. In the density of states with $I = 0$ and $1/2$, rotations make no contribution.

On this point we end the description of the basic postulates of the semimicroscopic method of calculation of nuclear level densities. What are the advantages of this method? By this method it is possible directly, and without recourse to assumptions of an especially mathematical nature, to calculate the density of nuclear states, and to relate the parameters describing nuclear motion at low excitation energies directly with the density of nuclear levels. It is not necessary to introduce any additional parameters in the transition to intermediate and higher excitation energies. Finally, the meth-

od permits taking into account the effect of collective nuclear motions on the density of levels. These items comprise the favorable aspects of the proposed method. However, before going over directly to the results of the calculations, we must dwell on such questions as the field of applicability of the semimicroscopic method and the accuracy which its use can provide in description of the density.

2. REGION OF APPLICABILITY AND ACCURACY OF RESULTS OBTAINED WITH THE SEMIMICROSCOPIC METHOD OF DENSITY CALCULATION

Let us start by considering some basic physical assumptions which form the basis of the semimicroscopic method of calculation of nuclear level densities. In calculation of the level density we take the parameters of the Hamiltonian to be those obtained in analysis of the properties of low-lying nuclear states.[23] This means that we are not taking into account changes in the mean field parameters (such as deformation) and the residual interaction constants with increasing excitation energy. This approximation is completely valid in nuclei close to semimagic and strongly deformed at excitation energies up to B_n. However, in nuclei of the transition region (for example, Nd, Ba, and Sm or Hg and Pt) the change in the mean-field parameters with excitation energy may turn out to be significant.[24] Also of interest here is the question of the effect on the density of states of the change in the correlation function C with increasing excitation energy. Calculation of the energy of multiphoton states has been carried out with the correlation function of the ground state of an even—even nucleus. At the same time it is known from calculations with the statistical model that with increase of excitation energy the correlation function C decreases and at some point goes to zero, which serves as the basis for the statement that a phase transition exists. If this is true, then we reduce somewhat the significance of the level density and increase that of C. For practical purposes the question is how strongly does the decrease in C affect the level density. From the microscopic point of view the decrease of the correlation function with increasing excitation energy is the result of the blocking effect. Strictly speaking, the correlation function depends on the structure of a many-quasiparticle state, i.e., on the number of quasiparticles, their components, and their quantum numbers. With increasing energy, C changes nonuniformly. Thus, for states with a given number of excited quasiparticles C *increases* with increasing excitation energy, approaching its value for the ground state. At the same time for a given excitation energy C will be different for different states: For states with a larger number of quasiparticles it will be smaller. Therefore, in discussing the correlation function at a given energy it is necessary to have in mind some average value of C, weighted over all possible states which lie near this energy. For energies at which the number of quasiparticles in excited states is small, the change of C will be large. There is a further indication that the blocking effect is somewhat overestimated. In describing excited states by means of quasiparticles, one makes use of wave functions which are not eigenfunctions of the operator of the number of particles. If

we correct this, i.e., if we project our wave functions on the space of the eigenfunctions both of the Hamiltonian H_{pair} and the operator of the number of particles, it turns out that the blocking effect is compensated to a significant degree—the effective value of C increases.[25] Therefore within the accuracy of the method (we will discuss this further below) we can use the approximation $C = \mathrm{const}$.

The next question is related to the inclusion of rotational states in the calculation. We will consider the quantum number K an integral of the motion, although this is not valid for states of intermediate and high excitation energies. Physically this is equivalent to the assumption that the various interactions (for example, the Coriolis interaction), although they mix states characterized by different K, nevertheless change the total number of states in some energy interval only weakly. It is evident that here we have an approximation equivalent to discarding the interaction of quasiparticles with phonons. The means described for taking into account rotations is very crude. It is furthest from our thoughts that in this way we can actually describe the structure of highly excited states with inclusion of rotation. The proposed method must rather be considered as a method of *effective* inclusion of rotation in the region where it still exists although in a greatly concealed form.[22]

If we separate the collective degrees of freedom and consider them independently from the noncollective ones (especially when this is done phenomenologically), we sooner or later encounter the problem of excess degrees of freedom of the nucleus. This can very substantially affect the level density. However, effects of this type[36] appear only at very high energies (several tens of MeV).

More important for us are the difficulties which have their source in the phonon language, which is used in formation of many-quasiparticle states. Strictly speaking, in constructing many-phonon states even from collective phonons it is necessary to recall the Pauli principle, since our phonons have a *fermion* structure. In particular, this leads to a peculiar limitation on the complexity of a many-phonon state. For example, in the region $A \approx 90$ it is meaningless to discuss eight-phonon or more complicated states of the nucleus,[26] and it is even more dangerous to use the language of noncollective phonons. Actually, in the chain of states described in Section 1 an appreciable number of excess states are obtained. Part of them appear as the result of violation of the Pauli principle, and these differ from each other only in the order of addition of the angular momenta of the quasiparticles comprising the phonons, although they have different energies. The number of such excess levels rises rapidly with excitation energy. At energies of the order of the neutron binding energy the number of such states is 30—50% in spherical nuclei, and in deformed nuclei it is appreciably smaller, as small as 15—25%. The relative number of excess states depends also on the spin and parity of the levels whose density is being calculated and in addition fluctuates appreciably as a function of the excitation energy. These difficulties limit very substantially the possibilities of our method.

In addition to the fundamental limitations which we have enumerated, the accuracy of the semimicroscopic method depends on the errors in determination of the constants of the Hamiltonian. We did not carry out any further fitting of its parameters, but took them the same as in the calculations of weakly excited states. The parameters of the Woods—Saxon potential for deformed nuclei can be found in the reviews of Gareev et al.,[27] those for spherical nuclei in Ref. 28, and the correlation functions and chemical potentials in Ref. 29. The constants of the quadrupole—quadrupole and octupole—octupole forces have been chosen so as to obtain the experimental energies of the lowest vibrational states in the random-phase approximation (see Eq. (2)). The other constants have no practical significance in the calculations, since they were taken to be very small. It is clear, however, that the parameters of the Hamiltonian are known to us only with limited accuracy. Thus, the pairing constants G_N and G_Z in spherical nuclei can be determined from the pairing energies with an accuracy of 10—15%. A check showed that a change of the parameters of the mean field and the pairing constants within reasonable limits in these nuclei leads to a change in level density by a factor of 1.5—2.0. In deformed nuclei, where the mean-field parameters, like the quantities G_N and G_Z, are determined with much greater accuracy, the deviations in the level density as the result of variation of these parameters are small.

The role of the moment of inertia $\mathcal{Y}$, which appears as an additional parameter on inclusion of rotation, is not so noticeable. Its value can be chosen equal to the solid-state value $\mathcal{Y} = \mathcal{Y}_{\mathrm{rig}} = 2mAR^2/5$. In most nuclei the results of calculations depend weakly on $\mathcal{Y}$. This is due to the fact that for low spins ($I \leqslant 5$) (and it is just these spins which occur in most resonances excited in capture of slow neutrons) the value of ρ does not change greatly with change of $\mathcal{Y}$. For example, with such values of I a decrease in the moment of inertia by a factor of two leads to a change in the density by 10—15%. The value of the moment of inertia is more important at high spins and consequently in calculation of the spin dependence $\rho(I)$. Since the solid-state value $\mathcal{Y}_{\mathrm{rig}}$ is the upper limit for the moment of inertia, a decrease in it will result in a reduction of the density, and for larger I the effect will be greater. This will lead to a more rapid falloff in the spin dependence of ρ, without changing its qualitative nature. Here we must consider such technical details as the maximum number of phonons which we take into account, the maximum spin of the phonon, and the interval of averaging.

The number of phonons which are taken into account in many-phonon states is limited for the following reasons. At a given excitation energy there is a maximum in the dependence of the number of states on the number of phonons, and this maximum shifts toward a larger number of phonons as the excitation energy increases. At the neutron binding energy this maximum in odd nuclei occurs at three-phonon or four-phonon states, and in even—even nuclei it occurs at four-phonon or five-phonon states. The contribution of more complex states is small (Table I). We note also that with increase of the complexity of the states the number of spurious states also increases.

TABLE I. Contribution of states with different numbers of phonons to the total number of states at various excitation energies.

Nucleus	Type of excited state	Contribution to total number of states, %				
		3 MeV	4 MeV	5 MeV	6 MeV	7 MeV
^{158}Gd	ω_g	32	9.4	2.3	0.5	0.1
	$\omega_{g_1} + \omega_{g_2}$	41	84	38	35	9
	$\omega_{g_1'} + \omega_{g_2'} + \omega_{g_3}$	27	6.6	54	44.5	45.9
	$\omega_{g_1''} + \omega_{g_2''} + \omega_{g_3'} + \omega_{g_4}$	0	0	4	19	31.8
	$\omega_{g_1'''} + \omega_{g_2'''} + \omega_{g_3''} + \omega_{g_4'} + \omega_{g_5}$	0	0	1.7	1	13.2
^{159}Gd	$\mathcal{E}(\nu) + \omega_{g_1} - \mathcal{E}_0$	66	34.6	12.6	1.1	0.2
	$\varepsilon_{\nu'} + \omega_{g_1'} + \omega_{g_2} - \mathcal{E}_0$	28.1	51.7	59	59	31.8
	$\varepsilon_{\nu''} + \omega_{g_1''} + \omega_{g_2'} + \omega_{g_3} - \mathcal{E}_0$	5.9	13.7	22.7	26.9	50.5
	$\varepsilon_{\nu'''} + \omega_{g_1'''} + \omega_{g_2''} + \omega_{g_3'} + \omega_{g_4} - \mathcal{E}_0$	0	0	5.7	13	17.5

The maximum angular momentum of a phonon depends on the mass-number region in which the nucleus is located, and on the structure of its single-particle spectrum. In deformed nuclei a value $K_{\max} = 7$ is chosen. In spherical nuclei for the regions $50 \leqslant A \leqslant 90$, $90 \leqslant A \leqslant 150$ and $190 \leqslant A \leqslant 210$ we have $I_{\max} = 6$, 8, and 9, respectively. Phonons with high angular momenta are important mainly in calculation of the spin dependence $\rho(I)$.

In order to calculate the density, we count the number of states in some energy interval near the excitation energy of interest. This interval is called the interval of averaging. It is obvious that the size of the interval will affect our numerical results. The interval of averaging should be sufficiently small that ρ can be considered constant over the interval. On the other hand, it should contain a sufficient number of states to smooth the random fluctuations in their distribution, i.e., it is necessary that $\Delta\mathcal{E} \times \rho \gg 1$. In odd spherical nuclei from the region $50 \leqslant A \leqslant 90$ we chose $\Delta\mathcal{E} = 300$ keV, from the region $90 \leqslant A \leqslant 150 - \Delta\mathcal{E} = 100$ keV, and in the isotopes of Hg and Pt—50 keV. For even—even nuclei from these same regions of mass number the values of $\Delta\mathcal{E}$ are respectively 100, 50, and 30 keV. In deformed nuclei, where the density of states is very high, $\Delta\mathcal{E} = 5—10$ keV.

In concluding this discussion, before turning to the direct results, we make the following observation. The semimicroscopic method of calculation of the density is convenient at energies of the order of the neutron binding energy and below, i.e., for $\mathcal{E} \leqslant 7—9$ MeV. At higher energies the results appear to us unreliable. In addition, with increasing excitation energy our results approach the statistical results, which require much less effort to obtain. In spite of the absence of a special fitting of the parameters, as will be seen below, we obtain a good description of the experimental data. Nevertheless the accuracy of our results must be estimated as a factor in the range 1.5—2.0 in deformed nuclei and 2—4 in spherical nuclei. This is the accuracy which applies to all of the results which follow.

3. LEVEL DENSITY AT THE NEUTRON BINDING ENERGY. DEPENDENCE OF THE DENSITY ON MASS NUMBER

The method described in Section 1 has been used by

us primarily for calculation of the density of excited states near the neutron binding energy, since it is for just this energy that the density of levels has been measured in most nuclei. The experimental data have been taken, as a rule, from Refs. 30—38. We note that in some cases there is an appreciable spread between the results of different experimental groups. For z-odd, even—even, and odd—odd nuclei the experiments measure the combined density of states for two spins, and therefore the theoretical data in these nuclei are also given for two spins. They were obtained from the following formula:

$$D = \{\rho(I + 1/2) + \rho(I - 1/2)\}^{-1}.$$

Here D is the average distance between levels with spins $I - 1/2$ and $I + 1/2$; I is the spin of the target nucleus. The theoretical results and the experimental data are given in Tables II—IV. The experimental and theoretical results as a whole agree satisfactorily. The calculations satisfactorily reflect the variation of D in the transition from element to element and from one isotope to another. Particularly good agreement is observed for deformed nuclei, where the difference between D_{theor} and D_{exp} in most cases does not exceed 50%. The differences in spherical nuclei are found to be greater. Here we single out nuclei having one nucleon less than a magic number, and also the isotopes of mercury and platinum. The reasons for the substantial deviations of the theoretical results from the experimental values in these nuclei lie in the large number of spurious states, and for near-magic nuclei in addition the contribution of vibrational excitations is overestimated.

The differences between D_{theor} and D_{exp} have a systematic nature also in the transition region of nuclei for the isotopes of Nd and Sm. It is well known that there is a lack of rigidity of the shape of these nuclei with respect to β and γ vibrations, which leads to a rich spectrum of excited states characterized for this reason by the different equilibrium shapes of the nucleus. We have not taken this fact into account, and the nuclei have been considered spherical.

Døssing and Jensen[36] have attempted to describe nuclei of the transition region $A \approx 140-150$ in terms of the statistical model with phenomenological inclusion of rotation (see Section 1). It turned out that treatment of these nuclei as deformed leads to an appreciable excess of the theoretical density value over the experimental value. At the same time, on considering them as spherical, Døssing and Jensen[36] found, as in our work, sharply reduced values of D_{theor}. In these calculations the mean field was described by a Woods—Saxon potential.

In comparison with results of Ref. 36, our calculations describe the experimental data better on the average. Thus, for the rare earth elements the statistical-model calculations systematically give a level density 3—4 times lower. In the opinion of the authors, this difference can be compensated by inclusion of vibrational excitations. This statement, which is based on estimates of Ref. 22, is in agreement with the results of our investigations. However, the calculations made in terms of the statistical model with use of the Nilsson potential,[37] give results extremely close to ours, al-

TABLE II. Experimental and theoretical values of average distance between $1/2^+$ levels at the neutron binding energy B_n.

Compound nucleus	B_n, MeV	D, experiment, keV			D, theory keV
		[30, 33—35]	[31]	[32]	
^{51}Ti	6.38	18±6	—	120	200
^{53}Cr	8.17	46±7	47	44	100
^{55}Cr	6.25	48$^{+37}_{-24}$	66	23.5	400
^{55}Fe	9.30	21±4	20	25	30
^{57}Fe	7.64	21±4	25	29	14
^{59}Ni	9.00	21±6	22	27	15
^{61}Ni	7.82	21±4	17	23	17
^{63}Ni	7.14	19.5	19	19.5	21
^{65}Ni	6.40	28.5	28	28.5	36
^{65}Zn	7.99	3.4±0.9	3.6	1.8	6.9
^{67}Zn	7.05	5.6±1.9	6.0	5.0	17
^{69}Zn	6.50	20	10	—	18
^{71}Ge	7.41	2.0±0.8	1.33	1.7	2.7
^{73}Ge	6.78	3.9±1.5	1.55	2.1	4.8
^{75}Ge	6.49	8.5$^{+4.7}_{-4.3}$	3.9	8.5	5.0
^{77}Ge	6.03	8.0±0.8	4.2	8.0	5.6
^{75}Se	8.03	0.2$^{+0.6}_{-0.1}$	0.37	0.25	1.0
^{77}Se	7.41	1.2±0.6	0.70	1.2	2.9
^{79}Se	6.97	4.5±1.0	1.0	3.7	3.8
^{81}Se	6.71	1.6±0.6	1.2	4.3	5.0
^{83}Se	5.99	6.9±1.1	6.7	7.0	1.1
^{85}Sr	8.48	0.35±0.12	0.4	—	1.6
^{87}Sr	8.44	2.1±1.0	1.0	—	2.1
^{89}Sr	6.39	12±2	12	55	10
^{91}Zr	7.19	3.3±0.8	5.0	4.5	10
^{93}Zr	6.75	3.4±1.1	2.5	1.2	1.8
^{95}Zr	6.47	3.3±0.9	2.4	2.4	1.8
^{97}Zr	5.57	1.1±0.3	—	—	5.3
^{97}Mo	6.82	1.2±0.5	0.29	0.29	0.98
^{99}Mo	5.92	0,790$^{+0.740}_{-0.395}$	0.19	—	2
^{101}Mo	5.39	0,400±0.075	0.18	0.180	4.3
^{113}Gd	6.54	0.200	0.198	0.200	1.1
^{113}Sn	7.74	0.140±0.05	0.02	0.108	0.045
^{115}Sn	7.54	0.320±0.09	0.30	0.150	0.336
^{117}Sn	6.94	0.250±0.04	0.25	0.18	0.962
^{119}Sn	6.48	0.730±0.180	0.60	0.18	1.3
^{121}Sn	6.18	0.240±0.05	0.18	0.20	3.2
^{123}Sn	5.93	0.400	1.7	0.4±0.2	4.8
^{125}Sn	5.77	0.250	2.5	0.40	6.7
^{123}Te	6.94	0.130±0.008	0.132	0.036	0.13
^{129}Te	6.12	0.55	0.262	—	0.95
^{131}Te	5.89	5.7+0.8	0.872	3.5±0.6	0.9
^{131}Ba	7.63	0.120	0.055	—	0.036
^{135}Ba	7.20	0.380	0.14	—	0.08
^{137}Ba	6.95	3.8$^{+2.8}_{-1.9}$	0.60	0.80	0.224
^{139}Ba	4.72	9.6±3.4	10	10	50
^{137}Ce	7.84	0.058±0.012	0.058	—	0.032
^{139}Ce	7.49	0.200	—	—	0.076
^{141}Ce	5.44	3.0±1.0	3.0	—	50
^{143}Ce	5.10	1.0±0.2	1.0	1.0	12
^{143}Nd	6.10	1.0±0.25	0.415	—	2.2
^{145}Nd	5.97	0.520±0.07	0.537	—	2.2
^{147}Nd	5.14	0.310±0.043	0.211	—	3.2
^{149}Nd	4.94	0.200±0.021	—	—	2.6
^{145}Sm	6.90	0.200±0.04	—	—	3
^{149}Sm	5.86	0.090±0.015	—	—	1.3
^{153}Sm	5.89	0.0518±0.0015	0.052	—	0.063
^{157}Gd	6.35	0.047±0.004	0.047	—	0.042
^{159}Gd	5.94	0.085±0.009	0.085	—	0.067
^{163}Dy	6.25	0.042±0.006	0.072	0.130±0.05	0.056
^{165}Dy	5.71	0.080±0.026	0.20	0.070	0.130
^{167}Er	6.44	0.0384±0.00032	0.038	—	0.057
^{169}Er	5.99	0.0953±0.0017	0.095	—	0.12
^{173}Yb	6.48	0.0703±0.0026	0.063	—	0.042
^{175}Yb	5.83	0.162±0.018	0.164	—	0.11
^{177}Yb	5.53	0.185±0.019	0.185	—	0.16
^{179}Hf	6.07	0.055±0.008	0.064	>0.0039	0.058
^{185}W	5.75	0.093±0.019	0.089	0.09	0.078
^{195}Pt	6.13	0.27±0.07	—	—	0.11
^{197}Pt	5.85	0.38±0.1	—	—	0.19
^{199}Pt	5.57	0.34±0.07	—	0.048	0.18
^{199}Hg	6.65	0.10±0.03	0.09	0.099	0.084
^{201}Hg	6.23	1.3±0.1	1.3	2.2	0.11
^{203}Hg	5.99	2.4	—	2.4	0.28
^{205}Pb	6.73	2.8±0.7	2.7	—	0.23
^{207}Pb	6.73	24±4	50	50	0.34
^{231}Th	5.12	0.011	0.0077	0.0077	0.018
^{233}Th	4.79	0.0124±0.0011	0.0167	0.0167	0.022
^{233}U	5.74	0.0142±0.0036	0.0042	0.0041	0.0059
^{235}U	5.31	0.01803±0.0073	0.0123	0.0106	0.011
^{237}U	5.12	0.027±0.009	0.0154	0.0173	0.0091
^{239}U	4.78	0.0181±0.0023	0.0208	0.0208	0.026
^{239}Pu	5.40	0.016±0.005	0.0117	0.0095	0.006
^{241}Pu	5.24	0.014±0.002	0.0125	0.0127	0.010
^{243}Pu	5.04	—	—	0.017	0.023
^{245}Cm	5.69	0.020±0.006	0.0137	0.0148	0.0063
^{247}Cm	5.16	—	0.038	0.0408	0.013
^{249}Cm	4.71	—	0.035	0.035	0.056
^{253}Cf	4.79	—	—	0.016	0.027

TABLE III. Experimental and theoretical values of average distance D between levels, for two spins of even—even nuclei at excitation energy B_n.

| Compound nucleus	B_n, MeV	$j\pi$	D, experiment, eV			D, theory, eV
			[30, 33—35]	[31]	[32]	
^{50}Ti	10.94	3⁻, 4⁻	$(3.6\pm0.9)\cdot10^3$	$6.0\cdot10^3$	—	$10.3\cdot10^3$
^{54}Cr	9.72	1⁻, 2⁻	$(3.2\pm1.1)\cdot10^3$	$5.7\cdot10^3$	—	$7.1\cdot10^3$
^{58}Fe	10.04	0⁻, 1⁻	$(5.9\pm1.5)\cdot10^3$	$1.5\cdot10^3$	—	$10\cdot10^3$
^{62}Ni	10.60	1⁻, 2⁻	$(2.3\pm0.4)\cdot10^3$	$1.4\cdot10^3$	—	$1.7\cdot10^3$
^{68}Zn	10.20	2⁻, 3⁻	720 ± 190	600	—	$2.7\cdot10^3$
^{74}Ge	10.20	4⁺, 5⁺	77 ± 9	62	—	280
^{78}Se	10.49	0⁻, 1⁻	150 ± 4	120	100	420
^{88}Sr	11.10	4⁺, 5⁺	210 ± 8	250	—	450
^{92}Zr	8.64	2⁺, 3⁺	250 ± 50	110	315	260
^{96}Mo	9.16	2⁺, 3⁺	100 ± 40	102	—	100
^{98}Mo	8.64	2⁺, 3⁺	120 ± 60	80	170	190
^{100}Ru	9.67	2⁺, 3⁺	200	34	55	80
^{102}Ru	9.21	2⁺, 3⁺	15 ± 4	18	16	90
^{106}Pd	9.55	2⁺, 3⁺	11.1 ± 1.7	9	13	80
^{112}Cd	9.40	0⁺, 1⁺	34 ± 6	26	26	130
^{114}Cd	9.05	0⁺, 1⁺	27 ± 3	25	25	210
^{116}Sn	9.56	0⁺, 1⁺	50 ± 20	—	50	330
^{118}Sn	9.33	0⁺, 1⁺	65 ± 15	45	25	230
^{120}Sn	9.11	0⁺, 1⁺	62 ± 12	70	30	150
^{124}Te	9.41	0⁺, 1⁺	33 ± 9	26	39 ± 14	20
^{126}Te	9.09	0⁺, 1⁺	46 ± 11	38	65 ± 25	34
^{132}Xe	8.93	1⁺, 2⁺	31 ± 1	35	31 ± 16	20
^{136}Xe	7.88	1⁺, 2⁺	500	—	—	63
^{136}Ba	9.23	1⁺, 2⁺	35 ± 9	36	51	20
^{138}Ba	8.54	1⁺, 2⁺	460^{+250}_{-230}	230	200	310
^{144}Nd	7.83	3⁻, 4⁻	19 ± 9	36	72	140
^{146}Nd	7.56	3⁻, 4⁻	25 ± 9	19	33	210
^{148}Sm	8.14	3⁻, 4⁻	7.9 ± 1.3	7.3	8.0	100
^{156}Gd	8.53	1⁻, 2⁻	1.99 ± 0.32	1.9	2.1	2.0
^{158}Gd	7.93	1⁻, 2⁻	6.3 ± 0.6	6.0	12	4.8
^{162}Dy	8.20	2⁺, 3⁺	2.55 ± 0.38	2.9	2.1	3.9
^{164}Dy	7.66	2⁺, 3⁺	9.6 ± 1.6	9.6	11.0	12
^{168}Er	7.77	3⁺, 4⁺	4.0 ± 0.4	4.1	3.0	7.2
^{172}Yb	8.14	0⁺, 1⁺	5.79	6.5	—	6.6
^{174}Yb	7.44	2⁻, 3⁻	7.81 ± 0.93	8.4	—	6.5
^{178}Hf	7.62	3⁻, 4⁻	3.2 ± 0.2	2.4	2.9	3.1
^{180}Hf	7.33	4⁺, 5⁺	5.8 ± 0.5	4.4	4.0	4.5
^{184}W	7.42	0⁻, 1⁻	15.8 ± 2.0	12.0	15.0	16.0
^{196}Pt	7.93	0⁻, 1⁻	19.3 ± 3.6	12	18	39
^{200}Hg	8.03	0⁻, 1⁻	84 ± 18	75	70	20
^{202}Hg	7.76	1⁻, 2⁻	110 ± 20	90	100	25
^{230}Th	6.79	2⁺, 3⁺	0.58	0.6	0.41	0.7
^{234}U	6.84	2⁺, 3⁺	0.99 ± 0.08	0.61	0.6	0.83
^{236}U	6.47	3⁻, 4⁻	0.67 ± 0.13	0.53	0.5	0.6
^{238}U	6.14	0⁺, 1⁺	—	—	2.5	6.5
^{240}Pu	6.53	0⁺, 1⁺	2.3 ± 0.6	2.4	2.25	2.1
^{242}Pu	6.30	2⁺, 3⁺	1.17 ± 0.17	—	0.65	1.2
^{246}Cm	6.45	3⁺, 4⁺	—	1.5	1.55	1
^{248}Cm	6.21	4⁺, 5⁺	—	—	1.33	0.8
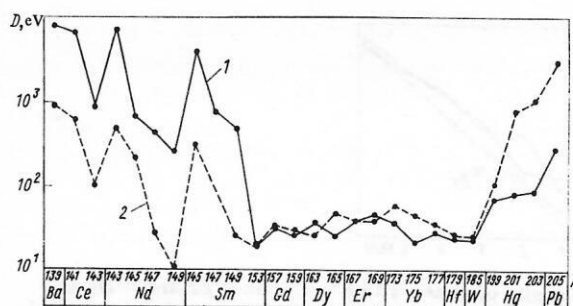
FIG. 1. Average distance D between levels with $I^\pi = 1/2^+$ at excitation energy $\mathcal{E} = 6.5$ MeV: 1—theoretical curve, 2—counting of experimental levels at $\mathcal{E} = B_n$ according to Eq. (8).

and the increase in D with the approach to magic nuclei. The calculations confirm that the density of highly excited states depends on the density of single-particle states near the energy of the Fermi surface, i.e., shell effects strongly influence the level density at excitation energies of the order of B_n. The strong dependence on mass number appears also in the parameter a extracted from the experimental and theoretical values of D.[11]

Thus, the semimicroscopic method gives a completely satisfactory quantitative description of the level density at the neutron binding energy. This is an additional indication that other results obtained with this model are also correct.

4. DEPENDENCE OF NUCLEAR LEVEL DENSITY ON EXCITATION ENERGY

The statistical model of the nucleus with inclusion of pairing, in the approximation of small angular momenta, gives an energy dependence $\rho(\mathcal{E}, I)$ similar to Eq. (8) but with the parameters a and σ depending on $\mathcal{E}$, and in order to find them it is necessary to solve several equations.[2]

The calculations of $\rho(\mathcal{E})$ carried out by us give similar results on the whole. Thus, in Fig. 2 we have shown the dependence of the density of states with $K^\pi = 1/2^+$ on energy in the nuclei ^{165}Dy and ^{167}Er. Here the dashed lines show the results of a calculation with Eq. (8), where the

though the contribution of vibrational levels has not been taken into account in them.

The neutron binding energy takes on different values for different nuclei, and this makes it difficult to determine the dependence of D on A. In order to trace the dependence of D on A in pure form, Vertebnyi et al.[34] make a comparison of D in different nuclei for the same excitation energy. We calculated D for $1/2^+$ states in N-odd nuclei from the region $140 \leqslant A \leqslant 205$ at an energy of 6.5 MeV. The corresponding experimental data were converted to an energy 6.5 MeV by the well known formula

$$\rho = \frac{\sqrt{\pi}}{12} \frac{\exp\{2\sqrt{U_{ef}a}\}}{a^{1/4}U_{ef}^{5/4}} \frac{(2I+1)\exp\{-(I+1/2)^2/2\sigma^2\}}{2\sqrt{2\pi}\,\sigma^3}, \quad (8)$$

where $\sigma^2 = 0.089(aU_{ef})^{1/2}A^{2/3}$, I is the spin of the target nucleus; $U_{ef} = \mathcal{E} - \delta$; $\mathcal{E}$ is the excitation energy; δ takes into account the pairing energy.[2] The parameter a was chosen from the experimental data for $\mathcal{E} = B_n$. The results of calculation in accordance with Eq. (8) for $\mathcal{E} = 6.5$ MeV are given in Fig. 1, from which we can see that the theory correctly reflects the rapid rise of D in the transition from deformed nuclei to spherical nuclei

TABLE IV. Experimental and theoretical values of D for two spins of z-odd and odd—odd nuclei at the neutron binding energy B_n.

| Compound nucleus	B_n, MeV	I^π	D, experiment, eV			D, theory, eV
			[30, 33—35]	[31]	[32]	
^{151}Eu	6.44	2⁺, 3⁺	1.3 ± 0.4	1.1	1.4 ± 0.4	1.3
^{160}Tb	6.38	1⁺, 2⁺	4.3 ± 0.78	4.2	3.9 ± 0.6	3.0
^{166}Ho	6.24	3⁻, 4⁻	5.67 ± 0.74	5.5	6.1 ± 1.2	2.4
^{170}Tm	6.59	0⁺, 1⁺	6.6 ± 1.3	7.3	6.0 ± 1.5	5.5
^{176}Lu	6.29	3⁺, 4⁺	3.61 ± 0.62	3.0	3.7 ± 0.7	2.4
^{177}Lu	6.89	13/2⁻, 15/2⁻	2.37 ± 0.27	2.3	2.3 ± 0.4	2.1
^{181}Ta	7.64	15/2⁺, 17/2⁺	1.5	—	>0.2	0.8
^{182}Ta	6.06	3⁺, 4⁺	4.33 ± 0.51	4.3	4.4 ± 0.4	3.5
^{186}Re	6.18	2⁺, 3⁺	3.2 ± 0.6	3.3	3.8 ± 0.8	5.4
^{232}Pa	5.56	1⁻, 2⁻	0.44 ± 0.06	—	0.41	0.7
^{234}Pa	5.20	1⁻, 2⁻	1.03 ± 0.15	—	0.69	0.9
^{238}Np	5.48	2⁺, 3⁺	0.72 ± 0.08	0.67	0.69	0.9
^{242}Am	5.58	2⁻, 3⁻	0.58 ± 0.09	—	0.77	0.2
^{243}Am	6.38	9/2⁻, 11/2⁻	—	0.6	<0.67	0.3
^{244}Am	5.36	2⁻, 3⁻	1.5 ± 0.3	—	0.68	0.25
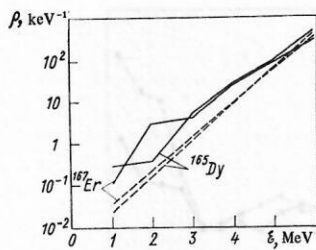
FIG. 2. Density of states with $K^\pi = 1/2^+$ in ^{167}Er and ^{165}Dy as a function of $\mathcal{E}$: solid lines—theoretical curve; dashed lines—calculation with Eq. (8) with a parameter a extracted from experimental data at $\mathcal{E} = B_n$.

FIG. 3. Density of states with $K^\pi = 1/2^+$ for ^{239}U and with $K^\pi = 0^+$ for ^{238}U as a function of $\mathcal{E}$: solid lines—theoretical curve; dashed lines—experimental data. [38]

parameter a in the calculation was chosen in each nucleus from the experimental density at the neutron binding energy and was taken as energy-independent. If we do not consider the sharp breaks in the calculated curves at low energies (this will be discussed below), the two types of dependences are extremely similar, the difference lying mainly in the absolute value of the density. At the same time our calculations are in good agreement with experiment. This is seen from comparison of the theoretical and experimental results on the dependence $\rho(\mathcal{E})$ in the uranium isotopes. Vorotnikov, [38] by analysis of the experimental data on neutron and radiative widths in terms of the statistical theory of nuclear reactions, obtained data on the energy dependence of the density of states with $K^\pi = 0^+$ for ^{238}U and with $K^\pi = 1/2^+$ for ^{239}U. The agreement of the experimental data obtained in this way and our calculations is shown in Fig. 3. An appreciable discrepancy is observed at energies below 3 MeV for ^{239}U and 5 MeV for ^{238}U. It can be seen that at an energy of the order of 8 MeV the dependence $\rho(\mathcal{E})$ for ^{238}U and ^{239}U comes together. This occurs in spite of the fact that we have not taken into account the change in the gap with increasing excitation energy. The same thing is found in the description of the total density of levels as a function of energy.

The experimental data existing for the nucleus ^{230}Th practically coincide with the calculation (Fig. 4). The disagreement observed between theory and experiment at energies below 4 MeV is explained by use of the solid-state value for the moment of inertia in this energy region. It is removed if we take for $\mathcal{Y}$ the value obtained in terms of the cranking model with pairing. [4] In the figure we have shown a calculation according to the statistical model, [2] which was carried out with inclusion of pairing; the parameters of the single-particle potential and the pairing-interaction constants were taken the same as in our calculations. In obtaining the $\rho(\mathcal{E})$ curve in terms of the statistical model the program of Zheregi et al. [40] was used. The predictions of statistics differ considerably from the experimental data in absolute value, being smaller by a factor of 20–30. Thus, in the regions of deformed nuclei our results are significantly better than the statistical calculations and satisfactorily describe the experiments (see Figs. 3 and 4).

The statistical theory essentially describes the average nuclear characteristics and the smoothed functional dependences (although it is possible to raise the question in this theory of determining deviations from the average). At the same time all combinatorial calculations
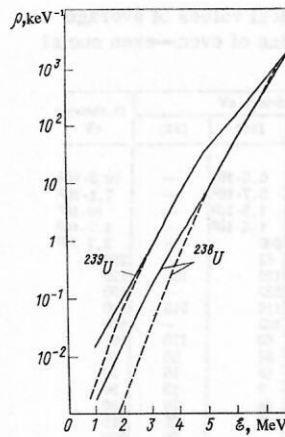
indicate the existence of appreciable deviations and fluctuations in the function $\rho(\mathcal{E})$. Some of these fluctuations originate as a result of the shell structure of the single-particle spectrum, and as a rule, this part appears in nuclei close to semimagic. Such fluctuations of a regular nature were pointed out by Hillman and Grover[7] and by Maruyama. [41] In other cases the fluctuations are associated with the existence of an energy threshold for excitations with a given number of particles, the cause of which is pairing. [6] These fluctuations should be smoothed by the blocking effect. Of course, the residual interactions, which have not been taken into account, will smooth the fluctuations, so that their existence is partly due to the limitations of the theoretical model.

The semimicroscopic method permits this question to be studied with allowance for the effect of collective states on fluctuations. For this purpose we shall construct for certain nuclei histograms of the number of levels in a definite energy interval as a function of the excitation energy. The main conclusion which can be drawn from the histograms shown in Figs. 5–11 is that the fluctuations remain rather strong in some cases. The clearest example of this behavior is in Fig. 5, which shows the number of states with $I^\pi = 1/2^+$ in a 200-keV interval for ^{143}Nd. There is a sharp peak in the density at energy 6.0–6.6 MeV. For comparison we have shown in the same figure the curve $\rho(\mathcal{E})$ calculated with
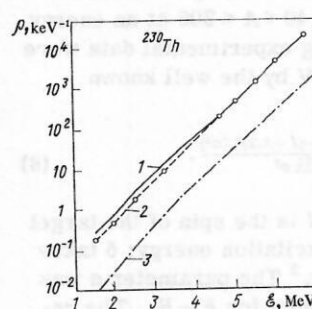

FIG. 4. Total level density for ^{230}Th as a function of $\mathcal{E}$: 1—theoretical curve with $\mathcal{Y} = \mathcal{Y}_{\text{rig}}$; 2—theory with $\mathcal{Y} = \mathcal{Y}_{\text{crank}}$; 3—theory according to the statistical model[40]; points—experimental data from Ref. 39.
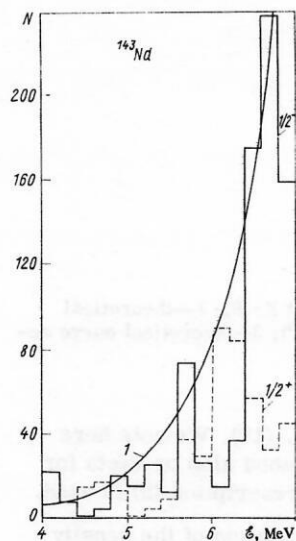
388   Sov. J. Part. Nucl., Vol. 7, No. 4, Oct.–Dec. 1976
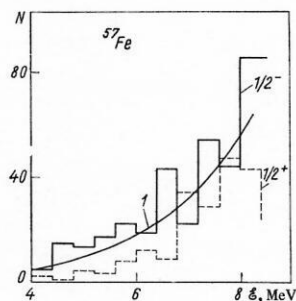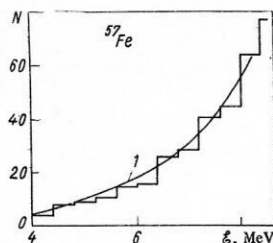
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FIG. 7. Histogram of half the sum of the numbers of levels with $I^\pi = 1/2^+$ and $1/2^-$ in a 400-keV interval in ^{57}Fe: 1—theoretical curve according to the statistical model.[40]

FIG. 5. Histogram of number of levels with $I^\pi = 1/2^+$ and $1/2^-$ in a 200-keV interval for ^{143}Nd: 1—theoretical curve according to the statistical model.

the program of Ref. 40, i.e., in terms of the statistical model, with the same spectrum and constants G_N and G_Z. We consider that the calculations are in agreement with ours, although the agreement is rather rough. Fluctuations of states of different parity appear at various energies; we can say that there is a peculiar compensation of fluctuations of the number of states with identical angular momenta but different parities (Fig. 6 and 7). In these figures we have shown histograms of the number of states in a 400-keV interval with $I^\pi = 1/2^+$ and $I^\pi = 1/2^-$ (see Fig. 6) and $[N(1/2^+) + N(1/2^-)]/2$ (see Fig. 7) in the nucleus ^{57}Fe. The curve obtained in the framework of the statistical model with the program of Ref. 40 agrees astonishingly well with the envelope of the histogram in Fig. 7. We note that the statistical model assumes $N^+ = N^-$. A similar situation exists also in ^{143}Nd, although here the agreement of the statistical calculation and the histogram of half the sum of the numbers of states with $I^\pi = 1/2^\pm$ is poorer than for ^{57}Fe. Note that the significant difference between the results of the statistical model and our calculations, which existed in the deformed nuclei, is practically absent in spherical nuclei, although here the contribution of collective motions, in our opinion, is significant (for more detail see Section 7).

FIG. 6. Histograms of the number of levels with $I^\pi = 1/2^+$ and $1/2^-$ in a 400-keV interval for ^{57}Fe: 1—theoretical curve according to the statistical model.[40]

The fluctuations discussed appear most clearly in nuclei with a low level density.[1] As the density increases the fluctuations decrease and in deformed nuclei are appreciable only at extremely low energies (it is just this which explains the breaks in the curves of the density of states with $K^\pi = 1/2^+$ in ^{165}Dy and ^{167}Er in Fig. 2). An example is given by the histogram of the number of levels with $K^\pi = 1/2^+$ in a 50-keV interval in ^{157}Gd (Fig. 8). However, in the same nucleus for states with high spins ($K^\pi = 21/2^+$, $17/2^+$), whose density is much lower than for $1/2^+$, the fluctuations are much more noticeable (Fig. 9). Fluctuations can appear much more strongly if we restrict ourselves to some definite class of states, for example, states with a given angular momentum and a given number of quasiparticles. In this case the fluctuations can substantially change the behavior of various nuclear characteristics. A good example of this situation is the explanation proposed in Ref. 17 for the local peak at $\mathscr{E} = 6.2$ MeV in the $E1$ strength function of ^{238}U. Just in this energy region there is also a local peak in the density of two-quasi-particle 1^- states (Fig. 10).

We note, however, that the fluctuations which we are discussing can in some cases be an additional source of discrepancy between theory and experiment.

5. DEPENDENCE OF LEVEL DENSITY ON SPIN AND ITS PROJECTION
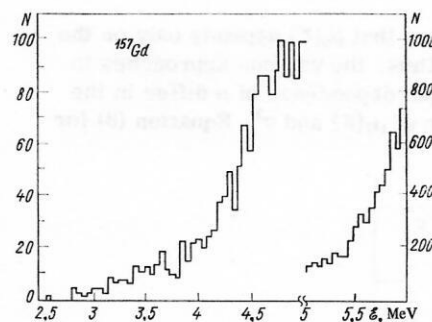
Up to the present time there have been practically no

FIG. 8. Histogram of number of levels with $K^\pi = 1/2^+$ in a 50-keV interval in ^{157}Gd.

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[1] It must be pointed out that an additional source of fluctuations in spherical nuclei is the absence in our model of splitting of many-phonon states in intermediate angular momentum.
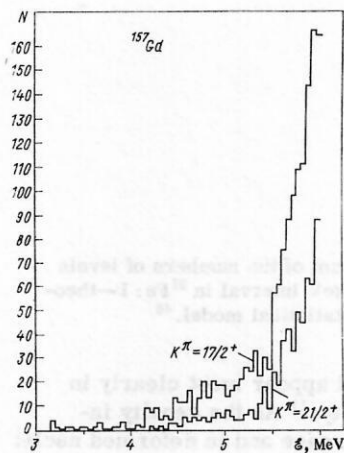
FIG. 9. Histograms of the number of levels with $K^{\pi} = 17/2^+$ and $21/2^+$ in a 50-keV interval in ^{157}Gd.

direct experimental data on the behavior of the level density as a function of spin. This is due to the large number of experimental difficulties which arise in identification of the spins of highly excited states. There is as yet only some indirect information on the nature of the dependence $\rho(I)$ (more precisely, on the quantity σ^2). This information was obtained by analysis of data on (α, α')-, (α, p)-, (p, α)-, (α, n)-, and (n, α) reactions for the group of light spherical nuclei.[42-44] Therefore the question of the correctness of the various theoretical predictions regarding the behavior of $\rho(I)$ has not yet found an experimental solution.

Some of the current variants of nuclear-structure theory turn out to be in a state to provide an analytical form of the dependence of ρ on the excitation energy $\mathscr{E}$ and angular-momentum projection M. In order to determine from these data the dependence of ρ on the angular momentum I, the procedure proposed by Bethe[1] is ordinarily used:

$$\rho(\mathscr{E}, I) = \rho(\mathscr{E}, M = I) - \rho(\mathscr{E}, M = I+1). \quad (9)$$

The most general form of the function $\rho(\mathscr{E}, I)$ is then found to be

$$\rho(\mathscr{E}, I) = \rho_0(\mathscr{E})(2I+1) \exp\{-I(I+1)/2\sigma^2\}. \quad (10)$$

It must be emphasized that $\rho_0(\mathscr{E})$ depends only on the excitation energy. Thus, the various approaches to description of the spin dependence of ρ differ in the means of calculation of $\rho_0(\mathscr{E})$ and σ^2. Equation (8) for
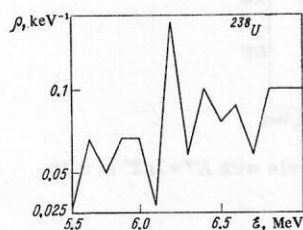
FIG. 10. Density of two-quasiparticle states with $K^{\pi} = 1^-$ in ^{238}U as a function of energy: the peak in the density at $\mathscr{E} = 6.2$ MeV coincides with the peak in the E1 strength function measured experimentally.[17]
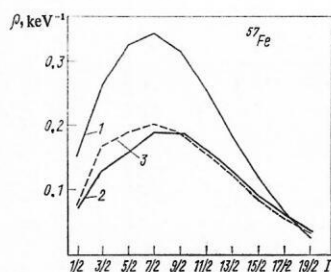
FIG. 11. The function $\rho(I)$ for ^{57}Fe at $\mathscr{E} = B_n$: 1—theoretical curve $\rho(I^-)$; 2—theoretical curve $\rho(I^+)$; 3—theoretical curve according to the statistical model.[40]

$\rho(\mathscr{E}, I)$ is a particular case of Eq. (10). We note here that the spectral-distribution method also predicts for $\rho(\mathscr{E}, I)$ the form (10) if Bethe's prescription (9) is used.

Combinatorial methods of calculation of the density naturally do not provide an analytic expression for $\rho(I)$. We have calculated the spin dependence in the nuclei ^{57}Fe, ^{58}Fe, ^{157}Gd, and ^{158}Gd; we shall discuss the results given by our method for the spin dependence of the density in the case of these nuclei.

Figures 11 and 12 show the spin dependence in the Fe isotopes. We note first of all that it is different for states of different parity. Leaving the detailed discussion of this fact to Section 6, we note here only that the statistical model assumes identical density values for states with identical spins and different parities and, consequently, identical curves for $\rho(I^+)$ and $\rho(I^-)$. Our calculations do not support this assumption. The calculations of the statistical model with pairing[40] with the same mean-field parameters and constants G_N and G_Z give the spin dependences shown in Fig. 11 by curve 3 and in Fig. 12 by curve 5. While for ^{57}Fe we can note an agreement of the theoretical curve $\rho(I^+)$ and the statistical curve if only for $I \geqslant 5$, on the other hand for ^{58}Fe there is no such agreement. The curve $[\rho(I^-) + \rho(I^+)]/2$ turns out to be somewhat closer to the statistical curve for ^{58}Fe, but even these curves do not agree. Are our curves fitted by a dependence similar to Eq. (10)? This question was examined in the case of the ^{58}Fe nucleus and the Gd isotopes. The $\rho(I^+)$ curve for ^{58}Fe is satisfactorily described by the function (10) for $\sigma = 4$. However, such agreement is not achieved for the $\rho(I^-)$ curve; for the condition of agreement of the function (10)
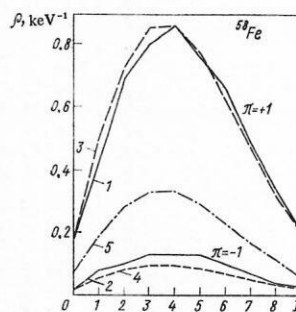

FIG. 12. The function $\rho(I)$ for ^{58}Fe at $\mathscr{E} = B_n$: 1—theoretical curve $\rho(I^+)$; 2—theoretical curve $\rho(I^-)$; 3—fit to $\rho(I^+)$ with Eq. (10); 4—fit to $\rho(I^-)$ with Eq. (10); 5—theoretical dependence $\rho(I)$ according to the statistical model.[40]
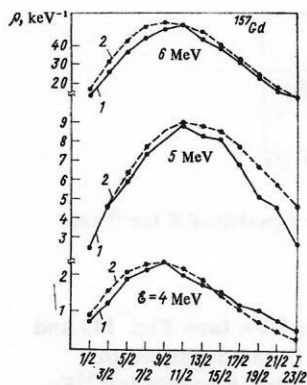
FIG. 13. The dependence $\rho(I^+)$ for ^{157}Gd at various $\mathscr{E}$: 1—theoretical curves; 2—fits with Eq. (10).

with the theoretical value at the point $I = 0$ and for high spins an appreciable disagreement of the two curves is obtained in the region of the maximum. In deformed nuclei the spin dependence is satisfactorily fitted by the function (10) at various excitation energies (Figs. 13 and 14) if the parameter a is chosen from the value of the density at the maximum of the $\rho(I)$ curve at a given energy. Note that the parameter a, and also therefore σ, change with increase of $\mathscr{E}$ in this case. Calculations for ^{157}Gd on the basis of the statistical model[40] give similar results regarding the shape of the $\rho(I)$ curve, but in absolute value the statistical results are much lower than ours, as occurred also for ^{230}Th (see Section 4). The spectral-distribution method, which was used to study the spin dependence for the nucleus ^{63}Cu, and the shell calculations in the same nucleus[8] agree as a whole with the shape of the $\rho(I)$ curve, which is given by Eq. (10).

Let us turn to discussion of the nature of the density dependence on the quantum number K. In Figs. 15 and 16 we have plotted the $\rho(K^+)$ curves calculated by us for ^{157}Gd and ^{158}Gd for various values of $\mathscr{E}$. While for $\mathscr{E} = 2$ MeV fluctuations are observed in the behavior of $\rho(K)$, on the other hand, at higher energies they practically disappear. The function $\rho(K)$ behaves smoothly and falls off appreciably for large K. At these energies ($\mathscr{E} \geq 4$ MeV) the function is satisfactorily approximated by the formula

$$\rho(\mathscr{E}, K) = \rho_0(\mathscr{E}) \exp[-K^2/(2\sigma^2)]. \tag{11}$$
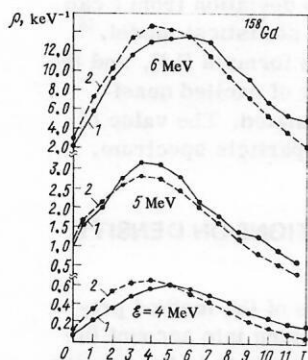
FIG. 14. The dependence $\rho(I^+)$ for ^{158}Gd at different $\mathscr{E}$: 1—theoretical curves; 2—fit with Eq. (10).
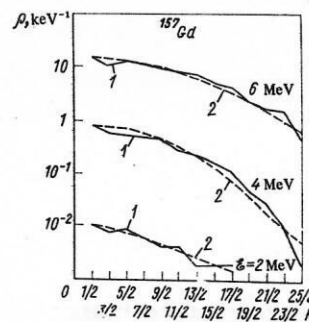
FIG. 15. The dependence $\rho(K^+)$ for ^{157}Gd at various $\mathscr{E}$: 1—theoretical curves; 2—fit with Eq. (11).

The corresponding curves are given in Figs. 15 and 16. Of course, the parameters $\rho_0(\mathscr{E})$ and σ^2 were specially fitted in each individual case. Thus, the dependence $\rho(K)$ obtained in our calculations and in the spectral-distribution method is close to Gaussian.

We note an interesting fact regarding the relation between $\rho(I = K)$ and $\rho(I)$. In terms of the statistical approach for small values of I, Bjørnholm et al.[22] established the existence of the following regularity:

$$\rho(\mathscr{E}, I \pm 1/2) \approx (I + 1/2)\rho(\mathscr{E}, I = K). \tag{12}$$

Our calculations confirm this conclusion. This can be seen as an illustration of the dependence $\rho(\mathscr{E}, I^\pi = 5^-)$ in the nucleus ^{158}Gd (Fig. 17). For energies $\mathscr{E} \geq 5$ MeV the curve calculated by the semimicroscopic method practically coincides with the curve calculated on the basis of Eq. (12).

6. DEPENDENCE OF NUCLEAR LEVEL DENSITY ON LEVEL PARITY

In discussing the question of the relation of the densities of levels with different parity in the nuclear spectrum, as a rule, we recall the classical discussion of Ericson.[2] He obtained a formula for the probability P_- of encountering in the spectrum of excitations with a given number of particles a level with negative parity if we know the relative number of levels with negative parity[2)] p_- in an unfilled shell and the number of excited particles ν:

$$P_- = [1 - (1 - 2p_-)^\nu]/2. \tag{13}$$

It follows from Eq. (13) that P_- rather rapidly approaches 0.5 both with increase of p_- and with increase of the number of excited particles. This has served as the basis for the assumption usually used in the statistical model: $P_- = 0.5$.

However, Eq. (13) refers to the complete spectrum of states with a given number of excited particles. As the result of nonuniformities in the single-particle spectrum and the presence in the nuclear spectrum at a

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[2)] For a given excitation energy only some limited number of single-particle levels mainly determine the structure of the spectrum. If there are r such levels, then $p_- = r_-/r$, where r_- is the number of single-particle states with negative parity among the r levels (for spherical nuclei it is necessary to take into account degeneracy in the angular-momentum projection).
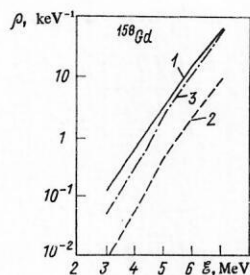
FIG. 17. Dependence of ρ on $\mathcal{E}$ for different states of ^{158}Gd: 1—$I^\pi = 5^-$; 2—$I^\pi = K^\pi = 5^-$; 3—theoretical curve according to Eq. (12).

FIG. 18. The ratio $D(K^+)/D(K^-)$ as a function of K for ^{157}Gd and ^{158}Gd at $\mathcal{E} = B_n$.

given energy of states associated with excitation of various numbers of particles, large departures from this value of P_- are possible. They will appear most strongly in spherical nuclei and will decrease with increasing excitation energy. The first factor is due to the shell structure of the single-particle spectrum of spherical nuclei and to the fact that in the main shells states of definite parity are dominant and the value of p_- is small. In particular, it can be the case that a state of the opposite parity in the single-particle spectrum will lie high in energy, and at $\mathcal{E} \sim B_n$ only a small number of states will take part in formation of the structure. Therefore the spectrum of odd nuclei should be dominated by levels whose parity coincides with the basic parity of the unfilled shell. In even—even nuclei states of positive parity should dominate. The collective motions will strengthen the effect of parity (for more detail see Section 7).

Thus, deviations from the rule $P_- = P_+$ should be observed. In practice the question reduces to how strong these deviations are. Hillman and Grover[7] studied this question in the case of the nuclei ^{114}Cd and ^{166}Ho. They compared the total density of states of a given parity at excitation energies from zero up to 10—20 MeV. The maximum observed ratio $R = P_+/P_-$ was 2.5 (for ^{114}Cd at $\mathcal{E} = 1.5$ MeV). With increasing energy the fluctuations of R decreased and did not exceed 20%. For ^{166}Ho the deviations of R from unity were still smaller.

Our calculations demonstrate a more clearly expressed effect of parity, particularly in nuclei with $A \sim 60$. This is quite evident from comparison of the histograms for states with $I^\pi = 1/2^+, 1/2^-$ in ^{57}Fe nuclei (see Fig. 6) and ^{143}Nd nuclei (see Fig. 5), and from

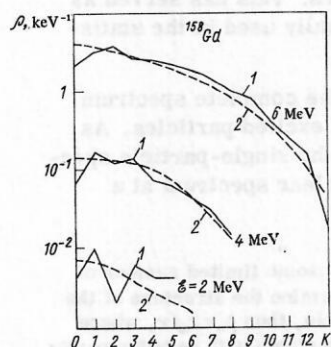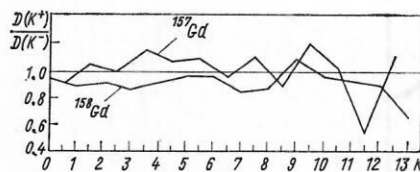FIG. 16. A dependence $\rho(K^+)$ for ^{158}Gd at various $\mathcal{E}$: 1—theoretical curves; 2—fit by Eq. (11).

curves of the spin dependence for ^{57}Fe (see Fig. 11) and ^{58}Fe (see Fig. 12). A difference in the densities for different parity shows up particularly strongly for ^{58}Fe. Here the maximum value of $R(I) = \rho(I^+)/\rho(I^-)$ is 8 (for $I = 4$) and the minimum value if $R(I) = 4$ (for small spins). For ^{57}Fe the parity effect is weaker, and here we have $R_{max}(I) = 0.5$. We note that in this nucleus, in agreement with the dominant parity of the unfilled neutron shell, $\rho(I^-) > \rho(I^+)$. A similar situation is observed for ^{143}Nd. In deformed nuclei, where as a consequence of the destruction of the shell structure of the single-particle spectrum the single-particle states of different parity are mixed and $p_- \approx 0.5$, the parity effect should not be observed on the average. This is confirmed by calculations of the ratio $D(K^+)/D(K^-)$ for ^{157}Gd and ^{158}Gd (Fig. 18).

The density fluctuations existing in the spectrum of excitations with a given spin can lead in individual parts of the spectrum to violation of the formulated rules. This is also shown by the histograms of the number of levels with $I^\pi = 1/2^\pm$ for ^{57}Fe and ^{143}Nd. Since the parity effect is directly related to the shell structure of the spectrum of single-particle states, the ratio R will depend strongly on such characteristics as the average distance between single-particle levels in the unfilled shell, the distance between the main shells, and also the location of the chemical potential inside a shell, i.e., its degree of filling. Changes in these quantities will lead to a weakening of the parity effect at the neutron binding energy in heavy nuclei. A specific reason for this weakening is the well known compression of the single-particle spectrum with increasing mass number A. As a result, already for Te we have $R(I) \approx 1.2$, and for the isotopes of Hg $R(I) \approx 1$. The strong dependence of the parity effect on the location in the shell of a level of opposite parity is demonstrated also by model calculations in terms of the spectral-distribution method.[8]

We note that the ratio R and its deviation from I can be calculated also in terms of the statistical model.[45] This has been done with Ericson's formula (13), and the dependence of the average number of excited quasiparticles on energy has been calculated. The value of p_- was estimated from the single-particle spectrum.

7. EFFECT OF COLLECTIVE MOTIONS ON DENSITY OF NUCLEAR LEVELS

The principal distinctive feature of the method proposed by us is the possibility of taking into account the contribution of collective nuclear motions to the density of states. The results presented above indicate the excellent possibilities of the method and, in particular,

TABLE V. Effect of vibrational states on level density at the neutron binding energy B_n.

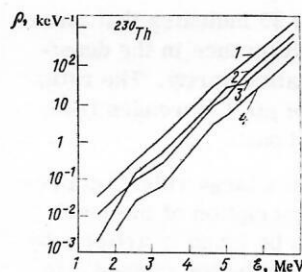| Compound nucleus	B_n, MeV	$I^\pi (K^\pi)$	D, eV		Compound nucleus	B_n, MeV	$I^\pi (K^\pi)$	D, eV	
			$\kappa^\lambda = \kappa^\lambda_{exp}$	$\kappa^\lambda = 0$				$\kappa^\lambda = \kappa^\lambda_{exp}$	$\kappa^\lambda = 0$
57Fe	7.642	1/2+	13.6·10³	200·10³	165Dy	5.715	1/2+	133	213
58Fe	10.043	0-, 1-	10.0·10³	60·10³	164Dy	7.657	2-, 3-	19.6	48.0
97Mo	6.816	1/2+	980	1.7·10³	199Hg	6.653	1/2+	84.2	135
96Mo	9.157	2+, 3+	98	304	200Hg	8.027	0-, 1-	19.5	36
123Te	6.943	1/2+	145	267	239U	4.8	1/2+	26.6	37.3
124Te	9.408	0+, 1+	19.6	30.2					

FIG. 19. Total density of levels with $I^\pi = 2^+$ and 3^+ as a function of $\mathcal{E}$ for ^{230}Th: 1—with inclusion of vibrational and rotational motion; 2—with inclusion only of rotational motion; 3—with inclusion only of vibrational motion; 4—without inclusion of collective motions.

the good quantitative agreement of the theoretical and experimental results at the neutron binding energy. A natural question arises: What role do nuclear collective motions play here?

It is rather simple to find out numerically the contribution of vibrations and rotations. For calculations of the phonon energies it is sufficient to set $\kappa^\lambda = 0$, and the collective vibrational states will disappear. It is still simpler to remove rotations: It is necessary to restrict oneself to calculation of the bases of the rotational bands. In Tables V and VI we compare calculations of the level density at the neutron binding energy with and without inclusion of collective motions for several nuclei from different mass-number regions. These calculations show the following: The contribution of vibrations is most significant in nuclei with mass number $A \approx 50$–60; here they increase the density by a factor of 8–10. With increasing A the significance of vibrations decreases, and already in the mercury isotopes they increase the density by only a factor of 1.5–2.0. In deformed nuclei for spins $I > 1/2$ rotation further increases the density. Its contribution for small spins can be estimated at once from the approximate formula (12), and it turns out that as the result of rotation the density increases by a factor of 1.5–3. For higher spins the rotational states increase the density by a factor of 5–7 (see for example ^{177}Lu and ^{181}Ta).

Collective motions exert a considerable influence not only on the absolute value of the level density; it is important to take them into account also in calculation of the dependence of the density on spin and excitation energy. It is true that vibrations and rotations apparently do not greatly change the shape of the $\rho(\mathcal{E})$ curve. The combined density of levels with $I^\pi = 2^+, 3^+$ for ^{230}Th in the energy interval 1–8 MeV was calculated both with

inclusion of vibrations and rotations and with various branches of the collective excitations turned off (Fig. 19). The entire group of curves differ practically only by a shift along the density scale. We note that the contribution of rotations and vibrations in this nucleus are approximately identical.

The role of collective motions in the energy dependence of the density appears mainly in the smoothing of fluctuations. Therefore in the histograms (see Figs. 5–9) we do not observe those sharp and regular spikes of density which were found by Hillman and Grover[7] and Maruyama.[41] The fluctuations, of course, remain, but they acquire a random nature. In nuclei with a large density of states and a large number of collective degrees of freedom the fluctuations are quite insignificant (see Fig. 8).

The spin dependence curve $\rho(I)$ can change its shape when vibrations are turned off. Thus, the $\rho(I^+)$ curve for ^{58}Fe, as was pointed out in Section 5, is satisfactorily fitted by the function (10) for $\sigma = 4$. On turning off vibrations not only is the absolute value of $\rho(I^+)$ changed considerable, but the fit to it by the function (10) also becomes much poorer. The results of these calculations are given in Fig. 20. They agree only at small and large values of I, and at the maximum the function (10) has a value 1.5 times smaller ($\sigma = 4.6$). At the same time the $\rho(I^-)$ curve disagrees to an identical degree with the functional dependence (10) both with and without inclusion of collective motions (see Figs. 12 and
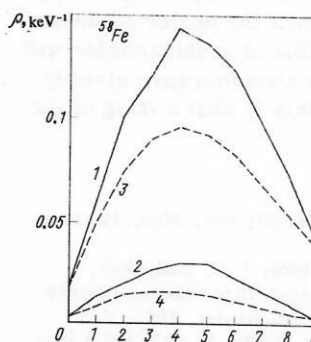
TABLE VI. Effect of rotational motion on average distance between levels at the neutron binding energy B_n.

Compound nucleus	B_n, MeV	I^π	D, eV		
			Experiment	$I = K$	With inclusion of rotation
156Gd	8.527	1-, 2-	1.99±0.32	3.5	2.0
158Gd	7.929	4-, 2-	6.3±0.6	8.7	4.8
162Dy	8.204	2+, 3+	2.55±0.38	11.1	3.9
164Dy	7.657	2-, 3-	9.6±1.6	19.6	11.8
168Er	7.770	3+, 4+	4.0±0.4	26.6	7.2
174Yb	7.440	2-, 3-	7.81±0.93	13.6	6.5
177Lu	6.890	13/2-, 15/2-	2.37±0.27	14.5	2.1
178Hf	7.620	3-, 4-	3.2±0.2	11.1	3.1
180Hf	7.330	4+, 5+	5.8±0.5	18	4.5
181Ta	7.640	15/2+, 17/2+	1.5	4.1	0.8
236U	6.467	3-, 4-	0.67±0.13	1.4	0.6

FIG. 20. The dependence $\rho(I^\pi)$ for ^{58}Fe at $\mathcal{E} = B_n$ ($\kappa^\lambda = 0$); 1—theoretical curve $\rho(I^+)$; 2—theoretical curve $\rho(I^-)$; 3 and 4—fits of curves 1 and 2, respectively, by Eq. (10).

20). Comparison of Figs. 12 and 20 indicates that turning on vibrations increases the difference in the densities of states of positive and negative parity. The ratio $R(I)$ at the point of the maxima of $\rho(I^{\pm})$ increases from 6 to 8 when vibrations are turned on.

Thus, our calculations indicate a large role of collective excitations in the correct description of the level density. A similar statement can be found in articles by authors who used the spectral-distribution method.[8] In the statistical model until recently there were no numerical evaluations of the effect of the long-range components of the residual nuclear forces on the level density. The recently published articles by Ignatyuk[46,47] estimate the importance of the long-range components in different ways. In Ref. 46 it is asserted that the effect of vibrations is significant, while in Ref. 47 for nuclei with $A \approx 50-60$ the contribution of vibrations is estimated as a factor of 1.5—1.8. There is a large quantitative difference from our results, but Ignatyuk's results apparently cannot be considered as final.

CONCLUSION

The results set forth permit as a whole a positive evaluation of the possibility of nonstatistical methods of calculation of level density, and in particular, of the semimicroscopic method. One of the strongest features of these methods is the detailed nature of the study of the nuclear spectrum, which permits description and explanation of the departure from the averaged dependences predicted by the statistical model, which are necessarily rather crude. An additional value of the semimicroscopic method is the possibility of taking into account the effect of collective motions on the density of nuclear excitations. Their role is significant in description of the absolute value of the density and in the functional dependence of ρ on the excitation energy, spin, and parity of the states. The effect of parity and the fluctuations of the level density, discussed in the review, until recently have not been considered in terms of traditional statistical models, although they are significant in a number of nuclear processes.

On the other hand, the good description of the experimental data in terms of the semimicroscopic method permits one to hope that the model itself on which the method is based will work satisfactorily at excitation energies up to the nucleon binding energy. Of course, in order to describe the structure of the states correctly, it is necessary to take into account the neglected interactions, first of all the interaction of quasiparticles with phonons. The first steps in this direction have already been taken,[21,48] but as a whole this is still a thing of the future.

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